DOCUMENT RESUME

ED 098 061 SE 018 190

AUTHOR Evans, Diane
TITLE Learning Activity Package, Algebra 93-94, LAPs 12-22.
INSTITUTION Ninety Six High School, S. C.
PUB DATE [73]
NOTE 164p.; See ED 069 504 for related document

EDRS PRICE MF-$0.75 HC-$7.80 PLUS POSTAGE
DESCRIPTORS *Algebra; Analytic Geometry; Curriculum; *Individualized Instruction; *Instructional Materials; Mathematics Education; Number Systems; Objectives; *Secondary School Mathematics; Set Theory; Teacher Developed Materials; Teaching Guides; Units of Study (Subject Fields)
IDENTIFIERS *Learning Activity Package

ABSTRACT

A set of 11 teacher-prepared Learning Activity Packages (LAPs) in beginning algebra, these units cover sets, properties of operations, operations over real numbers, open expressions, solution sets of equations and inequalities, equations and inequalities with two variables, solution sets of equations with two variables, exponents, factoring and polynomials, functions, and equations and their applications. Each unit contains a rationale for the material; a list of behavioral objectives; a list of resources including texts (with reading assignments and problem sets specified), tape recordings, commercial games, filmstrips, and transparencies; a problem set for student self-evaluation; suggestions for advanced study; and references. (DT)

LEARNING ACTIVITY PACKAGE

SETS

BEST COPY AVAILABLE

ALGEBRA 93-94

LAP NUMBER 12

REVIEWED BY

WRITTEN BY Diane Evans

91572

3

- 018 190
1 of 11
OK BS

RATIONALE

The purpose of this LAP is to introduce the fundamental theorems of exponents and radicals. At this stage you will not be expected to prove these laws, but will discover them through observing existing patterns. Familiarity with these theorems is an important prerequisite for the learning of factoring, extension of the concept of function, and in using scientific notation.

A wide variety of experiences will be provided to enable you to associate the basic theorems of exponents to their application either in mathematics or science. The laws will be extended from natural number exponents--to negative exponents. Rational and real exponents will be left to a later date. Scientific notation will be used in demonstrating application of exponents.

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given any number written in exponential form, write it as a product where the factors are alike.
2. Given any number expressed in exponential form, name the base and the exponent.
3. Use the product of powers property ($\forall x \in R\ \forall m\ n \in N\ x^m \cdot x^n = x^{m+n}$) to rename any given product of powers so that no base is used more than once. (Simplify).
4. Given any rational number, and a base, write the rational number in exponential form using the given base.
5. Given any number expressed in exponential form (power of a base), write it as a decimal numeral.
6. Given two or more monomial expressions of the form $a^m \cdot a^n$, $(a^m)^n$, $\frac{a^m}{a^n}$, $(\frac{a}{b})^n$, or $(ab)^m$, use the laws of exponents together with the associative and commutative properties of multiplication to rename it as an equivalent expression.

RESOURCES

<u>Objectives 1, 2, 3, 4</u>

Nichols, read pp. 312-314, Ex. 2, 4 every other letter, 3 a, c pp. 314-315.

Vanatta, read pp. 69, 113-114, Ex. 1-18 page 114.

Dolciani, read p. 203, Ex. 1-24 page 204.

Payne, read pp. 257-259, Ex. 1-25 odd, 33-41 odd pages 259-260.

Pearson, read pp. 340, Ex. 1-5, 6 a,b,c,j,k,l, 7 a,b,c, 8 a,b,i,j,l, 10 page 341.

Introduction to Exponents frames 27-29 (Obj. 1)
frames 10-26, 30-32 (Obj. 2)
frames 1-9 (Obj. 3)
frames 33-35 (Obj. 4)

* Appendix I

Objective 5

Nichols, read pp. 312-313, Ex. 1 a-j page 314.

Objective 6

Nichols, read pp. 315-319, Ex. 1-3 every other letter page 316;
1,2,5,6 every other letter page 317;
1,2 every other letter page 318;
1-10 page 319.

Vanatta, read pp. 114-116, Ex. 1-16 page 116;
1-12 top page 117;
1-23 even page 117;

Dolciani, read pp. 204-205, 215-217, Ex. 1-16 even page 205; 1-10 page 206; 1-24 even top page 218; 1-10 pages 218-219.

Payne, read pp. 260-265, Ex. pages 262-264 every number divisible by 4; 1-41 odd page 266.

Wooton, read pp. 315-319, Ex. 1-45 odd page 271.

Pearson, read pp. 342-343, 347-349, Ex. 1 every other letter, 2 page 342; 1, 2, 3 every other letter page 343; 1,2,3 every other letter, 4,5,6 pages 345-346.

Introduction to Exponents frames 80-141
197-205
218-220

* required

SELF EVALUATION

OBJECTIVE 1. Solve by the COMPARISON method.

1. $3x - 10 = y;\ y = 4x$
2. $x = 12 + 2y;\ x = 3 + 3y$
3. $x + y = 10;\ 2x + 2y = 20$
4. $\frac{1}{2}x = y;\ y + \frac{3}{15}x = 7$
5. $x + y = \frac{25}{2} \qquad \frac{x}{2} + \frac{y}{2} = 5$

OBJECTIVE 2 Solve by SUBSTITUTION method.

6. $7x + 9y = 10;\ x + y = 2$
7. $x + y = 20;\ y = 2x + 5$
8. $3(x + 2) + 3y = 21;\ x + 2y = 8$
9. $3x + 2y = 15;\ -x = 2 - 4y$
10. $2x - 3y = 17;\ x + 4y = 3$

OBJECTIVE 3. Solve by ADDITION method. Check by substituting your solution in each problem.

11. $x + y = 5;\ 2x - y = 7$
12. $8x - 3y = 15;\ 13x - 3y = 15$
13. $4x + 3y = 14;\ 9x - 2y = 14$
14. $2x + 3y = 12.4;\ 4x + -6y + 3y = -5.8$
15. $3sx + 2ty = -5st \qquad 4sx + -5ty = 24st$

OBJECTIVE 4. Find the EQUATIONS and solve.

(One Variable)

16. John has twice as many nickels as quarters and 3 fewer dimes than quarters. The sum of the values of the coins is $1.95. Find how many of each kind of coin he has.

17. A solution of salt and water weighs 100 lbs and it is 10% salt. How much water must evaporate to leave the solution at 30% salt concentration?

18. Two cars start together in opposite directions, one at 40 mph, the other at 50 mph. How long will it be before they are 300 miles apart?

Solve 19-23, using Two Variables.

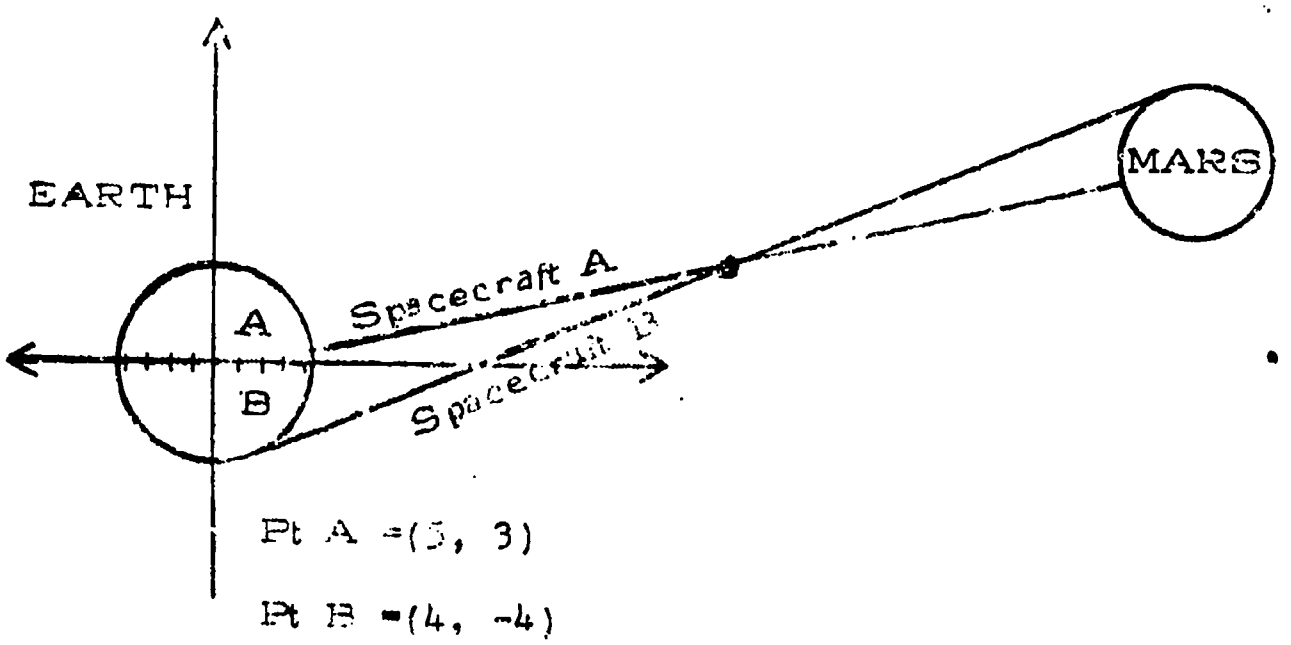
19. The sum of two numbers is 19. Their difference is 1.

20. The sum of two numbers is 20. Twice one is 3 times the other.

21. A rectangle is twice as long as it is wide. The sum of length and width is 9.

22. John's age now is 2 less than twice his sister Sue's age. In five years John's age will equal 3 times Sue's age now.

23. A plane flies 360 mph with the wind and 270 mph against. what is the speed of the plane in still air? What is the wind speed?



Pt A =(3, 3)

Pt B =(4, -4)

Units are in thousands of kilometers.

The spacecraft have only ONE opportunity for a mid-course rendezvous for supplies and fueling.

1. Two spacecraft A and B are going to Mars. Spacecraft A contains men and light equipment. Spacecraft B contains fuel, heavy equipment and life-support supplies.

   Question: At what point do the ships rendezvous?

   Note: Spacecraft A path: $x - 2y = -1$

   Spacecraft B path: $3x - 4y = 28$

2. A man exerting a force of 150 pounds and using a lever 6 feet long would be able to life a weight of how many pounds if he placed the fulcrum 2 feet from the weight?

3. A man in an automobile is traveling 5 times as fast as a boy on a bicycle. The time required by the boy in going 40 miles is 3 hours greater than that required by the man going 50 miles. What is the rate of travel of the boy?..of the man?

4. Job Problems (These problems are best done on a one-day-total-cost job basis) e.g.

   If John does the job in 3 days and Sue in 4 days, how long will it take them to do it together?

5. Vanatta, page 252, nos. 9, 10.

6. Dolciani, page 377, nos. 19, 20.

REFERENCES

Vanatta(abbreviation)

Vanatta, Glen D., Goodwin A. Wilson, Algebra One, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Nichols (abbreviation)

Nichols, Eugene D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Ginn and Company, 1964.

Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis, Algebra One, Harcourt, Brace and World, Inc., 1969.

Wooton

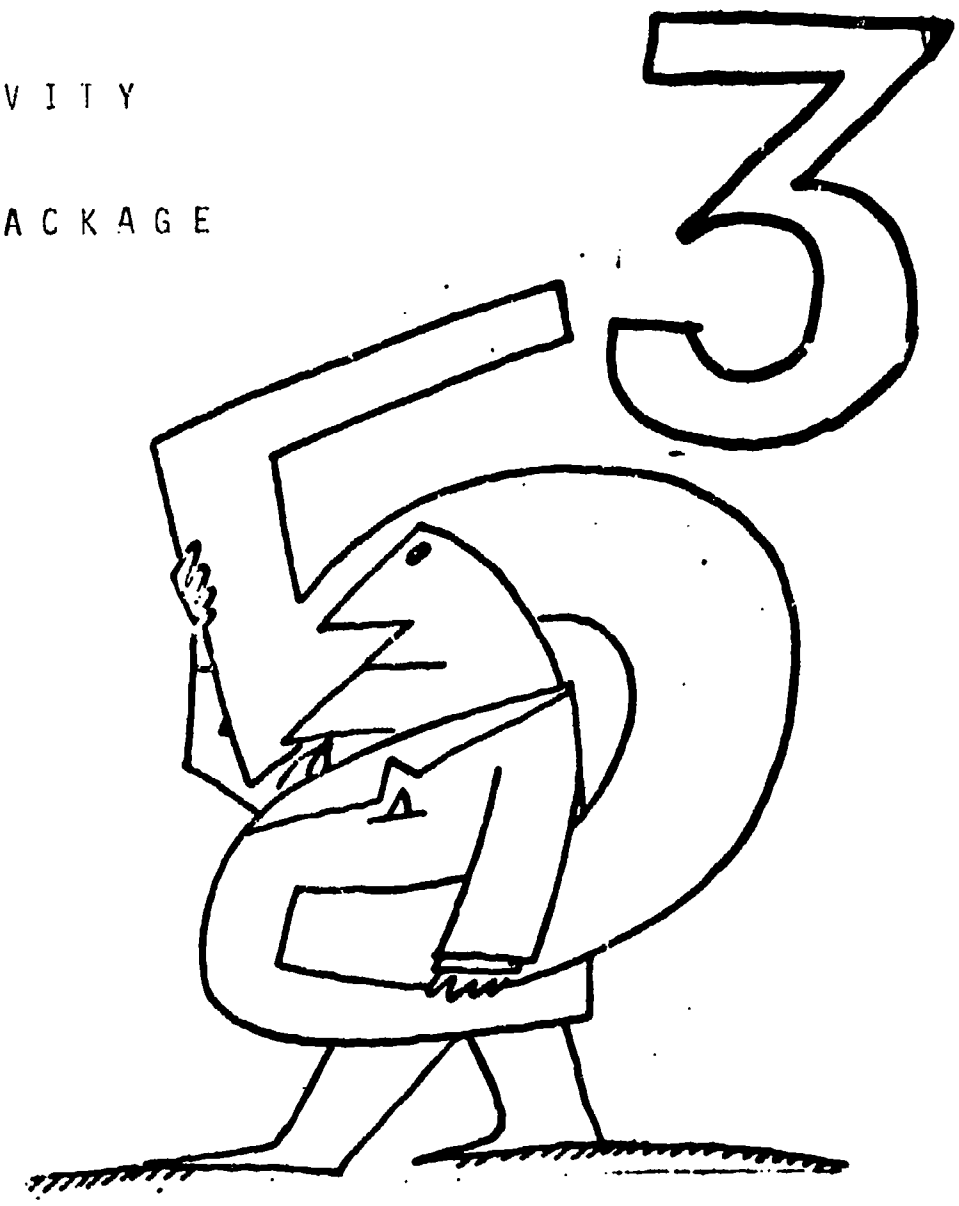
Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics, Algebra 1, Houghton, Mifflin Company, 1967.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

Wollensak Teaching Tapes: C-3809

LEARNING ACTIVITY PACKAGE



EXPONENTS

Algebra 93-94

REVIEWED BY J. S. Ritchie

LAP NUMBER 19

WRITTEN BY Diane Evans

RATIONALE

In your previous LAPs you have been solving applied problems by using linear equations and inequalities involving one variable. Actually, most applied problems can be solved in this manner, depending upon your ingenuity! There are instances where it is preferable to use two variables rather than one. This requires that you be able to solve systems of linear equations and inequalities.

In LAP 8, you had some experience in finding solution sets to systems of linear equations and inequalities, through the use of graphing. You also found that the "graphing technique" was of limited value since your results were only approximations of the correct solutions.

In this LAP, you will be learning more precise techniques of computing the solution sets for systems of linear equations and inequalities. This will enable you to solve applied problems, using two variables rather than one!

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given a pair of linear equations in two variables, compute their solution set using the COMPARISON method.

2. Given a pair of linear equations in two variables, compute their solution set using the SUBSTITUTION method.

3. Given a pair of linear equations in two variables, compute their solution set using the ADDITION method.

4. Given a word problem, TRANSLATE it into an OPEN mathematical sentence (or sentences) and SOLVE for the UNKNOWN (or unknowns).

RESOURCES

Obj. 1

Nichols, read pp. 293-297, Ex. 1 all parts pages 297-298.

Games: Graphing Pictures nos. 11, 6, 22

Obj. 2

Vanatta, read pp. 249-250, Ex. 1-8 pages 250-251.

Dolciani, read pp. 378, Ex. 1-18 even page 378.

Nichols, read pp. 299-300, Ex. 1,2 page 300.

Payne, read p. 235, Ex. 1-18 even page 236.

Wooton, read pp. 240-242, Ex. 1-18 even page 243.

Pearson, read pp. 477, Ex. 1,2 page 478.

Games: Graphing Pictures: nos. 11, 6, 22.

Obj. 3

Vanatta, read pp. 246-248, Ex. 1-4 page 245; 5-10 page 246; 1-4 page 248.

Dolciani, read pp. 370-371, 374-375; Ex. 1-18 even page 372; 1-18 even page 376.

Nichols, read pp. 301-303, Ex. 1,2 page 303.

Payne, read pp. 227-229, Ex. 1-10 p. 230.

Wooton, read pp. 236-238, 1-31 even page 239.

Obj. 4

Vanatta, read p. 253, Ex. 1-7, 11-15 pages 253-254.

* Dolciani, read pp. 372-373, Ex. 1,3,5,6 page 373; 1-4 page 374; 2,3,4 p. 379; 1-3 page 311; 1-3 page 385.

Wooton , read pp. 244, 247-248, Ex. 1-20 p. 245; 1-22 even pages 250-251.

Pearson, read pp. 482-483, Ex. 1-3, 6, 17, 21 pp. 484-486.

Wollensak tape C-3809

Nichols, read pp. 228-232, 305; Ex. 1-18 even pages 232-234; Ex. 11,12 p. 238; 4 p. 403.

* required

OBJECTIVE

1 I. Write the [illegible] in the word
[illegible] of the four whose names
[illegible] between 2 and 11.
[illegible] boundary
[illegible] between 2 and 12.
[illegible] by loss

2 II. Re[illegible] [illegible] method.
[illegible] there here than 4}
[illegible] [illegible] Ca. [illegible]}
[illegible] [illegible] [illegible] }
[illegible] than 10}
[illegible]

3 III. Rewrite [illegible] [illegible] [illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

4 IV. Match [illegible]
not in [illegible]

[illegible] 17. {1, [illegible]

___ 18.

___ 19.

5, 6 V.

___

___

___ 2[illegible]

7 VI.

___ 2[illegible]

___

___ 2[illegible]

___ 30.

___ 31.

8 VII.

___

BEHAVIORAL

9. Describe

10. Give a
are p

11. Give a
e

12. Give

and interaction

15.
be

16.
be

OBJECTIVES

11,12 J. ... ... true or false.

... ... and the set ...

... ...

... and the set of ... joint sets.

... 2, 1 + 1, 1 + 0} ,

... {0}

1. ... following pairs

... 3, 0}

... B = {1, 3, 5, 7...}

... {2, 3, 5, 7...}

... {50, 51...99,100}

... = {2, 3, 5, 10}

13,14,15 ... true or false.

... the

16

23.

__ 24.

The Shaded Area is

a. (A ∪ B) ∪ C

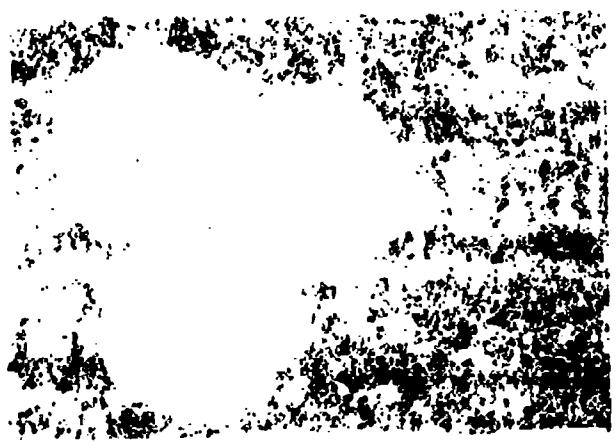
b. (A ∩ B) ∪ C

c. A ∪ C

d. (A ∪ B) ∩ C

e. none of these

__ .



The Shaded Area is

a. $(\overline{A \cup B}) \cup C$

b. $A \cap (\overline{B \cup C})$

c. $(A \cap \overline{B}) \cup \overline{C}$

9, 10 VIII. [illegible] following statements are true [illegible]

__ . [illegible] member of B is also a member

__ .

__ .

__ . [illegible] set above is only one subset of [illegible] elements.

__ . [illegible] of the set of natural numbers

__ .

IF YO[illegible] [illegible], TAKE THE LAP TEST.
CONSULT [illegible].

The [illegible]

1- Take either kingdom and draw a Venn diagram showing [illegible] particular species. [illegible] information [illegible]

2- [illegible] there are three primary [illegible] and then blended in [illegible] diagram of the three [illegible] are blended. Use set [illegible] obtained.

[illegible] construct a Venn Diagram [illegible]

Holt, Rinehart

and Winston, Inc., [illegible].

Pearson, Helen [illegible], and Allen, Frank B. Modern Algebra A

[illegible].

[illegible] and Co., 1964.

[illegible], Francis G.

[illegible]

[illegible], Science and Mathematics, [illegible].

Dolciani, Mary [illegible], Berman, Simon, Allman, Gustave J., Freilich, Edwin F.,

[illegible], Kaskosz, Stanley, [illegible], Modern School

[illegible].

[illegible], Houghton Mifflin Company, [illegible].

[illegible], Albert and Sexton, Inc., [illegible].

[illegible], Mary K. Reed, [illegible], Fehlbaum, Julius

[illegible] Company, 1965.

[illegible].

[illegible], Harold [illegible], Howard [illegible]. Merrill

[illegible].

and [illegible]

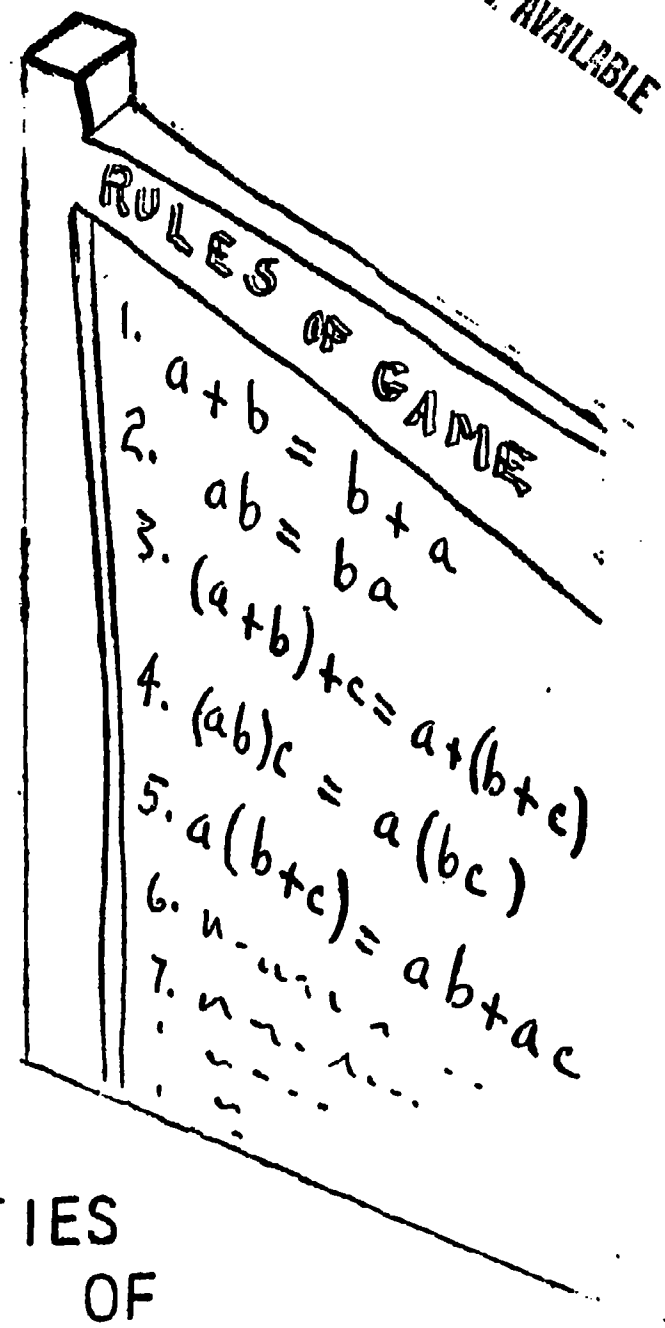
[illegible] 0-3452

LAP

LEARNING

ACTIVITY

PACKAGE



PROPERTIES OF OPERATIONS

Ninety Six High School
Ninety Six, S.C.

ALGEBRA 93 - 94

REVIEWED BY _____

LAP NUMBER 2-13

WRITTEN BY Diane Evans

RATIONALE (The LAP's Purpose)

Mathematics can be thought of as a game in which you perform moves by specific rules. For example:

WHAT NUMBER IS EQUAL TO 4 + 3 x 5?

You may work it out this way:

4 + 3 = 7 x 5 = 35

or you may compute it this way:

3 x 5 = 15 and 4 + 15 = 19

As you can see there are two possible answers!! Obviously, both ways cannot be correct because 35 is not equal to 19! The expression 4 + 3 x 5 must have only one meaning! It is customary to use parentheses, which are mathematical punctuation marks, to make the meaning of such phrases clear.

In this LAP you will not only learn how parentheses are used but also about the "order of operations" - that is the order in which certain operations are to be performed. Though you have studied the properties of operations in the past, you will review them again because of their importance in learning mathematics.

Behavioral Objectives

Upon completion of your prescribed course of study, you will be able to:

1. Write the simplest name for any numerical expression which involves the use of grouping symbols and order of operation.

2. Given any mathematical sentence, identify which of the following properties (if any) are being illustrated.

   a) The Commutative Property of Addition (CPA)

   b) The Commutative Property of Multiplication (CPM)

   c) The Associative Property of Addition (APA)

   d) The Associative Property of Multiplication (APM)

   e) "Symmetric Property of Equality" (SPE)

   f) "Distributive Property of Multiplication over Addition" (DPMA)

3. Given any set, determine if it is closed with respect to a given operation

4. Given any mathematical sentence, identify which of these properties (if any) are being illustrated.

   a) Multiplicative identity

   b) Property of one for Division (P1D)

   c) Additive identity

   d) Property of Zero for Multiplication (PZM)

5. Given any mathematical sentence, identify which of the following properties is being illustrated:

   a. multiplicative inverse
   b. additive inverse

6. Given any mathematical sentence involving one operation, write an equivalent sentence using the inverse operation.

7. Given any word phrase, of the type in Appendix I, translate it into an equivalent mathematical phrase.

8. Correctly write a mathematical sentence of the type in Appendix II which would be used to solve a given word problem.

NOTE: (EOL means every other letter.)

Objective 1

Vanatta, read pp. 47-48, Ex. 1-4 page 47.
Dolciani, read p. 23, Ex. 18-20, 30-34 page 24.
Nichols, read pp. 31-33, Ex. 1-25 odd pages 33-34; 2 EOL page 37.
Payne, read page 18, Ex. 1-5 page 18.
Wooton, read pp. 10-16, Ex. 1-10, 25-32 pages 13-14; 1-10 odd, 17-29 odd pages 15-16; 1-10 page 51.
Pearson, read pages 52-59, Ex. 1-25 odd page 55; 1-3 EOL pages 56-57; 1-5 EOL page 58.

Objective 2

Vanatta, read pp. 27-33; Ex. 2, 7, 10 page 29; 13, 18, 19, 24 page 34; 14, 15 page 37; 14 page 30; 6, 20 page 34.
Dolciani, read page 69, 73-76; Ex. 1-26 page 74; 15, 16 pages 100-101; 1, 6, 7 page 70; 2 page 98.
Nichols, read pages 34-37, 40-46; Ex. 1 page 37; 1 EOL, 2 pages 40-41; 1, 2 EOL, 3 pages 42-43.
Payne, read pages 31-38, Ex. 1, 3 page 33; 1-51 odd pages 35-37.
Wooton, read pages 48-52, 55-59, 71-76; Ex. 11-28 pages 50-51; 1-6, 7-23 odd pages 57-58.
Pearson, read pp. 139-145, 166-169; Ex. 1-14 pages 141-142; 1-3, 4 EOL, 5, 6 pages 144-145; 1-4 EOL, 5-7 pages 168-169.
Wollensak C-3453 The Commutative Property
C-3454 The Associative Property
C-3455 The Distributive Property

Objective 3

Dolciani, read pages 70-71, Ex. 1-14 oral p. 72; 1-12 written page 73.
Nichols, read pages 37-39, Ex. odds page 39; 4 a, c, e page 51; 8 (state why or why not) page 53.
Payne, read page 31; Ex. 62-73 pages 37-38.
Wooton, read page 47, Ex. 1-10 pages 49-50.
Pearson, read pages 135-137, Ex. 1-12 pages 136-137.
Wollensak C-3456 The Closure Property

Objective 4, 5

Vanatta, read page 27 and study 2-5, page 28 study 2-5; Ex. 1, 4, 8, 9 page 29; 12, 13 page 30; 15-17, 21, 22 page 34; 17-19 page 37.
Dolciani, read pages 77, 121, 138; Ex. 33-42 page 141; 1-10 oral page 122.
Pearson, read pages 162-166, 178-180, 204; Ex. 1, 2 EOL, 3, 5-7 EOL, 9 pages 164-166; 2 a, b, e, f, g, 5 a-h, page 165, 3 a-e, 4 a-d, g-i page 296.
Wollensak C-3459 Identity Element
C-3451 The Inverse Element

Objective 6

Nichols, read pages 46-48, Ex. 1-3 EOL pages 47-48.

Objective 7

Dolciani, read p. 51, Ex. 1-22 pages 52-53; 1-25 even page 54; 3, 6, 13, 14 page 55.
Nichols, read pages 48-49, Ex. 1-21 page 49.
Wooton, Ex. 11-24 page 13; 11-16 page 16.
Pearson, read pages 145-147, Ex. 10-13 page 147.
* Appendix I

Objective 8

Vanatta, read pages 154-156, Ex. 1-10, 12-15 pages 156-157.
Dolciani, read pages 57-58, Ex. (write equation only) 1-14 odd pages 57-58; 1-14 page 18.
Payne, read pages 65-66, Ex. 21-23 pages 65-66.
Wooton, read pages 29-33, Ex. 31-38 page 33 (equation only).
Pearson, read pages 157-159, Ex. (write equations only) 1-19 odd pages 158-159; 5-15 odd pages 176-177.
* Appendix II
Wollensak C-3801 Open Phrase, Open Sentence
C-3803 Open Sentence: Solution
C-3809 Reading Written Problems

Games

Equations by Layman Allen

* required

1. What is the cost of n pencils at 3 cents each?
2. What is the cost of x articles at y dollars each?
3. How far can a boy run in h hours at the rate of 6 miles per hour?
4. The sum of two numbers is 7 and one of them is x; what is the other number?
5. Represent in terms of x two numbers that have the ratio 3:4.
6. If x represents the sum of two numbers and one of them is 5, what is the other?
7. What is the total weight of n boys weighing y pounds each?
8. The sum of two numbers is x and one of the numbers is 5. What is the other number?
9. How many cents are there in d dollars?
10. Mary is n years old now. How old was she 3 years ago?
11. How many inches are there in x feet and five inches?
12. A man had x dollars and spent y dollars. How much did he have left?
13. If n represents a certain number, represent the next larger consecutive number.
14. A parcel weighs t pounds and a smaller parcel weighs $\frac{2}{3}$ as much. What is the weight of the smaller parcel?
15. What is the average weight of two boys who weigh x pounds and y pounds each respectively?
16. How much salt remains when x pounds have been used from a bag containing a pounds?
17. What is the perimeter of a square one side of which is s?
18. The difference between two numbers is 2 and the smaller number is n. What is the other number?
19. One part of t is w. What is the other part?
20. Elizabeth's age is now 7 years. How old will she be in n years
21. x = 5y. Upon what does the value of x depend?
22. What is the perimeter of a triangle whose sides are a, b, and c?
23. What is the perimeter of a rectangle whose length is l and whose width is w?
24. What is the area of a rectangle whose base is b and whose height is h?
25. How many inches are there in y yards, f feet and i inches?

Write the mathematical sentence which would be used to solve each word problem.

1. Five times a certain number is 105.

2. The greater of two numbers is twice the smaller and their sum is 48.

3. Mary is 5 times as old as her brother and their combined ages total 18 years.

4. One number is 4 times another and their difference is 24.

5. A man walks 6 hours at a certain rate and then proceeds 3 hours at twice his former rate. If he walked 24 miles in all, at what rate did he start walking?

6. The length of a rectangle is three times its width and its perimeter is 56 feet.

7. The sum of three numbers is 56. The second number is 3 times the first and the third number is 4 times the first. What are the numbers?

8. The sum of the three angles of any triangle is 180 degrees. In a certain triangle ABC, angle A is twice as large as angle C and angle B is three times as large as angle A.

9. Two numbers have the ratio 5:6 and their sum is 88. Find the numbers.

10. Separate 92 into two parts such that one part is three times the other.

11. One number is three times another. Six times the larger diminished by twice the smaller is 48.

12. When a quart of cream cost four times as much as a quart of milk, 5 quarts of milk and 3 quarts of cream cost $2.72, what is the cost of each per quart?

C. Applications

1. If you mix 4 grams of salt and 64 grams of water, what percent of the total solution is salt?

2. Compound X is composed of elements Y and Z in the ratio of 3:2. If you had 50 grams of element Y, how many grams of element Z would you need to utilize all of Y into making compound X?

Self-Evaluation Test

Behavioral Objectives

1 I. Write the simplest answer for each of the following:

____1. $56 \div 2 \times 3 \div \frac{1}{2}$

____2. $(\frac{3}{4} - \frac{1}{3})(3 + 9) + 3 \quad [(5 - 3) \div 6]$

____3. $15 - 3 \times 2 + (18 - 5) \cdot 5$

____4. $3 \cdot 4 + 2 + 5 \cdot 3 - 3 \cdot 7$

____5. $5 \cdot 0 + 4 - 2 \cdot 2$

____6. $[5 \cdot (2) + 11] \div 7$

____7. $4 \times \{78 - [5 \cdot 4 + (15 \div 3)] \times 3\} \cdot 2$

____8. $28 \div 2 - 2 \times 6 \div 3 + 3$

____9. $(7 + 5)^2 - 6$

____10. $7 \times 8 - 6$

2 II. For each given sentence, write the name of the property illustrated. (Use abbreviations such as CPA, etc.)

____11. $(46 + 21) + 7 = 46 + (21 + 7)$

____12. $16 \cdot (5 + 7) = (5 + 7) \cdot 16$

____13. $4 \cdot (7 + 13) = 4 \cdot (13 + 7)$

____14. $15 \cdot (6 \cdot 2) = (15 \cdot 6) \cdot 2$

____15. If $a + b = c$ then $c = a + b$

____16. $4 \cdot [x + (2 + 5)] = 4 \cdot x + 4 \cdot (2 + 5)$

____17. $ax + ay = a \cdot (x + y)$

3 III. Each of the following statements is either true or false.

____ 18. The set $\{0, \frac{1}{2}, 1\}$ is closed under multiplication.

____ 19. The set of odd natural numbers is closed under the operation of multiplying by the factor 2.

____ 20. The set of even natural numbers is closed under addition.

____ 21. The set $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots\}$ is closed under addition.

____ 22. The set of all prime numbers is closed under addition.

4 IV. For each given sentence, write the name of the property illustrated. (Use abbreviations such as P1 M, etc.)

____ 23. If $3 + y = n$ then n is a natural number.

____ 24. $y \cdot (5 - 4) = y$

____ 25. $5 \cdot (x - x) = 0$

____ 26. $x + 0 = x$

____ 27. $7 + 0 = 7$

____ 28. $6 \cdot 1 = 6$

6 V. For each statement, write a correct related problem using the inverse operation.

_____ 29. $11 - 4 = 7$

_____ 30. $\frac{1}{10} + \frac{1}{2} = \frac{3}{5}$

_____ 31. $51 \div 3 = 17$

_____ 32. $\frac{15}{23} \times \frac{7}{5} = \frac{3}{4}$

7 VI. For each phrase write a correct mathematical phrase.

_____ 33. The sum of four times n and 7.

_____ 34. 15m decreased by one-half x.

_____ 35. Four y divided by the sum t and x.

_____ 36. The quotient of seven x and fifteen.

_____ 37. Three times the difference of 2 and q.

_____ 38. The product of 5 and x increased by t.

_____ 39. Seven less than four p.

_____ 40. Four times the sum of n and 7.

8 VII. Translate each word sentence into an equivalent open mathematical sentence. (Do not solve)

_____ 41. The difference of three times a number (w) and six is the number increased by four.

_____ 42. The quotient of seven and the sum of a number (x) and eight plus five is twenty-three.

_____ 43. One-fifth of a certain man's lifetime (y) spent in childhood, plus one-third of his life served in the armed services, totals two eight years he spent as a bum and the two-fifths of his life as a millionaire.

5 VIII. For each given sentence, write the name of the property illustrated.

_____ 44. $2 \cdot \frac{1}{2} = 1$

_____ 45. $6 + (-6) = 0$

_____ 46. $\frac{2}{3} \cdot \frac{3}{2} = 1$

_____ 47. $(-7) + 7 = 0$

_____ 48. $8 \cdot \frac{1}{8} = 1$

1. Dolciani, Modern Algebra, Page 90 nos. 21-31. (Work at least six problems)

2. Dolciani, Modern Algebra, page 95 nos. 17-21. (Work at least 4 problems)

3. Dolciani, Modern School Mathematics, page 52 nos. 17-20. (Work all problems) .

4. Research the concept of field - Select a system of numbers and determine if the system is a field. Write a report on your findings, giving reasons for your conclusions.

5. Prepare a bulletin board showing all the properties and their relationships to the following sets of numbers: naturals, wholes, integers, and rationals.

References

1. Vanatta, Glen D. and Goodwin, A. Wilson, Algebra One: A Modern Course, Charles E. Merrill Publishing Co., 1966.

2. Dolciani, Mary D., Berman, Simon L., and Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

3. Nichols, Eugene D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1966.

4. Pearson, Helen R., and Allen, Frank B., Modern Algebra A Logical Approach, Ginn and Company, 1964.

5. Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis G. Algebra One, Harcourt, Brace and World, Inc., 1969.

6. Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics Algebra 1, Houghton, Mifflin Company, 1967.

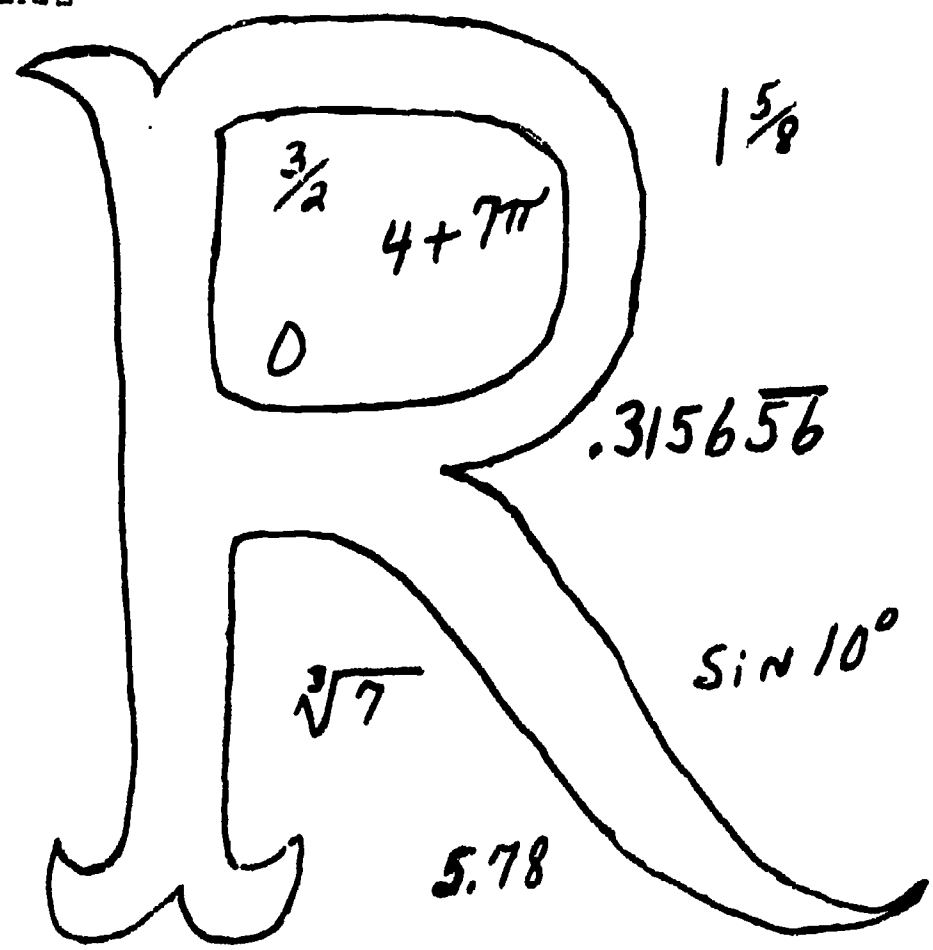
7. Wollensak Teaching Tapes

Wollensak C-3451 The Commutative Property
C-3454 The Associative Property
C-3455 The Distributive Property
C-3456 The Closure Property
C-3459 Identity Element
C-3451 The Inverse Element
C-3801 Open Phrase, Open Sentence
C-3803 Open Sentence: Solution
C-3809 Reading Written Problems

8. Equations by Layman Allen

LEARNING ACTIVITY PACKAGE

OPERATIONS OVER REAL NUMBERS



Ninety Six High School - Ninety Six, S.C.

ALGEBRA 93-94

REVIEWED BY

LAP NUMBER 14

WRITTEN BY Diane Evans

R A T I O N A L E

You have studied many sets of numbers through your mathematical career. The first set you discussed was the set of "NATURAL NUMBERS" (1, 2, 3, . . .). You then added zero and the set became the "WHOLE NUMBERS", after which you extended the set to include additive inverses and the set became the "INTEGERS". When you finally added the multiplicative inverses and arrived at the "RATIONAL" numbers, it appeared as though you were finished!

In this LAP you will extend the set of "RATIONAL" numbers. We will call the numbers that we ADD, the "IRRATIONAL" numbers. The set then becomes the "REAL" number system. Once you have at your disposal knowledge of the complete set of REAL numbers you will be equipped to investigate the basic concepts of elementary Algebra!

Behavioral Objective

After having completed your prescribed course of study, you will be able to:

1. Write or identify the definition of the sets of natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.

2. Given any number, determine if it is a member of the set of

   a. natural numbers
   b. whole numbers
   c. integers
   d. rational numbers
   e. irrational numbers
   f. real numbers

3. Given any statement involving relationships among the sets of natural, whole, integer, rational, irrational, and real numbers, determine if it is true or false.

4. Determine if each of the sets of natural, whole, integer, and rational numbers is a field. If a set is not a field, state the properties that do not apply, or answer questions, or by completing a chart like the one in Appendix I.

5. Given a pair of integers a and b, determine whether $a < b$, $a = b$, or $a > b$.

6. Given two or more rational numbers, compute their sum, difference, quotient, and/or product.

RESOURCES

NOTE: EOL means every other letter.

Obj. 1

Vanatta, read pages 22, 80-82, Ex. 1-5 page 83.
Wooton, read pages 22, 316, 425, Ex. write the definitions of the terms in Obj. one.
Wollensak Tape C-3458: The Real Number System

Obj. 2

Payne, read pages 52-54, Ex. 1-12 page 54; 1-15, 32-41 page 55.
Vanatta, read pages 79-83, Ex. 14 page 84.
Wollensak Tape C-3458: The Real Number System

Obj. 3

Vanatta, read pages 79-83, Ex. 13 page 83.
Payne, read pages 52-54, Ex. 13-20 page 54; 16-30 page 55.
Wollensak Tape C-3458: The Real Number System

RESOURCES (cont')

Obj. 4

* Appendix 1
Vanatta, read pages 28-39, 85, Ex. 2, 3, 5 page 86.
Nichols, read pages 104-105, Ex. 1-4 pages 105-106.
Wollensak Tapes C-3458: The Real Number System
C-3453: The Commutative Property
C-3454: The Associative Property
C-3456: The Closure Property
C-3457: The Inverse Elements
C-3459: The Identity Element

Obj. 5

Nichols, read pages 55-59, Ex. 1-5 EOL pages 59-61; 1-2 EOL page 62
Wooton, read pages 58, Ex. 1-49 odd pages 9-10.
Payne, read pages 56-57, Ex. 1-61 odd pages 57-58.
Pearson, read pages 37-38, Ex. 1-6 EOL pages 38-39.

Obj. 6

Vanatta, read pages 90-106, Ex. 1-13 pages 92-93; 1-24 pages 95; 1-16 page 99; 1-20 odd (bottom) pages 100-101; 1-26 page 104; 1-18 page 106; 21-25 page 107.
Dolciani, read pages 125-126, 128-130, 133-135, 138-140, Ex. 1-16 page 126; 1-12 page 130; 1-11 page 135; 1-12 page 140; 1-8 page 141.
Nichols, read pages 62-63, 85-88, 92-94, 96-98, Ex. 1-2 EOL pages 64-65; 6 a-n page 68; 3 a-z page 89; 1 a-y page 95; 2 all page 99.
Payne, read pages 66-69, 73-74, 75-77, 79-80, 82-83; Ex. 1-51 even page 69; 1-34 page 74; 1-30 page 77-78; 10-24 page 81; 1-12 page 83.
Arithmetic of Directed Numbers, A programmed unit.
Wollensak Tapes C-3331 Directed Numbers: Addition
C-3332 Directed Numbers: Subtraction
C-3333 Directed Numbers: Multiplication
C-3334 Directed Numbers: Division
Filmstrips:
Comparing Fractions: Adding and Subtraction
Multiplying Fractions
Multiplication of Signed Numbers
Dividing Fractions
Game: The Conversion Game

* required

SELF-EVALUATION 1

1 I. Write the definition or set for each of the following:

1. natural numbers

2. whole numbers

3. integers

4. rational numbers

5. irrational numbers

6. real numbers

2 II. Identify the following numbers as elements of naturals (N), wholes (W), integers (I), rationals (Q), irrationals (Z), or Reals (R). List all the sets that contain each number.

____ 7. 28

____ 8. $\sqrt{9}$

____ 9. $\frac{3}{4}$

____ 10. $\sqrt{7}$

____ 11. $1.\overline{3}$

____ 12. .010010001...

____ 13. 0

____ 14. -18

____ 15. $-\frac{7}{8}$

3 III. True or False.

_____ 16. The natural numbers are a subset of the whole numbers.

_____ 17. The integers are a subset of the natural numbers.

_____ 18. The whole numbers are a subset of the rationals.

_____ 19. The rational numbers contain the integers and the fractions.

_____ 20. The natural numbers are a subset of the rational numbers.

_____ 21. The integers are not a subset of the rational numbers.

3 IV. True or False.

N = set of natural numbers
W = set of whole numbers
I = set of integers
Q = set of rational numbers
Z = set of irrational numbers
R = set of real numbers

_____ 22. $Q \subset R$

_____ 23. $Q \cup Z = R$

_____ 24. $W \subset N$

_____ 25. $Z \subset R$

_____ 26. $N \subset R$

4 V. Is each of the following sets a field? If no, write the properties necessary to make it a field.

_____ 27. whole numbers

_____ 28. integers

_____ 29. natural numbers

_____ 30. rational numbers

5 VI. In each blank write <, >, or = to make a true statement.

31. 7 ____ 2

32. -10 ____ 5

33. 0 ____ 18

34. 8 + 1 ____ 9

35. 0 ____ -17

36. -2 ____ 2

37. 3 · 5 ____ -15

38. -5 ____ 3

39. -7 ____ -9

40. 10 ____ -10

6. VII. Work the following:

_____ 41. $-\frac{2}{3} + -\frac{4}{5} =$

_____ 42. $-\frac{2}{3} \cdot \frac{4}{5}$

_____ 43. $-\frac{4}{5} + -\frac{2}{3}$

_____ 44. $-\frac{6}{5} + \frac{3}{4}$

_____ 45. $-\frac{2}{3} + \frac{4}{5}$

_____ 46. $-\frac{4}{7} + -\frac{3}{4}$

_____ 47. $-\frac{2}{5} \cdot \frac{3}{4}$

_____ 48. $-\frac{1}{6} + -\frac{2}{3}$

_____ 49. $\frac{1}{4} - \frac{1}{7}$

_____ 50. $\frac{2}{5} + -\frac{4}{5}$

_____ 51. -8×-7

_____ 52. $9 + -3$

_____ 53. $-6 - 9$

_____ 54. $28 + -7$

_____ 55. $-18 + -9$

_____ 56. $36 \cdot -2$

_____ 57. $-6 + -12$

_____ 58. $3 - -7$

_____ 59. $-18 \div -2$

_____ 60. $12 + -13$

4 VII. True or False.

_____ 61. The set of natural numbers form a field with operations addition and multiplication.

_____ 62. Every element in a field has a multiplicative inverse.

_____ 63. In a field addition and multiplication are associative.

_____ 64. The integers do not have multiplicative inverses.

_____ 65. The natural numbers have all the properties except additive inverses.

_____ 66. The irrational numbers do not have an additive identity.

_____ 67. The rational numbers form a field.

_____ 68. Every integer has an additive inverse.

_____ 69. Addition is associative.

_____ 70. Subtraction is commutative.

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, YOU MAY TAKE YOUR PROGRESS TEST. CONSULT YOUR TEACHER FIRST.

SECTION 2

Behavioral Objectives

After having completed your prescribed course of study, you will be able to:

7. Write the simplest name for any given phrase involving a combination of addition, subtraction, multiplication, and/or division.

8. Given an open sentence with at least one unknown and a universal set which is a subset of the reals, determine the set of all replacements for the unknown(s) that will make that sentence true. (That is the solution set for the sentences.) You will determine the solution set by observation only.

9. Given any rational number of the form $\frac{a}{b}$, express it in decimal form and state whether it is a terminating or repeating decimal.

10. Given any rational number expressed in decimal form, write it in the form $\frac{a}{b}$ where a is a whole number and b is a natural number.

11. Given any pair of rational numbers, name the number midway between them.

12. Given any word phrase like the ones in Appendix I, translate it into an equivalent mathematical phrase.

RESOURCES

Obj. 7

Vanatta, Ex. 9-25 page 107.
Nichols, read pages 31-32, 101-102, Ex. 1 page 103.
Wooten, Ex. 1-20 page 99; 23-30 page 115.
Payne, Ex. 45-48 and 55-66 page 84.
Pearson, Ex. 7, 12, 15 page 265; 9 page 267.

Obj. 8

Nichols, Ex. 5-7 page 90; 2 page 95; 5 page 101, 2 EOL page 104.
Wooten, Ex. 9-12 page 109.
Payne, Ex. 1-12 page 72; 36-44 and 49-54 page 84.
Pearson, Ex. 12, 13 page 226; 7 page 230.

Obj. 9

Dolciani, read pages 400-402, Ex. 1-12 even page 403.
Nichols, read pages 30-31, 65-77, Ex. 1 a, c, e, g, h, i page 77, 1-4, 5-6 EOL pages 67-68.
Wooten, read pages 422-423, Ex. 1-8 page 426.
Pearson, read pages 268-269, Ex. 58-63 page 30.

RESOURCES (cont')

Obj. 10

Dolciani, read pp. 400-402, Ex. 13-20 p. 403.
Nichols, read page 68-70, Ex. 1-3 EOL page 70.
Wooten, read page 424-425, Ex. 9-19 odd p. 426.
Payne, read p. 30, Ex. 64-69 page 30.

Obj. 11

Dolciani, read p. 398, Ex. 15-20 p. 400.
Nichols, read p. 71, Ex. 1 p. 71.
Wooten, read p. 2, Ex. 21-22 p. 3, 19-22 p. 115.
Pearson, read pp. 35-36, Ex. 1-4 p. 36.

Obj. 12

Nichols, read p. 79, 107, Ex. 1-19 pp. 79-80; 1-22 p. 108.
Wooten, read p. 10, Ex. 11-23 odd p. 13; 31-38 p. 33.
Payne, read p. 128, Ex. 1-35 odd pp. 128-130.
Pearson, read p. 145, Ex. 10, 12-14 p. 147; 6-11 p. 226.
Wollensak Tape C-3801: Open Phrase, Open Sentence
* Appendix II

* Required

SELF-EVALUATION 2

7 I. Express the following fractions as decimals and state if they are repeating or terminating.

1. $\frac{4}{9}$

2. $\frac{3}{8}$

3. $\frac{2}{11}$

4. $\frac{2}{7}$

8 II. Express the following decimals as fractions.

5. .12

6. $.\overline{274}$

7. $.5\overline{3}$

8. .684

9. $.\overline{73}$

10. .82

9 III. Find the rational number midway between the following:

11. $9\frac{1}{2}$ and $11\frac{1}{3}$

12. 2.19 and 1.11

13. -3.12 and 3.76

14. $\frac{1}{6}$ and $\frac{3}{24}$

15. $\frac{3}{4}$ and $\frac{15}{16}$

10 IV. Write the mathematical phrase of each word phrase.

_____ 16. sum of 17 and x

_____ 17. 3 more than x

_____ 18. the square of the sum of 3 and x is

_____ 19. three times the square of x

_____ 20. the quotient of x divided by 2 y

_____ 21. ____ is the next consecutive odd integer after x

_____ 22. t is an integer, give the next three consecutive integers

_____ 23. number of feet in 7t yards

_____ 24. number of quarts in (a + 3t) gallons

_____ 25. worth in cents of y eight-cent stamps

_____ 26. the reciprocal of 2m

_____ 27. the sum of the reciprocals of x and y

_____ 28. If Dave is 2n years older than Suzie and Suzie is 3x + 1 years old, how old is Dave?

_____ 29. The reciprocal of the sum of x and y

_____ 30. 12 increased by n-4

_____ 31. If Ed was x + 1 years old 3 years ago, how old is he now?

7 V. Perform the indicated operation (remember the order of operation from a previous LAP.)

_____ 31. $-4 + 7 - (-3)$

_____ 32. $2 - (-4) + 3$

_____ 33. $-3 \times 4 - 6$

_____ 34. $17 \times (-6) + 17 \times (-4)$

_____ 35. $9 - 12 + 7 + 6 - 4$

_____ 36. $2 \times (-3) + (-2) \times \frac{1}{2} + 12 + (-6)$

8 VI. In the following write the number(s) that make each statement true.

_____ 37. $3 \cdot 7 + -4 = x$

_____ 38. $P - 6 = -12$

_____ 39. $\frac{1 + x}{2} = -2$

_____ 40. $3x + 2 = -13$

_____ 41. $x \cdot -4 = -24$

APPENDIX I (Obj. 4)

Put an x by each property that holds for the given sets of numbers.
Put a circle (0) by each property that does not hold. Do not leave a blank.

PROPERTY	NATURALS	WHOLES	INTEGERS	RATIONALS	REALS
Closure for +					
Closure for x					
Commutative +					
Commutative x					
Associative +					
Associative x					
Distributive					
Add. Identity					
Mult.Identity					
Add. Inverses					
Mult.Inverses					

APPENDIX II (Obj. 12)

IN THIS SECTION you will learn to translate from a word phrase to a mathematical expression. This will help you when you later solve word problems. You are to fill EACH blank below with a mathematical expression. YOU ARE NOT TO DO ANY COMPUTING. (The first two problems have been completed for you as examples.)

1. The sum of 3 and the product of 2 and 6 is $3+2(6)$
   (You should not write 15 or 3 + 12, since that requires computing!)
2. Three more than the square of x is 4^2+3
3. The sum of 5 and 9 is _____
4. The sum of $\frac{2}{3}$ and -6 is _____
5. The sum of 17 and x is _____
6. 5 more than 7 is _____
7. 18 increased by 12 is _____
8. x more than 10 is _____
9. 3 more than x is _____
10. The sum of $2x$ and $5 + 3x$ is _____
11. $5t$ increased by $5 - x$ is _____
12. The square of the sum of 3 and 4 is _____
13. The sum of the squares of 3 and 4 is _____
14. The square of the sum of $2x$ and $3y$ is _____
15. The sum of the squares of 5 and m is _____
16. Three times the square of x is _____
17. The square of the product of 3 and x is _____
18. The quotient of 17 divided by 6 is _____
19. The quotient of x divided by $3t$ is _____

20. The square of the opposite of 5 is _____

21. The opposite of the square of 5 is _____

22. The square of the opposite of x is _____

23. The opposite of the square of x is _____

24. Monday and _____ are consecutive days of the week.

25. Tuesday, _____ and Thursday are consecutive days of the week.

26. 1, 23, -62, and -14 are integers. -15, -16, -17, ____, and -19 are consecutive integers.

27. If x is an integer, then x, $x + 1$, and _____ are consecutive integers.

28. If y is an integer, then $y - 2$, $y - 1$, y, $y + 1$, _____, and $y + 3$ are consecutive integers.

29. If t is an integer, then $3t$ is an integer. Also, $3t$, $3t + 1$, _____, _____, and $3t + 4$ are consecutive integers.

30. -3, 0, 5, 7, and 212 are integers. -8, 0, 2, 16, -40, and 18 are even integers. If k is an even integer, then $k + 8$ is an _____ integer. 18, 20, _____, and 24 are consecutive even integers. If x is an even integer, x and _____ are consecutive even integers. If t is an even integer, then $t - 2$, t, $t + 2$, _____, and $t + 6$ are consecutive even integers.

31. 7 is an odd integer. 3, 5, 7, ____, and 11 are consecutive odd integers.

32. If m is an odd integer, then m and _____ are consecutive odd integers.

33. If r is an odd integer, then r, $r + 2$, _____, and $r + 6$ are consecutive odd integers.

34. The average of 6 and 4 is: _____

The average of 5, 82, 16, 93, and 74 is: _____

The average of a, b, c, and d is: _____

35. 5 less than 7 is _____.

36. 7 less than 5 is _____.

37. 3 less than x is _____.

38. x less than 3 is _____.

39. 10 decreased by 2 is _____.

40. 15 decreased by 5 is _____.

41. 7 decreased by x is _____.

42. 3t decreased by 5n is _____.

43. 7r less than 10t is _____.

44. 4 less than t + 5a is _____.

45. The absolute value of the sum of 3x and 2y is _____.

46. The sum of the absolute values of 3x and 3y is _____.

47. 5 times the sum of 6 and 2 is _____.

48. The product of 8 and the sum of 2 and t is _____.

49. The product of the sum 5 and 2t and the sum of 9 and 7t is _____.

50. The quotient of 8x divided by the sum of t and 3 is _____.

51. The quotient of the sum of 3 and p divided by the product of 3 and p is _____.

52. The reciprocal of 7 is _____.

53. The reciprocal of m is _____.

54. The sum of the reciprocals of 7 and m is _____.

55. The reciprocal of the sum of 7 and m is _____.

56. 7 nickels are worth _____ cents.

57. 42 nickels are worth _____ cents.

58. x nickels are worth _____ cents.

59. c + 2 nickels are worth _____ cents.

60. 3x nickels are worth _____ cents.

ADVANCED STUDY

1. Draw a chart showing in Venn diagram form the set of real numbers and all its subsets.

2. Prove $-(a - b) = -a + b$ Hint: $(a - b)$ is the additive inverse of $-(a - b)$.

3. Work all of the following:

a. $\left(-\frac{2}{5} + \frac{3}{4}\right) \cdot \frac{1}{5}$

b. $\left(\frac{3}{4} + \frac{1}{2}\right) + -\frac{2}{3}$

c. $\left[\left(\frac{-3}{4} + \frac{2}{3}\right) + \frac{1}{7}\right] \cdot \frac{2}{5}$

d. $\left(\frac{-3}{2} + \frac{2}{5} + \frac{1}{4}\right) + \frac{3}{2}$

e. $\left(\frac{-2}{3} - \frac{-3}{4}\right) + \left(\frac{-2}{3} \cdot \frac{6}{5}\right)$

4. Prove the integers of the form $3n + 1$ are not closed under addition.

5. Show examples to illustrate the following.

   a. two irrational numbers whose difference is irrational

   b. two irrational numbers whose product is irrational

   c. two irrational numbers whose quotient is irrational

   d. two irrational numbers whose quotient is rational

   e. two irrational numbers whose product is rational

6. Prove the set of irrational numbers is not a field.

7. Determine if the set of irrational numbers is a field by listing the field properties, giving an example of each using irrational numbers, and explaining whether or not each property applies.

8. Complete our number system by studying the complex numbers. Read pages 474-476 in Vanatta and work exercises 1-24 on pages 476-477.

REFERENCES

Vanatta, Glen D. Algebra One, Charles E. Merrill Publishing Co., 1966.

Dolciani, Mary P. Berman, Simon L., and Freilich, Julius, Modern Algebra, Book One, Houghton Mifflin Co., 1965.

Nichols, Eugene D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Payne, Joseph N., Zamboni, Floyd F., Lankford Jr., Francis G., Harcourt, Brace and World, Inc., 1969.

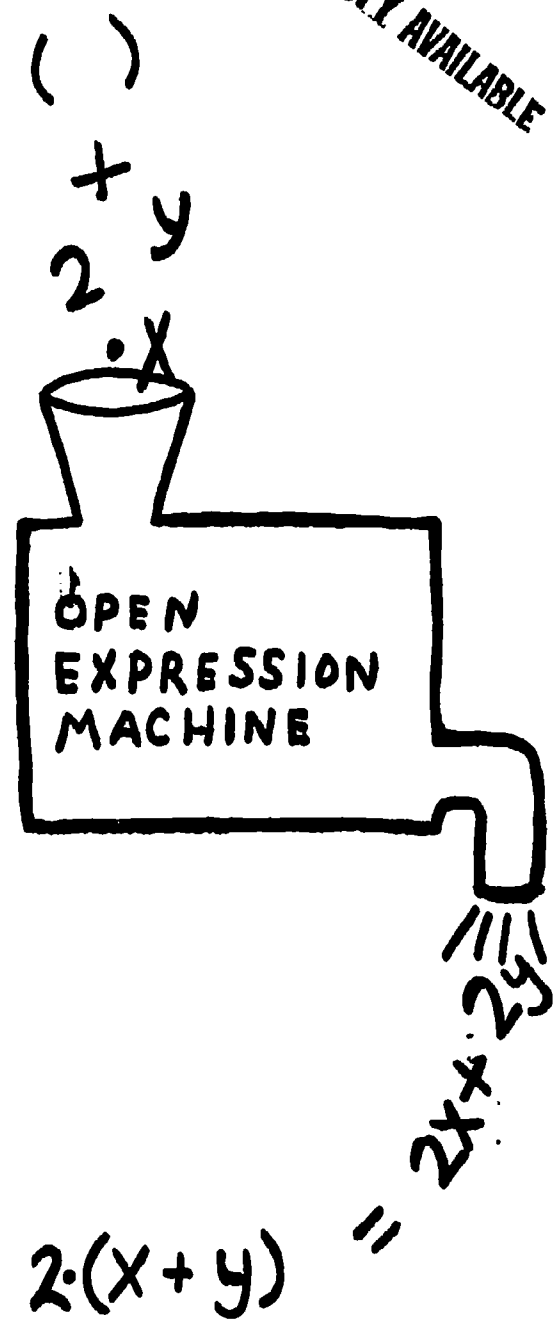
Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Jurgensen, Ray C., Donnally, Alfred J., Houghton Mifflin Co., 1967.

Pearson, Helen R., Allen, Frank B., Modern Algebra, A Logical Approach, Ginn and Company, 1964.

Nichols, Eugene D., Arithmetic of Directed Numbers, Holt, Rinehart and Winston, 1962.

Wollensak Teaching Tape - C-3458 The Real Number System
C-3453 The Commutative Property
C-3454 The Associative Property
C-3456 The Closure Property
C-3457 The Inverse Elements
C-3801 Open Phrase, Open Sentence
C-3459 The Identity Element
C-3331 Directed Numbers: Addition
C-3332 Directed Numbers: Subtraction
C-3333 Directed Numbers: Multiplication
C-3334 Directed Numbers: Division

LEARNING ACTIVITY PACKAGE



OPEN EXPRESSIONS

Ninety Six High School · Ninety Six, S.C.

ALGEBRA 93-94

REVIEWED BY

LAP NUMBER 15

WRITTEN BY Diane Evans

R A T I O N A L E

One of the most important concepts in the study of Algebra is that of a variable. In this LAP you will study terms and expressions, most of which contain one or more variables. Using many of the previously introduced properties and definitions, you will learn to apply new theorems, listed on the next page, which are concerned with equivalent expressions.

You will develop the ability to judge whether two given expressions are equivalent. This skill is necessary in solving equations. While you are developing this skill, you will begin to learn how to prove theorems.

PROPERTIES AND THEOREMS

For every number x, y, and z, the following apply:

Distributive property of multiplication over addition

$x(y + z) = xy + xz$

$xy + xz = x(y + z)$

$(y + z)x = yx + zx$

Distributive property multiplication over subtraction

$x(y - z) = xy - xz$

$(y - z)x = yx - zx$

$xy - xz = x(y - z)$

Multiplication by -1 $x(-1) = -x$

Division by -1 $\frac{x}{-1} = -x$

Opposite of x - y $-(x - y) = y - x$

Opposite of x + y $-(x + y) = -x + -y$

$(-x)y = -(xy)$

Some additional theorems to be covered in this LAP

$-(-x) = x$

$-(-x)y = xy$

$(-x)(-y) = xy$

THEOREMS TO BE DEVELOPED IN THIS LAP

$\forall_x \forall_y \neq o \forall_z \quad \frac{xz}{y} = \left(\frac{x}{y}\right)(z)$

$\forall_x \forall_y \neq o \forall_z \quad \frac{xz}{y} = (x)\left(\frac{z}{y}\right)$

$\forall_x \forall_y \neq o \forall_z \quad \frac{xz}{y} = \left(\frac{1}{y}\right)(xz)$

$\forall_{x \neq o} \forall_{y \neq o} \quad \left(\frac{1}{x}\right)\left(\frac{1}{y}\right) = \frac{1}{xy}$

$\forall_x \forall_{y \neq o} \forall_r \forall_{s \neq o} \quad \left(\frac{x}{y}\right)\left(\frac{r}{s}\right) = \frac{xr}{ys}$

(Multiplicative Identity Theorem) $\forall_x \forall_{y \neq o} \forall_{r \neq o} \quad \frac{xr}{yr} = \frac{x}{y}$

(Addition Theorem) $\forall_x \forall_{y \neq o} \forall_r \forall_{s \neq o} \quad \frac{x}{y} + \frac{r}{s} = \frac{xs + ry}{ys}$

(Subtraction Theorem) $\forall_x \forall_{y \neq o} \forall_r \forall_{s \neq o} \quad \frac{x}{y} - \frac{r}{s} = \frac{xs - ry}{ys}$

(Division Theorem) $\forall_x \forall_{y \neq o} \forall_{r \neq o} \forall_{s \neq o} \quad \frac{\frac{x}{y}}{\frac{r}{s}} = \frac{x}{y} \cdot \frac{s}{r}$

$\forall_x \forall_{y \neq o} \quad \frac{-x}{y} = -\frac{x}{y}$

$\forall_x \forall_{y \neq o} \quad \frac{x}{-y} = -\frac{x}{y}$

$\forall_x \forall_{y \neq o} \quad \frac{-x}{-y} = \frac{x}{y}$

$\forall_x \forall_{y \neq o} \quad -\left(\frac{x}{-y}\right) = \frac{x}{y}$

$\forall_x \forall_{y \neq o} \quad -\left(\frac{-x}{y}\right) = \frac{x}{y}$

$\forall_x \forall_y \forall_{r \neq o} \forall_{s \neq o} \quad \frac{x(-y)}{r(-s)} = \frac{xy}{rs}$

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given any polynomial, classify it as

   a. monomial
   b. binomial
   c. trinomial

2. Given any open expression and replacements for the variables, compute the value of the expression.

3. Given a pair of expressions, determine whether or not they are equivalent.

4. Using the appropriate properties, definitions, and theorems, write equivalent expressions for any given expression.

5. Given a pair of rational expressions, write a single equivalent expression that names their product.

6. Given a rational expression, use the multiplicative identity theorem to write a single equivalent expression where the numerator and denominator have no common factors.

7. Given a pair of rational expressions, write a single equivalent expression that names their sum.

RESOURCES

Obj. 1, 2

Vanatta, #1 read p. 67-71, Ex. 4 p. 71: #2 read p. 72, Ex. 1-27 p. 73.
Dolciani, Modern Algebra, Bk. 1, #1 ___: #2 read pp. 36-37, Ex. 1-9 written p. 37, 37-46 p. 43.
Nichols, #1 and 2, read pp. 119-122, Ex. 1, 2 pp. 121-124.
Wooton, #1 and 2, read pp. 52-54, Ex. 1-19 oral p. 54.
Payne, #1 and 2, read pp. 307-309, Ex. 1-10, and 18-27, pp 309-310.
Pearson, #1 and 2, read pp. 240-241, Ex. 1,2 p. 241, and 3 p. 243.

Obj. 3

Nichols, read pp. 124-132, Ex. 6, 7 pp. 127-128; 1-12 pp. 130-131; 4 p. 132.
Payne, read pp. 86-87, Ex. 1-19 pp. 87-88.

Obj. 4

Vanatta, read pp. 67-71, Ex. 5 p. 71, nos. 1-20 even p. 125, no. 3 p. 75.

Dolciani, pp. ___, Ex. 1-30 even p. 79.

Nichols, read pp. 133-137, Ex. 2-6 pp. 135-136.

Wooton, read pp. 55-57, Ex. 7-24 p. 58, 1-28 p. 59.

Payne, read pp. 88-90, Ex. 1-20 p. 90, 39-43 p. 92.

Pearson, read pp. 242-243, Ex. 1-27 p. 242, nos. 1, 2 p. 243

Obj. 5

Vanatta, read pages 320-321, Ex. 1-5, 7-10 page 322.

Dolciani, MA, read pages 292, Ex. 1-20 even (oral) page 293.

Nichols, read pages 146-150, Ex. 1,2 p. 148, 3a,c,d,f,g,i page 148; 3a,c,e,g,i, 4 page 150.

Payne, read pp. 386-394, Ex. 1-9, 11-31 odd, pages 387-388; 1-11,15, 17,19 pages 394-396.

Wooton, MSM, read pp. 320-323, 328-330, Ex. 1-10(written) pp. 323-324; 1-10 p. 329; 1-7,9,16 p. 331.

Pearson, read pp. 397-400, 401-402; Ex. 1,2,4abdfhkmq page 398; 1adfhkl[illegible] page 400; 1-4, 7, 8, 10, 12, 16, 20-22 pages 402-403.

Obj. 6

Nichols, read page 151, Ex. 1acdeg, 2 page 152.

Payne, read pp. 405-408, Ex. 1-3, 5-21 odd, 22, 25 page 408.

Obj. 7

Vanatta, read pp. 236-330, 331-332, 333-335, Ex. 1,2,3,6,8 page 328; 1,4,5,7,9,10,11,12,16 page 330; 1,3,4,6,8,9,10,11,13,15 page 333; 1,3,5,6,8,9,10,12,14,15,23,24 page 335.

Dolciani, MA, read pp. 298-300, Ex. 1-14 even pages 298-299; 1-16 even, 20 p. 301.

Nichols, read pp. 152-153, Ex. 1a,c,e,g,i,j,l page 153.

Wooton, MSM, read pp. 332-336, Ex. 7,12 page 334; 1,3,5,7,10,11,13,15 pages 337-338.

Payne, read pp. 398-403, Ex. 1,4,5,7,12,13,15,16,18,20,23,27,28,32,35, 36,39,48,50,54,60,62,64,66 pages 403-405.

Pearson, read pp. 403-405, Ex. 2,4,7,8,12 page 405.

SELF-EVALUATION

OBJ. 1

Classify the following as monomials, binomials, or trinomials.

_____ 1. x^5

_____ 2. $x^4 + 6x^2$

_____ 3. $5 - 4x + 2y$

_____ 4. $456x^4y^5$

_____ 5. $4x^3y^2z^5 + 6$

OBJ. 2

Determine the value of each expression if the replacement for x is 5 and the replacement for y is -3.

_____ 6. $5x^2$

_____ 7. $x + y$

_____ 8. $4x^2 + 3y$

_____ 9. $\frac{x^2 + y}{x}$

_____ 10. $\frac{x^3 - y^3}{y}$

OBJ. 3

Given the following pair of expressions, are they equivalent? Write Yes or No.

_____ 11. $x(-y)$ and $(-x)y$

_____ 12. $4 - x$ and $x - 4$

_____ 13. $-(a - b)$ and $b - a$

_____ 14. $-(a + b)$ and $b + a$

_____ 15. $(a + b)^3$ and $a^3 + b^3$

Obj. 4

Change each of the following expressions to equivalent expressions with the least number of terms possible.

16. $4x + 3x$

17. $7y - 0y$

18. $-5(x - 4y) + 5(x - y)$

19. $0 - 3d - 0 + 4$

20. $-\frac{3}{4}x - 7 + \frac{2}{5}y + \frac{5}{3}x + 5$

21. $(x^2 - 3x) - 2(2x^2 - 10x)$

22. $4xy - xy + 3yx + 4y$

23. $10r + m - 3n + 7n + m$

24. $3r + 5r - 6 - 7r + 4r$

25. $x - 2y - x + y$

26. $-(3x + y) + (2x - 9y)$

27. $-2(4a + 3b) - (5a + 6b)$

Obj. 5

For each of the following, write its equivalent in a single expression.

28. $\frac{1}{3} \cdot y$

29. $-\frac{4}{5} \cdot \frac{5}{3}$

30. $\frac{2}{3} \cdot \frac{5}{7}$

31. $\frac{6}{y^2} \cdot \frac{y}{2}$

32. $\frac{a+b}{2} \cdot \frac{8}{a-b}$

33. $\frac{5a^2}{21c^2} \cdot \frac{3bc^2}{5n^2} \cdot \frac{7e^2}{30b^3}$

Obj. 6

Simplify.

35. $\frac{2-a}{a} \cdot \frac{a}{3-a}$

36. $\frac{7}{6} \cdot \frac{-6}{6}$

37. $\frac{1}{xy} \cdot \frac{x(y-1)}{2}$

38. $\frac{2a}{a} \cdot \frac{b}{4}$

39. $\frac{4+a}{5} \cdot \frac{5}{4-a}$

40. $\frac{3xy}{7} \cdot \frac{7}{2y}$

Obj. 7

Compute the following:

41. $\frac{2}{3} + \frac{1}{7}$

42. $\frac{a}{b} \cdot \frac{x}{y}$

43. $\frac{3}{4} \cdot \frac{1}{2}$

44. $\frac{a}{7} + \frac{a}{3}$

45. $\frac{2x}{5y} + \frac{x}{2y}$

46. $\frac{3}{x} + \frac{4}{y}$

Answer true or false to the following:

Obj. 5 _____ 47. $\frac{3}{5} \cdot \frac{4}{5} = \frac{12}{5}$

_____ 48. $(-\frac{3}{4})(-2) = \frac{(-3)(-2)}{4}$

_____ 49. $\frac{3}{x} \cdot \frac{2}{y} = \frac{3y}{2x}$

_____ 50. $(-x) \cdot y \cdot (-\frac{1}{y}) = x$

6 _____ 51. $\frac{x-3}{x-2} = \frac{3}{2}$

_____ 52. $\frac{2}{-3} \cdot \frac{-3}{5} = \frac{2}{5}$

_____ 53. $\frac{3m}{7+m} = \frac{3}{7}$

6 _____ 54. $\frac{4y}{5y} = \frac{4}{5}$

_____ 55. $\frac{(-7)x2}{3x(-7)} = \frac{2}{5}$

_____ 56. $\frac{3-x}{4-x} = \frac{3}{4}$

7 _____ 57. $\frac{2}{4} + \frac{5}{3} = \frac{7}{7} = 1$

_____ 58. $\frac{3}{2x} + \frac{4}{3x} = \frac{17}{6x}$

_____ 59. $\frac{2}{x} + \frac{1}{y} = \frac{3}{xy}$

_____ 60. $\frac{x}{3} + \frac{k}{4} = \frac{4x+3k}{12}$

_____ 61. $\frac{-2}{3} + \frac{4}{7} = \frac{2}{21}$

_____ 62. $\frac{-3k}{2} + \frac{-k}{4} = \frac{-7k}{4}$

If you have satisfactorily completed your work, take the Progress Test. Consult your teacher first.

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Given a pair of rational expressions, write a single equivalent expression that names their difference.

9. Given a pair of rational expressions, write a single equivalent expression that names their quotient.

10. Given a pair of rational expressions which involves the additive inverse of an expression, write a single expression equivalent to it.

11. Given any complex rational expression, use the appropriate properties, theorems, and definitions to write a single expression equivalent to it.

12. Given a word phrase, change it to an equivalent mathematical phrase.

RESOURCES

Obj. 8

Vanatta, read pp. 326-335, Ex. 4,5,7,9-12, page 328; 2,3,6,8,13,14,15, 17,18 page 330; 2,5,7,12,14 page 333; 2,4,7,11,13,16,18,20,21,25 page 335.

Dolciani, MA, read pp. 298-300, Ex. 1-14 odd pages 298-299; 1-16 odd, 19, 21, 22, page 301.

Wooton, MSM, read pp. 332-336, Ex. 1-6,9,11 page 334; 4,8,9,12,14, 16,17,18 pages 337-338.

Payne, read pp. 398-403, Ex. 8,9,11,21,22,24,29,30,31,33,38,41,44, 56,60,61,63,65 pages 403-405.

Pearson, read pp. 403-405, Ex. 3,5,6,9-11 page 405.

Obj. 9

Vanatta, read pp. 323-324, Ex. 1-15 pp. 324-325.

Dolciani, MA, read p. 295, Ex. 1-12 p. 295; 1-4 pp. 296-297.

Nichols, read pp. 154-155, Ex. 1 a,b,d,e,g,i,k,l, and 2 a,b,e,f,h, i,j,l,m pages 156-160.

Payne, read pp. 396-397, Ex. 1-22 even page 397.

Pearson, read pp. 401-402, Ex. 5,9,13,14,15,17,18 pages 402-403.

Obj. 10

Nichols, read pp. 154-155, Ex. 7 a,c,d,f,g,j,l; 8 a,b,e,d,g,h,i,l, m,o,p,q,s,u,v; 10 a,c,d,g,j,k,o,q, pages 158-160.

Obj. 11

Vanatta, read pp. 341-342, Ex. 1-7, 10, 12, 13 page 343.

Dolciani, MA, read p. 304, ex. 1-8, 11, 12, 15, 16, 23, page 305.

Nichols, read pp. 161-163, Ex. 9 a,b,c,e,f,g,h,i,j,k,l,m,n,o,r,t; 11 a,c,d,e,f,g,n,p,q,r,s,i,l,m; 12 a,c,e,f, pages 159-161. Ex. 1 a,c,f,h,j,m,n,p; 2 a,c,d,f,g,h,k,l,m,o,p,q,s,t,v,x,y,z, a',b',c',e',g',i',j',k',l', pages 163-164.

Payne, read pages 409, Ex. 1-11, 19, 21, 23, 25, 27, 29, 35 pages 410-411.

Pearson, read pages 406-407 (examples 7 and 8), Ex. 46 a,c,e,f,h, j page 407.

Obj. 12

Vanatta, pp. ___, Ex. 1-6 p. 74, 1 p. 75, 1 p. 77.

Dolciani, read pp. ___, Ex. 1-24 p. 42.

Nichols, read page 165, Ex. 1 all parts; 2 a, c, d, f, g, k, n, o; 3 a, c, f, h, i, l, m, n, q, r, u, w, x, z, a', c' pages 165-167.

Wollensak Teaching Tapes C-3801: Open Phrase
C-3802: Open Sentence

* Appendix

* Required (turn in to teacher)

OBJ. I. For each of the following expressions, write its equivalent in a single expression.

8 (1) $\frac{5}{8} - \frac{2}{3}$ (2) $\frac{1}{2} - \frac{2n-1}{n}$ (3) $\frac{x}{y} - \frac{-2}{3}$

(4) $\frac{5b}{3y} - \frac{3b}{4y}$ (5) $\frac{2}{x} - 5$ (6) $\frac{-3}{x} - \frac{-2}{y}$

9 (7) $x + \frac{2}{3}$ (8) $\frac{a}{b} + \frac{1}{2b}$ (9) $\frac{-3}{4} + \frac{m}{n}$

(10) $\frac{2}{3} + \frac{5}{6}$ (11) $\frac{-a}{b} + \frac{c}{-d}$ (12) $\frac{3}{2} + \frac{x-y}{2}$

10 (13) $-\frac{-5}{-3}$ (14) $-\frac{-5x}{3x}$ (15) $-\frac{4w}{-r}$

11 (16) $\frac{\frac{12}{6}}{x}$ (17) $\frac{\frac{x}{y}+1}{\frac{x}{y}-1}$ (18) $\frac{\frac{x+y}{x-y}}{\frac{x-y}{x+y}}$

(19) $\frac{\frac{a}{x}+\frac{a}{y}}{\frac{b}{x}-\frac{b}{y}}$ (20) $\frac{2-\frac{1}{x}}{4-\frac{1}{x}}$ (21) $\frac{x}{1+\frac{1}{x}}$

II. True or False.

8 _____ 22. $\frac{1}{y} - 2 = \frac{1-2}{y} = \frac{-1}{y}$

_____ 23. $\frac{a}{2} - \frac{b}{8} = \frac{a-b}{6}$

_____ 24. $\frac{-x}{3} - \frac{y}{6} = \frac{-2x-y}{6}$

_____ 25. $\frac{3a}{2b} - \frac{5b}{6a} = \frac{9a^2-5b^2}{6ab}$

_____ 26. $\frac{a}{2} - \frac{-a}{3} = \frac{5a}{6}$

_____ 27. $\frac{-2}{3} - \frac{4}{2} = \frac{-6}{5}$

_____ 28. $\frac{3}{4} - \frac{-2}{3} = \frac{17}{12}$

9 _____ 29. $\frac{3}{4} \div \frac{a}{b} = \frac{3a}{4b}$

_____ 30. $\frac{x}{y} \div \frac{a}{b} = \frac{xb}{ya}$

_____ 31. $\frac{3}{5} \div \frac{x}{y} = \frac{5x}{3y}$

10 _____ 32. $\frac{(-x)3}{y(-7)} = \frac{-3x}{7y}$

_____ 33. $-\frac{1}{r - s} = \frac{1}{s - r}$

_____ 34. $-\frac{1}{x + y} = \frac{1}{-x - y}$

_____ 35. $-\frac{-3x}{-2y} = \frac{3x}{2y}$

11 _____ 36. $\frac{-\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$

_____ 37. $\frac{\frac{a - b}{a + b}}{\frac{a + b}{a - b}} = 1$

_____ 38. $\frac{\frac{a + b}{2}}{\frac{6}{a + b}} = \frac{1}{3}$

_____ 39. $\frac{\frac{x}{y} - \frac{3}{2}}{\frac{x}{y} + \frac{3}{2}} = \frac{2x - 3y}{2x + 3y}$

_____ 40. $\frac{1}{1 + \frac{1}{x}} = \frac{x}{x + 1}$

12 III. Change the following word phrases to equivalent mathematical phrase.

_____ 41. The product of seven and the sum of some number and five.

_____ 42. The sum of x and twice y.

_____ 43. Twenty more dollars less than one-third of last years salary.

_____ 44. The difference of 4r and 6 multiplied by the sum of 4r and 6.

_____ 45. Number of feet in 10y inches.

_____ 46. The number of pints in 3y quarts.

_____ 47. Worth in cents of (4y - 1) nickels.

_____ 48. Number of inches in the perimeter of a square with x feet for the length of the side.

If you have satisfactorily completed your work, you may take your LAP TEST. Consult your teacher first.

APPENDIX
(to be turned in to the teacher)

1. 3 dimes are worth _____ cents.

2. x dimes are worth _____ cents.

3. 7x dimes are worth _____ cents.

4. $\frac{x}{2}$ dimes are worth _____ cents.

5. x - 4 dimes are worth _____ cents.

6. 7 3-cent stamps are worth _____ cents.

7. k + 4 3-cent stamps are worth _____ cents.

8. 5t 7-cent stamps are worth _____ cents.

9. If I have 3 nickels and 4 dimes and 2 quarters, then I have _____ coins worth _____ cents.

10. If I have 4 nickels and x dimes and 3 quarters, then I have _____ coins worth _____ cents.

11. If I have x nickels and 3x dimes and x + 2 quarters, then I _____ have coins worth _____ cents.

12. Al is 12 years old. 5 years ago he was _____ years old and 8 years from now he will be _____ years old. 3 times his present age is _____. Bill is 4 years younger than Al. Bill is _____ years old.

13. Ed is x years old. 3 years from now he will be _____ years old and 2 years ago he was _____ years old. Dave is 4 times as old as Ed is now. Dave is _____ years old. 2 years ago he was _____ years old. Hal is 2 years younger than Ed. Hal is _____ years old. In 5 years he will be _____ years old. Sam is 6 years older than Dave. Sam is _____ years old.

I. Complete the following proofs by writing the correct reason in the blank space provided.

(1) Prove: $\forall_x \forall_y \neq 0 \forall_r \forall_s \neq 0 \left(\frac{x}{y}\right)\left(\frac{r}{s}\right) = \frac{xr}{ys}$

Proof: Statements	Reasons
a. $\left(\frac{x}{y}\right)\left(\frac{r}{s}\right) = \left[x\left(\frac{1}{y}\right)\right]\left[r\left(\frac{1}{s}\right)\right]$	_____
b. $= x\left[\left(\frac{1}{y}\right)r\right]\left(\frac{1}{s}\right)$	_____
c. $= x\left[r\left(\frac{1}{y}\right)\right]\left(\frac{1}{s}\right)$	_____
d. $= (xr)\left(\frac{1}{y} \cdot \frac{1}{s}\right)$	_____
e. $= (xr)\left(\frac{1}{ys}\right)$	_____
f. $= \frac{xr}{ys}$	_____

(2) Prove: $\forall_x \forall_r \forall_y \neq 0 \forall_s \neq 0 \frac{x}{y} + \frac{r}{s} = \frac{xs + ry}{ys}$

Proof: Statements	Reasons
a. $\frac{x}{y} + \frac{r}{s} = \frac{xs}{ys} + \frac{ry}{sy}$	_____
b. $= \frac{xs}{ys} + \frac{ry}{ys}$	_____
c. $= (xs)\left(\frac{1}{ys}\right) + (ry)\left(\frac{1}{ys}\right)$	_____
d. $= (xs + ry)\left(\frac{1}{ys}\right)$	_____
e. $= \frac{xs + ry}{ys}$	_____

(3) Prove: $\forall_x \forall_y \neq 0 \forall_r \forall_s \neq 0 \frac{x}{y} - \frac{r}{s} = \frac{xs - ry}{ys}$

Proof: Statements	Reasons
a. $\frac{x}{y} - \frac{r}{s} = \frac{xs}{ys} - \frac{ry}{sy}$	_____
b. $= \frac{xs}{ys} + \left(-\frac{rs}{sy}\right)$	_____
c. $= \frac{xs}{ys} + -\frac{(rs)}{sy}$	_____
d. $= \frac{xs}{ys} + -\frac{(rs)}{ys}$	_____
e. $= \frac{xs + -(rs)}{ys}$	_____
f. $= \frac{xs - rs}{ys}$	_____

II. Work Problems 1-16 page 325, Venatta.

III. Dolciani, p. 319, Just for Fun.

IV. Dolciani, pp. 328-330, Extra for Experts.

V. Work the following:

(1) $5x - \dfrac{3}{5x - \dfrac{3}{5x}}$

(2) $\left[\dfrac{2x + 1}{x} - \dfrac{x}{2x + 1}\right] \cdot \left[\dfrac{5x - 1}{x} + \dfrac{x}{5x - 1}\right]$

(3) $\left[\dfrac{4}{x + 1} + \dfrac{3}{x - 2}\right] \cdot \dfrac{x + 3}{7x - 5}$

(4) $\dfrac{2 + \dfrac{5}{a + 2b}}{\dfrac{3a}{a + 2b}}$

VI. Prepare a chart using a Venn diagram showing the relationships among polynomials, monomials, binomials, and trinomials.

VII. Dolciani, Modern Algebra, Bk. 1, work any ten problems from 1-15 on pages 43-44.

VIII. Nichols, page 132, number 7.

IX. Nichols, page 141, number 3.

REFERENCES

Nichols (abbreviation)

Nichols, Eugene, D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Ginn and Company, 1964.

Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis, Algebra One, Harccurt, Brace and World, Inc., 1969.

Wooton, MSM (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics, Algebra 1.

Dolciani, MA (abbreviation)

Dolciani, Mary P., Berman, Simon L., Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

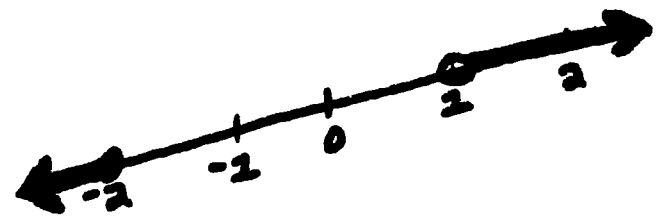
Vanatta (abbreviation)

Vanatta, Glen D., Goodwin, A. Wilson, Algebra One, A Modern Course, Charles E. Merrill Publishing Inc., 1966.

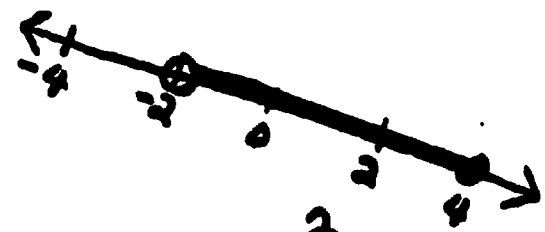
Wollensak teaching tape C-3801 - Open Phrase .
C-3802 - Open Sentence.

LEARNING ACTIVITY PACKAGE

$X+2=5$



$2x+6 \leq 12$



$-3x+4=6x-2$ $\neq -$

SOLUTION SETS OF EQUATIONS AND INEQUALITIES

Ninety Six High School · Ninety Six, S.C.

Algebra 93-94

REVIEWED BY

LAP NUMBER 16

WRITTEN BY

RATIONALE

In daily life you most often express yourself in English sentences. Because of the importance of clean and effective communication, a great deal of time in school is spent studying the English language.

In mathematics, ideas are expressed in a combination of English sentences and special <u>mathematical sentences</u>. Mathematical sentences consist of mathematical symbols rather than WORDS. An example of a mathematical sentence is $3x + 5 = 9$. An understanding of the types and properties of mathematical sentences is essential to your advancing in mathematics.

SECTION I

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given any mathematical sentence, classify it as being true, false, or neither.

2. Given any linear equation where the solution is dependent on the addition and/or the subtraction property determine the solution set showing all steps and giving reasons.

3. Given any linear equation where the solution is dependent on the multiplication and/or division property, determine the solution set showing all steps and giving reasons.

4. Given any linear equation whose solution is dependent upon a combined use of addition, subtraction, multiplication, and/or division properties; determine the solution set showing all steps and giving reasons. Appendix I will be completed and turned in to the teacher.

RESOURCES

Obj. 1

Nichols, read pp. 174-176, Ex. 1-21 odd pages 175-176.

Payne, read pp. 20-22, 101-104, 110-112, 114-116, Ex. 1-12 page 22; 1-10 page 104; 11-20 pages 111-112; 1-18 page 116.

Wooton, read pp. 29-33, Ex. 1-9 page 33.

Pearson, read pp. 57-59, Ex. 3 page 58.

Obj. 2

Vanatta, read pp. 47-52, Ex. 1, 2, 6, 7, 13, 14, 17, 18, 19, page 53.

Dolciani, read pp. 80-82, Ex. 1-30 odd p. 83.

Nichols, read pp. 188-191, 177, Ex. 1-48 every 4th problem pages 190-191; 1-10 page 177.

Payne, read pp. 101-104, 110-112, Ex. 11-20 pages 104-105; 1-10 page 111.

Wooton, read pp. 116-118, Ex. 1-10 page 119.

Pearson, read pages 151-152, Ex. 1 a, f, h page 152; 3 d, e, f, g, h, i, m, n, o, s, t page 152.

Transparency: Properties of Equality (3M)

Games: Equations

Obj. 3

Vanatta, read pp. 47-52, Ex. 3,4,5,8-12,15,16, and 20 page 53.

Dolciani, read pp. 83-84, Ex. 1-20 even pages 84-85.

Nichols, read pp. 191-192, Ex. 1-41 odd p. 192; 11-29 odd pages 177-178.

Wooton, read pp. 116-120, Ex. 11-26(written) pages 119-120.

Payne, read pp. 105-107, Ex. 1-23 odd p. 106.

Pearson, read pp. 150-153, Ex. 1 b,c,d,e,i,j; 2 and 3 a,b,c,j,k, l,p,q,r; and 4 pages 152-153.

Games: Equations

Obj. 4

Vanatta, read pp. 53-54, Ex. 5-20 p. 55.

Dolciani, read pp. 86-87, 91-93, Ex. 1-4, 11-15, 29-33 page 88; 1-26 odd (written) p. 93.

Nichols, read pp. 193-196, 178-180; Ex. 1-45 (every fourth one) pp. 195-196; 1-39 every fourth one pp. 179-180.

Wooton, read pp. 116-120, 131-134, Ex. 27-55 odd page 120; 1-39 odd page 134.

Payne, read pp. 107-110, pp. 112-114, Ex. 1-55 every 4th one pages 109-110; 1-39 every fourth one pages 113-114.

Pearson, read pp. 154-156, pp. 174-175, Ex. 1 EOL, 3,6,9 pages 155-156; 1 a,c,e,g,i,h; 2 a,c,e,g; 3 a,c,e,g; 4 a,c,e,g,i page 175.

* Appendix I

Audio Tapes: C-3801 Open Phrase, Open Sentence

C-3803 Open Sentence: Solution

Filmstrip: Proof in Algebra: Solving Equations

Games: Equations

* REQUIRED

Obj.

1 I. Classify each sentence into one of the following categories:

T if the sentence is true.

F if the sentence is false.

R if at least one replacement, but not every replacement, for the variable or variables will result in a true statement.

N if the sentence is neither true or false and there is no replacement for the variable or variables which will result in a true statement.

E if the sentence is neither true or false and every replacement for the variable or variables will result in a true sentence.

____a. $x + 1 = 2x$

____b. $2 = 1.4$

____c. $9 = 3 \times 3$

____d. $12 = 6 \times 6$

____e. $1y + 11 = 10$

____f. $1\% = .1$

____g. $a = 2a$

____h. $c + 1 = c$

____i. $7r - r = 6r$

____j. $d^2 = -4$

____k. $\frac{1}{8} = .125$

____l. $\frac{1}{3} = 33\frac{1}{3}\%$

____m. $\frac{3n}{n} = 3$

____n. $-(x - y) = y - x$

____o. $-1(x - 4) = 4 - x$

____p. $x - y = y - x$

____q. $-2(x - y) = (y - x) \cdot 2$

2 II. Determine the solution sets. Show all steps and give reasons for parts b and c.

a. $x - 3 = 6$

b. $x + 2 = 5$

c. $y - 14 = 4$

d. $12 = b - 1$

e. $26 = x + 16$

f. $t + \frac{1}{5} = 3\frac{3}{5}$

g. $2.5 = r - 1.5$

h. $.05 + x = 3.5$

i. $x + 2.34 = 3.06$

3 III. Determine the solution sets. No denominator is zero. Show all steps and give reasons for parts a and g.

a. $2a = 22$

b. $3x = 5$

c. $.3m - 3$

d. $\frac{1}{2}r = 25$

e. $4 = \frac{m}{4}$

f. $\frac{1}{2}y = 60$

g. $\frac{2}{3}a = 4$

h. $\frac{3}{a} = 4$

i. $\frac{22}{7} = \frac{3y}{7}$

4 IV. Determine the solution sets if the universal set is the set of real numbers. No denominator is zero. Show all steps and give reasons for parts a and c only.

a. $3u + 5 = 1$

b. $2w + 3 = 5$

c. $\frac{3x}{2} - 6 = 7$

d. $18x + 11 = 9x - 70$

e. $\frac{1}{2} + \frac{1}{3}x = 1$

f. $3n + 50 = -10$

g. $\frac{p + 1}{2} = 2$

h. $7(z - 1) - 2(2z - 3) = 0$

i. $3x - 7 = -(7 - 3x)$

j. $\frac{4x + 7}{3} = \frac{4}{3}x + 7$

k. $\frac{x - 3}{x} = \frac{1}{4}$

l. $\frac{7}{y + 2} = \frac{11}{y}$

m. $\frac{2x + 11}{4} = \frac{3x - 7}{5}$

n. $8x + 91 = -5x - 17$

4 V. Write the reason for each step in the following:

(1) $6x + 1 = 9$

$6x + 1 - 1 = 9 - 1$

$6x + 0 = 9 - 1$

$6x = 9 - 1$

$6x = 8$

$\frac{6x}{6} = \frac{8}{6}$

$1 \cdot x = \frac{8}{6}$

$x = \frac{8}{6}$

$x = 1\frac{1}{3}$

equation

a. _____

b. _____

c. _____

d. _____

e. _____

f. _____

g. _____

h. _____

(2) $\frac{3x}{2} - 1 = 4$ equation

$\frac{3x}{2} - 1 + 1 = 4 + 1$	a. _____
$\frac{3x}{2} + 0 = 4 + 1$	b. _____
$\frac{3x}{2} = 4 + 1$	c. _____
$\frac{3x}{2} = 5$	d. _____
$\frac{3x}{2} \cdot 2 = 5 \cdot 2$	e. _____
$3x \cdot 1 = 5 \cdot 2$	f. _____
$3x = 5 \cdot 2$	g. _____
$3x = 10$	h. _____
$\frac{3x}{3} = \frac{10}{3}$	i. _____
$1 \cdot x = \frac{10}{3}$	j. _____
$x = \frac{10}{3}$	l. _____
$x = 2\frac{1}{3}$	m. _____

If you have satisfactorily completed your work, you may take the Progress Test. CONSULT YOUR TEACHER FIRST.

SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

5. Given any mathematical sentence involving absolute value, determine the solution set.

6. Given any verbal problem, translate it into an equivalent mathematical sentence and find its solution set.

7. Given any statement using the properties of inequalities, determine if it is true or false. Appendix II will be completed and turned in to the teacher.

8. Given any inequality whose Universal set is the set of real numbers, determine and/or graph the solution set on the number line.

9. Given any pair of polynomials, write their product.

RESOURCES

Obj. 5

Nichols, read pp. 186-188; Ex. 1-29 odd page 188.

Payne, read pp. 148-151; Ex. 1-23 odd page 149.

Wooton, read pp. 165-168, Ex. 1-23 odd page 168.

Pearson, read pp. 217-219, page 224 Ex. 3, Ex. 6 page 219; 13 page 226.

Obj. 6

Vanatta, read ___, Ex. 1-14 page 74.

Dolciani, read page 92, Ex. 1-8 page 94.

Nichols, read pp. 180-182, pp. 214-217, pp. 221-223, Ex. 1-10 pages 181-182; 1-9 odd pages 216-217; 1-8 odd pages 222-223.

Wooton, read pp. 120-123, pp. 124-130, pp. 131-133, Ex. 1-6 pages 122-123; 1-27 odd pages 128-130; 1-21 odd pages 134-136.

Payne, read pp. 132-136; Ex. 1-9 odd pp. 133-134; 1-15 odd pages 134-135; 1-8 odd pages 136-137.

Pearson, read pp. 157-159, Ex. 1-19 odd pages 158-159; 5-17 odd pages 176-178.

Audio Tapes: C - 3809 Reading Written Problems

Obj. 7

Vanatta, read pp. 55-58, Ex. ___.

Dolciani, read pp. 159-163, Ex. ___.

* Appendix II

Transparencies: Properties of Inequality

(* required)

Obj. 8

Vanatta, read pp. 55-58, Ex. 1-10 page 58.

Dolciani, read pp. 159-162, Ex. 1-10, 14-16 page 163.

Nichols, read pp. 182-185, pp. 241-246, Ex. 1 a,c,e,g,i,k pages 183-185; 2 a,c,e,g and 3 a,c,e,g pages 183-185; 1 a,c,e,g,i, k,m,o,q,r and 2 a,c,e,g and 3 a,c,e page 244.

Wooton, read pp. 157-159, Ex. 1-10 page 159.

Payne, read pp. 117-122, p. 124 exercise 5; Ex. 1-19 odd pp. 119-122, 23-29 odd pages 119-122; 11-25 odd page 126.

Pearson, read pp. 72-74, Ex. 1 c,d,h,i,j and 4 a,c,d,g,i pages 73-74.

Audio Tapes: C-3805 The Compound Sentence
C-3806 Inequality and Equality Sentences

Filmstrip: Graphs of Inequalities in One Variable

Transparencies: Properties of Inequality

Obj. 9

Vanatta, read pp. 289-290, 293-294, Ex. 1-20 even p. 290; 1-30 odd pp. 294-295.

Dolciani, read pp. 206-207, 209, Ex. 1-6 oral page 207; 1-20 even page 210.

Nichols, read pp. 199-201, Ex. 1 a,c,e,g,i,k and 2 a,c,e,g,i,k,m, o,q,s,u,w,y page 201.

Wooton, read 272-275, 277-279, Ex. 1-23 odd pages 274-275; 13-39 odd page 279.

Payne, read pp. 313-317, Ex. 1-29 odd p. 315.

Pearson, read pp.170-173, Ex. 1, 5. 7 pages 172-173.

SELF-EVALUATION 2

Obj. 5

I. Solve the following:

(a) $|x| = 8$

(b) $|x - 3| = 4$

(c) $|3t + 1| = 7$

(d) $|6 - 2x| = 2$

(e) $|\frac{1}{2}x + 4| - 2 = 3$

(f) $\left|\frac{2 + x}{3}\right| = 2$

Obj. 6

II. Write the equation used to solve each verbal problem and solve the problem. Show your work.

(a) The sum of a number and 1 is equal to the product of 3 and the number. What is the number?

(b) Multiplying a no. by 3 gives the same result as adding 4 to the number. What is the no.?

(c) Taking one-half of a number gives the same result as adding 5 to the number. What is the no.?

(d) How long is a rectangular plot if its length is 9 ft. longer than its width, and its perimeter is 94 ft.?

(e) The difference between the length and the width of a rectangle is 11 inches. What is the length and the width of the rectangel if its perimeter is equal to 26 inches?

Obj. 7

III. TRUE OR FALSE.

_____ 1. If $x < 6$, then $x + 2 > 6 + 2$.

_____ 2. If $x < 5$ and $c < 0$, then $x \cdot c < 5 \cdot c$.

_____ 3. If $K > 6$ and $c > 0$, then $K \cdot c > 6 \cdot c$.

_____ 4. If $4 < 12$ and $-2 < 0$, then $4 \div -2 > 12 \div -2$.

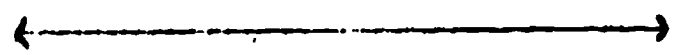
_____ 5. If $7 < K$, then $7 - 6 < K - 6$.

_____ 6. If $8 > m$, then $8 - 7 < m - 7$.

_____ 7. If $T < 4$ and $c < 0$, then $Tc < 4c$.

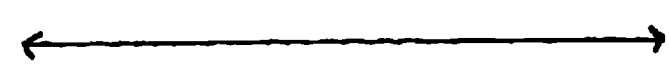
_____ 8. If $k > 7$ and $c < 0$, then $k \cdot c < 7 \cdot c$.

8 IV. On the number line, graph the solution set of each inequality. The universal set is the set of all real numbers.

(a) $p > 3$ 

(b) $-3 \leq a \geq 2$ 

(c) $-2 \leq x \leq -\frac{1}{2}$ 

(d) $x < -3$ or $x \geq 4$ 

(e) $a \leq -2$ or $a \geq 0$ 

8 V. Solve and graph the solution sets of the following:

(1) $3x + 6 < 33$

(2) $-6x + 1 \geq -11$ 

(3) $\frac{-4x}{2} - 6 \leq -12$ 

(4) $\frac{2x}{3} - 6 > 0$ 

9 VI. For each of the following expressions, write an expression which is equivalent to it and which does not contain parenthesis.

a. $-3(2a - 5m + 4n)$

b. $(a + 4n)(a - n)$

c. $(4 + 5s)(5s - 4)$

d. $-(x - 1)(1 - x)$

e. $(2a + 3)(2a + 3)$

f. $(2a + 3)(2a - 3)$

If you have satisfactorily completed your work, take the LAP Test. CONSULT YOUR TEACHER FIRST.

APPENDIX 1

1. Write an explanation of the addition property of equality.

2. Write an explanation of the subtraction property of equality.

3. Write an explanation of the multiplication property of equality.

4. Write an explanation of the division property of equality.

5. Write the reason for each in the following.

(1) $2x = 10$ — equation

$\frac{2x}{2} = \frac{10}{2}$ — _____

$1 \cdot x = \frac{10}{2}$ — _____

$x = \frac{10}{2}$ — _____

$x = 5$ — _____

(2) $\frac{x}{3} = 10$ — equation

$\frac{x}{3} \cdot 3 = 10 \cdot 3$ — _____

$x \cdot 1 = 10 \cdot 3$ — _____

$x = 10 \cdot 3$ — _____

$x = 30$ — _____

(3) $x + 2 = 9$ — equation

$x+2-2 = 9-2$ — _____

$x + 0 = 9-2$ — _____

$x = 9-2$ — _____

$x = 7$ — _____

(4) $y - 3 = 7$ — equation

$y-3+3 = 7+3$

$y + 0 = 7 + 3$

$y = 7 + 3$

$y = 10$

(5) $3x + 6 = 33$ — equation

$3x + 6 - 6 = 33 - 6$

$3x + 0 = 33 - 6$

$3x = 33 - 6$

$3x = 27$

$\frac{3x}{3} = \frac{27}{3}$

$1 \cdot x = \frac{27}{3}$

$x = \frac{27}{3}$

$x = 9$

(6) $\frac{-3n}{4} - 2 = 4$ — equation

$\frac{-3n}{4} - 2 + 2 = 4 + 2$

$\frac{-3n}{4} + 0 = 4 + 2$

$\frac{-3n}{4} = 4 + 2$

$\frac{-3n}{4} = 6$

$\frac{-3n}{4} \cdot 4 = 6 \cdot 4$

$-3n \cdot 1 = 6 \cdot 4$

$-3n = 6 \cdot 4$

$-3n = 24$

$\frac{-3n}{-3} = \frac{24}{-3}$

$1 \cdot n = \frac{24}{-3}$

$n = \frac{24}{-3}$

$n = -8$

APPENDIX 2

1. Explain the following:

   A. If $a < b$, then $a + c < b + c$: and $a > b$, then $a + c > b + c$.

   B. If $a > b$, then $a - c > b - c$; and $a < b$, then $a - c < b - c$.

   C. If $a > b$ and $c > 0$, then $ac > bc$; and $a < b$ and $c > 0$, then $ac < bc$.

   D. If $a > b$ and $c < 0$, then $ac < bc$; and $a < b$ and $c < 0$, then $ac > bc$.

   E. If $a < b$ and $c > 0$, then $a + c < b + c$.

   F. If $a > b$ and $c < 0$, then $a + c < b + c$.

2. True or False.

   _____ 1. If $-2x < 8$, then $x < -4$.

   _____ 2. $3x < 9$, then $x > 3$.

   _____ 3. $x + 3 < 6$, then $x < 3$.

   _____ 4. If $-\frac{x}{6} > 2$, then $x < -12$.

   _____ 5. If $x - 3 < 5$, then $x > 8$.

   _____ 6. If $-6x > 12$, then $x > -2$.

   _____ 7. If $\frac{x}{3} < 9$, then $x > 27$.

   _____ 8. If $6x < 18$, then $x < 3$.

   _____ 9. If $x - 8 < 28$, then $x < 20$.

   _____ 10. If $2x + 9 < 19$, then $x < 5$.

ADVANCED STUDY

I. Work the following. Show your work.

1. A car starts out at a point 3 miles outside of town at a rate of 50 mph. How long will it take for the car to be 353 miles from the town? (d = rt)

   Total distance d_t = rate times time (rt) plus the 3 miles out of the town.

   $$353 = 50(t) + 3$$

   solve for t

2. An airplane travels 702 miles from its point of origin to its destination, it made one stop to pick up passengers and then flew at a rate of 310 mph. for 2 hours to arrive at its terminal point.

   Given the equation $d_t = d_1 + d_2$

   But $d_2 = R \cdot t = 310(2)$ So:

   $702 = d_1 + 310(2)$ solve for d_1

II. Work the following, showing your work.

1. The total electrical resistance in a series circuit is equal to the sum of the individual resistances. If the first resistor has a rating of two ohms (a measure of resistance), and the second resistors rating is 7 ohms, then what is the ohm rating of the third resistor, given the total circuit resistance is 10 ohms?

   Write the equation.

   Solve the ohm rating of the third resistor.

2. The total electrical resistance in a parallel circuit may be found by equating the reciprocal of the total resistance to the sum of the reciprocals of the individual resistances.

   $r_1 = 3$, $r_2 = 6$, $r_3 = ?$ Total resistance is 1 ohm

   Write the equation, then solve for r_3.

3. The focal length of a lens can be found by dividing the product of the image distance (d_i) and the object distance (d_o), by their sum.

   Write the equation, if the focal distance is 1 and the object distance is 2.

   Solve the equation for the image distance.

ADVANCED STUDY (cont')

III. Wooton, read pp. 160-163, Ex. 1-26 even (written) pp. 164-165.

IV. Wooton, Ex. 10-18 page 174 any 4 problems.

V. Dolciani, Ex. 23-32 page 163 any 5 problems.

VI. Dolciani, read 164-163, Ex. 1-20 any 8 problems.

VII. Dolciani, page 168 any 5 problems.

REFERENCES

Nichols (abbreviation)

Nichols, Eugene, D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Ginn and Company, 1964.

Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis, Algebra One, Harcourt, Brace and World, Inc., 1969.

Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenback, Edwin Fr., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics, Algebra 1.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

Vanatta (abbreviation)

Vanatta, Glen D., Goodwin, A. Wilson, Algebra One, A Modern Course, Charles E. Merrill Publishing Co., 1966.

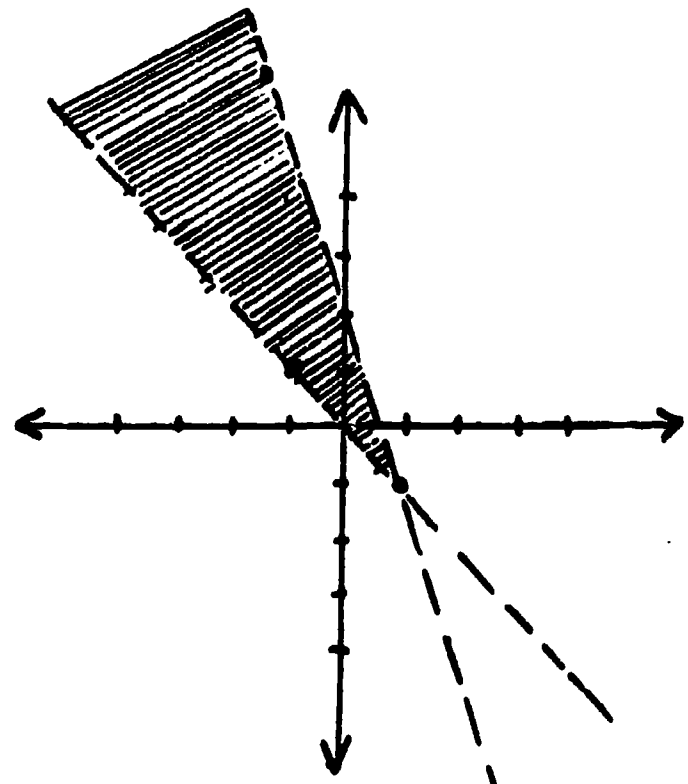
Wollensak teaching tapes C-3801, C-3803, C-3805, C-3806, and C-3809

Transparencies: 3M Properties of Equality
Properties of Inequality

Filmstrips: Proof in Algebra: Solving Equations
Graphs of Inequalities in One Variable

Games: Equations by Layman Allen

LEARNING ACTIVITY PACKAGE



EQUATIONS AND INEQUALITIES WITH TWO VARIABLES

Ninety Six High School · Ninety Six, S.C. ·

Algebra 93-94

REVIEWED BY

LAP NUMBER 17

WRITTEN BY Diane Evans

RATIONALE

EQUATIONS OF TWO VARIABLES

Graphs are not new to your. In your study of history, geography, and science, many relationships were made clear by graphing. For example, temperatures in relation to altitude might be indicated by means of a graph. Here the temperature depends upon the altitude. When a quantity depends upon another so that corresponding values can be determined, a graph of their corresponding values can be made.

You have learned how some physical problems can be translated into equations and inequalities. You will continue to learn about word problems in this LAP. You will also learn how to set some of these ideas in a pictorial manner. Their notions should be more meaningful to you through graphing.

Since an equation or inequality represents a relationship of variables, we can associate a point with each pair of values, of this relationship.

We shall study graphs, which will help us gain insight into relationships described by equations and inequalities.

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given an equation in two variables and an ordered pair of numbers for replacements of those variables, tell whether the resulting equation is true or false.

2. Identify or define the following:

   A. Cartesian coordinate system
   B. Descartes
   C. abscissa
   D. ordinate
   E. origin

3. Given an ordered pair of real numbers, locate the point on a coordinate system corresponding to that ordered pair.

4. Given an equation of two variables put it in standard form.

5. Given an equation in two variables, name at least three ordered pairs of real numbers that are members of the solution set.

6. Given an equation or inequality of two variables, graph it.

RESOURCES

Obj. 1

Dolciani, read pp. 333-335, Ex. 1-10 oral p. 335.

Nichols, read pp. 259-260, Ex. 1-12 pages 260-261.

Wooton, read pp. 189-192, Ex. 1-10 page 192.

Pearson, read pp. 428-431, Ex. 1-3 page 431.

C. Algebra (programmed) Frames 65-90.

C. Geometry (programmed) Frames 1-83.

Obj. 2

Dolciani, read pp. 365, 337-338, Ex. 1-32 page 339.

Vanatta, read pp. 191-195, write definitions in Obj. 2.

Wooton, read pp. 194-195, Ex. 1-6 oral pages 195-196; write definitions in Obj. 2.

Obj. 3

Vanatta, read pp. 193-195, Ex. 1 page 196.

Dolciani, read pp. 337-338, Ex. 1-12 page 340.

Nichols, read pp. 263-264, Ex. 1-6 pages 264-266.

Wooton, read pp. 194-195, Ex. 1-14 (written) pages 196-197.

Payne, read pp. 167-170, Ex. 1-14 pages 170-172.

Pearson, read pp. 431-432, Ex. 1-4 page 432; 1-5 pages 436-437.

C. Algebra (programmed) read Unit 1 - Book 3, Ex; Frames 1-65.

C. Geometry (programmed) Frames 128-181; 329-338.

Obj. 4

Vanatta, read pp. 246-248, Ex. ___.

Nichols, read pp. 261-262, Ex. 1-16 page 262.

Wooton, read pp. 197-200, Ex. 1-6 page 201.

Obj. 5

Nichols, read pp. 266-267, problems assigned in next objective.

Wooton, read pp. 197-200, Ex. 7-12 page 201.

C. Algebra (programmed) read Unit 1, Book 3, Ex - Frames 66-98.

Obj. 6.

Nichols, read pp. 266-267, 276-280, Ex. 1-12 pages 267-268; 1-4 pages 280-281.

* Payne, read pp. 178-180, 205-207, Ex. 14-21 page 181; 1-10 even page 207.

Pearson, read pp. 448-451, 487-489, Ex. 1-2 page 452; 1-2 page 488; 1-7 parts a and b only page 490.

C. Algebra (programmed) read Unit 1, Book 3, Ex. frames 99-132 Unit 1; frames 1-124 Unit 2.

C. Geometry (programmed) read Unit 2, Book 3, Ex. frames 182-291 and 324-379.

* required

SELF-EVALUATION 1

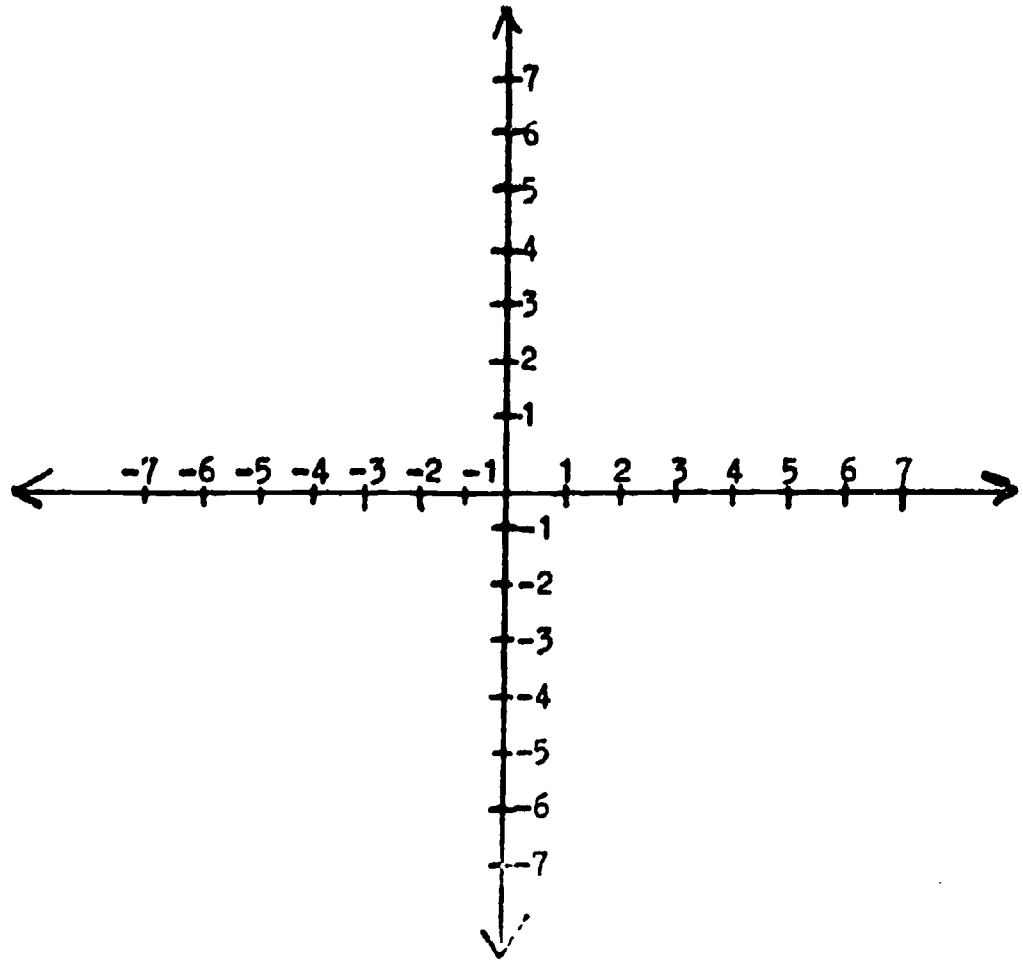
Obj.

1 I. Next to the equations listed below, there are 3 ordered pairs of numbers; tell whether or not it satisfies the equations.

1. $x + y = 7$; (6, 1) ; (-10, 3) ; (6.99, .01)
2. $2x + 3y = 6$; (0, 2) ; (2, 0) ; $(1, \frac{4}{3})$
3. $3m = 2n + 4$; (0, -2) ; (-2, 0) ; (-5, -2)
4. $\frac{1}{2}|x + y| = \frac{1}{3}|x - y|$; (0, 0) ; $(6, -\frac{6}{5})$; (6, 30)
5. $2a = 3|b| - 1$; $(0, \frac{1}{3})$; $(0, -\frac{1}{3})$; (-4, -3)

3 II. Graph the following ordered pairs on the coordinate system to the right.

6. (5, 3)
7. $(-2, 3\frac{1}{2})$
8. (-4, -6)
9. $(2\frac{4}{5}, -3)$
10. (0,-4)
11. (-2,0)
12. (0,2)
13. (0,0)
14. (5,0)



2 III. Define the following terms.

15. abscissa

16. origin

17. Descartes

18. Cartesian coordinate system

19. ordinate

4 IV. For each equation below, find an equivalent equation in standard form.

20. $2x = -3 - 8y$

21. $\frac{x - y}{x + y} = 3$

22. $3x - 2y + (-3) = 2(3y - 6x) + 4$

23. $\frac{4x + 2}{6} = \frac{-3x + 6y}{-2}$

24. $\frac{-2}{3x + 7y} = \frac{-4}{5x - 2}$

5 V. Which of the ordered pairs listed to the right are members of the solution set of the equations on the left. (There may be more than one answer for each equation).

25. $2x - 3y = 12$

26. $2x + 3y - 1 = x + y$

27. $\frac{x + y}{3} = 2x$

a) (3, -1)

b) $(\frac{3}{2}, -3)$

c) (0, 0)

d) (6, 0)

e) (1, 5)

f) (1, 0)

6 VI. Graph each of the following sentences. The universal set in each case is the set of real numbers. (Use the graph paper provided)

28. $y = 2x + 6$

29. $2x + 3y > 1$

30. $\frac{2(3 - 3x)}{y + 1} = -3$

31. $2x - y = 4$

32. $x - y \leq 3$

33. $2x - y > -4$

If you have satisfactorily completed your work, take the Progress Test. CONSULT YOUR TEACHER FIRST.

SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

7. Given a system of equations in two variables, graph their solution set.

8. Given the graph of a pair of equations in two variables, tell whether they are:

   A. dependent
   B. inconsistent
   C. independent
   D. consistent

   and if they are independent, name the point of intersection.

9. Given a system of equations and/or inequalities in two variables, graph their solution set.

10. Given a word problem, translate it into an open mathematical sentence and solve for the unknown.

RESOURCES

Obj. 7

Vanatta, read pp. 233-234, Ex. 1-8 page 241.

Dolciani, read pp. 267-269, Ex. 1-8 page 369.

Nichols, read pp. 268-270, Ex. 1-13 pages 270-271.

Wooton, read pp. 223-225, Ex. 7-15 page 228.

Payne, read pp. 219-221, Ex. 1-10 pages 222-223.

Pearson, read pp. 465-466, Ex. 1-3 page 466.

C. Algebra (programmed) read Unit 3, Book 3 (includes obj. 9) Ex. frames 8-105.

C. Geometry (programmed) frames 181-191; 324-279.

Obj. 8

Vanatta, read pp. 233-234, Ex. given for obj. 7.

Dolcinai, read pp. 367-369, Ex. given in obj. 7.

Nichols, read pp. 271-275, Ex. 1-10 page 276.

Payne, read pp. 234-235, Ex. 1-18 odd page 243 (do not use slope; use graph method)

Pearson, read pp. 480-481, Ex. 1-9 page 482.

Obj. 9

Dolciani, read pp. 350-352, 379-380, Ex. 1-12 even written page 352; 9-11 page 370; 5-14 even page 380.

Nichols, read pp. 281-284, Ex. 1-3 page 284.

Wooton, read pp. 252, Ex. 1-12 page 253.

Payne, read pp. 214-222, 244-245, Ex. 11-14 page 223; 1-10 page 246.

Pearson, read pp. 490-491, Ex. 1-2 page 491.

C. Geometry (programmed) frames 324-477.

C. Algebra (programmed) (same as obj. 7)

Obj. 10

Vanatta, read pp. 154-158, 164-166, 169-170, 214-216, 235-237, Ex. 1, 2,8,10,14 page 156; 1-9 page 159; 1,2 pages 166-167; 17 page 177; 11 page 175; 6 page 178; 1,2,4,5,10 page 170.

Dolciani, read pp. 166-171, 172-175, 178-180, 182-183, 310, Ex. 1,2, 4,10,19 page 168; 6,7 page 167; 1,4,7,10,13 bottom page 171; 4, 5,6 page 177; 1-3 pages 180-181; 2-5 page 183; 1,3-5 page 311.

Nichols, read pp. 223-228, Ex. 1-14 even page 224-225; 1-9 even, 11 pages 227-228.

Wooton, read pp. 169-172, 353-355, Ex. 1-36 even pages 173-176; 1-30 even pages 356-358.

Payne, read pp. 128-136, 139, Ex. 1-9 pp. 133-134; 1-15 even pages 134-135; 1, 3, 5 page 136; 1-4 pages 139-140.

Pearson, read pp. 251-252, 296-299, Ex. 1-30 even pages 252-253.

SELF-EVALUATION 2

Obj.

7 I. Graph each system of equations and name the point intersection. (approximately) Use the graph paper provided.

1. $2x - y = 0$
   $2x + y = -4$

2. $3x + y = 10$
   $2x - y = 1$

3. $4x = 2y$
   $2x - y = 2$

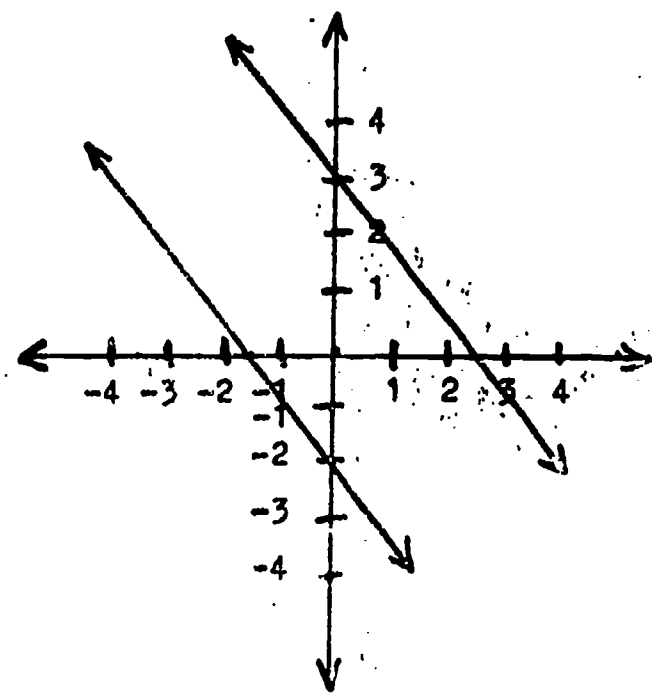
4. $3x + 5y = 4$
   $12 - 9x = 5y$

5. $2x + 3y = 8$
   $x + y = 3$

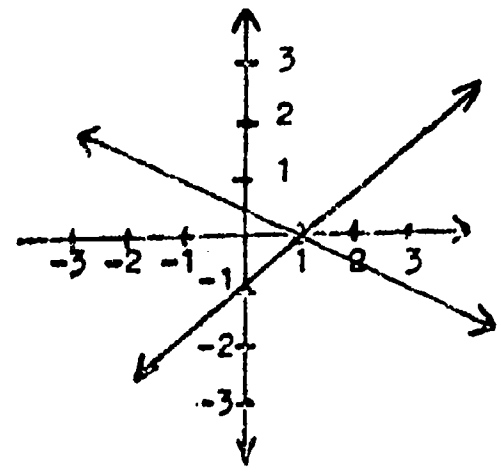
6. $x + y = 1$
   $y = -x$

8 II. Categorize the following graphs of pairs of equations as being (a) dependent (b) inconsistent (c) consistent (d) independent and if they are independent, name the point of intersection.

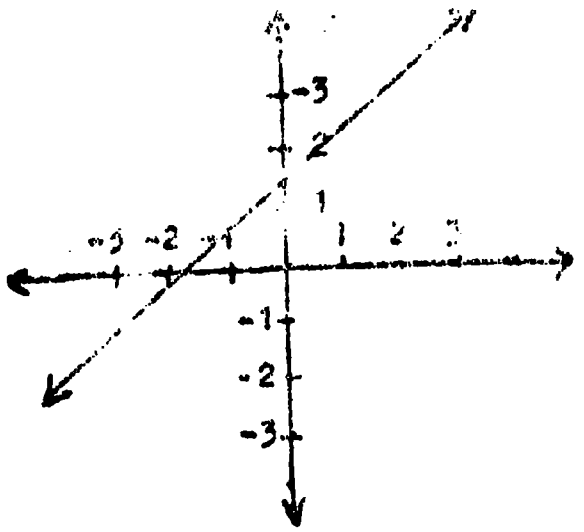
_____ 7.



_____ 8.



_____ 9.



9 III. Graph the following systems. Use the graph paper provided.

10. $3x = 2 - y$
$3y + 3x = 0$

11. $2x > y$
$3x + 5y = y$

12. $2x + y = 6$
$x + y = y + 3$

13. $3x < 2 - y$
$3y + 3x > 0$

10 IV. Work the following problems. SHOW YOUR WORK.

14. Two men start out from the same city and travel in opposite directions. One travels north at an average rate of 35 mph and the other man travels south at 40 mph. In how many hours will they be 250 miles apart?

15. The sum of four consecutive odd integers is 152. What are the integers?

16. Jim and Joe ride their motorbikes in opposite directions from Joe's house on the highway. They start at the same time. We find them 19 miles apart 15 minutes later. The average speed of Joe's bike is 8 miles per hour less than the average speed of Jim's bike. Determine the average speed of Joe's bike.

17. In Sue's bank she has some dimes and some nickels. She has two more dimes than she has nickels. In all she has $1.30. How many dimes and how many nickels has she?

18. How much water must be added to 16 pounds of a 25% salt solution to reduce it to a 15% solution?

If you have satisfactorily completed your work, take the LAP Test. CONSULT YOUR TEACHER FIRST.

APPENDIX

Work the following problems. Show your work and turn it in to your teacher with this sheet.

1. Tony broke his bank and found he had $9.35 in nickels and dimes. The bank contained ten more dimes than nickels. How many nickels and how many dimes did he have?

   nickels _____ dimes _____

2. Mr. James weighs 30 pounds more than his son. His son weighs twice as much as Mrs. James. Their combined weight is 495 pounds. How much does Mr. James weigh?

   _____

3. Jim and John went hunting and shot 21 rabbits in all. John shot three less rabbits than Jim. How many did each boy shoot?

4. A man purchases some three-cent stamps and some one-cent stamps for $3.05. There are 19 more three-cent stamps than one-cent stamps. How many of each kind does he buy?

   number of 3¢ _____

   number of 1¢ _____

5. At a certain time two airplanes start from the same airport and travel in opposite directions at 300 miles an hour and 250 miles an hour respectively. In how many hours will they be 1375 miles apart?

   _____

6. At a certain time a train leaves New York going to Albany traveling at 75 mph. At the same time a train leaves Albany going to New York traveling at 50 mph. In how many hours will they meet if New York is 375 miles from Albany?

   _____

7. John left Greenville traveling to Atlanta driving 40 mph. At the same time Sam left Atlanta traveling to Greenville driving 50 mph. In how many hours will they meet if Greenville is 190 miles from Atlanta?

   _____

8. How much water must be added to a barrel containing 48 pounds 10% brine to obtain a 6% brine?

   _____

9. How many ounces of water must be added to 80 ounces of a 5% acid solution to produce a 2% acid solution?

_____

I. **Mixture problem from chemistry:**

What quantities of gold 80% and 20% pure should be mixed to give 12 grams of 70% pure gold?

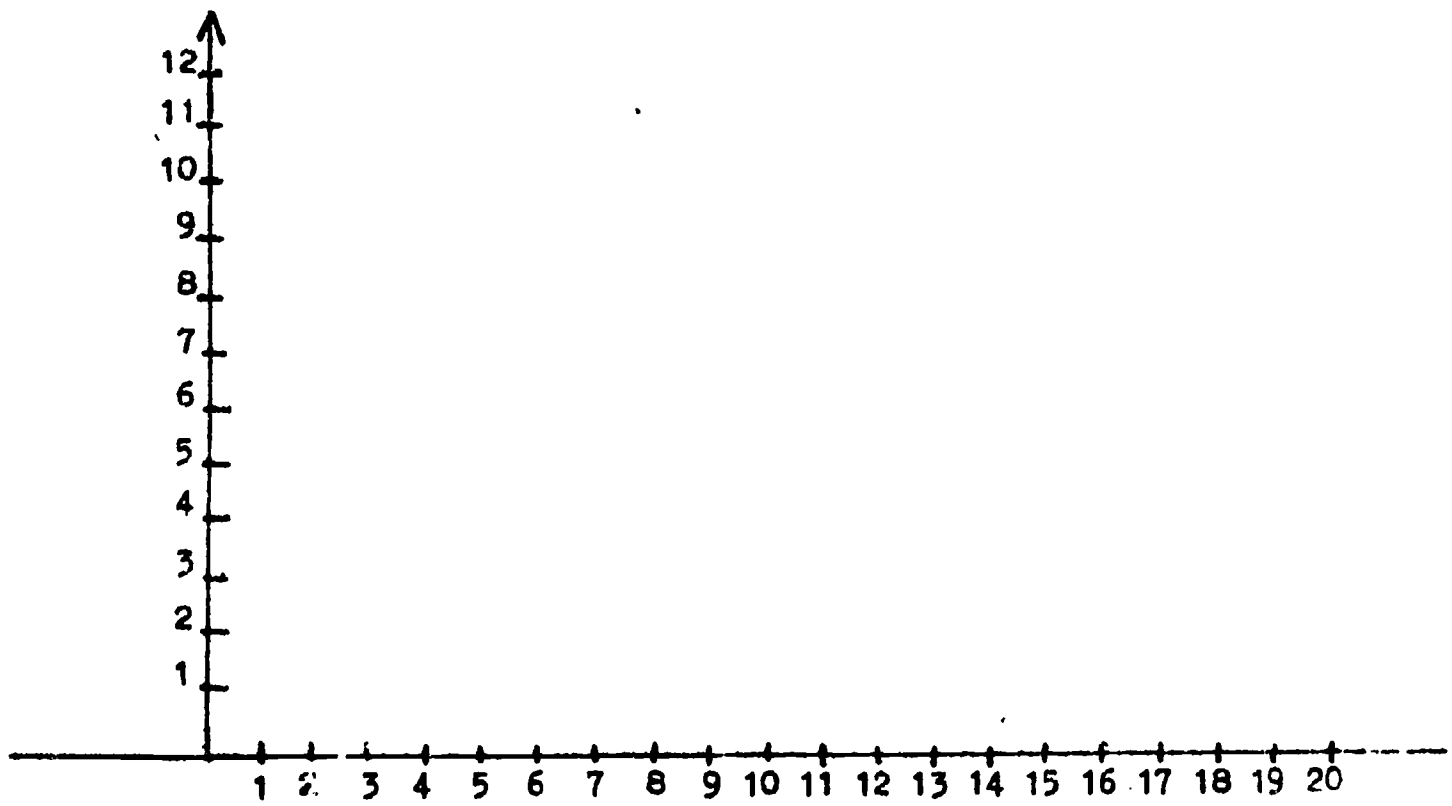
Let x = 80% pure gold
y = 20% pure gold

.4x = gm of gold in 80%
.6y = gm of gold in 20%

So the two equations are

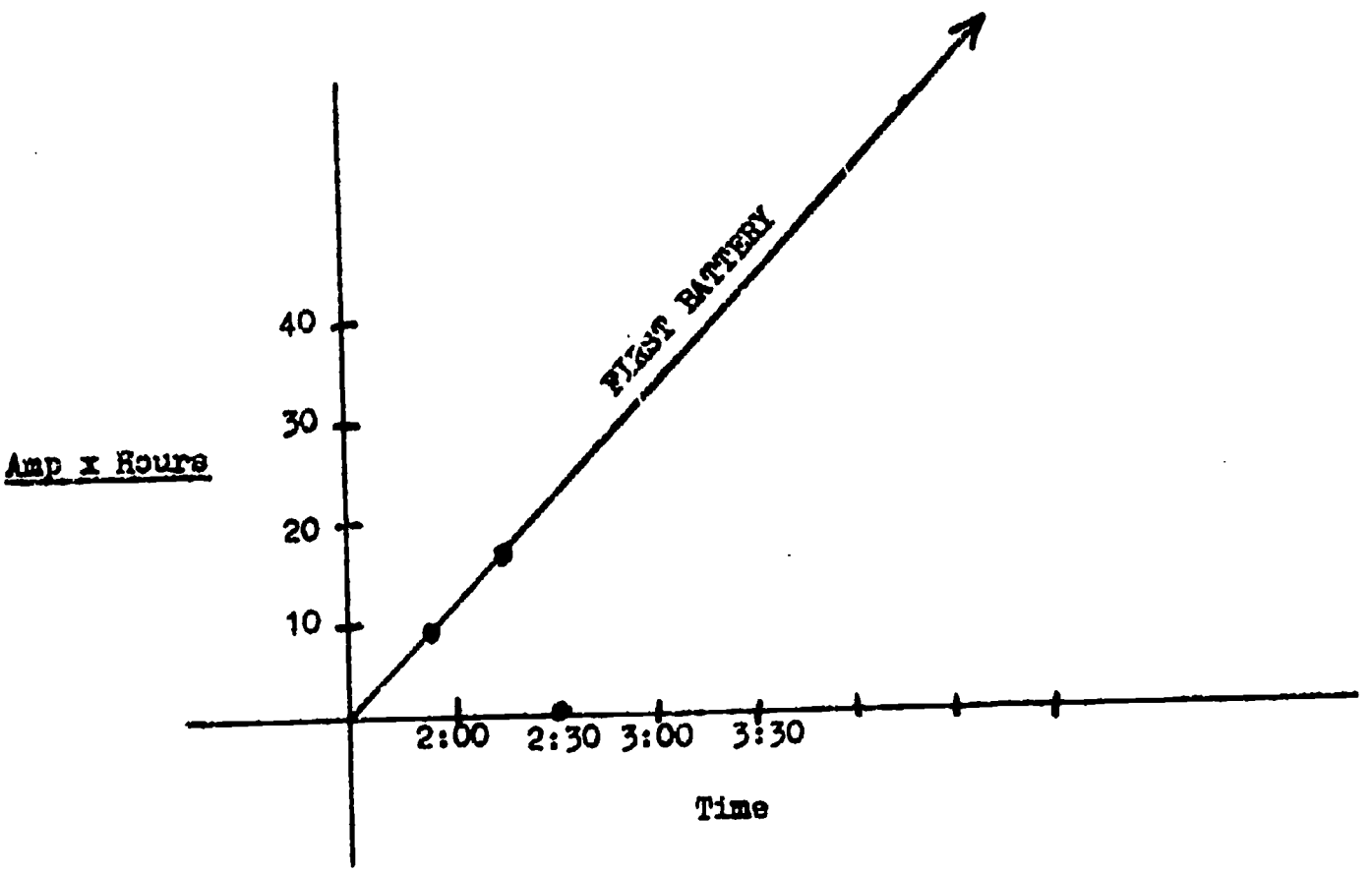
$$x + y = 12$$
$$.3x + .2y = 12\ (.7)$$

Graph to find x and y



A 12 volt D.C. generator can charge a battery at the rate of 20 amperes which is 20 coulombs of charge per second. It starts charging a new battery at 1:30 P.M.

Another D.C. 12 volt generator can charge a battery at the rate of 40 amperes. It starts charging a similar battery at 2:30 P.M. When will both batteries have the same charge? What will the charge be?



Let (0,0) be time to = 1:30 with 30 min. intervals. Plot the second battery and find the time of equal charge, and the amount of charge?

ADVANCED STUDY (cont')

III. Work any 5 of the following problems:

A. Dolciani, page 172, numbers 14-18.

B. Nichols, page 227. nos. 7-9.

IV. Work any 5 of the following:

A. Dolciani, page 182, numbers 13,14,16; page 191, numbers 56,58.

B. Nichols page 176, number 14; page 177, number 4

V. Work any 6 of the following:

A. Dolciani, page 184, numbers 13,14; page 311, numbers 9,10; page 318, numbers 1-5.

B. Vanatta, page 176, numbers 9, 20; page 178, number 7.

VI. Payne, page 244, numbers 23-26.

Payne, page 247, numbers 1-6.

VII. Payne, read pp. 250-252, Ex. 1-4 pages 252-253.

Nichols (abbreviation)

Nichols, Eugene, D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Ginn and Company, 1964.

Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis, Algebra One, Harcourt, Brace and World, Inc., 1969.

Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenback, Edwin Fr., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics, Algebra 1, Houghton Mifflin Company, 1967.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

Vanatta (abbreviation)

Vanatta, Glen D., Goodwin, A. Wilson, Algebra One, A Modern Course, Charles E. Merrill Publishing Co., 1966.

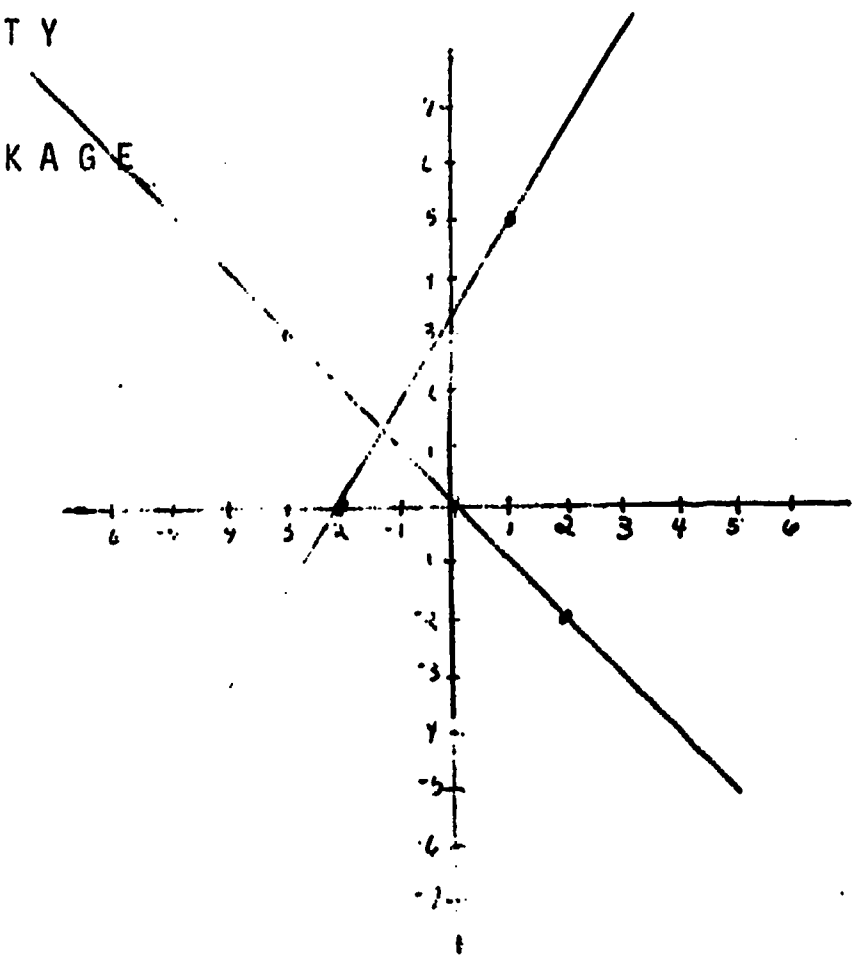
Programed Algebra (abbreviation)

Heimer, Ralph T., Kocher, Frank., and Lottes, John, J., A Program in Contemporary Algebra, Book 3, Equations and Inequalities in Two Variables, Holt, Rinehart and Winston, Inc. New York, N. Y., 1963

Programed Geometry (abbreviation)

Nichols, Eugene D., Wilson, Robert., Garland, Henry., An Introduction to Coordinate Geometry, Holt, Rinehart and Winston, Inc.

LEARNING ACTIVITY PACKAGE



SOLUTION SETS OF EQUATIONS WITH TWO VARIABLES

Ninety Six High School · Ninety Six, S.C.

Algebra 93-94

REVIEWED BY J. S. Pistole

LAP NUMBER 18

WRITTEN BY Marie B Evans

Instructions:

I. Read Rationale

II. Read BEHAVIORAL OBJECTIVES.

III. Resources

A. All work must be done [illegible] pencil only.

B. Keep your notebook up [illegible] may ask for it at any time [illegible]

C. Work all the Exercises [illegible] for each goal. Always [illegible] notebook [illegible] your teacher.

IV. Self-Evaluation

A. Must be taken at completion [illegible] each section.

B. Does not affect your grade [illegible]

V. Advanced Study

A. To be done only after [illegible] been satisfactorily completed.

B. Must be approved by teacher.

VI. Progress Test and LAP Test

A. Teacher graded

B. Recycling may take place [illegible] is not satisfactory.

DO NOT LOSE YOUR LAP. If you do, [illegible]

Rationale (The LAP's Purpose)

A natural place to begin [illegible] branch of mathematics is in the theory of sets. Sets can be thought of as the building blocks of mathematics. Before [illegible] to any form of higher mathematics [illegible] familiar with what a set is, [illegible] with sets, and how sets can be used.

In this LAP you will be given a mathematical way of the subject of sets, including the basic notation associated with sets. [illegible] concepts you will study are subsets, equality and matching sets, operations on sets, [illegible] sets, and graphing sets.

Section 1

BEHAVIORAL OBJECTIVES : At the completion of your [illegible] of study, you will be able to:

1. Given any set stated in words, write [illegible] identify it in set notation.

2. Given any set written in the [illegible] method, rewrite or identify it in the [illegible] or roster method.

3. Given any set written in the [illegible] method, rewrite or identify it in the description method.

4. Given a particular set and a list of elements, decide which are elements of that set and which are not.

5. Given any sets, tell which are infinite and which are finite.

6. Given a rule for a set (description method), decide whether or not the result is the empty set - $\emptyset$, { }

7. Given a list of numbers, [illegible] those are prime & those that are composite.

8. Given any two sets, decide whether or not they are matching sets (one to one correspondence).

Reading

Objective 1

Vanatta, read pp. 8-12, Ex. 1 page
Nichols, read pp. 1-3, Ex. 6 page and 9
Wollensak tape C-3461 Introduction to Sets

Objectives 2,3

Dolciani, read pp. 10-14, Ex. 1-6 and 7-
13, 16 written (roster only) page 14.

Objectives 4,5,6

Payne, read pages 1-4 and the
pages 4-6.
Nichols, read pages 1-3, Ex.
Dolciani, read pages 12-14 Ex.
18, 19, page 27
Wooton, read page 16-18, ex.
Pearson, read pages 144, Ex.
Introduction to Sets, Frames
Wollensak C-3012, Prime Numbers

Objective 7

Payne, read problems 16, 17 on page
Nichols, read 4-6, Ex. 2-6 page 6
Wollensak C-3010, Prime Numbers

Objective 8

Payne, read pages 12-14, Ex. 14,15
Nichols, read pages 7, 8, Ex.
Dolciani, read pages 12-14 Ex.
Wooton, read pages 20-23, Ex. 27,
Introduction to Sets Frames 226-

SELF-EVALUATION 1

OBJECTIVE

1 I. Match each exponential form on the left with its equivalent product on the right.

____ 1. 6^4	A. 2 x 2 x 2
____ 2. 3^2	B. 4 x 6
____ 3. 2^3	C. 3 x 3
____ 4. 4^6	D. 6 x 6 x 6 x 6
	E. 4 x 4 x 4 x 4 x 4 x 4

2 II. In each of the following, circle the exponent and underline the base.

(5) 6^4

(6) a^4

(7) b^x

(8) x^a

4 III.A.Write each of the following in exponential form using 2 as the base.

_____ 9. 64

_____ 10. 32

_____ 11. 4

_____ 12. 16

B. Write the following in exponential form using 4 as the base.

_____ 13. 16

_____ 14. 4

_____ 15. 64

5 IV. Write the following as decimal numerals.

_____ 16. 3^4	_____ 19. $(\frac{2}{3})^4$
_____ 17. -5^2	_____ 20. $(-5)^2$
_____ 18. 7^2	_____ 21. -4^3
	_____ 22. $(-4)^3$

V. Simplify the following.

3 _____ 23. $3^4 \cdot 3^2$

_____ 24. $x^3 \cdot x^4 \cdot x$

_____ 25. $a^2b^3ab^4$

_____ 26. $x^2y^3x^4y^2$

6 _____ 27. $(r^2)^3$

_____ 28. $(a^5)^3$

_____ 29. $(3ab^3)^2$

_____ 30. $(4xy)^2$

_____ 31. $(3a^2m^3)^2$

_____ 32. $\left(\frac{a^2}{b^3}\right)^2$

_____ 33. $\frac{c^2m^3}{cm^4}$

_____ 34. $\frac{18\ x^3y^2}{3xy}$

_____ 35. $\left(\frac{a^3}{b^4}\right)^2$

_____ 36. $\left(\frac{-x}{y}\right)^3$

_____ 37. $\frac{-48r^5s^7}{-4r^2s^4}$

_____ 38. $(r^5)^3$

_____ 39. $(3\ r^2s^3)^4$

_____ 40. $\frac{-15rs^4}{3rs}$

If you have satisfactorily completed your work, take the Progress Test. Consult your teacher first.

SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

7. Given any non-zero rational expression involving exponents, write equivalent expressions using only positive exponents.
8. Use the distributive property to name the product of a monomial and a polynomial.
9. Given any positive number, express it in scientific notation.
10. Given a number expressed in scientific notation, express it as a decimal numeral.
11. Given two or more numbers expressed in scientific notation, find the indicated sum, difference, product or quotient.
12. Given a verbal problem involving very large or very small numbers, express the numbers in scientific notation and find the solution of the problem.

RESOURCES

Objective 7

Nichols, read pp. 320-324, Ex. 1-15 even page 322; 1-11 even top page 323; 1, 2 every other letter bottom page 323; 3 a,b,d,f, g,h,j,n,p,w,r, 6 a,b,e,i,j pages 335-336.

Vanatta, read pp. 117-119, Ex. 1-40 even pages 119-120.

Dolciani, read pp. 232-233, Ex. 1-14 page 233.

Payne, read pp. 267-268, 273-274, Ex. 1-9 page 269.

Wooton, read pp. 324-326, Ex. 1-45 odd pages 327-328.

Pearson, read pp. 347-349, Ex. 1 page 349; 3-7 page 350.

Introduction to Exponents frames 142-196.

Objective 8

Vanatta, read pp. 120-121, 137, Ex. 1-20 even page 121; 1-20 even pages 137-138.

Objective 8 (cont')

Dolciani, read pp. 206-207, Ex. 1-14 page 207.

Wooton, read pp. 272-274, Ex. 1-18 oral page 274 ALSO,

(a) $2x(3x^2 + 2x - 5)$
(b) $2a(a^2 - 3a + 2$
(c) $3y(2y^2 + y - 3)$
(d) $5d(6 - d + 2d^2)$
(e) $xy(x - 2xy + y^2)$

Pearson, read page 350, Ex. 8 page 350.

Objectives 9, 10, 11, 12

Nichols, read pages 339-341, Ex. 1-3 pages 340-341.

Dolciani, read pp. 376-377, Ex. 1-17 page 278.

Payne, read p. 268, Ex. 25-55 page 269; 27-32 page 260.

Pearson, read pages 350-351, Ex. 1-10 pages 351-353.

Introduction to Exponents Frames 56-78 (Obj. 9)
Frames 79 (Obj. 12)

* Appendix 2

* Nichols Ex. 4 pages 340-341.

* required

SELF-EVALUATION 2

Objective

7 I. Write the following using only positive exponents and simplify.

1. $5^9 \cdot 5^{-4}$
2. $\frac{x^3y}{x^4}$
3. $2x^{-3}$
4. $\frac{3x^{-2}}{a^{-1}b^2}$
5. $\frac{2x^{-2}}{8y^{-3}}$
6. $\frac{2x^{-6}}{-8x^4}$
7. $x^{-2}y^4$
8. $\frac{x^3y^4}{x^{-4}y^{-2}}$
9. $\frac{5x^2}{r^{-3}}$
10. $3x^{-4}$
11. $\frac{6xy}{-2^{-1}}$
12. $\frac{3b}{a^{-2}c^4}$

8 II. Simplify the following:

13. $3x(2x - 3y + 4c)$
14. $a^2(3a - 2b + c)$
15. $(3xy)(2x^2y^3)$
16. $(2x^3y^4)(3xy^4)$
17. $3x^2y(2x + 3y + 4xy)$
18. $-2a^3b(a^4b - a^3b^2 + 2a^2b^4 - b^5)$
19. $3x^2y(5 - 2xy^4 + 3x^2y^3 - y^5)$

9 III. Express each in scientific notation.

20. 68.5 = _____
21. .205 = _____
22. .0024 = _____
23. 136,000,000,000 = _____
24. .0000000612 = _____

SELF-EVALUATION 2 (cont')

10 IV. Express each as a decimal numeral.

25. $3.2 \times 10^4 =$ _____

26. $2.9 \times 10^{-4} =$ _____

27. $3.1 \times 10^2 =$ _____

28. $6.7 \times 10^{-8} =$ _____

11 V. Simplify, leaving the answer in scientific notation.

29. $(4.5 \times 10^2) + (3.6 \times 10^3) =$ _____

30. $(3.7 \times 10^5) - (2.3 \times 10^2) =$ _____

31. $(6.2 \times 10^5) \cdot (2.1 \times 10^3) =$ _____

32. $\dfrac{5.4 \times 10^3}{.2 \times 10^2} =$ _____

33. $\dfrac{14 \times 10^4 \times 2 \times 10^{-6}}{7 \times 10^{-2}} =$ _____

34. $\dfrac{3 \times 10^{-6} \times 21 \times 10^4}{9 \times 10^{-4}} =$ _____

12 VI. Solve each problem.

35. Give, in scientific notation, the number of minutes in a year. (1 year = 365 days)

36. The speed of sound at sea level is 760 mph. Give this speed in feet per second written in scientific notation.

37. Spaceships travel at speeds of 18,000 mph. How many miles per second is this?

38. The sun is 93,000,000 miles away from earth. How far is this in feet? Express in scientific notation.

IF you have satisfactorily completed your work, take the LAP TEST. Consult your teacher first.

APPENDIX I

Objective

1 I. Write the following as a product where the factors are alike.

A. 7^4

B. 10^2

C. 8^6

D. 9^3

E. 6

2 II. In each of the following, name the base and exponent.

A. 7^4 base _____ exponent _____

B. a^9 base _____ exponent _____

C. 2 base _____ exponent _____

D. x^3 base _____ exponent _____

E. 5^2 base _____ exponent _____

4 III. Write each number on the left in exponential form using the number on the right as the base. Example $27 = 3 \times 3 \times 3 = 3^3$

A. 16 Use 2 as base _____

B. 9 Use 3 as base _____

C. 64 Use 4 as base _____

D. 64 Use 2 as base _____

E. 64 Use 8 as base _____

APPENDIX II

OBJECTIVE

9 **I. Express the following in scientific notation.**

1. 3,000,000,000 =

2. 463,000,000,000 =

3. .049 =

4. .0000000000061 =

10 **II. Write the following as decimal numerals.**

1. 3.42×10^{6}

2. 5.12×10^{-5} =

3. 7.412×10^{3} =

4. 3.216×10^{-7} =

5. 6.014×10^{4} =

11 **III. Compute the following:**

1. $3 \times 10^{4} \times 6 \times 10^{6}$

2. $\dfrac{6.8 \times 10^{4}}{3.4 \times 10^{6}}$

3. $\dfrac{3 \times 10^{7} \times 15 \times 10^{-2}}{9 \times 10^{4}}$

4. $\dfrac{10 \times 10^{4} \times 2 \times 10^{7}}{5 \times 10^{8} \times 2 \times 10}$

5. $(4.5 \times 10^{2}) + (3.6 \times 10^{3})$

6. $(3.7 \times 10^{4}) - (2.3 \times 10^{2})$

7. $(4.1 \times 10^{6}) + (2.4 \times 10^{4})$

8. $(6.1 \times 10^{3}) - (5.3 \times 10^{2})$

9. $(7.6 \times 10^{10}) + (5.6 \times 10^{8})$

10. $(4.3 \times 10^{8}) - (3.2 \times 10^{6})$

ADVANCED STUDY

I. Payne, read pp. 257-259, Ex. 43, 45, 47 pages 259-260.

II. Nichols, read pp. 332-335, Ex. 1, 2, 3 c,e,i,m,p, 4 pages 335-336.

III. Write a mathematical formula for the volume of a cube of edge X. The volume of a cube is equal to the product of the length, width, and height. Given a cube, write the formula for its volume using exponents. What happens to volume if you should double the length of the edge?

IV 1. Light travels at a speed of three hundred million meters per second. How far is the sun from the earth (meters) if it takes 8 minutes for light to travel from the sun to the earth? Express in scientific notation.

2. A Radar beam is directed toward the moon and the reflected beam is received 2.6×10^0 seconds later. The beam travels at 1.86×10 mile per sec. How far is the moon from the earth? Express in scientific notation.

REFERENCES

Nichols (abbreviation)

Nichols, Eugene, D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Ginn and Company, 1964.

Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis, Algebra One, Harcourt, Brace and World, Inc., 1969.

Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenback, Edwin Fr., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics, Algebra 1.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

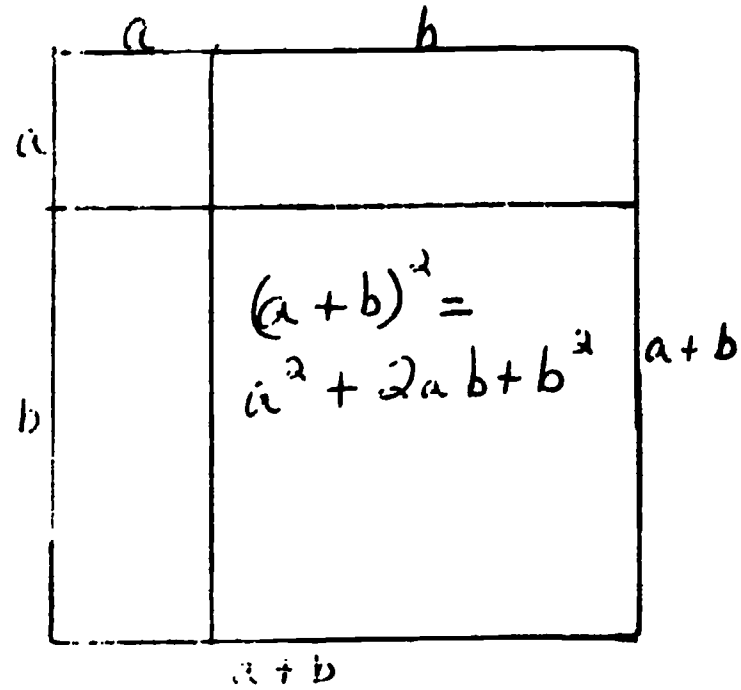
Vanatta (abbreviation)

Vanatta, Glen D., Goodwin, A. Wilson, Algebra One, Charles E. Merrill Publishing Co., 1966.

Odom, Mary Margaret, Nichols, Eugene D., consulting Editor, Introduction to Exponents, A Programmed Unit, Holt, Rinehart and Winston, Inc.

LAP

LEARNING ACTIVITY PACKAGE



FACTORING AND POLYNOMIALS

Ninety Six High School - Ninety Six, S.C.

Algebra 93-94

REVIEWED BY

LAP NUMBER 20

WRITTEN BY Diane Evans

RATIONALE

In arithmetic, before you could solve practical problems, you had to be able to perform the fundamental operations with numbers. You needed to know the addition combinations before you could find the total cost of a number of items. In order to find the cost of several pounds of an item at a given price per pound, you needed to know how to multiply. Before you could work problems containing fractions and decimals, you had to learn the operations with those special types of numbers.

In algebra we will be dealing largely with polynomials. You must learn to perform the basic operations with polynomials before you can use them in applications. In this LAP you will learn to use polynomials in addition, subtraction, multiplication, and division. You will also learn to solve equations involving polynomials.

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given an algebraic phrase, identify the coefficients, factors, terms and degree of the phrase.
2. Given a polynomial of 1, 2, or 3 terms, determine if it is a monomial, binomial, or trinomial.
3. Given a polynomial, write it in descending or ascending order.
4. Given any pair of polynomials compute their
   a. sum
   b. difference
   c. product
   d. quotient

RESOURCES

OBJECTIVES 1,2

Nichols, read pp. 119-120, 199-200, 384-385, Ex. 1 page 201.

Vanatta, read pp. 67-71, Ex. 2,3,4 page 71.

Payne, read pp. 307-310, Ex. 1-27 pages 309-310.

Wooton, read pp. 52-53, Ex. 1-19 odd oral page 54; 19-24 page 55.

* Appendix I parts I-III

OBJECTIVES 3,4

Vanatta, read pp. 133-135, 137-139, 140-143, Ex. 1-15 even p. 134; 1-12, 13, 15, 16, 19 page 136; 1-24 even p. 137; 1-24 even p. 139; 1-20 every 4th problem p. 140; 1-30 even p. 141; 1-20 even p. 144.

Dolciani, read pp. 198, 200-201, 2-3, 2-9, 219-222; Ex. 1-20 odd written p. 199, 1-8 p. 201; 1-9 page 202; 1-20 even p. 210; 1-14 even p. 207; 25-28, 33-40 page 210; 9-27 even p. 220; 1,3,6,8,12,15,18,24,25,29 p. 223.

Wooton, read pp. 310-323, Ex. 19-28,33,35 page 59; 17-49 odd pages 86-87; 1-41 odd pages 105-106; 1-23 odd pages 274-275; 1-39 odd p. 279; 1-17 even bottom p. 314; 1-29 odd pp. 319-320.

* Appendix I parts IV, V

* required

SELF-EVALUATION 1

OBJ.

1 I. For the polynomial $2a^3 + 4a^2b^3 + 9a^2c^4$ state each of the following:

a. The degree of polynomial

b. The degree of the polynomial with respect to a

c. The degree of the polynomial with respect to b

d. The degree of the polynomial with respect to c

e. The coefficient of a^3

f. The number of terms in the polynomial

2 II. Classify each of the following on either a monomial, a binomial, or a trinomial (all letters are variables).

1. $x + y$
2. $4m$
3. $-\frac{1}{2}mxy$
4. $2x - y - z$
5. 26
6. $4.5a - 1.2b + 3.6c$
7. $\frac{1}{3} - \frac{1}{4} + \frac{1}{5}s$
8. $3xyz - 2ab$

3 III. Express the polynomial in ascending order of b and then in descending order of a

$3a^2b^2 - 4a^3b^4 + 2ab^3 + 5a^4b$

3,4 IV. Find each sum and arrange in order of decreasing degree in n.

1. $(3n^3 + 5 - 2n) + (n^2 - 6n - 8)$
2. $(2m^3n - 3m^2n^2) + (4m^2n^2 - mn^3) + (-mn^3 - 7m^3n)$

OBJ.

SELF-EVALUATION 1 (cont')

4a **V. Add**

(1) $3x^2+6x+4$
$\underline{-2x^2-6x+1}$

(2) $3xy-6x^2+3y$
$\underline{4xy+2x^2-4y}$

(3) $5x^2-3x+1$
$\underline{2x^2-6x-4}$

(4) $(7x^2+6x+1)+(-4x^2-3x-6)$ =

(5) $(-3x+6y-3)+(4x-2y-7)$ =

VI. Subtract

(1) $3x^2+6x-1$
$\underline{2x^2-4x+6}$

(2) $7x+3y-6$
$\underline{-2x-4y+1}$

(3) $8x+6y-7$
$\underline{-2x+6y+7}$

(4) $(6x+7y-2) - (8x+6y-7)$ =

(5) $(2x^2+7x-3) - (6x^2+3x+1)$ =

4a,b VII. Simplify

1. $(3x+2y-1) + (4x+6y) - (2x+3y+2)$ =

2. $(5x^2-6x+1) - (4x^2+2x+1) + (6x^2+9x-2)$ =

3. $(4xy-6x+7) + (2xy+6x-2) - (4xy+3x-7)$ =

4c VIII. Multiply

_____ 1. $(x+1)(x+5)$

_____ 2. $(2x-6)(2x+7)$

_____ 3. $(5y+8)(4y-3)$

_____ 4. $(7z-3)(6z+2)$

_____ 5. $(8x+4)(8x-4)$

_____ 6. $(2x^2+1)(3x^2-5)$

_____ 7. $(4x^2+1)(2x^2-9)$

4c IX. Multiply

1. $x+6$ / $x-9$

2. $2x-3$ / $x+1$

3. $7x-7$ / $2x+3$

4. $4x^2+1$ / $2x^2-6$

5. $6x^2-7$ / $2x^2+1$

6. $3x^2+6x-9$ / $2x+3$

7. $2xy+3x-1$ / $2x+4$

8. $-3x^2+2x-6$ / $x+4$

9. $(x^2-2x+1)(2x^2+4)$

10. $(2x^2-3x+2)(3x^2+4x+3)$

11. $x^2-7x+12 \div x-4$

12. $6x^3-x^2+3x-20 \div 3x+4$

13. $30x^2-28x+8 \div 5x-3$

14. $2x+4 \overline{)\,2x^2-9x-34}$

15. $x^5+32 \div x+2$

16. $7x-2 \overline{)\,14x^3+38x^2-5x+9}$

If you have satisfactorily completed your prescribed course of study, take the PROGRESS TEST. CONSULT YOUR TEACHER FIRST.

SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

5. Write the prime factorization of any given composite number.
6. Given any polynomial, express it in factored from when the polynomial:
   a. has a common monomial factor
   b. is written as the difference of two squares
   c. is a perfect square trinomial
   d. is of the form $x^2 + (a + b)x + ab$
7. Given a polynomial of the form $ax^2 + bx + c$, express it in factored form.
8. Given a quadratic equation, determine the solution set by factoring.

RESOURCES

OBJECTIVE 5

Vanatta, read pp. 286-289, Ex. 1-10 page 289.

Payne, read pp. 324-325, Ex. 22-41 odd page 325.

Wooton, read pp. 280-282, Ex. 1-33 odd page 283.

OBJECTIVE 6

Vanatta read pp. 289-292, 293-297, 299-300, 301-305; Ex. 1-5 p. 290; 1-22 even page 292; 1-30 even p. 297; 1-14 even page 305.

Nichols, read pp. 359-360, 355-356, 381-384, Ex. 1-2 every other letter pages 360-361; 1-59 odd p. 356; 1-45 odd p. 384.

Payne, read pp. 326-335, 338; Ex. 1-15 even, 31-45 even pages 326-328; 1-18 odd pages 329-330; 1-40 odd pages 331-333; 1-24 odd pp. 334-335; 1-12 odd pages 338-339.

Wooton, read pp. 282-287, Ex. 1-53 odd page 286; 1-50 odd p. 287.

Pearson, read pp. 247-249; 373-386, 392-393; Ex. 1-4 every other letter p. 248-249, 4,6,10,12 EOL pages 246-247; 1-10 odd p. 374; 2-4 EOL p. 377; 1,2,4 EOL pp. 380-381; 1-4 EOL p. 382; 1-12 EOL pp. 385-386; 1-31 EOL p. 393.

(CONT')

OBJECTIVE 7

Vanatta, read pp. 297-300, Ex. 1-16 even p. 300.

Nichols, read pp. 357-359, Ex. 1-59 odd pp. 358-359.

Payne, read pp. 335-339, Ex. 1-35 odd p. 337.

Pearson, read pp. 387-388, 395-396, Ex. 1-7 even, 8 every other letter, page 388.

OBJECTIVE 8

Nichols, read pp. 361-363, Ex. 1-25 odd page 363.

Payne, page 340, 358-359, Ex. 1-19 odd pages 340-341; 1-15 odd pages 360-361; 1-26 odd page 368.

Pearson, read pp. 389-395, 592-593, Ex. 1, 2 every other letter pages 390-391; 1-2 EOL page 593.

SELF-EVALUATION 2

OBJ. 5

I. Find the prime factors of the following:

1. 78 = _____

2. 143 = _____

3. 833 = _____

4. 180 = _____

6

II. Express in factored form.

1. $7x + 14y$

2. $-2x^2 + 4y^2$

3. $6xy - 3ax + 9xb$

4. $9d^2 - 1$

5. $r^2 - 9s^2$

6. $a^2 - 4b^2$

7. $9a^2 - 81b^2$

8. $x^2 - 42x + 49$

9. $x^2 - 12x + 36$

10. $16x^2 + 32xy + 4y^2$

7

III. Find the factors.

1. $x^2 - 3x - 10$

2. $c^4 - 2c^2 - 63$

3. $18 + 3x - 10x^2$

4. $6y^2 - 17y + 12$

5. $8x^2 - 10xy + 3y^2$

6. $6x^2 - 5x - 21$

7. $45x^2 + 320x + 35$

8. $9x^2 + 6x - 8$

9. $15y^2 - y - 2$

10. $30x^2 + 39x - 9$

8 IV. Solve the following by factoring.

1. $x^2 - 25 = 0$
2. $x^2 - 5x + 6 = 0$
3. $x^2 - 2x = 15$
4. $x^2 - 8 = 7x$
5. $2x^2 - 5x + 3 = 0$
6. $6t^2 - 5t + 1 = 0$
7. $6y^2 - 25y + 25 = 0$
8. $9x^2 - 49 = 0$
9. $x^2 - 3x = 10$
10. $x^2 = 3x$

If you have satisfactorily completed your prescribed course of study, take the LAP TEST. CONSULT YOUR TEACHER FIRST.

APPENDIX I

OSJ.
2

I. Write the definition for each of the following.

1. polynomial

2. monomial

3. binomial

4. trinomial

II. Tell if each of the following is a monomial, binomial, or trinomial.

_____1. $2x+3y$

_____2. $6xyz$

_____3. $4x^2y^2$

_____4. $8x-7xyz$

_____5. $3xyz+2kl+1$

_____6. $3xyzl$

III. Give the degree of each of the following and identify the coefficien

_____1. $3x^2y^3+3xy^3+2x^4y^2$

_____2. $6x^2+7x^3+9y^4$

_____3. $2xy \cdot 3x^2+6x^2y^3$

_____4. $5x^2+3xy+9xy^2$

_____5. $3x+7y+9xy$

IV. Rewrite the following in decending order of powers of x.

_____1. $3xy+2x^2+6x^4-3x^3y$

_____2. $7x^2y-2x^5y+3x^4+x$

_____3. $x^3-6x^6+6x^9-2x$

_____4. $3xy^2+4x^2y+7x^3y^4$

_____5. $x^5y-6xy^3+2x^3y^2$

_____6. $x^7y^5-3x^4y^2+8x^2y^3-2xy$

V. Write each of the polynomials in part IV in ascending order of powers of x.

ADVANCED STUDY

I. Work the following:

1. $2x^5 + 9x^2 - 2x^3 - 5x^4 - 7x + 3 \div 2x^2 - 3x + 1$

2. multiply: $(3x^3 - 6x^2 + 9x - 6)(2x^2 - 3x - 9)$

3. divide: $9x^{N+2} - 6x^{N+1} + 24x^N \div -3x^N$

II. Show how synthetic division works and work the following using synthetic division.

1. $x^3 - 3x^2 + 5x - 6 \div x - 2$

2. $x^3 - 6x^2 - 100 \div x - 5$

3. $\ldots + 25 \div x + 5$

III. Work any three of the following:

1. $2x - (3x + 7) + 5(2x - 2) = 8x + 1$

2. $(2x \pm 1)(2x - 6) = -3x(-2x \pm 4) + 6$

3. $\frac{\ldots}{2} + \frac{3x - 4}{5} = \frac{11 + 2x}{3}$

4. $\frac{3x - 4}{10} - \frac{6x + 2}{5} = \frac{x - 2}{2} + \frac{2x + 3}{4}$

IV. Nichols, read pp. 361-373, Ex. 1,2 EOL page 364; 1 p. 373; 1,2 EOL page 366.

V. Payne, read pp. 358-373, Ex. 1-27 odd p. 363; 1-25 odd page 369-370.

REFERENCES

Vanatta (abbreviation)

Vanatta, Glen D., Algebra One: A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965.

Nichols (abbreviation)

Nichols, Eugene D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 1, Houghton Mifflin Co., 1967.

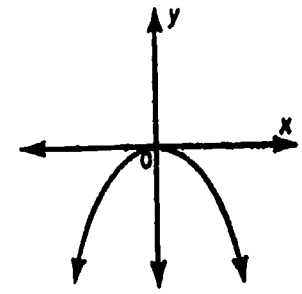
Payne (abbreviation)

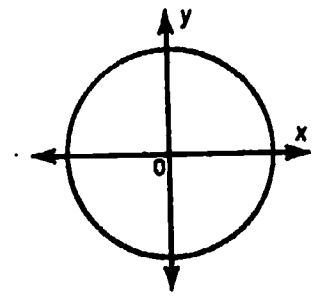
Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis G., Algebra One, Harcourt, Brace and World, 1969.

Pearson (abbreviation)

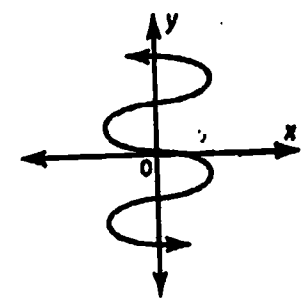
Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, 1964.

LEARNING ACTIVITY PACKAGE

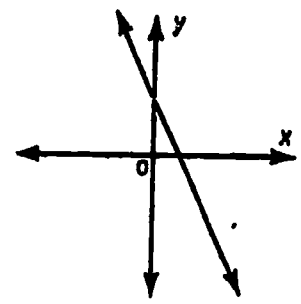




RELATIONS



AND



FUNCTIONS

Ninety Six High School
Ninety Six, S.C.

Algebra 93-94

REVIEWED BY D. Patelis

LAP NUMBER 21

WRITTEN BY Diane Evans

RATIONALE

The words RELATION and FUNCTION in mathematics are probably new to you! Consider the OPERATION of ADDITION with which you have worked for most of your school years . . it is <u>not only</u> a RELATION <u>but also</u> a FUNCTION! There are many things which you have studied and which you will study in your future mathematics courses that are relations and functions.

Throughout mathematics we pair numbers and obtain set of ordered pairs which are relations. These ordered pairs and graphing (which you have previously studied) will serve as a basis for the study of relations and functions.

In this LAP you will be concerned primarily with the meaning of relation and function. Graphing relations and functions will be stressed in order to give you experience in actually working with these ideas which are basic to future courses in mathematics and science!

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given two finite sets, list the ordered pairs which belong to their Cartesian Set (Cartesian Product).
2. Given two subsets of the real numbers, graph their Cartesian Set on the coordinate plane.
3. Given a relation defined by a rule of correspondence, name the ordered pairs which belong to this relation.
4. Given a relation, name:
   a. its domain
   b. its range
5. Given a relation, determine whether or not that relation is a function where the relation is defined by:
   a. a set of ordered pairs
   b. a graph
   c. a rule or correspondence
6. Given a relation, name its inverse.

RESOURCES

Objectives 1, 2

Nichols, read pp. 393-395, Ex. 1,3 page 394; 1, 3, 5, 6 page 395.

Wooton, read pp. 377-379, Ex. 19, 20 page 381.

Pearson, read pp. 431-435, 441-442, Ex. 1, 3, 5 page 432; 1 a, c, f pages 436-437; 1, 3, 5 pages 442-443.

Objective 3

Nichols, read pp. 396-397, Ex. page 398 1 - a, c, d, f
2 - a, c, d, f
3 - a, c, d, f

Payne, read pp. 174-175, Ex. 4, 5 page 176; 1, 4, 5, 8, 10, 11 pages 180-181.

Pearson, read page 542-543 Ex. 1, 2, 3, 5, 8a,b,c page 544.

Objectives 4, 5

Nichols, read pp. 398-399, Ex. pages 399-404 1 - a,b,d,f,g,h,j,k
3 - a,b,d,f,g,h,j,k
4 - a,b,d,f,g,h,j,k
5, 8a,c,e,g
9 a thru n
10 a thru j

** Payne, read pp. 175-179, 466-467; Ex. 1, 2, 10, 11, 12, 15, 17, 18-21 pages 176-177; 28 page 181; 1, 2, 4, 5, 7, 9-12, 13, 14, 21-30, 33, 35 page 467.

Pearson, read pp. 545-550, Ex. 1a,b, 2a,b, 5a, 6a pages 545-547; 1-20 pages 550-551.

* Appendix I

Objective 6

Nichols, read pp. 404-407, Ex. 1 a, f, 1, 2 a, c, e, h, 1 pages 407-408.

Payne, read pp. 481-483, Ex. 1, 2, 6, 7, 11-14, 15-17, 19, 21, 24 pages 483-484.

* required

** recommended

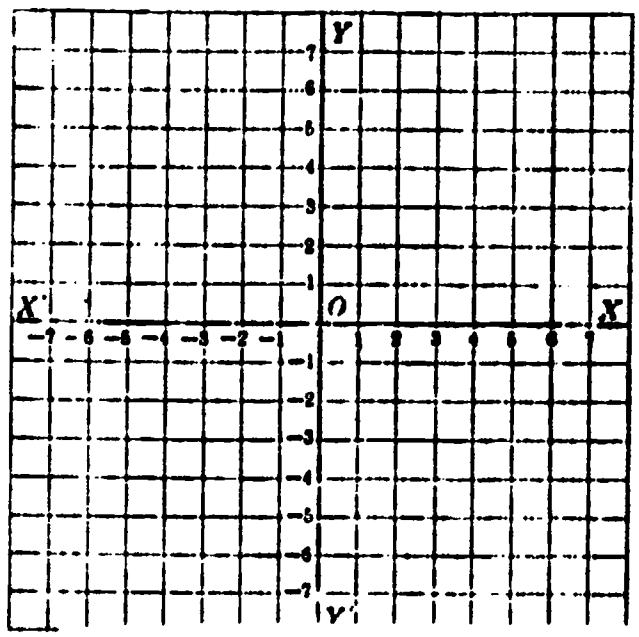
OVER.

I. Given A = {2, 3, 5} and B = {3, 5}

1. Find A x B

2. Find B x A

II. Graph A x B from Example I



III. 1. If the Universal set is the set of real numbers, which one of the following ordered pairs belong to the solution set of the relation y = 2x - 1.

(a) (0,-1)

(b) $(\frac{3}{4}, \frac{1}{2})$

(c) (10, 19)

(d) $(\frac{5}{8}, \frac{1}{4})$

(e) all of these

2. If the Universal set is the set of real numbers, which of the following belongs to the solution set of the relation 2x - 3y = 1?

(a) all ordered pairs in the coordinate plane

(b) (5,3)

(c) $(10, \frac{19}{3})$

(d) (4,2)

(e) none of these

4 **IV. List the domain and range of the following.**

1. (2,1) (2,3) (3,4) (5,6) (7,6) DOMAIN RANGE

2. $y = 2x + 1$

3.

x	3	-2	-7	0	-7
y	4	6	8	2	4

4. $y = x^2$

5a **V. Which of the following relations are functions?**

(a) {(-1,1)(0,0)(0,1)(1,2)(1,3)}

(b) {(1,3)(3,17)(2,3)(3,2)}

(c) {(-2,1)(-1,2)(0,0)(1,2)(2,1)}

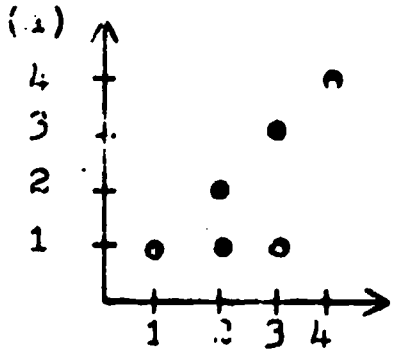
(d) {(1,1)(1,2)(1,3)(1,4)}

(e) none of these

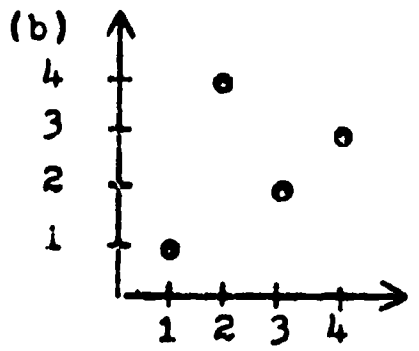
Continued on the following page.

5h VI. From the graphs of some relations, tell which of these are functions by answering YES and NO for those which are not functions.

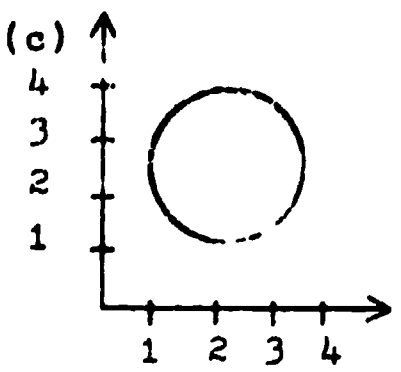
(a)



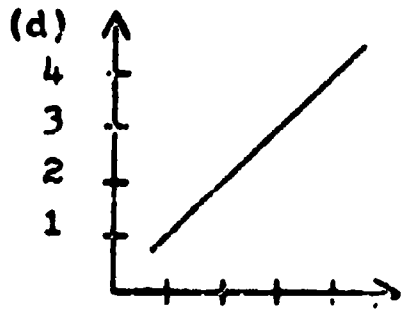
(b)



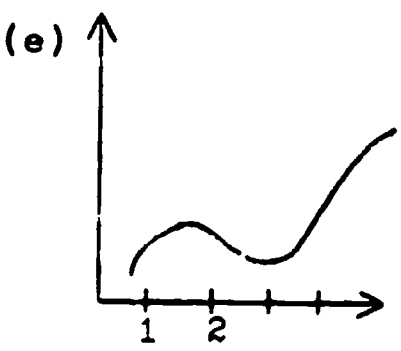
(c)



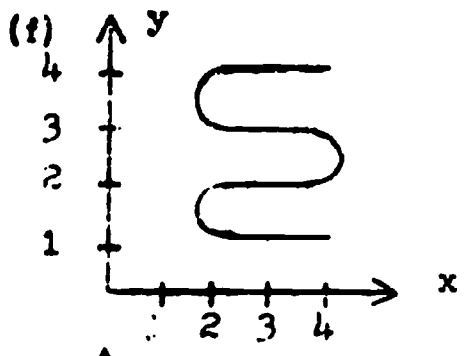
(d)



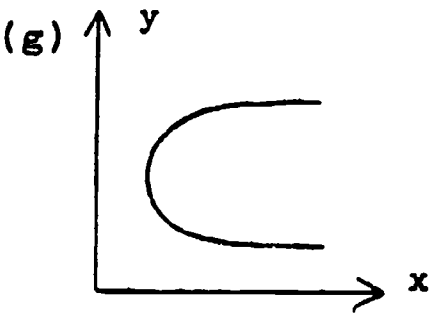
(e)



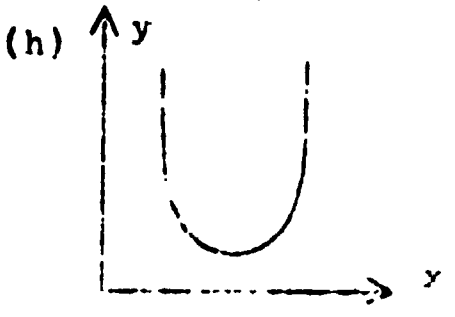
(f)



(g)



(h)



(i)

5c VII. Which of the following relations is a function?

(a) $y \leq 2x + 1$

(b) $x = -3$

(c) $y = 2x + 1$

(d) $y = x$

(e) $y > x + 2$

(f) $y = -2$

6 VIII. Write the inverse of each of the following.

(1) $\{(-1,2), (2,1), (3,2), (4,7)\}$ = _____

(2) $y = x + 5$ _____

(3) $2x + 3y = -1$ _____

(4) $y = 2x^2$ _____

(5) $(3,1)(-2,4)(-6,8)(4,-2)$ _____

(6) $5x = 1 - y$ _____

If you have satisfactorily completed your work, take the Progress test. Consult your teacher first.

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

7. Given a function and a real number, compute the value of the function at the given number.
8. Given a function, name whether it is a linear function or a quadratic function.
9. Given a linear function, construct its graph.
10. Given a linear function, determine its slope.
11. Given a quadratic function of the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$, construct its graph.

RESOURCES

Objectives 7, 8

Nichols, read pp. 408-409, Ex. 1 a,b,e,f,h,i, 2 top p. 410.

Payne, read pp. 179-181, 349-353, 471-472; Ex. 11-13 page 181; 1-5, 7, 10 page 473; 1-10 middle page 353; 5 page 183; 1, 2 checkpoint page 183.

Pearson, read pp. 554-555, Ex. 1 a,b,c,d,g, 3 a,b,c - page 555.

* Appendix II

Objectives 9, 10

Vanatta, read pp. 199-204, Ex. 1-5 pages 204-205.

Nichols, read pp. 410-412, Ex. 1 a,b,c,f, 2, 3 a,b,d,e,g, 4 pages 410-411; 1 a,b,c,d,f,g, 2 a,c,e,f,i,1 page 412.

Payne, read pp. 178-180, 185-188, 471-472, Ex. 14,16,18,20 page 181; 1,2,5,8,12,27 page 189; 13,15 page 473

Dolciani, read pp. 346-348, Ex. 11-18 page 348.

Objective 11

Nichols, read page 413, Ex. 1 a-f, 2 a,d,g, 3a, [illegible], 4 pp. 413-415.

Payne, read pp. 349-353, Ex. 1-3, 7, 8, 11, 21 pages 353-354.

Wooton, read pages 394-399, Ex. 1-12 page 399.

OBJ.
7 I. For each of the following functions, find the value indicated.

_____ 1. Find f(2) for f(x) = 6x + 1

_____ 2. Find f(-3) for f(x) = 2x - 1

_____ 3. Find f(0) for $f(x) = \frac{x + 1}{6x}$

_____ 4. Find f(30) for $f(x) = x^2$

_____ 5. Find f(-10) for $f(x) = \frac{8}{4} - \frac{2x}{}$

8 II. Determine if each of the following equations is linear or quadratic.

_____ 1. $y = x$

_____ 2. $y = 2x + 1$

_____ 3. $y^3 = x^4 + 2 + 4x^2$

_____ 4. $y = x^2 + 2$

_____ 5. $3x + 2y = 6$

9 III. Graph the following linear functions. Use the graph paper that follows.

1. $y = 2x$
2. $y - 2 = \frac{1}{3}x + 3$
3. $y = -3$
4. $2x + 4y = 8$
5. $3y = -2x + 6$
6. $x = 5$

10 IV. Give the slope of each of the following linear functions.

_____ 1. $2x + 3y = 4$

_____ 2. $y = 6x - 1$

_____ 3. $y - 2 = \frac{1}{3}x + 3$

_____ 4. $y = 6$

_____ 5. $x = -2$

11

V. Graph the following quadratic functions. Use the graph paper that follows.

1. $f(x) = x^2 + 2x + 1$

2. $f(x) = x^2 - 2$

3. $f(x) = x^2 + x - 6$

If you have satisfactorily completed your work, take the LAP TEST. Consult your teacher first.

ADVANCED STUDY

1. Payne, Ex. 39, 40 page 184.
2. Payne, read pp. 193-195, Ex. 1-20 even page 195.
3. Dolciani, read pp. 346-348, Ex. 1-10 page 348.
4. Pearson, read pp. 559-560, Ex. 1-5 page 560.

I. Write the domain and range of the following.

DOMAIN RANGE

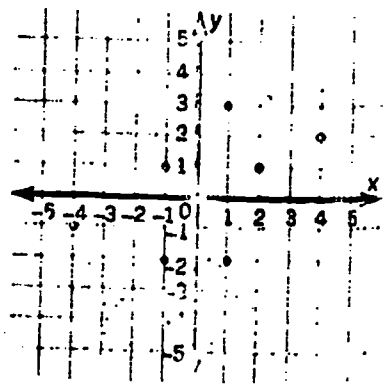
1. (8,1)(7,2)(-3,1)(7,-6)

2. $y = x^2$

3.

x	y
-8	5
0	2
-6	7
2	5

4. y is equal to twice x

5. 

II. Determine if the following is a function. Write F if it is a function. If it is not a function, write R for relation only.

____ 1. (3,6)(2,4)(-4,2)(-6,4)

____ 2.

x	-6	-2	-4	6
y	8	1	3	7

____ 3. y = x + 1

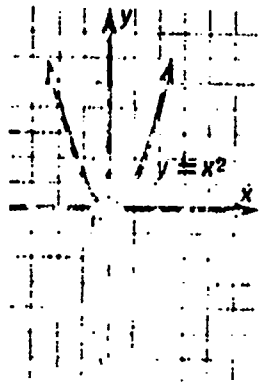
____ 4. (3,-3)(4,-4)(5,-6)(7,-8)(3,-9)

____ 5.

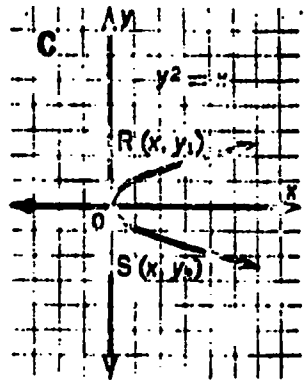
x	y
0	7
7	0
3	5
9	4
5	3

____ 6. 

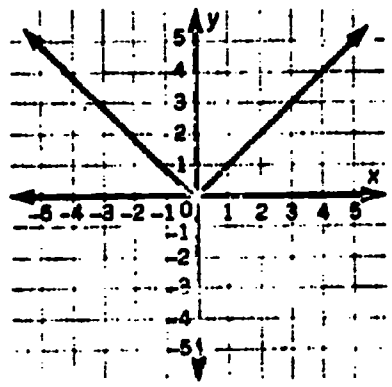
____ 7.



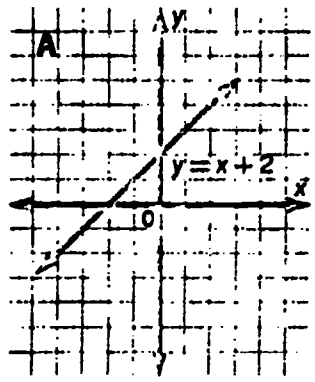
____ 8.



____ 9.



____ 10.



APPENDIX II

1. Define linear function.

2. Define quadratic function.

3. Determine if the following are linear or quadratic.

_____ a. $y = x$

_____ b. $y^2 = 4x + 2$

_____ c. $y = 3x^2 + 2x + 1$

_____ d. $y = 3x + 2$

_____ e. $x^2 + y^2 = 25$

_____ f. $2x + 3y = 7$

APPENDIX III

For each of the following linear functions, rewrite each in slope-intercept form, state the slope, and y-intercept, and graph each. (USE THE GRAPH PAPER THAT FOLLOWS.)

(1) $2x + 3y = -6$ _____ Slope = _____ y-int. _____

(2) $2y = -4x + 8$ _____ Slope = _____ y-int. _____

(3) $y = -x$ _____ Slope = _____ y-int. _____

(4) $-18x - 6y = 18$ _____ Slope = _____ y-int. _____

(5) $3x = 6y - 12$ _____ Slope = _____ y-int. _____

REFERENCES

Vanatta (abbreviation)

Vanatta, Glen D., Algebra One: A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965.

Nichols (abbreviation)

Nichols, Eugene D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenbach, Edwi F., Sharron, Sidney, Modern School Mathematics, Algebra 1, Houghton Mifflin Co., 1967.

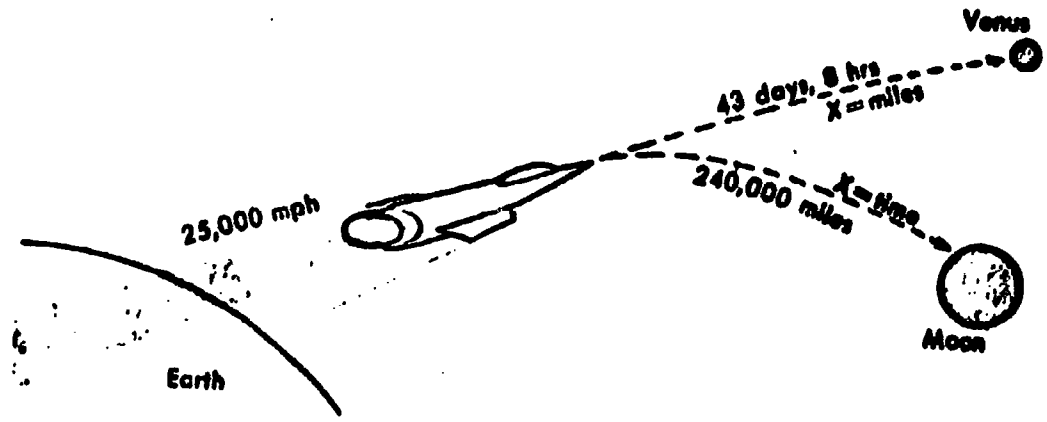
Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis G., Algebra One, Harcourt, Brace and World, 1969.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, 1964.

LEARNING ACTIVITY PACKAGE



Equations And Their Applications

Ninety Six High School · Ninety Six, S.C.

ALGEBRA 93 - 94

REVIEWED BY _____

LAP NUMBER 22

WRITTEN BY Mrs. Diane Evans

42673

Suppose you and your family are driving to another city to visit some friends. At what time will you arrive? The answer to this question depends on a number of things: distance to be traveled, average speed, and hour of departure. Moreover, the problem may be complicated by the possibility that you may encounter unforseeable delays caused by road construction or unusually heavy traffic.

In a situation like the one described, the exact time of arrival is probably not critical. But, there are many situations that do require a very careful consideration of time and other factors. Think of the precise calculations that are involved in launching and controlling a rocket so that a satellite will be put into orbit around the moon or land on the moon in a specific spot. A business man often must make computations that will be the basis for very important decisions.

These are only a couple of the everyday situations to which mathematics must be applied. Somtimes the mathematics required is only simple arithmetic, but as civilization grows more complex we find an increasing need for algebra and higher mathematics. In this LAP, you will learn how to use algebra to solve ordinary problems that arise often.

Section 1 Problems With Number Relations

BEHAVIORAL OBJECTIVES:

1. Given any word problem involving number relations, you will be able to write and/or identify an equation and/or determine its solution.

2. Given any word problem involving consecutive integers, you will be able to write and/or identify an equation for it and/or determine its solution.

RESOURCES

Objective 1 (work at least three problems from each group.)

Dolciani, Book 1, read pages 166-167; Ex. 1,2,4,10,19 page 168.

Nichols, Book 1, read pages 214-216; Ex. 6-10 page 217.

Vanatta, Book 1, read pages 154-156; Ex. 1,2,8,10,14 page 156.

Objective 2 (work at least 9 problems)

Dolciani, Book 1, read pages 170-171; Ex. 1,4,7,10,13 bottom page 171.

Nichols, Book 1, read pages 225-227; Ex. 1,2,4 page 227.

Vanatta, Book 1, read pages 157-158; Ex. 1-6, 8,9 page 159.

Obj. 1

I. Work the following.

_____ 1. [illegible] buttons. If Mary has 15 more than twice as many [illegible] and together they have 975, how many does Jane have?

_____ 2. In Sue's bank she has some nickels and some dimes. She has two more dimes than she has nickels. In all she has $1.10. How many nickels and how many dimes does she have?

_____ 3. The sum of 12 and 1 times a number is 6. Find the number.

_____ 4. Seven more than five times a number is 27. Find the number.

_____ 5. The sum of twice a number and 6 equals the sum of three times the number and 9. Find the number.

_____ 6. One number is 5 less than two times another. If their sum is 1, find the numbers.

_____ 7. Taking one-half of a number gives the same result as subtracting three from the number. Find the number.

_____ 8. Multiplying a number by 3 and adding 5 to the product gives the same result as multiplying the number by -2. What is the number?

II. Work the following. Show your work.

_____ 1. The sum of two consecutive integers is -11. Find the two integers.

_____ 2. Find three consecutive integers if the sum of the first and the third is 73.

_____ 3. The larger of [illegible] consecutive even integers is 3 less than one-half [illegible] the smaller. Find the numbers.

_____ 4. Find three consecutive odd integers whose sum is 33. Find the numbers.

_____ 5. The sum of four consecutive integers is 102. Find the integers.

_____ 6. There are two [illegible]tive integers such that two times the first plus the second, equals 58. Find the numbers.

_____ 7. Find two even consecutive integers that three times the smaller equals six more than the larger.

_____ 8. Find five consecutive integers such that the largest is twice the smallest.

_____ 9. The product of two consecutive integers is 6 more than the square of the smaller. Find the integers.

If you have satisfactorily completed your work, take the Progress Test. Consult your teacher first.

Section 2 Motion Problems

BEHAVIORAL OBJECTIVES:

3. Given any simple motion problem using the distance formula, you will be able to write and/or identify its equation and/or determine its solution.

4. Given any of the following three types of motion problems: 1. meeting, 2. overtaking, and 3. round-trips, you will be able to write and/or identify an equation for it and/or determine its solution.

RESOURCES

Objective 3

Vanatta, Book 1, read pages 160-161; Ex. 1,2,3,6,7,8 pages 161-162

Objective 4 (work all problems)

1. Meeting Problems: Dolciani, Book 1, read pages 178-180; Ex. 1, 2,5 pages 180-181.

   Vanatta, Book 1, read pages 164-166, Ex. 1,2 pages 166-167.

2. Overtake Problems: Dolciani, Book 1, Ex. 5,6,7,8 pages 180-181
   Nichols, Book 1, read page 226; Ex. 12 page 227.
   Vanatta, Book 1, read pages 164-166; Ex. 3, 4 page 167

3. Round-trip problems: Dolciani, Book 1, Ex. 9-12 pages 180-181
   Nichols, Book 1, Ex. 10 page 227.

EVALUATION

OBJ.

3 I. Work the following. Show your work. Put your work on the back of this sheet or on an extra sheet.

_____ 1. A train can travel 500 miles in 6 hours. Find its average speed.

_____ 2. A family wishes to travel about 8 hours per day on a vacation. If they drive at an average rate 50 miles per hour, what distance will they cover in 4 days?

_____ 3. A tourist travels at a rate of 45 miles per hour. How long will it take him to travel 135 miles?

_____ 4. A train averages 50 miles per hour for 100 miles in open country. In populated areas it averages 40 miles per hour for the next 100 miles. What is his average speed?

4 II. Work the following. Show your work.

_____ 1. Two men start out from the same city. One travels due north at an average rate of 35 m.p.h. and the other travels directly south at a rate of 40 m.p.h. In how many hours will they be 250 miles apart?

_____ 2. One train starts from Charleston going to Atlanta at a rate of 50 miles per hour. At the same time a train starts at Atlanta going to Charleston at a rate of 45 miles per hour. If Charleston is 285 miles from Atlanta, how long will it be before the trains pass?

_____ 3. Mr. Jones left home at 7 a.m. and drove at an average rate of 40 m.p.h. At what time did he overtake his father?

_____ 4. A new car leaves Detroit traveling at an average speed of 45 m.p.h. One and one-half hours later another car leaves Detroit on the same route. If the second car catches up with the first car in 4½ hours, find the speed of the second car.

_____ 5. Having 6 hours at his disposal, a man decided to ride into the country with a friend and walk back. If the friend drove at an average speed of 33m.p.h., and the man could walk 3 m.p.h., how many miles could he ride and have time to walk back?

_____ 6. Jack walked to Jim's house at 3 m.p.h. He borrowed Jim's bicycle and rode home at 15 m.p.h. If the entire trip took 6 hours, how far is it to Jim's house?

If you have satisfactorily completed your work take the Progress Test, consult your teacher first.

Section 3: Mixture Problems

BEHAVIORAL OBJECTIVES:

5. Given any mixture problem, you will be able to write and/or identify an equation and/or determine its solution.

6. Given any geometry word problem involving 1. perimeter, 2. area, 3. supplementary angles and 4. complementary angles, you will be able to write and/or identify an equation for it and/or write its solution.

RESOURCES

Objective 5 (work all problems)

1. Simple mixture problems:

   Dolciani, Book 1, Ex. 2,3,4,5,8,9,12, pages 183-184
   Nichols, Book 1, Ex. 1,3,4, page 216
   Vanatta, Book 1, Ex. 17, page 177

2. Solution problems:

   Dolciani, Ex. 1,3,4,5,13 page 31:
   Nichols, Ex. 10,11,12 page 238
   Vanatta, page 175 nos. 11 and page 178 no. 6

Objective 6 (work all problems)

1. Perimeter problems: Vanatta, Ex. 1,2,10 page 170
   Dolciani, Ex. 6,7,16 page 168

2. Area problems: Dolciani, Ex. 12, 13 pages 168-169
   Nichols, Ex. 1 page 232

3. Supp. angles: Vanatta, Ex. 5,8 page 170
   Dolciani, Ex. 5,6 page 177

4. Comp. angles: Vanatta, Ex. 4,8 page 170
   Dolciani, Ex. 1,4 page 177

Also for Supp. and Comp. angles work Nichols page 234 no. 28.

OBJ.
5 I ork the following. Show your work.

_____ 1. Tea worth 95¢ a pound is to be mixed with tea worth $1.25 a pound to make 20 pounds of a mixture worth $1.15 a pound. How many pounds of each would you use?

_____ 2. A candy store has on hand 25 pounds of candy worth $1.90 a pound. How many pounds of $1.50 candy shall they mix with it so that the mixture can sell at $1.75 a pound?

_____ 3. Ticket sales to a French Club play brought in a total of $77 for 136 tickets sold. If this included student tickets at 50¢ each and adult tickets at 75¢ each, how many of each kind were sold?

_____ 4. Fred paid $2.45 for 50 stamps. He bought air mail stamps at 8 cents each, as well as some 5-cent stamps and some 4-cent stamps. If he bought 5 times as many 5-cent stamps as air mail stamps, how many of each kind did he buy?

_____ 5. How much water must be added to 16 pounds of a 25% salt solution to reduce it to a 15% solution?

_____ 6. A druggest has 40 ounces of a 10% acid solution. How much pure acid must be added to make it a 40% solution?

_____ 7. How much water must be evaporated from 60 pounds of a 5% starch solution to make a 20% solution?

6 II. ork the following. Show your work.

_____ 1. The length of a rectangle exceeds twice its width by 7 feet. The perimeter of the rectangle is 74 feet. Find its dimensions.

_____ 2. The area of a 20-foot square equals the area of a rectangle 25 feet long. How wide is the rectangle?

_____ 3. Find two supplementary angles if one is four times the other.

_____ 4. Find two complementary angles if one is 30^{0} more than the other.

_____ 5. How large is an angle if the measure of its supplement is 15^{0} less than four times the measure of its complement?

If you have done your work satisfactorily, you may either go into advanced study or to progress test. There is a progress test on this section and then the LAP Test.

ADVANCED STUDY

Section 1:

Work any 5 of the following problems.

Dolciani page 172 nos. 14, 15, 16, 17, 18

Nichols page 227 nos. 7, 8, 9

Section 2:

Work any 5 of the following problems.

Dolciani page 182 nos. 13, 14, 16; page 191 nos. 56, 58

Nichols page 176 nos. 14; page 177 no. 4

Section 3:

Work any 10 from group 1 or any 10 from group 2 or any 10 from groups 1 and 2.

1. Dolciani page 184 nos. 13, 14; page 311 nos. 9,10; page 318 nos. 1, 2, 3, 4, 5

   Vanatta page 176 nos. 9, 20; page 178 no. 7

2. Vanatta page 179 no. 27; page 171 no. 6

   Dolciani page 212 nos. 1,2,3,5,6,7,11; page 177 nos. 9,11,12

   Nichols page 234 nos. 21,22,23,24