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ABSTRACT

The following elementary mathematics curriculum development projects are described in this report: Madison Project, Arithmetic Project for Teachers, Nuffield Project, Individually Prescribed Instruction - Mathematics (IPI), Individualized Mathematics System (IMS), Patterns in Arithmetic (PIA), Minnesota Mathematics and Science Teaching Project (Minnemast), Unified Science and Mathematics for Elementary Schools (USMES), and Developing Mathematical Processes (DMP). A set format is followed for each report to make for easier comparisons of programs. A brief one-page summary of basic information is given at the start of each project report. It includes such items as developer names, publishers, dates, format, content, uniqueness, uses, length, target audience, and aids for teachers. The first major section, Goals and Rationale, spotlights theoretical considerations. The next section discusses content, while the third part is devoted to classroom action, meaning the type of interaction between teacher and students and among students. The fourth section, Implementation, gives details about what a school system need provide in order to use the program and includes costs. The Program Development and Evaluation section includes comments from independent observers as well as from the developers. (LS)

CURRICULUM DEVELOPMENT IN ELEMENTARY MATHEMATICS:

9 PROGRAMS

FAR WEST LABORATORY FOR EDUCATIONAL RESEARCH AND DEVELOPMENT

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INTRODUCTION

The curriculum development projects in elementary mathematics that are described in this book are part of the government-financed educational reform movement which began in the United States in the 1950's and continues today. The reform began with attempts to modernize curriculum (particularly to make early schooling a better preparation for later academic studies): to inject new knowledge into timeworn texts and dated pedagogies. Later the reform movement assumed an overriding concern for the problems of teaching children of wide-ranging achievement levels in desegregated classrooms.

Eight of the reports in this book analyze American curricula developed under funding from the Office of Education, the National Institute of Education, or the National Science Foundation. The ninth report describes the concurrent reform of the mathematics curriculum in the United Kingdom, undertaken by the Nuffield Foundation and the National Schools Council.

All of the projects described here were influenced by the experience of the early curriculum reform known as "the new math." That reform was an attempt to broaden and deepen children's understanding of mathematics beyond rote arithmetic. Curriculum developers believed that if children were having trouble with arithmetic it was because they didn't understand what they were doing when they performed algorithms for addition, subtraction, multiplication, and division. The new math was intended to give children the concepts which mathematicians identified as underlying arithmetic operations. The mathematicians developers also intended that even grade-school children should master and appreciate mathematics as a mode of thought and as a foundation for later intellectual endeavor rather than just as a set of tools for workaday computation.

The first injection of the new math into elementary classrooms revealed that its designers generally overestimated both children's and teachers' capacity to understand abstract math concepts. The programs that followed the first math reforms--among them some of the programs reported here--thus attempted to correct the apparent failings of the new math: by returning to more conventional arithmetic, by rethinking the manner and sequence in which the newer and more powerful concepts were presented to children, or by couching arithmetic learning in terms that would seem more natural and relevant to children.

The Prerequisite Problem: Preparing Teachers

However, the most immediate drawback of the new math was its difficulty for teachers themselves. If teachers couldn't grasp it and didn't accept its benefits, how could they teach it to kids? Thus the issue of teacher preparation for teaching new curricula was the single most critical problem addressed by the nine curriculum projects reported here. The projects attempted solutions that range from updating and deepening the mathematical education of teachers themselves by means of inservice instruction, to reducing the influence of the teacher in the instructional process and placing main reliance on the curriculum materials--making them "self-instructional" for pupils.

These same two solutions--improving the teacher or bypassing the teacher--are the major alternatives that have been tried as answers to the problem of individualizing instruction within racially and culturally mixed classrooms. Thus the strategy of individualizing by means of a prepackaged, diagnostic-prescriptive system is a hallmark of some of the curricula described here; others seek to individualize by giving the teacher the responsibility to assess individual learning levels as well as the flexibility to provide more diverse learning materials and activities in the classroom.

Implementation and Results

As all of these programs were part of the research and development movement in public education, their developers shared a confidence in the importance of applying research findings and new knowledge to the practical problems of classroom instruction, and of testing new curriculum products scientifically and revising them according to field-test findings before release to the public. As it has gained experience, the R&D movement has learned that its instruments for evaluating--that is, certifying--its products have not been as scientific as had been believed. Developers' evaluation results are often confusing, or ambiguous, or based on such small or select samples that they are not convincing. Furthermore, the process of introducing a new curriculum to teachers and children is a complex, unpredictable enterprise that affects evaluation; in public school rooms it simply is not possible to control all of the many variables that powerfully affect new curriculum experiments. In broadest terms, this means that developers, even during field testing, have not been able to assure that their programs are implemented in the manner intended, and they certainly cannot control implementation after the field-test stage.

A problem of implementation for all nine of these programs (especially for those projecting a more decisive and improvisatory role for

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the teacher) has been the provision of adequate inservice preparation. By and large, school districts adopting any of these new programs have not provided the length or depth of teacher preparation and ongoing assistance developers had planned. In most cases, more teacher preparation has been needed, not less, if an ordinary classroom teacher is to 1) approach the degree of competence in math that was possessed by the developers and by teachers who trial-tested the new lessons, or 2) manage the diverse array of materials introduced by the new programs.

A related but subtler aspect of the implementation evaluation problem derives from the emphasis program developers have placed on teaching children to *think*. Defining "thinking" as a whole series or clusters of interrelated intellectual processes, or as problem solving, developers of almost all the projects described in this book have tried to design lessons and to sequence them so that deeper thinking processes--not just computation operations--would be stimulated in children. A significant aspect of most of the programs described in this book is the *manner* in which the curriculum content and teaching strategy stimulate cognitive development or provide experiences in problem solving. There is wide variation in the thinking styles of project developers, so that, though they all may agree on the common goal of teaching children to think, they go about it in very different ways. Some programs call for preconceived, ordered, spelled-out lessons, strongly directed by the teacher. Others believe students should be led to make choices, explore, improvise, make mistakes, and learn from them. A teacher's understanding of and agreement with the developer's point of view about children's thinking probably strongly influences her implementation of the program, but this is not a factor that is taken into account in most program evaluations.

These and many other problems of implementation mean that program evaluators rarely are able to study truly comparable groups of children, taught by truly comparable teachers, in truly comparable settings. The result is that there is as yet little convincing evidence about whether children studying a new curriculum learn math better or faster than similar children studying other programs.

Uses For These Reports

This is not the book, then, in which the school superintendent, the mathematics curriculum specialist, or the classroom teacher can find a government-certified, guaranteed, foolproof elementary mathematics program. Schoolpeople still must base program selections upon their own educational goals for children, their own assessment of their children's needs in math, their district's resources, and their beliefs about how children and *teachers* learn.

The writers hope that these reports--sampled from time to time, not read all in a lump--will sharpen such judgments. Teachers, curriculum coordinators, principals--their heads full of kids and classrooms and lessons that do and don't work--should critique and adapt the work of university scholars and educational technologists. One way to begin is to bore in on existing programs' goals, rationale, knowledge content, and teaching strategy, searching for the heart of a curriculum underneath the words in promotional brochures and the appealing design and packaging of texts and apparatus. *Is the program's teaching strategy consistent with its goals? Are its goals our goals? Can our teachers do this program? Will they? What help will they need? Can we afford it? Who else has tried it? What evidence is there that it works?* The section headings of these reports, generating such questions, might serve as agenda items in framing one's own local program, tailored to one's own goals and resources. If classroom teachers play an influential role in it, such a process may well result in adoption and adaptation of several curriculum programs within a school or a district: acknowledgment of many teachers' experienced-based belief that no one program works with all kids.

The format of these reports was designed to make comparisons among programs easy for the reader. Thus the first section of each report, *Goals and Rationale*, is intended to spotlight theoretical similarities and differences among programs. In these sections one observes two major, apparently contradictory curriculum theories: that of the cognitive-developmental learning theorists influenced by Dewey and Piaget and that of the behaviorists influenced by Skinner. From the second section, *Content*, the reader can discern which of the programs have a priority for introducing new elementary mathematics *topics*, and which are committed to the basic arithmetic curriculum with some updating. One can also select out programs that have similar content (like the Madison Project and the Arithmetic Project or IPI and IMS), and then compare their other aspects.

The third section, entitled *Classroom Action*, highlights the style of interactions between teacher and students and among students. *Implementation*, the fourth section in all the reports, provides detail about what a school district needs to provide in order to use the program. Here one can compare classroom organization requirements, various forms of inservice (Do you teach teachers more mathematics and learning theory or do you teach them how to apply a prepackaged program?), and costs. In the fifth section, *Program Development and Evaluation*, are short sketches of the history of development and the manner and results of the developer's evaluation, as well as comments from independent observers. *Summary* sections, at the beginning of each report provide thumbnail descriptions of the programs and basic information about content, style, and availability.

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Curriculum Projects and Related Trends

Aside from their use in the process of analysis and decision making, the reports can be read also as case studies of the process of curriculum development, an educational art *own* science still in its infancy. As a group of case studies, the book is a representative rather than a comprehensive collection of recent elementary math curriculum developments. Several other developments are of equal interest. CSMP (Comprehensive School Mathematics Program) being developed at CEMREL, St. Louis, pursues the problem of designing a complete and completely new K-6 curriculum, including student materials and teacher preparation. Project One at the Education Development Center, Newton, Massachusetts, is seeking to combine television and naturalistic math experiences in urban classrooms. Project PLATO at the University of Illinois Curriculum Laboratory is designing topics which can induce children to use computers to learn problem-solving skills. A Canadian development, Project Mathematics, influenced by Nuffield, has been translated into a text series available in the U.S. from Winston Press. IPI math is being revised, to incorporate manipulative materials, at the University of Pittsburgh's Learning Research and Development Center.

As these more recent R&D projects have learned from the earlier generation of projects reported in this book, so have the commercial textbook publishers. Careful analysis of publishers' new series shows the influence of Nuffield and Madison, of behavioral objectives, and of the R&D programs such as Minnemast that combine science and math.

Several of the projects described in this book respond to demands of stronger guarantees about the performance of the teacher: developers have structured the lessons around behavioral objectives that clearly state what will constitute a child's and a teacher's competency at every level of instruction. These schedules of competency statements are compatible with the influential trend, in colleges and universities, toward competency-based teacher education programs. In these programs, a list of behaviorally-stated competencies in classroom management and in teaching in several curriculum areas rather than a series of courses constitutes criteria for graduation. Some states are beginning to require periodic re-certification of practicing teachers on the basis of similar competency lists. The reports of IPI, IMS, DMP, and PIA included here presage and complement the competency-based approach to teacher education.

Those projects among these nine that have placed priority on re-educating teachers in mathematics by means of their own workshop-style learning, have also had an impact on preservice and inservice education. Math lab, active-learning inservice programs have evolved into a new institution for the continuing education of teachers--the teachers' center. In these informal learning centers stressing voluntary attendance, teachers are encouraged to participate as actively and concretely as their students, to self-assess and self-prescribe their needs for continuing education, and to request, design, and even teach the courses presented in the center. In well-established centers some teachers have gained enough expertise and confidence to design their own lessons and materials. As this kind of work on the part of teachers takes root in widely distant and differing settings, and teachers' home-made curriculum products are exchanged and adapted, the idea of the teacher as curriculum developer attains practicality and promise.

THE MADISON PROJECT

INTRODUCTION

> *I began the year with a large table at the rear of the room set up as an exploratory area in mathematics. . . . Weighing and balancing was an instant hit, greatly in demand. I noticed that even as children went by the table, they would quickly balance a pencil against a handful of beans, a container of milk against the metal washers. . . . After several months of this as free play (with occasional questions from me when I came by) I added task cards which asked, "How many beans balance one button?" and so forth. . . .*

This is the start of a report by Patricia Post, a third-grade teacher at P.S. 29, New York City, on her first four months of teaching Madison math. Looking back in January on what had happened since the *Madison Project* inservice workshop she had taken during two full weeks of August, Pat assessed what she had learned about Madison activities, herself, and her 29 students.

> *The greatest shapes have been made on five geoboards! After six weeks of their availability for free play, I produced dot paper. Instant success: hundreds of shapes recorded. . . . I have been working in groups of 11 to 15, following the task cards accompanying the geoboards. . . . Some of the boys in my class who most resist reading, who contribute little conversationally, have the most acute comments about shapes on the geoboard. It seems they are quite willing to discuss (this) because the material is so interesting to them. . . .*

This class had scored between the fourth and eleventh percentile in arithmetic and reading in New York City achievement tests at the start of the school year. All of these third graders came from homes where Spanish was spoken. They had not spoken English until they started school.

> *I began using boxes of very small split peas and dried baby limas, assigning fairly large two-place numbers to be added and subtracted. As I hoped, the children wearied of the tiny materials and wanted to do something to make their counting easier and faster. Various ideas were forthcoming: paste some together to make bunches, wrap them in paper and put rubber bands around them, and so*

forth. Since the custodian was gazing at me frostily, I produced a large box of tongue depressors and suggested pasting 10 kidney beans on each depressor. . . . A large pretzel box became the Bank and small peas were traded in for tens. Our adding and subtracting moved more rapidly, and I saw understanding of place value sooner than in "better" classes of other years.

Pat had also taken a Madison Project course given as inservice the previous school year. Teachers had learned and practiced lessons which combined arithmetic with basic ideas of algebra and geometry.

I have tried to follow the (abstract) lessons as learned in the inservice course. Began with Pebbles in the Bag, discovered linguistic confusion about "more" and "less," have continued with this game intermittently. . . . Doing number line work, leading to discovery of negative numbers, they first called them "behind zero ones." One of my ordinarily uninterested boys began calling them "negative one," "negative two," and so forth, after my use of this term for Pebbles in the Bag answers. . . .

This is the beginning of my fourth year of teaching. Using the math workshop approach and Madison Project ideas I am having so much fun this year that I can't believe it. My class is third or fourth (in supposed ability) out of five third grades (in the school). But they and I don't know it!

Patricia Post illustrates the Madison Project's aim of improving mathematics instruction by educating teachers rather than by producing new texts or work materials for students. The project director, Robert B. Davis, says they do this because they believe students don't learn textbooks; rather they learn the things that are valued in the culture they're in. He believes that students will learn mathematics if they can be in a classroom culture where mathematical things and thinking are part of the action.

BASIC INFORMATION

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Program name: The Madison Project

Format: Collections of lessons on modern math topics and active-learning or math lab approach, disseminated by inservice courses.

Uniqueness: Topics integrating algebra and analytic geometry with arithmetic. Dissemination by inservice training rather than texts. Teaching style stressing "discovery" experiences and concrete materials.

Content: Number and algebra (signed numbers, Cartesian coordinates, variables, bases other than 10, functions), measurement, geometry, logic.

Suggested use: Enrichment to the basic arithmetic curriculum.

Target audience: Students of all abilities, primary through junior high.

Length of use: Teacher's discretion.

Aids for teachers: Films of fully worked-out lessons being taught in classrooms. Student's kit and teacher's startup kit of manipulative materials. Inservice training courses, "live" or packaged.

Date of publication: 1964 through 1972.

Director/Developer: Robert B. Davis directed The Madison Project when he was a Syracuse University professor. He is now director of the University of Illinois Curriculum Laboratory, 1210 W. Springfield, Urbana, Ill. 61801.

Distributor: The film series for teachers, *A Concrete Approach to Introductory Ideas in Mathematics*, is from Houghton Mifflin. The teacher's startup kit of math workshop materials (*Maths Minilab*) comes from Selective Educational Equipment, Inc., 3 Bridge St., Newton, Mass. 02195; student's *Shoebox Kits* from H&M Associates, Box 1107, Danbury, Conn. 06810; Davis' teacher texts *Discovery in Mathematics* and *Explorations in Mathematics* are from Addison-Wesley. Madison Project teacher's manuals, films, and developer-conducted workshops are available from the Madison Project, Curriculum Laboratory, University of Illinois, Urbana, Ill.

1. GOALS AND RATIONALE

1.1 Goals

The seed of the Madison Project was Syracuse University professors' experience teaching in public school classrooms in order to improve their math courses for prospective teachers. As the project grew into the development of a series of teaching episodes in modern math topics, its bent was always to improve teachers rather than to design products. The teacher was seen as the most powerful influence for introducing new mathematics topics and for changing the setting and style in which children learn.

The project conceived of mathematics as a *process* for understanding and organizing experience rather than as a *collection* of knowledge and procedures. Thus it called for a classroom in which the student can have real-life rather than textbook experiences and can use his insights about these experiences, with the teacher's leadership, to formulate and organize mathematical ideas. Since so few American schools provide such settings for students, and so few teachers have mastered a style which fosters the student's independence but does not leave him entirely on his own, an essential goal of the Madison Project was to give school people examples of new ways and to help them change if they wished.

The project also intended to contribute to a general broadening of the traditional school curriculum, which has offered only arbitrary, "narrow slices" of knowledge, in the view of Madison Project director Robert B. Davis. Mathematics education ought to teach children to "learn how to learn," not just how to compute. Davis predicted the automation of arithmetic by low-cost calculating machines. He maintained that today so much knowledge is produced so fast that the average mind can't take it all in, so remembering will be a less important skill for people than investigative methods of thought.

At the start the project shared with "new" physics, "new" grammar, "new" social studies, *et al*, the goal of revealing the structure of a discipline and methods for obtaining "interpretive knowledge" rather than facts, so that eventually ordinary citizens would be able to understand better and share in decisions of our technological society.

The Madison Project also wanted to prepare all kinds of students, not just the college-prone, for all kinds of mathematical tasks in later schooling.

> There are three kinds of mathematics: *concrete*, *computational*, and *generalization*. All three must be taught at all levels. . . . In this way, hopefully, all learners can learn not only how to solve it but

> *why* it works. . . . Those who are mathematical have always been and will continue to be able to build their own bridges of understanding. . . . Methods for teaching the others--the average or the slow--require the use of manipulative materials. The children need to see for themselves and do it over and over before they can internalize any working concept.[1]

In his 1967 teacher text, *Explorations in Mathematics*, Davis offered this list of specific *cognitive* objectives for mathematical growth in an individual student over the years:

1. Develop ability to discover patterns in abstract situations.

2. Develop independent exploratory behavior that goes beyond anything the teacher suggested . . . and sees open-ended possibilities where others would see only the assigned task.

3. Master important techniques.

4. Know basic mathematical facts; for example, the fact that $^{-}1 \times {}^{-}1 = {}^{+}1$.

5. Acquire a set of mental symbols which he can manipulate in order to try out mathematical ideas; for instance, the above notation for negative and positive numbers.

6. See math in daily life and in a natural relation to other school subjects.

7. Learn the really fundamental mathematical ideas; such as variable, function, graph, matrix, isomorphism, and so on, early enough in life so that they can serve as the foundation for subsequent learning. This includes using some of these ideas as "systematic apparatus" for doing arithmetic computation.

Davis also spelled out *attitudes* which the project aimed to impart to teachers and students:

1. Belief that mathematics is discoverable, not given; and *discoverable by me*.

2. Ability to assess his own ability honestly.

3. Valuing of abstract rational analysis.

4. Valuing of the shrewd guess.

5. Feeling that math is fun, challenging, and rewarding.

1.2 Rationale

Mathematics is not the destination. It's the trip. The single proposition on which the Madison Project rests is that math is *not* "a collection of facts, definitions, algorithms, or explicit procedures which mathematicians in the past have already arrived at." All of these will be used in doing math, Davis wrote, but the doing itself is the important thing, not its "result," or "the answer."

> The theory is sometimes explicitly stated either that students lack the ability to get beyond "facts" or else that, whatever your goals, "you must begin with facts." It is the Madison Project's contention that neither of these statements is true. Quite the contrary: students *can* move beyond "facts" and deal with "processes," and many students perform better (and enjoy school more) if the school program focuses on "reasonable tasks"--that is, on processes--and deals with facts incidentally as they relate to these processes.[2]

The project-designed lessons and activities stress stories, games, and toy-like learning materials because of developers' belief that children should learn new ideas in a context in which they use the ideas as tools to do mental work that is intrinsically interesting to them. Such an approach is in conformity with developmental learning theory influenced by Piaget. Providing concrete, play-like, "developmental" activities preceding abstraction and practice of skills, is thought not only to improve children's attitude to learning but to increase their mastery of skills.

> At least four studies have found that classes spending about 75 percent of the mathematics time on "developmental activities" score higher on achievement tests related to problem solving, computation, and concepts than those spending 75 percent of their time on drill-related activities. . . . The children who spent most of their time working on developmental activities were better in computation than those spending most of their time practicing computation. What are the developmental activities? . . . discussions of the whys and hows of the topic of study; pupil reports and explanations of the approaches that have been developed; pupil and teacher demonstrations of significant ideas being studied; small-group and individual handling, inspecting, analyzing and arranging visual and manipulative materials; individual or small group exploration to find alternative means of finding solutions to mathematics exercises; solving and inventing puzzles and games related to the topic of study; and engaging in laboratory activities related to the topic.[3]

The Madison Project rejected the curriculum development approach that sought to construct "hierarchies" or "continuums" of "skills." In a conversation with mathematics professors Marilyn Suydam and C. Alan Riedesel, Davis explained his reasons for doubting that there is one essential body of knowledge all students should learn, or one optimum order to learn the parts in:

> (I have experience) that people can do what they have to do when they really want to. Not all university professors would agree to my view--some would say, "Well, now, we can't teach anything to a kid who doesn't know thus and such." Personally, I have yet to find that essential "thus and such," the real *sine qua non* . . . except that you do have to be able to count to get very far in arithmetic. . . . Zacharias once said, if you'd allow infinite lists of behavioral objectives, maybe you could do it . . . (but) very often the most important ones can't be stated behaviorally. . . .
>
> A lot of math lab objectives are experience objectives. The (class) should have this many experiences with the geoboard; now, one student learns area, somebody else learns the triangular shape, and somebody else just played with it, but maybe the next time he'll get some insight. Now that begins to make some honest sense.[4]

Thus the Madison Project placed little emphasis on sequencing lessons or testing for mastery.

2. CONTENT AND MATERIALS

2.1 Content Focus

Robert B. Davis thinks of Madison math topics as "yeasty" additives to the arithmetic curriculum, claiming their small injection of newness can produce big changes in the traditional program. At the start of the project the "yeast" was compounded of modern math subject matter--topics combining *arithmetic, axiomatic algebra,* and *coordinate geometry*--plus the discovery method--the teacher leading the whole class to develop mathematical ideas from stories, games, and board work. Later the project added an active-learning or math lab approach, in which children get their first introduction to mathematical concepts by experimenting with concrete objects and apparatus. Algebra and geometry are meant to give the student more various and powerful mental tools with which to do arithmetic.

> Some who have heard that we must make arithmetic *meaningful* cannot understand why children exposed to the *abstract* mathematics of the Madison Project take to it with such evident enthusiasm. The answer may be quite simple. The children do think more creatively when the ideas are *meaningful*, but the meanings do not have to be concrete! If we are careful the child can enjoy it every bit as much as if it were more concrete. . . . A derivation is much more fun to fifth graders than a problem on percent markup in retailing. . . . It is easy and natural for children to handle abstractions--it is we adults who worry about taxes and double-entry bookkeeping.[5]

Davis maintains that Madison subject matter is essentially conservative because it includes accepted basics such as *measurement* and *geometry* (emphasizing work with coordinates and shapes rather than with theorems and proofs), or treats newly important areas of mathematics such as *logic*, *probability*, and *statistics*.

The Project emphasized ties between math and science; for instance, it recommended many of the teaching units and apparatus developed by the Elementary Science Study (published by McGraw-Hill), as well as combinations of nature study and math.

2.2 Content and Organization of the Subdivisions

There is no one Madison curriculum, but rather several different collections of "informal learning experiences" or lessons. These are thought of as versatile supplements to be introduced into "nooks and crannies" of whatever basic arithmetic program the teacher is using. These collections are available as films of full length lessons, in packaged inservice training courses, and through school district-sponsored inservice workshops for teachers. They are:

1. Primary course for preschool through grade 2, emphasizing counting.

2. Course of discovery exercises unifying arithmetic with algebra, geometry, and physical science, which can be used between grades 2 and 8. or for older students, depending on the school.

3. Discovery course above plus active-learning lessons stressing manipulative materials, small-group work, and individualized instruction. This grouping of supplementary lessons became the main thrust of the Madison Project.

4. Ninth grade course for college capable students.

Following is a list of topics which have been developed into the lessons which comprise the various collections:

- Counting experiences with counters and with graphs
- Variables
- Open sentences and truth sets
- Signed numbers (positive and negative numbers, integers)
- Coordinate geometry (Cartesian): truth sets and linear graphs
- Place value numerals using counters
- Using the number line for solving equations, practice with fractions, concept of identity
- Arithmetic with signed numbers
- Practicing variables, open sentences, and signed numbers by means of quadratic equations
- Mathematical logic: truth tables and inference schemes, many-valued logics, two-valued logic
- Functions
- Mapping or transformations
- Matrices
- Arithmetic with Dienes multibase arithmetic blocks
- Arithmetic with Cuisenaire rods
- Geometry with geoboards, mirrors, concrete materials
- Measurement: length, height, volume, area
- Empirical statistics: average and variance
- Measurement and arithmetic from maps and timetables

This list should be thought of as representative rather than all-encompassing, for the major means of dissemination of Madison "learning experiences" is the inservice workshop, and no two workshops cover exactly the same topics. Nor are all Madison-recommended topics the original work of Madison staff. They recommend units, apparatus, and exercises developed by other educators and curriculum developers, as well as new ideas worked out by students themselves, always giving credit to the originators. Madison Project content thus includes ideas and apparatus developed by England's Nuffield Project, the Elementary Science Study, the School Mathematics Study Group, the University of Illinois Center for School Mathematics and its Arithmetic Project, Caleb Gattegno, Zoltan Dienes, Burt Kaufman, Robert Wirtz, and many others.

Madison Project materials compare with other mathematics curricula about the same way a cookbook compares with a line of TV dinners. Madison sets forth a distinctive approach to teaching plus selected, "kitchen tested" recipes but places the responsibility for the finished "dish" on the teacher. By contrast, other curricula provide lessons ready-made and packaged and ask the teacher only to "heat and serve."

If Madison materials are like a cookbook, the project itself is rather like a cooking school, intent on the gradual introduction of a whole new mathematics cuisine into American schools. At the start the teacher may learn to prepare and then serve to her students just one or two new Madison recipes. As she gains experience she tries out more and more new recipes, adds them to her basic entrees, and gradually transforms the mathematics bill of fare. The content of Madison math and the way it is organized are expected to be different in every class.

Sequencing. A recommended sequencing of the signed numbers, algebra, and Cartesian geometry topics is found in *Inservice Course I* and in *Discovery in Mathematics* and *Explorations in Mathematics*, the teacher texts written by Davis. All Madison teaching--of teachers as well as of children--is characterized by spiral sequencing of topics, which Davis calls "the light touch."

> A subject is not pursued too heavily within a single session, but recurs from time to time and in various guises, until it becomes familiar. . . . Thus we get a sequence: very rough ideas, rough ideas, moderately refined ideas, more minutely detailed ideas. . . . We advocate a sequence wherein the child first gets experience, then (as a result) develops intuitive ideas, and finally strives for explicit words and symbols to describe his experience.[6]

This sequence is ordinarily spread over several encounters with a topic such as Cartesian coordinates. The teacher first chooses a topic that is in keeping with the basic arithmetic text and the students' abilities and interests. Having introduced it in an opening learning experience, she will return to it from time to time and introduce new activities which review and extend the topic. Or she may decide to put a topic aside for a time, finding that children don't grasp the concept.

Davis assumes that any good teacher will learn how to sequence. This won't be perfect but it will be no worse, Davis believes, than the mistakes that are made when tightly structured, graded textbooks are applied across the board.

2.3 Materials Provided

Student. All of the Madison Project materials are designed for the teacher to adapt to the needs of students. Robert Davis' Shoebox Kits are a set of six different apparatus and games, each with task cards which the teacher can use to make work projects for individual students or small groups. The apparatus are disks, geoboards, centimeter blocks, weights and springs, the peg game, and the tower puzzle. Don Cohen's *Maths Minilab* is a starter set of manipulative materials for a teacher at the intermediate grade level. It can furnish a variety of math projects for individuals or small groups and also serve as a model of materials the teacher should gather or make in quantity.

Following are some of the things the *MiniLab* contains: beans, clay, rubber bands, Cuisenaire rods, color cubes, base-3 blocks, geoboard, plastic mirror, ruler, tape measure, pattern for a homemade slide rule, compass, protractor, map, map measurer, railroad timetable. This kit also provides task cards, stating open-ended problems for students, and a guide for teachers, which cues the task cards and the apparatus to math topics in the basic curriculum.

Addison-Wesley's *Discovery* and *Explorations* Student Discussion Guides, accompanying the teacher texts mentioned above, are designed to be used during whole-class or small-group lessons led by the teacher. These books contain discussions, stories, games, and exercises on topics interrelating arithmetic, algebra, and geometry. The student pages are reproduced in the teacher texts.

Teacher. The *Discovery* and *Explorations* books and Cohen's *Inquiry in Mathematics Via the Geo-board* (Walker, New York, 1967) are Madison Project textbooks for teachers. However, inservice workshops sponsored by school districts, colleges, and/or the National Science Foundation have been the major means for spreading Madison Project curriculum innovations. Such workshops are taught by teachers who have used Madison methods.

In the absence of experienced Madison Project teachers to lead workshops, there are available 2 kinds of filmed inservice courses. The first is Houghton-Mifflin's *The Madison Project Films*, a 12-film series subtitled "A Concrete Approach to Introductory Ideas in Mathematics." These 16mm black and white films, lasting from 5 to 15 minutes each, show classroom teaching in primary grades, in which washers, beansticks, number lines, geoboards and other simple manipulative objects are used to convey concepts in place value, addition, subtraction, multiplication, division, fractions, and area. The teacher's handbook accompanying the film series provides Robert Davis' commentary on the Madison Project approach in general and the filmed lessons in particular.

Another packaged, film-based inservice course on the Madison Project is that produced by the project staff: *Supplementary Modern Mathematics*. It is also a series of 16mm black and white films showing classroom lessons, with a teacher's workbook providing commentary on the lessons shown, plus discussions and exercises for the teachers themselves. The topics developed in this film series are in algebra, Cartesian geometry, signed numbers, and fractions. A group of as many as 30 teachers can manage this course by itself with the help of a discussion leader who has participated in a Madison Project workshop. The course requires from 10 to 15 1 or 2-hour sessions. There is also an advanced course, *Supplementary Modern Mathematics II*. They are available from the Curriculum Laboratory, University of Illinois.

These packaged courses should be used only by teachers who accept the Madison Project's basic approach of informal, discovery-style teaching, and who will invest effort to teach themselves.

The films which are used in the inservice courses are excerpts from the videotapes of more than 60 complete classroom lessons made by the Madison Project in the early 1960's. The uncut films also are available from the Curriculum Laboratory at Illinois. Many of these lessons are also available as tape recordings.

2.4 Materials Not Provided

The teacher is responsible for gathering the concrete objects, counters, and math apparatus she will need. As mentioned above, the *Mathe Minilab* kit provides a model of what to gather for intermediate grades. Besides duplicating what's in the kit (if she wants to use activities for many students or a whole class at once), the teacher must add graph paper, construction paper, timers, toothpicks, milk cartons, a math balance, beans, dice, games, and much, much more.

Children need to experience a richness of things, not only to use in their work, but also to help them learn to sort out that which is pertinent from the mass, Cohen says. He urges teachers to furnish their classrooms with interesting and provocative "junk," and to welcome children's additions of whatever they find fascinating. (But not to treat it as junk; rather to store it in an orderly way so that it is attractive and accessible to students.) Such materials should be varied in composition, surface, finish, color, thickness, so that they raise questions "of observation, of structure, and of feel."

> These are not immediately *mathematical* questions, but if children are to be able to *see the math in their activities, the pattern and order which are implicit,* they need the experience of "seeing through the noise," (that is, the diversity and the distractions) to the immediate problem at hand. Something which is all metal or all wood . . . paper cutouts which are all one color or all one texture . . . tend to restrict a child's observations to a narrow field.[7]

3. CLASSROOM ACTION

3.1 Teaching-Learning Strategy

"Do and then discuss" is a nutshell definition of the Madison Project theory of instruction. Project staff believe traditional teaching is too verbal. It makes *talk about* math a focus of attention, whereas the project puts the emphasis on "doing something active that embodies the relevant mathematics."

> In the eyes of some teachers we do not present "lessons." What we do instead is to suggest to the children one or more mathematical tasks, and then work with them, unobtrusively, as they devise their own methods for tackling the tasks. (We begin with tasks rather than with definitions because) we try to have the students learn concepts in context. Every mathematical concept or technique was developed to aid in attacking some kind of problem. When we tear the concept out of this discussion and attempt to state it *in vacuo*, we render the concept unintelligible. . . . The concept unfolds naturally (if we help the student investigate) the nature of the task.
>
> We try to see that the mathematical tasks possess intrinsic motivation: the task itself cries out to be done. Examples might be finding a key word in a crossword puzzle or finding a long-sought piece in a jigsaw puzzle. We make very little use of extrinsic rewards--indeed, some research appears to indicate that extrinsic rewards can stand in the way of genuine creativity.
>
> Passive roles, such as listening to a lecture or reading exposition, are usually avoided. The "active role," however, may refer to mental activity as well as to physical activity. The child who leaves class with a look of puzzled involvement is playing an active role quite as much as the student who is making a measurement with a meter stick.[3]

A characteristic Madison technique is to get students' participation by asking them to make up the problems. Students name the amounts on the checks and bills in Postman Stories and the prices of the pets in Petshop Stories, both of which are used to teach signed numbers. They specify the numbers in the ordered pairs they plot on graphs.

The Madison Project is a foremost advocate of discovery learning. Here is what it means to the project staff:

> In every lesson we try to have opportunities for discovery lurking just beneath the surface . . . sometimes an essential part of the lesson but often going beyond (it). The point here is to get the children in the habit of "looking for patterns" whenever they are working in science or mathematics; the discovery of such patterns is, after all, the main device by which science moves forward.

> We try to avoid an authoritarian atmosphere . . . (and) provide *autonomous decision procedures*, whereby a student can distinguish true statements from false statements *without recourse to the teacher or to books*. For the very young child, the process of counting often serves. Is it true that 3 + 4 = 7? If the child can count reliably . . . he can settle for himself the truth or falsity of the statement.[9]

Discovery learning depends on the self-confidence that such self-deciding procedures engender in children; it is impeded by children's fear of failure.

> Whenever possible, withhold value judgments. Students become conservative if they feel they are being judged; they become more creative when they feel they are being appreciated.[10]

> . . . What is important is the creative originality of (children's) own thinking in relation to their own experience. We have witnessed many original mathematical inventions or creations by children . . . The main factor was that the teachers *respected* the children, that they *believed in the child's ability to think for himself* . . .[11]

The project does not gloss over the difficulties of assuring that children learn in a nondirective classroom. It acknowledges the danger that math activities can be done just for activity's sake, with little or no math arising out of them.

> If we leave the children too much on their own, we may be failing our responsibility. The class may drift aimlessly, or the children may learn far less than they might have learned. Yet if we steer too much, the children lose initiative. . . . There probably is no general answer which applies to all teachers and all classes at all times. What we can do is to try to observe ourselves and see which error we make more often--allowing too much drifting or imposing too much adult interference. We can avoid too much teacher talk and too little child response. . . . We can also remember (that) children cannot choose to do something they have never even heard about. We must acquaint them with many attractive alternatives if they are to exercise any right of selection.

The Madison Project materials should not be identified with either the "drill it into 'em" point of view, or with its apparent negation, the "wait until they are older and ready for harsh abstraction" point of view.[12]

Whether or not developers' scripts for discovery learning ever get played out in the classroom depends almost entirely on the teacher; first, how she *sees* her role *vis a vis* the students, and then how well she learns to play it. Students will not play their parts as explorers and discoverers if their teacher is being a Rule Giver and Answer Certifier, nor if she is simple a buddy saying "do your own thing." The discovery-style teacher must be a keen observer, a supportive and stimulating leader with lots of ideas, and an authoritative resource.

Lesson format. A pattern offered by the Madison Project to teachers to help them develop their discovery technique goes like this:

> 1. First begin (if necessary) by recalling those key words, experiences, notations, etc., from previous lessons that will be crucial in today's lesson.
>
> 2. Second, do something. Have the children actually carry out some activity or happening.
>
> 3. Thirdly, as the occasion arises, and as it becomes appropriate, discuss what the class has just done.
>
> Avoid asking children to discuss things they have never done. . . . A definition is more nearly the place to end up rather than the place to start.
>
> The project would not begin an elementary school class on the idea of functions with a definition of "function." It would, and does, begin with Warwick Sawyer's procedure of having some children make up a rule (such as "whatever number you tell me, I'll double it and add seven"), which is kept secret but used (to work out a table) such as:

□	△
3	13
4	15
5	17
10	27

> The children who don't know the secret use the table to attempt to guess the rule and even to write an algebraic formula to express it, such as: $(\square \times 2) + 7 = \triangle$.[13]

Teachers attempting discovery for the first time may rely mainly on the playful but educational qualities of good manipulative materials, rather than trying algebraic games like the one discussed above. But Donald Cohen cautions, "Materials alone will never create for children a situation which is rich for learning." The teacher must provide equally important ingredients for discovery:

> There must exist a certain amount of order: a place to keep materials, a notebook for recording results, a time for clean-up, a limited noise level, a sharing of responsibility, and an awareness (of what each child is learning from each day's task). The whole class or part of it should come together for discussions, for lectures, for excursions, and for reporting to each other; but much of the work should be independent and should be performed individually or in small teams. Eventually a collection of well developed and illustrated writings (related to the working experience) should be the joint product of small groups or of whole groups.[14]

3.2 <u>Typical Lessons</u>

The Pebbles in the Bag game is a way of introducing the idea of signed numbers as early as first or second grade or possibly as late as junior high. The following description of the way to play the game and what it's supposed to teach is adapted from the project's *Inservice Training Course I*, which is designed to be self-taught by a group of teachers using instructional notebooks keyed to a series of films which show the topics being taught in a classroom:

> We have a bag containing a large number of pebbles. We do not know how many pebbles are in the bag at the outset. We never use this number. (This is not a counting problem.) We have a large number of pebbles *not* in the bag. We begin the game by having some child say "Go!" *This establishes our arbitrary reference point.* We have another child put as many stones as he likes into the bag, and we write the number on the blackboard: let's say it's four. We say, "Are there *more* stones in the bag than there were when Jack said 'Go,' or are there less? . . . How many more?" Another child takes two stones out of the bag and we write that number on the board. We now have on the board: 4 - 2. "Are there more stones in the bag now than when Jack said 'Go,' or less?" "How many less? How do we write this?" We now have on the board: $4 - 2 = {}^{+}2$. "How do we *read* this?" "Positive two" is written with a small plus sign written above the middle of the line so that we will not confuse it with the sign for addition. ("Negative three" would be written similarly, as ${}^{-}3$. It avoids confusion if we use two different

symbols and two different words to express two different mathematical concepts.) . . . Continue the game with a different student choosing either to put pebbles in the bag or take some out, specifying how many. Keep a record on the board of each transaction with pebbles and have the class say whether it results in more or less total pebbles--expressed as "positive" or "negative" numbers. When a new student says "go" this is a new arbitrary reference point and the start of a new game.

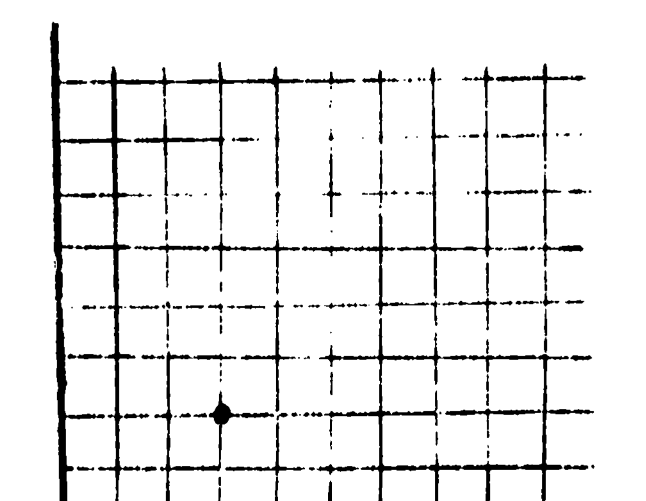
The topic of signed numbers is extremely important as one gets into work in algebra and science, and can even be important for work in arithmetic. There are many possible tie-ins with other school work; for example, temperatures below zero are essentially expressed as negative numbers . . .[15]

This learning experience is ordinarily followed by drawing pictures of or constructing real *number lines*, always extending in both directions from a point children choose as zero. Then can come introduction of *crossed number lines*--one horizontal and one vertical--and children naming *ordered pairs* of numbers (for instance, "3, 2") to describe points on a *graph* between the crossed number lines.

The number line gives us a valuable way to picture numbers. From another point of view, it lets us use numbers to describe points on a line.

By crossing two number lines, we can use an ordered pair of numbers to describe points in the plane. This invaluable device was first thought of by Rene Descartes (1596-1650) and is the foundation of much modern work in mathematics (what is variously called "Cartesian geometry," or "analytic geometry," or "coordinate geometry").

The point on this graph is called (3, 2). The first number in the pair refers to the horizontal number line. The second number refers to the vertical.



> We count from the heavy lines (known as "axes"), and we count "city blocks" rather than "intersections."[16]

These topics, which interweave with each other, also combine with previous learning experiences about *open sentences, variables and truth sets* to make another topic, *graphs for truth sets,* or *linear equations.*

The aim of these experiences is to give students opportunities to discover patterns in number relationships. Of course, teachers have to see these patterns for themselves before they can teach them.

Another typical Madison learning experience uses the timetable of any railroad. This activity is taken from task cards in Donald Cohen's *Maths Minilab.*

> Find the distance between two big cities. Check the scheduled running time and see if you can figure out the train's average speed on that run. Try it for another two cities.
>
> Find the fares between two big cities. How much does it cost per mile to go by train? Is the cost per mile the same between two other cities?[17]

3.3 Evaluation of Students

The Madison Project recognizes that if a school requires a teacher to guarantee that every child in class shall pass some quite specific examination, the teacher probably will resort to rote teaching. ("Don't think; just do it *exactly* like this . . .") Tests thus are seen as "a dangerously narrow goal" which the teacher has to approach by a "dangerously narrow path." If the teacher must test, then any standard instrument will do for arithmetic, the project director, Robert Davis says. What is tested in this way should be truly a "basic minimal core," not including long division. When you get into "the breadth that converts arithmetic into mathematics," you need to evaluate by observing students' classroom activity and their graphing and written reports of experiments and experiences, Davis says.

The Madison topics in algebra, coordinate geometry, functions, and the rest, could be evaluated by using exercises from the *Discovery* and *Explorations* teacher texts, which contain answers. Inservice courses also contain exercises and answers on these topics.

> How could you use pieces of string on a number line drawn on the blackboard, to indicate addition (for example, 2 + 3 = 5)? How could you use the string to locate 1/2? 1/3? 1/4? 2-1/2?

Using only integers, can you locate five points on the graph for the truth set of the open sentence $(1 \times \square) + 3 = \Delta$?

Do the formulas $(\square + 2) \times 3 = \Delta$ and $(3 \times \square) + 6 = \Delta$ represent the same function, or not?[18]

Diagnostic instruments. Present-day tests aiming to measure achievement are far from the Madison Project concept of education, which is influenced by Piaget's work on the stages of children's cognitive growth. The project worked with Cornell's Center for Research in Education to devise instruments for assessing the child's cognitive *processes* rather than his mastery of *content*. They developed an unstructured classroom interview between a mathematician or teacher and a child, similar to the Piagetian clinical interviews. (In England the Nuffield Project also has developed a Piagetian interview for diagnosing pupils' learning stages rather than for measuring achievement.) The interviewer engages the child in arithmetic problems or activities, and the two talk informally about the work. Interviews so far have been with children who are having specific problems, and typically the interview starts with a topic the student knows and has confidence in and proceeds to a skill he can do only by rote, or which he fails to do. The interviewer noncommittally observes the work the child is doing until the student himself becomes aware of a mistake or doesn't know how to go further. Only then does the interviewer point out the child's misunderstanding and gives a hint as to the correct approach. In this way the interview serves to pinpoint the child's difficulties and also to teach him at the precise time when he is aware of his problem.

Several such interviews have been videotaped in order to provide accurate observations which can be referred to over and over again. These have brought to light unsuspected holes in students' mathematical thinking. While standard protocols for conducting interviews have not been developed, the videotapes show techniques an experienced teacher could adapt. A paper by Robert Davis and Rhonda Greenstein is an example of the diagnosis-plus-teaching that can be done. ("Jennifer," *New York State Mathematics Teachers Journal*, 18 [3], June, 1969.)

The videotapes also reveal that children's knowledge of mathematics is "extraordinarily complex and often much different from what we supposed," writes Herbert Ginsburg of the Cornell center. The "startling contradictions, unsuspected strengths and weaknesses" which have shown up on videotape help to explain why standardized individualized instruction curricula do not yet work as well as expected, Ginsburg claims. "Standard tests focus merely on the child's ability to come up with correct answers."[19] Correct answers have never been the goal of the Madison Project. "A correct answer doesn't prove the student understands the algorithm he has performed; a mistake doesn't tell why he made it," says Ezra Heitowit, a mathematician who conducted several videotape interviews. "To see what children do on paper is not enough. It doesn't tell what they're thinking." Further discussion of these clinical interviews is presented in Ginsburg's *The Myth of the Deprived Child* (Prentice-Hall, 1972), and in Stanley Erlwanger's article, "Benny's Conception of Rules and Answers in IPI Mathematics" in *The Journal of Children's Mathematical Behavior* for November 1973.

4. IMPLEMENTATION: REQUIREMENTS AND COSTS

4.1 School Facilities and Arrangements

The Madison Project approach and materials are particularly suited to nongraded schools and to departmentalized teaching in which there are math specialist teachers, but they are usable also in traditionally graded schools with one teacher to each class. Some Madison teachers like the idea of a separate room in the school to be designed as a mathematics environment and presided over by a math specialist teacher, but others say that active-learning materials for math should be in an activity area in the classroom itself so that all subjects are interrelated and children can move according to their own interests from one kind of schoolwork to another. The project recommends Edith Biggs' and James MacLean's book *Freedom to Learn* (Addison-Wesley, 1969) for its illustrations and instructions on how to furnish and arrange an active-learning classroom. The math center should be a place where students can work with apparatus and real materials--including messy things like sand and water--and where they can move around and talk with one another. It must be equipped with ample, versatile storage equipment.

4.2 Student Prerequisites

At the primary level the only prerequisite for starting to learn some Madison topics is the ability to count. This is true also for intermediate-level children in classes where the whole class works together under the direct supervision of the teacher. However, intermediate-level children who are set to work on their own in an individualized math lab approach must have reading ability in order to follow the "task cards" which take the place of the teacher telling them what to do. Beyond reading ability, they must have reasoning ability and a well-developed concept of sequence, which enables them to follow standardized directions.

4.3 Teacher Prerequisites

There are two qualities a teacher must have or get before she can use Madison topics and teaching approach. The first is understanding of the mathematics behind the topics and activities, and the second is a disposition to be informal, improvisational, and "open" in her relationship with students. Madison director Robert Davis conceives of the teacher as an artist with a personal, unique vision and discipline, rather than as a technician who implements someone else's plan. Davis makes a distinction between what he calls "ought" people and "growth" people--his own kind.

> From the "ought" people you get the feeling that an abstract ideal exists, and they are primarily concerned that things must be much more orderly and right . . . By contrast, the "growth" people think of a child . . . building cognitive structures in his mind from quite personal interpretations of his unique . . . experiences, and seeking out and fitting in new information . . . I'm not arguing for the truth of my assumptions. I just think people ought to identify their assumptions. I'm not going to be able to say, "Now go practice multiplication tables." Every time I say that to kids they don't do it. But there are teachers who go on the assumption that kids do exactly what they're told, and those teachers seem able to make their approach work.[20]

Such teachers are unlikely to be able to make Madison's approach work.

But "growth" people won't be able to make Madison work either unless they (a) know the math, (b) can select appropriate learning activities and sequence them into their school's curriculum, and (c) have mastered some techniques of discovery teaching and individualizing instruction. The Madison Project no longer focuses exclusively on the "discovery dialogues" which were its hallmark in its early years, but these are still an important component of the program. Whether these teacher-and-students discussions and games on abstract topics are conducted with the whole class or in small groups, a special assortment of talents is required in the teacher. This is a combination of mathematical knowledge that is deeper than just the surface of the lesson, plus a perception of how children are thinking, that may be intuitive in some teachers and the product of long or highly concentrated experience in others. Some observers believe that the abstract lessons, especially the advanced ones, should be taught by math specialist teachers. Harrison Geiselmann, professor of mathematics and education at Cornell University, believes "it takes quite a talented teacher with knowledge of math so that he knows what lines to pursue, what not to pursue; what will be fruitful with students, and what won't be." However, many teachers without specialized background in mathematics learn to do Madison's basic courses of abstract lessons well. Lee DeBarros, a Walnut Creek, California teacher, who has taught as a math specialist, says nonspecialist teachers can teach abstract math in the discovery style but it requires forethought, concentration, and a flair for improvisation:

> The nice thing about discovery is it's logical. It's just a matter of wondering with students about the consequences of certain assumptions. You start where the children are. You have to think hard; it's hard to do that. And it is hard to encompass a class of 30 kids. I found that, working with a whole class at once, some kids were invisible. You have to be some kind of virtuoso to bring everyone into the discussion.

Finding out that the Madison abstract exercises were not every teacher's and student's cup of tea resulted in the project's broadening its topic coverage and its teaching style to include "math workshop" lessons and small-group or individual projects. This kind of teaching also assumes teachers of intelligence, who have at least a beginning grasp of the mathematics beyond computation, some understanding of how to individualize, and a firm set of math goals to work toward. This is a large assumption, as attested by several observers of the active-learning movement.

Barry Barnes, an early childhood education specialist at the Far West Laboratory, says:

> A basic problem is teachers' lack of confidence. Unlike organic reading, which teachers are willing to try because they feel pretty confident about their skills in teaching reading, the math activity center makes teachers exceedingly uncomfortable because they are not confident about their own math ability. They must have more structure for their teaching in math than they need in reading.

Lois Knowles, professor of education at the University of Missouri, told the National Council of Teachers of Mathematics convention in April 1971:

> I have yet to see (in classrooms) much progress in planning activities suited to children's level of development. We still find teachers taking bits and pieces of math--no matter what children are ready for. Teachers aren't really listening to children. Children don't have a chance to tell us what they know and don't know.

And J. Fred Weaver of the University of Wisconsin warns that *experience, activity,* and *[illegible]* are becoming "shibboleths":

> The crucial factor associated with experience and activity is *[illegible]*. No leadership is

> required to generate a hodgepodge of "interesting" diverse experiences and activities whose intents and purposes are ill-defined . . . But strong leadership is needed to suggest promising activities and experiences that are *appropriate* for the attainment of particular *mathematical* goals. . . .[21]

Support from administrators. A school administration could not realistically expect teachers on their own to provide this leadership. Thus a change to active-learning mathematics requires an administrative commitment to substantial inservice training for teachers, writes Lola May, the math consultant and writer:

> Mathematics can no longer be taught by a "show-and-tell" artist who is nothing more than a textbook wired for sound. . . . (Teachers) need first-hand experience, and this means (they) must work with the materials and learn the same way the students learn. Someone has to demonstrate how to direct learners and how to ask open-ended questions . . . School systems must provide this type of inservice training, and schools of education must incorporate this type of learning in their methods classes.
>
> Methods of evaluation must also be changed. Little questions with little answers are no longer enough. Teachers have to learn to evaluate . . . what a learner says and does (rather than test scores. This) requires a change of thinking on the part of teachers and administrators. Then it must be sold to parents. . . .[22]

Inservice training. The major effort of the Madison Project now is to help school districts retrain teachers. The typical method of Madison inservice training is a summer workshop of from two to four weeks, with college credit, sponsored by the National Science Foundation. Teaching is by former Madison Project staff and classroom teachers who have been through previous Madison workshops. The workshops have two focuses: (a) starting "math-illiterate" teachers to learn some abstract algebra, Cartesian geometry, functions, negative numbers, and the like--by means of the same lessons and games that Madison designed to teach children; and (b) introducing informal, "math lab" or active-learning methods by having teachers work with manipulative materials. A summer workshop may be followed by regular after-school or Saturday morning meetings in the ensuing year or by a Madison consultant visiting each teacher participant's classroom from time to time. Teachers who participate in Madison workshops are expected to pass on the new ideas to their colleagues. The most successful ones are invited to teach subsequent Madison workshops.

From 1967 to 1972 the California State Department of Education provided two-week summer workshops for elementary teachers as part of the statewide "Miller math" program to improve mathematics instruction. These workshops stressed work with manipulative materials and provided each participant with $100 worth of materials to take back to his school. They were based on Madison Project workshops developed by the project and the school districts of Chicago, St. Louis, Philadelphia, New York, San Diego, and Los Angeles. The Miller math workshops were taught by teachers rather than by college professors

When the state-supported program ended, a nonprofit group, Center for the Improvement of Mathematics Education (CIME), was formed to provide Madison-style summer workshops in several California cities. CIME's address is P.O. Box 81594, San Diego, Ca. 92138.

The Madison math lab style also survives in that part of the new teachers' center movement which emphasizes inservice courses taught by classroom teachers, promoting active-learning methods and materials, and developmental learning theory.

Other forms of inservice training made available by the Madison Project are its films of classroom lessons and its packaged inservice training courses, described in Chapter 2.

4.4 Background and Training of Other Classroom Personnel

Many American teachers with experience in active-learning mathematics are convinced that the teacher with 25 or more students needs extra adults in the classroom in order to manage this approach. This is particularly so when students lack reading proficiency, or need a lot of teacher control. If nonmathematically-inclined students are to learn from activities instead of from memorization and drill, they must be closely observed to make sure that they follow directions, understand the mathematical meaning of their experiences, and translate understandings into skills. Volunteers or paraprofessionals assisting the teacher in mathematics should understand what the activity is supposed to teach, have confidence in the discovery/active-learning approach, and have patience with students.

4.5 Cost of Materials, Equipment, Services

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Manipulative materials: Such things as Cuisenaire rods, Dienes Multibase blocks, counters, math balance, cartons, string jars, maps, geoboards, math puzzles and games, mirrors, tape measure.		User can buy some equipment, some prototype kits*, can collect and make most of his own stuff.		

*Discussion: Robert Davis' *Shoebox Kits* are available at $17.25 for four or more sets from Math Media Division, H and M Associates. Box 1107, Danbury Conn. 06810. (There are six different apparatus in each set.) Donald Cohen's prototype kit, *Math Minilab*, is available at $32.50 from Selective Educational Equipment, 3 Bridge St., Newton, Mass. 02195. Contents are described in Chapter 2.

Recommended Supplementary Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Inservice training workshop conducted by Madison Project		Madison Project, Curriculum Laboratory, University of Illinois		
The Madison Project Films series of 12 plus teachers' guide		Houghton-Mifflin	$1,521 sale $ 194 rental $ 1.50 each guide	
Freedom to Learn by Edith Biggs and James MacLean		Addison-Wesley	$6.95	
Nuffield Project teacher's guide		John Wiley	$1.50-$2.50 each	
Teacher texts: *Discovery in Mathematics*, grades 2-8; *Explorations in Mathematics*, grades 5-9	1 per teacher	Addison-Wesley	$8.00 each	Reusable
Student discussion guides *Discovery*, grades 2-8; *Explorations* grades 5-9	1 per student	Addison-Wesley	$3.00 $2.80	Reusable
Supplementary Modern Mathematics inservice courses: teacher's manuals plus film excerpts	1 packaged course per 30 teachers	Madison Project, Curriculum Laboratory, University of Illinois	$140 each course (including books)	Films on rental basis
60 films of lessons taught by Madison staff: 16mm black and white, sound		Madison Project	$4.00-$6.00 each	Rental

4.6 Demonstration Sites

Professor Davis' office at University of Illinois Curriculum Laboratory can suggest particular teachers or schools in all parts of the country which demonstrate use of Madison topics and/or an informal, active-learning style. In California the Center for the Improvement of Mathematics Education can recommend classrooms of teachers from all over the state who have taken part in summer workshops, a la Madison active-learning math. Write Leonard M. Warren, director, CIME, P.O. Box 81594, San Diego, Ca. 92138.

5. PROGRAM DEVELOPMENT AND EVALUATION

5.1 Program Development

The Madison Project began as university professors' experiments in elementary classroom teaching and gradually joined the nationwide thrust for math teaching that is at once more mathematical and more childlike. Madison's modern topics and its naturalistic approach do not represent a steady refinement of one technique but a blend of experiences with many. Its history was one of creating materials and methods, combining outside ideas with their own, sharing their own ideas with others, and giving up or changing things that didn't work or didn't transfer well to new situations. Thus the present shape of Madison math is different from the first outline.

Leonard Warren, director of California's Specialized Teacher Project (the so-called "Miller math" inservice workshops) which grew out of Madison Project training in San Diego and Los Angeles, comments on the major change in the project:

> Original Madison math was essentially dialogue between teacher and students by someone fairly sophisticated in math. Davis came to believe that this is not something the average teacher can do. You need a richer background in math to continue to ask provocative questions that make discovery go.

The project's evaluation studies and experience with inservice training showed that students need physical as well as abstract experience.

> In the summer of 1964 a class of supposedly culturally deprived children in Chicago, who were in fact very bright, were assembled from grades 4, 5, and 6 and combined into a single class to serve as a demonstration class, via closed-circuit T.V., for the inservice

> workshops in Chicago (which were held summers and Saturdays for two years). Morale was beginning to wear thin. . . . The teacher had been standing at the chalkboard and often dominating class discussions. He now broke the class up into groups of three or four children each, seated around tables, using physical materials much of the time, with each group working independently of the others. Morale improved dramatically.
>
> (The children) taught the project that the teacher should not stand and thereby dominate the room, but rather sit and work with students as equals; that the teacher should only occasionally address the entire class, but usually talk privately with two or three children at a time; that children should sit at tables in groups of three or four, working together, but not necessarily on the same tasks as other groups; and that much of what the children do should involve the manipulation of physical objects more mathematically suggestive than a pen or pencil or a piece of chalk.[23]

The project began in November 1956 in an "underachieving" seventh grade at the Madison School in a low-income neighborhood of Syracuse, New York, under Office of Education sponsorship. It gained classroom experience in middle-class and in privileged suburban schools, and tested the new lessons it had devised on teachers in "inner city" schools in St. Louis, Chicago, New York, and Philadelphia.

The project never intended to create a comprehensive curriculum because it lacked the resources to innovate the entire elementary mathematics program. "Rather than carry over much unsuitable material into a 'new program'," Davis wrote, "the project approached the curriculum as one might approach urban renewal. Most of the city was left untouched. Only in spots, where it was possible to make definite improvements, was the curriculum tampered with . . ."[24]

The first step in creating a new lesson was to identify a mathematical concept, such as *variable*, as being of high priority.

> In working with children in the classroom, specialist teachers try alternative methods of letting children work with variables--seeking always *processes*. In the course of doing this, other topics will appear which turn out to be intertwined with the classroom work on the original topic--for instance, open sentences, truth set, number line. The topic is retained only if suitable experiences for children can be devised . . .[25]

More than 100 specialist teachers were involved in creating the original Madison topics, teaching them in several different classrooms, and then giving them to a different teacher to try out. The project assembled its successful lessons into four distinct supplementary courses, available not as texts for students but as films of lessons for teachers.

The Office of Education supported an implementation phase (1961-67) to spread these new lessons by means of big inservice training workshops like those the project had designed with the school districts of St. Louis and Chicago. Two packaged, self-administered inservice courses were written. They used excerpts from the films of classroom lessons.

During this time, the project's association with Syracuse and with Webster College, Missouri, produced new preservice courses in mathematics. Up to the present, the project continues to focus on re-education of teachers with National Science Foundation support. Since Robert Davis' move to the University of Illinois, where he is director of the Curriculum Laboratory (1210 W. Springfield, Urbana, Ill. 61801), continuing Madison activities are centered there. The Laboratory's PLATO project for computer-assisted instruction in mathematics has translated a number of Madison topics in graphing to PLATO. Thus they are available to teachers and students who have access to computer terminals in the PLATO system.

5.2 Developer's Evaluation

The Madison Project did not measure itself against standards of traditional mathematics programs because it views its own work as "more than new routes to old goals." Its work is to put forth examples of new goals.

> The project has sought to produce certain actual changes in schools, (but it has not) sought to prove that these changes were desirable . . . These are things that you do, and then allow people to view them, and to build upon them, and to form their own judgments.[26]

The project considers traditional testing measures to be antithetical to its goals because they take too narrow a view of mathematics and because new materials and lessons are taught so differently in different schools. Comparing Madison lessons with other curricula was never attempted; first because Madison materials are supplementary, but more importantly because in the field of math education there is no commonly agreed-upon body of goals and, Davis claims, the "relatively trivial" goals which have been set down get to be overemphasized. All of this is not to say that measurements of project performance should never be made, Davis added, but rather that "one should balance the estimated gain in information against the probable costs."

With those cautions in mind the Madison Project undertook a variety of tests of its own work:

> (These consist of) careful observation by mathematicians, teachers, administrators and clinical psychologists; "viability" testing in the hands of a variety of teachers of varying qualifications, and with a wide variety of students; confirmation of appropriateness through viewing of films by relevant panels of professionals; tape-recording lessons by a variety of teachers and allowing a panel to study these recordings; following the same students for up to five years in the program in order to observe cumulative effects; and tape-recorded interviews of students conducted by a clinical psychologist.[27]

The most formal study of effects of Madison Project teaching was conducted in 1965 (before the movement toward active learning with physical materials) by J. Robert Cleary of Educational Testing Service. He matched three seventh grade classes in three different upper-middle-class communities. Two of the classes had had Madison Project instruction for several years. He chose 45 items of the new mathematics test of the Stanford Achievement High School Test Battery, to see "how Madison Project students operated on fairly difficult materials of both traditional and more contemporary content but with the more traditional notation." He hypothesized that the Madison Project seventh grade students "would perform as well as a sample of ninth grade students taking some variety of modern math in similar schools." This hypothesis was proved. Cleary also reported that the students out-performed the non-Madison class in "all items dealing with algebraic knowledge, graphic interpretation of functions linear and nonlinear, and other mathematical topics," while the non-Madison students equalled or surpassed the Madison groups only in arithmetic items or "items requiring formula substitution skill."[28]

A less formal study was conducted to investigate the use of Madison lessons by many different teachers. Their lessons were tape recorded and a copy of each tape was sent to a panel of 60 mathematicians, teachers, and psychologists, who were not told in advance what to observe but focussed on whatever they chose. Each panelist prepared a report on each lesson, and these analyses were studied by the project. The most significant result of this study was the indication that Madison Project materials are more successful in the hands of "child-centered" teachers than when taught by teachers concerned with "the way things *ought* to be" (one teacher-panelist referred to lessons by such teachers as "more orders from the Giant People!"). However, Davis adds, there is evidence that "rather rigid, compulsive teachers, if they can relax somewhat, can teach project materials very well indeed, especially with children who tend to misbehave."[29]

A third study consisted of a clinical psychologist's interviews with individual sixth and seventh grade children, in an attempt to find out why children in these grades in all kinds of schools seem to perform less well than they had in earlier grades. Children were not aware that the study was concerned with mathematics. The psychologist studied tape recordings of the interviews and reported his findings.

> The most striking result, which emerged rather clearly, was that the children liked those subjects which involved physical activity and opportunities to talk to other children, and disliked those subjects which involved sitting still, and which offered no opportunities to talk with friends. . . . The children were quite articulate and quite explicit. They disparaged subjects where "all you do is sit and read and write." They liked orchestra, chorus, physical education, laboratory work, and art work; they disliked Latin, modern languages, social studies, English, and mathematics.
>
> Perhaps the most important fact is that the data collected in the Herbert Barrett study *did*, indeed, form the basis for a decision which has been implemented: the project moved further away from an exclusively paper-and-pencil approach and has come to make extensive use of physical materials and "mathematics laboratories" *at all grade levels, K-9, and also in college courses for prospective teachers.*[30]

The project cites its collection of films of actual classroom teaching as descriptive evaluation. The lessons recorded in this way are "fully worked out," not just directions and exercises in a text. The teacher can see first-hand how the Madison topics work and make her own evaluation of their effectiveness with herself as teacher.

The project cites its experience in "inner city" schools, starting with St. Louis' Banneker district, to prove that Madison Materials are "every bit as viable" with poor, nonwhite children as with suburban, economically privileged children, *provided that teachers have inservice training.* Other big districts in which the National Science Foundation sponsored Madison teaching training workshops are New York, Philadelphia, Los Angeles, and San Diego.

Chicago's NSF-sponsored training project in 1964-66 was the prototype for the others, and it is cited by Davis for illustrating "the large amount of feedback data" the project was able to acquire by developing its curriculum in the classroom and by remaining open to influences from students.

5.3 Evaluation Results

"Properly educated teachers using a proper selection of Madison Project materials" can achieve any one or more of the following goals, Robert Davis reported to the Office of Education in 1967:

1. Building an improved understanding of certain commonplace topics in arithmetic, such as placevalue numerals, algorithms, fractions, etc.

2. Arousing an interest in school (or in mathematics) among children (and, for that matter, teachers) who have not lately exhibited a very likely involvement or an eager enthusiasm. (This includes elementary education majors who believe they hate mathematics, etc.)

3. Providing a basic foundation for unifying arithmetic, algebra, and geometry in grades two through nine.

4. Providing a basic foundation for relating mathematics to science (and even to such subjects as history and music).

5. Providing a program to allow more talented children to move ahead more quickly in mathematics.

6. Providing materials and ideas which enable teachers to change their mathematics classes from a textbook-dominated approach to a more flexible "mathematics laboratory" approach--including small-group work and individualization of instruction.[31]

All of these accomplishments were proved by one or more of the studies described in the pages on evaluation in the OE report, or by studies by other investigators, which are reported in the next section.

5.4 Independent Analyses of the Program

Evidence from the California "Miller math project" showed that students of second grade teachers with Madison-style training scored significantly higher than control groups on measures of both comprehension and computation and on the Modern Math Understanding Test. Students of fifth grade teachers in the Madison-style math workshop program performed significantly better than control groups on nine of ten scales selected from the National Longitudinal Study of Mathematical Abilities--computation plus understanding of number operation, geometry, probability, and graphing. The research was conducted from 1968 to 1972 by the

California Institute of Technology for the California state department of education Specialized Teacher Project, which had provided the two-week math workshop inservice training to more than two thousand elementary school teachers throughout California. The studies also showed that substantial improvement in pupils' scores was obtained when teachers attended the summer workshops two years in a row. Further, the evaluation ascertained that the math workshop inservice resulted in pupil gains in low socioeconomic and minority-group communities which were equal to the gains of pupils in mainly middle class white schools.[32]

Similar results were achieved in a 1965 study of Indiana fifth graders. Charles D. Hopkins investigated what happens to a child's proficiency in "traditional arithmetic" when time is diverted to the study of Madison topics. He found that "the students perform better even on the traditional topics (which are receiving less emphasis)." This was true for low achievers in math as well as high achievers. In his comment about this study Davis reported that it was made independently of the Madison Project but that the teacher had studied with the project for several years. "The project has never claimed, and does not claim, that untrained teachers can make effective use of project ideas. It is for this reason that the project's efforts at dissemination are directed almost entirely toward *teacher* education."[33]

5.5 Project Funding

The Madison Project curriculum development was supported by Syracuse University and Webster College, and by the Bureau of Research of the U.S. Office of Education. Its films were developed with funds from the Course Content Improvement Section of the National Science Foundation. Inservice training workshops for teachers in Madison topics and methods have been supported by NSF and by the school districts themselves.

5.6 Project Staff

Robert B. Davis, Professor of Mathematics and Education at Syracuse University and Webster College, now Director of the University of Illinois Curriculum Laboratory, was the originator of the project. His ideas about mathematics, about children, and about pedagogy--and his experiences--have shaped the project most strongly, and these in turn have been shaped by the many university mathematicians and educators and the more than one hundred math specialist teachers who have joined the project from time to time to help work out topics, to teach inservice workshops and to work as resident consultants in school districts. The primary-level materials are largely the work of Beryl S. Cochran. The main "assembled" curriculum consisting of Madison topics plus activities with manipulative materials, and adaptable for children from grades 2 through 8, is attributed to Donald Cohen, Gerald Glynn, and Louise Daffron, all Madison staff members. St. Louis Banneker district personnel are credited with moving Madison learning experiences into "inner city" classrooms. Among these, Katherine Vaughn and Katie Reynolds have continued to be associated with the project as teacher instructors. Chicago workshop designers were Evelyn Carlson and Bernice Antoine. Katharine Kharas was associated with the project in designing new courses in teacher education at Webster College.

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THE ARITHMETIC PROJECT COURSE FOR TEACHERS

INTRODUCTION

Discussion Leader: *Here's how some of the jumps looked for the rule* $\square \rightarrow 2 \times \square - 5$:

Participant B: *The jumps are different lengths and they go in opposite directions.*

Participant D: *I noticed that you have two jumps of one space, two jumps of two spaces, but then only one jump of four spaces.*

Discussion Leader: *If the pattern were preserved, where should the jump from 1 land?*

Participant D: *Would it be four spaces to the left of 1?*

Discussion Leader: *Yes, it would. How long do you predict the jump from 13 will be?. . .*

The above is a sample discussion among a group of elementary teachers participating with a discussion leader in the first session of the Arithmetic Project Course for Teachers. They have just seen a 33-minute black and white film (copyright--1967) of a Project staff member teaching a class of fifth-graders from Medford, Mass., to work out some number line problems on the chalkboard.

The impact of 20 such teacher seminars was described by one participant as follows:

> *This is a program that 'allows' the child to develop math concepts and principles in his mind, not a program that puts math concepts in his mind....I feel that the films are necessary in presenting the material to us [teachers]--otherwise I never would have believed it....Math can be fun and it can be easy and this program brings that idea across--and I love it!*

The *Arithmetic Project Course* is a series of films plus homework assignments for teachers (or prospective teachers), presenting a variety of fresh ways to teach math concepts and giving participants practice in designing learning experience for their own pupils. The topics were developed by David A. Page and his colleagues at the University of Illinois Curriculum Laboratory, starting in 1958, and the Course for teachers was developed by Page and associates at the Education Development Center in Newton, Mass.

The topics are not a complete arithmetic curriculum even for one grade, but rather supplementary ideas which can enliven and broaden the mathematics instruction in elementary school. The ideas are meant to be adapted during the Course for use in the classes of the participating teachers.

It has been more than 15 years since the Arithmetic Project began, and in that time other pedagogical emphases have come to the fore: arranging classrooms and schedules less formally, provisioning instruction with more concrete and naturalistic, child-relevant materials; breaking up instruction into small groups or individual tasks. These newer emphases on methods are not inconsistent with the stress of the Arithmetic Project on interesting and mathematically valid content. And the Course's main purpose--to expand and enliven the mathematical repertoire of teachers--is still relevant to a priority need of American education: to upgrade and renew the learning and teaching ability of elementary teachers.

Nevertheless, it is important to note that the Arithmetic Project Course began in the era in which university scientists and scholars became interested in developing sound and intellectually provocative ways to introduce their disciplines to young children. In this search, mathematicians tended to assume the readiness of children to learn powerful, authentic concepts if only they would encounter them in stimulating ways. The questions of what cognitive foundations were needed in order for children to cope with the ideas being introduced were not always confronted. These are questions which have become priorities--indeed, obsessions--as teachers increasingly work in situations in which they cannot assume a given level of achievement among all the pupils within a classroom.

BASIC INFORMATION

Program name: The Arithmetic Project Course for Teachers (formerly The University of Illinois Arithmetic Project)

Format: Series of films, discussion notes, and written lessons for teachers disseminated by inservice and preservice courses.

Uniqueness: Self-contained course which can be given by a school district without specially trained personnel. Topics are math concepts of significance to mathematicians and capable of being understood by children.

Content: Some ten topics developed by the Project, including transformations (about half of the course), equations, "maneuvers on lattices," and "greatest-integer function."

Suggested use: An inservice or preservice course introducing mathematical topics to teachers and helping them to adapt them for children.

Target audience: Teachers of elementary mathematics in grades 1-6.

Length of use: Twenty weekly sessions of 2 hours each offered consecutively or in 2 10-week courses. A 15-week preservice course is also available.

Aids for trainers: Twenty books (one per session) containing homework assignments, summaries of problems in the film, and supplements. Materials for staff include one *Guide for Course Leaders*, containing discussion notes on films and homework, and a *Corrector's Guide*. The project offers participating districts and colleges a 24-hour "hotline" and consultant services in addition to contact with local users.

Date of publication: Originally published in 1968; revised 1973.

Director/Developer: Professor David A. Page, Department of Mathematics, University of Illinois at Chicago Circle, Box 4348, Chicago, Ill. 60680. Mr. Jack Churchill, Associate Director, Arithmetic Project, Education Development Center, Inc., 55 Chapel St., Newton, Mass. 02160. (617) 969-7100.

Publisher: Education Distribution Center, 39 Chapel St., Newton, Mass. 02160.

1. GOALS AND RATIONALE

1.1 Goals

The Arithmetic Project was a spin-off from the high school mathematics curriculum development led by the late Max Beberman at the University of Illinois Curriculum Laboratory in the late 1950's.

David Page, director of the Project, believed that you could not determine what should happen at the secondary level until you had some understanding of the mathematical capabilities of elementary school children. While he was sure that competent mathematicians could produce spectacular results with elementary classes, Page wanted to see what ordinary teachers could do after a little training. He thought that with proper background even ordinary teachers could "allow children to pursue mathematics beyond the usual limits of elementary school."

The Arithmetic Project Course was developed in an attempt to substantially change daily classroom mathematics teaching. By increasing the teacher's knowledge of math and experimentation with new teaching strategies, it was hoped that she would be freed from reliance on the textbook and be able to teach more powerful and wide-ranging mathematics ideas.

> The central theme of the Project is that the study of mathematics should be an adventure, requiring and deserving hard work. Children who grasp some of the inherent fascination of real mathematics while they are in elementary school are well on the way to success in further study of mathematics and science. Students who are not to continue a formal study of mathematics deserve a taste of the subject that is at least appealing.
>
> However, the Project is not attempting to develop a systematic curriculum for any grade level, in the view that determining an adequate curriculum is not possible until more alternatives exist to choose among. What is needed are frameworks that provide day-to-day, "here-is-something-to-try" ideas for the classroom. The emphasis is on things that the teacher can begin working with as quickly as possible.*

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*All quotations are from introductory, course, and publicity material issued by the Project.

The term "new mathematics" is not used by the developer. Instead, ". . .the Project seeks novel ways of doing old mathematics--new structures or schemes within which can be found large numbers of interrelated problems revealing significant mathematical ideas."

As a result of taking the Course, state the developers, teachers "will uncover through their own efforts and with other teachers, some of the basic ideas of mathematics. At the same time, they will learn how to present these ideas effectively to children."

Actually the Course confines itself to a relatively small number of topics that were developed by the University of Illinois Arithmetic Project between 1958 and 1968 and to demonstrating "in an efficient, concrete way" how to teach them.

Rather than exposing the trainee to prepared student materials, the Course tries to focus attention on building a framework of ideas which the teacher can adapt and deploy. By increasing her knowledge of mathematics it is thought the teacher will no longer need to rely so heavily on the textbook and can adapt topics to personal teaching style and to local syllabus and procedures.

Course objectives are stated for the teachers rather than for children, in the belief that more competent and confident teachers will produce increased learning by their students. Course objectives are as follows:

1. To give them [teachers] ideas for teaching math.
2. give them topics and problems they can use with students,
3. show them ways of creating problems on their own,
4. give them a deeper understanding of some of the most important math concepts in the elementary school curriculum,
5. build their confidence in their own mathematical ability,
6. and at the same time help students develop skills in reasoning and computation.

1.2 Rationale

Arithmetic Project developers state that the topics were selected because they. . ."have aroused the interests of children, teachers, and mathematicians across the nation," and "all of the topics present fundamental ideas of mathematics in ways that are exciting to children and teachers alike."

It is held that children are excited by "scientific enterprise within a limited universe," which is actually what occurs when a child works with these topics, and that students love wrestling with abstract math if they can succeed. Furthermore, the developers have observed that "things are so devoid of interest and spark in the elementary classroom as far as mathematics is concerned" that when certain topics are introduced the excitement and interest of the students in abstract ideas becomes self-evident.

Fundamental to this Course is the notion that ordinary students can and should gain experience with significant mathematical *ideas*, not just rote skills. Just as fundamental is the notion that ordinary teachers can learn and teach this kind of mathematics.

2. CONTENT AND MATERIALS

2.1 Content Focus

Topics (or as the developer sometimes calls them "intermediate inventions") were designed to meet criteria of interest and accessibility to children and significance to mathematicians. The topics eventually selected for the Course were those which engaged "the interest and imagination of children by providing a diversity of problems that are not too hard for children to solve and which reveal some of the basic ideas of mathematics." The topics included are:

Number line jumping rules (functions or transformations).

Equations (identities, with one or two roots or no roots).

Jumping rules in the plane (transformations in two dimensions).

Negative numbers.

Maneuvers on lattices.

Lower and upper brackets (the greatest-integer function).

Artificial operations (properties of binary operations).

Commutativity and associativity of addition and multiplication.

Distributivity of multiplication over addition.

Subtraction and division as inverses to the operations of addition and multiplication.

The developers do not claim that these are a comprehensive selection representing all or most of the important concepts in elementary mathematics. (Some participants have in fact expressed disappointment that more than half the Course is devoted to transformations or "line jumping rules.") The developers believed that it was important for teachers to gain experience with some new topics in order to take part in worthwhile discussions about a truly broad and encompassing math curriculum. Thus the developers did not attempt to provide fresh "inventions" for all the concepts in the arithmetic syllabus. They thought it more important to provide a teaching *framework* in which children can wrestle with basic mathematical ideas and have some success.

The Project presents its own system of notation--arrows, brackets, boxes, parentheses, and the like--which may be different from that familiar to teachers and their pupils, and thus confusing to learners whose symbolic mathematical foundations are shaky.

2.2 Content and Organization of the Course

The Course is usually given as 20 weekly sessions but this is not mandatory. Each session includes a film, followed by discussion of the film, and discussion of the written homework lessons handed in by the participants.

It is possible to arrange to give the Course in 2 parts of 10 sessions each. Developers recommend that teachers take the first part before the second because topics are arranged sequentially. A 15-week preservice Course is also available.

The following chart indicates the topics scheduled for each session. The *Supplements* referred to are sections of the lesson booklets containing further exposition on the topics and suggestions as to how teachers can adapt topics to different teaching strategies and/or grade levels.

The written lessons are completed by the teacher between sessions. They are handed in, discussed by the group, and corrected by a "Corrector" with the help of extensive *Corrector's Guides*. The homework is discussed again at a subsequent meeting. Whether the *written lessons* are done before or after viewing the film is at the discretion of the leader.

While each film demonstrates how a topic may be taught at a particular grade level, teachers are expected to discuss and experiment, with the aid of the *Supplements*, teaching these same topics at differing levels.

2.3 Materials Provided

Materials for the complete inservice Course include 20 *films* of classroom lessons; each film must be individually scheduled and returned the day following each session.

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Each participant is supplied with 20 *booklets*, 1 for each session. The booklets contain *written lessons* to be done between meetings, *examples of the problems in the film* and *Supplements* containing further ideas, mathematical exposition and bibliographies.

Each leader is given a *Guide for Course Leaders* containing *general instructions, Written Lesson Discussion notes* and *Film Discussion Notes*. Each corrector is supplied with a *Corrector's Guide* with instructions for correcting the written lessons.

The Project also offers participating districts "contact with others who have used the Course," a 24-hour "hotline," and consultant services. Consultants will help set up the Course or conduct a session.

Participants may elect to take the Course in 2 10-week segments. The 15-session preservice Course covers most of the material in the complete inservice Course, but many of the films are omitted.

1
Introduction to frames and number line jumping rules
Film: **A First Class With Number Line Rules and Lower Brackets** Lee Osburn, Grade 5
Supplement: Answers to Common Questions About the Course

2
Consecutive jumps; distances jumped, competing number line rules
Film: **Which Rule Wins?** Phyllis R. Klein, Grade 3
Supplement: Computing With Positive and Negative Numbers

3
Parentheses and "multiplying before you add", standstill points
Film: **Standstill Points** David A. Page, Grade 5
Supplement: Answers to Common Questions About the Film "Standstill Points"

4
Effects of using rules in different orders
Film: **Three A's, Three B's, and One C** David A. Page, Grade 5
Supplement: Dividing by Zero

5
Introduction to maneuvers on lattices
Film: **A Seven-Fold Lattice** Francis X. Corcoran, Grade 5
Supplement: Maneuvers on Lattices

6
Frame equations; midpoints; some wrong answers, absolute value
Film: **Counting With Dots** David A. Page, Grade 2
Supplement: Ways to Find How Many

7
Lower brackets
Film: **Lower and Upper Brackets** Carol Daniel, Grade 4
Supplement: Using Centimeter Blocks to Introduce Prime Numbers to a Third Grade

8
Lower brackets and upper brackets
Film: **Inequalities With Lower Brackets** Francis X. Corcoran, Grade 5
Supplement: Arithmetic With Frames

9
Maneuvers on lattices, continued
Film: **A Periodic Lattice** Phyllis R. Klein, Grade 5
Supplement: More Suggestions for Lattices

10
"Surrounding" With Centimeter Blocks
Film: **Surface Area With Blocks** Phyllis R. Klein, Grade 1
Supplement: Using Blocks to Introduce Other Bases of Numeration to a Fourth Grade

11
Artificial operations; competing rules
Film: **Some Artificial Operations** Phyllis R. Klein, Grade 4
Supplement: Well-Adjusted Trapezoids

12
More work with artificial operations
Film: **Frames and Number Line Jumping Rules** Lee Osburn, Grade 5
Supplement: Functions

13
Graphing equations with lower or upper brackets
Film: **Graphing With Square Brackets** David A. Page, Grade 5
Supplement: Graphing Number Line Jumping Rules, Part One

14
Two points to two points on a number line
Film: **Competing Number Line Rules** David A. Page, Grade 5
Supplement: Graphing Number Line Jumping Rules, Part Two

15
Rules moving two points; composition of number line rules
Film: **Rules Moving Two Points** David A. Page, Grade 5
Supplement: Examples of Questions Dealing With [illegible]

16
Composition, continued
Film: **Introduction to Composition** Marie L. Hermann, Grade 5
Supplement: More Problems With Composition of Number Line Rules

17
Simultaneous equations; points and lines in a plane
Film: **Graphing Absolute Value Equations** Marie L. Hermann, Grade 2
Supplement: Graphing Simultaneous Equations

18
Number plane jumping rules
Film: **Jumping Rules in the Plane, Part I** Lee Osburn, Grade 6
Supplement: Composing Number Line Rules to Move Two Points to Two Points

19
Number plane rules, continued
Film: **Jumping Rules in the Plane, Part II** Lee Osburn, Grade 6
Supplement: More Work With Number Plane Rules

20
(Discussion of last lesson)
Film: **Rotations in the Plane** David A. Page, Grade 5
Supplement: Hybrid Rules: Jumping Rules from the Line to the Plane and from the Plane to the Line

2.4 Materials Not Provided

The Course provides no materials for children.

Groups presenting the training must provide a film projector and suitable room.

3. CLASSROOM ACTION

3.1 Teaching-Learning Strategy

Project films and written lessons for teachers emphasize the teacher assisting the children to develop a mathematical idea by means of a sequence of problems which the teacher makes up to suit his or her own class. The films show the classroom teacher writing a problem on the chalkboard and asking students for an answer. After individual students have responded to several variations of the same idea, the teacher asks the class if anyone can suggest a pattern among the problems they have been doing. Once the students have discovered the pattern and discussed it, the teacher suggests more intricate versions of the same idea.

The Course does not mandate this style of teaching but rather suggests that all of the topics in the Course can be adapted to an individual teacher's own style, once the teacher truly understands the topic. Although all of the films show whole-class teaching, using only words and symbols on the chalkboard--little work with concrete objects--the developers consider the mathematical ideas translatable to small-group teaching and to instruction that uses concrete objects.

The Course's most pressing requirement is that participating teachers spend several hours a week doing assigned homework problems, writing their own similar problems, and then as a group discussing their work. This rather considerable intellectual stretching is intended to give teachers the strength and versatility to adapt the Course topics to the needs of their own pupils and to their own teaching style.

The Course instructions to discussion leaders underscore the developers' faith that ordinary teachers--*if* they are stimulated to think hard about mathematics and conquer their fear of making mistakes--*will* be able to fashion these basic topics into specific lessons suiting their own pupils.

> The notion of presenting a [mathematical] idea through a series of problems is a subtle one. It does not come all at once. There is no formula for doing it, and no general instructions are really of much help. . . .

The real value in having teachers begin early to write problems is that writing problems, even imperfect ones, helps teachers get started in their classes. Trying these ideas with their students is crucial if teachers are to learn about them and learn effective ways to teach them.

The Course would not be appropriate for use by teacher educators who do not share the developers' confidence in the learning capacity of motivated teachers and the developers' reliance on discussion and problem sharing by a group of colearners rather than on lectures by a mathematical expert.

Because the course is designed to be useful without expert mathematical guidance, the discussions are of particular importance. They enable the whole group to profit from the knowledge of those who have had more experience with mathematics, and from the intuition of those with special aptitude for the material. Any reasonable sized group will possess a diversity of backgrounds and talents. Exploiting this diversity for the benefit of everyone is the discussion leader's task. It is not an easy one, but it can be exciting and rewarding.

One of the first things you the discussion leader should do is to be sure you have a class to teach. If your normal duties do not include teaching regularly, you should arrange during the institute to have a class of children to work with on these topics three or four times a week. The course will be immensely more effective if you are genuinely sharing with the other participants the process of learning how to teach these ideas to children.

3.2 Typical Lesson

A typical session of the Course is organized as follows:

Introduction to film	2 minutes
View film	25-45 minutes
Discuss film	10-15 minutes
Discuss written lesson to be handed in	15-25 minutes
Return of corrected lesson (from previous week) and discussion	10-15 minutes
Talk about participants' classroom experiences	0-20 minutes

While the Project does not advocate any specific teaching style, the following materials about the film "Standstill Points," specified for the third week of the Course, show the format used throughout. We include here only the first page of each section of the booklet a trainee receives in each session.

Trainees use the sheet labeled *Summary of Problems in the Film* (Sample 1) while viewing the film; it is intended to help facilitate subsequent discussion. The *Film Discussion Notes* (Sample 2) are for the use of the discussion leaders. These notes are based on questions which teachers taking the Course have asked, and are designed to alert the leaders to possible areas of discussion.

The *written lesson* (Sample 3) parallels the topic presented in the film and provides trainees an opportunity to work out for themselves problems similar to those seen in the film. These lessons may be done either before or after viewing the film.

The returned homework is corrected and subsequently discussed, with particular attention to common points of error or apparent confusion.

The *Supplement* (Sample 4) is intended to provide the teacher with additional resources for implementing the topic in her classroom or for adapting the topic to a particular teaching method or different grade level. Some *Supplements* do not parallel a film topic but present enrichment material.

Sample 1

Summary of Problems in the Film "Standstill Points"

5th Grade, James Russell Lowell School, Watertown, Massachusetts
Teacher: David A. Page

Here is a jumping rule: □ ⟶ 3 × □

Start at 4; make one jump. Where do you land? (12)

Make a jump from 2. Where do you land? From 0? From $\frac{1}{2}$?

How long is the jump from 4 to 12? From 2 to 6? From $\frac{1}{2}$ to $1\frac{1}{2}$? From 0?

Take a jump from z2. Land?

Now a jump from z$3\frac{1}{2}$. Land?

[z2 means "negative 2" or "minus 2" on a number line.]

Using □ ⟶ 3 × □ − 5 :

Who can tell where to start so that you land on 1? (2)

How long is that jump?

Who can tell where to start to land on 10?

Find another place to start besides 2 where there is a jump that is 1 unit long. (0 is suggested; this jump lands at z5 and that is the companion of the jump from 5 to 10) (Answer: 3)

Try starting at z2. Land? (z11)

How long is the jump from z2?

Find a jump where you get back to the same place. ($2\frac{1}{2}$)
(Wrong answers: $1\frac{1}{2}$, 4)

New Rule: □ ⟶ 3 × □ − 19

Where is the place that you start from to land right back where you started? ($9\frac{1}{2}$) (Wrong answers computed: 100, 38, $6\frac{1}{3}$, 9, $9\frac{1}{3}$, $1\frac{1}{2}$.)

□ ⟶ 3 × □ − 117

According to your method, what would be the place that you start from in order to get right back there? ($58\frac{1}{2}$)

Sample 2

Film Discussion Notes "Standstill Points"

Preliminary information:

This class is a heterogeneous fifth grade from the James Russell Lowell School in Watertown, Massachusetts. The teacher is David A. Page. Before this film, he had met with the class three or four times. The filming took place in March. [Film running time: 45 min.]

The discussion that follows occurred in a previous institute. It is intended to alert the moderator to possible questions. Most of these answers were given by participants.

Q: Fairly early in the film the class was doing things like $\square \longrightarrow 3 \times \square - 19$, and somebody gave as her explanation: "You just take the number on the right and cut it in half, and you stay right there." But, when the teacher was going around asking people what the standstill point was, Terry said 38. What was going on there?

A: Terry multiplied 19 by 2 instead of dividing it by 2.

Q: Nancy's answer was $6\frac{1}{3}$. Where did that come from?

A: Possibly she divided the 19 by 3.

Q: "Why is the standstill point one less than the number? How come it works?" (The person who asked this question did not state the question clearly. What she really wanted to know was why you can find the standstill point by dividing the last number in the rule by the number which is one less than the multiplier in that rule.)

A: Since we are looking for a place to start so that we will land at the same place, we can say that for the standstill point the starting number will equal the landing number. In this particular case the rule $\square \longrightarrow 3 \times \square - 19$ may be rewritten as $\square \longrightarrow \square + 2 \times \square - 19$. If we can find a starting number for $\square$ so that $2 \times \square - 19$ is zero, then that starting number must be a standstill point for the rule. (Why?)

Sample 3

WRITTEN LESSON

I. SOME NOTES ABOUT PARENTHESES, AND ABOUT WHAT TO DO WHEN THEY AREN'T THERE

What is $3 + 5 \times 6$? Is it 33 or 48 ?

If you have $10 + 3 + 1$, it doesn't matter whether you do $10 + 3$ first, or $3 + 1$:

$$(10 + 3) + 1 = 14$$

$$10 + (3 + 1) = 14$$

Now do these:

1. $100 + (1 \times 2) =$

2. $(100 + 1) \times 2 =$

3. $(3 \times 3) + 3 =$

4. $3 \times (3 + 3) =$

5. $(5 \times 5) \times 5 =$

6. $5 \times (5 \times 5) =$

Sample 4

Supplement

Answers to Questions About the Film "Standstill Points"

Q: How did the teacher introduce negative numbers, and why were they labelled "z" numbers?

A: This class was introduced to negative numbers on an earlier day by working with the rule $\square \longrightarrow \square - 5$ and making successive jumps. Soon the rule required moving to the other side of zero.

When a jump had been made into the region of the number line below zero, this class was asked what it would like to call the numbers there. Nancie said "zero-one, zero-two, zero-three," which she suggested might be written 01, 02, 03. The teacher took the word "zero" and used the abbreviations z1, z2, z3, thereby avoiding the confusion with decimals that might have occurred with Nancie's symbolism. He worked with this terminology for two class hours. The third day he told the children that henceforth he would call the numbers below zero negative numbers and write -1, -2, -3, and so on.

Other classes have selected the letter b (for "below") as a symbol: b1, b2, b3, or 1b, 2b, 3b, and so forth.

Many children have heard about negative numbers, if only from other children. Notice that one of the virtues of working with these unconventional symbols is that they can help make clear the distinction between a "$-$" used for the operation of subtraction, and a "$-$" used to mean "the opposite of". A child who has done z10 $-$ z5 is in a better position to see what is going on when he first faces $-10 - (-5)$.

3.3 Evaluation of Students

There are no materials or discussion regarding evaluation of students.

(See 5.3 - Program Evaluation).

3.4 Out-of-Class Preparation

Each participant is given homework assignments which parallel the topics presented on film. These written lessons, which take about two hours to complete, are to be done between class meetings. They are marked by a "corrector" and discussed at the subsequent session. The *Supplements* contain many opportunities for optional math practice.

The whole Course implies that a teacher will do considerable out-of-class preparation making up her own sequence of problems based on what she has learned in the Course.

The teacher is greatly encouraged, even expected, to "build on the materials, to do her own invention and try out things in her own class, things not specifically taught in the institute." It is also hoped that the teacher will share with the other trainees accounts of problems and successes.

4. IMPLEMENTATION: REQUIREMENTS AND COSTS

4.1 School Facilities and Arrangements

The Course is designed to provide a school district or a teacher education institution with everything its own personnel will need to conduct instruction. The people who conduct the Course do not need to take special training or to hire consultants.

The films and mimeographed materials are intended to provide not only the materials needed by the participants but also the instruction needed by the local discussion leader, who conducts the weekly sessions, and by the "corrector," who reads and comments on the written lessons turned in by participants. One leader and one corrector are usually appointed for each ten participants. However, in some cases, the tasks of discussion leader and "corrector" are performed by the same person. If the films are rented, the Course sponsor must schedule the showing of each film so that it can be returned the following day.

The Project offers consultant services to interested colleges and school districts; Project staff will help set up the Course or teach a session. Assistance is available 24 hours a day by telephone.

4.2 Student Prerequisites

The Arithmetic Project topics are supplementary, not in any particular sequence, and not assigned to a particular grade level. Teachers participating in the Course are expected to translate topics to practical teaching situations in their own classrooms--grades 1 through 6.

Each topic is illustrated by means of a film showing classroom teaching of the topic with a specific class. Most of these films show fifth-grade pupils, but that does not mean that the developers consider these topics appropriate only or mainly for fifth graders. The developers expect participant teachers to know the mathematical abilities of their students and, through the experience of their own learning of the topics, to select appropriate ones and present them appropriately to their own pupils.

While skill and experience in assessing students' learning levels is not a prerequisite, the Course does demand a teacher's conviction that such assessments are vital and realistic aspects of the teaching act, and willingness to work hard to learn this skill. Through its requirement that teachers solve homework problems and write problems for their students, the Course aims to give participants insight into the prerequisite skills and concepts for each topic, and practice in designing learning experiences for their own pupils that will guide the pupils' grasp of the topic.

4.3 Teacher Prerequisites

Participants in the inservice Course should be concurrently teaching a mathematics class for elementary pupils so that they can experiment with and practice skills gained from the sessions. Teachers should be prepared to spend about two hours a week completing homework assignments.

4.4 Background and Training of Other Personnel

The discussion leaders and correctors who conduct institutes using the Course should have a "better than average background in math," and be "interested in helping others improve their mathematics teaching." Previous experience with the Course or with modern math is not essential.

The discussion leader should be teaching a class of elementary students on whom to try out the problem sequences presented in the Course, thus gaining understanding of the task of transferring the topics to a particular classroom. Both the discussion leader and the corrector should actively participate in the Course by doing all the written lessons.

The discussion leader should be familiar with the district syllabus and the textbooks or programs being used by the teacher participants so as to be able to help teachers adapt topics to the curriculum.

The discussion leader's experience and talent as a facilitator of teachers' learning seem indispensable to the success of this Course. The discussion leader's priority task is defined as drawing out ideas from participants, not putting in facts. Similarly, the corrector's task is defined as helping participants learn from their mistakes, not certifying the level of their achievement.

4.5 Cost of Materials, Equipment, Services

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Film rentals*	1 film per institute session	EDC		
Materials for trainees: Twenty books (one per session) containing homework assignments, film summaries, mathematical supplements.	1 per trainee per session	EDC	Cost varies from $50 per teacher for 20-session institutes of 30 teachers to less than $22 per teacher for institutes with 150 participants. Full set of 20 films together with leader and participant materials may be purchased for $3,200; purchasers receive 40% discount from $20 price for additional sets of participant materials. Seasonal discounts may apply.	
Materials for staff: One *Guide for Course Leaders*, containing discussion notes on films and homework, and 18 *Corrector's Guides*.	1 per staff member per session			
Staff: Discussion leaders and correctors	Varies from 1 per 10 trainees to 1 per 30 trainees	User	User arranges stipend, assigns staff, or recruits volunteers	
16mm film projector with separate loudspeaker		User		

Recommended Supplementary Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Consultant services		EDC has names of leaders of previous institutes	User must pay stipend and transportation	

*Each film must be returned the day following each session for normal leased program. Films may also be purchased or obtained on lease-to-purchase plan.

4.6 Demonstration Sites

The following is a partial list of school systems, colleges, and universities who have used the Arithmetic Project Course. Further information can be obtained from the Education Development Center.

Alaska:	Anchorage Borough School District Anchorage, Ala.
Connecticut:	EDC Bridgeport Pilot Communities Project Bridgeport, Conn.
Illinois:	University of Illinois Champaign, Ill.
Malaysia:	Seameo Regional Centre for Education in Science and Mathematics Penang, Malaysia
Montana:	Western Montana College Dillon, Mont.
Nebraska:	Chadron State ` lege Chadron, Neb.
North Carolina:	University of North Carolina Chapel Hill, N.C.

5. PROGRAM DEVELOPMENT AND EVALUATION

5.1 Program Development

In an attempt to upgrade elementary level mathematics teaching, David A. Page and mathematician colleagues at the University of Illinois Curriculum Laboratory in 1958 began inventing and developing new topics in mathematics for the elementary grades. The products sought were not more "new math" but rather fresh and interesting ways to present arithmetic basics. The project was located, however, at the curriculum laboratory directed by Max Beberman, "father of the new math." The project was supported by the Carnegie Corporation of New York.

Five years later, in 1963, the Arithmetic Project was invited by the Education Development Center to develop Course materials for teachers that would enable nonspecialist teachers to teach topics in their own classrooms. This second phase was funded by the National Science Foundation.

A number of topics were tried out in classroom situations, and of these, ten were selected for inclusion in the Course for teachers. The basis for selection was that they were mathematically rich, could be taught by teachers with an average background in math, and were found "to engage the interest and imagination of children [and] are not too hard for children to solve. . ."

To introduce its materials to teachers, the Project prepared an inservice Course that was self-contained and could be given without expert mathematical guidance. Completed materials were first published in 1968; a revised edition was released in 1973. The Course, formerly called the University of Illinois Arithmetic Project, later became The Arithmetic Project Course for Teachers.

5.2 Developer's Evaluation

No formal evaluation has been made that shows whether or not the Courses for teachers are successful in transmitting their ideas to the trainees or whether there is transfer from the Course to the classroom. This may be a serious lack in the minds of evaluation-conscious administrators. Project staff at EDC report personal observations that more than one-half of the trainees do use the topics in the classroom on at least an occasional basis.

A questionnaire is given to the discussion leaders of all Courses. They are asked whether or not trainees are using in their classrooms the topics and methods discussed in the Arithmetic Project Course.

From the responses of the leaders, it appears that the extent to which a teacher uses Project topics in the classroom depends on the interests of both teacher and students. The questionnaires state that from one-half to "almost all" have tried some of the ideas in the classroom. It is the developer's belief that teachers who complete the Course "might be able to devote a fourth or more of their arithmetic class time to pursuing Project materials."

One attempt has been made to see whether students increase their knowledge of mathematics when taught the topics and methods of the Project. Standardized pre- and posttests (Stanford Achievement Test and Metropolitan Achievement Test) were given to elementary pupils taught by Project staff members (not teacher trainees) to elementary pupils in Watertown, Massachusetts in 1968. The developer states the test showed that the computational skills of the students increased an average of one full grade compared to the two control groups. They emphasize, however, that standardized tests are not available to test the topics developed by the Project.

5.3 Comments on the Program (Participants)

At the end of each Course the leaders are asked for comments concerning the Course. While most of the responses describe the Course as "very good" and "excellent" there are some specific criticisms. A number of participants stated that not enough attention was paid to primary teachers (K-2) and that the appeal of the Course was limited. Regarding the mathematics involved, one leader wrote: "The course assumes a certain amount of basic mathematics knowledge. It seems to me it is actually a *second* course for most teachers." Other participants stated they often could not see the mathematical rationale involved and questioned using a newly contrived terminology for well-known concepts.

The films are occasionally criticized as being too long, having little relevance to the topic of the week, and being redundant about the same topic. However, many leaders felt the films demonstrated an effective method of teaching and had a "definite, positive impact on teachers."

Despite such criticisms of the Course, many teachers speak positively of their experience using in their classrooms the methods they have seen in Project films. "Third graders became enthusiastic with the idea of playing a game in mathematics," "The students were all excited," "The sixth graders have developed a new attitude toward math--not all drudgery--they now feel math can be fun." These are typical of the comments made.

Their own successful experience with mathematics gives many participants a better understanding of the problems children are encountering, and a new-found confidence in their own ability.

5.4 Project Funding

The University of Illinois Arithmetic Project was initially supported by the Carnegie Corporation of New York. Subsequent funding was obtained from the National Science Foundation and the Ford Foundation.

5.5 Project Staff

David A. Page, professor of mathematics, University of Illinois at Chicago Circle was the originator of the Project.

Jack Churchill, Associate Director, University of Illinois Arithmetic Project, Education Development Center, 55 Chapel St., Newton, Mass. 02160 is presently in charge of the Project and the person to whom further inquiries should be directed.

NUFFIELD MATHEMATICS PROJECT

and

MATHEMATICS FOR SCHOOLS

66/67

INTRODUCTION

> *A group of 10-year-olds had collected a number of bird and animal skulls and wanted to measure the capacities of the brain cavities. They devised a method of measuring them--filling the skull cavities with sand--and then had to make a cubic container for measuring the sand. A cubic inch worked for the cat and rabbit skull, but not for the bird's skull, so they worked out a quarter-inch container. . . . Few adults would have been able to devise as elegant and simple a solution to a difficult question of measurement.*
>
> *A teacher. . .said she began by deciding to base all teaching on the premise that no child should be asked to accept a mathematical truth on her authority, which meant that she had to arrange matters so children could learn from themselves. . . . She discovered that whenever possible it was best to use material from the immediate environment: leaves from trees were a better "apparatus" for understanding perimeter and area than rectangles, so her pupils fitted string along the edges of leaves, and got the area by laying the leaves on flat pieces of paper marked off in square units. After a time, she estimated, the children were spending about a third of the time experimenting with materials, a third of the time discussing what they found with each other and the teacher, and a third practising skills. She found little difference in their computation work. They began detailed explorations: "From a study of making polygons rigid, came an interest in bridges and towers; from tesselation with hexagons came an interest in bees, patchwook quilting, and modern architecture. . . ."*

Joseph Featherstone described the teaching of mathematics in English primary schools in a series of articles in *The New Republic* in 1967 and 1968, from which the paragraphs above are quoted.[1] His and other observers' reports of the active-learning innovations in England have deeply influenced American schools. Now two mathematics programs which have grown out of the English informal schools movement are available in the United States: the *Nuffield Mathematics Teaching Project*, and *Mathematics for Schools*. Both programs are designed to alter traditional mathematics so that children can see the mathematical implications of everyday situations and conduct mathematical investigations. Real-life objects are used, and problems are posed stemming from children's own interests, in the belief that school work of this kind makes children purposeful and self-directing.

Both programs are described in this one report so that their similarities and differences can stand out. The Nuffield publications are aimed at the teacher and are not "potted lessons," in Featherstone's phrase, but rather explanations of math topics, plus suggestions that teachers can use to design lessons. Mathematics for Schools is a complete, K-6 mathematics curriculum. There are detailed instructions that tell the teacher what to do throughout each lesson, and exercise books for the students.

Teachers using either program will need to adapt them to American ways--or perhaps to learn some English ways. In either case, they will find that active-learning math demands far more preparation time, organization, flexibility, attentiveness, and responsiveness to students as individuals, than does a conventional curriculum.

BASIC INFORMATION

Program names: Nuffield Mathematics Project *and* Mathematics for Schools

Format:

Nuffield: Series of 13 guidebooks for teachers, organized into 3 parallel "streams" running from age 5 to 13: computation, algebra, and geometry. Also guides for supplementary topics, 3 sets of activity cards, and 20 project manuals for upper elementary and junior high students.

Mathematics for Schools: Six manuals for teachers containing lessons for age 5 to 13, and 12 student workbooks.

Uniqueness: Traditional and modern math topics presented through children's experiences with manipulative objects and activities in their own surroundings. This is the active-learning approach identified with the informal, "integrated day" English primary schools.

Content: Sets, counting, arithmetic, operations, measurement, integers, geometry, algebra, fractions, decimals, statistics, probability, and functions.

Suggested use:

Nuffield: Teacher uses guidebooks and inservice training to design lessons which fit into the standard curriculum, or to design whole new curriculum.

Mathematics for Schools: Complete mathematics curriculum.

Target audience: For both programs, students of all ability levels, ages 5-13.

Length of use:

Nuffield: At teacher's discretion.

Mathematics for Schools: Math period daily throughout eighth grade.

Aids for teachers: Both programs include a series of teachers' guides. In England inservice training accompanies introduction of these programs. In the United States training in active-learning approach to mathematics is strongly urged.

Date of publication:

Nuffield: Publication began in 1967; not all materials are yet available in the United States.

Mathematics for Schools: 1970-74.

Directors/Developers:

Nuffield: The Nuffield Foundation and the Schools Council, Nuffield Lodge, Regents Park, London N.W.1, England.

Mathematics for Schools: Addison-Wesley Publishers Ltd., West End House, 11 Hills Place, London, England. The late Harold Fletcher was senior author of the program.

Publisher:

Nuffield: John Wiley & Sons, Inc., 605 Third Ave., New York, N.Y. 10016.

Mathematics for Schools: Addison-Wesley Publishing Co., International Division Headquarters South St., Reading, Mass. 01867.

1. GOALS AND RATIONALE

1.1 Goals

"The Victorian clerk, sitting on a stool in a counting house, kept his ledgers meticulously," the Nuffield developers recall. "He wrote in beautiful copperplate, his immaculate figures were neatly underlined, and his calculations were always accurate. . . . Elementary education as it then existed encouraged the growth of these skills."* In the twentieth century, when "the pace of life began to quicken," elementary schools took on the responsibility of teaching speed in addition to neatness and accuracy, but the arithmetic was basically the same.

Now, however, the age of computers renders both the Victorian and the twentieth century clerk nearly obsolete, and there is need for "people who can assess situations, who can formulate and solve problems." The value of mathematics for the average person thus is to gain not computation skill but intellectual power. Even more, "Mathematics offers a way of ordering all experience" because it reveals pattern and relationships--aesthetic and philosophical insights. Elementary-school mathematics today ought to be a fuller thing, then; not just arithmetic. And it should be pleasurable, not dreaded. If these changes are made in early school years, mathematics can be opened up to all students, not just those who may become professional mathematicians, scientists, and engineers.

Both the Nuffield Mathematics Project and Mathematics for Schools intend to broaden the content of the mathematics taught in the elementary school, and to change the manner in which students experience math. The two programs emphasize an active-learning approach. What the student should gain from active, naturalistic, modern math is summed up by Edith E. Biggs, one of the originators of this approach.

> Our aims. . .are to give our students (1) the opportunity to think for themselves, (2) the opportunity to appreciate the order and pattern which is the essence of mathematics, not only in the man-made but in the natural world, as well, and (3) the needed skills.[2]

Joseph Featherstone lists the active-learning teacher's criteria for a good thinker: "Confidence, concentration, and an ability to make informed rather than haphazard guesses and estimates; mental habits of synthesizing ideas and making analogies; the capacity to communicate thoughts and feelings in various ways."[3]

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*Unless otherwise indicated, all quotations are drawn from materials issued by the developer.

To these cognitive aims are added the intention to make math practical and bound up with real-life experiences and interests of youngsters. In this way developers hope to change children's attitudes toward math from boredom, fear, and feelings of inadequacy to enjoyment, purposefulness, and confidence--what Harold Fletcher, senior author of Mathematics for Schools, called an "I'll have a go at it!" attitude.

The English educators' use of the word *freedom* to mean *informality* is frequently misunderstood by Americans, as is pointed out by Featherstone in his comments on American adaptations of the British active-learning approach.

> Letting children talk and move about is helpful in establishing a setting in which the teacher can find out about students; it helps children to learn actively, to get the habit of framing purposes independently, using their own judgment. But this freedom is a means to an end, not a goal in itself. . . . Informality is pointless unless it leads to intellectual stimulation. Many children in [American] 'free' schools are not happy, and one suspects that part of the reason is that they are bored with their own lack of intellectual progress.[4]

Both Nuffield and Mathematics for Schools recognize the ability of young children to grasp the "purity and order" of mathematics--its abstractness--if encouraged to do so. Edith Biggs says that making this aspect of mathematics available to average as well as bright children can result in the early discovery of unusual creativity and persistence in some students who do not show other signs of high intelligence. In practice, however, teachers who themselves lack mathematical confidence tend to "condemn children to the eternally mundane--the postman and the candy shop," Featherstone observed.[5] Thus Harold Fletcher, putting together Mathematics for Schools after his experience in the Nuffield Project, was determined to stress math's patterns more than its practicality. "I prefer *mathematics* to the *results* of mathematics," Fletcher declared.[6]

Preparing students for mathematics in British secondary schools is a high priority for developers of both programs. First, students must be enabled to pass the national examinations for entrance into secondary school. But, of equal importance, they must build deep conceptual foundations during their middle years for the advanced algebra and geometry they will encounter in secondary school.

Although the programs share many similarities, they differ in the stress they place on individualized learning. Because the Nuffield Project set a goal of helping teachers identify every student's individual learning stage and style and designed a learning program to fit, it offers a teacher suggestions rather than set lessons. In contrast, Mathematics for Schools did not attempt this complete individualization, and provides instead a standard curriculum with options for flexible pacing.

1.2 Rationale

By dedicating its series of teacher's guides to the Swiss psychologist Jean Piaget, the Nuffield Project acknowledges that it borrows its fundamental approach from his theories about the stages of growth which children pass through in developing thinking power. As an adaptation of Nuffield, Mathematics for Schools is based on Piagetian theory too.

In both programs the curriculum units for early years are designed to engage and stimulate children who are in the stage of *intuitive thinking*, in which they gather information from the appearances of objects or events rather than by testing their real nature. By experiencing, understanding, and assimilating a rich variety of things, places, and happenings, children prepare themselves for the stage of *concrete operations*, in which they can work investigatively with problems that involve the use of objects they can manipulate. According to Piaget, each child's movement through these stages is individual but the general pattern is that it takes until the age of 11 or 12 for the child to be able to learn from abstracted information right away instead of from concrete experience followed by concepts. Piaget's theory is the basis for both programs' emphasis on children's play-like work with a great variety of materials--sand and water, objects for counting, shapes, devices for measuring--and on the teacher's *discussing* the meanings of children's own experiences, rather than *telling* them rules and formulas.

In the Nuffield Project Piaget's theory is also used as the rationale for recommending that teachers regularly diagnose each child's level of learning and custom design his learning activities. Additionally, Piaget is the basis for their series of diagnostic guidebooks. In contrast, Mathematics for Schools sets forth a standard curriculum without any diagnostic tests. This may indicate that these authors attach less importance to the teacher's trying to chart a Piagetian learning path for each child separately.

Both programs share the modern mathematician's rationale for teaching children more math than is comprehended in arithmetic, both because it is inherently interesting to children and because arithmetic alone is no longer adequate preparation for later life.

2. CONTENT AND MATERIALS

2.1 Content Focus

Nuffield Project writers describe the mathematical content of their work as "drawing together the best of the new and the best of the old."

However, they emphasize that they intend the teacher to focus on *how* children learn, not *what* they learn, and in any event it is left to the teacher and headteacher to select which of the Nuffield topics to teach. Nuffield developers accept the real possibility that the school people will leave out most of the new stuff, and use the guides only for "enrichment" at first. Be that as it may, the following list of topics covered in the Nuffield guides conveys their intention to gradually modernize the traditional curriculum, not only in methodology but also in content.

Topics for ages 5 to 9 in Nuffield guides:

Number work: sets, counting, addition, multiplication, number line, subtraction, and modular arithmetic.

"Environmental" work: weight and volume, length and area.

Geometrical work: "shape and size," straight lines, angles, verticality, horizontality, perspective, symmetry, and patterns.

Pictorial representation and graphs: open sentences, truth sets, Cartesian coordinates, and graphs.

Topics for ages 9 to 13 in Nuffield guides:

Arithmetic: positive and negative numbers (integers), subtraction, division, fractions, decimals, indices, and large numbers.

Geometry: rigid and non-rigid shapes, symmetry, rotations, reflections, translations, tesellations, two-dimensional patterns, vectors, invariants, relationships (squares, cubes, circles), similarity and ratio, and topology.

Algebra: symbolic forms of arithmetic and geometry, graphs of inequalities, intersection of two graphs, graphs using integers, mechanics, speed and gradient, and functions.

Probability and Statistics.

A booklet designed to present the rationale for modern math to parents gives Nuffield definitions of the elementary level topics as well as the Nuffield designers' plan of how the various topics at various levels reinforce each other. This is *Your Child and Mathematics* by Professor W. H. Cockcroft, chairman of the Project's consultative committee.

> All of these [arithmetic, geometry, and algebra] are woven together; none can stand separately. . . . The aim is to teach children to see mathematics as a unified

way of thinking about the world, not as a separate collection of technical subjects. . . . In each of them hand, eye, and thought all have a part to play.

This booklet, written for traditional-arithmetic-minded parents, should be indispensable to teachers. Together with the index of topics, *The Story So Far*, it can serve as a syllabus of the mathematics covered in the guides for ages 5-11. Cockcroft makes clear that the ideal use of the material in the guides is to fit it into a strong, comprehensive mathematics framework of a decidedly modern design.

A good deal of emphasis is given throughout the guides to the importance and the nature of the geometry which is presented.

> As you will see, this [active-learning] approach brings ideas previously treated in geometry lessons at the age of 11 or 12 to a much lower age group. Of course, to do this, the treatment of the work must change. . . . One does not expect primary school children to be proving "theorems." One does expect them to be developing, from practical experience, an understanding of basic theoretical geometrical concepts.

In middle school years (ages 9 to 13), "this marriage of practical geometrical work with theoretical work" will be accomplished through "the theoretical interpretation of turning, moving, reflecting, and so on." Cockcroft also explains the reason for introducing algebra to children: it is a symbolic system to be used to express the patterns, and the rules that can be made about these patterns, in *both* arithmetic and geometry.

> It is not the object of the Nuffield Project to give highly abstract symbolic algebraic work to middle school children, but it is [rather] to see that they leave middle school aware of many of the algebraic patterns present in all their work. . . .

Teachers using Nuffield guides frequently neglect the theoretical emphases which were intended by the curriculum designers, probably because the teachers do not themselves understand the abstractness of modern mathematics. The late Harold Fletcher attempted to remedy this not only by educating teachers (his courses for teachers were famous throughout England) but by combining his Nuffield and his teacher education experiences into a structural curriculum series. This series, Mathematics for Schools, covers basically the same topics as have been listed for the Nuffield guides.

2.2 Content and Organization of the Subdivisions

"Streams" in the Nuffield guides. The nondirectiveness of Nuffield's presentation should not be interpreted to mean that the guides themselves have no structure. There is a sophisticated, *intended* framework on which suggested activities are to be hung. Whether teachers use this subtle framework or one similar, or construct their own, depends on their own experience in teaching and in mathematics and on the curriculum structure and teacher-supporting resources of the schools in which they work. The guides assume the existence in England of teachers' centres, where the Nuffield topics are explained by a mathematics consultant, discussed, changed, fleshed out, tailored to individual classrooms, and practiced.

The guides are organized into three "streams," which run from beginning through advanced levels: Computation and Structure (Books 1-5), Shape and Size (Books 1-5), Graphs Leading to Algebra (Books 1-3). Computation is symbolized by a circle, Shape by a triangle, and Graphs by a square. All guides except Book 5 in Shape are available in the United States; publishers expect this guide to be available in late 1974. The material in each stream rises and broadens in difficulty and complexity from age 5 till age 13. Thus, in general, topics are meant to be taught in the order in which they appear in a book, and books within a stream are to be taught in order. However, the three streams have to be intermingled. Suggestions for doing this are offered by means of references from one topic to other topics in different books. Intermingling is complicated by the fact that the streams do not begin at the same age level, and they advance at differing rates. Thus, *Graphs 2* presnets more difficult material than *Computation 2* or *Shape 2*.

The following is a chart of the topics which appear as teaching units in the Nuffield streams.

	Computation & Structure	Shape & Size	Graphs Leading to Algebra
Book 1	relations, sorting, 1 to 1 correspondence, conservation of number, ordering, counting, numerals, number strip, addition, mapping, presubtraction (ages 5-7)	sand and water play; picture, pattern, and model making; music, movement; bricks, constructional play (ages 5-7)	block graphs: interpreting, using for computation and for science reports; graphs showing sets, inequalities, computation, measurement (ages 5-10)
Book 2	development of natural numbers, length, weight, capacity, addition, place value, time, money (ages 6-8)	3-dimensional space: volume and capacity; 2-dimensional space: symmetry and regular shapes, area, right angles and half right angles, perpendicular and parallel lines (ages 6-9)	coordinates, open sentences and truth sets; graphs of inequalities, intersections, and coordinates using the integers; open sentences and graphs (ages 8-13)
Book 3	addition tables and problems, commutativity, associativity, subtraction, multiplication, simple sharing, factors and primes, fractions (ages 7-11)	area, volume, parallels and angles, circles, tesellations, reflectional symmetry, regular polygons, translations (ages 7-11)	rational numbers, simultaneous equations, simple linear programming (ages 11-13)
Book 4	extension of place value, modular arithmetic, integers, application of integers, large numbers and indices (ages 10-13)	2-dimensional patterns, vectors, invariants, similarity, relations--area to volume, etc., model making, enlargements (ages 11-13)	
Book 5	addition of decimal numbers, rational numbers as equivalence classes of ordered pairs of natural numbers and as points on a number line, ordering and four fundamental operations (ages 10-13)	Publishers expect this guide to be available for distribution in late 1974.	

A Nuffield unit. Nuffield guides address the teacher, not the student directly. Each guide is divided into sections; each section is devoted to a single mathematical topic. These sections are similar to units in an American textbook series. A section explains the mathematics of the topic to be taught and presents suggestions for activities through which students can first *experience* the working of the concept and then *analyze* what is happening and discover patterns. The suggestions include (a) descriptions of how to introduce topics, pose problems, and initiate student activities; (b) assignments which can be copied onto cards and handed out to individuals or groups of students to do by themselves; (c) ideas to be covered in teacher/student discussions; and (d) reproductions of pictures, stories, graphs, and sums done by children in Nuffield classrooms.

The series also includes a set of "weaving" guides which are like supplementary or enrichment units in an American curriculum. General guides present an overall view of the series and explain how elements interrelate. Modules, each consisting of a short teacher guide and student task card, cover selected supplementary topics for 11- to 13-year-olds. The curriculum also includes three sets of problems for this age group.

Units and levels in Mathematics for Schools. The Fletcher writing group labeled their work an "integrated series" to distinguish it from the Nuffield Guides' three streams and do-it-yourself approach. Mathematics for Schools arranges lessons into Level I (corresponding to the nongraded English "infant school"--ages 5 through 7) and Level II ("junior school" and first two years of secondary school--ages 7 through 13). Within Level II there are five sublevels. The lists on the following pages show the topics treated within each level and make clear the spiraling sequencing and the interweaving of computation, algebra, and geometry, similar to that in Nuffield. Each title in a list is the name of a unit (called a *section* in the Teacher's Resource Book). A unit may contain from one to ten lessons, each consisting of developmental activity, discussions, and exercise pages. The exercises for several units are combined into booklets called *Children's Books*, of which there are seven in Level I, and ten in Level II.

Units for primary level (ages 5-7) in Mathematics for Schools (listed in the order in which they appear in the teacher's manual):

Introduction to Sets
Sets and Subsets
Solid Shapes
The Idea of Matching
Cardinal Number: 2, 4, 3, 5, 1
Ordering Cardinal Numbers: 1-5
Cardinal Number and Sequential Patterns: 0, 6, 7, 8, 9, 10
Cardinal Number: 1-10
Solid Shape
Pre-Measurement Activities: Length (meters)
Introduction to Addition (number line, mapping)
Basic Addition Facts: Totals to 10
Comparison: Taking Away, Adding On
2-Digit Numbers: Introduction
Measurement Activities: Capacity, Mass
Counting On (Addition by counting on a number line)
Counting On and Addition
Counting Back and "Taking Away"
"Taking Away" and Addition: Exchange and Coin Recognition (vertical addition and subtraction)
"Sets Of" (commutative property of multiplication)
Sharing
Algebraic Relations: Open Sentences and Truth Sets
Addition: Commutative and Associative
Measurement Activities (height, length, mass, capacity, time)
Solids: Volume, Faces, Plane Shapes
Algebraic Relations: Open Sentences and Truth Sets
Plane Shapes: Covering Surfaces
Algebraic Relations: Inequalities
Plane Shapes: Conservation, Vertices, Edges
Open Sentences and Truth Sets
Symmetry

Level II (ages 7-13) in Mathematics for Schools:

Book 1

Tallying and Addition
Difference and "Take Away"
Enrichment: Number Facts
Addresses and Regions
Measurement: Length (m and cm) and Mass (g)
Multiplication
Measurement: Time
Sharing
Symmetry
Addition: Tens and Units

Book 2

Addition and Difference: Tens and Units
Measurement: Area (cm^2), Capacity, Volume, and Mass (g)
Multiplication
Angles and Direction
Addresses and Regions
Sharing
Shapes: Properties of Plane Shapes

Book 3

Statistics
Addition: Hundreds, Tens, and Units
Statistics
Measurement: Area (cm^2)
Difference: Hundreds, Tens, and Units
Measurement: Mass (g and kg)
Multiplication
Angles
Number Patterns

Book 4

Addition and Difference Involving Money
Measurement Height and Length
Multiplication
Angles
Division
Introduction to Probability
Fractions
Algebraic Relations
Shapes: Circle
Introduction to Decimals

Book 5

Bases
Statistics
Addition and Difference
Fractions: Addition
Measurement: Accuracy
Algebraic Relations
Sorting: Classifying Shapes
Multiplication
Fractions: Multiplication
Measurement Area (cm^2)

Book 6

Flow Charts
Division
Shapes: Solid and Plane
Fractions: Multiplication
Probability
Measurement: Volume (cm^3)
Pattern
Decimals
Algebraic Relations
Translations, Reflections, and Rotations

Book 7

Surveying
Decimals
Division
Vectors
Fractions
Probability
Decimals
Shapes: Circles and Discs
Algebraic Relations
Flow Charts

Book 8

Integers
Fractions
Transformations
Decimals and Percentages
Algebraic Relations
Measurement: Volume
Statistics
Proportion
Time and Speed
Similarity
Integers

Book 9

Algebra: The Laws of Arithmetic
Pythagoras' Theorem and Square Roots
Integers
Vectors
Enrichment: Computation
Punched Cards
Symmetry
Money
Measurement: Significance and Index Notation
Plane Sahpes

Book 10

Relations and Functions
Integers Rationals, and Reals
Flow Charts
Eastings and Northings
Probability
Measurement: Volume, Mass, and Relative Density
Enrichment: Computation
Similarity
Formulas and Equations
Indices, Slide Rules, and Logarithms

2.3 Materials Provided

Student. *Problems--Green Set, Problems--Red Set,* and *Problems--Purple Set.* These are sets of activity cards with accompanying teacher's book giving answers, discussions of the problems, and ideas for follow-up. In general, these are mental puzzles which can be done with paper and pencil rather than with manipulative materials. They are meant for students of 11 years and older.

Work cards. Each of 20 modules (see description under teacher in this section) includes a set of about 20 cards. Students complete paper-and-pencil and active-learning exercises individually or in small groups.

Several kinds of apparatus have been designed by the Nuffield Project, notably the Multiboard, a collection of number strips, cubes, colored washers, pegboard, rubber bands, slide rule, Napier's rods, and 144 square. This must be obtained from science and math equipment suppliers rather than from the project.

The 13 student books in Mathematics for Schools are not texts but workbooks containing only exercise pages. All instruction is carried out by the teacher. In Level I (ages five-seven) students write in 7 exercise books. Level II has 10 exercise books, all of which are stiff-cover and meant to be reused. Children write exercises on separate pieces of paper. These books have little narrative instruction and thus do not depend heavily upon the student's reading ability. Lively, richly detailed cartoons illustrate the exercises and instruct the student; for instance, at the end of the exercise pages for each lesson a cartoon of a child holding his hand up signals the student to stop work and see the teacher. This is to prevent the child from beginning new written work before the teacher's introduction of developmental activities.

Teacher. *Teachers' Guides.* The 13 Nuffield guides for teachers are small softcover booklets with both black and white and full color illustrations of children's work. These exuberant and charming samples of Nuffield results should be powerful motivation to American teachers to cope with the Englishness of the books and to undertake the work that must be done to translate the topics into American classroom lessons Teachers should study the whole series of guides to get a feel for the sequencing of topics and intermingling of streams. Then they can pick individual guides from which to develop lessons.

Weaving guides. These eight booklets are of the same design as the teaching guides; they are like supplementary or enrichment units in an American curriculum: How to Build a Pond, Desk Calculators, Probability and Statistics, Mathematics with Everything, Computers and Young Children, Logic, Nuffield Geometry, and Environmental Geometry. (Because the American publishers consider the last book, Environmental Geometry, too expensive to distribute, it will only be available until the current supply is exhausted.)

General Guides. *I Do, and I Understand* (how to change over to active learning) and *Your Child and Mathematics* are introductory booklets which should be read in preparation for serious work with Nuffield. Other general guides which teachers may find useful include *The Story So Far* (an index to materials covered in the three streams--Computation 1-3, Shape 1-3, and Graphs 1-2), *Math: The First Three Years* and *Math: The Later Primary Years* (general description of mathematical activities which teachers should encourage), *Maths with Everything* (explains how teachers can provide valuable math experiences for 5- to 7-year-olds), and *Into Secondary School* (how Nuffield mathematics can be used with 11- to 13-year olds). Film summaries of three of these books, *I Do, and I Understand; Maths with Everything; and Into Secondary School*, are available.

Evaluation Guides. Two primary level evaluation guides, *Check up 1 and 2*, are available.

Modules. Modules consist of about 20 work cards for students and a short teacher's booklet. The booklet contains drawings of the cards in addition to background information and explanation. The modules, intended as supplementary material for 11- to 13-year-olds, may be used in any order with groups or individual students. Modules currently available in the United States are: *Speed and Gradient 1, Decimals 1, Number Patterns 1, Symmetry,* and *Angles, Courses and Bearings.* The publishers have ordered 15 additional modules; they are not yet available for distribution.

There are six *Teacher's Resource Books* in Mathematics for Schools: one for Level I and five for Level II. They provide the organization for teaching through all nine years of the curriculum, as well as the planning for each daily lesson. They are softcover, 11 x 8½ inch books which include small reproductions of the children's exercise pages.

2.4 Materials Not Provided

The everyday objects called for by the Nuffield guides and the Mathematics for Schools Teacher's Resource Book must be gathered, organized, and efficiently stored by the teacher. The following list gives only an inkling of the kinds of things the teacher will need.

> Sand, water, dried peas, acorns, nails, matchboxes, straws, pipe cleaners, cubes, cylinders, colored beads, balls, bricks, Cuisenaire rods, Dienes blocks, balances, equalizers, rulers, compasses, protractors, string, jars, boxes, scissors, puzzles, plasticene, timers, clocks, and thermometers.

Everything is carefully selected for its teaching purpose. In *I Do, and I Understand* the Nuffield developers caution:

> "Setting the children free" does not mean starting a riot with a roomful of junk for ammunition. . . . Storage is the important issue here. All materials should be adequately stored in suitable containers, clearly labelled, in a precise position in the classroom. . . . Expense is involved [in gathering measuring devices], but the criterion must always be that of quality. Inadequate tools only lead to frustration, and one really good pair of scales is a far better value than five inaccurate ones.

The activity cards suggested in the Nuffield guides must be prepared by the teacher, personalizing for her own students the ideas suggested in the book. Eventually these assignments are supposed to arise entirely from the teacher's observation of each child's activity. The task assigned must always involve more than active measurement or "mental agility"; it must make the child look for patterns, consult with the teacher, form judgments, and make decisions.

The Nuffield developers also recommend having some conventional mathematics texts and workbooks in the classroom as source books for the teacher and references for students. Teachers who believe that students need drill can use exercises from such books.

3. CLASSROOM ACTION

3.1 Teaching-Learning Strategy

Both the Nuffield Project and Mathematics for Schools try to give the student a concrete experience of a mathematics concept and then to help him capture it in the form of an idea which he uses over and over, in later learning and in different situations. "Active-learning" is their catchword for this process. Both programs intend to move children gradually beyond concrete experiences and their concomitant computational skills into recognizing and working with the abstract patterns which are the essence of mathematics. Mathematics for Schools moves more assertively in this direction because its lessons and curriculum are spelled out, while Nuffield guides only suggest what the teacher should do.

Nuffield stages in learning. In their introductory guide, *I Do, and I Understand*, the Nuffield developers sketch the "discovery" line of development for a child learning mathematics: "experimentation → thinking → communication." They reject "demonstration → explanation → memory → practice" because "memory, although a useful tool, is clearly fickle" unless it is linked with *a conceptual understanding* already fixed in the child's mind, and "practice is necessary, but there is a significant difference between practice that is mere repetition, and practice that reinforces *[illegible]*." (Emphasis added.)

> Mathematical activity . . . can derive from the most commonplace objects if and when powers of observation are developed . . . . Initially the role of the teacher is to help the child to acquire acute powers of observation and to assess the possibilities that lie within the most commonplace objects and events . . . then . . . to provide interesting materials to stimulate further work . . . . The situation must be carefully structured by the teacher if the children are to make real discoveries.
>
> Whenever new materials are introduced there seem to be three separate stages through which children must pass. At first the child needs a period of free experimentation with the material . . . . The second stage involves the introduction of the necessary vocabulary related to the particular materials . . . best introduced through teacher/child discussion while the materials are actually being handled. The third stage sees the emergence of a problem--probably some question that has arisen during the discussion. This sequence seems to arise naturally . . . . It is representative of an unobtrusive, yet carefully structured, situation.

It is vital that the materials and situation give rise to a problem that is natural and important to the children, so that they will wish to solve it for its own sake and not to earn a reward. After the problem is met, children size it up and look for ways to find the answer, working first with concrete things and then with the abstract skills they have learned. This is the *thinking* stage of Nuffield's learning scheme. It is powered mainly by the teacher's discussion with the students.

> The role of the teacher today is not to stop children talking but rather to ensure that there is something very worthwhile for them to talk about . . . . The quality of the discussion will be directly dependent upon the quality of the teacher/class relationship . . . . If discussion is to foster not only language but thought and reasoning, then it needs to take place in much smaller groups [or] between a teacher and a small group of children.

Finally the students record what they find out: the *communication* stage.

> [The] time comes when the children feel the urge to communicate . . . . Sometimes they get stuck for lack of an adequate vocabulary. Here the role of the teacher [is to] infiltrate the necessary vocabulary into his responses [so that] the child hears words in the context of an enjoyable experience.

Reports may be written in journals, or kept in folders. Some reports are not words but drawings and graphs. It is important that children get to keep their own work so that mathematics becomes part of their value experiences.

All of the Nuffield guides enhance the reputation of the English for subtlety and understatement. This leads to some uncertainty in the teacher about the practical matters of *doing* the theory, as the developers acknowledge in a characteristic understatement. "There is sometimes a little concern among teachers as to the kind of materials to provide, and when to introduce them." This is a significant drawback of the guides and the cause for Nuffield emphasis on teachers' inservice training: If a learning situation is not carefully structured, the problems the teacher asks the students to solve will not be meaningful or profitable.

Mathematics for Schools stages in learning. Experience with Nuffield guides led Harold Fletcher and the Mathematics for Schools authors to specify precisely what situations and materials to use, to prescribe steps for every lesson, and to arrange lessons in sequence. They particularly considered Nuffield's *thinking* stage too ambiguous, and so they spelled out ways to translate each set of concrete experiences into mathematical terminology, and they provided paper-and-pencil exercises to make sure that children consolidate the insights they gain. The Teacher's Resource Book sets out this diagram for a lesson.

REAL SITUATIONS	DOING	DISCUSSION	PRACTICE	NEW SITUATIONS	GENERALIZATIONS
Concrete Materials	Activities, planned and spontaneous		Textbook studies & activities	Applications	

This pattern will not work unless the teacher follows these requirements:

> You should introduce the number operations and the associated facts only after much discovery-activity and discussion using a variety of concrete materials. "Getting to the sums" too soon can often impede rather than enhance mathematical progress.
>
> After discovery, you must give the children plenty of practice and time to develop and consolidate their understanding of mathematical concepts. It does not follow that children always remember what they discover.
>
> You should encourage the children to constantly seek environmental situations appropriate to the [topic] under study. Such situations may arise spontaneously, or they may be set up and guided by you, but they must have meaning for the children, for they will learn little mathematics that is not real to them.

> You must not allow the children's mathematical progress to be held up by lack of ability to verbalize. [If the children understand the language of mathematics] they will make progress in other areas of study.

The last proviso suggests that the Fletcher group does not place a priority on Nuffield's *communication* stage. Mathematics for Schools does not suggest that children write extemporaneously about their math work. Children's discoveries are recorded in their exercise pages. The "Follow-Up Activities" suggested in the Teacher's Resource Book as applications of the concept learned tend to be [nonverbal] exercises, projects, and games, as are the "Enrichment Activities".

3.2 Typical Lessons.

Nuffield. Here is a description of the material on length presented at the start of *Computation and Structure 3*. This guide is recommended for "early years in the junior school"; that is, for children around eight years old, or those who have developed concepts of "longer than," "shorter than," "higher or taller than," "near and far." The length of teaching time for working on this material is not suggested; nor is there a plan for breaking it up into daily lessons.

> Early experiences of length will best be carried out by children using any sort of measure that suitably comes to hand: lengths of paper, book-lengths, knitting needles, matchboxes, strides, spans [hand-breadths] and finger lengths; something the child is familiar with and which he understands. We shall, of course, be working towards the need for, and the discovery of, *standard units*.

The use of fingers, hands, arms, feet, and strides as common but approximate measures is explained and diagrammed in a chart of digits, palms, spans, cubits, and fathoms, which the teacher can copy for her class. Teachers are told that with these historical units of measurement "we can begin to train children to estimate before actually measuring . . . [and] let them discover themselves . . . that we can never measure anything *exactly*."

Assignment cards ("task cards," "job cards") are shown. Such tasks are to be undertaken as individual or small group work or to be directed by the teacher if reading ability is limited.

1. Measure your neck, wrist, and waist. What will you use to do this? How many "wrist measurements" will go round your neck? (Estimate first.) How many neck measurements will go round your waist? (Estimate first.) See if your measurements are different from your partner's.

2 Measure the length of the desk using (a) cubits, (b) spans, (c) palms, (d) digits. First check with the chart that you know what these are. Which did you find gave you the best measurement? Which took the longest to do?

3. Which human units would you use to measure: the height of a giraffe? the height of a horse? a mouse from nose to tail? the length of your garden? Write down some more things you could measure and say which units you would use.

The guide reiterates to the teacher that these exercises are used to show children that human measuring units vary in different people so that there is a need for fixed standard units. It mentions that Piaget found that children naturally proceed from measuring with whatever is convenient to using standard feet and inches. There are a few paragraphs which the teacher can adapt to tell her students about the history of introducing standard measurements in Britain. Now the teacher is ready to introduce standard units and assign practical measuring tasks in the room or schoolyard with foot and yard rulers and a trundle wheel (a wheel with a circumference of one yard, meter, etc., which clicks each time it makes a complete turn).

Next the guide suggests the teacher move from this real-world experience to abstracting experiences like those on the following assignment card: making ordered pairs of numbers out of yards and feet.

YD	FT
1	3
2	6
3	9
4	12
5	
6	
7	
8	
9	
10	
	33
	36
	39
	42
	45

Complete this table.

Can you explain what you were doing?

Can you use this table to change yards to feet or feet to yards?

A way to make a graph illustrating the conversion of feet and yards is illustrated, but teachers are cautioned not to get anything but whole feet and whole yards, for children are not yet ready for fractions and decimals. They are ready for work with rulers that show inches, however.

> They will want to find the length of the "bits and pieces" which are left over at the end. This is the time to make the inch rulers available (without other sub-divisions if possible). . . . Some foot and yard rulers, now with inch divisions, will also be helpful at this stage. . . .

Several activity cards are shown which call for children to measure desks, tables, bookshelves, and corridors and express measurements in two ways; for instance, "5 ft 9 in or 69 in."

Piaget is cited again in explaining that young children do not understand conservation of length when objects are not straight. Several activity cards are shown which call for children to measure zig-zags, spirals, curves, their own feet, objects they find in the classroom, and finally circumference and diameter of objects.

> Here the child will need an intermediate model of the object he is measuring--in measuring round a tin lid he will take a piece of string or tape measure to acquire the appropriate length and match this in turn against his ruler. Calipers can be introduced at this stage for measuring diameters. . . .

With this much experience behind them children will want to be more precise with their measuring, and foot rulers with half and quarter inches can be introduced. But fractions are to be taught now only as a means of appreciating *meanings* of fractional parts, not as operations with numbers.

The unit continues with a lot more suggestions for practical work measuring distances, shapes and heights, and making ordered pairs and tables of feet and inches (like the one on yards and feet). Throughout the pages are found full-page reproductions of students' work on this topic. The unit then proceeds to treat the topic of *weight* in the same manner.

Where this unit appears within the year's mathematics curriculum, how it is related to the students' work in computation and other math topics, what materials are given to the students and what situations are posed, whether some parts of the unit are taught at one time and others later, or whether they are taught at all--these are all decisions for the teacher.

A unit in Mathematics for Schools. A unit called "Measurement Activities: Length" appears about one-third through the Level I (ages five through seven) Teacher's Resource Book. It covers portions of the same content as was described in the Nuffield unit above, but the organization of the material for the teacher is quite different, starting with the manner of stating objectives:

> To enable children to use arbitrary (nonstandard) units for measuring the property of length. To enable children to understand and use the metre for measuring length.

A paragraph of mathematics background notes that there are three basic ideas in measurement: "the choice of a unit, comparison, and counting."

There follows a brief sketch of the history of measurement from parts of the human body. As in the Nuffield guide, the teacher is told that the children's experience with many random units of measurement is an important preliminary to their appreciation of the need for a standard unit.

Materials needed for the lessons are listed. General activities which will refresh what the children learned in a unit called "Pre-Measurement" are suggested. There is an optical illusion diagram which the teacher can copy and present as a special activity to emphasize estimating, and a suggestion that children will discover the idea of fractions as "bits" left over in measuring.

After this general introduction, two separate lessons are explained; first, "Purpose"; then "Preliminary Activities," "Teaching the Pages" (meaning the exercises in the Children's Book), and "Follow-up Activities." The pages of exercises from the Children's Book are reproduced. The first lesson is about nonstandard units and gives children experience with strides, reach, arm, foot, and handspan. Children are to measure these units using strips of paper, compare their strips with their friends', and record their findings. Feet are traced and measured and compared, and the foot measures are used to measure other lengths, such as those of the classroom and corridor.

When the teacher believes the children understand these activities, she "teaches the pages." There are three pages of exercises devoted to this lesson in Children's Book 3. First: "Compare your measurements with those of your friends: whose is longest?" Drawings illustrate reach, stride, foot, and span. The suggested measuring device is string. The children work in small groups and record their discoveries by writing down the name of the person who has the longest measurement. The second exercise directs: "Draw a picture of your foot. Compare its length with your friends' feet.' The last says: "Measure the class shop [store] with pictures of your foot."

In the "Follow-Up Activities" the children apply their knowledge to new objects. For example, they measure the widths and lengths of additional classroom objects using tracings of their own outstretched arms, legs, and fingers. In a final discussion the teacher asks these questions:

> Would you use your span or your reach to measure the length of a classroom wall?. . . your stride or your arm to measure the length of the playground? How would you measure around a football?

The second lesson introduces the "metre stick" and compares it with the children's body measurements. Finally children use the meter to measure the classroom and playground and compare this with their body measurements as expressed in meters.

The small unit of measurement--centimeter rather than inch--and height and nonstraight lines are the subjects of a unit which appears many weeks later in Level I. Activities on circumference and diameter similar to the Nuffield suggestions are delayed until Level II (ages 7 to 13).

3.3 Evaluation of Students

There are no written tests for either program. Nuffield guides emphasize that students should keep journals and record their work in pictures, graphs, and stories. *I Do, and I Understand* explains the importance of teachers' keeping a general record of the class and individual records on each student's interests, projects, achievements, attitudes, and difficulties. The Project is innovating an entirely new system of making clinical observations of children doing active mathematical tasks, not for the purpose of measuring achievement but for diagnosing levels of understanding so that the teacher can design individualized lessons. A series of check-up guides is being developed and field tested in collaboration with psychologists from Piaget's Geneva institute. The first of these, *Checking Up 1* and *Checking Up 2* go along with the beginning books in the three streams.

Teachers of Mathematics for Schools evaluate student progress by observing their participation in discussions and activities and by checking their work in their exercise books.

3.4 Out-of-Class Preparation.

In both programs, teaching success depends on thorough preparation and organization. The teacher must begin with the understanding that active learning is for her, not just for the students. Any math program which attempts to teach the real mathematics behind arithmetic memorization and drill requires a teacher who understands the meanings behind rituals. Most elementary teachers will have to come upon these meanings in the same way children do: by work with concrete materials. Once this content is mastered, the work of preparing the classroom and identifying the learning characteristics of every student can begin. The teacher using the Nuffield guides will need to prepare activity cards, as has been mentioned, in addition to selecting what lessons to use and furnishing the classroom.

4.0 IMPLEMENTATION: REQUIREMENTS AND COSTS

4.1 School Facilities and Arrangements.

Both the Nuffield Project and Mathematics for Schools are designed for informal or open classrooms in the style of the innovative British primary schools. *Open* describes the use of classroom space as well as the scheduling of classroom time. Instead of fixed rows of desks there are tables and workbenches that can be moved about, comfortable chairs placed in quiet corners for reading, laboratory-style areas for math and science, cupboards, blackboards, and portable screens serving as room dividers. Students move around throughout this area, working in pairs or in small groups. The classroom day is not divided into fixed segments; instead, time is budgeted each day to activities that children and teacher judge most interesting and productive. Suggestions and illustrations for rearranging the classroom furniture and schedules gradually over a period of weeks or months are given in *I Do, and I Understand,* and in Edith Biggs' and James MacLean's book on active mathematics, *Freedom to Learn*.

Both Nuffield and Mathematics for Schools materials are designed for nongraded classes, in which students move ahead from their individual starting points and at their own paces, regardless of age. However, Mathematics for Schools has been used successfully in traditionally graded American schools. Both programs were developed to be used by average teachers (with inservice training, to be sure) rather than math specialists; the developers state that they prefer this use because generalist teachers can more easily relate mathematics learning to language, science, and art.

4.2 Student Prerequisites.

Children must be accustomed to working productively in an active, open classroom. If they have not had such experience, this style must be introduced gradually, perhaps one afternoon a week, or an hour a day.

4.3 Teacher Prerequisites.

Although both programs give teachers explanations of the mathematical background for each unit, this is not likely to be sufficient for the average elementary school teacher who feels inadequate in math and dislikes it to boot. In England most teachers starting out in either Nuffield guides or Mathematics for Schools have access to inservice training and to classroom assistance from the headteacher and the government mathematics adviser. American teachers will need a good math background and/or a workshop course in modern math and active learning. Such workshops have been pioneered by the Madison Project of Syracuse University and Webster College. They may be conducted by school districts or offered by university extension departments.

A second prerequisite is the teacher's belief that real-life experiences are indeed the best way to learn, and her willingness to provide this kind of learning by doing extra work. Nuffield should not be attempted unless, as Joseph Featherstone stated, "the teachers really believe that children can learn a great deal by themselves and that most often their own choices reflect their needs."[7]

Organizational ability is a prerequisite frequently overlooked by Americans trying to adapt the English methods. Gathering together materials, keeping them orderly (most children will not work their way through messes of junk), organizing separate lessons for different groups of children, keeping track of individual students' progress--these are management tasks which may be unfamiliar to both traditional and "free" teachers in America.

A fourth prerequisite has to do with the relationship between teacher and students. The "child centeredness" of English informal classrooms is not the same as that of the American "progressive" school of the 30's and 40's. Featherstone stresses the importance of teachers in active learning classrooms using their "natural legitimate authority" as adults.

> Actually, in a proper informal setting, as John Dewey pointed out, adults ought to become more important: ". . . Basing education upon personal experience may mean more multiplied and more intimate contacts between the mature and the immature than ever existed in the traditional schools, and *consequently more rather than less guidance*."[8]

4.4 Cost of Materials, Equipment, Services

Nuffield

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Everyday objects, measuring devices, manipulative materials	teacher's discretion	teacher		
13 teacher's guides*	1 set per school	Wiley	$2.55-$4.00	Reusable

Recommended Supplementary Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Problems: Green, Purple, Red Set (task cards plus teacher's book) (ages 11-13)	1 set per class	Wiley	$3.20-$3.56	Reusable
Modules (ages 11-13)	1 per class	"	$2.75-$3.85	Reusable
"Weaving" guides (selected)	1 per teacher	"	$1.30-$4.12	Reusable
General guides (selected)	1 per teacher or school depending on particular guide	School district	$1.30-$4.12	Reusable
Evaluation guides (primary level)	1 per teacher	"	$2.25, $3.15	Reusable
Films (*I Do, I Understand; Maths with Everything; Into Secondary School*)		University of California Extension Media Center, Berkeley, Ca.	$9.00-$16.00 per day	Rental

*After studying the whole set of guides, individual teachers can order extra copies of those particular guides which they intend to use extensively. The last guide in the Graphs series will not be available in the United States until late 1974.

Mathematics For Schools

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Children's books, Level I	7 per student (ages 5-7)	Addison-Wesley	$.80 each	Consumable
Children's books, Level II	10 per student (ages 7-13)	"	$1.08 each	Reusable
Everyday objects, measuring devices, concrete materials	as prescribed in manual	teacher		
Teacher's Resource Book (1 for Level I, 5 for II)	1 per teacher	Addison-Wesley	$4.76-$5.20	Reusable

Recommended Supplementary Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Freedom to Learn	1 per school	Addison-Wesley	$8.95	Reusable
Inservice training		school district		

4.5 Community Relations.

The need for interpreting the active-learning approach to parents is recognized by the Nuffield developers. *Your Child and Mathematics* is designed to explain to English parents the reasons for the modern math topics as well as for the informal approach to learning. Many American parents are interested in the British innovations and are eager to see them adapted in the United States. School people may need to interpret to some of these parents the importance of gradual introduction of the new methods to teachers and children, and that the active learning approach may not be suitable for all teachers or all students. Other parents will be most concerned with their children's mastery of traditional computational skills, and they will need to be persuaded that an active-learning approach can accomplish this.

5.0 PROGRAM DEVELOPMENT AND EVALUATION

5.1 Program Development

The Nuffield Mathematics Project began in 1964 to build a comprehensive math program that would combine modern math topics with the new ideas for teaching that were gaining hold in England's primary schools. Active learning or "laboratory mathematics" was already being practiced in these informal, nongraded, flexibly scheduled classrooms, and the approach was being spread through teachers' seminars by Edith Biggs, Harold Fletcher, Leonard Sealey, and others. The Project was an effort to organize this experience and make it available as topical guidebooks supported by inservice training centers.

The Project is the combined effort of the Nuffield Foundation and the Schools' Council. The latter is composed of representatives from all the educational organizations in Great Britain--associations of teachers, headteachers, college teachers, and mathematics advisers from the national ministry of education. The Project commissioned teams of teachers, advisers, and professors to prepare the guides under the direction of Professor Geoffrey Matthews and a national consultative committee of mathematicians. The Schools' Council set up teachers' centers to give teachers their own active-learning experiences in the new math topics. The evaluation guides are prepared by a team from Piaget's institute in Geneva, Switzerland.

The late Harold Fletcher, the mathematics adviser to Staffordshire schools, was a member of the Nuffield writing team from 1965 through 1967, when he began to work on a Nuffield-style comprehensive curriculum for Addison-Wesley International. The assistant author of Mathematics for Schools is Ruth Walker, headteacher of a school in which Fletcher worked with teachers using the Nuffield approach and topics.

5.2 Developers' Evaluations

The Nuffield guides were written in trial versions and tested in more than 250 schools in 14 areas of England. They were revised on the basis of these trials and then published. Research from the trials is published by the Schools' Council in its Field Report Number 4.

Mathematics for Schools can be considered a still further revision and adaptation of the Nuffield materials. It was tried out for two years in England before publication. During the school year 1970-71 Addison-Wesley's office in Menlo Park, California, gave the primary level mathematics materials to 30 teachers throughout the United States. They taught the program for a year in grades one and two, and then were asked to fill out questionnaires on their impressions of the curriculum, students' progress, difficulties in using the materials, and the like. Because of the small sales volume, Addison-Wesley does not expect to prepare an analysis of the data.

5.3 Independent Analyses of the Programs

David Rappaport of Northeastern Illinois State College, Chicago, is one of scores of Americans who have observed the English open schools with an eye to translating their practices for American classrooms. His evaluation of the Nuffield activities he observed, and of the mathematics teaching in Ruth Walker's school under Fletcher's guidance, are published in *The Elementary School Journal* issues of March 1971 and October 1970, respectively. Among his comments on the Nuffield guides are the following:

> The guides are, with few exceptions, superb. Every American elementary school teacher would profit by reading [them] and using them as source material. The examples of children's work could very well be the basis for overcoming teachers' fear of . . . trying out laboratory techniques. . . . The guides do explain the mathematics concepts in a developmental manner. Teachers who still lack an understanding of the new mathematics . . . could find the Nuffield guides an excellent method of learning and understanding [it].[9]

Rappaport cautions, however, that the teacher must have good preparation and understanding of how the guides work before using them. He observed both good and bad teaching by teachers using the guides in England, and he comments that the good teachers knew "when to capitalize on children's discoveries to direct them to new efforts," while poor teachers "did not understand the mathematics and were unable to develop the next step by themselves."[10]

Some English mathematics educators observe that English teachers, lacking strong mastery of mathematics theory, overstress the environmental applications for math and neglect the purely structural mathematics in Nuffield. Mathematics should not be presented to children solely as a way to solve practical problems in the environment.

> Math is artificial. Environmental maths is just as unsatisfactory as textbook maths if the questions you pose to children are the type which cause them to ask, "Who wants to know?" The environment is not necessarily intrinsically mathematical. We must make the maths environment of students more precise by structuring it so that it can be explored and so that children ask questions about mathematics, not just about its fringe benefits. We want them to be interested in mathematics itself, not just what you can do with it.[11]

There is also the problem that the teacher must work out a way to develop the material in the guides into lessons that fit into the school's basic mathematics curriculum, or to devise a new curriculum. Mathematics for Schools is in itself evidence that in England there are educators who doubt that the Nuffield guides can serve as framework for a complete curriculum in math. In the United States, where principals and teachers do not usually design their own courses of study, there are understandably even more doubts. Questions arise on two grounds: first, that the Nuffield guides are so subtly and permissively presented that only teachers who are experienced in mathematics and in individualizing instruction can develop a whole curriculum from them; and second, that Piagetian theory does not provide adequate guidelines in math. On this second point Robert B. Davis, director of the Madison Project in Mathematics, which has worked closely with Nuffield, wrote in 1967:

> How adequate are [Piaget's] "clearly-defined developmental stages" in providing us with curriculum guidelines? The answer seems to be that this method has great promise for the future, but that this promise has not been realized as yet. . . .
>
> It appears that Piaget has focused attention on a very particular selection of tasks--such as his famous "conservation" tasks in pouring water--and it is by no means clear that these tasks, taken together, form an adequate and appropriate set of "pegs" on which we can hang the mathematics curriculum. Many important aspects of mathematics remain untouched, and in the case of some others the analogies with Piaget's tasks may be misleading rather than illuminating.[12]

For school people impressed by these English materials but looking for definitive answers to questions like those above, there is little solid data to refer to. In his 1971 comparisons of informal teaching in English and American schools Joseph Featherstone comments:

> We could all proceed more wisely if we had better notions of how to evaluate learning in informal settings, but we don't. . . . If the British lack rigor [in evaluation], we [Americans] lack many examples of good practice; far too many of our school systems have emphasized conventional measurement and ignored children's learning, forgetting the principle that children and teachers do not get any heavier for being weighed.
>
> On measurable achievement in conventional tests, children in formal British schools do slightly better than children in informal schools, though uniformly the differences are very slight. This is not surprising: formal schools teach children to take tests. The surprising thing to me is that test results are so similar. . . . There is no evidence that reducing the amount of formal control over students impairs conventional academic skills. On the other hand, it is plainly impossible to make inflated claims for informal teaching in terms of conventional test scores.[13]

Thus we need new kinds of tests in order to assess active-learning mathematics. But tests are "a side issue" Featherstone says. "We need different values too."[14] It all goes back to what long-range goals communities and parents set for their children. Is it important that in adulthood mathematics be more than a breadwinning skill, more than a tool? Is it expected that mathematics be also an art, adding to the student's enjoyment of his own mind and of his environment?

American experience of these two active-learning math curricula from England is still very much in the concrete operations stage. We need to learn by *doing* them, not by reading the data.

FOOTNOTES

*1. Featherstone, Joseph. Schools for learning. *The New Republic*, December 21, 1968, pp. 18, 20.

2. Biggs, Edith. Mathematics laboratories and teachers' centres--the mathematics revolution in Britain. *The Arithmetic Teacher*, 2, (4), May 1968, pp. 405-406.

*3. Featherstone, Joseph. The British and us. *The New Republic*, September 11, 1971, p. 22.

*4. Featherstone, Joseph. Tempering a fad. *The New Republic*, September 25, 1971, p. 19.

5. Featherstone. Schools for learning, p. 18.

6. Rappaport, David. Beyond Nuffield. *The Elementary School Journal*, 71, (1), October 1970, p. 24.

*7. Featherstone, Joseph. Schools for children. *The New Republic*, August 19, 1967, p. 20.

8. Featherstone. Tempering a fad, p. 19.

9. Rappaport, David. The Nuffield mathematics project. *The Elementary School Journal*, 71, (6), March 1971, p. 306.

10. Rappaport. The Nuffield mathematics project, p. 306.

11. Fielker, David, Director, Abbey Wood Teachers' Center, London, personal communication.

12. Davis, Robert. *The Changing Curriculum: Mathematics*. Washington: ASCD, 1967, pp. 35, 38.

13. Featherstone. The British and us, p. 23.

14. Featherstone, The British and us, p. 23.

*The series of articles have been combined in Featherstone, Joseph. *Schools Where Children Learn*. Liveright, N.Y.: 1971.

INDIVIDUALLY PRESCRIBED INSTRUCTION-MATHEMATICS (IPI MATH)

INTRODUCTION

Math period begins in a mixed-age, intermediate-level class in a modern open-space school. Ten-year-old John goes to the math center and finds his IPI folder. He takes out the short pretest for the unit called "Level C, Division," which he took during yesterday's math period. The test has been scored by the teacher's aide, and John now sees that he got all the problems relating to the first "skill" (lesson) correct, but he made many mistakes on the questions testing his knowledge of the other three "skills" in the unit. So John goes to the aide and tells her he needs the Student Booklet for Level C, Division, Skill 2. He takes this booklet to his teacher and tells her he needs a "prescription." When she finishes helping one of John's classmates, the teacher conferences with John, referring to his pretest, and then she assigns three pages of lessons in the Student Booklet. John returns to his seat and works by himself in the booklet for the rest of the math period. John's classmates are working in a similar manner.

Individually Prescribed Instruction-Mathematics is a reorganization of the traditional elementary mathematics curriculum to "individualize" each student's progress through the curriculum, thereby accommodating many levels of student ability and achievement within the same classroom. Textbooks are replaced by programmed booklets--one for each lesson. These "self-teaching" booklets are the major source of instruction. The teacher does not present lessons to the whole class or even to small groups at one time but rather assigns each child's individual study program, monitors progress, and provides individual tutoring when a student gets stuck.

The hallmark of the individualization movement, of which IPI is a leading example, is its reliance on a carefully constructed sequence or "continuum" of "skills"--from simple to difficult--building to mastery of whatever subject is being studied. Skills are called "behavioral objectives" because the student will demonstrate a specified, observable behavior or performance to prove mastery. Tests of mastery are included with every unit, each test keyed precisely to the objectives being taught. Children are not assigned to new work until they have passed the tests for the prerequisite skills. Conversely, the student may skip a lesson if his test shows he already knows the skill. By this method both quick and slow students are thought to be able to proceed through the same subject matter without hampering each others' progress.

IPI's distinction is its embellishment of programmed instruction--providing the opportunity for "continuous progress" rather than slotting a student into a grade based on his age or accomplishment; and managing instruction in schools with very diverse student populations. Like other programmed curricula, IPI relies on behaviorist strategies of breaking up a training episode into small, simple segments so that the student's frequent experience of success can act as a spur, or "reinforcement," for further learning. This strategy is thought to be especially appropriate for pupils who have had frequent experiences of school failure.

BASIC INFORMATION

Program name: Individually Prescribed Instruction-Mathematics (IPI Math)

Format: A sequence of 359 instructional objectives with a student skill booklet for each.

Uniqueness: Completely self-paced instructional system of elementary mathematics based on behaviorally specified math "skills." Children's work is assigned through "prescriptions" which the teacher writes for each child separately, based on frequent testing to assess mastery of skills.

Content: A continuum of objectives grouped into ten learning areas: Numeration/Place Value, Addition/Subtraction, Multiplication, Division, Fractions, Money, Time, Systems of Measurement, Geometry, and Applications.

Suggested use: Complete curriculum for grades 1 through 6.

Target audience: Students of all abilities, grades 1-6.

Length of use: Daily math period for six years.

Aids for teachers: Set of training materials including three training manuals, two student case studies on audio cassettes, and a filmstrip with accompanying tape.

Date of publication: 1972.

Director/Developer: Robert Glaser/Learning Research and Development Center (LRDC), University of Pittsburgh, 160 North Craig St., Pittsburgh, Pa. 15213, and Robert Scanlon/Research for Better Schools, Inc. (RBS), 1700 Market St., Philadelphia, Pa. 19103.

Publisher: Appleton-Century-Crofts, 440 Park Ave. So., New York, N.Y. 10016.

1. GOALS AND RATIONALE

1.1 Goals

How can one teacher effectively teach 30 or more children, all with different experiences, abilities, needs, cultural backgrounds, and rates and styles of learning? For years schools have addressed the problem of different learning speeds by "tracking" children into fast, average, and slow groups, within schools or within classrooms. However, tracking presents the injustice of being a self-fulfilling prophecy for slow-starting students and it frequently results in *de facto* segregation of minority group students. IPI developers attacked the problem by restructuring the elementary classroom for "individualization." They designed both an instructional program and a management system which permits each child to work separately--as quickly or as slowly as he needs--within a heterogeneously grouped classroom.

IPI attempts to respond to children's differences in terms of their learning rates only: the content and method of instruction are the same for everyone, based on the theory of programmed instruction. Changing the content of the mathematics which children learn, or loosening their reliance on textbooks, are not goals of IPI developers. Nor do IPI developers evidence concern for problem-solving behavior or for math activities arising out of children's interests and natural environment. The motivation for learning textbook arithmetic is expected to come entirely from the student's experience of previous success--as measured by frequent tests. Success on these tests is assumed to function as a powerful reward. Developers aim to guarantee success by breaking the curriculum into very small steps and insuring that no student ever has to take a step that is wider than his stride.

This mastery of mathematics in small doses is an attempt to halt the dismal history of failure for those children who have difficulty learning arithmetic--who fall further and further behind with each year of school, and who steadily deepen their conviction that they themselves are failures. At the same time, IPI aims to loosen the brake which slow-learning children place on average and high-achieving children. The developers intend that their individualized program will enable students of all achievement levels within the same classroom to experience success in math, liking for math, and respect for their own abilities as students.

1.2 Rationale

Teaching machines and programmed textbooks, stemming from the work of B. F. Skinner in the 1950's, had indicated that a student could progress with little or no outside help through a sequence of learning experiences *if* they were arranged in order of gradually increasing difficulty and *if* the student could progress at his own pace. According to this theory the student would be continually rewarded by the warm feeling of success, however small, and this would act as a powerful enough motivation to encourage him to tackle even difficult (or boring) work. Most important, he would not be discouraged by repeated failure. IPI developers had evidence that low-achieving as well as high-achieving students could learn the same body of elementary mathematics so long as it was presented in this way.

This programming theory rests on the traditional assumption that there is one "body of knowledge" and that even young children can grasp the logic of its division into separate subject matter areas--mathematics, science, language, etc. Programming also depends on a new assumption that learning technologists know how to break up a body of knowledge into discrete component bits, and that they can arrange these pieces into sequences of learning which are equally efficient for all learners. The *content* of the training episode is seen as having less importance than the method of transmitting it.

The combination of these theories results in the IPI rationale that all children, regardless of their aptitude, inclination, or past achievement, can master the same curriculum, using the same instructional method and materials, provided simply that each student is allowed to progress at his own rate of speed.

However, IPI expands the theory of programmed instruction. The use of programmed texts had permitted individualization within a grade level. Our schools have been organized into grade levels so that certain skills and topics are supposed to be covered in the first grade, others in the second grade, and so on. Thus even if programmed instruction allowed students to progress at different rates through the same grade, they all had to start at the same point at the beginning of the next grade. IPI developers therefore used systems theory to expand programming. They developed curriculum sequences that ignored grade level boundaries and extended instruction through all the elementary school years. With such "continuous progress" the grading or grouping of students loses importance. There is no need for special promotion, retention, homogeneous ability grouping, or other student sorting schemes based on achievement.

In actually developing a total individualized curriculum, IPI applied the principles of programmed instruction:

1. The objectives to be achieved were spelled out in terms of desired student behaviors. (Given two common fractions less than or equal to 1, the student renames each fraction using the least common denominator for the given pair and writes $>$, $<$, or $=$ between the given fractions to show their relationship. LIMIT: Given fractions having denominators $\leq$ 50.)

2. The objectives were sequenced into a hierarchy with each behavior building on the one before it.

3. Instructional materials were developed so students could learn with little or no outside help. The booklets do the teaching. Learning is measurable because it is defined in terms of each student being able to demonstrate a carefully stated behavior under carefully stated conditions.

4. Methods of diagnosis were developed so that the point at which the student enters the sequence will be most appropriate for him.

5. Each student works at his own pace.

6. Since objectives spell out what a student must do ("demonstrate a behavior") to indicate mastery, booklets were written as self-contained lessons explaining the required behavior and requiring him to practice it.

7. Students are to receive immediate feedback on all work done by taking frequent criterion-referenced tests. These were written and included as an integral part of the curriculum, each test keyed to the specific behaviors just studied. From the tests, teachers also are to receive the constant feedback they need to make new assignments for each individual student. Teacher training materials were written to teach the management system of the curriculum.

8. Materials and procedures were modified on the basis of feedback from users to improve effectiveness.

2. CONTENT AND MATERIALS

2.1 Content Focus

IPI Math includes all those mathematical concepts which typically form the program for first through sixth grades. The content is divided into ten areas. Five of the areas deal directly with numbers and operations; two deal with measurement; another with money; one covers geometry; the remaining area is applications, which includes work on sets, functions, graphs, and word problems which require more than one operation. The major emphasis is on number and operations. Two-thirds of all the objectives fall in these five areas.

The ten areas and the number of objectives at each level are shown in Table 1. There are no units in the areas that are blank. Six units are divided into Part I and Part II, and are marked as such (e.g., A - Numeration/Place Value).

AREA	A	B	C	D	E	F	G
NUMERATION/ PLACE VALUE	15	9	14	5	6	7	4
ADDITION/ SUBTRACTION	17	12	13	10	4	4	6
MULTIPLICATION		4	7	9	7	4	3
DIVISION		3	4	7	9	5	6
FRACTIONS	3	3	6	7	11	8	8
MONEY	1	1	5	5			
TIME	1	3	6	4	4	2	
SYSTEMS OF MEASUREMENT		3	6	6	5	5	6
GEOMETRY		3	2	4	6	4	2
APPLICATIONS		3	8	9	5	4	6

Table 1

2.2 Content and Organization of the Subdivisions

The ten content areas of IPI Math are developed at seven levels of difficulty, A-G. Each "unit" contains one specific content area at a defined level; for example, Multiplication-D. Each unit is composed of a carefully delineated, hierarchically arranged sequence of skills to be mastered. These skills are stated as performance objectives.

A unit may contain from 1 to 17 skills. Each skill is presented in a consumable student booklet designed to teach 1 instructional objective. The back cover of the booklet is the "skillsheet description page," listing the objective and the contents of the booklet. The front cover is the "prescription form," on which the teacher records the specific assignments made.

There are four kinds of pages in each student booklet:

1. Review pages (marked "r"). These review a skill previously taught which is essential to mastery of this objective.

2. Teaching pages ("t"). These introduce new skills.

3. Summary pages ("s"). These include all the behaviors a student needs to master the skill.

4. Curriculum Embedded Tests (CET). There are two in each booklet. They are miniature posttests which measure the mastery of the skill.

It is rare for a student to do every sheet within a booklet. Rather, he does only what is necessary for mastery of that single skill. When he has mastered all the skills within a unit, this is recorded on his IPI student profile. Since IPI is a carefully constructed continuum of skills, with each unit building on what came previously, the learner proceeds in the system in a set order, completing all work at one level before moving on to more advanced levels of any one area. For instance, the sample student profile form in Table 2 shows the sequence in which this student's instruction will progress.

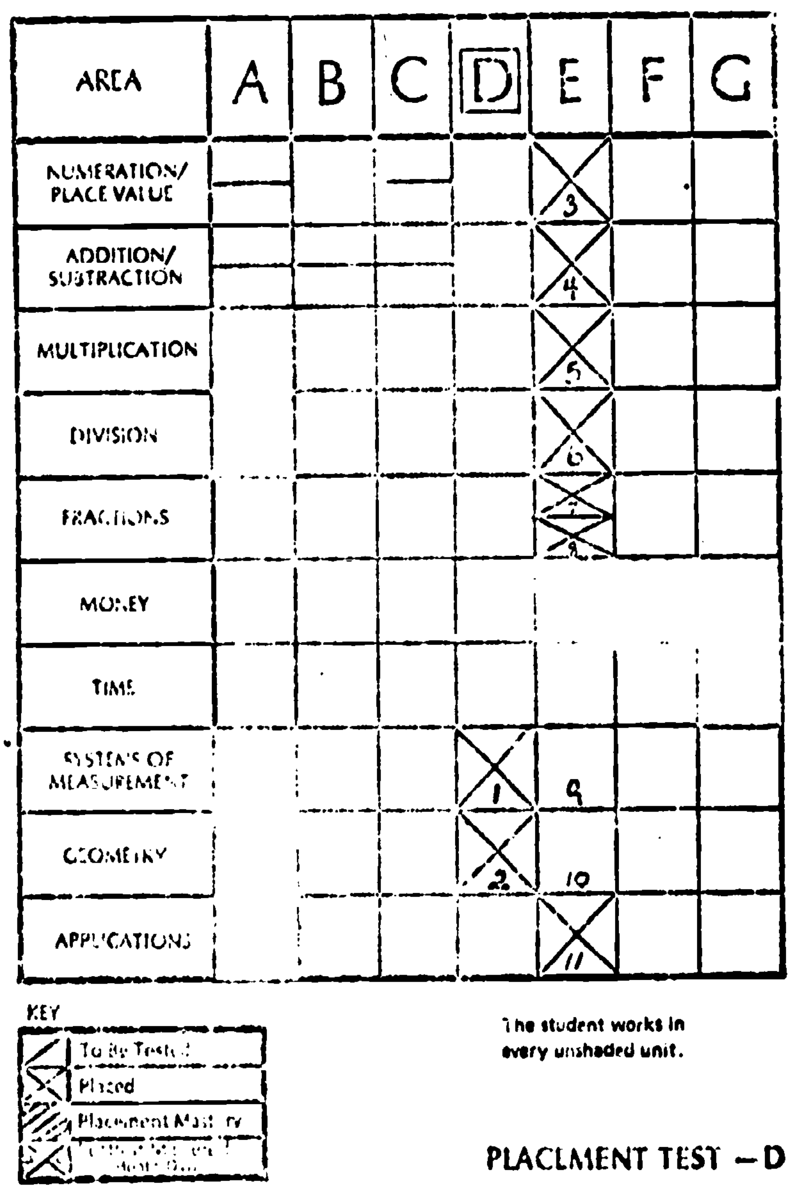


Table 2

2.3 Materials Provided

The student materials consist of 359 student booklets, 1 for each instructional objective in the system. There must be enough booklets for each child to have a fresh book for each new skill. In addition, there are placement tests, pretests, and posttests, all consumable, and 27 optional audio cassettes for lessons in Levels A and B.

The student booklet is the primary instructional tool. If the learner has difficulty in mastering a skill from this, the teacher must supplement his instruction so that he can achieve mastery before going on to the next booklet in the sequence.

There are also materials designed to train teachers in the procedures of IPI Mathematics and in techniques for individualizing learning. They consist of a filmstrip, "Individualized Instruction and IPI," with an accompanying cassette tape, a second audio cassette, "Identifying Instructional Objectives," which presents two student case studies, and two training manuals, Volume 1, *Diagnosing and Prescribing for Individualized Instruction,* and Volume 2, *Managing Individualized Instruction.*

2.4 Materials Not Provided

The developers say that supplementary textbooks or workbooks, teacher-made skillsheets, audiovisual materials, and manipulative materials may be needed. They should be keyed to the specific instructional objectives in the IPI continuum. None of these is provided.

3. CLASSROOM ACTION

3.1 Teaching-Learning Strategy

IPI is a generalized treatment--that is, anyone ought to be able to follow the same basic procedures for preventing or curing individual learning failures. Since it is based on the theory that the most common reason for failure is that the student is told to learn new things before he knows how to do the old things--to divide before he can multiply--the first strategy is to find out on a continuing basis what the student now knows and doesn't know. This is called *diagnosis* and is done by tests provided as an integral part of the IPI program. The second strategy is to assign work which will produce the skills which were diagnosed as missing. This is called *prescription* and is accomplished by the teacher matching the pupil's test results to the IPI continuum of learning objectives. The third strategy is the *instruction* itself. For the most part this is to be accomplished by the student booklets--like a textbook which has been divided into separate booklets for each skill. Within one classroom every student can be doing a different lesson. The teacher adds instruction which she herself devises if the student booklet fails to produce mastery, as measured by the posttests in the booklet. When both the student and his teacher become familiar with the IPI system, the student is supposed to be able to move through the continuum at his own pace.

Six major steps are used by the teacher in planning each student's learning sequence and guiding him through it.

1. The teacher must learn how to place students in the IPI continuum of skills. The IPI placement test is the first step. It is a general test, given at the beginning of the school year, which places a student in the *unit* and *level* at which he will begin work. There is a placement test for all but the first level (Level A). Although teacher judgment is important in deciding which placement test to administer, a general range is indicated:

Grade	Level
1	B-when ready
2	B, C
3	B, C, D
4	C, D, E
5	D, E, F
6	D, E, F

The developers acknowledge that the system may be difficult for primary children. Reading is a prerequisite for using the student booklets and taking tests. Cassette tapes are available at Levels A & B for children who can't read. The system must be understood before a child can proceed independently. The teacher's manual suggests that the teacher assign student booklets at the A level and guide a pupil's work in them, using them as a model to teach how the system operates. In addition, vocabulary work is necessary. Students should be given placement test B as they complete the A booklets or cassette tapes.

2. After placement, in a specific unit and level, an IPI pretest is given to measure the student's proficiency in each skill in that level.

3. Next, the prescription of the student's learning program within the level is made. The teacher uses the pretest results to assign specific work in the student booklet--whatever skills the test shows to be lacking. In addition, a learning setting is defined so that a student is told whether to work in independent study, with a peer tutor, or in a small group.

4. Now the prescribed program must be implemented. Usually, a student begins work independently, freeing the teacher for tutoring and evaluating other students' progress. The student booklets comprise the major instructional method. Although teacher guidance may be necessary during a pupil's introduction to the program, students are to learn primarily from reading instructions and answering questions on the prescribed pages in the booklet. Most of the time students are expected to work independently in their booklets.

5. The teacher evaluates student progress on an ongoing basis, using the feedback from the student's daily work to write further prescriptions in the unit.

6. The final step is measuring the student's mastery of the unit objective. When all the lessons prescribed in a unit have been completed as indicated (including successful completion of the Curriculum Embedded Tests contained in each student booklet), the appropriate unit posttest is given to find out whether the learner is ready to move on to the next unit.

Once they have learned the system, primary children are expected to work in the student booklets in the same way as older children. In fact, for all students, regardless of age, student booklet instruction is flexible only as to the pace at which they work. Booklets do not provide different activities for individual learning styles at any level.

If paper-and-pencil exercises in the booklets do not produce mastery of an objective, then other instructional materials are recommended. None are provided by the system. The additional materials may be other texts or workbooks, teacher-made exercise sheets, and audiovisual materials. Manipulative devices are also suggested for each level. They are listed on the front of the student booklet with which they are to be used. A caution is stated with respect to manipulatives: "Remember, manipulative aids are only helpful if students know how to use them. Students who don't understand their purpose will treat them as toys!"

Free-time activities should also be available for students who are waiting for scoring or for new prescriptions. Activities are to be created by the teacher to supplement or reinforce skills learned in the student booklets. Suggested free-time activities include puzzles, games, open-ended material. Another caution is made: "In selecting free-time activities it is wise to consider both the noise level and the amount of movement involved."

3.2 Typical Lesson

It is math period for a class of 30 fifth graders. At the beginning of the math time, two students pass out the IPI Math folders to each student in the class. Before work begins, the teacher introduces to the entire class a new fraction math game she has set out for free-time activity. After that, the students open their folders and get to work.

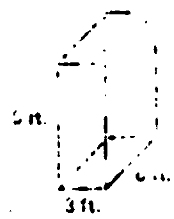
Five students are ready for pretests in new units. Two are working in units at Level D, two at Level E, and one at Level F. Following the written test instructions, the student working in Level D-Multiplication-Pretest begins completing single digit multiplication problems; when these are complete, he moves to six word problems. The next section asks him to complete multiplication sentences and to develop a new sentence for each part of an array. The final problems increase in difficulty; the student multiplies a single digit and a three-place numeral and finds the product when three numerals are multiplied. Two other students have prescriptions which call for posttests, one for Division-Level E, the other in Geometry-Level D. These seven go off to the school materials center to get what they need and then return to class and begin work.

Eight students find in their folders new prescriptions assigning more work in the booklets in which they were working yesterday. They begin work independently. Three other students return to work on material they had started in the last math period. They are working in different booklets and none is having difficulty. When they are done, they will score them if they know how or they will go to the aide to have them scored.

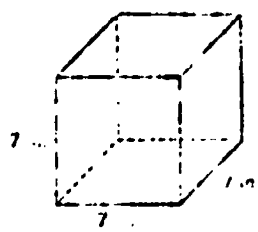
Peer tutoring is indicated on the prescriptions for two pairs of students. A small group of four, all working on E-Geometry-4, is going to watch a film loop on triangles in the library. When they return to class, they will work on a teacher-made skillsheet. The remaining students are working with the teacher in measurement, after which they continue work separately in their student booklets for G-Systems of Measurement-4. One student has just completed the following page.

CET II

[illegible]



[illegible]



[illegible]

When the teacher has finished the measurement work with the small group, she answers questions of students who need help and writes new prescriptions for those who have put up small flags on their desks to show they are ready. Two students who were stuck had gone to play the new fraction game while waiting for the teacher to be free.

At the end of the period, students hand in their folders, putting them in one of three boxes. One box is for folders of students who need the teacher to give them new prescriptions for the next day; one is for those still working on their prescription; one for those who have completed work on tests or in booklets that need scoring.

3.3 Evaluation of Students

Ongoing evaluation of students' learning is an essential component of IPI. In addition to the placement tests and pretests, which determine the student's prescription, IPI provides Curriculum Embedded Tests (CET) to keep track of the student's progress as he works in one skill. There are two CET's in each student booklet, and they are prescribed when the teacher decides that a student can probably master a skill. The posttest is used to measure all the skills in one unit. It is similar to the pretest, but not identical.

3.4 Out-of-Class Preparation

Teacher. In daily planning, the teacher is primarily concerned with management rather than preparing lectures, demonstrations, or designing activities for students. If she has organized the use of time, materials, supervision, and space before class begins, the teacher can give a lot of instructional assistance to individual students. In order to do this, the teacher needs to write all needed prescriptions before class, prepare all supplementary materials and organize all supplementary texts and needed manipulative aids so that they are accessible to students. She must also plan students' activities--who will be working independently, who with peer tutors, who in teacher-directed activities--and organize the space, if needed, to accommodate the activities. In addition, the teacher attends planning sessions scheduled by the administrator at least once a week.

Student. There is no student homework in IPI.

3.5 Role of Other Classroom Personnel

Teacher aides are essential to the functioning of IPI. During class they are expected to score and record the skillsheets and tests that are not scored or recorded by the students themselves; help students read prescriptions or skillsheet instructions; assist students in obtaining materials; help with classroom management if the teacher requests. Outside of class, aides keep student files current; prepare any materials needed by the faculty for planning sessions; organize, inventory, and order IPI instructional materials. Aides do not tutor or teach students. A guide, *Aiding IPI Mathematics - A Manual for Teacher Aides in IPI Mathematics*, clearly defines an aide's responsibilities.

4. IMPLEMENTATION: REQUIREMENTS AND COSTS

4.1 School Facilities and Arrangements

IPI can be used in any school setting (self-contained classrooms, open-pod schools, etc.) and student grouping arrangement (graded or non-graded classroom). Some place is needed for storage of IPI materials, but whether this be in the individual classroom or in a central place for use by all classes is a school decision.

4.2 Student Prerequisites

Reading ability is a prerequisite for a student to use IPI, except where audio cassettes are provided at Levels A and B.

4.3 Teacher Prerequisites

IPI teachers do not need special mathematics background. However, they must receive training in the use of the program. This training is left to individual school administrators. IPI has produced two training manuals, a filmstrip with an accompanying audiotape, and two case studies on cassettes for teacher inservice.

The film strip and tape introduce teachers to IPI's view of individualization and workings of the IPI system. The manuals, aside from presenting detailed information on how to use IPI Math, attempt to trouble-shoot problems that may occur.

4.4 Background and Training of Other Classroom Personnel

Teacher aides. Training for teacher aides is essential. The responsibility for this lies with the school administrator; a training manual is available.

Administrators. Training of the school principal is required when IPI is introduced. The publisher pays for a three-day training session provided at various sites throughout the United States, but principals must provide for their own transportation and accommodations. The number of administrators eligible for training from one school or district depends on the number of students using IPI Math, as follows:

Enrollment	Trainees
150-400	1
401-800	2
801-1200	3

4.5 Cost of Materials, Equipment, Services

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
The following indicates the IPI Math program for a minimum enrollment of 150 students per school:				
Student Booklets	50 per student	Appleton-Century-Crofts	$7.85 per student per year	Yearly
Pretests	20 per student			
Posttests	10 per student			
Placement Tests	3 per student			
Answer Keys	1 per school			
Continuum - Booklet areas	10 per school			
Student Folder	1 per student			
Additional materials - These prices apply only to schools which enroll more than 150 students:				
Student Booklets	For each skill	Appleton-Century-Crofts	$1.25 per package of 10	
Pretests	For each unit		$ .85 per package of 10	
Posttests	For each unit		$ .85 per package of 10	
Answer Keys			$2.00 per level	
Pretest, posttest, and CET combined			$1.50 per level	
Placement Test - Levels B through G			$1.00 per level	

Recommended Supplementary Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Cassettes	1 set per school	Appleton-Century-Crofts	$125.00 per set of 27 cassettes	Reusable
A "Training Kit" including single copies of items listed below may be purchased for $10.00. Individual prices are as follows:				
Teaching in IPI Mathematics Volume 1: Diagnosing & Prescribing for Individualized Instruction	1 per teacher	"	$2.50	Reusable
Volume 2: Managing Individualized Instruction	1 per teacher	"	$2.50	Reusable
Volume 3: Aiding IPI Mathematics, A Manual for Teacher Aides in IPI Math	1 per aide	"	$2.25	Reusable
Media Kit: 2 cassettes, 1 film loop	1 per school	"	$1.75 per cassette $ .50 for film loop	Reusable

Additional costs are involved for supplementary manipulative materials and audiovisual materials. In addition, storage of materials may necessitate building special shelving.

4.6 Demonstration Sites

The following are schools where IPI training for administrators is conducted.

California:	Mariners Elementary School 2100 Mariners Drive Newport Beach, Ca. Earl Bjelland, Principal phone: (714) 646-4835
Florida:	Oakland Terrace Elementary 2010 West 12th Street Panama City, Fla. Paul Boswell, Principal phone: (904) 763-2252
Georgia:	Lake Park Elementary School Lake Park Valdosta, Ga. Charles Bethea, Principal phone: (912) 559-5153
New Hampshire:	Paul A. Smith Elementary School Lawndale Road Franklin, N.H. Robert Ross, Principal phone: (603) 934-4144
New Jersey:	Allenwood Elementary School Allenwood Road Allenwood, N.J. John Gasparini, Principal phone: (201) 223-9858
Texas:	Lincoln Elementary School 1319 E. Lovett Edinburg, Tex. Andrew Lopez, Principal phone: (512) 383-4994
Washington:	Midland Elementary School 2300 105th Street, East Tacoma, Wash. I. B. Eliason, Principal phone: (206) LE7-0211, Ext. 297

5. PROGRAM DEVELOPMENT AND EVALUATION

5.1 Program Development

During the school year 1963-64, the Learning Research and Development Center (LRDC), at the University of Pittsburgh, and the Baldwin-Whitehall Public Schools of suburban Pittsburgh initiated an experiment to investigate the feasibility of converting an entire K-6 school to a system of individualized instruction. The passage of Title IV of the Cooperative Research Act provided the funding needed for beginning this cooperative project at the Oakleaf School. LRDC served as the major initiator of IPI Math products, installing them in Oakleaf during the 1965-66 school year. Appleton-Century-Crofts Publishers secured a copyright on the early version of the IPI Math materials as they were being tried out in the Oakleaf School, and also contributed expertise to the later production of materials.

In 1966, Research for Better Schools was founded as the regional educational laboratory for Eastern Pennsylvania, Delaware, and New Jersey. One of its initial efforts was to take IPI Math from its beginning stages in Oakleaf School to its installation in more than 300 schools around the country.

In the 1966-67 school year, IPI Math was instituted in five demonstration and development schools. Staff from these schools were trained in the summer of 1966, and during the school year several thousand visitors saw IPI being used in these five schools. RBS staff visited teachers in the demonstration schools at least once a week for feedback on changes required in the materials. Information was gathered from student performance on IPI tests, classroom observations of procedures and attitudes, parent, student, and teacher interviews, time needed for students to move through the program, and standardized tests.

During the summer of 1967, several activities were undertaken by RBS, LRDC, and Appleton-Century-Crofts. The materials for use in the 1967-68 school year were being published by ACC. These represented the first major revision of the IPI materials. They reflected many revisions in sequencing of skill objectives and in format, which had been made by staff members of LRDC and RBS, along with teachers in the 5 demonstration schools. Also RBS began a pilot test of the materials in 15 other schools.

Materials were continually revised according to feedback received from operation of the program in the pilot schools. Changes were always tried out first in the 5 demonstration schools, where almost continuous contact with RBS staff was maintained. When changes were effective, they were sent out to the 15 pilot schools, where less frequent contact was maintained. Changes in the materials continued until the final commercialization of IPI Math in September of 1972.

A new version of IPI, Individualized Mathematics, incorporates and stresses the use of manipulative materials, including Dienes blocks and number lines, at all levels. Because addition of manipulatives increases the cost of the program, developers plan to prescribe some manipulatives and make others optional. While inclusion of manipulatives is the major difference between IPI and Individualized Mathematics, developers have also attempted to break the tight sequencing of IPI by providing alternative sequences of units. Although teacher training materials have not yet been developed, a representative from Research for Better Schools says that teacher materials will be based on a teaching mode similar to that used in the programmed student booklets. During 1973, materials were field tested in several first and second grades; because additional funding was not received, further development and field testing has been discontinued at RBS. A representative from Learning Research and Development Center notes that a publisher is interested in distributing the program, but no firm plans for commercial dissemination have yet been made.

5.2 Developer's Evaluation

The IPI evaluation program has been geared to assessing the objectives, operation, and degree of implementation of IPI in the demonstration and pilot schools. In the early stages of IPI, evaluators were also functioning as developers and disseminators of the program. Their role in refining the curriculum and management system was difficult to isolate from their role in developing it. RBS published the second progress report on IPI in March of 1971. This contains references to more than 30 studies conducted from 1966 to 1971 by LRDC, RBS, and participating school personnel, whose goal was the improvement of IPI Mathematics.

Several sources were used to gather information. The first was data from student performance on all IPI tests--placement tests, pretests, posttests, and Curriculum Embedded Tests. Data on students' rates of learning was collected by recording the time required for students to progress from point to point in the sequenced continuum. Classroom observation was used to examine teacher, pupil, and teacher aide implementation of desired procedures. Conventional instruments were also used, such as student attitude inventories, parent interviews, pupil interviews, teacher ratings. In addition, results of standardized achievement tests were sometimes used.

5.3 Evaluation Results

Since one feature of IPI is that it is being continuously modified on the basis of student performance data, it is not intended to become a fixed program which can be given a final assessment. Therefore, any evaluation is seen by IPI as a description of what results have been produced at a particular stage of the program. Changes in the system have resulted in a general increase over time in student performance on IPI objectives. This is seen as an indication of IPI effectiveness.

Reporting results of student performance on standardized tests is seen as merely describing how IPI students working in a particular type of school in a particular type of community do on whichever tests are typically given in that school. They are not seen as an evaluation of IPI effectiveness because standardized tests do not measure student mastery as defined by the instructional objectives of IPI Math. RBS reported more than 25 studies which compared standardized test results of IPI students with non-IPI students in the same schools. There is no consistent pattern of IPI or non-IPI students performing better on the skills measured by standardized tests.

RBS did report that their findings in the affective area indicated that IPI students have a positive attitude toward school and learning, and demonstrate a change in social behavior. Also, parent reactions were reported to be positive.

In an overall sense, IPI is able to claim that by using the system, students will achieve mastery of the instructional objectives as defined by the system.

5.4 Independent Analyses of the Program

Because of IPI's school-wide scope, its renown, and its reliance on programming and systems theory, IPI has been the subject of intense interest and many independent studies and commentaries. Those studies that involved comparisons on standardized tests in general indicated no significant differences between IPI and non-IPI students. Findings in the affective area varied, with some reporting IPI students' self-concepts seem higher (e.g., Sandvick - An Evaluation of IPI [Math] Procedures, Carmen School, 1968-70, Waukegan School District 61, Illinois), and others reporting the contrary (e.g., Ms. Karin R. Myers - The Self Concept of Students in Individually Prescribed Instruction, Center for Innovation in Teaching the Handicapped, Indiana University, April, 1972.)

Most prominent have been analyses which express concern over the effects of the IPI system. One argument has been that the interpretation of individualization by IPI has concentrated on individualizing the pace at which students move through the system while little attention has been given to children's varying learning *styles*.

Rodney Tillman, Dean of the School of Education at George Washington University, opposes the IPI program for being based on a "what they should be taught" approach, rather than on a child-centered approach based on "observation of how children learn."

> The actual name of the program leaves me with concern. Usually we associate prescriptions with sickness, and while it may be helpful to "prescribe" for those unable to function in a normal manner the prescription approach for all children leaves much to be desired.[1]

A challenge of IPI's assumptions and performance arises from a University of Illinois study in which fifth- and sixth-grade Urbana students, who had used the IPI program since first grade, were given in-depth clinical interviews in the style of Piaget to assess their understanding of basic arithmetic concepts. Stanley Erlwanger concluded that these students, who are successful according to IPI criteria, reveal a basic misunderstanding of arithmetic.

> The insistence in IPI that the objectives in mathematics be defined in precise behavioral terms has produced a narrowly prescribed mathematics program with a corresponding testing program that rewards correct answers only, regardless of how they were obtained, thus allowing undesirable concepts to develop. . . Through an over-reliance by the teacher and pupil on the adequacy of IPI, and through the highly independent study by the pupil, the teacher is prevented by her perception of her role from understanding how the pupil learns and what he thinks. The rigidity of the IPI structure and its programmed mode of instruction discourages the use of enrichment material, and tends to develop in the pupil an inflexible rule-oriented attitude toward mathematics, in which rules that conflict with intuition are considered "magical" and the quest for answers "a wild goose chase."[2]

Eugene D. Nichols, Director of the Department of Mathematics Education at Florida State University, comments on the problem-solving goal of education in relation to individualized systems.

> There are two essential ingredients in the educational process which are necessary for teaching individuals to face novel problem situations: (1) the face-to-face discovery process--a back-and-forth encounter between a mature mind and a developing mind, and (2) a "room for disagreement and questioning" attitude on the part of the learner. No individual system in existence today has these features built into it. Furthermore it cannot, because a mind-to-mind confrontation leads to the unexpected, and that cannot be mapped out in advance.[3]

Alvin Hertzberg and Edward Stone, elementary school principals who are proponents of the open education methods practiced in British primary schools, analyzed the IPI approach in *Schools Are for Children* in 1971:

> Just as the textbook sequence will not fit each child, the programmed sequence will not fit each child. At its worst, this mode of instruction pays little attention to principles of child development; at its best, it directs its energies to the realization of an achievement goal without taking into account many other

> vital interests and attitudes of the child, and without proper concern for the many individualized ways of learning.
>
> Underlying the concept of a fixed scope and sequence in curriculum are assumptions that all children must be exposed to a set body of knowledge, that there is a basic amount of information to cover, that there are required skills which must be taught in a certain order, and that all children should learn the same things in the same way, and often in the same amount of time. But are these assumptions valid.[4]

It is important to note that developers have initiated a new version of IPI. In this version, Individualized Mathematics, they have attempted to break the tight structure of IPI by providing alternative sequences of units. The new program also incorporates the use of manipulatives.

5.5 Project Funding

The following information was obtained from a January, 1972 report done by the American Institutes for Research in the Behavioral Sciences under contract to the Office of Education.

Funds for the development of IPI Math have come from four basic sources:

1. U.S. Office of Education, through funding to the University of Pittsburgh Learning Research and Development Center, and through funding to the regional laboratory for Pennsylvania, Delaware, and New Jersey, Research for Better Schools, Inc.

2. The University of Pittsburgh, with additional grant and contract support from the Andrew W. Mellon Educational and Charitable Trust, the Carnegie Corporation of New York, the Ford Foundation, and the Office of Naval Research.

3. Funds from the Baldwin-Whitehall School District.

4. Considerable developmental monies provided by the Appleton-Century-Crofts Publishing Company.

5.6 Project Staff

The Learning Research and Development Center of the University of Pittsburgh (LRDC) and Research for Better Schools, Inc. in Philadelphia (RBS) have cooperated in the development and progress of IPI since

June 1966. Robert Glaser of LRDC was the major source of the developmental concepts of IPI. Other key personnel at LRDC include: John Bolvin, Director of the IPI Project, C. L. Linvall, Associate Director, William W. Cooley, Co-director, and Glen Heathers, Lauren Resnick, Richard C. Cox, Joseph I. Lipson, John L. Yeager, and Richard L. Ferguson.

At RBS, major responsibility for IPI has been held by James W. Becker and Robert C. Scanlon, Executive Director and Program Director, respectively. Both of these men received their initial contact with IPI while employed by public schools in the Pittsburgh area.

Also, personnel of the Baldwin-Whitehall School District of suburban Pittsburgh cooperated in the development of IPI, including W. R. Paynter, Superintendent.

FOOTNOTES

1. Tillman, Rodney. Do schools need IPI? no! *Educational Leadership*, 29, (6), March 1972, pp. 496-498.

2. Erlwanger, S.H. Benny's conception of rules and answers in IPI mathematics. *The Journal of Children's Mathematical Behavior*, (Urbana, Ill.: University of Illinois Curriculum Laboratory), 2, (1), November 1973, pp. 5-26.

3. Nichols, Eugene D. Is individualization the answer? *Educational Technology*, XII, (3), March 1972, p. 56.

4. Hertzberg, Alvin, and Stone, Edward. *Schools Are for Children*, New York: Schocken, 1971, p. 37.

INDIVIDUALIZED MATHEMATICS SYSTEM (IMS)

BEST COPY AVAILABLE

INTRODUCTION

Nine-year-old Jane is in a multi-age-grouped class of 30. At the beginning of math period she gets her IMS math folder and returns to her seat. Inside she finds the posttest on Division-Level IV, which she was working on yesterday. She has almost completed it and gets to work. When she's done, she takes the test to the place in the room where the answer keys are kept and corrects it. There are six skills in this unit and Jane achieves the mastery score indicated for all six skills. She marks her score on the proper form in her folder and checks her profile chart to see what unit comes next. It's Fractions-Level IV. Jane finds the pretest for that unit. She has learned how to work independently within the IMS system, but after she takes the pretest she will ask her teacher for a new "prescription form" assigning work in fractions. These assignments are presented not in a text or a workbook but on several large laminated worksheets contained in the Fractions IV "skill folder". . . . Among the self-teaching lessons are several activities with simple manipulative materials to be used according to directions. Another part of Jane's fractions prescription will be a "seminar"--a group project and discussion with other pupils working in the same unit and the teacher. Jane's classmates are all at different stages in the system, but working through similar cycles.

Individualized Mathematics System is an outgrowth of the curriculum developments of the 1960's which produced *Individually Prescribed Instruction*. IMS is a complete program reorganizing the traditional elementary mathematics curriculum into a sequence of 393 behavioral objectives and providing instructional materials and tests so that students can progress through the system at their own rate--not affected by their grade level or by the progress of other students. But IMS has several features which distinguish it from IPI, its predecessor. There is a variety of instructional materials including paper-and-pencil exercises but also activity lessons using manipulative materials. The lessons are not in consumable booklets but on approximately four thousand large, illustrated, laminated worksheets, which are reusable by other students. Materials were not designed to be "teacher-proof"; the developers stressed that the teacher was to be not only the manager of the children's pace through the system but also the provider for each child's individual style of learning.

Developed by educational technologists at the regional educational research laboratory in Durham, North Carolina, and field tested in Carolinas and Virginia classrooms, IMS is a complicated, comprehensive mathematics program. It prescribes a precise instructional sequence to be followed, but also attempts to inject more teacher-pupil and pupil-pupil interaction into the programmed curriculum.

BASIC INFORMATION

Program name: Individualized Mathematics System (IMS)

Format: Series of 3,994 instructional activities organized into 11 content strands. Each strand is divided into 9 levels of difficulty.

Uniqueness: Individualized instructional system. Plastic laminated worksheets used at Levels II-IX. (First-level lessons are in 11 consumable booklets.) Use of teacher-supplied manipulative materials with some lessons.

Content: Concepts and operations of mathematics organized into strands of numeration, addition, subtraction, multiplication, division, fractions, applications, money, time, measurement, and geometry.

Suggested use: Complete curriculum for grades 1 through 6.

Target audience: Students of all abilities, grades 1-6.

Length of use: Daily use, 30 to 45 minutes, for six years.

Aids for teachers: Systems Management Guide. Inservice training course.

Date of publication: 1973.

Director/Developer: The late Frank Emmerling was the principal developer. After May, 1970, development was completed under the direction of James W. Knight at the Center for Individualized Instructional Systems, a division of the National Laboratory for Higher Education, Mutual Plaza, Durham, N.C. 27701. (919) 688-8057.

Publisher: Ginn and Company, Division of Xerox Educational Services, 1901 Spring Street, Lexington, Mass. 02173.

1. GOALS AND RATIONALE

1.1 Goals

IMS comes out of the "break the lockstep" movement for individualized instruction, which sought to give the feelings of success to both slow and quick learners in the same classroom by differentiating the pace at which they learned but not the quality of what they learned. *Individually Prescribed Instruction--IPI*--(see page 101) is the immediate forerunner of IMS.

Believing that low achievers are held back by feelings of failure as much as by low ability, the pioneers in individualization sought to design instructional programs which would give these students experiences of success and would prove to their teachers that so-called slow students could master the same content as average and bright students. Each student was to be freed not only from his fellow students but also from dependence on the teacher. He was to work through the curriculum at his own speed, doing a new lesson only when a short test proved he had mastered the prerequisite material. It was expected that if the student gained his experience of success by comparison with his past work, not by comparison with his fellow students, he would greatly increase his incidence of success and his motivation to continue learning.

The first individualizers took for granted that there is a single body of mathematics which all children should master, and that the curriculum writers understood the mental processes of mathematics well enough to be able to specify a "hierarchy of competencies" which all learners should pass through--each at his own speed. IMS developers shared these beliefs. However, IMS developers judged that IPI lessons didn't provide students enough variety in style of learning, so they sought to add more choices as to *how* skills could be learned. They did not leave open to teachers or students the choice of *what* was to be learned or *when*; all students were expected to master the same content in the same order.

Early experiments with individualization showed that a class pursuing many individual learning paths could become a wilderness in which each solitary learner trudged on within sight and earshot of companions but isolated by his singular task. Thus another major goal of IMS was to bring the student into closer touch with the teacher, who could act as a tutor, and to help the teacher bring individual students' paths together from time to time by means of group instruction and partner or small-group projects.

IMS developers set out to improve the model established by IPI in the following ways:

1. To reduce the verbal content of students' lesson materials,

2. to make the printed materials attractive to children and relevant to their everyday experiences,

3. to include a variety of instructional styles offering different avenues to learning,

4. to give teachers a more creative role,

5. to develop efficient installation plans and classroom management techniques, and

6. to lower costs substantially.

Using IMS, students are to (a) master the learning objectives in the sequenced continuum, (b) proceed through the system at a self-paced rate, (c) become responsible for their own learning and achieve an enhanced self-concept. The mathematics to be learned is similar to that presented in traditional math programs--mainly number concepts.

1.2 Rationale

The thinking which produced IMS is a combination of recent ideas about the way people learn coming from two divergent directions--behaviorist and developmental learning theory. First of all IMS is based on the precepts of behavioral psychology and systems theory. Educators and learning psychologists committed to these ideas believe that any teaching task can be accomplished by breaking subject matter down, ordering it into a sequence of component bits, and administering each bit to each child separately. This way the student digests only one bit at a time--and in his own good time--and receives a reward (success) with almost every bite. This procedure is expected to guarantee that the student never encounters a learning task that his mind is unprepared for and also that the student forms a habit of success and a self-image as a learner. Systems technology, derived from very large and complex engineering and management enterprises, is applied to education as a way of keeping the separate learning bits firmly attached to a whole six-year curriculum, as well as enabling one teacher to keep track of the separate learning paths of a classroom of students all working on different lessons. The expected pay-off for this whole procedure is that the slow student should be able to master the same knowledge as the bright one, not a watered-down version.

Although IMS is basically an application of behaviorist programming and systems theory to elementary mathematics, it also reflects recognition of some limitations in IPI (IMS' forerunner) and a consequent attempt to compensate for them. IMS developers believed that children need to like math for its own sake, not just for the reward when they are successful. One key to enjoyment of learning was thought to be variety in ways to learn--more than paper-and-pencil exercises in a workbook. Although they believed that each child should work at his own pace, IMS developers did not believe the student should work by himself all the time. Most important, they believed young children need to understand math concepts through impractical experiences with concrete objects before they are able to work with abstract symbols like numbers and letters. In these respects IMS developers were influenced not only by their own and teachers' observations of children's work with IPI but also by the learning theories of developmentalists Jean Piaget and Zoltan Dienes, which began to influence American educators during the years following the first enthusiasm for programmed learning.

Piaget theorized that the child's most *efficient* (not just enjoyable) path to abstract logical thinking is his gradually developing understanding of his interactions with objects, happenings, and people in his immediately surrounding environment. Dienes designed materials that seem like toys but embody math concepts and thus can be used by teachers to stimulate the growth of conceptual foundations for later work with abstract numbers.

IMS developers added such experiences and materials to the lessons in their continuum. However, it is important to recognize that IMS uses materials to illustrate, activate, and make more interesting lessons already prescribed, whereas other interpreters of Piaget consider that there is no one continuum of behavioral objectives appropriate for all children, and that prescribed schemes of objectives do not adequately allow for the variety, complexity, and sophistication of children's thinking.

2. CONTENT AND MATERIALS

2.1 Content Focus

IMS presents a mathematics curriculum for grades 1 through 6 which is comparable in content to modern textbook series. IMS arranges the content into 11 topics, called "strands," 7 of which deal directly with number and operations. Three of the remaining topics--measurement, money, and time--also are based in number concepts. (Both English and metric measurement systems are taught.) The last topic is geometry. Sets, logic, statistics, probability, functions, graphs, and algebra--modern math topics taught in some recent elementary math curriculums--are absent. The same content is taught to all students.

2.2 Content and Organization of the Subdivisions

The 11 topics in IMS are Numeration, Addition, Subtraction, Multiplication, Division, Fractions, Applications, Money, Time, Measurement, and Geometry. All are introduced at the first-grade level and taught in the order listed above. Within each topic, a sequence of progressively more difficult skills is to be learned, 393 in all. (These are not the same objectives on which IPI is based, for IMS developers' intent to reduce the verbal content of IPI lessons led them to reexamine IPI's mathematical content as well.) These skills (stated as behavioral objectives) are organized into 9 levels of difficulty. Several topic skills at the same level comprise a unit; for instance, Multiplication, Level V. All of the Level I units are taught in order to the whole class at the same time. Beginning with Level II, students' progress through units is self-paced but prescribed by the teacher on the basis of placement tests and unit pretests and posttests. In Levels II through IX a student may vary the order in which topics are studied only if the diagnostic tests show he already possesses mastery of some units.

Since IMS is an uninterrupted sequence of skills, and each topic builds on the one above (see Table 1), the student starts filling in the knowledge gaps which appear on his profile chart always by working on the unit which is located in the topmost lefthand square in his chart. For instance, the Student Profile Form below shows that this child will be assigned to work on Division, Level III. After he masters this he will move to Multiplication, Level IV and work down the IV column, skipping Money, Time, and Measurement but doing Geometry, before starting Level V at the top, Numeration. The program is always to be used in this sequence.

IMS	I	II	III	IV	V	VI	VII	VIII	IX
NUMERATION	X	X	X	X					
ADDITION	X	X	X	X	X				
SUBTRACTION	X	X	X	X					
MULTIPLICATION	X	X	X						
DIVISION	X	X							
FRACTIONS	X	X	X						
APPLICATIONS	X	X	X						
MONEY	X	X	X	X	X				
TIME	X	X	X	X					
SYSTEMS OF MEASUREMENT	X	X	X	X					
GEOMETRY	X	X	X						

Table 1

2.3 Materials Provided

Student. Level I materials consist of 11 consumable paperback Skill Booklets, 1 for each topic. These are workbooks for student practice of skills taught by the teacher to the whole class. For Levels II through IX, there are 357 Skill Folders, each containing from 6 to 20 separate instructional pages. There are 4 different kinds of pages in each folder:

1. Guidelines. These state the objective to be learned, list the workpages and their content, and indicate the vocabulary that should be learned and special materials that are needed. Guidelines are a summary of the folder's contents.

2. Workpages. There are three different kinds of workpages: teaching pages present the concept; practice pages provide for using the concept; extension pages provide for using the concept in combination with a previously learned skill.

3. Check-tests. There are two in each Skill Folder. They are miniature posttests.

4. Activity pages. These are assignments for projects or games giving the student experience with a skill. Activities may be for one child, partners, or a group.

The worksheets are color-coded so that each topic is identified by one color throughout the system. Each worksheet is plastic laminated. and students mark on the plastic with special IMS pencils. After the student completes and checks his work with the answer keys, and records results, he wipes the sheet clean and returns it to the storage cart. Thus one set of materials stored centrally is sufficient for a whole class. Included in the central supply are consumable placement tests, and unit pretests and posttests. Each child has a folder of his own, which contains his pupil profile, prescription form, and work that is current.

Cartoons and drawings are used wherever possible, both to make the program appealing to children and to minimize dependence on reading skill.

Teacher. The teacher's basic resource is the Systems Management Guide, a 176-page bound volume describing the complete program and procedures and including samples of student materials and forms. The Guide also contains directions for teaching the lessons, called "seminars," which comprise all the instruction at Level I, and which provide a large-group mode of instruction, offering reinforcement and practice, at the end of each unit in Levels II-IX. The teacher receives duplicating masters for student profiles, prescriptions, and class profiles. A mobile cart, to store materials for 100 students, is available.

2.4 Materials Not Provided

Manipulative materials are called for throughout IMS. All materials are to be provided by the teacher. A suggested materials list divides materials into four classes:

1. Supplies: general office and school supplies, such as crayons, tape, rulers, scissors, magnets, pipe cleaners, etc.

2. Math materials to buy: specifically math-oriented materials, such as blocks, centimeter rods, play money, inch cubes, etc. Suggested vendors are noted, and those materials essential to IMS are starred.

3. Materials to make: includes number cards used in activities, number lines, transparencies for overhead projector, etc.

4. Materials to bring from home: coat hangers, egg cartons, paper cups, toys, etc.

3. CLASSROOM ACTION

3.1 Teaching-Learning Strategy

The necessity to allow for the varying rate and state of ease with which fast, slow, and average students within the same classroom will master the same objectives causes the abandonment of the technique of whole-class lessons conducted by the teacher, and the installation of educational technology to handle individualized instruction. This technology consists of individual lessons presented by "self-teaching" printed materials in a packet, not bound in a book, *plus* a battery of short tests telling when each student is to study which lesson. In this system the teacher acts to teach students how to use the system efficiently and as a backup to the system. The procedures are as follows:

1. The *Placement Test* determines the level at which each student should begin instruction in each topic. The student takes the placement test only once, when he enters the program. The

placement test is a printed 32-page, consumable booklet divided into Part I and Part II. Each part has a 10- to 12-minute test for each of the 11 topics. Each student takes *either* Part I or Part II of each test. Teachers are supposed to correct the placement tests themselves, thereby gathering from answers to individual test items more precise diagnostic information than the test scores alone reveal. The scores do tell the student's general competency level in each topic, and this information is plotted on a *Student Profile Form* for each student.

2. Working from the profile, the teacher assigns each student the *Pre-test* for the first unit which shows up as unmastered on his profile. For instance, in the profile on page 109, that pretest assigned would be Multiplication, Level IV. The pretest results show which skills, if any, within the unit the student already knows, and thus can skip.

3. *Prescriptions* are made from the pretest results. A prescription starts with an assignment to learn the first skill missed on the pretest, by studying a series of *workpages*. As the manager of the child's learning, the teacher is responsible for writing the prescription, although in the upper grades students themselves can learn to do this. The teacher is expected to know how each child learns best; how big a chunk of workpages to prescribe at one time; and what kinds of workpages will be most effective.

4. Work in a *Skill Folder* now gets underway. There are different kinds of workpages appearing as separate, unbound sheets within each folder: teaching pages, practice pages, extension pages, checkup tests, and activity pages. The latter are projects for one or more children, providing experience with a particular skill just learned. The student scores his own workpages from the answer key in the IMS supply cart, and records his results on his prescription form.

5. Sometime during his work in a unit the student participates in a *skill seminar*, a task for pairs or teams of students all working on the same unit. The task is followed by a teacher-led group discussion about the generalizations that can be derived from the task.

6. When the student successfully completes all the checkup tests for all the skills the teacher has prescribed for unit mastery, he can take the unit *Post-test*. If the test shows incomplete mastery, the teacher sends the student back to workpages in the Skill Folders, prescribes peer tutoring, or tutors him herself.

When mastery is achieved, the child records this on his profile and begins the cycle over again by taking the pretest for the next unmastered unit on his profile.

IMS intends to acknowledge different learning styles and stages in children, and so offers five different learning styles in the lessons appearing on the worksheets. Teachers should know which of these styles best fits the student in order to prescribe the most suitable workpages.

Lesson Style 1	The child works by manipulating real objects. Workpages of this sort use drawings rather than words to indicate to the child what is needed. Most of these lessons appear in Level I.
Lesson Style 2	The child works with perceptual materials. Drawings of objects, charts, or various shapes and forms are presented on workpages as representations of numbers.
Lesson Style 3	The child works abstractly. The workpages in this category present numbers and symbols.
Lesson Style 4	The child participates in activities--on his own, with a partner, or in a small group--which require him to use a mathematical skill to play a game or carry out a project.
Lesson Style 5	The child participates in teacher-directed group projects, games, and discussions, which require him to use the skill and then to think and generalize about it in words.

Descriptions of IMS maintain that the curriculum provides opportunity for "open-ended" or "problem-solving" activities, but the behavioral objectives in the continuum do not stipulate such kinds of learning. Activities described as open-ended are not presented as challenges to the child's own inventiveness but as tasks leading to a given behavioral objective although they do offer some options as to the manner in which the objective will be reached; for instance, the numbers which will be used. The program does not encourage students or the teacher to devise problems from the local environment or student interests which could be solved by the use of math.

Although the IMS developers strongly stressed the need for the teacher to use his or her own instructional ideas and knowledge about individual students to enrich and supplement the system, the Ginn published version gives no special encouragement to nonsystem activities. If a student comes up short on a posttest; for instance, the only means of help suggested are a return to the workpages, peer or teacher tutoring. Even though the IMS developers believed that too rigid interpretation of the program would cause student boredom and that teachers should not assume the materials would do the teaching for them, the publisher's promotional materials convey the impression that the system is considered all-providing, and teachers' own ideas or judgments should be brought into play only as a last resort.

The exception to this impression appears in Level I, in which the manner of instruction is turned around: group projects and teacher-led "seminars" emphasizing work with manipulative materials come first, and individual student work in booklets follows. These seminar lessons are outlined in the *Systems Management Guide*. By the end of Level I students are expected to be ready for transition to the pretest-prescription-workpages-posttest cycle of instruction.

Worksheets provide a consistent set of pictorial rather than verbal instructions. Thus it is believed that students with reading problems will not be penalized in their learning of mathematics.

3.2 Typical Lesson

Math period begins for a class of 30 8-, 9-, and 10-year-olds. Each child gets his or her math folder and begins to work.

Four children are all working in Numeration, Level IV, each with a prescription assigning different workpages within this unit. Nevertheless, as all have been working several days in the unit, this is a good time for the teacher to pull them together for a "unit seminar"--a team game explained in the Systems Management Guide. The students take turns drawing number cards (numbered from 0 to 250) from 5 packs of 3 cards each. They place the cards from each pack in order and record the order for each pack. After they finish this task the teacher conducts a discussion with the children around questions suggested in the Guide: "How did you decide if the numbers were in order? Which digit should you look at first?" Etc.

Meanwhile, two students who are ready for posttests, one for Fractions IV and the other for Money V, go to the materials cart to get the tests and take them. Four other students who yesterday scored mastery on various posttests are taking pretests for the next units on their profiles.

Six students are working on laminated worksheets they started several days ago. They are all working singly, each in a different Skill Folder. Two girls are scoring their posttests, one in Numeration

V and the other in Division IV. The latter discovers she has not passed and goes to the teacher for a prescription calling for a repeat of some workpages and then a session with the teacher. The other girl passes her Numeration V posttest and begins the IMS cycle again with the next unit on her profile, Multiplication V.

Seven other students received new prescriptions today in units ranging from Levels II to VII. They all work independently. For instance, Jim is doing a "skill 2" worksheet in Measurement III, which calls for him to use a ruler to measure distances between points on a "pirate treasure map" and then convert the total distance from "start" to "treasure" into miles, according to the scale given on the map. Jim's best friend, Peter, is taking a checkup test on "skill 2" in Money V. The worksheet shows pictures of items with pricetags on them, and adjacent pictures of coins, and asks how many coins are needed to purchase the items. Two more students have prescriptions to work together on a game matching multiplication factor cards (for instance, the card 68 x 34 is matched with the card 34 x 68). This is an activity page for "skill 2" in the Multiplication III unit.

After conducting the Numeration seminar the teacher tutors two children in Multiplication IV.

3.3 Evaluation of Students

The entire IMS system depends on testing, which is called for at each small step along the continuum. All test items are derived from the behavioral objectives in the IMS continuum. In addition to the placement test, pretests and posttests, there are two or three checkup tests within each Skill Folder. They are supposed to indicate whether a child is understanding the workpages in a unit as he goes along. If he falls short on the checkup he can go back immediately and redo the workpages. If the student passes the checkup tests for all the skills in a unit, he can go on to the posttest. Mastery scores are indicated on each test: /5/6/ indicates that the test has six items and the student must have five correct in order to pass.

3.4 Out-of-Class Preparation

Teacher. What the teacher has to do before class falls into the areas of organization and instruction. In terms of organization, the teacher must do what is necessary for each child to have his work clearly laid out. That means scoring any pages or tests that children can't score themselves, writing all necessary prescriptions, arranging needed materials. Instructional tasks include planning for group seminars for students working in the same unit at the same time, and for tutoring.

Teachers involved in field testing said that IMS takes an enormous amount of work at first, when students are starting to learn the system.

This preparation time eases up as the year progresses and older students assume responsibility for much of the mechanics of the system. However, for younger children, more work is needed.

Student. There is no student homework in IMS.

3.5 Role of Other Classroom Personnel

Although Ginn makes no mention of aides in its promotional materials, IMS developers strongly recommended aides without requiring them. These developers suggested seven ways in which an aide could help:

1. Assisting in the administration of tests;
2. assisting in the scoring of tests;
3. assisting in the recording of test results;
4. helping to train students in the procedures of the system;
5. keeping weekly placement charts;
6. keeping track of supplies;
7. ordering materials.

Help may be essential when the program is starting. After the program is underway, older children assume routine responsibilities. If younger children are not able to do their own scoring and recording, help may be necessary to keep the teacher from getting too bogged down in the mechanics of the system.

4. IMPLEMENTATION: REQUIREMENTS AND COSTS

4.1 School Facilities and Arrangements

Since IMS is individualized, it is particularly suited for nongraded classrooms. It accommodates a wide spread of ages and ability levels within a single classroom. It can be used equally well in self-contained classrooms or open-pod schools. It is not suited for teachers practicing "open education," which calls for each teacher to create naturalistic curriculum materials from children's own experiences and interests.

School scheduling will depend on how much IMS material is available in the school. Materials are sold in "Level Boxes" (all the tests and Skill Folders for all the topics in a single level). A school with 100 students in grade 3, whose students are performing at national norms, needs three boxes of Level II materials for the third graders, four boxes of Level III, three boxes of Level IV, and two of Level V. This supply can be stored on a mobile cart available from the publisher. If this set is shared among several classrooms, each class schedules math at a different hour.

4.2 Student Prerequisites

There are no special student prerequisites for IMS. A student begins the program at the unit indicated by the placement test.

4.3 Teacher Prerequisites

IMS teachers do not need mathematics background. However, they must receive training in the use of the program. The publisher provides attendance at a training workshop for two or three teachers in a school adopting IMS. These teachers are to teach the rest of the staff in their school. Each receives a training kit sufficient to train ten other teachers in the school.

Training and instructions focus on management of the system's mechanics. No emphasis is placed on teacher presentation of lessons except for beginners, and in the group seminars which terminate each unit. The workpages are assumed to handle all instructional tasks except for instances of a student not passing a checkup or posttest.

4.4 Background and Training of Other Classroom Personnel

While the publisher makes no recommendation for aides, presumably if they are used, they should be trained along with teachers using the system.

4.5 Cost of Materials, Equipment, Services

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Level boxes (Containing 1 of each folder in the level, 96 pre- and posttests, pencils, record form masters)	Depends on grade level and number of students. (For groups of 100 students, number of boxes varies from 9-15 for each grade 1-6)	Ginn and Co.	($118.40-$179.52)	Reusable
Systems Management Guide	Shared by several teachers	"	$6.00	Reusable
Answer Keys	Shared by several teachers	"	$9.20	Reusable
Placement Tests	1 set per 35 students	"	$16.18	Yearly

4.6 Demonstration Sites

The following is a partial list of IMS users.

California:	Lee Mathson School San Jose, Ca.
Indiana:	Riverside Elementary School Jeffersonville, Ind.
Massachusetts:	Andover Public Schools Massachusetts
Minnesota:	Roosevelt Elementary School South St. Paul, Minn.
North Carolina:	Winston-Salem/Forsyth County Schools Winston-Salem, N.C.
Texas:	Marlin Independent School District Marlin, Tex.
	Juvenile Achievement Center School Waco, Tex.

5. PROGRAM DEVELOPMENT AND EVALUATION

5.1 Program Development

IMS is an outgrowth of an earlier systems approach to mathematics curriculum, *Individually Prescribed Instruction (IPI)*, developed by the Learning Research and Development Center, University of Pittsburgh. In 1968 the elementary and secondary school division of the Regional Educational Laboratory for the Carolinas and Virginia in Durham, North Carolina, began the project of making IPI manageable in a classroom setting. The Laboratory is now the National Laboratory for Higher Education and the completion of IMS was undertaken by a separate division, the Center for Individualized Instructional Systems.

Some of the original staff of IMS had enthusiastically worked with IPI, but they wanted to change it in several ways: (a) from sole reliance on paper-and-pencil exerices to provision of several lesson styles; (b) to reduce the program's dependency on student reading ability; (c) to give teachers a broader range of teaching activities for each behavioral objective; and (d) to improve classroom efficiency and cost effectiveness of the system. As these aspects of IPI were considered for change, the new format of IMS began to emerge. The program which started as an installation of IPI became a major developmental effort as it attempted to correct problems teachers experienced with IPI.

IMS was a cooperative venture with school administrators and teachers from the beginning. In order to keep the program responsive to schools' needs, and to continue financing for evaluation and dissemination after the Office of Education grant for development expired, the Consortium for Individualized Instructional Systems was established in 1970. The Consortium included 34 schools in the Carolinas and Virgina, plus the state departments of education in these states.

Besides evaluating IMS the Consortium undertook to develop IMS-II, an extension of the existing IMS into grades 7-9. The basic philosophy for IMS-II is the same; the format differs in that skills are clustered into learning booklets, and lessons need not be taught in any one prescribed sequence. The complete junior high program is to be ready by fall of 1975.

5.2 Developer's Evaluation

Preliminary testing was carried out during the 1968-69 school year with 2,400 pupils in four schools. Eight schools and about 1,000 second- and third-grade pupils began using IMS on an experimental basis during the second half of the 1969-70 school year.

During the 1970-71 school year, more than 5,000 pupils in 23 schools in the Consortium field tested IMS at all grade levels. The IMS Formative Evaluation Plan was drawn up in August of 1970 to determine whether IMS had achieved the following goals:

1. Curriculum adequacy. The provision of a comprehensive set of mathematics objectives suitable for a wide spectrum of pupil aptitudes.

2. Materials effectiveness. The provision of attractive and effective learning materials and teaching aids which incorporate various alternative means of achieving curriculum goals.

3. Cost-effectiveness. Achievement of low cost per pupil compared with other available mathematics systems with similar structure.

   Specific areas to be considered in this respect are:

   a. Actual production costs and adequacy of reusable materials.

   b. The extent to which students can and do assume responsibility for operation of the system (thus reducing or eliminating the need for paraprofessional personnel in the classroom).

c. The cost and effectiveness of teacher training required to implement the system.

4. Learning effectiveness. Pupil achievement and progress within the system comparable with or superior to that obtainable under conventional teaching conditions.

Information about different aspects of the system was obtained from questionnaires and surveys concerning teachers' opinions, from reports by "experts" in the field of mathematics and from training records gathered at IMS teacher workshops. In addition, student test scores from four schools were collected on the Metropolitan Achievement Test (MAT) in grades 1 and 2 and the Iowa Test of Basic Skills (ITBS)--Mathematics in grades 3 through 6. Teachers were polled often about materials they felt ought to be revised or which received unsatisfactory student responses or reactions. Only Levels I through VI were available for student use at the time of this evaluation.

5.3 Evaluation Results

The major concern of the evaluation was to obtain revision data to improve the system. The evaluation was not chiefly concerned with comparing what is learned in IMS with what might be learned in a conventional curriculum. However, grade level scores on the standardized tests showed that IMS students who had been six months or more below grade level at the beginning of the year gained approximately one year. Average and brighter students did not achieve a year's growth. This outcome was attributed to the fact that brighter students spent most of their time mastering below-grade-level topics formerly learned too superficially to achieve mastery on the IMS tests.[1]

In 1971-72 a follow-up test of students who had worked exclusively in IMS for two years was carried out by the developers. A group of 453 fourth-, fifth-, and sixth-graders in three Title I schools in the Carolinas and Virginia were tested on the Iowa Test of Basic Skills, and their scores were compared with the average scores of children in the southeast region. The study concluded that over the two-year period IMS pupils "made conceptual gains quite consistent with (or slightly above) children in the region."[2]

Changes in the design of the junior high program (IMS-II) may possibly indicate the developers' own judgment that the structure of IMS-I is too rigid, although this is nowhere stated. The junior high program now being developed presents groups of skills together in "learning situations" instead of teaching one skill at a time as in IMS-I. Developers say the expanded lessons make it possible to emphasize relationships among math topics. Grouping topics together means that strictly prescribed sequencing is not possible, and students are not required to prove mastery in one topic before undertaking another. Students are largely free to develop their own sequences of study. These seem significant departures from the strict programming theory of IMS-I. Developers attribute these departures to the fact that they are now designing for older students. Many observers would argue that seeing relationships among math topics and forming their own sequence for learning skills is just as appropriate for elementary as for junior high students.

5.4 Independent Analyses of the Program

None were obtained.

5.5 Project Funding

From 1968-71 funding was provided from the Office of Education grant to the Laboratory for the Carolinas and Virginia. In 1971 support was provided by school districts and state education departments in those states which were using IMS.

5.6 Project Staff

IMS was developed by the Center for Individualized Instructional Systems, a division of the National Laboratory for Higher Education. Dr. Frank C. Emmerling, who died in May, 1970, was the prime developer of IMS. J.W. Knight directed work to complete the project. Members of the staff were Edward Bruchak, T. Jeffrey Cartier, Jerrie P. Charlesworth, Evelyne Graham, William U. Harris, Kenneth B. Hoyle, Ellen M. Ironside, Daniel C. Morton, Jack E. Nance, and Audrey N. Walker. Fred E. Holdredge was the director of evaluation and development, assisted by Robert B. Frary and Victoria Fuller.

FOOTNOTES

1. Frary, Robert B. *Formative Evaluation of the Individualized Mathematics System (IMS)*. Durham, N.C.: Center for Individualized Instructional Systems, 1970. ERIC ED 059 096.

2. Fuller, Victoria. *Evaluation of IMS-I, Follow-up Report*. Durham, N.C.: Center for Individualized Instructional Systems, 1972.

PATTERNS IN ARITHMETIC (PIA)

INTRODUCTION

For fifteen minutes to half an hour each week, students and teachers view televised *Patterns in Arithmetic* programs. The lessons serve two functions: they introduce new topics to pupils; at the same time they teach the topic and model teaching methods to the teacher. The televised instruction is thus designed for both teachers and students. TV programs do not replace the classroom teacher, who retains responsibility for daily instruction. On those days when students and teachers watch the televised lessons, the teacher first prepares the students for the broadcast, watches it with the students, and then discusses what they have seen. For the remainder of the period, and for the entire mathematics period when programs are not seen, students work with math manipulatives and in PIA student exercise books. The teacher's manual contains lesson plans coordinated with the TV programs.

PIA was developed between 1959 and 1969 when most teachers had little background in new math but were required to introduce it into their classrooms. Developers thought TV would make it possible for large numbers of teachers and students to learn modern mathematics together quickly and easily. Developers soon realized that PIA was not a program that would capture an audience over a long time period. The districts that did adopt the program used it to introduce "new math" to their students and teachers; after using it for a year or two most districts switched to other mathematics curricula. In the 1972 Wisconsin R&D Center *Basic Program Plan* developers note:

> After teachers have used it (PIA) for one to three years and have mastered the "modern" mathematics concepts, they tend to discontinue using the program. Professor Henry Van Engen, the principal investigator and primary PIA developer hypothesized that this pattern of use would occur. PIA is a mass educational approach and is incompatible with the instructional programming model for the individual student. Schools do not have the equipment or money to purchase the video tapes for more flexible individualized use.[1]

By 1972 developers at Wisconsin Research and Development Center were concentrating on a new program, Developing Mathematical Processes (DMP), a mathematics curriculum for schools using Individually Guided Education (IGE). A report on DMP is included in this book. Distribution of PIA was turned over to the National Instructional Television Center. A field representative for NIT says that the program has never achieved widespread use. During the 1973-74 school year, the program was purchased by only seven school districts or state departments of education.

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BASIC INFORMATION

Program name: Patterns in Arithmetic (PIA)

Format: PIA consists of 333 15-minute television programs which introduce the main topics of a 1-6 curriculum in modern math.

Uniqueness: Television master teachers provide a model for teachers while introducing new topics to children. Teachers design weekly instruction around one or two 15-minute TV lessons.

Content: Arithmetic in a modern math context. Fundamental strands include: sets, number, numeration system, operations, mathematical sentences, measurement, and geometry.

Suggested use: Complete curriculum and inservice program for grades one through six.

Target audience: Students of all abilities in grades one through six; elementary teachers, one through six.

Length of use: Four to five hours per week. The TV lessons occur only twice a week, for about 15 minutes each time. These are lessons which introduce new concepts and are followed by daily instruction directed by the classroom teacher. Districts tend to use PIA for a year or two to introduce "new math" and then switch to another mathematics curriculum.

Aids for teachers: Teacher's manual for each grade level.

Date of publication: Fall 1969 by television broadcast, fall 1971 by videotape reels.

Director/Developer: Professor Henry Van Engen, University of Wisconsin, Wisconsin Research and Development Center for Cognitive Learning, 1404 Regent St., Madison, Wis. 53706. (608) 262-4901. Developed in cooperation with The Wisconsin School of the Air, WHA-TV.

Distributor: National Instructional Television Center (NIT), Box A, Bloomington, Ind. 47401. NIT regional offices: Arlington, Va.; Wauwatosa, Wis.; San Mateo, Ca.; and Atlanta, Ga. Programs may also be available through local educational television stations.

1. GOALS AND RATIONALE

1.1 Goals

The developers of Patterns in Arithmetic set out to construct a complete curriculum in elementary mathematics that would present the modern ideas of the "revolution" in mathematics since World War II.

> In spirit and in body, the mathematics courses (following World War II) were not those needed for the further industrial and scientific growth of the nation. The spirit of the old elementary mathematics was too heavily loaded with computational devices with too little emphasis on the fundamental ideas of mathematics.[2]

The developers also sought to apply the findings of research about how children learn math; for instance, the finding that spiral organization of subject matter through several years is more conducive to learning than long-term concentration on a single skill at a time; and the finding that a variety of applications of an idea or a skill--including physical manipulations--in which mastery is not immediately required, is more effective than drill.

Thirdly, the developers sought to provide major, immediate implementation of the new curriculum by insuring that districts would train teachers in new math concepts and methods. Television could simultaneously teach the students *and* provide inservice training for their teachers. The developers believed it could re-educate the huge staffs of big-city school districts and the scattered staffs of small or rural districts, "communicating the newer ideas in mathematics. . .and demonstrating a change in the spirit of teaching the subject." This change in spirit was defined as letting children find out that mathematics can be enjoyed.

PIA is expected to produce both cognitive and affective results in students; that is, to improve their *learning* in math and their *feelings* about learning. First of all, the developers intended that children experience arithmetic as "new ideas": sets, natural numbers, functions, integers, decimals as a numeration system. Students should learn to compute--addition, subtraction, multiplication, division--in association with these math concepts rather than as rote calculation; and they will learn to apply computational skills in verbal situations, and to translate verbal problems into mathematical sentences. Children should encounter geometry in early years as experience with shape and symmetry; learn to measure length, area, and volume; and get an introduction to number theory and probability.

The developers placed importance on the affective responses of students to the curriculum. They believe that television is "intensely" interesting and stimulating to children and thus that televised instruction will in itself heighten children's initial interest in math. They expect the discovery and conceptual approach will make math enjoyable to children. "Children will enjoy this arithmetic because it is an arithmetic of ideas. You will enjoy teaching it for the same reason."[2]

1.2 Rationale

The developers of PIA believed that the old methods of teaching arithmetic were too drill-oriented and children were not being given enough opportunity to learn mathematical concepts. Developers directed their thinking toward determining what children should learn about mathematics and how to stimulate them to learn concepts faster and retain them longer.

Certain learning principles generated by basic research at the Wisconsin Research and Development Center were crucial in construction of the program. It was determined that a spiral organization of content was more conducive to learning arithmetic than a nonspiraled one. Thus, PIA does not spend more than two consecutive weeks on any one idea or skill. Instruction in the skills of addition, subtraction, multiplication, and particularly division is reintroduced throughout as many as three or four years. This method is counter to the actual practice of many classroom teachers who like to teach all of the skills of addition of two- and three-digit numbers before considering other skills.

Research is continuously being carried out on questions such as the effectiveness of sequential learning. Results have indicated, for example, that the idea of one-to-one correspondence (matching) is more fundamental than the idea of counting for young children. The program lessons reflect this finding in the introduction of one-to-one correspondence in the first lesson in first grade, before counting is presented. Thus, during the early stages of the program, PIA uses those problems which have been found the easiest. The harder problems are presented only when the children have mastered the basic concept.

2. CONTENT AND MATERIALS

2.1 Content Focus

Patterns in Arithmetic presents both arithmetic and geometry in all six elementary grades. Nine key concepts thread throughout the six-year course. They are presented in lessons which use television and classroom demonstrations, classroom discussions of examples of the concept from the students' daily lives, and workbook exercises of both nonverbal and verbal problems. The key concepts and the topics which comprise them are as follows:

1. Sets. Sets, one-to-one correspondence, transitivity, numerousness, conservation. The set idea is used in teaching addition, subtraction, fractions, and geometric figures.

2. Number systems. The natural numbers, the positive rational numbers, zero.

3. Numeration systems. The decimal system. (Other systems are introduced as enrichment for gifted pupils.)

4. Operations. Addition, subtraction, multiplication, division, and the concepts associated with computing.

5. The mathematical sentence. Mathematics is a language with unique ways to express its ideas. Emphasis on formulating sentences clearly and on translating verbal problems into mathematical sentences.

6. Measurement. Basic ideas of linear measurement, area, and volume. This is taught "as a key link between our physical and our social environments."

7. Geometry. Early intuitive exploration of similarity, congruence; later these are approached from transformations in the plane.

8. Number theory. Prime and composite numbers, prime factorization.

9. Probability. Elementary ideas.

2.2 Content and Organization of the Subdivisions

PIA's nine basic concepts are arranged in a spiral sequence. Pupils encounter the same ideas and practice the same skills many times in increasingly complex settings. A brief summary of the content of each grade level shows how several concepts are taught at each grade level.

Grade 1: Natural order of numbers is taught through 99; addition through 10; monetary system; linear measurement; geometrical concepts of curve, triangle; mathematical sentences are introduced.

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Grade 2: Order of numbers is extended through 999; addition and subtraction through 999; special properties of zero; multiplication using facts to 18; geometrical concepts are extended; linear measurement emphasizing the inch and foot; perimeters of polygons; mathematical sentences extended to problem solving with larger numbers.

Grade 3: Order of numbers extended to 9999; base 5 and base 8 number systems introduced; addition and subtraction extended to deriving sums of two and three numbers less than 1000; concept of factors introduced; special terms of multiplication; mathematical sentences used in multiplication, division, addition, and subtraction problems; geometrical concepts of similarity and congruence for polygons, angles, and circles; idea of function introduced.

Grade 4: Order of numbers extended through millions; division with remainders; fraction explained using model; linear and liquid measures extended; geometrical concepts of parallel and perpendicular lines.

Grade 5: Order of numbers extended to decimals and fractions; long division, including techniques for shortening; rationals in percentages; fractions greater than 1; addition and subtraction of fractions with unequal denominators.

Grade 6: Place value extended through ten-thousandths for decimals; base 2 number system presented in relation to computers; remainder in division related to fractions; addition and subtraction of fractions and decimals extended; multiplication and division of fractions and decimals thoroughly developed; negative whole numbers reviewed; solutions of equalities and inequalities graphed; area formulas for triangles and parallelograms presented; measurement of volumes and angles introduced; probability introduced; bar, line, and circle graphs interpreted.

The titles of the 30 lessons which comprise the entire first year also demonstrate the use of a variety of concepts at the same level of difficulty.

1. One-to-One Correspondence
2. Transitivity of "As Many As"
3. The Numbers from One to Four
4. Ordering the Numbers from One to Four
5. Conservation of Numerousness 1

6. Transitivity of "More Than" and "Fewer Than"
7. Conservation of Numerousness 11
8. Introduction to Addition
9. The Numbers from Five to Seven
10. The Numbers from Eight to Ten
11. Ordering the Numbers from One to Ten
12. Addition Combinations 1
13. Addition Combinations 11
14. Geometry: Open and Closed Curves
15. Addition Combinations 111
16. Geometry: Points and Curves
17. Addition Combinations IV
18. Geometry: Betweenness
19. Sets of Ten
20. Numeration: Eleven Through Nineteen
21. Numeration: Twenty Through Ninety-Nine
22. Numeration: Order of Ten Through Ninety-Nine
23. Our Monetary System
24. Introduction to Subtraction
25. Subtraction Combinations 1
26. Introduction to Measurement
27. Subtraction Combinations 11
28. Standard Units of Measurement: Inch, Foot
29. Geometry: Names for Common Curves
30. Ordinal Numbers

Midyear Check-up Exercises
End-of-Year Check-up Exercises

Unlike other televised instruction programs, Patterns in Arithmetic is constructed as a self-contained curriculum. Henry Van Engen, primary PIA developer, served as a major consultant to the Scott Foresman publishing company. Because of this relationship, one field representative says, PIA and the Scott Foresman math textbooks have many similarities. Although some teachers have combined the two programs, PIA is not intended to be used in conjunction with any textbook series.

2.3 Materials Provided

Student. A 15-minute lesson taught by a teacher on television introduces each new topic of instruction. There are 30 TV lessons for first graders, 48 for second graders, 64 for third graders, 63 for fourth graders, and 64 each for fifth and sixth graders: 333 in total over the six-year curriculum. The older children see televised lessons twice a week; the younger ones only once a week. Since the curriculum is intended to be taught every day in periods of about 45 minutes, the televised lesson serves as an introduction rather than as the lesson itself, which remains the responsibility of the classroom teacher. A television monitor is needed for every group of 30 or fewer students using PIA (see Implementation section). If the televised lessons are shown on classroom videotape players rather than by broadcast, the player can be mounted on a portable cart to serve several classrooms at different times during the day.

Each student has his own pupil exercise book. The book provides activities which extend the concepts introduced in the TV lessons, plus exercises for practicing computation.

Preview materials. Preview materials consist of one or more representative lessons from the series, one copy of the Teacher's Manual, and other related print materials. These materials are available upon request, free of charge, from the distributor, NIT. A handling charge of $5.00 per lesson is made when materials other than the standard preview package are requested.

Teacher. There is a separate Teacher's Manual for each grade level. For each lesson the manual devotes two or three pages to a list of lesson objectives, brief mathematical background on the concept (for the teacher), directions for preparing children for the telecast, a description of the materials to be used by students during the telecast, a description of the television program, instructions for classroom discussion or activities immediately following the telecast, several suggestions for follow-up activities, and instructions for directing pupils' work in exercise books.

2.4 Materials Not Provided

Student activities require the use of some materials usually provided in an ordinary classroom (flash cards) and others that teachers may have to provide (buttons, beads). Some materials considered essential for each grade level are not provided. For instance, for Grade 1 these materials include the following: flannelboard or magnetic board; cut-outs or magnets of small animals; geometric shapes; numerals, and sets of ten; addition flash cards (to 10); subtraction flash cards (to minuend 10); play money; several sets of number cards and sets; symbols; number line; objects which can be readily bundled into sets of 10 or 100; place value chart; foot rulers; plastic or wood geometric shapes; wire or plastic plane figures; and a counting chart.

3. CLASSROOM ACTION

3.1 Teaching-Learning Strategy

PIA was designed to maximize participation by and interaction between teachers and students even though it relies on televised instruction to introduce new mathematical topics. Responsibility for teaching rests with the classroom teacher, following the example set by the master teacher on television and the directions given in the Teacher's Manual. The manual [illegible] the special role of the classroom teacher:

> Remember that the telecasts are not intended to replace the classroom teacher. They are intended to help introduce and demonstrate new mathematical ideas. You, the teacher, are still the most important element in. . .a sound and meaningful mathematics program.*

The teaching-learning strategy is described as "discovery." This description should be interpreted to mean that the student's experience with demonstrations, discussions, and work with manipulative materials should give him real-world understanding of the abstract concepts which are presented.

> Practice to reinforce basic skills should come only after pupils have an understanding of the processes underlying the computational techniques. By using concrete materials, such as small everyday objects, and semi-abstract methods of computation, such as tally charts, students can actually see and feel what is happening.

Students do not generate or verbalize concepts by themselves. These are presented by the television instructor or the classroom teacher.

The classroom teacher's demonstrations and discussions with pupils as the core of the instruction are an indication that the curriculum is intended for whole-class teaching rather than for individualized instruction. Developers themselves state: "PIA is a mass educational approach and is incompatible with the instructional programming model for the individual student." Attempts to adapt the program for individualization involve videotape players and school purchase or rental of videotapes rather than educational station broadcasts.

3.2 Typical Lesson

Before the television lesson which introduces a topic, the classroom teacher sees that all materials needed during the telecast are on the students' desks. The teacher gives the students a brief overview of what the lesson will cover and points out specific things students should watch for. Suggestions for pre-telecast activities and each lesson's objectives are presented in the Teacher's Manual. These objectives are listed under "what the student can do," and they define the skills and concepts which the student should get from the lesson. To illustrate, in the first-grade lesson, Number 11, entitled "Ordering the Numbers from 1 to 10," the following student objectives are listed:

1. The student can order sets of one through ten objects.

2. The student can order the numbers 1 through 10.

*Unless otherwise indicated, all quotations are drawn from program materials issued by the developer.

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3. The student can identify the position of the numbers 2 through 9 between two other numbers; for example, 6 lies between 5 and 7, between 3 and 9, etc.

Lesson Number 14, "Geometry: Open and Closed Curves," lists these objectives for the first-grade pupil:

1. The student can identify a simple closed curve by tracing it from a starting point back to that point.

2. The student can identify a simple open curve by showing that it cannot be traced from a starting point back to that point without backtracking.

During the telecast, the teacher shown in the TV lesson will often ask the students to respond to questions, to count, to notice things in their classroom, etc. The classroom teacher is supposed to encourage the students to respond.

After the telecast, the classroom teacher initiates a discussion of questions that may have arisen. The manual suggests discussions and demonstrations for the teacher to conduct to emphasize the main ideas presented on television. Follow-up activities in the manual and exercise pages from the pupil workbook provide activities for the classroom teacher to assign in order to complete the students' mastery of the topic.

To illustrate student and teacher activities, Lesson 3 from the first grade unit, "The Numbers from One to Four," is detailed:

Before this lesson, the students will be able to count to 4 and to write the Arabic numerals from 1 through 4. They may practice this skill by tracing numerals on worksheets, copying calendars, writing their room numbers, etc. During the telecast, the TV teacher demonstrates the ideas of the Arabic symbol (1), the number word (one), and the Roman numeral (I) by using a puppet and pieces of candy. She asks the children to write the Arabic symbol with their fingers and say the number name along with her. She then reviews the number names which have been presented. She shows the children number cards and asks them how many pieces of candy each card represents. As a final review she plays a game with a puppet which picks out the correct number card for the objects she shows him.

After the telecast, the students cut out numerals and number words from their Exercise Book and paste them on heavy paper, thus making their own number cards. The classroom teacher then repeats the game that the TV teacher played with the puppet. She holds up objects and the students respond with either the numeral or the number name, depending on what she asks for. Roman numerals are not stressed for slower students. There are three more exercise sheets in this lesson and five follow-up exercises, which the teacher can use in following days to reinforce the lesson. The follow-up exercises which are categorized as "highly recommended" are intended to help all students meet the basic objective of the lesson. The two activities designated as "optional" may be used as time allows. The following are "highly recommended" follow-up activities for this lesson:

Each student should make a chart which includes a set of objects, the Arabic numeral, number word, and Roman numeral for each number 1-4. This chart may be continued after other numbers are taught. The set of objects should be drawn or made of pictures cut from magazines.

Write a numeral or number word on the board and ask your students to form the set from their collection of objects.

Those follow-up activities which are "optional" for this lesson:

Use felt letters and numerals to designate number words and numerals for sets placed on the flannel board. Place a set of objects on the flannel board and call on a student to select the corresponding number.

Using different small objects, glue several sets which illustrate the same number on a chart.

3.3 Evaluation of Students

There are several points during the year at which the teacher can test the students to determine their mastery. Evaluation sheets provided in the Teacher's Manual can be duplicated for testing purposes. However, the developers state that assessment need not always be formal. A simple, oral question asked of a child can often yield enough information to determine whether he understands an idea.

Four check-up exercises are provided in the Teacher's Manual for Grade 1. The mid-year and end-of-year checkups are given by the TV teacher during lessons 16 and 30; the first and third check-up may or may not be given by the classroom teacher, at her discretion.

Teachers are instructed to tally the number of incorrect answers for each question in order to determine those areas in which the whole class needs the most help. Each evaluation question lists the program number in which the concept was presented. No special method of reporting student performance is required.

3.4 Out-of-Class Preparation

Teacher. To prepare for a telecast lesson, the teacher should read the material in the Teacher's Manual. It is preferable for the teacher to read two or three lessons ahead of the current lesson to be aware of the sequence of development. The teacher is also responsible for gathering manipulative materials and seeing that the television set is in working order before the lesson and that all children can see the set easily.

Student. The teacher may assign students out-of-class work depending upon her evaluation of their progress. The student may also have projects from time to time that supplement his classroom activities.

4. IMPLEMENTATION: REQUIREMENTS AND COSTS

4.1 School Facilities and Arrangements

PIA was designed for use in traditional one-teacher classrooms. There should be no more than 30 pupils in the group viewing the TV lesson because it is immediately followed by instruction led by the classroom teacher, not suitable for groups of more than 30 pupils. If there are several classes at the same grade level, students can be ability-grouped for their math lessons. In this way, pre- and post-broadcast instruction can be directed toward each group's level. The developers state that a student who has mastered certain concepts at his own grade level should be encouraged to work with the next grade during the programs on those concepts.

The curriculum was designed for transmission over educational television networks or school districts' closed circuit television stations. Either method may present time scheduling problems for teachers. In one district using PIA on closed circuit the first year all teachers reported that program scheduling was erratic and subject to transmission failures. If a teacher missed one or more programs, either because of her own schedule or a station power failure, there was no opportunity to have them repeated. If a teacher missed too many lessons, she finally dropped the program.

The teachers of upper grades commented that their students would be gaining more from the course if they had participated in the earlier levels. There was also considerable juggling of television schedules to permit all six levels to be broadcast each week. This suggests it may be easier to introduce PIA one year at a time for both educational and technical reasons.

Solution to the problems created by broadcasting may be found in the adaptation of the PIA curriculum to videotape classroom players. A project by the Northwest Regional Educational Laboratory of Portland, Oregon, adapting PIA for use on classroom videotape classroom players. VTR enables each classroom teacher to schedule the televised lessons according to the progress of the class or of individual students within it. Since this involves purchase of costly tapes and equipment, it is probably not a viable solution in most districts.

Personnel requirements and training. PIA requires one teacher for each class of 25 to 30 studen . This need not be a math specialist. All teachers participating in the program receive inservice training by observing the television teacher's lessons and by studying the discussion of new math concepts and instructions for classroom activities which are provided in the Teacher's Manual.

Equipment needed. For educational television station or closed circuit broadcasts each classroom must be furnished with a 21-inch television set, a Teacher's Manual, one pupil exercise book for each student, and a large assortment of manipulative materials. Classrooms using the curriculum on videotape reels need a videotape player and a television monitor, preferably mounted on a wheeled equipment cart, plus the Teacher's Manual, exercise books, and materials.

4.2 Student Prerequisites

Since Patterns in Arithmetic is a sequential six-year course, it is desirable that students begin the program at first grade. The developers state that at each grade level students of average ability should find all of the material within their grasp. However, results of the first (1966-67) evaluative testing across a varied student population indicated middle and higher socioeconomic group students were more successful in the program than were low-income children. Transfer and late-entering students within the broadcasting area are expected to have little difficulty in keeping up with materials since they are presumed to have studied the curriculum in the previous school.

4.3 Teacher Prerequisites

PIA was designed to be taught by teachers who are not math specialists and without inservice training. The detailed instructions in the Teacher's Manual provide explanations of the mathematics concepts. The TV teacher is expected to provide a model for the classroom methods. The Teacher's Manual and pupil exercise book are supposed to provide ample activities for students.

The teacher has to assemble many materials for students' classroom work. Because the television programs are paced for "average" students, the teacher will need to offer some students remedial work and others enrichment. Such assignments can be taken from the optional activities listed for each lesson.

4.4 Background and Training of Other Classroom Personnel

No additional personnel are required.

4.5 Cost of Materials, Equipment, Services

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Television lessons	333 programs (six grade levels)	Educational TV station or NIT	1-Educational TV station's school service determines charge based on number of participating districts	Fees paid every year
			2-NIT charges a basic fee of $32.00 per program plus $1.40 for each 10,000 pupils viewing the series. (For each 10,000 above 250,000, additional charge is reduced to $.50). Since developer accepts no royalties, total purchaser's bill is reduced by 15%.	Fees paid every year
			3-NIT will reproduce programs on purchaser's tape for a charge of $5-$6 per program. NIT notes that this option is more expensive unless purchasers plan to show the programs for 4-5 years.	Unlimited use
Exercises For Pupils	1 per student per year	ETV or NIT	1-9 $1.00-$1.50 each 10-499 $.70-$.90 500+ $.63-$.81	Yearly
Teacher's Manual	1 per teacher per year	ETV or NIT	1-9 $2.50-$4.00 each 10-499 $1.80-$3.00 500+ $1.62-$2.70	Reusable
Math activity materials (rulers, rods, graphs, etc.)		Classroom teacher		

4.6 Demonstration Sites

During 1973-74 PIA was purchased by the following school districts or state departments of education:

Las Vegas, Nevada; Monroe, Louisiana; Little Rock, Arkansas; Salt Lake City, Utah; Wisconsin; South Carolina; and Kentucky.

5. PROGRAM DEVELOPMENT AND EVALUATION

5.1 Program Development

In 1959, the University of Wisconsin received a Ford Foundation grant "to establish an imaginative program for the improvement of schools in Wisconsin." During 1960, a math program using the concept of instructional television was developed for grade 4. A grant from the National Science Foundation in 1961 enabled work to extend the program to grades 5 and 6. Then, in 1964, the Office of Education established the Research and Development Center for Cognitive Learning at the University of Wisconsin. One of its tasks was to develop a six-year program in mathematics based on the past experience and to subject it to extensive field testing. In 1965-66 lessons for grades 2 and 4 were developed, and grades 1 and 3 were field tested. Field testing of grade 2 was carried on in 1967-68.

A developmental year for the PIA program, grades 1 and 3, went through the following series of stages. First each lesson was planned according to content, method, and television presentation. From 30 to 100 teachers in the Madison, Wisconsin, viewing area participated in the course through the educational television station. Every four to eight weeks the teachers evaluated the program either in writing or by meeting with the R&D staff members. When necessary, lessons were revised and rewritten according to the teachers' comments. A mid-year and end-of-year achievement test was given to all participating students and results were reported to the teachers. At the end of the year, the lessons were edited and bound as a pupil exercise book.

5.2 Developer's Evaluation

During the 1966-67 school year, 675 first graders and 760 third graders in Wisconsin and Alabama participated in a summative evaluation of PIA. Three achievement tests, one designed specifically for the PIA program by Educational Testing Service, were used to gauge computation skills and conceptual mastery. These tests included the ETS Cooperative Primary Test, the California Achievement Test, and the Stanford Achievement Test. Participating classes were divided into two groups to minimize testing. Both groups took the PIA-designed tests, but the standard test on computation was given to one group and the concepts test was given to the other group. Classes from large communities were classified into high, middle, or low socioeconomic categories. Teachers were surveyed to determine their attitudes about how much they had learned, how they liked the curriculum, how they thought children responded, how suitable the curriculum is for high, average, and low performing students, etc.

In 1970-71 nearly 400 teachers in New York, Illinois, Oregon, Vermont, and Virginia participated in a study designed to test the ways in which PIA affects teachers. Tests and questionnaires were devised to measure changes in mathematical knowledge, knowledge of PIA-specific content, and attitudes toward teaching arithmetic.

5.3 Results of Evaluation

The study group of Wisconsin and Alabama first graders participating in the 1966-67 evaluation of PIA compared favorably with the norms group on achievement tests measuring both computation and concepts. The third-grade study group did better than the norm on the standardized concepts test but not as well as the norms group on the computation test. Developers attribute the low rating in computation to the Wisconsin group, whose students also scored low on pretests. Although the third graders showed considerable progress during the year, it was not sufficient to carry them to the norms group achievement level.

Data analyzed by socioeconomic class indicated that the first grade program favored high and middle groups over low. Third grade students in the high socioeconomic group achieved more than middle and low socioeconomic groups.

Results of the student and teacher attitude inventories showed that both, in general, were pleased with TV arithmetic. Teachers felt that the concepts were appropriate and reasonably placed and that the inservice training was effective. Students indicated that the TV teacher helped them to learn arithmetic and that they enjoyed working with the exercises.

Results of the 1970-71 summative evaluation of the ways in which PIA affects teachers indicate that "PIA can be used effectively as inservice education, particularly for those teachers with relatively lower initial knowledge of the basic mathematics which underlies a contemporary elementary school mathematics program. PIA does not seem to change teachers attitudes, however; nor is it beneficial in increasing knowledge of concepts not specifically related to PIA."[3]

The developers found that television instruction is more effective when used on a local closed circuit basis than when broadcast over an ETV network. Closed circuit lessons can be repeated and paced according to the classes' abilities, they said. They also stressed that the program should not be used unless truly adequate facilities (only one television set per 30 pupils) are available. The developers conclude:

> Mass communication techniques are effective in providing both sound instruction for elementary school children and inservice training for elementary school teachers. Teachers' comments. . .that for the first time in ten years of teaching their children are able to understand a concept are excellent indications that for the first time in ten years of teaching the teacher understands the concept.[4]

5.4 Independent Analyses of the Program

No independent analysis of the program is available.

5.5 Project Funding

PIA has received grants from the U.S. Office of Education, the National Science Foundation, and the Ford Foundation.

Distributor. The program is distributed by National Instructional Television Center, Box A, Bloomington, Indiana 47401. All materials are available from NIT and all inquiries should be directed to them.

5.6 Project Staff

PIA was developed by the Wisconsin Research and Development Center for Cognitive Learning at the University of Wisconsin. Henry Van Engen, Professor of Education and Mathematics at the University of Wisconsin, was project director.

FOOTNOTES

1. Wisconsin Research and Development Center for Cognitive Learning. *Basic Program Plan*. Madison, Wis.: Wisconsin Research and Development Center for Cognitive Learning, University of Wisconsin, 1972, p. 115.

2. Van Engen, Henry, and Robert B. Parr. Using Mass Communication Media to Improve Arithmetic Instruction. *Audio Visual Instruction*, February 1969.

3. Wisconsin Research and Development Center for Cognitive Learning. *Evaluation of Patterns in Arithmetic in Grades 1-4, 1970-71: Effects on Teachers*. Madison, Wis.: Wisconsin Research and Development Center for Cognitive Learning, University of Wisconsin, 1972.

4. Van Engen, and Parr. *Audio Visual Instruction*, p. 38.

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT (MINNEMAST)

INTRODUCTION

Each second grade child has a plastic cylinder with a centimeter tape attached, into which he puts water and navy beans. He observes and records in his workbook the changes in height of the column of water after the beans have stood overnight in the water. A subsequent activity is to dry the beans and make new measurements. Students use balances made in a previous unit to compare the weight of wet beans and dry beans. The teacher suggests that children observe the operation of the swelling phenomenon in their mother's cooking of oatmeal and rice.

These activities are part of a second grade unit in *Minnemast* (*Minnesota Mathematics and Science Teaching Project*), a K-3 curriculum which encompasses both mathematics and science. The developers have coordinated teaching of the two disciplines since they believe each needs the other in order to make itself completely understood. The math/science "togetherness" is accomplished through careful sequencing of the curriculum's 29 units--some in math, some in science, some both. The same processes are encountered in different subject matters at many levels.

The basic goal of the project is to teach students to think. James Werntz, former project director, explains that the lessons, which use a wide array of manipulatives, were prepared to "direct the thinking of children into desired approaches to the subject. Teachers serve as guides, leading the students toward specific discoveries, following a. . .route that we know they're going to follow." The amount of direction or information children will need for their investigation depends upon their maturity, ability and prior experience.

Minnemast developers were in the forefront of educators who defined thinking and problem solving in terms of intellectual "processes" common to both mathematical and scientific investigations. Long before participants at the 1967 Cambridge Conference on the Correlation of Science and Mathematics in the Schools suggested an activity-based unified program for elementary school, University of Minnesota physicists and mathematicians had begun to develop and field test Minnemast. USMES, a program incorporating many of the ideas presented at the conference, is described in this book.

Although the project began as a K-6 curriculum development, only the K-3 materials were completed before funding was cut off. The developer provides a booklet of recommendations about other science and math curricula which can complete the objectives started in Minnemast.

In 1969, Roger Jones, then associate director of Minnemast noted:

> I think that this project in a sense is typical of many of the projects around the country today that started several years ago. The thinking has changed and become even more radical since then, and the "old" projects are all behind the times. They are miles ahead of what's actually going on in the school, but not in terms of what people are thinking of doing in schools today. In the sense of a really free, open experimental school in which there's no grading and the children sort of do what they want and follow their interests and have lots of things available but no direct guidance, Minnemast as well as the other projects still have a long way to go.

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BASIC INFORMATION

Program name: Minnesota Mathematics and Science Teaching Project (Minnemast)

Format: Complete mathematics and science program for kindergarten through third grade with suggestions for continuing program in intermediate grades. The 29 units, some in math, some in science, and some both, are intertwined and spirally sequenced so that concepts are introduced in simple form and later reinforced and elaborated.

Philosophy: An active-learning mathematics curriculum coordinated with science units and emphasizing the contribution of both disciplines toward the child's development of logical processes. Teaching strategy relies on use of a large number of manipulatives and laboratory investigations.

Content: Concepts and processes common to mathematics and science. Subjects included are real numbers and geometry in math; science concepts of systems, interaction, change, reversibility, invariance; and science topics of space, time, matter, force and field, life. Processes are observation, experimentation, generalization.

Suggested use: Complete K-3 mathematics and science curriculum.

Target audience: Students of all ability levels, grades K-3.

Length of use: K-3 with recommendations for the intermediate grades. Developers expect the program to be used daily for one class period (20-45 minutes). Each unit takes about 4-8 weeks to complete.

Aids for teachers: Teachers' manuals for each unit, complete lesson plans, student workbooks, kit of manipulative materials.

Date of publication: 1964-70.

Director/Developer: Alan Humphreys, Associate Professor of Elementary Education/Minnesota School Mathematics and Science (Minnemath) Center, University of Minnesota, 720 Washington Ave. S.E., Minneapolis, Minn. 55455. (612) 373-3522. Founding director was Paul C. Rosenbloom, and James Werntz was director during a major development phase.

Publisher: Minnemast, 720 Washington Ave. S.E., Minneapolis, Minn. 55455.

1. GOALS AND RATIONALE

1.1 Goals

Developers of the Minnesota Mathematics and Science Teaching Project, Minnemast, believe that most students suffer from a condition they call "atrophied thought process." This degeneration occurred, in the developer's view, because students had been taught to produce ready answers to problems; they had not been urged to apply thought to the solution of these problems. They noted that if young children are expected only to react, it is difficult, if not impossible, to later teach them to think. Program initiators therefore sought to develop a curriculum which would help elementary school children learn to produce reasonable solutions to problems. They believed that math and science were the best media for teaching a rational approach to problem solving:

> The objective of Minnemast is to help the children develop the intellectual tools of rational inquiry and not necessarily to prepare them to be scientists or mathematicians. The tools of rational inquiry apply not only in science and mathematics and other studies, but are powerful ways that lead to understanding the experiences of our daily lives as well. Briefly, our main objective is to guide children in learning to think.*

Paul C. Rosenbloom, the project's initiator, indicated these specific program goals:

1. Supplying children with effective, efficient procedures for arriving at rational conclusions and with motivation to apply these procedures in scientific and other appropriate contexts.

2. Presenting mathematics and science as part of the continuing human endeavor to make sense of the universe and man's place in it.

3. Presenting mathematics and science as creative and ever-changing disciplines, in order to close the gap between science as it is taught in our schools and science as it applies to current research, research scholars and everyday life.

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* All quotations are drawn from materials issued by the developer.

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4. Supplying experiences in the processes of the physical and biological sciences and skills in mathematical techniques as are needed by everyone to function effectively in society.

5. Providing a sound foundation for the training of future professional mathematicians and scientists.

1.2 Rationale

Minnemast has been shaped by many influences, according to developers, but two educational theories have been of special importance. First is Jerome Bruner's theory that a curriculum should provide a coherent pattern rather than isolated bits of knowledge. Bruner believes that a child learns more and retains longer those ideas and skills which fit together in a unified structure. In Minnemast, a pattern is provided by a "spiral curriculum" where fundamental concepts reappear in increasingly complex situations.

The second major influence on Minnemast is that of Jean Piaget, whose analysis of the stages of the child's intellectual development indicates that learning attributable to a later stage of thinking can only occur after completion of earlier stages. In Piaget's work, the preschool and primary student uses his experiences with "concrete" phenomena to build ever more elaborate conceptual structures leading to abstract thought. Piaget asserts that all humans develop thinking in stages, and skills associated with later abstract stages of thought cannot be lastingly learned until the person has assimilated the meaning of his early, simple experiences. Accordingly, Minnemast does not attempt to teach children techniques involving symbolic thought--such as arithmetic algorithms--until their development has reached the stage of real comprehension of concepts--not just rote memory. Minnemast provides many experiences (for instance, in manipulating, classifying, ordering) to aid the child's transition from "pre-operational" to "operational thought," which occurs around the age of six or seven in most children. Then the curriculum offers operational experiences to prepare the child for the logical stage around the early teens, when generalization, abstraction, and deductive thinking become natural ways of organizing experience.

Developmental theory argues strongly against the effectiveness of traditional instruction, which relies on learning by rote memory. Minnemast developers reject science teaching which demands mastery of facts rather than understanding of operations and concepts. This follows both from their beliefs about how children learn and their belief about the nature of science. It is not an encyclopedic collection of facts, Minnemast developers believe. James Werntz, a former director of the project, has said:

> As an overall objective we must attempt to give children an understanding of the essential intellectual content of science--the operation of science, if you

> will. And we must gear our programs to provide them early enough and often enough and so concretely that they find a place in the immediately accessible intellectual toolbox of all children. . . . It is important to structure presentations so that drawing of conclusions by the teacher is discouraged; and to organize the material carefully, so as to direct the thinking of children into desired approaches to the subject.

Another important feature of the Minnemast rationale is the commitment to a coordinated mathematics and science curriculum. This coordination implies that each subject can support and reinforce the other where appropriate, with common techniques and concepts being deliberately sought and exploited. The developers give several reasons for combining math and science. For example:

> It is natural to teach mathematics with applications and illustrations from science, and to teach science when you can make use of its mathematical framework The description of Newton's law of gravity or the growth rate of a plant is so simple and precise in mathematical terms that words seem cumbersome in comparison. Similarly, the abstract idea of vectors or of the real number system can be made much clearer through physical illustrations from the sciences.
>
> In addition, mathematics and science have relied heavily on each other throughout the course of their common development. The very backbone of much theoretical science has evolved, in a sense, as a branch of applied mathematics and would not exist today without it.
>
> The boundaries between science and mathematics are not always very well defined. Many aspects of the two disciplines overlap to such an extent that they are of equal importance to mathematicians and scientists. Thinking about mathematics and science as distinct disciplines is not necessarily the most fruitful approach for the mathematician or the scientist, to say nothing of the layman. It seems quite reasonable to avoid making a strong distinction in the mind of a child. . . . Breaking the bonds that join mathematics and science together probably harms that child's appreciation and understanding of both subjects as much as it weakens the creative union between the two.

This integration of science and mathematics is seen as only the beginning of a possible full integration of other disciplines.

2. CONTENT AND MATERIALS

2.1 Content Focus

Minnemast combines subject matter from both math and science in its K-3 curriculum. Mathematics subjects covered include number line, number theory, arithmetic, continuity, probability and statistics, sets and groups, functions, measurement (metric), shapes and configurations, and Euclidean geometry. Science subjects covered are: systems, interaction, change, reversibility, invariance, space, time, matter, force and field, and life. Although the entire curriculum seeks to weave mathematics and science together, some units emphasize one subject more than the other. In some units, developers caution, the union of the two subjects may not be apparent simply because we traditionally consider certain topics as either math or science, when in fact, they are basic to both mathematical and scientific understanding.

> For instance, from the title and initial inspection of the first unit, Watching and Wondering, one might conclude that only science is treated. The project developers feel, however, that watching (careful observation) and wondering (asking questions) are just as basic to the thinking of a mathematician as to that of a scientist. A topic which may seem important only to mathematicians, such as graphing, is just as necessary for a scientist.

The units are based on skills and processes such as observing, describing, classifying, comparing, ordering, measuring, and computing. These skills and processes are arranged in a spiral curriculum in which they are touched on over and over again but at more sophisticated levels in succeeding units. Because Minnemast uses a spiral format and because many lessons require experience with previous lessons, random deletion and rearrangement of material is not recommended.

2.2 Content and Organization of the Subdivisions

Minnemast consits of 29 units designed for kindergarten through third grade students. There are 7 units each for kindergarten, first grade, and third grade, and 8 for second grade. Following are lists of titles of the units. After some of the titles are sketches of the unit content.

Kindergarten

1. *Watching and Wondering*

2. *Curves and Shapes*

3. *Describing and Classifying.* Sets.

4. *Using Our Senses*

5. *Introducing Measurement.* Length, area, volume, and time duration are introduced as properties that can be compared and ordered. All work is non-numerical. Children first see a need to measure a particular property and then develop methods of comparing each of two objects using that property.

6. *Numeration.* Children first use a set of objects, then tally marks, and finally numerals to represent the number of objects in a set. Through comparing different length rods, children perform pre-counting, pre-addition, and pre-subtraction exercises.

7. *Introducing Symmetry*

Grade One

8. *Observing Properties*

9. *Numbers and Counting.* Set comparison, numbers, numerals, and counting.

10. *Describing Locations.* Children learn to make and read a variety of simple maps, and to describe verbally where something is. Two mathematical concepts are involved. One, "locus," reviews and extents set concepts; "frame of reference" provides a foundation for later mathematics work with graphing.

11. *Introducing Addition and Subtraction.* Sets, number line, and place value.

12. *Measurement with Reference Units.* Length, area, volume, and time.

13. *Interpretations of Addition and Subtraction.* Measurement, sets, and number line. Children learn to use an addition slide rule to add and subtract larger numbers and numbers other than counting numbers.

14. *Exploring Symmetrical Patterns*

Grade Two

15. *Investigating Systems*

16. *Numbers and Measuring.* Numerals to 999, fractions, negative numbers base four, ordering of lengths including diameter and circumference.

17. *Introducing Multiplication and Division.* Multiplication is represented as repeated addition by jumps on the number line, combinations of equivalent sets or arrays, and related scales on parallel number lines. Division is presented as breaking up a set of things into equivalent subsets.

18. *Scaling and Representation.* Maps and scale models.

19. *Comparing Changes.* Children observe changes that occur and measure and record these changes. Experiences with plants, volume and weight, and temperature changes lead the children to explore relationships among variables and to represent them graphically.

20. *Using Larger Numbers.* Addition and subtraction of two- and three-digit numbers. Graphing, place value, and measurement are included.

21. *Angles and Sp[illegible].* This unit integrates some scientific ideas with th[illegible] of "pure" geometry. Angles are defined and measured. Angles and their properties are used to describe a variety of natural and geometrical objects. Children are introduced to regular polygons and symmetry, as well as to some three-dimensional concepts in geometry. Geometric congruence and similarity are used to make some size comparisons and to introduce the idea of proportionality. Optional projects, such as making periscopes and sundials, are suggested.

22. *Parts and Pieces.* Rational numbers.

23. *Conditions Affecting Life*

24. *Change and Calculation.* Simulation of a computer.

25. *Multiplication and Motion.* Differences and similarities between multiplication and addition; commutative, associative, and distributive laws. The activities provide a foundation for the study of the relationship between a sloped linear graph and multiplication.

26. *What are Things Made Of?*

27. *Numbers and Their Properties.* Multiplication of numbers written in base ten and other bases. Field properties of closure, identity, inverse, associativity, commutativity, and distributivity. Some work with equations, patterns, negative integers, and exponents.

28. *Geometry and Geography (Mapping).* Using simple geometrical ideas, the children review and extend map-making. They use elementary surveying techniques and some simple coordinate systems. By constructing maps of their own, the children begin to see mapping as a transformation. Children study different kinds of maps. Through optical projection, they learn how the curved surface of the earth can be represented on a plane. They use longitude and latitude for the global coordinate system.

29. *Natural Systems.* Animal locomotion, plant, wind, and river.

2.3 Materials Provided

Student. Pupil exercise books have been developed for the first through third grade units. Kits containing manipulative materials are packaged for classes of 30 students.

Students use the following three items during kindergarten, first, and second grade Minnemast units:

> Minnebar: wooden rods varying in length from 1 to 12 cube units. These differ from Cuisenaire Rods because unit divisions are indicated on each rod.
>
> Property blocks: similar to attribute blocks of other projects but modified to fit the program. There are 48 blocks having three shapes (square, triangle, and circle), two sizes, two thicknesses, and four colors (red, yellow, green, and blue) for each shape.
>
> Addition slide rule: introduced in first grade, this tool enables the children to add and subtract larger numbers before learning the standard algorithms, and also to add and subtract numbers other than counting numbers.

Teacher. The project provides separate teachers' manuals for each Minnemast unit. The manuals contain a suggested teaching schedule, a list of materials needed for teaching the unit, notes to the teacher, a list of unit objectives, and detailed lesson plans.

Two enrichment booklets are also available from Minnemast. The first, *Adventures in Science and Mathematics*, is a series of narratives about the lives of famous scientists and mathematicians. *Living Things in Field and Classroom* suggests to the teacher ways of coordinating activities in the classroom and out-of-doors in the study of plants and animals.

Another Minnemast publication, *Overview*, explains the program philosophy and provides summaries of each of the units. *Minnemast Recommendations for Math and Science in the Intermediate Grades* offers suggestions of math and science units or programs which can correspond to the content and approach of Minnemast.

2.4 Materials Not Provided

Most units require a number of items which are not included in the kits but are easily obtainable from grocery, hardware, or scientific firms. For example, the teacher is expected to provide plastic cups, shells, seeds, corn meal, and mealworms.

3. CLASSROOM ACTION

3.1 Teaching-Learning Strategy

Lessons are introduced in a variety of ways including games, demonstrations, experiments, or stories. During the course of one unit children may work by themselves in small or large groups. Minnemast activities are designed to place the student in situations where he can *hypothesize possible solutions, propose methods of checking the hypothesis, carry out experiments,* and *decide whether a hypothesis seems plausible in the light of experimental results.*

Although the materials have been designed so that the entire class works on one unit at a time, teachers may select specific lesson activities for individual students. According to developers, children can continue to work on Minnemast activities as long or as in much depth as their abilities and interests permit; they expect that every pupil will be capable of performing some work on a problem and gaining the basic knowledge needed to move on to new units. The units include supplementary materials for more able students. Robert Jones, former associate director of Minnemast, said:

> . . .Teachers have found that they can, by selecting the materials, make some things easier for the children who are having more trouble. They may use other materials for those that are doing well. We try to provide material on different levels and then expect the teachers to do some selecting on the basis of student need.

Minnemast units are designed to be used in a specific sequence because the curriculum is a spiral one; concepts presented in early units are reinforced and extended in later units. Instructions for teaching a unit vary. Some activities are outlined in specific sequences, others are described in general terms.

In Minnemast, the teacher serves as a director and guide for student learning. According to developers:

> . . .the teachers should try to teach in such a way that children determine the answers for themselves through experimentation. . . . It is easy for a teacher to answer a question, but very hard to give the student some way to find an answer for himself. . . . If a teacher succeeds in the latter, she comes close to educating. But if she resorts to the former, she is missing the point we are trying to make.

Thus, teachers are expected to help students formulate appropriate questions and discover ways to test hypotheses; teachers are not to lead students to the "right" answer. The specific amount of direction or information children will need depends upon their maturity, ability, and prior experience.

3.2 Typical Lesson

Unit 12, Measurement with Reference Units, is designed to be used by first grade students. Developers suggest that this unit be taught daily for 2½ months. The unit is divided into four sections: measurement of length, area, volume, and time. The following lesson, Measuring Volume by Water Displacement, occurs midway in the unit. In this unit and others measurement serves as the link between math and science.

The teacher begins this two-day lesson by showing the class a glass of water and a piece of plasticine. After asking the question, "What will happen if we put the plasticine into the water?" and eliciting the response that the water level will rise, she drops the plasticine into the water. Next, a student alters the shape of the plasticine and the teacher again asks the class what will happen when the plasticine is dropped into the water. She also queries, "Will it rise the same amount as before?". Rather than telling the children the correct answer, the teacher asks the students to suggest an experiment to check the answer. Experiments and reshaping of the plasticine continue until it is clear to the children that the shape of the plasticine does not determine the amount of water that is displaced.

The teacher then conducts two demonstrations to illustrate that the volume of water displaced is the same as the volume of the plasticine. In preparation for a fourth demonstration, the teacher fills a tall container half full of water and marks the water level with a magic marker or rubber band. She then shows the students a dozen small objects such as marbles, pebbles, or washers and small glasses called "Minneglasses" and asks, "How could we find the volume of these objects?" and "Could we use this water somehow?". The teacher then guides the discussion so that children see that they can place the objects in the water, note the new water level, and measure the amount of the increase using the Minneglass. Next, teacher and students calibrate the larger container using Minneglass units. These calibrations can be used to measure the volume of objects.

After these demonstrations are completed, pairs of students are given a tray, a cylindrical container, water, a Minneglass, and a piece of plasticine. The students are directed to calibrate the cylinders, find the volume of the piece of plasticine, and measure the volume of other objects. As a concluding activity the class discusses a Minnemast story, "The Crow and the Pitcher"; the tale illustrates how a cleaver crow uses displacement to quench his thirst.

3.3 Evaluation of Students

Minnemast does not provide tests or other instruments specifically designed to measure student performance. Teachers may evaluate pupil progress using a list of learning objectives contained in each unit. Experiments and worksheets are meant to allow the child to check his own development of understanding and skill.

3.4 Out-of-Class Preparation

The teacher's guide for each unit contains detailed information and lesson plans. In addition to reviewing this material, the teacher may wish to read background material listed in the bibliography.

In preparation for each lesson, teachers will need to collect, arrange, and organize many materials for demonstration and student experimentation. Minnemast activities also require that teachers give a great deal of thought to classroom organization and grouping of students.

4. IMPLEMENTATION: REQUIREMENTS AND COSTS

4.1 School Facilities and Arrangements

The program is designed for use in any classroom in which desks and chairs can be moved so as to form large open areas, and which has counters or shelves where simple apparatus can be left between classes. There should be shelf space for sorting equipment. Some schools have found it convenient to provide one Minnemast room where the kits for all classes can be stored and be easily assessible as needed.

No particular form of school or classroom organization is required for Minnemast. It was designed to be used for whole-class teaching in which all children could be working on the same activities but at varying levels of sophistication. The teacher who has already mastered the problem of breaking up a class into smaller instructional groups can find Minnemast a valuable curriculum, if she picks and chooses activities she considers appropriate. The program is not designed as an individualized program or as a vehicle to assist the individualization of a classroom. Because it was designed for whole class teaching, it may appear unattractively teacher-directive to teachers committed to open classroom teaching. If they can look at the substance of lessons rather than format, experienced teachers may find it a rich curriculum resource since its activities for children are based in developmental learning theory.

Because the program does differ from traditional ones and because there are no traditional assessment instruments, administrative assistance and support is important to the successful implementation of the program. Developers recommend that administrators become acquainted with the program through observing the program in action, working with the materials, and possibly teaching small groups of students.

Most districts have chosen to implement the program one year at a time, beginning with kindergarten. Developers suggest that schools follow this pattern but see little difficulty with a school introducing Minnemast all at once in kindergarten through grade 2, and introducing third grade materials the following year. Developers believe that students new to a Minnemast school will have little difficulty entering the curriculum.

4.2 <u>Student Prerequisites</u>

Minnemast units and activities are carefully sequenced, proceeding from simple to complex activities. The curriculum is a spiraling one; skills needed for activities are often presented in the early units. Because of this organization, developers recommend that teachers follow the prescribed order of lessons and units. Teacher notes for each unit refer teachers to earlier lessons which are prerequisite. Teachers whose classes are new to the program may wish to provide additional introductory activities or adapt supplementary activities.

4.3 <u>Teacher Prerequisites</u>

Developers originally intended that inservice training in the Minnemast philosophy and mode of teaching would be an integral part of the program. Therefore, teacher manuals spell out in detail *what* teachers should do but do not explain *why* they should do it. If teachers do not understand the rationale behind the program, they are likely to follow directions unthinkingly and the program will take on a highly teacher-directive style entirely contrary to the developers' intent. Although Minnemast developers conducted courses to introduce teachers to Minnemast in the past, these efforts were curtailed because of funding shortages. Occasionally, Minnemast staff members teach workshops on a consultant basis. Interested groups should contact the developer. Over the years, the National Science Foundation has also sponsored several workshops in Minnemast across the United States. At the very least, teachers planning to implement the program should plan to attend a course concentrating on active-learning or math workshop styles of teaching before using the program. Such courses are available in most colleges, extension departments, and teachers' centers.

4.4 Cost of Materials, Equipment, Services

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Student's manuals Grade 1 Grade 2 Grade 3	1 set per student	Minnemast	30 manuals: $127 $215 $325	Yearly
Printed aids	1 set per class	"	$10-$21	Yearly
Classroom kit for each grade	1 per class	"	$75-$240	Reusable with refill kit
Refill kit for each grade	1 per class	"	$18.50-$80.00	Yearly
Minnebars	1 unit per student grades K-2	"	$13.50 (5 unit pkg.) $26.50 (10 unit pkg.)	Reusable
Property blocks	1 unit per student grades K-1	"	$39.50 (5 unit pkg.) $78.00 (10 unit pkg.)	Reusable
Slide rules	1 per student grades 1-2	"	$2.25 (5 unit pkg.) $4.00 (10 unit pkg.)	Reusable
Teacher Manuals (1 per unit)	1 per teacher	"	$3.50-$5.00 each	Reusable
Overview booklet	1 per school	"	$1.00	Reusable

Recommended Supplementary Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Adventures in Science and Mathematics (handbook)	1 per teacher	Minnemast	$5.00	Reusable
Living Things in Field and Classroom (handbook)	1 per teacher	"	$3.00	"
Minnemast Recommendations for Science and Math in the Intermediate Grades	1 per school	"	$1.00	"

4.5 Demonstration Sites

The following is a partial list of schools where Minnemast has been used for more than one year.

Florida:	University of West Florida Pensacola, Fla.
Illinois:	Evergreen Park School District Evergreen Park, Ill.
Louisiana:	Rapides Parish School District Alexandria, La.
Massachusetts:	South Hadley School District South Hadley, Mass.
Minnesota:	Burnsville Public Schools Burnsville, Minn.
Mississippi:	Leflore City School District Leflore City, Miss.
Missouri:	Kirkville R-III School District Kirkville, Mo.
New Jersey:	Newton Public Schools Newton, N.J.
New York:	Union Free School District Oceanside, N.Y.
Texas:	Keene Adventist Keene, Tex.
Virginia:	Falls Church School District Falls Church, Va.
Washington:	Edmonds School District Lynnwood, Wash.
Wisconsin:	Alverno College Elementary Milwaukee, Wis.

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5. PROGRAM DEVELOPMENT AND EVALUATION

5.1 Program Development

In the early 1960's many educators, scientists, and mathematicians began urging the development of elementary school curricula which would integrate science and mathematics. Minnemast was the earliest of the programs planned to accomplish that purpose. In 1964 Minnemast's first director, Paul Rosenbloom, stated, "We are possibly the only ones attempting a full-fledged coordination of the curriculum." Many of Minnemast's initiators later influenced the development of other mathematics and science curricula. Robert Karplus later the director of SCIS (Science Curriculum Improvement Study), directed the initial stages of the development of science units. James Werntz, Jr., professor of physics, was director of Minnemast from 1965 to 1972. He later participated in the development of USMES, another combined mathematics/science program which was developed in response to suggestions growing out of the 1967 Cambridge Conference on the Correlation of Science and Mathematics in the Schools. (USMES is described in a separate report in this book.)

Minnemast originally began as a mathematics program, Minnemath, at the University of Minnesota. In 1962 the project received its initial National Science Foundation support: teams of more than 60 mathematicians, scientists, and educators began developing independent mathematics and science materials for kindergarten through sixth grade. The units were revised after classroom trials, and developers began combining the two subjects into a coordinated mathematics/science series. Minnemast developers had originally envisioned a complete K-6 program, but because of funding shortages, development was terminated in 1969. The task of weaving the mathematics and science units together was accomplished only through third grade before funding was discontinued.

The project is no longer active at the University of Minnesota, but a Minnemast office remains as distributor of the materials, and Alan Humphreys and Thomas Post of the University's department of elementary education are in touch with school people using Minnemast or considering adoption.

5.2 Developer's Evaluation

During the course of Minnemast development, the curriculum was field tested in classrooms across the nation. The materials were revised on the basis of teacher and observer comments and results of student tests of acquisition of certain skills.

Data on student performance on selected arithmetic achievement tests were collected, but a final report was not prepared. Similarly, a summative evaluation of the curriculum could not be completed because of the premature termination of the project.

Developers note that they continue to receive orders for replacement or implementation materials. This continuing interest in Minnemast is impressive when one considers that because of its incomplete status, Minnemast was never commercially published as were other major science projects of its era (Science--A Process Approach, Elementary Science Study, SCIS). The Minnemast Center at the University of Minnesota continued to distribute all printed materials but entered into contracts with various distributors to market the classroom kits and manipulatives. Difficulty in assembling materials from three different suppliers was for several years a handicap to implementation. In spite of this drawback and regardless of the fact that the program has never received commercial promotion, developers say it continues to sell by word of mouth.

5.3 Project Funding

Minnemast received funding from the National Science Foundation from 1962-69. Supplementary support was given by the Louis W. and Maud Hill Family Foundation of St. Paul, The School Mathematics Study Group, The U.S. Office of Education, and the National Institute of Health. The project uses money from the sale of materials to support program dissemination.

5.4 Project Staff

The project was initiated by Professor Paul C. Rosenbloom of the University of Minnesota. Dr. James Werntz, Jr., professor of physics, University of Minnesota, was director from 1965-72. The program is presently directed by Alan Humphreys, associate professor of elementary education.

UNIFIED SCIENCE AND MATHEMATICS FOR ELEMENTARY SCHOOLS (USMES)

INTRODUCTION

Faced with the challenge: "Find a way to design or make changes in things that you use or wear so that they will be a good fit," one USMES (Unified Science and Mathematics for Elementary Schools) sixth-grade class decided to design and make carpentry aprons which students could wear while they worked in their school's workshop. Small groups of children polled other students in the school to gather suggestions for apron design, collected appropriate measurements, graphed the measurement data, and decided on five apron sizes. The class then set out to produce the aprons; they purchased the necessary materials and divided the production tasks among groups of students. When the aprons were completed, the class held an apron sale and sold more than 60 aprons.

Throughout the unit, students practiced or learned new skills in math, science, social studies, and language arts. In planning the design and construction of the aprons the students used sampling, measurement, and computational skills. Development and solution of their specific problem, making the aprons, required that students use scientific inquiry methods. Social studies concepts were practiced as the class worked in groups, disseminated information, and considered individual differences and similarities; during group reports students improved language arts skills.

Developers expect that each class using an USMES unit will approach that "challenge" in a different manner; for instance, one class built comfortable chairs in conjunction with the unit described above. Because activities differ according to student choice, different classes are expected to learn different things. What all children are expected to learn from USMES challenges is the process of organizing their thoughts and personal and material resources, and the processes of tracking down information and learning the skills needed to accomplish a practical objective.

USMES is a supplementary program weaving together elementary math, science, language arts, and social studies by posing challenges to a class to solve long-range problems about their school, neighborhood, or classroom. The challenges are intended to be strongly motivating to children by emphasizing children's choices, to provide concrete examples of abstract concepts in math and science, and to give children experience in the objective, practical approach needed for real-life decision making.

The first challenges were suggested in the report of the 1967 Cambridge Conference on the Correlation of Science and Mathematics in the schools. Those challenges were developed and added to by professors and teachers who had attended the conference, and in 1970 they became the nucleus of a development project funded by the National Science Foundation at the Education Development Center (EDC) in Newton, Massachusetts.

The materials for each unit are records of the ways that widely varying groups of students responded as they investigated the same challenge, and of the skills they learned. Units include teachers' journals detailing ways they adapted challenges to suit class environment, interests, and learning needs. Twelve units are available in 1974, 7 are being classroom tested, and about 10 more ideas are being considered for development during 1974-75. In all, developers at EDC plan to complete 32 units by 1978.

BASIC INFORMATION

Program name: Unified Science and Mathematics for Elementary Schools (USMES)

Format: Independent teaching units integrating mathematics, physical science, language arts, and social studies, and posing real-life problems (called "challenges") for the class to solve.

Uniqueness: Long-term problems involving the whole class in adult-style research and development; individual students pursue investigative and decision-making tasks.

Content: Problem statements or "challenges" involving interdisciplinary work in mathematics, science, language arts, and social studies. Every class's work is different because it is shaped by the distinct interests and needs of the individual school environment.

Suggested use: Developers recommend that one-fourth to one-third of the total school program be devoted to work on USMES. Units are designed to complement and enrich, not replace, the regular math, science, language arts, and social studies curricula.

Target audience: Students of all socioeconomic, cultural, and intellectual backgrounds in kindergarten through eighth grade.

Length of use: Usually 45-60 hours for three to eight months for each unit.

Aids for teachers and students:

Teacher's Resource Book for each unit--background materials, discussions of classroom management, and descriptions of activities which previous classes have undertaken.

"How To" cards--short sequences of directed-learning task cards for students to use when they need a particular skill in order to work on a challenge

The USMES Guide--a program overview for long-range planning.

Design Lab--a workshop, provided by the school, where students and teachers make the equipment they need for researching and solving challenges.

Design Lab Manual--describes Lab specifications and place of Lab in total school program.

Background Papers--background information for teachers on a variety of topics that may arise during the course of a unit.

Date of publication: Twelve units published by EDC in 1974; 17 more in testing or development stages. Materials mentioned in this report are available from the developer. Interested schools are urged to contact the developer before using the units, because USMES should be used only in conjunction with workshops designed to introduce teachers and administrators to the required teaching approach.

Director/Developer: Earle L. Lomon, Professor of Physics, Massachusetts Institute of Technology. Education Development Center, 55 Chapel St., Newton, Mass. 02160.

Publisher: Unified Science and Mathematics for Elementary School Project, Education Development Center, 55 Chapel St., Newton, Mass. 02160.

1. GOALS AND RATIONALE

1.1 Goals

USMES grew out of resolutions adopted at the 1967 Cambridge Conference on the Correlation of Science and Mathematics in the Schools, a gathering of scientists, mathematicians, and educators believing that in our technological society even ordinary citizens need the kind of schooling that will result in scientific understanding: "not so much the mastery of techniques, which rapidly become outmoded, but the ability and habit of thinking through specific problems."

USMES designers aim to convince students of the usefulness of science and mathematics in our society by involving them in research and development projects which require the use of math, science, social science, and language arts skills. Casting these academic subjects in practical situations meaningful to children is done not just to build skills more efficiently, but more importantly to give practice in responsible, self-determined use of skills. Decisions which adults have to make require the ability to understand a situation--to observe, organize, quantify, predict, and control. Acquiring these abilities and being able to apply them thoughtfully are the deeper purposes of the USMES program.

Earle Lomon, director of the USMES development group, states that emphasis on real-life problems not only draws social studies and language into the math and science learning, but also conveys to children the self-respect that comes from working on matters that have importance for adults as well as for children. Lomon wants children to have experience changing a bit of their society because he believes in this way they will gain confidence in the political process. Lomon explains:

> There are important subjects--things kids can do something about, can act on directly. This type of experience provides them with actual political experience. It involves them with adults and with adult problems in a way that directly prepares them for being adults in this society. . . . We're not putting out kits or units of "magic materials"; rather we're raising vital problems and trying to find out what kids can do with them. . . . We consider the unit a success only if it leads many students to pursue the problems until some considerable progress has been made toward its solution.*

*Unless otherwise indicated, all quotations are drawn from materials issued by the developer or from conversations with the developer.

The program is organized to require group work on problems rather than individual investigations, so that children will get experience in planning, working, communicating, and decision making in groups--other essential skills for political maturity.

Since USMES differs from traditional textbook learning, it usually requires a change in teaching style. The developers say that they consider the project's effect on teaching style as important as the development of new units; much of the staff's energy has gone into organizing pre- and inservice teacher training. Developers anticipate that teachers who use USMES will become so adept at working with students on problem-solving experiences that they will be able to create their own USMES-type projects.

1.2 Rationale

The rationale for teaching children problem solving through mathematics and science grows out of a belief that children learn through experience, and that they will learn far more from a real experience than from a contrived one. USMES developers see four important educational advantages to basing USMES units on real-life problem-solving activities. First, motivation is provided by a student's expectation of bringing about a change that will be useful to him and his school. He can be proud of his accomplishments, and more importantly, he will grow to appreciate the power of applied intelligence.

> . . .Science and mathematics thinking has [an] immediate payoff, especially for the elementary school student. . . a child can make his own observations and organize them, then make his own predictions and check them. Thus he can directly appreciate the power of the scientific style of thought. In mathematics, a child can be led to see for himself how focus on essential concepts can make hard problems easy and bring seemingly impossible problems within range. . . . This means that science and mathematics . . .are ideal vehicles for the primary message of our educational process: Thinking is worthwhile.[1]

Second, high standards for a student's work come out of his own need for success and correctness in order to attain a goal important to him.

Third, a problem-solving approach requires that the problem (or "challenge") be analyzed by the students themselves so that they can decide which aspects of the problem they need to investigate. The developers believe that one of the most useful skills students--and adults--can learn is where to begin to tackle a problem.

Finally, problems leading to real projects do not have artificial data or requirements, nor are they overly simplified. As they work on real problems, therefore, students are more likely to develop all the aspects of good problem solving. The developers have identified these as: *observation, quantification, simplification of the problem, applying judgment, formulation and trial of successive models for change, acquisition of needed skills,* and *development of a critical faculty.* Unlike science and mathematics programs in which each unit centers on one basic concept, USMES units are based on problem-solving work that requires the student to learn many skills and concepts as he works toward a solution.

In most instances the student is expected to learn through observing the results of his own and classmates' experiences. Students are encouraged to work things out for themselves cooperatively and to learn from needs they discover as they proceed toward solutions of the problems. Thus the USMES view of discovery learning is in between "guided discovery," in which everything the student is supposed to discover has been planted in the activities ahead of time by the teacher or the materials (like a treasure hunt), and "messing about" in which no culminating concept or "light-bulb" experience is required to flow out of the random activity. To the extent that students accept a challenge and with help from the teacher pursue its solution, developers expect students to gain in responsibility for their own learning. They should also begin to sense their potential as learners and their own power to affect their social environment.

2. CONTENT AND MATERIALS

2.1 Content Focus

All USMES units focus primarily on the processes included in problem solving. The learning experience always includes the steps of *deciding to seek a solution, evolving a set of plans, carrying out specific tasks, analyzing results,* and *recommending changes* or *producing new products.*

The USMES units emphasize:

observing
data gathering
hypothesizing
interviewing
designing and building test apparatus
controlling variables
statistical analysis
recording data accurately and efficiently
making and disseminating products
testing procedures
writing evaluations
improving small-group dynamics

In order to pursue these activities, the students must blend concepts, skills, and knowledge from mathematics, natural and social science, and language arts. Developers caution, however, that the curriculum is intended to be supplementary; students cannot learn everything they need to know through USMES:

> USMES, or a similar program, can enrich each subject area and therefore can be allotted time normally given to each part of the school program. On the other hand, it will not fulfill every cognitive and affective need; learning is best attained through a mixture of modes and strategies. Furthermore, the openness of USMES activities implies that other more structured programs may be needed to fill in gaps, or teach the more formal aspects of the disciplines which are within the cognitive range of children in grades 1-8.

The developers emphasize that an USMES unit takes a different direction and shape in every classroom in which it is used--the unit becomes unique to that class. However, because the units which are available for use in schools have grown out of many classroom experiences with the same topic, the developers predict the subjects and skills which students can learn as they work on the unit. Each Teacher's Resource Book contains an index of the activities that may be undertaken. For instance, in the Lunch Line Unit (used successfully in grades 2-6), this index includes counting, timing, and graphing activities; organization of groups and discussion of tasks; making scale drawings and models; studying nutrition; making lunchroom posters and slogans; writing, administering, and reporting on questionnaires, interviews, and surveys; discussion and presentation of recommended improvements; and trial of recommended improvements.

Each of the units integrates aspects of mathematics with social science, physical science, and language arts. The unit, Play Area Design, for example, might include activities from physics (mass and springs, pendulums, friction and stress, centrifugal force); biology and ecology (animals and plants to be displaced or included in the new environment, exercise and human health, drainage); mathematics (computation, geometry, cost analysis, scale models and mapping); community relations; economics; geography; child development; population; land use; and whatever else is needed to develop a local solution to the challenge of designing a playground. In general, units contain abundant opportunities for mathematical, language arts, social and physical science activities. There are fewer opportunities for experiences in the biological sciences in the units currently available for classroom use. However, three new units now in development (School Zoo, Nature Trails, and Growing Plants) emphasize biology.

2.2 Content and Organization of the Curriculum

The curriculum consists of units that are organized around a problem (called a "challenge") for students to solve. The twelve units that are ready for classroom use are described below. Developers do not specify particular grade levels for each of the units. They say that teachers should review the units and then decide how to adapt them to fit their particular group of students.

1. *Pedestrian Crossings* (The challenge is, "Recommend and try to have a change made which would improve the safety and convenience of a pedestrian crossing near your school.")

   One of a series of units originally suggested at the 1967 Cambridge Conference on the Correlation of Science and Mathematics in the Schools. Children collect a great deal of data under different conditions, make comparisons, draw conclusions, and recommend improvements. They may decide to design and carry out a field investigation to measure the performance of various pedestrian crossings under different types of control.

2. *Describing People* ("Find out what is the best information to put in a description so that a person can be quickly and easily identified.")

   Student's own concern for self-identification is extended into a broader search for a *systematic* way to identify a person by recognizing certain physical characteristics. For instance, what kinds of information are the most efficient for finding one person in a crowded lunchroom? Primary classes have been especially interested in this problem.

3. *Burglar Alarm Design* ("Build a burglar alarm which will give adequate warning.")

   A practical design problem which requires the exploration of many different concepts in electricity. Students learn about basic circuit components and characteristics from the Elementary Science Study (ESS) unit, "Batteries and Bulbs," or by working through the USMES "How To" cards including: "How to Make Simple Electric Circuits," "How to Check a Circuit by Tracing the Path of the Electricity," "How to Make Good Electrical Connections," "How to Find Out What Things to Use in an Electric Circuit," and "How to Make a Battery Holder and Bulb Socket."

4. *Dice Design* ("Construct practical shapes which can be used as dice to make a fair decision between two or among four. . .choices.")

   Design activities, primarily in spatial geometry, blended with testing activities including probability and statistics. Understanding of functions and graphing are needed to solve the problem of whether a student-constructed shape is a "fair" die.

Each Teacher's Resource Book contains descriptions of available Background Papers written by USMES staff, consultants, and teachers. Teachers use these to understand basic concepts and to organize materials as they make decisions about how to proceed with the unit. The Background Papers also suggest additional experiences or explain in detail activities recommended in the Teacher Logs. Descriptions of the following Background Papers are included in the Dice Design Resource Book: Fair and Regular Polyhedra, Making Polyhedra, Solids Made of Equilateral Triangles, The Five Regular Solids, Semi-regular Solids, Mass Production of Equilateral Triangles and Squares, Thumbtack Experiments (probability), Coin Games (ranges and probability), and Geometric Comparison of Ratios.

5. *Lunch Lines* ("Recommend and try to have changes made which would improve the service in your lunchroom.")

   Students are motivated by their own daily lunchroom experiences and by the possibility of real changes in a school service coming out of their efforts. They make detailed observations of the present conditions in the lunchroom; then hypothesize and test improvements in the problem areas (serving arrangements, change-making, garbage collection, traffic flow, table arrangement, milk distribution, noise, dismissal schedules, etc.).

6. *Soft Drink Design* ("Invent a new soft drink which would be popular and produced at a low cost.")

   Students may start with opinion polls to determine favorite drinks, or they may conduct blindfold-tasting tests to explore taste factors. The information from these factor analyses is combined in the invention of a new soft drink. Aspects of the problem which often arise are three-dimensional data representation, random sampling, ecology, nutrition, advertising and consumer attitudes, production procedures, and market research.

7. *Designing for Human Proportions* ("Find a way to design or make changes in things that you use or wear so that they will be a good fit.")

   Possible class challenges: "Design chairs which would be comfortable for students in your class. Determine how many sizes of Design Lab aprons should be made for students in your school for comfort and reasonable cost."

   This unit grew out of activities that originated in classes developing the Describing People Unit. Students first use their own body measurement data to devise a measuring system, including a set of standard sizes for each age group. Instruments suitable for making different body measurements are designed and constructed in a school workshop.

Analysis of the data determines how many sizes are needed; consideration of the trade-off of cost for comfort is an important aspect of this decision. In some classes this work has led into investigation of rates of growth. It is hoped that the students will carry out their projects to the point of actual construction of furniture and clothing, making successive improvements and refinements over a period of trials. Follow-up activities might include studies on production and marketing, origin and conservation of raw materials, home and school design, or consumer research on other items.

8. *Consumer Research-Product Testing* ("Determine which brand of a product is the best buy for a certain purpose.")

Which tape sticks the best and lasts the longest for the price? Which paper towelling is the best for a combination of wiping and absorbing? Which pencils should the school buy? Often the work begins with investigations of claims made on TV commercials. The balance of quality vs. price that is acceptable to the customer is tested. Students exchange ideas about which factors are relevant, suitability of tests, etc. Socioeconomic questions arise from comparative shopping (costs in different neighborhoods, taste preferences). Students may decide to design and produce a better product of their own, produce ads and commercials.

9. *Weather Predictions* ("What will the weather be this afternoon. . . tomorrow? Find out what information helps you most in accurately predicting the weather.")

Students investigate weather conditions and the effect that they have on people's lives. Some students may build weather instruments; others may check past records or develop ways to record current data. Correlation of observations with predictions should begin early in the unit. Students may wish to issue forecasts or hold competitions to predict the weather for a special event.

10. *Play Area Design and Use* ("Recommend and try to have changes made which would improve the design or use of your school's play area.")

Students tackle problems of playground improvements or expansion. Small groups measure the area and equipment, survey students in the school to determine age group needs, visit other play areas, make scale drawings or construct models, and investigate the financial aspects of the problem. After the problem has been fully investigated, recommendations are made to the school administration; if possible, actual improvements are made in the play area.

11. *Traffic Flow* ("Recommend and try to have a new road design or a system for rerouting traffic accepted so that cars and trucks can move safely at a reasonable speed through a busy intersection near your school.")

Students investigate traffic patterns at a nearby intersection. After determining factors such as speed of cars, adherence to traffic regulations, and length of entrance and exit lanes, the students design a new system, taking into account safety, cost, and minimum use of land.

12. *Electromagnetic Device Design* ("Design a good electromagnet for a specific purpose.")

    Frequently this unit is used in conjunction with the Burglar Alarm Unit since an electromagnet is a way for students to make a signal operate in a second circuit when the first circuit is broken. During the unit children investigate the variables affecting the strength of the electromagnet as they search for a way to build a strong or lightweight electromagnet.

The following seven units underwent classroom development during 1973-74. During the 1974-75 school year they will undergo trial implementation.

1. *Bicycle Transportation:* While attempting to make cycling a convenient and safe way to travel, students investigate safe routes or plan new ones, set up a safety course, or develop security arrangements for bicycle parking.

2. *Ways to Learn:* Students investigate individual variables in rates and styles of learning.

3. *Classroom Design:* Students determine what factors in their classroom environment are conducive or detrimental to learning and recommend changes to the teacher and school administrators.

4. *Manufacturing:* Students wrestle with decisions any small scale entrepreneur might face: what to produce, what method is best, how cost should be determined, and how the item should be marketed. Students might consider candles, skate scooters, electric games, bookends, etc.

5. *School Zoo:* Students keep records, make cages, measure food and water consumption, etc., for classroom pets.

6. *Orientation:* How can students make the transition to a new school or community easier for a newcomer? Children conduct surveys to discover what problems exist, collect information newcomers may need, and provide services to help new arrivals.

7. *Advertising:* Students investigate the best ways to advertise a product or idea and conduct surveys and experiments to determine the effectiveness of the different approaches. Students might launch an advertising campaign in conjunction with another USMES unit (for example, Manufacturing or Soft Drink Design).

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Ten additional topics *Design Lab Design, Finding Your Way, Mass Media, Nature Trails, Energy, Growing Plants, Games for Indoor Recess, School Rules and Decision Making, Classroom Management*, and *Recycling* plus *School Supplies/School Store* are being considered for development during the 1974-75 school year. Those units which are adopted will be field tested in 1975-76.

2.3 Materials Provided

Student. The project director has commented:

> We're not putting out kits or units of magic materials; rather we're raising problems and trying to find out what kids can really do. It's very important that the curriculum be open in how the kids actually follow their *own* lead, and we give them no equipment kits, just a Design Lab that is a general shop plus some measuring and testing equipment and the "How To" cards to help when they get stuck. Our purpose is to keep the learning environment really their own.

"How To" cards are short sets of instructions to help students solve problems that may arise during the course of a unit; they are the only written materials which USMES has developed for students. USMES staff cautions that the cards should not be used as a sequence or set of programmed "task" cards; they should not be introduced at the start of the unit or outside the context of the student's open investigation of a practical problem. Earle Lomon points out that the cards are *not* for use to motivate bored children ("Why don't you try this?"), but only when a child *asks* for them ("I'm stuck, and I need to know 'how to'. . . .").

Teacher. USMES teacher materials consist of the USMES Guide, Teacher's Resource Books, a Design Lab Manual, and Background Papers.

The USMES Guide is an overview of the program; developers suggest that it be used for long-range USMES-centered curriculum planning. To relate USMES units to elements of the regular classroom curriculum, developers have prepared charts which delineate major activities in USMES units and show skills, processes, and concepts which are emphasized in the units. The Guide also contains basic information about each of the units, the Design Lab, a list of "How To" cards, and an annotated list of the available Background Papers.

Every USMES unit has its own *Teacher's Resource Book* which contains all the materials needed for beginning a class project on the challenge posed by the unit. Two or three logs are included, made by teachers whose classes pursued the challenge. These logs are anecdotal, detailed, journal-like descriptions of a class's work on the unit. All the activities undertaken are described, including student work, teacher preparation and suggestions, and problems which arose. Many actual examples of student work are included. The logs are the heart of the USMES materials. For those unable to attend a teacher's workshop or visit an USMES classroom, reading the logs is the best way to understand the flavor and scope of the program. Teacher Resource Books also contain a description of USMES philosophy and approach, information about the unit, references to materials relevant to the unit, and charts which indicate the skills, concepts, and processes that students have learned and practiced in USMES.

The *Design Lab Manual* includes information about cost, scheduling, safety, staffing, training, and an inventory of tools and supplies for a classroom or school workshop in which children can make equipment to pursue their challenge.

The *Background Papers* provide information for teachers on a variety of topics that might arise during the course of an USMES unit. Some of the Papers available in connection with one unit, Pedestrian Crossings, are: "Traffic Flow at Pedestrian Crossings," "Notes on the Use of Histograms for Pedestrian Crossings Problem," "Notes on Data Handling," and "Using Scatter Graphs to Spot Trends."

2.4 Materials Not Provided

Design Lab. The Design Lab may take many forms; it may be a corner of a classroom, a movable cart, or a separate classroom containing tools and materials used for construction and testing. Developers describe the Lab as a place where "A student is free to build his own apparatus according to his own theories, making whatever mistakes he is bound to make, and benefiting from those mistakes, thereby arriving at improved designs."

Although some USMES activities can be conducted successfully without a Design Lab, a comprehensive program requires one. Cost and staffing of the Design Lab (both borne by the school) are discussed in Section 4. A complete inventory of suggested tools and supplies can be found in the Design Lab Manual and in the USMES Guide. All are available from department, hardware, electronic, stationery stores, and lumber yards.

3. CLASSROOM ACTION

3.1 Teaching-Learning Strategy

USMES requires an active-learning approach; the student gains information and skills through a wide variety of activities which he chooses to do in order to solve the long-range, practical problems posed by USMES units. The teacher suggests, coordinates, and extends these activities for individual students, for small groups, and for the class as a whole.

USMES is intended to be a supplementary curriculum; students are not expected to cover certain subject matter in a specified amount of time. Developers suggest that USMES sessions be held at least 2 or 3 times a week for a total of 45-60 hours over a 3- to 8-month period However, some classes may spend an entire year on one unit. Because there are no set deadlines for completion of a unit, teachers can afford to let students make mistakes. This freedom to make mistakes allows students to fully investigate a problem and to learn through trial of various solutions. Developers assume that teachers will cover skills and concepts not learned through USMES during other parts of the day.

The specific learning experiences which occur as a class works on a unit will obviously differ from student to student and class to class. The Teacher's Resource Book offers suggestions on several ways to approach a single unit as well as detailed Teacher Logs describing how different classes pursued challenges. The flow chart on the following page suggests some activities which might take place in the classroom.

Obviously, USMES activities require that the teacher be skillful in classroom management. USMES publications describe these additional teacher responsibilities:

1. Introduce the challenge in a meaningful way that not only allows the children to relate it to their particular situation but also opens up general avenues of approach.

2. Act as a coordinator and collaborator. Assist, not direct, individuals or groups of students as they investigate different aspects of the problem.

3. Hold USMES sessions at least two or three times a week so that the children have a chance to become involved in the challenge and carry out in-depth investigations.

4. Provide the tools and supplies necessary for initial hands-on work in the classroom or make arrangements for the children to work in the Design Lab.

5. Be patient in letting the children make their own mistakes and find their own way. Offer assistance or point out sources of help for specific information, such as the "How To" cards, only when a child reaches the point of frustration in his approach to the problem.

6. Provide frequent opportunities for group reports and student exchanges of ideas in class discussions. In most cases, students will, by their critical examination of procedures, improve or set new directions in their investigations.

3. FLOW CHART

Pedestrian Crossings

Challenge: Recommend and try to have a change made which would improve the safety and convenience of a pedestrian crossing near your school.

Main Activities:

Class Discussion: Are nearby crossings safe? Do you have to wait too long? Are traffic controls needed?

↓

Observation of controlled & uncontrolled pedestrian crossings.

↓

Class Discussion: Report on observations: list hazards, controls. Measurements needed to study possible improvements.

↓

Design & construction of measuring instruments. ⇄ Classroom simulation of measuring activities.

↓

Collection of data at intersections.

↓

- Speed of cars (distance, time).
- Timing (crossing times, gap times).
- Counting (rate of flow of traffic)

↓

Class Discussion: Conclusions that can be drawn from measurements, graphs, accuracy of data, correlation of data, new data needed. Recommendation for control.

↓

- Preparation of bar graphs, line graphs, histograms. ⇄ Collection of new and different data.
- Construction of scale layouts, traffic signals, etc.
- Production of films. Other activities.

↓

Class Discussion: Final recommendation(s) for improvement. Plans for class report on findings.

↓

- Tryout of suggested improvement.
- Report writing.

Optional Follow-up Activities:

- USMES unit: Traffic Flow.
- Alternative transportation methods, city economics.
- Study of costs of controls

7. If necessary, ask appropriate questions to stimulate the student's thinking so that he will increase the depth of his investigations or analysis of data.

During the teacher's informal conversations with students, she identifies the inadequacies in experience and concept that are hampering progress and suggests an activity leading to needed skills, facts, or ideas. Through observation, listening, and questioning, she assesses the student's absorption of the work into his total experience. USMES requires teachers with belief and experience in this manner of working with children.

Although students may spend most of their time working individually or in pairs or small groups, the fact that the whole class is engaged in working together on the same unit makes it necessary for them to plan together, to discuss what they're finding, and to take into account and learn from each other's experience. USMES encourages students to teach and help one another. The student is also seen as a self-teacher because at each step he is encouraged to decide what work he wants to do and to carry it out himself with the support of his classmates, his teacher, and the Design Lab coordinator.

3.2 Typical Work on a Unit

Following is a description of student activities, many of which are represented on the flow chart in the preceding section.

Challenge: Recommend and try to have a change made which would improve the safety and convenience of a pedestrian crossing near your school.

One sixth grade teacher initiated this unit with a discussion of routes students took to get to school, and what problems they had crossing streets. Singling out a nearby crossing that did not have a signal light, the class came up with factors which made crossing difficult. With the goal of determining how to make the crossing safe, the entire class initially visited the crossing and made maps of the intersection. Then, one group spent about three weeks making a scale model of the area. In order to accomplish this they used information from "How To" cards on scaling, measured the crossing, drew a blueprint, and finally made the model in the Design Lab from Tri-wall and papier maché. (Individual children next decided to make their own models. The group that made the original model taught other students scaling procedures. These students used the information to build small reproductions of the original model.)

During the model-building, four small groups were going to the intersection once a day for about 45 minutes to measure gap times between cars traveling through the intersection. Two groups recorded information on cars going in one direction, two groups did the same for cars traveling the other way. After subtracting arrival times to get gap times, students tallied up times, and made bar graphs. The four groups then reported to the class that 70 percent of the gap times were less than five seconds. This information led them to realize that the crossing could be considered safe only if it took people less than five seconds to cross. The four groups next timed students as they crossed and discovered that the average crossing time was eight and one-third seconds. Another group timed the signal light at another nearby intersection to determine the amount of time allowed for crossing.

While at the intersection, children discovered that many cars did not signal before turning. Another group was formed to record the number of cars that signaled compared to the number that did not. Data collected over a five-day period showed that 41 percent of the cars did not signal. The children drew and held "Please use your blinkers" signs at the intersection and reported to the class that their three-day investigation showed that 95 percent of the drivers signaled when the signs were held. A school safety officer was invited to attend a class presentation in which students used graphs, maps, and the scale model to present their findings and recommendations.

For the next several months the class continued working on the unit. They wrote letters to the state traffic department, constructed traffic lights, made trundle wheels, compared crossing times at four different places, and made a video tape and scale models of first graders and cars as a part of a traffic safety program they developed for primary students. As a final project the students investigated school driveway and parking area problems and made recommendations to the principal.

3.3 Evaluation of Students

The evaluation of students working on an USMES unit is done by the teacher. The developers give no specific guidelines for evaluation because they believe that the work in each class will be different, that what each student will learn from his work will be different, and that each teacher should be free to approach the work in any way that will be beneficial to the students. As the teacher diagnoses what skills and information a child needs to proceed with what he wants to accomplish, she is evaluating as well. As she helps students prepare reports she will recognize what they have and have not learned.

Teacher Resource Books for each unit contain examples of the developer's conception of good class work and lists of concepts and skills used in each major activity; additional charts relate activities to subject matter. Although developers emphasize that each student and class will pursue each unit in a different manner, these charts can be used as general checklists to aid the teacher in student evaluation. The students themselves become evaluators of their own progress in this program; they decide what they need to learn in order to accomplish a specific goal, and if they do not learn it, they are unable to proceed.

3.4 Out-of-Class Preparation

Prior to beginning a unit the teacher must read the materials which the developers provide. These include a Teacher's Resource Book, "How To" cards, the USMES Guide, and Background Papers. She must consider how her own class will approach the challenge, what skills they will need to develop, the local factors which will affect the solutions children try, etc. Also she must work out with the Design Lab manager the procedures and likely materials her students will need in the following months.

3.5 Role of Other Classroom Personnel

No assistant teachers or aides are required for the classroom management of an USMES unit, but they might be very helpful. The Design Lab manager (who might be the classroom teacher, another school or district staff member, or a community volunteer) is a very important person in the successful use of an USMES unit. His or her role is essentially that of a teacher; he must help students figure out (not tell them) what materials and equipment they will need. Students may need a variety of things, including measuring equipment (for example, trundle wheels for measuring crosswalks), or ways to represent data (making scale models), or devices to construct (various polyhedra for the Dice Design Unit). The Design Lab manager must share the USMES learning philosophy and must be especially sensitive to helping students learn to be responsible for their own learning. The USMES Guide explains the manager's role:

> The Design Lab manager or teacher provides an open atmosphere for the students. They are not forced into preconceived avenues of endeavor, which might preclude the exploration of their own ideas. Experience in the Design Lab should be rewarding and meaningful to the student and help him learn to be inventive, to be scientifically curious, and to work with others.

4. IMPLEMENTATION: REQUIREMENTS AND COSTS

4.1 School Facilities and Arrangements

Because USMES integrates many subject areas and because it requires an active-learning approach, it demands a greater change in the school environment than a more traditional program would.

> . . .it is not the relatively simple matter of taking out an old curriculum in one or more subject areas and replacing it with USMES. Rather, there is a need

> to rethink the whole program, deciding how to combine USMES with other curricula to enhance the effectiveness of all components. . . .The payoff can, however, be very large; instead of merely improving pieces of the curriculum, there is the opportunity to fundamentally alter and improve the educational system. . . .

USMES may be used in a variety of classroom arrangements. While it is suitable for traditionally graded, self-contained classrooms or homogeneous grouping, it is also an excellent supplementary or "core" curriculum for nongraded, individualized groups, for "family" or "vertical" classes, or for other forms of flexible and heterogeneous groupings. Because each challenge contains many different aspects, which can be approached on different levels, the terms of a single challenge can be made suitable for children of different preparation and ages.

While specific scheduling is left up to the individual teacher or school, the developers recommend that about one-fourth to one-third of the total school program be devoted to work on USMES. Each challenge is designed to represent from 45 to 60 hours over a 3- to 8-month period. Teachers may choose to have students work on a single challenge before moving to a new one or on a number of related challenges at one time.

Full implementation of USMES requires installation of a Design Lab (see Section 2.4). The Lab may occupy one corner of a classroom, be a movable cart, or a full-scale shop in a separate classroom. Optimally, a full- or half-time staff person or volunteer is useful to operate the Lab, but it can be handled by the regular classroom teacher.

For some of the units (for example, Pedestrian Crossings, Traffic Interchange, Play Area Design, and Weather Prediction), students need to gather data out-of-doors or off school grounds. Adequate provisions must be made for safety, supervision, and transportation.

Because USMES differs from traditional programs, it is difficult for one teacher to implement alone; administrative support is almost always necessary. Developers note:

> . . .except in rare cases the teacher still needs the support of the administration. This is especially important in USMES because the activities of students range beyond the classroom and need to be coordinated from grade to grade.
>
> Although many teachers with sufficient classroom autonomy have very successfully used USMES when no one else in the school was involved, the cooperation of the principal and district administrators is needed to have an USMES program broadly implemented in a

school or a district. The principal is the key element in reassuring teachers that USMES is an integral part of the school program and can be used on an everyday basis. He is responsible for providing space for a Design Lab and arranging for its management. Arrangements among teachers and aides to logistically support the activities of the students in and out of the classroom and Design Lab need his approval and possibly his initiative. Laying out a coordinated long-range program for the introduction of USMES at all grade levels needs the kind of planning and information handling that the principal is in the best position to provide.

4.2 Student Prerequisites

Developers say that "a sufficient background for USMES units is much less than that required in the more traditional educational context in which the student is expected to proceed rapidly along a predetermined route. . .the [USMES] student may. . .acquire the skills and concepts when needed in his search for some solution to the problem." The following is a list of prerequisite skills which appears in the Teacher's Resource Book for the Pedestrian Crossings unit.

1. Students who can count can make a start on the quantitative aspects of the unit. Graphing skills may be learned as the need for them arises.

2. Measurement skills may be learned as each new activity is begun, and improved when additional or new data are required.

3. An ability to divide by small one-digit numbers is sufficient for making calculations for scale diagrams. Young children can convert their measurements to "blocks" on graph paper.

4. Sets of data can be compared graphically and by subtracting medians (halfway values) and ranges; the calculation of averages is not necessary.

USMES activities are designed to help children learn and practice greater self-direction. However, in classes where students do not already possess some self-control, or where teachers do not have experience engaging students in open-education type activities, organizing students for USMES activities may be difficult.

4.3 Teacher Prerequisites

Providing an environment where students can explore and seek their own solutions to relevant problems may be a big order for most teachers. USMES developers, realizing that their program demands a change in both teacher and student behavior, suggest that teachers attend a workshop before implementing the program. USMES developers offer summer "Resource Personnel Workshops" to train district personnel, who in turn train local teachers in USMES implementation. Participating districts must agree to provide release time for teacher training and meetings during the year, Design Lab space, materials, and staff. USMES furnishes a complete set of written materials; audio recordings and slides are available on a rental or purchase basis. Districts differ in the ways that they choose to train teachers and to implement the program; inservice course length varies from several weeks to a semester. District personnel are trained to conduct workshops aimed at giving teachers both experience working with units and an understanding of the teacher's role in USMES. During the courses, teachers work on adult challenges such as designing ways to improve the teachers' lounge, auditorium, or office. The workshop staff models the teacher's role in USMES, while teachers experience for themselves the ways their students will approach unit activities. Although a short period of time is spent discussing USMES mode of teaching and learning, participants are expected to gain this understanding through their work on the adult challenge or, whenever possible, through working with children on small segments of several USMES units.

Teachers who trial-test USMES units attend 8-10 workshops taught by USMES staff. Travel and subsistence expenses for participants who attend more than one workshop are covered by the project. In addition, trial-test teachers are given a complete set of written materials and a 25-dollar petty cash fund for materials not found in the Design Lab. Teachers are required to try out the challenges in their school to agree to have observers visit their class, and to write reports on their students' work. School administrators are required to permit teachers to spend at least three hours a week on USMES and to provide a Design Lab.

USMES preservice courses have been offered since 1971; inservice courses since 1972. Up to the spring of 1974 nearly 20 colleges, scattered around the United States, had offered USMES-related courses. Developers point out that the number of colleges offering USMES courses is constantly increasing; an additional 26 institutions have submitted proposals to NSF, requesting consideration for USMES implementation funding.

4.4 Background and Training of Other Classroom Personnel

The Design Lab manager must be able to plan with the USMES teachers so that the Lab is stocked with materials appropriate to the units being used and so that the ideas the students explore in the Lab are fully integrated with classroom work. He or she needs to have a sound knowledge of mechanics, carpentry, and design and an imaginative, problem-solving approach to setting up and running the Lab. Essentially the Lab manager is a coordinator who organizes and facilitates students' activities.

USMES recommends that Lab managers participate in a five-day training workshop held concurrently with a workshop for teachers. However, because managers are often not appointed until after the school year begins, they may be unable to attend summer workshops. Developers therefore suggest that teachers be trained as Design Lab managers so that they will be able either to handle the Lab themselves or to train others, either district personnel or community volunteers, to do this important job.

4.5 Cost of Materials, Equipment, Services

The developers note that the following materials are still under development and may be modified.

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
USMES Guide	1 per teacher	USMES Project	$2.00	Reusable
USMES Design Lab Manual	1 per teacher plus manager	"	$2.00	Reusable
Teacher's Resource Books	1 per teacher for each unit taught	"	$2.00 ea.	Reusable
Background Papers (complete)	1 set per school	"	$7.50	Reusable
"How To" cards (excluding Design Lab Cards)	1 set per class	"	$7.50	Reusable
"How To" cards for Design Lab	1 set per class	"	$2.50	Reusable
Workshop Resource Book	1 per district instructor	"	undetermined	Reusable
Slide-Tape Show (35 slides)	(Rental for inservice workshop or community presentation)	"	loan at no charge, or purchase-$20	
Videotapes of classroom trials, presentations, and panel discussions	(Rental for inservice workshop or community presentation)	"	undetermined	

Many different arrangements have been made for providing materials and staff in USMES Design Labs. Some districts have small workshop areas in each classroom; others have set aside a separate room. Since some USMES activities can be successfully completed without a Lab, some teachers have begun the program using only classroom facilities and materials. Labs have been run with no budget; tools were donated and the materials scrounged. Some have been funded from federal aid programs, or privately funded by local citizen groups or merchants. Lab managers have included assistant superintendents, vice-principals, parents, janitors, science specialists, or retired people who were hired by the school district. In some schools volunteers--usually retired men, parents, and community workers--manage the Lab.

Thus, cost of the Design Lab will differ from school to school. USMES developers say that in the first year of operation, the Lab usually costs between $850 and $1,000. The cost of replacement of consumable supplies usually runs $200-$300 for each succeeding year. When teachers or volunteers do not serve as Lab manager, the salary of a half-to-full-time manager must be budgeted.

4.6 Demonstration Schools

Persons interested in seeing schools where USMES is being used should contact Christopher Hale at Unified Science and Mathematics for Elementary Schools Project, Education Development Center, 55 Chapel St., Newton, Mass. 02160. (617) 969-7100.

There are demonstration and field test sites in:

California:	Carmel, Ca. Los Gatos, Ca. Marina, Ca. Monterey, Ca.
Colorado:	Boulder, Colo.
Georgia:	Athens, Ga.
Illinois:	Chicago, Ill. Urbana, Ill.
Iowa:	Iowa City, Iowa
Massachusetts:	Arlington, Mass. Boston, Mass. Lexington, Mass. Watertown, Mass.

Michigan:	East Lansing, Mich. Eaton Rapids, Mich. Howell, Mich. Okemos, Mich. Lansing, Mich.
Minnesota:	Minneapolis, Minn. Owatonna, Minn.
New Hampshire:	Durham, N.H. Epsom, N.H.
New Jersey:	Plainfield, N.J.
South Carolina:	Charleston, S.C.
District of Columbia:	Washington, D.C.

4.7 Community Relations

Because USMES challenges involve work toward practical solutions to real problems, students often have active contact with the outside community. Usually this happens when they write letters or ask informed persons for information or techniques they need to collect and/or assess data. In every case, the response from communities has been favorable, even eager. Parents and neighbors have been approving and interested in their children's attempts to solve traffic safety problems, to conduct consumer research, and to design their own tools and equipment. Many schools have had parents and neighbors volunteer time, skills, and materials to classrooms using USMES. The teacher and principal using USMES materials must help children develop tact and awareness of the way decisions in the school and community are made, as they present their solutions and requests to the people empowered to carry them out.

The developers have prepared materials for school or district presentations to community groups and parents. Meeting logs explain how these groups have participated in short experiences to acquaint them with USMES.

5. PROGRAM DEVELOPMENT AND EVALUATION

5.1 Program Development

At the 1967 Cambridge Conference on the Correlation of Science and Mathematics in the Schools a group of 30 scientists and mathematicians defined educational goals and discussed the implementation of an integrated mathematics, science, and social studies curriculum for elementary school students. USMES grew directly from suggestions recorded in the report of the conference: *Goals for the Correlation of Elementary Science and Mathematics* (Houghton Mifflin Company, Boston, Mass., 1969). In a series of meetings over the next two years several people who had attended the conference or were stimulated by its report sharpened the USMES philosophy and goals and planned the activities to be undertaken in the first 18 months of development. They added appendices to the "idea" material of the Goals report and tried out some of the ideas in a few classrooms. Their proposal was backed by EDC, and in January 1970 it was funded by the National Science Foundation.

USMES staff members carry out the directives of a group of more than 30 advisors; 19 of these advisors form a planning committee which meets two or three times a year. Suggestions from the 1967 conference and from planning committee meetings, or those made by teachers and students, may become the basis for new units. Usually the suggestion is in the form of a general challenge (for example: "Design, or redesign, a playground for your school.") that includes both general questions ("Where? Cost? Number of users? Ten years from now?") and specific questions ("What equipment? Location of objects? Use by age level? Effect on animal and plant ecology of the area?"). Earle Lomon commented on this process:

> We, as a staff, or the Planning Committee come up with lots of ideas (for example, land use for parking, location of new schools in the community, prevention of molds, water pollution), but for an idea to be considered at all it must contain a strong challenge, and students in the development classrooms must find it exciting and important to work on. Also, it must contain opportunities for a large variety of research activities.

The proposed challenge is explored by teachers, students, and USMES staff and consultants at a two-week summer workshop. Then trial-test teachers develop the challenge with students in their own classrooms during the following school year. These teachers write reports and keep logs of each class meeting. A paid observer also attends the class frequently and takes notes. An USMES consultant may work in the classroom from time to time. The following summer, or in some cases, after two years of trial, the unit is tentatively adopted or dropped on the basis of its success in at least ten classrooms of varied socioeconomic backgrounds and locations. If a unit is tentatively adopted, a complete Teacher's Resource Book, primarily made up of class notes from teachers and students who have been working with the unit, is compiled. Sets of "How To" cards are written as they are needed during classroom trials and are revised whenever the content or wording is found to be unsatisfactory. Some students have enjoyed editing their own sets of cards for specific needs that came up during their work. Development of this kind has taken place with teachers and students in schools throughout the United States. A new unit is then field tested and evaluated during use by a new set of teachers during the next school year. Constant re-evaluation and direction comes from the planning committee meetings.

Twelve units have been completed and are available for classroom use. An additional 7 units underwent classroom development during 1973-74; 10 more are being considered for development during 1974-75. In all, the developers plan to complete 32 units by 1978.

5.2 Developer's Evaluation

Developers and many users of USMES are convinced that students learn more from the USMES style curriculum than they would from a more traditional one. Their contention, however, is difficult to prove. An USMES evaluator notes, "It is very difficult to find evaluation instruments that really do justice to the type of things that children learn in USMES. Everyone involved in open education is struggling with what type of program evaluation to use." Since 1971 USMES evaluators have attempted to find solutions--tests and observation instruments that will effectively measure student gains. To date, the evaluation has been done in three parts: achievement tests, problem-solving tests, and classroom observation. Two sub-tests of the *Stanford Achievement Tests of Arithmetical Computation and Reading Comprehension* (paragraph meaning) are given students in experimental and control classrooms. The purpose of giving these tests is to show that the students spending time working on USMES gain as much in mathematics and reading as students involved in more traditional modes of instruction.

Two Boston University professors, Bernard Shapiro and Mary Shann, working with University personnel, have come up with three *problem-solving tests*. In the Notebook Problem Test, individual children are asked to decide which of three notebooks the school principal should order for student use. A group of five students is challenged to design a playground on an open piece of land in the Play Area Design Problem. The Picnic Test is a third test which asks students to make plans for a class outing.

The Boston University evaluators have also devised a classroom *observation scale* to record the type of activities taking place in USMES and control classrooms. Their findings help to shed light on different organizational patterns and interactions which might evolve because of USMES.

In addition to these evaluation instruments, there is program monitoring by observers who are present in classrooms in which a unit is being developed (during the first year) or field tested (during the second year). Too, evaluators interview a sampling of teachers, principals, administrators, and resource team leaders in participating districts. Finally, teachers involved in program development are paid to fill out a report on each class session devoted to USMES work and a monthly report on the overall activities related to the unit. The Teacher Logs included in the Teacher's Resource Book for each unit are compiled from these various reports. Developers use information from these observations and interviews to revise the units and teacher-training strategies.

5.3 Evaluation Results

What have Boston University's studies shown? Do USMES students perform better on achievement tests than non-USMES students? Are they more creative problem solvers? Do they interact differently with other students and the teacher in classroom situations?

Data gathered from *Stanford Achievement Tests of Arithmetical Computation and Reading Comprehension* administered to students in 23 experimental and 23 control classrooms during the 1972-73 school year indicate, "There was no consistent evidence that exposure to USMES either facilitated or impeded growth in the basic skills of reading and arithmetic; comparison of USMES with non-USMES samples yielded no strong trend in favor of either group." Developers explain that these findings illustrate that students are able to learn the problem-solving process while absorbing basic facts, skills, and concepts of math and reading as quickly as students who are not involved in USMES. They caution, however, that the results are not conclusive because of the small sample size. Additional data were collected during the 1973-74 school year, but have not yet been analyzed.

USMES developers are also encouraged by the results of the Notebook Problem Test.

> Children were asked to examine three different spiral-bound notebooks (differing in dimension, number of pages, quality of paper, number of lines per page, price, and so on) and recommend which should be ordered in quantity for student use. Normal classroom tools such as pencils, pens, rulers, and erasers were made available to the children to use during their investigation. In the pretest there were no discernible differences between the USMES and the control classes on the two dimensions analyzed: a) whether any of the reasons given for the choice was based on factors that were measurable within the test situation, and b) whether the choice was based on personal opinion, a suggested test, or a performed test. In the posttest however, every USMES class altered its scores to reflect a) predominantly quantifiable reasons for the choice, and b) higher levels of proof involving suggested or performed tests.

One USMES staff member commented that "there were all sorts of problems with the methodology," and one would anticipate difficulties in measuring USMES problem-solving type learning; but the results are nevertheless impressive because they overwhelmingly favor the USMES group.

The evaluators came up with some interesting findings concerning students' behavior in USMES classes compared to non-USMES classes. These findings are based on data collected during the 1972-73 school year by trained observers using a specially designed interaction scale. A summary of their conclusions from the USMES Guide states:

> In general, USMES classes were found to be involved in whole group activities as often as the control classes, but students in USMES classes were much more likely to be involved in small group work while students in the control classes were more likely to be working individually. There was also some evidence that USMES classes changed structure (e.g., whole group work to small group work to whole group) more often during the one hour observation period than the control classes. While both the USMES and the control classes utilized large group instruction to much the same extent, the kinds of interactions differed from the two kinds of classes. Students in the USMES classes contributed new ideas much more often than students in control classes. On the other hand, student verbalizations in control classes took the form of answers to specific questions posed by the teacher or random comments much more often than in USMES classes. In addition, there was somewhat more debating and arguing points in USMES classes. There were no clear-cut differences between USMES and control classes on the number of times students reiterated ideas or made presentations to the whole class. When the classes were involved in small group work, the USMES classes were characterized by much more child-child interaction while the control classes were characterized by much more child-teacher interaction.

Teacher's Logs, anecdotal summaries of teachers' experiences with USMES, are included in the unit materials. These logs are a valuable resource for those who wish to conduct their own subjective evaluation of USMES.

5.4 Independent Analyses of the Program

Developers are unaware of any independent analyses of the program. However, several participating districts are currently discussing ways to include an assessment of the effect of USMES on student learning in their ongoing evaluation programs. USMES staff may aid in these efforts.

5.5 Project Funding

The National Science Foundation has funded the entire USMES program. USMES is a working project of Education Development Center, Newton, Massachusetts. The developers expect to continue development of USMES units under NSF funding through 1978. Where USMES workshops have been held in connection with preservice courses (as at California State College in Bakersfield) or inservice courses (as in the Lansing, Michigan, School District) part or most of the cost has been the responsibility of the college or school district involved.

5.6 Project Staff

Project Director, Earle Lomon, professor of physics, Massachusetts Institute of Technology. Other key personnel include: Christopher Hale, Project Manager; Betty Beck, Associate Director for Development; Thomas Brown, Associate Director for Implementation; Charles Donahoe, Design Lab Coordinator; Carolyn Arbetter, Editor for Implementation/Evaluation; and Ray Brady, Editor for Development.

FOOTNOTES

1. Cambridge Conference on the Correlation of Science and Mathematics in the Schools. *Goals for the Correlation of Elementary Science and Mathematics.* Boston, Mass.: Houghton Mifflin Company, 1969, p. 6.

DEVELOPING MATHEMATICAL PROCESSES (DMP)

INTRODUCTION

*Once there was a yummy place to live called the Land of Nibble. In the Land of Nibble everything could be nibbled on. The candy-cane tree had delicious peppermint canes. The ice-cream bush had 93 different flavors. The lollipop forest contained yummy all-month suckers. There was a giant cupcake on top of a rock, but only the bravest people got to taste it. Popsicle Creek flowed through the Land of Nibble. . . . One day Martin the Monstrous Munch took a walk. He had a huge appetite and wanted to eat everything in sight. His friend the Piffle Bird took a walk too. You can tell where each one went and what he ate by looking carefully at the tracks.**

Following the instructions in the Developing Mathematical Processes (DMP) Guide for Level 2, the teacher has read this story to a small group of students who have, according to the DMP assessment inventory, mastered the necessary prerequisites for this lesson's objectives--to choose points and construct a path between them. The teacher shows how to draw the two paths on the maps in the students' workbooks. As this group completes this assignment one by one, the teacher or classroom aide may teach another group of children working on a different lesson.

DMP is a new program being developed by the Wisconsin Research and Development Center as the mathematics component of Individually Guided Education (IGE). The latter is a total system of education; along with its curricular components, it includes an organizational plan for rearranging a school from self-contained, graded classrooms into clusters of multiage-grouped children and several staff members with roles ranging from master teacher to student teachers and aides. IGE is an attempt to combine several reforms: nongraded classes, individualized instruction, team teaching, "accountability," and shared decision making among a differentiated staff and administrators. Similarly, DMP combines several reforms: a developmental theoretical base, active-learning methods, objectives-based curriculum structure, and an individualized diagnostic/prescriptive system.

*All quotations are from materials issued by the developers or from conversations with developers or teachers.

Teachers introducing DMP in an IGE school are expected to receive a great deal of implementation support from the program developers and from district administration. Developers say the program works just as well in non-IGE schools, provided that teachers receive inservice training, additional staff, and strong administrative backing.

Without such preparation and support a teacher may find the program unusually demanding of time and energy and perhaps ambiguous because of emphases on both developmental learning theory and objectives-based lessons. Developers comment that most non-IGE teachers with traditional teaching styles implement the program slowly, accepting one part at first (for example, the manipulative materials) and gradually adding other aspects. The complete program at any one level may take two to three years to implement fully. Teachers with successful experience in active-learning math and/or open-classroom teaching may have evolved their own ways of structuring the classroom and curriculum to respond to varied student needs and thus may find DMP confining.

BASIC INFORMATION

Program name: Developing Mathematical Processes (DMP)

Format: Developmental Edition is composed of eight levels for use in kindergarten through sixth grade. Number of levels may differ in the commercial edition.

Emphasis: DMP was developed as the mathematics component of the IGE system for nongraded schools. It incorporates both a sequencing of behavioral objectives and an active-learning approach. Manipulatives are an integral part of the program. DMP stresses student assessment and grouping for individualizing of instruction.

Content: Computation, geometry, probability, and statistics; all based on measurement.

Suggested use: Complete kindergarten through sixth grade curriculum.

Target audience: Students of all abilities, grades K-6. Although the program was developed for use in conjunction with IGE, it is suitable for self-contained classrooms.

Length of use: Daily use for at least one and one-half hours per week for students in their first year of school and two and one-half hours per week in later years.

Aids for teachers and students: Teachers receive one or two guides for each level, and a package of materials including game boards, game directions, story and picture cards, assessment materials, and an answer book. Student materials include consumable workbooks, non-consumable textbooks for Level 5 and above, test booklets, and a classroom kit of manipulatives.

Date of publication: Levels K-2, 1974. Levels 3-4 are due to be published by September 1975; Levels 5-6 by September 1976.

Director/Developer: Wisconsin Research and Development Center for Cognitive Learning, The University of Wisconsin, 1404 Regent St., Madison, Wis. 53706. (608) 262-4901.

Publisher: Rand McNally and Company, DMP Project Customer Service, P. O. Box 7600, Chicago, Ill. 60680.

1. GOALS AND RATIONALE

1.1 Goals

DMP developers expect that graduates of their K-6 mathematics program will have a command and understanding of the relevance of math, that they will be able to perceive, pose, and solve mathematical problems based on relationships and patterns among objects and phenomena in their environment--and that they will have fun doing it. Developers anticipate that experience working with concrete objects and with mathematical processes in DMP will prepare students to work abstractly, to "examine, identify the structural properties and relationships, and logically validate mathematical assertions."

Developers see their program differing from traditional programs in three major ways: the entire program, including not only arithmetic, but also geometry, probability, and statistics, is based on measurement; students are involved in "active" learning; and the teacher role is altered to stress student assessment and classroom organization for individualizing learning.

The program intends to allow for individual progress by means of a scheme of objectives-based lessons and competency tests to insure that each child moves at his own pace and does not attempt new work until he has mastered its prerequisite. DMP attempts to combine an active-learning approach with this individualization system. (The rationale underlying active learning is discussed in the Nuffield and Madison Project reports in this book, and the rationale for objectives-based individualization is presented briefly in the IPI report.) The developers claim, "Not until DMP has a serious effort been made to incorporate this [active] learning approach in a carefully sequenced, complete program of mathematics instruction."

1.2 Rationale

DMP developers accept the rationale that children should encounter mathematics not as a collection of facts and rules but as a system which people use to solve real problems. Their brochure announces, ". . .Children have for too long accepted math as an isolated subject unrelated to other aspects of their lives. . . . The program [DMP] helps the child understand at the outset that mathematics and the application of mathematical concepts have relevance both to his own environment and to his everyday life."

They chose to center the program around measurement because it provides relevant, everyday math activities for children. But they also believe a measurement approach is sound mathematically. They use measurement as the means for having children investigate the attributes of objects. They present measurement as the practical guise in which children can apply basic thinking processes: describing and classifying, comparing and ordering, equalizing, joining and separating, and grouping and partitioning. These processes underlie the concepts and skills of the math curriculum in the DMP rationale.

Following Piaget, DMP designers deem experiences with concrete manipulative objects essential for building children's conceptual understanding of math, but they decry the haphazard use of math workshop materials. *Carefully chosen* manipulatives, used in *structured* activities, based on developmental theories are required, they say:

> DMP's activity approach to math is rather different from that usually found in traditional classrooms. It should be clear, too, that activity-centered math is not turning children loose to riot; nor is it hit-or-miss random learning, with a haphazardly conducted instructional program. In fact, just the opposite is true. DMP's activities are organized and sequenced with great care, so that skills needed at a certain point have already been mastered in prior activities.

The centrality of the developer-designed structure differentiates DMP from some other programs (for instance, the Nuffield and Madison Projects) following an active-learning approach, which calls for the teacher to shape and pace lessons using the curriculum only as a guide. DMP uses only those math workshop materials specified in the curriculum and uses them only to teach specific behavioral objectives. The same manipulatives are used repeatedly so that mastery can be developed gradually, but also so that "fooling-around time" is greatly reduced. Other developmental programs advise teachers to provide children with a great variety of naturalistic but mathematically rich materials and to encourage children to explore them freely, following explorations with generalizations and skill learning. DMP designers believe that teachers need the security provided by explicit directions as to diagnosis and prescription, and detailed lesson construction, in order to make experience with materials result in demonstrable skills and sound ideas.

The commitment to teacher security and to "accountability" (which is a hallmark of IGE) appears to be the source of DMP's reliance on behavioral objectives. Writing such "competency-based" goals for instruction in advance, as measures of student and teacher performance, does not necessarily connote belief in behaviorist learning theory. Behavioral statements can be seen simply as ways to make learning goals clear and public. In practice, however, behavioral objectives do tend to shape and pace a program in advance. They may pose questions for thoughtful teachers as to how to reconcile the need to respond to children's idiosyncratic, developing learning with the need to meet present objectives.

2. CONTENT AND MATERIALS

2.1 Content Focus

Developing Mathematical Processes is to be a complete mathematics program for kindergarten through sixth grade. The developmental edition contains books for eight levels. Although the developers advocate flexibility in assigning levels to specific grades, there is an approximate correlation: Level 1 is for kindergarten; Level 2 and part of Level 3--first grade; part of Level 3 and all of Level 4--second grade; Level 5--third grade; Level 6--fourth grade; Level 7--fifth grade; and Level 8--sixth grade.

A major deviation from standard programs is that in the early grades the concepts of number and mathematical sentences are presented as ways to represent measurement situations. Thus equals ("=") is used to represent "weighs the same as," "is the same height as," "has the same area as," "holds the same amount as," in addition to the usual interpretation, "is the same number as." Consequently, although the content is primarily arithmetic, there is an unusually heavy emphasis on measurement, with both metric and English units being used. There is also some geometry at each level, and statistical procedures are introduced for grades 4 through 6. Developers specifically rejected set theory, believing it inappropriate for young children, and preferring the concept of measurement as a basis for arithmetic.

Physical objects are used to introduce new topics (for instance, a balance for equality and inequality, shapes for angle measurement, and toothpicks and rubber bands for place value). Children are trained to use the objects as models for the mathematical topics. In addition to physical objects, poems and stories are used to introduce concepts and to pose problems. Developers valued the topics presented in previous active-learning curricula, but sought to develop the basic ideas in greater detail.

DMP presents problem solving in mathematics as the application of basic thinking processes to attributes of objects. The basic processes, adapted from those conceived by the Science--A Process Approach program, are describing and classifying, comparing and ordering, equalizing, joining and separating, and grouping and partitioning. The process of *describing and classifying* is taught throughout the curriculum in activities like counting, describing shapes, using units to measure weight, describing location by coordinates, using fractions to describe areas, organizing data by means of a graph, interpreting two-thirds as two divided by three, and using negative numbers to represent movements. *Comparing and ordering* are applied to attributes of length, time, weight, capacity, area, angles, whole numbers, fractions. and decimals. *Equalizing* is the process of adding or subtracting pairs of weights, lengths, and numbers. In geometry, areas and angles are *joined and* *separated*; the arithmetic operations of addition and subtraction are taught as abstractions of the same processes. Similarly, the operations of multiplication and division are symbolic representations of the process of *grouping and partitioning*.

Two other processes in the program, representation and validation, are special processes which aid problem solving in conjunction with the basic ones. *Representation* is the process by which concrete attributes are expressed gradually in more abstract ways. Thus, the attribute of length can first be represented physically by a piece of string, pictorially by a graph, and finally symbolically using units such as centimeters. All of the attributes and processes are first introduced concretely through physical representation, then through pictorial representation, and finally symbolically. DMP stresses that children should *validate* their statements; in particular, they should validate arithmetic solutions.

2.2 Content and Organization of the Subdivisions

Each of the 8 levels is divided into topics (there are 96 topics in the Developmental Edition--the number may vary in the final edition). The 11 topics which comprise Level 2, Developmental Edition, are listed below with a summary of the content in each topic:

TOPIC 2.1 TWO-DIMENSIONAL SHAPE

The child learns to describe and classify regions (including faces of solids) on the attribute of shape.

TOPIC 2.2 COMPARING AND ORDERING ON WEIGHT

The child directly compares and orders real objects on weight using a balance beam.

TOPIC 2.3 WRITING NUMBERS

The child learns to write the numbers 0-10 and practices writing them in a variety of situations.

TOPIC 2.4 COMPARING AND ORDERING EVENTS ON TIME

The child compares and orders events on time of duration and time of occurrence.

TOPIC 2.5 ASSIGNING MEASUREMENTS

The child uses arbitrary units to represent lengths or weights of objects and he assigns a number and unit. Then he compares and orders objects using these measurements.

TOPIC 2.6 PATHS

The child describes closed paths in terms of number and length of sides (triangle, rectangle, and square). He is introduced to the geoboard as a simple way to make paths.

TOPIC 2.7 COMPARISON SENTENCES

The relationship between two sets or two objects (on a given attribute) is represented by a sentence involving = or ≠ (for example: $5 \neq 7$, $6 = 6$, $A \neq B$). The process of validating is introduced.

TOPIC 2.8 COMPARING AND ORDERING ON CAPACITY

The child directly compares and orders the capacities of various containers by pouring from one to another. Also he learns to represent the capacity of a given container with arbitrary units and he assigns a measurement.

TOPIC 2.9 ORDER SENTENCES

The relationship between two sets or two objects (on a given attribute) is examined further. Now the child not only decides if the two are equal, but also, if they are not equal, he decides which is larger and writes an order sentence (for example: $5 < 7$, $6 = 6$, $A > B$). The child also learns to validate given order sentences.

TOPIC 2.10 MOVEMENT AND DIRECTION

Simple maps are examined here. The children follow simple oral or written directions involving movement on a given path or between given points. They also learn to give such directions.

TOPIC 2.11 THE NUMBERS 0-20

The numbers 11-20 are introduced as representing the numerousness of sets of that many members. The children learn to recognize and to write these numbers and to count such sets. The numbers 0-10 are reviewed.

Within each topic are several "activities" designed to teach specific behavioral objectives. (Additional activities within the topic are designed to review previously learned skills and to prepare for future topics.) As an example, the behavioral objectives for Topic 2.9 are:

1. Given an order sentence, reads it. (reads order sentence)

2. Given two objects or sets, chooses an appropriate order sentence. (chooses order sentence)

3. Given two objects or sets, writes an appropriate order sentence. (writes order sentence)

4. Given an order sentence, validates it physically or pictorially. (validates order sentence)

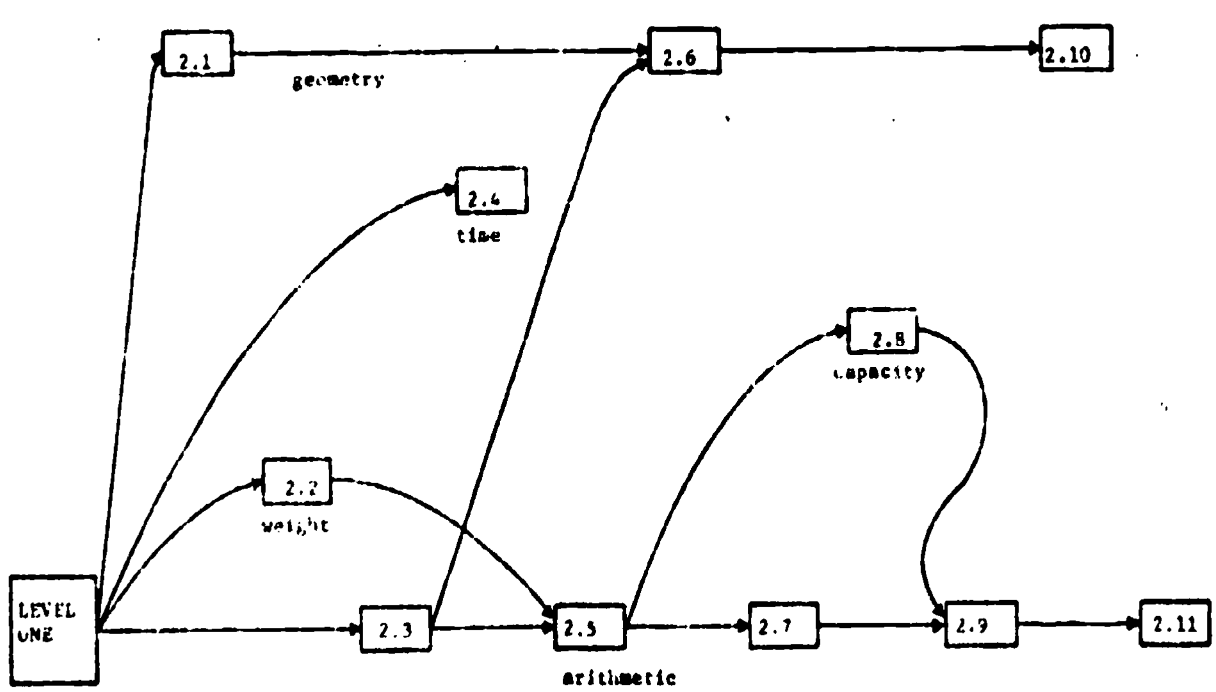
5. Given an open order sentence, completes it. (completes open order sentence)

There are 14 "activities," or daily lessons, under this topic. Six of the activities are classified as regular, 3 activities are optional, and 5 serve as alternative ways to teach 2 lessons. Each activity is explained to the teacher in terms of the materials needed, vocabulary to be introduced, behavioral objectives to be reinforced, the type or classroom organization required for each sequence within the activity, and teacher preparation.

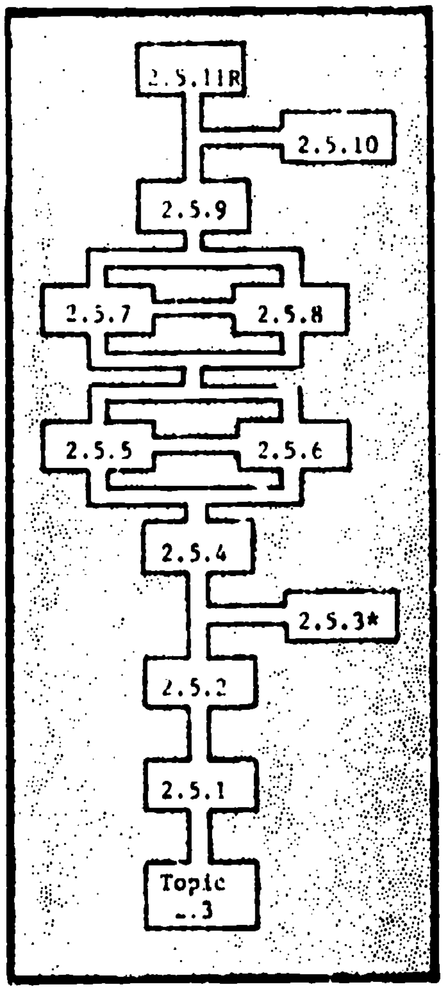
The program specifies the order in which topics are to be taught. There is some choice the teacher can make regarding order; for instance, in most cases the geometry topics are independent of the arithmetical ones. All children in the class cover the same content; however, optional and alternative activities are included. The teacher can use these optional activities instead of regular ones if she thinks them preferable for the whole class.

The chart below shows the suggested sequencing for Level 2. If one topic is to the left of another and an arrow connects them, then mastery of the objectives under the left-most topic is prerequisite to the mastery of those under the one on the right.

TOPIC SEQUENCE CHART



Activities within topics are also carefully sequenced, as this diagram illustrates:



*: an activity that contains one or more additional suggestions for your use

R: an activity that reviews an objective listed in a previous topic

Developers explain:

> Activities at a lower level on the diagram are usually prerequisite to those at a higher level. If there is only one activity at a level, all children working on the objectives should engage in that activity. If there is more than one, they are alternate activities and you may choose to do either or both.

2.3 Materials Provided

Student. There are several consumable notebooks for each level, as well as nonconsumable student textbooks for Level 5 and above. Consumable student test booklets include a Placement Inventory for each level and Topic Inventories for each topic within a level. A classroom materials kit containing mostly nonconsumable items is also available. Items include: adding machine tape, balance beams, counting chips, blank dice, geoboards, rubber bands, toothpicks, Unifex cubes, and washers.

Teacher. The Teachers' Guide is thought by the developers to be the most important piece of material in the package. There is one guide and sometimes two for each level. The guides contain both assessment and activity suggestions as well as sequencing options. In addition to the guide, the teacher gets a package of materials including game boards, game directions, cards for station activities, story cards, and picture cards. The package also contains assessment materials. An answer book to students' workbooks is available.

2.4 Materials Not Provided

A great number of physical materials which teachers may or may not have in the classroom are not included but are needed for many activities. These materials include: buckets, cups, funnels, play money, rice, tongue depressors, bottles, cans, tops, clay, paint, ditto masters, and felt-tip pens.

3. CLASSROOM ACTION

3.1 Teaching-Learning Strategy

The typical DMP activity is begun by the teacher either leading the children in a discussion or demonstrating a problem-solving strategy. This usually takes place in a large group, though sometimes the teacher introduces materials to smaller groups. Next the children work in groups or individually at structured tasks, either with physical objects or workbook pages.

The children are sometimes asked to make predictions or to invent stories which model mathematical statements, but in general they work at answering questions posed in the book in a prescribed way. Teachers can choose to substitute optional lessons or ones they invent themselves for regular ones, and children can choose which material they use for validation, but most activities and materials are specified in the DMP Teachers' Guide.

The teacher is asked to determine which children are meeting the behavioral objectives and to provide special activities for those children who have not yet mastered them. The objectives have been set out by the DMP developers to move from a concrete to a symbolic level. Thus, when a child has advanced to an objective on the symbolic level, the teacher is instructed to discourage him from returning to physical objects.

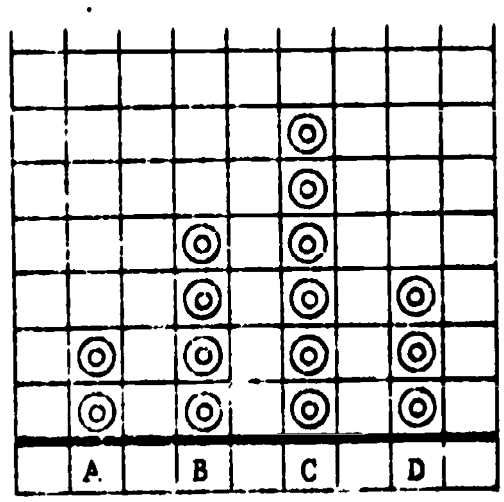
Developers have established a series of 24 behavioral objectives to provide a basis on which teachers can evaluate their own performance. (They were originally designed to evaluate the effectiveness of inservice training and are included at the end of Section 5.3, Evaluation

Results.) These objectives help illustrate some of the differences between DMP and traditional mathematics programs. Some of the objectives suggest the *active-learning* element of DMP: "The teacher moves from group to group, acting as a resource person," "The teacher allows students to move purposefully about the room," and "The teacher allows students to interact verbally while working." Other objectives show the emphasis which DMP developers place on *assessment*: "Using appropriate instruments, the teacher assesses students and completes records," and "The teacher states the roles of placement and topic inventories." Too, the objectives illustrate the role of the teacher as manager of *individualized instruction*: "On the basis of information gathered, the teacher forms instructional groups based on achievement," "When presented with a student who has not mastered an objective, the teacher can choose an activity that will help the student," and "When given information on student achievement, the teacher classifies students on the basis of prerequisite behaviors needed to start a new topic."

3.2 Typical Lesson

The objectives of Activity 2.5.8 ⓐ, a lesson giving students practice in comparing and ordering weight measurements by graphing, are to teach students to represent weight physically, to assign with measurement, and to use weight measurements to compare and order. Before the lesson the teacher prepares a set of four cartons for each group of four children in the class. Each carton is filled with objects which weigh the equivalent of one to ten small washers and each is labeled A, B, C, or D inside of the lid. Each group of four cartons is marked with the same color for easy identification.

At the outset of this activity the students are divided into groups of four; each group is given a piece of graph paper, four cartons, crayons, a balance beam, and ten small washers. Students label columns on the graph paper A, B, C, D. Each student weighs one carton with small washers on the balance beam, lifts the lid to view the letter, and records the carton weight by drawing washers in the appropriate column.



After all groups have finished weighing their four cartons, both graphs and cartons are traded with another group and students are instructed to find the letter for each new carton without looking inside the lid. The Teachers' Guide states: "They do this by choosing one of the cartons, weighing it with small washers on the balance beam and then finding the bar on the graph that corresponds to the weight of the carton." It is unclear whether the teacher tells the students that they should do this. (If she does not, there is a likelihood that students will determine the appropriate order through alternative methods like simply lifting them to determine which is heavier.) Children validate their own answers by lifting the carton lids.

At the conclusion of the lesson the teacher is directed to discuss the graphs with the children. Students are to use their graphs to answer questions such as the following: "Which carton is the heaviest?", "Which carton is the lightest?", "Are any two cartons the same weight?", "How many washers did carton A weigh?" Teachers are also instructed to discuss ordering the weights for the cartons from heaviest to lightest and vice versa.

The interaction afforded by group work is thought to be essential by the developers. Although students do not move on to new work until they have mastered the prerequisite skills, DMP is not intended to be used as a completely individualized program with each child working at a separate pace. In at least the first year of implementation a teacher may have difficulty managing DMP in a classroom where children are not grouped according to mathematical ability because she may have to prepare and teach two or more different activities every day. Where children do differ greatly, it may be practical for the teacher to try DMP with only one group within the classroom.

3.3 Evaluation of Students

DMP judges student success by mastery of the stated behavioral objectives. The Teachers' Guide advises teachers to observe children's behavior to see if they are mastering the topic objectives. Some of the objectives are assessed solely through such classroom observations rather than by means of paper-and-pencil tests.

Group and sometimes individually administered tests (Topic Inventories) are used to assess many of the objectives. These tests are provided by DMP and closely resemble the workbook pages for the related topic. The teacher is provided with a form to rate each child's performance on the topic test. The teacher indicates the number of correct responses and then refers to a chart which determines whether the child is to receive a rating of M (mastery), P (making progress), or N (needs considerable help) for each behavioral objective. Children are not to proceed to a topic for which the specific objective is requisite if they have an N rating on that objective.

Teachers are provided with both individual progress sheets and group record cards on which students' performance on each objective can be recorded. In addition to the assessments for students' mastery, Placement Inventories are provided for each grade to determine which level students should be using and which topics they have already mastered. Results of the Placement Inventory can be used to group students if the teacher wants to have students working on different topics.

3.4 Out-of-Class Preparation

Teacher. Although teachers sometimes can choose an activity which takes less preparation, almost every activity involves the collection and setting out of materials; many require the preparation of special materials. In addition, the teacher must read and work through the day's activity in the Teachers' Guide. Naturally the amount of preparation that is required is multiplied if the teacher has grouped students to be working on different topics.

Student. Student homework assignments have not been written into the DMP program.

3.5 Role of Other Classroom Personnel

Teacher aides. Teacher aides are very important to the DMP program; in fact they are probably essential in the first year of implementation. Not only are they needed to help prepare and gather materials, but they help the teachers assess children's mastery of the objectives by direct observation.

Advisor. Developers suggest that, whenever possible, a DMP local coordinator be appointed to assist teachers with implementation. The coordinator should be available to answer teacher questions, to provide inservice training, and to give demonstration lessons. In schools using IGE, the "unit leader" may serve as DMP coordinator. Developers anticipate that the last levels (grades 5 and 6) will be released in 1976. Until that time, schools will not be able to implement the program at all grade levels, and so developers suggest that districts should provide a coordinator for several years, throughout the initial adoption period. After 1976, they say, a coordinator may only be required for the initial implementation year.

4. IMPLEMENTATION: REQUIREMENTS AND COSTS

4.1 School Facilities and Arrangements

Most classrooms are physically suitable for DMP implementation. It is important that there be tables or movable desks so that the children can work in groups with physical objects. Ample storage for the materials is needed within the classroom.

DMP was developed as a part of IGE, a system that calls for differentiated staffing to work with students of varying ages and abilities and to perform different instructional tasks. Developers believe that teachers adapt the program successfully for use in self-contained classrooms. According to Project Coordinator James Moser, most of the teachers who used the program in 1973-74 were not in IGE schools.

However, successful implementation requires that many of the elements that are part of IGE exist in any school using DMP: the program works best when several teachers in a school are using it, when there is strong administrative support, teacher aides, inservice training, alternative means of evaluating achievement, and a math coordinator. Because the program demands a great deal of teacher time, developers also advocate a preparation period for teachers.

4.2 Student Prerequisites

There are no special student prerequisites for DMP. Students begin the program according to their performance on Placement Inventories. If students within a class differ widely in ability, teachers can begin using DMP with only one group of students. Student progress is guided by assessment instruments provided to the teacher.

4.3 Teacher Prerequisites

Although no special subject matter background is required to teach DMP, special training in both mathematical content and implementation of the program is highly recommended. It is questionable whether the ordinary traditional teacher can orchestrate assessments, lessons in differentiated groups, manipulative materials, and workbooks unless she has had intensive inservice training and advisory help in the classroom. It may be necessary for a traditional teacher to see a master teacher using the materials and lessons with children, in order to grasp the developmental intent of the program. The teacher whose training and experience are entirely traditional may tend to focus on the sequence of behavioral objectives and to use the manipulatives to train for "competency" instead of focussing on the learning *experiences*, elaborated by work with apparatus, which children gradually transform into mental concepts. The traditional teacher with insufficient inservice might also omit the enrichment ideas (which seem to offer the most creative experiences to children) and require all children to do the same lesson at the same time.

Developers themselves do not expect that most teachers will be able to implement a full DMP program during the first year. A teacher may concentrate on making assessments of students by watching their activities rather than by tests, or on teaching to small groups during the first year, adding other aspects of the program later. Full implementation may take two or three years, according to Project Coordinator Jim Moser.

Three-day inservice workshops are available for school district personnel and college professors who have been designated as local DMP coordinators in various parts of the United States. DMP publishers, Rand McNally and Company, cover the costs of the inservice workshops as well as travel expenses. The workshops prepare the local coordinators to provide inservice for teachers. DMP is currently developing pamphlets, films, and tapes which the coordinators can use to train local teachers.

The usual pattern of inservice training for teachers includes a college course of varying length or two days of inservice workshops before the school year begins. During the school year, biweekly after-school follow-up sessions are recommended for teachers and DMP coordinators. Fewer sessions are recommended for the second semester of implementation.

For information regarding coordinators and coordinator training sessions contact: Mary Montgomery, Wisconsin Research and Development Center for Cognitive Learning, 1025 W. Johnson St., Madison, Wis. 53706.

4.4 Background and Training of Other Classroom Personnel

Teacher aides. Teacher aides should be present at the same inservice sessions as teachers. See Section 4.3.

Administration. The DMP developers recommend that principals attend inservice meetings with teachers. See Section 4.3.

Advisors. See Section 4.3 for inservice information for local DMP coordinators.

4.5 Cost of Materials, Equipment, Services

Required Items	Quantity Needed	Source	Cost Per Item	Replacement Rate
Complete kit for teachers and 32 students	1 per class	Rand McNally and Company		
Level K			$198	Reusable (student workbooks, yearly)
Level 1			$340	
Level 2			$386	
(Levels 3-4 will be available in 1975)				
(Levels 5-6 will be available in 1976)				
Miscellaneous materials (see Section 2.4)		School storeroom or local stores		
Inservice (see Section 4.4)			Depends on local arrangements; may be free	

4.6 Demonstration Sites

The following is a partial list of DMP users, including both IGE and non-IGE schools.

California:	Dickson School Compton, Ca.
	Stipe School San Jose, Ca.
	Highlands School San Mateo, Ca.
Colorado:	Sun Valley School Lakewood, Colo.
Connecticut:	Mill Road School New Haven, Conn.
Illinois:	Carrie Busey School Champaign, Ill.
Indiana:	Walt Disney School Mishawaka, Ind.
Iowa:	Hoover School Dubuque, Iowa
Maryland:	Kensington School Kensington, Md.
Nebraska:	Oakdale School Omaha, Neb.
New York:	St. Mary's Dunkirk, N.Y.
	Denton Avenue School New Hyde Park, N.Y.
Ohio:	Green Valley School Parma, Ohio
Pennsylvania:	Union Terrace School Allentown, Pa.

5. PROGRAM DEVELOPMENT AND EVALUATION

5.1 Program Development

DMP is one component in the Individually Guided Education (IGE) program under development since 1964 by the Wisconsin Research and Development Center for Cognitive Learning. The Wisconsin IGE model (there is also a similar plan called IGE disseminated by /I/D/E/A) is a total system of education including an organizational scheme, staff training, and curricula. The organizational plan, called "multiunit school," is an alternative to a school of traditional, self-contained classrooms. Instead, students in a three- to four-year age span are grouped in nongraded clusters of 100 to 150. Within each cluster, 8 or 9 adults (including a lead teacher, 3 or 4 staff teachers, 1 teacher aide, 1 instructional secretary, and 1 intern) are responsible for planning, carrying out, and evaluating each child's instructional program. School goals are defined by an instructional improvement committee that includes unit leaders and the building principal.

Although changing the school organization is the core of IGE, Wisconsin also sees need for individualized curriculum materials. In addition to DMP, the Center has developed a reading program to be used as a part of IGE. The reading program is a compendium of suggestions for teachers to use portions of other available reading programs.

At the same time as the multiunit model was being tried, researchers at the Wisconsin R&D Center were investigating processes of instruction in math. Finding no curricula that they deemed adequate for elementary children, in light of new knowledge about how children develop mathematical concepts, the math research group decided to design its own program. When the Center administration decided that its curriculum programs should serve IGE, the math research group became an integral part of IGE. The strategy for individualization in IGE is based on behavioral objectives. The DMP staff began its curriculum design by delineating objectives in mathematics and then sequencing these objectives using models developed by other educators. In 1971, after competitive bidding, Rand McNally was selected as publisher for DMP. Rand McNally assisted with field testing and implementation of K-1 materials during 1971-72 in IGE and non-IGE schools. Nationwide field tests were conducted in 1972-73 and 1973-74. The first K-2 commercial editions were published in 1974; editions for grades 3 and 4 are scheduled for publication in 1975; the 5-6 editions for 1976.

5.2 Developer's Evaluation

A field test of the first two levels of DMP was conducted in 8 schools in 1971-72; 41 teachers and 1,500 students were included in the study. Four of the schools had conventional organization and were located in large urban areas (Milwaukee and Chicago). The other four schools were IGE schools in Wisconsin cities (Milwaukee, Green Bay, Sparta, and Galesville).

"The purpose of the field test was (a) to determine the effectiveness of the instructional program in terms of student achievement, (b) to gauge the impact of an inservice program on teacher performance, and (c) to document the usability of the program."

Student mastery of the DMP behavioral objectives was determined at three times during the year. Classes were randomly selected and visited at these times to see what proportion of students had met specific objectives. This was determined by administering program tests and by the teachers' rating of students on Topic Inventories and teacher observation schedules.

In order to evaluate the success of inservice training, teachers were observed biweekly by coordinators, who filled in an observation schedule at each visit. The observation schedule allowed developers to rate teachers on 24 specific performance objectives. In addition, teachers completed a questionnaire and were interviewed.

5.3 Evaluation Results

Student mastery is defined to be a rating of M (mastery) on at least 80 percent of the objectives and a rating of P (making progress) on the rest. The table below shows the results of this evaluation:

Percentage of Students Attaining the Specified Mastery Level*

School Type	K	Grade 1	Mean
Urban	82	43	63
Nonurban	75	81	78
Mean	78	62	70

*M ratings on 80 percent of the objectives; P ratings on the remaining objectives.

The developers attribute the relatively low levels of mastery to the early stage of the development of instructional and assessment materials.

Teachers were rated on 24 performance objectives. A teacher was said to have mastered an objective if she was observed exhibiting it 75 percent of the time. The following table shows the percent of teachers who achieved mastery of the objectives.

DMP TEACHER PERFORMANCE OBJECTIVES
AND PERCENTAGE OF TEACHER MASTERY

		Objective	Percentage
Providing Instruction	1	The teacher chooses activities that help students achieve the objectives of DMP.	92
	2	The teacher provides the printed, manipulative, or other materials needed for the activity.	95
	3	The teacher identifies the problem or the objective of the activity, providing an appropriate focus.	95
	4	During the opening or closing of an activity, the teacher states the relationship of the activity to previous work.	47
	5	During the opening of an activity, the teacher explains the activity clearly and in a well-organized manner.	89
	6	During the closing of an activity, the teacher displays and discusses student work.	50
	7	The teacher uses student ideas.	82 (95)*
	8	The teacher does not negatively criticize a student's work.	87 (100)
	9	The teacher responds to student statements by asking for validation or justification of the mathematical ideas expressed.	63
	10	The teacher asks questions and leads discussion, rather than lecturing.	92
	11	Given an activity that requires students to work individually, in pairs, etc., the teacher organizes the students.	95
	12	The teacher moves from group to group, acting as a resource person.	95
	13	The teacher allows students to move purposefully about the room.	95
	14	The teacher allows students to interact verbally while working.	95
	15	The teacher arranges furnishings and materials as recommended.	95
	16	The teacher demonstrates mastery of the DMP objectives being studied by the students.	92
	17	The teacher describes the mathematical processes being used.	58
Managing Instruction	18	Using appropriate instruments, the teacher assesses students and completes records.	71 (80)
	19	The teacher states the roles of placement and topic inventories.	66 (85)
	20	On the basis of information gathered, the teacher forms instructional groups based on achievement.	58
	21	When presented with a student who has not mastered an objective, the teacher can choose an activity that will help the student	39
	22	The teacher redirects individual students when they finish.	18
	23	When given information on student achievement, the teacher classifies students on the basis of prerequisite behaviors needed to start a new topic.	55
	24	The teacher identifies the various options that are made available in each topic of the Teacher's Guide.	58 (80)

*Numbers in parentheses indicate percentage of nonurban teachers where appreciably different.

Evaluators conclude that the inservice training was effective in helping teachers implement its activity approach to learning; participating teachers reached the criterion level on 13 of the 17 objectives related to providing instruction (instructional materials used by the teacher, the teacher's verbal behavior, the classroom organization, and the teacher's knowledge of mathematics). The program was not judged successful in training teachers to manage DMP instruction to provide for individual student differences. None of the non-IGE teachers attained the criterion level on objectives relating to managing instruction; IGE teachers reached the criterion level for only 3 of the 7 objectives. It was expected that IGE teachers would perform better than non-IGE on these objectives both because they had received previous exposure to use of assessment and management information during IGE inservice and because they taught in schools where arrangements had been made for student grouping. IGE teachers also did a better job of using student ideas (Objective 7), and they criticized student contributions less frequently (Objective 8) than did non-IGE teachers.

Developers caution that one should not draw definitive conclusions from this study because of the small sample size. Information gathered from nearly one hundred schools during 1973 suggests that DMP works as well, and sometimes better, in non-IGE schools. A formal analysis of the data has not been published. Future evaluation plans call for comparative evaluation of DMP students and those learning through other math programs.

Data gathered from teachers,questionnaires and interviews indicate that both teachers and students were enthusiastic about the program even though "the program cannot be implemented. . .without an expenditure of faculty effort and staff resources which goes beyond the conventional elementary mathematics program."

5.4 Independent Analyses of the Program

No independent review exists of the DMP program at this time. Observation of several classroom teachers who were trying DMP for the first time, and who were receiving no in-school advisory help, led observers to speculate that in order to use DMP successfully, teachers need previous experience or background in developmental learning theory, active-learning math or science, or open classroom teaching. (However, if teachers already have such prerequisites, they may find this program too constricting.)

Those teachers observed stressed the workbook exercises, and their approach to manipulatives was that students could use materials only in the prescribed, teacher-demonstrated way to answer the teacher's questions. Experimentation with materials was discouraged. One teacher said she doesn't allow students to touch the materials before she explains their use, since "it would get too chaotic."

The teachers were using the standard state texts in conjunction with the DMP materials in order to prepare students for state achievement tests. Consequently, the students were working at very advanced symbolic levels before they had had the concrete experiences which DMP intends to be prerequisite. However, the teachers commented that students enjoyed the work with DMP much more than they enjoyed the work in the texts.

5.5 Project Funding

The development of DMP was funded by a grant from the U.S. Office of Education, Department of Health, Education, and Welfare, augmented with funds from the National Institute of Education and the National Science Foundation.

5.6 Project Staff

DMP was developed by the staff of the Analysis of Mathematical Instruction Project at the Wisconsin Research and Development Center for Cognitive Learning as part of the IGE program. Principal Investigators were Thomas A. Romberg and John G. Harvey, Project Coordinator is James M. Moser, and Implementation Coordinator is Mary E. Montgomery.