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ABSTRACT

The guide is arranged in vertical columns relating algebra curriculum concepts to curriculum performance objectives, career concepts and career performance objectives, suggested teaching methods, and resource materials. Career information on a variety of occupations includes comments on what a person in the occupation does, the level of education required, approximate salary range, approximate number of people in the field, and employment opportunities. Space is provided for teacher's additions, deletions, notes, and criticisms which will be useful when the guide is revised. Audiovisual source information also is provided. (NH)

careers

Career - Curriculum Guide

CAREER EDUCATION CENTER
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ALGEBRA 1 ALGEBRA 2

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HARLANDALE INDEPENDENT SCHOOL DISTRICT

SAN ANTONIO, TEXAS

ALGEBRA I

CURRICULUM GUIDE

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A C K N O W L E D G E M E N T S

Appreciation is expressed to the following teachers who contributed to the research and development of this curriculum guide.

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Preface

Meaningful existence is the goal of life in today's world. Living takes on meaning when it produces a sense of self-satisfaction. The primary task of education must be to provide each individual with skills necessary to reach his goal.

When children enter school, they bring with them natural inquisitiveness concerning the world around them. Normal curiosity can be the nucleus which links reality to formal training if it is properly developed. A sense of continuity must be established which places education in the correct perspective. Communities must become classrooms and teachers resource persons. Skills such as listening, problem solving, following directions, independent thinking and rational judgement then can merge into daily living procedures.

In classrooms especially designed to form a bridge between school and the world of work, experiences must be developed. On campus performance in job tasks and skills, following a planned sequence of onsite visitation, will fuse information into reality. Practical relationships developed with those outside the formal school setting will provide an invaluable carry-over of learned skills.

Search for a rewarding life vocation is never easy. Without preparation it becomes a game of chance. With a deliberate, sequential, and planned program of development, decisions can be made based upon informed and educated judgements.

A full range career education program, K-12, will offer opportunities for participants to enter employment immediately upon completion of training, post secondary vocational-technical education, and/or a four-year college career preparatory program.

C. N. Boggess

C. N. Boggess, Superintendent
Harlandale Independent School District

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P H I L O S O P H Y

The educational needs of any community are somewhat unique. This was certain to have been one of the guiding principles used when our forefathers set up local control for public schools. Accordingly, the philosophy of the Harlandale school system is to serve the educational needs of all its citizens as evidenced by adult classes, government sponsored retraining programs, vocational courses, etc. The mathematics department follows this philosophy in planning a program best suited to the needs of our students.

The past decade has proven the need for the new emphasis on the importance of the study of mathematics. Usefulness of mathematics in many fields of learning and endeavor, long thought to be free of mathematics, is now an accepted reality. Also, basic principles must be understood in order that mathematical systems can be devised to describe new human or mechanical activities, as they come to pass.

As mathematics continues to grow this must, of necessity, result in the addition of new symbols, terms, topics, and new approaches. The changing times will make some older topics and methods obsolete. To meet the mathematical needs of our students and to assist teachers in their instruction, the mathematics department has prepared this mathematics guide. Any given faculty consists of personnel with different degrees of training, experience, local tenor, and understanding of student needs. Hence, the desirability for some guiding criteria. In addition, we feel all students should consider career planning as a major facet of their education. Thus it follows that they will need some exposure to the different mathematical requirements of the varied career fields. In part, it is the purpose of this guide to assist the teacher in providing appropriate instruction to meet such needs.

The department feels that the most important basic guide for any mathematics course is the textbook, and careful care is taken in the selection of this book. Not only is the textbook an important guide for the teacher but it is also desirable for the student to learn the use of a textbook as a guide and important tool for learning.

Therefore, the plan of this mathematics guide is not to rewrite the textbook but to improve on it. Generally, the plan is to implement, where desirable, the textbook coverage, describe supplementary material that is needed and make suggestions on methods, procedures, order of coverage, etc.

Mathematics is a thoughtful, creative and intellectually stimulating subject. The enthusiasm and interest of the teacher in the subject is the best atmosphere for creating student enthusiasm for mathematics. This guide is planned to help foster this enthusiasm and in no way infringes on the academic freedom of the teacher.

It is hoped that the guide will prove helpful in understanding the basic standards, improving instruction, and developing the desired uniformity for the classes in each area of study. Finally, the guide should serve as the nucleus for a continuing effort to improve mathematics instruction.

Mr. Hamilton C. Dupont - Head of Math Department
Harlandale Independent School District

The audio-visual materials listed in this guide have been assigned catalogue numbers by the Harlandale Independent School District audio-visual department or the Education Service Center, Region 20, San Antonio, Texas.

ALGEBRA I

The following outline is built upon a "conceptual ladder" (concepts from 'easiest to hardest') for Algebra I. The outline corresponds to the outline found in the curriculum concepts of this guide. The page numbers refer to the present textbook being used in Algebra I by the Harlandale Independent School District. (Dolciani, Mary P., Modern School Mathematics, Algebra I. Boston: Houghton Mifflin, 1970.)

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IX. Radicals
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ALGEBRA I

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
I. Introduction to Algebra I A. Sets 1. Definition 2. Operation with Sets	I. THE STUDENT SHOULD BE ABLE TO: A. Correctly specify 80% of the sets on a written exercise by a roster, listing the names of the elements; or by a rule, describing the element; or (if the elements are real numbers) by a graph, locating the elements as points on a number line.	
B. Real Numbers 1. Binary Operations	B. 1. Give an oral definition which, in the judgement of the teacher, will show an understanding of a binary operation.	
2. Basic Properties	2. Correctly identify in a written exercise 80% of the problems (elementary algebraic proof) which use the basic properties such as reflexive, symmetric, transitive, commutative, associative and distributive properties.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
I. A. 1. Introduce sets by discussing sets in general: eg. set of golf clubs; set of dishes; set of tools. Gradually make the change to mathematical sets, making sure that a set or element of the set is well-defined. Give examples of sets and let the student name the elements. 2. These should also be introduced - (symbols, when possible, should be used) a. subsets e. infinite sets b. equal sets f. empty sets c. equivalent g. union of sets d. finite sets h. intersection of sets B. 1. Write the definition of a binary operation (as it applies to addition and multiplication) on the board. Use examples to illustrate. Stress that subtraction is the addition of an additive inverse, and division is the multiplication of a multiplicative inverse. Use examples to illustrate. 2. Each property should be written on the board and explained fully. Examples should be used to illustrate each property. Assignments should be made so that students can identify each step in solving the problem, and the property used. A list of all the properties should be kept by the student.	<u>A Way of Thinking about Numbers</u>; 16mm film -- 8015 <u>Sets, Crows and Infinity</u>; 16mm film -- 4806 <u>Algebra Visual Overhead Transparencies</u>; transparencies -- TP-63 thru TP-92 This set of tranparencies should prove helpful throughout the study of Algebra I.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
3. Equivalence Relations	3. Fill-in correctly 80% of the blanks in mathematical sentences, on a written exercise, the correct equivalence relation ($=$, $\neq$, $\leq$, $<$, $\nless$, $\geq$, $>$, $\ngtr$).	
4. Even and Odd Numbers	4. Make an oral statement which, in the judgement of the teacher, will identify a number as even or odd.	
5. Consecutive Integers	5. Find a set of four consecutive integers (also even or odd) when given vital information about two members of the set on a written exercise.	
II. Signed Numbers A. The Number Line 1. Coordinates	II. THE STUDENT SHOULD BE ABLE TO: A. 1. Numerically name on a written exercise nine out of ten coordinates of the points on a number line when letters of the alphabet are used to name the coordinates.	Concept Relationship of positive and negative numbers to the formation of compounds as studied by the chemist. Performance Objective Write correctly four out of five chemical formulas when presented with certain elements and their corresponding valences as would be used by the chemist. General information Students wishing to obtain extra activities should be encouraged to research careers in chemistry. They may wish to compile and analyze material on specific chemical careers.
2. Negative of a Number	2. State orally the negative of any number when presented with any positive number.	
3. Absolute Value	3. Correctly determine the absolute value of 90% of the real numbers on a written exercise.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
3. Equivalence relations should be explained, written on the board, and used in mathematical sentences.		
4. Define even and odd numbers. Write the set of even number and the set of odd numbers on the board. Stress that if a is even, then a+1 is odd; and that if b is odd, then b+1 is the next even number.		
5. It should be pointed out to the student that if a is an integer, a+1 is the next consecutive integer greater than a; a-1 is the next consecutive integer less than a. If a is even or odd, then a+2 is the next greater even or odd number.		
II. A. 1. With the use of a number line, demonstrate positive and negative numbers. Have students come to the board and graph coordinates assigned by the teacher.	Positive and Negative Numbers; 16mm film -- 8326 Absolute Value; filmstrip -- Z-46 Meaning of Signed Numbers and How to Add Them; filmstrip -- Z-43	
2. Show a number and its negative on the number line, pointing out that they are the same distance from the origin. Stress that the negative of a number, when added to the number itself, will give you 0. (additive inverse)	Subtraction of Signed Numbers; filmstrip -- Z-44 Multiplication of Signed Numbers; filmstrip -- Z-45	
3. Point out that the absolute value of a number tells you only the distance from the origin.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
4. Operations on the Number Line	4. Correctly add, subtract, multiply, and divide 80% of the problems on a written exercise concerning integers.	Examples: 1. Chemists (approximately 137,000 were employed in the United States in 1970 with salaries ranging from $9,400 to $24,000). 2. Chemical analyst 3. Chemical engineers 4. Chemical mixers 5. Chemical oceanographers 6. Chemical operators 7. Chemical technicians High school students wishing a career in chemistry should acquire all math and science courses available in high school. College training requires at least a bachelor's degree with a major in chemistry. Graduate work is essential for many positions, especially in research and college teaching.
B. Basic Properties of the Real Numbers	B.	<u>Teaching Activity</u> Practice writing chemical formulas when presented with certain elements and the corresponding valences (positive or negative).
1. Closure of the Operation	1. Determine correctly 80% of the problems on a written exercise concerning whether a given set is closed under a given operation.	Examples: 1. Sodium - Na^{+1} and chlorine - Cl^{-1} combine to form NaCl (table salt). Stress to the student that the combination of the two valences (+1, -1) in the compound must be zero.
2. Associativity Commutativity	2. Correctly write associative or commutative beside 80% of the problems on a written exercise in order to identify the property used in an elementary algebraic proof.	2. Hydrogen - H^{+1}; and Oxygen - O^{-2} combine to form H_2O (water). Stress to the student that two

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
4. Show how to add and subtract on the number line. Illustrate to the students that subtraction is the addition of the additive inverse. Send students to the board (those who you think are capable) to work problems. Make a chart which illustrates the results. Addition: (+) + (+)=(+); (-) + (-)=(-); (+) + (-)= Sign of the largest absolute value; (-) + (+)= Sign of the largest absolute value Subtraction: Same as Addition when you use the additive inverse of the second number. Multiplication and Division (+) x (+) or (+) ÷ (+) = (+) (-) x (-) or (-) ÷ (-) = (+) (+) x (-) or (+) ÷ (-) = (-) (-) x (+) or (-) ÷ (+) = (-) It is suggested that the career teaching activity be used at this time. B. 1. The axioms of closure should be introduced and explained. Problems should be worked on the board that show if a set is closed or not closed under the operation. 2. These properties have been introduced. The teacher should have each student refer to their list of properties. Other properties that should be covered are: a. the properties of 0, 1, and -1 b. additive properties of equality		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		hydrogen atoms have a "combined valence" 2 X (+1) = +2. (+2) plus the (-2) valence of the Oxygen, (+2) + (-2), will make 0.
3. Additive Inverse	3. State orally to the class the additive inverse of any real number.	
4. Multiplicative Inverse	4. State orally to the class the multiplicative inverse of any real number.	
5. Additive and Multiplicative Identities	5. Show in a written exercise one example which clarifies that when one multiplies the multiplicative inverse of a number by that number one receives an answer of one; or that when the additive inverse of a number is added to a number one receives an answer of zero.	
III. Simple Equations and Sentences	III. THE STUDENT SHOULD BE ABLE TO:	Concept Relationship of algebraic equations and bookkeeping. Performance Objective Explain orally, to the satisfaction of the teacher how the bookkeeper uses the properties of equations in finding the simple interest rate on a loan made to a company for which he is working. General Information Students wishing extra activities should be encouraged to research the 1.34 million career force found in the bookkeeping field (paying \$4,000 to \$6,000 a year). They may wish to compile and analyze materials on specific bookkeeping careers.
A. Variables 1. Definition 2. Use	A. Define a variable in such a manner that, in the judgement of the teacher, one will present a clear meaning of the x in 3x + 5 = 11.	
B. Equations 1. Solutions by Use of the Basic Properties	B. 1. Solve correctly 80% of the equations on a written exercise containing one variable by using either separately or in combination the addition, multiplication, and division properties of equations.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
Examples should be worked by the teacher using each of the properties and then have the students name the properties used in algebraic proofs.		
3. State the axiom of additive inverse. Work problems using it.		
4. Same as above.		
5. Have students work problems showing the additive and multiplicative identities. Some problems should be worked orally to achieve speed and confidence.		
III.		
A.		
1. Introduce an expression such as 3n first; n ϵ {real numbers}. Then let the student realize that n can have many values. Call n a variable and define a variable.	<u>Equations - Number Sentences</u>; 16mm film -- 4180 <u>Algebra - Intro. Equations</u>; 16mm film -- 305	
2. Use simple equations (like $3x + 5 = 11$) to show the value of the variable in the specific equations used.	<u>Proof in Algebra: Solving Equations</u>; filmstrip -- Z-33	
B.		
1. Let students go to the board and solve equations (the students learn from each other) or use the overhead projector to solve some problems. (Refer to the properties used.)	<u>Equations with Fractions</u>; filmstrip -- Z-34	
It is suggested that the career teaching activity be used at this time.	<u>The Opening Phase of Bookkeeping</u>; filmstrip -- BB-63	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
2. Translation of Verbal Sentences	2. Intuitively replace 80% of the English word problems on a written exercise with algebraic sentences that will correctly solve the "word problem."	Examples: 1. Bookkeeping clerks 2. Accounting clerks 3. Bookkeeping machine operator 4. Bookkeeping machine servicemen High school students wishing a career in bookkeeping should concentrate on business courses in their study with emphasis on math courses. College training is particularly helpful to those seeking advancement in bookkeeping. Teaching Activity Derive and use a simple interest formula as used by a bookkeeper. Example: Given the equation $RA(N+1)=2PC$ where R=Simple interest rate; P=Number of payment periods in one year; C=Finance charge in dollars; A=Amount borrowed; N=Number of payments in contract; solve the equation for R. Use the resulting equation $R = \frac{2PC}{A(N+1)}$ in working the following problem: The company for which you are doing bookkeeping has borrowed $2,600 for the purchase of a truck. The loan is to be repaid by 24 monthly payments. The interest rate for this loan is $6.00 per

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
2. Solution of "word problems" comes from experience. Understanding of the word phrase is extremely important.	<u>The Recording Phase of Bookkeeping</u>; filmstrip -- BB-64 <u>The Closing Phase of Bookkeeping</u>; filmstrip -- BB-65	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		hundred per year. How would you find the simple interest rate? Answer: $R = \frac{2 \times 12 \times 312.00}{(2{,}600)(24 + 1)} = \frac{748{,}800}{65{,}000} = 11.52\%$
IV. Order Relations A. Symbols	IV. THE STUDENT SHOULD BE ABLE TO: A. Read orally, to the satisfaction of the teacher (within two seconds) each of the following symbols: ($=$, $\neq$, $\leq$, $<$, $\nless$, $\geq$, $>$, $\ngtr$).	Concept Relationship of inequalities to stress forces as studied by the architect.
B. Inequalities 1. Axioms	B. 1. Write correctly from memory 80% of the axioms of inequality (addition, multiplication, and division).	Performance Objective Explain orally, to the satisfaction of the teacher, the use of inequalities in an architects study of shear stress on a beam. General Information
2. Solutions	2. Solve correctly 80% of the inequalities on a written exercise which contains such problems as $2x - 3(5 - x) \geq 0$.	Students wishing extra activities should be encouraged to research other careers in architecture. They may wish to compile and analyze material on specific architectural careers. Examples: 1. Architect, construction 2. Architect, landscaping (salaries range from \$7,000 to over \$25,000 a year) A solid background in math is needed for the high school student wishing to go into architecture. Requirements for an architect generally require graduation from an accredited professional school followed by 3 years of pratical experience in

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
IV. A. Have the students list all the symbols with their meanings. Let them use them in examples. B. 1. Using the overhead projector (or on the board) let the student copy a list of all the axioms of inequality. Point out the fact that they are very similar to the basic properties already learned. Use them in mathematical sentences. 2. Students can get practice by working in groups to help each other, or by going to the board. (Problems should be worked). It is suggested that the career teaching activity be used at this time.	Equations and Inequalities; filmstrip -- Z-30 Equations and Equivalent Equations; filmstrip -- Z-31 Solving Inequalities; filmstrip -- Z-37 Mathematics in Architecture Series; colored slides -- CS-1	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		an architect's office. A license (acquired by test) is required by all states. Teaching Activity (Teacher note: This career activity has been simplified for Algebra I presentation.) Calculate the maximum shear stress that an architect will consider acceptable in construction using a concrete beam. In consideration of the physical strength of his architecture an architect sets a limit of 3 on the shear stress of the beam is represented by l/h where l represents the load the beam will be supporting and h represents the height or depth of the beam. Therefore l/h must be less than 3. Determine if a concrete beam (15 feet long, 180 inches; cross section 10 inches by 15 inches) which is to support 450 pounds will meet the architects' specifications. Answer: $l/h < 3$ $450/180 < 3$ $2.5 < 3$ The beam will provide the support.
V. Polynomials A. Definition	V. THE STUDENT SHOULD BE ABLE TO: A. Write a definition which will, in the judgement of the teacher, shows an understanding of the relationship between a monomial and a polynomial.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
V. A. Define a monomial: Stress the fact that a monomial (using any variable, say x) is an expression in the form ax^2, where a is the numerical coefficient and n is the degree. Point out that the addition of two or more monomials make a polynomial. Refer to a polynomial with 2		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
B. Addition, Subtraction, Multiplication, or Division	B. Correctly add, subtract, multiply, and divide 80% of the problems on a written exercise which concern polynomials.	
VI. Special Products and Their Related Factors A. Law of Exponents	VI. THE STUDENT SHOULD BE ABLE TO: A. Correctly solve 80% of the problems on a written exercise which involve the Laws of Exponents for Multiplication; 1) $a^m \times a^n = a^{m+n}$, 2) $(a^m)^n = a^{mn}$, 3) $(ab)^m = a^m b^m$ and Division $(a^m \div a^n = a^{m-n})$.	<u>Concept</u> Relationship of the Law of Exponents (scientific notation) to the work of the astronomer. <u>Performance Objective</u> Explain orally, to the satisfaction of the teacher, how the astronomer uses scientific notation in his study of mass of the stars and planets. <u>General Information</u> Students wishing to go into astronomy should take all possible courses in math and science in high school. Although astronomy is a small field, the person with an advanced degree (especially Ph.D.) has no trouble finding employment. <u>Teaching Activity</u> Astronomers must often use exponents in expressing the large numbers with which they must deal. Astronomers have
B. Products of Binomials	B. Multiply correctly 80% of the problems on a written exercise which concern two binomials by standard multiplication procedure or by using a shortened form such as the "Foil Method".	
C. Monomial Factors	C. Derive correctly 80% of the greatest common monomial factors from a set of polynomials on a written exercise.	
D. Binomial Factors	D. Factor at the board, to the satisfaction of the teacher, (within thirty seconds) the difference of two squares or a trinomial square.	
E. Polynomial Factors	E. Combine the different forms of factoring in order to factor polynomials correctly in working 80% of the problems on a written exercise (grouping, monomial factoring, and trinomial factoring by inspection of all possible factors).	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
terms as a binomial and with 3 terms a trinomial. B. Review the 4 basic operations with real numbers. Work problems using the overhead projector. Use transparencies which show addition, subtraction, multiplication and division of polynomials. Have students work on the board or in groups for mutual assistance.		
VI. A. Prove the laws of exponents using real numbers. This gives the student a clear picture of what he is doing when he uses the laws of exponents and <u>why</u>! (Suggestion--Use career teaching activity) B. The foil method is an excellent way of remembering (First, Outer, Inner, Last). Working problems by the student is a must! C. Explain that factoring is division by a monomial. (can be checked by multiplication) D. Review squares. The student must have an understanding of square roots before attempting difference of squares. Use transparencies if possible. E. Let the students make a list of all the possible ways of factoring (from the easiest to the most complicated). Let them use the list in "eliminating methods" until they find one that applies to the problem.	<u>Big Numbers - Little Numbers</u>; 16mm film -- 4796 <u>Measuring in Astronomy - How Big, How Far</u>; 16mm film -- 8839	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		estimated that the mass of the sun is 1.98×10^{33} grams. How many times greater is the mass of the sun than the mass of the earth? $\frac{1.98 \times 10^{33}}{6 \times 10^{21}} = .33 \times 10^{12} =$ 3.3×10^{11} times greater
VII. Rational Expressions A. Definition	VII. THE STUDENT SHOULD BE ABLE TO: A. State orally a definition which, in the judgement of the teacher, shows an understanding of a rational expressions in terms of two polynomials.	Concept Relationship of proportion to calculating reductions and expansions of sizes of illustrations in the printing industry. Performance Objectives Explain orally, to the satisfaction of the teacher, how a printer proportionally determines the dimensions of a picture to be made smaller.
B. Simplifications	B. Simplify correctly 80% of the problems on a written exercise containing algebraic fractions such as $\frac{a^3 - 36ab^2}{a^4 - 4a^3b - 12a^2b^2}$	
C. Operations	C. Correctly add, subtract, multiply, and divide 80% of the proportions from a written problem set containing rational expressions.	General Information Students wishing extra activities should be encouraged to research other careers in printing. They may wish to compile and analyze material on specific and related printing careers.
D. Ratios 1. Proportions 2. Equations	D. Intuitively write and solve 80% of the proportions from a written problem set containing English word problems which contain information concerning two equal ratios.	
E. Complex	E. Simplify and rewrite correctly, 80% of the problems on a written exercise, complex rational expressions in the form of standard rational expressions.	Examples: Approximately 400,000 were employed in the printing industry in 1970. Some of the specific occupations involved were: 1. bookbinders 2. compositors 3. electrotypers 4. photoengravers

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
VII. A. Work problems on the board to illustrate rational numbers to the student. Stress (for simplicity), that rational expressions are ratios (fractions) involving polynomials. B. Review the laws of exponents and work problems on the board. Let students work problems themselves. C. This is just a combination of add., sub., mult., and div., of polynomials. These have been covered, but should be reviewed again and problems worked for the class. D. Understanding the written concept should be the first objective. Have students read aloud and explain in their own words what is written. It is suggested that the career teaching activity be used at this time. E. Work problems on the board or overhead projector for the class to observe. Stress division as the use of the multiplicative inverse (reciprocal). Apply what has been learned to solve problems. Students should work at the board, groups, or by themselves to gain experience (use transparencies or films if possible).	<u>Ratio and Proportion in Mathematics</u>; 16mm film -- 4496	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		5. pressmen 6. sterotypers 7. mailers 8. linotype operators High school students wishing to enter the printing industry should follow a basic curriculum such as basic mathematics (in many cases elementary algebra) with a thorough knowledge of spelling, punctuation, and the fundamentals of grammar. An apprenticeship program lasting from 4 to 6 years is necessary after graduation. Apprenticeship may be aided by taking print shop offered by many high schools. <u>Teaching Activity</u> The printer often has to use algebraic formulas involving proportions to increase or reduce the size of illustrations to fit on a certain amount of space. Example: A printer has a photograph which measures 16 inches by 28 inches. This illustration is to be reduced to a width of 5 inches. What will the reduced length of the phtograph be? The printer must have a workable knowledge of the proportion formula (A:B= C:D ; AD = BC). Substituting in this formula we obtain 16/28 = 5/L; 16L = 140; L = 8.75 inches. The new dimensions of the illustration will be 5 X 8.75 inches.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
VIII. Functions and Their Graphs A. Graph of a Function 1. Ordered Pairs 2. Coordinate Plane B. Definitions 1. Domain 2. Range C. Linear Function 1. Slope of a Line 2. Intercept	VIII. THE STUDENT SHOULD BE ABLE TO: A. 1. Identify orally, to the satisfaction of the teacher, an ordered pair of numbers (x, y) in terms of identifying the x as the first coordinate and y as the second coordinate. 2. Locate correctly 80% of the points on a coordinate plane when given the first and second coordinates on a written exercise. B. 1. Define, to the satisfaction of the teacher, the domain of a function in terms of the set of all the first components of the ordered pairs in the function. 2. Define, to the satisfaction of the teacher, the range of a function in terms of the set of all the second components of the ordered pairs in the function. C. 1. Find correctly 80% of the slopes of lines on a written problem set by placing vertical change over horizontal change $(m = \frac{y_2 - y_1}{x_2 - x_1})$ when presented the graph of a line on a coordinate system. 2. Correctly write 80% of the y-intercepts of lines when presented with linear equations in the form y = mx + b.	Concept Relationship of systems of equations and the quality expert at a dairy. Performance Objective Explain orally, to the satisfaction of the teacher, how a quality control expert in a dairy decides on a certain mixture to keep the butter fat level in milk at seven percent. General Information Students wishing extra activities should be encouraged to research other careers in agriculture as well as the dairy industry. They may wish to compile and analyze material on specific careers. The agriculture industry is undergoing a tremendous revolution. Jobs are tending to become more specialized with more training needed. Examples: 1. Soil scientists and conservatists (salaries range from $6,938 to $20,815 in 1971). 2. Agriculture economists 3. Agriculture engineers 4. Agronomists 5. Veterinarians 6. Entomologists 7. Farm managers The careers listed above require a solid background in math and science for the high school

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
VIII.		
A. 1. Introduce the coordinate plane. Identify points on the plane by using the correct ordered pair of numbers.	Algebra-Relations, Functions and Variation; 16mm film -- 4015 Introduction to Functions; filmstrip -- Z-41	
2. Let students locate points on the plane orally.		
B. 1. Define a function and stress that the domain is the X coordinate in the set of ordered pairs.		
2. Practice work should be given in representing functions. (Given the domain, find the range) and graphing these functions on the coordinate plane previously covered. Students should be sent to the board to graph problems. Stress that the range of the function is the y coordinate in the set of ordered pairs.		
C. 1. Define a linear function (use the form $ax + by = c$ to refer to linear equations in 2 variables. Illustrate the slope of a line. Make a chart to illustrate how the slope determines the direction of a line (using positive and negative slopes).	Graphing Linear Equations; 16mm film -- 4226 Our Soil; filmstrip -- PR-703, (AA-51)	
2. Explain the equation $y = mx + b$ (taken from $ax + by = c$). Graph equations and show that b always determines where the line cuts the y axis.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
3. Linear Graph	3. Correctly graph 80% of the linear equations on a written exercise by using ordered pairs which solve the equation or by using the slope and y-intercept of equations.	student. Most of the above careers require a bachelor of science degree with some requiring more schooling (veterinarian). Teaching Activity The exact butter fat in a dairy's milk is highly important to profit and the quality of milk. The cream which comes from a dairy's separator contains 30% butter fat. The dairy's quality control expert must determine how much of the cream is to be mixed with milk containing 4% butter fat to obtain 100 gallons of milk containing 7 percent butter fat. Answer: x = the number of gallons to be taken of 30% cream. y = the number of gallons to be taken of 4% milk. $x + y = 100$ gallons 30% of $x = .3x$ = number of gallons of fat in the 30% cream used. 4% of $y = .04y$ = number of gallons of fat in the 4% milk used. $.3x + .04y$ = total number of fat in the mixture. 7% of $100 = 7$ = total number of gallons of fat in the mixture. Therefore: $.3x + .04y = 7$ $x + y = 100$ or
D. Systems of Linear Equations	D.	
1. Definition	1. Define, to the satisfaction of the teacher, a system of linear equations as a set of linear equations which appear on a coordinate plane.	
2. Solution by Graphs	2. Solve correctly 80% of the problems on a written exercise concerning a system of linear equations by finding their intersection on a coordinate plane (the intersection should be identified as an ordered pair).	
3. Solution by Addition or Subtraction	3. Solve correctly 80% of the problems on a written exercise concerning a system of linear equations by adding or subtracting two equations in order to eliminate one variable (the resulting equation will be solved by the basic properties).	
4. Solution by Substitution	4. Solve correctly 80% of the problems on a written exercise concerning a system of linear equations by the substitution method.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
3. Have the students graph linear equations given the slope and the y-intercept. Also graph problems by finding at least 3 ordered pairs that solve the equation. Compare the two methods of graphing using one equation for both. (Send students to the board to graph sample equations.) (This helps others, since they will try to correct any mistakes.)	<u>Common Solution to Two Linear Equations</u>; filmstrip -- Z-38	
D.	<u>Solving Two Linear Equations Algebraically</u>; filmstrip -- Z-39	
1. Stress that a coordinate plane contains an infinite number of linear equations.		
2. Graph examples of linear systems. Find their intersecting point (an ordered pair). Let the students work some examples.		
3. Review addition and subtraction of polynomials. Review the basic properties to solve the resulting equation. Illustrate the method by working some examples. Stress again that the values of <u>X</u> and <u>Y</u> make an ordered pair which are roots of both equations.		
4. Review the substitution method. (Use the basic properties). Work problems using this method. Stress the fact that the values of x and y make an ordered pair which are roots of both equations.		
It is suggested that the career teaching activity be used at this time.		

CURRICULUM CONCEPTS	"CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		$\begin{array}{r} .3x + .04y = 7 \\ \underline{-.04(\;x + \;\;y = 100)} \\ .26x \qquad = 3 \end{array}$ $x = 11.54$ gallons of 30% cream $\begin{array}{r} .3x + .04y = 7 \\ \underline{.3x + .3\,y = 30} \\ .26y = 23 \end{array}$ $y = 88.46$ gallons of 4% milk
IX. Radicals A. Definition	IX. THE STUDENT SHOULD BE ABLE TO: A. Define a radical, to the satisfaction of the teacher, in terms of $\sqrt{a^2} = a$ where $a \in$ the set of real numbers.	<u>Concept</u> Relationship of square roots and automobile accidents as investigated by a police officer. <u>Performance Objective</u> Explain orally, to the satisfaction of the teacher the use of square roots in a police officers job. <u>General Information</u> Students wishing extra activities should be encouraged to research other careers in police work. They may wish to compile and analyze material on specific police careers. Examples: 1. Guards and watchmen (approximately 200,000 were employed in 1970 with salaries ranging from $3,848 to $15,000 yearly) 2. FBI agents (approximately 7,900 were employed in 1970 with salaries ranging from $10,869 to $23,000 yearly)
B. Simplification	B. Simplify correctly 80% of the problems on a written exercise containing radicals to the point that 1. the radicand has no factor that is a perfect square, 2. the radicand does not contain a fraction, and 3. no radical appears in a denominator.	
C. Squares and Square Root Approximation 1. Tables	1. Orally find, to the satisfaction of the teacher, the squares and square roots from a standard square root table containing natural numbers from one to one-hundred.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
IX. A. Define square root. Show that each positive number has 2 distinct square roots. Stress that the radical sign ($\sqrt{\ }$) denotes the positive square root of a number, and that $(\sqrt{a})^2 = a$. Show by examples that $\sqrt{a^2} = a$ for any non-negative a, and that $\sqrt{a^2} = -a$ for any negative a. Let the class discuss this problem: If a is neither known to be positive nor negative, then what is $\sqrt{a^2}$ equal to? Illustrate that since a^2 has to be positive for any number a, and that $(\|a\|)^2$ is also non-negative for any number a, then $\sqrt{a^2} = \|a\|$.	*Finding Square Roots*; filmstrip -- AA-74 *Policeman*; cassette tape -- Cas. T-29	
B. Work problems for the class to the point that 1. the radicand has no factor that is a perfect square, 2. the radicand does not contain a fraction, 3. no radical appears in the denominator.		
C. (Suggestion--Use career teaching activity) 1. Assign the students this task. Let them make a chart of the numbers 1-100, showing their squares and square roots. Tell them to learn it! Have an oral contest (boy vs. girls, or any other grouping, etc.), to practice finding squares and square roots.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
2. Approximation of Square Roots	2. Complete 80% of the problems on a written exercise which contain square root approximation to at least two decimal places by using Euclid's method or Newton's iteration method.	3. Police officers (approximately 330,000 full-time were employed in 1970 with salaries ranging from \$8,500 for a new officer to \$23,000 for such positions as chiefs or commissioners. Because of specialization in police work it should be emphasized that a high school student wishing to go into police work should take college prepartory courses. FBI agents are required to have a law degree. Teaching Activity Determine the speed of an automobile on dry asphalt pavement from the skid marks at the scene of an accident. Example: At the scene of an automobile accident one car leaves skid marks on the asphalt pavement which measure 65 feet. The formula S(speed) = $5.5\sqrt{d(\text{length of skid}) \times f(\text{resistance})}$ may be used in this problem. A police officer can find from his police manual that dry pavement has a resistance of 0.75. Substituting we find S = $\sqrt{5.5\ 65 \times .75}$ = $5.5\sqrt{49}$ = 5.5 X 7 = 38.5 miles per hour.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
2. Show that $\frac{(x-h)^2}{a} + \frac{(y-k)^2}{b} = 1$ is equivalent to $\frac{x^2}{a} + \frac{y^2}{b} = 1$, when $h = 0$ and $k = 0$. Stress that (h,k) denotes the center and that it is symmetric to both axis. Illustrate how to find the x,y-intercepts. Make an assignment.		
D. 1. Illustrate a graph at the board. Point out and discuss the branches, center, and asymtotes of a hyperbola. Use the overhead and different graphs in order to let the students identify orally the above mentioned terms.		
2. Show that $\frac{(x-h)^2}{a} - \frac{(y-k)^2}{b} = 1$ is equivalent to $\frac{x^2}{a} - \frac{y^2}{b} = 1$ when $h = 0$ and $k = 0$. Illustrate by working a few examples how to determine the intercepts and the equations for the asymptotes. Assign some problems.		
E. 1. Briefly review how to graph a linear equation using the slope-intercept form. State that both equations must be graphed on the same set of axes and the points of intersection, if any, will determine the solution set. Assign some problems. 2. Illustrate at the board how to solve correctly a system of quadratic and linear equations by substitution. Send the students to the board for drills, while the teacher observes and corrects mistakes.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
VII. Exponential and Logarithmic Functions A. Exponential Function 1. Properties of Exponents	VII. THE STUDENT SHOULD BE ABLE TO: A. 1. Correctly solve 85% of the problems on a written exercise which involve the Laws of Exponents for Multiplication; 1) $a^m X a^n = a^{m+n}$, 2) $(a^m)^n = a^{mn}$, 3) $(ab)^m = a^m b^m$ and Division $(a^m \div a^n = a^{m-n})$.	Concept Relationship of logarithmic computation to plant growth as studied by the botanist. Performance Objective Calculate, to the satisfaction of the teacher, the work done by a botanist in order to determine the "growth curve" of a population of bateria. General Information Students wishing extra activities should be encouraged to research other careers in life science. They may wish to compile and analyze material on work done by the life scientist. Examples: Botanists Zoologists Microbiologists Agronomists Anatomists Biochemists Biological Oceanographer Biophysicists Ecologists Embryologists Entomologists Geneticists Horticulturists Husbandry Nutritionists Pathologists Pharmacologists Physiologists
2. Exponential Function $(y = a^x)$	2. Graph 80% of the functions on a written exercise of the form $y = a^x$ where $a<1$, $a = 1$, $a>1$.	
3. e	3. Define orally, to the satisfaction of the teacher, the value of e in the function $y = e^x$ as 2.71828.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
VII. A. 1. Place the laws of exponents for multiplication and division on the board and demonstrate each with examples using positive and negative numbers and/or variables. Recite an oral exercise before making an assignment. 2. Define an exponential function as a function in the form $\{(x,y): f(x) = a^x, x,y \in R\}$ on a coordinate plane. Graph at least three examples of exponential functions using a coordinate chart to find the values for y. It should be pointed out that the graph of $y = 0$ would result in a constant function where its value is always +1. Also mention the fact that when $x = 0$ the y-intercept is +1. The student should be familiar in estimating small radicals and be able to plot ordered pairs consisting of decimals and fractions. 3. In the function $y = e^x$, the value of e as 2.71828 should be stated and verified by the teacher with the means of a diagram.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
B. Logarithmic Function 1. Definition	B. 1. Define correctly on a written exam the logarithmic function y = log x as the inverse of the exponential function $y = a^x$.	Approximately 180,000 persons were employed in the life sciences in 1970. Salaries ranged from $6,548 to more than $26,100. High school students wishing a career in life science should acquire all available science courses with strong emphasis on math. Students wishing a career in life science should obtain an advanced degree (possibly a PhD.) in their particular field of interest. A bachelor's degree may be adequate for some positions, but opportunities for promotion are few without graduate training.
2. Graphing	2. Correctly graph 80% of the logarithmic functions on a written exercise.	<u>Teaching Activity</u> A botanist has established that in the growth of a certain population of bateria 40 members were found at 1pm. and 5C members at 2pm. His job is to determine the number at 3pm. The "growth curve" of such a colony of bateria is expressed by the mathematical law: $\ln N/N_o = kt$, where N_o is the number of bateria at 1pm., N is the number at 2pm., and k is the "growth constant" of the bateria being studied. His job is to determine the number at 3pm.
3. Properties	3. Correctly solve 80% of the problems on a written exercise illustrating the following properties of exponents: 1) log ab = log a + log b, a>0, b<0; 2) log a/b = log a - log b, a>b, b>0; and 3) If a>0 and n ε R, then $\log a^n$ = nlog a.	Let: N_o = 40 members at 1pm. N = 50 members at 2pm. t = 1 hour

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
B. 1. Review the inverse function. The student should be able to visualize from a graph that the logarithmic function with base "b" specified by $\{(x,y): y - \log_b x, b>0, b\neq 1\}$ was obtained by interchanging the components of the ordered pairs of an exponential f. It should be demonstrated what happens when $b>0$ and $b = 1$. The teacher writes an approved definition for a logarithmic function from a student. 2. Illustrate how to graph a logarithmic function with a few examples on the board. Stress that the domain can only be the set of all positive numbers. Define logarithm and antilogarithm. Graphs are suggested to help in defining logarithm. For easier reading expand the x-scale. 3. Go over the previous laws of exponents since they are the basis for the laws of logarithms. The following examples are helpful: (1) $\log 30 = \log 6.5 = \log 6 + \log 5$ or $\log 2 + \log 15$ (2) $\log 4 = \log 120/30 = \log 120 - \log 30$ or $\log 16 - \log 4$ (3) $\log 16 = \log 2^4 = 4 \log 2$ or $2 \log 4$ Try small numbers at first. It is suggested that the career teaching activity be used at this time.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
C. Computations with Base 10 Logarithms	C.	since $\ln N/N_0 = kt$ $\ln 50/40 = k(1)$ $\ln 1.25 = k$ $k = .3552$ Then $\ln N/20 = (.3552)(2)$ at 3pm. $\ln N - \ln 20 = .7104$ $\ln N = 2.9957 + .7104$ $\ln N = 3.7061$ $N = 41$ at 3pm. (nearest unit)
1. Characteristic and Mantissa	1. Define correctly on a written exercise, to the satisfaction of the teacher, the characteristic and mantissa as they relate to exponents.	
2. Logarithm Tables	2. Determine correctly 80% of the logarithms of a list of ten numbers by using tables and 80% of the antilogarithms of a list of ten logarithms by using tables.	
3. Interpolation	3. Interpolate to find correctly to two decimal places 80% of the problems on a written exercise to find logarithms and antilogarithms.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
C. 1. Before defining characteristic and mantissa, scientific notation should be fully covered. It should be brought to the attention of the student that a common base of 10 will be used. Stress that a logarithm consists of two parts (characteristic and mantissa). Define each term orally and mathematically. Cover negative characteristics. 2. Introduce the common logarithm table. Show the student where to find the number and mantissa. Characteristics are found by inspection. Illustrate how to find the logarithm of a number between 1 and 10 first. Then use numbers greater than ten (where the student needs to express it in scientific notation. Recite an oral exercise where each student has the opportunity to participate in finding logarithms. Follow the same procedures in finding antilogarithms. 3. Use any method (preferably assign a problem with four or more significant digits and another one where the mantissa does not appear in the table) to show the student that an entry cannot be made in the table. Work a few examples using the process of linear interpolation to find logarithms and antilogarithms of numbers with more than three significant digits.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
4. Computational procedures	4. Multiply, divide, and raise to a power correctly 80% of the problems on a written exercise by using logarithms.	
VIII. Matrices A. Definition	VIII. THE STUDENT SHOULD BE ABLE TO: A. Define on a written exercise, to the satisfaction of the teacher, the terms determinant and matrix.	Concept Relationship of matrices to the study of velocity and acceleration by a physicist. Performance Objective Calculate the original velocity and acceleration of a uniformly acceleration particle by using "Cramer's Rule" on a written problem in which the needed matrix is given. General Information Students wishing extra activities should be encouraged to research careers in physics. They may wish to compile and analyze material on work done in the field of physics. Examples: Approximately 48,000 physicists were employed in the United States in 1970. Most physicists specialize in one or more branches of science. Starting salaries averaged about $9,000 in 1970. About 10% of the physicists in 1970 earned $25,000 or more.
B. Properties	B. Calculate correctly 80% of the determinants on a written exercise by using six properties that will simplify their expansion (properties placed on the board by the teacher).	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
4. Review the product, quotient, and power properties of logarithms. Emphasize that such properties should be used to shorten work. Suggest to the student to express the problem in logarithmic form before computing. Leave examples on the board for reference.		
VIII. A. Clearly define matrix (matrices) orally and write it on the board giving two examples such as: (Define a determinant as a real number which is associated with a square matrix to constitute a function.) B. Place on the board the six properties that will simplify the expansion of any determinant. While the properties are on the board, the teacher can verify all six properties and demonstrate such properties with good example problems. The properties are as follows:		
"1) If any two rows or any two columns of a determinant are interchanged, the resulting determinant is the negative of the original determinant. 2) If two rows or two columns of a determinant are identical, the determinant is 0. 3) If all the rows and columns of a determinant are interchanged in order, the resulting determinant equals the original one. 4) If the elements of one row or one column are multiplied by the real number k, the resulting determinant is k times the original one.	Dolciani, Mary P., <u>Modern School Mathematics</u>, <u>Algebra II</u>. Boston: Houghton Mifflin, 1963, pp. 563-564.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
C. Addition and Multiplication	C. Correctly state with 80% accuracy on a written exercise whether two matrices can be added or multiplied, and if so perform the operations.	High school students wishing to become physicists should obtain all advanced math and science courses available in high school. A bachelor's degree in physics is usually required as a minimum requirement for entrance into a physics career. Graduate training is required for many entry positions and is helpful for advancement.
D. Determinant of a Matrix 1. Diagonal Method 2. Minoring	D. 1. Given a 2 X 2 or 3 X 3 matrix, correctly compute 80% of the determinants on a written exercise by using the "diagonal method". 2. Given a 2 X 2 or 3 X 3 matrix, correctly compute 80% of the determinants on a written exercise by minoring.	<u>Teaching Activity</u> A physicist is studying the final velocity of a particle. He knows that the velocity is related by the equation $V = V_o + at$, where a and V_o are constants. He also knows that $V = 20$ when $t = 5$, and $V = 35$ when $t = 10$. He needs to determine the original acceleration a of the given particle. He first determines the needed matrix. Since $V = V_o + at$ $20 = V_o + a5$ $35 = V_o + a10$ By Cramer's Rule
E. Solution of a System Using Determinants	E. Solve correctly 80% of the systems of matrices on a written exercise by using "Cramer's Rule".	$V_o = \dfrac{\begin{matrix}20 & 5\\ 35 & 10\end{matrix}}{\begin{matrix}1 & 5\\ 1 & 10\end{matrix}} = \dfrac{200 - 175}{10 - 5} = \dfrac{25}{5} = 5$ $a = \dfrac{\begin{matrix}1 & 20\\ 1 & 35\end{matrix}}{\begin{matrix}1 & 5\\ 1 & 10\end{matrix}} = \dfrac{35-20}{10-5} = \dfrac{15}{5} = 3$ Therefore the original velocity was 5 and the original acceleration was 3.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
5) If one row or one column has 0 for every element, the determinant is 0. 6) If each element of one row or one column is multiplied by a real number k and if the resulting products are then added to the corresponding elements of another row or another column, respectively, the resulting determinant equals the original one. C. Explain the meaning of dimension of a matrix and the square matrix. The student should be able to identify correctly the dimension of any matrix. Stress and illustrate some matrices that cannot be added or multiplied such as $\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 \\ 6 \end{bmatrix}$; $\begin{bmatrix} 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 5 & -1 \\ 2 & 3 & 4 \end{bmatrix}$. Work a few examples where the matrices can be added and multiplied. D. 1. Discuss and demonstrate the "diagonal method" while working a set of examples on the board up to the third order. 2. Define minor and illustrate the "minoring method" in expanding and simplifying matrices by working some problems on the board up to the third order. E. A short briefing on "Cramer" is recommended. Define coefficient matrix. Solve a system of linear equations in two variables (two equations) and another system in three variables (three equations). Explain each problem throughly and leave the example problems on the board for reference. It is suggested that the career teaching activity be used at this time.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
IX. Permutations and Combinations A. The Pascal Triangle	IX. THE STUDENT SHOULD BE ABLE TO: A. Compute the coefficents of the expansion of $(a + b)^5$ by using Pascal's Triangle.	Concept Relationship of permutations to the batting order of a baseball team as studied by a sports statisticians. Performance Objective Speculate by mathematical calculations the maximum number of possible batting orders on a baseball team having twenty players.
B. Permutations 1. Definition	B. 1. Define, to the satisfaction of the teacher, a permutation as an arrangement in a certain order of a set which can be counted.	General Information Students wishing extra activities should be encouraged to research other careers in statistics. They may wish to compile and analyze material on work done by the statistician.
2. Factorial	2. Calculate correctly 80% of the problems on a written exercise the number of permutations of any set with K members by finding K factorial.	Examples: Statisticians often work in the following subject areas: 1) Basic research in statistics 2) Biology
C. Combinations 1. Definition	C. 1. Define a combination, to the satisfaction of the teacher, as a certain set of n objects taken r at a time is a selection or r members from the set with no regard to ordering the chosen members.	3) Business 4) Demography 5) Economics 6) Education 7) Engineering 8) Health 9) Insurance
2. Symbol	2. State orally, to the satisfaction of the teacher, the meaning of $\binom{n}{r}$ as n elements taken r at a time.	10) Marketing and consumer research 11) Medicine 12) Operations and administration 13) Psychology and psychometry 14) Social sciences 15) Space sciences 16) (General) sciences

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
IX.		
A. Present a short lecture on the use and discovery of Pascal's Triangle. Tabulate the first five rows of the triangle. The student should identify the pattern being used. Let the student finish ten rows of the triangle. Work a few problems to illustrate how to apply the triangle. Assign at least three more problems.	Random Events; 16mm film -- 2263 Probability; 16mm film -- 8330 Population Statistics; filmstrip -- PR-734, (BB-70)	
B. 1. Define clearly the term permutation and write it on the board as any arrangement of the elements of a set in a definite order. 2. Demonstrate the meaning of factorial and emphasize that the number of permutations of a set containing "K" different elements is K!. Define circular permutations orally and mathematically as (k-1)!. Illustrate both types of permutations with examples.		
C. 1. Define a combination and write it on the board for the students. Illustrate diagram examples. 2. The teacher should state orally the meaning of $\binom{n}{r}$, C(n,r), and C_r as "n" elements taken "r" at a time. It is suggested that the career teaching activity be used at this time.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		Approximately 24,000 statisticians were employed in 1970 (more than one-third were women). Opportunities for employment tend to be favorable through the 1970's. Salaries in 1970 ranged from \$6,548 to \$14,192 (averages for private industry and government work). A bachelor's degree with a major in mathematics or statistics is an essential requirement for a position. Higher degrees are necessary for advancement. Teaching Activity A sports statistician often is involved in preparing new strategy for a baseball team. Casey Stengel (managed New York Yankees) often used different arrangements on his team formulated by a knowledge of mathematical permutations. In one problem a sports statistician is faced with determining the maximum number of possible line-ups on a team of twenty members. Answer: 20! = 2,432,902,008,-176,640,000 Approximately 2½ quintillion batting orders.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION. AND TEACHING ACTIVITIES
X. Progressions and Binomial Expansion A. Progressions 1. Definition 2. Formulas a. n^{th} Term b. Sum	X. THE STUDENT SHOULD BE ABLE TO: A. 1. Define correctly on an oral exercise, to the satisfaction of the teacher, an arithmetic progression as any sequence in which each term after the first is obtained by adding a fixed number to the preceding term. 2. a. Calculate correctly 80% of the n^{th} terms on a written exercise by using the formula $a_n = a_1 + (n - 1)d$. b. Calculate correctly 80% of the sums of a finite progression by using $S_n = \frac{n}{2} \; 2a_1 + (n-1)d$.	<u>Concept</u> Relationship of binomial expansion and compound interest to the banking business. <u>Performance Objective</u> Calculate the interest on an amount of money which compounds interest semiannually. <u>General Information</u> Students wishing extra activities should be encouraged to research other careers in banking. They may wish to compile and analyze material on work done by bank employees. Examples: 1) Clerks (approximately 500,000 employed in 1970; 9 out of 10 were women; salaries ranged in 1969 from \$70 to \$130 a week) 2) Tellers (approximately 150,000 employed in 1970; 9 out of 10 were women; salaries ranged from \$580 to about \$1,500 a month.
B. Infinite Geometric Progressions	B. State orally, to the satisfaction of the teacher, whether a geometric progression is divergent or convergent and state the limit.	High school graduation is usually adequate preparation for most clerical jobs in banks. For most jobs, courses in bookeeping, typing, business arithmetic, and office machine operation are desirable. Banks usually hire tellers who are high school graduates experienced in clerical work. College graduation is usually a requirement for management
C. Binomial Expansion	C. Expand correctly on a written exercise 80% of the problems involving (a+b) to the n^{th} power where n has values of 1, 2, 3, 4, and 5.	

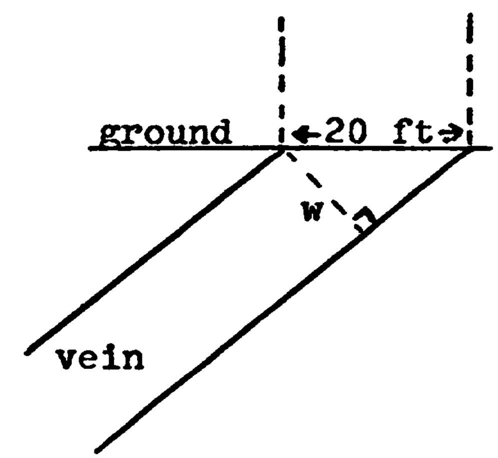
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
X. A. 1. Define arithmetic progression and write its definition on the board as any sequence in which each term after the first is obtained by adding a fixed number to the preceding term. 2. Before introducing any formulas, pass out a short quiz consisting of a few problems which can be solved without a formula and others in which a formula is needed for a much quicker result. Derive the formula a = a+ (n-1)d. Illustrate the formula with examples on the board. Introduce the summation sign Σ and its meaning. Show how important it is to use a formula in order to arrive at a solution. Derive the formula $S_n = \frac{n}{2}(a_1 + a_n)$ and then $S_n = \frac{n}{2}\{2a_1+(n-1)d\}$ Using the formula, work some examples on the board. B. Stress that an infinite sequence which has a limit is said to be convergent. An infinite sequence is said to be divergent (if it has no limit). Give examples (orally) of both types and a reasonable guess of the limit if the infinite sequence converges. C. Demonstrate the pattern displayed by the expansion of binomials. Two sample problems should be worked out. Assign some problems to the class.	Banker; cassette tape -- Cas. T-40	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		trainees. A business administration major in finance including accounting, economics, commercial law, political science, and statistics provides an excellent preparation to become a bank officer. Teaching Activity A bank employee must determine the amount of interest that will be paid a depositor on \$100 for two years at 6% interest semiannually. Answer: Total amount of money deposited at the end of 2 years will equal $P(1+r)^4$ where P = principal, r = interest for each period (3% in this instance), and 4 = number of periods paid. This compound interest formula (which is a binomial expansion) has been furnished by the mathematician. The bank employee usually uses a calculating machine to make the calculations. In other words he determines $\$100(1 + .06)^4$ which would equal \$126. Therefore the bank would pay the depositor \$26 for the use of his money for two years.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XI. Trigonometry A. Trigonometric Functions 1. Positive or Negative	XI. THE STUDENT SHOULD BE ABLE TO: A. 1. State orally, to the satisfaction of the teacher, that a positive angle indicates a counterclockwise rotation and that a negative angle indicates a clockwise rotation.	Concept Relationship of trigonometry to the career of a mining surveyor. Performance Objective Predict by trigonometric calculations the actual width of a vein of ore as studied by the mining surveyor. General Information Students wishing extra activities should be encouraged to research other careers in the mining industry. They may wish to compile and analyze material on work done by the mining industry. Employment in the mining industry is expected to make a slow decline in the 1970's even though increased technology will produce more output. However, job opportunities are great for a person with proper training. Examples: 1)Mineralogists 6)Crushers 2)Stratigraphers 7)Mechanics 3)Surveyors 8)Equipment operators 4)Prospecting computers 9)Managerial workers 5)Drillers Approximately 620,000 wage and salary workers were employed by the mining industry in 1970. High school students wishing
2. Values for Special Angles	2. Write correctly from memory 85% of the values of the sine, cosine, and tangent of 30, 60, and 90 degree angles.	
3. Approximate Values	3. Write correctly 85% of the approximate values of problems on a written exercise concerning the sine, cosine, and tangent of angles in the first quadrant using tables.	
4. Graphs	4. Graph correctly, to the satisfaction of the teacher, the sine, cosine, and tangent functions.	

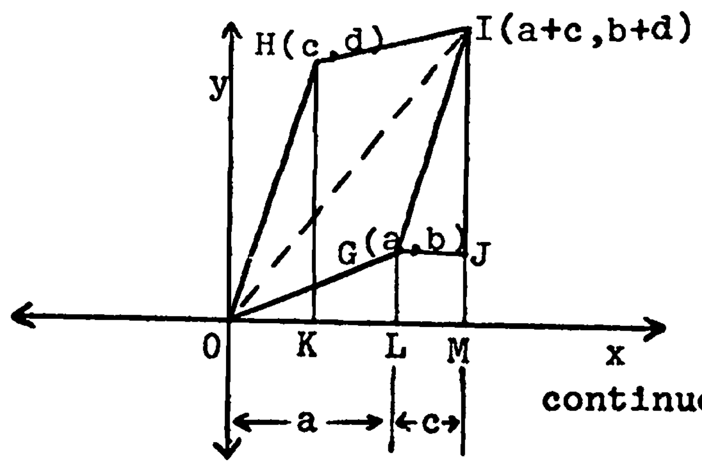
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XI. A. 1. Mention the fact that a positive angle indicates a counterclockwise rotation and that a negative angle indicates a clockwise rotation. Draw some angles on the board with an indicated rotation and let the student state whether the angle is positive or negative. 2. Review the 30, 60, 90 theorem and the relations of sine, cosine, and tangent. Reconsider the unit circle. The student should make a list of the values of the sine, cosine, and tangents of 30, 60, and 90 degree angles. The student should memorize the list of values. Daily short tests (oral tests) are suggested. 3. Present the Cartesian coordinate plane and illustrate how to use the table of values of trigonometric functions for angles in degrees by working some examples orally. Make an assignment including a few problems in which interpolation is required. 4. The teacher should graph and explain the sine function. The graph for the cosine and tangent functions should be left to the students. It is suggested that the career teaching activity be used at this time.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
B. Trigonometric Equations 1. Graphic Solution 2. Algebraic Solution	B. 1. Correctly graph 80% of the trigonometric equations on a written exercise. 2. Correctly solve 80% of the trigonometric equations on a written exercise by using algebraic methods.	a career in the mining industry should seek guidance from a resource person employed by the mining industry. The educational standards are as varied as the many jobs offered. Teaching Activity In the discovery of a vein of ore a mining surveyor observes after careful study that the vein makes an angle of 30% with horizontal ground. He also notices that the width of the vein exposed at the surface is 20 feet. His job is to predict the actual width of the vein underground. $\sin 30^0 = \frac{w}{20'}$ $.5 = \frac{w}{20'}$ $w = 10'$ (width underground) 

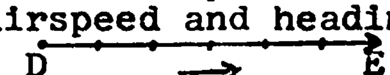
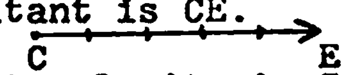
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
B. 1. Illustrate the graphs of trigonometric equations by working examples which demonstrate the different steps. Make an assignment. 2. Demonstrate that the properties and axioms over R should be applied in order to solve trigonometric equations by the algebraic method.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XII. Vectors A. System of Vectors 1. Graphical Representation	XII. THE STUDENT SHOULD BE ABLE TO: A. 1. Correctly graph 80% of the vectors on a written exercise by representing ordered pairs on a plane in such a manner that if one starts at $p(x_1,y_1)$ and moves to $q(x_2,y_2)$ then the motion can be expressed by the vector (x_2-x_1,y_2-y_1).	Concept Relationship of vector subtraction to a pilot's career. Performance Objective Calculate the resultant vector when given the direction and speed of a jet liner flying against a wind as determined by a pilot. General Information Students wishing extra activities should be encouraged to research other careers in the airline industry. They may wish to compile and analyze material on work done by the airline industry.
2. Equality of Vectors	2. State orally, to the satisfaction of the teacher, that the vector (a,b) is equal to the vector (c,d) if and only if a = c, and b = d.	Examples: 1) Pilots and copilots (approximately 27,000 employed in 1970 by scheduled airlines; 1,600 were employed in 1970 by supplemental airlines; and 2,500 were employed in 1970 by the Federal Government.
B. Operations with Vectors 1. Addition	B. 1. Correctly add 80% of the problems on a written exercise concerning vectors such as (a,b) plus (c,d) equals (a + c, b + d).	Salaries averaged about \$30,000 a year on domestic scheduled airlines and \$37,000 a year on international operations. 2) Flight engineers 3) Stewardesses - Specified qualifications (19 to 27 years old, 5 feet 2 inches to 5 feet 9 inches tall, weight not to exceed 140 pounds, and must be in excellent health). There are several thousand job

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
XII. A. 1. Define a <u>vector</u> as any directed <u>line segment</u> in the plane or space. It should be demonstrated that a vector begins at one end-point called the <u>initial point</u> and ends at a second end-point called the <u>terminal point</u>. Define the <u>norm</u> as <u>the length of the vector</u>. Illustrate the above underlined words with a graph diagram. Use about four sets of ordered pairs to graph their vectors.	<u>Airport Workers</u>; <u>filmstrip</u> -- BB-61	
2. Stress that two vectors are equivalent only if they have the same norm and direction. Use graphs to demonstrate equal and unequal vectors (having a different norm and length, same norm, but different direction, and a different norm with the same direction).		
B. 1. State the definition (a,b) + (c,d)= (a+c, b+d) and verify with the following interpretation:		



continued

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		openings in this field each year. Salaries ranged in 1970 from \$523 to \$800 per month. 4) Mechanics 5) Airline dispathers 6) Air traffic controllers 7) Ground radio operators 8) Traffic agents and clerks High school students wishing a career with the airlines industry must prove proficiency
2. Subtraction	2. Correctly subtract 80% of the problems on a written exercise concerning vectors such as (a,b) minus (c,d) equals (a-c, b-d).	in their chosen field. A pilot must have a solid background in math with a degree (usually in math) from college. Traffic controllers and flight engineers also require a broad
3. Multiplication	3. Correctly multiply 80% of the problems on a written exercise concerning the product of a scalar (a) and a vector (x,y).	background in mathematics. Stewardesses, traffic agents, and clerks must master the proper use of language (foreign helpful). Teaching Activity A pilot whose air speed is 600 mph. flies in easterly winds
4. Basic Vectors	4. Define orally, to the satisfaction of the teacher, the basic vectors (i) as the vector (1, 0) and (j) as the vector (0, 1).	blowing at 100 mph. The pilot must determine the resultant velocity of his plane if its heading is 90^0. Answer: 1) At C draw $\overrightarrow{CD}$ to represent wind velocity.  2) At D draw $\overrightarrow{DE}$ to represent the plane's airspeed and heading.  3) The resultant is $\overrightarrow{CE}$.  The resultant velocity is 500 mph. due east.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
(1) Find the fourth vector R. (2) $\Delta OKH=\Delta GJI$, $\therefore$ GJ=OK=c; IJ=HK=d (3) OL=a (4) Since OM is the x-component of $\overrightarrow{OI}$, then OM=OL+LM; OM=a+c; similarly IM=b+d Show with examples on the board how to add vectors both graphically and algebraically. It is suggested that the career teaching activity be used at this time. 2. A similar geometric interpretation as in XII, B(1) can be used to verify that vectors such as (a,b) minus (c,d) equals (a-c,b-d). Work examples on the board both graphically and algebraically on how to subtract vectors. 3. Warn the student that complex numbers and two dimensional vectors have the same rules for addition but completely different rules for multiplication. Point out that if (x,y) is a vector and "a" a scalar, the product a(x,y) is defined to be the vector (ax,ay). Examples: 3(1,4) = (3,12) or -2(4,-1)= (-8+2). 4. Define the basic vectors (i) as the vector (1,0) and (j) as the vector (0,1). These are vectors of length 1 drawn along the positive directions of the coordinate axes.		

ALGEBRA II

CURRICULUM GUIDE

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A C K N O W L E D G E M E N T S

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ALGEBRA II

The following outline is built upon a "conceptual ladder" (concepts from 'easiest to hardest') for Algebra II. The outline corresponds to the outline found in the curriculum concerts of this guide. The page numbers refer to the present textbook being used in Algebra II by the Harlandale Independent School District. (Dolciani, Mary P., Modern School Mathematics, Algebra II. Boston: Houghton Mifflin, 1971.)

I. Real Number System
   A. Closure - pp. 19, 20, [illegible], 52
   B. Factoring - pp. 259-272
   C. Graphing - pp. 5-12
II. Functions
   A. Definition - p. 149
      1. Domain - pp. 149-153
      2. Range - pp. 149-153
   B. Graph of a Linear Function - pp. 160-163
   C. Special Functions
      1. Absolute Value - not in present book
      2. Polynomial - p. 353
      3. Greatest Integer - not in present book
      4. Inverse of a Function - pp. 404-407
   D. Quadratic Functions - pp. 353-361
III. Systems of Linear Equations and Inequalities
   A. Systems of Linear Equations in Two Variables
      1. Graphic Solution - pp. 160-163
      2. Linear Combinations - p. 203
   B. The Equation $ax + by + cz + d = 0$ - pp. 223-226
   C. Systems of Linear Equations in Three Variables
      1. Graphic Solution - pp. 235-238
      2. Linear Combinations - pp. 235-241
IV. Complex Number System
   A. Complex Numbers
      1. Properties - pp. 364-365
      2. Binary Operations - pp. 369-374
      3. Standard Form of a Complex Number - p.369
   B. Complex Conjugate
      1. Definition - pp. 371-372
      2. Properties - pp. 371-374
   C. Absolute Value
      1. Definition - p. 585
      2. Properties - p. 585
   D. Graphical Representation
      1. Definition of Axes - pp. 584-585
      2. Properties - pp. 591-594

V. Quadratic Equations and Inequalties
   A. Solution of Quadratic Equations
      1. Graphic Solutions - pp. 353-358
      2. Factoring - pp. 265-269
      3. Completing the Square - pp. 337-341
      4. Formula Development - pp. 338-339
      5. Quadratic Formula - pp. 339-341
   B. Root-Coefficent Relations
      1. Sum of Roots - pp. 343-345
      2. Product of Roots - pp. 343-345
   C. Role of the Discriminant - pp. 374-378
      1. Positive
      2. Zero
      3. Negative
   D. Quadratic Inequalities in One Variable
      1. Graphic Solution - pp. 362-364
      2. Solution by Factoring - not in present book
      3. Solution by Completing the Square - not in present book
   E. Equations Transformable to Quadratic Equations
      1. Equations Involving Rational Expressions - pp. 287-297
      2. Equations of Higher Degree - pp. 381-385

VI. Quadratic Equations in Two Variables
   A. Circle
      1. The Equation $(x-a)^2 + (y-k)^2 = r^2$ by Completing the Square - p. 442
      2. The Equation $x^2 + y^2 + dx + ey + f = 0$ - p. 443
   B. Parabolas
      1. Definition of General Properties - pp. 444-445
      2. Equation Forms - pp. 445-448
   C. Ellipse
      1. Definition of General Properties - pp. 449-451
      2. Equation Forms - p. 452
   D. Hyperbola
      1. Definition of General Properties - pp. 453-454
      2. Equation Forms - pp. 455-457
   E. Systems of Equations Involving Quadratics
      1. Graphic Solutions - pp. 463-465
      2. Substitution and Combination - pp. 465-467

VII. Exponential and Logarithmic Functions
   A. Exponential Function
      1. Properties of Exponents - pp. 251-256
      2. Expnential Function $(y = a^x)$ - pp. 407-410
      3. e - p. 427
   B. Logarithmic Function
      1. Definition - pp. 407-408

2. Graphing - pp. 407-408
3. Properties - pp. 420-427

C. Computations with Base 10 Logarithms
1. Characteristic and Mantissa - p. 415
2. Logarithm Tables - pp. 670-671
3. Interpolation - pp. 417-419
4. Computational Procedures - pp. 420-427

VIII. Matrices
A. Definition - pp. 633, 649
B. Properties - not in present book
C. Addition and Multiplication - pp. 634-648
Determinant of a Matrix
1. Diagonal Method - pp. 649-650
2. Minoring - pp. 651-653
E. Solution of a System Using Determinants - pp. 658-661

IX. Permutations and Combinations
A. The Pascal Triangle - pp. 614-616
B. Permutations
1. Definition - p. 602
2. Factorial - p. 602
C. Combinations
1. Definition - p. 608
2. Symbol - p. 608

X. Progressions and Binomial Expansion
A. Progressions
1. Definition - pp. 105-106
2. Formulas
a. n^{th} Term - pp. 109-111
b. Sum - pp. 116-117
B. Infinite Geometric Progressions - pp. 133-141
C. Binomial Expansion - pp. 612-616

XI. Trigonometry
A. Trigonometric Functions
1. Positive or Negative - pp. 483-484
2. Values for Special Angles - pp. 672-676
3. Approximate Values - pp. 672-676
4. Graphs - pp. 506-508, 513-514
B. Trigonometric Equations
1. Graphic Solution - pp. 509-512, 517
2. Algebraic Solution - pp. 526-555

XII. Vectors
- A. System of Vectors
  1. Graphical Representation - pp. 591-592
  2. Equality of Vectors - not in present book
- B. Operations with Vectors
  1. Addition - p. 593
  2. Subtraction - not in present book
  3. Multiplication - not in present book
  4. Basic Vectors - not in present book

ALGEBRA II

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
I. Real Number System A. Closure	I. THE STUDENT SHOULD BE ABLE TO: A. Correctly add, subtract, multiply, divide, raise to a power, or take a root of 80% of the problems on a written exercise which contains a subset of the real numbers in order to determine if the set is closed under the stated operation.	
B. Factoring	B. Correctly factor 85% of the problems on a written exercise by combining the different forms of factoring.	
C. Graphing	C. Correctly graph on a number line (prepared by the student) 80% of the sets of real numbers which are contained on a written exercise.	
II. Functions A. Definition	II. THE STUDENT SHOULD BE ABLE TO: A. Orally define, to the satisfaction of the teacher, a function as a set of ordered pairs of which no two have the same first element.	Concept Relationship of the domain of a quadratic function to the number of new cars sold by a car manufacturer. Performance Objective Determine through mathematical calculations, to the satisfaction of the teacher, the highest number of new cars which would be sold according to the number of extras on each car. General Information Students wishing extra

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
I. A. Review each of the following briefly: 1. All four operations of directed numbers (and their properties) 2. Working with exponents (and their properties) 3. Working with square root approximation (Newton's and Euclid's methods should be explained) 4. Determining if sets are closed under any operation for any $x \in \{R\}$ Examples illustrating each of the above should be worked. Students should work an assignment that contains examples of each of the above. B. Review all methods of factoring, illustrating examples for each. Students should work an assignment that combines all forms of factoring. C. Review graphing of sets on the number line. Work some problems on the board. Make an assignment.		
II. A. Define a function as a set of ordered pairs (x,y) none of which have the same first element. Illustrate samples of functions on the coordinate plane. 1. Stress that the domain is the x coordinate in the ordered pair (x,y). Give examples. 2. Stress that the range of the function is the y coordinate of the ordered pair (x,y) after the rule of the function has been performed. Work examples given the domain. Allow the students to find the range of sample problems.	<u>Algebra-Relations, Functions and Variation</u>; <u>16mm film -- 4015</u> <u>Competition and Profit Motive</u>; filmstrip -- <u>TP-139</u>	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
B. Graph of a Linear Function	B. Correctly graph 90% of the linear equations on a written exercise by using ordered pairs which solve the equation or by using the slope and y-intercept of equations.	activities should be encouraged to research other careers in the car manufacturing industry. They may wish to compile and analyze material on work done in the car manufacturing industry. Examples: 1. Professional and Technical Occupations 2. Administrative, Clerical, and Related Occupations 3. Plant Occupations 4. Machining Occupations 5. Inspection Occupations 6. Assembling Occupations 7. Finishing Occupations 8. Maintenance Occupations
C. Special Functions 1. Absolute Value 2. Polynomial Functions 3. Greatest Integer 4. Inverse of a Function	C. Define mathematically, to the satisfaction of the teacher, special functions as presented on the board by the teacher (1. Absolute value function, 2. Polynomial function, 3. Greatest integer function, and 4. Inverse of a function)	The automobile industry employed 810,000 persons in 1970. This number fluctuates greatly according to economic growth. Average earnings of production workers are among the highest in manufacturing (average of \$4.23 an hour). High school students wishing to enter the automobile industry should investigate particular job requirements. A course in auto mechanics should prove very helpful to a student preferring the mechanics of the industry. Students wishing either administrative or professional type jobs need at least a bachelor's degree in an applicable field.
D. Quadratic Functions	D. Correctly graph 80% of the problems on a written exercise concerning quadratic functions of the form $f(x) = a(x-h)^2 + K$, $a \neq 0$.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
B. Work examples on the board which illustrate the following: 1. Graphing by the substitution method (ordered pairs) 2. Graphing by the use of the slope and y-intercept, Assign equations to be graphed using both methods. C. 1. Briefly review the absolute value of a number a, $a \in \{R\}$. State that a function in the form $g(x) = \|x\|$ is called an absolute value function. Work some sample problems. (Stress that the value of $g(x)$ will always be positive). 2. Clearly define a polynomial function as $g(x) = a(x-h)^2 + k$. Illustrate it with the use of some sample problems. 3. Define $f(x) = x$ as the greatest integer function, where x is used to denote the greatest integer equal to or less than x. Orally find the greatest integer for any x, $x \in \{R\}$. 4. Define an inverse function as f^{-1}. Emphasize that the (-1) is not an exponent. Explain the inverse function of x, $f^{-1}(x)$ is the interchanging of the component of every ordered pair of $f(x)$. D. State that the polynomial function $g(x) = a(x-h)$ is referred to as a quadratic function. These points should be covered fully before attempting the graph. 1) $(x-h) = 0$ denotes the equation for axes of symmetry. 2) (h,k) are the coordinates of the vertex.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		<u>Teaching Activity</u> Management experts employed by car manufactures have determined that the number of new cars sold highly correlates to the number of extras on the car. Experience is usually the main ingredient for determining which extras will sell a car. A car manufacturer knows that if x is the number of extras he puts on his car, then from past experience he knows he can expect to sell $f(x) = 40 + 8x - x^2$ cars during the model year. The car which he wishes to sell comes with anywhere from 0 to 8 extras (such as power steering, radio, air-conditioning, and so on). Therefore ~~the~~ domain of this function is the set $x \epsilon \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. $f(0) = 40$ $f(5) = 55$ $f(1) = 47$ $f(6) = 52$ $f(2) = 52$ $f(7) = 47$ $f(3) = 55$ $f(8) = 40$ $f(4) = 56$ The range of this function is the set $f(x) \epsilon \{40, 47, 52, 55, 56\}$. The above calculations indicate to the car manufacturer that he can expect to sell the highest number of cars which contain 4 extras.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
3) "a" determines whether the function contains a maximum or minimum point. The teacher should work some sample problems and then assign some to the students. It is suggested that the career teaching activity be used at this time.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
III. Systems of Linear Equations and Inequalities A. Systems of Linear Equations in Two Variables 1. Graphic Solution	III. THE STUDENT SHOULD BE ABLE TO: A. 1. Solve correctly 85% of the problems on a written exercise concerning a system of linear equations by finding their intersection on a coordinate plane (the intersection should be identified as an ordered pair).	Concept Relationship of inequalities to the production of television sets as studied by the economist. Performance Objective Determine and calculate the work an economist must perform in order to determine the number of each of two television models a manufacturer should produce for the greatest profit. General Information Students wishing extra activities should be encouraged to research other careers in the economic field. They may wish to compile and analyze material on work done by the economist. Economics is the largest of the basic social science fields. Approximately 33,000 economists were employed in 1970 with salaries ranging from \$6,548 as a start to over \$23,000 yearly. High school students wishing a career in economics should acquire all the math courses available to them. A bachelor's degree with a major in economics is sufficient for many beginning jobs. Graduate work is very important for advancement. Teaching Activity A television production firm
2. Linear Combinations	2. Solve correctly 80% of the problems on a written exercise concerning a system of linear equations by adding or subtracting to eliminate one variable and by the "substitution method".	
B. The Equation $ax + by + cz + d = 0$	B. Identify orally, to the satisfaction of the teacher, ordered triplets (x, y, z) in terms of the coordinates on a three-dimensional coordinate system.	
C. Systems of Linear Equations in Three Variables 1. Graphic Solution	C. 1. Graphically represent the solution set (as a plane) of a linear equation (three variables) on a three-dimensional coordinate system.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
III.	<u>Graphing Linear Equations</u>; 16mm film -- 4226	
A.	<u>Basic Economic Concept</u>; filmstrip -- TP-133	
It is suggested that the career teaching activity be used at this time. 1. Review graphing of linear equations by the ordered pairs, or the slope and y-intercept methods. Stress that the point (ordered pair) where the two equations intersect, is a root of both equations. Assign some problems. Check results algebraically. 2. Introduce the addition-subtraction method and the substitution method as "algebraic" methods of solving systems of linear equations. Work examples using both methods. Send students to the board to practice. The teacher can then correct their mistakes.	<u>Supply and Demand</u>; filmstrip -- TP-134 <u>Economics-Supply and Demand</u>; filmstrip -- PR-711	
B. Introduce the three axes (x-axis, y-axis, z-axis) as required by a three dimensional coordinate system. Preferably use a <u>model</u>. Present a few examples of ordered triplets which illustrates how to graph each correctly.		
C.		
1. Using a model, show the different octants, also the three different planes intersecting at the origin		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
2. Linear Combinations	2. Solve correctly 80% of the problems on a written exercise concerning a system of three-dimensional linear equations by adding the three equations in order to eliminate one variable and solving the resulting equations by using methods for linear equations in two variables.	makes (along with others) two types of color televisions: a portable model and a console model. The company has the equipment to manufacture any number of portable models up to (and including) 400 per month or any number of console models up to 300 per month. It takes 80 man-hours to produce a portable model, and 100 man-hours to produce console sets. The firm has up to 40,000 man-hours available for television production each month. If the profit gained on each portable model is \$50 and on each console model is \$70, find the number of each kind of set the firm should manufacture to gain the maximum profit each month. The firm has employed an economist to solve this problem. In solving the problem the economist may let x represent the number of portable sets and y represent the number of console sets that are manufactured each month. Therefore, $50x$ = monthly profit on portable sets $70x$ = monthly profit on console sets $50x + 70x$ = combined profit on both models

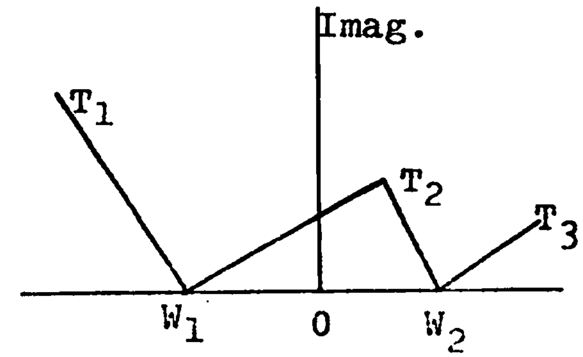
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
(xy-plane; yz-plane; xz-plane). Work different examples on the board and thoroughly illustrate how to solve and locate the x-intercept, y-intercept, and z-intercept on their respective axes. The triangle formed by the three intercepts will represent a partial graph of the solution set. Define the three different traces and discuss them. 2. Review with an example problem solving a system of equations in two variables. Stress that the same principles will be applied to a system of equations in three variables. Work a few example problems and explain clearly step by step. One of the example problems should be solved graphically so that the student can compare solution sets.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		Constraints of production: x and y $\geq$ 0 (company must produce both models); x $\leq$ 400 (greatest number of portable models company can produce each month); y $\leq$ 300 (greatest number of console models company can produce each month); 80x + 100y $\leq$ 40,000 (40,000 man-hours are available for production) Graphic solution of this problem gives corner points of (0, 0); (400, 0); (400, 80); (125, 300); and (0, 300). Using these values in the expression 50x + 70y the economist finds a maximum profit for (125, 300). Therefore, the manufacturer should produce 125 portable models and 300 console models for a maximum profit of $27,250.
IV. Complex Number System A. Complex Numbers 1. Properties	IV. THE STUDENT SHOULD BE ABLE TO: A. 1. Designate orally, to the satisfaction of the teacher, the complex number system by the symbol C which has the following properties: 1) addition and multiplication are defined; 2)each real number is a member of C, the additive identity of C is the zero of the reals, the mulplication identity of C is the one of the reals; 3)the set C contains a distinguished member i, with the property $i \times i = i^2 = -1$; 4)each element of C may be expressed in the form a + bi.	<u>Concept</u> Relationship of complex numbers to the career of a petroleum engineer. <u>Performance Objective</u> Calculate correctly, to the satisfaction of the teacher, a written problem which concerns finding a hidden gas well (one out of three) which is located midway between the centers of three storage tanks connected to the wells by pipes which form right isosceles triangles. <u>General Information</u> Students wishing extra

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
IV. A. 1. Solve and explain the equation $a+1=0$ over the set of all positive real numbers. Extend the domain to R and ask the class for the solution set. Present the equation $a^2+1=0$ over R. Question: Is there any number that will satisfy such an equation? At this point the teacher should introduce imaginary numbers and define the complex number $(a + bi)$. Illustrate why $i^2 = -1$.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
2. Binary Operations	2. Correctly add, subtract, multiply, and divide 80% of the complex numbers on a written exercise.	activities should be encouraged to research other careers in the petroleum industry. They may wish to compile and analyze material on work done by the petroleum worker.
3. Standard Form of a Complex Number	3. When presented with five complex numbers in standard form label correctly on a written exercise the real and imaginary parts of each number.	Examples: 1. Petroleum geologist 2. Drillers 3. Shooters 4. Rig builders 5. Derrickmen 6. Enginemen 7. Roustabouts 8. Pumpers 9. Gasoline plant engineer 10. Well puller
B. Complex Conjugate 1. Definition	B. 1. Write four out of five complex conjugates on a written exercise containing complex numbers in the form (a + bi).	
2. Properties	2. Prove correctly, to the satisfaction of the teacher, that the conjugate of the sum of two complex numbers is the sum of their conjugates; the conjugate of the product of two complex numbers is the product of their conjugates.	In 1970 approximately 266,800 persons were employed by the petroleum industry. High school students wishing a career in the petroleum industry should seek guidance from a resource person employed by the industry for educational requirements. Teaching Activity A petroleum engineer was assigned to reopen an abandoned 3 well gas field. He found two exposed caped gas wells but the third well had become buried and unknown in location. His only notes showed the well to be located midway between the centers of two of three storage tanks connected to the wells by pipes which form right isosceles triangles as shown on the next page:
C. Absolute Value 1. Definition	C. 1. Define absolute value of a complex number (a + bi) on a written exam as the principal square root of the sum of the squares a and b.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
2. Work a few examples at the board demonstrating how to add, subtract, multiply, and divide complex numbers using the familiar properties of sums and products in R. Make an assignment. 3. Using an example of a complex number in the form $(a + bi)$, identify the real part and the imaginary part of a complex number. Distinguish between a pure imaginary number and an imaginary number. B. 1. Define a complex conjugate and illustrate it with examples. 2. Use a formal proof in order to understand the meaning of the theorem. Let the student work some exercises similar to the following: 1) $(3 + 2i) + (5 + 3i) = 8 + 5i$ $(3 - 2i) + (5 - 3i) = 8 - 5i$ 2) $(3 + 2i)(5 + 3i) = 9 - 19i$ $(3 - 2i)(5 - 3i) = 9 - 19i$ Compare and describe the differences of the two examples. C. 1. Clearly define the absolute value of a complex number, $(a + bi)$, as the principal square root of the sum of the squares of a and b. e.g. If $(a + bi) = Z$, then $\|Z\| = \sqrt{a^2 + b^2}$.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
2. Properties	2. Prove on a written exercise, to the satisfaction of the teacher, that a complex number is zero if and only if the absolute value is zero; the product of a complex number and its conjugate is the square of the absolute value of the complex number; the absolute value of the product of two complex numbers is the product of the absolute values.	The length of the pipes and locations of the tanks was unknown. He must locate the third well. How? 
D. Graphical Representation 1. Definition of axes	D. 1. Graph correctly 80% of the complex numbers on a written exercise by plotting them on the coordinate plane.	He sets up a real axis using complex numbers through W_1 and W_2 with the centers of W_1W_2 as the origin. The imaginary axis would then be perpendicular to $\overline{W_1W_2}$ through the origin. If $OW_2 = 1$, then $OW_1 = -1$. Let tank T_2 have the complex number location $T_2 = a + bi$, then vector $W_2T_2 = (a + bi) - 1$. Hence, the vector $W_2T_3 = -i\{(a + bi) - 1\}$ produces a 90^0 clockwise rotation caused by multiplying by $-i$. Likewise,
2. Properties	2. Graph correctly 80% of the addition of complex numbers on a coordinate plane.	$\overrightarrow{W_1T_2} = (a + bi) - (-1)$, so that $\overrightarrow{W_1T_1} = i(a + bi + 1)$ produces a 90^0 counter clockwise rotation caused by multiplying by i. We then find $\overrightarrow{W_1T_1} + \overrightarrow{W_2T_3} = (-ia - bi^2 + i) + (ia + bi^2 + i) = 2i$ with the third well located at $\frac{1}{2}(2i)$ or i which is on the perpendicular bisector of $\overline{W_1W_2}$ a distance of OW_2 from 0.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

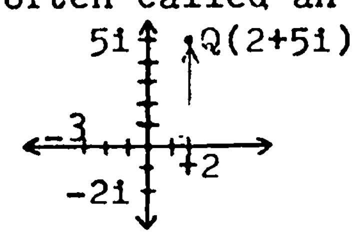
2. Preferably use an overhead projector to prove the stated theorems. Emphasize the importance of the theorems to higher mathematics. Have the students copy the proofs from the screen and use them as a study guide.

It is suggested that the career teaching activity be used at this time.

D.

1. Draw the coordinate plane. Stress that the abscissa becomes the axis of the reals, whereas the ordinate will be the axis for the pure imaginary numbers. Work a few examples as the one shown in the illustration below: (such a diagram is often called an argand diagram)

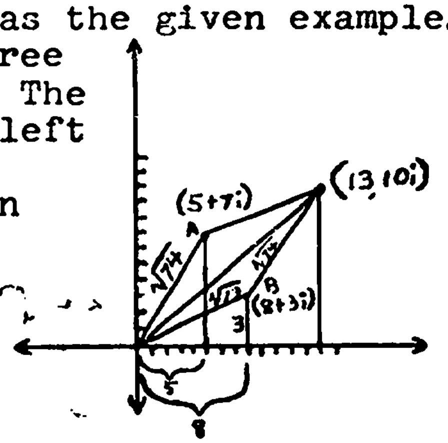
   To plot:
   Q(2+5i)



   Plot several complex numbers and let the students read the numbers from the graph.

2. Illustrate how to add complex numbers graphically such as the given example. Graph at least three similar examples. The graphs should be left on the board so the student can refer to them.

   graphically add:
   (5+7i)+(8+3i)



CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		It should also be noted that the electrical engineer often uses complex numbers (with j representing i) in his study of alternating current, however, the concept is considered too difficult at this time.
V. Quadratic Equations and Inequalities A. Solution of Quadratic Equations 1. Graphic Solutions	V. THE STUDENT SHOULD BE ABLE TO: A. 1. Correctly graph 80% of a list of quadratic equations on a written exercise.	Concept Relationship of a quadratic equation to a cable supported bridge being designed by a civil engineer. Performance Objective Correctly calculate the height of the towers supporting a bridge when given the quadratic equation of the supporting cables and the linear distance between the two supporting towers.
2. Factoring	2. Solve correctly 85% of the quadratic equations on a written exercise by factoring (if the quadratic equation can be factored in standard form).	General Information Students wishing extra activities should be encouraged to research other careers in civil engineering. Approximately 185,000 civil engineers were employed in the United States in 1970. They were
3. Completing the Square	3. Solve correctly 85% of the quadratic equations on a written exercise by completing the square.	employed by public utilities, railroads, educational institutions, iron and steel industry, and other major manufacturing industries. The employment outlook for the civil engineer is for continued growth through the 1970's. The average starting salary for the civil engineer in 1970 was $10,000. With

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
V. A. 1. Clearly define a quadratic equation and write its standard form ($Ax + By + C = 0$; where A, B, and C are integers). The student should realize that the given equation has to be transformed into an equivalent equation which expresses "y" in terms of "x". Use this equation to find the values of "y" corresponding to selected values of "x". Encourage the student to use a chart in finding the values of "y". A limit of seven points should be graphed. Emphasize that these points are just a few members of the graph. 2. Review all the methods on how to factor a quadratic trinomial. Stress and illustrate the theorem if $a \cdot b=0$, then either $a = 0$ or $b = 0$. Work as many example problems as required and point out the importance for the application of the above theorem. 3. The student should try to solve correctly an irreducible quadratic equation. Stress that not all quadratic equations are reducible and introduce the method of completing the square using the given example problem. No step should be omitted in the process and strongly point out why the entire equation is divided by A and Bx is divided by two.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION. AND TEACHING ACTIVITIES
4. Formula Development	4. Write, to the satisfaction of the teacher, the development of the quadratic formula by using the method of completing the square on the general quadratic equation $ax^2 + bx + c = 0$.	experience and training the salary rose to $25,600 or more. High school students wishing to enter civil engineering must obtain a strong background in math and science. A bachelor's degree in civil engineering is required for entry into civil engineering positions. Because of rapidly changing technology, an engineer must be willing to continue his education throughout his career.
5. Quadratic Formula	5. Solve correctly 85% of the quadratic equations on a written exercise by using the quadratic formula $(x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a})$.	<u>Teaching Activity</u> In constructing a bridge supported by cables (Golden Gate Bridge) a civil engineer knows that each cable is related to the quadratic equation $y = \frac{97}{882,000} x^2$.
B. Root-Coefficent Relations	B.	
1. Sum of Roots	1. Calculate correctly on a written exercise 80% of the sums of roots of quadratic equations in general form by using the coefficents $(r_1 + r_2 = -\frac{b}{a})$.	He also knows that the towers which support the cables must be 4,200 feet apart. His problem is to calculate the height of the supporting towers. In this case y represents the height of the cable above the midpoint of the cable and x represents the horizontal distance from the midpoint.
2. Product of roots	2. Calculate correctly on a written exercise 80% of the products of roots of quadratic equations in general form by using the coefficents $(r_1r_2 = \frac{c}{a})$.	The engineer determines that the towers will be 2,100 ft.(horizontal distance) from the midpoint of the cable. Substituting this

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
4. Send a student to the board to solve correctly an irreducible quadratic equation without omitting any steps. Using the standard form, $Ax + Bx + C = 0$, derive the quadratic formula using a parallel treatment with the given problem on the board. Verify that the formula is true by applying it in the given problem and compare results. 5. For better results emphasize that the quadratic equation should be in standard form, $Ax + Bx + C = 0$, before replacing any values in the quadratic formula. Illustrate how to use the quadratic formula by solving correctly a few example problems. It is suggested that the career teaching activity be used at this time. B. 1. Let three or four students solve different examples of quadratic equations at the board. After being solved, tell each student to add their solutions. Using the trial and error method, challenge the class for a way of finding the sum of the roots of a quadratic equation using the values of A, B, and C ($Ax = Bx = C = 0$). After coming out with the formula $r + r = -$, verify it. 2. Use the same method as above, except instead of adding multiply the roots.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
C. Role of the Discriminant 1. Positive 2. Zero 3. Negative	C. Calculate correctly on a written exercise 80% of the discriminants ($b^2 - 4ac$) of quadratic equations and write analytically the meaning of each, 1) two different real roots if $b^2-4ac>0$; 2) one double real root if $b^2 - 4ac = 0$; 3) two complex conjugate roots if $b^2 - 4ac < 0$.	value he obtains $y = \frac{97}{882,000}(2,100)^2$; $y = \frac{97}{882,000}(4,410,000)$; $y = 97 \times 5 = 485$ feet tall.
D. Quadratic Inequalities in One Variable	D.	
1. Graphic Solution	1. Correctly graph 80% of the quadratic inequalities on a written exercise.	
2. Solution by Factoring	2. Solve correctly 80% of a list of quadratic inequalities by factoring.	
3. Solution by Completing the Square	3. Solve correctly 80% of the quadratic inequalities by completing the square.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
C. Have three quadratic equations solved at the board. Equations such as: 1) $x^2 + 6x + 8 = 0$, 2) $2x^2 + 5x + 1 = 0$, 3) $2x^2 - 4x + 3 = 0$, where the discriminant is equal to zero, greater than zero and less than zero. Point out the difference in solutions and also the difference in the number, $b^2 - 4ac$, under the radical sign. Stress that the discriminant, $b^2 - 4ac$, will differentiate the nature of the roots of a quadratic equation. In determining correctly the discriminant, emphasize that the equation has to be in standard form, $Ax + Bx + C = 0$.		
D. 1. Briefly review how to graph quadratic equalities in one variable. Work sample problems, pointing out how factoring is used to solve each problem. One of the sample problems should have the > symbol and the other <. Assign problems. 2. Illustrate how to solve inequalities and stress the factoring of each problem. Send the students to the board for practice. The teacher should observe and make corrections, if any. 3. Reconsider the method of completing the square. Make an assignment. Let the students work in groups of not more than 5. The teacher serves as a supervisor for each group.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
E. Equations Transformable to Quadratic Equations	E.	
1. Equations Involving Rational Expressions	1. Solve correctly by quadratic methods 80% of the equations on a written exercise which contain rational expressions that transform to quadratic equations $(x + \frac{1}{x} = \frac{5}{2}$; $2x^2 - 10x + 2 = 0)$.	
2. Equations of Higher Degree	2. Factor and solve by quadratic methods 80% of the polynomials (higher degree than quadratic) on a written exercise which transform to quadratic equations.	
VI. Quadratic Equations in Two Variables A. Circle 1. The equation $(x-a)^2 + (y-k)^2 = r^2$ by completing the square	VI. THE STUDENT SHOULD BE ABLE TO: A. 1. Write in the form $(x-a)^2 + (x-k)^2 = r^2$ and graph 80% of the equations on a written exercise concerning circles.	<u>Concept</u> Relationship of an ellipse to the elliptical orbit of a space probe as studied by the space technologist. <u>Performance Objective</u> Calculate correctly on a written problem the perigee and apogee of an elliptical orbit of a space probe as studied by a space technologist. <u>General Information</u> Students wishing extra activities should be encouraged to research other careers in

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
E.		
1. Review the LCM and the appropriate axioms over R. The student should be aware of the existing restrictions, if any. For example: $X + 1/x = 5/2$. The LCM is 2x and the restriction is $x \neq 0$. Once the denominator is cleared by multiplying the entire equation by the LCM, solve correctly by any quadratic method (preferably by the quadratic formula). Work a few problems to illustrate how to solve such problems.		
2. Cover briefly the GCF and methods on how to factor quadratic equations in one variable. A few examples on the board with a good explanation for each problem will give the student a clear view of how to arrive at a correct solution for each problem.		
VI.		
A.		
1. Verify that in the form $(x-a)^2 + (y-k)^2 = r^2$, (a,k) denotes the coordinates of the center and r represents the radius. Make an assignment and encourage the students to use graphing paper.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
2. The Equation $x^2 + y^2 + dx + ey + f = 0$	2. Graph correctly 80% of the equations such as $(x^2+ y^2+dx+ey+f=0)$ on a written exercise which may be put in the form $(x-a)^2 + (y-k)^2 = r^2$ by completing the square.	space technology. At present the many job opportunities in space science tend to be very promising to the "projected year" of NASA (1990). There promises to be some new job openings as well as vacancies created by loss of personnel in space related careers. Examples: 1. Astronauts- (small number needed--salaries classified) 2. Space Technicians Many other careers are closely related to the space industry such as: engineers (electronic, electrical, aerospace, chemical, nuclear, mechanical, and industrial are among the larger classifications), mathematicians, geologists, physicists, biologists, and meteorologist.
B. Parabolas 1. Definition of General Properties	B. 1. Define orally, to the satisfaction of the teacher, directrix, focus, axis, vertex, and focal chord as related to a parabola.	
2. Equation Forms	2. Graph correctly 80% of the parabolas in the form: $x^2 = 2py$, $y^2 = 2px$, $(x-h)^2 = 2p(y-k)$, $x^2 + dx + ey + f = 0$, and $y^2 + dx + ey + f = 0$ on a written exercise.	High school students wishing to enter the space industry should prepare with all available math and science courses to prepare for the jobs of the future. In almost all cases a bachelor's degree is needed with a major in the applicable subject area. A masters and PhD degree are necessary in many fields.
C. Ellipse 1. Definition of General Properties	C. 1. Illustrate at the board, to the satisfaction of the teacher, the focus, major and minor axes as related to an ellipse.	<u>Teaching Activity</u> This activity has been simplified for presentation in Algebra II. A space probe which has been

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
2. Reconsider the method of completing the square. Transform $(x-a)^2+(y-k)^2=r^2$ into $x^2+y^2+dx+ey+f=0$, where $d=-2a$, $e=-2h$ and $f=a+h-r$. The students should recognize that any quadratic equation in the latter form can be transformed into the standard form $(x-a)^2+(y-k)^2=r^2$ by completing the square twice, once for x and once for y. Assign some problems. Check the transformations before graphing. B. 1. Identify from a graph the terms: directrix, focus, axis, vertex, and focal chord. Define each term and place their definitions on the board. 2. Demonstrate that any quadratic equation representing a parabola can be transformed into $(x-h)^2 = 2p(y-k)$, where (h,k) is the vertex and $y-k = 0$ is the equation for the axis of symmetry. Work a few examples on the board to illustrate the graphing. Make an assignment. C. 1. From a graph at the board, point out and discuss the focus (foci), major and minor axes as related to an ellipse. Using the overhead and different graphs, let the student identify orally the above mentioned terms. It is suggested that the career teaching activity be used at this time.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
2. Equation Forms	2. Graph correctly 80% of the ellipses on a written exercise in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$	rocketed into space has an elliptical orbit which has foci identified by the earth and the sun. The orbit has an eccentricity of 0.7. A space technologist has the job of calculating the closest approach (perigee) and farthest approach (apogee) of the space probe. Major axis = V_1V_2 e = .007 perigee = $V_1{}_1$ apogee = V_2F_1 $F_1C = \frac{1}{2}F_1F_2 = \frac{1}{2}(93 \times 10^6$ miles) $F_1C = 46.5 \times 10^6$ miles $V_1C = \frac{F_1C}{e}\ \frac{46.5\times10^6}{7\times10^{-3}} = 6.64 \times 10^9$ Therefore, V_1F_1 (perigee) = $V_1C - F_1C$ = 6,640,000,000 - 46,500,000= 6,593,500,000 miles And: V_2F_1 (apogee) = $V_1C + F_1C$ = 6,640,000,000 + 46,500,000= 6,686,500,000 miles
D. Hyperbola 1. Definition of General Properties	D. 1. Illustrate at the board, to the satisfaction of the teacher, the branches, center, and asymptotes of a hyperbola.	
2. Equation Forms	2. Graph correctly 80% of the hyperbolas on a written exercise in the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1,\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$	
E. Systems of Equations Involving Quadratics	E.	
1. Graphic Solutions	1. Plot and determine graphically 80% of the problems on a written exercise containing quadratics and linear equations.	
2. Substitution and Combination	2. Solve correctly 80% of the problems on a written exercise containing one quadratic and one linear equation on each problem by substitution of the vlaue of one variable of the linear equation into the quadratic equation.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
2. Work examples using both methods (Newton's is most of the time called the successive approximation method). Let students go to the board and practice them. Then let the individual students choose which method they would like to use. Example--find $\sqrt{40}$ correct to 2 places. Newton's Method We know $6<\sqrt{40}<7$ A) 1st approximation $\frac{6+7}{2} = \frac{13}{2} = 6.5$ B) Divide 40 by the 1st approximation $40 \div 6.5 = 6.1\underline{53}$ Thus $6.153<\sqrt{40}<6.5$ C) 2nd approximation $\frac{6.153 + 6.5}{2} = \frac{12.653}{2} = 6.3265$ D) Repeat step (B) using 2nd approximation $40 \div 6.3265 = 6.322$ thus $6.322<\sqrt{40}<6.3265$ Since both approximations agree on the two decimal places, $\sqrt{40} = 6.32$ correct to 2 decimal places. Euclid's Method Starting at the decimal point, pair digits to the left and to the right, with as many pairs to the right as the number of correct decimal places necessary. $\sqrt{40.00\ 00}$ A) Find the largest number which is a square root of 40 and place it above the 40. $\begin{array}{l} \ \ 6. \\ \sqrt{\underline{40}.\underline{00}.\underline{00}} \end{array}$		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

B)Square, subtract from 40, and bring down the next pair.

```
   6.
 √40.00.00
  36
   400
```

C)Double the 6 and place a question mark to the right of 12. Use this as the next divisor.

```
      6.
    √40.00.00
     36
12(?)]400
```

D)Estimate how many times 120 goes into 400. We try 3. Replace the question mark with 3. Place 3 next to 6. Multiply 123 by 3. Subtract; bring down the next pair.

```
    6.3
  √40.00 00
   36
123]400
    369
     3100
```

E)Repeat steps C and D, until the correct number of decimal places are obtained. Therefore; $\sqrt{40} = 6.32$ correct to 2 decimal places.

```
    6. 3  2
  √40.00 00
   36
123]400
    369
126?]3100
  (2)2524
      576
```

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
D. Pythagorean Theorem 1. Definition	D. 1. Orally state, to the satisfaction of the teacher, the Pythagorean Theorem ($a^2+b^2=c^2$) analogous to a right triangle.	
2. Application	2. Solve correctly 80% of the right triangles on a written exercise for the missing parts (hypotenuse or leg).	
E. Operations	E. Correctly add, subtract, multiply, and divide 80% of the problems on a written exercise containing radical expressions.	
X. Quadratics A. Definition	X. THE STUDENT SHOULD BE ABLE TO: A. Write, to the satisfaction of the teacher, the standard form of a quadratic equation ($ax^2 + bx + c = 0$; a, b, c ϵ of the set of real numbers, $a \neq 0$).	Concept Relationship of quadratic equations and the telephone equipment installer. Performance Objective Explain orally, to the satisfaction of the teacher, how a central office equipment installer employed by the telephone company can determine the number of telephones which can be connected to a swithboard that can make 325 connections by using the quadratic formula. General Information Students wishing extra activities should be encouraged to research other careers in the telephone industry. Examples: 1. Equipment installers
B. Solutions 1. Factoring	B. 1. Solve correctly 80% of the quadratic equations on a written exercise by factoring (if the quadratic equation can be factored in standard form).	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AID RESOURCE MATERIALS	TEACHER COMMENTS
D. 1. Prove the Pythagorean Theorem to the class first, then let the class go over it, and discuss it.	<u>Possibly So, Pythagoras</u>; 16mm film -- 8840	
2. Work problems which deal in solving the different missing parts of a right triangle.	<u>Pythagorean Triplets</u>; filmstrip -- AA-73	
E. All properties of square roots should be introduced first. Sample problems should be worked to illustrate all four operations with radicals to the class. Sample problems should include similar and non-similar radicals, conjugate expressions, and n^{th} roots.		
X. A. Introduce the standard form of a quadratic equation. Show that it is a quadratic polynomial (previously covered) where the multiplicative property of zero has been applied. Therefore; $ax^2 + bx + c = 0$. Stress the fact that this is one of those equations that we had termed irreducible before (since the left member could not be factored over the rationals). Write some quadratic equations on the board.	<u>Quadratic Equations and Their Solution</u>; 16mm film -- Z-32	
B. 1. Review all the methods of factoring previously covered. (Trial and error difference of squares, etc.) Work some problems in class. <u>Include some irreducible ones</u>.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
2. Completing the Square	2. Solve correctly 80% of the quadratic equations on a written exercise by completing the square	(approximately 22,000 installers were employed in 1970 with salaries ranging from \$2.50 to \$4.82 an hour) 2. Linemen and cable splicers (approximately 44,000 were employed in early 1970 with salaries ranging from \$2.51 to \$4.02 an hour)
3. Quadratic Formula	3. Solve correctly 80% of the quadratic equations on a written exercise by using the quadratic formula: $x = \frac{-b \pm \sqrt{b - 4ac}}{2a}$	3. Telephone and PBX installers and repairmen (approximately 102,000 were employed in 1970 with an average hourly wage of \$3.78) A high school education is required for almost all jobs related to the telephone industry. Courses in high school should stress math and electronics. Chances for advancement are excellent with more education. <u>Teaching Activity</u> Teacher note: (This career activity is simplified for presentation at this time). Many communities depend on a switchboard for telephone connections. In this case a telephone equipment installer faces the problem of determining the number of telephones which can be connected to a switchboard that can make 325 connections. The connections C which can be made through a switchboard to which n telephones are

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
2. Solve those equations which were found to be irreducible using standard forms of factoring, by means of completing the squares. Several sample problems should be demonstrated to the class. It is very important at this point that the students understand why $(2\frac{b}{a})^2$ is added to both sides of the equation to complete the square: it makes the left member a trinomial square.		
3. In a parallel illustration, using any sample of the standard form of a quadratic equation, ($ax^2 + bx + c = 0$) derive the quadratic formula by solving the sample problem using the completing the square method and at the same time performing the same operations on $ax^2 + bx + c = 0$. Illustrate to the class the use of the formula by using it on the sample problem and comparing answers. Assign the class problems to be solved by the use of the quadratic formula.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		connected is given by the equation $C = \frac{1}{2}n(n-1)$. Answer: $325 = \frac{1}{2}n(n-1)$ $325 = \frac{n^2}{2} - \frac{1}{2}n$ $325 = \frac{n^2 - n}{2}$ $n^2 - n = 650$ $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $n = \frac{-1 \pm \sqrt{1 - 4(1)(-650)}}{2(1)}$ $n = \frac{1 \pm \sqrt{1 + 2600}}{2}$ $n = \frac{1 \pm \sqrt{2601}}{2}$ $n = \frac{1 \pm 51}{2}$ 26, -25 telephones
XI. Trigonometry A. Ratios 1. Sine 2. Cosine 3. Tangent	XI. THE STUDENT SHOULD BE ABLE TO: A. Define, to the satisfaction of the teacher, the relationships between sides of triangles as sine, cosine, and tangent of angles in standard position.	<u>Concept</u> <u>Relationship of the sine function to the navigation of a submarine.</u> <u>Performance Objective</u> Explain, to the satisfaction of the teacher, how a navigator uses the sine function in a problem on submarine navigation. <u>General Information</u> Students wishing extra activities should be encouraged to research other careers in navigation.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XI. A. Introduce and explain the unit circle. Use the Pythagorean Theorem to prove the equation $x^2 + y^2 = 1$ and that a point (x, y) is on the unit circle if and only if the equation is true. Introduce initial side, terminal side, and standard position of angles. Define sine, cosine, and tangent, illustrating their relationship, for any angle. Stress and explain there points (for any angle y).		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		They may wish to compile and analyze material on specific navigation careers. Examples: 1. Aviation navigator 2. Navy navigator Students interested in navigation should be encouraged to take all possible math courses in high school. A college degree with a major in mathematics is preferred by many airlines. The armed forces will train men in preparing them to become navigators.
B. Values of Ratio for Special Angles 1. Quadrantal Angles 2. 30, 45, 60, Degree Angles	B. Write correctly from memory 80% of the problems on a written exercise concerning the values for sine, cosine, and tangent of 30, 45, and 60 degree angles as well as the quadrantal angles.	
C. Approximate Values of Ratios 1. Tables 2. Indirect Measure	C. 1. Write correctly 80% of the approximate values of problems on a written exercise concerning the sine, cosine, and tangent of angles in the first quadrant using tables. 2. Solve correctly 80% of the right triangles on a written exercise by using ratios (sine, cosine, and tangent).	<u>Teaching Activity</u> Calculate the depth of a submarine beneath the surface after it has traveled a certain distance at a certain diving angle. Example: Because of a failure in a depth detection device, a navigator on a submarine needs to calculate the depth of the ship after it has traveled 600 feet from where it submerged at a diving angle of 4^{o}. $\sin 4^{o} = d/600$ $d = 600 \sin 4^{o}$ $d = 600 \times .0698$ $d = 41.88$ ft. (depth of submarine)

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
1. $-1 \leq \sin y \leq 1$ and $-1 \leq \cos y \leq 1$ 2. $(\sin y)^2 + (\cos y)^2 = 1$ 3. $\tan y = \frac{\sin y}{\cos y}$ Assign problems to find the above relationships. B. The teacher should define quadrantal angles. With the class participating, the teacher should find the sine, cosine, and tangent of quadrantal angles as well as 30^0, 45^0, and 60^0 angles. A list should be made by each student of all the relationships above. This list should be studied and learned by the students. C. 1. The teacher should show the students how to use the tables which give sine, cosine, and tangents for angles. Example problems should be worked in class. 2. Introduce a right triangle, and by the use of the Pythagorean Theorem. Find: (for any angle y) A) tan y B) sin y In terms of(side adjacent C) cos y to y, side opposite to y, and hypotenuse. Examples in solving the triangle should be covered and some problems assigned to the class.		