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ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This unit introduces students to the Cartesian coordinate system and, in particular, to the graph of a linear equation. Ideas associated with Pascal's triangle, groups, and vector spaces are also introduced informally. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

MINNE

Math And Science Teaching project

UNIT XVI

SQUAREVILLE

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1965

MATHEMATICS

FOR THE

ELEMENTARY SCHOOL

UNIT XVI

Squareville

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We are deeply indebted to the many teachers who
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CONTENTS

Teacher's Introduction

Squareville introduces students to the Cartesian coordinate system and, in particular, to the graph of a linear equation. Ideas associated with Pascal's triangle, groups, and vector spaces are also introduced informally.

As you proceed through this unit, remember these points:

1. Streets run N and S; avenues run E and W. This is an arbitrary decision on the part of the authors and may not occur in real life situations.

2. When we say you may travel East, North, or North and East, we mean you must follow a street or an avenue. You do not go diagonally in a Northeast direction.

3. In real life, one would not be able to build a highway across town without causing many sociological disturbances such as right-of-way purchases, business displacements, and others. It is not our intention to deny reality, but for convenience sake do not dwell on a discussion of these points.

4. The story in the unit is not designed to be read in one sitting. Rather, each part of the story introduces a problem and asks the children to propose solutions. Then the story is read to compare their solutions with the ideas that are in the story. Once this is accomplished, the children are introduced to another problem situation and allowed to once again come up with their own conclusions. At no time should the children be allowed to think that the solutions of the story are the "final authority."

5. There is a danger that the teacher and the students may come to consider one point on the blackboard grid as always being the origin. Make sure that the point (0,0) is not always the lower left-hand corner. In general,

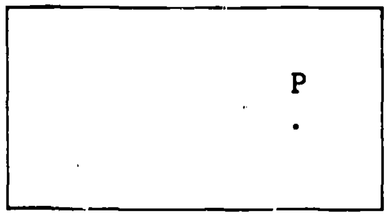
the origin should, for this unit, be picked somewhere in the lower left portion of the grid, but <u>not</u> on the edge of it.

6. Activities are designed to be led by the teachers. But the children should be allowed to work at their own rate. When they are finished, let them compare their answers to those of their partners or friends and debate their disagreements.

7. If the teacher will exploit it, this unit offers virtually unlimited opportunity for each child to become involved. Each child must be given as many chances to participate as possible. This can only be accomplished when the class is allowed to work in small groups. If the teacher is in a hurry to tell the answers or let the "sharp" kids give away the answers, the unit will not serve the needs of most students. Past experience has shown that most teachers and children enjoy this unit very much. So relax, become involved, and give it time to succeed.

Teacher Background, The Problem of Locating a Point

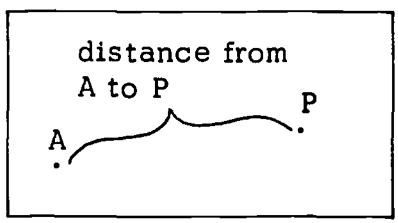
Suppose a point P was marked on a sheet of blank paper and you were asked to tell where that point was on the sheet of paper. What answer would you give?



Where is this point?

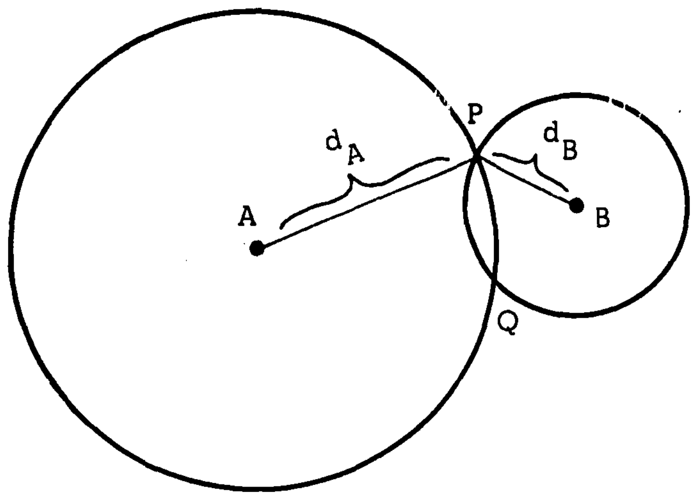
If you were a child you might put your finger tip on the point and say, "It's right here."

As an adult you might want to give a more sophisticated answer, and introduce a systematic way of locating points which would enable you to locate other points as well. For example, you might mark a special arbitrary point (call it A) on your paper. Then you could measure the distance from point A to point P and say that point P was a certain distance from point A.



This answer is insufficient, for there are an infinite number of other points on the plane all the same distance from point A. Construct a circle by placing a compass at A and using the distance from A to P as the radius. Every point on this circle is the same distance from point A. So your description of point P would also fit any other point on the circle.

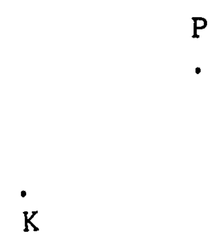
We might help the situation by picking a second special arbitrary point. Call it B. Let the distance from P to A be d_A, and the distance from P to B be d_B. Construct circles of radius d_A around A and of radius d_B around B.



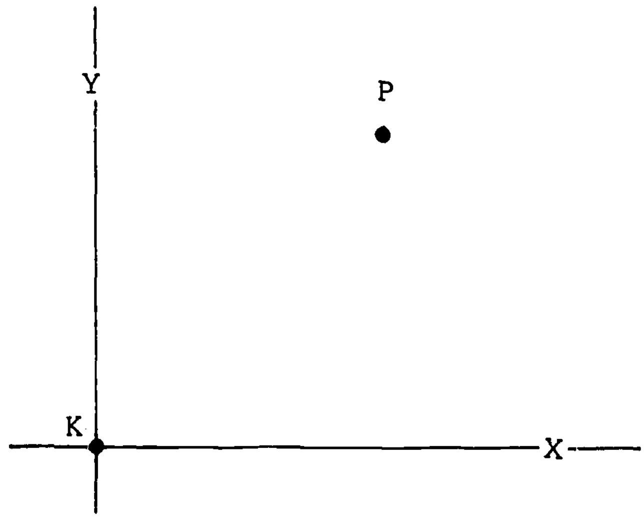
Now, P is at the intersection of the circles. But, unfortunately, so is Q. This is certainly an improvement over the previous situation, but it's not good enough. Even if we know A, B, d_A, and d_B, we wouldn't know where P was exactly.

What would happen if you located another point called C and constructed another circle? The three circles would intersect in only one point, the point P. So, by selecting 3 special points (A,B,C) and by knowing the distances from P to each of the points, we can uniquely locate P.

There is another much more common method of locating an arbitrary point P in a plane. Pick a special arbitrary point K in the plane.

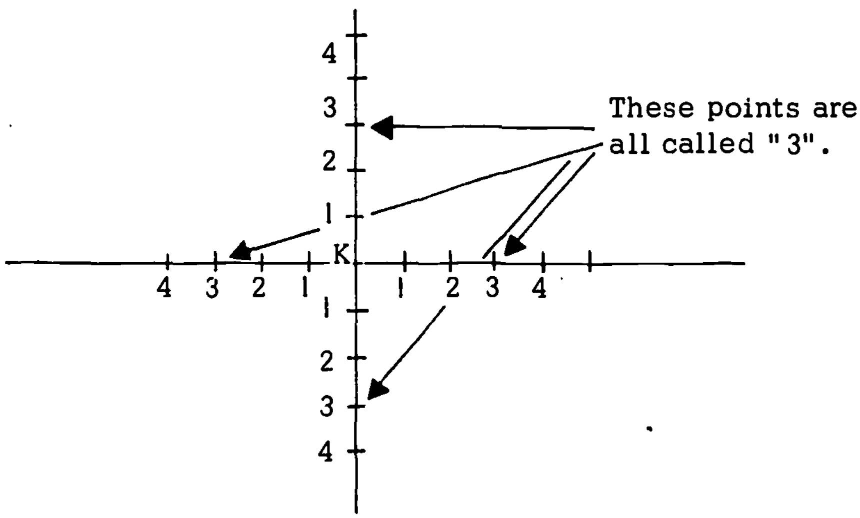


Then draw two different lines which intersect at the point K. The lines need not be perpendicular, but the lines used in the drawing below are. Call one line the X axis and the other the Y axis.

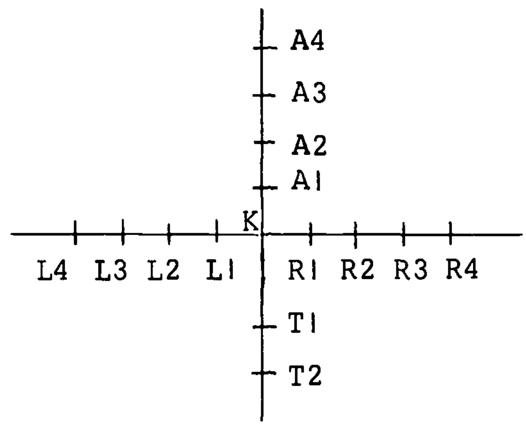


Once we decide on a unit of length, we can begin at K and lay off line segments on each line. Call the point K "0" on each axis and label the points sequentially along the X and the Y axis.

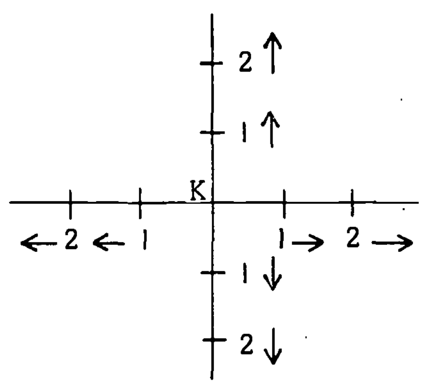
Almost immediately you may say, "Wait a minute. There is going to be some confusion here. Several of your points are marked with the same name."



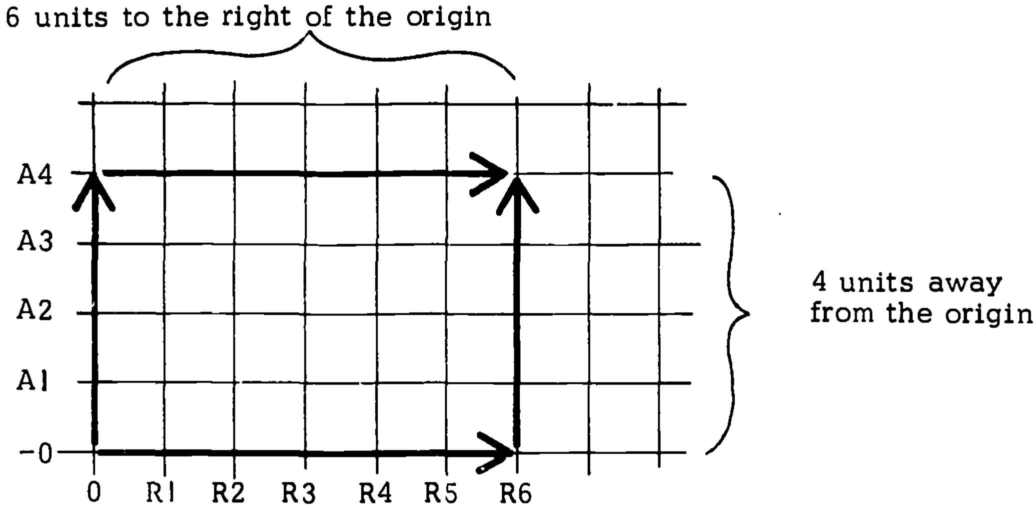
It is true that such labeling is ambiguous, so other identification marks are needed. One suggestion might be to think of directions on the X axis as being 'right' and 'left' and the direction on the Y axis could be 'away from' and 'toward'. Then we could label the four points which we have previously called "3" as "Right 3, Left 3, Away 3, Toward 3" respectively. A convenient shorthand could be R3, L3, A3, T3. We would then have:



Another way to indicate which "3" you were talking about would be to use arrows to indicate the position of the point from the origin.

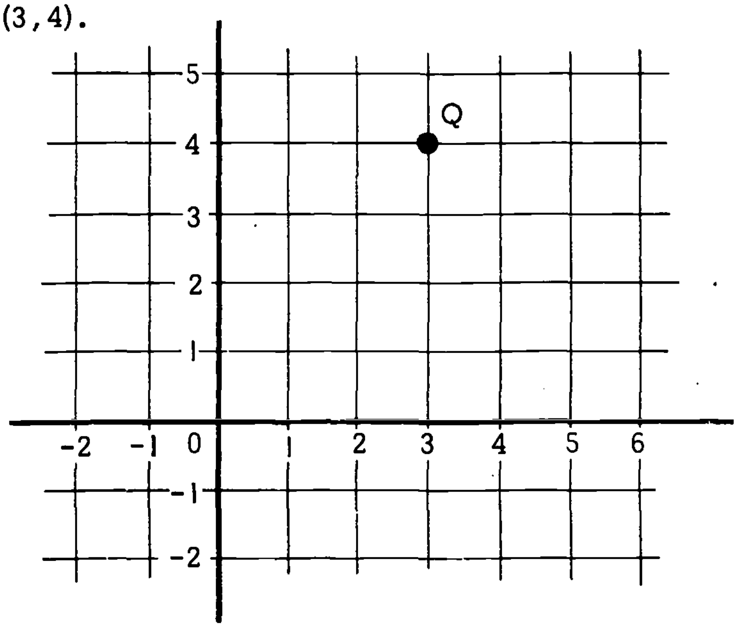


Once you have layed off and marked the two axes, you are in a position to say where P is located. The heavy arrows in this diagram indicate that P is 6 units to the right and 4 units away from the origin.



And so we give an address to the location of point P. The address is (6 right, 4 away from). You could write it as (R6, A4) or as (6$\longrightarrow$, 4$\uparrow$). The terminology doesn't matter as long as you can be sure that everyone else you communicate with will be able to understand what you mean.

For this reason, we agree, early in the unit, to name a street address and then an avenue address when locating a point. For example, the point Q in the diagram below would by convention have address (coordinates) (3,4).



THE STORY OF SQUAREVILLE

Mayor Brown was a very busy man. Someone was always telephoning him. But this didn't bother him nearly as much as all the driving he had to do in Squareville. It always took him so long to go from the City Hall to the Courthouse, which was way across town.

One night he came home in such a grouchy mood that he slammed the door behind him. The "B A N G" startled Tommy, who asked, "What's wrong, Dad?"

"It's that traffic situation again! Every time I go across town to the Courthouse I get caught in a traffic jam. It takes me longer every day," replied Mr. Brown.

"Just how far is it?" asked Tommy.

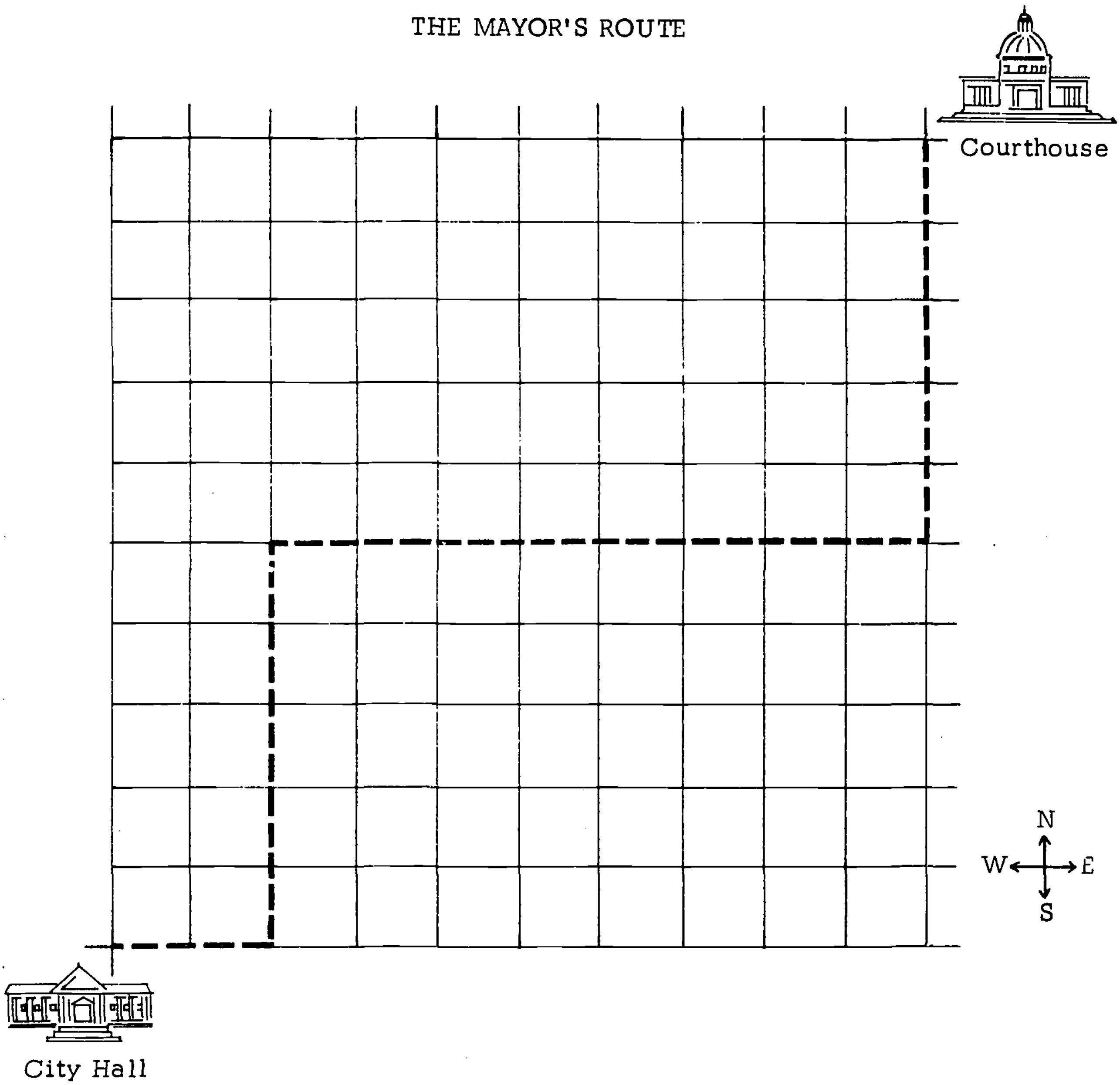
Mayor Brown stopped for a moment. "Come to think of it," he went on, "I really don't know. I drive it every day but I've never counted the blocks. You've aroused my curiosity. Why don't you get your map of Squareville and we'll count the blocks."

Tommy chuckled to himself as he went for his map. He'd soon have his Dad in a good mood.

He quickly found his map and handed it to his Dad. "Show me the route you take from City Hall to the Courthouse," he said.

"This is the route I take," said the Mayor. "It seems as if there are a million stop lights and big trucks. I've tried other routes, but I can't find a shorter one. Maybe you can suggest another route?"

Tommy began to study the map carefully.



Activity 1 - The Length of the Mayor's Route

1. Draw the Mayor's route on the large classroom map of Squareville, reminding the children that he is driving.

2. Have each child in the class "guess" how many blocks long the Mayor's route is. Insist that each child write his estimate on his own map of Squareville.

3. Have some child actually count the blocks along the Mayor's route. Each child should then record this numeral next to his estimate. A chart of the type shown below could easily be constructed.

Route	Estimated Length	Actual Length
Mayor's		

4. Have another child use a different color to show an alternate route from City Hall to the Courthouse. Make sure that these routes go along streets and avenues and do not cut across blocks.

5. Have each child estimate the length and fill in his chart. Ask questions such as: "Does one route appear to be longer than another? Why?"

6. Measure the length of the route and record the answer.

7. Follow this activity with Worksheet 1. Each child should have a map of Squareville.

Worksheet 1 Name________________________

1. Look at your map of Squareville.

2. Use the colors below to draw different routes from City Hall to the Courthouse. Give an estimate.

ROUTE	ESTIMATED LENGTH (in blocks)	ACTUAL LENGTH (in blocks)
RED		
GREEN		
BROWN		
BLUE		
ORANGE		

3. Now, take several routes North and East.

4. What did Tommy and his dad discover about the shortest routes (in blocks) from City Hall to the Courthouse? ________________________

____

5. What is the longest distance of any route when you go North and East or East and North? ________________________

____

Activity 2 - Naming Streets and Avenues

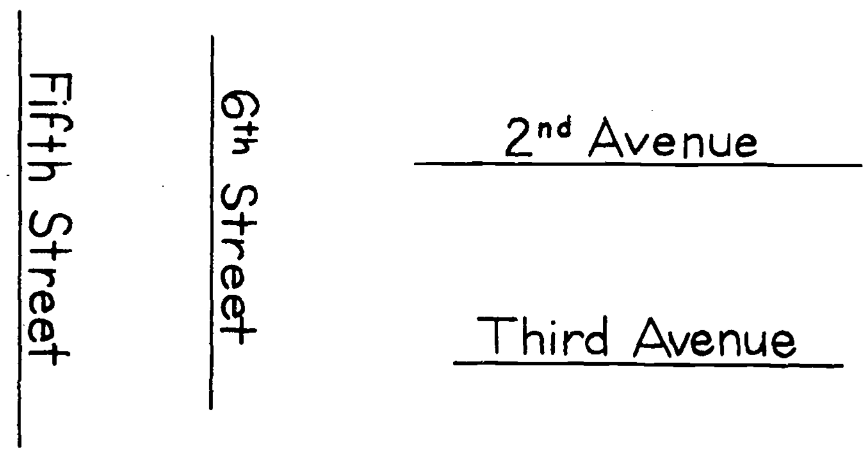
1. Draw each child's attention to the large map of Squareville.

2. Ask each child to take a small piece of paper and write on it the name of a street or an avenue. Ask them to use ordinal names, such as zero, first, second,..., tenth.

FIRST STREET or FOURTH AVENUE

Have each child put his name on his paper and drop it into a hat.

3. Make several draws. After each draw, have the child whose paper was drawn come to the chalkboard, draw a chalkline along his street or avenue, and label it.



Since there will probably be considerable disagreement as to what the name of any particular line should be, lead the children to a convenient manner of ordering - avenues running E to W, streets running N to S; streets increasing numerically to the right, and avenues increasing numerically upward. Explain that this is desirable for convenience sake - but is purely an arbitrary decision.

Activity 3 - Locating Intersections (Coordinates)

1. Let some child suggest a street address and an avenue address.
2. Ask each child to write that address on his paper. Have everyone agree to write the street address first.
3. Write the suggested address on the chalkboard.

Third Street and Fifth Avenue or 3rd St. and 5th Ave.

4. Have some child locate that intersection on the chalkboard map.
5. Have another child write the long address on the chalkboard map.
6. Ask if someone knows of a shorter or easier way to write

3rd Street and 5th Avenue

7. If there are no shorthand methods, suggest writing

(3, 5)

8. Review the idea that this means

3rd Street and 5th Avenue

and <u>not</u>

3rd Avenue and 5th Street

9. Have the children name several intersections of this form using only ordered pairs such as (3, 5).
10. Choose any two digits from 0-16. Arrange these numerals in one way, then reverse the order. An example would be: (3,6) and (6,3). Have these intersections located.

Activity 4 Find the Secret Intersection

This activity lends itself to small group participation thereby providing each student with a maximum opportunity to apply his knowledge about the coordinate system.

The activity can begin by designating one child from each group to be the leader. This child secretly chooses an intersection on Squareville. On a sheet of paper he writes the address of this intersection. Then he asks the other members of his group to guess the intersection he chose.

The three game situations which are mentioned in this activity should serve as a springboard for other ways of playing the game.

Method I

1. The leader picks a point and writes the address on a sheet of paper. Suppose he picks (5, 4).
2. He asks the students to find the secret intersection. These are the rules that must be followed.

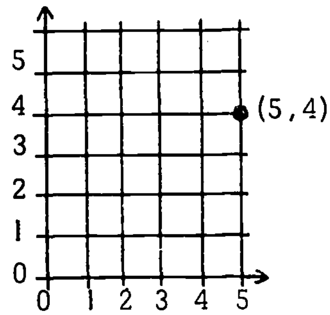
   "Pick any intersection. If you make the correct choice, you get to to be the leader. If you make the incorrect choice, some other child gets to make a choice."
3. The students then take turns guessing. They indicate their choice by asking something like, "Is the intersection (6,7)?"
4. When someone picks the correct intersection, the leader must verify the correctness of the choice by showing the address he wrote on his paper.

Method II

The student might attempt to find the secret intersection by asking questions that can be answered by asking questions that can be answered by either "Yes" or "No". Each "Yes" answer entitles the questioner to another turn.

Example

1. Suppose the intersection is at (5,4).



2. The student asks "Does the intersection lie north of 3rd Avenue?"

3. Since the intersection is actually north of 3rd Avenue, the answer is "Yes". If the secret intersection had been on or south of 3rd Avenue the answer would have been "No".

4. As the student continues to ask more "Yes" or "No" questions, he will develop his own technique for reducing the number of intersections which could qualify as the secret intersection.

5. The rules might be that the student who actually locates the intersection is the winner of the game.

Method III

1. One student is allowed to keep asking "Yes" or "No" questions until he locates the secret intersection. His score is the number of questions it took him to locate the point.

2. Then another student tries to locate a different point. He gets a score.

3. The winner is the student who asked the fewer questions. He gets 1 point for this part of the game.

4. The students locate more intersections. The first player to score 5 points wins the game.

Activity 5 Go Moko

The game "Go Moko" provides good practice in using coordinates.

1. Divide the class into two teams. Designate one team as the (X)'s and the other as the (0)'s. Use the chalkboard map of Squareville.

2. The object of the game is to gain control of any four successive intersections which lie in a straight line horizontally, vertically, or diagonally. The first team to do so wins the game.

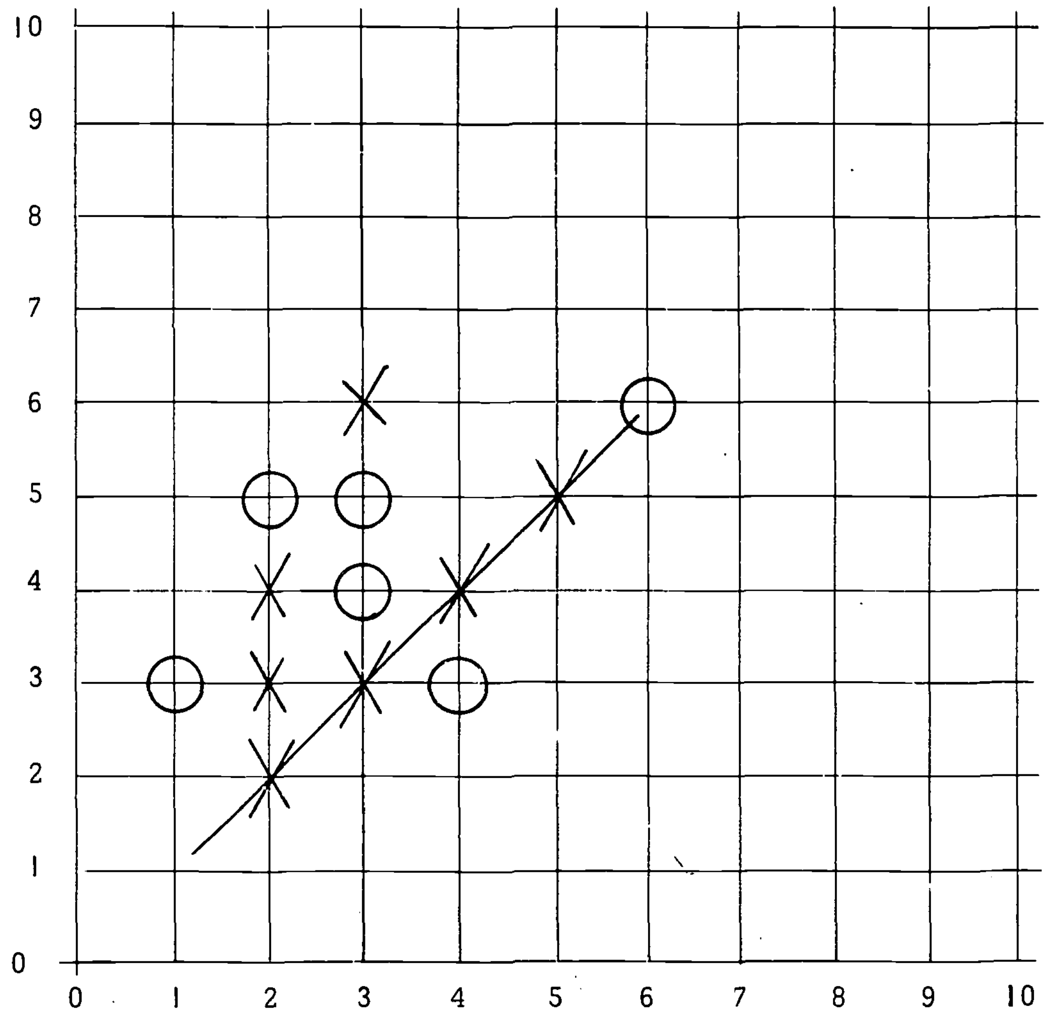
   The children from each team must take turns "calling out" intersection addresses, always naming the street address and then the avenue address. As each address is called the teacher marks that point with an X or an 0 depending on which team selected the intersection.

3. A team may lose its turn by:
   a) one member calling an intersection out of turn.
   b) one member conferring with another,
   c) calling an address improperly, (such as giving the avenue address before the street address).

4. After several games have been played a time limitation can be inserted into the activity. Each child can be given only a few seconds to announce his address. If he fails to announce it within the time limit, he loses his turn. The other team then proceeds to announce an address.

Teacher's Reference Sheet

Sample Scoresheet from a Game of "Go-Moko"



While explaining the game to the children, show such a sample scoresheet to the children. Here the X team has won because they have four (X)'s in a row at the points (2,2), (3,3), (4,4) and (5,5).

Ask the children if they could play this game without using Squareville, but simply by naming pairs of points. How would we then tell if the (X) team won?

Activity 6 Salvo

This game, Salvo, provides opportunity for additional practice in locating coordinates. Each player is given a grid on which he secretly marks the location of his three "ships". The opponents, in turn, fire a salvo of 3 shots, in an attempt to locate and "sink" the enemy's ships. A shot is fired by calling out or naming an intersection. An example would be (3,2).

Initially the game may be introduced by having the class play against the teacher trying to sink her ships. Later the game may be played by pairs of children at their desks. Other children may be involved by becoming "allies" of the opponents.

Salvo is really quite a sophisticated game. As the children become more familiar with the basic rules, see if they can work out rules of strategy. Some of these might involve where to place their boats and where to make shots to have the best chance of winning.

> Note: To start with it might be easier to use only part of the 10 x 10 Squareville grid, an area 5 units high and 10 units wide.

Procedure:

1. Have each player secretly mark, on his own map, the location of his three ships:
   a. a cruiser, covering 5 points on a line,
   b. a cargo ship, covering 3 points on a line,
   c. a destroyer, covering 2 points on a line.
2. The players alternate turns. One player begins by firing three shots (naming three different locations) at his opponent's ship. Since he is unaware of their position, he must fire his first shot in an exploratory manner.

3. Both players record all shots on their maps. This can be done by placing an X at the place where the shot hit.

4. If a hit is scored on any ship, the owner of that ship must immediately acknowledge the hit. He must also tell which of his ships was hit. If a player misfires and hits his own ship, he must announce this fact.

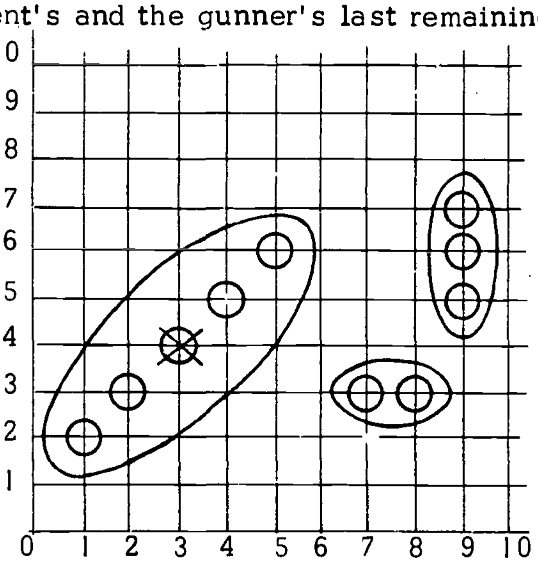
5. To sink a vessel, all points of that vessel must be hit. When a ship is sunk, the owner must announce the "sinking".

6. Sometimes opponents may discover that they have each placed ships on the same coordinates. In such cases, the player must decide if he wishes to fire upon himself as many times as necessary to sink his opponent. He could, for example, hit his own cruiser (which covers 5 points) four times and still have one point remaining. This could keep him afloat. The winner is the last person to have any ships remaining afloat.

7. The number and size of the ships is arbitrary. As children become expert at the game they may suggest new rules for the game.

8. A stalemate is reached when the firing of one last shot would result in the sinking of both the opponent's and the gunner's last remaining ship.

Player A's ships



 = denotes a hit at (3,4)

Activity 7 Finding Destinations

1. Give each child a 10 x 10 map of Squareville.

2. Comment to the children:

   "As mayor of Squareville, Mr. Brown is expected to travel to all parts of town. It is important that he knows how far it is from his office at (0,0) to any corner in Squareville. Since he travels only east, north, or a combination of north and east, he must quickly determine the driving distance."

3. Have one child come to the chalkboard map and use colored chalk to mark a corner on Squareville.

4. Then have each child predict the driving distance from (0,0) to this corner. Movement must be only east, north, or north and east. Each child should write his prediction on his Squareville map near the chosen corner.

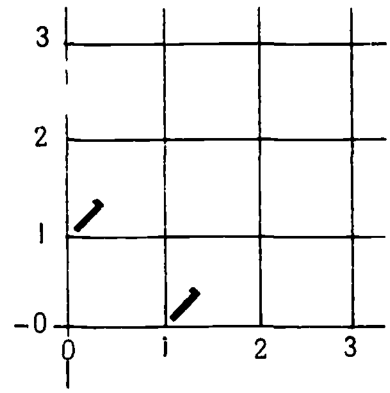
5. Have the children check their predictions by counting the blocks.

6. Comment to the children:

   "So far we have predicted and checked the driving distance from (0,0) to one corner on Squareville (at City Hall). Let us see if we can discover a method that works for all corners."

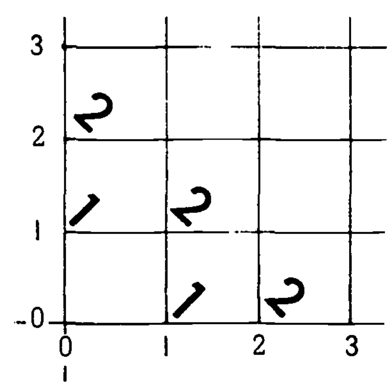
   "Suppose we begin at the corner of (0,0) and travel one block. Where would you be? (0,1) or (1,0)."

   "On the map of Squareville write a figure 1 on all the corners that are 1 block from (0,0)."



7. Now see where you would be if you started at (0, 0) and took a trip of two blocks. Remember, you can go only E, N, or N and E. Where would you be? (2, 0), (0, 2), (1, 1)

Place a 2 near those corners that are two blocks from (0, 0).

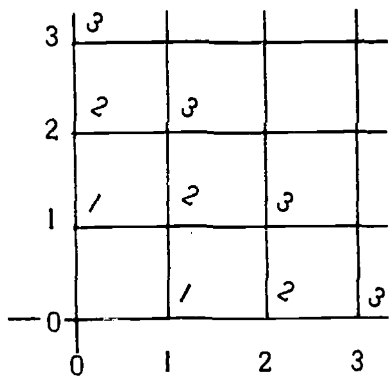


For chalkboard demonstration, use different colored chalk.

8. Have the children continue the activity, working independently, until they think they see a pattern. When they do, they should check this privately with the teacher.

Some of the patterns might be:

a. If the length of the route to each corner is correctly marked a pattern will emerge. All the numerals will fall in straight lines.



b. Choose any corner. Add the street address and the avenue address. The sum is the length of the route in blocks from (0,0).

Street Address + Avenue Address = Distance from (0,0)

c. Given a number of blocks to travel, the number of corners that can be reached is always 1 more than the block length of the route to be traveled.

Length of Route (in blocks)	Number of Corners that can be reached

Pupil's Copy

Worksheet 2 Name________________

Use your map of Squareville. Do you see a pattern? Fill in the chart if you need to.

Lengths of Route (in blocks)	Number of Destinations
4	
7	
6	
2	
9	

Problem 1

Suppose you are told to travel a given number of blocks. You are allowed to travel only E, N, or E and N. Do you have a quick way of determining how many different destination points there could be?

____

____

____

Problem 2

Pick any corner in Squareville. What is a quick way of determining what the driving distance is from (0,0) to your corner?

____

____

____

Teacher's Copy

Name________________________

Worksheet 2

Use your map of Squareville. Do you see a pattern? Fill in the chart if you need to.

Length of Route (in blocks)	Number of Destinations
4	
7	
6	
2	
9	

Problem 1

Suppose you are told to travel a given number of blocks. You are allowed to travel only E, N, or E and N. Do you have a quick way of determining how many different destination points there could be? The number of destination points is always 1 more than the route distance in blocks.

Problem 2

Pick any corner in Squareville. What is a quick way of determining what the driving distance is from (0,0) to your corner? The street number plus the avenue number gives you the length of the route in blocks.

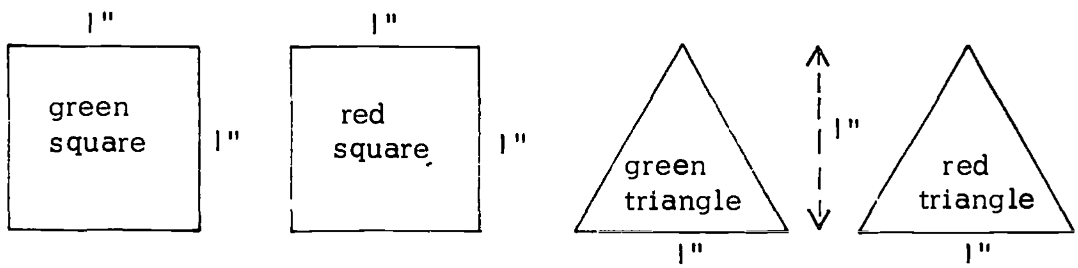
Teacher's Reference Sheet

The following student worksheets are intended to familiarize the student with locating and naming points in a coordinate system by having each student take "walks" from point to point on grid paper. It is very essential that each student comes to realize that he can assign the (0,0) address to any intersection on his grid. All other addresses are then assigned in relation to the (0,0) address.

Materials needed:

Each child should have the following materials:

1) about 20 of each of these form shapes constructed from tagboard or colored paper,



2) a box of crayons.

Procedure:

It is preferred that the students be allowed to work individually or in small groups as they do these worksheets. The directions are:

1) Imagine that you are going to take a walk on your grid. Use a color crayon to mark one intersection as your starting point and another intersection as your ending point.

2) Now take your walk, being sure to walk only on the grid lines taking "one step" at a time. A step is the distance from one intersection to the next along the same grid line.

3) As you take each step, do these two things:

   a) Use your color to mark that step on your grid.

   b) Pick up the shape that goes with that step. Here is the code you should follow:

      -1) For every step to the right ($\longrightarrow$), pick up a green square.

      -2) For every step to the left ($\longleftarrow$), pick up a red square.

      -3) For every step away from you ($\uparrow$), pick up a green triangle.

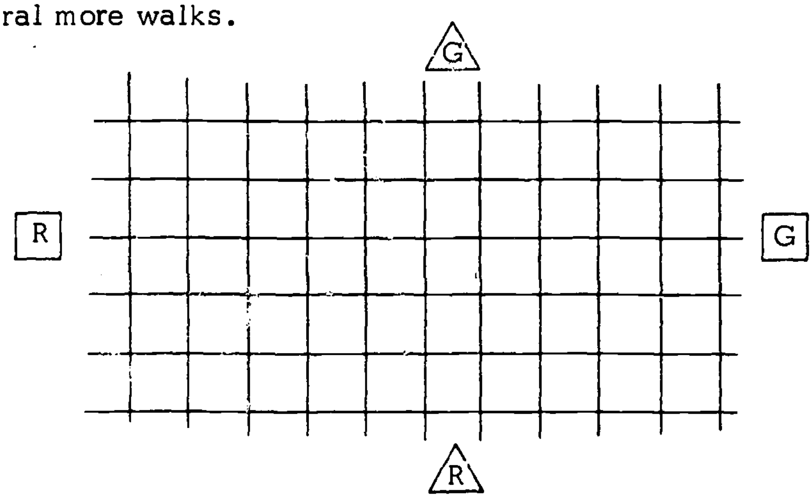
      -4) For every step toward you ($\downarrow$), pick up a red triangle.

4) When you have finished your walk, you will have the path marked in crayon and a handful of shapes. Count the pieces and give your walk that name.

   Example:

   6 $\boxed{R}$
   3 $\boxed{G}$
   4 $\triangle$R
   1 $\triangle$G

5) Take several more walks.



Worksheet 3 Name ____________________

1. Imagine that you are going to take a walk on your grid. Use a color crayon to mark one intersection as your starting point and another intersection as your ending point.

2. Now take your walk, being sure to walk only on the grid lines taking "one step" at a time. A step is the distance from one intersection to the next along the same grid line.

3. As you take each step, do these two things:

   a) Use your color to mark that step on your grid.

   b) Pick up the shape that goes with that step. Here is the code you should follow:

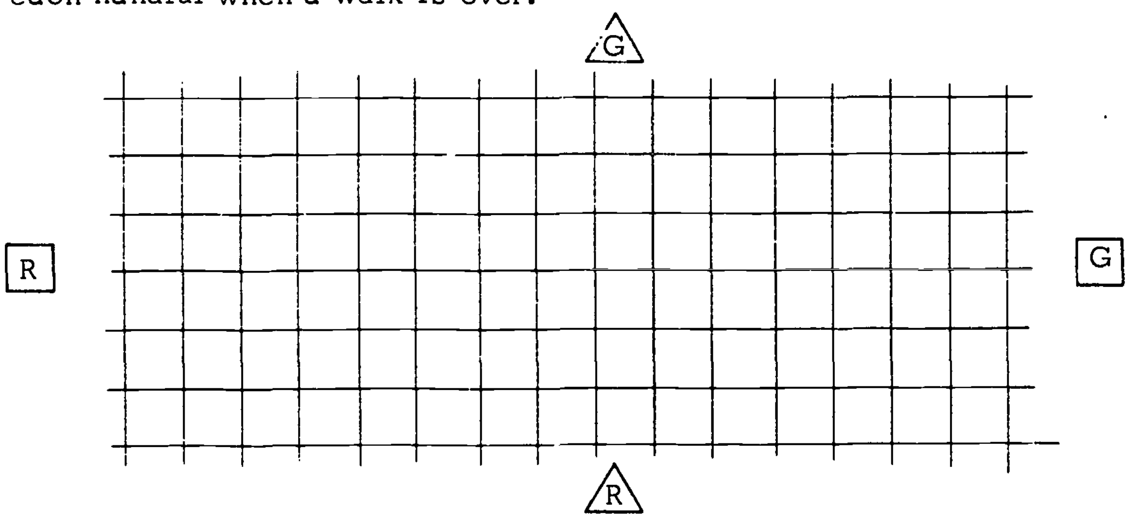
      1) For every step to the right (→), pick up a green square.

      2) For every step to the left (←), pick up a red square.

      3) For every step away from you (↑), pick up a green triangle.

      4) For every step toward you (↓), pick up a red triangle.

When you have finished your walk, you will have some squares and triangles. This handful describes the walk that you have just taken. Use a different crayon for each walk. Take many different walks. Save each handful when a walk is over.



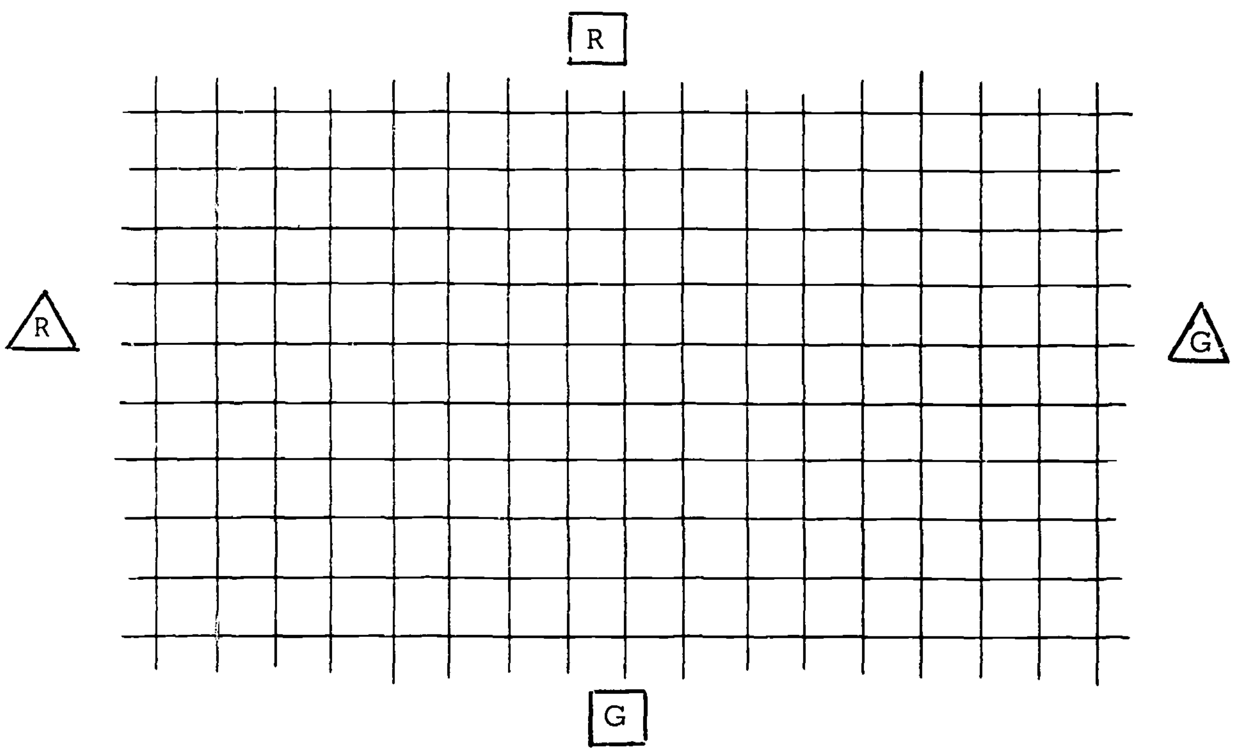
Worksheet 4 Name ____________________

1. Take four walks on the grid below. Use a different crayon to mark each route. Be sure to pick up the pieces which go with the steps you take.

   * Note: See the grid below for the new rules.

2. After you finish each walk, record the number of squares and triangles you have in this chart.

	crayons	△R	△G	□R	□G
First walk					
Second walk					
Third walk					
Fourth walk					



Worksheet 5 Name ____________________

1. Mark a starting point and an ending point on the grid below.

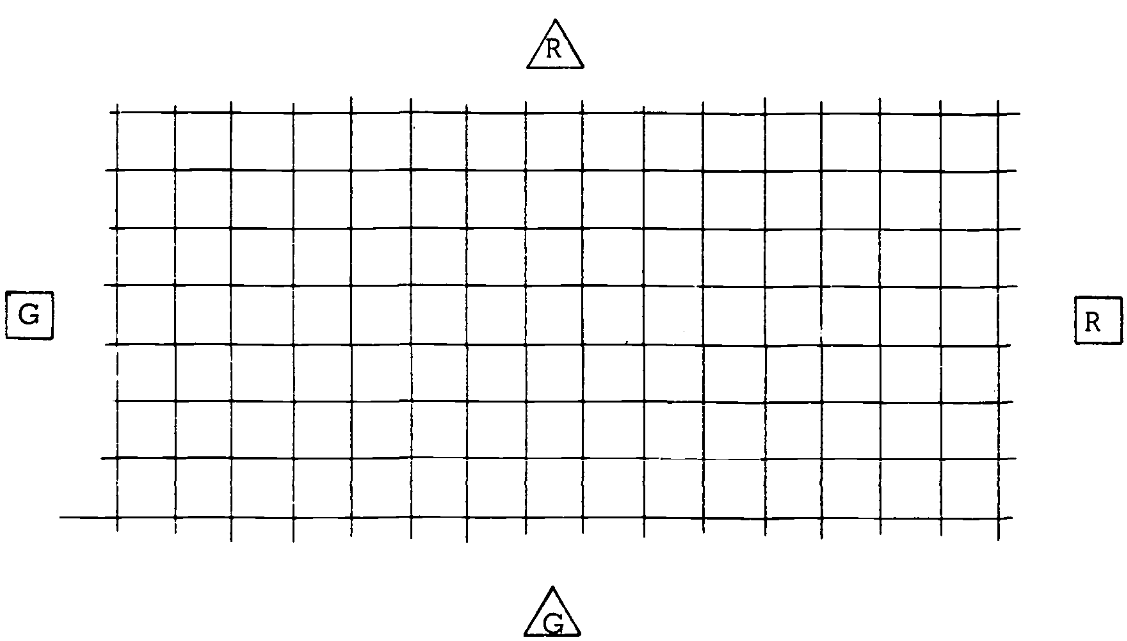
2. Choose a crayon and take a walk between the two points you chose. The grid shows the rule for picking up the pieces. Be sure to record your walk.

3. Then take another walk. Use the same starting point and the same ending point as for your first walk. Be sure to use a different crayon. Record your walk.

4. Take many other walks in this way.

color of walk	$\boxed{R}$	$\boxed{G}$	$\triangle$R	$\triangle$G

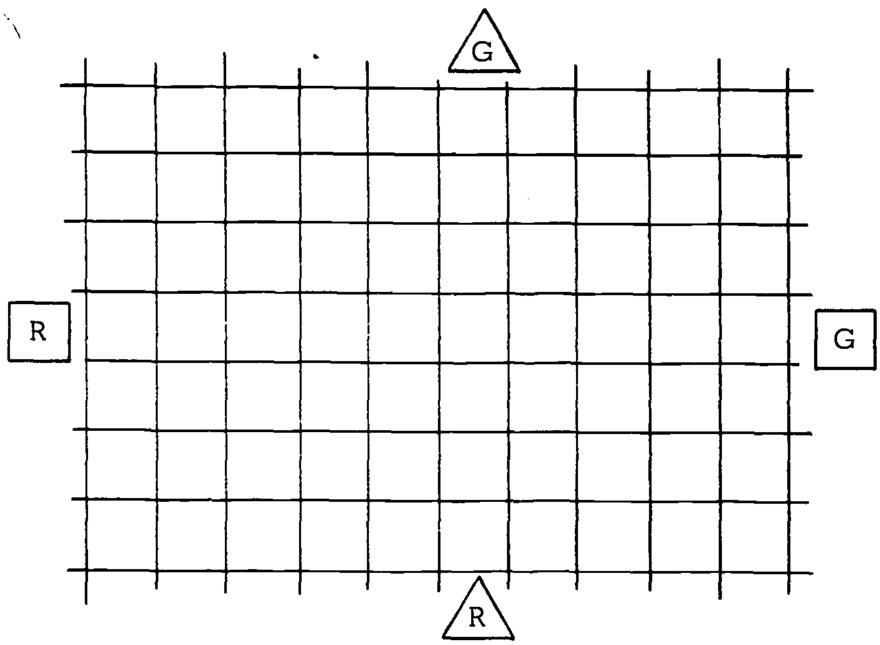
color of walk	$\boxed{R}$	$\boxed{G}$	$\triangle$R	$\triangle$G



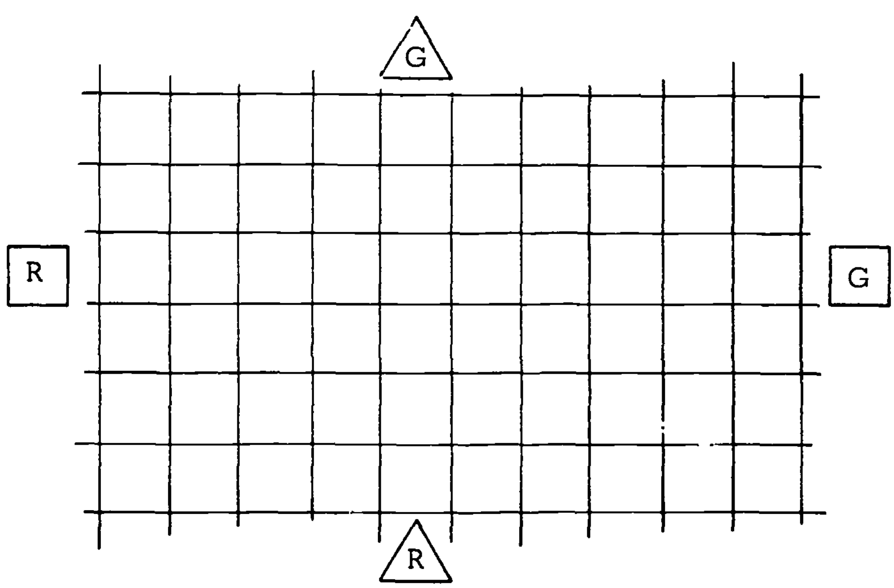
Worksheet 6 Name______________________

1. This worksheet contains information about four walks. Use this information to show the path of each walk. Show the walks on the proper grid.

a) Red Walk 5 $\boxed{G}$ 3 $\boxed{R}$ 3 △G 6 △R

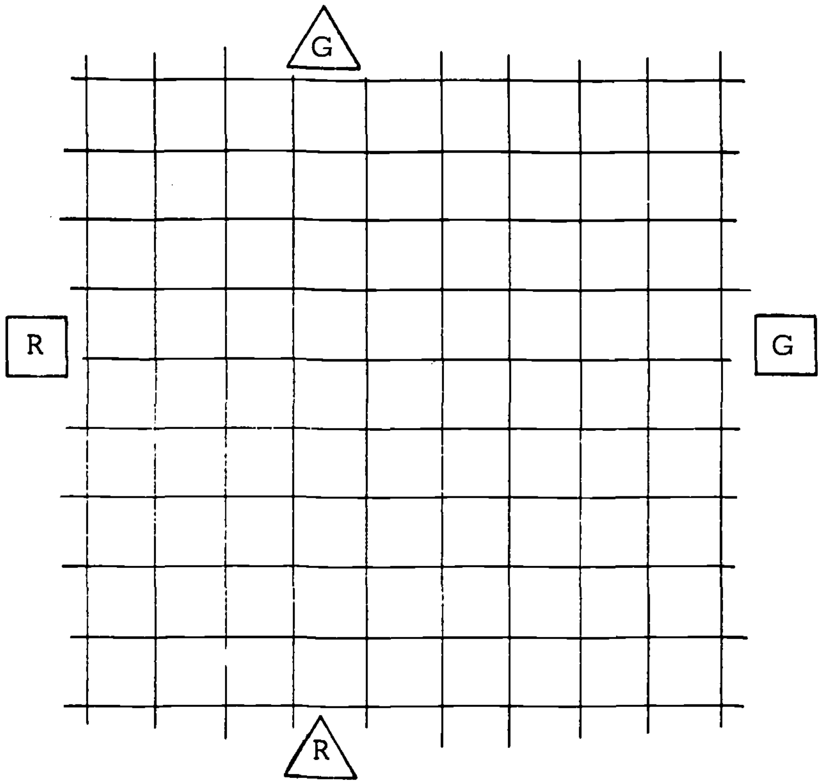


b) Blue Walk 3 $\boxed{G}$ 1 $\boxed{R}$ 0 △G 3 △R

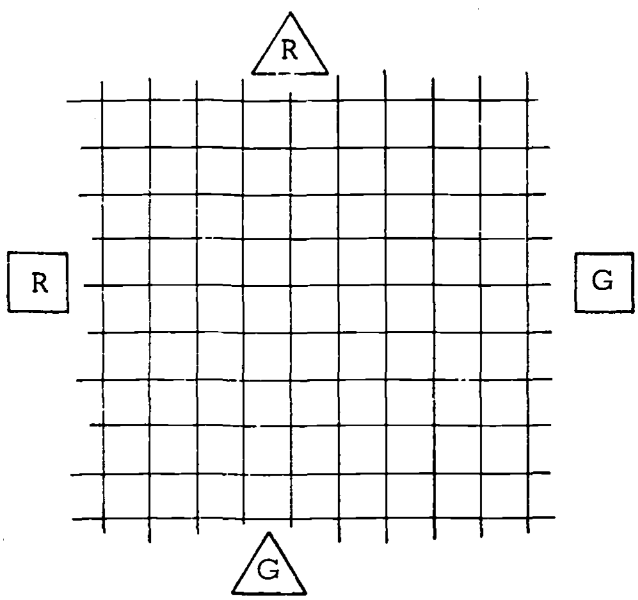


Name________________________

c) Green Walk 6 $\boxed{G}$ 4 $\boxed{R}$ 2 $\triangle G$ 5 $\triangle R$



d) Orange Walk 2 $\boxed{G}$ 0 $\boxed{R}$ 1 $\triangle G$ 4 $\triangle R$



Worksheet 7 Name ________________________

1. Mark a starting point on your grid. Take a walk, but be sure that at the end you have:

   2 more green squares than red squares

   and 3 more red triangles than green triangles.

   Fill in this chart for your walk.

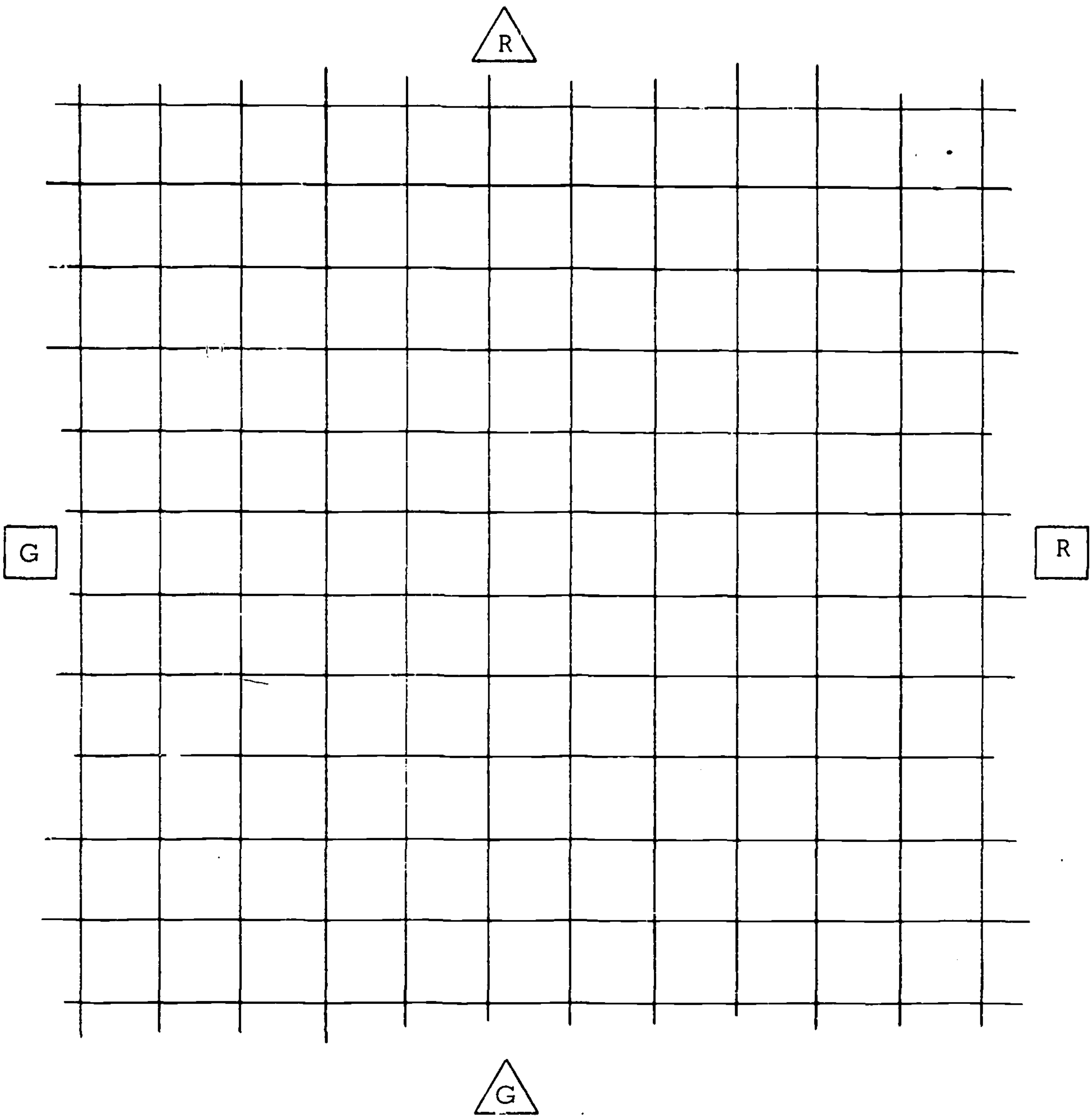
Color	$\boxed{R}$	$\boxed{G}$	△R	△G

2. Now try to draw some other walks that start from the same point and have the same kind of a handful (always 2 more green squares than red squares and 3 more red triangles than green triangles). Fill in this chart for those walks.

color of walk	$\boxed{R}$	$\boxed{G}$	△R	△G

3. Where do all these walks end? ________________________

Worksheet 7 continued Name ____________________

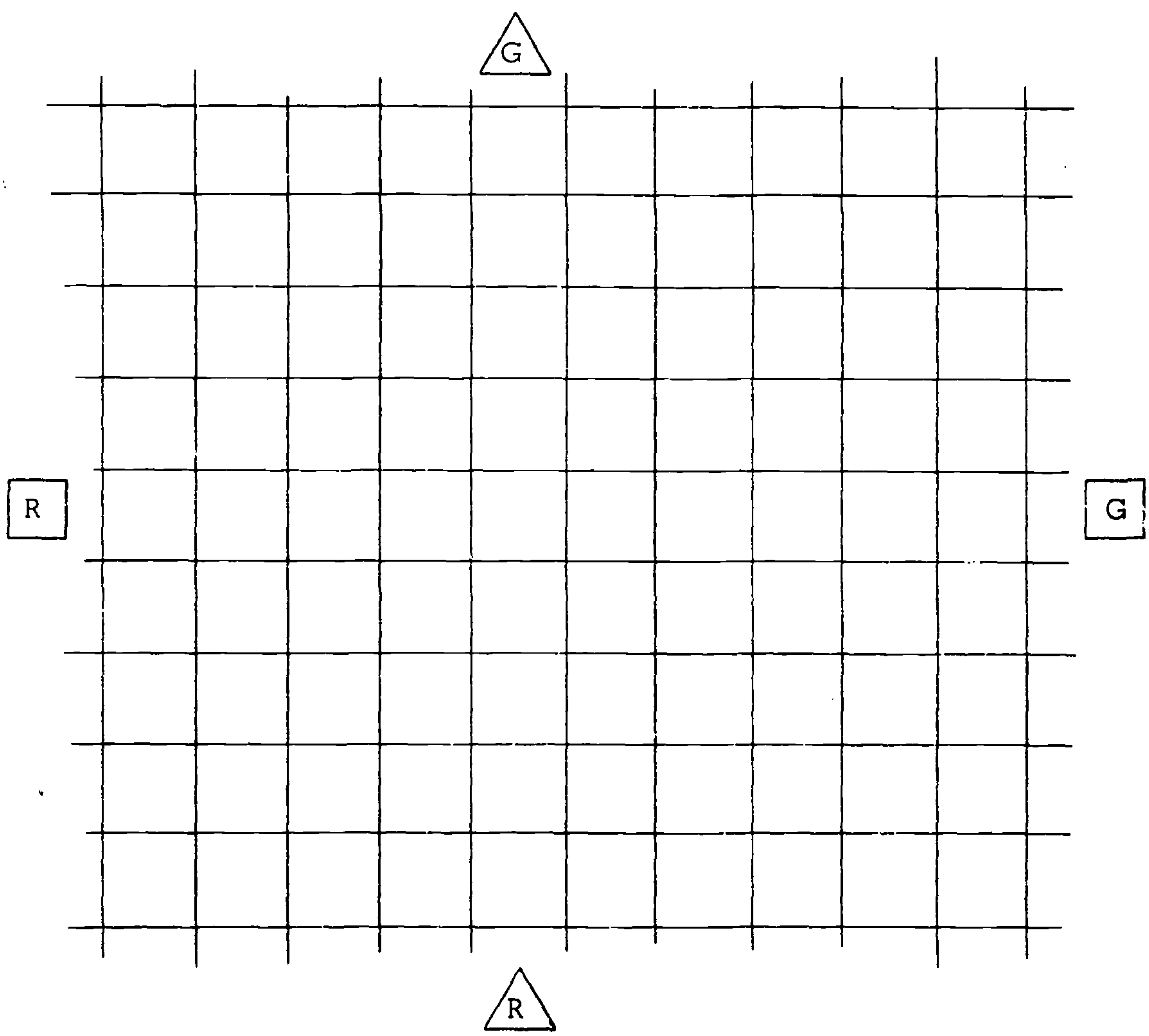


Worksheet 8 Name ____________________

Begin each walk in this chart at a different part of the grid at the bottom of this page.

color of walk	$\boxed{G}$	$\boxed{R}$	$\triangle G$	$\triangle R$
	3	4	3	0
	1	2	8	5
	5	6	5	2

Which walks have the same shape? ____________________



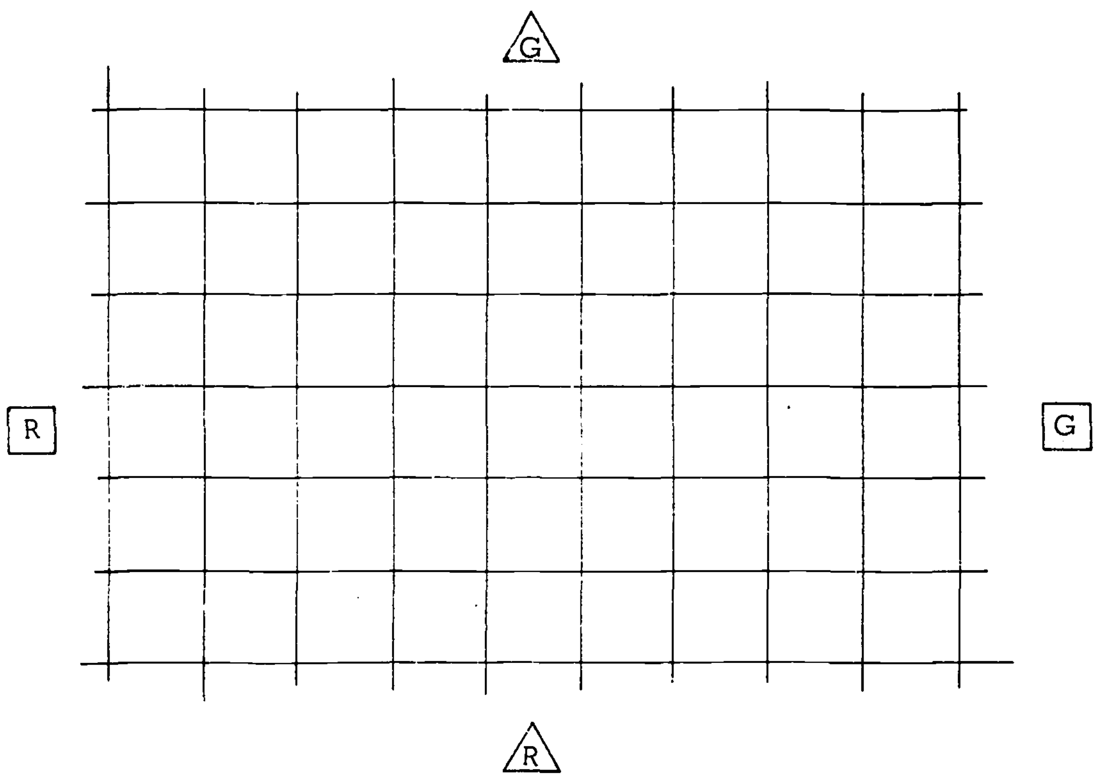
Worksheet 9 Name ______________________

1. Pick a starting point. Take a walk that has 1 green square and 1 red square. This walk does not have any triangles. How many steps are you from your starting point when you finish? __________

2. Now take another walk from a new starting point. This walk should have 3 green squares, 3 red squares, 2 green triangles and 2 red triangles. Where do you finish? Does anything special happen when you have the same number of both color squares? __________

____

of both color triangles? ______________________________



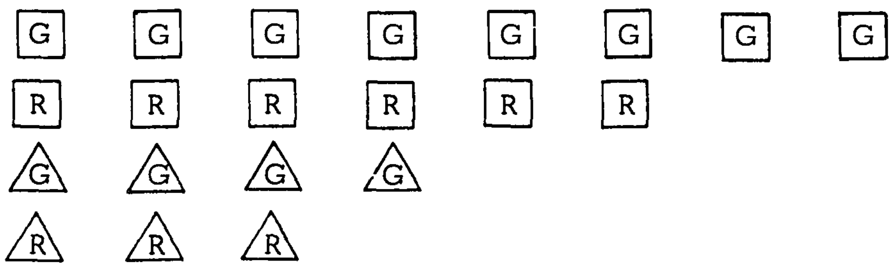
Worksheet 10 Name ____________________

1. Here is the record of a walk.

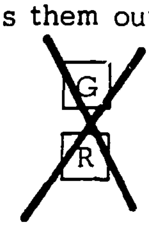
Yellow walk	8 G	6 R	4 G	3 R

2. We can call this the <u>yellow</u> walk or we can call it the <u>8 green square, 6 red square, 4 green triangle, 3 red triangle walk</u>. That is a long name, isn't it?

3. Suppose we record our information like this:



4. Now match a green square with a red square and a green triangle with a red triangle. Cross them out as you match them like this:



5. What shapes remain after you have finished your matching? ________

6. Here is another walk. Match its steps.

Blue walk	5 $\boxed{G}$	3 $\boxed{R}$	9 $\boxed{G}$	8 $\boxed{G}$

Worksheet 11 Name________________________

1. I took a walk that had

1 more red square than green square

and 3 more green triangles than red triangles.

Now, answer these questions.

a. How far did I walk to the left? __________

to the right? ______________

b. Away from me? ____________

toward me? _______________

2. Suppose we agree to describe each walk by

first describing the right and left directions and, then, giving the away from and toward directions.

3. I took a walk that had

3 more green squares than red squares and

2 more red triangles than green triangles.

Underline the correct answer:

Green squares mean steps to the (left) (right) (away from me) (toward me).

Red squares mean steps to the (left) (right) (away from me) (toward me).

Green triangles mean steps to the (left) (right) (away from me) (toward me).

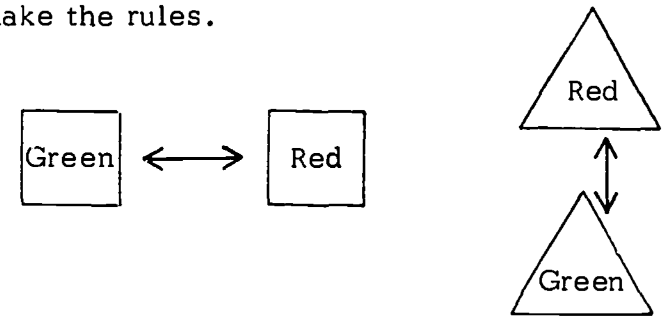
Red triangles mean (left) (right) (away from me) (toward me).

4. Can you be sure of the answer you gave? Yes No

Worksheet 12 Name________________________

1. Usually it is more fun if you can make your own rules, but on this worksheet I will make the rules.

RULES



2. I take a walk of

3 more red squares than green squares

and

4 more green triangles than red triangles.

A shorthand way of describing this walk is to write (3 red, 4 green).

3. When using shorthand we write the "left-right" steps first and then the "away from - toward you" steps.

4. Record some walks in this chart and then write the shorthand name for the walk.

R (square)	G (square)	R (triangle)	G (triangle)	Name of walk
3	4	2	0	(1 green, 2 red)

Worksheet 13 Name________________________

DO THIS WORKSHEET, THEN CHECK YOUR ANSWER WITH A FRIEND.

1. Make your own set of rules:

2. A (5 red, 2 green) walk has_____more red_____than green______.
and_____more green______than red_____.

3. A (5 red, 2 green) walk means 5 more steps to the right/left than to the right/left and 2 more steps away from/toward than away from/toward.

4.

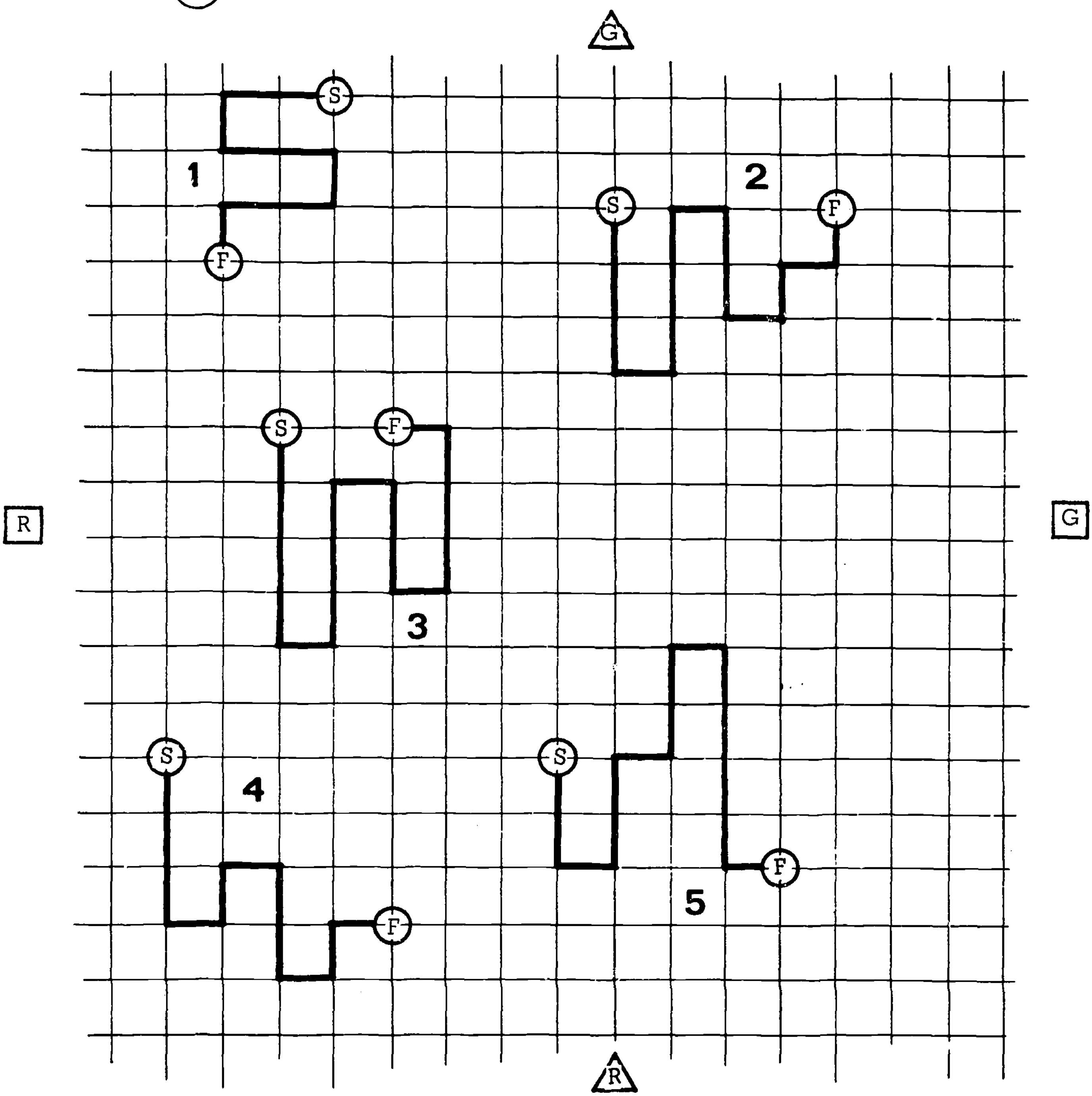
Walk	
(3 green, 4 green)	________steps more to the left/right than to the left/right ________steps more toward/away from you than toward/away from you.
(6 green, 0 red)	________steps more to the left/right than to the left/right ________steps more toward/away from you than toward/away from you.
(0, 0)	________steps more to the left/right than to the left/right ________steps more toward/away from than toward/away from you.

Worksheet 14 Name_______________

Many walks are pictured on this page. See the instructions on the next page.

 means start.

 means finish.



Worksheet 14 continued Name________________________

In the proper column draw a picture of the handful next to the number of the problem. Then in the other column write the name for that walk.

HANDFUL	NAME
1.	
2.	
3.	
4.	
5.	

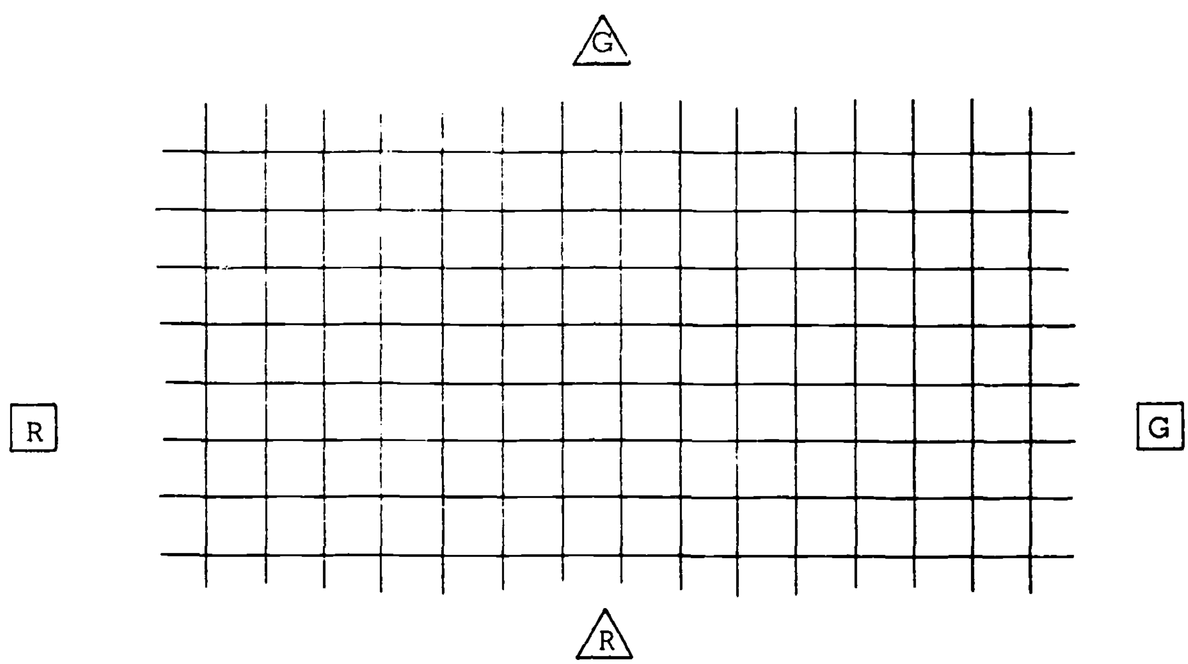
Worksheet 15 Name________________________

1. Pick the names and colors of three walks. List them on this chart.

2. Then work out the walks on the grid below, starting at different places each time.

3. Fill in the rest of the chart.

Color of walk	$\boxed{R}$	$\boxed{G}$	△R	△G	Name
					(,)
					(,)
					(,)

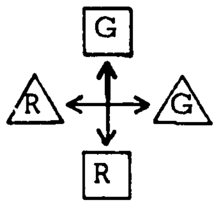


Are these walks alike in anyway? If you think they are, discuss it with your teacher or a friend.

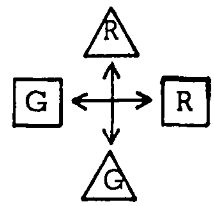
Worksheet 16 Name______________________

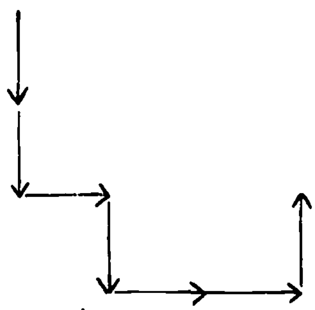
On this page each arrow will be one step. Name each walk.

a.

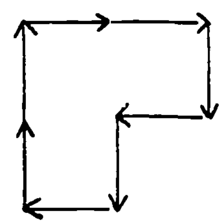


b.





(,)



(,)

2. Can you draw 2 different walks for each of these names below?

(2 red, 2 green)		
(6 red, 3 red)		
(0 red, 0 green)		

Part 2 - Story of Squareville

Mr. Brown's walk was a little springier and his smile a little broader as he entered his office the next day. He had a solution for getting from City Hall to the Courthouse. He would have the Street Department put up traffic lights along a special Mayor's Route. There would be no stopping along the way.

The more he thought about it, the more it pleased him. Why hadn't he thought of it before? He must tell the Street Engineer.

All the Street Department staff smiled in agreement when they heard his suggestion. They pulled down the map of Squareville and said, "Mark off the route, Mr. Mayor."

Mr. Brown suddenly realized that he hadn't decided on a route. He thought out loud, "The quickest route would involve the smallest number of turns. What route would have the fewest turns? Let's see...

> Use the large map of Squareville and discuss routes with few turns and why they are faster for the mayor. (He must slow down for turns. They should not think a route has a shorter driving distance because it has fewer turns.) Have the children choose several routes they feel would get the mayor to his office in the shortest time. Have them determine which routes the mayor could take involving only one turn.

From (0,0) E to (10,0) and N to (10,10) would involve just one turn and so would going from (0,0) N to (0,10) and E to (10,10). Either way the least number of turns would be <u>one</u>.

The next small number of turns would be two. Several routes would have two turns? I can easily see that the number of such routes would be very large."

At this time everyone was very interested in the number of different routes that could be taken between (0,0) and (10,10). They decided to have a contest for deciding the correct number. The winner would be invited to have lunch with the Mayor.

They soon found that there were more than any of them had expected. Here is how they determined the total number.

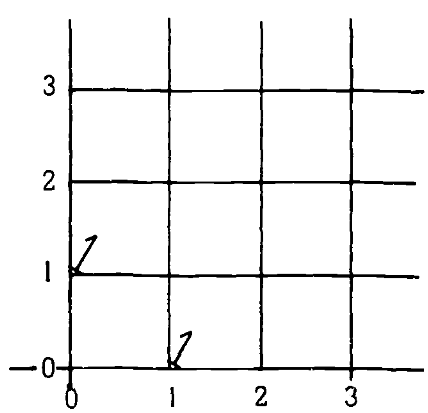
Activity 8 How Many Routes to the Courthouse?

1. Have each child look at his map of Squareville and predict the number of different routes there are between (0,0) and (10,10).
   These predictions should be written near the intersection of (10,10).
   Have the children compare their predictions.

2. Comment:
   "We could probably mark off all the different routes but that would take a long time. Let's see if we can find a pattern which will help us. Suppose we pick the corner that is nearest (0,0). What is its address?
   <u>(1,0) or (0,1), both corners are only 1 block from (0,0).</u>

   Now listen carefully!! How many different routes are there from (0,0) to the closest corner? Remember the question, 'How many routes, not how far?'"
   <u>There is only one route if we go east to (1,0).</u>
   <u>There is only one route if we go north to (0,1).</u>

3. Have each child write the number of routes in the appropriate intersection on their map.

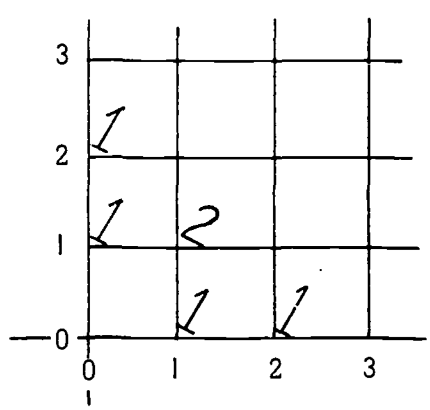


4. If you travel on the streets and avenues where is the <u>next</u> nearest corner? <u>(2,0), (0,2), (1,1)</u>. How far are these corners from (0,0)? <u>2 blocks</u>

5. Draw the following chart on the chalkboard and fill it out as the activity progresses.

Corner Address	Distance from (0,0) in blocks	Routes from (0,0)	Number of Routes to the Corner
(2,0)	2	EE	1
(0,2)	2	NN	1
(1,1)	2	EN or NE	2

6. Mark the number of routes on the Squareville map, as shown:



7. Continue with all corners 3 blocks from (0,0).

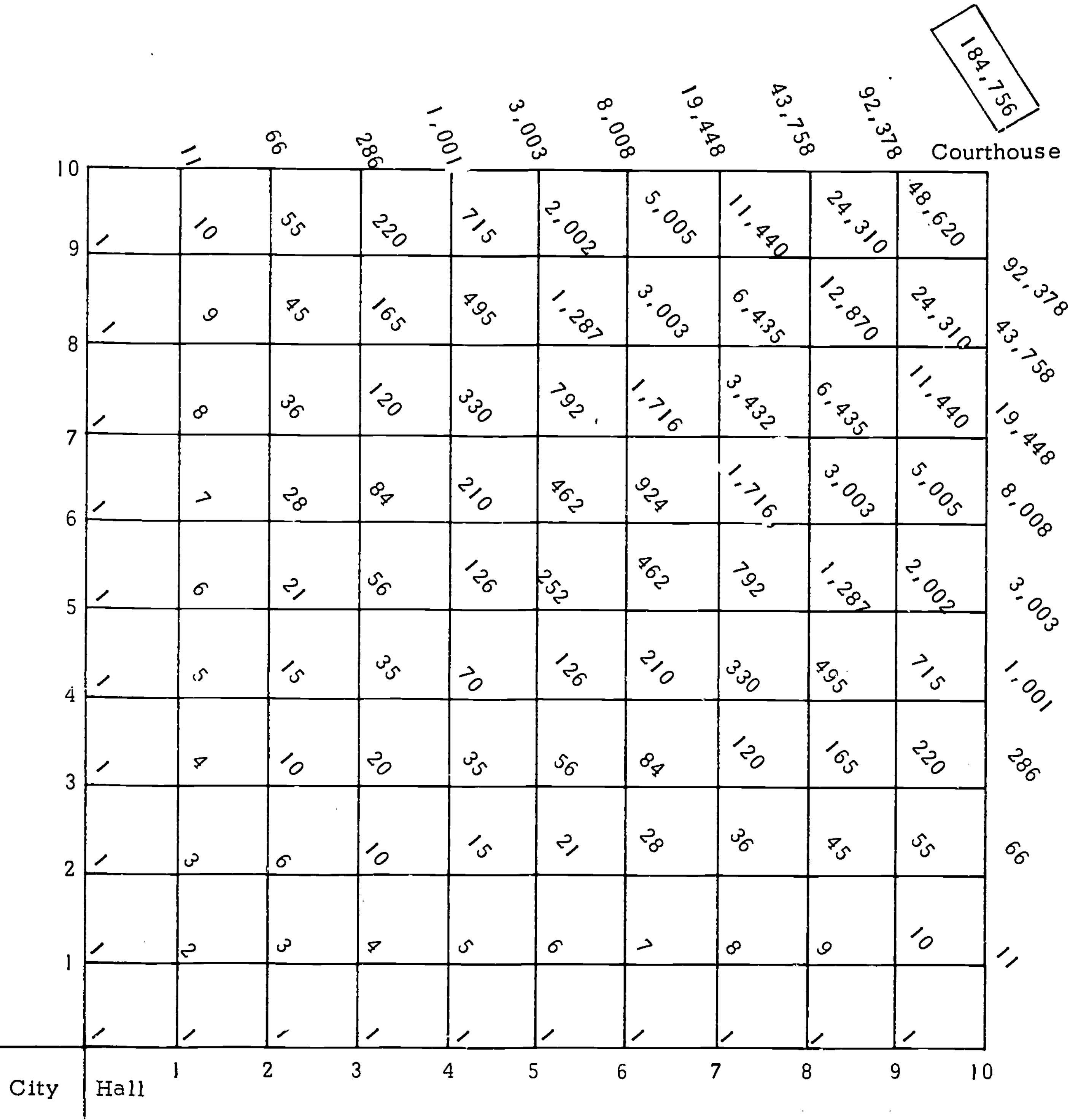
Corner Address	Distance from (0,0) in blocks	Routes from (0,0)	Number of Routes to the Corner
(3,0)	3	EEE	1
(0,3)	3	NNN	1
(2,1)	3	NEE EEN ENE	3
(1,2)	3	ENN NEN NNE	3

8. Encourage the children to continue their research. Many patterns will soon emerge. These patterns can be extended so as to determine the total number of different routes that can be taken between (0,0) and (10,10).

> Note: This search for a pattern must be an individual project. When a child feels he has discovered a pattern, he should consult with the teacher who can check his answer against the enclosed answer sheet and say either "Yes", or "No" to the child. It will then be the child's individual responsibility to continue from that point.
>
> At no time should any child give any other child any clues. Large numerals are involved in this work. Some children may not be able to complete the page.

Teacher's Reference Sheet

Number of Routes to Any Corner in Squareville



For your information, the pattern of numbers that the children generate is known as Pascal's triangle named after the famous mathematician. It occurs many times in varied contexts in mathematics.

For example, for any given numbers a and b,
$(a + b)^4 = (a + b)\ (a + b)\ (a + b)\ (a + b) = 1a^5 + 5a^4b + 10\ a^3b^2 + 5\ ab^4 + 1b^5$, and note that the numbers 1 - 5 - 10 - 10 - 5 - 1 occur as the 5th diagonal in the triangle. There are $\frac{(x + y)!}{x!\ y!}$ ways of getting from (0, 0) to (x, y).

[Note: $p! = p\ (p - 1)\ (p - 2) \ldots (2)\ (1)$, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$]

For example, there are $\frac{(3 + 4)!}{3!\ 4!} = \frac{7!}{3!\ 4!} = 35$ ways of getting from (0, 0) to (3, 4). Needless to say, this formula should not be given to the children.

Worksheet 17 Name________________________

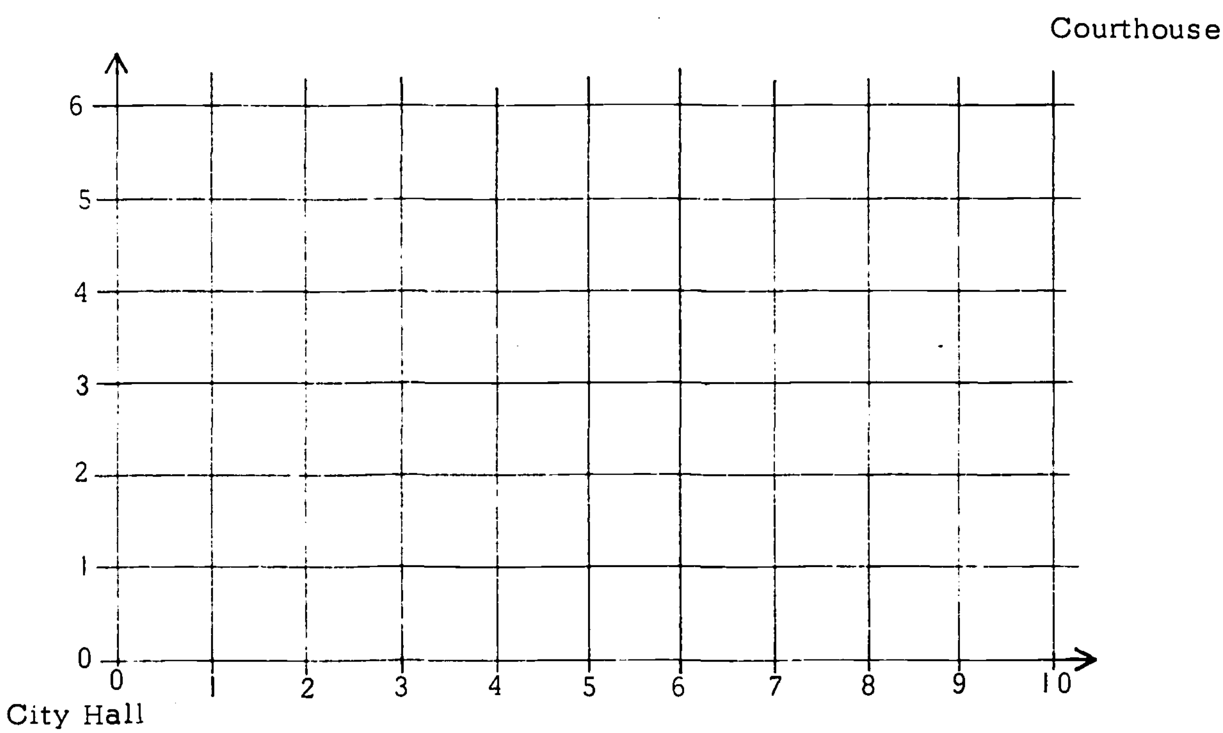
Mark these routes on the grid at the bottom of the page.

1. The mayor is lost. He is driving his car straight on one of the streets or avenues of Squareville. He saw one sign that said S = 2, A = 1, and the next one said S = 3, A = 1. On which street or avenue is the mayor driving? ____________________

2. If he saw a sign S = 2, A = 4 and then S = 2, A = 5, what would you tell him? ____________________

3. What would you tell the mayor if he saw S = 7, A = 3, and then S = 6, A = 3? ____________________ Is he going toward city hall or away from it? ____________________

4. Is the mayor going toward city hall or toward the courthouse if he sees the sign S = 3, A = 4 and then S = 3, A = 3? ____________________



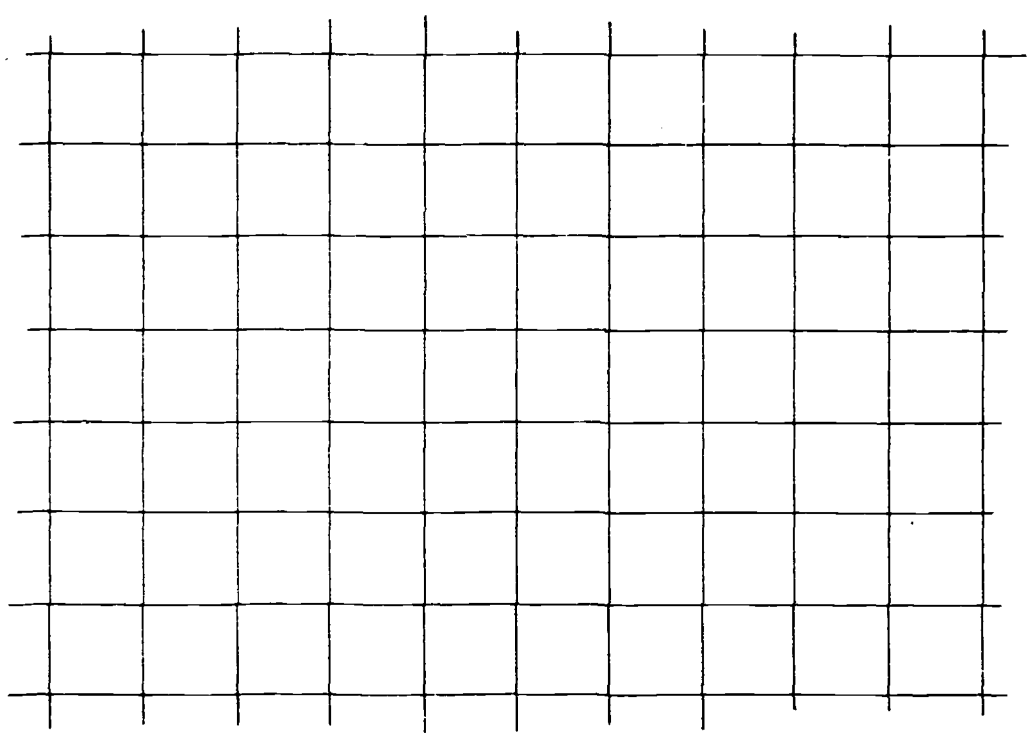
Worksheet 18 Name________________________

Pick any point and mark it on the grid at the bottom of the page. This point will be your home address for these problems. Remember that you must always follow the grid lines.

1. You are at home and have to hurry to a ball game at (7,5). If you ride your bicycle, what is the shortest route you could take? ___________ Put the number of blocks here. ______________

2. Your mother needs some milk for supper in a hurry! The store is on Third Street and Third Avenue. How many blocks must you go from your house to the store <u>and</u> <u>back</u>? ___________________

3. Your school is at (7,0). How many blocks must you walk to school each day? ________________



Name________________________

1. Anthony lives at (14,3). Cleo lives at (4,8). How many blocks does Anthony have to walk to get to Cleo's house? ____________________

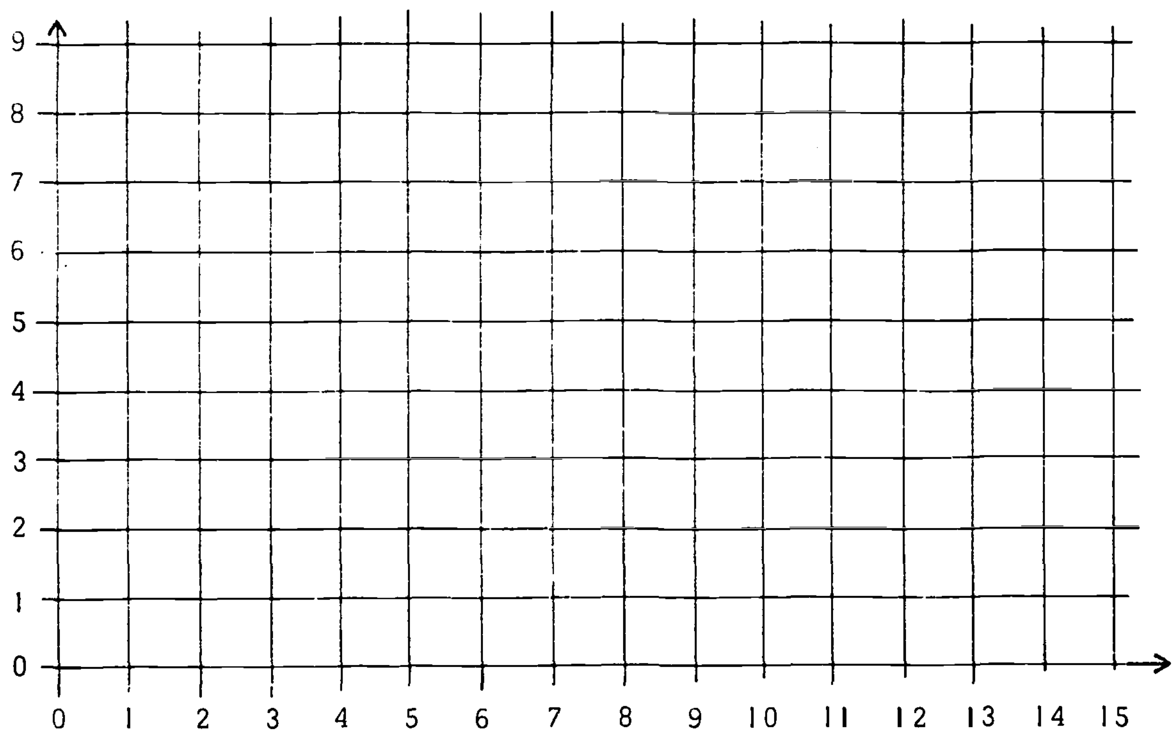
2. Jane lives at (3,5). She wants to visit Alice at (6,4) and Sally at (1,1) and then came back home. Try different routes between these three points on the grid at the bottom of the page. What is the length of the shortest route that connects all three houses? ___________ Is there more than one route that has this length? _________ How many? __________________

3. On Saturday, Jane had a birthday party. She invited Sally and Alice and Him who lives at (5,1). What is the shortest distance each one would have to walk from their house to Jane's.

Sally______________________

Alice______________________

Jim________________________



Worksheet 20 Name________________________

Use the grid at the bottom of the page to draw the routes you would follow in these problems.

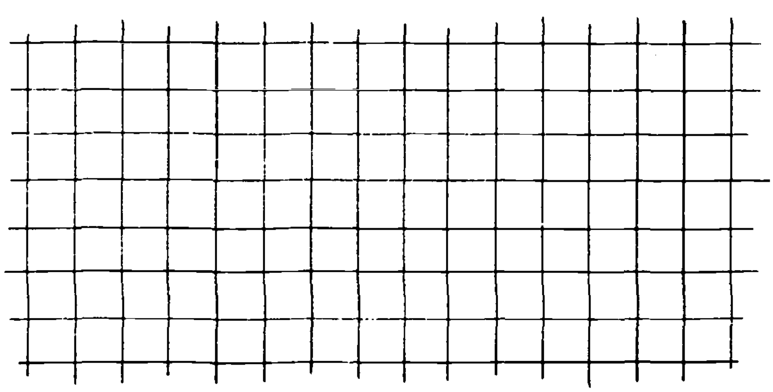
1. You want to drive from S = 3, A = 2, to S = 5, A = 4 by the shortest route. How many blocks long is that route? ____________________

2. How many possible shortest routes are there from S = 3, A = 2, to S = 5, A = 4? ____________________

3. Give each route a letter name and put that name in the left-hand column in this chart. Put the length of each route next to its letter name in the middle column. Put the number of corners you had to turn in the other column.

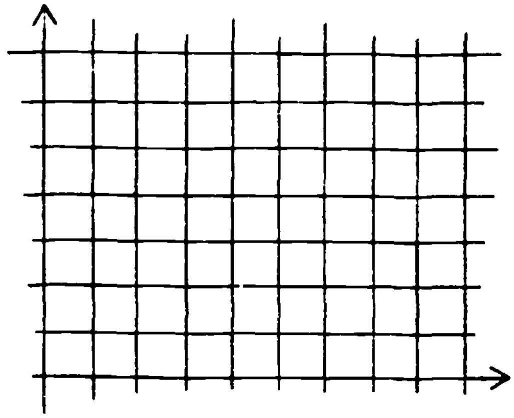
Route	Length of Route	Number of Corners

Which route has the fewest corners? __________ Which route has the most corners? ______________



Worksheet 21 Name________________________

1. Color the part of the grid that lies north of 3rd Avenue or on 3rd Avenue.



2. Write the names of a few intersections that lie in the colored part of the grid.

( ), ( ), ( ), ( ), ( )

3. In the names that you write, circle the avenue address. What do you notice about the above avenue addresses? ________________________

____

4. Look at every point in the colored section. Write the address of an intersection in the colored area that has an avenue address of

5________________________

7________________________

3________________________

1________________________

2________________________

5. One might say that all points in the colored area have avenue addresses that are (greater than or equal to) (less than or equal to) 3. Underline the correct answer.

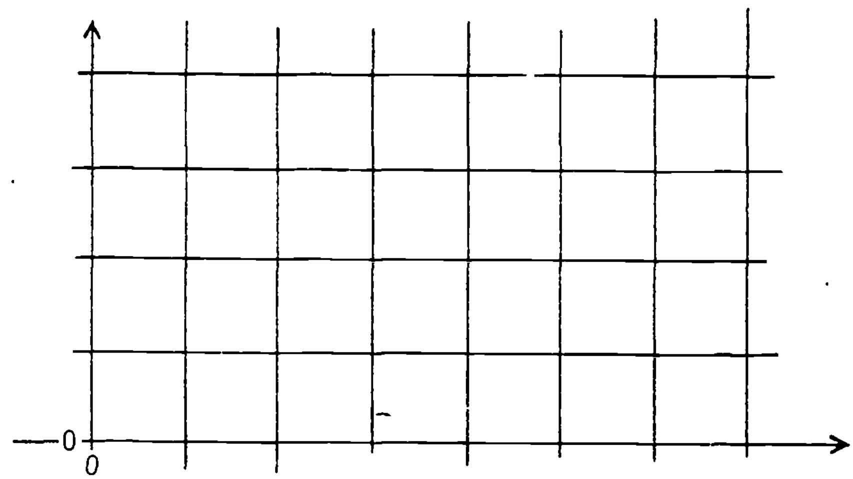
6. Instead of saying "the avenue address is greater than or equal to 3", we can use a shorthand and write "$A \geq 3$".

Worksheet 22 Name________________________

1. Choose a color. On the grid below color Second Avenue and all points north of it. Write the names of three avenues in the colored set.

____

We could also describe this colored set as the set of points such that $A \geq 2$ because the avenue address of any intersection in the set is always greater or equal to 2.



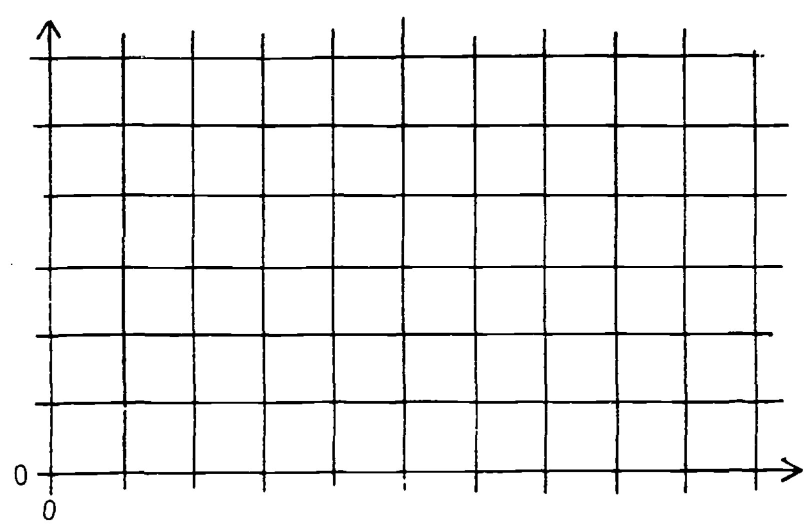
2. On this grid color the part where $A \geq 5$. Call this set F.

Choose another color. Color the area where $A \leq 5$. Call this set G.

Name some points that would be in both set F and set G. ____________

Name some points that would be in one set but not the other. ________

____



 Name________________________

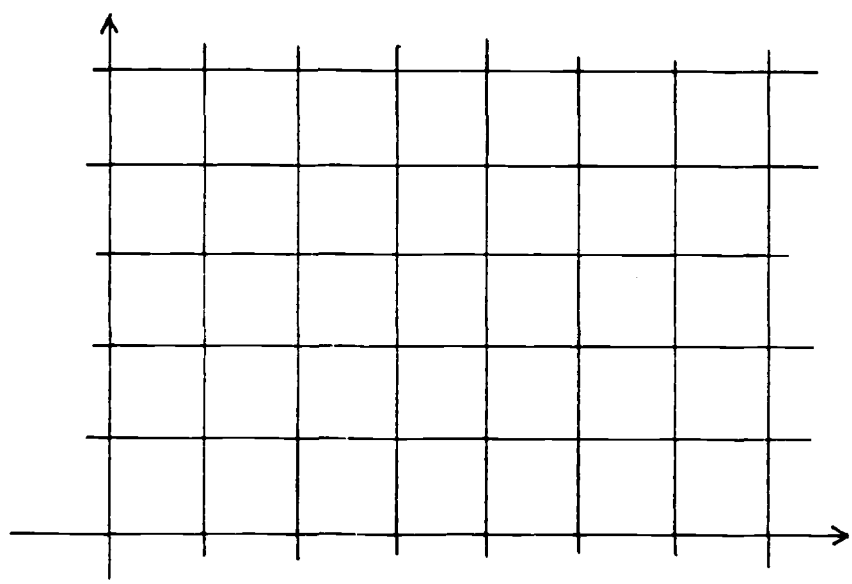
3. On this grid color the set where $S \geq 4$.

   With a new color, color the set of points where $S \leq 2$. What streets are in both sets? ____

   ____

   What streets are not in either set? ____

   ____



 Name________________________

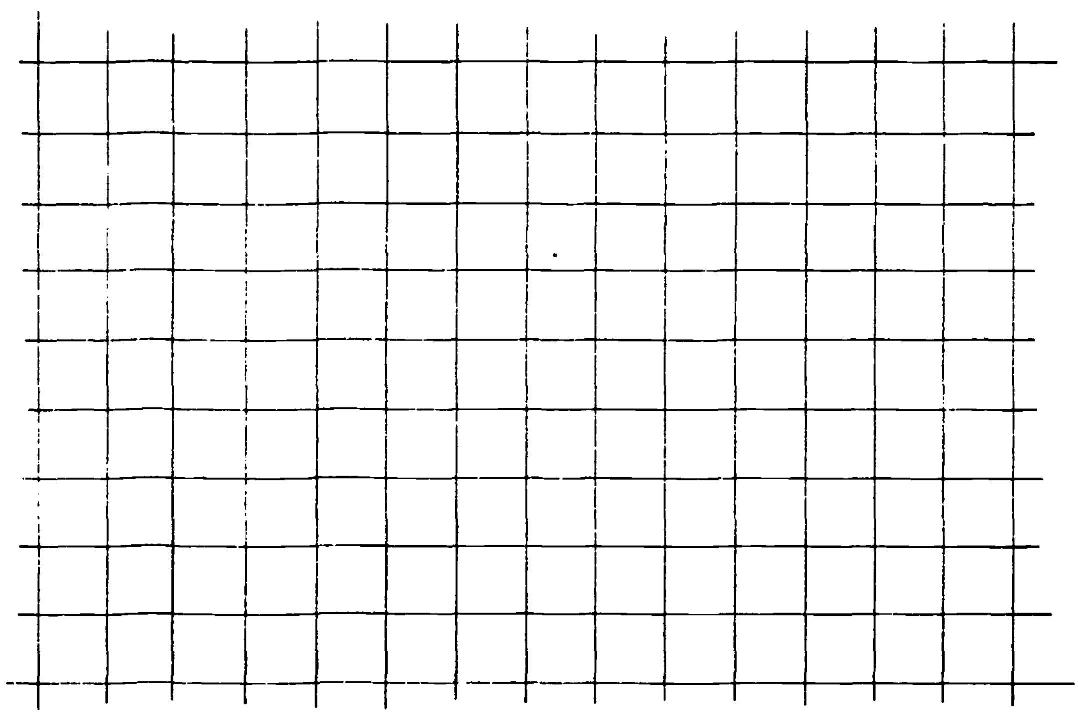
4. Color the set of points where $A \geq 3$. With a different crayon, color the set of points where $S \leq 2$. Name 4 points that are in both sets.

____

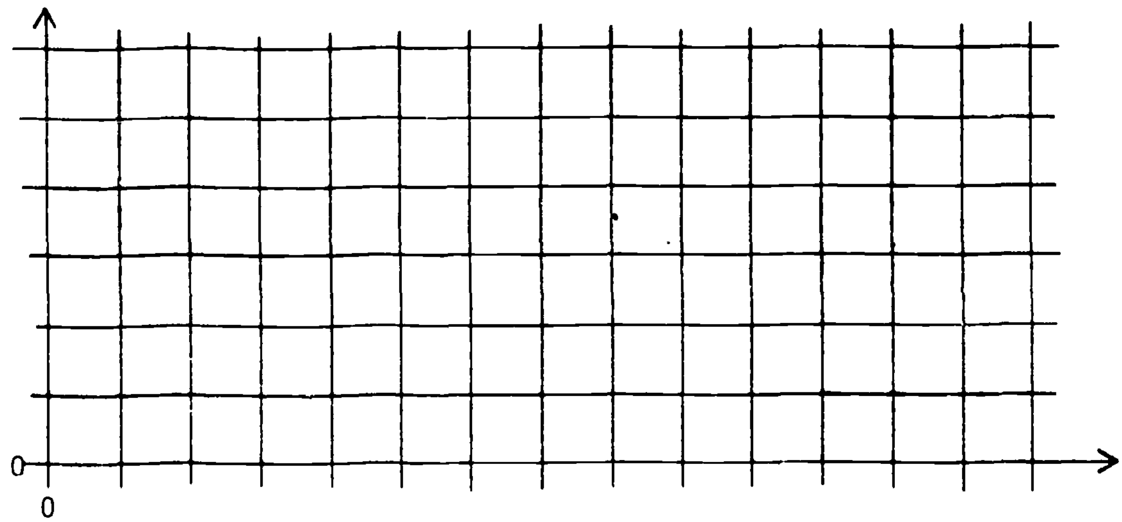
How could you tell these points were in both sets? ______________

____

5. Name 4 points that are in one of the colored sets but not in both of them. ____



Worksheet 23 Name________________________



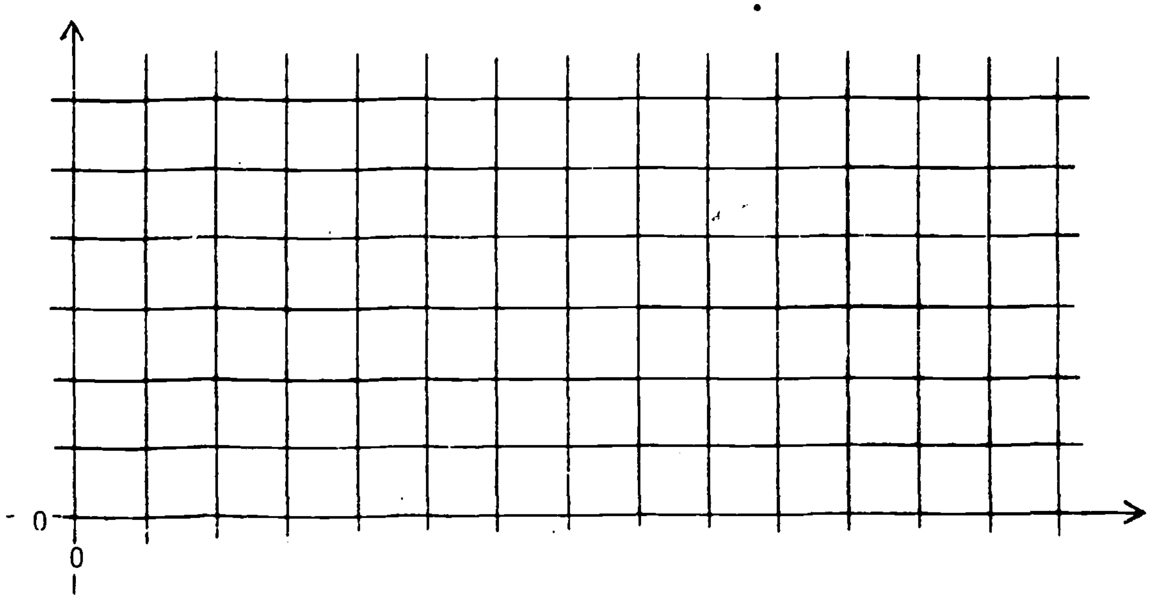
1. Color the part of the grid where all points have avenue addresses such that $A \geq 4$. Use a different crayon to color the set of all points whose street addresses are less than or equal to 5. The shorthand for this is $S \leq 5$.

2. On the graph below, mark with red any point in the set where $S \geq 3$.

3. Mark with blue any point in the set where $A \leq 2$.

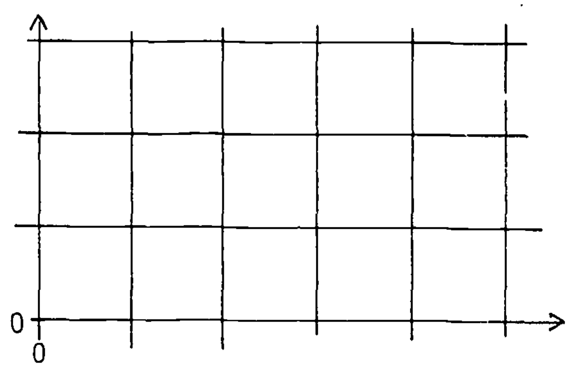
4. Mark with green any point in the set where $S \leq 6$.

5. Mark with purple any point in the set where $S \geq 1$.



Worksheet 24 Name______________________

1. Below is a grid. With one color mark all of Second Avenue. With the same crayon, color the part of the grid that is north of Second Avenue.



2. Use the same color to underline the names of the avenues that lie in the colored region.

0th Avenue   1st Avenue   2nd Avenue   3rd Avenue   4th Avenue   5th Avenue
6th Avenue   7th Avenue   8th Avenue   9th Avenue   10th Avenue

3. Use a pencil to circle the names of the avenues in problem 2 that do not lie in the colored region.

4. Now use a different crayon to color in the part of the grid that lies south of Second Avenue.

5. What color did you make Second Avenue? __________Check your answer with a friend to see if you are correct.

6. Does Second Avenue lie north of Second Avenue?   Yes   No

7. Does Second Avenue lie south of Second Avenue?   Yes   No

8. Name 5 points that lie north of Second Avenue?

(   ), (   ), (   ), (   ), (   )

9. Name 5 points that lie south of Second Avenue?

(   ), (   ), (   ), (   ), (   )

10. Name 5 points that lie neither north nor south of Second Avenue.

(   ), (   ), (   ), (   ), (   )

Worksheet 25 Name________________________

1. Color the area where $A \leq 7$. Call this set Q. Now use a different color for the area where $S \leq 5$. Call this set R. Name some points that are

   in Set Q________________________

   in Set R________________________

   in Set Q and Set R________________________

2. What streets would be in

   Set Q________________________

   Set R________________________

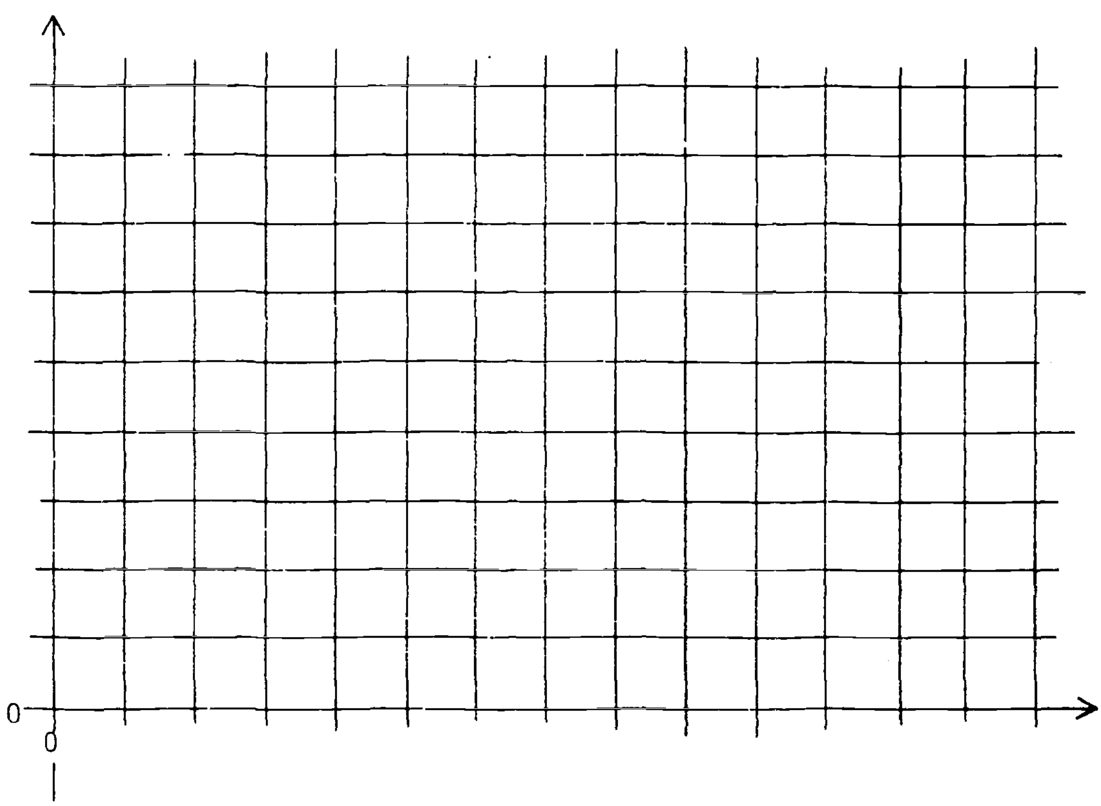
   What avenues would be in

   Set Q________________________

   Set R________________________

   both sets________________________

3. What streets would be in both sets?________________________



Worksheet 26 Name________________________

On this worksheet, you see many names of sets of points. Use extra sheets of paper to find the sets. Then put the name of three points included in each set next to the name of the set.

<u>Set</u>

$A \geq 6$ ________________________

$A \geq 5$ ________________________

$S \leq 3$ ________________________

$S \geq 4$, $A \leq 6$ ________________________

$S \geq 6$, $A \leq 5$ ________________________

$S = 4$, $A = 6$ ________________________

$S \leq 3$, $A \leq 3$ ________________________

$S \geq 4$, $A \leq 4$ ________________________

Part 3 - Story of Squareville

As time went by, Mr. Brown got his special Mayor's route. There were no stops to be made,and he was content - but only for a little while. Then he began to think, "There has to be a shortcut across town. I must find it!"

Every chance he had he thought about it, but no answer came. Every time he drove from City Hall to the Courthouse he looked for a shortcut and he still could not find a shorter way. Finally he decided to ask his son, Tommy.

Tommy had an idea almost before his Dad finished telling him his problem. "You've forgotten what you told me you used to do when you were late for school."

Mr. Brown knew just what Tommy had in mind. "Do you mean the times I cut across people's yards? Are you trying to say that we should build a highway straight across town?" he asked.

"Right!" was Tommy's reply.

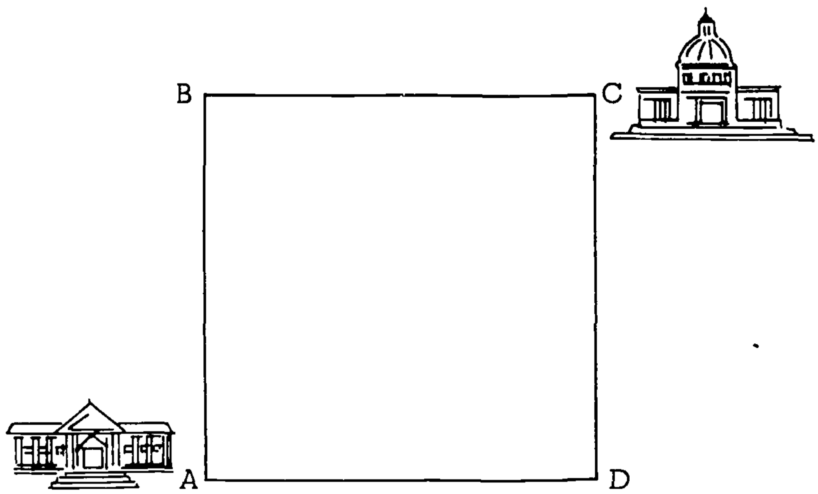
Elicit from the children reasons why Tommy's route would be shorter. Have a child draw the new route on the classroom map of Squareville, a diagonal from (0,0) to (10,10).

At this point, pass out Worksheet 27. This is designed to review the idea that the length of a diagonal of a square is greater than the length of one side but less than the combined length of two sides. Explain to the children that they will measure the old and new routes to see if the new one is really shorter. The square at the bottom shows the two quickest routes the mayor previously used to get from City Hall to the Courthouse.

Pupil's Copy

Worksheet 27 Name________________________

The lines below show the two shortest routes the mayor had been using before. You will not need the other streets and avenues to do this work.



1. Measure the length of line segment AB.
2. Mark a point on the line below the square. Begin at A and lay off the line segment AB.
3. Draw a diagonal in the square.
4. Measure its length.
5. Begin at point A on the line and mark off the lengths of the diagonal.
6. What can you say about the length of the diagonal? _______

____

____

Pupil's Copy

Name________________________

7. Measure line segment BC on the square. Begin at point B on the line and mark off the length of BC. Label your new point C.

8. Now what can you say about the length of the diagonal of a square? ____

____

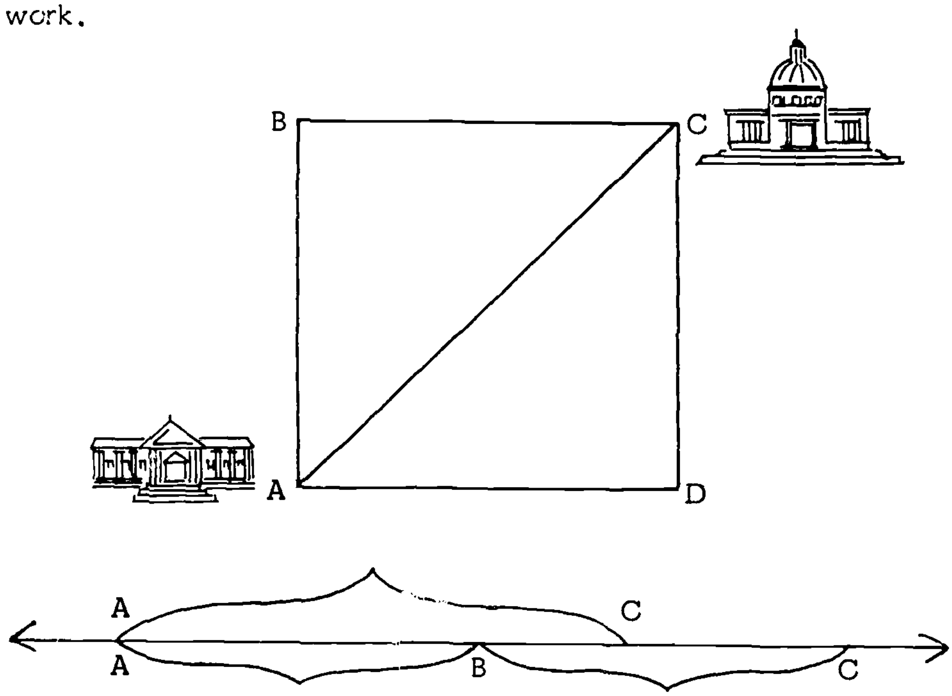
____

9. Would the mayor travel a shorter distance by going by the diagonal route or by his quickest old route? ____________

____

Worksheet 27

The lines below show the two shortest routes the mayor had been using before. You will not need the other streets and avenues to do this work.



1. Measure the length of line segment AB.
2. Mark a point A on the line below the square. Begin at A and lay off the line segment AB.
3. Draw a diagonal in the square.
4. Measure its length.
5. Begin at point A on the line and mark off the lengths of the diagonal.
6. What can you say about the length of the diagonal? The length of the diagonal of a square is greater than the length of one side of the square.

Worksheet 27 continued

7. Measure line segment BC on the square. Begin at point B on the line and mark off the length of BC. Label your new point C.

8. Now what can you say about the length of the diagonal of a square? The length of the diagonal of a square is greater than the length of one side of the square but less than the combined lengths of two sides.

9. Would the mayor travel a shorter distance by going by the diagonal route or by his quickest old route? The diagonal route is shorter.

Part 4 - Story of Squareville

Mr. Brown's mind was suddenly filled with the thought of a SUPER - HIGHWAY across town. He was so excited that he drew the route right on his map of Squareville.

> Let each child construct this proposed highway on his map of Squareville. Straightedges should be used. Questions such as these might be asked:
>
> 1. Where does the highway begin? <u>(0,0) City Hall</u>
> 2. Where does it end? <u>(10,10) Courthouse</u>
> 3. This new highway should be longer than <u>ten</u> blocks and less than <u>20</u> blocks.
> 4. How many street and avenue intersections does your new highway cross? <u>Including (0,0) and (10,10), the new highway should cross 11 intersections</u>.

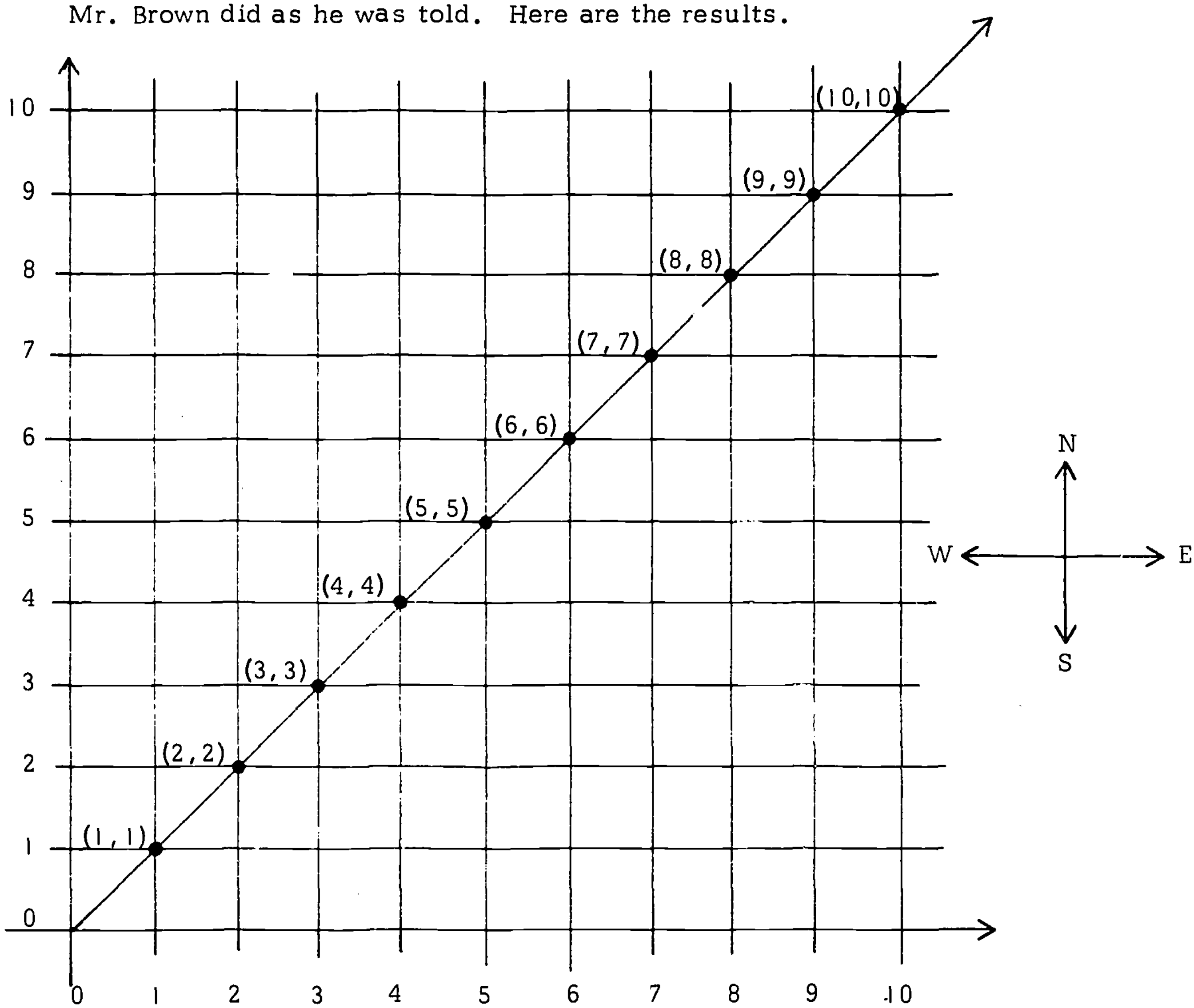
"This is a GREAT IDEA," exclaimed Mr. Brown. "So simple and so fast! But what shall we name the highway?"

This question puzzled Tommy. Naming the highway would be simple if it ran in the same direction as a street or an avenue. But no! This highway cut across town.

Suddenly he saw the answer. That was it! The new highway cut across the streets and avenues. "Dad!" he almost shouted. "Write in the address of every intersection on your new highway."

> Have the children write in the intersection addresses on their map. Have several children write in the addresses on the chalkboard map.
>
> | | | |
> |---|---|---|
> | Examples: | (4,4) | (6,6) |
> | | (5,5) | (2,2) |
>
> Ask the children if they notice anything about the names of the points this line crosses. Make sure they see that the street address is always the same as the avenue address.

Mr. Brown did as he was told. Here are the results.



Activity 9 Why (3,3), or (4,4), or ??

Question: Why is the street address equal to the avenue address at every point on the new highway?

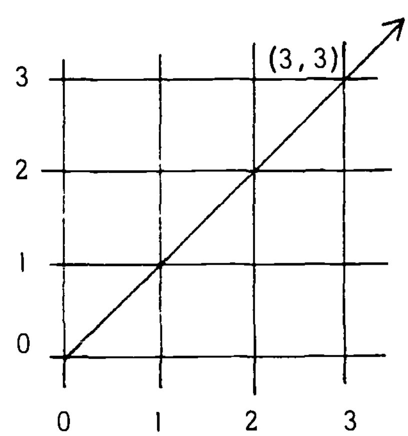
1. Place a dot at (0,0) on the chalkboard map of Squareville. Call this the Starting Point.

2. Then let some student take a SQUARE TRIP on Squareville. He must begin at (0,0) and end up at (0,0). His route must be a square. Use different colors.

3. After some investigation the children will discover that there could be ten different squares - all of which begin at the same point (0,0). The smallest square will have 1 unit sides while the largest will have 10 unit sides.

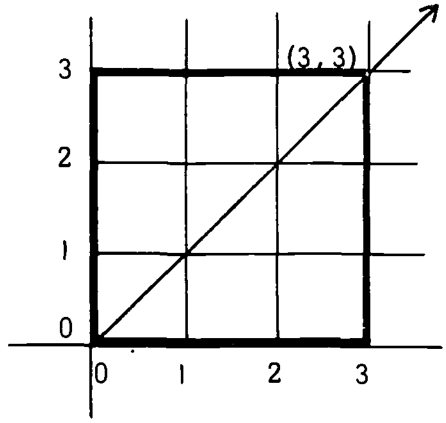
4. Erase the chalkboard map and draw in the new highway which runs from (0,0) to (10,10).

5. Have some child pick out an intersection on this highway and label its address.

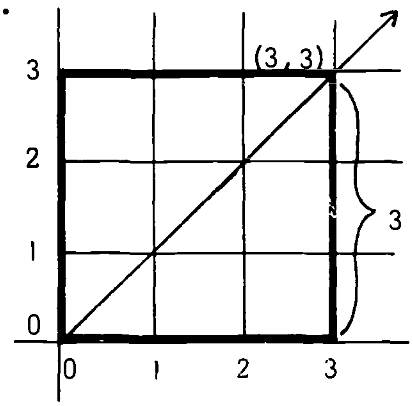


6. Repeat the question, "Why is the street address the same as the avenue address at this intersection?"

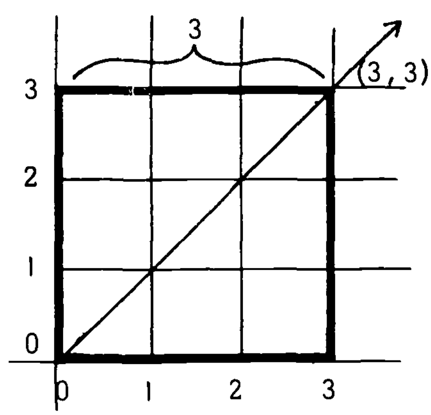
7. Some child could draw in the square which begins at (0,0) and passes through (3,3).



It becomes a simple matter to see that the length of each side in the above illustration is three units. Begin at (3,0) and count the units up to (3,3). It is 3 units.



Begin at (0,3) and count right to (3,3). It is 3 units.



Repeat the procedure with other squares. What do you find?

8. Find any intersection on 0 Avenue. Travel up that street to the new highway. Is there a quick way of telling how far it is from 0 Avenue to the new highway? <u>The distance along that street in blocks is always the same as the number of the street you use</u>.

9. Find any intersection on 0 Street. Travel on that avenue to the new highway. How many blocks will you need to travel? <u>Find the number of the avenue you are using. Your trip will be that many blocks long</u>.

10. Point to any intersection on the new highway and label it. Ask questions like these:

    a. How far must you travel south to reach 0 Avenue?

    b. What street number would you expect to find when you reached 0 Avenue?

    c. How far must you travel west to reach 0 Street?

    d. What avenue number would you expect to find when you reached 0 Street?

Worksheet 28 Name________________________

How Far to the Mayor's New Highway?

Use your map of the Mayor's new highway.

1. Choose any intersection on the new highway. Write the address in the chart.
2. Go south to 0 Avenue. How far is it? Record that number of blocks in the chart.
3. Next, go west from the same intersection on the new highway. How far is it to 0 Street? Write it in your chart.
4. Try a few more intersections.
5. Do you see any patterns in the chart? Why did they happen? ________

____

____

Intersection on the New Highway	Blocks South to 0 Avenue	Blocks West to 0 Street
(4,4)	4	4

6. What would be a good name for the new highway? ________________

____

Part 5 - Story of Squareville

The next morning Tommy showed his map to his friend, Ethelbert, and told him about the new highway. "I think we should call it Equality Boulevard. What do you think of that name?" he asked.

"That's great," Ethelbert replied, "because we know that at every intersection the avenue address equals the street address. It's all the same."

"Let's make a sign with that name," Tommy said. "That would be a good test of the name."

So Tommy and Ethelbert made this blank sign.

"Just write in the name of the highway," said Ethelbert.

Have the children make their own signs. Then let them discuss how they will write the name on their signs.

Remind them that if people are to read the sign in a hurry, the lettering must be large. This would result in a large sign and large signs are very costly.

Tommy printed in the name EQUALITY BOULEVARD

"That looks good to me," he told Ethelbert. But just to be sure why don't we test it outside. We will place it near a real street sign and see how it looks."

They hurried out carrying the sign and set it near the other sign on the corner. It looked good to them. They were satisfied.

"What have you written on that sign?" a man called from across the street. "I can't read it from here because the letters are so small."

Both boys looked at each other and smiled. Tommy answered the man's question. The printing had to be larger so people could read the sign.

"But try as they might, they couldn't design a sign that would be small enough in size and still have large letters."

They made this sign:

EQUALITY BOULEVARD

and this sign:

AVENUE ADDRESS = STREET ADDRESS

Using the map marked with Equality Boulevard, ask the children why the boys could make the sign:

AVENUE ADDRESS = STREET ADDRESS

Recall Worksheet 28 and the chart of distances to (0,0) from any intersection on the new highway. Before going on, make sure the children understand that for any intersection on Equality Boulevard the avenue number and street number will be the same or equal. Ask the children to think of simpler kinds of signs.

"A shorter way to do it would be like this," Tommy added.

$\boxed{A = S}$

"That should do it," exclaimed Tommy. These signs can have large letters. They are easy to make and will fit on any corner of Equality Boulevard. Wait until I show my Dad!"

"Just a minute! I've got another idea!" shouted Ethelbert. "If we can name Equality Boulevard with our shorthand, why couldn't we name the other streets and avenues that way too."

"Good idea," said Tommy. "Do you mean a sign on Fifth Avenue would look like this?

$\boxed{A = 5}$

And the sign for Third Street would look like this:

$\boxed{S = 3}$

"Right!" Ethelbert agrees. "And if we put them together on the corner of Fifth Avenue and Third Street, they would look like this:

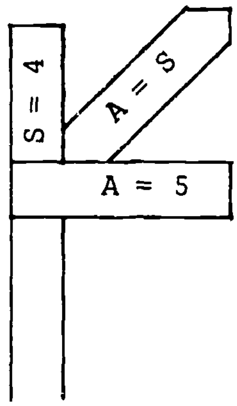
$\boxed{S = 3}$ $\boxed{A = 5}$

Have the children find several intersection and tell what the sign would say in this manner on the large map. Make sure you include some on the 0 axis so they are sure of the A = 0 or S = 0 notation.

Tommy showed the sign to his Dad when he came home that night.

"That's a good idea," said the Mayor, "and the people will be able to see them clearly. The only problem is that all the intersections of Equality Boulevard will have the same name. That would never do! No one could tell where he was."

Tommy thought for a moment and then drew these pictures for his father.



He said, "If we used signs like these, people would know exactly where they were."

"You're right Tommy," said the Mayor. "I'll talk to the Street Department tomorrow. I'm sure they'll like your idea."

Have the children do Worksheet 29, naming the intersections with the notation explained in the story.

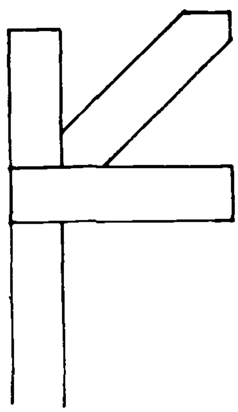
Teacher Background for Worksheets 29 and 30

Worksheet 29 is designed to give the children practice in the street sign notation. There is one sign which does not belong on Equality Boulevard. Discuss with the children why this sign could not be placed on any intersection of Equality Boulevard.

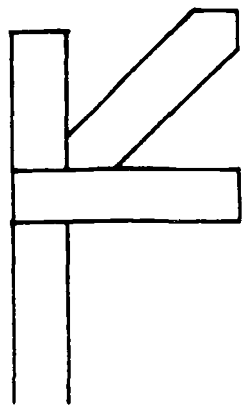
Through the use of Worksheet 30, the children should see that the rules that hold in naming intersections on Equality Boulevard hold also for its extension. You will also notice that the shape of the grid is rectangular rather than the usual square. The children should understand that, as long as the diagonal is still as many blocks from 0 Avenue as from 0 Street, it is still Equality Boulevard, no matter what the shape of the whole grid.

Worksheet 29 Name ________________________

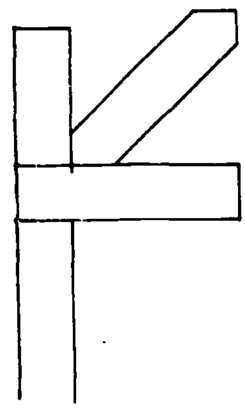
Print the shorthand name for the streets, avenues, and boulevards on these street signs. Use "S" and "A".



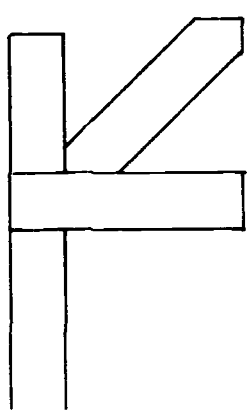
4th St., 4th Ave.
and Equality Blvd.



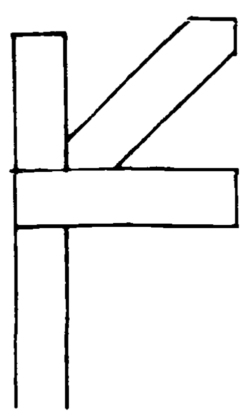
9th St., 9th Ave.
and Equality Blvd.



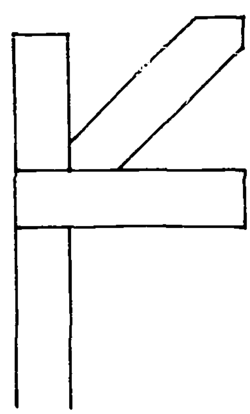
1st St., 1st Ave.
and Equality Blvd.



7th St., 9th Ave.
and Equality Blvd.



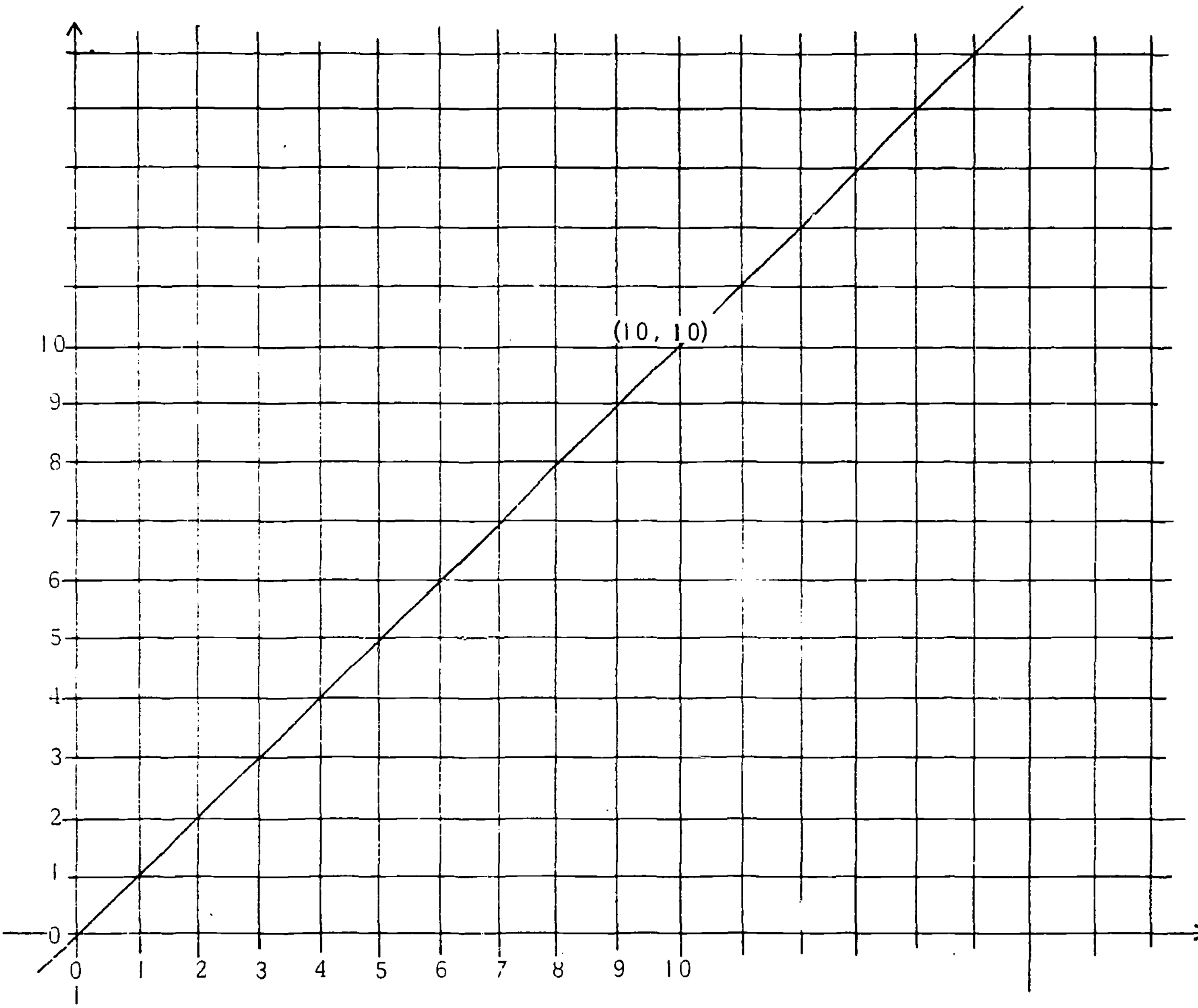
2nd St., 2nd Ave.
and Equality Blvd.



5th St., 5th Ave.
and Equality Blvd.

Worksheet 30 Name________________________

Label the new corners on Equality Boulevard.

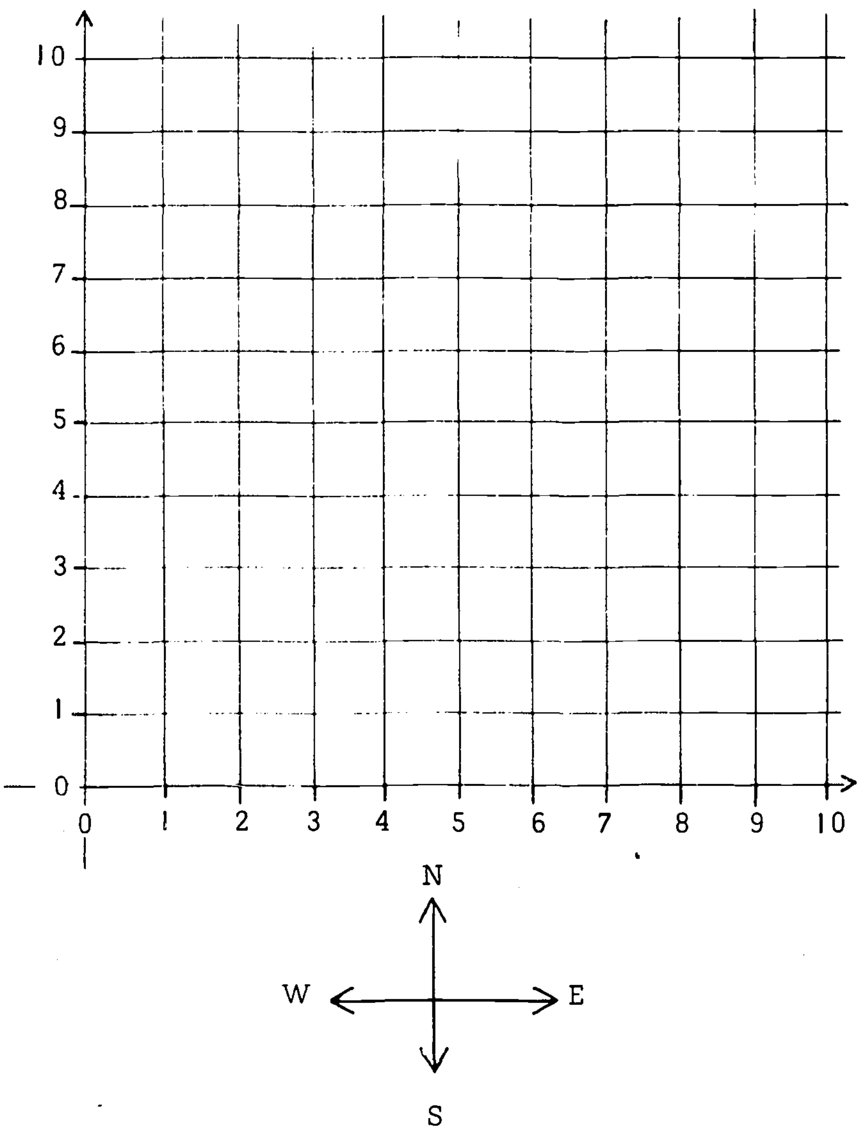


Worksheet 31 Name____________________

1. You are given a map and a chart. The chart lists several trips that are taken.

2. On the map, place a dot at the beginning of each trip, then make a heavy line with your pencil as you make each trip, finishing with a dot.

3. After you have completed all of the trips, darken in the insides of all the simple closed curves that you have drawn. What picture do you have?



TRIP

Start	Go	End
(2, 5)	1E, 3N	(3, 8)
(6, 7)	5S, 1W	(5, 2)
(2, 8)	3S	(2, 5)
(4, 1)	7N, 1W	(3, 8)
(7, 7)	5S, 1E	(8, 2)
(1, 1)	1E, 3N	______
______	1W, 7S	(1, 1)
(5, 8)	3E, 1S 1W	(7, 7)
(5, 2)	1S, 3E 1N	(8, 2)
(2, 4)	1E, 3S 1E	______
(6, 7)	1W, 1N	(5, 8)

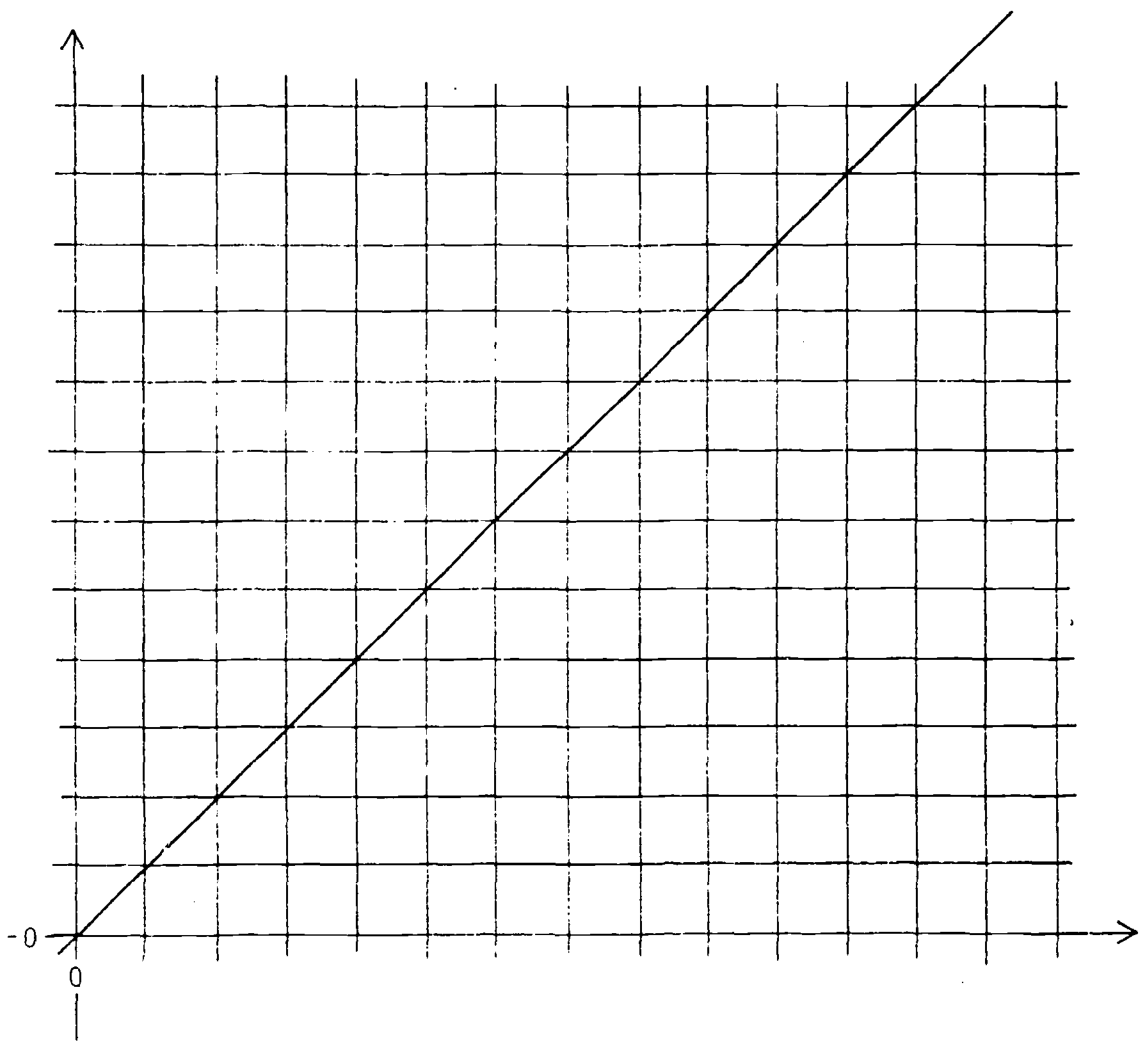
Worksheet 32 Name______________________

1. With your yellow crayon darken the part of Equality Boulevard from (0,0) to (7,7). How many intersections did you go through? (Don't count the starting and ending points.)______________________

2. With your red crayon, darken the S = 5 line. With your blue crayon, darken the A = 3 line. Do you see a triangle with one blue side, one red side and one yellow side? ______________________

   Name one point that is inside of the triangle. ______________

   Then color the interior of your triangle green.



Worksheet 33 Name________________________

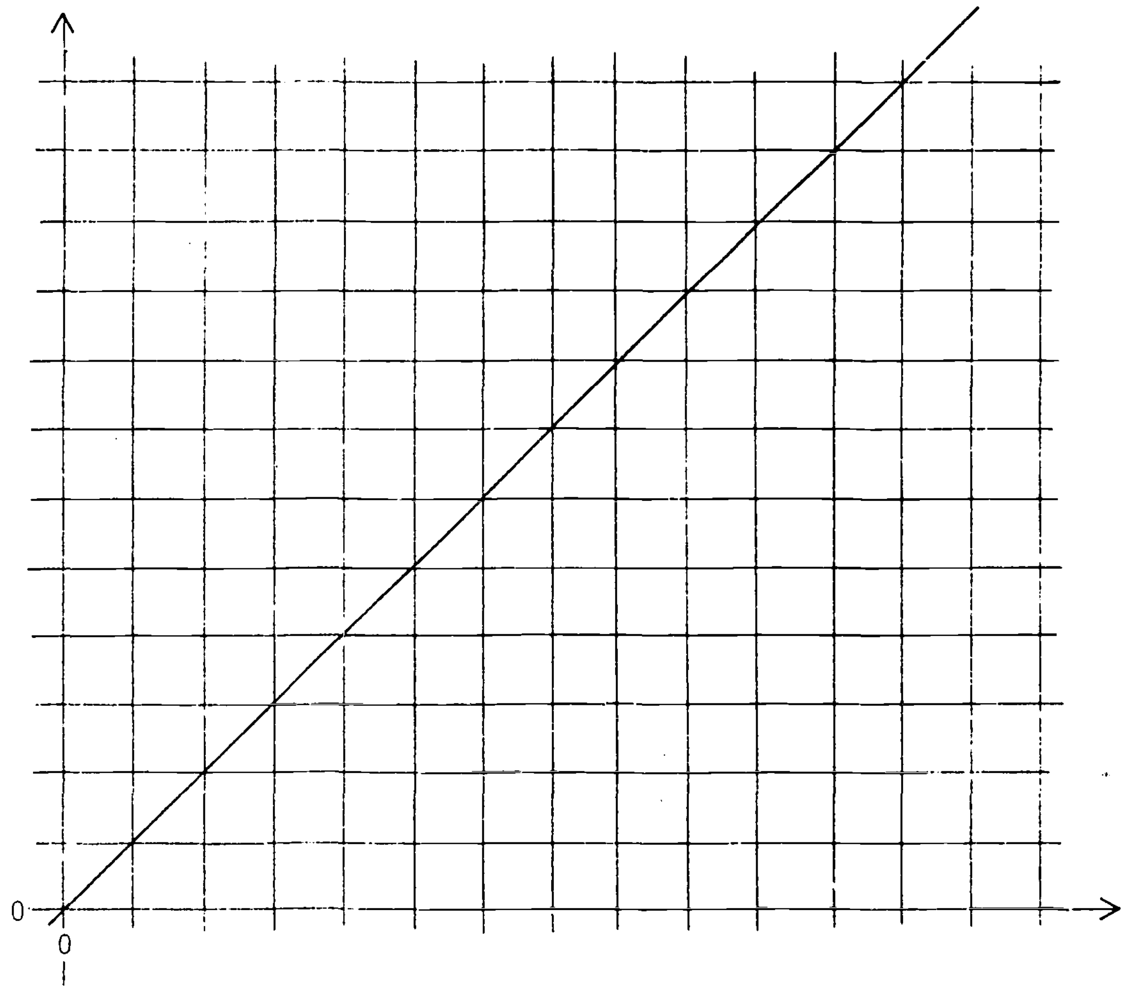
1. With one color darken any part of Equality Boulevard that is at least 5 blocks long.

2. Pick any street that crosses your colored part. Darken it with a different color.

3. Pick any avenue that crosses your colored part of Equality Boulevard. Color that avenue a different color.

4. Can you find a triangle that has 3 different colored sides? Color the interior of the triangle red.

5. Make another triangle on the other side of Equality Boulevard.



Worksheet 34 Name________________________

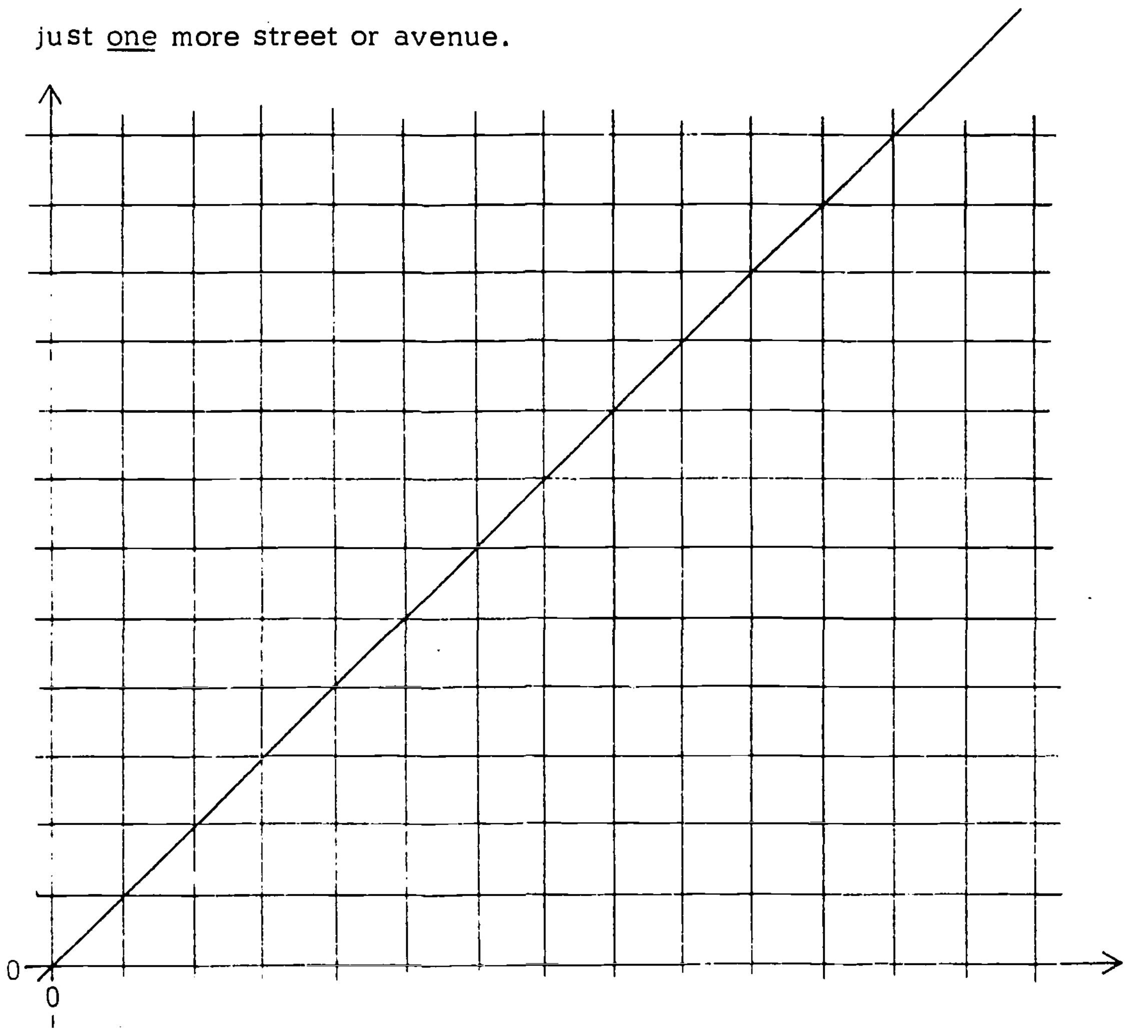
1. Darken the whole section of Equality Boulevard that you see on this worksheet.

2. Now darken S = 4 and A = 2.

3. Color the interior of this triangle blue.

4. What streets or avenues might you darken to make another triangle on the other side of Equality Boulevard that would be the same size and shape as the blue one? Do it on the map. ________________________

5. A tough one: Can you make yet another triangle? You may darken just one more street or avenue.



Worksheet 35 Name________________________

1. Color $A \leq 3$ red.

2. Color $A \geq 5$ blue.

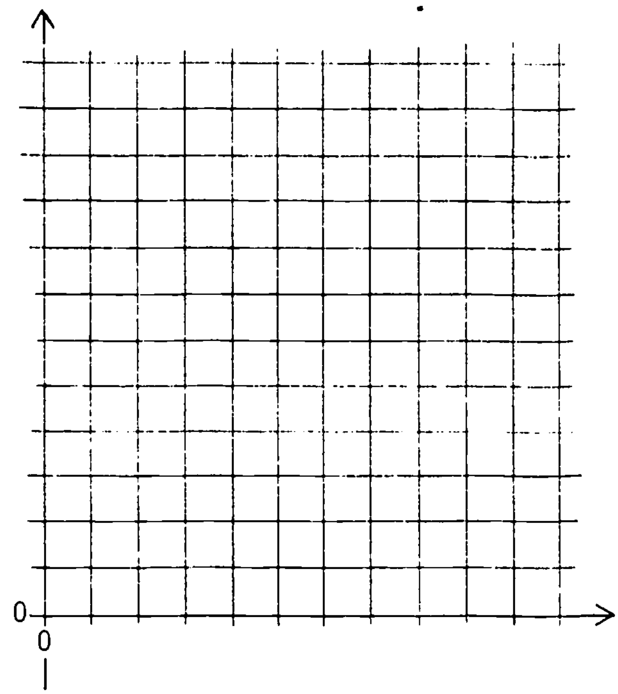
3. What color are these points?

   (2,1)________________

   (5,7)________________

   (7,2)________________

   (4,4)________________



4. Color $A \leq 4$ blue.

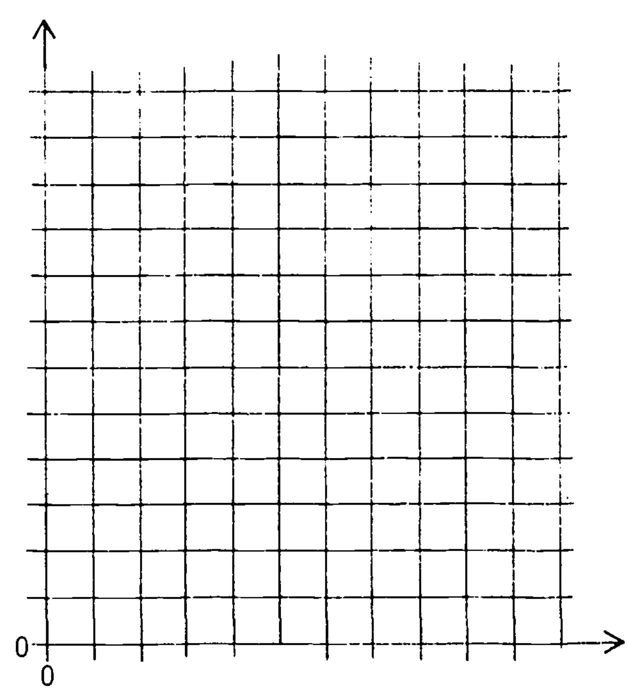
5. Color $S \leq 3$ yellow.

6. What color are these points?

   (7,2)________________

   (1,6)________________

   (2,2)________________



Worksheet 36 Name________________________

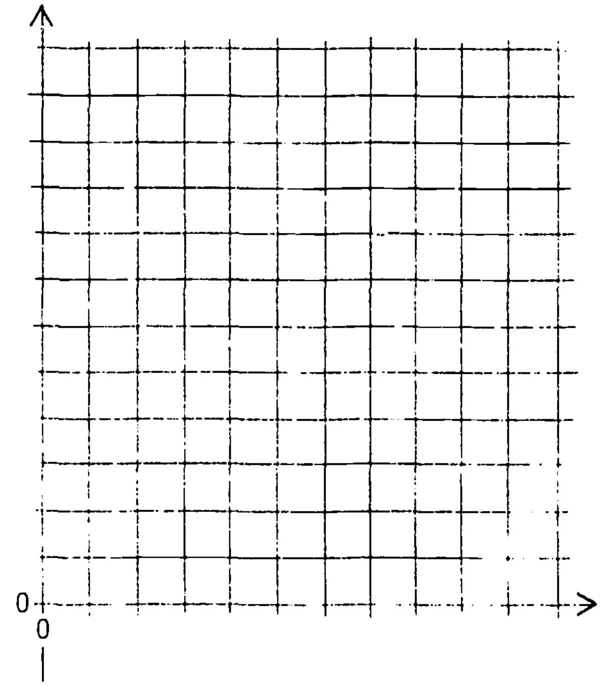
1. Color A $\geq$ S red.
2. Color S $\leq$ 3 yellow.
3. What colors are these points?

(2,1)________________________

(6,2)________________________

(7,8)________________________

(1,2)________________________



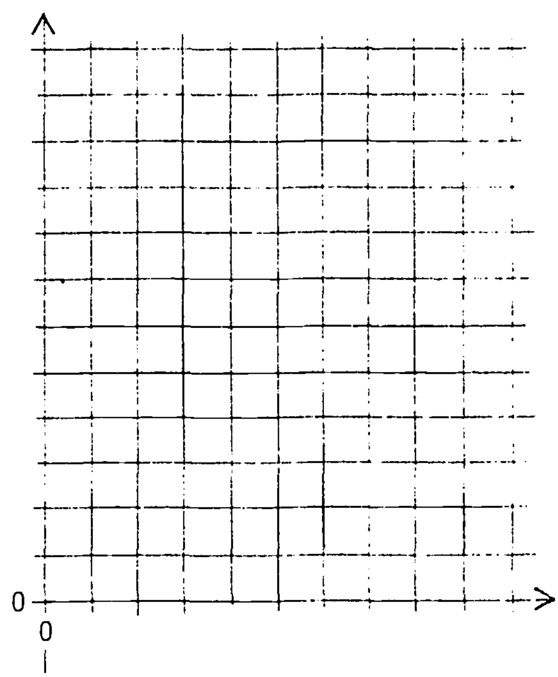
4. Color A $\leq$ S blue.
5. Color A $\geq$ 2 red.
6. What colors are these points?

(6,3)________________________

(3,1)________________________

(2,7)________________________

(7,9)________________________



A New Highway

You can imagine how pleased the people were to be able to use Equality Boulevard. In fact so many people used the highway that traffic was slower than ever.

"We need a larger highway!" everyone complained to the City Council. "We are worse off than we were before!"

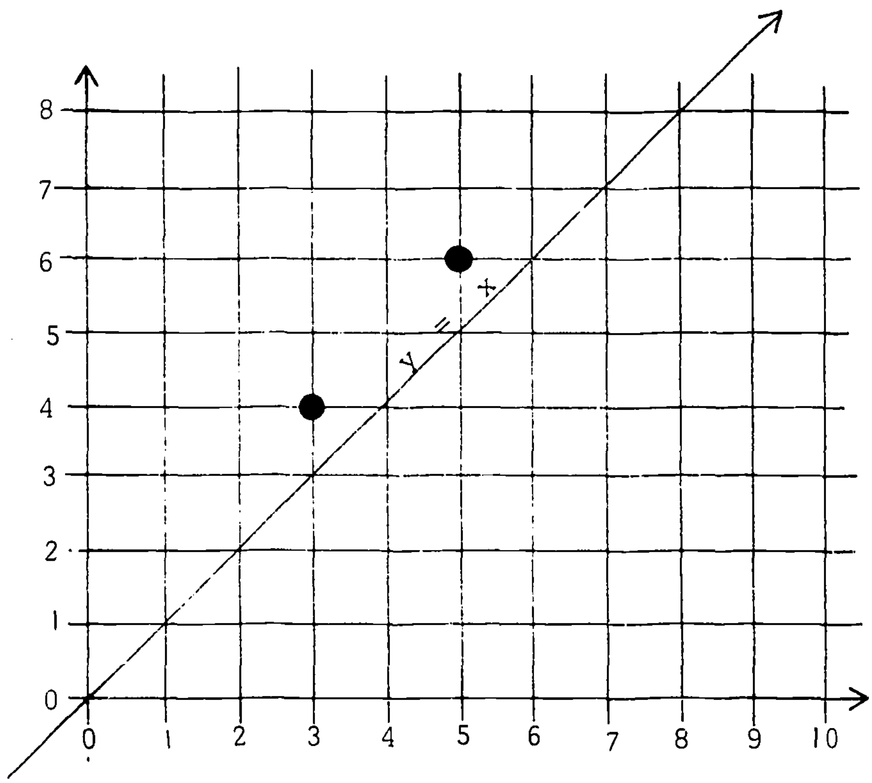
> Allow the children to indicate what they would suggest to help the Council solve its problem.

"All right," the City Council replied. "Since Mr. Brown is responsible for the idea of Equality Boulevard, we'll let him come up with an answer."

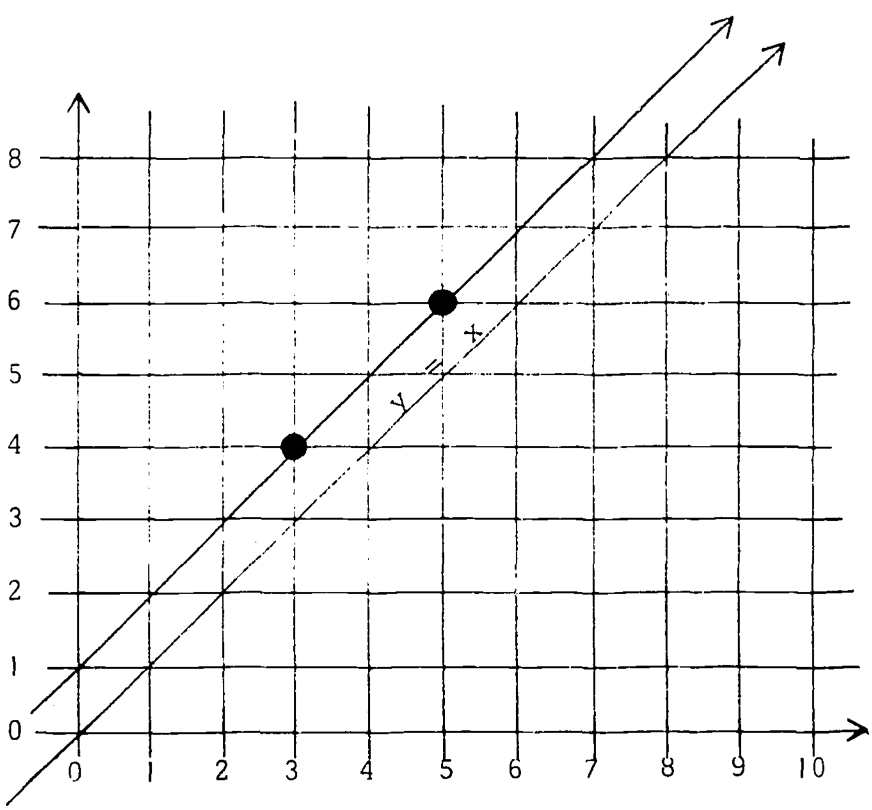
Mr. Brown was quick to answer, "Think about this. Instead of enlarging Equality Boulevard let's build another highway which runs parallel to it. We could build it any distance from Equality Boulevard, but I suggest we build the new highway so that it is always 1 block north of the present highway. Here, I'll show you the route on the map of Squareville."

> Allow each child to locate the intersections through which the new highway will pass. Then they should draw the route. (Remember - only 2 points are needed to determine a straight line.)

Mr. Brown began by marking only two intersections that were 1 block north of Equality Boulevard.



Then he drew a line which contained these two points.



"That line," he said, "should pass through all the intersections that are 1 block north of Equality Boulevard. There is your route for a new highway."

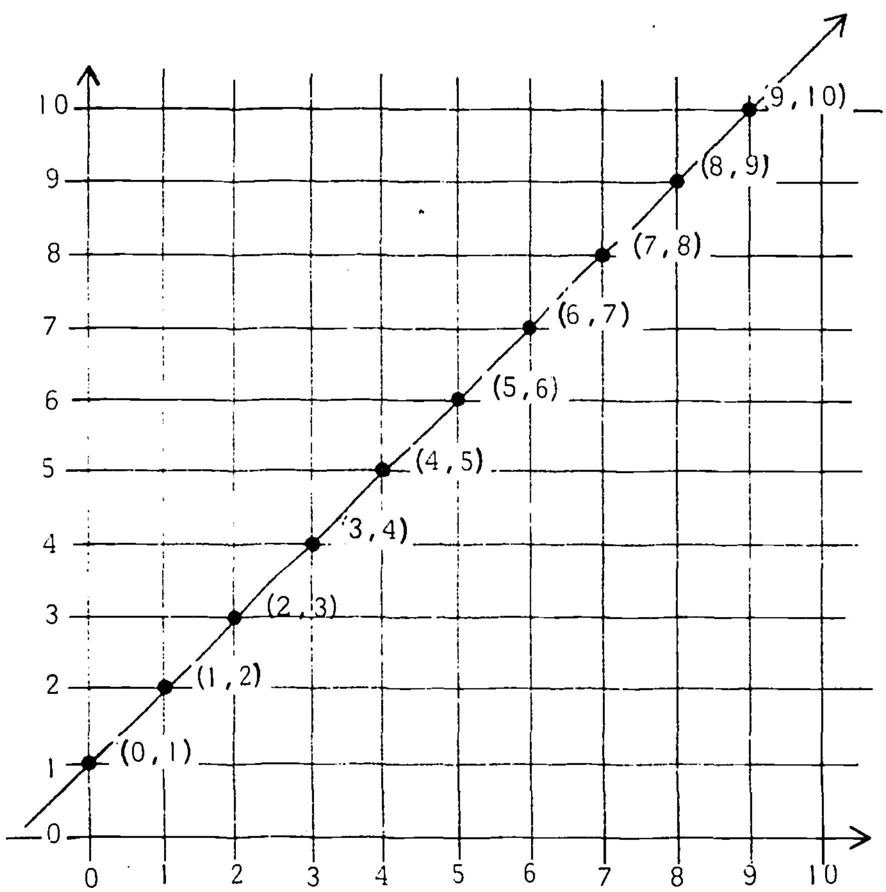
"And what do you propose to name it?" they asked.

Let the children suggest names.

Mr. Brown answered, "Maybe we should use the same method we used for naming Equality Boulevard. If you remember, we first labeled the addresses of all the corners on the highway."

Have children do this on their maps while someone does it on the chalkboard map.

Here are the addresses:



"Do you notice anything about these addresses?" he asked.

Discuss with the children the pattern in the chart, i.e., that the Avenue address is always one greater than the Street address.

When no one answered he suggested that they fill in this chart:

Avenue Address	Street Address
7	6
5	4
2	1
9	8

Avenue Address	Street Address + 1
7	6 + 1
5	4 + 1
2	1 + 1
9	8 + 1

"I get it," someone yelled. "The avenue address is always equal to one more than the street address."

"Good for you," Mr. Brown smiled as he added a 1 to each street address on the chart.

"Now," he said, "try putting that information onto a street sign."

Allow the children to design street signs.

Someone suggested this sign:

$$\boxed{\text{AVENUE ADDRESS} = \text{STREET ADDRESS} + 1}$$

Another person suggested:

$$\boxed{A = S + 1}$$

With this decision they were all very happy. The street engineer was told to build the highway and all their problems would be solved.

You might be interested to know that this was the first day in a long time that Mr. Brown didn't stay late at work. He could hardly wait to tell Tommy about the new highway.

By now the children have become quite familiar with naming coordinates on a grid and with the relationships of several sets of points to other such sets. During this unit, they have been primarily using Squareville as their grid and "intersections" as points on it.

They have also used A and S exclusively as letters to represent points.

It is important, at this point, that the children do not become <u>too</u> accustomed to using A and S in all work with a Cartesian coordinate system. If they are to generalize this learning to other work with coordinate systems, they must also see that points can be named by many different letters.

It would be appropriate here then to bring up the problem of naming streets in other towns. By asking the children their own addresses, they will be reminded that many streets are called, for example, "Boulevard" or "Place" and many do not have number names.

It could be suggested that mathematicians often use the letters X and Y to represent a general case. We could substitute X for S and Y for A and express the same ideas for any grid. It is not so important that the children use X and Y specifically; but it is important for them to see that the letter choice is arbitrary.

Map of Squareville