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ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This unit is designed to provide an adequate background for the presentation of the number line in the next unit. Elementary geometric concepts are presented (or reviewed) such as point, line, etc. Intersection is treated in order to establish the concept that "the interpretation of addition as union" is predicated on the presence of disjoint sets. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

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MINNE

Math

And

Science

Teaching project

UNIT VII

INTRODUCTION to the NUMBER LINE

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1965

MATHEMATICS

FOR THE

ELEMENTARY SCHOOL

UNIT VII

Introduction to the Number Line

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MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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We are deeply indebted to the many teachers who
used earlier versions of this material
and provided suggestions for
this revision

CONTENTS

★ Starring indicates content which is particularly important to the sequential development or evaluation of the program. We ask that all participating teachers try this starred material. It is expected that much of the remaining material will also be used; how much will depend on individual class needs and time available.

Purpose

The purpose of this unit is to provide adequate background for the presentation of number line in the next unit.

Elementary geometric concepts are presented (or reviewed) such as point, line, etc.

Intersection is treated in order to establish the concept that "the interpretation of addition as union" is predicated on the presence of disjoint sets, i.e., sets having no members in common. This is further accentuated by the game, "Sets Up". This topic would, in sequence, coincide with the consideration of intersection in the Science Unit - "Objects and their Properties".

A word of caution to the teacher is herein introduced. Whereas the operation of addition may be interpreted as the union of disjoint sets, this interpretation is limited to sets of counting numbers and is not the main interpretation of addition as presented in this program. Addition will be introduced in the next unit as an operation on the number line, which is an operation that can be performed on all numbers in the real number system, such as common fractions, decimal fractions, negative numbers, etc.

PART A

POINT, CURVE, LINE SEGMENT, RAY

Suggested Activities on Point

A point may be represented by a dot.

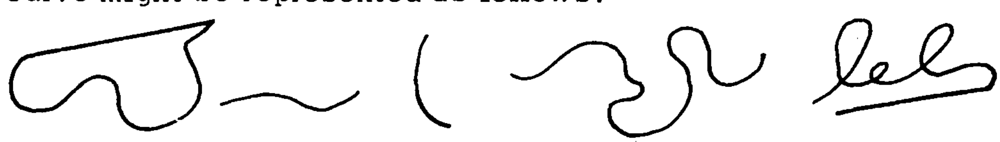
1. Demonstrate on chalkboard.

2. Have several children locate points on the board and make dots. (After they have done this, explain that they have used a dot to represent a point.)

3. Put a dot, and the word "point", on a bulletin board where other geometric figures can also be placed as they are introduced.

Suggested Activities on Curve, Line Segment

A curve might be represented as follows:



1. Draw some curves on the chalkboard in a fashion analogous to the above.

2. Help the children to see that there are two special points on each open curve which are called end-points and that in some way the curve connects these two points. Another way of saying this is that the curve is a picture of a path from one end-point to the other. If the curve is a closed curve then any one point on the curve can act as both end-points.

3. Have several children draw curves on the board. When someone draws a straight line, stop and go to #4. (If no one draws a straight line, after a reasonable time, the teacher should do so.)

★4. Explain that, usually, the term "line" is used to mean <u>straight line</u>. In everyday language, we use the word "curve" to mean something that is <u>not</u> straight. However, in mathematics, a line is a special kind of curve. To show that a line can continue on and on, it is sometimes shown this way:

(Draw this on the board and explain it to the children.)

5. Explain that we cannot, of course, draw a whole line on a sheet of paper or on the chalkboard because it keeps going on and on in both directions. So we make a <u>part</u> of a straight line. That part is called a <u>line segment</u> and is shown by placing end points at both ends of a section of a straight line. (Draw the following examples on the board.)

Examples: Here is line segment AB.

Here is line segment CD.

★ See note on page v.

6. Give the children a piece of 9 x 12 newsprint.

Have them fold it in half and number the pages 1, 2, 3, 4.

On page 1, tell them to use their rulers to draw some straight line segments (they may cross each other).

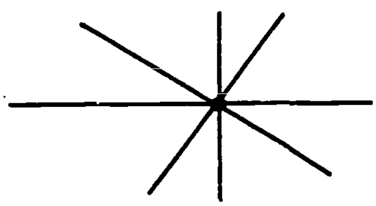
On page 2, have them draw some curves.

On page 3, have them locate a point and make a dot to represent the point. Then say, "Take your ruler and draw a line through the point."

Note:

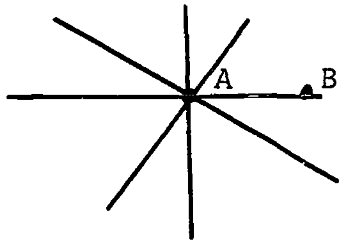
Some children might make two dots. They call one of them the point, and another they call the dot to represent the point. Check their papers and demonstrate on the board so that they understand that there should be just one dot which represents the point they have located.

On page 4, have the children locate a point and make a dot to represent the point. Then tell them to use their rulers and draw a set of lines through that point. All the lines must go through the same point:



(If they don't understand, demonstrate on the board. Allow them to experiment <u>before</u> you demonstrate, however.)

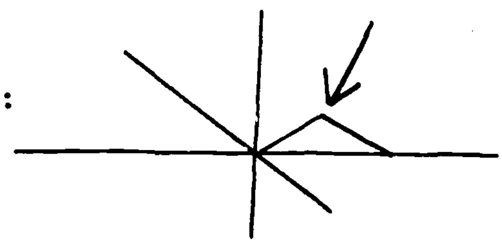
Have them refer to the figure they made on page 4. They are to choose one of the lines they made and locate another point on it.

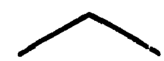


Call attention to the fact that they have made a line segment and have them label the two points that make the line segment - A and B.

Ask this question: "Can you make another line segment having the same end-points - A and B?" (No.)

Someone will probably do this:



Explain that they have drawn two line segments. 1  2

Demonstrate on the board enough times for children to realize that only one line segment can be drawn joining two given points.

7. Another day give the children each a piece of 9 x 12 newsprint.

Have them fold it as before to make four pages.

Have them number the pages.

On page 1, have them locate a point and draw four straight lines through the point.

On page 2, have them locate two points and label them A and B. Then have them draw a line that goes through both points.

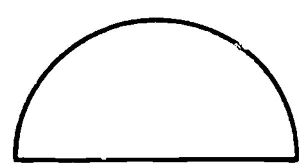
On page 3, have them locate two points and label them X and Y, and draw a straight path from X to Y. Ask the children what they have made. (A line segment, because it is a section of a straight line and has two end points.)

On page 4, have them locate 2 points and give them 2 different letter names. (Any letters they wish - they do not all need to be alike. This is so they can understand that the name given to the point or to the line segment is for convenience, and is not significant.) Tell them to draw a line segment by drawing a straight line connecting the two points. Call on a few children to tell the name of their line segment (i.e., line segment DG or PZ or RN, etc.)

Ask them if they can draw another line segment between the same two points that is different from the one they drew on their paper on page four. Accept all answers, then put 2 points and a connecting line segment on the chalkboard.

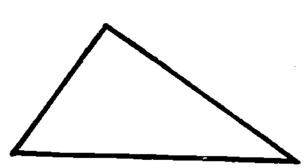
Ask them if they can draw another line segment between the points that is different from the one you drew. (The answer is "no" because there can be only 1 straight line between two points.) If some children say "yes" ask them to demonstrate on the board.

Some child might do this:

A  B

Explain that a line segment must be straight. Although many curves join two points, only one of them is a line segment.

Some child might do this:

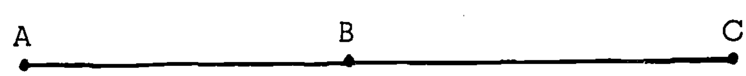
A  B

Explain that he has located another point and that this makes 3 line segments. He has one straight line path and one broken line path. The children should see then that there is no other way to draw line segment AB.

8. To prepare the children for Worksheets 1 and 2, be sure that they understand "pair". Put three dots on the board to represent 3 points. Do not put them in a straight line. Have someone else draw a line between a pair of points. Have someone else draw a line between another pair, etc. (They can make 3 lines connecting the pairs of points.) Do this enough times for the class to understand what is meant by "draw a line connecting a pair of points". The figure they make on the board should look like a triangle.

Then demonstrate what is meant by the directions, "Locate 3 points. Do not put them in a straight line."

Show what would happen if they were in a straight line:



It is impossible to see line segment AC as separate from AB and BC.

When children understand, give them Worksheet 1. This is not designed to be an independent worksheet. The teacher should circulate among the children to answer questions and assist. (The answers are 1 and 3.)

Make 2 dots to represent 2 points.

Draw as many line segments as you can, connecting pairs of these points. Use your ruler. How many did you draw?

____

Locate 3 points in this space. Do not locate them in a straight line.

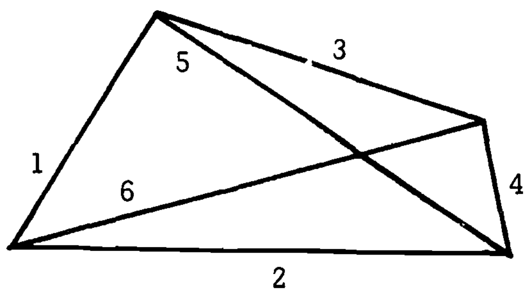
Draw as many line segments as you can, connecting pairs of these points. Use your ruler. How many did you draw?

____

Teacher Commentary
Worksheet 2

To prepare for Worksheet 2, have someone locate 4 points on the chalkboard. (Be sure they are not in a straight line.)

Call on different children to use a ruler and draw line segments between pairs of points. (It is possible to draw 6 line segments.)



Distribute Worksheet 2 when children appear to be ready for it. If sufficient time has been devoted to board practice, the children should be able to do this worksheet independently. (The answers are 6 and 10.)

Locate 4 points in this space. Make dots to represent the points. Do not locate them in a straight line.

Use your ruler and draw as many line segments as you can connecting pairs of these points. How many lines did you draw?

____

Locate 5 points in this space. Make dots to represent the points. Do not locate them in a straight line.

Use your ruler and draw as many line segments as you can connecting pairs of these points. How many lines did you draw?

____

Suggested Activities on Ray

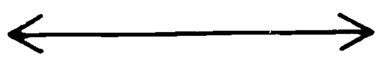
1. Draw a line segment on the board and have the children recall the identifying characteristics they have learned regarding it.

2. Next, draw a ray and ask the children to tell how it compares and contrasts with the line segment. Encourage them to tell what the picture of a ray reminds them of; e.g., light ray originating from the flashlight or the sun, etc.

3. To give the children practice in constructing rays, ask several children to go to the board and each make a ray pointing in a different direction.

4. Ask the children to estimate how many rays they can draw from one dot (or picture of a point). Let them experiment with a straightedge and draw as many rays as they can from one end-point. (Mathematically, an infinite number can be traced, but physically there will be a limitation to the number because of the space used by the pencil line.)

5. Draw a line on the chalkboard.



(The arrows are drawn to show that the line goes on and on in both directions, as far as you like. There isn't room on any sheet of paper to draw the whole line so the arrows are used. By general agreement the arrows are dropped in everyday use of "line".)

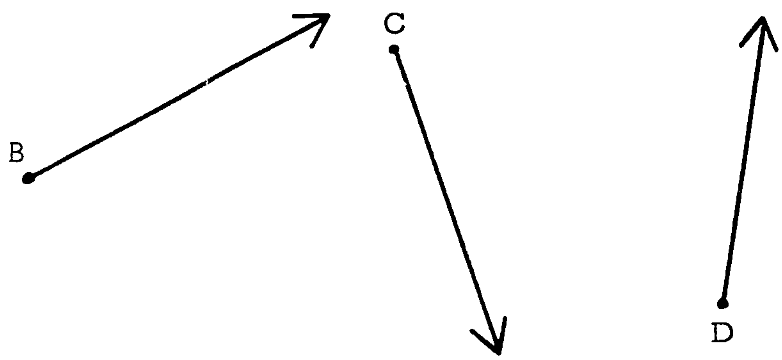
e.g.

We draw a ray like this:

A ray always includes one end point.

The arrow shows that the ray goes on in one direction.

Here are a few more examples of rays:



PART B

INTERSECTION AND UNION

Teacher Background

In the next unit we will introduce addition of real numbers. Before doing so, however, we will discuss a very useful interpretation of addition of non-negative integers or counting numbers. This interpretation allows us to determine the number of members of the union of two disjoint sets by adding the numbers of members of the respective sets. We should not say that addition is the union of disjoint sets. Although this is a very important use it is limited in application and for that reason addition is more adequately presented by the use of the number line. Nevertheless, the union concept is one that can be easily grasped by the child because of the multisensory motivation in actually "combining" or "putting together" sets of physical objects. If in textual material, then, it would be pictures of objects. These activities will make the operation as performed on the number line more meaningful to the child.

To apply this interpretation of addition of counting numbers to determining the number of members in the union of two sets, it is necessary that the sets have no common members. The concept of disjoint sets is dependent upon the understanding of the intersection of two sets, as well as empty set, as presented in earlier units. Therefore the following material is a brief treatment of union, intersection and disjoint sets.

Mathematicians use the symbol $\cup$ for union and $\cap$ for intersection. In the first grade we would avoid using the symbols and would use only the words union and intersection in writing these operations.

Intersection

The intersection of sets A and B is the set which contains those and only those elements which belong to both A and B.

Example:

$A = \{a, b, c, d, e\}$ $B = \{b, d, g\}$

The intersection of A and B could be named C such that

$$C = \{b, d\}.$$

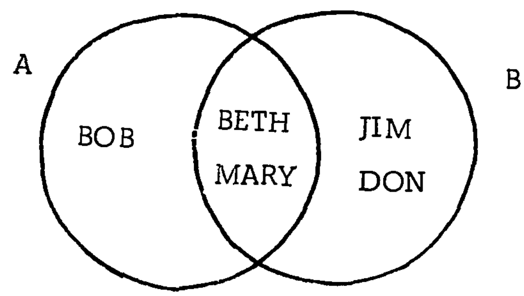
In the operation of <u>intersection</u> of two sets, another set is formed. In the intersection list only the elements common to both sets.

If $A = \{\text{Bob, Mary, Beth}\}$,

and if $B = \{\text{Jim, Mary, Beth, Don}\}$,

then $A \cap B = \{\text{Mary, Beth}\}$,

read: "A intersection B is a set whose members are Mary and Beth."



$A \cap B = \{\text{Mary, Beth}\}$

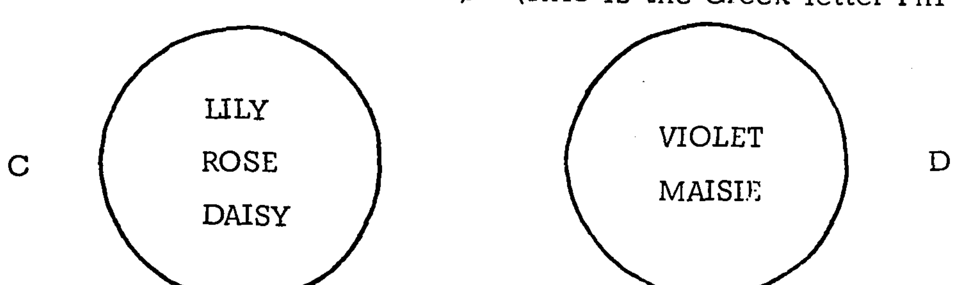
Below is an example of the intersection of two disjoint sets, that is two sets having no common elements.

$C = \{\text{Lily, Rose, Daisy}\}$

$D = \{\text{Violet, Maisie}\}$

$C \cap D = \{\ \}$

or $C \cap D = \emptyset$ (This is the Greek letter Phi [fē] .)



$C \cap D = \{\ \}$

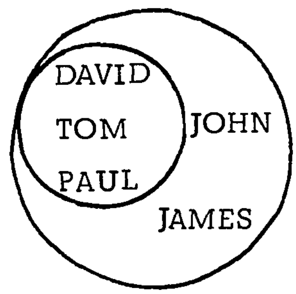
The intersection of two disjoint sets is the empty set.

Another example of intersection is the intersection of a set and one of its subsets.

E = {John, David, Tom, Paul, James}

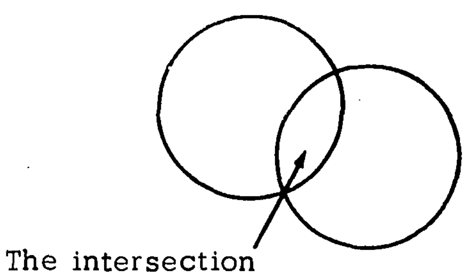
F = {David, Tom, Paul}

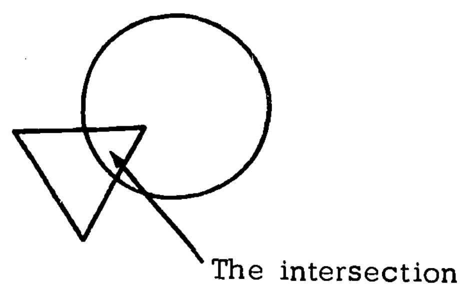
E ∩ F = {David, Tom, Paul}



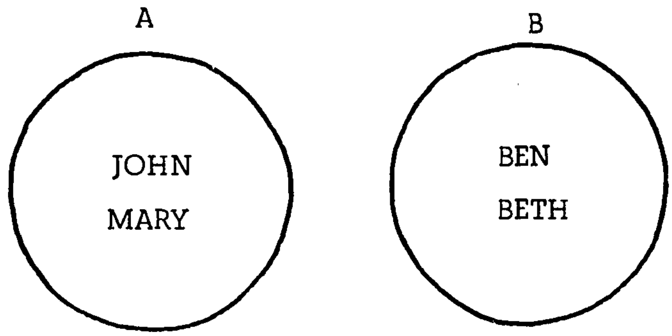
E ∩ F = {David, Tom, Paul}

Help them understand that any shape may be used to illustrate intersection:





Union



$A \cup B$ = {John, Mary, Ben, Beth}

The above is an example of the union of two sets having no common elements. Such sets, with no members in common, are called disjoint sets.

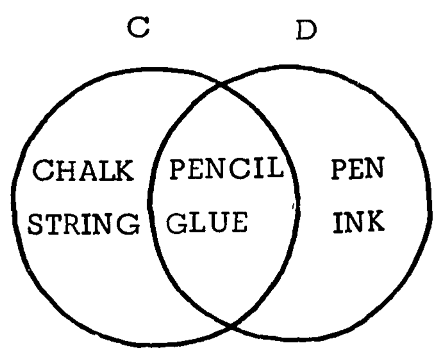
In the next illustration of union of two sets, we have two sets having some common elements.

C = {chalk, string, pencil, glue}

D = {pen, ink, glue, pencil}

$C \cup D$ = {chalk, string, pencil, glue, pen, ink}

Since a member is usually listed only once, we list pencil and glue just once in the union even though they appear in both sets.



$C \cup D$ = {chalk, string, pencil, glue, pen, ink}

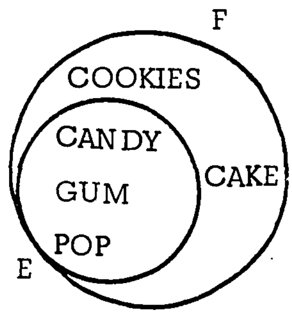
Another type of union is the union of a set and one of its subsets.

$$E = \{\text{candy, gum, pop}\}$$

$$F = \{\text{candy, gum, pop, cookies, cake}\}$$

$$E \cup F = \{\text{candy, gum, pop, cookies, cake}\}$$

Read: "E union F is a set whose members are candy, gum, pop, cookies, cake."



The teacher is also referred to the material on intersection and union at the end of Unit IV.

Suggestcd Activities on Union and Intersection

1. To introduce ★intersection of sets, have all children wearing blue form Set A.

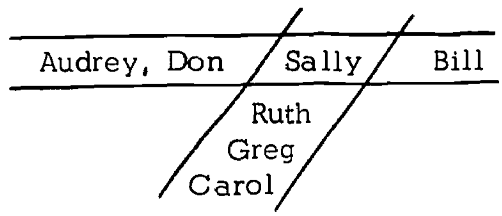
   A = { Audrey, Don, Sally, Bill }

   Have all children wearing red form Set B.

   B = { Carol, Greg, Ruth, Sally }

   Since Sally is in both sets, she is the only member of the set that is the intersection.

   Illustrate on chalkboard:

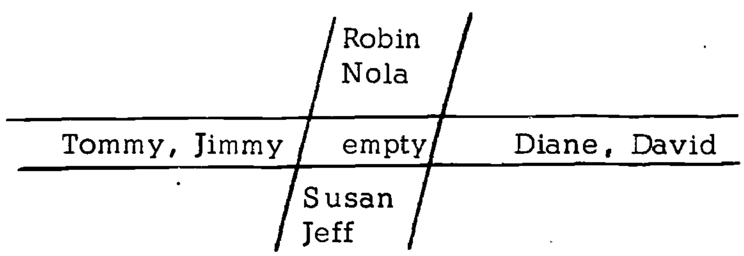


   The intersection is also a set. We can call it D.

   It is possible to have an empty set intersection; e.g., Set A is all children who have a baby brother or sister. Set B is all children who have no brothers or sisters at all.

   A = { Tommy, Jim, Diane, David }

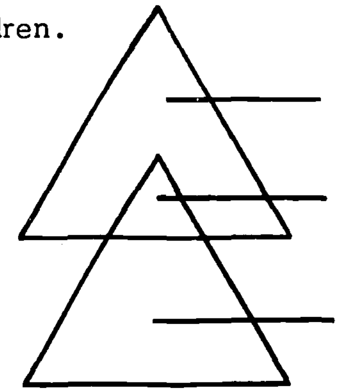
   B = { Robin, Nola, Susan, Jeff }



   Since no child is in both sets, the intersection is the empty set.

Another way to show the intersection is as follows:

Present it to the children.



2. Say, "Now we are going to unite sets to make new sets. All girls with plaid dresses (or pink or green, or any other means of identification) stand over here on this side of the room."

   "All boys with green (or any color you choose) shirts line up on the other side."

   "Now we have two sets - a set of girls with plaid dresses and a set of boys with green shirts."

   "Will the members of both sets move together over in this part of the room?"

   "Now we have made a ★union of sets and our new set is a set of girls with plaid dresses and boys with green shirts."

   Ask the question, "Why is (Mary) in the union of sets?" "Why is John in the union of sets?" "Why isn't Ruth in the union?" etc.

3. Have those children sit down. Ask the boys wearing tennis shoes to line up on one side of the room. Ask the boys wearing blue shirts to line up on the other side.

Ask the boys who belong to the union of these two sets to raise one hand. Ask the boys who belong to the intersection of these sets to raise both hands.

Ask how to arrange the two sets so that it is easier to see who is in which set. Have the children solve the problem.

"Which boys belong to the intersection of the set of boys with tennis shoes, and the set of boys with blue shirts? Can you think of a way to arrange these boys to <u>show</u> that they belong to both the set of boys with tennis shoes and the set of boys with blue shirts?"

(They might line up like this, or as in Figure 2 on the next page.)

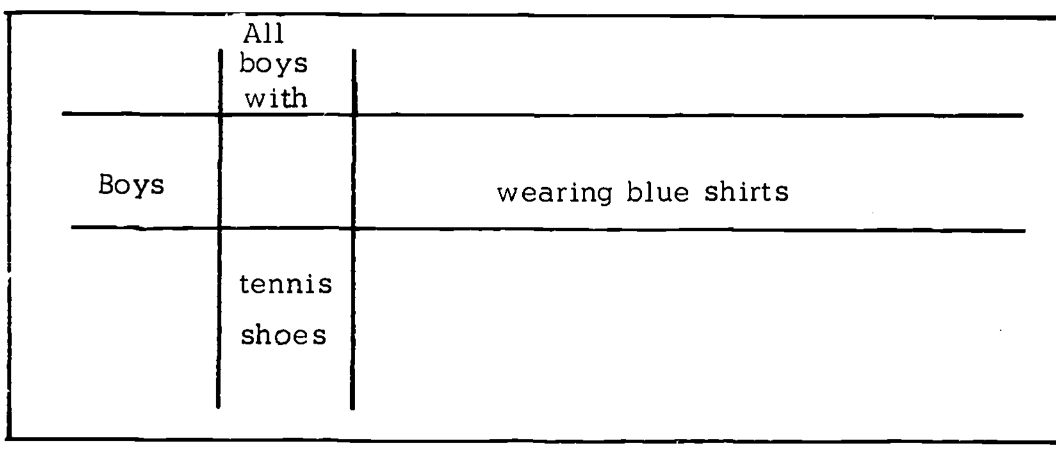


Figure 1

Here is another way to show the intersection of these sets. Draw a circle on the floor and have all boys wearing tennis shoes stand within this circle. Next, draw a second circle on the floor overlapping the first and ask, "Which boys should stand in the second circle? In the overlapping area?" Copy Figure 2 from the floor onto the chalkboard (as on this page.)

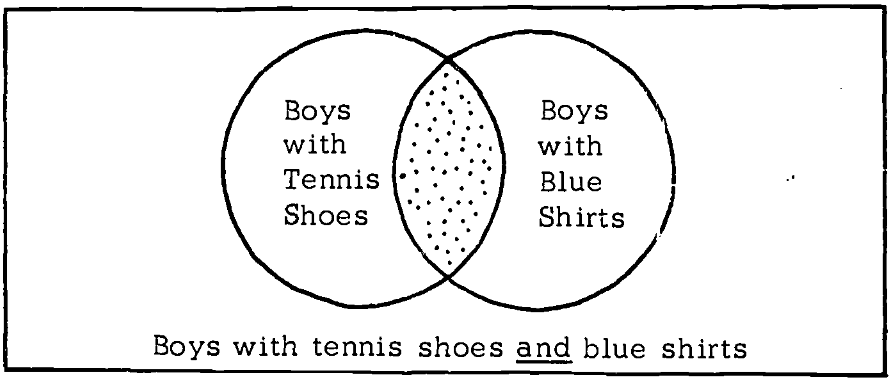


Figure 2

Point out that the boys are in the intersection of the sets, regardless of where they stand.

4. Repeat with variations; e.g., a set of girls with plaid dresses and girls with hair ribbons; boys having blue trousers compared with a set of boys having blond hair; set of children with blue eyes compared to set of children with dark hair.

   Do any children occupy the shaded area? If not, then the intersection of the two sets is <u>empty</u>. Deliberately choose some sets so that there will be a variety of relationships developed.

   Give each child with blue trousers an object, give each blue-eyed child another type of object; those with <u>both</u> objects are in the intersection - all children with one or both objects, form the union of the two sets.

5. Teach the song "Togetherness".

6. Do Worksheets 3, 4, ★5, 6.

Commentary on Song, "Togetherness"

A set of children wearing white intersects with a set of children wearing blue in a subset of children wearing both blue and white.

This song can be sung more meaningfully if the children gather into appropriate sets according to the color of their clothes. Two colors are used for each verse of the song.

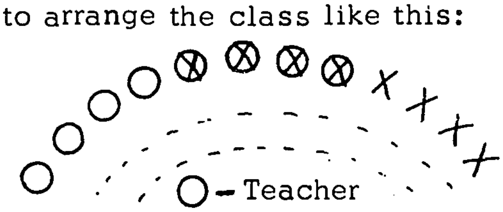
Suggested verses (it would not be difficult for a class to make more):

Those with white sing:	White and blue, blue and white, Without white we are not right.
Those with blue sing:	We are children wearing blue, Without blue we cannot do.
Those with both colors sing:	White and blue, blue and white, We're both blue and white and bright.

Another verse: We are children wearing brown,
We would frown without our brown.
We are children wearing pink,
We would sink without our pink.
Pink and brown, brown and pink,
We're both brown and pink - and think.

It is convenient to arrange the class like this:

O = blue



X = white

The blues and whites are standing, and the rest of the children sitting in front clapping in rhythm. Rather than playing the piano, the teacher preferably will be directing each "set" as they sing in turn.

TOGETHERNESS

Worksheet 3 Intersection of sets

On this worksheet there are some pictures of children. Among these children, there is a set of children with hats. There is also a set of children with shoes. Make a frame that will show the intersection of those two sets.

Fill in the frames at the bottom with the right names.

There are two committees to do jobs in the classroom. We call each one of these committees a set of children. The first set of children that have to water the plants is called "P". The second set of children that have to clean the blackboards is called "B".

If P has 5 members, shown in this manner;

$$P = \{Mary, Bill, Tom, Jim, Sally\}$$

and if B has 3 members, shown in this manner;

$$B = \{Bill, John, Sally\}$$

then the intersection of P and B is C such that

$$C = \{ \boxed{} \quad \boxed{} \}$$

Print in the boxes the names of the children who are in C - the intersection.

The sets below contain numbers as members.

$A = \{1, 3, 5, 7\}$ $C = \{2, 4, 6, 8\}$

$B = \{2, 4, 7\}$ $D = \{3, 4, 9\}$

Place numerals within the set marks to show the intersection of sets.

The intersection of A and C is $\{\quad\}$

The intersection of B and C is $\{\quad\}$

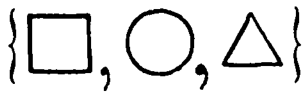
The intersection of A and D is $\{\quad\}$

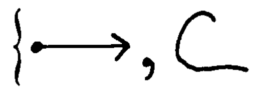
The intersection of C and D is $\{\quad\}$

The intersection of D and C is $\{\quad\}$

Worksheet 6 Intersection of Sets

The sets below contain geometric figures as members.

A =  C = 

B =  D = 

Draw figures within the set marks to show the intersection of sets:

The intersection of A and C is { }

The intersection of B and A is { }

The intersection of A and D is { }

The intersection of D and A is { }

Suggested Activities on Union

1. Have each child tell the number of members in the set of people belonging to his family.

   Make a diagram on the board showing the family of one or more of the children in the class. Suggest that all of them make their own. (They can use just heads to show the people if they can't print names.)

   Have a child tell how many adults there are in his family set. Call this set of adults A. How many children are in his family set? Call this set of children C. Illustrate on the chalkboard.

   A is a set of all adults in his family.
   A = {Mother, Father}
   C is a set of children in his family.
   C = {Ralph, Sue, Bill, Mary}

   Show the union of sets by combining two sets to make a total family set.

   The union of A and C is {Mother, Father, Ralph, Sue, Bill, Mary}.

   Then ask, "How many members are in Set A? (2) How many in Set C? (4) When we put these together (suggest that the family is going for a ride, or on a picnic, or eating dinner), how many are in the new set we made? (6)

   Illustrate on board:
   The number of members in Set A is 2.
   The number of members in Set C is 4.
   We will call our new set (every member of the family) F. The union of A and C is F. The number of members in F is 6.

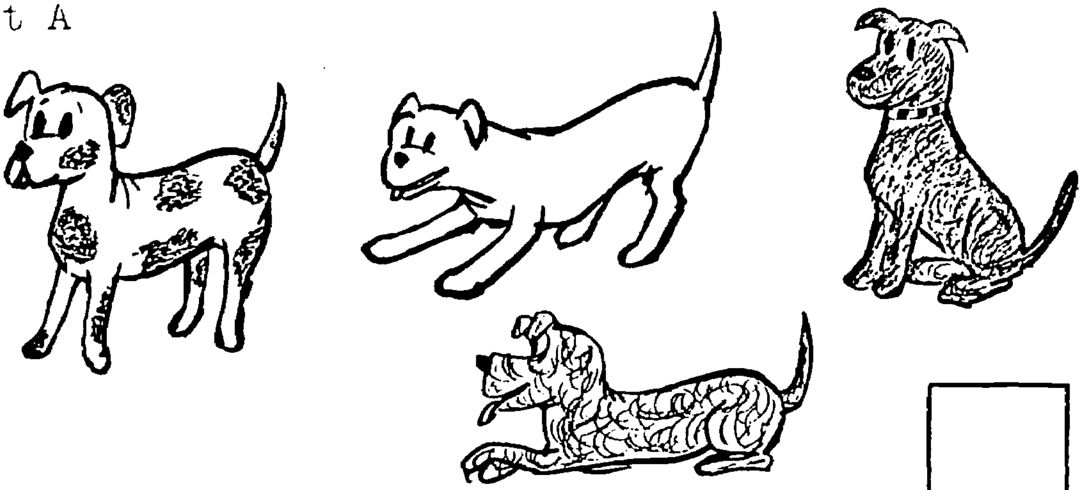
2. Using blocks, or other manipulative objects, have children form sets of different sizes, numerically, and then put two sets together to get the number of members in the union.

3. Have children do Worksheet ★7.

Worksheet 7A

Union

Set A



How many members are in this set?

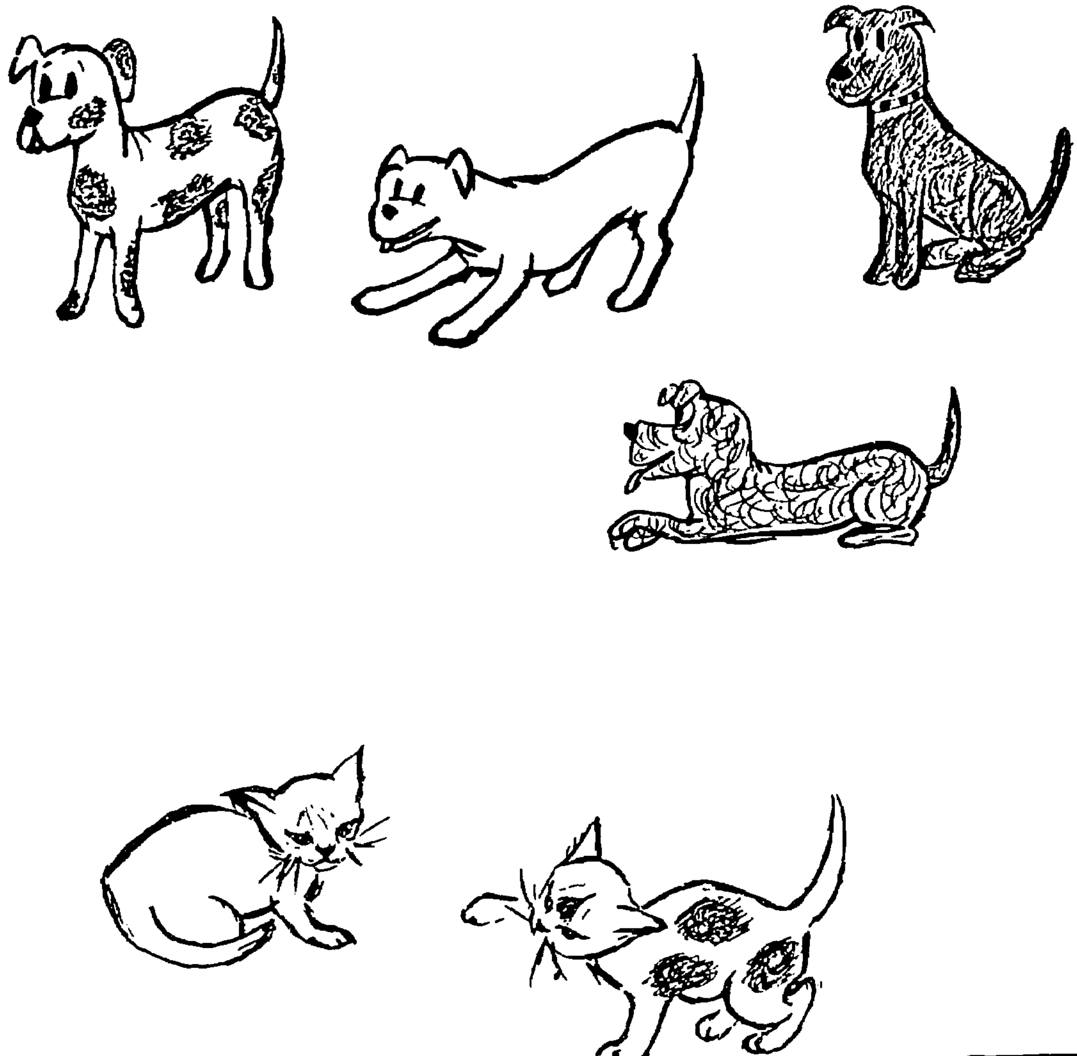
Set of Puppies

Set B

How many members are in this set?

Set of Kittens

Set C



How many members are in this <u>new</u> set?

This shows the union of A and B.

Teacher Background on Sets Up

Elementary school programs commonly introduce addition by way of set union. Since this interpretation of addition is not satisfactory for irrational numbers (such as $\sqrt{2}$ and π), or even for fractional numbers, the Minnemast mathematics program bases addition on the number line. However, in order to help children appreciate the generality of addition, we want them to understand the relationship between addition and set union. Although we do not want to introduce the term addition until the number line, "Sets Up" provides some experience to prepare for addition.

Before children learn, for example, that 2 and 3 is 5, they should learn that adding 2 blocks to 3 blocks gives 5 blocks, and adding 2 crayons to 3 crayons gives 5 crayons; that is, they should learn that 2 of anything added to 3 of anything always produces a set having 5 members. It should also be realized that in order for the number of the union to be the sum of the numbers of the sets being united, the intersection of those sets must be the empty set.

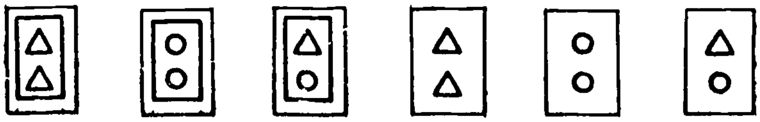
The purpose of "Sets Up" is to help children discover that the number of the union of two sets depends only on the numbers of the sets being united when the intersection of these two sets is the empty set; the number of the union does not depend in any other way on what the members of the sets happen to be. In seeking this discovery at this age level we do not expect sophisticated verbalization from the children.

While reviewing union, intersection, and counting, the activity also reviews circle and triangle.

★ <u>Sets Up</u>

Before trying the activity with students, it is suggested that the teacher first play it with herself to learn how it goes.

Needed are a deck of cards as described below:



RED	1	1	1	1	2	2	8
BLUE	2	2	2	2	2	1	11
ORANGE	2	2	2	0	0	0	6
GREEN	2	1	1	0	0	0	4
							29

These cards can be selected from a pack of Minnemast Color Form Cards.

The activity will be more interesting if this deck of 29 cards is shuffled before starting.

Search the deck for all the blue framed cards having 2 circles, and place them together. Next find the set of all blue framed cards having two triangles. How many members are there in the first set? In the second set?

To keep track of these numbers for further reference, make a chart of four columns as below:

Odd Numbered Set	Even Numbered Set	$\cup$	$\cap$
2	2		

Record the number of members in the first set in the column labeled "Odd Numbered Set", and record the number of members in the second set in the column for even numbered sets.

How many members in the union of these two sets? Place the appropriate numeral in the union ($\cup$) column. How many members in the intersection? Place the appropriate numeral in the intersection ($\cap$) column. If the intersection is the empty set, as in this example, put "0" in the intersection column.

Continue in the same manner through each of the following groups of sets (the first group is repeated here), recording the appropriate numerals in the chart each time.

Group A

1. All blue framed cards with 2 circles
2. All blue framed cards with 2 triangles

Group B

3. All orange framed cards with 2 circles
4. All orange framed cards with 2 triangles

Group C

5. All red framed cards with at least one circle
6. All red framed cards with at least one triangle

Group D

7. All plain cards with at least one blue circle
8. All plain cards with 2 blue triangles

Group E

9. All plain cards with two red circles
10. All plain cards with at least one red triangle

Group F

11. All green framed cards with at least one triangle
12. All green framed cards with at least one circle

Group G

13. All blue cards with both a triangle and a circle
14. All blue framed cards with both a triangle and a circle

Group H

15. All red framed cards

16. All orange framed cards with both a triangle and a circle

After completing the number chart for each group, study the patterns of numbers shown on the chart. Can you make any generalizations on the basis of this pattern of numbers?

The completed chart is shown below; check to be sure that you have counted and recorded accurately the number of members of each set, as well as those in the union and intersection.

Odd	Even	Union	Intersection
2	2	4	0
2	2	4	0
2	2	3	1
3	2	5	0
2	3	5	0
3	2	4	1
3	2	3	2
3	2	5	0

"Sets Up" for the children is played in the same manner with only a few modifications. Familiarize the children with the cards, making sure to distinguish between the plain cards and the framed cards. Then distribute this same deck of 29 cards to the class, giving each child one card. If the class is smaller than 29 pupils, give two cards to some children. If the class is larger than 29 pupils, add as many orange or green plain cards as needed to provide each child with one card; do not add cards of any other kind.

Next, ask the children holding the cards described in Group A, Number 1 to stand. Ask for suggestions of ways of recording this set of children; elicit the suggestion of listing the names of the children in the set. Make a chalkboard chart, as below. Record the names of the children - members of each set, rather than the number of members.

		∪	∩
Mary, Johnny	Tommy, Suzy	Mary, Johnny, Tommy, Suzy	

If colored chalk is available, it may be helpful to label the sets with the design they represent. For example, for set 1 the teacher might write (in blue)  : Mary, Johnny; for set 10 (in red), △ : Beth, George, Bill.

After recording the names of the children holding the cards described in Group A, 1 and 2, then ask all of the children in the union of these two sets to stand. Record their names in the appropriate column. Do the same for the intersection of these two sets.

Repeat this process, asking the children holding the cards described in each group, A - H, to stand and then record their names on the chart. The chart, when completed, will show the names of the children in each set, as well as the members of the union and intersection. The children may tend to confuse the terms "union" and "intersection" and a review of the terminology prior to the game may be profitable.

The next step is to add four columns to the chart. In these columns the <u>number</u> of members in the set will be recorded, rather than the names.

		∪	∩
2	2	4	0
2	2	4	0
2	2	3	1

Ask the children to count the members of each set, then the number in the union and intersection. Record these numbers on the chart, as the children give them. If the children have difficulty in determining the number of members, it may be helpful to reconstruct the sets by having the children stand once again. If this is done, have the class count the children, rather than the names.

After the second chart is complete, ask the class to observe the patterns of numbers. If no one sees a pattern, the teacher (or children) might suggest other sets (not necessarily using the cards). Check each generalization to see if it works for all existing observations. Consider new sets and see if the generalizations will also work for them; that is, see if it has predictive value.

Commentary on Worksheets 8 - 9

This exercise is designed to help children realize that the order of forming unions does not affect the number in the union.

In Worksheet 8, when we make a union of A and B, we have a new set (D) with 7 members. When we make a union of C and D we have a new set (E) with 10 members.

When we change the order and make a union of B and C, we have a new set (F) with 6 members. When we make a union of A and F we have a set G with 10 members.

Likewise, when we change the order again and make a union of A and C, we have a new set (H) with 7 members. When we make a union of B and H, we have set I with 10 members.

Be sure the children notice that E, I, and G are all the same set, so that the result is 10 members, no matter which order we use in forming the unions.

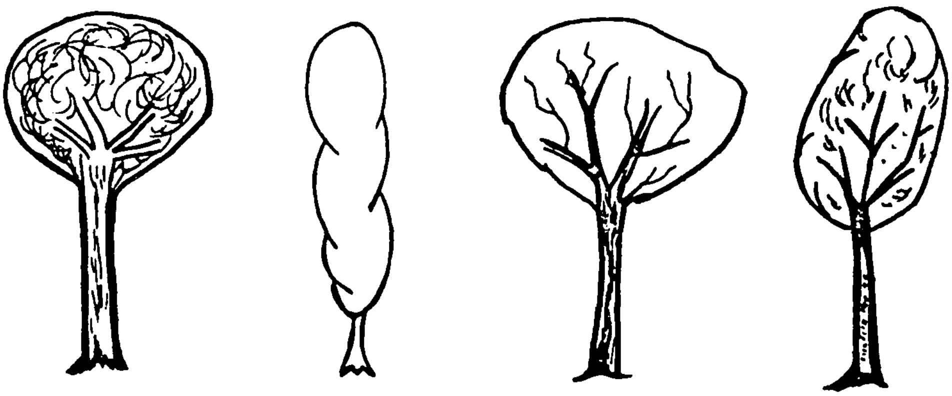
Worksheet 9 is similar to 8.

Worksheet 8A

Union

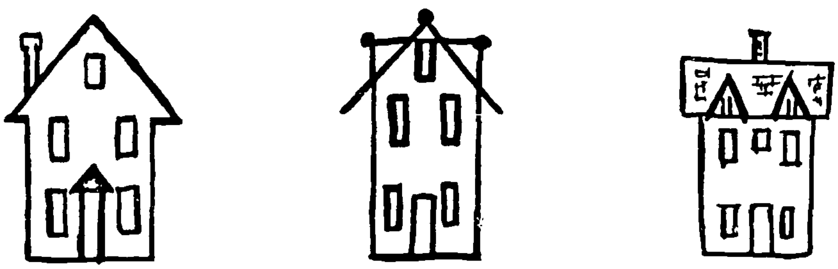
How many are there in each set? Put a triangle (△) in the box with the largest set.

Set A



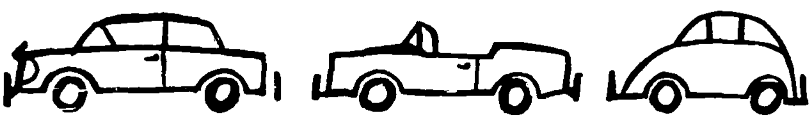
How many members are in this set?

Set B



How many members are in this set?

Set C



How many members are in this set?

Set D

How many members are in this new set?

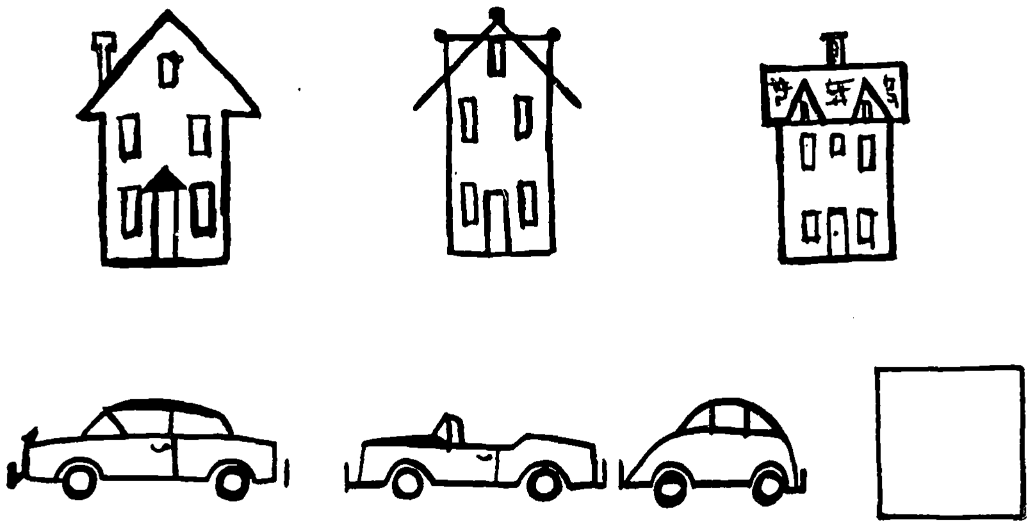
This Shows the union of A and B.

Set E

How many members are in this set?

This shows the union of C and D.

Set F



How many members are in this new set?

This shows the union of B and c.

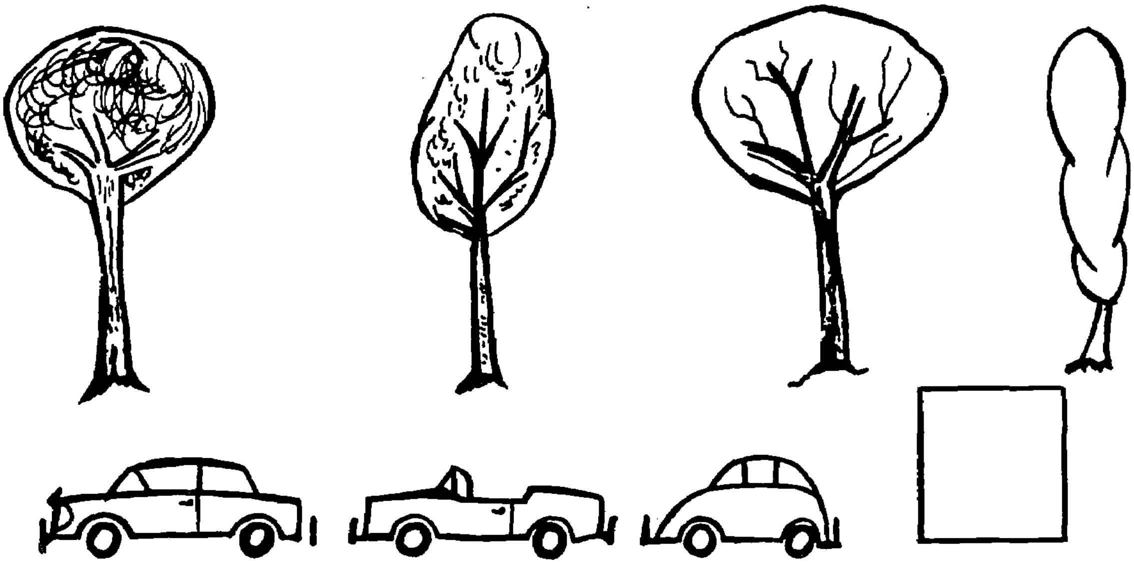
Set G

How many members are in this new set?

This shows the union of A and F.

Worksheet 8D   Union

Set H



How many members are in this new set?

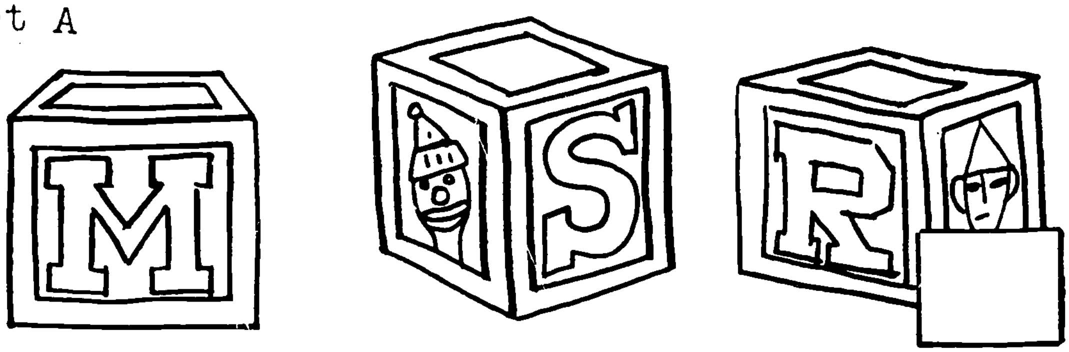
This shows the union of A and C.

Set I

How many members are in this new set?

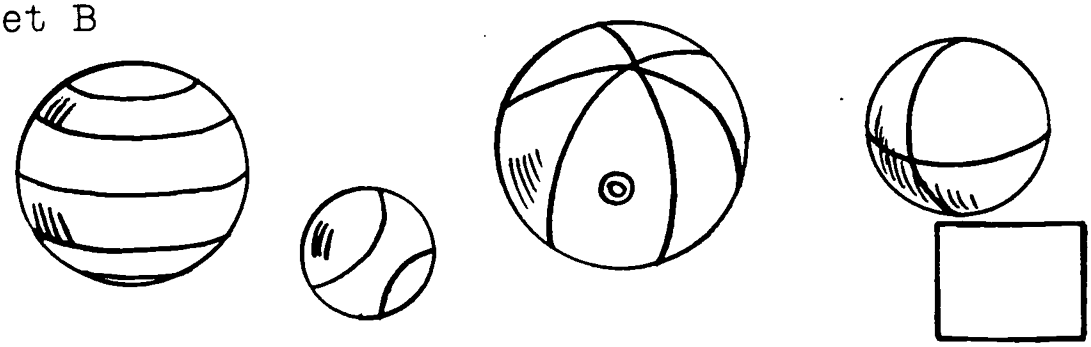
This shows the union of B and H

Set A



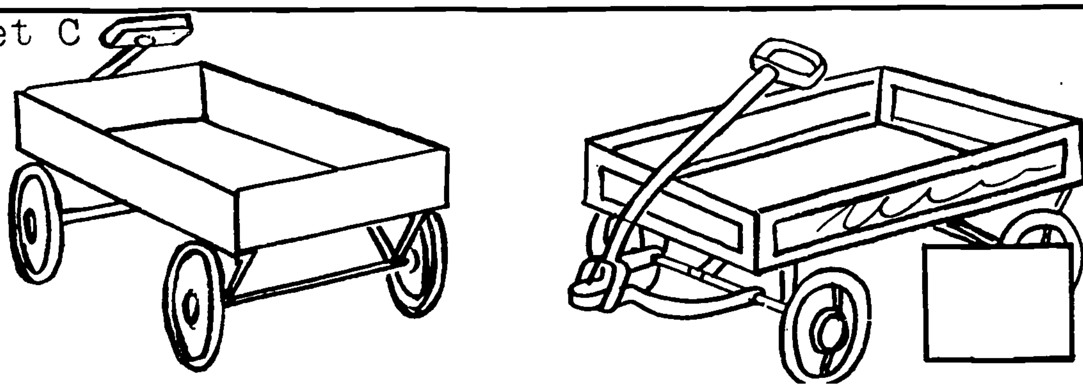
How many members are in this set of blocks?

Set B



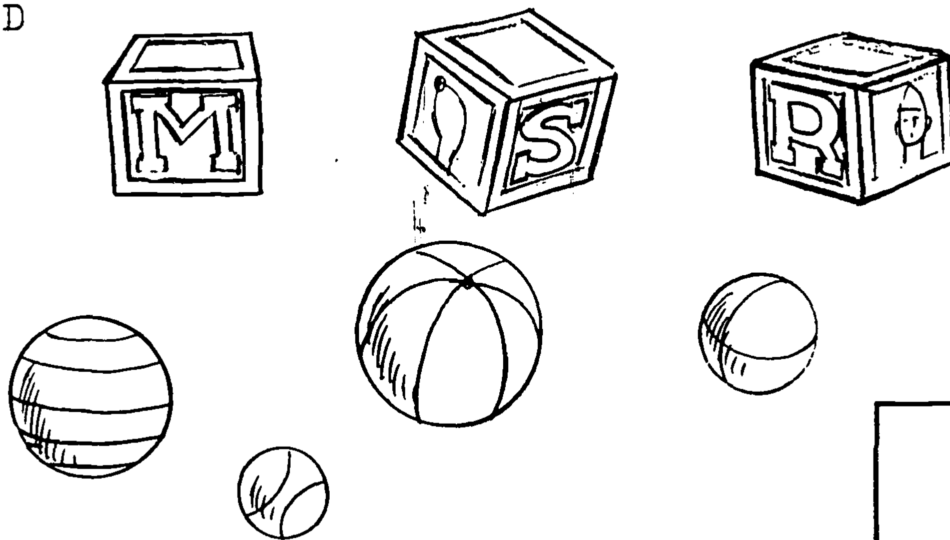
How many members are in this new set of balls?

Set C



How many members are in this set of wagons?

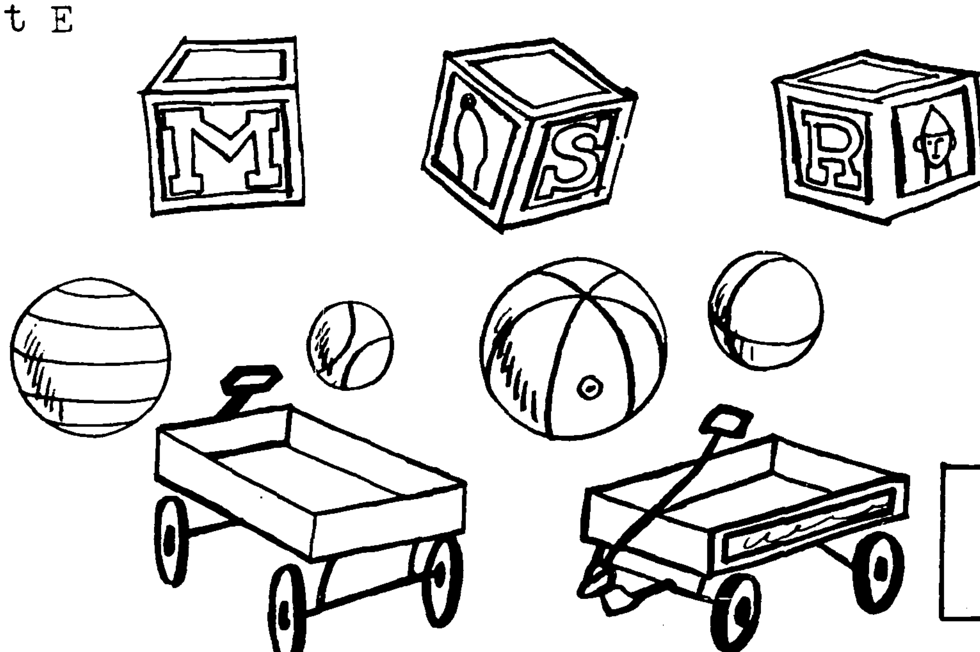
Set D



How many members are in this new set?

This shows the union of A and B.

Set E

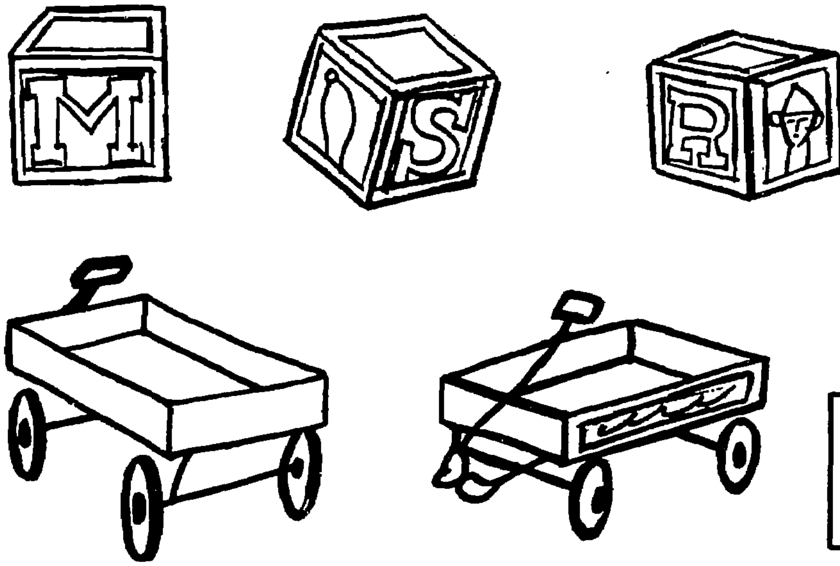


How many members are in this new set?

This shows the union of C and D.

Worksheet 9C Union

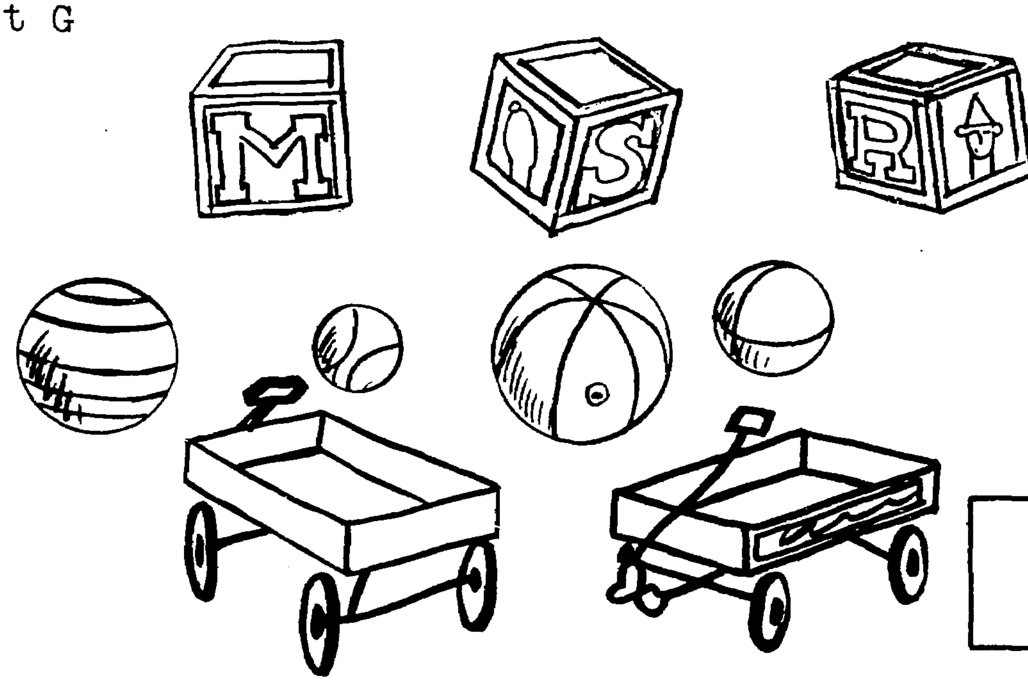
Set F



How many members are in this new set?

This shows the union of A and C.

Set G

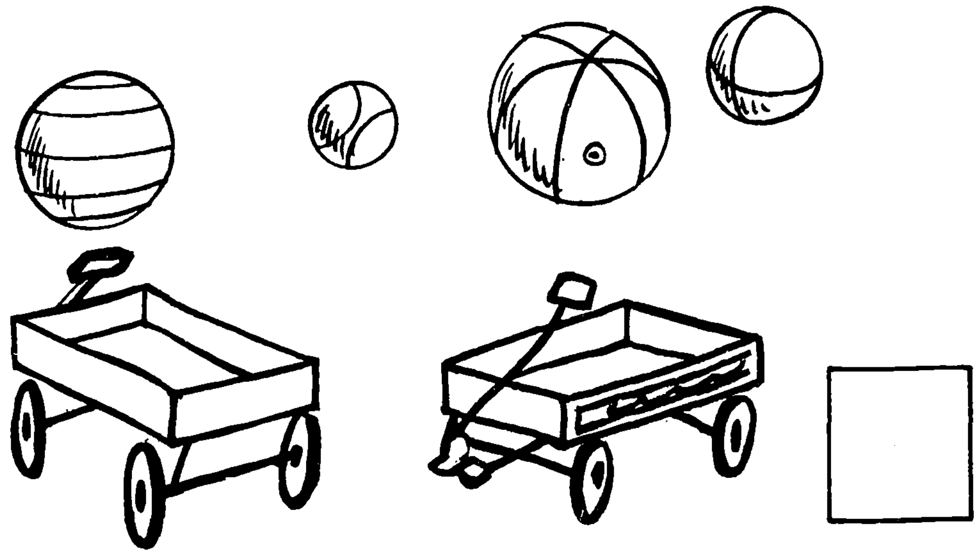


How many members are in this new set?

This shows the union of B and F.

Worksheet 9D Union

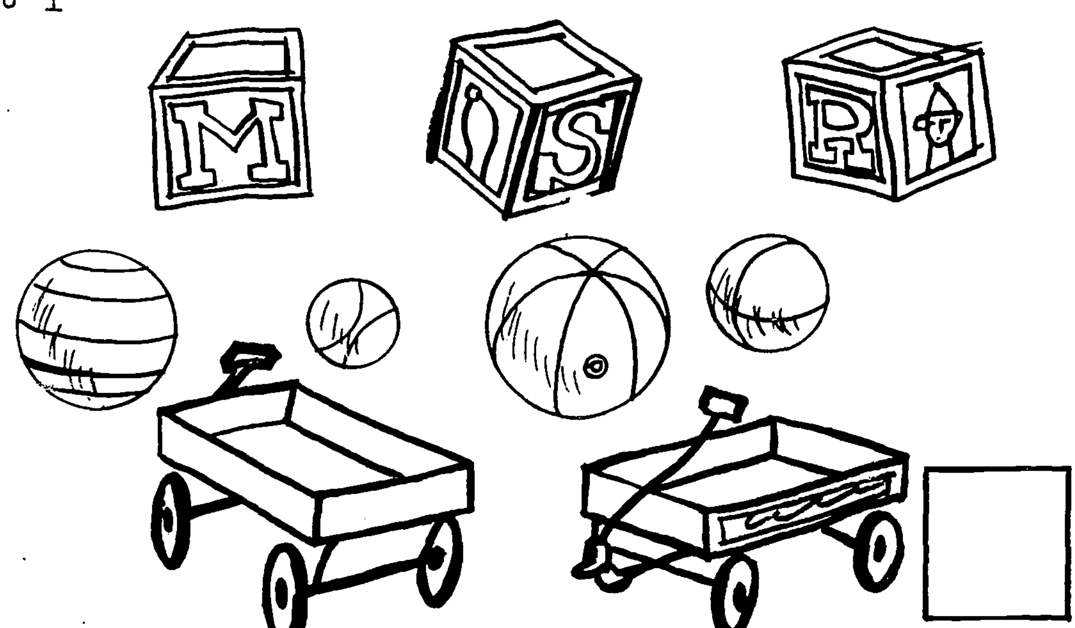
Set H



How many members are in this new set?

This shows the union of B and C.

Set I



How many members are in this new set?

This shows the union of A and H.