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ABSTRACT

During each of two school quarters, approximately 60 college students enrolled in a mathematics course were randomly assigned to an experimental group or a control group. The control group received instruction by the lecture method only; the experimental group received the same instruction, except that six computer-assisted instruction (CAI) units were substituted for six class lectures. All students were given a pretest and a posttest measuring attitude toward CAI, attitude toward mathematics, and achievement in the mathematical content of the CAI units. The following conclusions were drawn: 1) the experimental group's attitude toward CAI improved significantly fron pretest to posttest, but the control group's attitude toward CAI did not change significantly; 2) attitudes toward mathematics improved in both groups but this change reached significance level only in the control group. However, an analysis of covariance controlling on pre-test math attitude scores revealed no significant differences between the two groups in posttest scores measuring attitude toward mathematics. Finally, 3) both groups showed significant achievement gains in mathematics, and there was no significant difference between the achievement of the two groups. (Author/WCM)

Final Report

Project No. 2-G-020
Grant No. OEG-7-72-0022(509)

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CAI: OVERCOMING ATTITUDE BARRIERS OF PROSPECTIVE PRIMARY TEACHERS

March, 1973

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE

Office of Education

National Center for Educational Research and Development
(Regional Research Program)

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During each of two school quarters, approximately 60 college students enrolled in a mathematics course were randomly assigned to an experimental group or a control group. The control group received instruction by the lecture method only; the experimental group received the same instruction, except that six computer-assisted instruction (CAI) units were substituted for six class lectures. All students were given a pre-test and a post-test measuring attitude toward CAI, attitude toward mathematics, and achievement in the mathematical content of the CAI units.

The following conclusions were drawn:

1. The experimental group's attitude toward CAI improved significantly from pre-test to post-test, but the control group's attitude toward CAI did not change significantly.

2. Attitudes toward mathematics improved in both the experimental group and the control group, but this change reached significance level only in the control group. However, an analysis of covariance controlling on pre-test math attitude scores revealed no significant differences between the two groups in post-test scores measuring attitude toward mathematics.

3. Both the experimental group and the control group showed significant achievement gains in mathematics, and there was no significant difference between the achievement of the two groups.

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CAI: Overcoming Attitude Barriers of Prospective Primary Teachers

Lois H. Kockler

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INTRODUCTION

The use of computers as instructional aids in our educational institutions has become more common in recent years, to such an extent that "direct use of computers by teachers and students throughout the country is now an actuality" (11, p. 90). Since the area of mathematics is particularly well-suited to this mode of instruction, much of the early computer-assisted instruction (CAI) work was done in this field. On the primary level, computer programs that give the individual child practice with arithmetic problems appropriate for his own level have already been developed and tested with favorable results (6, 8, 10), and at higher educational levels computers have been used in many capacities to provide instruction in such diverse areas as foreign languages, mathematics and statistics, sciences, and social sciences (5). Furthermore, it seems likely that technological development will permit and encourage increased usage of computers as instructional aids in schools and colleges within the foreseeable future.

The Problem

Despite the expanded use of CAI and the probability of further expansion, there has been very little research in CAI that deals with the affective as well as the cognitive domain. This investigation, however, began with the idea that the quality of teaching ought to be judged not only on the amount of information the student has digested or on the skills he has gained, but on the student's attitudes as well. That is, if a student is unhappy with his classroom situation in mathematics, he may be discouraged from continuing his study in that field, or he may develop a dislike for mathematics or a fear of mathematics that causes him to avoid using the knowledge he has gained. When the students themselves are prospective primary teachers, their attitude toward mathematics assumed particular importance, since these students may transmit a negative attitude to another generation of scholars.

Furthermore, the attitudes of prospective primary teachers toward CAI would also be of special interest, since these students may have the opportunity to use the computer within their own classrooms at some time in their career. Yet Bishop (2), who researched programs of teacher education institutions in Missouri and adjoining states, concludes that there is currently very little instruction in CAI for future teachers, and a search of college catalogs indicates that this conclusion is

probably valid in other regions as well. Unfortunatly, if little instruction is being provided in the use of CAI, it is unlikely that these teachers will be psychologically and technologically prepared to integrate this media into their instructional strategies. Tobias (9) found among teachers a bias against terms describing newer forms of instructional media, causing him to advocate instruction in these media for teacher trainees. It seems reasonable to this investigator that such biases against CAI might be removed through the actual use of the computer in a student's academic program.

Purpose of the Study

Assuming that gains in mathematics achievement, gains in attitude toward mathematics, and gains in attitude toward CAI are worthy goals, particularly for prospective primary teachers, it is reasonable to ask whether the use of CAI as a part of a student's academic work will accomplish any or all of these goals. Thus the central objectives of this study were as follows:

1. To develop six CAI units. Each unit is an automated programmed instruction lesson that covers a topic normally included in Math 190, a course designed primarily for elementary education majors, at Iowa State University. The units were written in CPS (Conversational Programming System), and each unit provides approximately 30 minutes of instruction. For the experimental group (the CAI group), these six units were used as a replacement for six traditional classroom lectures covering the same topics.

2. To determine if the use of CAI as a part of an undergraduate mathematics course can change the student's attitude toward CAI.

3. To determine if the use of CAI as a part of an undergraduate mathematics course can change the student's attitude toward mathematics.

4. To compare gains of knowledge of mathematics made by CAI students with gains made by students in a conventional lecture situation (the control group).

METHODS AND PROCEDURES

The experiment was conducted in two parts; the first trial, which was used to evaluate and improve the computer programs and the testing instruments, and the replecation, which was conducted approximately six months later.

Experimental Procedure

Two hundred forty-three students enrolled in Math 190 in winter quarter, 1972, took the pre-tests in attitude toward CAI, attitude toward math, and achievement in math during the first class period of the quarter. By using a table of random numbers, a stratified random sample was drawn to form an experimental group and a control group. Each group consisted of 16 elementary education majors (15 females and one male), and 16 students not majoring in elementary education (10 females and six males). Although the groups were thus balanced by sex, no comparisons between sexes were made because of the relatively small number of male subjects. Students not selected for either the experimental group or the control group were required to take the post-tests also, but their scores were not used in the evaluation.

Students in the experimental group were told that on six specified class days during the quarter they were not to attend class. Instead, they were required to use the CAI unit covering the same topic as that day's class lecture. Since the programs could be used at any time, students were not limited to a particular day or class hour in which they could work a specific program. To make certain that students were actually using the programs, they were required to turn in the IBM sheet from the typewriter terminal after each lesson, but they were given assurance that the quality of their performance on the computer would have no effect on their course grade.

As a result of the first trial, one CAI unit was replaced by a new unit dealing with a different topic, and minor changes were made in the remaining five units and in the CAI attitudinal questionnaire. Except for these changes, however, the experimental procedure used during the first trial was repeated the following fall quarter, using an experimental group of 30 students (21 in elementary education and 9 not in elementary education), and a control group of 30 students (21 majoring in elementary education, and 9 not majoring in elementary education). By means of a stratified random sample, these students were selected from 135 students enrolled in Math 190.

Evaluation Instruments

The questionnaire measuring attitude toward CAI is a modified version of a questionnaire developed by Brown at Pennsylvania State University (7). Brown constructed his forty-item questionnaire largely on the basis of written comments of students and observations of students who had used CAI as a part of their coursework, and he reports the reliability of the instrument as .885 (7, p. 101). The author of the present paper, however, judged 15 questions on the Brown instrument to be inappropriate for this investigation, whereas four questions that the author wanted to ask were not included in the original questionnaire. The form of the questionnaire used in the trial run contains 25 items from Brown's questionnaire and 4 items that were constructed by the investigator. Since many items on this questionnaire are appropriate only for students who have experienced CAI, this form was used as a post-test for the experimental group, and a second form of the questionnaire was constructed by making appropriate changes in the wording of this post-test form, usually changing only the verb tense. This new form was used as a pre-test for all students, and as a post-test for the students in the control group.

After the trial run, an item analysis was done on the CAI questionnaire, and four items were eliminated because of the extremely low correlations with total score. Thus the CAI questionnaire used in the replication was a shortened form of that used during the first trial.

Each form of the CAI questionnaire lists five responses, "strongly disagree," "disagree," "uncertain," "agree," and "strongly agree," for each item, but some questions are worded positively, while others express a negative attitude toward CAI. Each item is scored on a five-point basis with items expressing a positive attitude toward CAI scored as follows: 1 point for marking "strongly disagree"; 2 points for marking "disagree"; 3 points for marking "uncertain"; 4 points for marking "agree"; and 5 points for marking "strongly agree." Scoring is reversed on items expressing a negative attitude toward CAI. On the questionnaire used in the first trial therefore, a theoretically neutral attitude would be represented by a score of 87 (3x29), and the possible extreme scores are 29, expressing a negative attitude toward CAI, and 145, expressing a positive attitude toward CAI. Since four questions were eliminated before the replication, however, a theoretically neutral attitude on the new questionnaire would be 75 (3x25), and possible extremes are 25 and 125.

To measure attitude toward mathematics, a scale developed by Aiken and Dreger in (1) was selected. This test consists of 20 items, 10 of which are positively worded and 10 negatively worded. Like the items on the Brown questionnaire, these are Likert items with five responses from "strongly disagree" to "strongly agree." Again, scoring is done on a five point basis so that the most negative attitude score is 20 (1x20), a neutral score is 60 (3x20), and the most positive attitude score is 100 (5x20). This questionnaire was used in both the first trial and in the replication.

Since an objective of this study was to measure very specific achievement, achievement in the material presented by the CAI units, the achievement test used in this experiment was constructed by the investigator. There are four items worth one point each that deal with each of the six CAI units, so scores could vary from 0 through 24. Although six items are multiple-choice questions, the remaining items are short-answer or completion items. A post-test form of the test was constructed by changing specific numbers or key words in the pre-test form.

RESULTS

As previously stated, this study was primarily concerned with changes in attitude toward CAI, changes in attitude toward mathematics, and changes in mathematics achievement that may result from the use of the CAI units as a part of a standard mathematics course. The first two hypotheses, then, deal with attitude toward CAI, and may be stated in the null form as follows:

1. There is no significant change in the control group's attitude toward CAI as a result of taking Math 190.

2. There is no significant change in the CAI group's attitude toward CAI as a result of taking Math 190.

For the first hypothesis, the calculated t-value obtained during the first trial was a non-significant 1.73, and the value calculated during the replication was a non-significant 1.12. In testing the second null hypothesis, however, the t-values obtained were 7.19 in the first trial and 6.97 in the replication. Both values are significant beyond the .01 level. In both trials, then, the experimental group's attitude toward CAI improved significantly, but the control group's attitude toward CAI did not change significantly.

The third hypothesis, also stated in the null form, is written as follows:

3. There is no significant difference in the control group's attitude toward CAI and the CAI group's attitude toward CAI when initial pre-test differences have been controlled.

To investigate this hypothesis, an analysis of covariance, shown in Tables 1 and 2, was done using two classifications for treatment (CAI or traditional instruction), and two classifications for curriculum (elementary education or not elementary education), and using the pre-test attitude toward CAI as the covariate. In both the first trial and the replication, the analysis reveals that differences between treatments are significant beyond the .01 level; that is, the CAI group and the control group held significantly different attitudes toward CAI at the end of the experiment. The F values associated with differences between curriculums and with interaction between treatment and curriculum, however, are not significant at the .05 level.

Table 1. Analysis of covariance of attitudes toward CAI using pre-test attitude toward CAI scores as a covariate: first trial data

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	3336.52	3336.52	27.89**
Curriculum	1	1.16	1.16	.01
Treatment x curriculum	1	26.26	26.26	.22
Residual	53	6340.56	119.63	
Total	56	9704.50		

Table 2. Analysis of covariance of attitudes toward CAI using pre-test attitude toward CAI scores as a covariate: replication data

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	3779.80	3779.80	42.13**
Curriculum	1	66.34	66.34	.74
Treatment x curriculum	1	.56	.56	.01
Residual	48	4305.98	89.71	
Total	51	8152.68		

**Significant at the .01 level.

During the first trial, both the CAI group and the control group had 32 students, but four CAI students and two control group students failed to complete the experiment. In the replication, four out of 30 CAI students and 3 out of 30 control group students failed to complete the experiment. Statistics are thus based on data collected from 58 students participating in the first trial and 53 students participating in the replication.

The second series of hypotheses, those dealing with attitudes toward mathematics, may be stated in the null form as follows:

4. There is no significant change in the control group's attitude toward mathematics as a result of taking Math 190.

5. There is no significant change in the CAI group's attitude toward mathematics as a result of taking Math 190.

6. There is no significant difference in the control group's attitude toward mathematics and the CAI group's attitude toward mathematics when initial pre-test differences have been controlled.

For the fourth hypothesis, the t-statistic calculated during the initial trial is 2.10, which is significant at the .05 level. During the replication, the calculated t-statistic for hypothesis 4 is 3.30, which is significant at the .05 level and at the .01 level. In both the first trial and the replication, however, the respective t values of 1.72 and 1.80 approach significance but fail to be significant at the .05 level. Hence the fifth null hypothesis cannot be rejected at the .05 level.

In hypothesis 6, treatments and curricula are as described in hypothesis 3, but the criterion variable is the post-test score on the attitude toward mathematics questionnaire, and the covariate is the pre-test score on the attitude toward mathematics questionnaire. In an analysis of covariance, presented in Tables 3 and 4, no significant differences are found between treatments. The effects of curriculum and of interaction between treatment and curriculum are also non-significant. Null hypothesis 6 can therefore not be rejected at the .05 level.

Hypotheses 7 and 8 are stated in the null form as follows:

7. There is no significant change in the control group 's achievement in mathematics as a result of taking Math 190.

8. There is no significant change in the CAI group's achievement in mathematics as a result of taking Math 190.

With hypothesis 7, the calculated t-statistic obtained during the first trial was 7.38, and the t-statistic obtained during the replication was 15.46; both were significant beyond the .01 level. Similarly, the values obtained for the t-statistic in hypothesis 8, (13.57 for the first trial and 15.49 for the replication), were significant beyond the .01 level. Both hypothesis 7 and hypothesis 8 were thus rejected at the .05 level and at the .01 level.

Table 3. Analysis of covariance of attitude toward mathematics using pre-test attitude toward mathematics scores as covariate: first trial results

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	4.85	4.85	.09
Curriculum	1	.86	.86	.02
Treatment x curriculum	1	144.20	144.20	2.55
Residual	53	3001.40	56.63	
Total	56	3151.31		

Table 4. Analysis of covariance of attitude toward mathematics using pre-test attitude toward mathematics scores as a covariate: replication

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	233.08	233.08	2.66
Curriculum	1	233.10	233.10	2.66
Treatment x cirriculum	1	1.36	1.36	.02
Residual	48	4198.52	87.47	
Total	51	4666.06		

The last hypothesis under investigation may be stated as follows:

9. There is no significant difference in the control group's achievement in mathematics and the CAI group's achievement in mathematics when initial pre-test differences have been controlled.

Again, an analysis of covariance was done using the post-test math achievement scores as the criterion variable and the pre-test math achievement scores as the covariate. The analysis,

printed in Tables 5 and 6, shows no significant differences between treatments, no significant differences between curriula, and no significant interaction between treatment and curriculum. Consequently, null hypothesis 9 cannot be rejected at the .05 level.

The means and standard deviations of each of the variables used in these hypotheses may be found in Tables 7 and 8.

Table 5. Analysis of covariance of mathematics achievement using pre-test mathematics achievement scores as a covariate: first trial data

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	22.34	22.34	1.24
Curriculum	1	10.89	10.89	.61
Treatment x curriculum	1	53.98	53.98	3.00
Residual	53	952.95	17.98	
Total	56	1040.16		

Table 6. Analysis of covariance of mathematics achievement using pre-test mathematics achievement scores as a covariate: replication data

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	2.77	2.77	.34
Curriculum	1	21.54	21.54	2.62
Treatment x curriculum	1	9.61	9.61	1.17
Residual	48	394.93	8.23	
Total	51	428.85		

Table 7. Means and standard deviations of the major variables in the study: first trial data

Variable	CAI group mean	CAI group standard deviation	Control group mean	Control group standard deviation
Pre-test CAI attitude	90.04	11.86	87.53	13.18
Post-test CAI attitude	107.68	10.92	91.03	14.23
Pre-test math attitude	59.32	19.76	50.03	20.90
Post-test math attitude	61.75	17.82	53.47	19.01
Pre-test math achievement	8.36	3.58	7.77	2.85
Post-test math achievement	16.64	3.76	15.07	5.30

Table 8. Means and standard deviations of the major variables in the study: replication data

Variable	CAI group mean	CAI group standard deviation	Control group mean	Control group standard deviation
Pre-test CAI attitude	74.46	13.91	76.07	13.04
Post-test CAI attitude	94.88	9.95	78.44	10.26
Pre-test math attitude	62.19	18.53	61.26	17.05
Post-test math attitude	65.27	16.51	68.78	15.68
Pre-test math achievement	7.35	2.48	7.00	2.39
Post-test math achievement	16.88	2.88	17.22	3.14

To examine the relationships among the variables, the correlation coefficient for each possible pair of the six variables used in this study was calculated for students in the CAI group and for students in the control group. These correlation matrices are presented in Tables 9-12.

Table 9. Correlation coefficients between listed variables for students in the CAI group: first trial data

	Pre-test attitude toward CAI	Post-test attitude toward CAI	Pre-test attitude toward math	Post-test attitude toward math	Pre-test achievement in math	Post-test achievement in math
Pre-test attitude toward CAI	1	.353	.050	.039	.081	-.289
Post-test attitude toward CAI		1	.148	.044	-.095	-.289
Pre-test attitude toward math			1	.926**	.574**	.515**
Post-test attitude toward math				1	.531**	.517**
Pre-test achievement in math					1	.614**
Post-test achievement in math						1

* Significant at the .05 level.

** Significant at the .01 level.

Table 10. Correlation coefficients between listed variables for students in the control group: first trial data

	Pre-test attitude toward CAI	Post-test attitude toward CAI	Pre-test attitude toward math	Post-test attitude toward math	Pre-test achievement in math	Post-test achievement in math
Pre-test attitude toward CAI	1	.675**	-.149	-.077	.167	-.028
Post-test attitude toward CAI		1	-.095	.029	.041	-.249
Pre-test attitude toward math			1	.904**	.512**	.314
Post-test attitude toward math				1	.600**	.295
Pre-test achievement in math					1	.229
Post-test achievement in math						1

* Significant at the .05 level.

** Significant at the .01 level.

Table 11. Correlation coefficients between listed variables for students in the CAI group: replication data

	Pre-test attitude toward CAI	Post-test attitude toward CAI	Pre-test attitude toward math	Post-test attitude toward math	Pre-test achievement in math	Post-test achievement in math
Pre-test attitude toward CAI	1	.249	.225	.027	-.355	-.311
Post-test attitude toward CAI		1	.527**	.533**	-.277	-.284
Pre-test attitude toward math			1	.882**	-.278	-.021
Post-test attitude toward math				1	-.281	-.003
Pre-test achievement in math					1	.320
Post-test achievement in math						1

* Significant at the .05 level.

** Significant at the .01 level.

Table 12. Correlation coefficients between listed variables for students in the control group: replication data

	Pre-test attitude toward CAI	Post-test attitude toward CAI	Pre-test attitude toward math	Post-test attitude toward math	Pre-test achievement in math	Post-test achievement in math
Pre-test attitude toward CAI	1	.552**	.197	.004	-.210	-.151
Post-test attitude toward CAI		1	.263	.200	.019	-.113
Pre-test attitude toward math			1	.741**	.429*	.072
Post-test attitude toward math				1	.393*	.142
Pre-test achievement in math					1	.251
Post-test achievement in math						1

* Significant at the .05 level.

** Significant at the .01 level.

CONCLUSIONS

The contention that a student's attitude toward CAI can be changed through the use of CAI as a part of his coursework was strongly supported by this study. In both trials of the experiment, attitudes toward CAI improved significantly in students who used CAI, but these attitudes failed to improve in students who did not use CAI (the control group). Thus exposure to CAI had a positive effect on student attitudes toward CAI.

Another test supporting this attitude change is the analysis of covariance, which revealed that the CAI group and the control group had significantly different attitudes toward CAI as measured by the post-test. (In this analysis, original pre-test differences in attitude toward CAI were controlled.) In other words, the CAI group and the control group had significantly different attitudes toward CAI at the end of the experiment; the CAI group's attitude toward CAI had changed significantly in the positive direction, while the control group's attitude had not changed significantly.

In the area of attitude toward mathematics, however, conclusions are less clear. Attitudes toward mathematics improved in both the CAI group and the control group, but this change in the control group's attitude fell slightly above significance at the .05 level in the first trial, and above significance at the .01 level in the replication. In both trials, however, the change in the CAI group's attitude fell below significance at the .05 level. Thus the control group's attitude toward mathematics improved significantly in both trials, and the CAI group's attitude toward mathematics improved in both trials, but did not improve significantly. Nevertheless, in both the first trial and the replication, an analysis of covariance controlling on pre-test attitudes toward mathematics scores revealed that the attitudes toward mathematics of the two groups were not significantly different. It would seem, then, that there is not enough evidence to say that CAI is less effective than traditional instruction in changing attitudes toward mathematics, but the hope that CAI would lead to a greater attitude gain was definitely not substantiated.

This experiment also found no significant difference in mathematics achievement between students who had used CAI and students who received traditional instruction. In both trials the CAI group and the control group each made significant gains in mathematics achievement between pre-test and post-test

scores, and the difference between the post-test scores of the CAI group and the post-test scores of the control group were non-significant. In assessing achievement, however, it should be noted that a typical student would complete each CAI unit in less time that the standard 50 minute class period. Thus CAI may be credited with producing achievement gains comparable to those resulting from traditional instruction in less time than was required by traditional instruction.

Recommendations for the classroom include the following:

1. The use of CAI as a part of a student's academic program does appear to be an effective means of improving his attitude toward CAI. In situations in which such improvement is an objective, this "hands-on" approach should definitely be considered.

2. Although students' attitudes toward mathematics did improve somewhat, these attitudes are still not very favorable, especially in the students participating in the first trial of the experiment. Attempts should be made to find ways of improving these student attitudes.

3. Computer-assisted instruction does appear to be a viable instructional strategy. Instructors should consider using CAI when it is appropriate for their educational objectives.

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APPENDIX A: CAI LESSON TOPICS

Units 1-6 were used in the first trial; units 1, 2, 3, 4, 5, and 7 were used in the replication.

I. CAI unit 1

- A. Converting base 12 numerals to base 10 numerals
- B. Counting in a base 12 numeration system
- C. Converting base 10 numerals to base 12 numerals
- D. Working with numerals of the form 23.4(twelve)
- E. Review of multiplying and adding fractions

II. CAI unit 2

- A. The concept of arbitrary operations
- B. Commutative property of arbitrary operations
- C. Properties of subtraction

III. CAI unit 3

- A. Properties of division
- B. Formal definition of division
- C. Quotients involving a zero

IV. CAI unit 4

- A. Definition of "divisible"
- B. Tests for divisibility by 2, 5, 4, 3, 9, 6, and 10
- C. Generality of the divisibility tests

V. CAI unit 5

- A. Definition of "prime"
- B. Review of square roots
- C. Method of determining whether a number is prime

VI. CAI unit 6

- A. Definition of "least common multiple"
- B. Three methods of finding the least common multiple
- C. Using the least common multiple in adding fractions

VII. CAI unit 7

- A. Definition of a mathematical "relation"
- B. The "reflexive" property of a relation
- C. The "symmetric" property of a relation
- D. The "transitive" property of a relation
- E. Recognition of an "equivalence relation"

APPENDIX B: PROGRAM LISTING

Routines Common to All Units

Each of the CAI units consists of a main program and two or three segments called "procedures." Within each procedure are the following two sets of commands: a set of declaration and initialization statements, and a group of statements that process the student's response. Because the declaration and initialization statements, (statements 58-69), and the processing statements, (statements 70-98.1), are identical in each procedure, these statements are printed separately on the following page, and are omitted from the remaining program listings.

```
58.             DECLARE  now LABEL , yes LABEL , no LABEL , dno LABEL , next LABEL , where LABEL ;
59.             DECLARE  fine LABEL , nblk LABEL , news CHAR(50) VAR;
60.             fine=ok;
61.             nblk=lprt;
62.             now=ahora;
63.             yes=ya;
64.             dno=dnein;
65.             no=nein;
66.             nrt=0;
67.             where=here;
68.             ir=1;
69.             iw=0;

70.     ahora:  PUT LIST('Reply:');
71.             READ INTO(reply) ;
71.1            e=reply;
72.             reply=upcase(reply);
73.             IF index(reply,' ')=0 THEN GO TO nblk;
74.             len=length(reply);
75.             IF len<=2 THEN GO TO nblk;
76.             news=substr(reply,1,1);
77.     cha:    DO M=2 TO len;
78.             IF substr(reply,M,1)=' ' THEN GO TO fine;
79.             news=news||substr(reply,M,1);
80.     ok:     END cha;
81.             reply=news;
82.     lprt:   DO K=1 TO ir;
83.             IF index(reply,ans(K))>0 THEN GO TO yes;
84.             END lprt;
85.     lwze:   IF iw=0 THEN GO TO no;
86.     lpno:   DO K=1 TO iw;
87.             KK=K;
88.             IF index(reply,wans(K))>0 THEN GO TO dno;
89.             END lpno;
90.             GO TO no;
91.     ya:     PUT LIST(cor);
92.             nrt=nrt+1;
93.             GO TO where;
94.     nein:   PUT LIST(unrc);
95.             GO TO where;
96.     dnein:  PUT LIST(diag(KK));
97.     here:   iw=0;
98.             ir=1;
98.1            GO TO next;
```

CAI UNIT 1

```
1.          DECLARE  TW1 ENTRY  EXT;
1.1         CALL TW1;
2.          DECLARE  TW2 ENTRY  EXT;
2.1         PUT LIST('                                                    Part 2: In case of malfunction, xeq 2 thru...');
2.2         PUT LIST('Which base twelve numeral would follow e?');
2.3         CALL TW2;
3.          DECLARE  TW3 ENTRY  EXT;
3.1         PUT LIST('                                                    Part 3: In case of malfunction, xeq 3 thru...');
3.2         PUT LIST('To solve 236(ten)=_____(twelve), we could proceed as follows:');
3.3         PUT LIST('236/144=1, remainder 92; 92/12=7, remainder 8; 8/1=8, remainder 0.');
3.4         PUT LIST('Thus 236(ten)=178(twelve). The 1,7, and 8 were the quotients in each of the steps above.');
3.5         PUT LIST('Convert 308(ten) to a base twelve numeral.');
3.6         CALL TW3;
4.          END ;

55.   TW1:  PROCEDURE ;
56.         DECLARE ans(2) CHAR(30) VAR,wans(5) CHAR(20) VAR, cor CHAR(254) VAR,diag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
57.         DECLARE  a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

69.1        PUT LIST('In a base n number system, the positions represented are 1,n,nxn,nxnxn,nxnxnxn, and so on.');
69.15       PUT LIST('In a base 12 number system, therefore, the positions represented are 1, 12, 12x12, _________, and so
on.');
69.2        PUT LIST('(Type the number that belongs in the blank--you may use the letter x to denote multiplication.)');
69.25       ans(1)='12X12X12X12';
69.3        ans(2)='TWELVEXTWELVEXTWELVEXTWELVE';
69.35       wans(1)='12X12X12';
69.4        wans(2)='12CUBED';
69.45       wans(3)='TWELVECUBED';
69.5        wans(4)='TWELVEXTWELVEXTWELVE';
69.52       wans(5)='1728';
69.55       cor='No, 12x12x12 is the correct answer.';
69.6        diag(1)='Yes, 12x12x12 is correct.';
69.65       diag(2)=diag(1);
69.7        diag(3)=diag(1);
69.75       diag(4)=diag(1);
69.77       diag(5)=diag(1);
69.8        unrc=cor;
69.85       ir=2;
69.9        iw=5;
69.93       next=k2;
69.95       GO TO now;

100.  k2:   nrt=0;
101.        PUT LIST('To change a base 10 numeral to a base 12 numeral, just multiply the place value of each position');
102.        PUT LIST('(1, 12, 144, 1728,...) by the numeral in that position; then add each of the products.  For example');
103.        PUT LIST('in the numeral 203(twelve), the place values-reading from left to right- are 144, 12, and 1.  Thus');
104.        PUT LIST('203(twelve)=(2x144)+(0x12)+(3x1)=288+0+3=291(ten).  What is the base 10 representation of 37(twelve)?
');
105.        ans(1)='43';
105.1       wans(1)='+';
105.2       d='Please record only the final form of the answer__';
105.3       diag(1)=d||' 43(ten)';
105.4       iw=1;
106.        cor='Fine.  37(twelve)=43(ten).';
107.        unrc='Not quite. 37(twelve)=(3x12)+(7x1)=36+7=43(ten).';
108.        next=k3;
109.        GO TO now;
```

```
110.    k3:     PUT LIST('Now change 315(twelve) to a base 10 numeral.');
111.            ans(1)='449';
111.1           diag(1)=d||' 449(ten)';
111.2           lw=1;
112.            cor='Right, 315(twelve)=449(ten).';
113.            unrc='No, in 315(twelve) the positions represented are 144, 12, and 1, so 315(twelve)=3x144+1x12+5x1=432+12+5=449
(ten)';
114.            next=k4;
115.            GO TO now;
116.    k4:     next=k6;
117.            IF nrt=2 THEN GO TO next;
118.            PUT LIST('Convert 147(twelve) to a base 10 numeral.');
119.            ans(1)='199';
119.1           diag(1)=d||' 199(ten)';
119.2           lw=1;
120.            cor='Good, 147(twelve)=199(ten)';
121.            unrc='No, 147(twelve)=1x144+4x12+7x1=144+48+7=199(ten).';
122.            next=k5;
123.            GO TO now;
124.    k5:     next=k6;
125.            IF nrt=2 THEN GO TO next;
126.            PUT LIST('Try one more: convert 270(twelve) to a base 10 numeral.');
127.            ans(1)='372';
127.1           diag(1)=d||' 372(ten)';
127.2           lw=1;
128.            cor='Fine, 270(twelve)=372(ten)';
129.            unrc='Not exactly: in 270(twelve) the positions represented are 144,12,and 1, so 270(twelve)=2x144+7x12+0x1=372(t
en)';
130.            GO TO now;
131.    k6:     PUT LIST('Counting in the base twelve system is a little strange, but it is easier if you think');
132.            PUT LIST('about a base 12 abacus. On a base ten abacus, we replace ten beads on any wire with a single');
133.            PUT LIST('bead on the wire to the left of the original wire; this means that there are never any more');
134.            PUT LIST('than nine beads on any wire, so the base ten system needs numerals to represent the whole');
135.            PUT LIST('numbers from 0 through 9. What is the largest number of beads that can be on any wire of a');
136.            PUT LIST('base 12 abacus?');
137.            ans(1)='11';
138.            ans(2)='ELEVEN';
139.            lr=2;
140.            cor='Yes, the answer is eleven.';
141.            a='No. On a base 12 abacus, 12 beads on a wire are replaced by 1 bead on the next wire,';
142.            b=' so there can be no more than 11 beads on any wire.';
143.            unrc=a||b;
144.            next=k7;
145.            GO TO now;
146.    k7:     PUT LIST('Thus a base twelve system needs numerals to represent the whole numbers from 0 through eleven. The');
147.            PUT LIST('base 12 numerals are the same as base 10 numerals from 0 through 9. Think about having 9 beads on
the');
148.            PUT LIST('one''s wire of an abacus, and then adding a single bead to the one''s wire. We now have ten beads on
the');
149.            PUT LIST('one''s wire, so we need a numeral to represent this arrangement. (We can''t use 10, since 10(twelve)
must');
150.            PUT LIST('mean 1x12 + 0x1 =12(ten).) Although almost any symbol could be used, we will use the letter t to
represent');
151.            PUT LIST('ten just because that symbol is easy to remember. Thus counting in the base twelve system goes as
follows:');
152.            PUT LIST('1,2,3,4,5,6,7,8,9,t...');
153.            PUT LIST('If you had to guess, which symbol do you think would be used for the number that follows t?');
```

```
153.5           ans(1)='E';
153.6           wans(1)='11';
154.            lw=1;
155.            cor='Yes, e will be used to represent eleven in the base twelve system.';
156.            diag(1)='No, 11 can''t be used, since 11(twelve)=1x12+1x1=13(ten). Instead, we will use the letter e.';
157.            unrc='Very interesting.';
158.            next=k8;
159.            nrt=0;
160.            GO TO now;
161.    k8:     next=k9;
162.            IF nrt=1 THEN GO TO next;
163.            IF index(reply,'11')>0 THEN GO TO next;
164.            unrc='Actually, though, we will use the letter e as the symbol for eleven in the base 12 system.';
165.            cor=reply||' could be used, but we will use the letter e instead.';
166.            IF length(reply)=1 THEN PUT LIST(cor); ELSE PUT LIST(unrc);
167.    k9:     END ;

 55.    TW2:    PROCEDURE ;
 56.            DECLARE ans(2) CHAR(30) VAR,wans(5) CHAR(20) VAR, cor CHAR(254) VAR,diag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
 57.            DECLARE  a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

 69.1           ans(1)='10';
 69.2           wans(1)='TEN';
 69.3           lw=1;
 69.4           cor='Right.';
 69.5           unrc='No, the answer should be 10(twelve).';
 69.6           diag(1)='No, "ten" means 10(ten); you want 10(twelve), which is read "one-zero".';
 69.7           next=k10;

200.    k10:    next=k11;
201.            IF nrt=1 THEN GO TO next;
202.            PUT LIST('Think of having eleven beads on the one''s wire of an abacus and then adding one more bead to');
203.            PUT LIST('the one''s wire. That makes twelve beads, so we replace all twelve by a single bead on the next');
204.            PUT LIST('wire. Thus we have one bead on the twelve''s wire, and nothing on the one''s wire. But the numeral');
205.            PUT LIST('that represents this arrangement must be one-zero--that is, 10(twelve).');
206.    k11:    PUT LIST('Thus our counting proceeds as follows:');
207.            PUT LIST('    1, 2, 3, 4, 5, 6, 7, 8, 9, t, e');
208.            PUT LIST('10,11,12,13,14,15,16,17,18,19,...');
209.            PUT LIST('Which numeral will suceed 19(twelve)?');
210.            ans(1)='1T';
211.            a='After this come the numerals 1e,20,21,22,...';
212.            cor='Good, 1t(twelve) follows 19(twelve). '||a;
213.            unrc='Not exactly: the answer is 1t(twelve).';
214.            next=k12;
215.            GO TO now;
216.    k12:    next=k13;
217.            IF index(reply,'1T')>0 THEN GO TO next;
218.            PUT LIST('You see 19(twelve) means 1-twelve + 9-one''s. If you add one to that number, the result will');
219.            PUT LIST('be 1-twelve + ten-one''s. Since t=ten, this number is represented by 1t(twelve).');
220.            PUT LIST(a);
221.    k13:    next=k16;
222.            IF nrt>1 THEN GO TO next;
223.            PUT LIST('After 22(twelve) come 23,24,25,26,27,28,29,2t,and_____.');
224.            ans(1)='2E';
225.            a='2e is followed by 30,31,32,33,34,35,36,37,38,39,3t,3e,and 40.';
226.            cor='Fine, 2e(twelve) is correct. '||a;
227.            unrc='The answer is 2e(twelve).';
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 228.              next=k14;
 229.              GO TO now;
 230.     k14:     next=k14A;
 231.              IF index(reply,'2E')>0 THEN GO TO next;
 232.              PUT LIST('2t(twelve) means 2-twelves + ten-ones; if you add 1 to this mess, you will get 2-twelves +eleven
ones,');
 233.              PUT LIST('which is represented as 2e(twelve).');
 234.              PUT LIST(a);
 234.5    k14A:    next=k16;
 234.6             IF nrt>1 THEN GO TO next;
 235.              PUT LIST('If we continue counting in this way, we will reach 49, 4t, 4e, _____(twelve).');
 236.              ans(1)='50';
 237.              cor='Very good, 50(twelve) comes after 4e(twelve).';
 238.              next=k15;
 239.              GO TO now;
 240.     k15:     next=k16;
 241.              IF nrt>1 THEN GO TO next;
 242.              PUT LIST('Since e=eleven, 4e(twelve) means (4x12) + (11x1). If we add 1 to this number, we then have');
 243.              PUT LIST('(4x12) + (12x1) = 5x12 =5-twelves + 0 one''s. But this is represented as 50(twelve).');
 244.     k16:     PUT LIST('Which numeral comes after 3te(twelve)?');
 245.              ans(1)='3E0';
 246.              cor='Fine, 3e0(twelve) is correct.';
 247.              unrc='No, the answer is 3e0.';
 248.              next=k17;
 249.              GO TO now;
 250.     k17:     next=k18;
 251.              IF index(reply,'3E0')>0 THEN GO TO next;
 252.              PUT LIST('3te means 3x144 + 10x12 + 11x1. If we add 1 more, this makes 3x144 + 10x12 + 12x1, which');
 253.              PUT LIST('is the same as 3x144 + 11x12 + 0x1=3e0(twelve).');
 254.     k18:     PUT LIST('Try one more: which numeral comes after 3ee(twelve)?');
 255.              ans(1)='400';
 256.              a='400(twelve) is correct.';
 257.              cor='Good, '||a;
 258.              unrc='Wrong! '||a;
 259.              next=k19;
 260.              GO TO now;
 261.     k19:     next=k20;
 262.              IF index(reply,'400')>0 THEN GO TO next;
 263.              PUT LIST('3ee means 3x144 + 11x12 + 11x1. Adding 1 to this number, we get 3x144 + 11x12 + 12x1, which');
 264.              PUT LIST('is equal to 3x144 + 12x12, which is really 4x144. Thus the answer is 400(twelve).');
 265.     k20:     PUT LIST('To change a base ten numeral to a base twelve numeral, follow the general rule for converting');
 266.              PUT LIST('a base 10 numeral to a base n numeral: divide the base ten numeral by the highest power of 12 that');
 267.              PUT LIST('is less than or equal to the base ten numeral; then divide the remainder by the next lower power of');
 268.              PUT LIST('12; continue this process for each lower power of 12, including the number 1.  The only thing
different');
 269.              PUT LIST('about this conversion is that a quotient of 10 or 11 (should you get one) must be changed to t or e.');
 270.              PUT LIST('Incidentally, do you know why 1 can be considered a power of 12? (Answer yes or no.)');
 271.              ans(1)='YES';
 272.              cor='Fine, then you can answer the following question: ';
 273.              a='1 can be considered a power of 12 because twelve with an exponent zero is equal to 1.';
 274.              unrc=a;
 275.              next=k21;
 276.              GO TO now;
 277.     k21:     next=k22;
 278.              IF index(reply,'NO')>0 THEN GO TO next;
 278.5            PUT LIST('Twelve with an exponent _____is equal to 1.');
 279.              ans(1)='0';
```

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280.              ans(2)='ZERO';
281.              ir=2;
282.              cor='Yes, '||a;
283.              unrc='No, '||a;
284.              GO TO now;
285.      k22:    END ;

 55.      TW3:    PROCEDURE ;
 56.              DECLARE ans(2) CHAR(30) VAR,wans(5) CHAR(20) VAR, cor CHAR(254) VAR,diag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
 57.              DECLARE  a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

 69.1             ans(1)='218';
 69.2             cor='Good, 308(ten)=218(twelve).';
 69.3             next=k23;
 69.4             unrc='No, 218(twelve) is the answer.';

300.      k23:    next=k24;
301.              IF nrt=1 THEN GO TO next;
302.              PUT LIST('The largest power of 12 that is less than or equal to 308 is 144, so the division process');
303.              PUT LIST('starts as follows: 308/144=2, remainder 20.');
304.              PUT LIST('Now divide the remainder by the next lower power of 12, which is 12 itself: 20/12=1 remainder 8');
305.              PUT LIST('Divide by the next lower power of 12:  8/1=8 remainder 0. Thus 308(ten)=218(twelve). The');
306.              PUT LIST('digits 2, 1, and 8 were the quotients in each of the three divisions.');
307.      k24:    PUT LIST('Convert 135(ten) to a base 12 numeral.');
308.              ans(1)='E3';
309.              cor='Right, 135(ten)=e3(twelve).';
310.              unrc='No, the division would proceed as follows:';
311.              next=k25;
312.              GO TO now;
313.      k25:    next=k26;
314.              IF index(reply,'E3')>0 THEN GO TO next;
315.              PUT LIST('135/12=11, remainder 3;  3/1=3, remainder 0.Thus the answer is e3(twelve), since e represents');
316.              PUT LIST('eleven in the base 12 system. A frequent mistake is to represent the numeral as 113(twelve), but');
317.              PUT LIST('113(twelve) would have to mean 1x144 + 1x12 + 3x1 =159(ten), which is incorrect.');
318.      k26:    next=k27;
319.              IF nrt>=2 THEN GO TO next;
320.              PUT LIST('Try one more. Convert 264(ten) to a base twelve numeral.');
321.              ans(1)='1T0';
322.              cor='Correct, 264(ten)=1t0(twelve).';
323.              unrc='No: 264/144=1 remainder 120; 120/12=10 remainder 0; 0/1=0 remainder 0. Hence 264(ten)=1t0(twelve).';
324.              GO TO now;
325.      k27:    PUT LIST('Now let''s consider numbers like 23.4(twelve).In a base 12 system, the value of each position');
326.              PUT LIST('should be 1/12 times the value of the position to the left. From left to right, then, the place
values');
327.              PUT LIST('of 23.4 are 12, 1, and_____.(Please answer with a number, not a word. Guess if you don''t know.');
328.              ans(1)='1/12';
329.              cor='Right, 1/12 is the place value of the next position.';
330.              unrc='No, the answer is 1/12 times the place value on the left, which equals 1/12 times 1, or 1/12.';
331.              next=k28;
332.              GO TO now;
333.      k28:    PUT LIST('What, then, is the base ten representation of 23.4(twelve)?');
334.              ans(1)='274/12';
335.              ans(2)='271/3';
336.              ir=2;
337.              cor='Very good, 23.4(twelve)=27 1/3.';
338.              unrc='Not quite: 23.4(twelve)=(2x12)+(3x1)+(4x 1/12)=24+3+ 4/12, or 27 1/3';
339.              next=k29;
340.              GO TO now;
```

```
341.    k29:    nrt=0;
342.            PUT LIST('In general, then, the positions represented in a base n numeral are as follows');
343.            PUT LIST('...nxnxnxn, nxnxn, nxn, n, 1, 1/n, 1/(nxn), 1/(nxnxn), 1/(nxnxnxn),...');
344.            PUT LIST('For example, .234(ten) really means 2x(1/10)+3x(1/(10x10))+4x(1/(10x10x10)); on the other hand,');
345.            PUT LIST('.234(five) means 2x(1/5) + 3x(1/(5x5)) + 4x(1/(5x5x5))=2/5 + 3/25 + 4/125 = 50/125 + 15/125 + 4/125');
346.            PUT LIST('=69/125, so .234(five)=69/125 (ten).');
347.            PUT LIST('What is the base ten representation of 3t.29(twelve)?');
348.            ans(1)='4633/144';
349.            ans(2)='4611/48';
350.            lr=2;
351.            cor='Very good, 3t.29(twelve)= 46 11/48 (ten).';
352.            unrc='No, 3t.29(twelve)=(3x12)+(10x1)+(2x 1/12)+(9x 1/144)=36+10+ 24/144 + 9/144=46 33/144 = 46 11/48 (ten).';
353.            next=k30;
354.            GO TO now;
355.    k30:    PUT LIST('Perform the following conversion: 142.51(six)=_____(ten).');
356.            ans(1)='6231/36';
357.            cor='Good, 142.51(six)=62 31/36(ten).';
358.            unrc='Not quite: 142.51(six)=(1x36)+(4x6)+(2x1)+(5x 1/6)+(1x 1/36)=36+24+2+30/36 +1/36=62 31/36 (ten).';
359.            next=k31;
360.            GO TO now;
361.    k31:    next=k32;
362.            IF nrt>0 THEN GO TO next;
363.            PUT LIST('Remember that to multiply fractions, you simply multiply the numerators and multiply the denominetors.'
);
364.            PUT LIST('For instance, 2/3 x 5/7 = (2x5)/(3x7) = 10/21. To add fractions, however, you must first change to a');
365.            PUT LIST('common denominator and then add the numerators. Thus 5/6 + 1/36 = (5x6)/(6x6) + 1/36 = 30/36 + 1/36 =
31/36.');
366.            PUT LIST('You get the 30/36 from 5/6 by multiplying the numerator and denominator of 5/6 by 6. This is legal,
because');
367.            PUT LIST('it is equivalent to multiplying by 6/6, which is 1, and multiplying by 1 doesn''t change the number''s
value.');
368.    k32:    PUT LIST('Try one more: What is the base ten representation of 32.13(four)?');
369.            ans(1)='147/16';
370.            cor='Good, 32.13(four)=14 7/16 (ten).';
371.            unrc='No, 32.13(four)=(3x4)+(2x1)+(1x1/4)+(3x 1/16)=12+2+ 4/16 + 3/16=14 7/16 (ten).';
372.            next=k33;
373.            GO TO now;
374.    k33:    PUT LIST('End of lesson--you may logout.');
375.            END ;
```

CAI UNIT 2

```
   1.          DECLARE  C1 ENTRY  EXT;
   1.1         CALL C1;
   2.          PUT LIST('                                                                    part 2: in case of malfunction, xeq 2
thru...');
   2.1         DECLARE  COM2 ENTRY  EXT;
   2.2         PUT LIST('The commutative property of addition on the whole numbers says that a+b=b+a for all whole numbers');
   2.3         PUT LIST('a and b, and the commutative property of set union says that AUB=BUA for all sets A and B. Then if
an');
   2.4         PUT LIST('arbitrary operation called * is commutative on the set of whole numbers, it must be true that_____
');
   2.5         PUT LIST('for all whole numbers a and b. (Your answer should be an algebraic equation in a and b.)');
   2.6         CALL COM2;
   3.          DECLARE  C3 ENTRY  EXT;
   3.1         PUT LIST('                                                                    part 3: in case of malfunction, xeq
3 thru...');
   3.2         PUT LIST('Hence (9-6)-2 is not equal to 9-(6-2), so subtraction is not associative.');
   3.3         PUT LIST('We have said that the set of whole numbers, W, is closed under addition because whenever a and b are
in W,');
   3.4         PUT LIST('then the sum a+b is also in W. The same thing isn''t true of subtraction, however, because both 2 and
9 are');
   3.5         PUT LIST('in W, but the difference_____is not in W.');
   3.6         CALL C3;
   4.          END ;

  55.   C1:     PROCEDURE ;
  56.           DECLARE ans(2) CHAR(30) VAR,wans(5) CHAR(20) VAR, cor CHAR(254) VAR,diag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
  57.           DECLARE  a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

  69.1          PUT LIST('Of course you are familiar with the ordinary operations of addition, subtraction, multiplication, and
division,');
  69.2          PUT LIST('but there are many more operations that can be defined on our number system. Let''s say, for example,
that each');
  69.3          PUT LIST('correct answer on page 1 of an exam is worth 2 points, and every correct answer on page 2 is worth 3
points.');
  69.4          PUT LIST('If a student has n correct answers on page 1 and m correct answers on page 2, his total score is
(2xn)+(3xm).');
  69.5          PUT LIST('This could be considered a new operation that we will call *. For any numbers n and m, then, n*m=2n+3m.
');
  69.6          PUT LIST('For instance, 25*10=(2x25)+(3x10)=50+30=80. What is the numeric value of 8*5? (Please give the answer
as a');
  69.7          PUT LIST('numeral, not an expression.)');
  69.8          ans(1)='31';
  69.85         wans(1)='+';
  69.87         d='Please give the final answer only. In this case, the answer is ';
  69.89         cor='Good, 8*5=31.';
  69.91         unrc='No, the answer is 31: n*m=(2xn)+(3xm), so 8*5=(2x8)+(3x5)=16+15=31.';
  69.93         diag(1)=d||'31; 8*5=(2x8)+(3x5)=16+15=31.';
  69.95         next=k2;

 100.   k2:     PUT LIST('Try another one: What is 35*2?');
 101.          ans(1)='76';
 102.          cor='Right, 35*2=76.';
 103.          unrc='No, 76 is correct: n*m=(2xn)+(3xm), so 35*2=(2x35)+(3x2)=70+6=76. Just substitute n=35, m=2 in the formula
n*m';
 104.          diag(1)=d||'76. Just substitute n=35, m=2 in the formula for n*m: 35*2=(2x35)+(3x2)=70+6=76.';
 105.          next=k3;
 106.          GO TO now;
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107.     k3:     next=k4;
108.             IF nrt=2 THEN GO TO next;
109.             PUT LIST('Try one more of this kind. What is the value of 30*10?');
110.             ans(1)='90';
111.             cor='Right, 30*10=90.';
112.             unrc='Not quite. n*m=(2xm)+(3xm), so using n=30 and m=10, we get 30*10=(2x30)+(3x10)=60+30=90: thus 30*10=90.';
113.             diag(1)=d||'90; n*m=(2xn)+(3xm), so 30*10=(2x30)+(3x10)=90.';
114.             GO TO now;
115.     k4:     PUT LIST('Let''s try an easier operation. Define n$m=(n+m)/2 for all whole numbers n and m. Hence for any
whole');
116.             PUT LIST('numbers n and m, the expression n$m is the arithmetic average of n and m. For example, 7$13=(7+13)/2=20
/2=10.');
117.             PUT LIST('What is 15$11?');
118.             ans(1)='13';
119.             wans(1)='/';
120.             cor='Fine, 15$11=13.';
121.             unrc='The answer should be 13: n$m=(n+m)/2, so 15$11=(15+11)/2=26/2=13.';
122.             diag(1)=d||'13: 15$11=(15+11)/2=26/2=13.';
123.             next=k5;
124.             nrt=0;
125.             GO TO now;
126.     k5:     next=k6;
127.             IF nrt=1 THEN GO TO next;
128.             PUT LIST('Find the value of 20$40.');
129.             ans(1)='30';
130.             cor='Very good, 20$40=30.';
131.             unrc='No, 20$40=(20+40)/2=60/2=30.';
132.             diag(1)=d||'30. 20$40=(20+40)/2=60/2=30.';
133.             GO TO now;
134.     k6:     PUT LIST('We could define the operation # as follows: let n#m=nm-1 for all natural numbers n and m. For
instance,');
135.             PUT LIST('9#8=(9x8)-1=72-1=71. For any pair of numbers, the "answer" is their product minus 1. What is the
value');
136.             PUT LIST('of 4#10?');
137.             ans(1)='39';
138.             wans(1)='-';
139.             cor='Good, 4#10=39.';
140.             nrt=0;
141.             unrc='Not exactly. If n#m=nm-1, then 4#10=(4x10)-1=40-1=39. Just substitute n=4, m=10 into the formula for n#m.';
142.             diag(1)=d||'39. Since n#m=nm-1, therefore 4#10=(4x10)-1=40-1=39.';
143.             next=k7;
144.             GO TO now;
145.     k7:     next=k8;
146.             IF nrt=1 THEN GO TO next;
147.             PUT LIST('Find 11#5.');
148.             ans(1)='54';
149.             cor='Correct, 11#5=54.';
150.             unrc='No, 11#5=(11x5)-1=55-1=54. Substitute n=11, m=5 in the equation n*m=nm-1.';
151.             diag(1)=d||'54. You see, 11#5=(11x5)-1=55-1=54.';
152.             GO TO now;
153.     k8:     PUT LIST('Since mathematicians sometimes have to work with exponents, the following operation is often useful:');
154.             PUT LIST('let n**m=n with an exponent m (that is, n raised to the mth power, or the product of m factors of n).
For');
155.             PUT LIST('instance, 2**3 means 2 to the third power, or 2x2x2, which is 8. Similarly, 5**2 means 5 to the second
power');
156.             PUT LIST('or 5x5, which is 25. (So 2**3=8 and 5**2=25.) What is the value of 3**4? (Type a numeral, not an
expression.)');
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157.              ans(1)='81';
158.              wans(1)='X';
159.              cor='Yes, 3**4=81.';
160.              diag(1)=d||'81: 3**4=3x3x3x3=81.';
161.              unrc='Wrong. 3**4 is 3 raised to the fourth power, which is 3x3x3x3, which is 81. Thus 3**4=81.';
162.              next=k9;
163.              nrt=0;
164.              GO TO now;
165.     k9:      next=k10;
166.              IF nrt=1 THEN GO TO next;
167.              PUT LIST('Find the value of 2**5.');
168.              ans(1)='32';
169.              cor='Right, 2**5=2x2x2x2x2=32.';
170.              diag(1)=d||'32, since 2**5=2x2x2x2x2=32.';
171.              unrc='Not exactly. 2**5 means 2 raised to the fifth power, which is 2x2x2x2x2, or 32. Hence 2**5=32.';
172.              GO TO now;
173.     k10:     next=k11;
174.              IF nrt>0 THEN GO TO next;
175.              PUT LIST('5**3 means 5 to the third power, or _____.(Type the numeric value, not an expression.)');
176.              ans(1)='125';
177.              cor='OK, 5**5=125.';
178.              diag(1)=d||'125. 5**3=5x5x5=125.';
179.              unrc='I''m afraid not: 5 to the third power means 5x5x5, (5 used as a factor 3 times) which is 125. Thus
5**3=125.';
180.              GO TO now;
181.     k11:     END ;

 55.     COM2:    PROCEDURE ;
 56.              DECLARE ans(2) CHAR(10) VAR,wans(5) CHAR(10) VAR, cor CHAR(200) VAR,diag(5) CHAR(200) VAR, unrc CHAR(200) VAR;
 57.              DECLARE  a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

 69.1             ans(1)='B*A';
 69.2             cor='Right, if * is commutative, then a*b=b*a.';
 69.3             unrc='No, if the operation * is commutative on the whole numbers, then a*b=b*a for all whole numbers a and b.';
 69.4             next=k12;

200.     k12:     PUT LIST('If we use the particular operation * described in problem 1 (that is, n*m=(2xn)+(3xm) for any');
201.              PUT LIST('whole numbers n and m), then 25*10=80. What is the numeric value of 10*25?');
202.              ans(1)='95';
203.              wans(1)='+';
204.              iw=1;
205.              cor='Yes, 10*25=95.';
206.              d='Please just type the final answer.';
207.              unrc='Not quite. If n*m=(2xn)+(3xm), then 10*25=(2x10)+(3x25)=20+75=95. Just substitute n=10, m=25 in the
formula n*m.';
208.              diag(1)=d||' 95, since 10*25=(2x10)+(3x25)=20+75=95. Substitute n=10, m=25 in the formula n*m.';
209.              next=k13;
210.              GO TO now;
211.     k13:     PUT LIST('Since 25*10 is not the same value as 10*25, we can therefore conclude that the operation * is not');
212.              PUT LIST('_____on the set of whole numbers.');
213.              ans(1)='COMMUTATIVE';
214.              wans(1)='ASSOCIATIVE';
215.              iw=1;
216.              cor='Good, we have shown * isn''t commutative on W.';
217.              unrc='No, since 10*25 doesn''t equal 25*10, we know that * isn''t commutative on the set of whole numbers.';
218.              a='No, we have shown * isn''t commutative. To show * isn''t associative we would find values';
219.              b=' such that (a*b)*c and a*(b*c) were not equal.';
220.              diag(1)=a||b;
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221.              next=k14;
222.              GO TO now;
223.     k14:     PUT LIST('To show that the operation $ is commutative, we must show that a$b=b$a for all whole numbers a and
b.');
224.              PUT LIST('Since n$m=(n+m)/2, we know that 19$31=(19+31)/2=25, and 31$19=(31+19)/2=25. Does the fact that
19$31=31$19');
225.              PUT LIST('prove that $ is commutative? (Answer yes or no.)');
226.              ans(1)='NO';
227.              cor='That''s right!';
228.              unrc='No!';
229.              next=k15;
230.              GO TO now;
231.     k15:     PUT LIST('The fact that 19$31=31$19 merely indicates that $ may be commutative--it doesn''t prove it, because');
232.              PUT LIST('"$ is commutative on W" means that a$b=b$a for ALL values of a and b in the set W, not just for the
specific');
233.              PUT LIST('values 19 and 31. For this operation, however, a$b=(a+b)/2, and b$a=(b+a)/2. Since (a+b)/2=(b+a)/2 for
all');
234.              PUT LIST('whole numbers a and b(by the commutative property of addition), we thus know that a$b=b$a for all
whole');
235.              PUT LIST('numbers a and b: hence $ is commutative on W. Now look at the operation #, where n#m=(nxm)-1. We
have');
236.              PUT LIST('already shown that 4#10=39. What is the value of 10#4?');
237.              ans(1)='39';
238.              wans(1)='-';
239.              iw=1;
240.              cor='Yes, 10#4=(10x4)-1=39.';
241.              unrc='No, if n#m=(nxm)-1, then 10#4=(10x4)-1=39.';
242.              diag(1)=d||' 39: 10#4=(10x4)-1=40-1=39.';
243.              next=k16;
244.              GO TO now;
245.     k16:     PUT LIST('Thus a#b=b#a for one pair of numbers; to show # is commutative on W, we must still show that a#b=b#a');
246.              PUT LIST('for all elements of W, however. Again, this is not hard, since a#b=(axb)-1, and b#a=(bxa)-1. Thus');
247.              PUT LIST('a#b=(axb)-1=(bxa)-1=b#a, so # is commutative on W.');
248.              PUT LIST('To see if the operation ** (previously defined) is commutative on W, we would want to see if a**b=_____
__');
249.              PUT LIST('for all whole numbers a and b.');
250.              ans(1)='B**A';
251.              cor='Fine, you are correct.';
252.              unrc='No, if ** is commutative on W, then a**b must equal b**a for all whole numbers a and b.';
253.              next=k17;
254.              GO TO now;
255.     k17:     PUT LIST('If n**m=n raised to the mth power, then 2**6=2x2x2x2x2x2=64. What is the numeric value of 6**2?');
256.              ans(1)='36';
257.              ans(2)='6X6';
258.              ir=2;
259.              cor='Right, 6**2=6x6=36.';
260.              unrc='Not quite: 6**2=6x6=36.';
261.              next=k18;
262.              GO TO now;
263.     k18:     PUT LIST('Thus 2**6 does not equal 6**2, so the operation ** is not commutative. Incidentally, 2**4=2x2x2x2=16,')
;
264.              PUT LIST('and 4**2=4x4=16, so it is possible that a**b=b**a for some, but not all, values of a and b. This is
why we');
265.              PUT LIST('can''t show an operation is commutative by showing a**b=b**a for some specific values of a and b.');
266.              PUT LIST('If subtraction is commutative, then it must be true that a-b=_____for all values of a and b.');
267.              ans(1)='B-A';
268.              cor='Correct.';
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 269.            unrc='No, if subtraction is commutative, then a-b=b-a for all values of a and b.';
 270.            next=k19;
 271.            GO TO now;
 272.    k19:    PUT LIST('This is not true, since 5-2 is not equal to 2-5, for instance; thus subtraction is not commutative');
 273.            PUT LIST('on the set of whole numbers.');
 274.            PUT LIST('If subtraction is associative, then it must be true that for all whole numbers a,b, and c, (a-b)-c=a-(b
-c).');
 275.            PUT LIST('For instance, (9-6)-2, which is 3-2, or 1 must equal 9-(6-2), which is_____.(State the final value
only.)');
 276.            ans(1)='5';
 277.            wans(1)='-';
 278.            iw=1;
 279.            cor='Yes, 9-(6-2)=9-(4)=5.';
 280.            diag(1)='Your answer should not contain a minus sign. Actually, 9-(6-2)=9-4=5, so the answer is 5.';
 281.            unrc='Not really. 9-(6-2)=9-4=5, so the answer is 5.';
 282.            next=k20;
 283.            GO TO now;
 284.    k20:    END ;

  55.    C3:     PROCEDURE ;
  56.            DECLARE ans(2) CHAR(30) VAR,wans(5) CHAR(20) VAR, cor CHAR(254) VAR,diag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
  57.            DECLARE  a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

  69.1            ans(1)='2-9';
  69.2            ans(2)='-7';
  69.3            ir=2;
  69.4            b=' Thus W is not closed under subtraction.';
  69.5            cor='Good, 2-9=-7 is not a whole number.'||b;
  69.6            unrc='No, the number 2-9=-7 is not a whole number.'||b;
  69.7            next=k21;

 300.    k21:    PUT LIST('If subtraction has an identity element, I, then I will have to satisfy the following two equations:');
 301.            PUT LIST('a-I=a and I-a=a for all whole numbers a. Can you name an element that works in both equations? (If
there is');
 302.            PUT LIST('such an element, name it--if not, type "no".)');
 303.            ans(1)='NO';
 304.            wans(1)='0';
 305.            wans(2)='ZERO';
 306.            iw=2;
 307.            b=' There is no element that always satisfies both equations.';
 308.            diag(1)='No, zero works in the first equation only.'||b;
 309.            diag(2)=diag(1);
 310.            cor='You are correct,'||b;
 311.            unrc='No,'||b;
 312.            next=k22;
 313.            GO TO now;
 314.    k22:    next=k23;
 315.            IF index(reply,'0')>0 THEN GO TO next;
 316.            IF index(reply,'ZERO')>0 THEN GO TO next;
 317.            PUT LIST('Is there a number, I, that always satisfies the equation a-I=a? If so, name it--if not, type "no."');
 318.            b=' The number 0 always satisfies this equation.';
 319.            diag(1)='Yes,'||b;
 320.            diag(2)=diag(1);
 320.5           iw=2;
 321.            unrc='No, but'||b;
 322.            cor='But there is such a number--the number 0.';
 323.            GO TO now;
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 324.     k23:    PUT LIST('Thus there isn''t an identity element for subtraction, because there is no element I such that ');
 325.             PUT LIST('a-I=a and I-a=a for all whole numbers a. Since a-0=a for all values of a, we do say that 0 is a right
identity');
 326.             PUT LIST('for subtraction. (A right identity because it is written on the right side.) Incidentally, you know
that a-0=0');
 327.             PUT LIST('because a-0 is defined as "the answer to the question ?+0=a", and you know that a+0=a for all whole
numbers a');
 328.             PUT LIST('because 0 is the _____.');
 329.             ans(1)='IDENTITY';
 330.             cor='Yes, 0 is the identity element for addition on W.';
 331.             unrc='No, 0 is the identity element for addition on W.';
 332.             next=k24;
 333.             GO TO now;
 334.     k24:    PUT LIST('End of lesson--you may logout.');
 335.             END ;
```

CAI UNIT 3

```
   1.            DECLARE  Z1 ENTRY  EXT;
   1.1           CALL Z1;
   2.            DECLARE  Z2 ENTRY  EXT;
   2.1           PUT LIST('                                                    Part 2: In case of malfunction, xeq 2 thru...');
   2.2           PUT LIST('If you know that 5=10/2, which multiplication fact do you also know?');
   2.3           CALL Z2;

  55.    Z1:     PROCEDURE ;
  56.            DECLARE ans(4) CHAR(30) VAR, cor CHAR(200) VAR, unrc CHAR(200) VAR;
  57.            DECLARE  a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

  69.1           PUT LIST('The operation of division has some properties of operations we have previously studied, but it also
falls');
  69.2           PUT LIST('to have some properties of previous operations. If division is to be commutative on the set of whole
numbers');
  69.3           PUT LIST('for instance, then for any whole numbers a and b it must be true that a/b=_____.');
  69.4           ans(1)='B/A';
  69.5           cor='That''s right!';
  69.6           unrc='No, if division is commutative, it must be true that a/b=b/a for all whole numbers a and b.';
  69.7           next=k2;

 100.    k2:     PUT LIST('Thus if division is commutative, it must be true that 3/2=_____.');
 101.           ans(1)='2/3';
 102.           cor='Good.';
 103.           unrc='No, if division is commutative, then 3/2 must equal 2/3';
 104.           next=k3;
 105.           GO TO now;
 106.    k3:    PUT LIST('Thus division is not commutative, because 3/2 is not equal to 2/3. ');
 107.           PUT LIST('If for all whole numbers a,b, and c it is true that (a/b)/c = a/(b/c), then we say division is_____
_.');
 108.           ans(1)='ASSOCIATIVE';
 109.           cor='Yes, you are correct.';
 110.           unrc='No, if (a/b)/c = a/(b/c) for all whole numbers a,b, and c, then division is said to be associative.';
 111.           next=k4;
 112.           GO TO now;
 113.    k4:    PUT LIST('Clearly, (8/2)/1=4/1=4. What is the value of 8/(2/1)?(Type a numeral, not an expression.)');
 114.           ans(1)='4';
 115.           cor='Right, 8/(2/1)=8/2=4.';
 116.           unrc='No, 8/(2/1)=8/2=4.';
 117.           next=k5;
 118.           GO TO now;
 119.    k5:    PUT LIST('Thus (8/2)/1 = 8/(2/1). Does this prove that division is associative?');
 120.           ans(1)='NO';
 121.           cor='Very good, one example doesn''t prove an operation is associative.';
 122.           unrc='No, a single example is never enough to show that an operation is associative.';
 123.           next=k6;
 124.           GO TO now;
 125.    k6:    PUT LIST('As a matter of fact, division is NOT associative, because (8/4)/2=2/2=1, but 8/(4/2)=8/2=4; hence');
 126.           PUT LIST('(8/4)/2 is not equal to 8/(4/2). If division is distributive over addition, then two equations must be
true:');
 127.           PUT LIST('a/(b+c)=a/b +a/c  and (b+c)/a=b/a + c/a. Try both of these equations with the values a=6,b=2, and
c=1.');
 128.           PUT LIST('Do these values work in both equations? Answer yes or no.');
 130.           ans(1)='NO';
 131.           cor='Correct: the second equation works for these values, but the first equation doesn''t.';
 132.           unrc='No, the second equation works, but the first equation doesn''t: 6/(2+1)=6/3=2, but 6/2 + 6/1 = 3+6=9.';
 133.           next=k7;
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```
134.              GO TO now;
135.     k7:      PUT LIST('Since 6/(2+1) does not equal 6/2 +6/1, we know that there is no such thing as the distributive
property');
136.              PUT LIST('of division over addition. The second equation, however, does hold true, and this property--');
137.              PUT LIST('(b+c)/a= b/a + c/a --is called the right distributive property of division over addition. It is
called');
138.              PUT LIST('the RIGHT distributive property, because the divisor, a, is written on the right.');
140.              PUT LIST('If there is an identity element, I, for division, it will have to satisfy the following two equations:'
);
141.              PUT LIST('I/a=a and a/I=a for all whole numbers a. Is there an element that satisfies both equations?(Yes or
no.)');
142.              ans(1)='NO';
143.              cor='Good, there is no element I such that I/a=a for all whole numbers a.';
144.              unrc='1 works in the second equation, but not in the first; there is no element that works in the first
equation.';
145.              next=k8;
146.              GO TO now;
147.     k8:      PUT LIST('Because 1 satisfies part of the requirements for being an identity element--namely, a/1=a for all
a--');
148.              PUT LIST('we say that 1 is a right identity element for division. (A right identity element, because it is
written');
149.              PUT LIST('on the right side of the division sign.) It is not, however, an identity element, because 1/a is not
equal');
150.              PUT LIST('to a for all values of a.');
151.              PUT LIST('Now consider the actual definition of division: If p and d are whole numbers with d not zero,');
152.              PUT LIST('then p/d is the answer to the question ?xd=p.  Thus 6/2, for instance, is the answer to the question
?x2=6.');
153.              PUT LIST('Similarly, 15/3 is the answer to the question_____.(Complete the sentence with a multiplication
sentence.');
154.              nrt=0;
155.              ir=4;
156.              ans(1)='?X3=15';
157.              ans(2)='3X?=15';
158.              ans(3)='15=?X3';
159.              ans(4)='15=3X?';
160.              cor='That''s right, 15/3 answers the question ?X3=15.';
161.              unrc='No, if p/d answers the question ?xd=p, then 15/3 should answer the question ?x3=15.';
162.              next=k9;
163.              GO TO now;
164.     k9:      PUT LIST('Which question does 35/7 answer?');
165.              ir=4;
166.              ans(1)='?X7=35';
167.              ans(2)='7X?=35';
168.              ans(3)='35=7X?';
169.              ans(4)='35=?X7';
170.              a='35/7 answers the question ?x7=35.';
171.              cor='Good, '||a;
172.              unrc='Not exactly: p/d answers the question ?xd=p, so '||a;
173.              next=k10;
174.              GO TO now;
175.     k10:     next=k12;
176.              IF nrt=2 THEN GO TO next;
177.              PUT LIST('Try another one: which question does 12/4 answer?');
178.              ans(1)='?X4=12';
179.              ans(2)='4X?=12';
180.              ans(3)='12=4X?';
181.              ans(4)='12=?X4';
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182.              lr=4;
183.              a='12/4 answers the question ?x4=12.';
184.              cor='Yes, '||a;
185.              unrc='No, if p/d answers the question ?xd=p, then '||a;
186.              next=k11;
187.              GO TO now;
188.     k11:     next=k12;
189.              IF nrt=2 THEN GO TO next;
190.              PUT LIST('Which question does 20/5 answer?');
191.              lr=4;
192.              ans(1)='?X5=20';
193.              ans(2)='5X?=20';
194.              ans(3)='20=?X5';
195.              ans(4)='20=5X?';
196.              cor='Right.';
197.              unrc='Look, p/d answers the question ?Xd=p; 15/3 answers ?x3=15; 12/4 answers ?x4=12; then 20/5 answers ?x5=20';
198.              GO TO now;
199.     k12:     END ;

 55.     Z2:      PROCEDURE ;
 56.              DECLARE ans(4) CHAR(10) VAR,wans(5) CHAR(10) VAR, cor CHAR(200) VAR,diag(5) CHAR(200) VAR, unrc CHAR(200) VAR;
 57.              DECLARE  a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

 69.1             lw=1;
 69.2             lr=4;
 69.3             ans(1)='2X5=10';
 69.4             ans(2)='5X2=10';
 69.5             ans(3)='10=2X5';
 69.6             ans(4)='10=5X2';
 69.7             wans(1)='?';
 69.72            cor='Right.';
 69.74            diag(1)='A multiplication fact shouldn''t contain a question mark. The answer should be 2x5=10.';
 69.76            unrc='No, if 10/5=2, then it must be true that 2x5=10.';
 69.78            next=k13;

200.     k13:     next=k14;
201.              IF nrt=1 THEN GO TO next;
202.              PUT LIST('If you know that 18/3=6, which multiplication fact do you also know?');
203.              lw=1;
204.              lr=4;
205.              ans(1)='3X6=18';
206.              ans(2)='6X3=18';
207.              ans(3)='18=3X6';
208.              ans(4)='18=6X3';
209.              cor='You are correct.';
210.              diag(1)='Multiplication FACTS don''t contain question marks. In this case, if 18/3=6, then 3x6=18: 3x6=18 is the
answer.';
211.              unrc='No, 18/3 answers ?x3=18. Since 18/3=6, 6 must answer this question. Thus 6x3=18, so "6x3=18" is the
answer.';
212.              GO TO now;
213.     k14:     PUT LIST('Zero has sometimes caused trouble in division. If we allow the expression 3/0, for instance, which
question');
214.              PUT LIST('would 3/0 have to answer?');
215.              lr=4;
216.              ans(1)='?X0=3';
217.              ans(2)='0X?=3';
218.              ans(3)='?=0X3';
```

```
219.            ans(4)='?=3X0';
220.            cor='You are correct!';
221.            unrc='Not exactly:';
222.            next=k15;
223.            GO TO now;
224.    k15:    PUT LIST('If 3/0 is defined at all, it must answer the question ?x0=3. But of course any number times zero is
zero,');
225.            PUT LIST('so there is no answer to this question. Thus division by 0 is undefined.');
226.            PUT LIST('If 0/3 is defined at all, which question will it have to answer?');
227.            ir=4;
228.            ans(1)='?X3=0';
229.            ans(2)='3X?=0';
230.            ans(3)='0=3X?';
231.            ans(4)='0=?X3';
232.            a=' 0/3 must answer the question ?x3=0.';
233.            unrc='No,'||a;
234.            cor='Fine,'||a;
235.            next=k16;
236.            GO TO now;
237.    k16:    PUT LIST('Is there an answer to this question?(Type yes or no.)');
238.            ans(1)='NO';
239.            wans(1)='YES';
240.            iw=1;
241.            cor='But there WOULD be an answer--zero answers this question, since 0x3=0. Thus 0/3 is defined: 0/3=0.';
242.            unrc=cor;
243.            diag(1)='Right, there would be an answer--zero answers this question, since 0x3=0. Thus 0/3 is defined: 0/3=0.';
244.            next=k17;
245.            GO TO now;
246.    k17:    PUT LIST('We have not yet looked at the expression 0/0. If we allow the expression 0/0, which question would');
247.            PUT LIST('it have to answer?');
248.            ir=4;
249.            ans(1)='?X0=0';
250.            ans(2)='0X?=0';
251.            ans(3)='0=0X?';
252.            ans(4)='0=?X0';
253.            a=' 0/0 must be the answer to the question ?x0=0.';
254.            cor='Yes,'||a;
255.            unrc='Not exactly:'||a;
256.            next=k18;
257.            GO TO now;
258.    k18:    PUT LIST('One number that answers this question is_____.(If you can think of a number that correctly answers
the');
259.            PUT LIST('question ?x0=0, then type that number-as a numeral, not a word. If not, type "Impossible.")');
260.            iw=5;
261.            ans(1)='IMPOSSIBLE';
262.            wans(2)='E';
263.            wans(1)='A';
264.            wans(3)='I';
265.            wans(4)='O';
266.            wans(5)='U';
267.            b=' 865 answers th's question, for instance, since 865x0=0.';
268.            cor='No, it isn''t impossible--'||b;
269.            unrc='You are absolutely correct:';
270.            diag(1)='I had wanted a numeral only, not a word answer.'||b;
270.1           diag(2)=diag(1);
270.2           diag(3)=diag(1);
270.4           diag(4)=diag(1);
```

```
270.5           diag(5)=diag(1);
271.            next=k19;
272.            GO TO now;
273.    k19:    next=k20;
274.            tot=index(reply,'A')+index(reply,'E')+index(reply,'I')+index(reply,'O')+index(reply,'U');
275.            IF tot>0 THEN GO TO next;
276.            PUT LIST(reply||' x0=0.');
276.5           a=reply;
277.            next=k21;
278.            GO TO next;
279.    k20:    a='865';
280.    k21:    nrt=0;
281.            PUT LIST('Now name another number that answers the question ?x0=0.');
282.            lw=5;
283.            b='127 also answers this question, for instance, since 127x0=0.';
284.            cor='It isn''t impossible:'||b;
285.            diag(1)='I had wanted a numeral rather than a word answer. '||b;
286.            diag(2)=diag(1);
287.            diag(3)=diag(1);
288.            diag(4)=diag(1);
289.            diag(5)=diag(1);
290.            unrc='Right,';
291.            next=k22;
292.            GO TO now;
293.    k22:    next=k23;
293.5           c=reply;
294.            tot=index(reply,'A')+index(reply,'E')+index(reply,'I')+index(reply,'O')+index(reply,'U');
295.            IF tot>0 THEN GO TO next;
295.5           next=sp;
296.            IF a=c THEN GO TO next;
297.            PUT LIST(c||' x0=0.');
298.            next=k24;
299.            GO TO next;
300.    sp:     PUT LIST('However, I wanted a different number, not '||a||' again.'||b);
301.    k23:    c='127';
302.    k24:    PUT LIST('Thus both '||a||' and '||c||' answer the question ?x0=0, so there is no single answer to this
question.');
303.            PUT LIST('In fact, any number will answer this question. That is why the expression 0/0 is undefined--it is
meaningless');
304.            PUT LIST('to talk about "the" answer to the question ?x0=0 when any number will answer this question.');
305.            PUT LIST('End of lesson--you may logout now.');
306.            END ;
```

CAI UNIT 4

```
   1.            DECLARE  DEC1 ENTRY  EXT;
   1.1           CALL DEC1;
   2.            DECLARE  DEC2 ENTRY  EXT;
   2.1           PUT LIST('                                                              Part 2: In case of malfunction, xeq
2 thru...');
   2.2           PUT LIST('Another useful test is the following: a number is divisible by 3 if and only if the sum of its');
   2.3           PUT LIST('digits is divisible by 3. For example, 171342 is divisible by 3, because the sum of its digits');
   2.4           PUT LIST('is 1+7+1+3+4+2=18, and 18 is divisible by 3. The number 4123 is not divisible by 3, since the sum');
   2.5           PUT LIST('of its digits is 4+1+2+3=10, and 10 is not divisible by 3. Is 25136 divisible by 3?');
   2.6           CALL DEC2;
   3.            DECLARE  DEC3 ENTRY  EXT;
   3.1           PUT LIST('                                                              Part 3: In case of malfunction, xeq
3 thru...');
   3.2           PUT LIST('Another test that students often generalize is the rule for division by 6. Do you believe, for
instance,');
   3.3           PUT LIST('that a number is divisible by 10 if and only if it is divisible by 2 and by 5?');
   3.4           CALL DEC3;
   3.5           END ;

  55.    DEC1:   PROCEDURE ;
  56.           DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
  57.           DECLARE  a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

  69.1          PUT LIST('If p and d are whole numbers with d not equal to zero, then p is divisible by d if the quotient p/d
is');
  69.2          PUT LIST('a whole number. 6 is divisible by 2, for instance, since 6/2 is a whole number, but 6 is not divisible'
);
  69.3          PUT LIST('by 4, since 6/4 is not a whole number.');
  69.4          PUT LIST('It is often possible to tell whether p is divisible by d without actually performing the division.
You');
  69.5          PUT LIST('probably already know the divisibility test for 2, for instance. Is 1432186 divisible by 2?');
  69.6          ans(1)='YES';
  69.7          cor='Right, 1432186 is divisible by 2.';
  69.8          unrc='Actually, 1432186 is divisible by 2.';
  69.82         next=k2;

 100.    k2:    PUT LIST('In general, a number is divisible by 2 if and only if it''s one''s digit is divisible by 2. Thus');
 101.           PUT LIST('1432186 is divisible by 2, because 6, the number in the one''s position, is divisible by 2.');
 102.           PUT LIST('The test for divisibility by 5 is very similar: a number is divisible by 5 if and only if it''s
one''s');
 103.           PUT LIST('digit is divisible by 5. For example 5 divides 1070 (since 5 divides 0) but 5 does not divide 259
(since');
 104.           PUT LIST('5 does not divide 9). Does 5 divide 55044?');
 105.           ans(1)='NO';
 106.           cor='Good, 5 won''t divide 55044 because 5 won''t divide 4.';
 107.           unrc='Wrong: 5 won''t divide the number in the one''s position (4), so 5 won''t divide the number 55044.';
 108.           next=k3;
 108.5          nrt=0;
 109.           GO TO now;
 110.    k3:    PUT LIST('Is 10536 divisible by 5?');
 111.           cor='Fine, 10535 isn''t divisible by 5.';
 112.           qrt=nrt;
 113.           nrt=0;
 114.           unrc='You are incorrect: what is the number in the one''s position of 10536?';
 115.           next=k4;
 116.           GO TO now;
 117.    k4:    next=k5;
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```
118.            qrt=qrt+nrt;
119.            IF nrt>0 THEN GO TO next;
120.            ans(1)='6';
121.            cor='Yes, 6 is in the one''s position of 10536, and 5 won''t divide 6; hence 5 won''t divide 10536.';
122.            unrc='No, 6 is in the one''s position of 10536, and 5 won''t divide 6; hence 5 won''t divide 10536.';
123.            GO TO now;
124.    k5:     next=k6;
125.            IF qrt=2 THEN GO TO next;
126.            PUT LIST('Is 24130 divisible by 5?');
127.            ans(1)='YES';
128.            cor='You are right, 24130 is divisible by 5.';
129.            unrc='Wrong: 24130 is divisible by 5 because 0 (the number in the one''s position) is divisible by 5.';
130.            GO TO now;
131.    k6:     nrt=0;
132.            PUT LIST('The test for divisibility by 4 is concerned not only with the last digit, but with the last two
digits');
133.            PUT LIST('of a number. The rule is this: a number is divisible by 4 if and only if the number named by the last
two');
134.            PUT LIST('digits is divisible by 4. For example, 21738 is not divisible by 4 because the number named by the
last');
135.            PUT LIST('two digits--38--is not divisible by 4. On the other hand, 21736 is divisible by 4, because 36--the
number');
136.            PUT LIST('named by the last two digits--is divisible by 4.  Is 612854 divisible by 4?');
137.            ans(1)='NO';
138.            cor='Good: 4 doesn''t divide 54, so 4 doesn''t divide 612854.';
139.            unrc='You are not correct. What is the number named by the last two digits in 612854?';
140.            next=k7;
141.            GO TO now;
142.    k7:     next=k8;
143.            qrt=nrt;
144.            IF nrt>0 THEN GO TO next;
145.            ans(1)='854';
146.            lw=1;
147.            wans(1)='54';
148.            cor='No, the answer is 54, and since 4 won''t divide 54, we know that 4 won''t divide 612854.';
149.            unrc=cor;
150.            diag(1)='Yes, 54 is correct--and since 4 won''t divide 54, we know that 4 won''t divide 612854.';
151.            next=k8;
152.            GO TO now;
153.    k8:     PUT LIST('To see why this test works, consider the number 631218. This can be written as 631200+18');
154.            PUT LIST('or (6312x100)+18. Obviously, 6312x100 is divisible by 4, so (6312x100 + 18) will be divisible by');
155.            PUT LIST('4 if and only if 18 is divisible by 4. In general, a number edcba could be written as (edcx100 +ba).');
156.            PUT LIST('Since edcx100 is always divisible by 4, the divisibility of (edcx100 + ba) depends only on the');
157.            PUT LIST('divisibility of ba. Thus the divisibility test for 4 involves only the last two digits.');
158.            PUT LIST('Is 41732 divisible by 4?');
159.            ans(1)='YES';
160.            cor='OK, 41732 is divisible by 4.';
161.            unrc='Wrong: what is the number named by the last two digits in 41732?';
162.            nrt=0;
163.            next=k9;
164.            GO TO now;
165.    k9:     qrt=qrt+nrt;
166.            next=k10;
167.            IF nrt>0 THEN GO TO next;
168.            ans(1)='732';
169.            wans(1)='32';
170.            lw=1;
```

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171.            cor='No, 32 is the number named by the last 2 digits in 41732. 4 divides 32, so 4 divides 41732.';
172.            unrc=cor;
173.            diag(1)='Good, 32 is correct; and since 4 divides 32, therefore 4 divides 41732.';
174.            GO TO now;
175.    k10:    next=k11;
176.            IF qrt=2 THEN GO TO next;
177.            PUT LIST('Does 4 divide 218572?');
178.            ans(1)='YES';
179.            cor='Good, 4 divides 72, so 4 divides 218572.';
180.            unrc='That is incorrect. 4 divides 72, so 4 divides 218572.';
181.            GO TO now;
182.    k11:    END ;

 55.    DEC2:   PROCEDURE ;
 56.            DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
 57.            DECLARE  a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

69.1            ans(1)='NO';
69.2            cor='Very good, 2+5+1+3+6=17, and 17 isn''t divisible by 3. Hence 25136 isn''t divisible by 3.';
69.3            unrc='That is incorrect. What is the sum of the digits in 25136?';
69.4            next=k12;

200.    k12:    qrt=nrt;
201.            next=k13;
202.            IF nrt>0 THEN GO TO next;
203.            ans(1)='17';
204.            cor='Yes, 2+5+1+3+6=17. 17 isn''t divisible by 3, so 25136 isn''t divisible by 3.';
205.            unrc='No, 2+5+1+3+6=17. 17, however, isn''t divisible by 3, so 25136 isn''t divisible by 3.';
206.            GO TO now;
207.    k13:    PUT LIST('Is 14269 divisible by 3?');
208.            ans(1)='NO';
209.            cor='Fine, 1+4+2+6+9=22; 22 isn''t divisible by 3, so 14269 isn''t divisible by 3.';
210.            unrc='Your answer is not right. What is the sum of the digits in 14269?';
211.            nrt=0;
212.            next=k14;
213.            GO TO now;
214.    k14:    qrt=qrt+nrt;
215.            next=k15;
216.            IF nrt>0 THEN GO TO next;
217.            ans(1)='22';
218.            cor='22 is correct--and 3 won''t divide 22, so 3 won''t divide 14269.';
219.            unrc='Nope, 1+4+2+6+9=22--and 3 won''t divide 22, so 3 won''t divide 14269.';
220.            GO TO now;
221.    k15:    next=k16;
222.            IF qrt=2 THEN GO TO next;
223.            PUT LIST('Try another--is 82764 divisible by 3?');
224.            ans(1)='YES';
225.            cor='Right, 82764 is divisible by 3.';
226.            unrc='That is incorrect. 8+2+7+6+4=27, and 3 divides 27, so 3 divides 82764.';
227.            GO TO now;
228.    k16:    PUT LIST('The test for divisibility by 9 is analogous to the test for 3: a number is divisible by 9 if');
229.            PUT LIST('and only if the sum of it''s digits is divisible by 9. Is 250713 divisible by 9?');
230.            ans(1)='YES';
231.            cor='OK, 2+5+0+7+1+3=18, and 9 divides 18, so 9 divides 250713.';
232.            unrc='That''s incorrect. What is the sum of the digits in 250713?';
233.            nrt=0;
234.            next=k17;
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235.            GO TO now;
236.    k17:    qrt=nrt;
237.            next=k19;
238.            IF nrt>0 THEN GO TO next;
239.            ans(1)='13';
240.            unrc='No, 2+5+0+7+1+3=18. 9 divides 18, so 9 divides 250713.';
241.            next=k18;
242.            GO TO now;
243.    k18:    PUT LIST('Is 41682 divisible by 9?');
244.            ans(1)='NO';
245.            cor='Right: 4+1+6+8+2=21, and 9 won''t divide 21, so 9 won''t divide 41682.';
246.            unrc='Not quite: 4+1+6+8+2=21, and 9 won''t divide 21, so 9 won''t divide 41682.';
247.            next=k19;
248.            GO TO now;
249.    k19:    PUT LIST('A final rule to be considered is the rule for divisibility by 6: a number is divisible by 6');
250.            PUT LIST('if and only if it is divisible by 2 and by 3. As an example, the number 35214 is divisible by 6');
251.            PUT LIST('because it is divisible by 2 (since it''s one''s digit is divisible by 2) and it is divisible by 3');
252.            PUT LIST('(since the sum of it''s digits is divisible by 3). Is 2726 divisible by 6?');
253.            ans(1)='NO';
254.            cor='Good, 2726 isn''t divisible by 6 because it isn''t divisible by 3.';
255.            unrc='No, 2726 isn''t divisible by 3, so it can''t be divisible by 6.';
256.            nrt=0;
257.            next=k20;
258.            GO TO now;
259.    k20:    next=k21;
260.            IF nrt>0 THEN GO TO next;
261.            PUT LIST('Try it again: is 8223 divisible by 6?');
262.            unrc='You are incorrect: 8223 isn''t divisible by 6 because it isn''t divisible by 2.';
263.            cor='Correct, 8223 isn''t divisible by 6 because it isn''t divisible by 2.';
264.            GO TO now;
265.    k21:    PUT LIST('Many students try to generalize these tests, but this must be done with caution. Consider');
266.            PUT LIST('the test for 3--the one that involves summing the digits. Will this test also work for divisibility');
267.            PUT LIST('by 7? That is, will a number be divisible by 7 if and only if the sum of it''s digits is divisible');
268.            PUT LIST('by 7? (Answer yes or no.)');
269.            ans(1)='NO';
270.            b=' 7 divides 14, for instance, but the sum of the digits is 1+4=5, and 7 won''t divide 5.';
271.            cor='Correct--the rule won''t work for 7.'||b;
272.            unrc='Wrong--the rule won''t work for 7.'||b;
272.5           next=k22;
273.            GO TO now;
274.    k22:    PUT LIST('In fact, the 3''s rule won''t work for any digits other than 3 and 9.');
275.            END ;

 55.    DEC3:   PROCEDURE ;
 56.            DECLARE ans(2) CHAR(20) VAR,wans(4) CHAR(20) VAR, cor CHAR(200) VAR,diag(4) CHAR(150) VAR, unrc CHAR(150) VAR;
 57.            DECLARE  a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

 69.1           ans(1)='YES';
 69.2           cor='Fine, this test works--but there is an easier way to state it.';
 69.3           unrc='Wrong! This test does work, although there is an easier way to state it.';
 69.4           next=k23;

300.    k23:    PUT LIST('A number that is divisible by 2 ends in a 0, 2, 4, 6, or 8, and a number that is divisible by 5');
301.            PUT LIST('ends in a 0 or a 5. Thus a number that is divisible by both 2 and 5 must end in the digit_____.');
302.            ans(1)='0';
303.            ans(2)='ZERO';
304.            ir=2;
```

```
            b=' a number that is divisible by both 2 and 5 ends in a 0.';
            cor='Yes, '||b;
            unrc='No, '||b;
            next=k24;
            GO TO now;
    k24:    PUT LIST('Thus a number is divisible by 10 if and only if it ends in a 0.');
            PUT LIST('Is it true that a number is divisible by 8 if and only if it is divisible by 2 and by 4?');
            ans(1)='NO';
            cor='You are right, this test does not work.';
            unrc='Unfortunately, this test won''t work.';
            next=k25;
            GO TO now;
    k25:    PUT LIST('For example, 12 is divisible by 2 and by 4, but not by 8. Incidentally, the reason this test');
            PUT LIST('won''t work for 2 and 4 is that 2 and 4 have a common factor.');
            PUT LIST('From the following list, name a number that is divisible by 3: 46312, 46313, 46314, 46315, 46316');
            ans(1)='46314';
            iw=4;
            wans(1)='46312';
            wans(2)='46313';
            wans(3)='46315';
            wans(4)='46316';
            diag(1)='No, the sum of the digits in 46312 is 16, and 3 won''t divide 16. The correct answer is 46314.';
            diag(2)='No, the sum of the digits in 46313 is 17, and 3 won''t divide 17. The correct answer is 46314.';
            diag(3)='No, the sum of the digits in 46315 is 19, and 3 won''t divide 19. The correct answer is 46314.';
            diag(4)='No, the sum of the digits in 46316 is 20, and 3 won''t divide 20. The correct answer is 46314.';
            unrc='No, the only correct answer is 46314.';
            cor='Right, the sum of the digits in 46314 is 18, and 3 divides 18, so 3 divides 46314.';
            next=k26;
            GO TO now;
    k26:    PUT LIST('From the same list, choose a number that is divisible by 4.');
            iw=4;
            ans(1)='46316';
            wans(4)='46314';
            cor='Good, 4 divides 16, so 4 divides 46316.';
            unrc='No, the number named by the last two digits must be divisible by 4, and this is true only of 46312 and
46316.';
            diag(1)='Good, 4 divides 12, so 4 divides 46312.';
            diag(2)='No, 13 (the number named by the last 2 digits) isn''t divisible by 4. The answer is 46312 or 46316.';
            diag(3)='No, 15 (the number named by the last 2 digits) isn''t divisible by 4. The answer is 46312 or 46316.';
            diag(4)='No, 14 (the number named by the last 2 digits) isn''t divisible by 4. The answer is 46312 or 46316.';
            next=k27;
            GO TO now;
    k27:    PUT LIST('Name a number from the list that is divisible by 6.');
            ans(1)='46314';
            iw=4;
            wans(4)='46316';
            cor='Yes, 46314 is divisible by both 2 and 3, so it is divisible by 6.';
            unrc='No, 46314 is the only listed number that is divisible by 2 and 3, so it is the only number divisible by
6.';
            diag(1)='No, 46312 isn''t divisible by 3, so it isn''t divisible by 6. The correct answer is 46314.';
            diag(2)='No, 46313 isn''t divisible by 2 or 3, so it isn''t divisible by 6. The correct answer is 46314.';
            diag(3)='No, 46315 isn''t divisible by 2 or 3, so it isn''t divisible by 6. The correct answer is 46314.';
            diag(4)='No, 46316 isn''t divisible by 3, so it isn''t divisible by 6. The correct answer is 46314.';
            next=k28;
            GO TO now;
    k28:    PUT LIST('End of lesson--you may logout.');
            END ;
```

CAI UNIT 5

```
   1.           DECLARE  PR1 ENTRY  EXT;
   1.1          CALL PR1;
   2.           DECLARE  PR2 ENTRY  EXT;
   2.1          PUT LIST('                                                                    part 2: In case of malfunction, xeq
2 thru...');
   2.2          PUT LIST('Hhat is the largest integer that is less than or equal to sqrt(140)?');
   2.3          CALL PR2;
   3.           DECLARE  PR3 ENTRY  EXT;
   3.1          PUT LIST('                                                                    part 3: In case of malfunction, xeq
3 thru...');
   3.2          PUT LIST('To see if 301 is prime, one should check the set of all primes that are less than or equal to
sqrt(301).');
   3.3          PUT LIST('What is the largest prime number that is less than or equal to sqrt(301)?');
   3.4          CALL PR3;
   4.           END ;

  55.    PR1:   PROCEDURE ;
  56.           DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(200) VAR, unrc CHAR(200) VAR;
  57.           DECLARE  a CHAR(130) VAR, b CHAR(130) VAR, reply CHAR(30) VAR;

  69.1          PUT LIST('You may remember that a prime number is a natural number that has exactly two factors. Alternately, we
could');
  69.2          PUT LIST('say that a prime number is a natural number greater than 1 that has no divisors other than 1 and
itself.');
  69.3          PUT LIST('Is 21 a prime number?');
  69.35         ans(1)='NO';
  69.4          a=' 21 isn''t prime, because it has 4 factors--1,3,7, and 21.';
  69.5          cor='Right,'||a;
  69.6          unrc='No,'||a;
  69.7          next=k2;
  69.8          GO TO now;

 100.    k2:    PUT LIST('Is 33 a prime number?');
 101.           a='33 isn''t prime because 3 and 11 are divisors of 33; 33 is divisible by natural numbers other than 1 and 33.';
 102.           cor='Good, '||a;
 103.           unrc='Unfortunately not: '||a;
 104.           next=k3;
 105.           GO TO now;
 106.    k3:    next=k6;
 107.           IF nrt=2 THEN GO TO next;
 108.           PUT LIST('Try again. Is 23 a prime number?');
 109.           ans(1)='YES';
 110.           a=' 23 is prime because it has no divisors other than 1 and 23.';
 111.           cor='Very good,'||a;
 112.           unrc='Wrong!'||a;
 113.           next=k4;
 114.           GO TO now;
 115.    k4:    next=k6;
 116.           IF nrt=2 THEN GO TO next;
 117.           PUT LIST('Just one more: Is 77 a prime number?');
 118.           ans(1)='NO';
 119.           cor='Correct, 77 isn''t a prime number.';
 120.           unrc='No, 77 is not a prime number.';
 121.           next=k5;
 122.           GO TO now;
 123.    k5:    PUT LIST('It is obvious that 77 is not prime because 77 is divisible by_____.(Type one number.)');
 124.           ans(1)='77';
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 125.            wans(1)='11';
 126.            wans(2)='1';
 127.            wans(3)='7';
 128.            b=' 77 fails to be prime because 7 and 11 are also divisors.';
 129.            cor='77 is divisible by 77, but so what? Any number is divisible by itself.'||b;
 129.5           iw=3;
 130.            diag(2)='77 is divisible by 1, but so what? Any number is divisible by 1.'||b;
 131.            diag(1)='Good, 77 fails to be prime because it is divisible by 11; it is also divisible by 7.';
 132.            diag(3)='Good, 77 fails to be prime because it is divisible by 7; it is also divisible by 11.';
 133.            next=k8;
 134.            GO TO now;
 135.    k6:     PUT LIST('For reasons that we will (hopefully) see later, we will be interested in square roots. In fact,
we''ll');
 136.            PUT LIST('be using the phrase "the square root of" so often that we will shorten it to "sqrt." Thus "the square
root');
 137.            PUT LIST('of 9" will be written "sqrt(9)" in our terminology. What is the value of sqrt(9)?');
 138.            ans(1)='3';
 139.            cor='Yes, sqrt(9)=3.';
 140.            nrt=0;
 141.            unrc='No, sqrt(9)="the square root of 9"=3.';
 142.            next=k7;
 143.            GO TO now;
 144.    k7:     next=k9;
 145.            IF nrt=1 THEN GO TO next;
 146.            PUT LIST('You see, sqrt(n) is the number which, when multiplied by itself, gives you n; that is, sqrt(n) is a
number');
 147.            PUT LIST('whose square is n. In this case, sqrt(9)=3, since 3 is a number which, when multiplied by itself,
gives');
 148.            PUT LIST('you 9. Similarly, sqrt(25)=5, since 5x5=25, and sqrt(100)=10, since 10x10=100. In general, then, ');
 149.            PUT LIST('(sqrt(n))x(sqrt(n))=n. What is the numerical value of sqrt(64)?');
 150.            ans(1)='8';
 151.            cor='O.K., sqrt(64)=8.';
 152.            unrc='Not quite: sqrt(64)=8, since 8x8=64. (8 is a number which, when multiplied by itself, gives you 64.)';
 153.            next=k8;
 154.            GO TO now;
 155.    k8:     next=k9;
 156.            IF nrt=1 THEN GO TO next;
 157.            PUT LIST('Try one more: what is the numeric value of sqrt(121)?');
 158.            ans(1)='11';
 159.            cor='All right, sqrt(121)=11.';
 160.            unrc='No, sqrt(121)=11. Sqrt(121) is a number whose square is 121; since 11x11=121, then, sqrt(121)=11.';
 161.            GO TO now;
 162.    k9:     PUT LIST('To decide whether 179 is prime or not, you are interested in discovering whether or not 179 has');
 163.            PUT LIST('any divisors other than 1 and 179. In this search for divisors, you would obviously not consider
numbers');
 164.            PUT LIST('that are larger than 179 as possible divisors of 179. In fact, you wouldn''t have to check any
numbers');
 165.            PUT LIST('that are larger than sqrt(179). By the way, what is the largest integer that is less than or equal to
sqrt(179)?');
 166.            ans(1)='13';
 167.            wans(1)='14';
 168.            iw=1;
 169.            cor='Very good, 13 is correct.';
 170.            diag(1)='You are close, but 14 is somewhat larger than sqrt(179). The correct answer is 13.';
 171.            unrc='No, 13 is the largest integer that is less than or equal to sqrt(179).';
 172.            next=k12;
 173.            GO TO now;
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 174.     k12:    nrt=0;
 175.             PUT LIST('Do you think you understand how to find the largest integer that is less than or equal to sqrt(n) for
a given n?');
 176.             ans(1)='YES';
 177.             cor='O.K., then try this one to make sure:';
 178.             unrc='O.K., to find the largest integer that is less than or equal to 253, for instance, we would first make a
reasonable';
 179.             next=k13;
 180.             GO TO now;
 181.     k13:    next=k14;
 182.             IF nrt=1 THEN GO TO next;
 183.             PUT LIST('guess. Since 100<253<400, we know that sqrt(100)<sqrt(253)<sqrt(400); therefore 10<sqrt(253)<20,
since');
 184.             PUT LIST('10=sqrt(100) and 20=sqrt(400). The number we are looking for, then, is somewhere between 10 and 20.');
 185.             PUT LIST('We might guess 15, but 15x15=225, and 225<253, so 15<sqrt(253). Also, 16x16=256, and 253<256, so
sqrt(253)<16.');
 186.             PUT LIST('Thus 225<253<256, which means sqrt(225)<sqrt(253)<sqrt(256); hence 15<sqrt(253)<16. This means that
the');
 187.             PUT LIST('largest integer that is less than or equal to sqrt(253) is 15. Always find the two integers that
sqrt(n) is');
 188.             PUT LIST('between--your "answer" is the smaller of these two.');
 189.     k14:    END ;

  55.    PR2:    PROCEDURE ;
  56.            DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(200) VAR, unrc CHAR(200) VAR;
  57.            DECLARE  a CHAR(130) VAR, b CHAR(130) VAR, reply CHAR(30) VAR;

  69.1           ans(1)='11';
  69.2           cor='You are right!';
  69.3           unrc='No, the answer is 11.';
  69.4           next=k15;
  69.5           GO TO now;

 200.     k15:    next=k16;
 201.             IF nrt=1 THEN GO TO next;
 202.             PUT LIST('You see, 11x11=121, and 12x12=144. But sqrt(121)<sqrt(140)<sqrt(144), so 11<sqrt(140)<12. Thus 11 is');
 203.             PUT LIST('the largest integer that is less than or equal to sqrt(140).');
 204.             PUT LIST('Try one more: what is the largest integer that is less than or equal to sqrt(203)?');
 205.             ans(1)='14';
 206.             cor='Right, 14 is the answer.';
 207.             unrc='No, the answer is 14. Since 14x14<203<15x15, therefore 14<sqrt(203)<15.';
 208.             GO TO now;
 209.     k16:    PUT LIST('Now let''s consider why, in deciding if n is prime, we can limit our search for factors to numbers
that');
 210.             PUT LIST('are less than or equal to sqrt(n).');
 211.             PUT LIST('if a and b are a pair of factors of n, (that is, axb=n), then at least one of the numbers a or b must
be less');
 212.             PUT LIST('than or equal to sqrt(n). After all, if a>sqrt(n) and b>sqrt(n), then axb>(sqrt(n))x(sqrt(n)), which
means');
 213.             PUT LIST('axb>n; hence a and b wouldn''t be factors of n.');
 214.             PUT LIST('Thus for each pair of factors of n, at least one member of the pair will be less than or equal to
sqrt(n).');
 215.             PUT LIST('This means that if n has any factors at all--other than 1 and n--it will have at least one factor
that');
 216.             PUT LIST('is less than or equal to sqrt(n). By this reasoning, then, if 179 has any factors, it must have a
factor');
 217.             PUT LIST('that is less than or equal to_____.');
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 218.              ans(1)='SQRT(179)';
 219.              ans(2)='SQRT179';
 220.              ir=2;
 221.              wans(1)='13';
 222.              wans(2)='14';
 223.              iw=3;
 224.              cor='Very good.'
 225.              unrc='No, if 179 has any factors, then it must have a factor that is less than or equal to sqrt(179).';
 226.              diag(1)='Very good.';
 227.              diag(2)='You are right, but you could be more exact:';
 228.              wans(3)='ROOT';
 229.              diag(3)='Please use the abbreviation sqrt in place of "square root." The answer is sqrt(173).';
 230.              next=k17;
 231.              GO TO now;
 232.     k17:     PUT LIST('Since 13 is the largest integer that is less than or equal to sqrt(179), then if 179 has any factors');
 233.              PUT LIST('at all, it must have a factor that is less than or equal to 13.');
 234.              PUT LIST('There is one fact that can cut our work even more: if n has a factor,f, (where f is not 1 or n) then n
has');
 235.              PUT LIST('a prime factor that is less than or equal to f. To see how this works, consider the number 1879, which
has');
 236.              PUT LIST('a factor, 297. 297 isn''t prime, but it can be factored into two numbers, 9x33. Again, 9 and 33
aren''t prime,');
 237.              PUT LIST('but 9 can be factored into 3x3, and 3 is prime. Thus 1879 has a prime factor, 3.');
 238.              PUT LIST('In general, if f is a factor of n, then either f is prime (so we''ve found our prime factor), or f can
be');
 239.              PUT LIST('factored into the product of two numbers, say g and h. If g or h is prime, we have again found our
prime');
 240.              PUT LIST('factor. If not, g and h can be factored, and so on. If the process is continued, a prime factor will
be found.');
 241.              PUT LIST('675 has a factor, 135. It therefore has a prime factor that is less than or equal to 135. Name this
factor.');
 242.              ans(1)='3';
 243.              ans(1)='5';
 244.              ans(2)='15';
 245.              wans(1)='3';
 246.              wans(2)='5';
 247.              ir=2;
 248.              iw=2;
 249.              cor='No, this is a factor of 675, but not a prime factor. The only prime factors are 3 and 5.';
 250.              diag(1)='Yes, 3 is a factor of 675. The other prime factor is 5.';
 251.              diag(2)='Yes, 5 is a factor of 675. The other prime factor is 3.';
 252.              unrc='No, the only prime factors of 675 are 3 and 5.';
 253.              next=k18;
 254.              GO TO now;
 255.     k18:     PUT LIST('Thus if a number n has any factors at all, then it must have a prime factor that is less than or
equal');
 256.              PUT LIST('to sqrt(n). To check on whether n is prime, then, WE NEED ONLY CHECK THE PRIME NUMBERS THAT ARE LESS
THAN');
 257.              PUT LIST('OR EQUAL TO SQRT(n).');
 258.              PUT LIST('To see if 179 is prime, then, check the primes that are less than or equal to sqrt(179): that is,
check');
 259.              PUT LIST('the numbers 2,3,5,7,11, and 13. If they are not factors of 179, then 179 HAS no factors (except 1 and
179.)');
 260.              PUT LIST('Are any of these numbers actually factors of 179?');
 261.              ans(1)='NO';
 262.              cor='Right--therefore 179 must be a prime number.';
 263.              unrc='No, none of these numbers are factors of 179: thus 179 is a prime number.';
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 264.              next=k19;
 265.              GO TO now;
 266.     k19:     END ;

  55.     PR3:     PROCEDURE ;
  56.              DECLARE ans(2) CHAR(20) VAR,wans(5) CHAR(20) VAR, cor CHAR(200) VAR,diag(5) CHAR(200) VAR, unrc CHAR(200) VAR;
  57.              DECLARE  a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

  69.1             ans(1)='17';
  69.2             cor='Correct.';
  69.3             unrc='No, 17x17<301<18x18, so 17<sqrt(301)<18; thus 17 is the largest integer that is < or = sqrt(301).';
  69.4             next=k20;
  69.5             GO TO now;

 300.     k20:     PUT LIST('To see if 301 is prime, one would check the numbers 2,3,5,7,11,13,and 17 to see if any of these are');
 302.              PUT LIST('factors of 301. If possible, name a number on this list that is a factor of 301. If there is none,
type "none".');
 303.              ans(1)='17';
 304.              wans(2)='NONE';
 305.              wans(1)='7';
 306.              cor='No, 17 isn''t a factor of 301. The answer should be 7.';
 307.              diag(1)='Very good, 301 is divisible by 7.';
 308.              diag(2)='But there is a factor in that list--tha number 7.';
 308.5            unrc='There is a factor in that list--the number 7 is a factor of 301.';
 309.              lw=2;
 310.              next=k21;
 311.              GO TO now;
 312.     k21:     PUT LIST('To see if 253 is a prime, one would divide by the numbers 2,3,5,7,11, and 13. If possible, name a');
 313.              PUT LIST('number on that list that is a factor of 253; otherwise, type "none."');
 314.              ans(1)='11';
 316.              cor='O.K., 11 is the only listed factor of 253.';
 317.              unrc='No, 11 is a factor of 253, since 253=11x23.';
 318.              next=k22;
 319.              GO TO now;
 320.     k22:     PUT LIST('List the set of all numbers that you should check to see if 91 is prime. Please list the numbers');
 321.              PUT LIST('in order, and separate them by commas.');
 321.5            ans(1)='9';
 322.              ans(2)='11';
 323.              wans(1)='2,3,5,7';
 324.              wans(2)='2,3,5,AND7';
 325.              wans(3)='2,3,SAND7';
 326.              lw=3;
 327.              cor='No, the primes that are less than or equal to sqrt(91) are 2,3,5,and 7';
 328.              lr=2;
 329.              diag(1)='Fine, the primes less than or equal to sqrt(91) are 2,3,5, and 7.';
 330.              diag(2)=diag(1);
 331.              diag(3)=diag(1);
 332.              unrc=cor;
 333.              next=k23;
 334.              GO TO now;
 335.     k23:     PUT LIST('Try to divide 91 by each of the factors 2,3,5, and 7. Is 91 a prime number?');
 335.1            ans(1)='NO';
 335.2            cor='Correct, 7 divides 91, so 91 isn''t prime.';
 335.3            unrc='No, 7 divides 91, so 91 isn''t prime.';
 335.4            next=k24;
 336.              GO TO now;
 337.     k24:     PUT LIST('Try this method (dividing by all the primes less than or equal to sqrt(n)) to see if 139 is prime.');
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338.              PUT LIST('Is it?');
339.              ans(1)='YES';
340.              b=' none of the primes 2,3,5,7,or 11 are divisors of 139: hence 139 is prime.';
341.              cor='Yes,'||b;
342.              unrc='No,'||b;
343.              next=k25;
344.              GO TO now;
345.     k25:     PUT LIST('Name the numbers from the following list that are prime numbers: 113, 115, 117. (If there are none,
type');
346.              PUT LIST('"none."');
347.              ans(1)='115';
347.5             iw=2;
348.              wans(1)='117';
349.              wans(2)='113';
350.              cor='No, 115 is divisible by 5:  only 113 is prime.';
351.              diag(1)='No, 117 is divisible by 3; only 113 is prime.';
352.              diag(2)='Yes, 113 is prime, because it can not be divided by 2,3,5, or 7.';
353.              unrc='No, 113 is prime, but it is the only listed number that is prime.';
354.              next=k26;
355.              GO TO now;
356.     k26:     IF reply='113' THEN score=1; ELSE score=0;
357.              PUT LIST('Name the numbers from the following list that are prime numbers:295, 297, 299. If there are none, type
"none."');
358.              ans(1)='NONE';
359.              b=' there are no primes on the list: 5 divides 295, 3 divides 297, and 13 divides 299.';
360.              cor='Correct,'||b;
370.              unrc='No, '||b;
371.              nrt=0;
372.              next=k27;
373.              GO TO now;
374.     k27:     next=k30;
375.              score=score+nrt;
376.              IF score=2 THEN GO TO next;
377.              PUT LIST('Name the numbers from the following list that are prime numbers: 287,289,291. If there are none, type
"none."');
378.              ans(1)='287';
379.              ans(2)='291';
380.              wans(1)='289';
380.5             wans(2)='NONE';
381.              ir=2;
382.              iw=2;
383.              cor='No, 287 is divisible by 7, 289 is divisible by 17, and 291 is divisible by 3: there are no primes on the
list.';
383.5             diag(2)='You are right, there are no primes on the list: 7 divides 287, 17 divides 289, and 3 divides 291.';
384.              unrc=cor;
385.              diag(1)=cor;
386.              next=k28;
387.              GO TO now;
388.     k28:     next=k30;
389.              IF index(reply,'NONE')>0 THEN score=score+1;
390.              IF score=2 THEN GO TO next;
391.              PUT LIST('Only one number from this list is prime--name it: 119, 121, 123, 125, 127, 129');
392.              iw=5;
393.              ans(1)='119';
394.              wans(1)='121';
395.              wans(2)='123';
396.              wans(3)='125';
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397.            wans(4)='127';
398.            wans(5)='129';
399.            cor='No, 119 is divisible by 7.';
400.            diag(1)='No, 121 is divisible by 11.';
401.            diag(2)='No, 123 is divisible by 3.';
402.            diag(3)='No, 125 is divisible by 5.';
403.            diag(4)='Very good.';
404.            diag(5)='No, 129 is divisible by 3.';
404.5           unrc='Your answer is unrecognized--you may have typed the letter l in place of the numeral 1, for instance.';
405.            next=k29;
406.            GO TO now;
407.    k29:    next=sp;
408.            tot=index(reply,'119')+index(reply,'121')+index(reply,'123')+index(reply,'125')+index(reply,'129');
409.            IF tot>0 THEN GO TO next;
409.5           next=k30;
410.            IF index(reply,'127')>0 THEN GO TO next;
411.    sp:     PUT LIST('Using the same list again, try to name the prime number.');
412.            lw=5;
413.            next=k29s;
414.            GO TO now;
415.    k29s:   IF index(reply,'127')=0 THEN PUT LIST('Actually, the prime number was 127.');
416.    k30:    PUT LIST('End of lesson--you may logout.');
417.            END ;
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CAI UNIT 6

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   1.            DECLARE  LCM1 ENTRY  EXT;
   1.2           CALL LCM1;
   2.            DECLARE  LCM2 ENTRY  EXT;
   2.1           PUT LIST('                                                    Part 2: In case of malfunction, xeq 2
thru...');
   2.2           PUT LIST('For example, consider LCM(12,63). 12=2x2x3, and 63=3x3x7. Thus LCM(12,63) must contain the factor 2x2
(or');
   2.3           PUT LIST('it wouldn''t be a multiple of 12), 3x3 (or it wouldn''t be a multiple of 63), and 7 (or it wouldn''t
be a');
   2.4           PUT LIST('multiple of 63). Hence LCM(12,63)=_____. Again, just type the final answer, not the factors.');
   2.5           CALL LCM2;
   3.            DECLARE  LCM3 ENTRY  EXT;
   3.1           PUT LIST('                                                    Part 3: In case of malfunction, xeq 3
thru...');
   3.2           PUT LIST('O.K., so the least common denominator is 36. Now complete the addition: 5/12 + 7/18 =_____.');
   3.3           CALL LCM3;
   4.            END ;

  55.    LCM1:   PROCEDURE ;
  56.            DECLARE ans(2) CHAR(20) VAR,wans(2) CHAR(20) VAR, cor CHAR(150) VAR,diag(2) CHAR(150) VAR, unrc CHAR(150) VAR;
  57.            DECLARE  a CHAR(150) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

  69.1           PUT LIST('The least common multiple of a and b--abbreviated LCM(a,b)--is the smallest non-zero number that is a
multiple');
  69.2           PUT LIST('of both a and b. The set of non-zero multiples of 12, for instance, contains the numbers 12,24,36,48,60
,72,...');
  69.3           PUT LIST('and the set of non-zero multiples of 18 contains 18,36,54,72,90,.... From this, it should be clear
that');
  69.4           PUT LIST('LCM(12,18)=_____.');
  69.5           ans(1)='36';
  69.6           cor='Fine, LCM(12,18)=36.';
  69.7           unrc='No, the smallest number that is a non-zero multiple of both 12 and 18 is 36.';
  69.8           next=k2;

 100.    k2:     PUT LIST('We could, then, find LCM(a,b) in this way: let A=the set of non-zero multiples of a, and let B=the set
of');
 101.           PUT LIST('non-zero multiples of b. Then LCM(a,b) is the smallest member of the _____of A and B.');
 102.           ans(1)='INTERSECTION';
 103.           cor='Correct.';
 104.           a='Not quite: LCM(a,b) is the smallest member that is in both A and B,';
 105.           unrc=a||' so it''s the smallest member of the intersection of A and B.';
 106.           next=k3;
 107.           GO TO now;
 108.    k3:    PUT LIST('Try another one. If the non-zero multiples of 30 are 30,60,90,120,150,180,210,240,270,... and the
non-zero');
 109.           PUT LIST('multiples of 24 are 24,48,72,96,120,144,168,192,216,240,264,..., then what is LCM(30,24)?');
 110.           ans(1)='120';
 111.           cor='Yes, LCM(30,24)=120.';
 112.           unrc='No, the smallest element that is in both sets of multiples is 120: thus LCM(30,24)=120.';
 113.           next=k4;
 114.           GO TO now;
 115.    k4:    PUT LIST('Unfortunately, this is not a very practical way to find LCM(a,b), because you may have to calculate a
lot');
 116.           PUT LIST('of multiples before you find one that is in both sets. Another--easier--way to find LCM(a,b) is to
factor');
 117.           PUT LIST('a and b into a product of primes, and then TAKE THE PRODUCT OF THE HIGHEST POWERS OF EACH OF THE
DIFFERENT');
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118.            PUT LIST('FACTORS THAT OCCUR IN THE PRIME FACTORIZATION OF EITHER NUMBER.');
119.            PUT LIST('As a start, let''s again find LCM(30,24). The prime factorization of 24 is 2x2x2x3, and the prime
factorization');
120.            PUT LIST('of 30 is_____. List the factors in order, smallest to largest, with the letter x as a times sign
between factors.');
121.            ans(1)='2X2';
122.            ans(2)='5X5';
123.            wans(1)='2X3X5';
124.            cor='Not exactly. 30, written as a product of primes, is 2x3x5.';
125.            diag(1)='Good, 30=2x3x5.';
126.            ir=2;
127.            iw=1;
128.            unrc=cor;
129.            next=k5;
130.            GO TO now;
131.            PUT LIST('All right, if 24=2x2x2x3, and 30=2x3x5, then "the different factors that occur in the prime factorizati
on of');
132.            PUT LIST('either number" are 2,3,and 5. The highest power of 2 in either number is 2x2x2 (which appears in 24),
and the');
133.            PUT LIST('highest power of 3 is simply 3; similarly, the highest power of 5 in either number is 5 to the first
power.');
134.            PUT LIST('Thus LCM(30,24)=2x2x2x3x5=120.');
135.    k5:     PUT LIST('Try one on your own: what is LCM(99,100)? Please don''t state the number in factored form--just the
final');
136.            PUT LIST('answer, which will be a single number.');
137.            ans(1)='9000';
138.            wans(1)='X';
138.5           wans(2)='900';
139.            iw=2;
140.            a='Please state the final answer, not a list of factors. The answer is';
141.            diag(1)=a||' 900.';
142.            cor='You are incorrect. Let''s analyze the way to get the correct answer.';
143.            unrc=cor;
144.            diag(2)='Good, LCM(90,100)=900.';
145.            next=k6;
146.            GO TO now;
147.    k6:     next=det;
148.            IF index(reply,'9000')>0 THEN GO TO next;
149.            next=k8;
150.            IF index(reply,'900')>0 THEN GO TO next;
151.    det:    PUT LIST('If 90=2x3x3x5, and 100=2x2x5x5, then the different prime factors occuring in either number are 2,3,and
5.');
152.            PUT LIST('The highest power of 2 that occurs in either number is 2 to the_____power.');
153.            ans(1)='SECOND';
154.            ans(2)='2ND';
155.            ir=2;
156.            cor='Yes, 2 to the second power is the highest power of 2 in either 90 or 100.';
157.            unrc='No, 2x2, or 2 to the second power, is the highest power of 2 in either 90 or 100.';
158.            next=k7;
159.            GO TO now;
160.    k7:     PUT LIST('Similarly, the highest power of 3 appearing in either number is 3x3, and the highest power of 5 is
5x5.');
161.            PUT LIST('Thus "the product of the highest powers of each of the different factors that occur in either number"
will be');
162.            PUT LIST('2x2x3x3x5x5=900. Hence LCM(90,100)=900.');
163.    k8:     PUT LIST('Now find LCM(8,30).');
164.            ans(1)='120';
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 165.            wans(1)='X';
 166.            lw=1;
 167.            diag(1)=a||'120: 2x2x2x3x5=120.';
 168.            cor='Fine, 2x2x2x3x5=120 is the LCM(8,30).';
 169.            unrc='Not exactly: 8=2x2x2, and 30=2x3x5, so again we are dealing with the factors 2,3, and 5. Now the highest
power of';
 170.            nrt=0;
 171.            next=k9;
 172.            GO TO now;
 172.5   k9:     next=k10;
 173.            IF nrt=1 THEN GO TO next;
 173.5           IF index(reply,'X')>0 THEN GO TO next;
 174.            PUT LIST('2 in either number is 2x2x2; the highest power of 3 is 3 to the first power, and the highest power of
5 is');
 175.            PUT LIST('5 to the first power. Then LCM(8,30)=_____. (Again, just type the final answer, not all the
factors.)');
 176.            alw=1;
 177.            unrc='No, LCM(8,30)=2x2x2x3x5=120--the product of the highest powers of the different factors present in either
number.';
 178.            GO TO now;
 179.    k10:    PUT LIST('You see, any multiple of n and m must contain the highest power of each of the prime factors appearing'
);
 180.            PUT LIST('in n (or else it wouldn''t be a multiple of n) and the highest power of each of the prime factors
appearing');
 181.            PUT LIST('in m (or else it wouldn''t be a multiple of m). The smallest multiple of both n and m should be the
number that');
 182.            PUT LIST('contains only these factors--hence the rule about taking the product of the highest power of each of
the prime');
 183.            PUT LIST('factors that occur in either number.');
 184.            END ;

  55.    LCM2:   PROCEDURE ;
  56.            DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
  57.            DECLARE  a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

  69.1            ans(1)='252';
  69.2            a='Please give the final answer,';
  69.3            wans(1)='X';
  69.4            lw=1;
  69.5            cor='Right, LCM(12,63)=252.';
  69.6            unrc='No, LCM(12,63)=2x2x3x3x7=252.';
  69.7            diag(1)=a||' 252: LCM(12,63)=2x2x3x3x7=252.';
  69.8            next=k11;
  69.9            GO TO now;

 200.    k11:    next=k12;
 201.            IF nrt=1 THEN GO TO next;
 202.            PUT LIST('Try one more by this method: find LCM(30,100).');
 203.            ans(1)='3000';
 204.            wans(2)='300';
 205.            lw=2;
 206.            diag(1)=a||' 300: 2x2x3x5x5=300.';
 207.            diag(2)='Good, LCM(30,100)=2x2x3x5x5=300.';
 208.            cor='No, LCM(30,100)=2x2x3x5x5=300.';
 209.            unrc=cor;
 210.            GO TO now;
 211.    k12:    PUT LIST('There is another method of finding the LCM(a,b) that works when you already know the GCD(a,b)--the ');
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212.              PUT LIST('greatest common divisor of a and b. The formula LCM(a,b)=(axb)/GCD(a,b) can then be used. For
example,');
213.              PUT LIST('GCD(12,63)=3. By the formula, then, LCM(12,63)=(12x63)/GCD(12,63) =(12x63)/3 = 4x63=252. (What you are
really');
214.              PUT LIST('doing when you divide by GCD(a,b) is removing all the factors that belong to both a and b--hence you
get rid of');
215.              PUT LIST('the unnecessary factors in the product axb.)');
216.              PUT LIST('Use this method to find LCM(72,120), given that GCD(72,120)=24. Again, just state the final answer.');
217.              ans(1)='360';
218.              wans(1)='/';
219.              wans(2)='X';
220.              lw=2;
221.              cor='Good, LCM(72,120)=(72x120)/24 =360.';
222.              diag(1)=a||' 360--(72x120)/24=360.';
223.              diag(2)=diag(1);
224.              unrc='Not quite: LCM(72,120)=(72x120)/GCD(72,120)=(72x120)/24 =360.';
225.              next=k13;
226.              GO TO now;
227.     k13:     PUT LIST('Use the same method to find LCM(90,300) given that GCD(90,300)=30.');
228.              ans(1)='9000';
229.              lw=3;
230.              diag(1)=a||' 900--(90x300)/30=900.';
231.              wans(1)='900';
235.              unrc='not exactly--LCM(90,300)=(90x300)/GCD(90,300)=(90x300)/30 =900.';
236.              cor=unrc;
237.              diag(2)=diag(1);
238.              diag(3)='Correct, LCM(90,300)=900.';
239.              next=k14;
240.              GO TO now;
241.     k14:     PUT LIST('Find LCM(294,252) given that GCD(294,252)=42.');
242.              ans(1)='1764';
243.              lw=2;
244.              cor='Yes, LCM(294,252)=(294x252)/42=1764.';
245.              unrc='No, LCM(294,252)=(294x252)/GCD(294,252) =(294x252)/42 =1764.';
246.              diag(1)=a||' 1764--(294x252)/42=1764.';
247.              diag(2)=diag(1);
248.              next=k15;
249.              nrt=0;
250.              GO TO now;
251.     k15:     next=k16;
252.              IF nrt=1 THEN GO TO next;
253.              PUT LIST('Just one more like this: find LCM(75,90), given that GCD(75,90)=15.');
254.              ans(1)='450';
255.              lw=2;
256.              cor='Fine, 450=LCM(75,90).';
257.              unrc='Not quite: LCM(75,90)=(75x90)/GCD(75,90) =(75x90)/15 =450.';
258.              diag(1)=a||' 450--(75x90)/15 =450.';
259.              diag(2)=diag(1);
260.              GO TO now;
261.     k16:     PUT LIST('The method used in the last few problems is useful when the numbers involved are extremely large--so
large');
262.              PUT LIST('that you would not want to factor the numbers into a product of primes. Instead, you can find GCD(a,b)
by');
263.              PUT LIST('using Euclid''s algorithm, and then use the formula LCM(a,b)=axb/GCD(a,b) to find the least common
multiple.');
264.              PUT LIST('By now you may be wondering why anybody would ever want to find LCM(a,b). Actually, least common
multiples are');
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 265.              PUT LIST('useful in adding (or subtracting) a certain class of numbers; least common multiples are useful in
adding and');
 266.              PUT LIST('subtracting_________.');
 267.              ans(1)='FRACTION';
 268.              cor='Right!';
 269.              unrc='I didn''t recognize your answer. Actually, least common multiples are used in adding fractions.';
 270.              next=k17;
 271.              GO TO now;
 272.     k17:     PUT LIST('For instance, to add 5/12 + 7/18, you are interested in finding the least common denominator--that
is,');
 273.              PUT LIST('the least common multiple of 12 and 18. But 12=2x2x3, and 18=2x3x3, so LCM(12,18)=_____.Type the final
answer.');
 274.              ans(1)='36';
 275.              wans(1)='X';
 276.              iw=1;
 277.              cor='Fine, LCM(12,18)=36.';
 278.              diag(1)=a||' 36--2x2x3x3=36.';
 279.              unrc='Actually, the answer is 36: 2x2x3x3=36.';
 280.              next=k18;
 281.              GO TO now;
 282.     k18:     END ;

  55.     LCM3:    PROCEDURE ;
  56.              DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
  57.              DECLARE  a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

  69.1             ans(1)='29/36';
  69.2             cor='Good, 29/36 is correct.';
  69.3             unrc='No, 5/12 + 7/18 = 15/36 + 14/36 = 29/36.';
  69.4             next=k19;

 300.     k19:     next=kk19;
 301.              IF nrt=1 THEN GO TO next;
 302.              PUT LIST('Remember that to add fractions that have the same denominator, you simply add the numerators and
keep');
 303.              PUT LIST('the denominator the same. For instance, 5/8 + 1/8 = (5+1)/8 = 6/8. If the denominators are not
equal,');
 304.              PUT LIST('however, it is necessary to first change to a common denominator, and the most efficient common
denominator');
 306.              PUT LIST('is the LCM of the denominators of the fractions. Remember that it is legal to multiply both numerator
and');
 307.              PUT LIST('denominator of a fraction by the same non-zero quantity: 2/5 = (2x6)/(5x6) = 12/30.');
 308.     kk19:    PUT LIST('The lowest common denominator that could be used to add 3/16 + 1/20 is_____.');
 309.              ans(1)='80';
 310.              cor='Correct, LCM(16,20)=80.';
 311.              unrc='Not exactly. The lowest common denominator is LCM(16,20), which is 2x2x2x2x5=80.';
 312.              score1=nrt;
 313.              nrt=0;
 314.              next=k20;
 315.              GO TO now;
 316.     k20:     PUT LIST('Thus 3/16 + 1/20 =_____. (Final answer only, please.)');
 317.              ans(1)='19/80';
 318.              cor='Fine, 3/16 + 1/20 = 19/80.';
 319.              score2=nrt;
 320.              nrt=0;
 321.              next=k21;
 322.              unrc='No, 3/16 + 1/20 = 15/80 + 4/80 = 19/80.';
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 323.              GO TO now;
 324.     k21:     score1=score1+nrt;
 325.              next=k22;
 326.              IF nrt=1 THEN GO TO next;
 327.              PUT LIST('Multiplying the numerator and denominator of a fraction by the same non-zero quantity doesn''t change
the');
 328.              PUT LIST('value of the fraction, because this multiplication is really multiplication by n/n, and n/n=1 for all
n not');
 329.              PUT LIST('equal to zero. In the previous problem, then, we wanted to use 80 as a denominator, so we could have
looked');
 330.              PUT LIST('at the problem as follows: 3/16 + 1/20 = ?/80 + ?/80. To get 80 from the denominator 16, it was
necessary to');
 331.              PUT LIST('multiply the denominator by 5; hence we must also multiply the numerator by 5, so 3/16 =(3x5)/(16x5)=15
/80.');
 332.              PUT LIST('Similarly, we must multiply numerator and denominator of 1/20 by 4: 1/20 =(1x4)/(20x4)= 4/80. Hence
the');
 333.              PUT LIST('addition problem is 3/16 + 1/20 = 15/80 + 4/80 = 19/80.');
 334.     k22:     PUT LIST('The lowest common denominator that could be used to add 1/36 + 1/45 is_____.');
 335.              ans(1)='180';
 336.              cor='Fine, LCM(36,45)=180.';
 337.              unrc='No, LCM(36,45)=2x2x3x3x5=180.';
 338.              next=k23;
 339.              nrt=0;
 340.              GO TO now;
 341.     k23:     score2=score2+nrt;
 342.              next=k30;
 343.              IF score1*(score1-1)*(score2-1)>0 THEN GO TO next;
 344.              PUT LIST('Thus 1/36 + 1/45 =_____.(Final answer only, please.)');
 345.              ans(1)='9/180';
 346.              ans(2)='1/20';
 347.              ir=2;
 348.              nrt=0;
 349.              cor='Very good, 1/36 + 1/45 = 9/180 = 1/20.';
 350.              unrc='No, 1/36 + 1/45 = 5/180 + 4/180 = (5+4)/180 = 9/180 = 1/20.';
 351.              next=k24;
 352.              GO TO now;
 353.     k24:     next=k30;
 354.              score1=score1+nrt;
 355.              IF score1*(score1-1)*(score2-1)>0 THEN GO TO next;
 356.              PUT LIST('Try another one: what is the smallest common denominator that one could use to add 7/60 + 8/75?');
 357.              ans(1)='300';
 358.              cor='Right, LCM(60,75)=300.';
 359.              unrc='I''m afraid not. LCM(60,75)=2x2x3x5x5=300.';
 360.              nrt=0;
 361.              next=k25;
 362.              GO TO now;
 363.     k25:     next=k30;
 364.              score2=score2+nrt;
 365.              IF score1*(score1-1)*(score2-1)>0 THEN GO TO next;
 366.              PUT LIST('Then 7/60 + 8/75 =_____.(Final answer, please.)');
 367.              ans(1)='67/300';
 368.              cor='Good, the answer is 67/300.';
 369.              unrc='Not quite: 7/60 + 8/75 = 35/300 + 32/300 = 67/300.';
 370.              next=k26;
 371.              nrt=0;
 372.              GO TO now;
 373.     k26:     next=k30;
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374.            score1=score1+nrt;
375.            IF score1=(score1-1)*(score2-1)>0 THEN GO TO next;
376.            PUT LIST('Try some new numbers. Name the least common denominator that could be used to add 2/105 + 9/70.');
377.            ans(1)='210';
378.            cor='O.K., LCM(105,70)=210.';
379.            unrc='No, LCM(105,70)=2x3x5x7=210.';
380.            next=k27;
381.            nrt=0;
382.            GO TO now;
383.    k27:    score1=score1+nrt;
384.            next=k30;
385.            IF score1=(score1-1)*(score2-1)>0 THEN GO TO next;
386.            PUT LIST('Hence 2/105 + 9/70 =_____.');
387.            ans(1)='31/210';
388.            cor='Good, 31/210 is right.';
389.            unrc='No, 2/105 + 9/70 = 4/210 + 27/210 = 31/210.';
390.            next=k28;
391.            nrt=0;
392.            GO TO now;
393.    k28:    score1=score1+nrt;
394.            next=k30;
395.            IF score1=(score1-1)*(score2-1)>0 THEN GO TO next;
396.            PUT LIST('The least common multiple of 70 and 28 is_____.');
397.            ans(1)='140';
398.            cor='Good, LCM(70,28)=140. Hence the least common denominator used in adding 3/70 + 5/28 is 140.';
399.            unrc='No, LCM(70,28)=2x2x5x7=140; thus the least common denominator used in adding 3/70 + 5/28 is 140.';
400.            next=k29;
401.            nrt=0;
402.            GO TO now;
403.    k29:    score2=score2+nrt;
404.            next=k30;
405.            IF score1=(score1-1)*(score2-1)>0 THEN GO TO next;
406.            PUT LIST('Just one more: 3/70 + 5/28 =_____.( Again, just the final answer.)');
407.            ans(1)='31/140';
408.            unrc='Not quite: 3/70 + 5/28 = 6/140 + 25/140 = 31/140.';
409.            cor='Good, 31/140 is correct.';
410.            GO TO now;
411.    k30:    PUT LIST('End of lesson--you may logout.');
412.            END ;
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CAI UNIT 7

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   1.          DECLARE  RST1 ENTRY  EXT;
   1.1         CALL RST1;
   2.          DECLARE  RST2 ENTRY  EXT;
   2.1         PUT LIST('                                                    Part 2. In case of malfunction, xeq 2
thru...');
   2.2         PUT LIST('Define a new relation R on the set of natural numbers as follows: xRy if x>y. Is this relation
symmetric?');
   2.3         CALL RST2;
   3.          DECLARE  RST3 ENTRY  EXT;
   3.1         PUT LIST('                                                    Part 3. In case of malfunction, xeq 3
thru...');
   3.2         PUT LIST('Let N be the set of natural numbers, and define a relation R with xRy if x is less than or equal to
y.');
   3.3         PUT LIST('Is R an equivalence relation?');
   3.4         CALL RST3;
   3.5         END ;

  55.    RST1:  PROCEDURE ;
  56.          DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
  57.          DECLARE  a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

  69.1         PUT LIST('Let A be the set containing the numbers 1,2, and 4. Then AXA, the Cartesian product of A with
itself,');
  69.15        PUT LIST('would contain the ordered pairs (1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(4,1),(4,2), and_____.');
  69.2         ans(1)='(4,4)';
  69.25        wans(1)='44';
  69.3         wans(2)='4,4';
  69.35        b=' the only missing ordered pair is (4,4).';
  69.4         cor='Yes,'||b;
  69.45        unrc='No,'||b;
  69.5         diag(1)='You seem to have forgotten the comma. Actually,'||b;
  69.55        diag(2)='You seem to have forgotten the parentheses. Actually,'||b;
  69.6         iw=2;
  69.65        next=k2;
  69.7         GO TO now;

 100.    k2:   PUT LIST('Anyway, a relation, R, in the set A has been defined as any subset of AXA. To say that');
 101.          PUT LIST('"x is related to y," abbreviated xRy, is the same as saying that (x,y) is in the relation R.');
 102.          PUT LIST('Temporarily, let R be defined as the set of all elements (x,y) in AXA for which y=2x.  Then (1,2) is');
 103.          PUT LIST('in R, (that is, 1R2), because 2=2 times 1. Name another ordered pair that is an element of R.');
 104.          ans(1)='(2,4)';
 105.          wans(1)='24';
 106.          wans(2)='2,4';
 107.          iw=2;
 108.          cor='Fine, (2,4) is the only other element of R.';
 109.          b=' (2,4) is the only other element, (x,y), in AXA for which y=2x.';
 110.          diag(1)='An ordered pair needs a comma between elements. Actually,'||b;
 111.          diag(2)='An ordered pair needs parentheses. Actually,'||b;
 112.          unrc='Not exactly:'||b;
 113.          next=k3;
 114.          GO TO now;
 115.    k3:   PUT LIST('A relation is said to be reflexive if xRx for every x in A; in other words, a relation is reflexive');
 115.5         PUT LIST('if every element in the set A "is related to" itself. Thus the relation defined in the previous');
 116.          PUT LIST('question is not reflexive, since (4,4) wasn''t one of the elements in R. (That is, the statement');
 117.          PUT LIST('4R4 is false, because the statement 4=2 times 4 is false.)');
 118.          PUT LIST('Now let N represent the natural numbers (1,2,3,4,5,6...), and define a relation R in N such that');
 119.          PUT LIST('xRy if x times y is a perfect square. For instance, 2R18, because 2 times 18 equals 36, and 36 is');
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120.            PUT LIST('a perfect square. (Perfect squares are numbers such as 1,4,9,16,25,36,49,...)');
121.            PUT LIST('Of the following ordered pairs name an ordered pair that is in R: (2,9), (3,27), (6,4)');
122.            ans(1)='(3,27)';
123.            cor='Very good, (3,27) is in R, since 3 times 27 is a perfect square.';
124.            wans(1)='(2,9)';
125.            wans(2)='(6,4)';
126.            diag(1)='No, (2,9) isn''t in R since 2x9=18, and 18 isn''t a perfect square. The correct answer is (3,27)';
127.            diag(2)='No, (6,4) isn''t in R since 6x4=24, and 24 isn''t a perfect square. The correct answer is (3,27).';
128.            unrc='No, (3,27) is the answer, since 3x27=81, and 81 is a perfect square.';
129.            lw=2;
130.            next=k4;
131.            GO TO now;
132.    k4:     nrt=0;
133.            PUT LIST('Is this last relation reflexive--that is, is xRx for every x in N?');
134.            ans(1)='YES';
135.            b=' x times x is always a perfect square, so xRx for all x in N. Thus R is reflexive in N.';
136.            cor='Fine,'||b;
137.            unrc='That''s incorrect,'||b;
138.            next=k5;
139.            GO TO now;
140.    k5:     PUT LIST('Again using the set N, of natural numbers, define a relation R such that xRy if x>y.');
141.            PUT LIST('Is the relation R reflexive in N?');
142.            ans(1)='NO';
143.            b=' R isn''t reflexive because x>x is not true for all x. In fact, the statement x>x is false for all x in N.';
144.            unrc='You are incorrect--'||b;
145.            cor='OK,'||b;
146.            next=k6;
147.            GO TO now;
148.    k6:     PUT LIST('Let S denote the set of students at Iowa State, and define a relation R such that xRy if x and y
have');
149.            PUT LIST('the same major. Is R reflexive in S?');
150.            ans(1)='YES';
151.            cor='Fine, R is reflexive since xRx is always true.';
152.            unrc='Wrong. Since a person always has the same major as himself, xRx is always true. Thus R is reflexive.';
153.            next=k7;
154.            GO TO now;
155.    k7:     next=k8;
156.            IF nrt>=3 THEN GO TO next;
157.            PUT LIST('Let N be the set of natural numbers, and let R be a relation such that xRy if (x+y)>5. Is R reflexive
in N?');
158.            ans(1)='NO';
159.            b=' since the statements 1R1 and 2R2 are false. Thus R isn''t reflexive, since xRx is false for some x.';
160.            cor='Correct: the answer is no,'||b;
161.            unrc='The answer should be no,'||b;
162.            GO TO now;
163.    k8:     PUT LIST('Another property some relations possess is the symmetric property: a relation is said to be');
164.            PUT LIST('symmetric in A if xRy implies yRx for all x,y in A. Please notice that this definition doesn''t');
165.            PUT LIST('require that xRy and yRx for all x and all y in A--it simply says that if xRy, THEN y must also');
166.            PUT LIST('be related to x. For example, define the relation R on the set of natural numbers as follows:');
167.            PUT LIST('xRy if x=y. Clearly, if xRy, then yRx (because x=y implies y=x), so this relation is _____.'
);
168.            ans(1)='SYM';
169.            cor='Yes, symmetric is correct.';
170.            unrc='No, R is said to be symmetric in the set N.';
171.            next=k9;
172.            GO TO now;
173.    k9:     END ;
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 55.    RST2:   PROCEDURE ;
 56.            DECLARE ans(3) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
 57.            DECLARE  a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

 69.1           ans(1)='NO';
 69.2           b='For instance, 7R2 because 7>2, but 2 isn''t related to 7, since 2 isn''t greater than 7.';
 69.3           cor='Correct,'||b;
 69.4           unrc='NO, R isn''t symmetric.'||b;
 69.5           next=k10;

200.    k10:   PUT LIST('Now define a relation R in the natural numbers such that xRy if x is a divisor of y.');
201.           PUT LIST('"x is a divisor of y" means that if one divides x into y, the remainder is zero. For instance,');
202.           PUT LIST('3 is a divisor of 15, since 3 divided into 15 yields 5 with remainder zero; thus 3R15.');
203.           PUT LIST('Is this relation symmetric in N?');
204.           b=' For instance 4 is a divisor of 20, but 20 isn''t a divisor of 4; Thus xRy doesn''t imply yRx.';
205.           cor='Fine, R isn''t symmetric.'||b;
206.           unrc='That''s incorrect, R isn''t symmetric.'||b;
207.           next=k11;
208.           GO TO now;
209.    k11:   PUT LIST('Let''s consider a relation defined in M, the set of all males. Define a relation R in M such that');
210.           PUT LIST('xRy if x is a brother of y. Is R symmetric in M?');
211.           ans(1)='YES';
212.           b='xRy implies yRx for all x,y in M, so R is symmetric in M.';
213.           cor='Good, '||b;
214.           unrc='Sorry, your answer is wrong. If x is a brother of y  then y is a brother of x--'||b;
215.           next=k12;
216.           GO TO now;
217.    k12:   PUT LIST('One might notice that if the previous relation had been defined in the set P of all people instead');
218.           PUT LIST('of the set of all males, it would not have been symmetric in P. Then xRy would not imply that yRx,
for');
219.           PUT LIST('it is possible that x is a brother of y and y is a sister of x.');
220.           next=k13;
221.           IF nrt>2 THEN GO TO next;
222.           PUT LIST('One more of this kind! Let P be the set of people, and define a relation R such that xRy if x likes
y.');
223.           PUT LIST('Is R symmetric in P?');
224.           ans(1)='NO';
225.           b=' It is possible that x likes y but y doesn''t like x. Thus xRy doesn''t imply yRx.';
226.           cor='Right, R isn''t symmetric, since'||b;
227.           unrc='No, R isn''t symmetric, since'||b;
228.           GO TO now;
229.    k13:   nrt=0;
230.           PUT LIST('Another property a relationship may have is the transitive property, defined as follows:');
231.           PUT LIST('A relation, R, is transitive in a set A if xRy and yRz imply that xRz for all x,y,z in A.');
231.5          PUT LIST('In other words, if you know that xRy and that yRz, then you automatically know that xRz.');
232.           PUT LIST('For example, let N be the set of natural numbers, and define the relation R in N such that');
233.           PUT LIST('xRy if x>y. This relation is transitive because if one knows that x>y and that y>z, then it');
234.           PUT LIST('is automatically true that_____.');
235.           ans(1)='X>Z';
236.           ans(2)='XRZ';
236.5          ir=3;
237.           ans(3)='Z<X';
238.           cor='Yes, x>y and y>z imply x>z; that is, xRz.';
239.           unrc='No, x>z is the answer. If x>y and y>z, then one knows that x>z.';
240.           next=k14;
241.           GO TO now;
242.    k14:   PUT LIST('Let P be the set of all people and define a relation R such that xRy if x is a friend of y.');
```

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243.            PUT LIST('Is R transitive in P?');
244.            cor='Good.';
245.            unrc='Your answer is incorrect.';
246.            ans(1)='NO';
247.            next=k15;
248.            GO TO now;
249.    k15:    PUT LIST('It could happen that x is a friend of y and y is a friend of z  but x isn''t a friend of z.');
250.            PUT LIST('Thus xRy and yRz won''t imply xRz  so R isn''t a transitive relation.');
252.            PUT LIST('Let N be the natural numbers  and define a relation R such that xRy if x=2y. Is R transitive in N?');
253.            ans(1)='NO';
254.            b='For example, 8R4 and 4R2, but 8 isn''t related to 2.';
255.            cor='You are right, R isn''t transitive. '||b;
256.            unrc='No, R isn''t transitive. '||b;
257.            next=k16;
258.            GO TO now;
259.    k16:    next=k17;
260.            IF nrt>2 THEN GO TO next;
261.            PUT LIST('Let P be the set of people taking math 190, and define xRy if x has the same instructor as y.');
262.            PUT LIST('Is R a transitive relation in P?');
263.            ans(1)='YES';
264.            cor='Fine, R is transitive.';
265.            unrc='But R IS transitive. If x is related to y and y is related to z  then x is related to z.';
266.            GO TO now;
267.    k17:    PUT LIST('A relation that is reflexive, symmetric and transitive is said to be an equivalence relation.');
268.            PUT LIST('The most famous of these relations is undoubtedly the "equals" relation, which may be defined as');
269.            PUT LIST('follows: xRy if x=y. (When you have time, you should satisfy yourself that this relation is actually');
270.            PUT LIST('an equivalence relation.)');
271.            END ;

 55.    RST3:   PROCEDURE ;
 56.            DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(150)VAR, unrc CHAR(150) VAR;
 57.            DECLARE  a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

 69.1           ans(1)='NO';
 69.2           cor='You are right.';
 69.3           unrc='Your answer is incorrect.';
 69.4           next=k18;

300.    k18:    PUT LIST('"Less than or equal to" isn''t an equivalence relation because it fails to have one of the');
301.            PUT LIST('properties of an equivalence relation. Which property does it fail to have?');
302.            ans(1)='SYM';
303.            wans(1)='REF';
304.            wans(2)='TRAN';
305.            iw=2;
306.            cor='OK, the relation isn''t symmetric, since x less than or equal to y doesn''t imply y less than or equal to
x.';
307.            diag(1)='No, the relation is reflexive since x is less than or equal to itself, but it isn''t symmetric.';
308.            diag(2)='No, the relation is transitive; it is not symmetric, however.';
309.            next=k19;
310.            GO TO now;
311.    k19:    PUT LIST('Try another one. Let R be a relation in N such that xRy if (x+y)=8. Is R an equivalence');
312.            PUT LIST('relation in N?');
313.            ans(1)='NO';
314.            cor='Fine, R isn''t an equivalence relation.';
315.            unrc='Wrong, R isn''t an equivalence relation.';
316.            next=k20;
317.            GO TO now;
```

```
 318.     k20:    PUT LIST('Name one property of an equivalence relation that R fails to have.');
 319.             ans(1)='REF';
 319.5            ans(2)='TRAN';
 319.6            ir=2;
 320.             cor='OK., R is neither reflexive nor transitive, although it is symmetric.';
 321.             unrc='No, the correct answers are reflexive and transitive--R is symmetric, however.';
 322.             next=k21;
 323.             GO TO now;
 324.     k21:    next=k22;
 326.             PUT LIST('Try another. Let P be the set of all people, and define a relation R such that xRy if x lives within');
 327.             PUT LIST('ten miles of y. This relation fails to have one of the properties of an equivalence relation; name');
 328.             PUT LIST('that property.');
 329.             ans(1)='TRAN';
 330.             cor='Great, this relation fails to be transitive.';
 330.1            wans(1)='REF';
 330.2            wans(2)='SYM';
 330.3            iw=2;
 330.4            diag(1)='But R is reflexive, since x always lives within 10 miles of himself. R isn''t transitive, however.';
 330.5            diag(2)='But R is symmetric-x lives within 10 miles of y implies y lives within 10 miles of x-but it isn''t
transitive.';
 331.             unrc='No, this relation fails to be transitive.';
 333.             GO TO now;
 334.     k22:    PUT LIST('For instance, x could live 8 miles north of y, and y could live 8 miles north of z. Then xRy and');
 335.             PUT LIST('yRz, but x and z do not live within ten miles of each other, so x isn''t related to z.');
 336.             next=k23;
 337.             IF nrt>3 THEN GO TO next;
 338.             PUT LIST('Just one more. In the set P of all people, define xRy if x is the mother of y. Name one of the');
 339.             PUT LIST('properties of an equivalence relation that R fails to have.');
 340.             cor='Either reflexive, symmetric, or transitive is correct for this one, since it fails to have all three
properties.';
 341.             unrc=cor;
 342.             GO TO now;
 343.     k23:    PUT LIST('End of lesson--you may logout.');
 344.             END ;
```

APPENDIX C: PRE-TEST QUESTIONNAIRE MEASURING ATTITUDE TOWARD CAI

Author: Brown, Bobby R. In: Mitzel, Harold E. and Brandon, George L. Experimentation with computer-assisted instruction in technical education. 1966.

All questions were used in the first trial. All questions except numbers 2, 4, 18, and 28 were used in the replication.

1. While taking computer-assisted instruction I would feel challenged to do my best work.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

2. While taking computer-assisted instruction I would be concerned that I might not be understanding the material.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

3. While taking computer-assisted instruction I would feel isolated and alone.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

4. I would feel uncertain as to my performance in the programmed instruction relative to the performance of others.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

5. While taking computer-assisted instruction I would find myself just trying to get through the material rather than trying to learn.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

6. Computer-assisted instruction should not be used in any form in the elementary school.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

7. Computer-assisted instruction could be used effectively in many college classes.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

8. In a situation where I am trying to learn something, it is important to me to know where I stand relative to others.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

9. Computer-assisted instruction would make this course more interesting.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

10. While taking computer-assisted instruction I would be more involved in running the machine than in understanding the material.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

11. I feel I could work at my own pace with computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

12. Computer-assisted instruction makes the learning too mechanical.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

13. I would feel as if I had a private tutor while on computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=sgree 5=strongly agree

14. While taking computer-assisted instruction I would be aware of efforts to suit the material specifically to me.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

15. While taking computer-assisted instruction I would find it difficult to concentrate on the course material because of the hardware.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

16. Computer-assisted instruction would be too confusing for grade school children.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

17. Computer-assisted instruction is an inefficient use of the student's time.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

18. While on computer-assisted instruction I would encounter mechanical malfunctions.

1=all the time 2=most of the time 3=some of the time
4=seldom 5=never

19. Computer-assisted instruction would make it possible for me to learn more quickly than traditional instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

20. I would feel frustrated by the computer-assisted instruction situation.

1=strongly disagree 2-disagree 3=uncertain
4=agree 5=strongly agree

21. The computer-assisted instruction approach is inflexible.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

22. Even otherwise interesting material would be boring when presented by computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

23. In view of the effort I put into it, I would be satisfied with what I had learned while using computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

24. In view of the amount I would learn, I would say computer-assisted instruction is superior to traditional instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

25. With a course such as the one I am taking, I would prefer computer-assisted instruction to traditional instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

26. I am not in favor of computer-assisted instruction because it is just another step toward depersonalized instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

27. Computer-assisted instruction is too fast.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

28. Typing experience is necessary in order to perform satisfactorily on computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

29. Computer-assisted instruction is boring.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

APPENDIX D: POST-TEST QUESTIONNAIRE MEASURING ATTITUDE TOWARD CAI

Author: Brown, Bobby R. In: Mitzel, Harold E. and Brandon, George L. Experimentation with computer-assisted instruction in technical education. 1966.

All questions were used in the first trial. All questions except numbers 2, 4, 18, and 28 were used in the replication.

1. While taking computer-assisted instruction I felt challenged to do my best work.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

2. While taking computer-assisted instruction I was concerned that I might not be understanding the material.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

3. While taking computer-assisted instruction I felt isolated and alone.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

4. I felt uncertain as to my performance in the programmed instruction relative to the performance of others.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

5. While taking computer-assisted instruction I found myself just trying to get through the material rather than trying to learn.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

6. Computer-assisted instruction should not be used in any form in the elementary school.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

7. Computer-assisted instruction could be used effectively in many college classes.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

8. In a situation where I am trying to learn something, it is important to me to know where I stand relative to others.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

9. Computer-assisted instruction made this course more interesting.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

10. While taking computer-assisted instruction I was more involved in running the machine than in understanding the material.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

11. I felt I could work at my own pace with computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

12. Computer-assisted instruction makes the learning too mechanical.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

13. I felt as if I had a private tutor while on computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

14. While taking computer-assisted instruction I was aware of efforts to suit the material specifically to me.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

15. While taking computer-assisted instruction I found it difficult to concentrate on the course material because of the hardware.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

16. Computer-assisted instruction would be too confusing for grade school children.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

17. Computer-assisted instruction is an inefficient use of the student's time.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

18. While on computer-assisted instruction I encountered mechanical malfunctions.

1=all the time 2=most of the time 3=some of the time
4=seldom 5=never

19. Computer-assisted instruction made it possible for me to learn more quickly than traditional instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

20. I felt frustrated by the computer-assisted instruction situation.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

21. The computer-assisted instruction approach is inflexible.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

22. Even otherwise interesting material would be boring when presented by computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

23. In view of the effort I put into it, I was satisfied with what I learned while using computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

24. In view of the amount I learned, I would say computer-assisted instruction is superior to traditional instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

25. With a course such as the one I am taking, I would prefer computer-assisted instruction to traditional instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

26. I am not in favor of computer-assisted instruction because it is just another step toward depersonalized instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

27. Computer-assisted instruction is too fast.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

28. Typing experience is necessary in order to perform easily on computer-assisted instruction.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

29. Computer-assisted instruction is boring.

1=strongly disagree 2=disagree 3=uncertain
4=agree 5=strongly agree

APPENDIX E: PRE-TEST MATHEMATICS ACHIEVEMENT MEASURE

Questions 13A, 14A, 15A, and 16A were used in the first trial; questions 13B, 14B, 15B, and 16B were used in the replication.

1. Using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, t, e to write numerals in the base 12 number system, what is the base 12 numeral immediately following after 59_{12}?

2. Write the base 10 numeral for $2t7_{12}$.

3. Write 21.34_6 as a base 10 numeral.

4. Write 268_{10} as a base 12 numeral.

5. Consider the operation * on the set of natural numbers where a*b = (a + b) - 2.

   a) Find the numeric value of 8*3

   b) Explain why the operation * is (or is not) commutative on the set of natural numbers.

6. State the commutative property of addition.

7. State a rule that can be used to tell whether a number is divisible by 5 without actually dividing the number by 5.

8. State a rule that can be used to tell whether a number is divisible by 4 without actually dividing the number by 4.

9. Consider the following statement: a number is divisible by n•m if and only if it is divisible by n and by m. If this statement is true for all whole numbers n and m, then write "true;" otherwise, find a pair of whole numbers (a value for n and a value for m) for which the statement is false.

10. Which pair of whole numbers does $\sqrt{78}$ lie between?

11. If 143 is prime, write "prime;" if not, name a number that divides 143 (other than 1 and 143).

12. The largest prime number that is less than or equal to 200 is _____.

13A. Use the set of all multiples of 8 and the set of all multiples of 12 to find the least common multiple of 8 and 12. Show your work.

13B. A relation that is reflexive, symmetric, and transitive is called an _____ relation.

14A. If $n \cdot m = 400$ and the greatest common divisor of n and m is 2, what is the least common multiple of n and m?

14B, 16B, 17B. Consider the given relations on the given sets. Place an R, S, and/or T in the space provided if the relation has the Reflexive, Symmetric, and/or Transitive properties:

relation	set on which reln. is defined
a is related to b if $a \cdot b$ is an even number	Natural numbers_____
A is related to B if A is a subset of B	Sets _____
a is related to b if a is a first cousin of b	People _____

15A. Find the least common multiple of 18 and 24.

16. To see if 101 is a prime number, it would be necessary to try to divide 101 by exactly four numbers. Name those four numbers.

17A. Add 1/18 + 5/24.

For each of the following questions, circle the letter corresponding to the correct response.

18. The statement $18 - 0 = 0 - 18$

a) is false.
b) is true because of the zero property of subtraction.
c) is true because of the commutative property of subtraction.
d) is true because of the associative property of subtraction.

19. The statement $(6 + 15) \div 3 = (6 \div 3) + (15 \div 3)$

a) is false
b) is true because of the commutative property of division.
c) is true because of the associative property of division.
d) is true because of the right distributive property of division over addition.

20. The statement $1 \div (3 + 5) = (1 \div 3) + (1 \div 5)$

a) is false.
b) is true because of the commutative property of division.
c) is true because of the associative property of division.
d) is true because of the right distributive property of division over addition.

21. The statement $(12 \div 4) \div 1 = 12 \div (4 \div 1)$

a) is true because of the commutative property of division.
b) is true because of the associative property of division.
c) is true because of the distributive property of division.
d) is true, but not for any of the above reasons.

22. The expression $0 \div 0$

a) is equal to 1, because a number divided by itself is always 1
b) is meaningless.
c) is equal to 0, since 0 divided by anything is always 0.
d) is equal to infinity.

23. To see that 864123 is divisible by 3 without actually dividing, one could notice that

a) the number in the one's position is divisible by 3.
b) the number names by the last three digits (123) is divisible by 3.
c) the sum of the digits is divisible by 3.
d) none of the above.

APPENDIX F: POST-TEST MATHEMATICS ACHIEVEMENT MEASURE

Questions 19A, 20A, 21A, and 22A were used in the first trial; questions 19B, 20B, 21B, and 22B were used in the replication.

Circle the letter corresponding to the correct response:

1. The statement $1 \div (4 + 5) = (1 \div 4) + (1 \div 5)$

   a) is false.
   b) is true because of the commutative property of division.
   c) is true because of the associative property of division.
   d) is true because of the right distributive property of division over addition.

2. The statement $(16 \div 4) \div 1 = 16 \div (4 \div 1)$

   a) is true because of the commutative property of division.
   b) is true because of the associative property of division.
   c) is true because of the right distributive property of division over addition.
   d) is true, but not for any of the above reasons.

3. The expression $0 \div 0$

   a) is equal to 1.
   b) is undefined.
   c) is equal to 0.
   d) is equal to infinity.

4. The statement $(6 + 15) \div 3 = (6 \div 3) + (15 \div 3)$

   a) is false
   b) is true because of the commutative property of division.
   c) is true because of the associative property of division.
   d) is true because of the right distributive property of division over addition.

5. The statement $1 - 0 = 0 - 1$

   a) is false.
   b) is true because of the zero property of subtraction.
   c) is true because of the commutative property of subtraction.
   d) is true because of the associative property of subtraction.

6. To see that 864123 is divisible by 3 without actually dividing, one could notice that

   a) the number in the one's position is divisible by 3.
   b) the number names by the last three digits is divisible by 3.
   c) the sum of the digits is divisible by 3.
   d) none of the above.

7. Write 23.32_4 as a base 10 numeral.

8. Consider the operation * on the set of natural numbers where a*b = 5a+b

   a) Find the numeric value of 8*3

   b) Explain why the operation * is (or is not) commutative on the set of whole numbers.

9. State the commutative property of multiplication.

10. Use the number 679 to explain <u>why</u> the divisibility test for 2 works as it does.

11. State a rule that can be used to tell whether a number is divisible by 6 without actually dividing the number by 6.

12. Consider the following statement: a number is divisible by n•m if and only if it is divisible by n and by m. If this statement is true for all natural numbers n and m, then write "true;" otherwise, find a pair of natural numbers (a value for n and a value for m) for which the statement is false.

13. Name the pair of consecutive whole numbers that $\sqrt{87}$ lies between.

14. If 247 is prime, write "prime." If not, name a number that divides 247 (other than 1 and 247).

15. The largest <u>prime</u> number that is less than or equal to 180 is _____.

16. Using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, t, e to write numerals in the base 12 numeration system, what is the base 12 numeral immediately following $5t9_{12}$?

17. Write the base 10 numeral for $e8_{12}$.

18. Write 136_{10} as a base 12 numeral.

19A. Use the set of all multiples of 10 and the set of all multiples of 12 to find the least common multiple of 10 and 12. Show your work (on the answer sheet).

19B. A relation that is reflexive, symmetric, and transitive is called an _____ relation.

20A. If $n \cdot m = 1200$ and the greatest common divisor of n and m is 4 what is the least common multiple of n and m?

20B, 21B, 22B. Consider the given relations on the given sets. Place an R, S, and/or T in the space provided if the relation has the Reflexive, Symmetric, and/or Transitive properties:

relation	set on which reln. is defined
a is related to b if a+b is an even number	Natural numbers _____
A is related to B if A is a proper subset of B	Sets _____
a is related to b if a is perpendicular to b	Lines _____

21A. Find the least common multiple of 24 and 36.

22A. Add 1/24 + 5/36.

23. To see if 157 is a prime number, it would be necessary to try to divide 157 by exactly five numbers. Name those 5 numbers.