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ABSTRACT

Designed for the student who has completed the geometry series in the Quinmester Program, this guidebook on minimum course content provides an investigation of challenging mathematical activities which are not usually developed in other mathematics courses. The content includes brainteasers, puzzles, and game theory. Overall course objectives are specified, a course outline is provided, performance objectives are listed, and references keyed to the performance objectives are provided. A sample posttest is included along with a 94-item annotated bibliography. (JP)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

MATHEMATICS: Geometric Game Strategy 5209.55

DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION • 1973

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

GEOMETRIC GAME STRATEGY

5209.55

(EXPERIMENTAL)

Written by

Edward M. Lasoff

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

An investigation of interesting and challenging mathematical activities which are not usually developed in other mathematics courses, including brainteasers, puzzles, and game theory. Designed for the student who has mastered the skills and concepts of Geometry 2.

TABLE OF CONTENTS

OVERALL GOALS

The student will

1. Grow in his ability to use patterns and symmetries.
2. Gain experience in creativity.
3. Devise and illustrate models through manual activities.
4. Experience many examples of logic and problem solving through verbal and non-verbal activities.

NOTE ON REFERENCES

There are no State Adopted References used in totality although appropriate material can be found in them. The diversity of the material used in this course is such that no present book contains everything. Abbreviations (See Annotated Bibliography) used are:

UM - University of Miami Library

UMCL - University of Miami Curriculum Laboratory (Merrick Building)

LH - Lindsey Hopkins Library (Downtown Miami)

DCPL - Dade County Public Library (any of the branches can order books from the main branch on Biscayne Blvd.)

The catalog numbers given from the University of Miami are Library of Congress call numbers. Listing of the Dewey decimal numbers would vary from library to library. The number of the item in the bibliography followed by the name of the author in parentheses will be used to identify the text in the outline, e.g., 53(Kaplan). The School Mathematics Study Group has voluminous amounts of material, as has been pointed out in the bibliography.

SUGGESTIONS

1. This course is being written as an elective. The instructor will have to tailor this course to fit the needs of his individual pupils.

2. In the course description the term "game theory" should be changed to "models" since the student's expertise at this level would not be adequate to treat game theory in any depth.

3. The course title should be changed to Geometric Illustrations and Puzzles.

4. There will be no state adopted text for the course. This course will be made up from many different books (See annotated bibliography) which can be obtained at the Lindsey Hopkins library, the Dade County Public Libraries, the instructor's high school library, the University of Miami curriculum Laboratory, and the University of Miami Library. In the suggested quinmester outline, some texts are stressed more than others. This is still up to the instructor to finally decide for his class. I also mentioned some books--Hoyle's Book of Rules, Encyclopaedia Britannica Dictionary--which I did not list in the bibliography but which an instructor could obtain. The instructor will have to prepare himself to instruct this course. It really can be fun.

5. I would like to suggest that as a result of this course, students will be better able to see relationships, patterns, and symmetries in this world. Hence, a student would be better prepared for any job which would require clear-thinking for problem-solving tasks (and the list of these jobs is endless). Such practical benefits of the course should be emphasized to the students throughout this course.

6. It will be up to the instructor as to how much basic geometry should be reviewed. (Bakst-Appendix II pp. 727-743) He will also have to determine how much basic geometry should be included on the posttest. The posttest should reflect the student's ability to describe certain games and to illustrate different constructions.

7. While the determination of a course grading system is up to the discretion of the teacher, I suggest incorporating such grading strategies as student experiments, presentations in class, projects, visitations, and papers. For example, I suggest:

    (1) Student oral presentations on mathematicians in the field of puzzles and mathematical recreations.

Suggestions (continued)

Mention should be made of 12(Bezuszka) Heritage Builders in the Arts & Sciences for making biographical sketch assignments.

(2) A project, e.g., each student delves into a particular phase of puzzles that he is interested in. The result would be a paper and/or class presentation. Projects also could be anything in art-collage, mobiles, tessisles, mosaics, pottery design, etc., to mechanical working models. The instructor should strive to get everyone involved in his own individual "thing." Let the artist draw; those useful with instruments design; and those who are mechanically skilled and manually skilled produce-clay, metal, glass, plastic, etc. There are many examples and enough materials for all types of backgrounds.

8. I have not mentioned overheads, periodicals, media resources, or other materials since I felt there were enough books to handle the material. There is a voluminous amount of media materials.

9. For advanced students: 6(Ball); 16(Cadwell); 18(Carroll); 21 (Davis); and 25(Domoryad) are some texts that would be interesting.

10. Many of the cited books will help the instructor in background material. Many suggestions found in Elementary School Teacher Textbooks will be found quite helpful in working with students who have trouble in clearly seeing and understanding the topics discussed in this course. 3(Allendoerffer); 17(Cameron); 19(Collier); 38(Garsten); 43(Hawley); 69(Minnesota Project); 75(Paling); 79(SMSG-Elementary); 81(Scott,Foresman Project); and 88(Mathematics Project: Book T).

11. Many of the newer editions extract puzzles, illustrations, etc. for older books and so several books repeat the same materials. The instructor must pull material together from these different books so as not to be too repetitious. He may not have these materials available to him immediately. I have given a tentative 50-day program so there is some choice for the instructor. Many of the ideas MAY TAKE LONGER than the time I've alloted in which case, CONTINUE THE ACTIVITY. Others will be too short, in which case the instructor should ALWAYS have something planned to fill out the class period.

12. Students may want to bring in their own puzzles.

Suggestions (continued)

13. Students could talk about puzzles (interlocking cardboard pieces) and actually put together a small puzzle. They could discuss why some people find it so difficult to do just that.

14. If the instructor would like to stress definitions, five definitions or so could be given each day with a small lecture followed by a laboratory period. Breaks between lessons should be inserted. These breaks go back to arithmetic puzzles mainly, but are interesting and very diverse. An encyclopedia and dictionary should be available for the student's use.

15. Facilities for duplicating processes will be needed.

16. For more advanced students, a project might be: writing Hilbert's Axioms and giving examples of each of Hilbert's Axioms. They could also compare Finite Projective and Analytic Construction of the Fano Geometry. 75(Stanton).

17. Different problem solving methods could be tried. For example: give a sheet with three or four puzzles per day. Allow students to work in groups solving-giving solutions the next day or so with discussions. Have students come up to the board and work out their solutions or have the group demonstrate their solution together. Ideas are limitless.

# MATERIALS FOR MODEL BUILDING
20(Cundy)

Sheets
- Cartridge paper
- Manila Folders
- Pasteboard, 4-sheet, 6-sheet, and 10-sheet
- Plain postcards
- Plywood
- Masonite
- 'Donnaconna' or plasterboard-thick, light layers for contour work
- Glass-lantern-slide cover-glasses
- Celluloid sheets or Perspex (offcuts can be bought cheaply)
- Sheet metal, cut from old tins and cans

Disks
- Wooden toy wheels
- Bases of broken plastic plates and beakers (easily cut off with a hack-saw and polished up with glass-paper)
- Cork mats
- Coins
- Lathe cores

Lines
- Colored twine
- Plastic (polyvinyl) thread
- Violin strings
- Fish lines
- Shirlastic
- Round elastic
- Plastic-covered wire from radio shops
- Copper wire strapping on crates
- Piano wire
- Steel knitting-needles (soften before machining)
- Wooden skewers
- Paint-brush handles from chain-stores (sometimes tubular plastic, which is very useful)
- Pencils

Strips
- Gumstrip
- Flat steel strapping from crates
- Cardboard strips
- 'Juneero' strips
- 'Meccano' strips
- Flat or I-section curtain rail from chain-stores
- Wooden laths

Materials for Model Building (continued)

Points

- Drawing pins
- Pins with colored heads
- Military marking pins
- Golf tees

Rings

- Steel washers
- Rings from haberdashery
- Curtain rings
- Quoits
- 'Halo' rings
- Wire circles from lampshades

Joints

- Metal rivets
- Nuts and bolts
- Solder
- Glue
- Balsa cement
- Perspex cement (solution of Perspex in ethylene dichloride)
- Florist's wire
- Sealing wax
- Paper fasteners
- Eyelets
- Cut rivets (heat the ends-violin strings)
- Nail-polish remover (acetone, for celluloid)
- Durofix (or solution of celluloid in amyl acetate)
- Glass cement (grind surfaces to be joined)
- Canada balsam

Tools

- Razor blades and steel rule
- Coping-saw
- Fine tenon-saw
- Hack-saw
- Wheelbrace and drills
- Compasses
- Eyelet punch
- Small clamp
- File
- Plane
- Hammer
- Screwdriver
- Shears and Scissors
- Wire-cutters
- Glass-cutters
- Glass-paper and emery cloth
- Household scouring powder
- Metal polish
- Soldering iron and cored solder

These are only suggestions for materials. Use what is at hand. Most models look better if painted. Paint metal strips with quick-drying dopes; cardboard with enamels (high gloss paints); flat diagrams with poster-paints.

Throughout the course, the instructor may actually want to build models. He can get the instructions from Cundy and Rolletts's Mathematical Models which contains examples in Plane Geometry, Polyhedra, Solid Geometry, and Mechanical Models. These models can be interspersed in appropriate places and students may want to try their hand with models other than just paper.

COURSE OUTLINE (50 Day Program)

UNIT I

1. Unicursal Drawings (One-Way Geometry)
   4(Bakst-Math. Puzzles and Pastimes) pp. 97-108 with illustrations

Performance Objectives

The student will

1. Identify geometric terms visually - edge, vertex, equilateral triangle, and regular octahedron.

2. Understand the objective: If the number of the edges which converge at every vertex of a geometric solid figure is even, then it is possible to travel along all the edges of the figure by traversing each edge only once.

3. Recognize the rules for the solution of such problems.

Materials

Paper, compasses, cardboard rulers

SUGGESTED TEACHING STRATEGIES

1. Have students maintain a notebook containing key ideas, concepts illustrations, and materials necessary for models.

2. Ask students if they can think of any such problems or make-up any of their own.

3. Pass out dittoed exercises on Unicursal Drawings. (Samples of exercises on following page.)

4. Give definitions of edge, vertex, equilateral triangle, regular octahedron, and use illustrations.

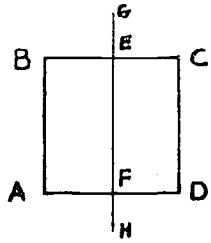
5. Show how this idea of unicursal drawing can be transferred to plane geometric figures such as "vertices"-points, and "edge"-line.

6. Illustrate unicursal drawings and the fact that some illustrations cannot be drawn unicursally. Illustrate the rules.

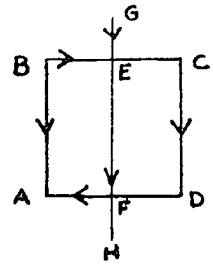
UNIT I
Exercises on Unicursal Drawings

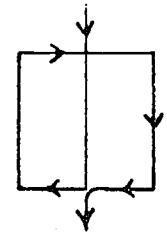
Illustrations:

1)



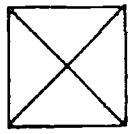
Problem



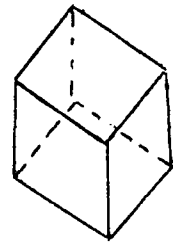


Solution

2)



Problem



Impossible Solution

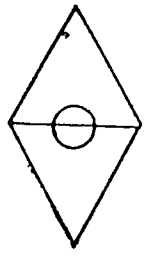
Exercises:

1)

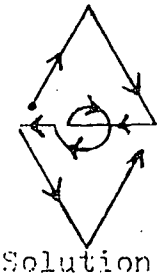
Problem

Solution

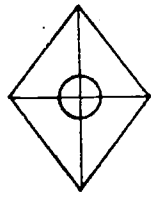
2)



Problem



Solution



Problem

Impossible Solution

2. Toothpicks 4(Bakst)

Performance Objectives

The student will

1. Experience trial and error exercises.

Materials

Toothpicks, paper.

Suggested Teaching Strategies

1. The instructor will give background 4(Bakst)-pp. 1-9.
2. Have student imagine the new figure. Exercises 1 to 36. 4(Bakst)- Solutions - pp. 181-188.
3. Have students solve these exercises, show solutions and discuss.
4. For advanced students - Bakst (chs. 10-13)

Sample Exercises (Bakst)

1. Place 3 toothpicks beside one another as shown in Figure 1. Move them around so they form a double of the figure.
2. With 4 toothpicks which have been placed alongside one another, construct the triple of this figure.
3. With 11 toothpicks obtain 1.
4. With 12 toothpicks construct a figure like the one shown in Figure 2. Now remove 2 toothpicks, leaving 2 squares.
5. In Figure 2 transpose 4 toothpicks to construct 3 squares.
6. Figure 3 contains 17 toothpicks. Remove 5 toothpicks, leaving 3 equal squares.
7. By removing 2 toothpicks from the design in Figure 3, make 6 squares.
8. Remove 4 toothpicks from the design in Figure 4 to make 5 squares.

Figure 1

Figure 2

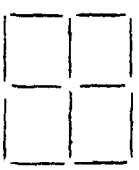
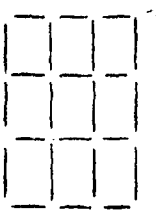


Figure 3

Figure 4



3. Hexaflexagons 33(Gardner-Math Puzz. & Div.) pp. 1-14

Performance Objectives

The student will

1. Construct a hexaflexagon.

2. See how a two-dimensional object can be "flexed" for three-dimensional results.

Materials
Ruler, paper, scissors, and crayons.

Suggested Teaching Strategies

1. Use examples (pp. 10-12) of 33(Gardner).

2. Have students make individual models singly or in groups of two forming unahexa models through heptahexa flexagons.

3. Students should be granted freedom to try and draw other arrangements.

4. Students can color triangles on forms to recognize differences.

4. Hexaflexagons (continued); Tictactoe 33(Gardner)

Performance Objectives

The student will

1. Learn how to play the Game of TICTACTOE.

Materials
Plastic three-dimensional tictactoe set, paper.

Suggested Teaching Strategies

1. Short explanation and history of tictactoe.

2. Cat's game and strategy of winning tictactoe.

3. Two-dimensional games using paper.

4. Three-dimensional games.

5. Problem for students Ex. 7, p. 26 "The sliced cube," answer on p. 33.

UNIT I

5. Tictactoe (continued)

Performance Objectives

The student will

1. Improve his space visualization.

Materials
Paper dittoed with diagrams.

Suggested Teaching Strategies

1. Review of two- and three-dimensional tictactoe.

2. Explanation of four-dimensional tictactoe - p. 42 33(Gardner) has a diagram. Imaginary hypercube set up by sectioning it into two-dimensional squares as diagrammed.

UNIT II

6. Topological Models 33 (Gardner)

Performance Objectives

The student will

1. Construct some basic topological shapes.

Materials
Paper, scissors.

Suggested Teaching Strategies

1. Construction of some of the 16 types of surfaces used in topology (Gardner - pp. 63-72) - shown on the blackboard.

2. Make paper models of each item on Table (p. 67) 33(Gardner). See pp. 68-69.

3. 8(Barr) will be useful for background and experiments. 30(Fuchs) offers explanation and colored illustrations.

7. Topological Models (continued)

Performance Objectives

The student will

1. Recognize a Moebius strip, a Klein bottle, and a torus.

Materials
Paper, scissors.

Suggested Teaching Strategies

1. Discussion about the use of these models.

2. Continuation of making models in Table, p. 67, 33(Gardner)

3. Elementary discussion about the Klein bottle; the torus; and the Moebius strip, including the results of its various cuttings.

8. Hex

Performance Objectives

The student will

1. Participate in the game of hex trying to master some of its strategies.

Materials
Checkerboards fixed for the game of hex, dittoed boards drawn on paper with small pieces of cardboard being used for men or other appropriate items.

Suggested Teaching Strategies

1. Lecture on background of Hex from 33(Gardner) p. 77.

2. Set up trial game and show moves on board.

3. Let students experiment in playing game.

4. Problems (p. 78) and solutions (p. 82), 33(Gardner).

9. Hex (continued)

10. Polyominoes 33(Gardner) Ch. 13 pp. 124-140

Performance Objectives

The student will

1. Participate in the construction of some polyominoes.

Materials
Paper, scissors.

Suggested Teaching Strategies

1. Lecture on background of Polyominoes, 39(Golomb).
2. Let students practice with their polyominoes.
3. Problem 33(Gardner) pp. 125, 127, Fig. 67.

UNIT III

11. Nim, Tac Tix 33(Gardner) Ch. 15, pp. 151-161

Performance Objectives

The student will

1. Participate in the playing of Nim and Tac Tix and learn about some of their basic strategies.

Materials
Dittoed paper for boards or other appropriate material.

Suggested Teaching Strategies

1. Use 33(Gardner) for background lecture.
2. Let students practice playing both games.

12. Nim, Tac Tix (continued)

13. Breaks 44(Heafford)

Performance Objectives

The student will

1. Participate in solving quizzes and puzzles involving Breaks.

Materials
Dittoed quizzes and puzzles.

Suggested Teaching Strategies

1. Pass out quizzes 1-6 to the students. A timer may be used and competition within the class may be initiated.
2. An almanac, encyclopaedia, and dictionary might be necessary.

14. Breaks (continued) Quizzes 7-11. 44(Heafford)

15. Breaks (continued) Quizzes 12-16. 44(Heafford)

UNIT IV

16. Optical Illusions 86(Steinhaus) Ch. 6, pp. 139-166

Performance Objectives

The student will

1. Observe what is meant by an optical illusion.

2. Construct straight line, circles, and symmetry examples of optical illusions.

3. Learn how to use the inversor (6 rods).

Materials
Inversor, compass, ruler, paper.

Suggested Teaching Strategies

1. The use of 86(Steinhaus) will help the instructor.

2. Other texts can be used which illustrate optical illusions. which the students can try to draw. 30(Fuchs); 5(Bakst, Chs. 28-29); and 72(Northrop, pp. 47-48, 64-76).

17. Optical Illusions (continued)

18. Breaks (continued) Quizzes 17-21. 44(Heafford)

19. Breaks (continued) Quizzes 22-26. 44(Heafford)

20. Tangram 20(Cundy and Rollett)

Performance Objectives

The student will

1. Construct his own tangram.

2. Observe different patterns which can be formed with the tangram.

Materials
Scissors, ruler, paper

Suggested Teaching Strategies

1. Discuss background of Tangram- (Chinese puzzle game considered thousands of years old and consisting of a square of thin material divided into 7 pieces, etc.).

2. Expand into a discussion about design and geometrical structures in architecture (modern skyscrapers and other buildings).

3. Have students see what designs they themselves can make up from the tangram.

UNIT V

21. Tessellations — 20(Cundy); 86(Steinhaus) Ch. 4 p. 75; 58(Kraitchik) Ch. 8 pp. 193-213

Performance Objectives

The student will

1. Observe pictures of mosaics and tessellations.

2. Try and construct paper tessellations.

Materials
Paper, scissors, paste, paints or crayons.

Suggested Teaching Strategies

1. Present background on Tessellations to students.

2. Models (elementary) drawn on board with steps on construction.

3. Provide books containing pictures of mosaics and tessellations so students may get ideas for their own tessellations.

4. Designs can be made on the tessellations themselves or if colored paper is available then strips of colored paper may be interwoven.
2(Alkema) would be an excellent book showing the student how designs are used in business enterprises today. Paper collage and montage could be tried by the students along with simple monofolds.

22. Tessellations (continued)

23. Geometric Shapes — 1(Adler) background; 60(Land, Ch. 11); 73(Ogilvy, Chs. 5-6); 78(Row); 81(Scott, Foresman); 92(Yates); 46(Horne)

Performance Objectives

The student will

1. Be able to name the basic geometric shapes.
2. Give examples for each shape from the real world.
3. Be able to draw or construct these shapes.
4. Be able to define these shapes in words.

Materials
Paper, dittoed sheets, tinker-toys, clay, etc.

Suggested Teaching Strategies

1. Present background lecture on the geometrical forms and examples from history. There are many texts that can be used.
2. 78(Row) and 46(Horne) have good suggestions for this lesson.
3. Definitions can be written on the board and discussed.
4. Time for student constructions of the basic shapes should be set aside. Use of clay models and photography can be used here. 60(Land).

24. Geometric Shapes (continued)

25. Breaks (continued) 44(Heafford) Quizzes 27-31

UNIT VI

26. Design in Paper Folding (Origami) 78(Row)

Performance Objectives

The student will

1. Practice folding paper into various designs.
2. Be able to make a paper model of a basic geometric design.

Materials
Paper, scissors, ruler, protractors.

Suggested Teaching Strategies

1. Use 78(Row) for background and 20(Cundy) for designs on folding and making geometric solids.

2. Explain use of the protractor and show how to measure angles for paper constructions.

27. Geo-board Activities

Performance Objectives

The student will

1. Show that he can set up the basic geometric designs on the geo-board and demonstrate them to his classmates or instructor.

Materials
Geo-board(pegboard), rubber bands, and peg(matches, etc.).

Suggested Teaching Strategies

1. Start with elementary shapes and increase in difficulty.

28. Breaks (continued) 44(Heafford) Quizzes 32-37

29. Four-Color Map Problem, Losing Area Experiment, Cutting Surfaces and Solids, Traveling Network Exercises, and Euler's Rules.

Performance Objectives 6(Ball) Ch. 8 pp. 22-241
49(Johnson and Glenn)
The student will pp. 210-224; 271-272

1. Be able to work the losing area experiment.

2. Show by drawing what happens when various surfaces are cut by planes.

Materials
Dittoed exercises, plastic geometric shapes.

Suggested Teaching Strategies

1. Use 49(Johnson and Glenn) for background material.

2. Students may practice drawing geographical maps to show only four colors are needed to differentiate the various states or countries.

3. Students may go to the board and try to visualize planes cutting through solid objects and guess resulting figures.

4. Work examples with Euler's Rules- remind students of beginning work in Unicursal drawing.

30. Breaks (continued) 44(Heafford) Quizzes 38-42
22(Degrazia); 64(Madachy)

Alphametics

UNIT VII

31. Brainteasers 26(Dudeney)

Performance Objectives

The student will

1. Participate in working out puzzles and discussing their solutions.

Materials
Paper.

Suggested Teaching Strategies

1. Present problems. Allow students to work out solutions by boardwork, in teams, or in any other convenient manner.

2. Discuss solutions and alternate ways of solving these problems.

3. Suggested exercises: 26(Dudeney) pp. 27,28,35,46,112,113.

32. Brainteasers (continued)

Suggested Teaching Strategies

1. Suggested exercises, 26(Dudeney) pp. 49,56,58-67,114-116.

33. Checkers Hoyle's Book of Rule for Checkers or any checker test

Performance Objectives

The student will

1. Demonstrate the basic rule of checkers.

Materials
Checkerboards and men.

Suggested Teaching Strategies

1. Utilize pp. 68-75, 117-119 26(Dudeney)

2. Present basic rules of checkers- show moves on board.

34. Checkers (continued)

Performance Objectives

The student will

1. Be able to play a game of checkers.

Materials
Ditto'd paper, checkerboards.

Suggested Teaching Strategies

1. Utilize pp. 127-137, 142-148 26(Dudeney)

2. Call off a game of checkers with the students playing the game. Illustrate one of the championship games played (found in Encyclopaedia Britannica or other test) by placing the numbers on the board. Strategy can be discussed as the moves are made or at a later time. Students may follow this illustration, if there is time, with their own game.

35. Checkers (continued)

Suggested Teaching Strategies

1. Allow students to play the entire period either with their own game or with games from different texts on checkers.

2. An introduction to timed moves using a second hand watch might be enjoyable for those students who demonstrate proficiency at the game. A small tournament might be held. As a starter, 15-second moves might be appropriate.

UNIT VIII

36. Graphing

Performance Objectives

The student will

1. Demonstrate how to graph in two-dimensional space.

2. Be able to define abscissa, ordinate, graph, mapping, coordinate, relation, function, vector, ordered pairs, ordered triples, and ordered n-tuples.

3. Demonstrate how to graph a three-dimensional space vector.

Materials
Paper, ruler, graph paper.

Suggested Teaching Strategies

1. Have students come to the board to show knowledge of abscissa, ordinate, coordinates, and ordered pairs by locating points.

2. Have students make up their own problems in two-dimensional space.

3. Discuss two, three, and n-dimensional space drawings.

4. Bring in descriptive and projective geometry at this time.

5. Ballistics, projectiles, rockets, and Apollo moon shots can be discussed in terms of trajectories which will involve parabolic curves and other geometric shapes.

6. Appropriate texts for this lesson are: 10(Beberman-Unit 4); 11(Bezuszka-Unit 3, Part 2, Lesson 6); 79(SMSG-see Bibliography); 51(Johnson); 55(Keedy).

37. Positional Games: Naval Battle 58(Kraitchik) Ch. 12

Performance Objectives

The student will

1. Demonstrate that he knows how to play the game of Naval Battle.

Materials
Paper.

Suggested Teaching Strategies

1. Strategy will be discussed by the instructor and rules (p. 283) will be explained.

2. Students will pick partners and try to learn this game.

38. Positional Games: Dominoes, Cards, Chinese Checkers

58(Kraitchik) Ch. 12 pp. 267-323

Performance Objectives

The student will

1. Demonstrate that he can play Chinese checkers and the game of dominoes.

Materials
Chinese checkerboards and marbles, sets of dominoes, deck of cards.

Suggested Teaching Strategies

1. Explain rules for dominoes and Chinese checkers. 58(Kraitchik) pp. 298,302,318; Hoyle's Book of Rules

2. Explain rules on how to guess two cards (ch. 12)

39. Positional Games: Magic Squares 58(Kraitchik) Ch. 7 pp. 142-192

Performance Objectives

The student will

1. Show that he can make a magic square.

Materials
Paper, plastic magic square puzzles (if possible), rulers.

Suggested Teaching Strategies

1. Teacher should explain the origin of magic squares and discuss the different types of magic squares.

2. Student should be introduced to Latin Squares and perhaps Greco-Roman Squares.

3. Instructor should give examples of methods of construction used in Lattices.

40. Breaks (continued) Quizzes 43-49 44(Heafford) 22(Degrazia) pp. 39-40 Solutions p. 121

Cryptograms

Suggested Teaching Strategies

1. Exercises can be dittoed for students.

Examples:

SEND	32 solutions possible	Ex. 6054
MORE		1720
GOLD		1934
MONEY		17508

AB+BA+B=AAB 19+91+9=119

ab	x	ac	=	acb		10	x 16	= 160
+		+		-		+	+	-
ad	x	e=		aaf		14	8	= 112

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          xxxxx                    84063
lxxxx)lxxxxxlxxl          15927)1338871401
      lxxxlx                     127416
        xxxll                     64711
        xxxxx                     63708
         lxxxxx                   100340
         xxxxx                     95562
           xxxxl                   47781
           xxxxl                   47781
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UNIT IX

41. Various Geometries — 85(Stanton) pp. 164-179; 29(Fawcett) p. 201

Performance Objectives

The student will

1. Realize that geometry is not restricted to Euclidean Geometry.

2. Recognize the names of the non-Euclidean geometries to date.

Materials
None

Suggested Teaching Strategies

1. Instructor demonstrates at the board the differences among the various geometries as exemplified by some of the rules.

2. Projective, finite, non-Euclidean, Absolute or Pan-geometry, Lobachevskian, Riemannian, and Topology should be compared. Hilbert's postulate should be mentioned.

42. Various Geometries (continued)

43. Review Session

Instructor should outline what has been covered during this nine week period. Student question and answer period should follow. Instructor should provide each student with study sheet containing the names of all games and definitions presented during the course.

44. Oral Reports

Students present their oral reports of mathematicians prominent in the field of Puzzles and Recreational areas. Student mechanical models or art work (collages, paper mache, busts, etc.) should also be presented at this time.

45. Posttest

UNIT X
OPTIONAL FIVE DAY UNIT

46. Braid Theory — 84(Spitznagel)

Take-off on group structures (Piet Hein). Game called Tangloid.

47. Shunting Problems — 58(Kraitchik) Ch. 9 pp. 214-226

Permutational Problems

Difficult Crossings

48. Knots — 86(Steinhaus) Ch. 12 p. 252; 76(Ransom)

49. Tower of Hanoi — 33(Gardner) Ch. 6 pp. 55-62.

50. Geometrical Arithmetic — 29(Fawcett)

Addition, Subtraction, Multiplication, Division, Involution, Evolution (square roots), Algebraic constructions, and Triangular numbers are topics to be shown through use of geometry with ruler and compass constructions.

POSTTEST

Sample I.

The student will:

1. Choose a game learned during the quinmester and write its rules and procedures of play. The instructor should be able to read the student's description and know that the student shows understanding and some mastery of the game.

2. Choose any one geometric solid and construct it with compass and ruler.

3. State Euler's equation.

4. State three examples of solids and give the number of edges and vertices for each solid. (Examples - tetrahedron, dodecahedron, icosahedron.)

5. Write his own idea about puzzles in general, perhaps illustrating with examples.

6. Draw a picture of a tangram.

7. Draw a picture of a tessellation.

8. Describe an optical illusion and illustrate with an example.

9. Give instructions for a paper folding example.

10. Pick three topological models and give a description of each, describing the results in one of the cases if that object is cut. (Hint: Use Moebius for the cutting unless you are sure of the other models.)

Sample II. (A majority of these questions will be based upon Euclidean Geometry.)

1. Choose the letter of the following if it is not a quadrilateral.

   a) Parallelogram b) Rectangle c) Square

   d) Trapezoid e) Tesseract

2. True or False (Place a T or F in the proper space.)

   1) ____ All squares are rectangles, but all rectangles are not squares.

   2) ____ All rectangles are parallelograms, but all parallelograms are not rectangles.

   3) ____ All parallelograms are quadrilaterals, but all quadrilaterals are not parallelograms.

3. Matching (Choices 1,2,3,4 may be used more than once.)

____ 1. Parallelogram	a. All opposite sides are parallel
____ 2. Rectangle	b. All sides are equal in length
____ 3. Square	c. All angles are right angles
____ 4. Trapezoid	d. All opposite sides are equal in length
____ 5. Tesseract	e. One pair of opposite sides are parallel
	f. No sides are parallel
	g. A fourth-dimensional object

4. Define line.

5. Define angle.

6. Define geometry.

7. Explain the meaning of congruent triangles.

8. List some of the differences between Euclidean and non-Euclidean geometries pointing out by illustrations main points of interest.

Posttest - Sample II (continued)

9. Name four non-Euclidean geometries.

10. Give an example from the first, second, third, and fourth dimensions in terms of geometry and illustrate and label.

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UMCL - University of Miami Curriculum Laboratory-School of Education

LH - Lindsey Hopkins Library

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