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ABSTRACT

The primary aim of this guide is to aid teachers in planning and preparing a senior high school mathematics course for students preparing for college work. It is divided into separate one-semester courses of seven chapters each. The first-semester course consists of a traditional approach to the introduction of trigonometry and trigonometric functions. The second-semester course represents a new approach, treating algebra, trigonometry, analytic geometry, and calculus in a unified manner rather than as four separate sections. Fundamental notions of the subject are unified into a sequence of topics beginning with the consideration of the real number system and the algebraic operations. Emphasis is placed on the importance of being able to visualize and graphically represent mathematical expressions. Ideas of algebra and geometry are presented in the study of linear, quadratic, and general polynomial functions. Permutations, combinations, and probability are treated as additional topics. For both courses, behavioral objectives are stated for each chapter and a set of abbreviated daily lesson plans is presented. (JP)

DE SOTO PARISH CURRICULUM GUIDE

Trigonometry

and

Advanced Math

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Issued by

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INTRODUCTION

This guide was prepared under the assumption that all students enrolled in this trigonometry course will continue their education. Therefore, the primary aim of this course is to prepare students for college work.

In preparing this teachers' guide, consideration was given to an outline of suggested topics to be covered in high school trigonometry which was compiled by the Louisiana Mathematics Advisory Committee. This outline was prepared for a two semester course in trigonometry, whereas this guide is prepared for a one semester course.

The teacher should not restrict the class activities to only those included in this guide, but should consider appropriate material for the individual class. Many of the problems suggested for homework and for tests may be too difficult for some students in the class, whereas supplementary exercises may have to be provided for the gifted students. The teacher should use his own initiative in assigning such problems.

GOALS FOR TRIGONOMETRY

1. To develop an understanding of the terminology and symbolisms used in trigonometry.
2. To develop concepts and skills essential for the study of higher mathematics courses.
3. To develop an interest in the history and growth of trigonometry.
4. To develop an understanding of how trigonometric concepts may be applied to solve common day problems.
5. To develop the ability to communicate accurately and effectively.
6. To develop an understanding of the relation of trigonometry to previous mathematics courses.
7. To develop an interest for the students to further their studies in higher mathematics courses.
8. To develop the skills and techniques necessary for problem solving.
9. To develop the ability to work neatly and to follow directions.

TIME BUDGET CHART

IV. Polar Coordinates (12 days)

A. Changing from one system of coordinates to the other

B. Coordinates and vectors

C. Distance between two points

D. Area of a triangle

E. Radian measure

F. Length of an arc

G. Linear and angular velocity

V. Complex Numbers (9 days)

A. Representing complex numbers by points in plane

B. Complex numbers and vectors

C. Adding vectors algebraically

D. Polar form of a complex number

VI. Fundamental Relations (14 days)

A. Identities

B. Reciprocal relations

C. Quotient relations

D. Pythagorean relations

E. Functions of angles in terms of another function of the angle

F. Simplifying trigonometric expressions

G. Proving identities

H. Trigonometric equations

VII. Functions of Two Angles (10 days)

A. Law of cosines

B. Law of sines

C. Cosine of the difference of two angles

D. Cosine of the sum of two angles

SEMESTER I

TRIGONOMETRY

Chapter I

Vectors

Behavioral Objectives:

1. The student will recognize vector quantities and equivalent vectors if he is given the description of a force, move, or velocity with direction.
2. The student will construct an equivalent vector, the opposite vector, and the vector representing a real number multiplied by the vector if he is given a vector.
3. Given three different vectors, the student will add and subtract the given vectors.
4. Given two triangles composed of vectors with two vectors of one equal to two vectors of the other, the student will prove the remaining vectors are equal.
5. Given examples involving three vectors, the student will demonstrate his understanding of the properties of operations by identifying the property used in each example.

Note: The student should demonstrate the ability to successfully perform four of the above behaviors.

Lesson Plan #1

Vector Quantities

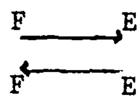
Aim: To teach the students to recognize equivalent vectors, to name vectors, and to recognize vector quantities.

Suggested Method: Questions and answers with lecture from pages 7-9, directed study.

Supplementary Materials: Straight-edge, notebook, and pencil.

Developmental Steps and Questions to be Asked:

1. What is meant by "vector"? D. A. Pictorial symbols that are used to express quantities of magnitude and direction. Discuss how a vector is represented.
2. Example: E $\longrightarrow$ F and A $\longrightarrow$ B are equivalent. Discuss the meaning of equivalent.
3. What are some quantities that may be represented by vectors? D. A. Forces, velocity with direction, etc.
4. Example: $\overrightarrow{FE}$ is opposite $\overrightarrow{EF}$

   Discuss the meaning of opposite.
5. What would you call a vector which represents a zero force? D. A. Null vector or zero vector.
6. Directed study.

Summary: Review the new terms introduced. Review the necessary conditions to have a vector quantity.

Suggested Problems:

All problems on pages 9-10.

Lesson Plan #2

Adding and Subtracting Vectors

Properties of Operations with Vectors

Aim: To teach the multiplication of a vector by a real number, to add and subtract vectors, and to present the properties of operations with vectors.

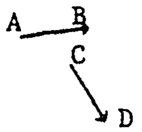
Suggested Method: Check and answer questions on homework, lecture, question and answer discussion, demonstration, directed study.

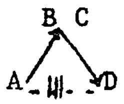
Supplementary Materials: Ruler, notebook, pencil.

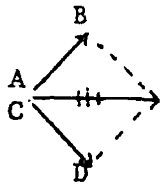
Developmental Steps and Questions:

1. Lecture on the material on pages 10-17 of the text.
2. What is the scalar in $-1/2\ \overrightarrow{AD}$? D. A. $-1/2$
3. What is the identity element in the addition of vectors?

   D. A. Null vector or zero vector.
4. Example: $3\ \overrightarrow{AB}$ = 

. Discuss.
5. Given: 

$\overrightarrow{AB} + \overrightarrow{CD} =$ 



   Discuss vector addition by both methods, triangular and parallelogram.
6. $3(\overrightarrow{AB} + \overrightarrow{CD}) = 3\ \overrightarrow{AB} + 3\ \overrightarrow{CD}$. Discuss the distributive properties of multiplication of a vector by a scalar.
7. How is subtraction defined in algebra? D. A. Addition of inverse. Explain how this pertains to vectors.
8. Directed study.

Summary: Review the terms introduced in today's lesson. Give the procedures for adding and subtracting vectors. Name the properties that hold for vectors, given on page 14. Assign all problems on pages 17-18.

Lesson Plan #3

Review on Vectors

Aim: To review vectors, including naming vectors, recognizing vector quantities, recognizing equivalent vectors, adding and subtracting vectors, and the properties of operations with vectors.

Suggested Method: Check and answer questions on homework, question and answer discussion, directed study.

Supplementary Materials: Ruler, notebook, pencil, overhead projector and materials for projector.

Developmental Steps and Questions:

Use some of the same questions as were used in lessons 1 and 2. Discussion on pages 7-18.

1. What two methods of adding vectors are used? D. A. Triangular and parallelogram. Review, using an example. Use a transparency.
2. Given: $\overrightarrow{AC} = \overrightarrow{EG}$

   $\overrightarrow{AB} = \overrightarrow{EF}$

   Prove: $\overrightarrow{BC} = \overrightarrow{FG}$

   Develop proof and discuss it with the help of the class.
3. Directed study.

Summary: None.

Suggested Problems:

Problems 1-5 on pages 18-19.

Note: Test for next class meeting on Chapter 1.

Lesson Plan #4

Vectors

Aim: To administer test for chapter I.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials Needed: Overhead projector, copies of test.

Suggested Problems:

Two or three problems such as #5, page 19.

One problem such as #2, page 18.

Two or three problems such as #6, page 18.

One problem such as #3, page 17.

One problem such as #1, page 9.

One problem such as #5, page 9.

Chapter II

Trigonometric Functions of Angles

Behavioral Objectives:

1. The student will demonstrate his knowledge of relations, functions, domains and ranges by writing down the domains and ranges for the functions if he is given a set of relations. Some of these relations will be functions.
2. The student will find the values of the six trigonometric functions if he is given a figure with a specific point in a plane. The six will be functions of the angle formed by the radius vector in standard position.
3. The student will demonstrate his knowledge of reciprocal functions by answering true or false statements about such.
4. The student will find the values of trigonometric functions by using the table of natural functions.
5. The student will find the angle if he is given the value of any trigonometric function. He will use the table of natural functions to do this.
6. The student will write down the reference angle for any positive or negative angle.
7. The student will find the missing sides and angles of a right triangle if he is given a right triangle with two sides given or one side and one acute angle given.

Note: The student will demonstrate the ability to successfully perform 5 of the above behaviors.

Lesson Plan #5

Angles (Plane)

Aim: To review the parts of an angle; to review the rectangular coordinate system; and to teach the meaning of standard position and coterminal angles.

Suggested Method: Demonstration, question and answer discussion, lecture, directed study.

Supplementary Materials: Graph paper, graph board, overhead projector, ruler.

Developmental Steps and Questions to be Asked:

1. Hand back the test and answer questions about it.
2. 

Name the initial and terminal sides.
3. Develop the rectangular coordinate system with help of the class.
4. How do we name the four quadrants? D. A. Starting from the upper right corner, name them I, II, III, IV, in a counterclockwise direction.
5. If $\phi = 40^\circ$ and $\theta = 400^\circ$, are they coterminal? D. A. Yes. Stress the meaning of coterminal
6. Name the ordinate and abscissa of the point (6, -5). D. A. -5, 6.
7. Directed study.

Summary: Review the terms from pages 20-23. Review the method of determining if two angles are coterminal. Demonstrate the formation of a positive and negative angle.

Suggested Problems:

Problems 1-6 on page 23.

Lesson Plan #6

Trigonometric Ratios, Relations, Functions, Domains and Ranges

Aim: To introduce the trigonometric functions and review relations, functions, domains, ranges, and Cartesian products.

Suggested Method: Check and answer questions on homework, question and answer discussion, directed study.

Supplementary Materials: Board compass, protractor, and graph paper, graph board.

Developmental Steps and Questions:

Lecture and discussion on pages 24-29.

1. A = $\{(2,3), (3,1), (4,5)\}$ Domain = (2,3,4)

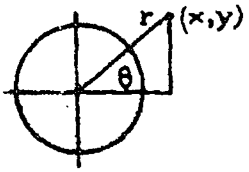
   Range = (3,1,5). Review the meaning of domain and range.

2. B = $\{(3,4), (5,0), (3,1), (4,6)\}$ Is B a function?

   D. A. No. Discuss this.

3. The Cartesian product for set A x A = $\{(1,1), (1,2), (2,1), (2,2)\}$

   A = $\{1,2\}$

4. 

   $\sin\theta = y/r$ $\quad$ $\text{ctn}\,\theta = x/y$

   $\cos\theta = x/r$ $\quad$ $\sec\theta = r/x$

   $\tan\theta = y/x$ $\quad$ $\csc\theta = r/y$

5. Directed study.

Summary: Review the meaning of the new terms. Show an example of Cartesian product. Discuss the number of elements in the Cartesian product for a given set.

Suggested Problems:

Problems 1-6 on pages 29.

Lesson Plan #7

Finding the Values of Functions

Aim: To teach the method of finding the values for the six trigonometric functions.

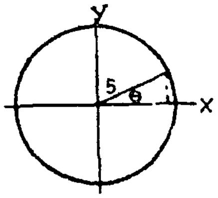
Suggested Method: Check and answer questions on homework, discussion, demonstration, directed study.

Supplementary Materials: Board compass, protractor and straightedge , prepared transparencies of the diagrams below.

Developmental Steps and Questions:

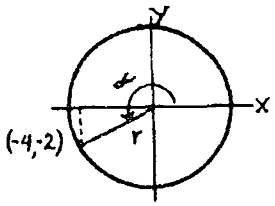
Discussion on pages 29-31.

1.



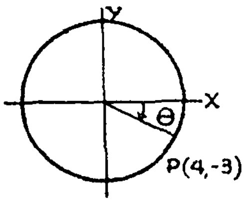
Using this as an example show how you would find the values of the six trigonometric functions. A prepared transparency of each of the diagrams would prove useful.

2.



Use this example to do the same as #1

3.



Use this example to let students solve for the values of the six trigonometric functions.

4. Directed study.

Summary: Review the definitions of the six functions. Review the signs for the six functions in each of the four quadrants.

Suggested Problems:

Problems 1-9 on pages 32.

Lesson Plan #8

Finding the Values of Functions

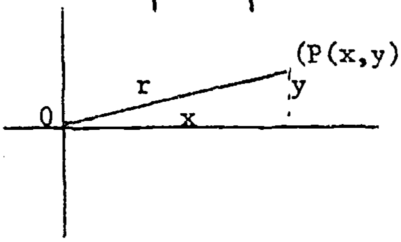
Aim: To review the method for finding the values of the six functions and to teach a method for proofs for related identities.

Suggested Method: Check and answer questions on homework, demonstration, question and answer discussion, directed study.

Supplementary Materials: Board compass, straightedge.

Developmental Steps and Questions: Discussion on pages 31-33.

1. Show that $|\sin \theta| \leq 1$ in the following example.



By making the appropriate substitutions develop an explanation (proof) for this problem. D. A. In text on page 32.

2. Explain the meaning for $\sin^2 \theta$. D. A. $\sin^2 \theta = (\sin \theta)^2$

3. Have students attempt a proof for $\sin^2 \theta = 1 - \cos^2 \theta$. Use the diagram above.

D. A. $(y/r)^2 = 1 - (x/r)^2$

$$y^2/r^2 = 1 - x^2/r^2$$

$$y^2/r^2 + x^2/r^2 = 1 - x^2/r^2 + x^2/r^2$$

$$\frac{y^2 + x^2}{r^2} = 1$$

$$r^2/r^2 = 1$$

$$1 = 1$$

4. If $\sec \theta = \csc \theta$, then θ is either a ______ or a ______ quadrant angle.

D. A. I or III.

5. Directed study.

Summary: Review the squares of the functions and the steps in proving some of these identities for this section.

Suggested Problems:

Problems 10-17 on page 33.

Note: Test next class meeting from pages 21-33.

Lesson Plan #9

Aim: To administer test on pages 21-33.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials Needed: Copies of test.

Suggested Problems:

One problem such as #1 on page 23.

One problem such as #2 on page 23.

One problem such as #3 and #4 on page 29.

Two problems such as #1-9 on page 32.

One problem such as #12 on page 33.

One problem such as #16 on page 33.

Lesson Plan #10

Reciprocal Functions

Aims: To teach the meaning of reciprocal functions and how they might be used in working problems.

Suggested Method: Demonstration, question and answer discussion, directed study.

Supplementary Materials: Test papers, prepared transparency used in plan #8.

Developmental Steps and Questions:

1. Hand back test papers and answer questions about missed problems.
2. Review the meaning of reciprocals from arithmetic and algebra.

   Example: (1/2, 5/2, x/y, x/6) Discussion on pages 33-34.
3. Show by substitution of X, Y, and R that the following pairs are reciprocal functions: (Use the prepared transparency from lesson 8.)

   sin θ - scs θ

   cos θ - sec θ

   tan θ - ctn θ
4. Have students consider the following questions:

   a. Is "tan θ ctn θ = 1" true for all values of θ, some values for θ, no values of θ? D. A. All.

   b. sin θ = 1/4 and cos θ = $4\sqrt{3}$. Is this statement true or false.

      D. A. False.
5. Directed study.

Summary: Review the pairs of reciprocal functions and the method for determining the answers to the true and false statements in the book on page 34.

Suggested Problems:

Problems 1-4 on page 34.

Lesson Plan #11

Functions of Quadrantal Angles

Aim: To teach the definitions of quadrantal angles and to show how we arrive at the values of the functions of quadrantal angles.

Suggested Method: Check and answer questions on homework, demonstration and directed study.

Supplementary Materials: Board compass, protractor, and straightedge.

Developmental Steps and Questions:

1. Draw examples of quadrantal angles on the board.

   (0°, 90°, 180°, 270°)

2. Have the students determine the values of the sin and cos functions for the above examples.

3. Discuss the meaning of the symbol "∞". Have the students give examples of "∞". (0/0, x/0, y/0).

4. What are some other examples of quadrantal angles?

   D. A. 360, 540, 450, any multiple of 90°.

5. Directed study.

Summary: Review the method for finding the values of the functions of quadrantal angles. Draw a table like the one on page 35 to show the values of the six functions.

Suggested Problems:

Problems 1, 2 on page 35.

Lesson Plan #12

<u>Values of the Functions of 30°, 45°, and 60° Angles</u>

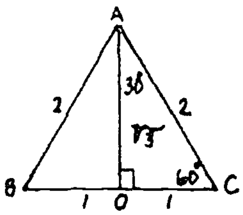
<u>Aim</u>: To teach the student a method for recalling the values of the functions of 30°, 45°, and 60° angles.

<u>Suggested Method</u>: Check and answer questions on homework, lecture, question and answer discussion, directed study.

<u>Supplementary Materials</u>: Straight edge.

<u>Developmental Steps and Questions</u>: Lecture on pages 36-39.

1. Draw an equilateral △ with the altitude and show the values of the functions of 30° and 60° angles.

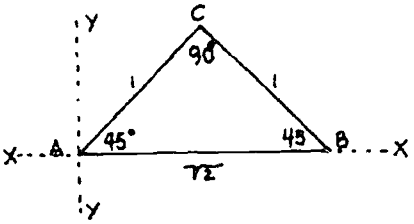


$AD = \sqrt{2^2 - 1^2}$

$AD = \sqrt{3}$

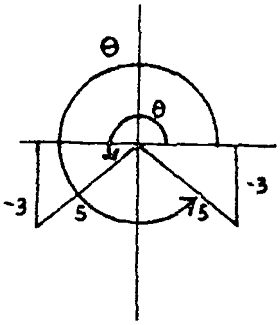
Put these angles in standard position.

2. Draw an isosceles △ and show the values of the functions of 45° angles.



Put the triangle on the set of axes such that the 45° angles is in standard position.

3. Find the other function values if sin θ = - 3/5.



$x^2 + y^2 = r^2$

$x^2 = 5^2 - (-3)^2$

$x^2 = 16$

$x = \pm 4$

Quadrant III, Quadrant IV

cos θ = -4/5 = 4/5

tan θ = 3/4 = -3/4

ctn θ = 4/3 = -4/3

sec θ = -5/4 = 5/4

csc θ = -5/3 = 5/3

4. Directed study.

<u>Summary</u>: Review the values of the functions for 30°, 45°, and 60° angles.

Review the signs of the functions in each quadrant and method for determining other function values when one is given.

Suggested Problems:

Problems 1-9 on pages 39-40.

Lesson Plan #13

Table of Natural Functions

Aim: To teach the student how to use the table of natural functions.

Suggested Method: Check and answer questions on homework, lecture and discussion, directed study, demonstration.

Supplementary Materials: none

Developmental Steps and Questions: Discussion with lecture on pages 40-41.

1. Discuss increasing and decreasing functions while having the students examine the tables on page 69 at the end of book.
2. Work several examples using the table in the back of the book. (sin 38° 20', tan 43°, 10', cos 78°, etc.)
3. Have students find the values of several functions of angles using the table.
4. sin θ = .5995. Show how the table can be used to find the angle if the value is given such as in this example. Have students work several examples like this.
5. Directed study.

Summary: Review why the table of natural functions was compiled and how we use it.

Suggested Problems:

Problems 1-25 on page 42.

Note: Test for next class meeting on pages 33-42.

Lesson Plan #14

Aim: To administer test on pages 33-42.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Material: Copies of test.

Suggested Problems:

One problem such as #1 on page 39.

One problem such as #3 on page 39.

One problem such as #7 on page 40.

Four or five problems such as problems 1-16 on page 42.

Two or three problems such as problems 17-25 on page 42.

Lesson Plan #15

Interpolation

Aim: To teach the student how to interpolate using the table of natural functions.

Suggested Method: Demonstration, directed study.

Supplementary Materials: Test papers.

Developmental Steps and Questions:

1. Hand back test papers and answer questions about missed problems.
2. Work an example such as #1 on page 42. Discuss the methods involved as you progress. Work a different example such as #3 on page 43.
3. Have students work one or two examples such as the above two.
4. Discuss the method of rounding off. This may vary from book to book.
5. Directed study.

Summary: Review the purpose for interpolating as well as the procedure and format.

Suggested Problems:

Problems 1-12 on page 43.

Lesson Plan #16

Reference Angles and How They are Used

Aim: To teach the student how to find the reference angle for a given angle. To teach the student how to use the reference angle to find the values of functions.

Suggested Method: Check and answer questions on homework, discussion, demonstration and directed study.

Supplementary Materials: Board compass and protractor.

Developmental Steps and Questions: Discuss on pages 44-47.

1. Discuss the meaning of reference angles. Show several examples. (70°, -30°, 600°, 135°, etc.)
2. Find sin 225°. Work this example by finding the reference angle and by using the table of natural functions.
3. Find the tan $(-25^\circ 20')$. Have the students find this value.
4. Directed study.

Summary: Review the meaning of reference angle and how to find the reference angle for a given angle. Review the method of using the reference angle to find the values of the functions.

Suggested Problems:

Problems 1-16 on page 44.

Problems 1-15 on page 48.

Lesson Plan #17

Rule for Finding the Values of Functions of Angles

Aim: To teach the student how to express a function of a negative angle in terms of the same function of a positive angle having the same magnitude. To teach the rule for finding the values of functions of angles.

Suggested Method: Check and answer questions on homework, discussion, demonstration, and directed study.

Supplementary Materials: Board compass and protractor, prepared transparency of the diagram, page 48.

Developmental Steps and Questions:

1. With the help of the class show how each of the following hold true from the figure on page 48: (A prepared transparency of the figure may be used.)

   $\sin(-\theta) = -\sin\theta$ $\qquad$ $\sec(-\theta) = \sec\theta$

   $\cos(-\theta) = \cos\theta$ $\qquad$ $\csc(-\theta) = -\csc\theta$

   $\tan(-\theta) = -\tan\theta$ $\qquad$ $\text{ctn}(-\theta) = -\text{ctn}\,\theta$

2. Using specific examples show how the above relations are used. Ex: $\sin(-40^\circ)$, $\tan(-20)$, $\cos(-175^\circ)$.
3. Have students read the rule on page 49. Discuss this rule with the class and use examples to show how it is used. Ex: $\cos 115^\circ$, $\csc 300^\circ$, $\text{ctn}\ 290^\circ 35'$.
4. Directed study.

Summary: Review the rule for finding the values of the functions of angles. Review all six functions of negative angles and how they may be written as the same function of a positive angle with the same magnitude.

Suggested Problems:

Problems 1-15 on page 49.

Problems 1-4 on page 50.

Lesson Plan #18

Functions of the Acute Angles of a Right Triangle

Aim: To teach the method of solving any right triangle for the functions of the acute angles.

Suggested Method: Check and answer questions on homework, discussion, demonstration, and directed study.

Supplementary Materials: Straightedge, compass, transparency of figure, page 50.

Developmental Steps and Questions:

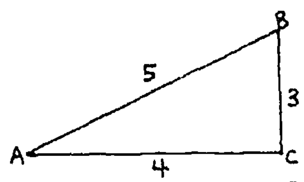
Discussion on pages 50-51.

1. Using the transparency of the figure on page 50, develop the proof for the following:

$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} \qquad \csc A = \frac{\text{hypotenuse}}{\text{opposite leg}}$$

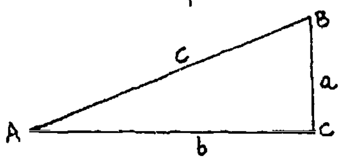
$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} \qquad \sec A = \frac{\text{hypotenuse}}{\text{adjacent leg}}$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} \qquad \text{ctn } A = \frac{\text{adjacent leg}}{\text{opposite leg}}$$

2. 

Use this example to find:

sin A, cos A, tan B, sec B, csc A, ctn A.

3. 

Use this example to prove:

$\sin A = \cos B$ or $\sin (90-A) = \cos A$

4. Directed study.

Summary: Review the definitions of the six functions in terms of the adjacent leg, opposite leg, and hypotenuse of a right triangle. Show how these may be used to find the values of the functions of the acute angles.

Suggested Problems: Problems 1-9 on pages 51-52.

Note: Test for next class meeting on pages 42-52.

Lesson Plan #19

Aim: To administer a test on pages 42-52.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test.

Suggested Problems:

Two problems such as #1-12 on page 43.

Five problems such as #1-16 on page 44.

One problem such as #1 on page 48.

One problem such as #4 on page 48.

One problem such as #11 on page 48.

Two problems such as #1 and #14 on page 49.

One problem such as #1 (i) on page 50.

One problem such as #3 (a) on page 50.

One problem such as #1 on page 51.

One problem such as #7 on page 51.

One problem such as #8 on page 52.

Lesson Plan #20

Inverse Use of Table of Natural Functions

Aim: To teach the inverse use of the table of natural functions.

Suggested Method: Discussion, demonstration, and directed study.

Supplementary Materials: Test papers.

Developmental Steps and Questions:

1. Hand back tests and discuss problems missed or any questions asked.
2. Discussion on pages 52-53.
3. Using $\sin \theta = .3338$ as an example, have the students find θ using the table on page 69.
4. $\tan A = .0825$

   Use this example to show the format for interpolation.
5. $\csc \theta = 1.112$. Let students use the format as in the above example to find θ. Discuss and answer questions that might arise.
6. Directed study.

Summary: Review the method for the inverse use of the table of natural functions.

Suggested Problems:

Problems 1-12 on page 53.

Lesson Plan #21

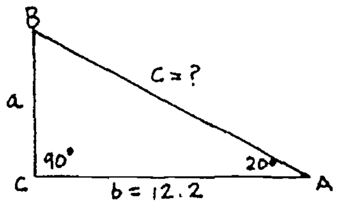
Finding the Sides and Angles of Right Triangles

Aim: To instruct the student in the proper procedure for finding sides and angles of right triangles.

Suggested Method: Check and answer questions on homework, discussion, demonstration and directed study.

Supplementary Materials: Board compass and straightedge , transparency of the example below.

Developmental Steps and Questions:

1. Discussion on page 53.
2. 

Use this example to demonstrate the method for finding C. Use steps 1-5 on page 53. (Transparency)
3. Given: c = 18 in. and a = 14 in. find B. Let students solve this example using the five steps involved.
4. Show an alternate way of solving the above example and point out how the best method is selected.
5. Directed study.

Summary: Review the parts of a right triangle and the necessary parts which must be given to solve for sides or angles. Review the method of selecting the best equation for solving the right triangle.

Suggested Problems:

Odd numbered problems 1-11 on page 54.

Lesson Plan #22

Review of Chapter 2

Aim: To review problems in chapter 2 with emphasis on the more difficult sections.

Suggested Method: Check and answer questions about homework, demonstration and directed study.

Supplementary Materials: Board compass, protractor, and straightedge.

Developmental Steps and Questions:

1. Solve and have students solve various problems from chapter 2 that students ask about.
2. Point out how some of the sections in chapter 2 are connected.
3. Directed study. (Use a large part of the period for this.)

Summary: Review the new terms that are found in the chapter and stress the importance of mastering this chapter.

Suggested Problems:

Problems 1-21 on pages 54-55.

Note: Test on chapter 2 for next class meeting.

Lesson Plan #23

Trigonometric Functions of Angles

Aim: To administer a test on chapter 2.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test.

Suggested Problems:

One problem such as #1 on page 54.

One problem such as #3 on page 54.

One problem such as #3 on page 55.

Two problems such as #4 (a) and #4 (f) on page 40.

Two problems such as #13 on page 55.

One problem such as #18 on page 55.

One problem such as #21 on page 55.

One problem such as #19 on page 55.

One problem such as #21 on page 42.

Chapter 3

Line Values and Graphs of Trigonometric Functions

Behavioral Objectives:

1. The student will draw the line segments representing the six trigonometric functions if he is given a unit circle on a set of axes with a given acute angle in standard position.
2. The student will construct the graphs for each of the six functions ($-360^{\circ} < \theta < 360^{\circ}$) if he is given the help of the table for natural functions.
3. The student will demonstrate his knowledge of the ranges for the six functions by writing such ranges if he is given the functions.
4. The student will demonstrate his knowledge of the graphs for the six functions by answering completion questions about such.

Note: The student should demonstrate the ability to successfully perform 3 of the above behaviors.

Lesson Plan #24

Line Values of the Functions of Acute Angles

Aim: To teach the student how to represent by line segments the functions of an acute angle.

Suggested Method: Demonstration, discussion and directed study.

Supplementary Materials: Board compass, straightedge , overhead projector with transparencies, test papers.

Developmental Steps and Questions:

1. Hand back test papers and discuss any questions.
2. Discuss page 57.
3. Draw an acute angle θ in standard position and construct the line segments which represent the trigonometric functions. Use overhead projector here.
4. Directed study. (Have students attempt the above step for themselves.)

Summary: Review the meanings of the new terms for this section. Discuss why the unit circle is used. Briefly review a method for remembering which line segment represents which function.

Suggested Problems:

Problems 1-3 on page 58.

Lesson Plan #25

Line Values of Functions in the Four Quadrants

Aim: To teach the student to recognize the line segments which represent the trigonometric functions for angles in each of the four quadrants.

Suggested Method: Check and answer questions on homework, discussion, demonstration, and directed study.

Supplementary Materials: Board compass, straightedge.

Developmental Steps and Questions:

1. Discuss pages 60 and 61.
2. Draw figures #2, 3, 4 page 60 on board. Have students help select the line segments which represent each of the functions. Give the sign for each of these.
3. Directed study.

Summary: Review the signs for each of the six functions in each quadrant. Review the range for each function.

Suggested Problems:

Problems 1-12 on page 61.

Note: Six weeks test announced for time after next.

Lesson Plan #26

Variations of the Six Functions

Aim: To teach the students a method for determining the range of each function.

Suggested Method: Check and answer questions on homework. Demonstration and directed study.

Supplementary Materials: Straightedge.

Developmental Steps and Questions:

1. Discussion on pages 62-64.
2. Take each function separately and let students help determine the range of values as the angle increases from 0° to 360°. Develop this by drawing angles which increase in size within each quadrant. Use the definitions of the six functions.
3. Have the students study the table on page 64, and compare it with the results from step 2 above.
4. Discuss the use and meaning "∞".
5. Spend about 15 minutes reviewing for six weeks test.
6. Directed study.

Summary: Review the range for each function and how it was determined.

Suggested Problems:

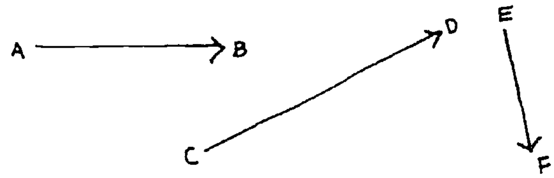
Problems 1-11 on page 65.

Note: Test for six weeks for next class meeting.

Lesson Plan #27

Trig (1st Six Weeks Test)

I. Use these vectors to show:



a. $\overrightarrow{AB} + \overrightarrow{EF}$

b. $-\overrightarrow{CO}$

c. $\overrightarrow{AB} + \overrightarrow{EF} + \overrightarrow{CO}$

d. $3\ \overrightarrow{AB}$

II. A vector that has zero magnitude and no direction is called ________.

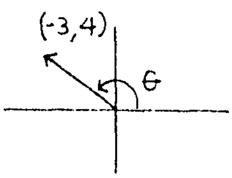
III. Name the quadrant in which each of these angles would terminate.

a. 80° b. 287° c. -115° d. 415° e. -500°

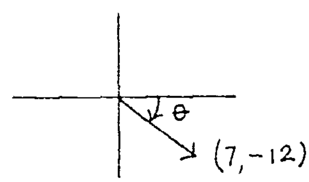
IV. If $U = \{2, 3, 4\}$, write out the elements of UXU.

V. Find the values of the six trigonometric functions in each of these:

a.



b.



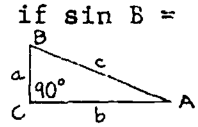
VI. What is the reciprocal of the sin function?, cos?, tan?

VII. If $\sin \theta = -3/5$ and $\cos \theta$ is negative, find the other 4 trigonometric functions.

VIII. Use table X to find:

a. $\tan 45^\circ 10'$ b. $\sin 36^\circ 14'$ 3. B if $\sin B = .0545$

IX. Given $A = 41^\circ$ and $c = 18$ in., find a



X. For an acute positive angle in the first quadrant, construct line segments equal to $\sin \theta$, $\cos \theta$, $\tan \theta$.

Lesson Plan #28

Graph of Sin θ

Aim: To teach the process for drawing the graph of $y = \sin \theta$.

Suggested Method: Check and answer questions on homework for page 65, demonstration, directed study.

Supplementary Materials: Graph board, straightedge , test papers.

Developmental Steps and Questions:

1. Hand back test papers and discuss questions about missed problems.
2. Discuss the graphs of functions pages 66-67.
3. Draw the graph of $y = \sin \theta$ on the graph board with the help of students. Discuss the selection of units for the axes. (Draw the graph for $-360^\circ < \theta < 360^\circ$)
4. Directed study.

Summary: Review the method for drawing the graph of sin θ. Review the range for sin θ and how it is used in drawing the graph.

Suggested Problems:

Have students draw the graph $y = \sin \theta$. $(-360 < \theta < 360)$

Problems 1-8 on page 68.

Lesson Plan #30

Graph of Cos θ

Aim: To teach the process for drawing the graph of $y = \cos\theta$.

Suggested Method: Check and answer questions on homework, demonstration and directed study.

Supplementary Materials: Graph board, straightedge.

Developmental Steps and Questions:

1. Draw the graph for $y = \cos\theta$ on the graph board with the help of students. $-360 < \theta < 360$
2. Ask students about the range for the cos function in terms of upper and lower limits of y.
3. Directed study.

Summary: Review the method for drawing $y = \cos\theta$. $-360° < \theta < 360°$.

Suggested Problems:

Have students draw the graph $y = \cos\theta$. $-360° < \theta < 360°$

Problems 1-7 on page 69.

Lesson Plan #31

Graphs of tan θ and ctn θ

Aim: To teach the student the method for drawing the graphs of $y = \tan\theta$ and $y = \text{ctn}\,\theta$.

Suggested Method: Check and answer questions on homework, demonstration, discussion, and directed study.

Supplementary Materials: Graph board, straightedge.

Developmental Steps and Questions:

1. Draw the graph for $y = \tan\theta$ $(-360° < \theta < 360°)$ on a graph board with the help of students. Discuss the very large values for tan θ as θ approaches 90°, 270°, -90°, and -270°. Show how the symbol "∞" may be used.
2. Have students draw the graph for $y = \text{ctn}\,\theta$. Discuss their graphs when they have finished.
3. Discuss the periods and ranges for the tan and ctn graphs. Also discuss how they are discontinuous curves.
4. Directed study.

Summary: Review the method for drawing the graphs for $y = \tan\theta$, and $y = \text{ctn}\,\theta$. Briefly discuss their periods and ranges. Point out that the tan θ increases in all quadrants and the ctn θ decreases in all quadrants as θ increases.

Suggested Problems:

Problems 1-9 on page 70.

Lesson Plan #32

Graphs of sec θ and csc θ

Aim: To teach the students the method for graphing $y = \sec \theta$ and $y = \csc \theta$.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Graph board, board compass, straightedge , overhead projector and transparencies.

Developmental Steps and Questions:

1. Have one of the better students work problem 8 on page 70 on the graph board. Some time will be required to answer questions about this problem.
2. Use overhead projector and transparencies to show the graphs for $y = \sec \theta$, and $y = \csc \theta$. The students should help develop these graphs.
3. Ask students about the ranges and periods for each function:

   D. A. period - 360°

   range - $+1$ to $+\infty$

   -1 to $-\infty$
4. Directed study if time allows.

Summary: Briefly review each graph including the periods and ranges for each function.

Suggested Problems:

Problems 1-3 on page 71.

Lesson Plan #33

Review of Chapter 3

Aim: To review any sections in chapter 3 which are giving students trouble.

Suggested Method: Check and answer questions on homework, demonstration and directed study.

Supplementary Materials: Graph board, board compass and protractor, straightedge.

Developmental Steps and Questions:

1. Question and answer discussion for each section in chapter 3 which students ask about. This will include working example problems as needed.
2. Allow most of the period for directed study on chapter review exercises on page 72.

Summary: Review the method for drawing line values for each of the trigonometric functions. Briefly discuss the graphs for the six functions. Review the new terms found in the chapter.

Suggested Problems:

Problems 1-15 on page 72.

Note: Test for next class meeting on chapter 3.

Lesson Plan #34

Test on Chapter 3

Aim: To administer test on chapter 3.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test.

Suggested Problems:

One problem such as #9 on page 72.

One problem such as #4 on page 61.

One problem such as #12 on page 61.

Problem 3 on page 65.

One problem such as #7 (a) on page 65.

One problem such as #9 on page 65.

One problem such as #7 on page 69.

One problem such as #7 on page 70.

One problem such as #8 on page 72.

Lesson Plan #35

Cumulative Review for Chapters 1-3

Aim: To review chapters 1-3 and show how they are related.

Suggested Method: Demonstration, lecture, directed study.

Supplementary Materials: Board compass, protractor and straightedge, test papers.

Developmental Steps:

1. Hand back tests and discuss problems missed. Use this time for re-teaching needed material.
2. Lecture on how the first three chapters are related. Include a summarizing statement for each chapter.
3. Directed study period for problems on pages 73-75.

Summary: none.

Suggested Problems:

Problems 1-37 on pages 73-75.

Chapter 4

Polar Coordinates

Behavioral Objectives:

1. The student will change rectangular coordinates of a point in a plane to polar coordinates.
2. The student will change polar coordinates of a point in a plane to rectangular coordinates.
3. Given the polar form of a vector, the student will sketch the vector on a set of coordinate axes.
4. The student will find the distance between two points in a plane if he is given the polar coordinates of these points. One of these points will be on the positive end of the x-axis.
5. The student will find the area of a triangle if he is given the polar coordinates of the three vertices. One of the vertices must be the origin and another will be on the positive end of the x-axis.
6. The student will change the measure of an angle to degrees, minutes, and seconds if he is given an angle in radian measure. He will also change the measure of an angle to radian measure if he is given the measure of an angle in degrees, minutes, and seconds.
7. The student will find the length of an arc of a circle if he is given the radius and the central angle subtending the arc.
8. The student will find the linear and angular velocity of a point on a circle if he is given the radius of the circle and the central angle subtending the arc through which the point moves.

Note: The student should demonstrate the ability to successfully perform 6 of the above behaviors.

Lesson Plan #36

Polar Coordinates

Aim: To introduce polar coordinates and to teach the student how to change from one system of coordinates to the other.

Suggested Method: Check and answer questions on homework, demonstration, lecture, and directed study.

Supplementary Materials: Board protractor and straightedge, transparency of the figure at the top of page 78, text.

Developmental Steps:

1. Introduce chapter 4 with a discussion of the polar axis and how a point may be represented by polar coordinates.
2. Use the definitions for the sin, tan, cos functions to develop the first 3 formulas on page 78. Use the Pythagorean theorem to develop #4 with the help of students and the figure at top of page 78. (Use a transparency of the figure.)

   $\sin\theta = y/r$ $\cos\theta = x/r$ $\tan\theta = y/x$

   $y = r\sin\theta$ $r = x\cos\theta$ $r^2 = x^2 + y^2$

3. Ex: $(4, 30^{\circ})$ Plot the point in the example and change to rectangular form.

   $x = r\cos\theta$ $y = r\sin\theta$

   $x = 4\cos 30^{\circ}$ $y = 4\sin 30^{\circ}$

   $x = 4\left(\frac{\sqrt{3}}{2}\right)$ $y = 4(1/2)$

   $x = 2\sqrt{3}$ $y = 2$

4. Ex: $(+1, 1)$ Change this point to polar coordinates. Use a diagram for this.

   $r^2 = x^2 + y^2$ $\tan\theta = 1/+1$

$r^2 = 1 + 1$ $\qquad\qquad$ $\tan \theta = +1$

$r = 2$ $\qquad\qquad$ $\theta = 45^\circ$

$$(\sqrt{2}, 45^\circ)$$

5. Directed study.

Summary: Review the meaning of new terms and review the methods for changing from one pair of coordinates to the other.

Suggested Problems: Problems 1-4 on pages 78-79.

Lesson Plan #37

Coordinates and Vectors

Aim: To teach the method for sketching the vectors represented by polar coordinates and finding the rectangular components for a given vector.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Board protractor and straightedge.

Developmental Steps:

1. Discuss how vectors may be represented by polar coordinates. Introduce the use of $r/\underline{\theta}$ to represent a vector which is centered and has the terminal point at (r, θ).
2. Ex: $2/\underline{30^\circ}$. Show how to find the rectangular components for this vector. Students should help find this.

$x = r \cos \theta$	$y = r \sin \theta$	
$x = 2 \cos 30^\circ$	$y = 2 \sin 30^\circ$	$(\sqrt{3}, 1)$
$x = 2 \left(\frac{\sqrt{3}}{2}\right)$	$y = 2\ (1/2)$	
$x = \sqrt{3}.$	$y = 1.$	

3. Directed study.

Summary: Review the method for representing vectors by using the polar coordinates of its terminal point. Review the formulas for finding its rectangular components.

Suggested Problems:

Problems 1-- on pages 80-81.

Lesson Plan #38

Distance Between Two Points

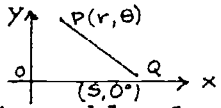
Aim: (a) To review the method for finding the distance between two points if the rectangular coordinates are given; (b) to teach the method of finding the distance between two points if two polar coordinates are given.

Suggested Method: Check and answer questions on homework, demonstration, questions and answer discussion, directed study.

Supplementary Materials: Straightedge.

Developmental Steps:

1. With help of the students develop the distance formula for two rectangular coordinates. (Pages 81-82)
2. Using (r, θ) and $(s, 0^{\circ})$ for the polar coordinates of two points p and q, have students help develop the formula for distance between them. (pages 82, 83)



3. $P = (4, 60^{\circ})$, $Q = (6, 0^{\circ})$. Solve this problem for distance PQ. Use for an example problem.

$$(PQ)^2 = r^2 + s^2 - 2rs\cos\theta$$
$$= 16 + 36 - 2(4)(6)(1/2)$$
$$= 28$$
$$\therefore |PQ| = \sqrt{28};\ |PQ| = 2\sqrt{7}.$$

4. Directed study.

Summary: Briefly review the formula for the distance between two points if given the polar coordinates.

Suggested Problems:

Problems 1-8 on page 84.

Lesson Plan #39

Area of a Triangle

Aim: To introduce the method for determining the area of a triangle (specially placed) if the polar coordinates are given.

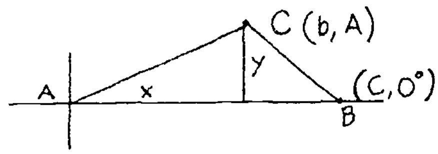
Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Straightedge, board protractor.

Developmental Steps:

1. With help of students, develop the formula for finding the area of a triangle

   K = 1/2 bc sin A



2. Using $(0,0^{\circ})$, $(8,30^{\circ})$, $(10,0^{\circ})$ as the vertices of a triangle, find the area of this triangle. Use this for an example problem. Have students help solve this problem.

   K = 1/2 bc sin A

   = 1/2(8) (10)(1/2)

   = 20

3. Directed study.

Summary: Review the method for finding the area of a triangle if polar coordinates are given. Point out the fact that it must be a specially placed triangle.

Suggested Problems:

Problems 10-16 on page 84.

Note: Test for next class meeting on pages 77-84.

Lesson Plan #40

Test on Pages 77-84

Aim: To administer a test on pages 77-84.

Suggested Method: Check and answer questions on homework, administer test.

Suggested Problems:

One problem such as #3 on page 79.

One problem such as #4 on page 79.

One problem such as #2 on page 80.

One problem such as #4 on page 81.

One problem such as #6 on page 81.

One problem such as #7 on page 81.

Two problems such as #6 on page 84.

Two problems such as #11 on page 84.

Lesson Plan #41

Radian Measure

Aim: To teach the process of changing from degrees to radians and from radians to degrees.

Suggested Method: Demonstration, directed study, lecture, question and answer discussion.

Supplementary Materials: Board compass, protractor, and straightedge.

Developmental Steps:

1. Hand back test papers and discuss problems.
2. Define a radian by using a circle with radius r on board.
3. Use circle with radius r to develop the relations:

   π radians = 180°

   1 radian = $\frac{180^\circ}{\pi}$ = $57^\circ 17'45''$

   $1^\circ = \frac{\pi}{180}$ radians

4. Discuss example problems 1-5 on pages 85-86. Introduce table IV, page 49, making sure each student can read it.
5. Have students solve the following problem: (Change $8^\circ 6'20'$ to radians). Call for answers and discuss questions that might arise.
6. Directed study.

Summary: Review new terms, discuss the process for changing from radians to degrees and vice versa.

Suggested Problems:

Problems 1-33 (odd numbers), on page 86.

Lesson Plan #42

Radian Measure

Aim: (a) To continue the study of radian measure (b) to simplify expressions involving a function of radians.

Suggested Method: Check and answer questions on homework, demonstration, question and answer discussion, directed study.

Supplementary Materials: Board compass and straightedge.

Developmental Steps:

1. Using the example sin $(\pi - \theta)$, simplify this to sin θ. Discuss how and why we make this change. (θ is acute) Use figure on board.
2. Have students attempt to simplify sin $(\frac{\pi}{2} + \theta)$. Call for answers and discuss the change to cos θ.
3. Directed study.

Summary: Review the method for simplifying expressions involving a function of radian measure.

Suggested Problems:

Even numbered problems 2-32 on page 86.

Problems 33-41 on pages 86, 87.

Lesson Plan #43

Length of an Arc

Aim: To teach the method for finding the length of an arc of a circle if the central angle and radius are known.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Board compass, protractor, and straightedge.

Developmental Steps:

1. Using a circle with radius r and central angle, θ, given in radians, develop the formula for finding the length of the subtended arc s.
   $s = r\,\theta$
2. With the help of students solve the formula for θ to find: $\theta = s/r$.
3. Find s when $r = 12$ and $\theta = 1/2\,\pi$. Using this for an example, have the students help solve this problem.
4. Directed study.

Summary: Briefly review the method for finding the length of an arc if the central angle and radius are given. Show how the central angle may be found if the arc and radius are given.

Suggested Problems:

Problems 2-14 on pages 87-88.

Lesson Plan #44

Linear and Angular Velocity

Aim: To introduce the terms linear and angular velocity and teach a method for finding each.

Suggested Method: Check and answer questions on homework, discussion, directed study.

Supplementary Materials: Board compass and straightedge.

Developmental Steps:

1. Review the formula $s = vt$, where s is distance, v is constant velocity and t is time.
2. Use the above formula to find $v = s/t$ which is the linear velocity of a body.
3. Using a circle with central angle θ and subtended arc s, introduce angular velocity in terms of θ being generated in time t. θ/t is angular velocity.
4. Develop the formula $v = r\omega$, where v is linear velocity, r is radius of circle and ω is angular velocity.
5. Use the formulas developed above to solve problem #1, page 89. Use this as an example.
6. Directed study.

Summary: Review the meanings of angular and linear velocity and how to find each.

Suggested Problems: Problems 2-8 on pages 89-90.

Note: Test next class meeting on pages 84-90.

Lesson Plan #45

Test on Pages 84-90

Aim: To administer test on pages 84-90.

Suggested Method: Check and answer questions on homework, administer a test.

Supplementary Materials: Compass, straightedge , copies of test.

Suggested Problems:

Three problems such as #4 on page 86.

Three problems such as #15 on page 86.

Two problems such as #25 on page 86.

Two problems such as #30 on page 86.

Three problems such as #34 on page 86.

Two problems such as #2 and #5 on page 87.

One problem such as #9 on page 87.

Two problems such as #1 on page 89.

One problem such as #7 on page 90. (top of page)

Lesson Plan #46

Chapter 4 Review

Aim: To review material in chapter four with emphasis on the more difficult sections.

Suggested Method: Discussion, demonstration, and directed study.

Supplementary Materials: Board compass, protractor, straightedge.

Developmental Steps:

1. Hand back test papers and discuss problems.
2. Review the method for changing from rectangular coordinates to polar coordinates and visa versa.
3. With help of the students, review the meaning of radians and how to change from radians to degrees.
4. Ask questions about linear and angular velocity. Have students work 2 or 3 problems such as #2, 6, and 8 on page 87.
5. Directed study.

Summary: None.

Suggested Problems:

Problems 1-20 on pages 90-91.

Note: Test on chapter 4 for next class meeting.

Lesson Plan #47

Test on Chapter 4

Aim: To administer test on chapter 4.

Suggested Method: Check and answer questions on homework, administer a test.

Supplementary Materials: Compass, straightedge, copies of test.

Suggested Problems:

One problem such as #3 on page 79.

One problem such as #4 on page 79.

One problem such as #4 on page 81.

One problem such as #6 on page 81.

One problem such as #4 on page 84.

One problem such as #10 on page 84.

Two problems such as #18 on page 86.

One problem such as #27 on page 86.

One problem such as #34 on page 86.

Two problems such as #3 on page 87.

One problem such as #11 on page 88.

One problem such as #2 on page 89.

Chapter V

Complex Numbers

Behavioral Objectives:

1. The student will demonstrate his knowledge of complex numbers by simplifying expressions involving the addition, subtraction, multiplication and division of two complex numbers.
2. The student will find the square root of a complex number if he is given a number of the form a + bi.
3. The students will find the imaginary roots of a quadratic equation with imaginary roots, by using the quadratic formula.
4. He will write the complex number representing a vector if he is given the vector in polar form, direction angle form, or the coordinates of the terminal point.
5. The student will add and subtract two or more vectors algebraically.
6. He will write a complex number in polar form if he is given the number in the form a + bi.

Note: The student should demonstrate the ability to perform five of the above six behaviors.

Lesson Plan #48

Complex Numbers

Aim: To introduce complex numbers and teach the student how to simplify expressions containing *i*.

Suggested Method: Lecture, question and answer discussion, demonstration, directed study.

Supplementary Materials: Overhead projector, screen and materials for the overhead projector, test papers.

Developmental Steps:

1. Hand back test papers and discuss problems.
2. Define a "complex number" after discussing why we have a need for them. Also define imaginary numbers and pure imaginary numbers. Ask students for examples of each. (Use overhead to write definitions.)
3. Example: When will $a + bi = 3 + 2i$?

   D. A. $a + bi = 3 + 2i$ if and only if $a = 3$ and $b = 2$.
4. With the help of the students, show the method for adding, multiplying and dividing imaginary numbers. (Use overhead projector)
5. Directed study.

Summary: Review the definitions of complex numbers, imaginary numbers, real numbers, pure imaginary numbers. Briefly explain the four fundamental operations for complex numbers.

Suggested Problems:

Odd numbered problems 1-23 on page 95.

Lesson Plan #49

Complex Numbers

Aim: To teach the student to find the square root of an imaginary number and to solve quadratic equations that have imaginary roots.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Overhead projector with its materials.

Developmental Steps:

1. Use $\sqrt{-16 + 30i}$ for an example to discuss the method for finding the square root of an imaginary number. Use the same method as given on page 94 of text.
2. Have students apply the quadratic formula to solve $4x^2 + 9 = 0$. Discuss the solution to this problem by using the overhead projector.
3. Directed study.

Summary: Review the method for finding square roots of imaginary numbers.

Suggested Problems:

Even numbered problems 2-24 and problems 25-32 on page 95.

Lesson Plan #50

Trig (2nd Six Weeks Test)

I. What is the range of the six functions?

II. True or false:

a. The period of the tan θ is 180°.

b. The period of the sin θ is 180°.

c. The sin curve is symmetric with respect to y - axis.

d. $\cos 38^\circ < \cos 35^\circ$.

e. $\sin 30^\circ = 1/2\sqrt{2}$

III. Draw the graph of $y = \cos\theta \quad -360^\circ \leq \theta \leq 360^\circ$

IV. Change to polar coordinates:

a. (3,0) b. (0, -2) c. (-1, 1) d. (-2, -4)

V. Change to rectangular coordinates.

a. $(3, 30^\circ)$ b. $(4, 0^\circ)$ c. $(5\sqrt{2}, 45^\circ)$

VI. Find the length and direction of vectors having these components.

a. $x = \sqrt{3}$, $y = -2$ b. $x = 0$, $y = 3$ c. $x = -1$, $y = \sqrt{7}$

VII. Find the distance between these pts.

a. $(6, 30^\circ)$, $(8, 0^\circ)$ b. $(6, 330^\circ)$, $(4, 0^\circ)$

VIII. Change to degrees, min., sec.

a. $\frac{2\pi}{3}$ b. $\frac{4\pi}{3}$ c. 1.7 rad.

IX. If θ is acute, pos. angle, simplify $\sin\left(\frac{3\pi}{2} - \theta\right)$

X. A bicycle has a 28 inch wheel. How many revolutions will the wheel make in one mile?

XI. Find the square root of -2i.

Lesson Plan #51

Complex Numbers Represented by Points in the Plane

Aim: To teach the representation of complex numbers by points in the plane.

Suggested Method: Check and answer questions on homework, discussion, directed study.

Supplementary Materials: graph board, straightedge.

Developmental Steps:

1. Introduce the complex plane by discussing the axis of reals and axis of imaginaries.
2. Define an "Argand diagram". D. A. The resulting figure when complex numbers are plotted.
3. Have students help plot several complex numbers such as (a) $3 + 4i$ (b) $-1 -2i$ (c) $3i$.
4. Discuss the meaning of pure imaginary numbers. Give examples. ($0 + 2i$, $0 - 5i$).
5. Graph the solution for $x^2 + 64 = 0$.
6. Directed study.

Summary: Review the meanings of new terms used in this section. Review the method for plotting complex numbers.

Suggested Problems:

Problems 1-4 on pages 96-97.

Lesson Plan #52

Complex Numbers and Vectors

Aim: To teach the method of representing vectors by complex numbers and how to sketch these on the complex plane.

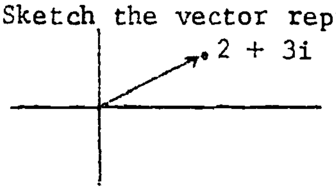
Suggested Method: Check and answer questions on homework, discussion, demonstration, and directed study.

Supplementary Materials: graph board, straight , board protractor.

Developmental Steps:

1. Point out the one-to-one correspondence between complex numbers and points in the plane. Point out how the length and direction of the vector may be found from the complex number corresponding to the vector.

2. Sketch the vector represented by 2 + 3i.



Have students help do this for (a) -2-2i (b) 0 + 3i (c) 5 - 0i.

3. With the help of students write the complex number corresponding to $\sqrt{2}\angle 60^\circ$.

$y = r \sin \theta$ $\qquad$ $x = r \cos \theta^\circ$

$y = \sqrt{2} \sin 60^\circ$ $\qquad$ $x = \sqrt{2} \cos 60^\circ$

$y = \sqrt{2} \left(\frac{\sqrt{3}}{2}\right)$ $\qquad$ $x = \sqrt{2}\,(1/2)$

$y = \frac{\sqrt{6}}{2}$ $\qquad$ $x = \frac{\sqrt{2}}{2}$

complex number $= \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2} i$

4. Directed study.

Summary: Review the method for representing vectors by complex numbers.

Suggested Problems:

Problems 1-3 on page 98.

Lesson Plan #53

Adding and Subtracting Vectors Algebraically

Aim: To teach the method for adding and subtracting vectors algebraically.

Suggested Method: Check and answer questions on homework, demonstration, discussion, directed study.

Supplementary Material: Straightedge, graph board, overhead projector with its materials.

Developmental Steps:

1. Have students study pages 98, 99, 100. (Allow ample time)
2. Using the figures on pages 98, 99, 100, develop the methods for adding and subtracting vectors. Use overhead projector.
3. Have students solve the following problems. Use overhead projector.
   a. $\overrightarrow{AB} = 8 \angle 60^\circ$

      $\overrightarrow{CD} = 10 \angle 0^\circ$

      Find the sum algebraically.

   b. Find the difference algebraically.
4. Help students start problem 9 on page 101. Use overhead projector.
5. Directed study.

Summary: Review the method for adding and subtracting vectors algebraically.

Suggested Problems:

Problems 1-7, 9, 10 on pages 100-101.

Note: Test for next class meeting on pages 93-101.

Lesson Plan #54

Aim: To administer a test on pages 93-101.

Suggested Method: Check and answer questions on homework. (Allow more time than usual.) Administer test.

Supplementary Materials: Straightedge, copies of test.

Suggested Problems:

One problem such as #1 on page 95.

Two problems such as #4 on page 95.

One problem such as #14 on page 95.

One problem such as #16 on page 95.

One problem such as #24 on page 95.

One problem such as #29 on page 95.

One problem such as #3 on page 97.

One problem such as #1 on page 98.

One problem such as #3 on page 98.

One problem such as #3 on page 100.

One problem such as #5 on page 101.

This will be an unusually long test. Some of the above may be deleted depending on the ability of your class.

Lesson Plan #55

Polar Form of a Complex Number

Aim: To teach the method of writing a complex number in polar form.

Suggested Method: Lecture, demonstration, directed study.

Supplementary Materials: Board protractor and straightedge, test papers.

Developmental Steps:

1. Hand back test papers and discuss problems.
2. Using the formulas $x = r \cos \theta$ and $y = r \sin \theta$, substitute into $x + yi$ to obtain $r (\cos \theta + i \sin \theta)$
3. Introduce the terms modulus and amplitude. Also show how the symbol $\text{cis}\ \theta$ may be used for $\cos \theta + i \sin \theta$.
4. Express $-3 + 4i$ in polar form. Have students help do this.

   $r = \sqrt{9 + 16} = 5$

   $\tan \theta = (-4/3)$

   $\tan \theta = -1.333$

   $\theta = 126^{\circ}52'$

   $-3 + 4i = 5 \text{ cis } 126^{\circ}52'$
5. Have students study example 2 on page 102. Help answer any question that students may have about it.
6. Directed study.

Summary: Review the meanings of new terms, and the method for writing complex numbers in polar form.

Suggested Problems:

Problems 1-16 on page 104.

Lesson Plan #56

Review of Chapter 5

Aim: To review chapter 5.

Suggested Method: Check and answer questions on homework, demonstration, question and answer discussion, directed study.

Supplementary Materials: Board protractor, graph board, straightedge.

Developmental Steps:

1. Review the meanings of new terms within the chapter.
2. Discuss the method for finding the square root of a complex number. Have students help do this.
3. Review the method for adding and subtracting vectors algebraically.
4. Discuss problems that students ask on any section in chapter.
5. Directed study.

Summary: None.

Suggested Problems:

Problems 1-12 on pages 104-105.

Note: Test next class meeting on chapter 5.

Lesson Plan #57

Test on Chapter 5

Aim: To administer a test on chapter 5.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Board protractor, straightedge , copies of test.

Suggested Problems:

One problem such as #1 on page 104.

One problem such as #2 on page 104.

One problem such as #4 on page 104.

One problem such as #7 on page 105.

One problem such as #10 on page 105.

One problem such as #8 on page 105.

Chapter 6

Fundamental Relations

Behavioral Objectives:

1. The student will demonstrate the ability to recall the fundamental relations by simplifying expressions that require substitutions of these relations.
2. The student will prove identities using fundamental relations.
3. The student will solve trigonometric equations using the fundamental relations and identities.
4. The student will solve pairs of trigonometric equations using the fundamental relations and identities.

Note: The student should demonstrate the ability to successfully perform three of the above behavioral objectives.

Lesson Plan #58

Fundamental Relations

Aim: To introduce the fundamental relations and the meaning of identities. To teach the method for expressing other functions of an angle in terms of a given function.

Suggested Method: Lecture, demonstration, discussion, directed study.

Supplementary Materials: Board compass, straightedge, test papers.

Developmental Steps:

1. Hand back test papers and discuss problems.
2. Introduce fundamental relations by discussing the definitions of the functions.
3. Define "identity" and "conditional equations". Point out the difference between the two.
4. With the help of the students, derive the reciprocal relations, quotient relations, and Pythagorean relations.
5. Express the other functions of θ in terms of $\sin \theta$. Have students help find these. (These may be found on page 109 of the text.)
6. Directed study.

Summary: Review the definitions of new terms found in this section. Review the fundamental relations that are to be memorized.

Suggested Problems:

Problems 1-12 on page 110.

Lesson Plan #59

Simplification of Trigonometric Expressions

Aim: To teach the method of simplifying a trigonometric expression.

Suggested Method: Check and answer questions on homework, demonstration, lecture, directed study.

Supplementary Materials: none.

Developmental Steps:

1. Define "trigonometric expression" and point out that an expression is simplified when it involves the least number of different functions.
2. Have students study examples given on pages 110-111 of the text. Answer any questions they might have about these.
3. With the help of students simplify $\frac{\tan^2 x}{1 + \tan^2 x}$.
4. Directed study.

Summary:

Review the meaning of trigonometric functions and when they are simplified. Review rules for simplifying these.

Suggested Problems:

Odd numbered problems 1-15 on page 111.

Lesson Plan #60

Simplification of Trigonometric Expressions

Aim: To reteach the method for simplifying trigonometric expressions.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: none.

Developmental Steps:

1. Have students put homework problems on board. Answer any questions that might arise.
2. With help of students simplify $\frac{\tan^2 x}{\sec^2 x} + \frac{\text{ctn}^2 x}{\csc^2 x}$.
3. Give a short quiz to see if the students have memorized the fundamental relations. (Have students write them.)
4. Directed study.

Summary: none.

Suggested Problems:

Even numbered problems 2-16 and 17-22 (all problems) on page 111.

Lesson Plan #61

Proving Identities

Aim: To teach the methods and procedure for proving identities.

Suggested Method: Check and answer questions on homework, questions and answer discussion, lecture, and directed study.

Supplementary Materials: none.

Developmental Steps:

1. Review the method for solving conditional equations. Ask students about the rules that are to be followed.
2. Introduce the rules to be followed for proving identities. Read and explain each rule given on page 113, text.
3. Have students study examples 1, 2, 3 on pages 113, 114 and 115 in the text.
4. Example: Prove $\cos\theta = \sin\theta \operatorname{ctn}\theta$

$$\begin{array}{r|l} \cos\theta & \sin\theta \operatorname{ctn}\theta \\ & \sin\theta \left(\dfrac{\cos\theta}{\sin\theta}\right) \\ \cos\theta & \cos\theta \end{array}$$

5. Directed study.

Summary: Review the rules and format for proving identities. Emphasize that no general method for proving identities can be given.

Suggested Problems:

Problems 1-11 on page 115.

Note: Test for next class meeting on page 106-115.

Lesson Plan #62

Test on Pages 106 - 115

Aim: To administer test on pages 106-115.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test.

Suggested Problems:

One problem such as #2 on page 110.

Two problems such as #6 on page 110.

One problem such as #2 on page 111.

One problem such as #12 on page 111.

One problem such as #15 on page 111.

One problem such as #18 on page 111.

One problem such as #2 on page 115.

One problem such as #9 on page 115.

Lesson Plan #63

Identities

Aim: To continue teaching the proving of identities.

Suggested Method: Demonstration, directed study.

Supplementary Materials: Test papers, overhead projector with its materials.

Developmental Steps:

1. Hand back test papers and discuss problems. Use overhead projector to discuss these problems.

2. Example: Prove $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$	
$(\cos^2\theta - \sin^2\theta)$	
$\cos^2\theta - \sin^2\theta$	$\cos^2\theta - \sin^2\theta$

3. Have students make suggestions for proving this.

Example: Prove

$\dfrac{1-\tan^2\theta}{1-\text{ctn}^2\theta} = 1 - \sec^2\theta$	Have students help prove this identity. (Ask about simplifying complex fractions.)
$\dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1-\dfrac{\cos^2\theta}{\sin^2\theta}}$	
$\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \cdot \dfrac{\sin^2\theta}{\sin^2\theta - \cos^2\theta}$	
$\dfrac{\sin^2\theta\cos^2\theta - \sin^4\theta}{\sin^2\theta\cos^2\theta - \cos^4\theta}$	
$\dfrac{\sin^2\theta(\cos^2\theta - \sin^2\theta)}{\cos^2\theta(\sin^2\theta - \cos^2\theta)}$	
$\dfrac{-\sin^2\theta}{\cos^2\theta}$	$-\tan^2\theta$
$-\tan^2\theta$	$-\tan^2\theta$

Summary: Review format and methods for proving identities.

Suggested Problems: Problems 12-28 on pages 115-116.

Lesson Plan #64

Identities

Aim: To continue teaching the proving of identities. These are the more complicated ones.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: none.

Developmental Steps:

1. Have students put homework problems on the board. Answer questions about these.
2. Example: Prove $\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \frac{1}{\csc\theta + \text{ctn}\,\theta}$

   Have students offer suggestions for proof of this.
3. Using the teacher's manual suggest ways to start several of the more difficult problems on page 116 of the text.
4. Directed study.

Summary: none.

Suggested Problems:

Problems 29-38 (attempt all of these. Some students will not be able to prove all of these.)

Lesson Plan #65

Trigonometric Equations

Aim: To teach the meaning of conditional trigonometric equations and the method used to solve these equations.

Suggested Method: Check and answer questions on homework, lecture on pages 116-119, demonstration, and directed study.

Supplementary Materials: Straightedge, board compass and protractor.

Developmental Steps:

1. Define a conditional trigonometric equation as given on page 116. Also include a definition of a solution for such an equation.
2. Have students study examples 1-6 on pages 117-118 of text. Answer any questions that students have about these.
3. Example: Solve $3 \tan^2 y = 1$ (Have students solve this example. Check their solutions after ample time is allowed for completion of it.)

   $\tan^2 y = 1/3$

   $\tan y = \pm \frac{\sqrt{3}}{3}$

   $y = 30^\circ, 150^\circ, 210^\circ, 330^\circ.$
4. Have the students review the rules given on page 119 of the text. Explain each of these.
5. Directed study.

Summary: Review the meanings of new terms introduced in this section. Review procedure for solving trigonometric equations.

Suggested Problems:

Problems 1-14 on page 119.

Lesson Plan #66

Trigonometric Equations

Aim: To continue teaching the method for solving conditional trigonometric equations.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Straightedge, board compass and protractor.

Developmental Steps:

1. Have the students put homework problems on the board. Answer any questions about these that students may have.
2. Example: $2 \sin^2 x - 3 \sin x + 1 = 0$

   $(2 \sin x - 1)(\sin x - 1) = 0$

$2 \sin x - 1 = 0$	$\sin x - 1 = 0$
$\sin x = 1/2$	$\sin x = 1$
$x = 30^\circ, 150^\circ$	$x = 90^\circ$

3. Directed study.

Summary: Review rules for solving trigonometric equations.

Suggested Problems:

Problems 15-32 on pages 119-120.

Note: Test for next class meeting on pages 115-120.

Lesson Plan #67

Test on Pages 115-120

Aim: To administer a test on pages 115-120.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test, straightedge, board compass and protractor.

Suggested Problems:

One problem such as #13 on page 115.

One problem such as #17 on page 115.

One problem such as #23 on page 116.

One problem such as #28 on page 116.

One problem such as #3 on page 119.

One problem such as #7 on page 119.

One problem such as #17 on page 119.

One problem such as #30 on page 120.

Lesson Plan #68

Trigonometric Equations

Aim: To continue teaching the method and procedure for solving trigonometric equations.

Suggested Method: Demonstration, directed study.

Supplementary Materials: Overhead projector with its materials, test papers, board compass and protractor.

Developmental Steps:

1. Hand back test papers and discuss problems. Use overhead projector to discuss these problems.

2. Example: $\sin \theta + \cos \theta = 1$

$$\sin \theta = 1 - \cos \theta$$

$$\sin^2\theta = 1 - 2 \cos \theta + \cos^2\theta$$

$$1 - \cos^2\theta = 1 - 2 \cos \theta + \cos^2\theta$$

$$2 \cos^2\theta - 2 \cos \theta = 0$$

$$2 \cos \theta (\cos \theta - 1) = 0$$

$2 \cos \theta = 0$	$\cos \theta - 1 = 0$
$\cos \theta = 0$	$\cos \theta = 1$
$\theta = 90^\circ, 270^\circ$	$\theta = 0^\circ$

For $\theta = 90^\circ$	For $\theta = 270^\circ$	For $\theta = 0^\circ$
$\sin 90^\circ + \cos 90^\circ = 1$	$\sin 270^\circ + \cos 270^\circ = 1$	$\sin 0^\circ + \cos \theta^\circ = 1$
$1 + 0 = 1$	$-1 + 0 = 1$	$0 + 1 = 1$
$1 = 1$	$-1 \neq 1$	$1 = 1$
90° is a solution	270° is no solution	0° is a solution

The students will help solve the example by answering questions or making suggestions.

Suggested Problems: Problems 33-40 on page 120.

Lesson Plan #69

Pairs of Trigonometric Equations

Aim: To teach the method and procedure for solving pairs of trigonometric equations.

Suggested Method: Demonstration, discussion on page 120, directed study.

Supplementary Materials: Board compass and protractor, straightedge.

Developmental Steps:

1. Have students put homework problems on the board. Answer any questions that students have about solving these equations. (Allow about one half the period if needed.)

2. Have students read and study page 120 of the text. Discuss the example problem on page 120.

3. Example: $\begin{cases} r = 2\sin\theta \\ r = \tan\theta \end{cases}$ With help of students solve this pair of equations.

$\tan\theta = 2\sin\theta$

$\dfrac{\sin\theta}{\cos\theta} = 2\sin\theta$

$\sin\theta = 2\sin\theta\cos\theta$

$\sin\theta - 2\sin\theta\cos\theta = 0$

$\sin\theta = 0$	$1 - 2\cos\theta = 0$
$\theta = 0^\circ, 180^\circ$	$2\cos\theta = 1$
For $\theta = 0^\circ$, $r = 0$	$\cos\theta = 1/2$
For $\theta = 180^\circ$, $r = 0$	$\theta = 60^\circ, 300^\circ$

For $\theta = 60^\circ$, $r = \sqrt{3}$

For $\theta = 300^\circ$, $r = -\sqrt{3}$

Summary: Review the method for solving pairs of trigonometric equations.

Suggested Problems: Problems 2-6 on top of page 121.

Lesson Plan #70

Review of Chapter 6

Aim: To review the topics in chapter 6 with emphasis on proving identities and solving trigonometric equations.

Suggested Method: Check and answer questions on homework, discussion, directed study.

Supplementary Materials: Overhead projector with its materials, board compass, protractor, and straightedge.

Developmental Steps:

1. Discuss the fundamental relations in chapter 6. Have students answer questions about these to see if they can recall the pythagorean relations, reciprocal relations, and the quotient relations.
2. Review the rules for proving identities given in the text on page 113.
3. Review the rules for solving trigonometric equations given in the text on page 119.
4. Answer questions about any specific exercises in chapter 6 that students find difficult.
5. Directed study.

Summary: none.

Suggested Problems:

Problems 1-15 on page 121.

Note: Test for next class meeting on chapter 6.

Lesson Plan #71

Test on Chapter 6

Aim: To administer test on chapter 6.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test, compass, protractor, straightedge.

Suggested Problems:

One problem such as #2 on page 110.

One problem such as #9 on page 110.

One problem such as #6 on page 111.

One problem such as #4 on page 115.

One problem such as #15 on page 115.

One problem such as #22 on page 116.

One problem such as #3 on page 119.

One problem such as #28 on page 119.

One problem such as #3 on page 121. (top)

Chapter 7

Functions of Two Angles

Behavioral Objectives:

1. The student will express a given function of an acute angle as a function of the complementary angle.
2. Using the functions of the angles 30°, 45°, 60°, and the addition and subtraction formulas, the student will find the sin, cos, and tan of the sum and difference of any two of these angles.
3. If the value of a function of an angle is given and the quadrant for the angle is given, the student will find the values of the sin, cos, and tan of half the angle and twice the angle.
4. The student will prove identities using the fundamental relations and the formulas in this chapter.

Note: The student should demonstrate the ability to successfully perform 3 of the above behavioral objectives.

Lesson Plan #72

Functions of Two Angles

Aim: To introduce the law of cosines, law of sines, the cosine of the difference of two angles, the cosine of the sum of two angles, and cofunctions.

Suggested Method: Discussion on pages 122-129, demonstration, directed study.

Supplementary Materials: Board protractor, and straightedge, test papers, transparencies of figures on pages 124 and 125.

Developmental Steps:

1. Hand back test papers and discuss problems.
2. Using the figures given in the text on page 124, discuss the proof for the law of cosines. A prepared transparency of these would be useful.
3. Using the figure on page 125 of the text, discuss the proof for the law of sines.
4. Discuss the formulas for the cosine of the sum and difference of two angles.
5. Discuss the cofunctions and how they are derived as given on pages 129 in text. Have students help derive some of these.
6. Directed study.

Summary: Review the law of cosines and sines, the cosine for the sum and difference of two angles, and the cofunctions.

Suggested Problems:

Problems 1-19 on page 129.

Lesson Plan #73

Functions of Two Angles

Aim: To introduce the formulas for the sine of the sum and difference of two angles, the tangent of the sum and difference of two angles, and show how the addition and subtraction formulas are applied to solve specific problems.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Straightedge, protractor.

Developmental Steps:

1. With the help of students derive the formulas for the sine of the sum and difference of two angles, and the tangent of the sum and difference of two angles.

2. Have students study the three example problems on pages 131 and 132 in text. Answer questions that they ask about these problems.

3. Example: Simplify $\cos(45^\circ + A)$

$$\cos(45^\circ + A) = \cos 45^\circ \cos A - \sin 45^\circ \sin A$$
$$= \frac{\sqrt{2}}{2} \cos A - \frac{\sqrt{2}}{2} \sin A$$
$$= \frac{\sqrt{2}}{2} (\cos A - \sin A)$$

4. Directed study.

Summary: Review the addition and subtraction formulas and how they may be applied to specific problems.

Suggested Problems:

Problems 1-5 on pages 132-133.

Learn the addition and subtraction formulas.

Lesson Plan #74

Functions of Two Angles

Aim: To continue the study of the addition and subtraction formulas and their applications.

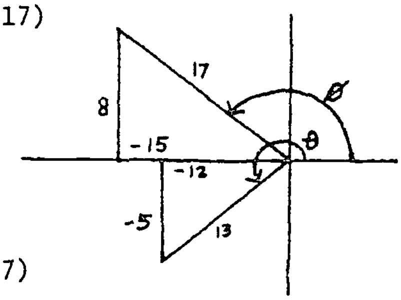
Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Straightedge, protractor, prepared transparency of the diagram below.

Developmental Steps:

1. Have students write all the addition and subtraction formulas. Take these papers up and grade for a quiz.
2. Example: Given $\sin\theta = -5/13$ in the third quadrant and $\tan\phi = -8/15$ in the second quadrant; find the sin, cos, and tan of $(\theta + \phi)$

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$= (-5/13)(-15/17) + (-12/13)(8/17)$$
$$= 75/221 - 96/221$$
$$= \boxed{-21/221}$$



$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$= (-12/13)(-15/17) - (-5/13)(8/17)$$
$$= 180/221 + 40/221$$
$$= \boxed{220/221}$$

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$
$$= \frac{(+5/12) + (8/15)}{1 - (5/12)(-8/15)}$$
$$= \frac{75 - 96}{180 + 40}$$
$$= \boxed{-21/220}$$

3. Directed study.

Lesson Plan #75

Functions of Two Angles

Aim: To continue the study of the addition and subtraction formulas and their applications.

Suggested Method: Check and answer questions on homework, demonstration, and directed study.

Supplementary Materials: Straightedge, protractor.

Developmental Steps:

1. Have students put some of the more difficult homework problems on the board. Discuss questions they have about these exercises.
2. Example:

Show that $\sin(A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C - \sin A \sin B \sin C$

$$\begin{aligned}\sin(A + B + C) &= \sin(A + B) + C \\ &= \sin(A + B)\cos C + \cos(A + B)\sin C \\ &= \cos C(\sin A \cos B + \cos A \sin B) \\ &\quad + \sin C(\cos A \cos B - \sin A \sin B) \\ &= \sin A \cos B \cos C + \cos A \sin B \cos C \\ &\quad + \cos A \cos B \sin C - \sin A \sin B \sin C\end{aligned}$$

Ask for student participation in this example.

3. Directed study.

Summary: none.

Suggested Problems:

Problems 11-15 on page 133.

Note: Test for next class meeting on pages 122-133.

Lesson Plan #76

Test on Pages 122-133

Aim: To administer a test on pages 122-133.

Suggested Method: Check and answer questions on homework, administer a test.

Supplementary Materials: Copies of test, protractor, straightedge.

Suggested Problems:

Three problems such as #1-12 on page 129.

One problem such as #17 on page 129.

One problem such as #19 on page 129.

One problem such as #1 on page 132.

One problem such as #4 on page 132.

Two problems such as #5(a) and 5(i) on page 133.

One problem such as #9 on page 133.

One problem such as #12 on page 133.

Lesson Plan #77

Functions of Twice an Angle and Half an Angle

Aim: To introduce the formulas for functions of twice an angle and half and angle and to teach their application.

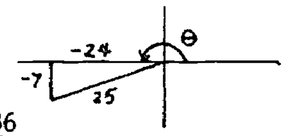
Suggested Method: Lecture on pages 134-136, demonstration, directed study.

Supplementary Materials: Test papers, protractor, straightedge.

Developmental Steps:

1. Hand back test papers and discuss the problems.
2. Have students study pages 134-135 of the text. With the help of students derive the formulas for the functions of twice an angle and half an angle.
3. Example: Given: $\sin \theta = -7/25$ in third quadrant.

   Find: $\tan 2\theta$



$$\tan 2\theta = \frac{2 \tan \theta}{1-\tan^2\theta} = \frac{2(7/24)}{1-49/576} = \frac{336}{527}$$

4. Directed study.

Summary: Review the formulas for the functions of twice an angle and the formulas for the functions of half angles.

Suggested Problems:

Problems 1-9 on page 136-137.

Lesson Plan #78

Functions of Twice an Angle and Half an Angle

Aim: To continue the study of the formulas for the functions of twice an angle and half an angle.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Board compass, protractor, straightedge.

Developmental Steps:

1. Example: Express $\sin 3\theta$ in terms of $\sin \theta$.

$$\begin{aligned}\sin 3\theta &= \sin(\theta + 2\theta)\\ &= \sin\theta \cos 2\theta + \cos\theta \sin 2\theta\\ &= \sin\theta\,(1 - 2\sin^2\theta) + \cos\theta\,(2\sin\theta\cos\theta)\\ &= \sin\theta - 2\sin^3\theta + 2\sin\theta\cos^2\theta\\ &= \sin\theta - 2\sin^3\theta + 2\sin\theta - 2\sin^3\theta\\ &= 3\sin\theta - 4\sin^3\theta\end{aligned}$$

Have students help solve this example by making suggestions.

2. Given right triangle ABC, $c = 90°$, show that

$$\sin 2A = \frac{2ab}{c^2}$$

$$\sin 2A = 2\sin A \cos A = 2(a/c)(b/c) = \frac{2ab}{c^2}$$

3. Directed study.

Summary: none.

Suggested Problems:

Problems 10-13 on page 137.

Lesson Plan #79

Identities

Aim: To teach the student how to prove identities involving the formulas of this chapter.

Suggested Method: Check and answer questions on homework, demonstration, lecture, directed study.

Supplementary Materials: straightedge.

Developmental Steps:

1. Review the rules for solving identities given on page 113 of the text.
2. Have students study examples #1 and #2 on pages 139-140 of text. Discuss any questions that arise.
3. Example: Prove $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$

$$\frac{2\sin\theta\cos\theta}{1 + 2\cos^2\theta - 1}$$

$$\frac{2\sin\theta\cos\theta}{2\cos^2\theta}$$

$$\frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \tan\theta$$

4. Directed study.

Summary: Review the new formulas studied in this chapter.

Suggested Problems:

Problems 1-8 on page 140.

Lesson Plan #80

Identities

Aim: To continue the study of identities involving the formulas in Chapter 7.

Suggested Method: Demonstration, directed study.

Supplementary Materials: None.

Developmental Steps:

1. Have students put homework problems on the board. Discuss any questions that arise.
2. Example: Prove $\frac{\sin 2\theta}{\sin \theta} + \frac{\cos 2\theta + 1}{\cos \theta} = 4 \cos \theta$

$$\frac{2\sin \theta \cos \theta}{\sin \theta} + \frac{2\cos^2\theta - 1 + 1}{\cos \theta}$$

$$2\cos \theta + 2\cos \theta$$

$$4 \cos \theta \quad \Big| \quad 4 \cos \theta$$

3. Directed study.

Summary: none.

Suggested Problems:

Problems 9-15 on page 141.

Note: Test for next class meeting on pages 134-141.

Lesson Plan #81

Test on Pages 134-141

Aim: To adminster test on pages 134-141.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test.

Suggested Problems:

One problem such as #1 on page 136.

One problem such as #3 on page 136.

One problem such as #8 on page 137.

One problem such as #13 on page 137.

One problem such as #1 on page 140.

One problem such as #5 on page 140.

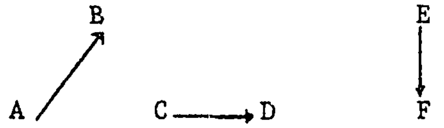
One problem such as #9 on page 141.

One problem such as #13 on page 141.

Lesson Plan #82

Trig (Midterm)

I. Use the vectors shown and construct:



a. $\overrightarrow{AB} + \overrightarrow{CD}$

b. $\overrightarrow{AB} - \overrightarrow{EF}$

c. $\overrightarrow{2CD}$

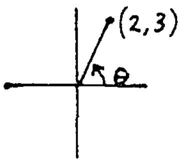
II. State what quadrant each angle is in:

a. 179° b. -45° c. 370° d. -370° e. 800°

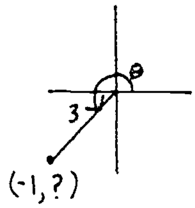
III. If U = 0,4,5 what are the members of UXU?

IV. Find the values of sin θ, cos θ, tan θ in each:

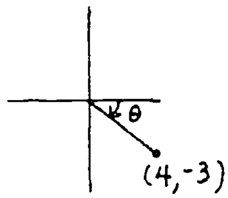
a.



b.



c.



V. True or False:

a. sin is positive in 1st and 4th quadrants.

b. sin and cos are cofunctions.

c. The period of the sin function is 180°.

d. tan and csc are cofunctions.

e. If sec (90° - A) = 4/5, then csc A = 3/5

VI. Use table to find:

a. sin38°20' b. sec 58°50' c. cos 36°43' d. B if tanB = .0825

e. B is cosB = .4924

VII. Write the range for all six trigonometric functions.

VIII. Change to polar coordinates:

a. (1,1) b. $(-\sqrt{3}, 1)$ c. (3,0)

IX. Find the distance between (6,30°) and (8,0°).

X. Change to degrees:

a. $\pi/2$ b. $\pi/5$ c. $3/4\pi$

XI. Simplify:

a. $\sqrt{-16}$ b. $\sqrt{-27a^2}$ c. $(3 - 2i)(2 - 3i)$ d. $\frac{\sqrt{5} + i}{\sqrt{2} - i}$ e. $3\sqrt{-32} + 2\sqrt{-8}$

XII. Prove the following identities:

a. $\tan\theta \sin\theta + \cos\theta = \sec\theta$

b. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

c. $\text{ctn}^2 B - \cos^2 B = \cos^2 B\,\text{ctn}^2 B$

XIII. Solve for all positive values less than 360°.

a. $2\sin\theta = 1$ b. $\text{ctn}^2\theta - 3 = 0$

XIV. Find the sin 75° using the functions of 30° and 45° and the addition formula.

SEMESTER II

ADVANCED MATHEMATICS

Introduction

Mathematics: Advanced Course represents a new approach, treating algebra, trigonometry, analytic geometry, and calculus in a unified manner rather than as four separate sections. Fundamental notions of the subjects are unified into a sequence of topics beginning with the consideration of the real number system and the algebraic operations. Emphasis is placed upon the importance of being able to visualize and graphically represent mathematical expressions. Ideas of algebra and geometry are presented in the study of linear, quadratic, and general polynomial functions. "Permutations, Combinations, and Probability" is treated as an additional topic.

As it is now generally assumed that college freshmen have had extensive work in algebra and some work in trigonometry, this unified course is an attempt to implement those inadequacies before the student enters college.

This guide will probably prove most helpful for teachers who have not taught this course before. However, even the experienced teacher will find suggestions and comments which should prove very useful. The guide covers the chapters listed topic by topic and attempts to emphasize the basic concepts of each.

Content Outline

I. The Real Number System (25 days)

A. Natural numbers

B. Equations

C. Rational numbers

D. Irrational numbers

E. Real numbers and the fundamental operations

F. Fundamental theorems on exponents

G. Special products and factoring

H. Highest common factor

I. Lowest common multiple

J. Fractions and the fundamental operations

K. Simplification of expressions containing radicals

L. Linear equations

II. Functions and Graphs (10 days)

A. Rectangular coordinates

B. Functional notation

C. Application of functions

D. Graphs of functions

E. Relationships of algebra and geometry

F. Distance formula

G. Mid-point of a segment

H. Slope of a line

I. Equation of a locus

III. The Linear Equation and the Straight Line (10 days)

A. The linear equation

B. Second order determinants

C. Inequalities

1. Properties

2. Absolute and conditional inequalities

3. Linear inequalities

4. Graphical solutions of linear inequalities

IV. The Quadratic Function and the Quadratic Equation (15 days)

A. Graphs of quadratic functions

B. The parabola

C. Solutions of quadratic equations

1. Graphing

2. Factoring

D. Complex numbers

1. Definition of complex numbers

2. Addition and multiplication of complex numbers

E. Quadratic formula

F. The factor theorem and its converse

G. Inequalities involving second degree polynomials

V. Other Special Types of Second Degree Equations (10 days)

A. Circles

B. Ellipses

C. Hyperbolas

D. Translation of axes

E. Simultaneous equations: Intersection of curves

VI. Polynomial Functions and Polynomial Equations (10 days)

A. Remainder theorem and factor theorem

B. Synthetic division

C. Graphs of polynomials

D. Roots

1. Location of real roots

2. Number of roots

3. Imaginary roots

4. Rational roots

E. Graphs of polynomials in factored form

F. Solutions of inequalities

VII. Permutations, Combinations, and Probability (8 days)

A. Permutations

B. Combinations

C. The Binomial theorem

D. Probability

General Objectives

1. To communicate effectively with others quantitatively.
2. To select the needed data to solve problems and to use it efficiently.
3. To state definitions precisely.
4. To reason deductively.
5. To understand the basic concept of relations and functions.
6. To state hypotheses explicitly.
7. To unify basic ideas of algebra, trigonometry, and analytic geometry.

CHAPTER 1

The Real Number System

NOTE

The time allotted for Chapter 1, Mathematics: Advanced Course, Elliott, Reynolds, Miles, is six weeks. However, this is a review chapter and depending on the ability and background of the students, it may be covered in a shorter time. A pre-test could be given covering the information contained in the chapter (which is actually a review of Algebra II) and from the results obtained, the time to be spent, as well as the facts to be retaught, could be determined.

The Real Number System

Behavioral Objectives:

1. Given any natural numbers, students will perform all operations.
2. Given whole, rational, irrational, and real numbers, students will identify each in terms of its properties.
3. Given special product formulas, student will recognize each and write its product without having to multiply the polynomials.
4. Given polynomials, students will factor in terms of special products.
5. Given any set of polynomials, students will find the highest common factor and lowest common multiple.
6. Given fractions having polynomials in one or both numerator and/or denominator, students will add, subtract, multiply, and divide them.
7. Given expressions containing radicals, students will simplify.
8. Given conditional equations in one unknown, students will find the solution sets.

Lesson Plan 1

Aim: To review the natural number system.

Suggested Method: Discussion, questions and answers. Directed study p. 1-3.

Supplementary Materials: Notebook, pencil.

Developmental Steps and Questions:

1. Show how the numbers are represented on the number line.
2. Show that the counting numbers are on the same side of the number line.
3. Show what happens when natural numbers are added and multiplied.
4. Show what happens when natural numbers are subtracted and divided.
5. What is the sum of any two natural numbers?
6. What is the product of any two natural numbers?
7. When is subtraction possible?
8. When is division possible?

Summary:

Review the position of numbers on the line.

Suggested Problems:

Tell the results of each:

Example: 5 + 6, name - (eleven), natural number.

1. 8 + 6
2. 5 x 6
3. 6 - 5
4. 5 - 6
5. 6 ÷ 3
6. 3 ÷ 6

Is the result a natural number in each exercise?

Lesson Plan 2

Rational Numbers

Aim: To show the need for rational numbers.

Suggested Method: Questions and answers, with student participation.

Supplementary Materials: Notebook, ruler, pencil.

Developmental Steps:

(Ask students to graph answers on the number line using natural numbers.)

1. Use five problems in addition.
2. Use five problems in subtraction.
3. Use two problems when $a > b$, and two with $a < b$.
4. Use four problems to illustrate division; two $\rightarrow a \div b = Q + R$ when $R = 0$

   two $\rightarrow a \div b = Q + R$ when $R \neq 0$.
5. Ask what must happen in order for all answers in 4 and 5 to be graphed.

Summary:

Review the rationals in terms of the number line.

Suggested Problems:

Page 3, problems 1-24, even numbers.

Lesson Plan 3

Irrational Numbers

Aim: To show that all numbers are not rational.

Suggested Method: Discussion, comparison, and questions with student participation.

Supplementary Materials: Notebook, ruler, pencil.

Developmental Steps:

1. Show that there are points on the line between the rationals.
2. Have students look for $x^2 = 2$ and similar examples on the line.
3. Show that $\sqrt{2}$ can not be written as a/b when $b \neq 0$.

Summary:

Compare rational and irrational points on the number line.

Suggested Problems:

Use problems from textbook, problems 1-7, page 5.

Lesson Plan 4

Absolute Value

Aim: To show that the absolute value of a number is always positive.

Suggested Method: Discussion, questions, illustrations.

Supplementary Materials: Notebook, paper, pencil.

Developmental Steps:

1. What do we mean by positive and negative direction with reference to the number lines?
2. Show the relationship between direction, negative and positive.
3. Review the meaning of absolute value.
4. What is the absolute value of +4 and -4?

Summary: Review the concept of absolute value.

Suggested Problems:

Textbook - exercises, pages 7 and 8.

Lesson Plan 5

Test

Aim: To administer a test on topics 1.1 - 1.8.

1. Using the number line graph the following:

   $5 + 6$; 3×4; $6 - 5$; $6 \div 3$; $3 \div 6$

2. Show that each of the following is a rational number.

   $-7 + 3$; 6×0; $14 \div (-4)$; 2.5×6.3; $\sqrt{9} + \sqrt{4}$

3. Show that the $\sqrt{3}$ is irrational.

4. Show that the absolute of every number is positive.

Lesson Plan 6

Real Numbers and the Fundamental Operations

Aim: To teach the properties of real numbers as they relate to the operations.

Suggested Method: Questions, discussion, example and students' participation.

Supplementary Materials: Notebook, pencil, paper.

Developmental Steps:

1. What is the sum of any two real numbers?
2. If a and b are real numbers, name the property: $a + b = b + a$.
3. Let a, b, and c be any real numbers; name the property $a + (b + c)$ and $(a + b) + c$.
4. If a and b are any two real numbers, name the property $a \times b$; $b \times a$.
5. If a, b, c are real numbers, name the properties $(a \times b)\, c = a\, (b \times c)$; $a\, (b + c) = ab + ac$.

Summary: Review the above concepts.

Suggested Problems:

Make five problems to illustrate each of the above concepts.

Lesson Plan 7

Subtraction

Aim: To show that the real number system is closed under subtraction, and that the commutative and associative laws do not apply to subtraction.

Suggested Method: Questions, discussion with student participation.

Supplementary Materials: Notebook, pencil, paper.

Developmental Steps:

1. Let a and b be any two real numbers with $a > b$, find their difference.
2. Interchange a and b in example 1 and find their difference. What happens?
3. Explain: $5 - (6 - 2) \neq (5 - 6) - 2$. Does the associative law hold for subtraction?

Summary: Review the concept of subtraction.

Suggested Problems:

Make problems to illustrate each concept, five problems for each.

Lesson Plan 8

Division

Aim: To show that division is not closed under the real numbers because $a \div b$ is not defined when $b = 0$, and that the commutative, associative, and distributive laws do not apply to division.

Suggested Methods: Questions and discussion.

Developmental Steps:

1. What is the function of zero in division?
2. Give the condition under which division is possible.
3. Give five examples of division $a \div b$ when $b \neq 0$. Let b = a factor such that $a = bk$, when k is the quotient.
4. Give an example letting the result of $b = 0$, example: $10 \div [(5 - 4) - 1]$.
5. Is $20 \div 5 = 5 \div 20$? What property does not apply to division?
6. Is $20 \div (8 \div 2) = (20 \div 8) \div 2$? Another property that does not apply is____.
7. Discuss the distributive law with respect to division.

Summary: Review the zero function in division.

Suggested Problems:

Make additional problems to illustrate 4, 5, 6, and 7, five problems for each.

Lesson Plan 9

Fundamental Theorems of Exponents

Aim: To show how exponents relate to multiplication and division.

Suggested Methods: Questions, discussion and illustrations.

Supplementary Materials: Notebook, pencil, paper.

Developmental Steps:

1. What are identities? Discuss. Review the fundamental theorems listed on page 10, text.
2. Substituting numerals for a, m, and n show that $a^m \cdot a^n = a^{m+n}$, with $a \neq 0$ and m and n denoting positive integers.

Substituting numerals for a, m, and n show that $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$, m and n are positive integers.

4. Show that $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ when $m < n$, if $a \neq 0$ and m and n are positive integers.
5. Show that $(a^m)^n = a^{mn}$, if the same thing holds for a, m and n as in each of the above.

Summary: Review the four fundamental theorems on exponents listed above.

Suggested Problems:

Use problems given text, page 10.

Lesson Plan 10

Test

Aim: To administer test on topics 1.9 and 1.10.

1. What is the sum of any two real numbers?
2. Name the property for each of the following:

   $a + b = b + a$; $a + (b + c) = (a + b) + c$; $a \times b = b \times a$; $a(b + c) = ab + ac$.
3. Using any numeral for a, b, and c show that subtraction is not commutative or associative.
4. Give the condition under which division is possible. Is division closed under the real numbers? Give a reason for your answer.
5. Perform the indicated operations.

   $x^4 \cdot x^3$; $(x^2)^3$; $(-2x)^3$; $(1/2)^3$; $\frac{8x^3}{12x}$; $\frac{3 \times 10^8}{1.5 \times 10^4}$

   Using numerals for letters show that with $a \neq 0$ and p and q positive integers

   a. $a^p a^q = a^{p+q}$; $\frac{a^p}{a^q} = a^{p-q}$ when $p > q$;

   b. $\frac{a^p}{a^q} = \frac{1}{a^{q-p}}$ when $q > p$;

   c. $(a^m)^n = a^{mn}$.

Lesson Plan 11

Special Products

Aim: To have students recognize the special product formulas and write products without having to multiply.

Suggested Methods: Discussion and questions.

Supplementary Materials: Notebook, paper, pencil.

Developmental Steps:

1. Multiply each of the special product formulas to make sure of multiplication (Students will do this).
2. Encourage the students to orally associate given problems with the proper special product formula.
3. Have the students write in words any two formulas.
4. Have the students write the product of at least one problem that will illustrate each special product formula.

Summary: Have the students translate orally each special product formula.

Suggested Problems:

Exercises, Page 11, numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, and 14.

Lesson Plan 12

Factoring

Aim: To show the relationship between special products and factoring.

Suggested Methods: Definition, questions and deductions.

Developmental Steps:

1. When is an expression prime?
2. When is a polynomial factored?
3. Define a factor.
4. Discuss the relation between factoring and the special product formulas.
5. Why don't we factor x^2-2?
6. Explain how an expression may be factored by grouping terms.

Summary: Define factoring in terms of the special products.

Suggested Problems:

Exercises, page 13, problems: 1, 2, 3, 6, 7, 16, 17, 18, 21, 26, 31, 36, 44, 46.

Lesson Plan 13

Highest Common Factors

Aim: To enable students to determine the highest common factor in two or more polynomials by finding the product of all their common prime factors.

Suggested Methods: Questions, definitions, examples.

Supplementary Materials: Notebook, pencil, paper.

Developmental Steps:

1. Define what is meant by common factor in mathematics.
2. Show that the H.C.F. is the product of the common prime factors.

Summary:

Review factoring and the concept of the highest common factor.

Suggested Problems:

Exercises, page 15, problems: 2, 4, 6, 7, 10, 12, 13, 15 and 16.

Lesson Plan 14

The Lowest Common Multiple

Aim: To enable students to find the smallest number that is a multiple of several numbers in one operation.

Suggested Methods: Discussion, question.

Supplementary Materials: Notebook, paper, pencil, overhead and transparencies.

Developmental Steps:

1. Explain and discuss multiple.
2. Show what is meant when we say one number is a multiple of another.
3. Give several examples to make sure of the concept. (Use overhead projector.)

   Example: a. $x^2 - 6x + 9$, $x^2 - 11x + 24$

   b. $2x^2 + 3x - 35$; $2x^2 + 19x + 45$

   c. etc.

Summary: Review the concept of a multiple. State that the L.C.M. of several polynomials is a polynomial of lowest degree that contains each of the given polynomials as a factor.

Suggested Problems:

Pages 14 and 15, problems: 1, 2, 4, 6, 7, 8, 12, 14, 16, 17, 18.

Test tomorrow, pages 11-15, text.

Lesson Plan 15

Test

Aim: To administer a test on topics 1.11 - 1.14.

Write the products:

1. $2\ (2x + 3y)$; $(3x + y)$; $(3x + y)\ (3x - y)$; $(x + 2y)\ (x^2 - 2xy + 4y^2)$; $(a + b - c)^2$; $(38)\ (42)$.

Factor:

2. $ax + bx - ay - by$; $27x^3 - 8$; $x^3 + 3x^2 - 4x - 12$; $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$.

Find the H.C.F.

3. (a) $1 - x^2,\ 1 - x^3,\ 1 + x - 2x^2$;

   (b) $ax + ay - x - y,\ 4a^2 + 3a - 7$; $a^2 - b - a + ab$.

Find the L.C.M. of each of the following sets of polynomial expressions:

4. $x^2 - 2ax + 2bx - 4ab,\ x^2 - 4b^2,\ x^2 - 4a^2$

   $12x - 8,\ 3x^2 + x - 2,\ x^2 - 1$;

5. $xy - ay - ab + bx,\ x^3 - 3ax^2 + 3a^2x - a^3,\ 4y^2 + 3by - b^2$.

Lesson Plan 16

Rational Fractions

Aim: To show that a fraction is meaningless when its denominator vanishes or becomes zero.

Suggested Methods: Definitions, discussions, examples.

Supplementary Materials: Notebook, paper, pencil.

Developmental Steps:

1. Define a rational number.
2. Show that the numerator and denominator of a fraction are the same as the dividend and divisor in division.
3. Discuss $\frac{x^2 + 8x + 1}{x - 3}$ as a rational fraction. What happens when $x = 3$?
4. Show that the sign of a fraction may be changed under certain conditions.
5. Discuss $\frac{-x^2 + 3x - 2}{x + 4} = \frac{x^2 - 3x + 2}{x + 4}$.
6. Discuss when a fraction is in its lowest terms.
7. Give an example of finding the L.C.D. of two or more fractions. Discuss it.

Summary:

Principles XIV and XV, pages 15 and 16 should be stated.

Suggested Problems:

Exercises, page 16-18.

Lesson Plan 17

Addition and Subtraction of Fractions

Aim: To develop the students' skill in adding and subtracting fractions.

Suggested Method: Questions with student participation.

Supplementary Materials:

Pencil, paper, notebook.

Developmental Steps:

1. What kind of fractions can be combined?
2. When are fractions "alike"?
3. What is the function of the numerator?
4. What is the function of the denominator?
5. What is the lowest common denominator?
6. Use examples to show how the addition and the subtraction of fractions can done.

Suggested Problems:

Exercises, page 18 and 19.

Lesson Plan 18

Multiplication and Division of Fractions

Aim: To develop the students' ability to multiply and to divide fractions.

Suggested Method: Discussion and question, with student participation.

Supplementary Materials: Paper, pencil, notebook, overhead, transparency.

Developmental Steps:

1. Explain why $\frac{x^2 - 2x - 35}{2x^3 - 3x^2} \times \frac{4x^3 - 9x}{x - 7} = \frac{2x^2 + 13x + 15}{x}$. Use a transparency.
2. Explain why $\frac{3x^2 + 13xy - 10y^2}{x^3 + 125y^3} \div \frac{6x^2 - 4xy}{x^2 - 5xy + 25y^2} = \frac{1}{2x}$. Use a transparency.
3. What happens to the numerator and the denominator of a comple fraction in division?

Summary: Review the concept of dividing out all common factors of numerators and denominators in the division of fractions.

Suggested Problems:

Exercises, page 21.

Lesson Plan 19

Simplification of Expressions Containing Radicals

Aim: To teach the students to write any radical in a variety of equivalent forms.

Suggested Method: Definitions, examples and student participation.

Supplementary Materials: Paper, pencil, notebook.

Developmental Steps:

1. Review the theorems on exponents listed on pages 21-23, text.
2. Show that $A^{m/n} = \sqrt[n]{A^m}$
3. Discuss $16^{1/3} = \sqrt[3]{16} = \sqrt[3]{8 \times 2} = 2\sqrt[3]{2}$
4. Show that $\sqrt[n]{k}$ is in simplest form if
   a. k contains no factors of the form A^n, where A is an integer.
   b. No fraction appears under the radical.
   c. When $k = A^m$, then the fraction m/n is in lowest terms.
5. Discuss examples such as those listed on page 24.

Summary: Review the concepts of XIX and XX, page 23.

Suggested Problems:

Exercises, pages 25-28, odd numbered problems. Give individual help as needed.

Test at the next class meeting, topics 1.15 - 1.20.

Lesson Plan 20

Test

Aim: To administer a test on topics 1.15 - 1.20.

Reduce to the lowest term:

1. $\frac{5x^3 - 5x^2y}{x^2 + 5xy - 6y^2}$; $\frac{a^2 + b^2 + c^2 + 2ab - 2bc - 2ac}{a^2 + b^2 - c^2 + 2ab}$

Perform the indicated operations:

2. $\frac{5}{x - 2y} - \frac{25x}{5x^2 - 9xy - 2y^2}$; $\frac{2x}{1 - x^2} + \frac{3x^2}{x^3 - 1} - \frac{4x^3}{1 - x^4}$

3. $\frac{x^2 - 2x - 35}{2x^3 - 3x^2} \times \frac{4x^3 - 9x}{x - 7}$; $\left(2 - \frac{2 + 8x - 3x^2}{9 - x^2}\right) \div \left(6 - \frac{14 + 7x}{3 + x}\right)$

Perform the indicated operations. Use positive exponents to write the results.

4. $\sqrt{50} - 5\sqrt{18} + 2\sqrt{32}$; $7/\sqrt{5} + 5\sqrt{45}$

Reduce to simplest form:

5. $\frac{\sqrt{2}}{\sqrt{2} - \sqrt{3}}$; $\frac{\sqrt{5} - 2}{a + \sqrt{2} + \sqrt{3}}$.

Lesson Plan 21

Conditional Equations

Aim: To show the solution set of equations.

Suggested Methods: Questions and student participation.

Supplementary Materials: Paper, pencil, notebook.

Developmental Steps:

1. Define an equation.
2. What do we mean by conditional?
3. When is an equation solved?
4. Show that an equation is not changed if both members are treated alike.
5. Show that if the degree of an equation is changed extraneous roots may appear or fewer roots may result.

Example: $x - 3 = 2$; multiply by $x - 3$.

$(x - 3)(x - 2) = 4(x - 3)$; divide by $x - 3$.

Solve these and other examples for the class.

Summary: Review the concept of an equation being conditional. Stress that checking is necessary to determine extraneous roots.

Suggested Problems:

Exercises, page 33.

Lesson Plan 22

Problems Solved by Linear Equations

Aim: To show how equations are applied to problem solving.

Suggested Methods: Discussion, questions, student participation.

Developmental Steps:

1. Show how to express unknown quantities in terms of letters. Explain examples 1, 2, and 3, pages 34 and 35.
2. Use problems 2, 4, and 13, page 37, as additional examples. Let students demonstrate how to work these.

Summary: Review solving examples of general conditional equations in one unknown.

Suggested Problems:

Pages 36 and 37, problems 1, 3, 5, 10, 14, 16, and 20.

CHAPTER 2

Functions and Graphs

Functions and Graphs

Behavioral Objectives:

1. Given an equation such as $y = 3x + 1$, students will plot the graph of the function.
2. Given an algebraic formula, students will show that it expresses some variable quantity as a function of other variable quantities.
3. Given a relation, students will draw its graph.
4. Given two points in a plane, students will find the distance between them.
5. Given a first degree equation, students will find the slope of the line it represents.
6. Given information about a locus, students will give the algebraic equation that represents it.

Lesson Plan 1

Aim: To make sure of the idea of relations and relationships.

Suggested Methods: Discussion, definitions.

Supplementary Materials: Coordinate paper, ruler, pencil, notebook.

Developmental Steps:

1. Define a constant (absolute and arbitrary).
2. Give several examples of each.
3. Show the existence of a relationship in terms of independent and dependent variables.
4. What is the domain? The range? What are ordered pairs?
5. Define function: What determines the number of ordered pairs in the function? $y = 3 - \frac{2}{5}x$ expresses y explicitly as a function of x, while $2x + 5y - 15 = 0$ defines y as an implicit function of x.

Summary: Review all definitions and concepts of a function.

Suggested Problems:

Exercises page 40-42, all problems.

Lesson Plan 2

Rectangular Coordinates

Aim: To show that a one to one correspondence can be established between the set of all ordered pairs of real numbers and points of a plane.

Suggested Methods: Discussion, questions, and illustrations.

Supplementary Materials: Graph paper, ruler, pencil, notebook.

Developmental Steps:

1. Draw a horizontal line._________, x axis.
2. Draw parallel lines above and below the x axis.
3. Choose a point zero on the x axis.
4. Fix a zero point directly above and below on the other lines, using the same scale.
5. Represent the set X of all real numbers (as indicated on page 43) on each of these lines. Write the ordered pairs.
6. Using the same concept draw a vertical line called the y axis and write the ordered pairs.
7. Define coordinate axes.
8. Show that the plane is divided into four quadrants.
9. Locate ordered pairs in each quadrant.

Summary: Review the concept of axis and coordinates.

Suggested Problems:

Exercises, page 46 and 47, all problems.

Lesson Plan 3

Application of Functions

Aim: To show that every algebraic formula is an equation which expresses some variable quantity as a function of other variable quantities.

Suggested Methods: Discussion

Supplementary Materials: Coordinate paper, pencil, notebook.

Developmental Steps:

1. Consider situations where one variable y is said to vary directly with another variable x. $y \propto x$ or $y = kx$.
2. Show that if x is doubled, y will be doubled also.
3. Show that the relationship can be written $y = kx$ where k is a constant $\neq 0$.
4. Show situations where two variables are said to vary inversely.
5. Explain the relationship as either $xy = k$ or $y = k/x$; k is the constant of variation.

Summary: Review kinds of variations.

Suggested Problems:

Exercises, pp. 50 and 51.

Lesson Plan 4

Denominate Quantities

Aim: To avoid the difficulty of using units of measurement.

Suggested Methods: Discussion with student participation.

Supplementary Materials: Ruler, paper, pencil, notebook.

Developmental Steps:

1. Discuss denominate quantities.
2. Show ways of representing denominate quantities.
3. Discuss standard units of measurement.
4. Discuss the relationship of lwh = V and the board foot.
5. Show that if an equation is expressed in a correct relationship, the resulting units may be converted to standard units of measurement.

Summary: Review standard units of measurement.

Suggested Problems:

Exercise, page 54.

Note: There will be a test tomorrow on topics 2.4 and 2.5.

Lesson Plan 5

Test

Aim: To evaluate topics 2.4 and 2.5.

Find the constant of variation for each of the following:

1. Given that y varies directly as x; y = 18 when x = 2.
2. Given that y varies jointly with x and z, and y = 24 when x = 6 and $z = 1/2$.
3. Given that S is directly proportional to T^2 and that S = -64 when T = 2.
4. The distance that a free falling body travels varies directly with the square of time. Find the distance traveled if k = 16 ft./sec.^{2}; t = 5 sec.
5. The density of a material is found by dividing the mass of the material by its volume. Find the density of water if 4 ft.^{3} of water has a mass of 250 lb.
6. At constant temperature the volume of a given mass of gas varies inversely with the pressure. Find the volume if the proportionality constant is 7200 lb. in. and the pressure is 12 lb/in^2.

Lesson Plan 6

Graphs of Functions

Aim: To draw the picture of a relationship.

Suggested Methods: Definitions and discussion of relations with student participation.

Supplementary Materials: Paper, pencil, coordinate paper, notebook.

Developmental Steps:

1. Define a relation.
2. Write equations for relations.
3. Graph equations of relations.
4. Demonstrate the definition of a function using a graph.
5. Show the graph of a non-function.

Summary: Review the concept - every function is a relation, but every relation is not a function.

Suggested Problems:

Exercises, pages 58 and 59, problems 1, 2, 4, 6, 8, 13, 14, 17, 21, 26, 27, and 29.

Lesson Plan 7

Distance Between Two Points on a Line Segment

Aim: To show that a line or a curve in a plane corresponds to an equation in two variables.

Suggested Methods: Lecture, questions and student participation.

Supplementary Materials: Coordinate paper, notebook, pencil.

Developmental Steps:

1. Review the Pythagorean Theorem.
2. Show that the distance between two points can be expressed using a right triangle.
3. Express distance on the x axis $(x_2 - x_1)$.
4. Express distance on the y axis $(y_2 - y_1)$.
5. Show that $(x_2 - x_1)$ and $(y_2 - y_1)$ are sides of a right triangle.
6. Show that distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
7. Discuss the mid-point formulas, page 61.

Summary: Review the distance formula as it relates to the right triangle.

Suggested Problems:

Exercises, page 65, problems 1-5.

Lesson Plan 8

The Slope of a Line

Aim: To show that the slope of any non-vertical or non-horizontal line is the ratio of the change in y to the change in x.

Suggested Methods: Discussion and demonstration.

Supplementary Materials: Coordinate paper, notebook, pencil, paper, graph board.

Developmental Steps:

1. Show the relationship between the tangent of the angle of inclination and the slope of a line using the right triangle. Demonstrate on a graph board.
2. Show that the line is the hypotenuse of a right triangle having sides $(y_2 - y_1)$ and $(x_2 - x_1)$.
3. Show that the slope is the ratio between $(y_2 - y_1)$ and $(x_2 - x_1)$.
4. Using the above what can be said about the slope of parallel and of perpendicular lines?

Summary: Compare the tangent of the angle of inclination of the line and the slope of the line.

Suggested Problems:

Exercises, page 63 - all problems.

Lesson Plan 9

Equation of a Locus

Aim: To translate the geometric definition of the locus into an algebraic form using a coordinate system.

Suggested Methods: Lecture and demonstration.

Supplementary Materials: Coordinate paper, notebook, pencil, ruler, paper.

Developmental Steps:

1. Show that a locus is a relation.
2. Show that the relation that satisfies a given equation is the locus of that equation.
3. Show that a locus is thought of as a point that moves along a certain path.
4. Discuss some examples such as those on pages 68-70.

Summary: Review the locus-point concept.

Suggested Problems: Exercises, pages 70 and 71, problems 2, 4, 6, 8, 10, 12, 14, 16 and 18.

Note: Test on topics 2.10 and 2.11 at the next class meeting.

Lesson Plan 10

Test

Aim: To evaluate topics 2.10 and 2.11.

Find the slope of the line through each of the following pairs of points:

(1) (2, 1), (7,4)

(2) (5,1), (2, 3)

(3) (13, 4), (0, 0)

(4) (-1, -1), (-6, 0)

(5) (3, -7), (0, -8)

(6) (-4, 0), (0, 3)

(7) At what point does the line through (7, 4) and (-3, 9) intersect the y - axis.

(8) Write the coordinates of the vertices of a regular hexagon, one of whose sides is a segment joining (0, 0) and (a, 0).

(9) The vertices of a right triangle are (-1, 1) (5, 7) and (k, 2). Find the value of k, if the vertex of the right angle is at (5, 7).

CHAPTER 3

The Linear Equation and the Straight Line

The Linear Equation and the Straight Line

Behavioral Objectives:

1. Given two points, students will write the equation of the line which they determine. They will graph the line.
2. Given a first degree equation, students will graph it.
3. Given a first degree equation and a point not on the line, students will find the distance from the point to the line.
4. Given two linear equations, students will find their solution using determinants.
5. Given an inequality, students will draw its graph.
6. Given an equation and inequality, students will show some analogies.
7. Given a linear inequality, the students will solve it.
8. Given a linear inequality to graph, students will show that its solution is a portion of the x-y plane.

Lesson Plan 1

Some Standard Forms of the Equation of the Straight Line

Aim: To show that if two points are known an equation of the line determined by them can be written.

Suggested Methods: Discussion with class participation.

Supplementary Materials: Ruler, coordinate paper, pencil, notebook.

Developmental Steps:

1. Show that if two points p_1 $(x_1\ y_1)$ and p_2 $(x_2\ y_2)$ are known a third point p (x,y) is on the line.
2. Write the equation of the line $p_1\ p_2$ using equal slopes of lines p_1p and p_1p_2.
3. Discuss $y - y_1 = m\ (x - x_1)$.
4. Discuss: $y = mx + y_1$; $x = x_1$; $x = k$ and $y = k$.

Summary: Review the two point form, point slope, and slope-intercept forms.

Suggested Problems:

Page 74, exercises 5, 7, 8, 9, 12, 17, 18, 20, 22 and 26.

Lesson Plan 2

The Linear Equation

Aim: To show that every first degree equation is a straight line.

Suggested Methods: Discussion, illustrations with class participation.

Supplementary Materials: Ruler, coordinate paper, pencil, notebook, graph board.

Developmental Steps:

1. Show that $Ax + By + C = 0$ represents a straight line.
2. Show that when $B \neq 0$; $y = -A/B\ x - C/B$.
3. Show what happens if $B = 0$.
4. $-A/B$ is the slope and $-C/B$ is the y-intercept, if $B \neq 0$.

Summary: Review first degree equations.

Suggested Problems: Exercises, page 75-77, problems 1, 3, 6, 7, 8, 10, 12, 13, and 18.

Lesson Plan 3

Distance From a Point to a Line

Aim: To show that if a linear equation is known and a point given the distance from the point to the line can be determined.

Suggested Methods: Discussion, illustrations.

Supplementary Materials: Ruler, coordinate paper, notebook, pencil, graph board.

Developmental Steps:

1. Graph any linear equation and take any point not on the line.
2. Using the graph and the given point form congruent triangles or similar triangles.
3. Solve for d in terms of similar triangle ratios.
4. Using $Ax_1 + By_1 + C = 0$ show that

$$d = \pm \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$$

Summary: Review linear equations and the distance formula.

Problems Suggested: Exercises, page 79, problems 1, 3, 5, 6, 7, and 9.

Lesson Plan 4

Second Order Determinants and Two Simultaneous Linear Equations

Aim: To show that a determinant is a unique arrangement.

Suggested Method: Discussion.

Supplementary Materials: Paper, pencil, notebook.

Developmental Steps:

1. Define a determinant of the second order.
2. Using $A_1x + b_1y = C_1$ and $A_2x + b_2y = C_2$ solve for x and y using deterinants.
3. Show the arrangements of the coefficients of the numerator and denominator of x and y.
4. Identify the principle diagonal.
5. What is the value of a second order determinant $\begin{vmatrix} a_1 b_1 \\ a_2 b_2 \end{vmatrix}$?
6. Discuss the geometric interpretation of the intersection of the pair of lines $A_1x + B_1y + C_1 + 0$ and $A_2x + B_2 + C_2 = 0$.

Summary: Review the arrangements of determinants and the principle diagonal.

State the unique solution of the above equations using determinants.

Suggested Problems:

Page 80-82, numbers 3, 4, 6, 7, 8, 9, and 10.

Test on topics 3.2 - 3.6 tomorrow.

Lesson Plan 5

Test

Aim: To evaluate topics 3.2 - 3.6.

Find the equation and draw the graph of each of the following:

1. A line through (3, -7) and (8, -2).
2. A line through (3, 2) and with slope -2.
3. A line through the points of intersection of $y = x^2 - 4$ and $2x - 3y + 9 = 0$.
4. Find the equation of the medians of the triangle with vertices at A (-5, -1), B (3, -4), and C (1, 6).
5. Find the distance between (3, 7) and $3x + 4y - 2 = 0$.
6. Find the distance between $4x + 3y - 12 = 0$ and $4x + 3y - 36 = 0$.
7. Using $A_1x + b_1y + C_1 = 0$ and $A_2x + b_2y + C_2 = 0$, solve for x and y and write in determinant form.
8. Using determinants solve $2x + 3y + 13 = 0$ and $6x + 5y + 15 = 0$.

Use any five problems.

Lesson Plan 6

Inequalities

Aim: To show that if a and b are real numbers, one of the following is true: $a = b$; $a < b$ or $a > b$.

Suggested Methods: Discussion with class participation.

Supplementary Materials: Paper, pencil, notebook.

Developmental Steps:

1. Let a and b represent real numbers and show that $a > b$ is positive.
2. Show that $a = b$ is zero.
3. Show that $a < b$ is negative.
4. What is the position of the point a in relation to b in each case? $a > b$; $a = b$; $a \not< b$?
5. Define the sense of an inequality.
6. Explain $a \leq x \leq b$.
7. Define the absolute value of a, 0, and -a.
8. Graph the absolute value a where a is any real number.

Summary: Review the inequality concept and absolute value.

Suggested Problems: Exercises, page 86, all problems 1-12.

Lesson Plan 7

Properties of Inequalities

Aim: To show some analogies between inequalities and equations.

Suggested Method: Discussion.

Supplementary Material: Paper, pencil, notebook.

Developmental Steps:

1. Define the sense of an inequality. (Review)
2. Show that the sense is not changed if the same number is added or subtracted from each of its members.
3. Compare this with solving an equation.
4. Show what happens when both members are multiplied or divided by the same positive number.
5. Is this true if we use a negative number? If not, why?

Summary: Review the properties of inequalities.

Suggested Problems: Exercises, page 89, all problems.

Lesson Plan 8

Linear Inequalities

Aim: To study the solutions and graphs of inequalities.

Suggested Methods: Discussion.

Supplementary Materials: Coordinate paper, pencil, notebook, graph board.

Developmental Steps:

1. Compare the solution of a linear equation and a linear inequality.

   Use a graph board or a transparency of a graph grid.

2. Compare the points on their graphs.

3. Which, if either, has an infinite set for its solution set?

Summary: Review solution sets of equations and inequalities.

Suggested Problems: Exercises, page 89, all problems.

Lesson Plan 9

Graphical Solutions of Linear Inequalities

Aim: To show that the graph of an inequality is a region of the x-y plane.

Suggested Method: Discussion and illustration.

Supplementary Materials: Coordinate paper, pencil, ruler, notebook.

Developmental Steps:

1. Graph $x = y$ where (x, y) are points in the plane.
2. Draw a graph of $x > y$.
3. Draw a graph of $x < y$.
4. Compare the three graphs.
5. Solve an inequality such as $3x - 2 > x + 4$ graphically.
6. Define and illustrate open and closed half planes.*

Summary: Review solving inequalities.

Suggested Problems: Exercises, page 91, all problems.

Note: Test tomorrow on topics 3.7 - 3.10.

* Vannatta, Carnahan and Fawcett, Advanced High School Mathematics. Columbus, Ohio: Charles E. Merrill Books, Inc., 1961, pages 191, 192.

Lesson Plan 10

Test

Aim: To evaluate topics 3.7 - 3.10.

1. Draw a diagram showing the locus of values of x; $-4 \leq x \leq 2$.
2. Draw a diagram showing the locus of values of x; $x \geq 3$.
3. Draw a diagram showing the locus of values of x; $/x/ \leq 3$.
4. Solve and graph $2x + 3 < 5x + 7$.
5. Solve and graph $x - 7 < 3x + 1$.
6. Solve and graph $2x - 1 < 3x + 2$.
7. Solve and graph $2x - 5 > 0$.

Solve and graph, showing the range of x:

8. $3x - 2 > x + 4$.
9. $2x + 2 < x + 1$.
10. $5 < 3x + 17$.

CHAPTER 4

Quadratic Functions and Quadratic Equations

Quadratic Functions and Quadratic Equations

Behavioral Objectives:

1. Given a quadratic equation in two variables,students will give a graphic solution.
2. Given a second degree equation which is a function, students will find the focus, directrex, and sketch the parabola.
3. Students will graph the solution and determine the nature of the roots of a quadratic equation.
4. Given a quadratic equation,students will solve it by:
   a. factoring
   b. by completing the square,
   c. by the quadratic formula.
5. Given a complex number students will identify its real and imaginary part.
6. Students will find the nature of the roots of a quadratic equation using the discriminant.
7. Given two complex numbers, students will show that the sum, difference, product, and quotient is a complex number, division by zero excluded.
8. Students will graph the solution of a given complex number.
9. Students will find the sum and product of the roots of a quadratic equation without solving.
10. Given a fractional equation students will reduce it to a quadratic and solve.
11. Students will graph and show the range of an inequality involving a polynomial of second degree.

Lesson Plan 1

Graphs of Quadratic Functions

Aim: To show the use of graphs in the solutions of quadratic equations.

Suggested Method: Discussion with class participation.

Supplementary Materials: Coordinate paper, ruler, notebook and pencil, transparencies of quadratic functions like figures 4.1, 4.2, 4.3, 4.4.

Developmental Steps:

1. Discuss a quadratic function.
2. Discuss a quadratic equation.
3. Using the function $y = ax^2 + bx + c$, discuss what happens when a is positive.
4. Discuss the conditions when a is negative.
5. Graph a function such as $y = x^2 - 4x + 4$. Discuss it.

Summary: Review quadratic equations in two variables.

Suggested Problems: Exercises, page 97 - odd problems.

Lesson Plan 2

The Parabola

Aim: To define a parabola.

Suggested Method: Discussion with class participation.

Supplementary Material: Coordinate paper, ruler, notebook, pencil, transparencies of figure 4.5, page 98, text.

Developmental Steps:

1. Define a parabola.
2. Show the focus and the directrix of the parabola. Use a prepared transparency of figure 4.5, page 98, text.
3. Define the vertex in terms of the focus and directrix.
4. Derive the formula from the definition of the parabola. Show that the locus is always equidistanct from a point called the focus and a line called the directrix using the prepared transparency.
5. When is the parabola in standard form?
6. When will the curve open to the right or left?

Summary: Review the definition and the formula for the standard form.

Suggested Problems: Exercises, page 100 and 101, problems 1, 3, 4, 6, 10, 13, and 15.

Lesson Plan 3

Graphic Solutions of Quadratic Equations

Aim: To determine the nature of the roots of equations graphically.

Suggested Method: Discussion with class participation.

Supplementary Materials: Coordinate paper, notebook, ruler, pencil.

Developmental Steps:

1. Show that hwere $y = ax^2 + bx + c$ crosses the x-axis the ordinates of the points are zero, the abscissas of these points are values of x that make $y = 0$.
2. Identify these as the real roots of $ax^2 + bx + c = 0$.
3. Show that in general, the graph of the function $y = ax^2 + bx + c$ will tell whether the quadratic equation $ax^2 + bx + c = 0$ has two, one, or no real roots.

Summary: Review the graph of a parabola.

Suggested Problems:

Exercises, page 102, all problems.

Lesson Plan 4

Solutions by Factoring

Aim: To show that a quadratic equation in one variable may be resolved into linear equations.

Suggested Methods: Discussion.

Supplementary Materials: Notebook, pencil, paper.

Developmental Steps:

1. Determine when a quadratic equation in one variable can be factored.
2. Show that $(rx + m)\ (sc + n) = 0$ is a quadratic equation in one variable.
3. Show what happens when either $(rx + m) = 0$ or $(sx + n) = 0$.
4. How many roots do we have in a quadratic equation?
5. Solve $6x^2 + 19x - 20 = 0$.

Summary: Review changing quadratic equations to linear equations equated to zero.

Suggested Problems:

Exercises, page 103, all problems.

Test at the next class meeting will be on topics 4.1 - 4.5.

Lesson Plan 5

Test

Aim: To evaluate topics 4.1 - 4.5.

Find the vertices and graph the following:

1. $y = x^2 + 4x + 4$.

2. $y + x^2 - 2x + 1$.

Draw the focus and directrix:

3. $y^2 = 4x$; $y^2 = -9x$.

4. Find the equation of the parabola: vertex at origin, focus at (3,0).

5. By graphs determine the nature of the roots:

   $x^2 - x - 6 = 0$; $2x^2 + 3x - 2 = 0$.

Lesson Plan 6

Complex Numbers

Aim: To show that the real number system is not sufficient to solve all quadratic equations.

Suggested Methods: Discussion.

Supplementary Materials: Pencil, paper and notebook.

Developmental Steps:

1. Define the imaginary unit, i, in terms of $i^2 = -1$.
2. Show that $\sqrt{-a} = i\sqrt{a}$.
3. What is the pattern of i^n?
4. Determine the value of i^n when $n = 9$.
5. Show what happens when we combine imaginary numbers.
6. Define a complex number and name its parts.
7. Solve an equation by completing the square. Solve one using the quadratic formula.

Summary: Review the imaginary unit and the complex number.

Suggested Problems:

Exercise, page 105, problems 2, 4, 6, 8, 10, 12, 14, 16.

Lesson Plan 7

Nature of the Roots of a Quadratic Equation

Aim: To show the function of the discriminant.

Suggested Method: Lecture, questions with class participation.

Supplementary Materials: Notebook, pencil, paper, transparencies used in lesson one may be used in the following discussion.

Developmental Steps:

1. Give a graphic solution to several quadratic equations. Discuss the nature of roots of each.
2. Substitute from the given equations in the discriminant $(b^2 - 4ac)$.
3. Observe what happens when $b^2 - 4ac < 0$; $b^2 - 4ac > 0$; $b^2 - 4ac = 0$. Does this agree with your graphical solutions?

Summary: Review the quadratic formula and the use of the discriminant.

Suggested Problems:

Exercises 2, 4, 6, 8, 10, 12, 14, 16, page 106.

Lesson Plan 8

Operations with Complex Numbers

Aim: To perform the operations of addition, subtraction, multiplication, and division using complex numbers.

Suggested Method: Discussion with class participation.

Supplementary Materials: Notebook, paper, pencil.

Developmental Steps:

1. Review the equality relationship of complex numbers.
2. State the theorem pertaining to the operations and complex numbers. Discuss it thoroughly.
3. Work an example involving each operation. (See page 108, text).
4. Have some students work an example of each type on the board.

Summary: Restate the theorem.

Suggested Problems:

All problems, pages 109, 110.

Lesson Plan 9

Graphical Representation of Complex Numbers

Aim: To show that each complex number has associated with it a pair of real numbers.

Suggested Method: Discussion, class participation.

Supplementary Material: Coordinate paper, notebook, pencil, ruler.

Developmental Steps:

1. Explain that a + bi may be written in the form (a,b). Discuss.
2. Modify the axes, showing real and imaginary.
3. Show that the procedure of plotting the points follows the same pattern as plotting points in the coordinate plane.
4. Graph p (3,2) and (3 + 2i). Discuss each.
5. Define vector. Relate a vector to a complex number using a graph. Work a vector sum such as exercise 26, page 111.

Summary: Review the graphical representation of complex numbers.

Suggested Problems:

Exercises, page 111, odd problems. Review for a test on topics 4.6-4.10.

Lesson Plan 10

Test

Aim: To evaluate topics 4.6 - 4.10.

Solve the following and check:

1. $4x^2 - 4x - 5 = 0$; $x^2 = 2x - 2$.

Determine the nature of the roots (using discriminant).

2. $4x^2 - 9 + 3 = 0$; $6x = x^2 + 10$.

3. Express as a complex number in the form a + bi:

   $(2 + 3i)\ (5 - 4i)$: $\frac{1}{(-1/2 + 3/2i)^2}$

Represent as points in a plane:

4. $2 - 3i$; $-4 -5i$

Find the length of the vectors:

5. $3 + 4i$; $-15 + 8i$.

Lesson Plan 11

Relations Between the Roots and the Coefficients of a Quadratic Equation

Aim: To show that certain facts may be learned without solving the equation.

Suggested Method: Lecture and illustrations.

Supplementary Materials: Paper, pencil, notebook.

Developmental Steps:

1. Use r, and r_2 to represent the roots of any quadratic equation.

   Then $(x - r_1)(x - r_2) = 0$ is a quadratic equation.

   Show that 2 is true regardless of the nature of the roots.

   We have $x^2 - (r_1 + r_2)x + r_1r_2 = 0$.

   Since $Ax^2 + bx + c = 0$.

   Then $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ and $x^2 - (r_1 + r_2)x + r_1r_2 = 0$, therefore

   $r_1r_2 = -\frac{b}{a}$ and $r_1r_2 = \frac{c}{a}$

2. Work some sample examples like the ones to be assigned.

Summary: Review the relations between the roots and the coefficients of a quadratic equation.

Suggested Problems: Odd numbered problems, pages 112 and 113.

Lesson Plan 12

Equations That May Be Reduced to Quadratics

Aim: To show the use of the multiplication theorem in solving fractional equations.

Suggested Method: Discussion, class participation.

Supplementary Materials: Paper, pencil, notebook.

Developmental Steps:

1. Show how fractional equations may be cleared using multiplication.
2. Show that equations containing radicals may sometimes be reduced to a quadratic equation. Discuss how to free such an equation of radicals. Work the example and check the values obtained.
3. Define extraneous roots.
4. How do you choose the correct root?
5. Solve an equation by using substitution to transform it to quadratic form.

Summary: Review least common multiples.

Suggested Problems:

Exercises,pages 114 and 115, even numbered problems. Page 116, exercises 2, 7, 10.

Lesson Plan 13

The Factor Theorem and Its Converse

Aim: To show what happens in factoring polynomials unrestricted to integers.

Suggested Method: Discussion with class participation.

Supplementary Materials: Notebook, paper, pencil.

Developmental Steps:

1. Prove: If r is a root of $ax^2 + bx + c = 0$, then $x - r$ is a factor of its left member; and conversely, if $x - r$ is a factor of $ax^2 + bx + c$, then r is a root of $ax^2 + bx + c = 0$.

   Show by definition of a root that $ar^2 + br + c = 0$. Then $ax^2 + bx + c =$

   $ax^2 + bx + c - (ar^2 + br + c) =$

   $a(x^2 - r^2) + b(x - r) = a(x - r)\ (x + r + b/a)$

   hence $x - r$ is a factor of $ax^2 + bx + c$.

2. Prove now that the converse is true. Why is the theorem important?

Summary: Review forming quadratic equations when the roots are given.

Suggested Problems:

Apply the factor theorem in exercises 1-10, article 4.11, page 112.

Lesson Plan 14

Inequalities Involving Polynomials of Second Degree

Aim: To show that graphs may be used to solve inequalities of this type.

Suggested Method: Discussion with class participation.

Supplementary Materials: Coordinate paper, notebook, ruler, pencil, paper, transparency grid.

Developmental Steps:

1. Solve a polynomial such as
   $x^2 + y^2 = 16$ graphically. Use a transparency grid.
2. Solve $x^2 + y^2 < 16$ graphically and compare the two and identify the points belonging to each. Use this transparency to overlay the one used in #1 above.
3. Compare the graphs: $x^2 + y^2 > 16$ and $x^2 + y^2 < 16$. Use transparencies.
4. Shade the section which applies to each.

Summary: Review the concept of inequalities.

Suggested Problems:

Exercises, page 121, odd problems. Test on topics 4.11 - 4.15 at the next class meeting.

Lesson Plan 15

Test

Aim: To evaluate topics 4.11 - 4.15.

Form equations with the given roots:

1. $1/2$, 2; $1 + \sqrt{2}$, $1 - \sqrt{2}$

Solve and check:

2. $5/x - 2/x + 1 = 17/20$; $\sqrt{3x + 4} + \sqrt{3x + 1} = 3$.

Solve graphically:

3. $x^2 + x > 2$.

4. $x^2 - x - 2 > 0$.

5. $x^2 < x + 6$.

CHAPTER 5

Other Special Types of Second Degree Equations

Other Special Types of Second Degree Equations

Behavioral Objectives:

1. Given the center and radius of a circle, students will write its equation.
2. Given an equation of a circle, students will find the center and radius of the circle.
3. Students will test a given equation for symmetry with respect to each coordinate axis and the origin; they will draw the curve.
4. Given an equation, students will determine its intercepts without plotting the graph of the equation.
5. Students will show the real number values of an equation and exclude other values.
6. Given the equation of an ellipse, students will draw its focus, and determine its foci, vertices and axes.
7. Given the equation of a hyperbola, students will find its locus, foci, vertices and axes, and asymptotes.
8. Given the general equation of a circle, students will translate axes and reduce to the general equation $x^2 + y^2 = r^2$.
9. Students will find the point or points of intersection of two simultaneous equations.
10. Given first and second degree curves, students will determine if they intersect at one, two, or no points.

Lesson Plan 1

The Circle

Aim: To show that the circle is a special type of second degree equation.

Suggested Method: Discussion with class participation.

Supplementary Material: Paper, pencil, and notebook.

Developmental Steps:

1. Define circle.
2. Write the general equation of the circle.
3. Show what happens when the center is at the origin.
4. Show that if two points are given which are end points of a diameter, the equation of the circle can be written.

Summary: Review finding the distance between two points.

Suggested Problems:

Exercises, page 123, all problems.

Lesson Plan 2

Discussion of the Equation of the Circle

Aim: To show that the general equation represents every circle.

Suggested Method: Discussion with class participation.

Supplementary Material: Coordinate paper, pencil, compass, notebook, ruler, transparency.

Developmental Steps:

1. Write the equation of the circle whose center is h, k with radius r passing through a point p (x,y).

   $(x - h)^2 + (y - k)^2 = r^2$; $x^2 + y^2 - 2hx - 2ky + h^2 + k^2 = r^2$.

2. Subtract r^2 from both sides and write in the form $x^2 + y^2 + dx + ey + f = 0$.

3. Compare 1 and 2 to demonstrate that D = 2h, e = -2k, and $f = h^2 + k^2 - r^2$.

4. Show that the center and radius of a circle can be found by completing the square.

Summary: Review solving quadratic equations, by completing the square.

Suggested Problem: Exercises, page 125-126, even numbered problems.

Lesson Plan 3

Properties of Graphs Which Represent Equations: Symmetry

Aim: To show that certain properties can simplify graphing.

Suggested Method: Discussion with class participation.

Supplementary Materials: Coordinate paper, ruler, pencil, notebook, graph board or transparencies prepared for 2, 3, 4, 5, below.

Developmental Steps:

1. Define symmetry.
2. Show an example of symmetry with respect to the x - axis. (Transparency)
3. Show symmetry with respect to the y - axis. (Transparency)
4. Demonstrate symmetry with respect to a line. (Transparency)
5. Show the axis of symmetry for a parabola. (Transparency)

Summary: Review symmetry with respect to x and y axis.

Suggested Problems: Exercises 2, 4, 6, 8, and 10, page 128.

Lesson Plan 4

Intercepts

Aim: To show that certain information about equations can be obtained without plotting their curves.

Suggested Method: Discussion.

Supplementary Materials: Coordinate paper, pencil, ruler, and notebook.

Developmental Steps:

1. Draw the graph of any equation cutting the x and y - axis. Using the equation substitute 0 for x and solve, then for y and solve. Compare with the points on the graph. (Use 1st and 2nd degree equations.)
2. Use equations that cut one, both and no axis.

Summary: Review solving equations with two variables.

Suggested Problems:

Exercises, page 129, all problems. Prepare for a test on topics 5.1 - 5.4.

Lesson Plan 5

Test

Aim: To evaluate topics 5.1 - 5.4.

1. Write the equations of the following circles:

   a. Center at (2, 4); radius 5.

   b. Center at (3, -5); radius 8.

2. Find the equation of the circle having (-6, -1) and (2,4) as the extremities of a diameter.

3. Find the center and radius of the circle $x^2 + y^2 + 10x - 6y - 18 = 0$.

4. Show that the line $4x + 3y = 25$ is tangent to the circle $x^2 + y^2 = 25$.

5. Find the intercepts and plot the graph of each:

   a. $x^2 + y^2 = 100$.

   b. $4x^2 + y^2 + 4x - 8 = 0$.

6. Test for symmetry with respect to each coordinate axis and the origin; draw the curve of each equation.

   a. $y^2 - 3x - 5 = 0$.

   b. $y = 4x^2$.

Lesson Plan 6

Excluded Values

Aim: To study equations whose coordinates are real numbers.

Suggested Method: Discussion with class participation.

Supplementary Material: Pencil, paper and notebook.

Developmental Steps:

1. Take the equation $y^2 = nx$ where n is any real number.
2. Y has real values when x is positive or 0. (y^2 can't be a negative number.)
3. Show that if an equation is solved for y in terms of x and gives rise to radicals of even order, the values of x that make the expression under such radicals negative must be excluded. Show the same is true process for y. ($y^2 = \sqrt{4x}$; $y = \sqrt{4x}$; $y = 2\sqrt{x}$; x cannot be negative; therefore all values of x that make a negative value under the radical are excluded.

Summary: Review solving equations with variables having limited values.

Suggested Problems:

Exercises, page 129, all problems.

Lesson Plan 7

The Ellipse

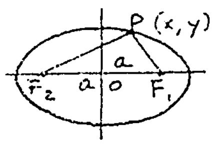
Aim: To show that the ellipse is a special second degree equation.

Suggested Method: Discussion with class participation.

Supplementary Material: Coordinate paper, pencil ruler, and notebook, transparencies.

Developmental Steps:

1. Define the ellipse.
2. Draw an ellipse and show its line of symmetry - the principle axis which passes through the foci. A transparency of an ellipse with overlays could be used here.
3. Show the vertices of the ellipse. (Overlay showing vertices)
4. Point out that the minor axis is a line which bisects the foci and is perpendicular to the principle axis. Sketch:



$F_1(c,0)$

$F_2(-c,0)$

Using the Pythagorean Theorem we have:

$\sqrt{(x - c)^2 + y^2} + \sqrt{(x + c)^2 + y^2} = 2a$; derive from this the standard equation of the ellipse. (A transparency of the sketch, fig. 5.3, p. 130, should be ready to use for this explanation.)

Summary: Review the standard equation of the ellipse and the definition of the ellipse.

Suggested Problems: Exercises, page 133-134, the odd problems.

Lesson Plan 8

The Hyperbola

Aim: To show that the hyperbola is a special second degree equation.

Suggested Method: Discussion with class participation.

Supplementary Material: Paper, pencil, notebook, prepared transparency of the hyperbola with overlays to show center, vertices, foci, etc.

Developmental Steps:

1. Define the hyperbola. (Use transparency.)
2. Compare the definition of the hyperbola with that of the ellipse.
3. Show that the hyperbola has a center, two vertices, and two foci, and is symmetrical about two axis. Point out a latus rectum.
4. Show that the hyperbola is not a closed curve but has two branches each opening outward and the two vertices lie between the foci and the center. The line segment joining the vertices is called the transverse axis of the hyperbola.
5. Using $\sqrt{(x + c)^2 + y^2} - \sqrt{(x - c)^2 + y^2} = \pm 2a$, derive the standard equation. (Use figure 5.5, p. 134 for a transparency. Get the information from it for the above equation.)

Summary: Review facts about the hyperbola.

Suggested Problems: Exercises 1-4, Parts a, c, d, page 138.

Lesson Plan 9

Asymptotes of the Hyperbola

Aim: To show the usefulness of asymptotes in sketching hyperbolas.

Suggested Method: Discussion with class participation.

Supplementary Materials: Coordinate paper, pencil, notebook, and ruler.

Developmental Steps:

1. Show that the lines through the vertices perpendicular to the transverse axis intersecting with lines perpendicular to the conjugate axis at distances b from the center, form a rectangle whose extended diagonals are the asymptotes of the hyperbola.
2. Show that knowing the vertex and using the asymptotes as guides, the graph of a hyperbola can be sketched.
3. Stress that the curve will not intersect the asymptotes. (Give examples.)
4. Show that the line $y = \frac{b}{a}x$ through the origin and (a,b) is an asymptote of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Summary: Review facts about the hyperbola.

Suggested Problems:

Exercises, pages 137-319, odd problems from 5-17.

Lesson Plan 10

Test

Aim: To evaluate topics 5.5 - 5.7.

1. Consider symmetry and excluded values; draw the curve of each.

   a. $x^2 + 2x + y^2 = 24$

   b, $x^2 + y^2 = 16$

2. Find the lengths of the axis, latera recta, and the coordinates of the foci of each of the following:

   a. $x^2 + 4y^2 = 16$

   b. $4x^2 + 9y^2 = 144$

3. Find the equation and draw the figure of the following ellipse: Foci at $(\pm 4, 0)$ vertices at $(\pm 5, 0)$.

4. Locate the vertices, foci, ends of the latera recta and draw the asymptotes and the curves of

   a. $\frac{x^2}{16} - \frac{y^2}{9} = 1.$

   b. $4x^2 - y^2 + 1 = 0.$

5. The point (x,y) moves so that its distance from the line y = 9/5 is 3/5 of its distance from the point (0,5). Find the equation of the locus.

Lesson Plan 11

Translation of Axes

Aim: To show that with new axes a curve can have a zero origin.

Suggested Method: Discussion with class participation.

Supplementary Materials: Paper, pencil, coordinate paper, ruler, notebook, transparency.

Developmental Steps:

1. Using the standard equation of the circle with center (h,k) when $h \neq 0$ and $k \neq 0$, construct a set of new axes through (h,k). Relate the new axes to the original axes using necessary addition or subtraction: $x = x^1 + h$ or $x = x^1 - h$; $y = y^1 + k$ or $y = y^1 - k$ $\therefore x^1 = x - h$ and $y^1 = y - k$.
2. Using $(x - h)^2 + (y - k)^2 = r^2$ let the point p $(h,k) = 0$; $x^2 + y^2 = r^2$ which results when axes are translated.
3. Show this to be true for other curves.

Summary: Review addition and subtraction of line segments in a plane.

Suggested Problems: Exercises, pages 141-142, problems 2 and 4.

Lesson Plan 12

More General Equations of the

Parabola, Ellipse, and Hyperbolas

Aim: To show that by translating axes we may derive the equation of the curves discussed under a less restricted choice of axes.

Suggested Method: Discussion with class participation.

Supplementary Materials: Coordinate paper, pencil, notebook and ruler.

Developmental Steps:

1. Show that the parabola whose vertex is (h,K), with axis on a line $y = k$ and foci f $(h + p, K)$ has an equation $(y - k) = 4p\ (x - h)$.
2. Show when p (h,k) is the center of an ellipse and the major axis is parallel to the x - axis the equation is $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$, when a and b are lengths of semi-major and semi-minor axes.
3. Show that if the major axis parallels the y - axis and has center (h,k), then $\frac{(y - k)^2}{a^2} + \frac{(y - h)^2}{b^2} = 1$ is the equation of the ellipse.
4. The hyperbolas with the same descriptions will be the difference of the above. That is $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ and

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

Summary: Review translation of axis.

Suggested Problems: Exercises, pages 144 and 145, problems 1, 3, and 5.

Lesson Plan 13

Simultaneous Equations; Intersection of Curves

Aim: To show that if two equations in x and y have one or more points in common they are simultaneous equations.

Suggested Methods: Discussion.

Supplementary Materials: Notebook, coordinate paper, pencil.

Developmental Steps:

1. Show that the locus of an equation consists of only those points that satisfy it.
2. Graph a 1st and 2nd degree equation and observe their intersection.
3. Solve the equations and compare with the graph for like points.

Summary: Review solving simultaneous equations.

Suggested Problems:

Exercises, pages146 and 147, problems, even numbers.

Lesson Plan 14

Intersection of First and Second Degree Curves

Aim: To show that a line and a comic section drawn on the same set of axes may or may not intersect.

Suggested Method: Discussion.

Supplementary Materials: Coordinate paper, pencil, notebook, ruler.

Developmental Steps:

1. Solve a first and second degree equation simultaneously and show that if they intersect the point of intersection satisfies both equations.
2. Show that the nature of the roots of the quadratic will determine the number of points that will be cut by the line.
3. Show that the line will cut the conic at most in two points if the roots of the quadratic are real and distinct.
4. Show what happens if the roots of the quadratic are imaginary; if they are real and equal, what happens?

Summary: Review the nature of roots.

Suggested Problems:

Exercises, pages 151-153, problems, even numbers.

Test on topics 5.8 - 5.11 at the next meeting.

Lesson Plan 15

Test

Aim: To evaluate topics 5.8 - 5.11.

1. Translate so that the new origin is at the point indicated. Draw both axes and the curve.

   a. $x^2 + y^2 - 2x - 4y - 2 - 0$ (1,2)

   b. $9x^2 + 4y^2 - 36x + 8y + 4 = 0$ - Translate to simplify.

2. Find the points of intersection and graph the following:

   a. $y^2 + 2x - 13 = 0$; $3x - 2y - 12 = 0$.

3. $2x + 5y = 10$; $y = \frac{8}{x^2 + 4}$.

4. Find the equation of the tangent to the parabola $x^2 - 4y = 0$ having a slope 2.

5. Show that the circles $x^2 + y^2 - 10x = 0$ and $x^2 + y^2 - 28x - 24y + 240 = 0$ are tangent to each other.

CHAPTER 6

Polynomial Functions and Polynomial Equations

Polynomial Functions and Polynomial Equations

Behavioral Objectives:

1. The student will find the remainder if a polynomial function f(x) is divided by (x - r) by the use of the remainder theorem.
2. The student will divide polynomials by using synthetic division.
3. The student will draw the graph of a polynomial function if he is given the function.
4. The student will form the equation of the lowest possible degree with integral coefficients, if he is given the roots of the equation.
5. The student will find all the rational roots of a polynomial equation if he is given the equation.

<u>Note</u>: The student should demonstrate the ability to successfully perform 4 of the above 5 behaviors.

Lesson Plan 1

Remainder and Factor Theorems

Aim: To teach the student how to solve problems using the remainder and factor theorems.

Suggested Method: Discussion, demonstration, and directed study.

Supplementary Materials: None

Developmental Steps:

1. Introduce the chapter by a discussion of pages 154 and 155 of the text. Discuss the meaning of a rational function and a polynomial equation.
2. Have the students read the remainder theorem and factor theorem in the text. Discuss each of these with them by giving examples.
3. Example: Find the remainder when

   $f(x) = x^3 - 2x^2 + 3x + 4$ is divided by $x - 3$.

   $f(3) = 3^3 - 2(3^2) + 3(3) + 4 = \underline{22}$
4. Example: Show that $x^n - a^n$ is divisible by $x + a$ when n is even:

   $f(x) = x^n - a^n$

   $f(-a) = (-a)^n - a^n$

   $= a^n - a^n$, if n is even

   $= 0$

   $\therefore x^n - a^n$ is divisible by $x + a$ when n is even.
5. Directed Study.

Suggested Problems:

Odd problems 1-18, on pages 156 and 157.

Lesson PLan 2

Synthetic Division

Aim: To teach the process of dividing by synthetic division.

Suggested Method: Check and answer questions on homework, discussion, demonstration, directed study.

Supplementary Materials: None

Developmental Steps:

1. Have students study pages 157-159 in the text. Discuss the rules given on page 158.

2. Example: By synthetic division, divide $x^3 - 2x^2 + 3x - 5$ by $x - 2$.

1	-2	+3	-5	2
	2	0	6	
1	0	3	1	

Have students prove this by actually performing the long division.

answer $x^2 + 3 + 1/x - 2$

3. Example: By synthetic division, divide $5x^4 - 10x^2 - 12x - 7$ by $x - 4$.

5	0	-10	-12	- 7	4
	20	80	280	1072	
5	20	70	268	1065	

answer $5x^3 + 20x^2 + 70x + 268 + \frac{1065}{5x^4 - 10x^2 - 12x - 7}$

4. Directed study.

Suggested Problems:

Problems 1-10, on pages 159-160.

Lesson Plan 3

Graphs of Polynomials

Aim: To teach a method of drawing the graph of polynomials equations.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: Overhead projector with its materials.

Developmental Steps:

1. Have students study pages 160-161 of the text before discussing the example given on page 160.
2. Point out how the remainder theorem is used in finding the y value for each value of x. Synthetic division is used to find this remainder.
3. Example: Draw the graph for $y = x^2 + 2x - 3$.

   The table of values may be drawn by using the transparencies and the overhead projector. Have the students help find these values. Point out the curve is continuous when you draw it in. (See note below)
4. Directed study.

Summary: Review the remainder theorem and the factor theorem. Review the importance of the critical points along the graph of a polynomial.

Suggested Problems:

Odd problems 3-17, on page 161.

Note: The teacher may discuss the process of finding the maximum and minimum points for the graph during this lesson. This will prove helpful in drawing the curve.

Lesson Plan 4

Location of Real Roots

Aim: To teach a method for locating between successive tenths real roots for an equation.

Suggested Method: Check and answer questions on homework, discussion on pages 161-163 of the text, demonstration, and directed study.

Supplementary Materials: Graph board.

Developmental Steps:

1. Discuss pages 161-162 of the text with students. Point out that the real roots of an equation are given by the abscissas of the points where the graph of the equation crosses the x-axis.
2. Discuss the principle given on the bottom of page 162 of the text. Have students join in this discussion by answering or asking questions.
3. Example: Locate between successive tenths, one real root of the equation $x^3 + 3x - 2 = 0$.

   $f(0) = -2$, $f(1) = 2$

   $f(1.5) = -0.375$, $f(0.6) = .061$

   Answer: Therefore one real root is between 0.5 and 0.6.

   Have students help in solving this example.
4. Directed study

Summary: Review the principle on page 162 of the text.

Suggested Problems:

Odd problems 3-21 on page 163.

Note: Test for next class meeting on pages 154-163.

Lesson Plan 5

Test

Aim: To administer a test on pages 154-163 of the text.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test.

Suggested Problems:

Two problems such as #1 on page 156.

One problem such as #5 on page 156.

One problem such as #3 on page 159.

One problem such as #5 on page 160.

One problem such as #10 on page 160.

One problem such as #1 on page 161.

One problem such as #9 on page 161.

One problem such as #3 on page 163.

One problem such as #5 on page 163.

Lesson Plan 6

Roots of Equations

Aim: To teach a method for forming equations from the roots of the equation.

Suggested Method: Discussion on pages 164-167 of the text, directed study.

Supplementary Materials: Test papers.

Developmental Steps:

1. Hand back test papers and answer questions about missed problems.
2. Have students study pages 164-167 of the text. Discuss each of the theorems given in this section. The proofs may be discussed for each theorem proved in the text. Be sure the students know what the theorems say.
3. Discuss the example problem given on page 116 of the text. Have the students arrive at the answer for themselves.
4. Example: Form the equation for the following roots: (1, -1, 2).

   $(x - 1)(x + 1)(x - 2) = 0$ Have students solve this example.

   Answer: $\boxed{x^3 = 2x^2 - x + 2 = 0}$
5. Directed study.

Summary: Review the fundamental theorem of algebra and point out how it is used to prove other theorems given in this section. Review the method for writing the equation from its roots.

Suggested Problems: Problems 1-16 onpages 167-168.

Lesson Plan 7

Roots of Equations

Aim: To continue the study of roots of equations, and how to find the equation if the roots are given.

Suggested Method: Check and answer questions on homework, demonstration and directed study.

Supplementary Materials: None.

Developmental Steps:

1. Point out that imaginary roots occur in conjugate pairs. Show how this may be used to work the example problem on page 167 of the text.
2. Example: Solve the equation $2x^4 + 6x^3 + 11x^2 + 12x + 5 = 0$ given that -1 is a double root.

   $(x + 1)(x + 1) = x^2 + 2x + 1$

   $2x^4 + 6x^3 + 11x^2 + 12x + 5 = 0$

   $\frac{2x^4 + 6x^3 + 11x^2 + 12x + 5}{x^2 + 2x + 1} = 2x^2 + 2x + 5$

   The roots for $2x^2 + 2x + 5 = 0$ are $\frac{-1 \pm 3i}{2}$

   Therefore the roots are $(-1, \frac{-1 + 3i}{2}, \frac{-1 - 3i}{2})$.

3. Directed study.

Summary: Review all the new terms found in this section.

Suggested Problems:

Problems 17-28 on page 168.

Lesson Plan 8

Rational Roots

Aim: To teach a method for finding all rational roots for a given equation.

Suggested Method: Check and answer questions on homework, discussion on pages 169-171 of the text, demonstration, directed study.

Supplementary Materials: None.

Developmental Steps:

1. Have students study pages 169-171 of the text. Discuss these theorems and example problems with the students. Have students answer questions about each.

2. Example: Find the rational roots for $2x^2 - x - 6 = 0$.

   integral factors of -6 are (1, -1, 2, -2, 3, -3, 6, -6)

   integral factors of 2 are (1, -1, 2, -2)

   The possible rational roots are:

   (1, -1, 2, - 2, 3, - 3, 6, - 6, 1/2, -1/2, 3/2, -3/2)

   Answer: Only 2 and -3/2 are roots for the equation.

3. Directed study.

Summary: Review the theorem and its corollary in this section, and the method used to determine the rational roots for a given equation.

Suggested Problems: Problems 1-17 on page 171.

Lesson Plan 9

Rational Roots

Aim: To continue the study of the method for finding the rational root of an equation.

Suggested Method: Check and answer questions on homework, demonstration, directed study.

Supplementary Materials: None.

Developmental Steps:

1. Have students put homework problems on the board. Discuss their solutions and answer any questions that students have.
2. Example: Find the rational roots for $x^3 - 8x^2 + 13x - 6 = 0$

   Have students solve this example in class.

   Possible rational roots are: (1, -1, 2, -2, 3, -3, 6, -6)

   1 is the only rational root.
3. Directed study.

Summary: None.

Suggested Problems:

Problems 18-32 on page 172.

Note: Test for class meeting on pages 164-172.

Lesson Plan 10

Test

Aim: To administer test on pages 164-172.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test.

Suggested Problems:

One problem such as #2 on page 167.

One problem such as #7 on page 167.

One problem such as #11 on page 168.

One problem such as #16 on page 168.

One problem such as #19 on page 168.

One problem such as #23 on page 168.

One problem such as #2 on page 171.

One problem such as #8 on page 171.

One problem such as #21 on page 172.

One problem such as #31 on page 172.

CHAPTER 7

Permutations, Combinations and Probability

Permutations, Combinations and Probability

Behavioral Objectives:

1. The student will find the number of permutations of n elements taken r at a time.
2. The student will find the number of permutations of n elements when some of the elements are alike.
3. The student will find the number of combinations of n elements taken r at a time.
4. The student will determine probabilities for mutually exclusive events, independent events and repeated trials.
5. The student will demonstrate his knowledge of the meanings of permutations and combinations by writing the permutations and combinations for 4 given elements taken three at a time.

Note: The student should demonstrate the ability to successfully perform 4 of the above 5 behaviors.

Lesson Plan 1

Permutations

Aim: To introduce permutations and teach the method of finding the number of permutations of n elements taking r at a time.

Suggested Method: Lecture, demonstration, and directed study.

Supplementary Materials: None.

Developmental Steps:

1. Define "permutation" and give examples using the five elements (a, b, c, d, e).
2. Discuss the meaning of n! Use examples such as $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, and $3! = 3 \cdot 2 \cdot 1$, and $n! = n(n-1)(n-2)\ldots(3 \times 2 \times 1)$.
3. Introduce the symbols $p(n,n)$, ($_nP_n$ may be used also) as a way of representing the number of permutations of n objects.
4. Discuss the three formulas:

   a. $P(n,n) = n!$

   b. $P(n,r) = n(n-1)(n-2)\ldots(n-r+1)$

   c. $P(n,r) = \dfrac{P(n,n)}{(n-r)!}$

5. Directed study.

Summary: Review the meaning of the new terms introduced in this section.

Review the formulas needed for solving exercises.

Suggested Problems:

Problems 1-5 and 7-9, page 373.

Lesson Plan 2

Permutations

Aim: To teach the method of determining the number of permutations when some objects are alike.

Suggested Method: Check and answer questions on homework, demonstration and directed study.

Supplementary Materials: Straightedge.

Developmental Steps:

1. Explain the example problem given in the text on pages 371 and 372. Use straightedge to help write out this example. Use this example to derive (with the help of students) the formula $P = \frac{n!}{s!}$. Point out that this is the formula for finding the number of permutations for n objects where s of them are alike.
2. Using the above formula, show that it can be generalized to find the number of permutations for n objects where s of them are alike and also t of them are alike. $P = \frac{n!}{s!\,t!}$
3. Example: Find the number of distinct permutations from the letters in the word "teeth". $P = \frac{n!}{s!\,t!} = \frac{5!}{2!\,2!} = 30.$
4. Directed study.

Summary: Review the formulas developed in this lesson.

Suggested Problems: Problems 6, 10-17, on pages 373 and 374.

Lesson Plan 3

Combinations

Aim: To introduce combinations and teach the method for finding the number of combinations for n objects taken r at a time.

Suggested Method: Check and answer questions on homework, demonstration and directed study.

Supplementary Materials: None.

Developmental Steps:

1. Define combination: A set of distinguishable objects in which the order or arrangement of the objects is not important.
2. Using the letters A, B, C write all the possible combinations taken 2 at a time. The students should help with this. (AB, AC, BC) Compare these combinations with the permutations for the three letters taken 2 at a time. (AB, BA, AC, CA, BC, CB)
3. Show how the formula $C(n,r) = \frac{p(n,r)}{r!} = \frac{n!}{r!(n-r)!}$ may be found from the permutations formula.
4. Example: Find the value of C (5,4).

   $C(5,4) = \frac{5!}{4!(5-4)!} = \frac{5.4!}{4!\ 1!} = 5.$
5. Directed study.

Summary: Review the meaning of combinations and the formula for finding the number of combinations for n objects taken r at a time.

Suggested Problems: Problems 1-8 on page 377.

Lesson Plan 4

Binomial Theorem

Aim: To teach the binomial theorem and how combinations may be used in the binomial theorem.

Suggested Method: Check and answer questions on homework, question and answer discussion, demonstration, directed study.

Supplementary Materials: Overhead projector with its materials.

Developmental Steps:

1. Have students study pages 374-376. Discuss the example on page 375 of the text. Use this example to help develop the binomial theorem. (Use overhead projector and transparencies to show the binomial theorem.)
2. By substitution into the binomial theorm show that $(a + b)^n =$
   $a^n + C(n,1)\ a^{n-1}b + C(n,2)\ a^{n-2}b^2$
   $+ \ldots + C\ )n,r)\ a^{n-r}b^r + \ldots +$
   $C\ (n,n-1)\ ab^{n-1} + C\ (n,n)b^n$.
3. Explain the development of the formula
   $C\ (n,1) + C\ (n,2) + \ldots + C\ (n,n) = 2^n - 1$ as given in the text on page 377.
4. Directed study.

Summary: Review the formulas developed in this lesson.

Suggested Problems: Problems 9-15 on page 377.

Note: Test for next class meeting on pages 369-377.

Lesson Plan 5

Test

Aim: To adminster a test on pages 369-377.

Suggested Method: Check and answer questions on homework, administer test.

Supplementary Materials: Copies of test.

Suggested Problems:

One problem such as #1 on page 373.

One problem such as #2 on page 373.

One problem such as #5 on page 373.

One problem such as #9 on page 373.

One problem such as #14 on page 373.

One problem such as #2 on page 377.

One problem such as #5 on page 377.

One problem such as #9 on page 377.

One problem such as #12 on page 377.

One problem such as #13 on page 377.

Lesson Plan 6

Probability

Aim: To introduce probability and teach the method for determining the probability of specific events.

Suggested Method: Discussion on pages 377-379 of text, demonstration, directed study.

Supplementary Materials: Test papers, overhead projector with a transparency for the 36 outcomes for a toss with a pair of dice.

Developmental Steps:

1. Hand back test papers and discuss test problems.
2. Discuss the formula $p = \frac{h}{h + f}$ as given in the text on page 378. Poin out the fact that the probability for an event happening is always a number between zero and one, inclusive.
3. After students have studied the example problems on page 378 and 379 of the text, discuss by asking questions about each. (Use overhead projector with its materials here.)
4. Directed study.

Summary: Review the new terms in this section and review the formula for finding the probability of an event.

Suggested Problems: Problems 1-8 on pages 379-380.

Lesson Plan 7

Mutually Exclusive Events, Independent Events and Repeated Trials

Aim: To teach the methods for finding probabilities involving mutually exclusive events, independent events and repeated trials.

Suggested Method: Check and answer questions on homework, discussion, directed demonstration study.

Supplementary Materials: None.

Developmental Steps:

1. Define mutually exclusive events and give an example.
2. Example: If 2 balls are drawn at random from a box containing 5 white balls and 4 red ones, what is the probability that both are the same color?

   Probability that both are white $= \frac{C(5,2)}{C(9,2)} = \frac{10}{36}$

   Probability that both are red $= \frac{C(4,2)}{C(9,2)} = \frac{6}{36}$

   Probability that both are the same color $= \frac{10}{36} + \frac{6}{36} = \frac{16}{36} = \frac{4}{9}$
3. Define "independent events" and give an example.
4. Have students study the example problems given on pages 383 and 384 of the text. Discuss the questions that are asked about each of these examples.
5. Use the example on page 385 of the text to show how the binomial expansion may be used to solve problems involving repeated trials.
6. Directed study.

Summary: Review the meanings of mutually exclusive events, independent events, and repeated trials.

Suggested Problems: Odd numbered problems 1-20 on page 385-386.

Lesson Plan 8

Review of Chapter 7

Aim: To continue the study of mutually exclusive events, independent events, and to review the more difficult topics in chapter 7.

Suggested Method: Check and answer questions on homework, question and answer discussion on chapter 7, and directed study.

Supplementary Materials: Overhead projector with its materials.

Developmental Steps:

1. Have students put several of the more difficult homework problems on the board. Have the students explain the problem using teacher help if necessary.
2. Discuss any section in chapter 7 that students are having trouble with. Example problems from these sections may be worked. Use the overhead projector to work these problems.
3. Directed study.

Summary: Review the meaning of the new terms for chapter 7. Also review the formulas for probability, permutations and combinations.

Suggested Problems: Even numbered problems 1-20 on pages 285-286.

Note: Test for next class meeting on chapter 7.

Lesson Plan 9

Test

Aim: To administer test on chapter 7.

Suggested Method: Check and answer questions on homework, administer test

Supplementary Materials: Copies of test.

Suggested Problems:

One problem such as #1 on page 373.

One problem such as #4 on page 373.

One problem such as #2 on page 377.

One problem such as #5 on page 377.

One problem such as #12 on page 377.

One problem such as #2 on page 379.

One problem such as #3 on page 379.

One problem such as #1 on page 385.

One problem such as #4 on page 385.

One problem such as #10 on page 386.

One problem such as #13 on page 386.