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ABSTRACT

This document is a first-person account by the mathematics coordinator at Temple University dealing with the segment of the TTT (Trainers of Teacher Trainers) project held at the Reynolds Elementary School. The stated purpose of this segment was to provide on-the-job experience in the area of elementary school mathematics. The basic program procedure is described as follows: TTT clinicians tried out contemporary teaching strategies and materials while working with fifth- and sixth-grade mathematics pupils; the classroom teachers also became involved in the planning and utilization of the mathematics laboratory; during the third year the mathematics laboratory moved to larger quarters, and the emphasis changed from assisting pupils to assisting teachers. The author defines the centralized mathematics laboratory which was used in this project as a room separate from the regular classroom, with pupils from all grades going to this laboratory for part of their mathematics instruction. Materials needed for such a laboratory are described in detail; suggestion for activities are also given. (JA)

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TEMPLE UNIVERSITY

of the Commonwealth System of Higher Education

COLLEGE OF EDUCATION

TRIPLE "T" PROJECT

MONOGRAPH SERIES

2

TTT DEVELOPS A MATH LABORATORY

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PREFACE

For the past four years, with financial support from the U. S. Office of Education, Temple University has been conducting a doctoral level program to prepare leadership personnel in the areas of mathematics and English education. The Trainers of Teacher Trainers program or "Triple T", as it has been called, has focused solely upon education in the urban environment.

The primary objective of the program was to provide teacher trainers and curriculum and instruction specialists with the insights and competencies necessary to provide leadership in inner-city education. This objective was achieved by a three-phase program: academic and professional experiences within the university; internships within the university and inner-city schools; realistic community experiences within the various urban communities of Philadelphia.

During its operational period, thirty-one doctoral students (clinicians) from elementary and secondary schools, colleges, universities, and social organizations were full-time participants. The majority of these were from minority groups. In addition to the student participants there were more than fifty college and school personnel and no less than one hundred community people who had an active involvement. The project indeed brought together, with singular purpose, representatives from the community, public schools, and various colleges of the University.

From the outset an integral part of the program was the creation of innovative curricular and instructional materials and projects, also a considerable number of papers were written and extensive research was conducted by the participants, an associated research professor, and the project director. The research efforts dealt with virtually every aspect of the project and at this point in time is nearing completion. The materials that follow in this publication, and others in the series, are a means of disseminating the results of TTT's efforts with the hope that others interested in similar problems can profit by the program's experiences. It is also hoped that several of TTT's innovative approaches would be of practical use to schools and teacher training institutions in the common quest to improve education and the training of teachers.

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Dr. Ann McPherson Wilderman is an Associate Professor of Mathematics Education at Temple University, Philadelphia, Pennsylvania. Prior to her appointment at Temple University she had considerable experience as an elementary school classroom teacher. Presently she is teaching Mathematics Education methods courses to pre-service and in-service graduates and undergraduates. She has conducted many workshops on elementary mathematics and the laboratory approach to teaching. Dr. Wilderman has published articles in various educational journals including: The Arithmetic Teacher, School Science and Mathematics, Audiovisual Instruction, and Highlights for Children. Dr. Wilderman is co-author of two current books for teachers, Metric Measure Simplified and Mathematics Card Games.

From 1970 to 1973, Dr. Wilderman coordinated the mathematics education phase of TTT. She worked with clinicians at Vaux Junior High School and Reynolds Elementary School as well as teaching the appropriate methods courses in mathematics education to the clinicians. She directed the clinicians in the establishment of three mathematics laboratories: (1) at the Martin Luther King Community Center; (2) Vaux Junior High School; (3) Reynolds Elementary School.

TTT DEVELOPS A MATH LABORATORY

INTRODUCTION . . .

From September of 1970 until the end of the school year in 1973, a segment of the TTT project (Trainers of Teacher Trainers) of Temple University was at the Reynolds Elementary School. With financial support from the United States Office of Education, this doctoral level program was designed to prepare personnel with awarenesses and indepth experiences in three areas: (1) the university's regular doctoral program; (2) on the job experience of a continuing nature in an inner-city public school; and (3) extensive experience in the urban environment.

At the Reynolds Elementary School various groups of TTT clinicians participated in the second phase of the program, i.e., the on the job experience in the area of elementary school mathematics.

Briefly, during the first year, the clinicians worked primarily with pupils, trying out contemporary teaching strategies and materials. The group met and worked from a closet, but were able to successfully teach fifth and sixth grade mathematics for four classes of pupils for the entire school year. During the second year space was available and the group had its own room which was developed into a mathematics laboratory. The clinicians each worked with a whole class of pupils on a daily basis for the entire school year incorporating the mathematics laboratory at least two days a week. The classroom teachers also got involved in the planning and utilization of the mathematics laboratory. During the third year, the mathematics laboratory was moved to larger quarters, i.e., two adjoining rooms with a folding door. The primary emphasis moved from the pupils to the teachers in that the group wanted to assist any teacher who wished to incorporate laboratory experiences into their ongoing mathematics program. Workshops were conducted on a regular basis during the lunch hour and after school one day a week. Teachers were encouraged to invite the clinicians into the classroom to work with pupils or to send small groups of pupils to the mathematics laboratory for a lesson experience.

As the mathematics coordinator from Temple University I worked closely with everyone involved in the TTT program at Reynolds Elementary School. Summarizing, I feel that TTT caused change in the school and school personnel in the following ways:

(1) <u>In terms of the Administration</u>: I feel that TTT helped them to become more aware of the need for an ongoing mathematics program throughout the school.

(2) <u>In terms of Teachers</u>: I feel that we were able to introduce them to the idea of a mathematics laboratory as a resource for supplementing their ongoing mathematics program in the self-contained classroom. The cooperating teacher, assigned to work with the TTT personnel, eventually was able to assume a leadership role in the school mathematics program and has been a vital resource person at the school since TTT clinicians are no longer at the school.

(3) In terms of Pupils: Since TTT was mainly concerned with staff development at the school, pupils benefited from what their teachers were able to learn. However, when the mathematics laboratory was established pupils came to the laboratory as a part of their classroom work and on their own when time permitted and thus reaped benefits.

(4) In terms of TTT Clinicians: The clinicians were able to establish good rapport with the teachers and principal through the mathematics laboratory. They were able to assume a leadership role in the school mathematics program and thus, advance their own knowledge of the functioning elementary school and the role mathematics plays in the total school program.

(5) In terms of Temple faculty: I was the only Temple faculty member directly involved at Reynolds and I feel that my constant contact with elementary inner-city teachers was an excellent on cite learning experience. I was also able to work closely with the TTT clinicians in introducing them to basic teaching strategies for elementary school mathematics and the vast number of manipulative materials available to them in working with pupils.

TTT's CONTRIBUTION TO REYNOLDS ELEMENTARY SCHOOL

As coordinator I look back on the three years at Reynolds Elementary School and feel that the most salient contribution of TTT was the introduction of a mathematics laboratory in the school and training those interested teachers how to utilize the materials in the laboratory in their own classrooms. This was not a project which took place quickly, rather it evolved as the clinicians and Temple University TTT were accepted at the school for what we had to offer. We did not force our project or ideas on anyone. In fact, the first year we just went about teaching the classes assigned to us, attended faculty meetings, helped with other functions at the school when asked, but did not talk about TTT unless we were asked.

When given a room during the second year we were able to invite teachers in to see what we were doing, but again, we did not make suggestions that they change "their" way of teaching mathematics. Only when asked, did we share ideas and make an occassional suggestion. During the third year, the teachers were more eager to learn about materials and new ideas and ways of teaching mathematics. The laboratory was attractive and pupils came in often to see what was going on.

Thus, I feel that we continued to work on a low key all the time we were there and maybe this had something to do with our small contribution to the school. Outsiders must first be accepted before they can make a contribution. Thus, we did not rush, but built up salient rapport gradually.

WHAT IS A MATHEMATICS LABORATORY?

After reading, visiting, and discussing various interpretations of a mathematics laboratory, the group of TTT representatives at Reynolds Elementary School decided that the laboratory would be an approach to learning as well as a physical setting for learning mathematics. Even though the room would always have a teacher and be filled with interesting manipulative materials, the difference from a regular self-contained classroom, would lie in the environment. A well structured program for each grade level would be maintained in a leisurely way, without pressure of tests and grades, allowing pupils to work on mathematical content and related materials meeting his own needs.

In our reading and research we found that although specific mathematics laboratories varied in type, style and organization, the following characteristics were common to all:

1. Essentially the mathematics laboratory should have a free, active, creative appearance which enhances the child's natural curious instincts and stimulates investigations which might never take place in the regular traditional classroom. For example, when a teacher-made balance scale is placed on a table in the mathematics laboratory with various objects near by, curious pupils will venture to the scale and begin to investigate. When pupils are ready, they will ask questions; thus, setting the stage for a unit on measurement.

2. Learning should be through experience: pupils exploring mathematical concepts through a variety of embodiments, and thus, developing a flexible mind are able to think and analyze a problem situation before actually solving a problem. For example, pupils thoroughly enjoy working with attribute materials. A first activity might be simply straightening up a set of "A Blocks" to see what

pieces there are in the set. A following activity might be to play a "one difference circle game". Other activities would follow as pupils made additional discoveries about the blocks and set relations. Later the teacher might suggest to the pupils that they investigate the "People Pieces". The beginning activities would be the same as for the "A Blocks", but pupils do not readily see the similarities between the two sets of materials. Concepts become clearer as pupils transfer knowledge from one set of materials to another -- the multi-embodiment approach advocated by Dienes.

3. In a child-centered laboratory the atmosphere should become one of resourcefulness, self-confidence, independence, patience and competence. Of course, these positive qualities are learned along with mathematics and aid the child to feel better about himself and the learning process. There are no "right" or "wrong" answers, but questions which guide the way to inductive discovery. Thus, many activities in the laboratory do not require written work. The teacher's goal is to provide each child the opportunity to stretch his mind as much as possible.

4. Pupils should learn to enjoy working by themselves or in small groups with their peers as they investigate a wide variety of interesting new ideas. Often peer discussion can be of more value to a pupil than a talk or an answer from the teacher. Pupils discuss from conviction and are willing to reflect upon the ideas and discoveries of other pupils. Group work and cooperation must be learned; it comes gradually as pupils learn to trust each other and are sure that the discussion is worthwhile to them.

5. The mathematics laboratory should provide for many different interests and abilities which seem to exist in every group. An initial investigation by a group and directed by the teacher may end, but for an individual pupil the investigation may have conjured up ideas in his mind which he wants to look at further. Thus, the pupil may work on ideas with materials as long as his interest is maintained.

6. Pupils should learn to appreciate and enjoy mathematics to a greater extent than ever before in the laboratory because they are encouraged to think for themselves, act for themselves, and actually take some responsibility for what they learn.

7. Experiences with a variety of materials incorporates the use of as many sensory stimuli as is possible in teaching. The old Chinese proverb says, "I hear and I forget, I see and I remember, I do and I understand". Pupils see, touch, and hear actively as they learn in a laboratory setting.

Thus, the atmosphere in a mathematics laboratory should shift the emphasis from the teacher to the child; from teaching to learning; from the adult world to the world of the child; and from the abstract to the concrete. Ultimately the child should be able to relate mathematical symbols and vocabulary to abstract models; to organize and manipulate symbols; to think creatively; perceive the structure of mathematics; and to have a more favorable attitude toward and appreciation for mathematics.

HOW WE BEGAN AT REYNOLDS

Deciding what laboratory setting is best for the pupils in a given school depends upon two factors: (1) the facilities available in the school; and (2) the methods, classroom setting, et.al to which the teachers and pupils have become accustomed. There were five possible mathematics laboratory organizations available for consideration at Reynolds Elementary School: The Corner Laboratory; (2) The Team Room Laboratory; (3) The Centralized Laboratory; (4) The Laboratory on Wheels; and (5) The Laboratory Period.

Briefly, The Corner Laboratory is an interest center in a self-contained classroom. The teacher supplies the center with mathematical teaching devices and activity cards designed to (1) develop, (2) reinforce, (3) vertically and horizontally enrich the learning of mathematics. The materials in the center should have a direct relationship to the mathematical topics being studied at the time. Thus, materials and activity cards would be changed as new topics are introduced to the class. For some pupils activities would be required as reinforcement of a topic being studied. For other pupils the activities would be voluntary as enrichment. It is also possible that pupils could, themselves, suggest activities and/or prepare materials to be utilized at the corner laboratory.

The Centralized Laboratory is a room, separate from the regular classroom. This room is supplied with an abundance of mathematical materials and activities for pupils. The Centralized Laboratory is directed by a mathematics specialist at the school who schedules classes and works with teachers to supplement the learning of mathematics in individual classrooms. The specialist is also coordinator of the school mathematics program helping teachers know what activities best supplement mathematics concepts being taught at various levels in the child's mathematical development.

The Team Room Laboratory is utilized by a departmentalized, open classroom, or team teaching school organization where pupils exchange classes or move about for instruction. Thus, the room or area where mathematics is taught would be equipped with manipulative materials and aids. This room becomes a laboratory for pupils during their mathematics class.

The Laboratory on Wheels is created when mathematics materials and activity cards are shared by a group of teachers. The materials are placed on a movable cart of some type and shared on a regular basis by a special group of teachers and pupils. If purchasing materials is a problem and a specialist is not available, a grade level or group of teachers can prepare a laboratory on wheels labeling all materials and activity cards. A schedule could be prepared and strictly adhered to, so that problems do not arise in sharing the materials.

The Laboratory Period is simply a class period set aside at regular intervals for exploration and discovery of mathematics ideas through manipulative materials. Ideally, large, more expensive materials would be purchased by the school and shared by classes through a resource center or library. Smaller items would be constructed by pupils and kept in individual classroom. The laboratory period offers little in the way of supplementing a coordinated mathematics program, since materials must be checked out from a central location and one or even two laboratory periods a week offers little motivation to the pupils.

Most schools should begin small without investing a large amount of money or time in materials until they are sure that the teachers are going to be willing to put time and energy into developing materials and activities.

At Reynolds Elementary School, TTT was providing funds for the purchase of materials. We were also acting as mathematics specialists at the school, thus, it was decided that we would combine the Centralized Laboratory with the Laboratory Period to meet the school needs. As mathematics specialists we could select the best materials for a Centralized Laboratory and since we were there at the school on a daily basis we had time to prepare many home-made aids and activity cards.

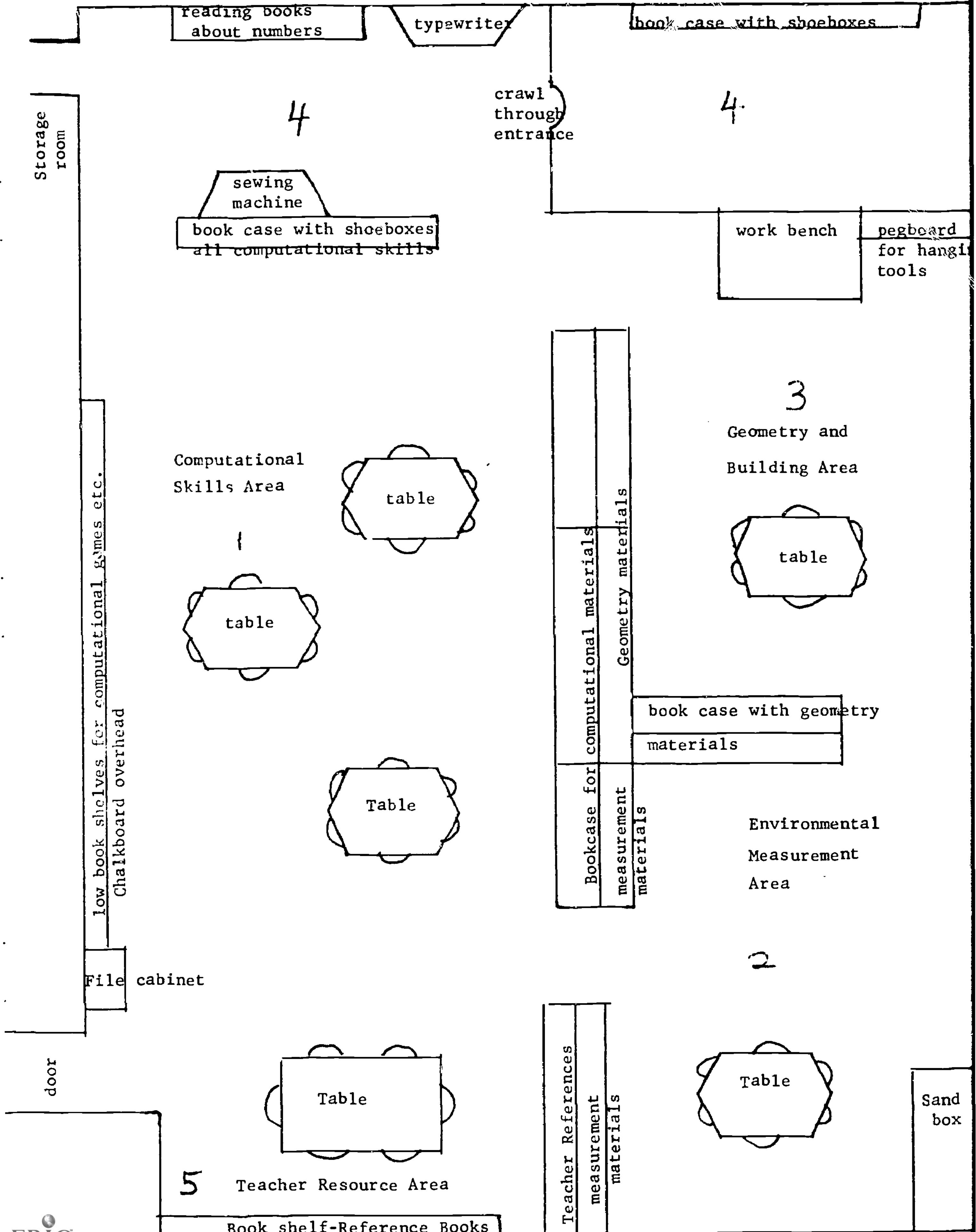
WHAT IS A CENTRALIZED MATHEMATICS LABORATORY?

A Centralized Mathematics Laboratory is set up in many schools where pupils from all grades, or one each for the upper and lower grades go to the laboratory for part of their mathematics instruction. The ideal mathematics laboratory program would include a specialist in the laboratory to plan and instruct pupils with the cooperation of the classroom teacher. In a centralized laboratory a specialist would (1) plan schedules; (2) take care of ordering and utilization of materials; (3) create activities; and (4) use his/her special training and talents to help teachers and pupils enjoy and understand mathematics. All of the mathematics materials are in one central room and concentrated efforts are made in the Centralized Laboratory to aid each pupil with his special needs. Time could also be set aside for teachers to explore and plan activities for their pupils in conjunction with the classroom learning of mathematics. It might also be possible for the mathematics specialist to have a mathematics club which meets weekly for special projects which interested pupils might wish to pursue.

At Reynolds we planned for about 6 weeks, making bookshelves, tables, and other hardware. We divided the room into the following areas:

1. computational skills area
2. environmental measurement
3. geometry and building
4. the quiet place
5. resource area for teachers

In each area commercial and home-made materials were stored on open shelves so that they could be seen and used by pupils at will. On the following page is a diagram of the Centralized Laboratory established at Reynolds Elementary School. Following is a list of the materials and educational supplies which we constructed and purchased for the laboratory.


reading books about numbers
typewriter
book case with shoeboxes
Storage room
4
crawl through entrance
4.
sewing machine
book case with shoeboxes all computational skills
work bench
pegboard for hanging tools
3
Geometry and Building Area
Computational Skills Area
table
1
table
low book shelves for computational games etc.
Chalkboard overhead
Bookcase for computational materials
Geometry materials
measurement materials
book case with geometry materials
Table
Environmental Measurement Area
2
File cabinet
door
Table
Teacher References
measurement materials
Table
Sand box
5
Teacher Resource Area
Book shelf-Reference Books

WHAT MATERIALS ARE NEEDED FOR A MATHEMATICS LABORATORY?

Materials for a Centralized Mathematics Laboratory fall into three categories: (1) physical equipment such as tables, bookshelves etc; (2) commercial materials such as teaching aids, blocks, etc. which are too complicated to construct; and (3) home-made materials such as activity cards, shoeboxes, card games etc.

PHYSICAL EQUIPMENT In a beginning laboratory two things are essential. (1) plenty of storage space, and (2) tables and chairs rather than traditional classroom desks.

The Centralized Laboratory is usually utilized by an entire school or if a laboratory is created for the primary grades and another for the upper grades, half of the pupil population. Thus, bookshelves, file cabinets and closet space are essential. The laboratory specialist will want to have the materials labeled and organized so that he/she can locate every item in the laboratory with ease. There may be some items which will not be out in the laboratory at all times. If these are expensive items, it may be necessary to have a closet with a lock on it. A file cabinet is needed for keeping records of groups and individual pupil progress and for filing activity sheets to accompany various materials in the laboratory.

As the Centralized Laboratory becomes established in the school other things such as carpets, curtains, and attractive bulletin boards may be added. The physical setting of the mathematics laboratory helps establish the environment and general atmosphere for the pupils. Thus, it should be planned carefully.

Physical equipment for the laboratory at Reynolds included the following:

(1) Five tables were constructed from triwall and pressed board. The tops of the tables were covered with plastic contact paper. Another table, in the teacher resource area, was a regular table from the school.

(2) The bookcases were constructed from triwall for storage of materials and also as room dividers. The bookcases were painted with pleasant colors and the shelves were covered with plastic coated contact paper.

(3) Two 9' x 15' carpets were purchased. One was for the Computational Skills Area and the other extended through the Environmental Measurement Area to the work bench in the Geometry and Building Area.

(4) Small throw rugs were placed in the Quiet Area for pupils to use when they wanted to work on the floor.

(5) Inexpensive curtains were made from clear plastic. Felt geometric shapes were stapled to them to add color.

(6) A work bench was purchased through a school supply house in the area.

(7) The sewing machine and typewriter were donated.

(8) The file cabinet was school property.

COMMERCIAL MATERIALS Manipulative materials abound on the educational market as well as in local department stores. This surge of mathematics materials made it necessary for us to evaluate our needs for the mathematics laboratory before any materials were purchased. At Reynolds Elementary School we used the following checklist questions when selecting commercial materials for the laboratory.

1. Is the aid educationally sound? We found that some aids were designed and manufactured by non-educators. As a result, they were not always based on principles that we felt were mathematically sound and did not comply with the general spiraling of ideas in the elementary school.

2. Is the aid versatile? Nothing enhances the value of an aid as much as if it can be used by the teacher and pupil to develop more than one mathematical idea. We were willing to invest in aids that could have several uses in the mathematics program, rather than in aids designed for one specific concept.

3. Will the aid arouse the pupil's interest? Brightly colored materials are always more appealing and motivating when trying to interest pupils in using an aid. We felt that an aid should look good so that pupils would be attracted to it, thus increasing its value and making our task of motivation easier.

4. Will it be easy to learn to use? Even if an aid has great potential value in the classroom or the mathematics laboratory, we did not want materials that required a great deal of pre-learning on the child's part.

5. Can the aid be used by pupils for independent work? If the directions for the use of an aid were simple and it looked easy to handle and use, we felt it was valuable for use by a group of pupils with minimal teacher participation. We felt that this was an extremely desirable attribute of an aid when one is desirous of developing independency on the part of pupils for their learning experience.

6. Will the "cuteness" of the product overshadow its value? Many products on the market today are designed to show an original approach to a problem, or a clever, different attack. While a new fresh approach is often quite useful, we felt that the very cleverness of the aid might tend to make it difficult for pupils to see the basic principle that the aid is attempting to illustrate. Therefore we avoided this type of aid.

7. Will it be easy to handle? If a product is large, bulky, cumbersome to handle, and has many parts that are easily lost or misplaced, we hesitated to purchase it. Who wants to work on a puzzle, knowing that some of the pieces are missing?

8. Will it be durable? Since most schools advocate active pupil involvement in the learning process, we felt aids should be constructed in such a way that groups of pupils using them will not break them quickly. The advent of strong, light plastics has greatly increased potential here.

9. Will it be easy to store? An aid that is too large to store easily and too large to carry from room to room was not considered because it would not be of much value to a teacher. We felt aids should be in convenient containers, easy to find, and easy to store.

10. Is the aid reasonably priced? Regardless of other things, a high price caused us to re-evaluate some items. The use of the item might be excellent, but exceeded the limited amount that we wanted to pay. Often we considered the possibility of constructing items that we felt were over priced and had only limited use.

Following is a list of the items purchased and constructed by TTT for Reynolds Elementary School according to the Area of use in the laboratory.

Educational Supplies and Materials were as follows:

AREA 1 - Computational Skills Area

Materials	Purpose
Multibase Block (Dienes bases 2, 3, 4, 5, 10)	classification, counting, 3-d visualization, place value (including other systems of numeration other than ten); addition, subtraction, multiplication and division of whole numbers (in any base); fractions; decimals; exponents; length; area; surface area; volume; measurement in meters and centimeters.
Wooden Abacus and squares	constructed in various bases for follow up with Dienes blocks (the Dowl sticks were just long enough for squares in the base being used)

Educational Supplies and Materials continued

Materials	Purpose
Plastic Abacus Wooden Abacus	place value; regrouping; renaming; addition; subtraction; multiplication; division.
Fraction Simplifier	renaming equivalent fractions; addition; subtraction; multiplication; division
Attribute Blocks	classification; relations between classes of objects; symmetry, mapping; shape, size, logic; difference; order; equivalence; pre-number ideas.
Cuisenaire Rods	classification; ordering; counting; equal to; greater than; less than; true/false statements; open sentences; transformations; odd and even numbers; prime, composite numbers; addition; subtraction, multiplication; division; properties; symmetry; measurement of length, area and volume; metric measures; functions.
Hundreds Board	counting; odd and even numbers; sequences; operation games; place value; primes and composites.
Flannel Board and pieces	classification; pre-number experiences; counting; number/numeral relations.
Colored Pegs and Peg Boards	counting; number operations; shapes; odd and even numbers.
Mathematical Balance	classification; counting; place value; operations with whole numbers; fractions; exponents; transform operations.
Dice	counting; addition, subtraction and multiplication games.

Educational Supplies and Materials continued

Materials	Purpose
Decks of Cards (homemade)	computation skill reinforcement; probability; counting.
Beans and other things to count and classify	counting; place value; operations on whole numbers; fractions; odd and even numbers; prime/composite numbers.
Shoebox activities such as winning touch; card games; board games	boxes covered with contact paper; including all materials and directions for an individual child or small group; to reinforce some phase of computation.
Around the World Games (Scott Foresman)	addition; subtraction; multiplication; division to reinforce computational skills.
S.R.A. Computational Kit	reinforce computational skills through individualized instruction and practice.
Unifix Cubes	counting; classification; place value; operations with whole numbers; fractions.

AREA 2 - Environmental Measurement Area

Materials	Purpose
2-pan Balance	Weight
Weighing Scale (with weights in grams)	comparison of weights; introduction to grams, kilograms; pounds and ounces.
Bathroom Scale	
Mathematical Balance	
Peas, beans, sand, rice, Blocks, balls, plasticine	
Weights and springs	
	Volume
Water, sand, beans, rice, jars, bottles; cans; marbles, milk cartons; paper cups; balls	comparisons of liquid and dry measure using liters and centimeter cubes.
Cubical Blocks and empty boxes	informal comparisons of balance.
Geoblocks	introduction of measurement vocabulary
Trundle Wheel	Length (area)
Geoboards	Investigation of area
Cuisenaire Rods	Study of lengths and areas in centimeters
Graph Paper	
Irregular shapes cut from construction paper	Recording measures of length and area To measure shapes.
Floor tiles (of various sizes)	To estimate and calculate linear measures
Rules such as 12" rulers; yard stick; tape measure	

Areas 2 - Environmental Measurement area continued

	Time
Pendulum	For estimations and calculations of time.
Clock Stop watch Electric Car	To count and learn to tell time.
	Temperature
Hot Plate	For boiling water
Thermometer	For melting ice cubes
	For recording inside and outside temperatures.
	Introduction to negative numbers.
Map measure	measuring length; approximations; estimating; scaling
Maps (from various areas of the U.S.A. and Canada)	length; area, distance, scaling, population
Balls	circumference; diameter; surface area; perimeter.
Protractor (full circle)	angle measure
Timetables (trains and airlines)	problem solving; planning
Play money	understanding of monetary values
Empty boxes of grocery store items	set up grocery story for monetary transactions

AREA 3 - Geometry and Building Area

Materials	Purpose
Geoboards	area; classification; points; lines; angles; transformations, rotations; flips.
Circular Geoboards	diameter; radius; central angles; inscribed angles; similarities; congruence; parallel lines; perpendicular lines; line designs.
Tangrams	problem solving; perception; angles; area; shapes; logic.
Tower of Hanoi	game of solitaire; exponential function derived from counting; problem solving.
Peg Game	game of solitaire; quadratric function derived from counting; problem solving.
Straws and pipe cleaners	edges, faces, ploygons; polyhedra vertices.
Geoblocks	counting; shapes; clope; surface area; volume; classification.
Soma Cube	problem solving; solitaire game
Peg Boards and golf tees	shapes; congruence; similarity angles; symmetry.
Mirrors and Pattern Blocks	reflections; rotations; translations; symmetry; angles; group theory; function fractions; elementary pattern construction.
Graph Paper	bar graphs; histograms; polynomies scaling.
String and yarn	line designs; fractions; similarity; congruence; area; open and closed curves; tessellations.

Area 3 - Geometry and Building Area continued

Materials	Purpose
Protractors (full circle)	angles; congruence.
Cans; boxes; bottles	coordinate geometry; symmetry.
Tic-tac-toe game	points; lines; problem solving game
Number line on the floor	point; line; computation and counting.
Tiles of all sizes	tessellations; estimation size.
Dials, spinners	rotation; angle size
Work tools, coping saws; coping blades; circle saw; hammers; screwdrivers; nails.	simple constructions; scaling.
Scrape wood and triwall	
Shoeboxes with games and activities for one or more children	board games; shapes; size; congruence similarity; constructions; patterns.

AREA 4 - The Quiet Place (An area for individual projects and reading)

Materials	Purpose
Simple picture books and story books relating to number ideas checked out monthly from the school library.	reading; relating mathematics to real world of the child.
Bulletin Board and Art Easel	allow pupils to display work; and add finishing touches to projects.
Typewriter	just for fun; never seriously used by pupils.
Sewing Machine	sewing covers for handmade books; making aprons; making curtains. (6th grade girls)

AREA 5 - Teacher Resource Area (used by teachers and clinicians for ideas, games and materials)

Books and Teacher's Guides

Addison Wesley Co. Reading, Mass.	Freedom to Learn by Edith Biggs and James R. MacLean (Canada Ltd., Ontario)
	Discovery in Mathematics students book and teachers guide
	Exploration in Mathematics students book and teachers guide
Arco Publishing Co. 219 Park Co. New York, N. Y. 10003	Let's Explore Mathematics series of books levels 1 to 4
	Children Explore Mathematics by L. G. Marsh
Creative Publications P. O. Box 328 Palo Alto, Calif. 94302	Aftermath series of 4 books of activities can also be ordered as dittomasters
Cuisenaire Co. Of America 12 Church Street New Rochelle, N. Y.	Using the Cuisenaire Rods by Jessica Davison
	Cube by David by David Fielker
	Tessellations by Joseph Mold
	Solid Models by Joseph Mold
	Circles by Joseph Mold
	Mathematical Awareness by John Trivett
	Notes on Mathematics in Primary Schools by the Assoc. of Teachers of Mathematics in England
	Mathematics and the Child by Frederique Papy
	Graphs and the Child by Frederique

Area 5 - Teacher Resource Area continued

Herder and Herder Co. New York, N. Y. 10016	Learning Logic and Logical Games by Z. P. Dienes
	Exploration of Space and Practical Measurement by Dienes and Golding
	Modern Mathematics for Young Children by Dienes and Golding
	Sets, Numbers and Powers by Dienes and Golding
	Fractions -- An Operational Approach by Dienes
	States and Operators by Dienes addition and multiplication in ditto masters
Harper and Row Publishers Elmsford, N. Y.	Activities Handbook for Stretchers and Shrinkers
	Activities Handbook for Motion Geometry
MacMillan Co. 866 Third Ave. N. Y., N. Y. 10022	How Children Learn Mathematics by Richard Copeland
Midwest Publication Co. P. O. Box 307 Birmingham, Michigan 48012	Cloudburst books 1 to 4
National Assoc of Independent Schools Four Liberty Square Boston, Mass. 02109	Learning to Think in a Math Lab by Charbonneau
National Council of Teachers of Mathematics Reston, Va.	Puzzles and Graphs by John Fujii also many other publications available
Prindle, Weber and Schmidt 53 State Street Boston, Mass. 02109	Laboratory Manual for Elementary Mathematics by Fitzgerald

Area 5 - Teacher Resource Area continued

W. B. Saunders Philadelphia, Pa.	Mathematics and the Elementary Teacher by Richard Copeland
Simon and Schuster New York, N. Y.	Fantasia Mathematicia by Clifton Fadiman
Scott Foresman Co. Glenview, Ill. 60025	Teaching Arithmetic Concepts to Pre-Primary Children by Gibb
	Activities in Mathematics series of 12 book and teachers guide of activities for upper elementary and junior high
	Geometry for Elementary Teachers by Ray Walsh
	Geoboards and Motion Geometry
Science Research Assoc. 259 East Erie St. Chicago, Ill. 60611	The Laboratory Approach to Mathematics by Kidd and others
University of Illinois Arithmetic Project 372 Main Street Watertown, Mass. 02172	Davis Page's books (3 in the series)
A. C. Vroman, Inc. 2085 #. Foothill Blvd. Pasadena, Calif. 91109	The Franklin Series (11 books of activities to supplement the teaching/learning process of mathematics)
Walker and Co. New York, New York	Let's Play Games by Michael Holt and Z. P. Dienes
Wiley and Sons, Inc. 605 Third Avenue New York, N. Y. 10016	The Nufflied Series (at least 22 individual books written as teachers manuals for the English Schools of the United Kingdom)

Other Materials which we collected for the Mathematics Laboratory

Beans, peas, rice etc. in jars of various sizes

Bolts

Buttons of various colors and sizes

Calendars for various years

Catsup, Mustard dispensers for pendulums

Colored construction paper and large chart paper

Colored toothpicks

Cubes of various colors

Different shaped objects from home

Different size funnels

Different size washers

Dot paper

Egg timers and egg cartons

Empty milk cartons (various sizes)

Glue

Golf tees

Graph paper - large and small

Long board for string, balls and cars

Magic markers (various colors)

Maps (from various states, airways, trains)

Medicine droppers

Mirrors (stainless steel if possible)

Nuts

Other materials continued

Peg board and pegs or golf tees

Pipe cleaners (various colors)

Poster board and oak tag

Rubber bands (all colors)

Sand

Scales (bathroom scales, grocery scales, spring scale)

Scissors

Slide Rule

String and balls

Tape measures

Timing devices (stop watch, egg timers)

Tongue depressors

Train or electric car and stop watch

Triwall and scrapes of wood

Weights and springs

Yard sticks and rulers both English and Metric

Yarn

HOMEMADE TEACHING AIDS AND MATERIALS Teachers often construct manipulative materials for their classroom when they find that funds are not available or that the aid could serve a better purpose when changed a little before use in their classroom. The clinicians constructed many varied aids for the mathematics laboratory and for their work with individual groups of pupils because they felt that a homemade device would serve the purpose best. The clinicians also found that pupils responded well to homemade materials and would often make construction of aids a part of the lesson. Thus, pupils could have the experience of making things, following directions, and have their own materials to use for practice later on during the day or at home. Following are representative items constructed by the clinicians and pupils:

1. Activity cards - The clinicians constructed many activity cards during the three year period at Reynolds Elementary School. These were written so that pupils could follow the directions and complete an activity or project during one class period. 5" x 8" index cards were used with the topic written in bold letters at the top and color coded according to the area of mathematics being studied. For example addition with whole numbers was coded "red", while subtraction was coded "blue". If several cards were to be completed in sequential order they were placed in order on a notebook ring. The cards told pupils what materials would be needed and how he would record his results. Records were kept of activity cards completed by pupils and suggestions were often made by clinicians of activities appropriate for individual pupils.

An example of an activity card series is as follows:

> LINEAR MEASURE
>
> Measure each item listed below to the nearest centimeter. Keep a record of your measures and record your findings in a graph.
>
> Width of your desk
> height of your desk from the floor
> length of your pencil
> width of your book

> LINEAR MEASURE
>
> Measure the length of a rubber band in centimeters as different masses are suspended from it. Graph your measures and write a description of relationships and trends which you found.

2. Shoeboxes - Shoeboxes were also a popular way of organizing activities for small groups of pupils or individuals. A shoebox was covered with colorful contact paper and a label was attached to one end to indicate the level and activity category. Inside the box the pupil would find all of the materials necessary to complete the activity. The directions were written or typed on 5" x 8" index card and attached to the inside lid of the shoebox. Many teachers began a small collection of shoebox activities in their classrooms as a result of seeing the ones in the laboratory.

They felt that this was a good way for individuals and small groups of pupils to use their spare time wisely. In fact, the teachers not only constructed shoebox activities for mathematics but science, social studies and language arts as well. An example of a shoe-box on fractions is as follows: circles, squares, rectangles, triangles, and diamond shapes cut from colorful poster board were placed in a shoebox. Each shape was cut into pieces, for example the circle was cut into fourths; while the rectangle was cut into thirds, the square was cut into halves and the triangle and diamond shape was cut into thirds and fourths. A die was also included in the box. It had fractions written on each side. The activity was for 2 or more pupils. The rules were written on an index card attached to the lid of the box telling pupils to take turns rolling the die and taking appropriate pieces from the box. The pupil to fit pieces together to make "one whole" first would be the winner.

3. Card Games Card games were also constructed by the clinicians and became very popular with pupils. The teachers felt that card games were useful since so many of them involved computations which pupils need to master before moving on to learning long division and fractions. Card games were constructed on 3" x 5" index cards with various colors of magic markers so that various sets of cards could be identified immediately. For example, a popular set of cards consisted of ordered pairs as follows on 3" x 5" index cards.

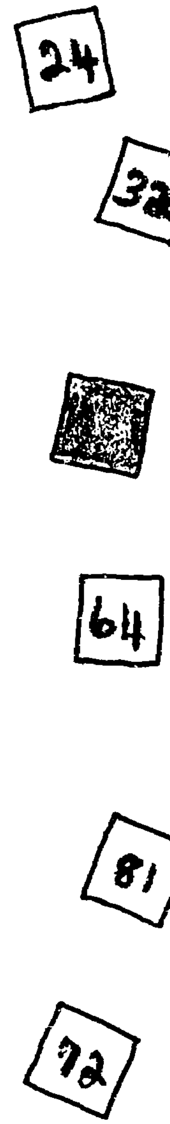
(9,9)	(9,8)	(9,7)	(9,6)	(9,5)	(9,4)	(9,3)	(9,2)	(9,1)
(8,8)	(8,7)	(8,6)	(8,5)	(8,4)	(8,3)	(8,2)		
(7,7)	(7,6)	(7,5)	(7,4)	(7,3)	(7,2)	(7,1)		
(6,6)	(6,5)	(6,4)	(6,3)	(6,2)	(6,1)			
(5,5)	(5,4)	(5,3)	(5,2)	(5,1)				
(4,4)	(4,3)	(4,2)	(4,1)					
(3,3)	(3,2)	(3,1)						
(2,2)	(2,1)							
(1,1)								

With these cards as the deck pupils could play "Fish", Rummy", "War" and other games with simple rules. The cards above could be used for addition, subtraction and multiplication of whole numbers. Many pupils made a deck of their own to play after school or in their free time in the classroom. Other card games were constructed for pupil use in the laboratory.

4. <u>Board Games</u> Pupils enjoy games such as checkers or chess and often played other board games in the mathematics laboratory. A popular game for the pupils was one called "The Winning Touch". The game was similar to <u>Scrable</u> but played on a basic addition or multiplication board. One inch squares were made with the sums or products written on them. Blank squares were also included. A 11" x 11" inch board was constructed as shown on the next page. Pupils each drew seven squares from the pile to begin the game. An additional square was drawn and placed on the board. Pupils took turns placing numerals on the board so that an edge of a square, being placed on the board, touched a square already on the board.

If the player did not have a numeral which could be used he could place a blank square from his hand or he had to draw a square from the pile. A player wins the game when he plays all of the numeral squares from his hand.

X	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										



PLANNING LABORATORY ACTIVITIES

After it was decided that the TTT clinicians would establish a Centralized Mathematics Laboratory at Reynolds Elementary School the group began to survey and order appropriate materials for the laboratory. As materials were ordered we began to plan activities. Some activities were short term or one-day activities, while others were long term or unit-type projects. Some were for individual pupils while others were for a small group of four to six pupils. Regardless of the duration we felt that each activity should accomplish certain pre-determined goals for the child.

1. The activity experience should always relate to a concept being developed in the regular mathematics program. For example, measurement was a unit taught at every grade level at Reynolds School. Thus, activity cards and the appropriate materials were gathered for an activity-unit on measurement. The following are examples:

a. For a teacher directed activity: Direct pupils to measure things around the room with ribbons of various lengths. Ask them to find out how tall things are; How long things are; How wide things are. Then as a group direct pupils to write a story about the things which they measured.

For older pupils scale drawings could be discussed and appropriate scales could be set up so that pupils, working in pairs, could make a scale drawing of a room or area.

A pupil activity: Find surface area using tiles of various sizes. Find out how many tiles it would take to cover the surface area of a table, a rug, and eventually the floor or the classroom or mathematics laboratory.

For the teacher or individual pupil: Depending upon the grade level, volume could be included by having containers and dried lima beans. Pupils could discuss estimating how many beans a container would hold and then fill the container and count the beans. This could lead to the study of various size measuring cups and beakers.

2. The activities planned should be at various levels of abstractness. This is so for a mathematics laboratory since various grade level groups will utilize the materials in the laboratory. Thus, for some activities very specific directions are needed. For other activities, no suggestions need to be made. Pupils can readily see from the materials, things which they can do. At Reynolds activity cards were purchased or written by the clinicians to accompany some materials in the laboratory. This made it easy for a teacher to study the versalitity of materials and for pupils to use materials in a constructive way rather than just for play.

3. The activities should each solve a problem. Activities were needed to lead the pupil from something which he knew to solving a problem which he had not solved before. For example, pupils were aware of traffic patterns around their school, but had probably never discussed traffic in their classroom. Therefore, an activity was developed which had pupils keeping a record of how many cars, trucks, etc. pass the corner of 24th and Jefferson (where the school is located) from 10:00 A.M. to 10:30 A.M. for a period of 5 days. After all of the data was collected, a graph was made and an experience story was written by the group describing their experience and making predictions about traffic patterns.

Thus, collecting the data, comparing the flow of traffic from day to day, drawing a graph, and writing their experience involved various stages of problem solving.

4. The materials used to teach a topic should vary so that the pupils can transfer knowledge gained and utilize what they are learning. For example, when helping pupils master the multiplication facts a dice game might be used; later the Magic Touch game might be introduced. Both activities involve the multiplication facts, but are very different and motivational for the child.

5. Varying the teaching strategy helps keep pupils interested and motivated and at the same time allows for social as well as educational progress. Sometimes pupils need to be directed by the teacher, but at other times a pupil may wish to work alone. For some pupils small group work is best, while others need to learn how to work in a group before educational goals can be accomplished. Also every pupil does not need every activity; whereas, some pupils may need additional work. This is also an area where the mathematics laboratory helps strengthen the regular mathematics program in a school. It is almost impossible for the self-contained classroom to supply all the materials needed to meet individual pupil needs in all subject areas.

INTEGRATING MATHEMATICS WITH THE OUTSIDE WORLD OF THE CHILD

In our discussions about how best to meet the needs of the pupils at Reynolds Elementary School, we tried to look at the total child, not just the child in terms of mathematics. This lead us to develop activities which often involved science, social studies, language arts and the streets around Reynolds Elementary School. In the world outside the school all subject areas are meshed into an environment in which the child spends most of his time and must survive. Yet inside the school, subjects are constantly compartmentalized. Pupils study mathematics separtely from social studies; likewise they write stories in language arts which are unrelated to any other academic discipline.

We planned activities which would require a clinician and a small group of pupils to go into the community for a survey, to a grocery store to study food prices or to a department store to make comparisons of display techniques. On one of the trips into the community the group stopped at Gino's (a popular hamburger shop). The property had recently been renovated and new cost signs had been installed. The pupils asked the store manager if they could have the old signs. So these signs were added to the mathematics laboratory. Pupils often "played" store, buying imaginary lunches and suppers. Much computation took place as a result of these signs and they were popular for several years until the prices were no longer realistic (according to the prices being charged after prices had been raised several times).

Another result of integrating subject areas through the mathematics laboratory came after pupils in a group took a walk through a six block area around the school to make a map of the area. Since the streets were relatively straight a map was not difficult. Then pupils decided to study the population one half square mile which lead to some startling discoveries about numbers. Graphs were made to show the number of men, women, boys, and girls in the area. Other charts were made to show the number of grocery stores, churches, bars, and package stores. Weeks later a sixth grade teacher who was not involved in the project, but had heard about it from various pupils, decided to use the study in her classroom to teach geography. Her pupils learned about latitude and longitude in terms of their own neighborhood and then related the information to a globe and flat maps. Her unit of study integrated the community, mathematics, and social studies.

Many other examples could be cited, but are not necessary because the main point is explicit in the two examples above; i.e., what counts is the child's overall education, not how much time is allotted to teaching science or mathematics et al. When an experience presented itself, regardless of where the group of pupils was or what they were studying at the time, emphasis was always placed on helping the child see that school subjects were a part of the real world and would help him to survive.

HOW DOES A MATHEMATICS LABORATORY FUNCTION?

The main objective of the laboratory at Reynolds Elementary School was to supplement the mathematics program planned by the teacher in a self-contained classroom. At Reynolds we developed a Centralized Laboratory combining its function with teaching pupils in the classroom. We decided on the following organization:

Each clinician would be responsible for developing and teaching mathematics to one group of pupils for an entire school year and would also assist another clinician with his group of pupils. Thus, each clinician would function as a leader and as an assistant each day. Another clinician was selected to be the resource person in the mathematics laboratory. Mrs. Gerri Myles' responsibilities as resource person centered in the laboratory as she developed materials and helped other clinicians with groups of pupils when they came to the laboratory.

1. For the Pupils: Each clinician was responsible for planning and executing a mathematics lesson each day. For example, Mr. Kevine taught Miss Seaman's first grade class mathematics each day with the assistance of Mr. Carter. Mr. Levine would then function as an assistant to Mr. Carter in teaching a third grade class later in the day. Mr. Levine divided his first grade class into two groups. Two days a week he would teach a mathematics lesson with half of the group in the classroom; while Mr. Carter took the other half of the group to the mathematics laboratory. Mrs. Myles was in the laboratory and would work with an individual or small group of pupils as requested by Mr. Levine and Mr. Carter. Miss Seaman, the regular classroom teacher, was encouraged to accompany the group of pupils to the laboratory. Sometimes she would just observe, other times she would get involved and work with a group or an individual pupil. There were also many materials in the Teacher Resource Area for her to brouse through if she wished.

Thus, Mr. Levine, Mr. Carter and Miss Seaman were all involved in the mathematics program of the pupils. Planning sessions were used to discuss the direction of the mathematics program and to plan for individual pupil needs. Since Miss Seaman was not familiar with the laboratory per se or materials, she was gradually introduced to its functions by obervation. She became involved when she felt that she could help and gradually became very active in working with her pupils in the laboratory setting. One day a week Mr. Levine and Mr. Carter kept the entire class in their regular classroom instead of taking a group to the mathematics laboratory. This period was usually used for paper/pencil activities. Miss Seaman was free of teaching responsibility during this period; however, as the school year progressed she, like other teachers, often came to the mathematics laboratory during this free period to talk to other clinicians, look at materials, or observe another grade level working in the laboratory. (The reason for this one day with no laboratory period for the children was because every child wanted equal time in the laboratory. So by having two days in the laboratory for each of the two groups, there was one day left over. Therefore, the clinicians felt that the day should be spent in the regular classroom rather than cause hard feelings among the pupils who wanted equal time in the laboratory).

2. For the Teachers: We found that the teachers at Reynolds felt insecure with mathematics in general and with the multitude of materials in the mathematics laboratory. Realizing this, the TTT clinicians did not force teachers to work with materials, but were available if a teacher wished to find out more about a particular item. For example, a third grade class has been studying shapes and had utilized the geoboard in the mathematics laboratory. Their classroom teacher had never used a geoboard and therefore, learned of its advantages as a teaching aid as she observed her pupils in the laboratory. Later she told someone that she used the idea of coordinate geometry, which her pupils had explored with the geoboard, to teach latitude and longitude on a large map in the classroom.

After the teachers had been coming to the mathematics laboratory for several months observing and working with small groups of pupils we began to have informal meetings with teachers to answer questions and to introduce them to more indepth ideas about various materials in the laboratory. For example, the teachers had seen pupils work with the Multibase Blocks in base ten with place value and regrouping for addition and subtraction. In order for the teachers to feel and understand the problems which the child might feel and to be aware of the questions which might be asked, we worked with the teachers using base five. Afterwards, we discussed the various problems related to teaching the four basic operations.

All of the teachers were receptive to the ideas and anxious to explore other materials.

Teachers were hesitant to take materials to their classrooms or to bring a class of pupils to the laboratory without the assistance of a TTT clinician. They seemed to fear that their pupils would move about in all directions and would not be under direct teacher supervision. As a result of this fear, groups of pupils never came to the laboratory unless a specific activity had been planned and several clinicians would be around to help direct small groups. It was only during the second and third years of TTT at Reynolds, that the teachers began to check out materials from the laboratory for use in their classrooms. Observing the materials which were checked out, it appeared that teachers were planning to use the materials with a whole class rather than to group pupils in small groups.

Teachers were also mainly concerned with computational skills. They seemed to feel that pupils needed reinforcement in this area above all others. The materials such as Cuisenaire Rods, card games, board games, Multibase Blocks etc. were popular with the teachers. However, when pupils came to the laboratory for an unstructured period, they were more likely to work in the Geometry and Building Area or the Measurement and Environment Area. Of course, computation would be a part of their activity as a tool rather than an end in itself. We discussed this with the teachers on many occasions, but they were uncertain how much computation was actually accomplished with measurement and geometry. Several examples of computation could be cited: (1) scale drawings; (2) constructing bird houses; (3) tabulating data; (4) measuring and working with estimating, etc.

Everyone benefitted from the laboratory; (1) the pupils, because much time and effort was spent on planning and meeting individual needs; (2) the regular classroom teachers, because they were able to see the laboratory in action without being threatened in any way; and (3) the clinicians, because they were directly responsible on a daily basis for the total mathematics program for a specific group of pupils.

<u>SUMMARY</u>

Many things happened during the three years TTT was at Reynolds Elementary School. The mathematics laboratory was just one salient contribution which I feel that we made to the school.

... In three years we moved from a closet to a two room complex setting up a Centralized Mathematics Laboratory useful to all teachers and pupils in the school.

... We had the opportunity to work with pupils from first grade through seventh grade in classrooms and in a laboratory setting.

... We were able to get teachers involved in using materials with their pupils without feeling that they might make a mistake or that the pupils might be disorderly.

... The principal was definitely proud of having a functioning mathematics laboratory in his school and often brought visitors to the room to see pupils and teachers at work.

... We were able to aid the principal in developing an on-going mathematics program in the school which correlated the activity approach with the traditonal textbook.

... Mr. Charles Amos, a teacher at the school assigned to work directly with TTT, gained a general knowledge of the mathematics laboratory and its function in the school. He is now in charge of the laboratory which is being utilized by the teachers in the school.

Summary continued

... The clinicians gained knowledge and experience from their involvement with the laboratory and working with both teachers and pupils at Reynolds. Most have utilized their knowledge to secure jobs in the field of education.

... As a Temple faculty member directly involved with TTT at Reynolds, I grew also, i.e., I was able to help set up a functioning mathematics laboratory in an inner-city school and work directly with the clinicians (TTT students and Temple doctoral students), teachers and pupils.

Ann McPherson Wilderman
Associate Professor
Mathematics Education