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AUTHOR Trabasso, Tom; Riley, Christine A.  
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## ABSTRACT

This discussion of transitive inferences (if A greater than B & B greater than C, then A greater than C) emphasizes an information processing analysis of logical thought. The two basic factors considered in such an analysis are (1) the task environment, including its structure, demands, decisions required, and information given; and (2) the individual as an information processor (his knowledge, limitations, etc.). In order to make transitive inferences a person must make several critical operations. He must know that the scale of comparison is transitive and must code task information. Research concerned with children's coding strategies is discussed to illustrate the importance of this operation. Young children (4-6 years) can make the inferences if they are forced to code in certain ways (if their attention is directed to the comparative relations among key elements of the problem). A second basic operation is memory storage. Two possible models of storage are identified: (1) a coordinate model in which each ordered pair of items is stored, and (2) a spatial integration model in which information is integrated as it accumulates into one representation which is stored for subsequent inferential thought. Experimental work with adult subjects indicating that spatial representations are constructed in transitivity tasks is described. (DP)

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An Information Processing Analysis of  
Transitive Inferences

Tom Trabasso and Christine A. Biley  
Princeton University

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The central question before us is: why do we study logic in a psychological context? One answer is that logic pervades human intellectual activity and to study its operation is a fundamental investigation of cognition. On this we probably all agree.

One could view certain forms of logic as a kind of competence model and use it as a norm against which to diagnose the presence or absence of logical abilities. Assuming that the adult possesses the highest form of competence, then one could investigate the development of intelligence as a series of approximations to this higher form. According to this view, the central concern is whether or not the child possesses a given logical ability or structure. The order in which these structures manifest themselves is of interest.

An alternative approach is to study how people perform logical tasks in order to discover what they do when forced to behave within the constraints imposed by the task. The focus is on what the person does, not on his success or failure. If one knows the cognitive processes that are determined by the requirements of the task environment, one could predict success or failure as well as diagnose it.

The latter position is the one we have developed in our study of reasoning. Our belief is that logical problems stress our information processing system and reveal much about the properties of this system: its structural and control processes. Our attitude is one of discovery rather than confirmation, induction rather than deduction.

In this paper, we hope to illustrate the value of an information processing analysis of a logical task. We have chosen the transitive inference problem because it is logically simple but psychologically complex, and because this problem has received considerable attention since Burt first used it in an intelligence test back in 1919. It is a well known task in Piagetian research. It has received considerable study in adults in what are known as "three-term series" problems. Recently, it has become a topic in psycholinguistic research which focuses on inferences made across sentences in text or connected discourse.

In formal terms, what is a transitive inference? A transitive inference is a logical operation of the form: if A is greater than B ( $A > B$ ) and if B is greater than C ( $B > C$ ), then A is greater than C ( $A > C$ ). We shall consider first the Piagetian view. Here, the failure of a child to form inferences before the stage of concrete operations (at about seven to eight years) is attributed to the lack of the logical grouping structure of addition of asymmetrical

relations. In other terms, the child is unable to coordinate the information about the two relations  $A > B$  and  $B > C$ . In order to combine these relations with a common term, the child must simultaneously conceive the relationship of the elements in the pair  $AD$  in terms of the direct ( $A > B$ ) and the inverse ( $B < A$ ) relationship. The preoperational child's inability to understand the reversibility of the ordered relationship  $A > B$  (and  $B > C$ ) prevents him from using  $B$  as a common term which is at the same time less than  $A$  and greater than  $C$ .

Let us begin by assuming that the logical operations described above are necessary to make a transitive inference. However, if we find that a child functions in a manner consistent with the formal logic model, this does not show that the underlying process is equivalent to that of formal logical operations. This point has been made repeatedly, most notably by Bruner (1966) and Flavell (1963).

Further, a logical explanation is insufficient. Logical descriptions correspond with only some properties of their referent, and a complete description would have to contain all properties. The properties we chose are those which together determine the behavior in question (Simon, 1972). Thus while a logical description helps us communicate about a process, symbolize its structure so as to better remember it, represent an abstract event, simplify and manipulate descriptions to form new ones, it may be necessary but certainly is not a sufficient description of the behavior in question.

To begin a information processing analysis of transitivity one has to take into account two major factors: (1) the task environment - its structure, its demands, the decisions it requires, the information it gives etc. and (2) the person as information processor - his knowledge, his limitations, his processes, etc.

With respect to transitivity, ability to perform a transitive inference presupposes that the child knows whether or not a relation is transitive. That is, if a child is to infer  $A > C$  from  $A > B$  and  $B > C$ , he has to know that the scale of comparison is transitive. Thus  $A$  is longer than  $C$  follows from  $A$  is longer than  $B$  and  $B$  is longer than  $C$ , but  $A$  prefers  $C$  does not follow from  $A$  prefers  $B$  and  $B$  prefers  $C$ .

Now consider a transitivity task where a child has to infer that stick  $A$  is longer than stick  $C$ . In order to make this inference symbolically, the child must remember the initial relations  $A > B$  and  $B > C$ . But then, one might ask, what is remembered? That is, how does the child code this



information.

Given that A is longer than B is the input, one might code the information:

1. A (is important)
2. A is long.
3. A is long; B is not long.
4. A is long; B is short.
5. A is longer than B and B is shorter than A.

Only (5) leads to the ordered set (A,B). Codes (1), (2), (3) and (4) would lead to success or failure depending upon the task. We (Riley and Trabasso, 1973) have found that children do not code anything at all about length relations when another, simpler code works. That is, we had four-year-old children initially learn a series of four comparisons  $A > B$ ,  $C < B$ ,  $C > D$  and  $E < D$  where they had to choose the element named by the relation (e.g. choose A when asked "which is longer?"). The children were smarter than we; they learned a simple rule that A, C and E are winners and B and D are losers. This corresponds to code (1) above (i.e. A is important).

In another task, when we asked only one comparative term (throughout:  $A > B$ ,  $B > C$ ,  $C > D$  and  $D > E$ , they couldn't learn the pairs much less draw inferences. Code (3) or (4) was used and lead to contradictions of the form identified by Piaget, namely labelling B both not long and long (or short and long).

It was only when we used both comparative terms within a pair, i.e. asking the child which is longer, A or B? and which is shorter, A or B? that they succeeded in both learning the ordered relation (A,B) and making inferences such as  $B > D$  (cf. Bryant and Trabasso, 1971; Lutkus and Trabasso, 1973).

A second critical operation is, given that the child has coded the relations, he must store it in memory. How it is stored or represented is a critical question because the operations upon this stored representation are what lead to correct answers in the test.

The nature of the representation in memory is critical since it determines the mental operations that will be performed on it. In the transitivity task, we can identify at least two possible representations in memory. In one, the person stores each ordered pair in memory. Then, when questioned: which is longer? (shorter?) he retrieves the critical pairs and coordinates them via middle terms, consistent with Piaget's analysis. We will call this the coordinate model.

An alternative representation could arise where the person integrates the information as it accrues into a

single representation and stores that in memory for subsequent inference making. That is, the person begins by finding the end pairs and uses them as "anchors". He then adds elements to the array as they occur until all elements are so ordered. When questioned, he isolates the critical elements, notes their order and answers the question. We will call this the spatial integration model.

These models lead to a simple experimental test in the transitivity task. Look at Table 1 in the Handout. Suppose there are six sticks of different lengths where 1 is the shortest and 6 is the longest. Following a procedure we have used extensively in showing a child's memory for the initial information is critical in making transitive inferences (Bryant and Trabasso, 1971; Lutkus and Trabasso, 1973; Riley and Trabasso, 1973), we first train subjects to make choices among adjacent pairs of sticks. The sticks are color coded and the subject can use only the colors to predict the length relation. On a given training trial, the subject is asked one of two questions: either "Which stick is longer?" or "Which stick is shorter?", and then is shown a pair of sticks of the same length but of different colors. The subject selects a color by pressing a panel in front of the stick. After he makes his choice, he receives feedback on its correctness. We record the time it takes him to make the choice.

After training each adjacent pair in a random order, we test the subject on all possible pairs without feedback. The matrix in Table 1 tells us three pieces of information:

1. The main diagonal (squares) gives us the speed of retrieving information on the adjacent training pairs.
2. The first row and last column entries (lines) tell us whether or not there are anchor effects, since these involve endpoint sticks.
3. The critical, off-diagonal entries are the inference tests (circles), and they involve inference steps of 1 or 2 units.

If subjects store only the diagonal (adjacent pair) information and coordinate it to answer questions on off-diagonal pairs, then we predict that those pairs with more inferential steps would take longer. If, on the other hand, subjects integrate the information into a spatial array and access this simple memory representation during testing, then we predict that the greater the distance between the sticks, the faster the time.

We tested these predictions on adult subjects by running three conditions. In one condition, subjects received both visual and verbal feedback after making a choice in training. That is, they were shown the sticks and

heard the relation stated (e.g. Red is longer than blue). In a second (verbal) condition, they were told the relation after a choice in training. No feedback of any sort was given during testing in the above conditions. In order to test whether the representation was indeed spatial, we ran a third group. Instead of training on pairs, we simply showed them the entire array of sticks, ordered 1 - 6. The subjects were tested with the sticks in full view via the same means as in the visual-plus-verbal and the verbal conditions. In all cases, we measured the reaction time in answering question.

There were 12 college students as subjects in each condition and there were four tests per pair.

Table 2 in the Handout gives the raw RT data on each question for each of the three conditions. Of particular importance are the underlined RTs for steps of 1 or 2.

In five out of six tables, you will note that the two-step RT is faster than the one-step RTs.

In order to see the relations more clearly, we scaled these data, finding a two-dimensional representation of all inter-pair distances. In this analysis, distance equals the reciprocal of RT so that faster times give longer distances.

Slide 1 shows the distances for the question longer? Note first of all how similar these distance plots are for all three conditions. The longest stick (number 6) is clearly further from the other five; the remaining five sticks are ordered 1,2,3,4,5 in distance.

Slide 2 shows the scaling results for the question shorter? Now we find the shortest stick (number 1) separated from the rest. The remaining sticks are generally ordered 2,3,4,5,6 with 6 separated slightly further away.

In our next analysis, we removed the longest stick (number 6) data and the shortest stick (number 1) data from the matrices with matching questions. We then collapsed statistically equivalent data points for distances of 0 (adjacent pairs), 1, 2, or 3 inferential steps. The mean RTs as a function of these distances are shown in Slide 3 for each condition. The data are remarkably similar across conditions. The adjacent pair or 0 step RTs are the longest despite the fact that these were the ones upon which the subjects were trained. RT decreases linearly as a function of step size, perfectly consistent with the spatial integration model. The visual and verbal feedback data are virtually identical with that obtained when subjects have the display in front of them, suggesting that spatial representations are constructed. The verbal RTs are longer and subjects indicated that they had trouble end-anchoring in training when the feedback was only verbal. Clearly, the

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coordination model finds no support in these data.

Thus, adults who are supposed to be in the formal operations stage do not perform such operations in transitivity tests. Rather they use their knowledge that length is transitive, isolate the extreme ends of the scale, order each pair and add single elements to a spatial array which is stored in memory for later use in answering "transitivity" questions. The integration of separate pieces of information into one unit conserves space and enables one to efficiently answer a wide variety of inferential questions.

We are currently carrying out these same experiments on eight to nine-year-old children. The results on the display and visual and verbal feedback groups are in and look very much like our adult data.

In overview then, our inquiry into transitive reasoning has lead us to discover a number of things. We have found that very young children (four to six years of age) can make transitive inferences if one assures that they are asked questions which direct their attention to the comparative relations among the elements. They may fail if the coding is inadequate, as determined by the task demands, or if they forget the original, ordered codes.

Our adult latency studies (and subsequent studies on concrete operational children) show that coordination or integration occurs spontaneously during training as subjects integrate ordered pair information into spatial arrays. This integration is an efficient representation for memory and inference making, a process not at all envisioned by logical operational analysis. These studies have convinced us of the value of viewing the human being as an information processor who is basically a good problem solver, using his limited capacities and extraordinary processes to operate on a variety of task environments. Logical processes are only one small part of these skills.



## References

- Bruner, J.S., Olver, R.R. And Greenfield, P.M. Studies in Cognitive Growth. New York: John Wiley and Sons, Inc., 1966.
- Bryant, P. E. and Trabasso, T. Transitive inferences and memory in young children. Nature, 1971, 232, 456-458.
- Flavell, J.H. The Developmental Psychology of Jean Piaget. New York: Van Nostrand Reinhold Company, 1963.
- Lutkus, A.D. and Trabasso, T. Transitive inferences in "preoperational" retardates. Unpublished paper, Princeton University, 1973.
- Riley, C.A. and Trabasso, T. Logical structure versus information processing in making inferences. Paper presented to the Society for Research in Child Development, March, 1973.
- Simon, H.A. On the development of the processor. in Farnham-Diggory, S. (Ed.) Information Processing in Children. New York: Academic Press Inc. 1972.

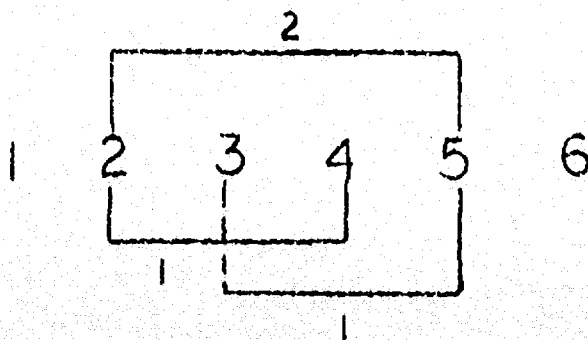
# TABLE 1

| TRAIN |   | TEST |   |   |   |   |
|-------|---|------|---|---|---|---|
|       |   | 2    | 3 | 4 | 5 | 6 |
| 1 < 2 | 1 | □    | — | — | — | — |
| 2 < 3 | 2 |      | □ | ① | ② | — |
| 3 < 4 | 3 |      |   | □ | ① | — |
| 4 < 5 | 4 |      |   |   | □ | — |
| 5 < 6 | 5 |      |   |   |   | □ |

□ = ADJACENT PAIRS (TRAINING PAIRS)

— = END ANCHORED PAIRS

① = INFERENCE PAIRS OF STEP 1



COORDINATE MODEL :  $RT(2,5) > RT(2,4), RT(3,5)$

INTEGRATION MODEL :  $RT(2,5) < RT(2,4), RT(3,5)$

TABLE 2

## VISUAL + VERBAL FEEDBACK

## SHORTER?

|   | 2   | 3    | 4           | 5           | 6    |
|---|-----|------|-------------|-------------|------|
| 1 | 955 | 926  | 852         | 791         | 843  |
| 2 |     | 1605 | <u>1204</u> | <u>1187</u> | 995  |
| 3 |     |      | 1285        | <u>1307</u> | 1110 |
| 4 |     |      |             | 1657        | 1209 |
| 5 |     |      |             |             | 1246 |

## LONGER?

|   | 2    | 3    | 4           | 5           | 6   |
|---|------|------|-------------|-------------|-----|
| 1 | 1189 | 1246 | 1207        | 1016        | 808 |
| 2 |      | 1685 | <u>1282</u> | <u>1022</u> | 884 |
| 3 |      |      | 1291        | <u>1184</u> | 837 |
| 4 |      |      |             | 1081        | 812 |
| 5 |      |      |             |             | 835 |

## DISPLAY

|   | 2   | 3    | 4           | 5           | 6    |
|---|-----|------|-------------|-------------|------|
| 1 | 803 | 940  | 795         | 763         | 742  |
| 2 |     | 1609 | <u>1279</u> | <u>1179</u> | 1055 |
| 3 |     |      | 1476        | <u>1471</u> | 1049 |
| 4 |     |      |             | 1501        | 1034 |
| 5 |     |      |             |             | 1256 |

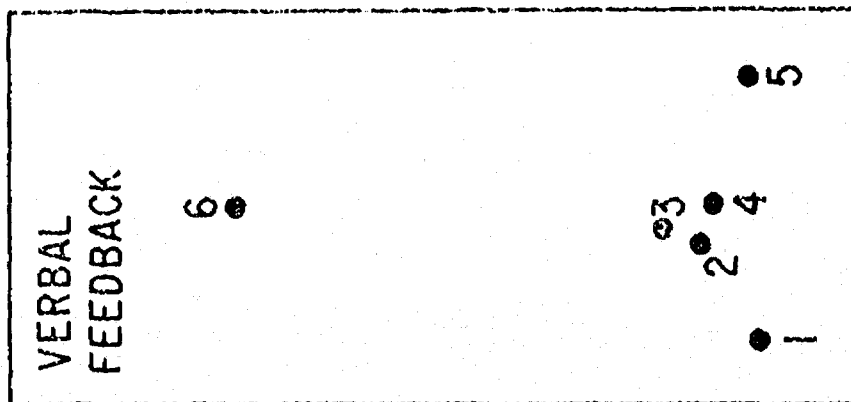
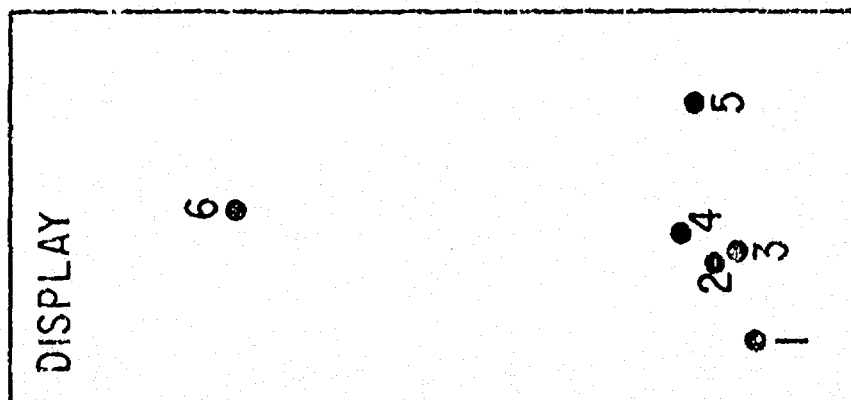
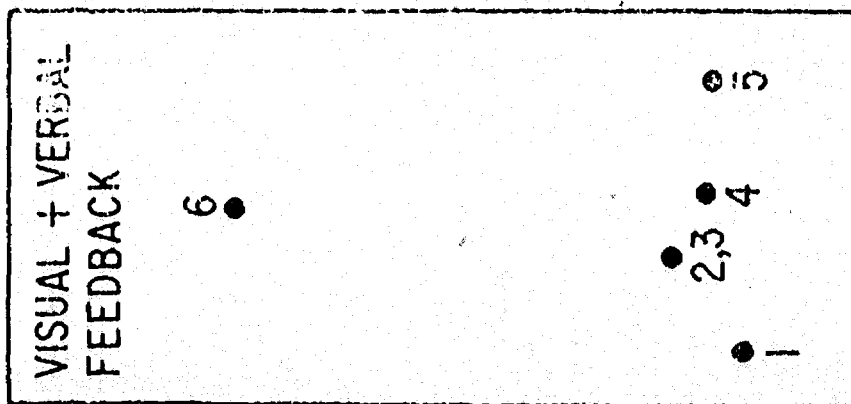
|   | 2    | 3    | 4           | 5           | 6   |
|---|------|------|-------------|-------------|-----|
| 1 | 1339 | 1231 | 1130        | 971         | 722 |
| 2 |      | 1453 | <u>1430</u> | <u>1143</u> | 799 |
| 3 |      |      | 1438        | <u>1178</u> | 754 |
| 4 |      |      |             | 1260        | 799 |
| 5 |      |      |             |             | 778 |

## VERBAL FEEDBACK

|   | 2    | 3    | 4           | 5           | 6    |
|---|------|------|-------------|-------------|------|
| 1 | 1029 | 1078 | 1079        | 1065        | 944  |
| 2 |      | 1544 | <u>1739</u> | <u>1469</u> | 1177 |
| 3 |      |      | 1784        | <u>1696</u> | 1242 |
| 4 |      |      |             | 1638        | 1482 |
| 5 |      |      |             |             | 1274 |

|   | 2    | 3    | 4           | 5           | 6    |
|---|------|------|-------------|-------------|------|
| 1 | 1442 | 1329 | 1292        | 1134        | 859  |
| 2 |      | 1645 | <u>1624</u> | <u>1432</u> | 1159 |
| 3 |      |      | 1749        | <u>1373</u> | 1049 |
| 4 |      |      |             | 1808        | 1866 |
| 5 |      |      |             |             | 887  |

LONGER ?





# SHORTER ?

VISUAL + VERBAL  
FEEDBACK

2 ●  
●4,5 3 ● 6 ●

!

DISPLAY

6 ●  
5 ● 4 ●  
2 ● 3 ●

!

VERBAL ●6  
FEEDBACK

2 ● 4 ● 3 ● 5 ●

!

