ABSTRACT
        This teacher guide is part of the materials prepared
for an individualized program for ninth-grade algebra and basic
mathematics students. Materials written for the program are to be
used with audiovisual lessons recorded on tape cassettes. For an
evaluation of the program, see ED 086 545. In this guide, the teacher
is provided with objectives for each topic area and guided to
materials written for a given topic. Three short criterion tests are
included for each topic covered. The content of this package centers
on the language of algebra. The concept of a variable is developed
and solution sets are found for simple equations. Provided is
practice in the use of exponents and quantifiers. This work was
prepared under an ESEA Title III contract. (JP)

ALGEBRA I

Package # 03-02

THE LANGUAGE OF ALGEBRA

Prepared by

Russ Thompson and Albert Fuller

Under a Grant From
ESEA, Title III
Nebraska State Department of Education
Jack Baillie, Administrator
to
Arnold Public Schools, Arnold, Nebraska

1

The Language of Algebra

There are many special symbols and terms used in Algebra. You will be introduced to many of them in this package — some you are already familiar with. You <u>could</u> learn to accomplish all of the operations of algebra without knowing some of the terms found in this package. You could also live a full life with a very limited vocabulary in the English language. However, a person can avoid many misunderstandings and can feel much more confident in himself if he knows the meanings of words used by a person he is conversing with.

Therefore, you are urged to take time to fully understand <u>each</u> of the symbols and terms introduced in this package. If you will do this, it will be much easier for you to talk to your friends and your teacher about algebra.

PACKAGE GOAL: to gain an adequate understanding of the language of algebra.

PACKAGE OBJECTIVES:

1. Given an expression that does not contain expon-
    ents and given the value of each variable that
    appears in it, state the number of terms in the
    expression and evaluate the expression.

2. Given an expression involving exponents and given
    the value of each variable that appears in it,
    evaluate the expression.

3. Given an expression in which some symbols of
    grouping have been omitted and given the value of
    each variable that appears in the expression,
    simplify the expression.

4. Given an open sentence and a replacement set,
    specify the solution set.

5. Given a statement containing a quantifier, indicate
    whether the statement is true or false and just-
    ify your answer.

6. Given a numerical relationship described in words.
    and described by symbols and numbers, show how
    the symbols and numbers description is obtained
    from the word description.

I. U. # 03-02-01

Variables

You will need to recall:

Names, or symbols, for numbers are called <u>numerical</u> <u>expressions</u>.
Examples: 3 + 1, 5 - 2 + 3, 4 X 8 - 3.

Changing the numeral by which a number is named in an expression does not change the value of the expression.

OBJECTIVES:

1. Given a mathematical expression, state the number of terms of the expression.

2. Given a variable expression and the value of each variable, evaluate the expression.

3. Given an expression that does not contain exponents and given the value of each variable that appears in it, state the number of terms in the expression and evaluate the expression.

ACTIVITIES:

Study: <u>S & M</u>: pages 31-33 (objectives 1,2, & 3)

Suggested exercises:
    page 34, written exercises, part A, odd.
    pages 34-35, problems, part A, odd.

Criterion Test 03-02-01-01

1. State the number of terms in each expression.

    (a) x - bc   (c) (x - 2c)(x + 2c)
    (b) (xy)(ab)   (d) (x - 2c) x + 1

2. Evaluate each of the following expressions.

    (a) $\frac{bx + c}{2x + a}$, if a is 5, b is 0, c is 4, and x is 3.5

    (b) 1/2h (b + c): Area of a trapezoid

        let h be 16 ft., b be 40 ft. and c be 15 ft.

3. State the number of terms in each expression and
   evaluate it. a is 2, b is 1, c is 3, x is 0, y is 4,
   and z is 1/2.

    (a) abcxyz   (b) (x + b)(y - 2a)   (c) a + b + c

Criterion Test 03-02-01-02

1. State the number of terms in each expression.

    (a) b - xy   (c) (x - y)(x + 1)
    (b) (ab)(cd)   (d) x ÷ 2(x + 1)

2. Evaluate each of the following expressions.

    (a) $\frac{x + 2c}{2b - a}$, if a is 8, b is 5, c is 7, and x is 4.

    (b) area of a triangle: 1/2bh.
         let b = 8.5 inches and h = 4 inches

3. State the number of terms in each expression and
   evaluate it. a is 2, b is 1, c is 3, x is 0, y is 4,
   and z is 1/2.

    (a) abxyz   (b) (c - b)(yz)   (c) ac - yz

6

Criterion Test 03-02-01-03

1. State the number of terms in each expression.

   (a) a + b (c + d)
   (b) (x + y)(3 + x)
   (c) (x - 4)(ab)
   (d) ab - xy + 3

2. Evaluate each of the following expressions.

   (a) $\dfrac{2x + a}{2b - c}$, if a is 5, b is 4, c is 3, and x is 0.

   (b) Perimeter of a rectangle: $2\ell + 2w$
        let ℓ be 4 yards and w be 1.5 yards

3. State the number of terms in each expression and
   evaluate it. a is 2, b is 1, c is 3, x is 0,
   y is 4, and z is 1/2.

   (a) acxyz
   (b) (y - x)(a - b)
   (c) cy - az

7

Answers to Criterion Tests

Test 03-02-01-01

    1. (a) 2   (b) 1   (c) 1   (d) 2

    2. (a) 1/3   (b) 440 sq. ft.

    3. (a) 1, 0   (b) 1, 0   (c) 3,6

Test 03-02-01-02

    1. (a) 2   (b) 1   (c) 1   (d) 2

    2. (a) 9   (b) 17 sq. in.

    3. (a) 1, 0   (b) 1, 4   (c) 2, 4

Test 03-02-01-03

    1. (a) 2   (b) 1   (c) 1   (d) 3

    2. (a) 1   (b) 11 yards

    3. (a) 1, 0   (b) 1, 4   (c) 2, 11

I. U. # 03-02-02

Exponents

You will need to recall:

A _positive_ _integer_ is a member of the set {1,2,3,4, ...}.

OBJECTIVES:

1. Given a term, state the coefficient, the base, and the exponent.

2. Given an expression in expanded form, write it in exponential form.

3. Given an expression involving exponents and given the value of each variable that appears in it, evaluate the expression.

ACTIVITIES:

Study: S & M: pages 36-38 (objectives 1-3)

If asked to define an exponent, you must be very careful what you say. An exponent does _not_ "tell how many times a number is multiplied by itself."

Consider 5^2. If I take 5 times itself twice (5 X 5 = 25 and 5 X 5 = 25), I get 25 and 25. This is incorrect. 5^2 = 25.

Consider 4^3. If I take 4 times itself three times (4 X 4 = 16, 4 X 4 = 16, 4 x 4 = 16), I get 16,16, and 16. This is incorrect. 4^3 = 4 X 4 X 4 = 64.

Suggested exercises:
   S & M: page 38; ex. 13-18 (objective 1)
           page 39; ex. 1-15 odd, (objective 2)
           page 39; ex. 17-27 odd, (objective 3)
           page 39; problems 1,3,5 (objective 3)

Criterion Test 03-02-02-01

1. For each term, state the coefficient, the base, and the exponent in that order.

   (a) $2xy^2$
   (b) $6y^4$
   (c) $3(x - y)^2$

2. Write in exponential form.

   (a) $c \cdot c \cdot c \cdot c$
   (b) the fourth power of $x + 3$.
   (c) $12 \cdot x \cdot y \cdot y \cdot y$

3. Evaluate each expression for the given values of the variables.

   (a) $2x^3$; 2
   (b) $2(xy)^2$; x is 5, y is 2.
   (c) $(3x)^2$; 4
   (d) Surface area of a prism: $2w^2 + 4\ell w$
       let ℓ be 4 ft. and w be 3 ft.

Criterion Test 03-02-02-02

1. For each term, state the coefficient, the base, and the exponent, in that order.

   (a) $3xy^3$
   (b) $7y^2$
   (c) $4(a + b)^3$

2. Write in exponential form.

   (a) $x \cdot x \cdot x \cdot x \cdot x \cdot x$
   (b) the cube of $x - 2$
   (c) $3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$

3. Evaluate each expression for the given values of the variables.

   (a) $3x^2$; 4
   (b) $2(xy)^3$; x is 2, y is 3.
   (c) $(2x^3)$; 3
   (d) Volume of a sphere: $4/3 \ \pi r^3$
       let r be 6 inches. $\pi \doteq 22/7$

11

Criterion Test 03-02-02-03

1. For each term, state the coefficient, the base, and the exponent, in that order.

   (a) $4xy^3$
   (b) $5y^3$
   (c) $2(x - y)^3$

2. Write in exponential form:

   (a) $x \cdot x \cdot x$
   (b) the square of $x + 8$
   (c) $3 \cdot x \cdot y \cdot y \cdot y \cdot z$

3. Evaluate each expression for the given values of the variables.

   (a) $3x^2$; 4
   (b) $2(xy)^3$; $x = 2$, $y = 3$
   (c) $(4x)^2$; 3
   (d) Volume of a right circular cylinder: $\pi r^2 h$
       let r be 4 inches, h be 35 inches, and $\pi \doteq 22/7$

Answers to Criterion Tests

Test 03-02-02-01

    1.   (a) 2, y, 2
          (b) 6, y, 4
          (c) 3, x - y, 2

    2.   (a) c^4
          (b) $(x + 3)^4$
          (c) $12 \cdot x \cdot y^3$

    3.   (a) 16
          (b) 200
          (c) 144
          (d) 66 sq. ft.

Test 03-02-02-02

    1.   (a) 3, y, 3
          (b) 7, y, 2
          (c) 4, a + b, 3

    2.   (a) x^6
          (b) $(x - 2)^3$
          (c) $3x^2y^4$

    3.   (a) 48
          (b) 432
          (c) 54
          (d) 905 1/7 cubic inches

Test 03-02-02-03

    1.   (a) 4, y, 3
          (b) 5, y, 3
          (c) 2, x - y, 3

    2.   (a) x^3
          (b) $(x + 8)^2$
          (c) $3xy^3z$

    3.   (a) 48
          (b) 432
          (c) 144
          (d) 1760 cubic inches

I. U. # 03-02-03

Order of Operations

OBJECTIVES:

1. Given a numerical expression in which some symbols of grouping have been omitted, simplify the expression.

2. Given an expression in which some symbols of grouping have been omitted and given the value of each variable that appears in the expression, simplify the expression.

ACTIVITIES:

Study:
S & M: pages 41-42. (objective 1 and 2).

Suggested exercises:
S & M: page 43; 1-19 odd (objective 1).
page 43; 25-37 odd (objective 2).

Criterion Test 03-02-03-01

1. Simplify each expression.

   (a) $17 + 5 (2 + 8) - 38$
   (b) $8 \cdot 2 - 2^2 + 3^2$
   (c) $4(9 + 1) - 35 \div 7$

2. Evaluate each expression if x is 2, y is 3, and z is 4.

   (a) $2y^2 - y + 1$
   (b) $\dfrac{x^2 + y - z}{y^2}$
   (c) $z^3 - xy + 5$

Criterion Test 03-02-03-02

1. Simplify each expression.

   (a) $14 + 3 (2 + 7) - 4$
   (b) $7 \cdot 3 - 3^2 \div 4 \div 2$
   (c) $3(6 + 3) - 21 \div 3$

2. Evaluate each expression if x is 2, y is 3, and z is 4.

   (a) $3x^2 - x - 2$
   (b) $\dfrac{2x^3 - 2xy + z}{xyz}$
   (c) $5x^2y + z$

Criterion Test 03-02-03-03

1. Simplify each expression.

   (a) $14 + 3(8 + 5) - 12$
   (b) $6 \cdot 3 - 3^2 \div 4^2$
   (c) $48 \div 8 + 3(8 + 2)$

2. Evaluate each expression is x is 2, y is 3, and x is 4.

   (a) $3x^2 + 3y^2 + z^2$
   (b) $\dfrac{x^2 + xy + 5}{3xz}$
   (c) $x^3 + xy + 2$

16

Answers to Criterion Tests

Test 03-02-03-01

    1. (a) 29
        (b) 21
        (c) 35

    2. (a) 16
        (b) 1/3
        (c) 63

Test 03-02-03-02

    1. (a) 37
        (b) 14
        (c) 20

    2. (a) 8
        (b) 1/3
        (c) 64

Test 03-02-03-03

    1. (a) 41
        (b) 25
        (c) 36

    2. (a) 55
        (b) 5/8
        (c) 16

I. U. # 03-01-04

Open Sentences

You need to recall:

The <u>domain</u> (replacement set) of a variable is the set of numbers that the variable may represent.

OBJECTIVES:

1. Given an open sentence and a replacement set, substitute members of the replacement set in the open sentence and tell which of the resulting statements are true.

2. Given an open sentence and a replacement set, specify the solution set of each equation by roster and each inequality by graph.

ACTIVITIES:

Study:
  <u>S & M</u>: pages 44-45 (objectives 1 and 2)

Suggested exercises:
  <u>S & M</u>: page 46; ex. 1-11, odd (objective 1)
  pages 46-47; ex. 13-35, odd (objective 2)

19

Criterion Test 03-02-04-01

1. Substitute each member of the given set in the
   open sentence and tell which of the resulting
   statements are true.

   (a) $x > 3$; $\{3,4,5\}$
   (b) $x \neq 5$; $\{3,4,5,6\}$
   (c) $y^2 > 2$; $\{0,1,2\}$

2. Specify the solution set of each equation by
   roster and each inequality by graph.

   (a) $x - 2 = 10$ {the whole numbers}
   (b) $x + 2 < 7$ {the positive integers}
   (c) $21 < 7x$ {the positive numbers}
   (d) $14 = 5x - 1$ {the real numbers}
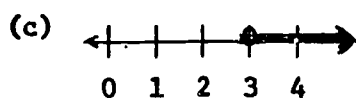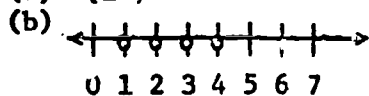

Criterion Test 03-02-04-02

1. Substitute each member of the given set in the
   open sentence and tell which of the resulting
   statements are true.

   (a) $y > 4$; $\{3,4,5\}$
   (b) $y \neq 7$; $\{6,7,8\}$
   (c) $x^2 > 3$; $\{1,2,3\}$

2. Specify the solution set of each equation by
   roster and each inequality by graph.

   (a) $x - 3 = 10$ {the whole numbers}
   (b) $3x > 21$ {the positive integers}
   (c) $x + 5 < 7$ {the positive integers}
   (d) $2x = 10$ {the real numbers}

Answers to Criterion Tests

Test 03-02-04-01

    1. (a)   true for 4 and 5
        (b)   true for 3,4, and 6
        (c)   true for 2

    2. (a)   {12}
        (b)

             0 1 2 3 4 5 6 7

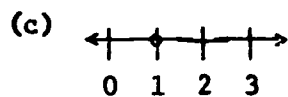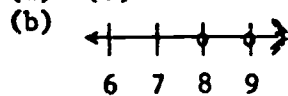   (c)

             0 1 2 3 4

        (d) {3}

Test 03-02-04-02

    1. (a)   true for 5
        (b)   true for 6 and 8
        (c)   true for 2 and 3

    2. (a)   {7}
        (b)

             6 7 8 9

        (c)

             0 1 2 3

        (d) {5}

Criterion Test 03-02-04-03

1. Substitute each member of the given set in the
   open sentence and tell which of the resulting
   statements are true.

   (a) $x < 4$; {3,4,5}
   (b) $x \neq 3$; {3,4,5}
   (c) $x^2 < 20$ {3,4,5}

2. Specify the solution set of each equation by
   roster and each inequality be graph.

   (a) $x + 2 = 8$ {the whole numbers}
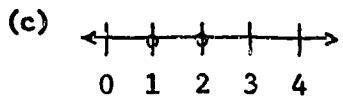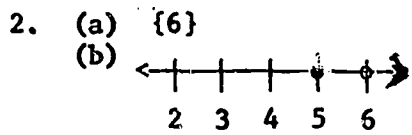   (b) $20 < 5x$ {the positive integers}
   (c) $7 > x + 4$ {the positive integers}
   (d) $5x + 2 = 17$ {the real numbers}

Answers to Criterion Tests (continued)

Test 03-02-04-03

1. (a) true for 3
   (b) true for 4 and 5
   (c) true for 3 and 4

2. (a) {6}
   (b)

       2 3 4 5 6

   (c)

       0 1 2 3 4

   (d) {3}

I. U. # 03-02-05

Quantifiers

OBJECTIVES:

1. Given a statement containing a quantifier,
   indicate whether the statement is true or
   false and justify your answer.

ACTIVITIES:

Study:
  S & M: pages 48 - 49   (Objective 1)

Suggested exercises:
  S & M: page 49; exercises 1 - 16   (Objective 1)
          page 50; exercises 1 - 8   (Objective 1)

Criterion Test 03-02-05-01

1. Indicate whether the statement is true or false and justify your answer.

   (a) All numbers are negative.

   (b) The square of each real number k is greater than k.

   (c) Any natural number n is less than 2n.

   (d) There is a number t such that $2t + 1 = 8$.

Criterion Test 03-02-05-02

1. Indicate whether the statement is true or false and justify your answer.

   (a) All cats are black.

   (b) Some numbers are not whole numbers.

   (c) There exists an integer y such that $2y + 1 > 6$.

   (d) All numbers are greater than 0.

Criterion Test 03-02-05-03

1. Indicate whether the statement is true or false and justify your answer.

   (a) Every polygon has four sides.

   (b) Some triangles have three sides congruent.

   (c) For every natural number s, $s + 2 > 2$.

   (d) There exists a number x such that $x^2 < x$.

Answers:

Criterion Test 03-02-05-01

1. (a) false; 5 is a number and 5 is not negative.

    (b) false; $\frac{1}{2}$ is a real number and $(\frac{1}{2})^2$ is not greater than $\frac{1}{2}$.

    (c) false; 0 is a natural number and 0 is not less than $2 \cdot 0$.

    (d) true; 3.5 is a number and $2(3.5) + 1 = 8$.

Criterion Test 03-02-05-02

1. (a) false; there is a white cat.

    (b) true; $\frac{1}{2}$ is a number but not a whole number.

    (c) true; 4 is an integer and $2 \cdot 4 + 1 > 6$.

    (d) false; -5 is a number that is not greater than 0.

Criterion Test 03-02-05-03

1. (a) false; a triangle is a polygon that does not have four sides.

    (b) true; an equilateral triangle has 3 sides congruent.

    (c) false; 0 is a natural number and $0 + 2$ is not greater than 2.

    (d) true; $\frac{1}{2}$ is a number and $(\frac{1}{2})^2$ is less than $\frac{1}{2}$.

I. U. # 03-02-06

Applying Mathematical

Expressions and Sentences

28

OBJECTIVES:

1. Given a word description of a number, write
   it in terms of a variable.

2. Given a numerical relationship described in
   words and described by symbols and numbers,
   show how the symbols and numbers description
   is obtained from the word description.

ACTIVITIES:

Study:
  S & M: pages 50 - 51   (Objectives 1 and 2)

Suggested exercises:
  S & M: pages 51 - 52; oral exercises 1 - 19, odd.
                              (Objective 1)

          page 53; exercises 1 - 10   (Objective 2)

          Please use the red answer book when
          checking your answers. The answers in
          the back of your book are not complete
          enough for these exercises. This applies
          to exercises on page 53 only.

Criterion Test 03-02-06-01

1. Express the number described in terms of n.

    (a) a number that is two less than n.
    (b) If Bob was n years old 5 years ago, how
         old is he now?
    (c) How much will n 8-cent stamps cost?
    (d) If n is an even number, what is the next
         larger even number?

2. Show how the given open sentence can be ob-
    tained from the word description.

    (a) k + 5 = 23
         Five years from now Jim will be 23 years old.

    (b) x + (x + 2) + (x + 4) = 24
         The sum of three consecutive even integers is 24.

Criterion Test 03-02-06-02

1. Express the number described in terms of n.

    (a) a number is five more than n.
    (b) If Laurie is n years old now, how old
         was she 5 years ago?
    (c) If n cigars cost $1.50, what does one
         cigar cost?
    (d) If n is an integer, what is the next smaller
         integer?

2. Show how the given open sentence can be obtained
    from the word description.

    (a) 2x + 1 = 15
         Barbara is 15. She is one year more than
         twice as old as Jimmy.

    (b) 20 - x = 15
         In a class of 20 students there are 15 girls.

Criterion Test 03-02-06-03

1. Express the number described in terms of n.

   (a) a number twice as large as n.
   (b) If Ruth is n years old now, how old will she
       be six years from now?
   (c) If a bottle of pop costs 15¢, what will ⊔
       bottles cost?
   (d) What is the number that is three less than
       the square of n?

2. Show how the given open sentence can be obtained
   from the word description.

   (a) $2\ell + 2(\ell - 5) = 120$
       A rectangle whose width is 5 less than its
       length has a perimeter of 120.

   (b) $x + 4 = 2(x - 16)$
       Four years from now Rick will be twice as
       old as he was sixteen years ago.

Answers:

Criterion Test 03-02-06-01

1. (a) n - 2
   (b) n + 5
   (c) 8nc
   (d) n + 2

2. (a) let k = Jim's age now. In five years he
        will be k + 5. He will be 23, so k + 5 = 23.
   (b) let x = the smaller integer. The other two
        would be x + 2 and x + 4. The sum would be
        x + (x + 2) + (x + 4). Since this sum is 24,
        x + (x + 2) + (x + 4) = 24.

Criterion Test 03-02-06-02

1. (a) n + 5
   (b) n - 8
   (c) $\frac{150}{n}$ ¢

   (d) n - 1

2. (a) let x = Jimmy's age. Then 2x would be twice
        his age. 2x + 1 would be one more than twice
        his age. Since 2x + 1 and 15 are both ex-
        pressions for Barbara's age, 2x + 1 = 15.
   (b) let x = the number of boys. Then 20 - x
        would be the number of girls. Since there are
        15 girls, 20 - x = 15.

Criterion Test 03-02-06-03

1. (a) 2n
   (b) n + 6
   (c) 15n ¢
   (d) $n^2 - 3$

2. (a) let ℓ = the length. Then ℓ - 5 would be
        the width. 2ℓ would be two lengths.
        $2(\ell - 5)$ would be two widths. $2\ell + 2(\ell - 5)$
        would be the perimeter. Since the perimeter
        is also 120, $2\ell + 2(\ell - 5) = 120$.
   (b) let x be Rick's age now. In four years he
        will be x + 4. Sixteen years ago he was x - 16.
        Twice his age then was 2(x - 16). Therefore,
        x + 4 = 2(x - 16).

                END OF PACKAGE. # 03-02