DOCUMENT RESUME

ED 089 989 SE 017 521

TITLE Mathematics Grades 4-6: A Teacher's Guide.
INSTITUTION Buffalo Public Schools, N. Y. Div. of Curriculum Evaluation and Development.
PUB DATE 68
NOTE 43p.

EDRS PRICE MF-$0.75 HC-$1.85 PLUS POSTAGE
DESCRIPTORS *Course Content; Curriculum; *Curriculum Guides; *Elementary School Mathematics; *Instruction; *Instructional Materials; Teaching Guides

ABSTRACT

This curriculum guide presents the outlines for course content in mathematics in grades 4-6. For each grade level general overviews are given of the goals and objectives of the course. A detailed explanation of the content outline includes suggestions as to method of presentation. The mathematical concepts are explained using a technically correct approach. This is intended primarily for the teacher, so that the foundations she builds with the pupils in an informal way are based on sound, accurate mathematical principles. (JP)

MATHEMATICS

A TEACHER'S GUIDE

GRADES 4 - 6

BUFFALO PUBLIC SCHOOLS   BUFFALO NEW YORK
DIVISION OF CURRICULUM EVALUATION AND DEVELOPMENT

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BUFFALO PUBLIC SCHOOLS
BUFFALO, NEW YORK

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FOREWORD

The Buffalo Public Schools have had a very successful mathematics program for many years. Our students have been achieving at a rate comparable to other public schools throughout the state and our teachers are to be complimented for maintaining this fine record.

New developments in the field of mathematics, however, demanded that teachers and administrators review the existing program and make necessary changes and adaptations.

A Curriculum Committee under the direction of Mr. Louis Scholl, Director of Mathematics, has done an excellent job in preparing this new tentative Elementary Mathematics Guide. I feel the Committee has made a successful transition to modern mathematics with its increased emphasis on reasoning and analysis and at the same time continuing emphasis on skills and understandings needed in our contemporary society.

This Guide is tentative. Comments and suggestions will be welcomed from teachers and parents throughout the school year. The Curriculum Committee will review these suggestions and make changes that are deemed necessary.

I ask all elementary teachers to work diligently in helping children achieve success with this new program.

Joseph Manch
Superintendent of Schools

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CONTENTS

OVERVIEW

As a child begins the work of fourth grade he possesses a very large pool of basic facts, concepts and skills. Such skills of course will have to be developed and insights deepened, but it is at this time that the child's power to think quantitatively, to discover principles, to generalize, and to reason independently is growing.

Thus the child needs a program which will stimulate these powers and a teacher who will guide their development. A course which relies chiefly upon the student's memory, and presents him with a set of largely unrelated facts and a set of mechanical tricks, will be insufficient to meet the demands of today s world. Some of the best minds of the present day, enormous amounts of time, money and experimentation all have been focused on this problem. The result is a curriculum which stresses meaning and understanding, and develops the pupil's powers to do quantitative reasoning.

The skillful teacher will seek to guide her pupils in discovering for themselves the basic generalizations and the detailed facts that are a part of this year's work. Her goal will be to see that the students develop a keen awareness of the structure and logic of mathematics, and gain a thorough knowledge and mastery of the basic facts and processes. Her own intellectual curiosity will be contagious and encourage her pupils to investigate ideas and to seek answers.

AIMS:

1. To develop an insight into the logic and structure of our mathematical system.
2. To develop an understanding of the basic properties of our number system.
3. To extend basic skills in computation.
4. To expand and deepen an understanding of fractions.
5. To develop basic geometric concepts and relationships.
6. To develop and apply number sentences to the solution of problems.
7. To continue to develop skill in estimating reasonable answers, and to do mental computation.
8. To guide and develop ability to analyze and organize sets of facts.

MATHEMATICS - GRADE FOUR

Review Mathematical Concepts Covered in Grades K - 3.

Vocabulary and Symbols

A. Reinforce vocabulary of previous years.

B. Introduce or review the terms:

additive inverse
associative law
braces
commutative law
disjoint sets
distributive law
empty set
equation
identity element
intersection of sets
line segment
one-to-one correspondence
open sentence
ray
region
simple closed curve
solution set
subset
union of sets
angle
arc
area
bar graph
bisect
concave
congruence
convex
diameter
edge
endpoint
equilateral triangle
hexagon
isosceles triangle
octagon
pentagon
perimeter
polygon
radius
right angle
average
cardinal number
decimal system
denominator
digit
dividend
divisor
expanded notation
factor
fraction
inequality
inverse operation
multiple
negative number
numeral
numerator
positive number
prime number
quotient

I. Sets and Set Notation

A. Use of braces to identify sets. The use of the letter N before the braces designates the cardinal number of the set.

B. Equal sets and equivalent sets.

C. The empty, or null, set with cardinal number zero.

D. Union, difference, and intersection of sets.

E. Disjoint sets.

F. Subsets.

II. Numeration Systems

A. Stress that numerals are symbols for numbers.

B. Review and extend the decimal place value system to include practice with numerals up to and including seven places, using expanded notation.

C. Group numerals of five digits and more into periods set off with commas.

D. Read numbers to nearest ten-thousand.

E. Teach and apply the commutative and associative laws for addition and multiplication.

F. Teach and apply the distributive law of multiplication over addition.

G. Develop and define zero as the identity element for addition, and one as the identity element for multiplication.

H. Extend the number system on the number line to include the negative integers.

III. Operations

A. Whole Numbers

1. Addition and Subtraction (with regrouping)

a. Review and practice for mastery the basic addition and subtraction facts.

b. Review and practice addition and subtraction with regrouping.

c. Add and subtract numbers in a variety of situations.

1) include two, three and four-digit numerals.
2) include three and four addends.
3) give practice in both horizontal and vertical form.

        4) include numerals representing money in dollars and cents, using the dollar sign and decimal point.

        5) teach addition of various combinations of one-digit positive and negative numbers.

    2. Multiplication and Division

        a. Define and use "Factor times factor equals product."

        b. Present and master the basic multiplication facts up through twelve times twelve.

        c. Introduce the conventional algorithm for multiplication of a two-digit number by a single digit number.

        d. Multiply two-digit numbers by ten, one hundred and one thousand, and also by multiples of ten up to ninety.

        e. Define and use "Dividend divided by divisor equals quotient."

        f. Introduce the conventional algorithm for division.

        g. Extend division to include a two-digit divisor with a three-digit dividend, with a one-digit quotient and remainder.

        h. Use the inverse operation property to check division problems by multiplication.

B. Fractions

    1. Introduce the terms numerator and denominator, in which the denominator indicates the number of equal parts into which the whole is divided, and the numerator tells how many of these equal parts we are considering.

    2. Cover addition and subtraction of like fractions.

    3. Find fractional parts of whole numbers.

    4. Introduce renaming of fractions using a number line.

    5. Introduce addition [illegible] having [illegible].

5. Rename an improper fraction as a mixed number.

6. Introduce inequalities using fractions on a number line.

IV. <u>Geometry</u>

A. Metric

1. Develop the concept of a standard unit of measure.
2. Continue to develop equivalent measurements for an interval of time. (year, leap year, century, decade)
3. Introduce second as a unit of time. (60 seconds = 1 minute).
4. Estimate heights, lengths and weights.
5. Measure to the nearest eighth inch.
6. Introduce the mile as a unit of measure. (5280 feet = 1 mile).
7. Find the sum of the lengths of the sides of a figure. (perimeter).
8. Use equivalent measurements of weight. (Introduce ton).
9. Continue the study of money.

B. Non-metric

1. Introduce concepts of point, line, and line segment.
2. Introduce point of intersection, endpoint, and vertex.
3. Introduce concept of ray, angle, and right angle.
4. Review open and closed figures.
5. Develop the concept of <u>inside</u>, <u>outside</u>, or <u>on</u> a figure.
6. Distinguish between concave and convex figures.
7. Discover properties of various polygons, and introduce rectangular prism.
8. Introduce the concept of line symmetry.
9. Extend the study of solids to the concepts of face and vertex.

V. Mathematical Sentences

A. Equations

1. Use letter symbols as variables in mathematical sentences.

2. Review equations using addition and subtraction.

3. Introduce simple equations which involve multiplication and division.

4. Use concept of replacement set (universal set) in solving mathematical sentences for the answer set (solution set).

B. Inequalities

1. Stress comparison using the concepts of greater than, and less than.

2. Use of number line to visualize comparison of numbers.

VI. Estimating and Mental Arithmetic

A. Round to the nearest tens, hundreds and thousands.

B. Estimate solutions in all types of problems before computation.

C. Estimate answers to judge reasonableness of computed answer.

VII. Problem Solving

A. Increase emphasis on estimating as an integral part of the problem solving process.

B. Continue to translate quantitative sentences into mathematical sentences using letters.

C. Use units of measure in various types of problems.

D. Find solutions for problems involving two steps.

E. Find averages.

INSTRUCTIONAL OUTLINE

The seven sections which follow are detailed explanations of the preceding seven sections of course content, and include suggestions as to method of teaching.

I. Sets and Set Notation

A set is simply a collection of objects or ideas. These objects or ideas are called members or elements of the set, and are enclosed in braces. For instance, the set of whole numbers less than ten is shown as $\{0,1,2,3,4,5,6,7,8,9\}$.

The number of members which a set contains is called the cardinal number of the set. This is indicated by the letter N before the braces. For example, the set $\{a,b,c,d\}$ has the cardinal number 4, (four members in the set), and this would be written $N\{a,b,c,d\} = 4$.

If the members of one set can be paired exactly in one-to-one correspondence with the members of another set, then these sets have the same cardinal number, and are called equivalent sets. An example of this would be matching a set of 30 children in a class with a set of 30 desks in the classroom. The set of children would be equivalent to the set of desks. We only speak of equal sets when the two sets are identical, that is, they are made up of exactly the same members. For instance, the set $\{a,b,c\}$ is equal to the set $\{a,c,b\}$.

The set with the cardinal number zero, that is, the empty (or null) set, is written $\{\ \}$, or sometimes the Greek symbol ϕ is used. An example of this set might be "the set of elephants living in our classroom." If we wish to speak only of certain members of a given set, for example the set of even whole numbers less than ten, we form the subset $\{0,2,4,6,8\}$ taken from the set of all whole numbers less than ten. To indicate a subset we use the symbol "$\subset$". Hence in this example we write $\{0,2,4,6,8\} \subset \{0,1,2,3,4,5,6,7,8,9\}$. We would read this "The set $\{0,2,4,6,8\}$ is a subset of (or is contained in) the set $\{0,1,2,3,4,5,6,7,8,9\}$. We might also form the subset of odd numbers less than ten $\{1,3,5,7,9\}$. In this instance the union of these two subsets, indicated by the symbol " U " would be our original set, that is,

$$\{0,2,4,6,8\} \cup \{1,3,5,7,9\} = \{0,1,2,3,4,5,6,7,8,9\} .$$

However in finding the union of two sets, we must be careful not to count any of the members of the sets twice. For instance the union of the set of odd whole numbers less than ten $\{1,3,5,7,9\}$, with the set of whole numbers less than ten that are exactly divisible by three $\{0,3,6,9\}$, would be the set $\{0,1,3,5,6,7,8\}$, that is

$$\{1,3,4,7,9\} \cup \{0,3,6,9\} = \{0,1,3,4,6,7,9\} .$$

Note that in the above example the members 3 and 9 are only counted once, although they appear in both sets. It may be noted that while the cardinal number of the first subset is 5, and the cardinal number of the second subset is 4, the cardinal number of the union of the two subsets is only 7, since there are exactly seven members in the final set. This will always happen when the two subsets are not disjoint, that is, they have some members in common.

The difference of two sets is analogous to subtraction. For instance, to find the difference of the following two sets - the set of whole numbers less than 10, and the set of odd whole numbers less than ten, that is,

$$\{0,1,2,3,4,5,6,7,8,9\} - \{1,3,5,7,9\} = \{0,2,4,6,8\} .$$

Note that to find the difference of two sets, it is more convenient if the one being subtracted is a subset of the other.

The intersection of two sets, indicated by the symbol " $\cap$ ", is defined as the set of only those members which are found in both sets. For instance, the intersection of the set of whole numbers less than ten with the set of even whole numbers greater than five and less than thirteen would be the set $\{6,8\}$, that is,

$$\{0,1,2,3,4,5,6,7,8,9\} \cap \{6,8,10,12\} = \{6,8\} .$$

As another example, the intersection of the set of odd numbers less than ten with the set of even numbers less than ten would be the empty set, since they have no members in common, that is,

$$\{1,3,5,7,9\} \cap \{0,2,4,6,8\} = \{\ \} .$$

II. Numeration Systems

Substantial work has previously been done on the concept of place value and the properties of the real number system. This work must now be extended and unified with the aim of appreciating the logic and structure of our man-made number system.

A brief study of the Roman numeral system will prove rewarding in noting the advantage of place value notation. With an example such as VIII and 5,111 and the use of expanded notation

$$5,111 = 5(1000) + 1(100) + 1(10) + 1(1),$$

the students can discuss the advantages of our decimal system. Points to be brought out would be the distinction between a number (idea - immaterial) and a numeral (symbol - material), and that in the example 5,111, the same digit one has 3 distinct values, whereas in the Roman numeral it has only one value. To make it easier to read large numbers we group them into periods of three digits, the comma serving as a visual aid in recognizing the groups.

The importance and daily usefulness of the basic properties of our number system were largely overlooked in traditional courses. The fact that the order of addends or factors does not alter the sum or product (commutative property) should be pointed out as an example of the structural beauty of a man-made system. The same is true of the associative property (the manner in which addends or factors are grouped does not change the sum or product) and the distributive property (the product of a sum of numbers by a given number may be found by multiplying that number times each addend and then adding the products.)

The distributive law of multiplication over addition enables us to solve a computation in which both the operation of addition and the operation of multiplication are found. The distributive law tells us that we may do such a problem in two different ways - first adding and then multiplying, or first multiplying and then adding. The children themselves should discover the law by doing such problems as the following:

$$5 \times (4 + 3) = 5 \times 7$$

$$(5 \times 4) + (5 \times 3)$$

$$20 \quad + \quad 15$$

$$35 \qquad = \quad 35$$

The children should be led to the discovery that in any multiplication involving partial products, the distributive law is used. For instance in multiplying 5 times 24, we first multiply 5 times 4 and then 5 times 20, and add these two partial products to obtain our answer.

The role of zero as the identity element in addition, and the role of one as the identity element in multiplication should be carefully developed. The term "identity element" is used because in each case if we apply the binary operation of addition or multiplication on any number paired with the proper identity element, our answer will be identically the same number we started with. That is, $n + 0 = n$ and $n \times 1 = n$.

III. Operations

Work in mastering the four fundamental operations should be developed using the "guided discovery" approach. Each algorithm should be carefully developed step-by-step with pupil suggestions and participation under the guidance of the teacher. Each step should be justified with a logical reason so that the entire process is seen to be logical and sensible rather than an arbitrary trick. For example in the long division algorithm, the concepts of place value and the distributive law can be brought out.

$$360 \div 15 \text{ may be written}$$

$$\begin{array}{r|l} & 20 + 4 + 0 \\ \cline{2-2} \underline{15} & 300 + 60 + 0 \end{array}$$

Here each of the addends is divided by the given divisor, or as the distributive law implies, the 15 has been distributed over each addend.

After the algorithms have been developed, practice is necessary. This should be carefully planned, and adapted to the needs of the pupils. Hours of meaningless drill may do more harm than good, whereas carefully planned practice exercises at the proper time are vitally necessary and extremely helpful.

Negative numbers are introduced in a visual and meaningful way at this grade level. Children will be curious to see what happens when we extend the number line to the left beyond zero. Simple addition of negative numbers using the number line is covered. Other concrete examples such as a number of degrees below zero on a thermometer, or money owed, can be used to make the concept of negative numbers meaningful. The pupils will supply many other examples from their own experiences.

It is to be noted that we never use the word "minus" to indicate a negative number, just as we never use the word "plus" to indicate a positive number. The words plus and minus are reserved to indicate the operations of addition and subtraction, whereas the words positive and negative are properties of a number, and indicate whether the particular number is greater than, or less than, zero. Zero is neither positive nor negative.

IV. Geometry

The study of the history of units of measurement is fascinating and rewarding. Class discussions on this topic will lead to a better understanding of what a "standard" unit of measure is.

The work in geometry should be concrete and visual. The physical world about us should supply countless examples of the geometric concepts being taught. However, the teacher at all times should be careful to use correct and accurate mathematical language.

When discussing such geometrical concepts as point, line or solid (e.g. cube), it should be pointed out that the drawings of a point and line are only representations of the concept. For instance, the drawing of a line will have some thickness, whereas a theoretical line has only one dimension, length. It should also be pointed out that a closed figure such as a triangle does not include the region enclosed by the triangle. The triangle consists only of the three line segments. Hence, instructions such as <u>inside</u> the triangle, <u>outside</u> the triangle, and <u>on</u> the triangle become accurate and precise.

V.. Mathematical Sentences

A mathematical sentence (sometimes called a number sentence) is a statement which describes a mathematical relationship involving numbers, such as $2 + 3 < 8$ or $n + 4 = 7$. If one or more of the numbers are missing, as in the second example above, the mathematical sentence is sometimes called an <u>open sentence</u>. The solution set (or answer set) to the open sentence consists of that number or numbers selected from a prechosen replacement set (sometimes called universal set) which will make the statement <u>true</u>. Open sentences may be either equations or inequalities. They are to be solved intuitively, that is, by trial and error and "educated guessing" without using formal rules. For instance, if $n - 18 = 41$, and the replacement set is the set of all numbers, a pupil may first substitute 50 for n, and test to see if this replacement will make a true statement of the open sentence. This will give $50 - 18 = 41$ which he sees is false, and hence he will try a larger replacement for n, until he finds that the correct solution set consists of the single number 59. Some pupils will discover for themselves the rules for the solution of equations - in this case, 18 is added to both members of the equation, giving the intermediate step $n - 18 + 18 = 41 + 18$. Such ingenuity by pupils should be accepted and encouraged.

Inequalities are extremely valuable in promoting meaning and understanding of number relationships. The concept of "is less than" and "is greater than" should be used constantly, and illustrated with various types of visual aids such as the number line. For instance, we may have the inequality $n + 5 < 8$ where the replacement set is the set of whole numbers. Here the solution set would be $\{0,1,2\}$.

In general the teacher supplies the mathematical sentence to be solved. However, some practice for pupils should be given in translating an English sentence into a mathematical sentence. For instance, the English sentence "After John gave six cents to Bill, he had eight cents left" would translate into the mathematical sentence J - 6 = 8. Pupils usually find this type of exercise difficult, but it is especially valuable for promoting understanding and helping them in reading comprehension.

VI. Estimating and Mental Arithmetic

One of the ways in which a pupil exhibits understanding is to be able to estimate intelligently. Often an unrealistic estimate by a pupil will betray a complete lack of any real understanding of some concept or type of problem. This will alert the teacher to the fact that the explanation and development of the concept has not met with success, and hence reteaching with new approaches and pupil participation are in order.

Mental problems are always valuable for learning number facts, for simple problem solving, and for increasing attention span and improving concentration.

VII. Problem Solving

In order to solve word problems the pupil must be able to translate the relationship expressed in the problem into a mathematical sentence. In order to do this successfully, the pupil must not only know the meaning of the words, but must be able to understand the relationship expressed. Usually it is understanding the mathematical relationship rather than word recognition, that causes difficulty. Where a pupil solves a word problem incorrectly, the trouble may lie in one or more of the four areas: not knowing the meaning of the words; inability to understand the relationship and express it in a mathematical sentence; inability to choose the correct operation; or errors in computation. The teacher must analyze and decide where the difficulty lies, and take necessary steps to help the pupil.

The guided discovery method for teaching problem solving is especially effective. The teacher would ask such questions as "What does the problem ask for?" "What numerical information is supplied to help you find the answer?" "How can we use this information?" "Let us write a mathematical sentence expressing a relationship."

Problem solving is one of the most difficult of topics, and is a real challenge to the creativeness and ingenuity of the teacher.

OVERVIEW

Grade five is a time of rapid expansion and correlation in a modern mathematics curriculum. The child's knowledge of the properties and processes of our number system, previously related primarily to whole numbers, will now be expanded to include the set of fractional numbers. Properties and relationships previously discovered in geometry will now be more thoroughly organized and channeled into the development of new work. The structural relationships between the set of whole numbers and the set of fractions are found at this level.

The goal of a skillful teacher using a modern course in mathematics is to teach in such a way as to bring out the various interrelationships in mathematical processes, so that the pupils will gain an appreciation and understanding of the beauty to be found in the logic and structure of this man-made system. The method of guided discovery and pupil participation will result in increased class interest, and will promote understanding and knowledge. Basic number facts will be learned more quickly and easily, and retained longer. Pupils will see and understand the reasons behind the basic arithmetic processes and procedures.

AIMS:

1. To develop an appreciation of our mathematical system by gaining insight into its structure and underlying logic.
2. To develop familiarity with the vocabulary and symbolism of mathematics.
3. To extend, by discovery, knowledge of our numeration system.
4. To build skills in operating on various sets of numbers in the real number system.
5. To review, discover and correlate basic facts of geometry.
6. To extend the study and usefulness of mathematical sentences.
7. To encourage the development of powers to estimate, to make quantitative judgments and to compute mentally.
8. To guide the child in the development of the powers of critical thinking, and of analyzing and organizing sets of data.

MATHEMATICS - GRADE FIVE

Review mathematical concepts covered in Grades K - 4.

Vocabulary and Symbols

A. Reinforce vocabulary of previous years.

B. Introduce or review the terms.

place-value system
expanded notation
commutative
associative
distributive
angle
right angle
ray
segment
arc

congruent
degree
perimeter
area
meter
centimeter
dimensions
simple closed curve
convex
concave

equation
universal set
replacement set
empty set
subset
open sentence
equality
solution set
union of sets
intersection of sets

overlapping sets
inverse operation
additive inverse
multiplicative inverse
reciprocal
partial product
multiple
positive
negative

equivalent fractions
unlike fractions
like fractions
simplest terms
common factor
prime number
composite number

numerator
denominator
common denominator
pentagon
hexagon
octagon
region
rectangular solid

I. Sets and Set Notation

- A. Set descriptions
- B. Equal sets and equivalent sets
- C. The empty set
- D. Union and intersection of sets
- E. Difference of sets
- F. Disjoint and overlapping sets
- G. Subsets
- H. Finite and infinite sets

II. Numeration System

- A. Extend the Hindu-Arabic numeration system through billions.
- B. Properties of the real number system
    1. Extend commutative, associative and distributive properties of the whole number system to the system of fractional numbers.
    2. Introduce the property that for any fractional number, there is another fractional number such that the product of the two numbers is equal to one. Such a number is called the multiplicative inverse or reciprocal.
    3. Develop the understanding of place value in base ten to include decimal notation to tenths and hundredths.
    4. Extend the number system on the number line to include negative integers.

III. Operations

A. Whole Numbers

1. Addition and subtraction
    - a. Master operations of addition and subtraction of whole numbers.
    - b. Identify subtraction of whole numbers as the inverse of addition.
    - c. Emphasize the meaning and application of regrouping in addition and subtraction, using expanded notation.
    - d. Develop negative integers using the number line.
        - 1) identify relationship of "is greater than" and "is less than" with various combinations of positive and negative numbers.
        - 2) addition of various combinations of positive and negative integers.
        - 3) develop the concept of additive inverse.
2. Multiplication and Division
    - a. Define and use "factor times factor equals product."
    - b. Review and master basic multiplication facts up through twelve times twelve.
    - c. Develop the concept of prime number, and multiple of a number.
    - d. Estimate quotients, developing the algorithm for dividing numbers named by two-, three- and four-digit numerals by one- and two-digit numerals.
    - e. Continue development of the conventional long division algorithm, with checking by inverse operation.
    - f. Show that the quotient of two whole numbers may be expressed as a fraction or a mixed number.

B. Fractions

1. Common Fractions

   a. Review terms numerator and denominator.

   b. Introduce two meanings of a fraction as

      1) one or more of the equal parts of the whole

      2) indicated division

   c. Define equal fractions.

   d. Develop the idea that when multiplying or dividing both the numerator and the denominator by the same number, the original fraction and the resulting fraction name the same value, since a form of the identity element 1 is being used as the multiplier, or as the divisor.

   e. Develop concepts of greater than and less than with fractional numbers.

   f. Express fractional numbers in simplest form. Simplest form is defined as having numerator and denominator relatively prime, that is, they have no common factor. Simplest form is obtained by dividing both the numerator and denominator by the same number, that is, divide the entire fraction by a form of the identity element.

   g. Find the least common denominator.

   h. Add and subtract fractional numbers with unlike denominators.

   i. Add and subtract numbers named in mixed form with regrouping.

   j. Multiply fractional numbers.

   k. Find a product when one or both factors are named in mixed form (distributive law).

   l. Introduce concept of the reciprocal of a number.

   m. Introduce complex fractions as a step toward division of fractional numbers.

   n. Introduce division of fractions.

2. Decimal Fractions

   a. Introduce the places to the right of the decimal point as another way of writing fractional numbers which have denominators of 10, 100, 1000.

   b. Develop the concept that the place value immediately to the right of units place has one-tenth the value of units place.

   c. Develop the concept that the decimal point locates the ones place in a decimal number.

   d. Write decimals for tenths, hundreths and thousandths.

   e. Change common fractions to decimal fractions and decimal fractions to common fractions.

   f. Match decimal fractions with points on a number line.

   g. Introduce addition and subtraction using decimals.

   h. Introduce the notation for money as a decimal notation.

IV. <u>Geometry</u>

A. Metric

1. Introduce the concept that the smaller the unit of measure the more precise the measurement.

2. Review concept of perimeter and area.

3. Discover and use formulas for perimeter of rectangle, square and triangle.

4. Discover and use formulas for determining area of a rectangular region.

5. Compare two different angles as to whether one is larger than, smaller than, or congruent to the other.

6. Introduce metric system including meter, centimeter, decimeter, millimeter.

B. <u>Non-Metric</u>

1. Develop familiarity with previously introduced plane and solid figures.

2. Review quadrilateral, pentagon and hexagon.

3. Review concept and symbolism for ray, line and line segment.

4. Review open and closed figures, and the concepts of inside, outside, or on a figure.

5. Discover properties of various plane and solid geometric figures, and study line symmetry.

6. Introduce the concept of perpendicular lines and right angles.

V. Mathematical Sentences

A. Equations

1. Use many different letters of the alphabet as variables in open number sentences.

2. Introduce the terms equation, universal set, solution set, and open and closed sentences.

3. Discuss the nature of a mathematical sentence, which may be a closed sentence, either true or false, or an open sentence (which has no truth value).

4. Find solution sets for simple equations.

B. Inequalities

1. Stress comparison of various numbers using the concepts of greater than, and less than, together with the symbols for these.

2. Use of number line to visualize comparison of numbers.

3. Extend concepts of greater than, and less than, to fractional numbers.

VI. Estimating and Mental Arithmetic

A. Rounding

1. Extend rounding of numbers using the number line as a visual aid.

2. Introduce the rule for rounding numbers and apply to rounding numbers to nearest tens, hundreds, and thousands.

B. Estimate answers to judge reasonableness of computed answer.

VII. Problem Solving

A. Extend verbal and written problems involving the four basic operations with money, measurements, and fractions, and including mental computations in the solution of one and two step problems.

B. Practice writing mathematical sentences to express the relationships given in verbal problems.

VIII. Graphs and Tables

A. Read and interpret bar graphs, line graphs, circle graphs, and pictographs.

B. Construct bar graphs.

INSTRUCTIONAL OUTLINE

The eight sections which follow are detailed explanations of the preceding eight sections of Course Content, and include suggestions as to methods of teaching.

I. Sets and Set Notation

See the instructional outline under this heading on page 10 of the Grade IV curriculum guide. The following explanations are given for the additional topics in Grade V.

Disjoint sets are those sets which have no members in common. Overlapping sets are sets which do have some members in common. An example of two disjoint sets would be the sets

$$\{a,b,c\} \quad \text{and} \quad \{x,y,z\}$$

An example of overlapping sets would be the sets

$$\{a,b,c\} \quad \text{and} \quad \{b,c,d\}$$

Hence the intersection of two disjoint sets would be the empty set, while the intersection of two overlapping sets would be the set of only those members which are found in both sets.

Finite sets are those sets which have a fixed number of members, that is, there is some cardinal number for the set. An infinite set has an endless number of members. For instance, an example of a finite set would be the set of pupils enrolled in your school, while an example of an infinite set would be the set of whole numbers, or the set of points on a line.

II. Numeration Systems

The concept of place value is now extended to decimal notation. When the child has recognized the fact that the value of each place is always one-tenth the value of the place to its immediate left, he can be led to discover that the place immediately to the right of the units place must also be one-tenth of a unit. In this way the concept of tenths, hundredths, and thousandths can be built up. The recognition of order and logic is essential.

The importance and daily usefulness of the basic properties of our number system were largely overlooked in traditional courses. The fact that the order of addends or factors does not alter the sum or product (commutative property) should be pointed out as an example of the structural beauty of a man-made system.

The same is true of the associative property (the manner in which addends or factors are grouped does not change the sum or product) and the distributive property (the product of a sum of numbers by a given number may be found by multiplying that number times each addend and then adding the products).

The distributive law of multiplication over addition enables us to solve a computation in which both the operation of addition and the operation of multiplication are found. The distributive law tells us that we may do such a problem in two different ways - first adding and then multiplying, or first multiplying and then adding. The children themselves should discover the law by doing such problems as the following:

$$6 \times (20 + 3) = 6 \times 23$$

$$(6 \times 20) + (6 \times 3) =$$

$$120 \quad + \quad 18 \quad =$$

$$138 \quad = 138$$

The children should be led to the discovery that in any multiplication involving partial products, the distributive law is used. For instance in multiplying 8 times 34, we first multiply 8 times 4 and then 8 times 30, and add these two partial products to obtain our answer.

The role of <u>zero</u> as the identity element in addition, and the role of <u>one</u> as the identity element in multiplication should be carefully developed. The term "identity element" is used because in each case if we apply the binary operation of addition or multiplication on any number paired with the proper identity element, our answer will be identically the same number we started with. That is, $n + 0 = n$ and $n \times 1 = n$.

They will find that these three principles hold for the set of fractions just as they did for the set of whole numbers. They will also discover that <u>zero</u> is the identity element in addition of fractions, and <u>one</u> is the identity element in multiplication of fractions.

III. <u>Operations</u>

The basic algorithms and tables to be memorized have been introduced for the set of whole numbers. Mastery of the four fundamental operations is now the goal. The reasoning behind the algorithms should be reviewed, and the logic and structure of each of the processes should be constantly brought to the attention of the pupil. The applications of the commutative, associative, and distributive laws should be pointed out at every opportunity.

The negative integers on the number line are introduced. Simple addition of various combinations of positive and negative integers using the number line is covered. This work with the negative integers is concrete and visual. Applications such as a number of degrees below zero on a thermometer, or number of yards gained or lost in a football game should be used to make the concept meaningful.

With regard to correct terminology, it should be noted that we always say "negative three" or "negative five to indicate a negative number, and never use the incorrect terminology "minus three" or "minus five". The word "minus" is reserved to indicate the operation of subtraction. In the same way we reserve the word "plus" to indicate the operation of addition, and use the terminology "positive three" or "positive five" when we wish to stress that a number is greater than zero. Zero is a unique number which is neither positive nor negative.

In selecting a common denominator in the addition or subtraction of fractions, the idea of the set of multiples of a number should be stressed. For instance, if we wish to add the fractional number three-fourths to the fractional number five-sixths, we would need to find the common denominator of four and six. We guide the children to discover that the set of multiples of four is

$$A = \{4, 8, 12, 16 \ldots\}$$

and the set of multiples of six is

$$B = \{6, 12, 18, 24 \ldots\}$$

The intersection of these two sets is the set of common multiples of four and six, that is

$$A \cap B = \{12, 24, 36 \ldots\}$$

Here the pupil can see that any of these common multiples can be used as a common denominator, but that it is easiest to use the smallest number of this set, 12, which we may call the least common multiple, or least common denominator.

When changing a common fraction to another fraction equal in value, we say that we are renaming the fraction. For instance, the fraction one-half may be renamed as two-fourths, three-sixths, four-eighths, etc. A visual method of illustrating this is to show that all these equal fractions locate the same point on the number line.

When renaming fractions, it is important to stress that this may only be done by multiplying the fraction by some form of the identity element. For instance, the fraction one-half may be renamed by multiplying it be the identity element (one) in the form two-halves, three-thirds, four-fourths, etc. to produce the equal fractions two-fourths, three-sixths, four-eighths etc. Another way of stating this basic principle of fractions (multiplying a fraction by the identity element one does not change the value of the fraction) is to state that if both numerator and denominator of a fraction are multiplied or divided by the same number (except zero), the value of the fraction is unchanged.

IV. Geometry

Metric geometry is concerned with that aspect of geometry which involves measurement, whereas non-metric geometry is concerned with those properties of geometric figures which do not involve size or measure.

At this stage children should begin to discover and use such simple formulas as those for perimeter and area of a square and rectangle.

Distinguish between plane figures and the regions they enclose. For instance, a rectangle consists of all the points of the four line segments which are the sides of the rectangle. The region enclosed by the rectangle is not considered to be part of the rectangle. When we find the area of a rectangle, we really mean that we are finding the area of the region enclosed by the rectangle.

An introduction to the metric system should include a discussion of its historical development together with a comparison to the English system noting the considerable advantages of the metric system.

At this time, we compare angles as to size. Two angles which are the same size are said to be congruent, that is, they will exactly coincide if one is placed on the other. It should be noted that the length of the rays which form the two sides of the angle do not affect the size of the angle.

V. Mathematical Sentences

Refer to the section under this heading on pages 14 - 15 of the Grade IV curriculum guide.

In this grade much use should be made of the number line to visualize equal fractions, and to compare unequal fractions. Mathematical sentences using the concept "is greater than" and "is less than" should be used.

VI. Estimating and Mental Arithmetic

One of the ways in which a pupil exhibits understanding is to be able to estimate intelligently. Often an unrealistic estimate by a pupil will betray a complete lack of any real understanding of some concept or type of problem. This will alert the teacher to the fact that the explanation and development of the concept has not met with success, and hence reteaching with new approaches and pupil participation is in order.

Mental problems are always valuable for learning number facts, for simple problem solving, and for increasing attention span and improving concentration.

VII. Problem Solving

In order to solve word problems the pupil must be able to translate the relationship expressed in the problem into a mathematical sentence. In order to do this successfully, the pupil must not only know the meaning of the words, but must be able to understand the relationship expressed. Usually it is understanding the mathematical relationship rather than word recognition, that causes difficulty. Where a pupil solves a word problem incorrectly, the trouble may lie in one or more of the four areas: not knowing the meaning of the words; inability to understand the relationship and express it in a mathematical sentence; inability to choose the correct operation; or errors in computation. The teacher must analyze and decide where the difficulty lies, and take necessary steps to help the pupil.

The guided discovery method for teaching problem solving is especially effective. The teacher would ask such questions as 'What does the problem ask for?" "What numerical information is supplied to help you find the answer?" "How can we use this information?" "Let us write a mathematical sentence expressing a relationship."

Problem solving is one of the most difficult of topics, and is a real challenge to the creativeness and ingenuity of the teacher.

VIII. Graphs and Tables

The emphasis should be on the reading and interpreting of the various types of graphs. It will be necessary to develop the idea of a number scale on a horizontal or vertical axis.

OVERVIEW

Grade Six is a time of continued exploration in mathematics. For most of your students it will be their seventh year of schooling in the subject and, in retrospect, the ground covered has been considerable. The first steps toward intellectual maturity are naturally the hardest, but now that curiosity has been awakened and insight into the power, logic and structure of our system has been gained, they will be expecting to make considerably more progress before the year is over.

The key to success in presenting the materials of a modern curriculum in mathematics lies in the art of guiding pupil discovery. Rote memorization of facts and procedures by themselves offer poor preparation for today's fast changing technological world. Experimentation has shown that when a person gains insight into the rationale of a concept, he will more easily assimilate it. Such insight comes from discovery and development of concepts under the guidance of a skilled teacher.

The successful teacher of mathematics desires to explore ideas with an open mind, and her intellectual curiosity is mirrored in her pupils. She realizes that an awareness of the beauty, logic and structure of a mathematical system permits flexibility of application, promotes retention of basic facts and procedures and instills a desire to seek further knowledge.

AIMS:

1. To expand the child's appreciation of our mathematical system by extending insights into its structure and underlying logic.
2. To extend familiarity with the vocabulary and symbolism of mathematics.
3. To extend a knowledge of the properties and structure of our numeration system.
4. To build skills in operating on the various sets of numbers in the real number system.
5. To review, discover and correlate basic facts of geometry and extend skills in related measurements and calculations.
6. To develop proficiency in using number sentences.
7. To develop skills in estimating, in mathematical judgment, and in computation.
8. To guide the development of the power to do analytical thinking.

MATHEMATICS - GRADE SIX

Review mathematical concepts covered in grades kindergarten through five.

Vocabulary and Symbolization

A. Reinforce vocabulary of previous years.

B. Introduce mathematical terms and symbols:

- addend
- additive inverse
- multiplicative inverse
- associative
- commutative
- distributive
- cardinal number
- disjoint
- expanded notation
- exponent
- factor
- identity element
- inequality
- union of sets
- intersection of sets
- inverse operation
- multiple
- metric system
- area
- angle
- arc
- bisect
- circle
- closed figure
- compass
- concave
- convex
- congruent
- diameter
- line segment
- ray
- polygon
- perpendicular
- radius
- region
- volume
- positive
- negative
- numeral
- open sentence
- place-value
- prime number
- composite number
- scalene triangle
- subset
- rational number
- whole number
- integer
- finite set
- infinite set
- vertex
- quadrilateral
- inequality
- equivalent

I. Sets and Set Notation

A. Equal and equivalent sets

B. Union and intersection of sets

C. Difference of sets, and the empty set

D. Disjoint and overlapping sets, and subsets

E. Finite and infinite sets

F. Identify the set of whole numbers, the set of integers, and the set of rational numbers.

II. Numeration System

A. Use expanded notation - introduce exponent, base, power

B. Properties of the real number system

1. The commutative property of addition and multiplication:

$$a + b = b + a, \qquad ab = ba$$

2. The associative property of addition and multiplication:

$$(a + b) + c = a + (b + c), \qquad (ab)c = a(bc)$$

3. The distributive property:

$$a(b + c) = ab + ac$$

4. The identity element in addition and multiplication:

$$a + 0 = a \qquad a \times 1 = a$$

5. Additive inverse and multiplicative inverse:

$$a + (-a) = 0 \qquad a \times \frac{1}{a} = 1$$

III. Operations

A. Whole Numbers

1. Addition and Subtraction

a. Reinforce concepts involved in using the algorithms for addition and subtraction.

b. Stress meaning in regrouping using expanded notation.

c. Extend addition and subtraction of denominate numbers.

d. Develop negative numbers using the number line

1) identify relationship of "is greater than" and "is less than" with various combinations of positive and negative numbers.

2) Addition of various combinations of positive and negative numbers.

3) Develop the concept of additive inverse.

2. Multiplication and Division

   a. Reinforce concepts involved in using the algorithms for multiplication and division.

   b. Introduce divisors named by three-digit numbers.

   c. Use rounding off techniques to estimate quotients.

   d. Introduce concept of prime factors.

B. Fractional Numbers

1. Common Fractions

   a. Review and extend work in addition, subtraction and multiplication.

   b. Review the concept that the numerator and denominator of a fraction may be multiplied or divided by the same number (except zero) without changing the value of the fraction.

   c. Review the concept of <u>greater than</u> and <u>less than</u> with fractions using the number line.

   d. Introduce division of fractional numbers, using the multiplicative inverse (reciprocal).

   e. Introduce negative fractions on the number line.

2. Decimal Fractions

   a. Introduce places to the right of the decimal point as another way of writing fractional numbers which have denominators of 10, 100, 1000, etc.

   b. Develop the concept that each place has a value of one-tenth of that place immediately preceding.

   c. Review and extend work in addition and subtraction.

   d. Round to the nearest whole number, tenth, hundredth or thousandth.

   e. Introduce multiplication and division of decimals.

   f. Discover a rule for placing the decimal point in the numeral for a product.

   g. Use estimation to aid in placing the decimal point correctly in a quotient.

3. Per cent
    a. Develop meaning of per cent.
    b. Change common and decimal fractions to per cent.
    c. Change per cent to common and decimal fractions.

IV. Geometry

A. Metric
1. Develop perimeter and area formulas for square and rectangle.
2. Develop formula for the volume of a rectangular prism.
3. Develop the concept of congruence with regard to line segments and angles.
4. Review concept of perpendicular and right angle.
5. Teach use of protractor.
6. Construction with compass and ruler of the bisector of a line segment and the bisector of an angle.
7. Construction of an angle equal to a given angle (copying an angle).
8. Continue study of metric system.

B. Non-metric
1. Review concept and symbolism for ray, line and line segment.
2. Introduce acute and obtuse angles.
3. Introduce parallel lines.
4. Review properties of various plane and solid figures, and study line symmetry.
5. Study properties of a circle, including radius, diameter, arc and chord.

V. Mathematical Sentences

A. Equations

  1. Practice writing mathematical sentences from English sentences.

  2. Solution of simple one-step equations.

B. Inequalities

  1. Write mathematical sentences involving inequalities.

  2. Use number line to visualize the inequality of various types of numbers, including negative integers, common fractions, and decimal fractions.

VI. Estimating and Mental Arithmetic

A. Rounding off

  1. Use number line to give visual meaning to rounding off.

  2. Round off decimal fractions to nearest tenth, hundredth, and thousandth.

B. Estimate answers to judge reasonableness of computed answer.

C. Give practice in mental arithmetic with various types of problems.

VII. Problem Solving

A. Give practice in various types of problems involving the four basic operations with integers, common fractions, and decimal fractions.

B. Introduce simple problems involving per cent.

VIII. Graphs and Tables

A. Read and interpret bar graphs, line graphs, circle graphs, and pictographs.

B. Construct bar graphs and line graphs.

INSTRUCTIONAL OUTLINE

The eight sections which follow are detailed explanations of the preceding eight sections of Course Content, and include suggestions as to methods of teaching.

I. Sets and Set Notation

See the instructional outline under this heading on page 10 of the Grade Four curriculum guide. The following explanations are given for the additional topics in Grades Five and Six.

Disjoint sets are those sets which have no members in common. Overlapping sets are sets which do have some members in common. An example of two disjoint sets would be the sets

$$\{a,b,c\} \quad \text{and} \quad \{x,y,z\}$$

An example of overlapping sets would be the sets

$$\{a,b,c\} \quad \text{and} \quad \{b,c,d\}$$

Hence the intersection of two disjoint sets would be the empty set while the intersection of two overlapping sets would be the set of only those members which are found in both sets.

Finite sets are those sets which have a fixed number of members, that is, there is some cardinal number for the set. An infinite set has an endless number of members. For instance, an example of a finite set would be the set of pupils enrolled in your school, while an example of an infinite set would be the set of whole numbers, or the set of points on a line.

II. Numeration Systems

The concept of place value has been extended to include decimals, but this concept must be reinforced and permanently fixed in the students' minds. The use of expanded notation is extended to include exponential form, for example,

$$567 = 5(100) + 6(10) + 7(1) = 5(10^2) + 6(10) + 7$$

In teaching exponents, the teacher might use the illustration

$$5^2 = 25$$

pointing out that 5 is called the base and 2 is called the exponent. The phrase "ours is a base ten system" should take an added meaning through the use of such notation.

The commutative, the associative, and the distributive principles should be developed by having the pupils themselves discover and develop them, with many numerical examples. These relationships can then be summarized as equations.

The variables a,b,c may represent any members of the set of rational numbers. Thus the open sentence

$$a + b = b + a$$

is a statement which says that two sums are equal even though the order of the addends is different. This is the commutative law. Likewise this law applies to the product of two factors, that is,

$$ab = ba$$

In the same manner through the use of parentheses

$$(a + b) + c = a + (b + c)$$

we may show that sums of three numbers are unchanged when the manner of grouping (associating) the addends is changed. This is the associative law. Likewise this law applies to the multiplication of three factors, as

$$(ab)c = a(bc)$$

The distributive law of multiplication over addition enables us to solve a computation in which both the operation of addition and the operation of multiplication are found. The distributive law tells us that we may do such a problem in two ways - first adding and then multiplying, or first multiplying and then adding.

Expressing this in terms of letters, we would have

$$a(b + c) = ab + ac$$

For each of the above laws - commutative, associative, and distributive - the pupils should be convinced of their reasonableness and logic by many varied numerical applications. For instance, in any multiplication involving partial products, the distributive law is used. As an example, if 83 is multiplied by 7, we first multiply 7 times 3 and then 7 times 80, and add these partial products, that is,

$$7 \times 83$$

$$7(80 + 3)$$

$$560 + 21$$

$$581$$

The role of zero as the identity element in addition, and the role of one as the identity element in multiplication should be carefully developed. The term "identity element" is used because in each case if we apply the binary operation of addition or multiplication on any number paired with the proper identity element, our answer will be identically the same number we started with. That is, $n + 0 = n$ and $n \times 1 = n$.

They will find that these three principles hold for the set of fractions just as they did for the set of whole numbers. They will also discover that zero is the identity element in addition of fractions, and one is the identity element in multiplication of fractions.

III. Operations

Refer to pages 25 - 27 under this heading in the Grade V Instructional Outline for the recommended methods and procedures in handling common fractions and negative integers.

In introducing negative fractions, the number line should be used to indicate which of two fractions is the greater. For example, any number (including a fraction) on the number line will be greater than any other number to its left on the number line. Operations such as addition need not be covered with negative fractions.

The rule for division of fractions "invert the divisor and multiply" may be developed logically in several different ways, one of which is shown below.

$$\frac{2}{5} \div \frac{3}{4}$$

This may be renamed using the fraction line as an indicated division:

$$\frac{\frac{2}{5}}{\frac{3}{4}}$$

We may now multiply by the identity element (one) without changing the value of the fraction:

$$\frac{\frac{2}{5}}{\frac{3}{4}} \times 1$$

We now rename the identity element, making use of the multiplicative inverse (reciprocal) of the denominator:

$$\frac{\frac{2}{5}}{\frac{3}{4}} \times \frac{\frac{4}{3}}{\frac{4}{3}}$$

We may now rewrite in an equivalent form:

$$\frac{\frac{2}{5} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}}$$

We now carry out the multiplication in the denominator:

$$\frac{\frac{2}{5} \times \frac{4}{3}}{1}$$

Since division of any dividend by the identity element (one) will result in the identical dividend we started with, we arrive at the form "invert and multiply":

$$\frac{2}{5} \times \frac{4}{3}$$

At this grade level, the concept of per cent is introduced. This should be on a very elementary level, with most of the emphasis on the meaning of per cent, using various visual aids. The concept that a per cent is another way of writing a fraction whose denominator is one hundred should be stressed.

IV. Geometry

Metric geometry is concerned with that aspect of geometry which involves measurement, whereas non-metric geometry is concerned with those properties of geometric figures which do not involve size or measure.

Distinguish between plane figures and the regions they enclose. For instance, a rectangle consists of all the points of the four line segments which are the sides of the rectangle. The region enclosed by the rectangle is not considered to be part of the rectangle. When we find the area of a rectangle, we really mean that we are finding the area of the region enclosed by the rectangle.

The metric system should be discussed, and its considerable advantages over the English system should be pointed out.

At this time, we compare angles as to size. Two angles which are the same size are said to be congruent, that is, they will exactly coincide if one is placed on the other. It should be noted that the length of the rays which form the two sides of the angle do not affect the size of the angle.

The use of compass and protractor is introduced at this time. The three basic compass constructions - bisection of an angle, bisection of a line segment, and construction of an angle equal to a given angle (copying an angle) are covered. It might be pointed out that after they have learned how to bisect an angle, this construction may be used to bisect a straight angle, that is, an angle of 180°. In bisecting this straight angle, they have constructed two adjacent angles of 90° each, that is, they have constructed a perpendicular to a line.

As enrichment work, pupils may find it stimulating and interesting to bisect the three angles of a triangle, and discover that the three bisectors will meet at a point which is the center of the inscribed circle. Likewise if they bisect the three sides of the triangle, they will find that these three bisectors meet in a point which is the center of the circumscribed circle.

Pupils should be encouraged to create various geometric designs using compass, ruler, and protractor.

The difference between a <u>line</u>, which extends to infinity in both directions, and a <u>line segment</u> which has a fixed length, should be pointed out. Another way of stating this is that a <u>line</u> has no endpoints, whereas a <u>line segment</u> has two endpoints. A <u>ray</u>, sometimes called a half-line, can then be covered. A ray has a beginning point, and extends to infinity in only one direction. A ray has only one endpoint, and is used in the study of angles, that is, two rays having a common endpoint form an angle.

The work in geometry should be visual, and should enlist pupil participation at every stage. Drawings and construction work by the pupils themselves will make the geometric concepts of perpendicular, parallel, symmetry and congruence meaningful.

V. Mathematical Sentences

Refer to the section under this same heading on pages 14 - 15 of the instructional outline for grade IV.

The pupils should understand that the letter or letters they use in mathematical sentences are <u>variables</u>, and represent <u>numbers</u>. The commutative, the associative, and the distributive laws can be written as mathematical sentences using letters.

In the solution of simple equations, the basic principle that whatever is done to one side of an equation should also be done to the other, should be stressed. For instance in the following example, we subtract seven from both sides of the equation in order to arrive at the solution:

$$\begin{array}{rcl} m + 7 & = & 12 \\ \underline{- \quad\; 7} & \underline{=} & \underline{7} \\ m & = & 5 \end{array}$$

Likewise we may divide both sides of an equation by the same number, in this case, four:

$$4b = 12$$

$$b = 3$$

Solution of simple inequalities can promote an understanding of the concepts of *is greater than*, and *is less than*. The number line should be used when comparing two numbers. It is easily seen that one number will be greater than another number if it is found to the right of the given number on the number line. For instance, it is easily seen that the number *three* is greater than the number *negative two*, since the point on the number line which is labeled 3 is to the right of the point which is labeled -2.

VI. Estimating and Mental Arithmetic

One of the ways in which a pupil exhibits understanding is to be able to estimate intelligently. Often an unrealistic estimate by a pupil will betray a complete lack of any real understanding of some concept or type of problem. This will alert the teacher to the fact that the explanation and development of the concept has not met with success, and hence reteaching with new approaches and pupil participation are in order.

Mental problems are always valuable for learning number facts, for simple problem solving, and for increasing attention span and improving concentration.

VII. Problem Solving

In order to solve word problems the pupil must be able to translate the relationship expressed in the problem into a mathematical sentence. In order to do this successfully, the pupil must not only know the meaning of the words, but must be able to understand the relationship expressed. Usually it is understanding the mathematical relationship rather than word recognition, that causes difficulty. Where a pupil solves a word problem incorrectly, the trouble may lie in one or more of the four areas: not knowing the meaning of the words; inability to understand the relationship and express it in a mathematical sentence; inability to choose the correct operation; or errors in computation. The teacher must analyze and decide where the difficulty lies, and take necessary steps to help the pupil.

The guided discovery method for teaching problem solving is especially effective. The teacher would ask such questions as "What does the problem ask for?" "What numerical information is supplied to help you find the answer?" "How can we use this information?" "Let us write a mathematical sentence expressing a relationship."

Problem solving is one of the most difficult of topics, and is a real challenge to the creativeness and ingenuity of the teacher.

VIII. Graphs and Tables

While the emphasis will be chiefly on the reading and interpretation of various types of graphs, pupils will learn to construct bar graphs and line graphs. They will learn the meaning of horizontal axis and vertical axis, and relate these to scale drawing.

There should be a discussion by the children as to when a line graph is more suitable than a bar graph, such as showing the temperature for a given period of time. Here every point on the line has meaning. This could be contrasted to making a graph showing the high temperature for the day for five successive days. Here a bar graph would be more suitable.