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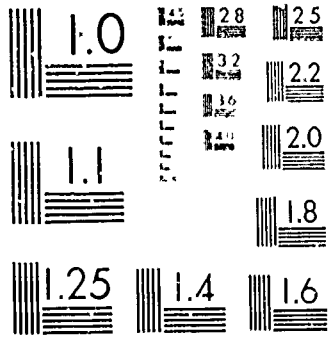

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ABSTRACT

An explicit and complete list of the Minimal Performance Objectives for Mathematics Education in Michigan public schools is contained in this document. The objectives are organized under the five major topics of Arithmetic, Measurement, Geometry, Algebra, and Probability and Statistics. Examples and comments are included. A summary of the rationale for setting these objectives and the method by which they were derived is also provided. (JP)

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EDUCATION & WELFARE
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State Board of Education

FOREWORD

Over two years ago, as a part of a program for educational accountability the Michigan Department of Education began the rather formidable task of developing pre-school through twelfth grade performance objectives for the various subject matter areas, and as Superintendent of Public Instruction, it is with a good deal of pleasure that I am able to report that the first results of our efforts—in the area of mathematics - are now available and are ready to be distributed throughout the state. We hope that this document containing a complete listing of the objectives will be a very useful reference to educators everywhere in Michigan, and we hope also that making these performance objectives available at the state level will help us to guarantee that all boys and girls in Michigan, regardless of where they live or what school they attend, will attain at least a minimal level of competency in mathematics skills, for without at least a minimal proficiency in mathematics, opportunities for persons in today's society are limited.

These performance objectives have been long in the preparation. Defining minimum levels of competency is always an extremely difficult and time-consuming task. The preparation of these sets of minimal competencies has been a particularly lengthy process in this case because we have involved large and representative groups of teachers and mathematics specialists throughout the state in preparing and reviewing them. We believe that if objectives issued from the state level are to have viability, this kind of massive involvement is absolutely essential. Thus, since we began with this task, these objectives have been considered by numerous advisory groups and councils, local curriculum committees, grade commissions, professional organizations, and various referent groups. We can say, therefore, that they do represent the consensual thinking of both mathematics educators and educators in general throughout the state.

Perhaps it should be stressed again that these objectives are designed to represent minimal levels of expectancies in mathematics. School districts will want their students to move far beyond these minimums.

It surely would be appropriate for me, at this point, to speak for the State Board of Education and thank the people who have been involved in this effort. I cannot possibly list all of those who have labored long and hard with this document, but the efforts of all of these people are greatly appreciated and the names of the people who have been mostly directly involved have been listed in the appendix of this publication. I do wish to extend our thanks to several mathematics groups that rendered the Department particularly meritorious service, the Task Force for the Development of Basic Mathematics Objectives for the State of Michigan, the Michigan Council of Teachers of Mathematics, the Detroit Area Council of Teachers of Mathematics, and the Greater Flint Council of Teachers of Mathematics. Also, I want to thank the two persons on the Department staff who have done most to oversee this job, Mr. Bob Sternberg, who, though not a mathematics specialist, served as a process person in organizing the initial group, and Dr. Richard E. Mignerey, the Department's math specialist who joined the staff after the task was begun, and has had the responsibility for editing and expediting the completion of this document.

John W. Porter
Superintendent
Public Instruction

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TABLE OF CONTENTS

	Page
ARITHMETIC	1
I. Number-Numeration	2
A. Pre-Number	2
B. Numeration	6
II. Whole Number	18
A. Addition	19
B. Subtraction	25
C. Multiplication	28
D. Division	33
III. Fractional Numbers	37
A. Meaning	38
B. Addition	42
C. Subtraction	46
D. Multiplication	49
IV. Decimal Fractions	53
A. Meaning	54
B. Addition and Subtraction	56
C. Multiplication	58
D. Division	61
V. Integers	63
VI. Ratio, Proportions and Percent	65
MEASUREMENT	72
I. Geometric	73
A. Linear	73
B. Area	76
C. Volume	79
D. Angle	81
II. Non-Geometric	82
A. Time	82
B. Money	84
C. Temperature	86
D. Weight	88
E. Liquid	89
GEOMETRY	90
I. Identification	91
A. Shapes	91
B. Points and Lines	92
C Congruence	93
D. Symetry	94
II Construction	95
ALGEBRA	96
PROBABILITY AND STATISTICS	103

INTRODUCTION

Among the many expectations communities hold for formal education is that their children will develop competencies in the field of mathematics. This desire is reflected in *The Common Goals of Michigan Education* as a basic skill for each student, "the ability to handle mathematical operations and concepts" (from *Goal III*, Student Learning). Such an ability has become thought of as absolutely essential for participation in our society today. It is with this mandate in mind that the *Minimal Performance Objectives for Mathematics* are offered here to the teachers and patrons of Michigan's elementary and secondary schools.

Implicit to the concept of equal educational opportunity is the existence of a *foundation program*, a program which speaks to the many areas of knowledge and skills necessary for personal and collective survival in a technological age. The performance objectives cited by this document are meant to be an integral part of that foundation program. The implication is strong that *all* students in Michigan have the right to acquire the mathematical skills set forth here. This, of course, does not imply that students will learn *only* these skills and concepts, since many will obtain mastery considerably in excess of the minimums established here; but that all youngsters will be assured at least this minimum foundation.

Objectives considered non-graded

It should be observed that the objectives for mathematics represent a *continuum* of progress rather than a graded sequence. Hence these objectives lend themselves to an individualized instructional program. It is assumed that the learners will be at various points of this continuum. An objective is timely whenever the learner gives evidence of readiness. Thus, some objectives can be introduced earlier or later than that time which has traditionally been considered appropriate.

Objects and diagrams

The objectives are designed to promote an active learner. The behaviors called for by the objectives ask the learner to manipulate the "teaching aids", rather than looking at pictures or observing a demonstration. For an object or diagram to serve its full purpose, it must be internalized by the student — to "see it" in his mind's eye.

The objectives expressed in three levels

The fact that an objective is in a given organizational level does not mean that it belongs exclusively to that level for instructional purposes. Some children in grades K-3 can and should achieve a significant amount of the material of the second and third level. *Indeed, instruction designed to treat only first level objectives before grade four, and second level objectives before grade seven would be an unfortunate distortion of the concept of minimal objectives.* Learning is an individual, personal business.

The objectives are organized in strands and sequences

The objectives are organized in strands (for example, whole number strand). Within a strand are one or more sequences of objectives (for example, addition or subtraction, and so on). These sets of sequenced objectives should not be taught as units in isolation from other sequences. In other words, the addition sequence should not be taught in complete isolation from its beginning to its end. Certain skills in the addition and subtraction sequence should be taught at the same time. The extent of "cross over" from one sequence to another, or from one strand to another should vary depending on the pattern of need of the students and the nature of the subject matter. As a child progresses through school he proceeds through certain strands and sequences simultaneously. Each year he masters some objectives in several strands.

The "Examples and Comments" column

The Performance Objectives have one or more examples of methods, techniques or models immediately following each objective in the right hand column. The purposes served by these examples and comments are:

- —They lend clarity to the objective
- —*They suggest teaching strategies*
- —They suggest sequences for teaching each objective
- —They suggest specific learning aids to each objective

Revision of Performance Objectives

Although much effort has been put into this selection and writing of these minimal performance objectives for mathematics, they will require revision and updating. To suggest revisions, additions, and/or deletions to the performance objectives, all that is necessary is for you to write your suggestions out in detail and send them to the Mathematics Specialist, General Education Services, Michigan Department of Education, P.O. Box 420, Lansing, MI 48902. Every suggested revision will be considered by an editorial revision team.

CONCEPT AREA:

ARITHMETIC

- I NUMBER-NUMERATION STRAND
  - A Pre-Number Sequence
  - B Numeration Sequence
- II WHOLE NUMBER STRAND
  - A Addition Sequence
  - B Subtraction Sequence
  - C Multiplication Sequence
  - D Division Sequence
- III COMMON FRACTION STRAND
  - A Meaning Sequence
  - B Addition Sequence
  - C Subtraction Sequence
  - D Multiplication Sequence
- IV DECIMAL FRACTION STRAND
  - A Meaning Sequence
  - B Addition & Subtraction Sequence
  - C Multiplication Sequence
  - D Division Sequence
- V INTEGER STRAND
- VI RATIO, PROPORTION & PERCENT STRAND

CONCEPT AREA: **MEASUREMENT**

- I GEOMETRIC STRAND
  - A Linear Sequence
  - B Area Sequence
  - C Volume Sequence
  - D Angle Sequence
- II NON-GEOMETRIC STRAND
  - A Time Sequence
  - B Money Sequence
  - C Temperature Sequence
  - D Weight Sequence
  - E Liquid Sequence

CONCEPT AREA: **GEOMETRY**

- I IDENTIFICATION STRAND
  - A Shape Sequence
  - B Point & Line Sequence
  - C Congruence Sequence
  - D Symmetry Sequence
- II CONSTRUCTION STRAND

CONCEPT AREA: **ALGEBRA**

- ALGEBRA STRAND

CONCEPT AREA: **PROBABILITY AND STATISTICS**

- PROBABILITY AND STATISTICS STRAND

QUESTIONS

I. The Writing Team

1. *How representative was the writing team?*

   There were thirty-three mathematics educators who participated; twenty-nine were extensively involved. Of those with continuous involvement, thirteen are teachers, nine are supervisors, two are administrators and five are professors working in teacher education. The teachers are equally divided among elementary, junior high schools and senior high schools. Twenty-three participants work continuously in public schools and most of the professors also have assignments which keep them in continuous contact with public schools. Geographically the four corners and center of the lower peninsula were represented. Size of districts varied from Swartz Creek and Rives to Detroit.

2. *Why was the writing team not composed solely of teachers? Aren't they closer to what is going on?*

   To assure a smooth flow of objectives K-10, it was necessary to include people who were familiar with programs at several grades as well as those with a strong understanding of program and achievement of students at particular grades. Accurate information concerning student performance on specific objectives was also desirable. Hence, we believe that the mix of teachers, supervisors, professors and administrators contributed to a better set of objectives.

II. The Mathematics Represented

1. *Is this "Modern" Mathematics?*

   These objectives are associated with late developments in mathematics education in its emphasis on meaning and understanding. They place a strong emphasis on the relationship between the physical world and numbers. They include topics such as equations and probability which were not in traditional programs. Since the objectives are basic, they tend to leave out some of the abstractions which have not been grasped by many students. These abstractions would generally be included in a quality program but experience seems to indicate that it is unrealistic to expect *all* students to grasp them. In this sense the objectives will appear less "modern" than has been typical in recent programs.

2. *Do these objectives describe a quality program?*

   No. They provide only a minimal program. Most students should be expected to learn more than this. Most students should have experiences in mathematics which go well beyond those suggested here. In addition, the writing group was asked to set "bench marks" which might be used in conjunction with 4th, 7th, and 10th grade testing. These were set to approximate the progress expected of slower students and as such are not a good gauge for approximating progress of students in a quality program.

3. *Do the objectives use "strange" mathematical terminology?*

   In order to assure that the objectives can be read and understood by a broad spectrum of parents and teachers, there has been a deliberate attempt to avoid specialized vocabulary. Occasionally, this would lead to such an ineffective way of stating a concept that we used special terms in spite of our good intentions. Usually this has been true only for objectives which would be used at or above the junior high school level where teachers are specialists and should be able to interpret them to parents.

III. The Objectives Themselves

1. *How were they chosen?*

   First of all, terminal objectives were selected on the basis that they should be useful to an *average citizen* and secondly that they should be achievable by *ninety percent of our students* before they leave us. Then developmental objectives were included if they were in a strand of concepts which a student would need to master in reaching the terminal objective. It was very obvious as we worked through the selection process that objectives which are necessary for all will be insufficient for most. The reader will be surprised by deletions, such as, the division of fractions, two cases of work with percent, sharply limited list of formulae, etc. This surprise should tend to emphasize the repeated reference to *minimal* vs quality programs.

2. *Why are the objectives not listed by grade level?*

   The group feels strongly supportive of the recent moves to individualized and non-graded programs. For a particular student it makes little sense to assign an objective to a grade level. They fit rather into developmental sequences being mastered by the student. Teachers should be guided by where students are in their mathematical experiences rather than by the grade they are in.

3. *Why didn't you supply simply terminal objectives? Why include the developmental ones and so many examples?*

   There was general agreement that these objectives would provide an opportunity to help K-8 teachers recognize good techniques and sequences as well as recognizing high priority tasks. If we listed only terminal objectives, there would be a tendency for teachers in the early elementary grades to feel unaffected when actually terminal objectives can be achieved only if suitable developmental work is done.

4. *How good are these objectives?*

   That question can only be answered through use and evaluation. It is assumed that they will be reexamined periodically. Time limitations on the work of the team served as both advantage and disadvantage. We were forced to arrive at choices but the opportunity for extensive study was limited. The writing groups generally tend to feel positive about the final product.

IV. Use of Objectives

1. *It is said by many that preparing statewide objectives will set mathematics teaching in Michigan back many years, wasn't the writing group concerned about this?*

   Yes, it was, and some did not participate because of that fear. However, the majority of the participants probably felt that the focus on preparing students for college in recent years has tended to ignore the students who needed a basic preparation for entering the work world. There was a feeling that many students whose education terminated with high school have not been prepared in mathematics. There was also a strong feeling that the preparation of basic objectives by a representative group, and basing tests on those objectives, is superior to the manner in which the present assessment program has operated. It was seen as a step in the right direction. Whether or not it will be a step in the right direction will depend upon the wisdom with which the objectives are used.

2. *Can a single set of objectives meet the needs of students in all of Michigan schools?*

   We do not think so. However, at the *minimal* (basic) level at which these objectives are written the needs of students throughout Michigan are probably quite similar. We believe it would be totally impossible to write objectives for a "quality" program that would fit all Michigan communities.

3. *Will all of these objectives be tested in the Michigan Assessment Program?*

   Probably not. Terminal objectives are preceded by many developmental objectives which are not likely to be tested. Some objectives would be costly and difficult to test. Even among the terminal objectives only a random sample could be tested at a single setting.

4. *Isn't there a danger that teachers will teach only for these objectives? When combined with state assessment, won't they push teachers to "teach the objectives"?*

   If an inexperienced teacher is handed these objectives without further guidance as to how they fit into the objectives of the local community, there is a distinct danger that they will become the program in that district. This will require districts to clarify their own objectives in mathematics if they have not already done so. Research tends to show that teaching for specific objectives increases effectiveness. The difficulty lies not in teaching for objectives but in the scope of the objectives that are selected.

USE AND PURPOSE OF THE RECORD OF INDIVIDUAL PROGRESS IN MATHEMATICS

The *Record of Individual Progress* in Mathematics is a suggested form for recording the progress of the individual child. It is not intended necessarily to replace other forms or other systems that schools have developed, but the form is keyed to the strands and sequences of this document and, as such, may be an appropriate model.

It is suggested that a cumulative record of individual progress be maintained as long as this child is in school. Its purpose is to facilitate instruction by providing a map, so to speak, of what the child has accomplished and some indication of the instruction that he has received.

The recording system suggested here should provide teachers, counselors and parents with a clear and current summary of the student's progress in mathematics from year to year.

Suggested Symbols:

A circle drawn in the box to indicate that extensive instruction has been given.

An X is placed in the circle when, in the teacher's judgment, the student has mastered the objective and is likely to be able to us it in the future.

To indicate the grade in which mastery was achieved.

RECORD OF INDIVIDUAL PROGRESS IN MATHEMATICS — ARITHMETIC

Name _____ Date of Birth _____

NUMBER-NUMERATION STRAND

A PRE-NUMBER SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28 • • •
B NUMERATION SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28 • • •

WHOLE NUMBER STRAND

A ADDITION SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
B SUBTRACTION SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21							
C MULTIPLICATION SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
D DIVISION SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20								

COMMON FRACTION STRAND

A MEANING SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26		
B ADDITION SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12																
C SUBTRACTION SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12																

RECORD OF INDIVIDUAL PROGRESS IN MATHEMATICS

ARITHMETIC

Name _____ Date of Birth _____

COMMON FRACTION STRAND (Continued)

D MULTIPLICATION SEQUENCE	1	2	3	4	5	6	7	8	9	10																					

DECIMAL FRACTION STRAND

A MEANING SEQUENCE	1	2	3	4	5	6	7	8	9	10	11																				
B ADDITION-SUBTRACTION SEQUENCE	1	2	3	4	5	6	7	8	9	10																					
C MULTIPLICATION SEQUENCE	1	2	3	4	5	6	7	8	9	10	11																				
D DIVISION SEQUENCE	1	2	3	4	5	6																									

INTEGER STRAND

INTEGER SEQUENCE	1	2																													

RATIO, PROPORTION, AND PERCENT STRAND

RATIO, PROPORTION AND PERCENT SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	•	•	•

RECORD OF INDIVIDUAL PROGRESS IN MATHEMATICS — MEASUREMENT

Name _____ Date of Birth _____

GEOMETRIC STRAND

I-A LINEAR SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
I-B AREA SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16										
I-C VOLUME SEQUENCE	1	2	3	4	5	6	7	8	9	10	11															
I-D ANGLE SEQUENCE	1	2	3	4																						

NON-GEOMETRIC STRAND

II-A TIME SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13													
II-B MONEY SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12														
II-C TEMPERATURE SEQUENCE	1	2	3	4	5																					
II-D WEIGHT SEQUENCE	1	2	3	4	5	6	7	8	9																	
II-E LIQUID SEQUENCE	1	2	3	4	5	6																				

RECORD OF INDIVIDUAL PROGRESS IN MATHEMATICS

GEOMETRY
ALGEBRA
PROBABILITY AND STATISTICS

Name _____ Date of Birth _____

GEOMETRY IDENTIFICATION STRAND

A SHAPE SEQUENCE	1	2	3	4	5	6	7																						
B POINTS & LINES SEQUENCE	1	2	3	4	5																								
C CONGRUENCE SEQUENCE	1	2																											
D SYMMETRY SEQUENCE	1	2																											

GEOMETRY CONSTRUCTION STRAND

CONSTRUCTION SEQUENCE	1	2	3	4	5	6	7	8																					

ALGEBRA STRAND

ALGEBRA SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	• • •

PROBABILITY AND STATISTICS STRAND

PROBABILITY & STATISTICS SEQUENCE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22							

ARITHMETIC

I NUMBER-NUMERATION

The ability to say number names in sequence does not indicate that children understand the concept of number. Number is an abstract idea. Many opportunities to manipulate objects are needed by young children to interact creatively in examining and organizing materials into logical relationships.

The *NUMBER-NUMERATION* Strand consists of two (2) Sequences:

a. Pre-Number Sequence

b. Numeration Sequence

The Pre-Number Sequence will form the basis for mathematics activities in preschool programs. All children must master the objectives in this foundation strand of NUMBER-NUMERATION to insure continued success in mathematics.

During the pre-number stage, number readiness can be developed if each child experiences sequences of:

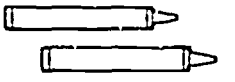
(1) Classifying the materials according to attributes and identifying the materials by *name*, by *function* or by *relationship*, and

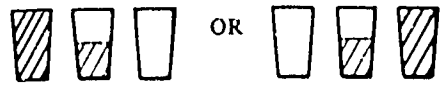
(2) Ordering the objects by various relationships and identifying the materials by *name*, by *function* or by *relationship*.

The Numeration Sequence covers the structural aspect of the decimal number system while its underlying number theory pervades the entire scope and sequence of arithmetic. Before introducing written symbols, many relationships basic to *numbers* must be experienced by each child. Sets are studied to give visual reality to number value and to allow concrete demonstrations of arithmetic operations. By matching sets, the child learns the relationships of "greater than", "less than", and "equal to".

K-3 ARITHMETIC

I Number-Numeration A. Pre-Number

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
CLASSIFICATION—MATCHING	
1. Given a set of three to five objects, two of which are alike, the learner will pick up two that are alike.	 crayons ball and block
2. Given a set of three to five objects, all but one of which are alike, the learner will pick up the different one.	
CLASSIFICATION—SORTING	
3. Using two kinds of objects in a large set, the learner will sort the objects into two sets according to characteristics.	Examples. 1. elbow and shell macaroni 2. bottle tops and checkers
CLASSIFICATION—ATTRIBUTES	
Color—Shape—Size	
4. Given a set of objects, the learner will pick up or point to those objects which are the same color.	Examples: 1. Red buttons 2. Blue buttons
5. Given a set of objects, the learner will pick up or point to those objects which are the same shape.	Examples: 1. Balls 2. Attribute blocks
6. Given a set of objects, the learner will pick up or point to these objects which are the same size.	Examples: 1. Large beads 2. Small beads
7. Given a set of objects of assorted colors, the learner, when shown an object of a specific color, will pick an object which is the same color from his set.	Teacher shows a red ball, the child picks red crayon from his set of crayons.
8. Given a set of two or more (a) circle (b) triangle (c) square or rectangle shaped objects and shown an object of one of those specific shapes, the learner will pick up an object which is the same shape.	(a) ball (b) triangularly folded paper hat (c) box or book
CLASSIFICATION—POSITION	
9. Given an object, the learner will hold it *up* and put it *down* as requested.	*Up* above his head *Down* on the floor
10. Given an object, the learner will place it in the following positions—*over* and *under; in front of* and *in back of*—in relation to a table or box as requested.	
CLASSIFICATION—SIZE	
11. Given two objects, the learner will pick up the one that is *bigger* or *smaller* as requested.	Big doll, Small doll
CLASSIFICATION—QUANTITY	
12. Given two sets of identical objects, one with many members and one with very few members, the learner will place the set having *more* in a box.	two strings of beads—or— two sets of jacks.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
CLASSIFICATION—TEXTURE	
13. Given an object, (a) smooth (b) rough (c) soft (d) hard, the learner, choosing from his own set of objects, will select an object that is of the same texture of the given one.	(a) plastic (b) sandpaper (c) cotton batting, velvet (d) rock, floor
CLASSIFICATION—PATTERNS	
14. Given a pattern using two objects different in color, the learner will duplicate the pattern selecting from a set of cubical beads.	blue bead red bead
15. Given a pattern using two objects different in *shape*, the learner will duplicate the pattern selecting from a set of beads.	
ORDERING	
16. Given a set of three clear plastic drinking glasses, one filled with sand; one empty; and one half filled with sand, the learner will arrange the glasses from "full to empty."	The "full to empty" can apply either to the sand or the air *Clear* glasses are specified because the children must be able to see through the containers. The *plastic* glass material is suggested as a safety factor to prevent the injury from broken tumblers. sand 
COUNTING	
17. Given the directions "count to five", the learner will say the names of the numbers one through five in the proper order.	This rote counting indicates *only* that the child can say the number words in the accepted order, not that he understands what a number is!
CLASSIFICATION—MONEY	
18. Given a set of three coins containing a penny, a nickel and a dime, the learner will pick up and name each one.	
CLASSIFICATION—ATTRIBUTES	
19. Given a set of ten objects varying in color and shape, the learner will pick out objectives having specific combinations of the two attributes.	Given: Colors, red, yellow, blue Shapes, squares, circles, triangles Possible Combinations: red circle shape, blue triangle shape, yellow square shape, etc.
20. Given a set of ten objects of varying shapes and textures, the learner will pick out objects having specific combinations of two attributes.	Given: Shapes, squares, circles, triangles Textures, rough (sandpaper) smooth (velvet) Possible Combinations: rough triangle shape, smooth circle shape, etc.
CLASSIFICATION—PATTERNS	
21. Given a series of three to five objects arranged in a pattern by color and/or shape, the learner will duplicate the pattern.	 red blue red green blue

PERFORMANCE OBJECTIVES

22. Given one series of three objects arranged in a pattern by color and/or shape and the first object of the second series, the learner will complete the second pattern series.

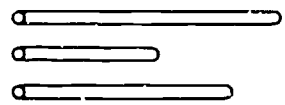
EXAMPLES AND COMMENTS

Given:

ORDERING

23. Given a collection of children of varying heights, the learner will choose the *tallest* or *shortest* child.

24. Given a collection of five objects of varying lengths, the learner will pick up the *longest* or the *shortest* as requested.



straws

CLASSIFICATION—ATTRIBUTES

POSITION

25. Given a small object such as a block, the learner will place the object *inside* a box.

26. Given a small object such as a block, the learner will place the object *outside* a box.

27. Given a small object such as a chalkboard eraser, the learner will place the object *on* a table or a shelf.

28. Given a small object such as a chalkboard eraser, the learner will take the object *off* the table or shelf.

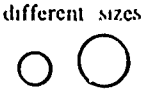
ORDERING

29. Given a set of five pictures of objects of various heights, the learner will arrange the pictures so that the objects are ordered from shortest to tallest.

30. Given a set of five pictures of objects of various sizes, the learner will arrange the pictures so that the objects are ordered from smallest to largest.

31. Given two objects of decidedly different weights, the learner will lift the objects and hand to the teacher the one that is heavier or the one that is lighter as requested.

(different sizes)



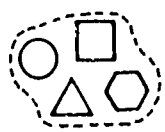
same type of material (wood)

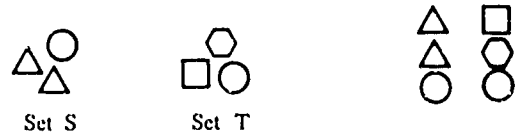
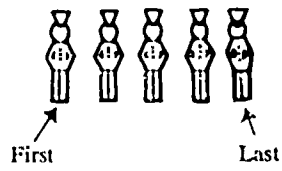
same size container

different material

NUMBER MEANING

32. Given a collection of from one to nine small objects and a length of string or yarn, the learner will place *all* the objects inside the closed curve formed by the yarn.



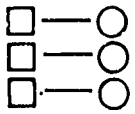
PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
33. Given a collection of from one to nine small objects and a length of yarn or string, the learner will place *some* of the objects inside the closed curve formed by the string.	
COUNTING	
34. Given the directions "count to ten", the learner will recite the number names from one through ten in the usual order.	This *rote* counting indicates only that the child can say the number words in the accepted order, not that he understands what a number is.
NUMBER MEANING	
35. Given a set formed by the learner from objects found in the classroom, the learner will describe the characteristic(s) (attributes of the set) such as: color and shape.	Color and shape 1. Red triangle shapes or 2. Things in my desk
36. Given an oral description of a set and a collection of objects, some of which belong to the set and some of which do not, the learner will pick up the objects that are members of the given set.	Set: My writing tools Given: pencil paper pin cushion
37. Given two sets, one set with one to three members and the other with eight to ten members, the learner, using only visual inspection, will place his hand on the set with *more* members.	
38. Given two equivalent sets of small objects (2 to 5 members), the learner will demonstrate a one-to-one matching by physically associating the objects of one set with the objects of the second set.	Child moves objects  Set S Set T
CLASSIFICATION—ATTRIBUTES	
POSITION	
39. Given five small toys, the learner will form a single line parade and then indicate the *first* toy and the *last* one.	Toy soldiers Toy cars Any assortment of little toy animals  First Last

PERFORMANCE OBJECTIVES

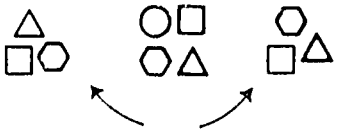
EXAMPLES AND COMMENTS

NUMBER MEANING

1. Given *pictures* of two sets, each having two to five members, the learner will pair the members of one set with the members of the other set by drawing lines and then will state whether or not the sets are equivalent.



2. Given sets having one to five small *objects* as members, the learner will point to the sets that have the same number of objects.



3. Given two sets consisting of different numbers (2-5) of *objects*, the learner will place a length of yarn around the set with *more* members.

4. Given two sets consisting of different numbers (2-5) of *objects*, the learner will place a length of yarn around the set with *fewer* members.

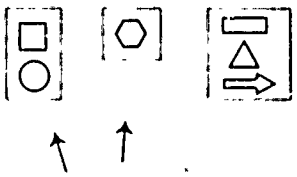
5. Given *pictures* of sets, the learner will pick out pairs of sets whose members cannot be matched one-to-one (non-equivalent); and then tell which set has *more* members.

MORE

non-equivalent sets

6. Given *pictures* of sets, the learner will pick out pairs of sets whose members cannot be matched one-to-one (non-equivalent); and then tell which has fewer members.

FEWER



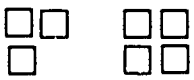
non-equivalent sets

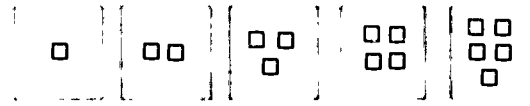
7. Given a set with less than ten *objects* the learner will make an equivalent set by using actual objects and then will draw a picture of the set.

8. Given a set of small *objects*, the learner (using his own collection of objects) will form another set that was exactly *one more* object than the original set.

Original Set:

New Set:

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
9. Given a set of small *objects*, the learner will draw a set with exactly *one more* object than the original set.	Original Set: New Set:
10. Given a set of small *objects*, the learner using his own collection of objects, will make a set with *one fewer* object than the original set.	Original Set: New Set:
11. Given a set of small *objects*, the learner will draw a *picture* of a set with exactly *one fewer* object than the original set.	Original Set: New Set:
ORDERING	
12. Given two sequentially ordered sets of *objects*, one of which has one more or one less than the other, the learner will form the set that comes next in order.	Given:  Result:
13. Given *pictures* of any three sets having from one to five members, the learner will arrange the *pictures* in sequential order, fewest to most.	
14. Given any three consecutive sets of *objects* consisting of one to five members, the learner will place a string around the set that has the *most* members.	
15. Given any three consecutive sets of *objects* consisting of one to five members the learner will place a string or yarn around the set that has the *fewest* members.	
16. Given any three consecutive sets of *objects* consisting of one to five members, the learner will place a loop of yarn around the set that is *between* the other two in the sequence.	
17. Given cut out *pictures* of any three sets (from one to five members), the learner will place the pictures of the sets in order, from that set with the *fewest* members to that set with the *most* members; then he will order the set pictures from *most* to *fewest*.	
18. Given five sets of *objects* consisting of one, two, three, and five members, the learner will place the sets in sequential order from the set with the fewest members to the set with the most members.	This is a pre-skill for the number line.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
19. Given *pictures* of sets consisting of one to five members, the learner will place the pictures in sequential order from the set with the fewest members to the set with the most members.	
20. Given *numeral cards* 1 through 5 and five sets of *objects* consisting of one, two, three, four and five members, the learner will place the sets in sequential order from the set with the fewest to the set with the most and then will place the numeral cards in front of the set having the number of members named by the numeral.	
NUMBER MEANING	
21. Given a set of *numeral cards*, 1 through 5 and the oral request for one of the numbers, the learner will pick out the proper numeral card.	Request: Three Response. Learner picks up [3]
COUNTING	
22. Given a set of *numeral cards*, 1 through 5 and the request to "count to 5" the learner will pick out the proper numeral card as he says the numbers in sequential order.	
23. Given a set of *objects* with 1-9 members, the learner will count the members of the set and state the cardinal number of that set.	
ORDERING	
24. Given *pictures* of sets from 1-9 in random order, the learner will arrange the sets in sequential order.	
NUMBER MEANING	
25. Given a collection of small *objects* and a loop of yarn, the learner will illustrate the empty set, and then draw or give an oral example, as requested.	Example. The set of pink elephants in the classroom.
26. Given a loop of yarn containing no members (empty set) and felt or sandpaper numerals (0-9) the learner will pick out the numeral for the cardinal number of the set and will say the number name.	
27. Given *pictures* of sets with 0-9 objects and number cards from 0-9 (using felt numerals, sandpaper numerals), the learner will match the right numeral with the picture of the set having the same of members.	
28. Given several sets of 0-9 *objects* with duplicates or triplicates of certain sets the learner will place loops of the same color yarn around the sets that have the same cardinal number.	

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
MEANINGFUL COUNTING	
29. Given dot pattern *cards* showing sets of 0-10 dots, the learner will count while pointing to the appropriate card.	Example. The child points to the blank card and says "zero" and then points to the card with one dot and says "one."
CLASSIFICATION—ATTRIBUTES	
30 Given an assorted set of ten *objects* of two to five colors or two to five different shapes, the learner will arrange the objects into subsets according to the various colors or shapes.	Given. Red, yellow and blue torn pieces of paper in the set. Result: Red Pieces Blue Pieces Yellow Pieces
NUMBER MEANING	
31. Given a set of 2 to 8 *objects*, the learner from his own group of objects will construct a set having *more members than* the original set.	Given: Result:
32. Given a set of 2 to 8 *objects*, the learner from his own group of objects will construct a set having fewer members than the original set.	Given: Result:
33. Given two written *numerals*, 9 or less, the learner will point to the number which is *greater in value than*, the other.	Given: 2 7 Request: Which number is greater than the other? Child points to 9
34. Given two written *numerals*, 9 or less, the learner will will point to the number which is *less in value than* the other.	Given: 3 8 Request: Which number is less than the other? Child points to 3
CLASSIFICATION—ATTRIBUTES	
35. Given a book, the learner will place this book beside another book.	OR
36. Given a sheet of paper, the learner, facing the paper, will point to the *left* or *right* side of the paper, on request.	Left Right

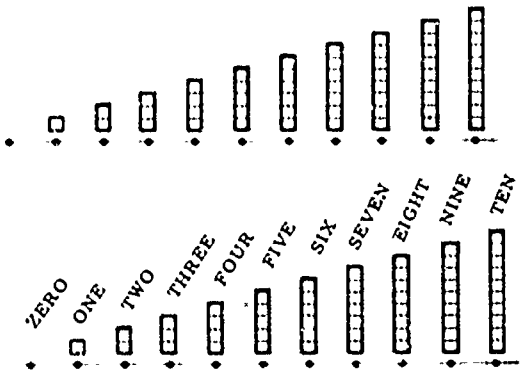
PERFORMANCE OBJECTIVES

EXAMPLES AND COMMENTS

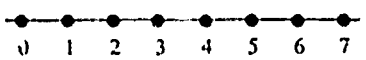
NUMBER MEANING

37. Given small blocks and a long line marked off in congruent segments each at least 3 inches long, the learner will construct the sets with from zero to ten members and will place the sets in order on the line.

38. Given small blocks and a long line marked off in congruent segments each at least 3 inches long, the learner will construct the sets with from one to ten members in increasing order on the line and will say, on request, the numbers represented by the sets.



39. Given a line marked with congruent segments and labeled from 0-10 the learner will touch any point, and say the number indicating that position on the number line.



40. Given a line marked with congruent segments and a set of numeral cards (0-10), the learner will place the appropriate numeral card below the appropriate point on the number line.

ORDERING

41. Given a line of five children, the learner will say the ordinal number for each child as he taps each child on the shoulder.

42. Given two ordinal numbers orally and a line of five children, the learner will touch the children who are in the positions named by the two ordinal numbers.

NUMBER MEANING

43. Given any three numbers, 0-10, the learner will tell which is the greatest and which is the least, on request.

Given: 8, 3, 5

Result: 8 is the greatest or 3 is the least.

ORDERING

44. Given two consecutive even or odd numbers 0-9, the learner will name the number that comes *between* the two given numbers.

45. Given a number from 1 to 8, the learner will name and write the numbers that come *before* and *after* the given number.

Step 1	Step 2
_____, 3	_____, 3, _____
6, _____	_____, 6, _____
_____, 2, _____	_____, 2, _____

Given: _____, 7, _____ Response: 6, 7, 8

PERFORMANCE OBJECTIVES

EXAMPLES AND COMMENTS

NUMBER MEANING

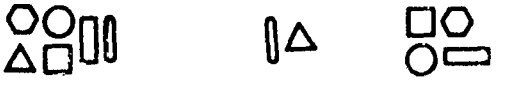
46. Given two sets of small *objects*, each with less than six members, the learner will name the number associated with each set, then move the objects into a single set and say the number for the newly formed set.

Given. Result:

"Three" "Four" "Seven"

47 Given a set of small *objects* with nine or fewer members, the learner will say the number for the given set, then form two subsets and say the number for each of the subsets.

Given: Results:

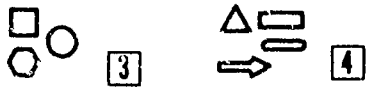


"Six" "Two" "Four"

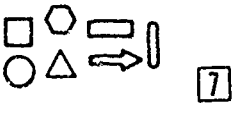
48. Given *numeral cards*, 0-9, and two sets of *objects*, each having four or fewer members the learner will (1) place the appropriate numeral cards beside each set, (2) move the sets together forming a new set, (3) name the new set with a numeral card, then (4) partition the new set into a pair of subsets and (5) associate with the pair of subsets.

Given 1 2 3 4 5 6 7 8 9

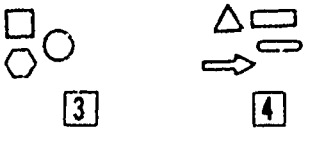
Step 1



Step 2-3

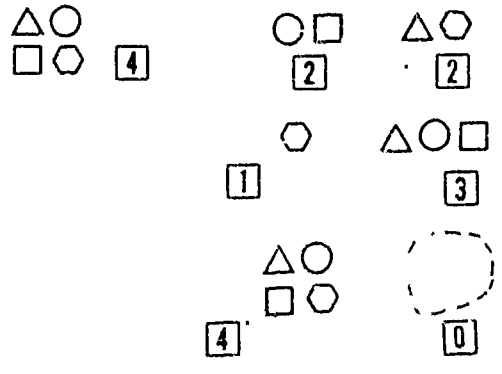


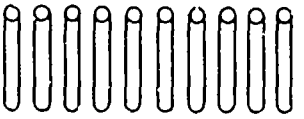
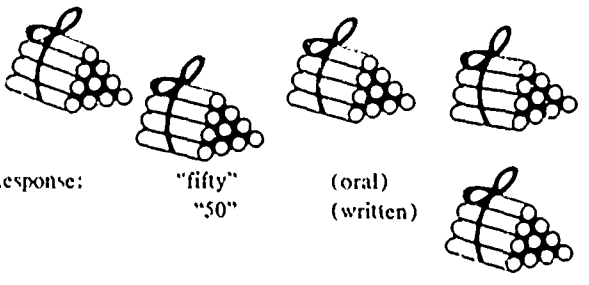
Result: (one possibility)



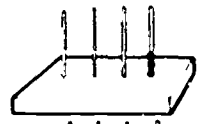
49. Given a set of nine or fewer small *objects* the learner will separate the given set into all possible pairs of subsets, say the number for each subset and then *write* the number for each subset.

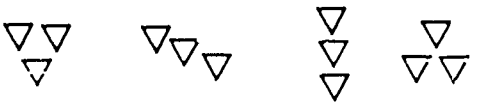
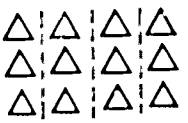
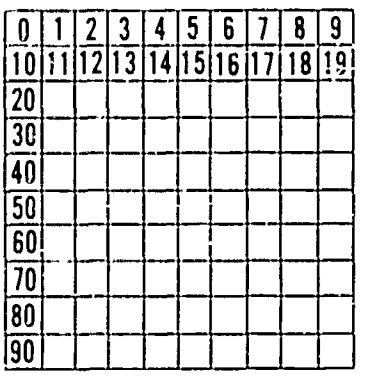
Given: Result.

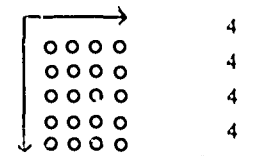
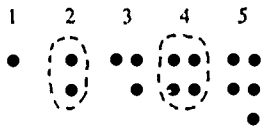


PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
NUMBER, MEANING	
50. Given a set of ten assorted *objects*, the learner will say the word "ten", while holding the collection in his hand and when he is asked "how many numbers in this set."	Straws, lollipops, sticks, beads, spoons
51. Given a set of 12 to 15 small *objects*, the learner will pick out ten objects on request.	Straws, lollipops, sticks, beads, spoons.
52. Given ten identical *objects* spaced out on a table, the learner will collect the set of ten, fasten them together, and indicate that he has one ten and no ones.	1)  2) 3) ___ tens ___ ones
53. Given a set of number word cards for zero through ten and a set of numeral cards for 0 through 10, the learner will say the number word, while matching the written number work with its numeral	Each child has two sets of colored numerals Envelopes: cards [1] [2] [3] . . . [10] Envelopes: cards [one] [two] [three] . . . [ten]
54. Given twenty, thirty, forty, up to ninety *objects*, the learner will form sets of ten and group and label sets of ten as 2 tens, 3 tens, 4 tens, . . . 9 tens.	Straws, lollipops, sticks, beads, spoons
COUNTING	
55. Given 10 bundles with ten straws in each bundle, the learner will count by 10's to 100, saying the number word while placing additional bundles of ten on the desk.	
56. Given cards with the numerals 10, 20, 30, . . ., 100, the learner will count by 10's to 100 saying the associated number word while picking up the proper number card.	
57. Given the spoken words 10, 20, 30, . . ., 100 in sequential order, the learner will write the given numerals.	
NUMBER MEANING	
58. Given 100 identical objects, the learner will make 10 bundles, each containing 10 objects, on request.	
59. Given a set of no more than ninety *objects* grouped by tens, the learner can say and write the numeral represented.	 Response: "fifty" (oral) "50" (written)

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
60. Given a numeral from the set 10, 20, . . . 90 and a supply of *objects*, the learner will form a set representing the given number.	Given 30 Response:
COUNTING	
61. Given orally any one of the following: 20, 30, 40, . . . 90, the learner will say the counting sequence of the following nine numbers.	Given: 30 Result: 31, 32, 33, 34, 35, 36, 37, 38, 39
NUMBER MEANING	
62. Given any number from the set 10, 11, 12, . . ., 99 and a supply of counters, the learner will represent the number with the counters.	Use straws, spoons, beads, paper strips, pegs, tongue depressors, toothpicks.
63. Given any number of objects greater than twenty and less than one hundred, the learner will form the objects into "tens" and "ones" and will label the results as ".... tens" and "..... ones".	
64. Given a set of tens and ones representing a number less than 100, the learner will say and write the numeral.	
ORDERING	
65. Given a set of sequentially ordered whole numbers within a decade less than 100, such as 31, 32, . . ., 40, printed on numeral cards the learner will pick out the number that comes immediately before or after a given number, as requested.	
NUMBER MEANING—EXPANDED NOTATION	
66. Given a two-digit numeral, the learner will be able to write it in expanded notation in two ways: as so many tens and so many ones, or as .... + ..., representing the value of the tens place plus the value of the ones place.	Given: 35 Response: 3 tens and 5 ones OR 30 + 5
67. Given two 2 digit numbers the learner will tell which number is greater and which number is less.	Given: 43 72 Student response: "43 is less" "72 is more"
MONEY	
68. Given a set of dimes, one to nine, the learner will state the value.	
69. Given two containers, one with pennies and one with dimes, the learner, when requested, will take out enough pennies and exchange the pennies for a dime.	

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
70. Given a set of dimes and pennies valued between 11 and 99 cents (from one dime, one penny to 9 dimes, 9 pennies), the learner will state the value.	Given 1 dime, 1 penny Response. 11¢ Given: 5 dimes, 2 pennies Response: 52¢ Given: 7 dimes, 9 pennies Response: 79¢
71. Given dimes and a dollar, the learner will pick up enough dimes to exchange for one dollar.	Given. dimes and one dollar Response pick up ten dimes to exchange for one dollar.
72. Given a three-digit number, the learner will arrange counters grouped to represent the hundreds, tens and ones, of the numeral.	
NUMBER MEANING—ABACUS	
73. Given an abacus, the learner will use counters on the right hand spindle to represent any one-digit numeral (1-9).	
74. Given an abacus, the learner will use one counter on the tens spindle plus 0-9 on the ones spindle to represent numbers 10-19.	10 11 12 etc.
75. Given an abacus the learner will move counters on the tens and ones spindle to illustrate counting through any two decades, i.e. 37 to 52.	37 40 52
76. Given an abacus, the learner will move counters on the hundreds, tens, and ones spindle to illustrate any three digit numeral dictated to him orally or in writing.	 "Show 324 on the abacus"

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
EQUIVALENT SETS	
77. Given 2 to 5 sets each with the same number of objects, the learner will join the sets and write the number that names the new set.	Use repeated addition  3 + 3 + 3 + 3 = 12 Arrange in rows and columns (array)  Each subset has 3 members
ORDERING	
78. Given a hundred chart with the first twenty numbers and multiples of 10, the learner will write in any portion of the chart as indicated by the teacher.	 Use chart having 10 by 10 grid (100 chart) Need 10 charts Used in this way the attention is drawn to the pattern of counting and place-value.
NUMBER MEANING—EXPANDED NOTATION	
79. Given any 3-digit numeral, the learner will write expanded notation, first by using place value words and then by using numerals.	243 2 hundreds 4 tens 3 ones 200 + 40 + 3
ORDERING	
80. Given a random list of 2-digit numerals, the learner will arrange them in ascending order.	Given 76, 42, 96, 34, 25 Result: 25, 34, 42, 76, 96
81. Given a random list of 2 and 3 digit numerals, the learner will arrange them in ascending order.	Given 123, 47, 91, 796, 31 Result: 31, 47, 91, 123, 796
NUMBER MEANING--COMPARISON	
82. Given two 3-digit numerals which have the same digits but in different positions, the learner will compare them to determine which is greater, which is less.	Given. 297 or 792 634 or 364

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
ARRAYS	
83. Given equivalent sets, the learner will form arrays by arranging the members into rows and columns and will then determine the cardinal number of the array.	 5 + 5 + 5 + 5 = 20
ORDERING	
84. Given a counting sequence of two to four numbers the learner will tell and write the next number in the sequence.	2, 4, 6, 8, ___ 5, 10, 15, ___ . 4, 8, 12, ___ .
NUMBER MEANING	
85. Given the counting numbers 1-10 the learner will indicate those that are multiples of 2.	
86. Given a set of *objects*, the learner will make another set that will have twice as *many* objects.	Given: Response:

BENCH MARK K-3 / 4-6

87. Given any 4-digit number, the learner will give the number that is 100 or 1000 more or less than it is without using formal addition or subtraction.	Given: 4823 Response: 100 more is 4923 100 less is 4723 1000 more is 5823 1000 less is 3823
88. Given a written Arabic numeral, the learner will read the number aloud.	Child sees: 7192 His oral response is "seven thousand one hundred ninety-two."
89. Given a number orally, the learner will write the Arabic numeral.	The teacher says, "Six thousand four hundred seventy-two." The child writes: 6472

PERFORMANCE OBJECTIVES

NUMBER MEANING

90. Given any 4-digit numeral, the learner will write expanded notation, first by using place value words and then by using numerals.

91. Given any numeral from 100 to 9999999 the learner will locate and separate the periods with commas.

92. The learner will be able to express in Roman Numerals any numeral from 1-25.

EXAMPLES AND COMMENTS

7192 7 *thousands* 1 *hundred(s)*
9 *tens* 2 *ones*

7000 + 100 + 90 + 2

Given: 654321 1032

Response: 654,321 1,032

Given 23 Response, XXIII

ARITHMETIC

II WHOLE NUMBERS

The WHOLE NUMBER Strand consists of four (4) separate Sequences:

A. Addition Sequence

B. Subtraction Sequence

C. Multiplication Sequence

D. Division Sequence

The importance and relevance of the skills in the above listed Sequences is common knowledge to all. Each skill included in each of these Sequences was carefully chosen. Skills that could distract, confuse or hamper the progress of the learner were omitted.

Any learner having difficulty with any of the Whole Number Computation skills could follow the appropriate Sequence, starting at the point where he fails to be successful, and continue at his own rate through the skills until he masters the particular skill area.

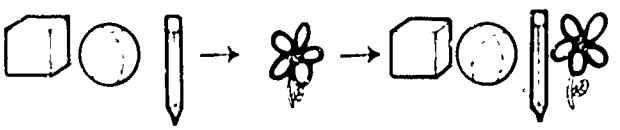
When drill is used to strengthen computation skills, it must be used only after understanding has been achieved. All drill must be appropriately timed and suited to the individual needs of each learner. Amount and duration of all drill and practice should be carefully planned to insure its usefulness. Without an understanding of the basic ideas and armed only with rote learning, the child has all avenues to success closed if memory fails.

PERFORMANCE OBJECTIVES

ADDITION—MEANING

1. Given two distinct (disjoint) sets of objects (totaling no more *than 10 members*), the learner will push the objects together and tell how many members are in the union of the sets.

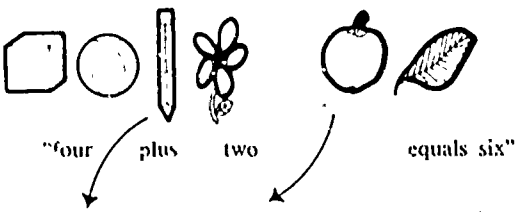
EXAMPLES AND COMMENTS



"FOUR"

ADDITION—BASIC FACTS

2. Given two distinct (disjoint) sets of objects totaling no more than 10 members, the learner will mentally form the union of the sets, and then orally and in writing tell the number sentence derived from that union.



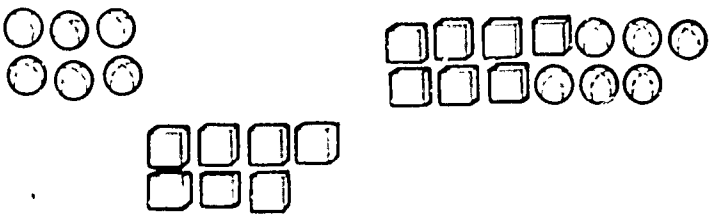
3. Given addition number sentences in horizontal form with sums missing (up through 10), the learner will write the missing sums in the number sentences, with or without the aid of the counters.

5 + 4 [9]

ADDITION—MEANING

4. Given two distinct (disjoint) sets of concrete objects (totaling *no more than 18 members*), the learner will push the objects together and tell how many members are in the union of the sets.

"six things and seven things are thirteen things"



5. Given two sets of pictures, felt objects, and so on, totaling no more than 18 members, the learner will mentally form the union of the sets, and then orally and in writing tell the number sentence derived from that union.

"six plus seven equals thirteen"

6 + 7 [13]

ADDITION—BASIC FACTS

6. Given addition number sentences with sums missing (*sums through 18*), the learner will write the missing sums in the number sentences, with or without the aid of the counters.

a) 8 + 7 [15]

b) 8 + (2 + 5) ☐

(8 + 2) + 5 ☐

10 + 5 [15]

c) 8 + 7 = 7 + 8 [15]

PERFORMANCE OBJECTIVES

ADDITION--VOCABULARY

7. Given an addition exercise with the answer, the learner will tell which number is the "sum".

8. Given addition facts written in either vertical or horizontal form, the learner will write the missing sums, with or without the use of aids.

ADDITION ALGORITHM—NO REGROUPING

9. Given an addition exercise involving a *two digit* number plus a *one digit* number requiring no regrouping (carrying), the learner will demonstrate the process of addition using objects.

10. Given addition exercises involving a *two digit* number plus a *one digit* number requiring no regrouping (carrying), the learner will find the sums with or without the use of aids.

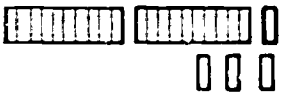
EXAMPLES AND COMMENTS

2 + 1 = 3

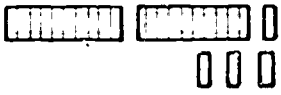
"Three is the *sum*."

$$\begin{array}{r} 5 \\ +\,7 \\ \hline 12 \end{array} \qquad 5 + 7 = \boxed{12}$$

$$\begin{array}{r} 21 \\ +\,3 \\ \hline \end{array}$$



Step I



Step II
Combine Objects and count them

$$\begin{array}{r} 21 \\ +\,3 \\ \hline 24 \end{array}$$

Step III

PERFORMANCE OBJECTIVES

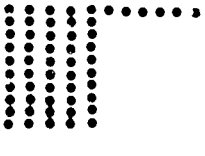
11. Given an addition exercise involving a two *digit* number plus a *two digit* number requiring no regrouping (carrying), the learner will demonstrate the process of addition using objects.

EXAMPLES AND COMMENTS

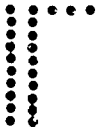
$$\begin{array}{r} 56 \\ +\ 23 \\ \hline \end{array}$$

Step I

56

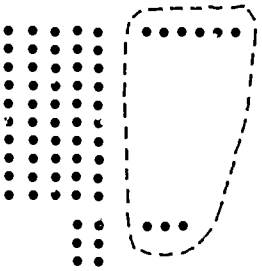


23



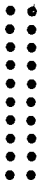
Step II

Combine the ones



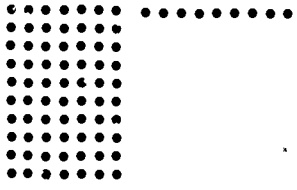
Add the ones column and write the sum:

$$\begin{array}{r} 56 \\ +23 \\ \hline 9 \end{array}$$



Step III

Combine the tens
Add the tens column and write the sum.



$$\begin{array}{r} 56 \\ +23 \\ \hline \boxed{79} \end{array}$$

ADDITION ALGORITHM—NO REGROUPING

12. Given addition exercises involving multiples of 10, the learner will write the sums.

2 ones + 4 ones 6 2 + 4 6

2 tens + 4 tens 6 tens: 20 + 40 60

13. Given addition problems involving a *two digit* number plus a *two digit* number requiring no regrouping (carrying), the learner will find the sums with or without the use of aids.

$$\begin{array}{r} 46 \\ +32 \\ \hline \boxed{78} \end{array}$$

46 + 32 $\boxed{78}$

ADDITION—PROBLEM SOLVING

14. Given addition word problems read by the teacher involving sums to 10, the learner will find the answer.

1. John has five pieces of candy and Mary has three pieces of candy. How many pieces of candy do the children have together?

K-3
BENCH MARK ———
4-6

PERFORMANCE OBJECTIVES

ADDITION ALGORITHM—WITH REGROUPING

15. Given an addition exercise involving a *two digit* number plus a *one digit* number requiring regrouping (carrying), the learner will demonstrate the regrouping process in addition using objects.

16. Given addition exercises involving a two digit number plus a one digit number requiring regrouping (carrying), the learner will find the sums, with or without the aid of counters.

EXAMPLES AND COMMENTS

$$\begin{array}{r} 47 \\ +\ 8 \\ \hline \end{array}$$

Child makes 4 bundles of 10, places the bundles in the tens pockets, and puts 7 singles in the ones pocket.

He adds 8 items to the ones pocket, then regroups the 15 ones into one bundle of 10 and 5 ones.

Techniques:

Renaming:

$$\begin{array}{r} 47 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 40 + 7 \\ \quad\ 8 \\ \hline 40 + 15 = \boxed{55} \end{array}$$

Partial Sums:

$$\begin{array}{r} 47 \\ +\ 8 \\ \hline 15 \\ 40 \\ \hline \boxed{55} \end{array}$$

Short Form:

$$\begin{array}{r} 47 \\ +\ 8 \\ \hline 55 \end{array}$$

Use Stern Blocks, or Dienes Multi Base Blocks, and so on.

1. Any of the following techniques are acceptable as a final form.

Renaming

$$\begin{array}{r} 59 \\ +\ 9 \\ \hline \end{array} \qquad \begin{array}{r} 50 + 9 \\ \quad\ 9 \\ \hline 50 + 18 \end{array} \qquad 68$$

Partial Sums

$$\begin{array}{r} 59 \\ +\ 9 \\ \hline 18 \\ 50 \\ \hline 68 \end{array}$$

Short Form

$$\begin{array}{r} 59 \\ +\ 9 \\ \hline 68 \end{array}$$

PERFORMANCE OBJECTIVES

17. Given an addition exercise involving *two 2-digit* numbers requiring regrouping (carrying), the learner will demonstrate the regrouping process in addition using aids.

18. Given *column addition exercises* involving *three single digit addends*, the learner will find the sums, with or without the aid of counters.

19. Given column addition exercises involving *three, one- or two-digit addends* with or without regrouping (renaming), the learner will find the sums, with or without the aid of counters.

ADDITION—PROBLEM SOLVING

20. Given addition word problems read by the teacher involving *sums to 18*, the learner will tell which operation to use, write an appropriate number sentence either vertically or horizontally and find the answer.

ADDITION ALGORITHM

21. Given addition exercises involving multiples of 100 (less than 1,000), the learner will write the sums.

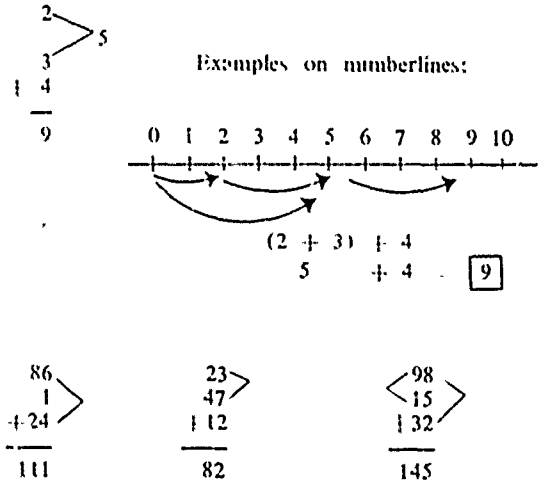
22. Given addition exercises involving a *three* digit number plus a *one, two* or *three digit* addends, with or without regrouping (carrying), the learner will write the sums, using any technique.

ESTIMATION

23. Given an addition exercise involving two digit addends, the learner will estimate the answer by rounding the addends to the closest multiple of ten.

EXAMPLES AND COMMENTS

1 See suggestions and techniques used in 9, 10, and 15.



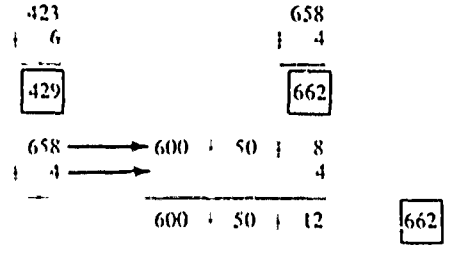
2 Susan had 8 cents in her bank and 7 cents in her purse. How much money does she have?

Student response: "You would ADD."

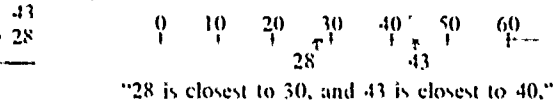
8 + 7 = 15 OR: 8 + 7 = 15

2 hundreds plus 5 hundreds equals 7 hundreds

200 + 500 = 700



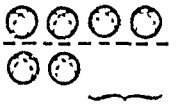
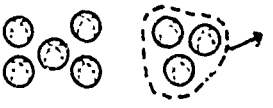
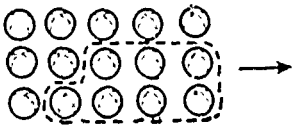
Step 1 Find the multiple of 10 that is closest to the given number.

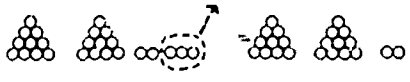


"28 is closest to 30, and 43 is closest to 40."

Step 2: 43 → 40
+ 28 → + 30
70 (approximately)

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
24. Given an addition exercise involving three digit addends, the learner will estimate the answer by rounding the addends to the closest multiple of ten or one hundred.	Use procedure listed in objective 23.
25. Having solved an addition exercise involving two digit addends, the learner will state the reasonableness of his answer after having compared his answer with his estimated answer.	Use procedure listed in objective 23. Give the children experiences rounding the numbers in particular problems both to the nearest ten and to the nearest hundred. Ask them to determine which estimate is closer to the actual sum.
ADDITION ALGORITHM	
26. Given addition problems involving 2 or 3 addends with three, four, five or six digits, with or without regrouping, the learner will find the sums, using any technique.	Any technique to correctly solve the problem is acceptable as a final form.
ADDITION FACTS	
27. Given the 100 addition facts, the learner will write or recite the sums.	The number of addition facts to be learned can be greatly reduced with application of the commutative law (changing the order in which the two numbers are added does not change the sum). The learner should not be given more than 20 facts per sitting. "Reasonable" time should be allowed and not limited too severely.
(*NO* 7-9 OBJECTIVES)	

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
SUBTRACTION—MEANING	
1. Given two sets with less than 10 objects each, the learner will supply the objects needed to make the sets equivalent.	 "How many more are needed?"
2. Given a set of 10 or less concrete objects, the learner will take away a given number of objects and state the number of objects that remain.	 "There are five balls left." Learner Response:
3. Given a subtraction situation involving objects, the learner will state an appropriate number sentence.	Learner Response: "six minus two equals four"
4. Given subtraction exercises with *combinations to 10*, the learner will use objects or pictures to illustrate the solution.	4 − 1 = [3]
5. Given subtraction exercises in horizontal form using numbers to 10, the learner will find the difference with or without the use of aids.	8 − 7 = [1] 6 − 3 [3]
6. Given a word problem read by the teacher involving addition or subtraction combinations to 10, the learner will identify the operation and find the sum or difference.	
SUBTRACTION—FACTS	
7. Given subtraction exercises with combinations to 18, the learner will use objects to find the difference.	15 − 7 [8] 
8. Given subtraction exercises in horizontal and vertical form using numbers to 18, the learner will find the difference with or without the use of aids.	14 − 6 [8] 14 − 6 — [8]
9. Given a set of objects or pictures showing a subtraction relationship with combinations to 18, the learner will write an appropriate number sentence.	12 − 7 [5]
10. Given two addends less than 10 and the sum, the learner will write the related subtraction fact	Given: 5 + 7 [12] Student writes: 12 − 7 = 5 12 − 5 = 7

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
SUBTRACTION—PROBLEM SOLVING	
11. Given a subtraction word problem read by the teacher involving combinations to 18, the learner will: 1) identify the operation, and 2) write an appropriate number sentence, and 3) find the answer.	Addition or subtraction number sentence is acceptable. Lou had 7 pennies, he lost 2 pennies. How many are left? Student Responses: Subtraction 7 − 2 = ☐ or Addition 2 + ☐ = 7 ☐ + 2 = 7
SUBTRACTION—VOCABULARY	
12. Given a subtraction exercise, the learner will identify the answer as the "difference".	7 − 3 = [4] "Four is the difference."
SUBTRACTION—MEANING	
13. Given two sets of objects one with more objects than the other, the learner will state how many more members it has.	
SUBTRACTION ALGORITHM—WITHOUT REGROUPING	
14. Given a subtraction example involving a two digit number minus a one digit number, the learner will demonstrate the solution using counters or objects. *No regrouping.*	Given: 25 − 3 = ☐ Student Response:  (bundled sticks)
15. Given a *two digit number*, the learner will subtract a *one digit number* with no regrouping (borrowing) with or without the use of aids.	24 − 3 — ☐
16. Given a two digit number, the learner will subtract a two digit number with no regrouping (borrowing).	86 −42 —

BENCH MARK K-3 / 4-6

SUBTRACTION ALGORITHM—WITH REGROUPING	
17 Given a subtraction example involving a *two digit number* minus a *one digit number*, the learner will demonstrate the process of regrouping using concrete objects.	Use place value pockets to show all regrouping.
18. Given a *two digit number*, the learner will subtract a *one digit number* with regrouping (borrowing) with or without the use of aids.	33 − 9 = 24 2 13 33 − 9 24 or 20 + 13 − 9 20 + 4 = [24]

PERFORMANCE OBJECTIVES

19. Given a *two digit number*, the learner will subtract a *two digit* number with regrouping (borrowing) with and without objects.

20. Given a *three digit number*, the learner will subtract a *two* or *three digit number*, with or without the use of concrete aids.

SUBTRACTION FACTS

21. Given a subtraction fact, the learner will write or recall the answer.

(NO OBJECTIVES FOR 7-9)

EXAMPLES AND COMMENTS

$$\begin{array}{r} 43 \\ -27 \\ \hline \end{array} \qquad \begin{array}{r} 3\ 13 \\ 43 \\ -27 \\ \hline \end{array} \qquad \text{or} \qquad \begin{array}{r} 30 + 13 \\ -20 + 7 \\ \hline \end{array}$$

Place value aids may be used in this process

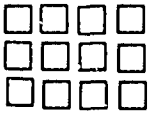
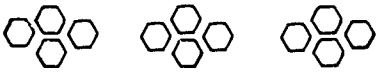
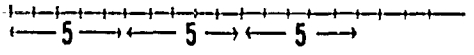
It is highly desirable that the subtraction facts be treated as *related addition facts*:

15 — 7 is the same as:

How much *must be added* to 7 in order to get 15?

Answer: 7 plus |||| ||||| = 15 .
8

The learner is not to be given more than 20 facts per setting. "Reasonable" time should be allowed and not limited too severely.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
MULTIPLICATION—MEANING	
1. Given a supply of identical objects and a repeated addition equation, the learner will arrange the objects to illustrate the equation.	4 + 4 + 4 = 12  3 fours = 12
2. Given equal sets of identical objects (up through 9), the learner will write a related repeated addition equation to describe it.	5 + 5 + 5 + 5 = 20 4 fives = 20
3. Given a collection of equivalent sets, the learner will write a multiplication sentence to describe it.	 3 sets of 4 3 × 4 = 12
4. Given a multiplication sentence, the learner will draw a diagram to demonstrate it.	Given: 4 × 2 = 8 Array Sets
5. Given a multiplication sentence, the learner will represent it as a repeated addition sentence. (Do not include zeroes or ones in repeated addition)	3 × 5 = 15  3 × 5 = 5 + 5 + 5 = 15
6 Given a repeated addition sentence, the learner will represent it as a multiplication sentence with its product.	"Seven children each paid 5¢ for milk" "5 + 5 + 5 + 5 + 5 + 5 + 5" 7 × 5 = 35

PERFORMANCE OBJECTIVES

EXAMPLES AND COMMENTS

MULTIPLICATION—FACTS

7. Given a basic fact, the learner will rename one of the numbers and will use the distributive property (multiply by parts) with or without the use of aids.

8×7

$8 \times 7 \begin{cases} 2 \times 7 & 14 \\ 2 \times 7 & 14 \\ 2 \times 7 & 14 \\ 2 \times 7 & 14 \end{cases}$

56

$(2 + 5)$
$\times \quad 8$
$16 + 40 \quad 56$

$(3 + 4)$
$\times \quad 8$
$24 + 32 \quad 56$

$(1 + 6)$
$\times \quad 8$
$8 + 48 \quad 56$

8. Given two whole numbers (0 . . . 9), the learner will determine the product

Any one of the methods below is acceptable

i. By arranging objects into sets
ii. By marking a number line
iii. By counting members of an array
iv. By number patterns on a grid
v. By multiplication facts on a grid
vi. By memory

MULTIPLICATION—PROPERTY

9. The learner will name the product of any number and one.

$6 \times 1 = \_\_$

$1 \times 3 = \_\_$

$4 \times \_\_ = 4$

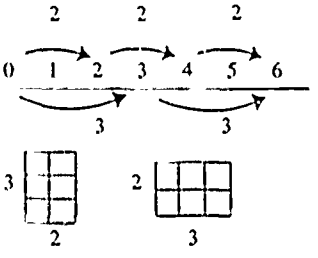
10. The learner will name the product of any number and zero.

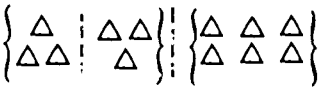
$3 \times 0 = \_\_$

$0 \times 6 = \_\_$

$5 \times \_\_ = 0$

11. Given two numbers, the learner will demonstrate that the order in which they are multiplied does not change the product.





MULTIPLICATION—ALGORITHM

12. Given a one-digit number and a power of 10, the learner will state the product.

$7 \times 10 \quad 10 + 10 + 10 + 10 + 10 + 10 + 10 \quad 70$

$7 \times 10 = \_\_ \qquad 7 \times 100 = \_\_$

$7 \times 1000 = \_\_$

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
13. Given a one-digit number and (10, 20, . . .), (100, 200, . . .), the learner will state the product.	$7 \times ?$ 7×20 $\quad 7 \times (2 \times 10)^*$ $(7 \times 2) \times 10$ $14 \times 10 = 140$ 7×200 $\quad 7 \times (2 \times 100)^*$ $(7 \times 2) \times 100$ $14 \times 100 = 1400$ *An application of the Associative Property
14. Given two whole numbers (one greater than 10), the learner will use the distributive property to find the product.	3×12 $\quad 12 \times 3$ $\quad 10 + 2 \times 3$ $30 + 6 = 36$

BENCH MARK K-3 / 4-6

15 Given a two-digit number to be multiplied by a one digit number, the learner will write the product, with or without aids.	32×6 Any one of the methods below is acceptable as the final form: a) 32×6 $\quad (30 + 2) \times 6$ $180 + 12 = 192$ b) 32×6 12 180 192 c) $32 \times 6 = 192$
MULTIPLICATION—PROBLEM SOLVING	
16 Given a verbal problem involving addition, subtraction or multiplication of two whole numbers, the learner will state what operation is to be performed.	"Joe sold 24 tacos each noon for 5 days. How many tacos did Joe sell?" Answer: "Multiplication"
17 Given a verbal problem involving addition, subtraction or multiplication, the learner will write a number sentence.	Using the verbal problem example from objective 16, the child will write $24 \times 5 = ?$ OR $5 \times 24 = ?$ since 3×4 can mean 3 sets of 4 or 4 sets of 3, depending on the point of view.
18 Given a verbal problem involving addition, subtraction or multiplication, the learner will find the answer.	$24 \times 5 = 120$ Refer to verbal problem example for objective 16. "Joe sold 120 tacos."

PERFORMANCE OBJECTIVES

EXAMPLES AND COMMENTS

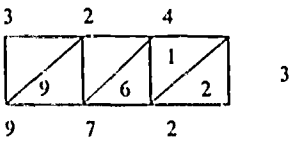
MULTIPLICATION—ALGORITHM

19. Given a three-digit number, the learner will multiply it by a one-digit number.

Any of the methods below is acceptable.

a) $324 \times 3 =$
324
324
$+ 324$
972

b) Lattice



c) $324 \times 3 = 300 \times 3 + 20 \times 3 + 4 \times 3$
$= 900 + 60 + 12$
$= 972$

d)
324
$\times \quad 3$
12
60
900
972

e)
324
$\times \quad 3$
972

MULTIPLICATION—ALGORITHM

20. Given two factors less than 100 that are multiples of 10, the learner will determine the product.

$3 \times 7 = 21$ and $30 \times 7 = 210$
and $30 \times 70 = 2100$

21. Given a two-digit and a one-digit number, the learner will estimate the product by rounding the two-digit number to the nearest ten and multiplying.

32×5 "The answer is close to 150."
$30 \times 5 = 150$

22. Given two factors of two digits each, the learner will estimate the product by rounding both to the nearest ten and multiplying.

32×16 "The answer is close to 600."
$30 \times 20 = 600$

23. Given two two-digit whole numbers, the learner will determine the product.

Any of the methods below are acceptable

a)
32
$\times \quad 15$
10
150
20
300
480

b)
32
$\times \quad 15$
160
320
480

c)
32
$\times \quad 15$
$32 \quad 32$
$10 \quad 5$
320
160
480

24. Given three digit multiples of 100 and two digit multiples of 10, the learner will find the product of any two of them.

30×400
$(3 \times 10) \cdot (4 \cdot 100)$
$(3 \cdot 4) \cdot (10 \cdot 100) = 12{,}000$

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
25. Given two whole numbers each less than 1000, the learner will estimate the product.	36×431 / 40×400 / 16000 } "The answer to 36×421 is about 16000."
26. Given any two factors less than 1000, the learner will determine the product.	327×5 / 327×25 / 327×251 Any method to correctly solve the problems is acceptable as a final form.
MULTIPLICATION—PROBLEM SOLVING	
27. Given a verbal problem involving the addition, subtraction or multiplication of whole numbers, the learner will: a) determine the operation to be used, and b) write appropriate number sentence, and c) find the answer.	"Our school has 17 rooms. Each room has 6 windows and each window has 18 panes of glass. In all there are _____ panes of glass in the rooms."

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
DIVISION—MEANING	
1. Given a whole number less than 10, the learner will make a table of multiples of that number with at least six entries, writing an equation for each multiple, as illustrated.	$1 \times 3 = 3$ $3 \times 3 = 9$ $2 \times 3 = 6$ $4 \times 3 = 12$ $3 \times 3 = 9$ $5 \times 3 = 15$ $4 \times 3 = 12$ or $6 \times 3 = 18$ $5 \times 3 = 15$ $7 \times 3 = 21$ $6 \times 3 = 18$ $8 \times 3 = 24$

BENCH MARK K-3 / 4-6

2. Given a group of objects, the learner will sort them into equivalent sets.	Given 50 chips put them into 5 equal stacks Given 18 toys show how many three children could be given so that they each would have an equal number of toys.
3. Given a set of objects to be partitioned into subsets of a given number, the learner will determine the number of subsets by constructing each subset.	Given: 50 chips—put 10 in each stack How many stacks?
4. Given a whole number less than 100, the learner will make a table of multiples of that number with at least six entries, writing an equation for each multiple, as illustrated.	$1 \times 12 = 12$ $5 \times 12 = 60$ $2 \times 12 = 24$ $6 \times 12 = 72$ $3 \times 12 = 36$ $7 \times 12 = 84$ $4 \times 12 = 48$ or $8 \times 12 = 96$ $5 \times 12 = 60$ $9 \times 12 = 108$ $6 \times 12 = 72$ $10 \times 12 = 120$
5. Given a sentence with one single digit, a missing factor, and a product (whole numbers), the learner will name the missing factor.	$3 \times \square = 21$ $5 \times \square = 50$
6. Given an open number sentence in multiplication using the basic facts only, the learner will supply any two factors to make the sentence true.	$\square \times \triangle = 9$ $\square \times \triangle = 12$ $\square \times \triangle = 9$ $\square \times \triangle = 12$ Possible Solution: Possible Solution: (1, 9) (1, 12) (12, 1) (9, 1) (3, 4) (4, 3) (3, 3) (2, 6) (6, 2)
DIVISION—FACTS	
7. Given a division fact, the learner will rewrite it as a multiplication fact.	$15 \div 3 = 5$, $5 \times 3 = 15$ $15 \div 5 =$, $5 \times$ = 15
8. Given two factors and a product, the learner will rewrite the sentence to show the related division facts.	$3 \times 7 = 21$ $21 \div 7 = 3$ $7 \times 3 = 21$ $21 \div 3 = 7$ $\begin{array}{r} 7 \\ \times 3 \\ \hline 21 \end{array}$ $\begin{array}{r} 3 \\ \times 7 \\ \hline 21 \end{array}$ $3\overline{)21}$ $7\overline{)21}$

PERFORMANCE OBJECTIVES

EXAMPLES AND COMMENTS

DIVISION—ALGORITHM

9. Given a one-digit division (factor) and a dividend (product) of less than 100, the learner will determine the quotient (missing factor) if there is no remainder.

Any of the methods below are acceptable.

a) $6\overline{)72}$

−12	2 × 6
60	
−12	2 × 6
48	
−12	2 × 6
36	
−12	2 × 6
24	
−12	2 × 6
−12	
−12	2 × 6
0	12

b) $6\overline{)72}$

30	5 × 6
42	
42	7 × 6
0	

c) $6\overline{)72}$

60	10 × 6
12	
12	
12	2 × 6
0	

d)
$$6\overline{)72}\quad \begin{array}{r} 12 \\ 60 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$$

e)
$$6\overline{)78}\quad \begin{array}{r} 13 \\ 3 \\ \hline 10 \\ 60 \\ \hline 18 \\ 18 \\ \hline 0 \end{array}$$

f)
$$6\overline{)78}\quad \begin{array}{r} 13 \\ 60 \\ \hline 18 \\ 18 \\ \hline 0 \end{array}$$

48

10. Given a division exercise with a single-digit divisor and a two-digit dividend, the learner will:
    a) Make a table of multiples of the divisor with at least 3 entries, and including the multiples just above and just below the dividend.
    b) Choose the largest entry in the table that has a product less than or equal to the dividend.
    c) Tell which number in the chosen multiplication equation is the quotient.

Given $7\overline{)58}$

5 × 7 = 35
6 × 7 = 42
7 × 7 = 49
8 × 7 = 56 ——— Must include
9 × 7 = 63

5 × 7 = 35
6 × 7 = 42
7 × 7 = 49
8 × 7 = 56 ——— Student's Choice
9 × 7 = 63

"8"
$7\overline{)58}$

DIVISION—ESTIMATION

11. Given a set of multiplication equations in which one factor is a multiple of 10 or 100, the learner will write related division equations.

Given		Therefore
3 × 7	21 ———	21 ÷ 3 = 7
3 × 70	210 ———	210 ÷ 3 = 70
3 × 700	2100 ———	2100 ÷ 3 = 700

12. Given a division exercise with a one-digit divisor and a three-digit dividend, the learner will:
    a) Write two equations: 10 times the divisor and 100 times the divisor
    b) Choose the largest of these numbers (10, 100) that results in a product less than or equal to the dividend.

$3\overline{)81}$

10 × 3 = 30
100 × 3 = 300

Student chooses 10

13. Given a division problem with a two or three digit dividend and a one digit divisor, the learner will find a multiple of 10 which when multiplied by the divisor will produce a product less than the dividend.

81 ÷ 3
10 × 3 = 30 — 10 is correct, but is *not* the largest
20 × 3 = 60 — 20 is the largest
30 × 3 = 90 — 30 is too large

810 ÷ 3
100 × 3 = 300 — 100 is correct, but is *not* the largest
200 × 3 = 600 — 200 is the largest
300 × 3 = 900 — 300 is too large

ARITHMETIC 4-6

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
DIVISION—ALGORITHM	
14. Given a single digit divisor and any dividend less than 100, the learner will determine the quotient and *the remainder.*	$3\overline{)59}$ 30 | 10×3 29 24 | 8×3 5 3 | 1×3 2 | $19 = 19$ R2 $3\overline{)\begin{array}{r}19\text{ R2}\\59\\30\\29\\27\\2\end{array}}$
15. Given an exercise with a dividend of four digits or less, and a one-digit divisor, the learner will determine the quotient.	$3\overline{)462}$ 300 | 100×3 162 90 | 30×3 72 60 | 20×3 12 12 | 4×3 154 $3\overline{)\begin{array}{r}154\\4\\50\\100\\462\\300\\162\\150\\12\\12\end{array}}$ $3\overline{)\begin{array}{r}162\text{ R1}\\487\\300\\187\\180\\7\\6\\1\end{array}}$
DIVISION—ESTIMATION	
16. Given a division exercise with a one-digit divisor and a two-digit dividend, the learner will estimate the tens digit in the quotient.	Given. Estimate by multiples of 10 $3\overline{)92}$ $3 \times 10 = 30$ $3 \times 20 = 60$ $3 \times 30 = 90$ $3 \times 40 = 120$ $90 < 92 < 120$ The estimate is 30 and the quotient is greater than 30. $3\overline{)81}$ $3 \times 10 = 30$ $3 \times 20 = 60$ $3 \times 30 = 90$ $60 < 81 < 90$ The estimate is 30 and the quotient is less than 30.

BENCH MARK $\frac{4\text{-}6}{7\text{-}9}$

17. Given a division exercise with a one or two digit divisor and a four digit dividend, the learner will: a) Write three equations: 10 times the divisor. 100 times the divisor. 1000 times the divisor. b) Choose the largest of these numbers (10, 100, 1000), that results in a product less than or equal to the dividend.	$16\overline{)5231}$ $10 \times 16 = 160$ $100 \times 16 = 1600$ $1000 \times 16 = 16000$ Student chooses 100.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
18. Given a division exercise with a three or four digit dividend and a one or two digit divisor, the learner will find a multiple of 10, 100 or 1000 which when multiplied by the divisor will produce a product less than the dividend.	9618 4 1000 × 4: 1000 is correct 2000 × 4: 2000 is correct 3000 × 4: 3000 is too large 9412 . 23 100 × 23: 100 is correct 200 × 23: 200 is correct 300 × 23: 300 is correct 400 × 23: 400 is the largest 500 × 23: 500 is too large

DIVISION—ALGORITHM

19. Determine the quotient for a two- three, or four digit dividend and a two digit divisor—with or without remainders.

All of the methods below are acceptable.

```
32 ⟌845
   640 | 20 × 32
   205
   160 |  5 × 32
    45
    32 |  1 × 32
    13 | 26    R13
```

```
32 ⟌768
   640 | 20 × 32
   128
   128 |  4 × 32
       | 24
    24
     4
    24
```

```
    26 R13
     6
    20
32 ⟌845
   640
   205
   192
    13
```

```
32 ⟌768
   640
   128
   128
```

```
    26 R13
32 ⟌845
   640
   205
   192
    13
```

```
    27
32 ⟌768
   640
   128
   128
```

20. Verbal Problems: Given a problem involving division of whole numbers, the learner will determine the dividend, divisor and solve for the quotient and the remainder.

"With 1500 seats in the auditorium and 50 students selling tickets, how many tickets should each student be given to sell?"

"A teacher shares 68 pencils among 22 students. How many remain after he shares them equally?"

ARITHMETIC

III FRACTIONAL NUMBERS

The FRACTIONAL NUMBER Strand consists of four (4) Skill Sequences:

A. Meaning

B. Addition

C. Subtraction

D. Multiplication

The rationale for the omission of a *Division* Sequence is the fact that the learner will seldom encounter a situation in daily life where he needs to use the algorithm for divison of common fractions. The multiplication algorithm can be used instead of the division algorithm to solve problems in practical everyday encounters:

"½ cup of sugar divided by 4" usually is solved by:

"¼ × ½" rather than by "½ ÷ 4"

The absence of the Division Sequence from this document does *not* mean that division of fractional numbers should be left out of the curriculum. It only means that it is not considered to be a minimum basic requirement for which *all* of the students in the State of Michigan should be held responsible.

The *Meaning* Sequence covers those concepts that are necessary to give the learner a complete understanding of the idea, and notation of fractions which he needs to work meaningfully and successfully with the algorithms of adding, subtracting and multiplying fractions.

This entire Strand is written on an "activity" basis. All the skills are task oriented and the learner will use models, concrete or pictorial, to progress from one skill to the next.

PERFORMANCE OBJECTIVES

CONGRUENCE

1. Given several objects, some divided into congruent parts, some divided into non-congruent parts, the learner will:
   a) determine which objects are divided such that *all* parts are congruent, and which objects are divided into non-congruent parts, by fitting the parts of each object on top of each other, and
   b) identify congruent parts.

HALVES—READINESS

2. Given several objects, some divided into 2 congruent parts, some divided into 2 non-congruent parts, the learner will:
   a) determine which objects have been divided into the 2 congruent parts, and
   b) tell the fraction name for each part upon request.

ONE-HALF

3. Given a rectangular piece of paper and the direction to indicate one-half, the learner will illustrate his understanding of the meaning of one-half by folding, coloring, and then cutting

FOURTHS—READINESS

4. Given several objects, some divided into 4 congruent parts, and some divided into four non-congruent parts, the learner will:
   a) determine which objects have been divided into the four congruent parts; and
   b) tell the fraction name for *each* part upon request.

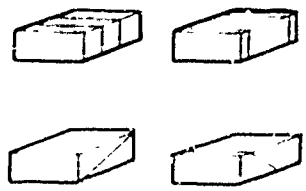
5. Given a rectangular piece of paper, the learner will make four fourths by folding, coloring, and then cutting.

THIRDS—READINESS

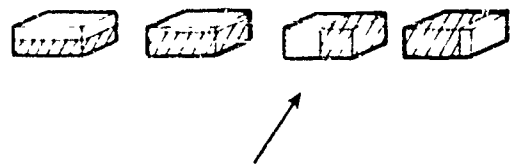
6. Given several objects, some divided into 3 congruent parts, some divided into 3 non-congruent parts, the learner will:
   a) determine which objects have been divided into the 3 congruent parts and
   b) tell the fraction name for each part upon request.

7. Given a rectangular piece of paper divided into thirds, the learner will make three thirds by folding, coloring, and then cutting.

EXAMPLES AND COMMENTS

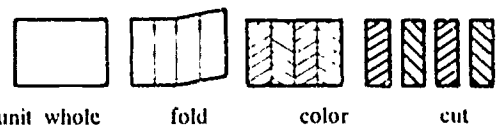


All parts are not congruent.

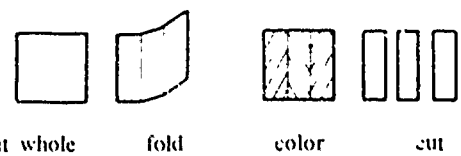


Student response: "This is one half."

Student response. "This is one fourth."



Student response "This is one third."



PERFORMANCE OBJECTIVES

DISTINGUISHING BETWEEN HALVES, THIRDS AND FOURTHS

8. Given several objects or pieces of cardboard of the same unit size and shape, some divided into fourths, and some divided into thirds, and one unit whole, the learner will name each of the parts appropriately as "one half" or "one third," or "one fourth," by comparing each part with the unit whole.

MAKING ONE WHOLE

9. Given objects or pieces of cardboard divided into two, three, or four congruent parts, the learner will orally relate that "two halves make *one* whole," and "three thirds make *one* whole," and "four fourths make *one* whole" as he moves the pieces together.

NUMERATOR–DENOMINATOR

10. Given a fraction, the learner will tell the meaning of the number above the bar (numerator) and the meaning of the number below the bar (denominator), with or without the aid of fractional cut-outs.

FRACTION SYMBOLS

11. Given illustrations of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of objects shaded, the learner will tell the correct fraction in each case orally.

FRACTIONS NAMING PART OF A SET

12. Given a set of *objects* separated into equivalent subsets, the learner will name each subset as the appropriate fractional part of the whole set.

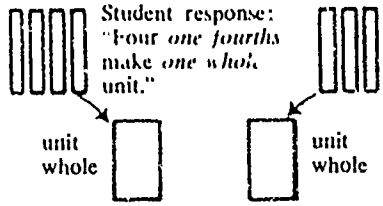
13. Given a set of *objects* the learner will separate the set into appropriate equivalent subsets as indicated by a given *unit* fraction ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and so on) provided that the denominators are less than 9 and the total number of objects is less than 21 and evenly divisible by the denominator of the fraction.

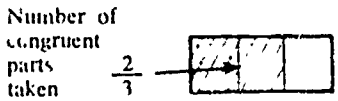
14. Given a set of objects and a *non unit* fraction (2/3, 2/4, 3/4 and so on) with a denominator of less than 6, with a total number of objects less than 16, the learner will:
    a) separate the objects into the number of equivalent subsets indicated by the denominator, and
    b) push over as many subsets as indicated by the numerator.

EXAMPLES AND COMMENTS

unit whole

Student response while picking up each part: "This is one half, this is one third, or this is one fourth."

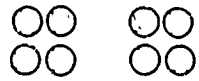




The terms numerator and denominator do not have to be memorized at this level.

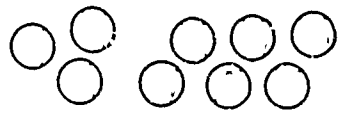
Total number of equal size parts.

Student responds orally: "One third is shaded."



(Use felt or magnetic cut outs.)

"One half" "One half"



"Move $\frac{1}{3}$ of the set of balls to the side."

PERFORMANCE OBJECTIVES

15. Given *pictures* of sets with ¼ or ⅓ of the objects shaded, the learner will name the shaded amount as a fraction, and write the name in fractional form.

16. Given a fraction and a *drawing* of a set, the learner will shade or circle a number of figures to illustrate that part of the set, provided that the denominators are less than 6, and the total number of objects is less than 16 and evenly divisable by the denominator.

17. Given a proper fraction with a denominator less than 6, the learner will explain its meaning by making a drawing or by using fractional cut-outs.

18. Given a diagram divided into congruent parts, with some parts shaded, the learner will name the shaded area by writing an appropriate fraction and by orally stating each name of the fraction.

19. Given any 5 fractional numbers with *like* denominators, in random order, the learner will write them in order. (halves, thirds, fourths, fifths, sixths, eighths, tenths); with or without the use of aids.

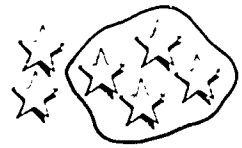
RENAMING FRACTIONS

20. Given a denominator, the learner will supply the correct numerator to make the value of the fraction equal to *one*, with or without the use of aids.

FINDING EQUIVALENT FRACTIONS

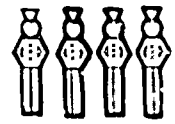
21. Given labeled fractional cut-offs, of the same unit whole with denominators of 2, 4, 8—or 2, 3, 6—the learner will find and write equivalent fractions.

EXAMPLES AND COMMENTS



Student Response: "Two thirds of the stars are circled" or "Four sixths of the stars are circled."

Directions:
"Circle 2/4 of the men" or "Circle ½ of the men."



Use graph paper, geoboard, or fractional cut outs.

Student writes: $\frac{3}{5}$

$\frac{2}{5}, \frac{1}{5}, \frac{4}{5}, \frac{5}{5}, \frac{3}{5}$ Given.

$\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$ Response:

$\rightarrow \frac{4}{4} \rightarrow$ 1 Whole

Unit Whole

$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

BENCH MARK 4-6 / 7-9

22. Given a fraction the learner will write a set of equivalent fractions with or without the use of fractional cut-outs.

$\frac{1}{2} = \frac{\quad}{4} = \frac{\quad}{8}$

PERFORMANCE OBJECTIVES

ORDERING FRACTIONS WITH LIKE AND UNLIKE DENOMINATORS

23. Given the fractional numbers ½, ⅓ and ¼ the learner will write them in order from the least to greatest, with or without the use of fractional cut-outs or numberline.

24. Given two fractional numbers with *unlike* denominators, the learner will tell which one is greater (denominators of 2, 3, 4, 6, or 8) with or without the use of aids.

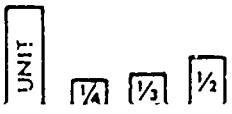
RENAMING MIXED NUMBERS AND IMPROPER FRACTIONS

25. Given fractional cut-outs including several *unit wholes*, and a mixed number with a denominator of 2, 3, 4, 6, or 8 and whole units up to 3, the learner will arrange the fractional cut-outs to show the change of the mixed form to a fraction and then write the answer.

26. Given fractional cut-outs including several *unit wholes*, and a fraction with a denominator of 2, 3, 4, 6, or 8 and a whole unit value of up to 3, the learner will arrange the fractional cut-outs to show the naming of the fraction to a mixed form, and then write the answer.

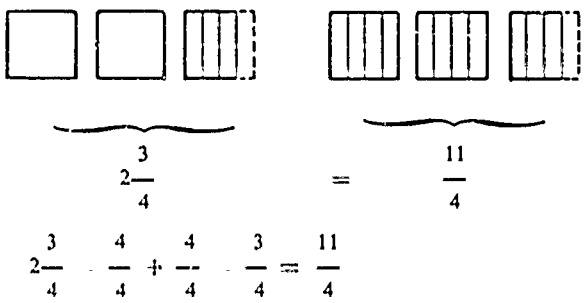
EXAMPLES AND COMMENTS

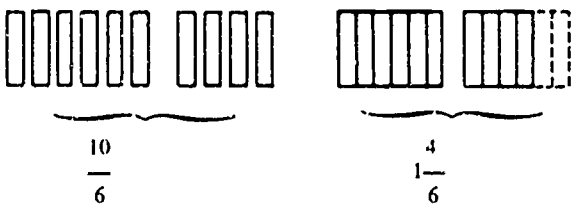
Take four congruent strips of paper each different color. Divide one into halves; one into thirds; one into fourths. Label, then cut out pieces. Using the fourth piece of paper as the unit segment, lay ¼ next to it and mark it, do likewise for ⅓ and ½. Make a "bar graph".



Given. "Compare ⅗ and ¾"

Response. "¾ is greater than ⅗"





PERFORMANCE OBJECTIVES

PROPER FRACTIONS—LIKE DENOMINATORS—NO REGROUPING

1. Given a set of labeled fractional cut-out parts, the learner will demonstrate the result of adding two fractional numbers with like denominators of 2, 3, 4, 6, or 8 with sums less than or equal to 1, by fitting the appropriate parts together, and then writing the sum.

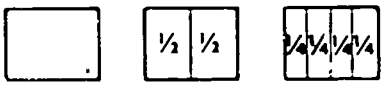
2. Given a set of labeled fractional cut-outs, the learner will show the result of adding fractional numbers with like denominators of 2, 3, 4, 6, or 8 with a sum greater than 1, by fitting the appropriate fractional parts together and then writing the sum.

3. Given two fractional numbers with like denominators having a sum less than or equal to 1, the learner will write the sum, with or without the use of fractional cut-out parts.

4. Given two fractional numbers with like denominators and a sum greater than 1, the learner will write the sum, with or without the use of fractional cut-out parts.

EXAMPLES AND COMMENTS

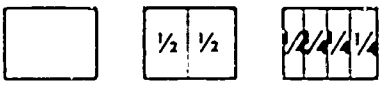
The fractional parts may be commercially prepared, teacher prepared, or student prepared. All the parts should be cut from *one common unit size*. The *unit whole* may be a rectangle or a circle but once the unit is established, the *same* size and shape unit should be used for all the exercises.



and so on

The learner should be able to use the cut outs interchangeably and this can only be done if all fractional parts originate from the *same unit whole*.

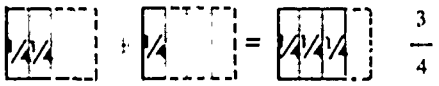
and so on



and so on

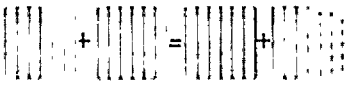
Problem: $\frac{2}{4} + \frac{1}{4}$?

Solution:



$\frac{3}{4}$

$\frac{3}{6} + \frac{5}{6}$?



$\frac{6}{6} + \frac{2}{6}$ $1\frac{2}{6}$

$\frac{3}{8} + \frac{3}{8}$ $\frac{6}{8}$

The answers may, but do *not* have to appear in simplest form.

$\frac{3}{4} + \frac{2}{4}$ $\frac{5}{4}$

$\frac{1}{2} + \frac{3}{2}$ $\frac{4}{2}$

The answers may, but do not have to appear in the mixed form.

The fractions of $\frac{5}{4}$ or $\frac{4}{2}$ are acceptable as answers as well as $1\frac{1}{4}$ or 2.

PERFORMANCE OBJECTIVES

MIXED NUMBERS—LIKE DENOMINATORS—NO REGROUPING

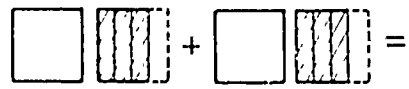
5. Given a set of labeled fractional cut-out parts including several unit wholes, the learner will demonstrate the result of adding two mixed numbers with like denominators of 2, 3, 4, 6, or 8, by fitting the appropriate parts together, and then writing the sum in "non-simplified" form.

6. Given a set of labeled fractional cut-out parts including several unit wholes, the learner will demonstrate the result of adding two mixed numbers with like denominators of 2, 3, 4, 6, or 8, by fitting the appropriate part together, and writing the sum as a whole number and a proper fraction.

7. Given two mixed numbers with like denominators, the learner will write the sum.

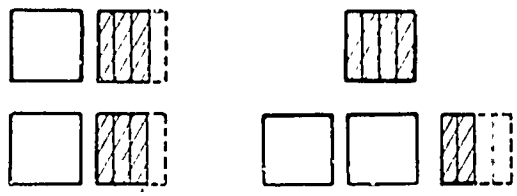
EXAMPLES AND COMMENTS

$1\frac{3}{4} + 1\frac{3}{4}$?



$2\frac{6}{4}$

$1\frac{3}{4} + 1\frac{3}{4}$?



$2 + \frac{4}{4} + \frac{2}{4} + 3\frac{2}{4}$

$2\frac{4}{6} + 3\frac{5}{6}$ $5\frac{9}{6}$ $5 + \frac{6}{6} + \frac{3}{6} = 6\frac{3}{6}$

Note: $6\frac{3}{6}$ can be simplified to $6\frac{1}{2}$,

but $6\frac{3}{6}$ is an acceptable answer.

BENCH MARK 4-6 / 7-9

PERFORMANCE OBJECTIVES

PROPER FRACTIONS—UNLIKE DENOMINATORS—NO REGROUPING

8. Given a set of labeled fractional cut-out parts, the learner will demonstrate the result of adding two (proper) fractional numbers, with unlike denominators of 2, 4, and 8—or 2, 3, and 6—with sums less than or equal to 1, by fitting the appropriate parts together, and then writing the sum.

PROPER FRACTIONS—UNLIKE DENOMINATORS—WITH REGROUPING

9. Given a set of labeled fractional cut-out parts, the learner will demonstrate the result of adding two (proper) fractional numbers with unlike denominators of 2, 4, and 8—or 2, 3, and 6—with sums more than 1, by fitting the appropriate parts together, and then writing the sum.

10. Given two (proper) fractional numbers with unlike denominators of 2, 4, and 8—or 2, 3, and 6—the learner will write the sum.

MIXED NUMBERS—UNLIKE DENOMINATORS

11. Given two mixed numbers with unlike denominators of 2, 4, and 8—or 2, 3, and 6—the learner will write the sum, with or without the use of fractional parts.

EXAMPLES AND COMMENTS

$\frac{1}{8} + \frac{2}{4} = ?$

$\frac{1}{8} + \frac{2}{4} = \frac{1}{8} + \frac{4}{8} = \frac{5}{8}$

Note: Always provide a unit whole to compare the fractional cutouts with.

$\frac{5}{6} + \frac{2}{3} = ?$

Note: Always provide a *unit whole* to compare the fractional parts with. The answer can be left in non-simplified form: $\frac{9}{6}$ can be written as

$\frac{6}{6} + \frac{3}{6}$ or $1\frac{3}{6}$, but is acceptable as $\frac{9}{6}$

$\frac{1}{2} + \frac{3}{8} \quad \frac{4}{8} + \frac{3}{8} = \frac{7}{8}$

Note: The skill of finding *Equivalent Fractions* can be found in Program Objectives #21 and 22 in Concept Area: FRACTIONAL NUMBERS: Meaning.

$3\frac{2}{3} \longrightarrow 3\frac{4}{6}$

$+ \; 5\frac{4}{6} \longrightarrow 5\frac{4}{6}$

$8\frac{8}{6} \qquad 8 + \frac{6}{6} + \frac{2}{6} = 9\frac{2}{6}$

$9\frac{2}{6}$ may be writen as $9\frac{1}{3}$

PERFORMANCE OBJECTIVES

12. Given two mixed numbers with like or unlike denominators of 2, 3, and 6—or 2, 4, and 8—the learner will find the sum, with or without the use of aids.

EXAMPLES AND COMMENTS

$$\begin{array}{rcl} 3\frac{4}{5} & \longrightarrow & 3\frac{12}{15} \\ +\ 2\frac{2}{3} & \longrightarrow & 2\frac{10}{15} \\ \hline & & 5\frac{22}{15} \end{array}$$

$$5 + \frac{15}{15} + \frac{7}{15} \qquad 6\frac{7}{15}$$

PERFORMANCE OBJECTIVES

PROPER FRACTIONS—LIKE DENOMINATORS, NO REGROUPING

1. Given a set of labeled fractional cut-out parts, the learner will demonstrate the result of subtracting two fractional numbers with like denominators of 2, 3, 4, 6, or 8 by arranging the appropriate parts, and then finding and writing the difference.

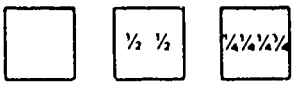
2. Given two common fractions with like denominators, the learner will subtract, with or without the use of fractional parts.

MIXED NUMBERS—LIKE DENOMINATORS, NO REGROUPING

3. Given a set of labeled fractional cut-outs, including several *unit wholes*, the learner will:
   a) demonstrate the result of subtracting a fractional number from a mixed number with like denominators of 2, 3, 4, 6, or 8 where no regrouping is necessary, by arranging the appropriate parts, and
   b) finding and writing the answer.

EXAMPLES AND COMMENTS

The fractional parts may be commercially prepared, teacher prepared, or student prepared. All the parts should be cut from *one common unit size*. The *unit whole* may be a rectangle or a circle, but once the unit is established, the *same* size and shape unit should be used for all the exercises



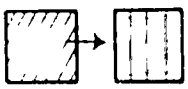
and so on

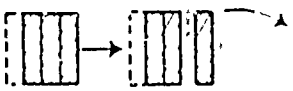
The learner should be able to use the cut outs interchangeably and this can only be done if all fractional parts originate from the *same unit whole*.

and so on

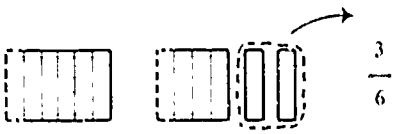
and so on

Problem: $\frac{3}{4} - \frac{1}{4} = ?$

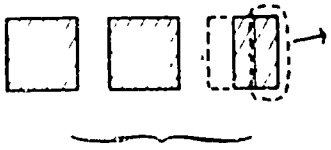




$\frac{5}{6} - \frac{2}{6}$



$2\frac{2}{3} - \frac{1}{3} = ?$



$2\frac{1}{3}$

PERFORMANCE OBJECTIVES

4. Given a mixed number and a fractional number with like denominators of 2, 3, 4, 6, or 8, where no regrouping is necessary, the learner will find the difference.

EXAMPLES AND COMMENTS

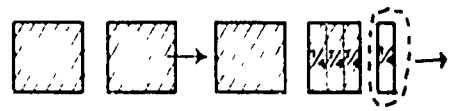
$3\frac{5}{8} - 1\frac{2}{8} = 2\frac{3}{8}$ OR

$$\begin{array}{r} 3\frac{5}{8} \\ -\,1\frac{2}{8} \\ \hline 2\frac{3}{8} \end{array}$$

WHOLE NUMBERS AND FRACTIONAL NUMBERS

5. Given several pieces of rectangular paper, and an exercise involving the subtraction of a whole number and a fractional number with a denominator of 2, 4, or 8, the learner will:
   a) divide one of the *whole units* into the appropriate number of equivalent parts, and
   b) demonstrate the result of subtracting a fractional number from a whole number by arranging the fractional parts to show the difference, and
   c) write the difference.

$2 - \frac{1}{4} = 1\frac{3}{4}$



Note: The student *starts* with the *unit whole*, then divides it into the appropriate parts by cutting it.

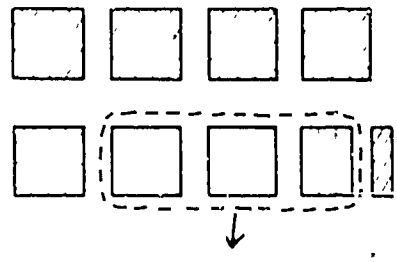
6. Given a whole number and a common fraction with a denominator of 2, 3, 4, 6, or 8, the learner will find the difference with or without the use of fractional parts.

$2 - \frac{3}{4} = 1\frac{1}{4}$ Note: Use ready made cutouts.

WHOLE NUMBERS AND MIXED NUMBERS

7. Given a whole number and a mixed number, the learner will find the difference with or without the use of fractional cut-outs.

$$\begin{array}{r} 4 \\ -\,2\frac{3}{4} \\ \hline 1\frac{1}{4} \end{array}$$

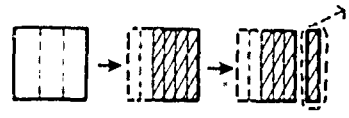


PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
MIXED NUMBERS AND MIXED NUMBERS—LIKE DENOMINATORS	
8. Given two mixed numbers with like denominators, the learner will find the difference.	$5\frac{7}{9} - 2\frac{3}{9} = 3\frac{4}{9}$

4-6

BENCH MARK

7-9

COMMON FRACTIONS—UNLIKE DENOMINATORS	
9. Given a set of fractional cut-outs and two fractions with unlike denominators of 2, 4, and 8—or 2, 3, and 6—the learner will show the result of the subtraction by arranging the appropriate fractional cut-outs, and by writing the difference.	$\frac{2}{3} - \frac{1}{6} = ?$ 
10. Given two fractions with unlike denominators of 2, 3, and 6—or 2, 4, and 8—the learner will subtract with or without the use of aids.	$\frac{1}{2} \rightarrow \frac{3}{6}$ $\quad -\frac{2}{6} \rightarrow -\frac{2}{6}$ $\quad \frac{1}{6}$ $\frac{6}{8} \rightarrow \frac{6}{8}$ $\quad -\frac{1}{2} \rightarrow -\frac{4}{8}$ $\quad \frac{2}{8}$
MIXED NUMBERS—UNLIKE DENOMINATORS—NO REGROUPING	
11. Given two mixed numbers with unlike denominators of 2, 3, and 6—or 2, 4, and 8—where no regrouping is necessary, the learner will find the difference.	$5\frac{5}{6} \rightarrow 5\frac{5}{6}$ $\quad -2\frac{2}{3} \rightarrow -2\frac{4}{6}$ $\quad 3\frac{1}{6}$
MIXED NUMBERS—UNLIKE DENOMINATORS	
12. Given two mixed numbers with unlike denominators of 2, 3, and 6—or 2, 4, and 8—the learner will find the difference.	$6\frac{1}{8} \rightarrow 6\frac{1}{8} \rightarrow 5\frac{8+1}{8} \rightarrow 5\frac{9}{8}$ $-2\frac{1}{2} \rightarrow -2\frac{4}{8} \rightarrow -2\frac{4}{8} \rightarrow 2\frac{4}{8}$ $3\frac{5}{8}$

PERFORMANCE OBJECTIVES

REPRESENTATION

1. Given a set of objects (multiples of 2 or 3 or 4, as appropriate less than 13) and the fraction ½ or ⅓ or ¼, the learner will:
   a) show that fraction of the set by moving the appropriate number of objects,
   b) write an appropriate number sentence using the word "of".

EXAMPLES AND COMMENTS

Given.

(a) $\frac{1}{2}$ of 6 3

(b) $\frac{1}{2}$ of 10 5

(c) $\frac{2}{3}$ of 6 2

(d) $\frac{1}{4}$ of 12 3

BENCH MARK K-3 / 4-6

SIMPLE COMPUTATION

2. Given a number sentence in the form: "Unit fraction 'of' whole number" (⅓ *of* 12) using denominators less than 9 and the whole number a multiple of the denominator, less than 40, the learner will:
   a) find the answer with or without the use of objects, and
   b) rewrite the number sentence in the form "fraction × whole number = product."

(a) $\frac{1}{2}$ of 8 4 $\frac{1}{2} \times 8$ or 4

Rewrite: $\frac{1}{2} \times 8 = 4$

(b) $\frac{1}{4}$ of 12 = 3

$\frac{1}{4} \times 12$ or 3

Rewrite: $\frac{1}{4} \times 12 = 3$

PERFORMANCE OBJECTIVES

USE OF MODELS

3. Given two unit fractions with denominators less than 5, the learner will shade a region representing the product and write the product.

COMPUTATION

4. Given two unit fractions with denominators less than 7, the learner will compute the product with or without the use of a model.

EXAMPLES AND COMMENTS

(a) Area Model $\frac{1}{3}$

$\frac{1}{2}$

$\frac{1}{2}$ of $\frac{1}{3}$ $\quad \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

(b) "Of" Model

(1) one unit

(2) $\frac{1}{3}$ $\quad \frac{1}{3}$

(3) $\frac{1}{2}$ of $\frac{1}{3}$ $\quad \frac{1}{2}$ $\quad \frac{1}{3}$

(4) $\frac{1}{2}$ of $\frac{1}{3}$ $\quad \frac{1}{6}$ $\quad \frac{1}{2}$ $\quad \frac{1}{3}$

(a) $\frac{1}{2} \cdot \frac{1}{3} \quad \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$

(b) $\frac{1}{5} \cdot \frac{1}{2} \quad \frac{1 \times 1}{5 \times 2} = \frac{1}{10}$

BENCH MARK $\frac{4\text{-}6}{7\text{-}9}$

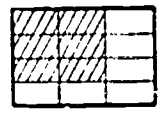
PERFORMANCE OBJECTIVES

EXAMPLES AND COMMENTS

REPRESENTATION

5. Given two proper fractions with denominators less than 5, the learner will construct and shade a region to represent the product.

(a) Area Model $\frac{3}{4}$ $\frac{2}{3}$



(b) "Of" Model $\frac{2}{4} \times \frac{2}{3}$

(1) establish a unit

(2) take 2/3 of the unit

(3) take 2/4 of the 2/3

(4) analyze the results

COMPUTATION

6. Given two (proper) fractions with denominators less than 7, the learner will compute the product.

(a) $\frac{1}{2} \times \frac{1}{3} \quad \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$

(b) $\frac{2}{3} \times \frac{1}{2} \quad \frac{2 \times 1}{3 \times 2} = \frac{2}{6}$

(c) $\frac{3}{4} \times \frac{2}{3} \quad \frac{3 \times 2}{4 \times 3} = \frac{6}{12}$

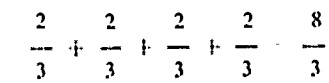
(d) $\frac{2}{5} \times \frac{2}{3} \quad \frac{2 \times 2}{5 \times 3} = \frac{4}{15}$

WHOLE NUMBERS AND FRACTIONS

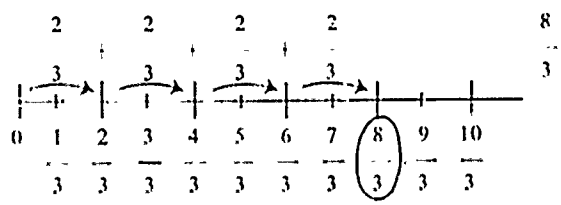
7. Given a whole number less than 5 and a proper fraction with denominator less than 7, the learner will illustrate the product.

(a) $4 \times \frac{2}{3}$

(b) $4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3}$



(c) $4 \times \frac{2}{3}$



PERFORMANCE OBJECTIVES

8. Given a whole number less than 5 and a proper fraction with denominator less than 7, the learner will compute the product.

9. Given a whole number less than 5 and a mixed number less than 5 and a denominator less than 7, the learner will compute the product.

VERBAL PROBLEMS

10 Given verbal problems involving fractional numbers, the learner will:
   a) identify the necessary operation to be used.
   b) write an arithmetic sentence for the situation.
   c) determine the desired result.

DIVISION OF FRACTIONS
THERE ARE *NO* MINIMAL CONCEPTS
OR SKILLS REQUIRED IN THIS AREA.

EXAMPLES AND COMMENTS

(a) $\frac{1}{4} \times 3 \quad \frac{1}{4} \times \frac{3}{1} = \frac{1 \times 3}{4 \cdot 1} \quad \frac{3}{4}$

(b) $7 \times \frac{3}{4} \quad \frac{7}{1} \times \frac{3}{4} = \frac{7 \times 3}{1 \times 4} \quad \frac{21}{4}$

(a) $4 \times 3\frac{1}{5} \quad 4 \times (3 + \frac{1}{5})$

$4 \times 3 + 4 \times \frac{1}{5}$

$12 + \frac{4}{5}$

$12\frac{4}{5}$

(b) $4 \times 3\frac{1}{5} \quad 4 \times \frac{16}{5} = \frac{4 \times 17}{5} \quad \frac{64}{5}$

Problems should start with multiplication, then be mixed with addition, and subtraction examples as well.

ARITHMETIC

IV DECIMAL FRACTIONS

The *DECIMAL FRACTIONS* Strand consists of four (4) Skill Sequences:

A. Meaning

B. Addition and Subtraction

C. Multiplication

D. Division

The Meaning Sequence consists of those skills that are considered basic and fundamental to the computational skills in decimals, common fractions and percents.

The objectives in the Addition, Subtraction, Multiplication and Division Sequences are limited to those skills required to solve everyday practical situations. Limits are set for the number of digits to the right of the decimal point. Limits ar also set in long division and in mulitiplication problems.

PERFORMANCE OBJECTIVES

PLACE VALUE

1. Given a place value chart and a numeral of no more than three decimal places, the learner will describe the value of each digit in the numeral.

TENTHS

2. Given a model of a fraction illustrating tenths, the learner will identify, name and write the decimal fraction illustrated.

HUNDREDTHS

3. Given a model of a fraction illustrating hundredths, the learner will identify, name and write the decimal fraction as illustrated.

TENTHS

4. Given a decimal fraction in tenths, the learner will construct a model that illustrates the decimal.

HUNDREDTHS

5. Given a decimal fraction in hundredths, the learner will construct a model that illustrates the decimal.

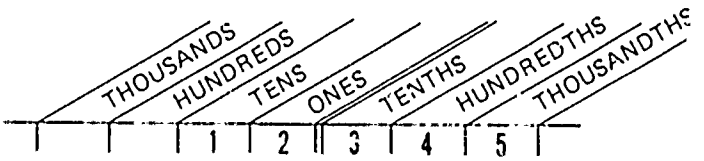
EQUIVALENT FRACTIONS

6. Given a decimal fraction, the learner will rename it as an equivalent decimal fraction.

PLACE VALUE

7. Given a decimal fraction of no more than three places, the learner will name the place value of each digit, without aides.

EXAMPLES AND COMMENTS

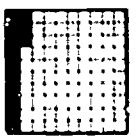


Student gives the value of each digit in the numeral:

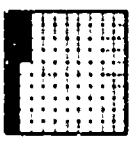
".3 is 3 $\times \frac{1}{10}$ or $\frac{3}{10}$ or three tenths."

"Each decimal place is *ten times* the place to the right, or one tenth of the place to the left."

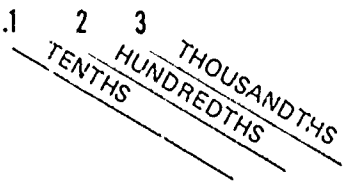
.3 3 tenths

.13  13 hundredths

.4

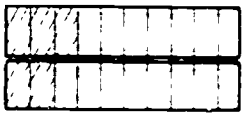
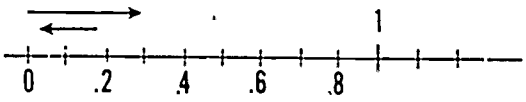
.14 

.5 = .50



BENCH MARK 4-6 / 7-8

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
ROUNDING	
8. Given a numeral with no more than three decimal places, the learner will round to the nearest whole number, tenths or hundredths as requested.	.656 ⟶ .66 .57 ⟶ .6
RENAMING	
9. Given a decimal fraction, the learner will rename as a common fraction.	$.5 = \frac{5}{10}$ $.25 = \frac{25}{100}$
10. Given a common fraction whose decimal equivalent terminates in thre. (3) places or less, the learner will rename the common fraction as a decimal fraction.	$\frac{1}{2} = \frac{5}{10}$ .5 $\frac{1}{4}$ $\frac{25}{100}$ .25
ORDERING	
11. Given a set of decimal fractions of no more than three (3) places, the learner will arrange the fractions in order from greatest to least or least to greatest as instructed.	Order .5, .25, .6, .05 *as* .6, .5, .25, .05 or .05, .25, .5, .6

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
ADDITION—TENTHS	
1. Given an addition decimal problem involving only tenths, the learner will find the sum using models.	.2 + 3 = ☐
SUBTRACTION—TENTHS	
2. Given a subtraction decimal problem involving only tenths, the learner will find the difference using models.	3 − 2 = ☐  
3. Given a verbal problem involving addition and subtraction of decimal numbers involving only tenths, the learner will find the answer.	Bob jogged three tenths of a mile on Sunday, and two tenths of a mile on Monday, how many miles did he jog all together?
ADDITION—HUNDREDTHS	
4. Given an addition problem involving tenths and hundredths, the learner will find the sum with or without the use of aids or models.	.4 + .13 = ☐ .156 + .48 = ☐
SUBTRACTION—HUNDREDTHS	
5. Given a subtraction problem involving tenths and hundredths, the learner will find the difference with or without the use of aids or models.	.48 − .06 = ☐ .6 − .04 = ☐
6 Given a verbal problem involving addition and subtraction of decimal numbers involving tenths and hundredths, the learner will find the answer.	Bob walks 0.7 miles to school, Jane walks 0.86 miles to school How much farther does Jane walk than Bob?
ADDITION	
7. Given a decimal addition problem in horizontal or vertical form involving whole numbers, tenths and hundredths, the learner will find the sum with or without aids or models.	4.06 + .4 = ☐ $24.05 + $1.25 ☐
SUBTRACTION	
8. Given a decimal subtraction problem in horizontal or vertical form with whole numbers, tenths, and hundredths, the learner will find the difference with or without aids or models.	1.25 − .3 ☐ $12.50 − $1.25 ☐

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
ADDITION AND SUBTRACTION	
9. Given an addition or subtraction decimal problem in horizontal or vertical form with no more than five (5) digits and no more than three (3) decimal places, the learner will find the sum or difference.	.6 + .25 + 1.5 .123 - .01
10. Given a verbal problem involving addition or subtraction of decimal numbers involving numbers with no more than five (5) digits and no more than three (3) decimal places, the learner will find the answer.	Jerry's paycheck was $398.04 and his Christmas bonus was seventy ($70) dollars. How much money did he receive?
NO 7-9 PROGRAM OBJECTIVES	

7-9 BENCH MARK

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
MEANING	
1. Given an exercise involving the multiplication of a whole number and a decimal less than 1, the learner will rewrite the exercise as a whole number times a common fraction.	$0.4 \times 8 \quad \frac{4}{10} \times 8$ $12 \times 0.75 \quad 12 \times \frac{75}{100}$ $237 \times 0.1 \quad 237 \times \frac{1}{10}$
ESTIMATION	
2. Given a multiplication exercise involving a whole number, and a decimal fraction that is close to ¼, ½, or ¾, the learner will approximate the decimal fraction as ¼, ½, or ¾, will make a convenient approximation of the whole number if necessary, and will multiply, arriving at an approximation of the product.	a) $38 \times .26$; 40 ; $\frac{1}{4}$; $\frac{1}{4} \times 40 \quad 10$ b) $74 \times .56$; 80 ; $\frac{1}{2}$; $\frac{1}{2} \times 80 \quad 40$ c) $110 \times .81$; 100 ; $\frac{3}{4}$; $\frac{3}{4} \times 100 = 75$
ALGORITHM	
3. Given a whole number and a one-place decimal fraction less than 1, the learner will: a) convert the decimal fraction to its common fraction form (with a denominator of 10), multiply and write the product in its decimal form. b) Find the product.	Example: 0.8×23 a) $\frac{8}{10} \times 23 \quad \frac{184}{10} \quad 18.4$ b) $23 \times .8 = 18.4$
4. Given a whole number and a two-place decimal fraction less than 1, the learner will: a) Convert the decimal fraction to its common fraction (with a denominator of 100), multiply and write the product in its decimal form. b) Find the product.	Example: 0.03×2 a) $\frac{3}{100} \times 2 \quad \frac{6}{100} \quad 0.06$ b) $.03 \times 2 = 0.06$
5. Given two one-place decimals fractions less than 1, the learner will: a) Convert both decimal fractions to their common fraction form (with denominators of 10), multiply the fractions and write the product in its decimal form. b) Find the product.	Example: 0.3×0.4 a) $\frac{3}{10} \times \frac{4}{10} \quad \frac{12}{100} \quad 0.12$ b) $0.4 \times 0.3 = 0.12$

PERFORMANCE OBJECTIVES

6. Given a one-place and a two-place decimal fraction, both less than 1, the learner will:
   a) Convert both decimal fractions to their common fraction form (with denominators of 10 and 100 respectively), multiply the fractions and write the product in its decimal form.
   b) Find the product.

7. Given two two-place decimal fractions both less than 1, the learner will:
   a) Convert both decimal fractions to their common fraction form (with denominators of 100), multiply the fractions and write the product in its decimal form.
   b) Find the product.

8. Given a decimal fraction and a whole number of 10 or a power of 10 (100, 1000, and so on), the learner will find the product by changing the value of the decimal number by placing the decimal point in the appropriate place value position.

ESTIMATION TWO

9. Given a whole number and a two-place decimal less than 1, the learner will round off the decimal to its nearest tenth or whole, and estimate the product by multiplying the rounded off decimal and the whole number.

EXAMPLES AND COMMENTS

Example 0.5 · 0.15

a) $\frac{5}{10} \cdot \frac{15}{100} = \frac{75}{1000}$ 0.075

b) 0.15
× 0.5
0.075

Example 0.07 × 0.08

a) $\frac{7}{100} \cdot \frac{8}{100} = \frac{56}{10{,}000}$ 0.0056

b) 0.08
× 0.07
0.0056

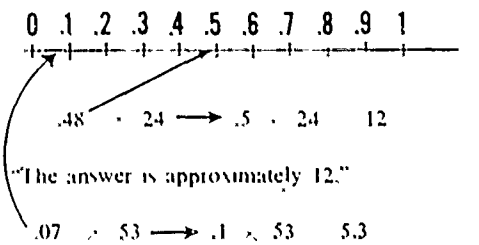
Example 10 × 0.43
100 × 0.43
1000 · 0.43

a) $10 \cdot \frac{43}{100} = \frac{430}{100}$ 4.3

$100 \cdot \frac{43}{100} = \frac{4300}{100}$ 43

$1000 \times \frac{43}{100} = \frac{43000}{100}$ 430

b) 10 × .43 4.3
100 · .43 43
1000 · .43 430



PERFORMANCE OBJECTIVES

EXAMPLES AND COMMENTS

ALGORITHM AND ESTIMATION

10. Given two one-place decimal fractions greater than 1, but less than 100, the learner will:
    a) estimate the product
    b) compute the product.

Example: 3.2×52.9

a) $3.2 \longrightarrow$ Rounded off: 3

$52.9 \longrightarrow$ {Rounded off: 53 / Rounded off: 50

"Since $3 \times 50 = 150$, the answer to 3.2×52.9 will be close to 150, and will have 3 digits in the whole number part of the answer."

b)
$$\begin{array}{r} 52.9 \\ \times\ 3.2 \\ \hline 1058 \\ 15870 \\ \hline 169.28 \end{array}$$

11. Given two two-place decimal fractions greater than 1, but less than 100, the learner will:
    a) estimate the product
    b) compute the product.

Example: 5.07×37.94

a) $5.07 \longrightarrow$ Rounded off: 5

$37.94 \longrightarrow$ {Rounded off: 38 / Rounded off: 40

"Since $5 \times 40 = 200$, the answer to 5.07×37.94 will be close to 200, and will have 3 digits in the whole number part of the answer."

b)
$$\begin{array}{r} 37.94 \\ \times\ 5.07 \\ \hline 26558 \\ 1897000 \\ \hline 192.3558 \end{array}$$

7-9 BENCH MARK

PERFORMANCE OBJECTIVES

PLACING DECIMAL POINT IN QUOTIENT

1. Given a division exercise of the form (Decimal Number ÷ Whole Number = Quotient), where the digits of the quotient are given, the learner will show the correct placement of the decimal point by successive multiplication trials, as follows:
   a) Arbitrarily place the decimal point in the quotient.
   b) Multiply that trial quotient by the divisor to see if that placement is reasonable.
   c) If that placement is not correct, place the decimal point in another position in the quotient, and multiply that trial quotient by the divisor to see if it is the correct placement.
   d) Repeat this process until the correct decimal point placement is found.

WHOLE NUMBER DIVISORS

2. Given a division exercise of the form: (Decimal number ÷ whole number) the learner will find the digits of the quotient by ignoring the decimal point in the dividend, then place the decimal point in the quotient such that: Quotient' × Divisor = Dividend

3. Given a division exercise using whole numbers (up to a four digit dividend and up to a two digit divisor), the learner will generate other exercises having the same answer, by multiplying or dividing the dividend and divisor by the same number.

CHANGING DIVISOR TO A WHOLE NUMBER

4. Given a division exercise of the form decimal number ÷ decimal number (up to four digits divided by up to two digits), the learner will change the divisor to a whole number by multiplying both divisor and dividend by the same number (10, 100, 1,000 and so on)

EXAMPLES AND COMMENTS

(a) $8 \overline{)3.44}$ Digits in quotient are 43
Student may try any of the following:
(43 × 8), (.43 × 8), (4.3 × 8), (0.043 × 8). He finds that 4.3 × 8 will give the correct dividend of 34.4

(b) $8 \overline{).344}$ Digits in Quotient are 43
Student may try any of the following:
(43 × 8), (.43 × 8), (4.3 × 8), (0.043 × 8). He finds that .43 × 8 will give the correct dividend of 3.44

(c) $8 \overline{)3.44}$ Digits in Quotient are 43
Student may try any of the following
(43 × 8), (.43 × 8), (4.3 × 8), (0.043 × 8). He finds that 043 × 8 will give the correct dividend of .344

Given $6 \overline{)1.26}$
Student divides: $6 \overline{)126}$
Digits of quotient: 21
Decimal point: 21
Because: 21 × 6 = 1.26

Given $3 \overline{)12}$ (4)

Student could write
Mult by 2: $6 \overline{)24}$ (4)
Mult. by 10. $30 \overline{)120}$ (4)
Mult by 100: $300 \overline{)1200}$ (4)
Divide by 3: $1 \overline{)4}$ (4)

Given $.06 \overline{)3.54}$
Student will write:
Mult by 100 $6 \overline{)354}$

Given: $5 \overline{).75}$
Student will write:
Mult by 10: $5 \overline{)7.50}$

PERFORMANCE OBJECTIVES

DIVIDING TWO DECIMALS

5. Given any decimal number division exercise (up to four digits divided by up to two digits), the learner will:
   a) Change the divisor to a whole number, where necessary,
   b) Determine the digits of the quotient by ignoring the decimal point in the divisor while dividing,
   c) Place the decimal point in the quotient such that: Quotient × Divisor = Dividend

6. Given any two decimal numbers (up to four digits divided by up to two digits), the learner will find the quotient.

EXAMPLES AND COMMENTS

Given: $.06\overline{)\,.359}$

(a) $6\overline{)\,35.9}$

(b) $6\overline{)\,35.9}$ (5.4)

(c) Since $5.4 \times 6 = 35.9$, the quotient is 5.4

Given: $27\overline{)\,.0918}$

Solution: $27\overline{)\,.918}$ (.034)

81

108

108

0

ARITHMETIC

V INTEGERS

Values above and below zero pervade daily life: temperature, game scores and economic records of profit and loss of credit and debit are frequently written in the form of integral values.

While other experiences with integers are interesting and necessary in specialized fields, only numberline relationships and addition of integers are included as minimal objectives.

PERFORMANCE OBJECTIVES

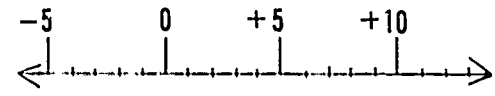
NUMBERLINE LOCATION

1. Given a set of integers and a numberline, the learner can locate the integers by pointing at the correct location on a numberline. (Using either a horizontal or vertical number line.)

ADDITION

2. Given two integers, the learner can correctly name the sum of any two integers.

EXAMPLES AND COMMENTS



Indicate the position of the following set of integers on the number line.

+ 3, − 5, − 2, + 1

− 6	+ − 5	− 11
− 5	+ + 4	− 1
+ 5	+ − 3	+ 2

Examples such as yardage gained or lost during a football game, temperature (above and below zero), and scoring in any card game where it is possible to "go in the hole" may be used as applications of addition of positive and negative integers.

ARITHMETIC

VI RATIO, PROPORTIONS AND PERCENT

People compare the size and number of objects and many other qantities throughout their lives. These comparisons are made more precise by using ratios, proportions and percents. Learners achieving the program objectives in this Sequence will be able to:

—write appropriate ratios,

—determine if two ratios are equivalent,

—prepare a table of equivalent pairs for a given ratio,

—determine the fourth member of a proportion (whole numbers) when the other three are given, and

—find a given percent of a whole number

Emphasis on measurement and manipulation of objects will be necessary to assist learners in achieving these objectives. The early work in this Sequence is closely related to that described in the Pre-Number Sequence.

PERFORMANCE OBJECTIVES

RATIO AND PROPORTION

1. Given a set of like objects (less than or equal to 10), the learner will pick out from a supply of like objects a set which has the same number of members.

2. Given the task of securing one object (pencils, crayons, or sheets of paper) for each member at his table, the learner will take the correct number from a supply which is remotely located in the room.

3. Given a set of unlike objects (less than or equal to 10), the learner will pick out from a supply of unlike objects a set which has the same number of members.

4. Given a pattern of two objects which differ only in shape or color, the learner will reproduce the pattern from a supply of objects.

5. Given a pattern of three objects which differ only in shape or color, the learner will reproduce the pattern from a supply of objects.

6. Given a picture of like objects (less than or equal to 10), the learner will circle from a second diagram or will draw a set which has the same number of members.

7. Given a picture of unlike objects (less than or equal to 10), the learner will circle from a second diagram of unlike objects or will draw a set which has the same number of members.

8. Given a picture of a pattern of two objects, the learner will reproduce the pattern by drawing.

9. Given a picture of three objects which differ in length or color, the learner can reproduce the pattern by drawing.

MANY TO ONE ACTIVITY

10. Given a supply of objects and the students at his table, the learner will take the correct number of objects from the supply to provide a specified number for each student (limit of 9 objects and 5 objects per student).

EXAMPLES AND COMMENTS

Given. Student Response: Beads

Given: Pencil Scissors Block

Student Response Bead Paste Jar Crayon

Given

Response:

Given:

Response:

Given. Balloons

Response:

Given.

Response:

Given:

Response:

Given: Y G R

Response: Y G R

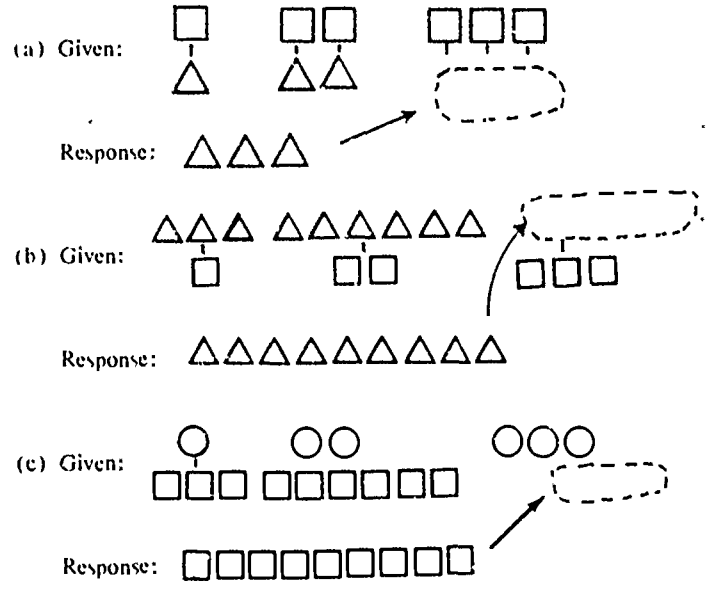
Four sheets of paper per table member
Three crayons per table member.
Two crackers per table member.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
ONE TO MANY ACTIVITY	
11. Given a set of objects and the students at his table, the learner will take the correct number of objects and deliver one object to each sub-group of specified size.	One sheet of construction paper for every two children. One basketball for every five children. One jump rope for every three children.
MANY TO MANY ACTIVITY	
12. Given a set of objects and the students at his table, the learner will take the correct number and deliver a specified number of objects to each subgroup of specified size.	Two scissors for three children. Five sheets of construction paper for two children.

BENCH MARK K-3 / 4-6

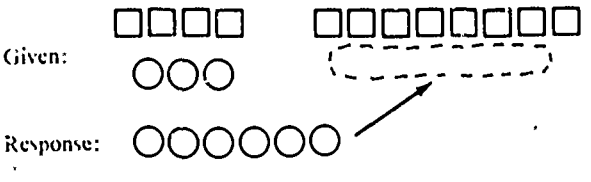
MANY-TO-ONE, ONE-TO-MANY, ONE-TO-ONE

13. Given sets of objects paired in a (a) one-to-one, (b) many-to-one, or (c) one-to-many ratio and part of another pair, the learner will complete that pair to maintain the pattern.



OBJECTS—MANY-TO-MANY

14. Given sets of two kinds of objects paired in a many-to-many ratio and part of another pair, the learner will complete that pair from a supply of objects to keep the ratio equivalent.



PERFORMANCE OBJECTIVES

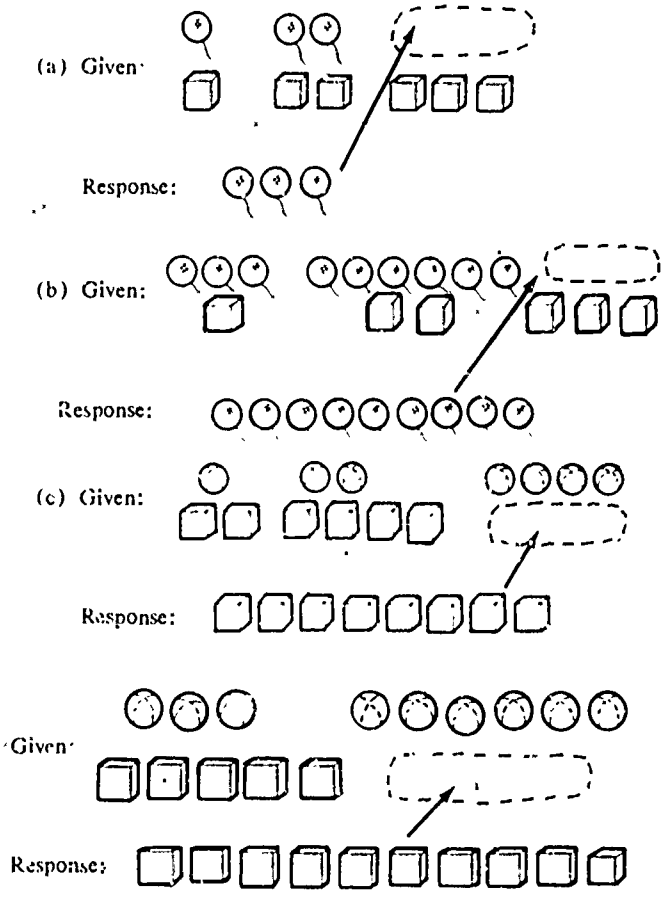
PICTURES—ONE-TO-ONE, MANY-TO-ONE, ONE-TO-MANY

15. Given a picture of sets paired in (a) a one-to-one, (b) a many-to-one, or (c) a one-to-many ratio and part of another pair, the learner will complete that pair by drawing to keep the ratio equivalent.

PICTURES—MANY-TO-MANY

16. Given a picture of paired sets of two kinds of objects showing a many-to-many ratio and one set of another pair, the learner will complete that pair by drawing to keep the ratio equivalent.

EXAMPLES AND COMMENTS



BENCH MARK 4-6 / 7-9

VERBAL—MANY-TO-ONE ACTIVITY

17. Given a written or verbal statement requiring selection of a certain number of items for each student at his table, the learner can determine how many objects he will need.

If there are four students at your table and each needs three pieces of paper, how many sheets should you get?

VERBAL—MANY-TO-ONE, ONE-TO-MANY

18. Given a written statement describing paired sets in (a) a many-to-one or (b) a one-to-many ratio and the number of members in one set of another pair, the learner will state the number of members needed for the missing set to keep the same proportion (equivalent ratio).

A person bought 9 cans of pop for $1. Another person bought 18 cans for $2. How many cans of pop can you buy for $3.

PERFORMANCE OBJECTIVES

EXAMPLES AND COMMENTS

VERBAL—MANY-TO-MANY

19. Given sets of objects paired in a (a) one-to-one, (b) many-to-one, or (c) one-to-many ratio and part of another pair, the learner will complete that pair to maintain the pattern.

A boy can cut the lawn of three houses in two hours. How many lawns can be cut in 6 hours?

WRITING RATIO

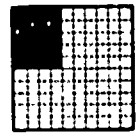
20. Given a picture of two sets or a subdivided region, the learner will write a ratio describing the indicated comparison.

Given:

Possible Responses:
4 suckers to 5 pennies
4 to 5
4: 5
$\frac{4 \text{ suckers}}{5 \text{ pennies}}$

PERCENT

21. Given a square subdivided into an array of 10 × 10 unit squares some of which are shaded, the learner will state the indicated ratio and per cent representing the shaded area.

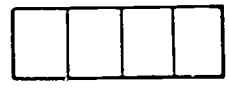
Given: 

Response:
20 to 100
or
20 or 20%
$\frac{20}{100}$

RATIO AND PROPORTION

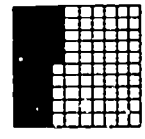
22. Given an unshaded square subdivided into an array of 10 × 10 unit squares, the learner will shade an amount to illustrate a given equivalent ratio or percent.

Shade the diagram to illustrate the ratio ¼

Given: 

Response:

Shade the diagram to illustrate the ratio $\frac{35}{100}$.



23. Given a ratio in the form a/b, the learner will draw a diagram illustrating the ratio and indicate by an arrow the order of comparison.

Given: $\frac{5}{2}$

or 5 balls to 2 boys

Student:

24. Given a ratio, the learner will display a set of equivalent ratios in tabular form.

$\frac{1}{2}$

1				
2				

25. Given a table of more than two equivalent ratios with (a) one value missing or (b) more than one value missing, the learner will complete the table.

(a)

gal. of paint	3	6	
cars painted	4	8	12

(b)

hours of work	2	4		12
dollars of pay	7	14	21	

Tables can be expanded.

PERFORMANCE OBJECTIVES

26. Given a problem that can be solved by ratio and proportion techniques, the learner will solve the problem by making a table of ratios.

EXAMPLES AND COMMENTS

A boat was partially painted with grey paint that was a mixture of 4 quarts of white paint and 12 quarts of black paint. How much black paint must be mixed with one quart of white paint to get a mixture that is the same shade of grey as the original batch?

white paint	1	2	4
black paint	3	6	12

EQUIVALENT RATIOS

27. Given two ratios, the learner will determine whether or not they are equivalent.

$\frac{4}{6} \stackrel{?}{=} \frac{6}{9}$

Techniques

(a) by making a table

(b) $\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$ or $\frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

(c) $\frac{2 \times 2}{3 \times 2} = \frac{2}{3}$ or $\frac{2 \times 3}{3 \times 3} = \frac{2}{3}$

(d) common denominator

(e) cross multiplication

NON-EQUIVALENT RATIOS

28. Given a relation in the form of a horizontal table, the learner will cross out the pair of numbers which need to be changed in order for the relation to be proportional.

Cross out the pair of numbers which do not belong.

2	4	6	~~8~~
3	6	9	~~16~~

USING A TABLE FOR SIMPLEST FORM

29. Given a ratio, the learner will write an equivalent ratio in lowest whole number terms.

$\frac{9 \text{ boys}}{12 \text{ girls}}$

		9
		12

Response: 3 or 3 to 4, $\frac{3}{4}$, 3:4

USING A TABLE FOR PROPORTION

30. Given three whole number parts of a proportional relation, the learner will use a table to determine the fourth part (the fourth part will be a whole number).

Hours	3	15
Miles	8	

Gallons of gasoline	20	30
Miles driven	340	

VERBAL—RATIO

31. Given a written statement involving the comparison of two whole numbers, the learner will write an appropriate ratio.

16 boys and 12 girls. What is the ratio of boys to girls?

Response: 16 to 12 or 4 to 3.

WRITTEN STATEMENT

32. Given a written statement involving proportionality, the learner will write equivalent ratios by supplying the missing whole number.

A race car is 18 feet long and 4 feet wide. A model of it is 9 inches long. How wide is it?

Other examples are cost per item, miles per hour, similar triangles and rate of work. Use tables.

PERFORMANCE OBJECTIVES

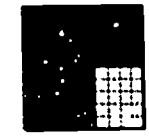
EXAMPLES AND COMMENTS

PERCENT

FRACTION—HUNDRED SQUARE—PER CENT

33. Given a fraction and a "hundred square", the learner will shade the appropriate area to represent the equivalent fraction and name the whole number percent involved.

Given: $\frac{4}{5}$

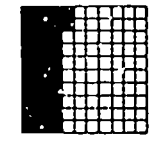


Response: 80%

PER CENT—HUNDRED SQUARE—DECIMAL

34. Given a whole number percent and "a hundred square", the learner will shade the appropriate area and name the decimal fraction involved.

Given 32%



Response: .32 or $\frac{32}{100}$

TABLE, FRACTION, DECIMAL—PER CENT

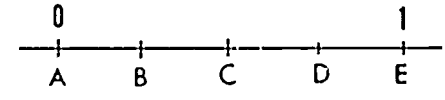
35. Given a partially completed table involving fractions with denominators of multiples of two and five, decimals and percents, the learner will complete the table.

Denominators of 2, 4, 8, 10, 20, 25, 50, 100.

Fraction	Decimal Fraction	Decimal	Percent
½	50/100		
		.10	
	30/100		
			25%

PER CENT—NUMBER LINE REGION

36. Given a percent, the learner can give the region on the number line in which it will fall if the number line is marked.



75% is ——. Response: Between A and D.

25% is —— Response: Between A and B.

PER CENT OF NUMBER

37. Given a whole number percent and a number, the learner will determine that percentage of the given number.

40% of 60

Possible technique:

1) 40% × 60 = .40 × 60

2)

40	
100	60

WRITTEN STATEMENT OF PERCENT OF A NUMBER

38. Given a simple written problem involving finding a percentage of a number, the learner will solve it.

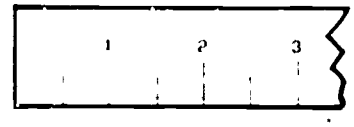
You may have 30% of the sales. The sales amounted to $80.

MEASUREMENT

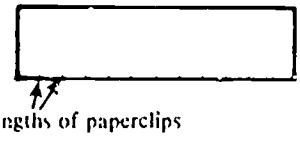
The measurement objectives are written to promote active participation by the learner, sometimes referred to as a "hands-on" approach to instruction. Each child should measure with ruler, scales, thermometer, cubic inch blocks, containers, cut-outs, and so on. Most of the activities in this strand are most effective when individually performed by each learner. This can be facilitated by developing a file of activities that children perform individually throughout the school year.

An important goal of the measurement objectives is the development of a "feel" and working or operational knowledge of the common measurement units. This "feel" should be developed by repeated manipulation of measuring instruments and unit models such as blocks, square units containers, and so on. Common units of the metric system are included because it is anticipated that the use of the metric system in this country will be greatly increased within the lifetime of today's school children.

MEASUREMENT K-3

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
COMPARISON IN MEASUREMENT	
1. Given two objects, the learner can identify the longer (taller) of the two.	Also covered in Pre-Number strand.
2. Given up to four sets of linear non-standard units and a given distance, the learner will use a train of each unit to measure the distance.	"My desk is 10 erasers long." "My desk is 6 pencils long." "My desk is 41 paper clips long."
3. Given objects to measure, the learner will use "caveman" units—parts of the body—to measure these objects.	Stress variability of measures from person to person. (This includes "laying off" a part of the body repeatedly.)
4. Given non-standard units of measure, the learner will be able to use these units to measure objects in the classroom.	Units: paper strips, string, sticks, 3"x5" card, tile, graph paper, a Cuisenaire rod. Objects: table top, desk surface.
5. Given rulers specially scaled in inches and half-inches, the learner will measure objects to the nearest inch.	(Too many marks on the ruler confuse young children.) Such rulers are commercially available or scale cardboard strips to the *half* inch. 
UNITS AND CONVERSION	
6. The learner will state the number of inches in a foot and the number of feet in a yard upon request.	This should be done from memory.

BENCH MARK K-3 / 4-6

COMPARISON IN MEASUREMENT	
7. Given lengths, the learner will compare them using arbitrary units of his own choice.	"This book is 5 thumbs long, and that book is 7 thumbs long."
8. Given a length of construction paper and an arbitrary unit, the student will construct a measuring instrument to measure length.	3" x 5" card as a unit. 
9. Given a yardstick, the learner will measure objects to the nearest foot or yard as requested.	
10. Given rulers specially scaled in quarter-inches or eighth-inches, the learner will measure objects to the nearest quarter unit.	Such rulers are commercially available.

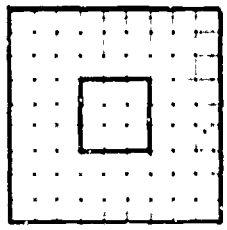
BENCH MARK 4-6 / 7-9

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
UNITS AND CONVERSION	
11. Given a standard yardstick, the learner will: (a) determine the number of inches in a given number of feet.	
12. Given a linear conversion table, the learner will convert inches, feet, yards and miles to common units.	Using the table, find the answers below 12 inches 1 foot 36 inches 3 feet 1 yard 4 feet = ? inches 5 yards ? inches
COMPUTATION	
13. Given exercises involving standard units of English measure, the learner will add and subtract, regrouping when necessary.	6 feet 3 inches + 5 feet 2 inches
COMPARISON IN MEASUREMENT	
14. Given rulers specially scaled in 1 16″, 1/10″, 1 cm, or 1 mm., the learner will measure objects to the nearest unit as requested.	Such rulers are commercially available.
15. Given a meter stick, the learner will measure objects to the nearest centimeter or meter upon request.	
UNITS AND CONVERSION	
16. Given a measure in inches, feet, or yards, the learner will convert the measure to the other two.	2 yards _ feet _ inches 18 inches __ yards 18 inches __ feet
17. Given a meter stick for conversion, the learner will state the relationships among meter, centimeter, and millimeter, and convert among the units.	
PERIMETER	
18. Given the following figures, the learner will find the perimeter of: 1) A rectangle (lengths of sides indicated)—formula may or may not be given). 2) A general polygon (lengths indicated). 3) A circle (diameter or radius indicated—formula and value of provided).	Using a string, the learner can make different figures (e.g., a triangle, a quadrilateral, a pentagon, a circle) and observe that the perimeter is constant, but the area changes.
THE NATURE OF MEASUREMENT	
19. Given an object, the learner will select a measuring instrument appropriate to the task of measuring the object.	Person—feet or centimeters. State—miles. Beetle—millimeters or fractions of an inch. Football field—feet or yards.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
COMPUTATION	
20. Given a length, expressed entirely in terms of one unit, the learner will multiply or divide the length by a whole number.	3 times 15 feet ___ feet?
WORD PROBLEMS	
21. Given word problems involving standard units of measure, the learner will solve the problems with or without aids.	
SCALE DRAWING	
22. Given scale drawings and a scale, the learner will determine the lengths of various parts of the actual object.	1) Given a 1:32 scale model of a car, the learner will determine the length of the actual car. 2) Given a 5:1 enlargement of a small beetle, the learner will determine the width of the beetle's body.
23. Given an object and a sheet of paper on which to represent the object, the learner will select an appropriate scale to use in making a scale drawing of the object.	If a giraffe is 14' high, find an appropriate scale for representing the giraffe on an 8½" by 11" piece of paper.
MAP READING	
24 Given a map with coordinates, the learner will locate places designated by pairs of coordinates.	1) Trout Lake is in section D-4, Locate Trout Lake. 2) Find Marquette on the Michigan map. 3) Locate Woodward Avenue on the Detroit City map.
25 Given a map, the learner will determine the straight-line distance between 2 points, using a ruler and the map scale.	1) Find the straight-line distance between Port Huron and Kalamazoo (given a Michigan map). 2) Find the distance (straight-line) between the Hilton Hotel and Tiger Stadium (given a Detroit map).
TABLE USE	
26. Given a table of data in common use, the learner will be able to locate items in the table.	Square root table Conversion tables Frequency tables Sales tax table Auto parts cost list (etc.) 1) Find the square root of 47. 2) How many inches are there in a centimeter? 3) What is the sales tax on a purchase of $4.58?

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
COMPARISON	
1. Given a region, the student will approximate the area by *covering* the region with units of area.	Measure a portion of the floor using sheets of different rectangular shapes or tiles.
2. Given a supply of square foot cutouts and a region, the learner will find the number of units it takes to cover the region by covering it.	Tiles, pieces of cardboard or plywood can be used as cutouts

BENCH MARK $\frac{\text{K-3}}{\text{4-6}}$

3. Given a supply of square inch cutouts and a region, the learner will find the number of units it takes to cover the region by covering it.	Blocks or plastic inch squares can be used instead of cutouts.
4. Given an arbitrary unit, the learner will approximate the area of a region in terms of the unit.	Use lima beans, kidney beans, oak leaves, triangles, and so on.
5. Given a region located on a piece of graph paper or some other grid, the learner will find the area by counting the number of square units.	
6. Given one cutout of standard units, the learner will find a given area by counting the number of units.	square inch square foot square yard square centimeter square meter
ESTIMATION OF AREAS	
7. Given a specific region, the learner will approximate the area of the region by placing a grid over the region and counting the square units.	A plastic sheet containing a grid would work
8. Given a polygon, the learner will estimate its area in square units.	 Techniques to name area Subdivide figure into triangles, squares, and rectangles and add each area to name total area. Enclose given figure in a rectangle and subtract areas of superfluous figures. Area of a right triangle is half the area of a rectangle (the hypotenuse is the diagonal of the rectangle).

PERFORMANCE OBJECTIVES

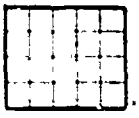
CONVERSION BETWEEN AREA MEASURES

9. Given physical models of a square yard and a square foot marked off in inches, the learner will demonstrate that 1 square foot = 144 square inches and 1 square yard = 9 square feet.

COMPUTATIONS OF AREAS

10. Given a physical model of a rectangle consisting of square units, the learner will describe the relationship between counting the square units and multiplying "length" times "width".

EXAMPLES AND COMMENTS



number of square units = 20

Possible Student Responses:

5 + 5 + 5 + 5 = 20

or: 4 rows of 5 = 20
or: 5 columns of 4 = 20
or: 4, 8, 12, 16, 20 (skip counting)
or: 5, 10, 15, 20 (skip counting)

BENCH MARK 4-6 / 7-9

11. Given a rectangle, (formula to be known), the learner will measure to the nearest whole unit and use the formula to find its area.

12. Given a triangle, (formula to be supplied), the learner will measure it to the nearest unit and find its area.

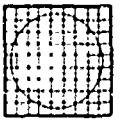
13. Given a box, the learner will find its surface area.

CIRCLES

14 Given a circle, the learner will approximate its area by covering the region with a grid.

EXAMPLES AND COMMENTS

Use inner area (all squares inside); outer area (all squares inside or cut by the curve).



Compare the approximate area *inside* the circle to the area of the entire square. Count the squares.

PERFORMANCE OBJECTIVES

15. Given: a) a circle

b) its center

c) the formula: $A = \pi R \times R$ or $A = \pi$

the learner will measure the radius to the nearest whole d) value for π 3.14[R] *or* π = 3-1/7 unit and find the area.

WORD PROBLEMS

16. Given word problems involving areas of rectangles (square included), triangles, and circles and the formulas for the triangle and circle (also the value of π), the learner will solve them.

EXAMPLES AND COMMENTS

MEASUREMENT K-3

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
MEANING OF VOLUME	
1. Given two different sized boxes and a set of non-standard units (blocks, beads, and so on), the learner will experimentally determine which box holds more.	Use blocks and rhombus shaped A-blocks in Attribute games. Pieces of 2"× 4" × 4" or 1" × 2" × 2".
2. Given a three-dimensional container and some sand, peas, beans, or water, the learner will use the media to demonstrate volume as the amount it takes to fill the container.	Half-gallon milk carton or shoe box.
3. Given a supply of cubical blocks, the learner will "build" a rectangular solid figure.	Use inch blocks.
COMPARISON OF RECTANGULAR CONTAINERS WITH VOLUME UNITS	A box similar to a shoe box filled with cubic units.
4. Given a box filled with cubic units, the learner will determine by counting the number of cubic units used.	
5. Given a box and a supply of cubical blocks, the learner will determine the volume of the box in terms of the blocks.	

BENCH MARK K-3 / 4-6

COMPARISON OF VOLUMES	
6. Given two or more containers, a cup, and sand or water, the learner will determine the volumes in terms of cups and compare them.	
7. Given a drawing of a rectangular solid divided into units (dimensions less than or equal to 5 units), the learner will name the number of units in it.	
ESTIMATION	
8. Given a box and a supply of cubic units which will not fill the box, the learner will: a) Estimate the volume b) Test, by using the cubic units, the correctness of the estimate of the volume.	

BENCH MARK 4-6 / 7-9

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
COMPUTATION OF VOLUME	
9. Given a solid marked off in cubic units or a drawing marked off in cubic units, the learner will describe the relationship between counting the units in a model or drawing and multiplying the "length" times "width" times "height".	
10. Given a rectangular box and the formula for the volume of a rectangular box, the learner will measure the box to the nearest inch and will use the formula to compute the volume.	Use the dimensions of the classroom in feet and yards; the dimensions of a textbook in inches.
WORD PROBLEMS	
11. Given the formulas and simple word problems involving rectangular boxes, the learner will solve the problems.	

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
COMPARISON OF ANGLES WITH UNIT ANGLES	
1. Given a unit angle (a wedge) and a drawing of an angle between 10 and 180 , the learner will approximate the measures of the given angle in terms of the unit angle.	Use wedges of 10° or 15° for the unit and angles of various sizes.

BENCH MARK 4-6 / 7-9

USE OF THE PROTRACTOR	
2. Given the degree as the standard unit angle, and a protractor marked off in degrees, the learner will read specific points on the protractor.	
3 Given a protractor marked off in degrees and a drawing of an angle between 0 and 180 , the learner will write the measure of the given angle accurate to within five degrees.	For the measurement of angles and drawing angles, it is suggested that they be angles on the 5° and 10° marks with some between 0° and 90° and some between 90° and 180°.
4 Given a protractor marked off in degrees and the measure of an angle between 0° and 180°, the learner will draw the angle accurate to within five degrees.	Directions to learner. "Draw an angle of 72°."

2 OF 2
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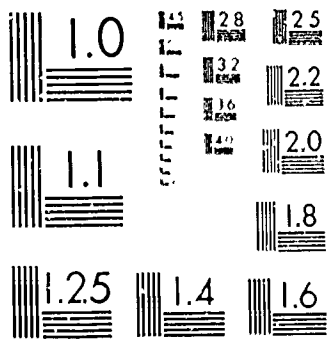
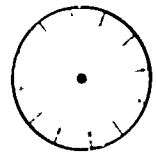
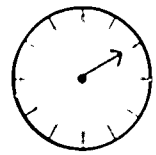
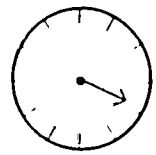
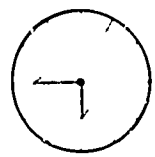


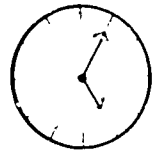
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS - 1963 - A

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
CALENDAR	
1. The learner will name orally the days of the week.	Use a calendar, a school week, and the learner's personal experience.
2. The learner will name orally the months of the year.	Use a calendar, a school year, and the learner's personal experience.
CLOCK	
3. Given a clock face, the learner will write numerals to 12 in the correct places on the clock face.	 Write '9' in the correct place in the clock face.
4. Given a clock face, the learner will point to midnight (or noon).	
5. Given a clock face with only an hour hand, the learner will state orally the time to the nearest indicated hour.	 The hour hand is pointing to __ o'clock.
6. Given the reading "__ o'clock" and a clock face with only an hour hand, the learner will move the hand to the appropriate time.	 Move the hour hand to 6 o'clock.
7. The learner will state orally the number of hours in a day.	

BENCH MARK K-3 / 4-6

8. Given a clock or a picture of a clock, the learner will tell time orally in whole hour, half hour, quarter hour, and minute intervals.	 Tell the time shown on the clock.
9. The learner will state orally the number of minutes in an hour.	

MEASUREMENT 4-6

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
NOTATION	
10. Given a clock face with hands on it, the learner will write the time in time notation.	 Write the time shown on the clock.
11. The learner will use A.M. and P.M. notation in writing time.	Three hours after midnight is what time?

BENCH MARK 4-6 / 7-9

CONVERSION	
12. Given a unit of time, the learner will convert to another indicated unit (hours, minutes, seconds, days, weeks, months, years).	1 hour 60 minutes 1 week 7 days and other reasonable conversions.
COMPUTATION	
13. Given two times to the nearest half hour, the learner will find the time interval.	How many hours between 10 A.M. and 2:30 P.M.?

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
VALUE	
1. Given five different U.S. coins, the learner can order them, or any subset of them, from least to most (in value) or most to least (in value).	Play-money may be used.
2. Given an odd number of 3 or more quarters, the learner will verbally indicate that the sum is closest to a given dollar.	Acceptable response: 7 quarters is nearest to 2 dollars.
3. Given three to five different amounts of money all less than or equal to $5.00, the learner will indicate verbally the greatest and the least.	Given 73¢, $1.73, $1.00, $2.93 and $4.53, state verbally the greatest amount.

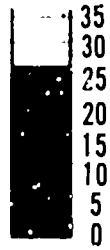
BENCH MARK K-3 / 4-6

RECOGNIZE DENOMINATIONS	
4. Given any of the following U.S. coins: penny; nickel; dime; quarter; half-dollar; dollar; the learner will name them and compare as requested.	Responses: 1 nickel — 5 pennies 1 dime — 10 pennies 1 quarter — 25 pennies 1 half-dollar — 50 pennies, 5 dimes, 10 nickels, 2 quarters 1 dollar — 100 pennies, 10 dimes, 20 nickels, 4 quarters
5. Given real money, pictures of money or play money less than or equal to $20, the learner will write the given money value using symbols with dollar sign and decimal.	
MAKE CHANGE	
6. Given a total purchase value less than $20, the learner will count out the change starting with the purchase value.	Bill Brown bought tennis shoes for $11.95. With the tax the total bill was $12.43. He paid with a $20 bill. What change did he get?
SOLVE PROBLEMS	
7. Given two money values, the learner will add or subtract using dollar and cents notation with or without the use of aids.	$4.15 + 2.23 = $6.38 — Suggested aid –play money
8. Given two money values, the learner will approximate the sum or difference to the nearest dollar or ten cents.	$1.23 is approximately $1.20 + $ .27 is approximately $ .30 Therefore, the sum is approximately $1.50.
9. Given verbal problems consisting of one or two operations involving money values less than or equal to $20, the learner will solve the problems.	One step problem: One plastic race car cost $.75, how much would three cost? Two step problem: One pencil cost $.05. If a student bought six pencils, how much change would he receive from a $1 bill?

MEASUREMENT 7-9

BENCH MARK 4-6 / 7-9

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
MAKE CHANGE	
10. Given a list of items with their price, the learner will select those items he could buy with a certain amount of money.	Provide a catalog, auto parts list, newspaper, or other source of items.
11. Given an expressed amount of money, the learner will multiply or divide the given amount by a positive integer.	$3.50 × 2 = $7.00 $6.42 : 2 $3.21
12. Given $100 and purchases totaling $100 or less, the learner will make change for the $100.	

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
READING A SCALE 1. Given a Fahrenheit or centigrade thermometer, the learner will read and record temperature to the nearest degree.	 State the temperature reading on this thermometer.
2. Given a Fahrenheit or centigrade thermometer, the learner will indicate verbally whether the temperature is: a) above: freezing, zero, 100 b) below: boiling, zero, 100 c) between: combinations of two of these.	 Is the temperature reading on the thermometer above freezing? zero? 100°?

BENCH MARK K-3 / 4-6

READING A SCALE 3. Given a Fahrenheit or centigrade thermometer the learner will read and record temperatures to the nearest degree, using degree ( ) symbol. **READING A TABLE OR GRAPH** 4. Given a table or a graph with the high and low temperatures expressed in Fahrenheit for each day of the week, the learner will identify the days with the lowest and highest temperatures and will state what the temperatures were.	Freezing and boiling points should be discussed. Day / High / Low S 43° 18° M 48° 23° T 39° 20° W 47° 25° Th 51° 29° F 45° 24° S 46° 22°

BENCH MARK 4-6 / 7-9

MEASUREMENT 7-9

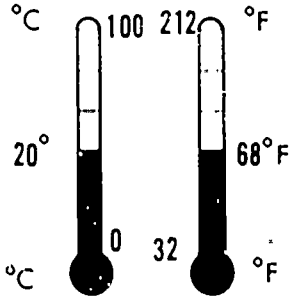
PERFORMANCE OBJECTIVES

CONVERSION

5. Given a nomograph consisting of Fahrenheit and centigrade scales, the learner will convert Fahrenheit temperatures to centigrade and conversely.

EXAMPLES AND COMMENTS

The nomograph will consist of the two temperature scales, properly lined up side by side.



Use a string to assist in reading across the two scales.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
BALANCE	
1. Given two objects of different weights and a balance, the learner will name the heavier (lighter) object on request.	Use a balance or make a balance with a ruler and fulcrum.
2. Given an object, a non-standard unit and a balance, the learner will indicate the object's weight to the nearest unit.	Metal washers of the same size may be used as a non-standard unit.
BENCH MARK K-3 / 4-6	
3. Given a device for measuring weight, the learner will find the weight of an object.	Devices for measuring weight may include a bathroom scale, a doctor's scale, a pan balance, and a spring balance.
4. Given a balance or scale, a standard metric unit, a standard English unit, and an object, the learner will indicate in writing the weight of the object using each unit.	Use a pan balance and weights.
CONVERSION	
5. Given a weight in pounds, the learner will convert pounds to ounces.	3 pounds ? ounces 2½ pounds ? ounces
BENCH MARK 4-6 / 7-9	
6. Given a table of equivalent weights, the learner will convert from one unit to another within the same system.	Convert within the English system *or* within the metric system, not between the two systems.
SOLVE PROBLEMS	
7. Given a measurement involving two units in the same system, the learner will multiply the measurement by a positive whole number and regroup as necessary.	2 lb 6 oz. × 3 6 lb. 18 oz. 7 lb. 2 oz.
8. Given an addition or subtraction problem involving weights with regrouping of units, the learner will write the sum or difference in simplest form. The learner may use a table to convert.	3 lb. 12 oz. + 4 lb. 10 oz. 7 lb 22 oz. 8 lb. 6 oz. 6 kgm. − 300 gm. 5 kgm. 700 gm.
9. Given a word problem involving standard units of weight measure, the learner will find the solution, using a table of equivalent weight if necessary.	If a road has weight restrictions on it of 10,000 pounds during the spring thaw, how many tons are allowed on this road?

MEASUREMENT K-3

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
UNIT	
1. Given five containers, water, and a one cup unit, the learner will determine experimentally the capacity of each container to the nearest cup.	Use different sized containers.
2. Given standard liquid measures, the learner will determine experimentally the number of cups in a pint.	
BENCH MARK K-3 / 4-6	
CONVERSION	
3. The learner will state the number of pints in a quart, quarts in a gallon, ounces in a cup, upon request.	
BENCH MARK 4-6 / 7-9	
4. Given a liquid measure and a table of equivalent measures, the learner will convert to an indicated equivalent measure.	3 gal. ___ pts. 2 qts. ___ cups
PROBLEMS	
5. Given addition, subtraction and multiplication problems involving regrouping of liquid units, the learner will compute the answer in the simplest form.	3 gal. 2 qt. + 2 gal. 3 qt. = ___ 5 gal. 1 qt. − 2 gal. 3 qt. = ___ 4 × 325 cc. = ___ l. ___ cc. See the parallel problems under weight.
6. Given a word problem involving liquid units of measure, the learner will solve the problem.	Each of five jars hold one and a half quarts of liquid How many gallons do these five jars hold together?

GEOMETRY

Every child lives in a world of relative place and shape. From this geometry should arise. The geometric form of physical objectives and the relative location of objects are the essence of intuitive geometry. From the first day in school children should have the opportunity to handle and contemplate geometric forms, discuss their properties and be encouraged to generalize their findings. Abstractions such as point, line, plane and space will become meaningful only after many experiences with objects providing models for the concepts. Consequently, emphasis is on student contact with geometric shapes and relations.

The objectives included here are those which are not associated with measurement. Metric objectives appear in the Geometric strand under Measurement.

There are many experiences which are excellent for motivational and background purposes that are deleted here because of the minimal nature of these objectives.

GEOMETRY K-3

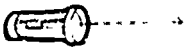
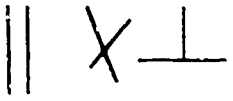
PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
SHAPES	
1. Given models of circular, square, rectangular and triangular regions, the learner will identify and name each model.	Put models on a desk and have the student name each. Use paper or flannel cutouts. Find objects in the classroom that will serve as models. Use a set of tangrams with the square, rectangle, and triangle shapes serving as models.
2. Given an assortment of cutout shapes including squares, triangles, rectangles and circles of various sizes randomly arranged, the learner will select a given shape as requested.	The student will pick up the designated objects from a set of assorted cutouts.
3. Given pictures of various shapes, the learner will identify circles, triangles, squares and rectangles as requested.	(a) Draw an X on the circle. (b) Mark the larger rectangle. (c) Draw an X on the triangle.

BENCH MARK K-3 / 4-6

4. Given a set of quadrilaterals, the learner will identify and name a parallelogram, a square, and a rectangle.	

BENCH MARK 4-6 / 7-9

5. Given a square, a rectangle, a parallelogram and a triangle, the learner will describe the properties of each.	Four congruent sides, four right angles, opposite sides parallel. Opposite sides congruent, opposite sides parallel, four congruent (right) angles. Opposite sides parallel, opposite sides congruent (opposite angles congruent). Closed figure, three sides.
6. Given a circle and its related parts, the learner will identify the center, radius, diameter, semicircle and circumference.	
7. Given models of cubes, prisms, spheres, cylinders, cones and pyramids, the learner will identify and name each.	The learner should pick up objects from an assortment and name them.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
POINTS AND LINES	
1. Given models of broken lines, curved lines, and straight line segments, the learner will identify and name each.	Use pipe cleaner models. Use a clothes line or sash cord to represent a straight or a curved line. Could also use jump rope. Design string pictures with yarn and glue.
2. Given models of points, lines, line segments, and rays, the learner will identify and name each.	To illustrate a line, use string that has been rolled into balls from each end. A flashlight or a ray of sunlight can illustrate a ray.  Use clothespins on a clothes line as a model of points. Fold a paper twice so that the creases intersect. The intersection of the creases is a point. The Big Dipper or other star clusters will also illustrate points.
3. Given models of lines or line segments, some of which are parallel, intersecting and perpendicular, the learner will identify and name each.	Cite examples from the classroom. Have students suggest examples: Parallel—railroad tracks, opposite edges of table, desk, sheet of paper. Perpendicular—corner of desk, sheet of paper. Circle examples of intersecting lines: 
4. Given a simple closed curve, the learner will identify the *exterior* and the *interior regions*.	Color the interior of a given simple closed curve red and the exterior blue.
5. Given the description of a plane, and a part of a plane, the learner will identify surfaces which represent a plane or part of a plane.	Name objects in the classroom which represent a part of a plane or plane such as floors, sheets of paper, blackboards, etc.

NO 7-9 PROGRAM OBJECTIVES

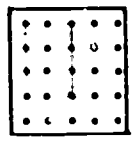
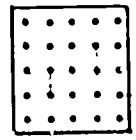
PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
CONGRUENCE	
1. Given pairs of congruent and non-congruent line segments or angles, the learner will be able to identify them as congruent or not congruent.	Student selects pairs by matching figures in some manner (trace and overlay, cutouts, measure).
2. Given pairs of congruent and non-congruent triangles or polygons, the learner will identify them as congruent or not congruent.	Match congruent cutouts or trace and overlay

4-6 GEOMETRY

I Identification

D. Symmetry

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
SYMMETRY 1. Given a description of symmetry, the learner will identify examples of symmetry in nature and indicate the point and line symmetries in each example.	Have students bring in leaves, or pictures of butterflies, spiderwebs, snowflakes, etc when appropriate. Fold over a sugar maple leaf or a picture of a butterfly.
BENCH MARK 4-6 / 7-9	
2. Given a variety of symmetrical plane figures, the learner will identify lines and points of symmetry in each figure.	Make some ink blobs on a paper and fold it over or fold paper and make cutouts. Investigate lines of symmetry by using mirror cards. Fold paper representations of symmetrical objects along their axes of symmetry. A H W Y C D

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
CONSTRUCTION	
1. Given several lengths of string, the learner will construct open and closed curves.	Geoboard  Geoboard  String
2. Given two (2) points on a piece of paper and a straight edge, the learner will construct a straight line through the points.	
3. Given a line segment, the learner will use paper folding to find the midpoint of the line segment.	Folding a piece of string may also be used to exhibit the midpoint of the segment. Identify the bisector of the line segment as a point.
4. Given a straight edge, the learner will draw a right angle.	Another method. Fold a piece of paper, then fold it once again so that the creases coincide.
5. Given a string and pencil, or a compass), the learner will construct a circle.	Or use a string and chalk and make the circle on the chalk board.

4-6

BENCH MARK ———

7-9

6. Given a model of the intersection of a line and plane, the learner will identify the intersection as a point.	Arrow in a target.
7. Given a model of the intersection of two planes, the learner will identify the intersection as a line.	Intersection of the walls in the classroom.
8. Given a model of non-intersecting planes, the learner will identify them as parallel planes.	Table top and floor.

ALGEBRA

Work with equations and inequalities enables learners to apply formulas and solve problems. Facility with formulas is a skill useful in many job classifications and occasionally important in everyday life. Preparation and interpretation of graphs is another skill, associated with algebra, which frequently is useful to everyone

The major thrust of these objectives is the development of skills in using equations and graphs. They are objectives which are generally present in that portion of algebra included in K-8 mathematics. They are not intended to be a set of objectives for a first-year course in algebra.

ALGEBRA K-3

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
EQUATIONS	
1. Given a statement of equality involving addition or subtraction facts and a place-holder for the sum or difference, the learner will supply the sum or difference.	1. 5 + 3 □ 2. 5 − 3 □

K-3
BENCH MARK
4-6

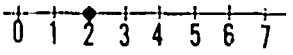
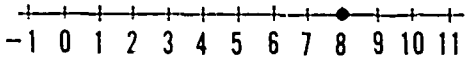
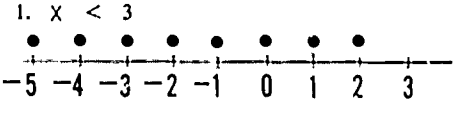
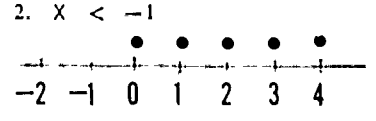
2. Given a statement of equality involving addition, subtraction, or multiplication facts and containing a place-holder or letter, the learner will find the missing number.	1. 5 + □ = 7 2. □ · 6 = 8 3. 7 − □ = 3 4. □ ÷ 6 = 2 5. 5 · 4 = □ 6. 5 · □ = 25 7. □ · 3 = 12 8. 8 + 7 = n 9. 6 · n = 11 10. n − 3 = 10 11. n ÷ 9 = 7 12. 5 · n = 3 13. n ÷ 8 = 8 14. 5n = 20
3. Given two numerical phrases, the learner will compare correctly the expressions by using = or ≠.	5 + 9 ( ) 14 5 · 3 ( ) 16 8 − 7 ( ) 2 × 3
SYMBOLS	
4. Given a pair of whole numbers or number phrases less than 1000, the learner will supply the appropriate symbol of equality or inequality, <, or = or >.	1. 3 ( ) 4 2. 6 + 2 ( ) 3 3. 5 + 7 ( ) 12 4. 8 + 3 ( ) 7 + 5 5. 517 ( ) 240
EQUATIONS	
5. Given an equation involving one or zero, the learner will complete the sentence.	1. 5 · 0 = □ 2. 7 · □ = 0 3. 7 × 1 = □ 4. 8 × □ = 8 5. 6 + 0 = □ 6. 7 + □ = 7

4-6 ALGEBRA

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
EQUATIONS	
6. Given a number sentence, the learner will indicate whether the sentence is true or false.	1. $4 \times 7 \stackrel{?}{=} 29$ 2. $3 + 12 \stackrel{?}{=} 15$
7. Given numerical statements involving the distributive property and a placeholder, the learner will insert the missing value.	$5 \times (10 + 3)$ $5 \times 10 + 5 \times \square$ $(10 + 4) \times 6$ $(\ ) \times 6 + 4 \times 6$ $(\ ) \times 10 + (\ ) \times 6 - 3 \times (10 + 6)$
FUNCTION	
8. Given a whole number input and a rule, the learner will name the output.	Methods. Function machine Tables. Examples: 1) Input 6, Rule Add 2. 2) Input 9, Rule Subtract 3. 3) Input 5, Rule Multiply by 6.
9. Given a table of whole number inputs and outputs related by a single operation, the learner will state the rule.	Function machine. Input | Output 3 | 6 4 | 7 0 | 3 8 | 11
FUNCTION	
10. Given a verbal exercise involving a number in a single operation, the learner will write a related algebraic expression.	1. A number added to 4. $(n + 4)$ 2. Apples cost a nickel each. How much will n apples cost? $(5n)$

4-6
BENCH MARK
7-9

EQUATIONS	
11. Given an equation involving addition, subtraction, multiplication, or division of whole numbers and involving a variable, the learner will find the value of the variable.	Methods: 1. Trial and error 2 Intuition 3. Number facts 4. Inverse operations Examples 1. $8 : 4 \quad n$ 2. $24 : n \quad 6$ 3. $n : 8 \quad 9$ 4. $\frac{x}{3} \quad 6$ 5. $18 : x \quad 3$ 6. $8 + 7 \quad n$ 7. $6 + n \quad 11$ 8. $n - 3 \quad 10$ 9. $n - 9 \quad 7$ 10. $5 - n \quad 3$ 11. $n - 8 \quad 4$ 12. $5n \quad 20$

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
12. Given an equation and a possible solution, the learner will determine if the solution is correct by substitution.	Checking is important. 1. $4x + 8 = 12$; $x = 1$ Ck. $4(1) + 8 = 12$ $12 = 12$ True 2. $3y = 6 = 9$; $y = 4$ Ck. $3(4) = 6 = 9$ $12 - 6 = 9$ $6 = 9$ False
EQUATIONS 13. Given a linear equation of the form $ax + b = c$, where a, b, c, and x are whole numbers, and the solution is a whole number, the learner will be able to find the solution.	Methods: 1. Trial and error 2. Balance beam 3. Axioms of equality may be used. 4. The learner may use the "cover-up" technique. Examples: 1. $4x + 8 = 12$ 2. $3y - 6 = 9$ 3. $11n + 10 = 21$ 4. $16 - 5a = 11$
14. Given a linear equation and its solution, the learner will represent the solution on a number line.	A prerequisite skill is the plotting of numbers on a number line. Examples: 1. $2x = 4$; $x = 2$  2. $x + 9 = 17$; $x = 8$ 
INEQUALITIES 15. Given an inequality of the form $x < a$ or $x > a$, the learner will find numbers that make the inequality true and graph the numbers on the number line.	Examples: 1. $x < 3$  2. $x < -1$ 
INEQUALITIES 16. Given a linear inequality of the form $ax < b$ or $ax > b$, where a and b are whole numbers, and b is a multiple of a, the learner will solve the inequality.	Methods: 1. Trial and error 2. The balance beam is a good teaching aid. Remarks. The solution set will be a continuous set (i.e., a set of real numbers). Examples: 1. 2 4 2. 4 12 3. 6 18 4. 10 10
EXPONENTS 17. An expression of the form r^n, r is a whole number and n is 2, 3, or 4, the learner will write r^n as r, r, r...r (n factors).	1. 2^4 ? $2 \times 2 \times 2$ 2. 5^3 ?

7-9 ALGEBRA

PERFORMANCE OBJECTIVES

18. Given an expression of the form r · r · r · r, four factors or less with a whole number base, the learner will write the product in the form r^n.

EXAMPLES AND COMMENTS

1. 10 · 10 · 10 ? 10^3
2. 3 · 3 · 3 · ?

EXPONENTS

19. Given an expression of the form r^n, r is a variable and n, is 2, 3, or 4, the learner will write r^n as r · r · r · . . . r (n factors).

1. x^3 ? x · x · x
   answer
2. y^4 ? y · y · y · y
   answer

20. Given an expression of the form r · r · r · r, four factors or less, with the base a variable, the learner will write the product in the form r^n.

1. x · x · x · x ? x^4
2. r · r · r = r^3

21. Given an expression of the form $n^{\square} = a$, where the exponent will be 2, 3 or 4 and n is a whole number less than 11, the learner will supply the missing exponent.

1. $2^{\square} = 8$
2. $3^{\square} = 81$

FUNCTION

22. Given a formula relating two quantities, the learner will make a table and describe how changing one quantity affects the other.

Given: Result:

1) 20 Lw

L	W
1	20
2	10
4	5
5	4
10	2
20	1

As L increases w increases.

Given: Result:

x 3y + 2

x	y
5	1
8	2
11	3
14	4

As x increases y increases.

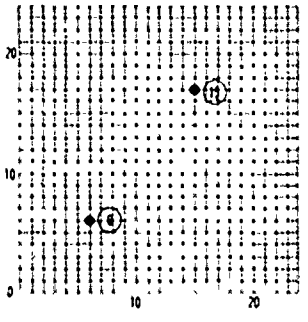
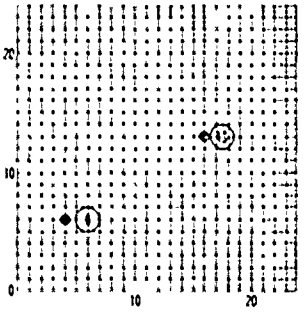
EQUATIONS

23. Given a statement about number which can be expressed as an algebraic sentence, involving a single operation on whole numbers, the learner will write the equation.

Given. Twice a number added to 6 is equal to 12.
Result: 2 n + 6 = 12

24. Given a common algebraic expression representing area, volume, etc., of degree · 2 and the value for each of the variables, the learner will evaluate the expression.

1. LW L 5 W 3
2. $2\pi r$ π 3.14 r 4
3. πr^2 π 3.14 r = 2

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
25. Given a formula and values for all the variables except one, the learner will find the missing value provided that the given values and solution are whole numbers.	1. d rt 2. v LWH 3. a LW 4. p 2L + 2W
SQUARES AND SQUARE ROOT	
26. Given a whole number less than or equal to 100, the learner will tell whether the given number is or is not a perfect square and give the square root of the perfect square.	1. Is 98 a perfect square? 2. Find the square root of 16 if it is a perfect square. 3. Trial and error is an acceptable method.
27. Given a square root table, the learner will be able to find the square root of a specified whole number 100.	Remark. Use of tables is included in the measurement strand Example. Find the square root of 38 correct to the nearest tenth.
GRAPHING	
28. Given an ordered pair of integers and a coordinate grid with axes indicated, the learner will plot the point represented by the ordered pair.	Useful activities include the geoboard, graphing pictures, the Battleship game, and Tic-Tac-Toe. Examples: 1. (3, −8) 2. (0, 5) 3. (3, 7) 4. (−4, −6)
29. Given one coordinate of a point and a graph of the point, the learner will supply the missing coordinate.	Find the missing coordinates on the graphs below.  

7-9 ALGEBRA

PERFORMANCE OBJECTIVES

EXAMPLES AND COMMENTS

GRAPHING

30. Given a table of values representing a linear relation, the learner will graph the straight line.

Remark:
Three points are sufficient for the graph.

Examples:

Plot the points in the table and the straight line determined by the points.

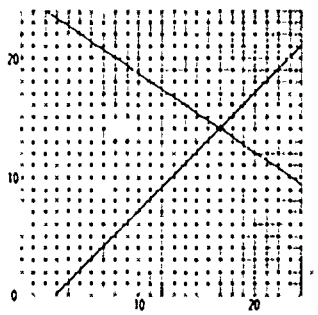
x	0	1	2	3	4	5	6
y	4	7	10	13	16	19	22

x	1	2	3	4	5	6	7
y	3	2	1	0	−1	−2	−3

Physical Example: time versus distance.

31. Given the graph of two lines which intersect, the learner will give the coordinates of the point of intersection if the coordinates are integers.

Name the point of intersection of the two lines.



PROBABILITY AND STATISTICS

Chance and uncertainty are a part of everyone's life. Frequently, probability and statistics utilize numbers to describe that chance and uncertainty. Probabilistic statements appear regularly in the news, and information is frequently presented with the assumption that common statistical terms are understood.

Grasp of the concepts included in these objectives is greatly strengthened if the concepts are taught experimentally with the learner actively involved in rolling dice, counting outcomes, collecting and organizing data, making graphs and other similar activities.

K-3 PROBABILITY AND STATISTICS

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
TALLYING	
1. Given data to record, the learner will use tally marks in recording the data.	(a) Recording the number of students who have brought lunch money. (b) Recording the number of walkers and the number of bus riders in the classroom. (c) Keeping track of the number of birds seen on a nature walk.
MAKING TABLES	
2. Given a set of data, consisting of number pairs, the learner will make a simple table.	Note. It is highly desirable that students collect their own data. (a) Attendance by date September 1–26 2–30 3–29 4– (b) Time-temperature table. (c) Input-output table.
	Time | 8:00 | 9:00 | 10:00 Temp | 48° | 52° |
3. Using information describing a familiar situation, the learner will use unit markers to assemble a bar graph unit by unit.	(a) A class places, on a chart, one by one, a sticker after the month in which they were born. J □ □ □ F □ M A (b) Type of pets students have. (c) Number of robins seen by day of the week.
4. Given a set of strips of paper of various lengths, the learner will match them with objects, and use the strips to form a bar graph.	(a) Heights of students. (b) Lengths of feet.
5. Given a sequentially numbered set of points, the learner will be able to connect them in order.	Dot-to-dot puzzles.

K-3
BENCH MARK ——
4-6

6. Given a set of data, the learner will tabulate it and organize it in a frequency table.	(a) Tabulating and recording test scores. (b) Weights of students in a school. (c) Student ages by months. (d) Makes of cars in the parking lot.
GRAPHS	
7. Given a set of objects of various lengths and a measuring instrument, the learner will measure the objects and use these measurements to make a bar graph.	(a) Measure a set of sticks. (b) Measure the lengths of shoes in the class Measuring instruments: ruler, meter stick, etc.

PROBABILITY AND STATISTICS 4-6

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
8. Given data to be graphed, the learner will choose an appropriate scale for the space provided.	Students should make a variety of bar graphs, both horizontal and vertical. Suggested data: (a) Stopping distance of cars. (b) Distance of planets from the sun. (c) Speed of vehicles. (d) Number of hits by ball players. (e) 1" represents 1'. (f) 1" represents 100 lb. (g) 1 square on graph paper represents 1" or 1 cm. (h) etc.
9. Given a coordinate system using whole numbers and/or letters and given 3 pairs of coordinates, the learner will locate the points.	(a) points towns on maps (b) tic-tac-toe (c) battleship
10. Given points plotted on a coordinate grid, the learner will connect the points in a designated sequence.	(a) Connecting dots to form pictures. (b) Connecting dots to form geometric figures. (c) Using objective 9, the learner can construct his own dot-to-dot or interpret a given dot-to-dot.

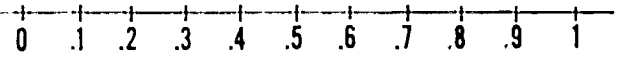
4-6

BENCH MARK ———

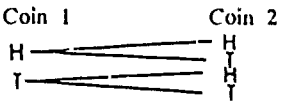
7-9

11. Given data, the learner will make bar graphs representing the data.	(a) Make a bar graph comparing the weights of common cars. (b) Make a bar graph showing the stopping distances of a car at different speeds.
12. Given a bar graph, the learner will be able to answer questions comparing the data.	(a) Given suitable graphs, answer questions such as what is the longest distance? (b) What is the rainfall in Mienigan? (c) What is the difference in weight between a black bear and a grizzly bear?
13. Given data, the learner will make line graphs representing the data.	(a) 24-hour (on the hour) temperatures. (b) Interest related to fixed principal for different rates. (c) Growth of an individual—height or weight compared to ages. (d) Change in weight over a week. (e) Resale value over time (depreciation) of a car (f) Resale value over time of a guitar.
14. Given a line graph, the learner will be able to answer questions comparing data.	Given suitable graphs, answer questions such as, (a) In which year did Al Kaline hit the most home runs? (b) Between which hours did the greatest temperature drop occur? (c) In which month did Bill earn the least money?

7-9 PROBABILITY AND STATISTICS

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
PROBABILITY	
15. Given a probabilistic situation, the learner will know that an event can be assigned a numerical probability between 0 to 1, inclusive, and will indicate appropriate probabilities for selected events.	The probability scale consists of a portion of a number line, beginning with 0 and terminating at 1. In between, it is helpful to scale it in tenths. The learner can indicate probabilities directly on the scale.  (a) The probability of getting neither a head nor a tail when tossing a coin. P = 0 (b) The probability of seeing a pterodactyl in the sky. P = 0 (c) The probability of obtaining a tail when a fair coin is tossed. P = .5 (d) The probability of obtaining a selected card from a 52-card deck. $P = \frac{1}{52}$
16. Given a fair die, coin, or other suitable devices, the learner will be able to determine the probability of selected *simple* events.	(a) The probability of getting a 6 when tossing a die. $P = \frac{1}{6}$ (b) The probability of a head when tossing a coin. P = 0.5 (c) The probability of a thumb tack landing point up. (d) The probability of drawing a "jack" from a 52-card deck. (e) The probability of drawing a "black queen" from a 52-card deck.
17. Given an experiment in probability with all events equally likely, the learner will find the probability of a designated *simple* event.	(a) What are the chances that a 6 will be rolled on a die? (b) If there are 32 people in your classroom and a name is selected at random, what is the chance that it will be your name? (c) If there are 8 persons in each of 4 rows, what is the probability that the name of a person in row 2 will be drawn.
18. Given an experiment in probability where all events are not equally likely, the learner will find the probability of a designated *simple* event.	(a) There are three green marbles and five black marbles in a jar. What is the probability of drawing a black marble? (b) A spinner is divided into 15 sections of equal size. Five of these sections are red, six are green and four are blue. What is the probability that the spinner will stop in a blue section?
19. Given the results of an experiment performed a given number of times, the learner will predict the number of times a particular event will occur if the experiment is performed many times.	

PROBABILITY AND STATISTICS 7-9

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
20. Given Event A, which can happen in "a" ways and Event B, which can happen "b" ways, the learner will determine the number of possible combinations of A and B, (a-b).	Should be preceded by Cartesian product in grades 4-6. (a) If a coin is tossed 2 times, how many possible outcomes are there?  (b) The local drive-in restaurant features 3 milk shakes and 5 sandwiches. How many different snacks (consisting of a milk shake and a sandwich) could the drive-in serve? (c) How many possible outcomes are there if a die is thrown and a coin is flipped at the same time? (d) A girl has 3 skirts and 4 blouses, how many possible outfits does she have?
MEASURES OF CENTRAL TENDENCY	
21. Given a set of up to 30 whole numbers, the learner will find the mean (the average).	(a) Find the mean (average) score of the quizzes in your class. (b) Find the average number of points scored by last year's basketball team in each game.
22. Given a set of up to 30 whole numbers, the learner will be able to find the median (middle score).	(a) Sidney Stryker went bowling several times last month. He bowled ten lines and his scores were 104, 150, 153, 201, 164, 179, 141, 129, 207. Find the median score. (153) (b) Twelve people were checked to see the amount of money they were carrying. The results were $5.63, $7.89, $15.42, $5.81, $12.08, $21.93, $25.00, $11.11, $3.92, 9.90, 13.58. Find the median amount. ($11.11).

TASK FORCE FOR THE DEVELOPMENT OF BASIC MATHEMATICS OBJECTIVES FOR THE STATE OF MICHIGAN

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