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ABSTRACT

Theory and research regarding four levels of concept attainment and three uses of concepts as specified by the model of conceptual learning and development (CLD) are described. Subtests, utilizing three-dimensional stimuli, were developed to assess each level of attainment and each use of the concept equilateral triangle. The tests were administered individually to 40 children at each of seven grade levels: preschool, kindergarten, second, fourth, sixth, eighth, and tenth. Major predictions were confirmed: (1) mastery of the concept levels and the uses increased with increased age, (2) the four levels were attained according to an invariant sequence, and (3) the higher the level of concept attainment the more effective was the use of the attained concept. Test construction, administration, item difficulty, etc. is described. (Author/RC)

AN INDIVIDUALLY ADMINISTERED TEST TO ASSESS LEVEL OF ATTAINMENT AND USE OF THE CONCEPT EQUILATERAL TRIANGLE

RESEARCH
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AN INDIVIDUALLY ADMINISTERED TEST TO ASSESS LEVEL OF ATTAINMENT AND USE OF THE CONCEPT EQUILATERAL TRIANGLE

by

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Statement of Focus

Individually Guided Education (IGE) is a new comprehensive system of elementary education. The following components of the IGE system are in varying stages of development and implementation: a new organization for instruction and related administrative arrangements; a model of instructional programing for the individual student; and curriculum components in prereading, reading, mathematics, motivation, and environmental education. The development of other curriculum components, of a system for managing instruction by computer, and of instructional strategies is needed to complete the system. Continuing programmatic research is required to provide a sound knowledge base for the components under development and for improved second generation components. Finally, systematic implementation is essential so that the products will function properly in the IGE schools.

The Center plans and carries out the research, development, and implementation components of its IGE program in this sequence: (1) identify the needs and delimit the component problem area; (2) assess the possible constraints—financial resources and availability of staff; (3) formulate general plans and specific procedures for solving the problems; (4) secure and allocate human and material resources to carry out the plans; (5) provide for effective communication among personnel and efficient management of activities and resources; and (6) evaluate the effectiveness of each activity and its contribution to the total program and correct any difficulties through feedback mechanisms and appropriate management techniques.

A self-renewing system of elementary education is projected in each participating elementary school, i.e., one which is less dependent on external sources for direction and is more responsive to the needs of the children attending each particular school. In the IGE schools, Center-developed and other curriculum products compatible with the Center's instructional programing model will lead to higher student achievement and self-direction in learning and in conduct and also to higher morale and job satisfaction among educational personnel. Each developmental product makes its unique contribution to IGE as it is implemented in the schools. The various research components add to the knowledge of Center practitioners, developers, and theorists.

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Abstract

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Theory and research regarding four levels of concept attainment and three uses of concepts as specified by the model of conceptual learning and development (CLD) are described. Subtests, utilizing three-dimensional stimuli, were developed to assess each level of attainment and each use of the concept <u>equilateral triangle</u>. The tests were administered individually to 40 children at each of seven grade levels: preschool, kindergarten, second, fourth, sixth, eighth, and tenth.

Major predictions were confirmed: (1) mastery of the concept levels and the uses increased with increased age, (2) the four levels were attained according to an invariant sequence, and (3) the higher the level of concept attainment the more effective was the use of the attained concept.

I
Introduction

Individuals at all levels of human development are constantly learning new concepts and extending and using old concepts in new situations. It is apparent, however, that an individual's level of mastery of a particular concept will differ depending upon his experiences with concept instances and his ability to perform the cognitive operations. For example, a four-year-old child and a biologist may both have a concept of tree, but although both may perform equally well when asked to identify a few obvious examples and nonexamples of tree their concepts differ markedly. Despite the large difference in level of understanding, concepts are the fundamental agents of thought for human beings from early childhood through adulthood.

We and others have completed a substantial amount of research on concept learning during the past two decades. Two types of research have been conducted, one dealing with the internal and external conditions of concept learning and another type involving the behavioral analysis of learning concepts related to various subject-matter fields. Sufficient knowledge has accrued so that Klausmeier, Ghatala, and Frayer (in press) were able to formulate a model that specifies and describes the cognitive operations involved in the attainment of concepts at specifiable levels of mastery by individuals whose abilities change in predictable ways with age. (We use "age" as a shorthand term to indicate the product of learning and maturation; age, per se, is not considered a determining factor of how well individuals can perform.) This study was undertaken to ascertain the extent to which children's performances coincide with the main predictions based on the model. A description of the model precedes the report of the empirical study.

An analytical descriptive model of conceptual learning and development (CLD model) was initially formulated by Klausmeier (1971) and later described more fully by Klausmeier, Ghatala, and Frayer (in press). The model defines four levels of concept attainment and the possible uses and extensions of attained concepts, specifies the cognitive operations involved in learning concepts at each of the four levels, and postulates internal and external conditions of learning related to the specified levels. The levels of concept mastery, the operations, and the conditions of learning have been identified through behavioral analyses of concept learning tasks and through empirical research in laboratory and school settings carried out at the Wisconsin Research and Development Center for Cognitive Learning and other research laboratories. The CLD model is intimately related to a conception of the nature of concepts and the associated experimentation with subjects ranging from preschool children to young adults.

The Nature of Concepts

The word concept is used by Klausmeier, Ghatala, and Frayer (in press) to designate mental constructs of individuals and also identifiable public entities that comprise part of the substance of the various disciplines. Thus, concept is used appropriately in two different contexts just as many other English words are. A concept is defined as ordered information about the properties of one or more things--objects, events, or processes--that enables any particular thing or class of things to be differentiated from, and also related to, other things or classes of things.

In connection with concepts as mental constructs it is noted that each maturing individual attains concepts according to his unique learning experiences and maturational pattern. In turn, the concepts he attains are used in his thinking about the physical and social world.

Concepts as public entities are defined as organized information corresponding to the meaning of words. Carroll (1964) related concepts, words, and word meanings in the following way. Words in a language can be thought of as a series of spoken or written entities. There are meanings for words that can be thought of as a standard of communicative behavior that is shared by those who speak a language. Finally, there are concepts--that is, the classes of experiences formed in individuals either independently of language processes or in close dependence on language processes. Putting the three together, Carroll stated: "A 'meaning' of a word is, therefore, a societally-standardized concept, and when we say that a word stands for or names a concept it is understood that we are speaking of concepts that are shared among members of a speech community" (Carroll, 1964, p. 187).

When starting a large programmatic research effort dealing with concept learning and instruction, Klausmeier, Davis, Ramsay, Fredrick, and Davies (1965) formulated a conception of concept in terms of defining attributes which they identified as common to many concepts from various disciplines. Klausmeier, Ghatala, and Frayer (in press) carried the definition further by specifying eight attributes of concepts: learnability, usability, validity, generality, power, structure, instance numerousness, and instance perceptibility. Other researchers and subject-matter specialists are also treating concepts in terms of defining attributes. For example, Flavell (1970) indicated that a formal definition of concept in terms of its defining attributes is useful in specifying what concepts are and are not and also in identifying the great variability among concepts. Markle and Tiemann (1969) and Tennyson and Boutwell (1971) have shown that the external conditions of concept learning can be delineated through research that starts with a systematic analysis of the attributes of the particular concepts used in the research. Scholars at the Wisconsin R & D Center demonstrated that analysis of concepts in terms of their relevant and irrelevant attributes is useful in clarifying the meanings of the concepts drawn from four disciplines: language arts--Golub, Fredrick, Nelson, and Frayer (1971); mathematics--Romberg, Steitz, and Frayer (1971); science--Voelker, Sorenson, and Frayer (1971); and social studies--Tabachnick, Weible, and Frayer (1970).

The CLD model deals primarily with concepts represented by words that can be defined in terms of attributes, although some concepts are defined on other bases, including through the use of synonyms and antonyms. Further, not all words potentially definable in terms of attributes are so defined, even in unabridged dictionaries. Therefore, the researcher and also the developer of curriculum materials must ascertain the defining attributes independently or cooperatively with scholars from the various disciplines.

Cognitive Operations and Levels of Concept Attainment

The structure of the model is shown in Figure 1. The various parts deal with four levels at which individuals may attain the same concept, the operations involved at each level, the use and extension of concepts, and acquiring the name for the concept and the concept attributes.

Concept Attainment Levels

A unique feature of the model is that it specifies four levels of attaining the same concept, rather than postulating that a concept is attained at its final level of mastery the first time it is learned. This provides the long-term developmental context of the model; that is, it provides the basis for explaining the changes that occur in the level of mastery of concepts attained by the same individual across long time intervals.

Attainment of a concept at the concrete level is inferred when the individual cognizes an object that he has experienced on a prior occasion. Attainment of a concept at the identity level is inferred when the individual cognizes an object as the same one previously encountered when observed from a different spatiotemporal perspective or sensed in a different modality such as hearing or seeing. The attainment of a concept at the classificatory level is inferred when the individual treats at least two instances of the same set of things as equivalent even though he cannot name the attributes that are common to them. Attainment of a concept at the formal level is inferred when the individual can name the concept, discriminate and name the societally accepted defining attributes and values, and accurately designate certain instances as belonging to the set and others as not belonging to the set.

Attaining the concept at each higher level successively is postulated to be the normative pattern by which individuals attain concepts under two conditions. First, the concept is of the kind for which there are actual perceptible instances or representations of instances, and second, the individual has experiences with the instances starting in early childhood. For example, the individual will have successively formed a concept of tree at the concrete, identity, and classificatory levels before he describes and treats tree and various subclasses

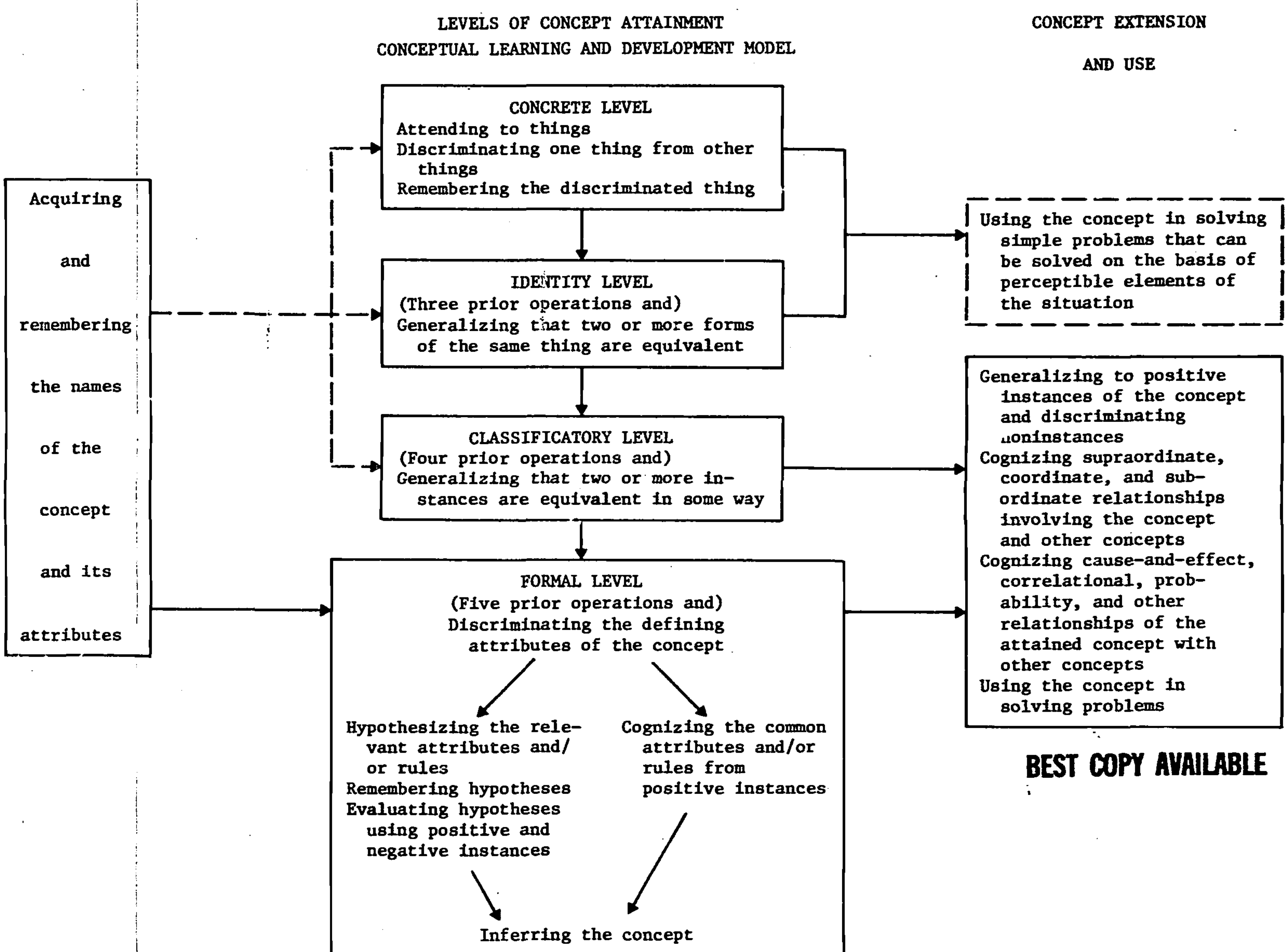

Figure 1. Cognitive operations in concept learning.

of trees formally in terms of their defining attributes. Classes of concepts that are not attainable at all four of the successive levels because of the nature of the concepts or because of the learning experiences of the individual are identified by Klausmeier, Ghatala, and Frayer (in press).

Cognitive Operations

Figure 1 also indicates the operations involved in attaining each level of a concept. This feature of the model provides the context for explaining short-term learning phenomena and also for identifying the changes that occur across time as new operations emerge and make possible the attainment at the successively higher levels.

In the CLD model the term operations is used much as Guilford (1967) uses the term, rather than as Piagetians use it. Guilford defines the operations of cognition, memory, productive thinking, and evaluation formally and also operationally in terms of test performances. He states that cognition must be related to the products cognized and defines cognition formally as follows:

> Cognition is awareness, immediate discovery or rediscovery, or recognition of information in various forms; comprehension or understanding. . . . The most general term, awareness, emphasizes having active information at the moment or in the present . . . the term, recognition, is applied to knowing the same particular on a second encounter . . . if cognition is practically instantaneous, call it recognition; if it comes with a slight delay, call it "immediate discovery." [Guilford, 1967, pp. 203-204]

According to Guilford, awareness, recognition, and immediate discovery apply generally to two products at the lower levels in his taxonomy, namely, units of information and classes. On the other hand, comprehension, which Guilford used synonymously with understanding, applies to the higher level products of relations and systems. Thus, cognition of principles, sequences, patterns, or structures involves comprehension, rather than mere awareness, recognition, or immediate discovery.

Operations at the lower levels of attainment. The first step in attaining a concept at the concrete level is attending to an object and representing it internally (Woodruff, 1961). Gagne (1970) indicates that as the individual attends to an object he discriminates it from other objects. Woodruff (1961) calls the outcome of these attending and discriminating operations a "concrete concept," a mental image of some real object experienced directly by the sense organs. The infant, for example, attends to a large red ball and a white plastic bottle, discriminates each one on a nonanalytic perceptual basis, maintains an internal representation of each, and cognizes each of the objects when experienced later.

Whereas the attainment of a concept at the concrete level involves only the discrimination of an object from other objects, attainment at the identity level involves both discriminating various forms of the same object from other objects and generalizing the forms as equivalent. The new and critical operation is generalizing. For example, the child attaining the identity level of dog generalizes that the family poodle is the same poodle when seen from straight ahead, from the side, and from various angles.

The additional operation required for the attainment of a concept at the classificatory level is generalizing that different instances are equivalent in some way. The lower limit in attaining a concept at the classificatory level is generalizing that at least two different instances are equivalent in some way. The individual is still at the classificatory level when he correctly classifies a large number of instances as examples and others as nonexamples but is unable to describe the basis for his grouping in terms of the defining attributes of the concept. Henley (cited in Deese, 1967), like many other researchers, reported that individuals can group things without being able to describe the basis of the grouping.

Operations at the formal level of attainment. Two sets of operations are involved in learning concepts at the formal level as shown in Figure 1. One set of operations includes discriminating and naming the defining attributes (Fredrick & Klausmeier, 1968; Kalish, 1966; Klausmeier & Meinke, 1968; Lynch, 1966), hypothesizing the attributes that define the concept (Levine, 1963, 1967), remembering hypotheses (Ghatala, 1972b; Williams, 1971), evaluating hypotheses (Bruner, Goodnow, & Austin, 1956), and inferring the concept. These operations go beyond those involved in attaining classificatory concepts and occur when the individual infers the defining attributes by using information from positive and negative instances of the concept. The attribute information may be given to the individual verbally or he may secure it by attending to the positive and negative instances.

The preceding operations characterize individuals who cognize the information potentially available to them from actual positive and negative instances, or from verbally presented descriptions of positive and negative instances. These individuals apparently reason like this: Instance 1 has land surrounded by water. It is a member of the class. Instance 2 has land but is not surrounded by water. It is not a member of the class. Therefore, lands surrounded by water belong to the class, and lands not surrounded by water do not. Being surrounded by water is a defining attribute of the concept. This individual can now properly classify newly encountered instances, based on experiences with only one positive and one negative instance of the concept.

The second set of operations given at the formal level in Figure 1 includes discriminating and naming the defining attributes, cognizing the common attributes and/or rules from only positive instances, and inferring the concept. According to Tagatz (1967), elementary school children up to about age 12 carry out these operations. They are not able to utilize information well from negative instances or to hypothesize and evaluate the defining attributes.

The CLD model is considered applicable to the learning of concepts from verbal descriptions, although the operations at the formal level shown in Figure 1 are more directly related to learning inductively from attending to and observing instances rather than from being given a definition of the concept and some examples of it. An individual may initially attain a concept at a low level of mastery after being given the concept name, its defining attributes, and a verbal description of an instance or two as is frequently done in classroom settings. His task thereafter is to properly generalize to positive instances when they are encountered and to discriminate negative instances. This requires transfer of learning and thereby use of the operations specified in the model at the formal level: hypothesizing whether the instance does or does not belong to the concept and evaluating the hypothesis in terms of the attributes given in the definition. Prerequisite to these two operations are discriminating the attributes of the concept and knowing their labels.

Operations of attending and remembering. Only a brief mention has thus far been made of attending and remembering. Ghatala (1972a) recently reviewed the literature dealing with attention in concept learning which shows that attending to environmental phenomena is requisite for subsequent discrimination of elements in the environment. Only recently have experiments been conducted that explain why an individual attends to certain elements and not others and how he organizes what he observes.

Concerning memory, Atkinson and Shiffrin (1968) postulate three memory systems--a long-term store, a short-term store, and a sensory-information register. There is ample evidence that in adults the predominant mode of information storage in both the short- and long-term systems is the verbal-linguistic mode. However, other modes of storage must be possible since adults are able to recognize smells, tastes, and visual stimuli which have not been verbally encoded. Also, a nonlinguistic store is presumed to be essential for preverbal children to learn concepts at the concrete, identity, and classificatory levels. Bruner (1964) discusses the nonlinguistic features of memory in terms of the enactive and ikonic representation of sensory experiences.

Acquiring Appropriate Labels

The importance of language in concept learning is emphasized by Bruner (1964), Carroll (1964), and Vygotsky (1962), among others. The broken line in Figure 1 indicates that the labels may be learned at any of the first three levels, while the solid line leading to the formal level indicates that verbal labels or other symbols are essential to attainment at that level. Thus, language is central in the learning of concepts, and the points at which labels of concepts and labels of attributes are associated is now clarified.

Children who have somewhat similar sensory experiences and instruction regarding dogs might acquire the name dog and higher-level meanings of it according to a sequence like this: A young child first encounters a dog. The child's mother points to the dog and says dog. The child then says dog and associates the name with his concept of the dog at this concrete level. Next, the child develops a concept of the same dog at the identity level by experiencing it in different locations and situations. His mother repeats the name at various times to the child when the dog is present; the child says the word repeatedly. The word dog now represents the child's identity-level concept of the dog. Subsequently, the child encounters other dogs, generalizes that they are equivalent in some way, and associates the name dog with whatever similarities he has noted. The word now represents his class of things called dogs. At the formal level and usually with instruction, the more mature child discriminates the intrinsic and societally accepted attributes of the class of things called dogs and also learns the names of the attributes. Now the child's concept of dog, his mental construct, and the societally accepted definition of the word dog become alike.

Both horizontal and vertical transfer are implied by concept utilization, and further learning is presumed to occur as the individual extends knowledge about an attained concept through using it. The individual who has attained a concept at the classificatory or formal level may use it in four ways as shown at the right of Figure 1--in generalizing to new instances, cognizing supraordinate-subordinate relations, cognizing various other relationships among concepts, and in generalizing to problem-solving situations. It is not implied that attainment of every concept at the classificatory and formal levels must be followed with all the uses. Little research has been completed regarding any of the uses of attained concepts; however, Ausubel's (1963) constructs of correlative and derivative subsumption are intended to explain how the individual relates concepts to one another. Similarly, Gagné (1970) postulates that having prerequisite concepts is an essential condition of rule learning and problem solving.

Additional Features of the CLD Model

The CLD model is more heavily oriented toward learning than toward development in that it implies that all the concepts held by any individual are learned; they do not emerge simply with maturation. In this context it is similar to four theories of concept learning generated by American experimental psychologists and reviewed by Bourne, Ekstrand, and Dominowski (1971): theory of associations (Bourne & Restle, 1959), theory of hypotheses (Levine, 1966; Trabasso & Bower, 1968), theory of mediation (Osgood, 1953), and theory of information processing (Hunt, 1962). Also, in agreement with these theories, the model specifies that the attainment of concepts is potentially explainable in terms of principles of learning. Despite some differences in terminology, the CLD model, like Hunt's, represents an information-processing approach to learning. The CLD model differs from the four theories just mentioned in that it describes different levels in the attainment of the same concept and specifies the operations essential to attaining concepts at the successively higher levels. While some of the operations are postulated to be common to more than one level, these operations at the successively higher levels are carried out on more highly differentiated and abstract properties of actual concept instances or on verbal descriptions of instances and attributes.

The CLD model is similar to Gagné's (1970) cumulative learning model in that both provide a framework for studying the internal and external conditions of learning. It also differs in two regards. Whereas Gagné describes seven forms of learning, ranging from the simplest learning through rule learning and problem solving, in the CLD model only one form of learning, concept learning, is analyzed according to its several constituent cognitive behaviors at each of four levels. Gagné also postulated a linear vertical learning hierarchy extending from signal learning through problem solving. The CLD model, as shown in Figure 1, indicates that a concept when learned at the classificatory or the formal level may be used in cognizing supraordinate-subordinate relations among the concept and other attained concepts, in understanding relations among concepts such as incorporated in principles and laws, and in problem solving. Thus, the CLD model departs from the straight linear learning hierarchy postulated by Gagné.

Possibly different from the preceding learning theories and more in agreement with Piaget (1970), the CLD model presumes that the new operations at each successive level involve qualitative changes in operating on instances and attributes of concepts, not merely additions to or modification of prior operations. Further, the operations that continue from one level to the next are carried out on more highly differentiated and abstract concept attributes. While the model does not postulate a stage concept associated with age levels as does Piaget, qualitative differences in thinking of the kinds pointed to by Kagan (1966) and Bruner, Olver, Greenfield, et al. (1966) are recognized. Also, Bruner's (1964) conceptualization of enactive, ikonic, and symbolic representation is accepted as a satisfactory global explanation of how experiences are represented and stored.

The roles of language and directed learning experiences are recognized as of central importance in attaining concepts at the classificatory and formal levels. The cross-cultural studies of Bruner, Olver, Greenfield, et al. (1966) support the directed-experiences point of view (cf. Goodnow, 1969). Also, Bruner's (1964) intermediate position that specifies how language facilitates thinking, rather than being essential to thinking (Luria, 1961) or being dependent on thought (Inhelder & Piaget, 1964), appears valid for the present model. Accepting directed experience as critical in concept attainment deemphasizes a maturational-readiness viewpoint, such as that expressed by Gesell (1928, 1945). While it is accepted that certain cognitive operations emerge with education and experience, this conception does not espouse a behaviorist-environmentalist point of view regarding learning to the extent that either Gagné (1970) or Staats (1971) does.

II
Validating the Levels of Concept Attainment

Four successive levels of concept attainment and four uses of concepts have been described in the previous section. The four levels are concrete, identity, classificatory, and formal. According to the CLD model, individuals normally attain the successive levels sequentially as new and higher-level cognitive operations emerge with learning and maturation. Further, an individual may use a concept attained at the classificatory level or the formal level in any of four ways: identifying examples and nonexamples of the concept, cognizing supraordinate-subordinate relationships when the concept is part of a taxonomy, cognizing cause-and-effect and other relationships when the concept or its attributes are incorporated in a principle, and solving problems when the informational basis of the problem utilizes the concept.

Theory and research regarding each of the levels of concept mastery and related cognitive operations, the uses of concepts, and the internal and external conditions of concept learning have been presented in the preceding section. This knowledge and behavioral analyses based on it have led to the central proposition that an individual, over a period of years, attains many concepts at the four successively higher levels. Furthermore, an individual may use concepts attained at the classificatory level and the formal level in the four ways which have been described. A concept attained at the formal level can be used more effectively than one attained only at the classificatory level. Empirical validation of these propositions is essential in establishing the robustness of the CLD model as a guide for research and also for the design of instruction.

In this section we report a cross-sectional study in which 280 children of preschool through high school age participated. The materials and procedures were developed specifically to assess each child's level of attainment of the concept equilateral triangle and his use of the concept.

Participating Children

Two hundred eighty children from the same small Wisconsin city participated in the study. Forty subjects were drawn from each of seven grades: preschool, kindergarten, second, fourth, sixth, eighth, and tenth. The 40 preschool subjects attended a nursery school and the 240 subjects of school age attended the public schools. The subjects were included in the study on the basis of grade level rather than chronological age. Intact classrooms were sampled, so the number of boys and girls in each group is not exactly the same. The number of boys and girls, the mean age, and the age range at each grade level are shown in Table 1.

It should be noted that the difference in mean age between the preschool and kindergarten groups is only 15 months, while the difference between the other groups is roughly two years. One might therefore expect smaller differences in attainment between these two younger groups in comparison with the other groups.

The difference in age range within each group merits brief attention. The age range varies substantially for the six school-age groups. The smallest range is 12 months for kindergarten subjects. The largest range is 27 months for fourth-grade subjects. The oldest subject in the fourth grade was older (11-9) than the youngest subject in the sixth grade (11-7). There is a similar overlap between the sixth- and eighth-grade groups, but none between the other groups. The larger range results primarily from a few subjects having taken more than one year to complete a grade at some point in their schooling. The youngest ages for the six school-age groups reflect school laws and regulations which permit a child to enter kindergarten only if his birthday comes on or before a certain day. Thus, for each of the six school-age groups, the youngest child is always six to nine months past the date

TABLE 1
NUMBER OF MALES AND FEMALES, MEAN AGE, AND AGE RANGE AT EACH GRADE LEVEL

Grade	Males	Females	Mean Age (in years and months)	Age Range (in years and months)
Pre	23	17	5-0	4-1 to 5-9
K	23	17	6-3	5-6 to 6-9
2nd	23	17	8-2	7-9 to 8-9
4th	15	25	10-3	9-6 to 11-9
6th	25	15	12-2	11-7 to 13-0
8th	17	23	14-1	12-8 to 15-0
10th	14	26	16-0	15-6 to 17-2

of his birthday, i.e., 5-6, 7-9, 9-6, 11-7, 12-8, and 15-6; the variability in age ranges at different grade levels is therefore due to variability in the age of the oldest subjects rather than of the youngest subjects.

Test Development

A subtest was developed to assess each of the four levels of concept attainment and three of the four uses. Because of the difficulty in devising a test to assess the use of a concept in identifying examples and nonexamples which would be distinct from the test to assess attainment of the classificatory level, this use of the concept was not separately assessed. Therefore, seven subtests were developed. The tests required specially constructed materials and were administered individually.

Criteria for Tests

To develop the tests of concept attainment and utilization, the behaviors involved in attaining the concept were analyzed and then materials and instructions to assess the behaviors were developed. The test items went through expert review while under development. The entire battery was then tried out on a small scale before it was used in this study.

A few criteria in addition to the usual ones of reliability, objectivity, and usability were set up to guide the development. First, the materials and instructions had to permit assessment of subjects of preschool through high school age. It was presumed that not all subjects of preschool age would attain a given concept at the concrete level and that not all high school subjects would attain it at the formal level. Second, to test for attainment at the concrete, identity, and classificatory levels the particular concept selected had to have perceptible instances or representations thereof. Third, the concept had to be definable by publicly accepted attributes in order to test attainment at the formal level. In this connection it is noted that many concepts are definable in terms of attributes even though this method of definition is often not used in abridged dictionaries. Fourth, the concept selected for a battery should be relatable to the subject matter which the pupils encounter in school. This is in line with the proposition that directed experience, including instruction in school, is a powerful determinant of the particular concepts attained by individuals and also of their level of attainment and use. Further, since much instruction in school deals with concepts, the CLD model should be applicable to the design of instruction, and the subtests, when fully validated, should be usable in assessing the level of conceptual development in school-age children. Fifth, the particular concept had to be part of a taxonomy in order to test its use in cognizing supraordinate-subordinate relationships. Finally, the concept had to be usable in cognizing principles and in problem solving. Here, the concept may be usable in solving simple problems that can be solved on a perceptible basis without being used first in understanding a principle, or it may be used first in understanding a principle and then in solving more complex problems.

Three of many concepts that meet these criteria are equilateral triangle from the field of mathematics, noun from the field of English, and tree from the field of science. The concept equilateral triangle was selected for the first battery of tests to be developed and administered.

Test Materials

Four sets of 36 three-dimensional blocks were constructed according to the specifications given in Table 2. Although three-dimensional, none of the blocks were cubes or pyramids. The blocks varied along four dimensions: the shape formed by the edges of the two larger surface areas--equilateral triangle, right triangle, or square; color of all six surface areas--blue, red, or yellow; length of the edges of the two surface areas--longer or shorter (thus the area of the surface appeared larger or smaller); and thickness--thicker or thinner. In subsequent

TABLE 2
SPECIFICATIONS FOR BLOCKS AND DESCRIPTIVE LETTERS

Block Number	Shape of Surface Area	Color	Thickness[a] (in millimeters)	Length of Side[b] (in millimeters)
1	Equilateral Triangle	Blue	20-K	100 -L
2	Equilateral Triangle	Blue	10-N	100 -L
3	Equilateral Triangle	Blue	20-K	66.7 -S
4	Equilateral Triangle	Blue	10-N	66.7 -S
5	Equilateral Triangle	Red	20-K	100 -L
6	Equilateral Triangle	Red	10-N	100 -L
7	Equilateral Triangle	Red	20-K	66.7 -S
8	Equilateral Triangle	Red	10-N	66.7 -S
9	Equilateral Triangle	Yellow	20-K	100 -L
10	Equilateral Triangle	Yellow	10-N	100 -L
11	Equilateral Triangle	Yellow	20-K	66.7 -S
12	Equilateral Triangle	Yellow	10-N	66.7 -S
13	Right Triangle	Blue	20-K	87.87-L
14	Right Triangle	Blue	10-N	87.87-L
15	Right Triangle	Blue	20-K	58.58-S
16	Right Triangle	Blue	10-N	58.58-S
17	Right Triangle	Red	20-K	87.87-L
18	Right Triangle	Red	10-N	87.87-L
19	Right Triangle	Red	20-K	58.58-S
20	Right Triangle	Red	10-N	58.58-S
21	Right Triangle	Yellow	20-K	87.87-L
22	Right Triangle	Yellow	10-N	87.87-L
23	Right Triangle	Yellow	20-K	58.58-S
24	Right Triangle	Yellow	10-N	58.58-S
25	Square	Blue	20-K	75 -L
26	Square	Blue	10-N	75 -L
27	Square	Blue	20-K	50 -S
28	Square	Blue	10-N	50 -S
29	Square	Red	20-K	75 -L
30	Square	Red	10-N	75 -L
31	Square	Red	20-K	50 -S
32	Square	Red	10-N	50 -S
33	Square	Yellow	20-K	75 -L
34	Square	Yellow	10-N	75 -L
35	Square	Yellow	20-K	50 -S
36	Square	Yellow	10-N	50 -S

[a] K-thicker, N-thinner.

[b] L-longer side, S-shorter side.

descriptions of test items the following abbreviations are used to designate any particular block:

E - surface shape of equilateral triangle
R - surface shape of right triangle
S - surface shape of square

B - blue
Y - yellow
R - red

N - thinner block
K - thicker block

S - smaller surface area
L - larger surface area

Thus, EBNS designates a small, thin, blue block having two sets of surface edges that correspond to an equilateral triangle; RYKL is a large, thick, yellow block having two sets of surface edges corresponding to a right triangle; and SRKL designates a large, thick, red block having two sets of edges corresponding to a square. Hereafter, blocks are referred to as equilateral triangle, right triangle, or square even though they were three-dimensional as noted.

Other materials used in the study were four drawings, a six-inch ruler, and a Masonite screen held in a vertical position by a wooden base. The screen was used to conceal blocks for periods of time during administration of the items to assess attainment at the concrete and identity levels.

Test Administration, Scoring and Passing Criteria

Seven subtests were developed to assess the various levels of attainment and utilization of the concept equilateral triangle. These subtests were designated as follows: (1) Concrete, (2) Identity, (3) Classificatory, (4) Formal, (5) Supraordinate-Subordinate, (6) Principle, and (7) Problem Solving. Only the first five subtests were administered to subjects at the preschool level; all seven were administered to subjects at the other six grade levels. The subtests were administered in a unique random order to each child.

Four experimenters, three male and one female, administered the subtests to each subject individually. Each subject was tested in a private room and was seated at a table with the experimenter across from him. All instructions were stated in the easiest possible vocabulary so that the youngest subjects would understand what they were to do. The opening instructions were informal in that they were not given verbatim, but they did conform to the following general format:

> "Hi! My name is ___________. I work at the University at Madison. At the University we're interested in finding out how children learn. To help us find out about learning, I'd like you to tell me the things you notice or already know about some blocks I'll show you. OK? First, I'd like to write down your name and age. What is your name? How old are you?"

Next, a warm-up exercise was administered. The purpose of the warm-up exercise was to familiarize the subject with the 36 blocks. The experimenter placed all 36 blocks in a random array in front of the subject and said, "These are the blocks we will use today. You can pick up any of the blocks you want to so you can see what they are like."

When the subject indicated that he had finished examining the blocks, the first subtest was administered. If the first subtest was Classificatory, all 36 blocks were left on the table. If the first subtest was Supraordinate-Subordinate, squares were removed from the table and put in a box out of the subject's view; if the first subtest was Concrete, Identity, Formal, Principle, or Problem Solving, all blocks were removed from the table and put in a box out of the subject's view.

The instructions to the subjects and the criteria for passing necessarily varied somewhat for each subtest because of the purpose of the test. Therefore, in the following descriptions of each subtest an indication of the purpose of the test, a brief description of the test and its administration, and the criteria for passing are given.

Concrete Level

According to the CLD model, attainment of a concept at the concrete level is inferred when the individual cognizes an object that he has encountered before. To assess attainment of the concept equilateral triangle at the concrete level, a target example block was presented for five seconds and then removed from sight for 15 seconds. Then the target example was presented again in the same orientation in an array with nonexample blocks. The subject's task was to choose the block he had seen before. The array was left in view until the subject made a choice.

The subtest consisted of five items which utilized the following targets and nonexamples (items are numbered consecutively for the first four subtests; the letters preceding the number indicate the subtest to which the item belongs):

Co 1 - Target ERNS. Two nonexamples RYKS, SBNL
Co 2 - Target EYNS. Four nonexamples EBNL, EBKS, RYNL, RYKS
Co 3 - Target EBNL. Six nonexamples EBKL, EBNS, ERNL, EYNL, RBNL, SBNL
Co 4 - Target ERKL. Six nonexamples EBKL, ERNL, ERKS, EYKL, RRKL, SRKL
Co 5 - Target EBKS. Six nonexamples EBKL, EBNS, ERKS, EYKS, RBKS, SBKS

The number of dimensions on which the nonexample blocks differed from the target block, and the total number of nonexample blocks varied across the first three items. These variations were introduced to manipulate the hypothesized difficulty level of the items. Based on previous research, Co 1 was hypothesized to be easiest, Co 2 more difficult, and Co 3-5 most, and possibly equally, difficult. For Co 1 there were two nonexamples, each differing on three attributes from the target. For Co 2 there were four nonexamples, each differing on two attributes from the target. For Co 3-5 there were six nonexamples, each differing on only one attribute from the target. The placement of the target in relation to the nonexamples was systematically alternated (center, right, left, etc.).

Between trials an intentional interval of approximately 30 seconds was introduced during which the experimenter engaged the subject in friendly conversation. The purpose of the interval was to minimize interference between trials due to memory of previous blocks.

The criterion for passing the concrete level was Co 1 correct as well as any three of the remaining four items. The allowance of one error was judged to be reasonable in terms of any possible waning of attention for whatever reason during the five-second interval when the target was displayed. It was inferred that the subject who met this criterion attended, discriminated, and remembered; that is, he manifested the operations associated with this level.

Identity Level

Assessment of attainment at the identity level proceeded with instructions identical to those for the concrete level. However, only three items were used, and during the test display the target block was placed in a different orientation with respect to the subject than when it was initially displayed.

It may be recalled that the identity level is inferred when the individual cognizes an object as the same one previously encountered when observed from a different perspective or sensed in a different modality. Whereas for the concrete level nonexamples were placed in a regular orientation with the base of the blocks on the same horizontal line, at the identity level the nonexamples were placed in various orientations so that the target block would not be conspicuous by its irregular orientation. As at the concrete level the placement of the target among the nonexamples was systematically alternated. Again, a 30-second interval was allowed between items.

The same blocks were used in the three items here as in Co 3-5 of the concrete level. Thus, there were six nonexamples for each item, and each nonexample differed on one attribute from the target block. The blocks used for each item and the orientation of the target blocks are specified below.

Id 6 - Target EBNL. Nonexamples EBKL, EBNS, ERNL, EYNL, RBNL, SBNL. Presentation: The target block was presented six inches from the edge of the table nearest the subject and placed such that its equilateral area faced up from the table. Test: In the recognition set the target block was 18 inches from the edge of the table nearest the subject, again placed such that its equilateral area faced up from the table. Nonexample EBNS was six inches from the edge of the table nearest the subject.
Id 7 - Target ERKL. Nonexamples EBKL, ERNL, ERKS, EYKL, RRKL, SRKL. Presentation: The target block was presented six inches from the edge of the table nearest the subject and placed such that its equilateral area faced up from the table. Test: The target block was 18 inches from the edge of the table nearest the subject and placed such that its equilateral area faced directly toward the subject; the block rested on a base of the triangle.
Id 8 - Target EBKS. Nonexamples EBKL, EBNS, ERKS, EYKS, RBKS, SBKS. Presentation: The target block was presented six inches from the edge

of the table nearest the subject and placed such that its equilateral area faced up from the table. Test: The target block was 18 inches from the edge of the table nearest the subject, placed such that a side of the triangle faced directly toward the subject; the block rested on a base of the triangle.

The variations in perspective were cumulative over items. That is, Id 6 varied distance between the target presentation and the test display, Id 7 varied distance and uprightness, and Id 8 varied distance, uprightness, and orientation. The intention of these cumulative variations was to manipulate the hypothesized difficulty of the items, making Id 6 the easiest and Id 8 the most difficult.

The criterion for passing the identity level was Id 6 and one of the last two items correct. This criterion was intended to be parallel in degree of stringency to that for the concrete level so that various comparisons could be made across attainment levels by age groups and by individual subjects.

It was inferred that subjects who met the criterion generalized that the target block was the same block when observed from a different perspective. This generalizing capability is the primary differentiator between the concrete and the identity levels of attainment as specified by the CLD model. It is presumed that in accordance with the CLD model the subject who passed the identity level, in comparison with one who passed only the concrete level, also discriminated and remembered either more perceptible properties of the targets or less obvious abstract properties of the targets.

Classificatory Level

Earlier it was indicated that attainment at the classificatory level is inferred when the individual responds to at least two different instances of the same class as equivalent. He may not be able to describe the basis of his classification, and he may not have the name of the concept in his spoken vocabulary. In addition to the operations at the identity level, the individual who passes this level is inferred to generalize that at least two different instances are equivalent in some way--in the case of equilateral triangle, equivalent in shape.

Two sorting items were administered at this level to ascertain whether the subject could identify examples and nonexamples of equilateral triangle. A warm-up item preceded the two test items to determine how the subject would sort in the absence of any instruction regarding the basis of the sorting. Item Cl 9, which followed the warm-up item, was administered to see whether the subject could sort consistently and exhaustively, given examples of the concept. Item Cl 10 was designed to determine whether the subject could sort consistently and exhaustively, given both examples and nonexamples of the concept. The materials and instructions for the items are now given.

Cl 9 - Target examples EBKL, EYKL, ERKS, EYNS. Examples to be identified were the other eight equilateral blocks distributed randomly among the 12 right-triangle blocks and the 12 square blocks. Procedure: The experimenter showed the subject the four targets for 30 seconds, the subject viewed the blocks, and the experimenter said, "Think about how all these blocks [experimenter pointing to the four target blocks] are alike. Can you see how they are all alike? Now find as many blocks as you can that are like all of these blocks [experimenter again pointing to target blocks] in some way and put them over here." The experimenter then removed the target blocks from sight. After the subject sorted the blocks the experimenter asked, "Why did you pick these blocks?" If the subject did not give an interpretable answer to this question, the experimenter asked a prompting question, "How are these blocks like the ones I showed you before?" The experimenter then replaced all blocks which the subject had sorted to the area at the left of the subject.

Cl 10 - Target examples EBKL, EYKL, ERKS, and EYNS as in Cl 9. Target nonexamples RBKL, SRKS, SYNL. Examples to be identified were the other eight equilateral triangle blocks distributed randomly among the remaining 11 right-triangle blocks and 10 square blocks. Procedure: The experimenter showed the target sets of examples and nonexamples and the experimenter said, "Think about how all these blocks [experimenter pointing to the first group of blocks] are alike. Can you see how all these blocks are alike? All of these blocks [experimenter pointing to the second group of blocks] are different from these blocks [experimenter again pointing to the first group of blocks] in some way. Can you see how these blocks [experimenter pointing to the second group of blocks] are different from these blocks? Now find as many blocks as you can that are like all of these blocks [experimenter pointing to the first group of blocks] but not like these other blocks [experimenter pointing to the second group of blocks]." The experimenter then removed the target blocks from sight. After the subject sorted the blocks, the experimenter asked the same questions as in Item Cl 9.

These items were readily scored for the number of equilateral blocks included, the number of equilateral blocks omitted (errors), and the number of other blocks included (errors). The experimenter recorded which blocks the subject picked and also his verbatim answers to the questions. The answers to the questions were examined but were not scored. The vocabulary used in the answers varied greatly and the scorer could not infer whether the subject had more appropriate terminology in his spoken vocabulary than that which he used in answering the question.

The criterion for passing this level was either of the two items correct. Correctness was defined as identifying at least seven of the eight equilateral blocks and excluding all the other blocks except one.

It was predicted that both Cl 9 and Cl 10 would be relatively easier for older subjects than for younger subjects. According to the CLD model, the younger subjects might not attend to or might become confused by the relatively long instructions, might not readily discriminate the relevant attribute of shape, and might attend to the nonrelevant attributes of color, thickness, and largeness. A further prediction was that older subjects would perform better on Cl 10 than on Cl 9 and the younger subjects poorer on Cl 10 than on Cl 9 because the younger subjects would not be able to use the information potentially available in the nonexamples and the older subjects would.

Formal Level

According to the CLD model, an individual is judged to have attained a concept at the formal level when he has the name for the concept, can define it in terms of its attributes, can discriminate and name the attributes, and can differentiate between examples and nonexamples of the concept in terms of the defining attributes. Items were constructed for each of these purposes. In contrast to the prior levels of attainment, the model calls for explicit terminology at this level. The items assessing the various aspects of the formal level of mastery were as follows:

Fo 11 - Naming the Concept. Targets SBKL, EBKL, RBKL. Procedure: The experimenter said, "See the shape of this block [experimenter showing block SBKL]? Its name is a square. [The square was removed from sight.] Now look at these two blocks [experimenter showing blocks EBKL and RBKL]. They each have different shapes. What name can you give this block [experimenter pointing to block EBKL] that is different from the name for this block [experimenter pointing to block RBKL]?" If the subject did not give the response "equilateral triangle" or an acceptable equivalent, the experimenter prompted, "What else can it be called?" If the subject said "triangle," the experimenter prompted further, "Can you tell me more? What kind of triangle?"

Fo 12 - Defining the Concept. This question was omitted if the subject did not give an acceptable answer to Fo 11. If the subject did give an acceptable concept name, that name was used in asking this question: "What must something have for you to call it a ___________?" If the subject did not mention equal sides or equal angles, the experimenter prompted, "What else must something have for you to call it a ___________?"

Fo 13 - Evaluating Instances. These questions were omitted if the subject did not give an acceptable answer to Fo 11. If the subject did give an acceptable concept name, that name was used in asking these questions.

a. The experimenter showed block EBNL and asked, "Would you call this a ___________? Why (or Why not)?"
b. The experimenter showed block

RBNL and asked, "Would you call this a ____________? Why (or Why not)?"

c. The experimenter showed block SBNL and asked, "Would you call this a ____________? Why (or Why not)?"

d. The experimenter showed block EBKS and asked, "Would you call this a ____________? Why (or Why not)?"

Fo 14 - Discriminating and Naming Attributes.

a. The experimenter showed blocks EBKL and SBKL and said, "See these blocks? Can you tell me the way they look different from one another?" If the subject did not say, "One has three sides and the other has four sides," or "One has three angles, the other has four angles," the following prompt was given: "Can you tell me another way they look different from one another?"

b. The experimenter showed blocks EBKL and RBKL and said, "Now see these blocks? Can you tell me the way they look different from one another?" If the subject did not say, "One has equal sides and the other has unequal sides," or "One has equal angles, the other has unequal angles," the following prompt was given: "Can you tell me another way they look different from one another?"

A passing score for the subtest was described in this manner. The concept name had to be given (Fo 11); two of the three items (Fo 12, Fo 14a, Fo 14b) calling for a definition and for discriminating and naming the attributes had to be correct; and both items (Fo 13a and d) involving identifying and evaluating the examples, and one of the two items (Fo 13b and c) for the nonexamples had to be correct. These criteria may seem easy in terms of the definition of attainment at the formal level. It is noted, however, that the three-dimensional blocks were not equilateral triangles, right triangles, and squares, even though the edges of surfaces pointed to by the experimenter formed the shapes. This may have confused older subjects who had already learned the difference between plane and three-dimensional figures. Also, the experimenter and the scorer could not be completely certain that a subject did not have the name "equilateral triangle" or an acceptable substitute for it in his speaking vocabulary, even with the prompting that was done. Acceptable substitute names were "equilateral," "equal sided," "equal sides," "equal angled," "equal edges," "equal points," and "equal corners."

Supraordinate-Subordinate Relationships

A possible misconception about the uses of an attained concept may be prevented by considering the uses in more detail. A reexamination of Figure 1 shows that the uses of concepts are diagramed as parallel to attainment at the classificatory and formal levels. Further, there are no arrows joining the uses (as there are for the four levels) which would specify a linear sequence for them. Thus, the use of an attained concept in cognizing supraordinate-subordinate relationships is not presumed to be more or less difficult than its use in cognizing a principle or in solving a problem. One properly infers, however, that a concept attained at the formal level may be used in all three ways more effectively than one attained only at the classificatory level. Further, according to the CLD model, concepts attained only to the concrete or identity level cannot be used in cognizing supraordinate-subordinate relationships or in understanding principles. Concepts attained at lower levels may possibly be used in solving easy problems. For example, a child may have a concept of <u>equilateral triangle</u> at the identity level but not at the classificatory level and be able to find the distance around an equilateral triangle that has a side one inch long. In line with this discussion, the items of the three subtests which follow are not numbered consecutively.

The four items for this use of the concept <u>equilateral triangle</u> were designed to ascertain (<u>a</u>) whether the total set of 12 examples of equilateral triangles and 12 examples of right triangles belonged to the supraordinate class <u>triangle</u>; (<u>b</u>) whether the subject could classify 12 examples as belonging to the subclass <u>equilateral triangle</u> and 12 as belonging to the subclass <u>right triangle</u>; (<u>c</u>) whether the subject knew the attribute values that differentiated between the subclasses <u>equilateral triangle</u> and <u>right triangle</u>; and (<u>d</u>) whether the subject knew that the set of 12 equilateral triangles were triangles and that the set of 12 right triangles were triangles.

The blocks that were presented to each subject and the questions that were asked follow.

Su 1 - The experimenter showed the 12 equilateral-triangle blocks and the 12 right-triangle blocks and asked, "What name can be given to all of these blocks? How are they all alike?"

Su 2 - The experimenter placed the 12 equilateral-triangle blocks in one group and the 12 right-triangle blocks in another group and asked these questions:

a. "Can you give me one name for these blocks [experimenter pointing to the equilateral-triangle blocks]? What name can you call these [experimenter pointing to the right-triangle blocks]?" The experimenter prompted to get the correct names for the two kinds of triangles. If the subject answered simply, "triangle," he was asked, "Can you tell me more?"

b. "How are these two groups of blocks different from one another [experimenter pointing to the 12 equilateral-triangle blocks, then to the 12 right-triangle blocks]?"

c. "Can these blocks be called triangles [experimenter pointing to the equilateral-triangle blocks]? Why (or Why not)? Can these blocks be called triangles [experimenter pointing to the right-triangle blocks]? Why (or Why not)?"

Questions Su 2b and c were asked only if the subject gave the answer "triangle" to Question Su 1.

The passing criterion here was Su 1 correct and two correct of Su 2a, b, and c.

Relationships in Statements of Principles

There is one key principle, or axiom, pertaining to equilateral triangle and four subsidiary ones that follow from it. The key axiom and the subsidiaries are these:

1. Equilateral triangles are similar in shape.
2. If the three angles of a triangle are equal, the three sides of the triangle are equal.
3. If the three sides of a triangle are equal, the three angles of the triangle are equal.
4. A line that bisects an angle of an equilateral triangle is perpendicular to and bisects the side opposite that angle.
5. The perimeter of an equilateral triangle is three times the length of one side.

Each of the principles involves either a relationship among other concepts and the concept of equilateral triangle or a relationship among the attributes of equilateral triangle and other concepts.

Four items were constructed to assess cognition of the first three principles. Items Pri 1 and 2, which follow, assess cognition of the principle that "equilateral triangles are similar in shape." Items Pri 3 and 4 assess the next two principles in the list.

Pri 1 - The experimenter showed a drawing of an equilateral triangle and said, "Here is a picture of a block. If each of the sides was made this much longer [experimenter indicating length of approximately one inch with thumb and index finger], would the shape be the same or different? Why?" If the subject said, "same," but did not give an adequate justification, this prompt was given: "Can you tell me more about why it would be the same shape?"

Pri 2 - The experimenter showed equilateral-triangle block EYKL and said, "Let's imagine drawing a three-sided figure on one side of the block [experimenter gesturing to indicate a three-sided figure on one side of the block]. The sides of the figure you draw will each be this long [experimenter gesturing to indicate length of the side of the block]. Tell me about the figure you just imagined drawing. Would the figure be the same shape as this block or different? Why? Would the figure be the same size as this block or different? Why?"

Pri 3 - The experimenter showed a drawing of a right triangle and said, "Here is a picture of a block. These corners [experimenter pointing to the three angles] are called angles. These angles are not all the same size. Suppose you drew a picture of another block and had all three angles the same size. What can you tell me about the sides of the figure in that drawing?"

Pri 4 - The experimenter showed right-triangle block RBNL and said, "Look at this block. Does it have any angles that are the same size as the angles of a square? Now think of a block that has three sides that are all equal in length. Would this block have any angles that are the same size as the angles of a square?" If the subject answered, "No," the experimenter asked, "How would the size of the angles be different from those in a square?" If the subject answered, "Yes," the experimenter asked, "How many angles would be the same size as the angles of a square?"

The criterion for passing the Principle subtest was one of the first two items and one of the last two items correct.

Problem Solving

The CLD model indicates that a concept attained at either the classificatory or formal level may be used in solving problems. There is no empirical evidence that would indicate that a concept must first be used in a principle before it can be used in problem solving. Thus, simple problems may be solved on perceptible bases without the use of a principle.

Five items were constructed to assess the use of the concept equilateral triangle in solving problems. One item called for the use of a ruler. The imprecise measurements of many subjects made this item unscorable. The other four items follow.

Pro 1 - This item started with the question, "Suppose I wanted to draw a three-sided figure and I wanted all the angles to be equal. If I made the first side five inches long, how long should I make the second side? How long should I make the third side? How did you decide how long to make the sides?"

Pro 2 - The experimenter showed a drawing of two right triangles and said, "Could you make a square by putting these two figures together in some way? How could you tell that they would (would not) make a square?" The experimenter then showed drawings of two equilateral triangles and said, "Could you make a square by putting these two figures together in some way? How could you tell that they would (would not) make a square?"

Pro 3 - The experimenter presented equilateral block EBNS and said, "The distance around the edge of this block [experimenter tracing around the edge of the block with his finger] is found by adding the length of the three sides. If the length of this side [experimenter pointing to one of the sides] is two inches, what is the distance around the edge of the block? How did you find your answer?"

Pro 4 - The experimenter presented equilateral block ERNS and said, "The total number of degrees in these three angles [experimenter pointing to the three angles] is 180 degrees. How many degrees do you think are in each one of these? Why?"

Items Pro 3 and 4 appear to be readily solvable by an older subject using the information provided directly in the items--perceiving the equality of sides or angles and performing simple computations. One cannot reliably infer that the subject used any of the principles even when considering the verbal responses to the "why" and "how" questions as previously stated. The responses are considered appropriate, however, for relating level of attainment as measured by the first four attainment subtests to problem-solving performance.

The criterion established for passing was any three of the four exercises correct. This criterion may appear somewhat less stringent than that for the Supraordinate-Subordinate subtest where Su 1 and two of Su 2a, b, and c had to be correct. It also might appear less stringent than that for the Principle subtest in which one of each set of two items was required for passing. A lower proportion of the total was required for Principle than for the other two uses because of the kinds of exercises used to measure the principles. As will be noted in Section III, a much higher percentage of the subjects passed the Problem-Solving subtest than either of the other two subtests.

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III
Results

The results are presented in accordance with the several sets of predictions implied by the CLD model. First, it was predicted that subjects would attain the four levels of attainment consecutively. Therefore, within a grade group, the percentage of children passing each successive level should decrease, but the percentage of children passing a given level should increase as a function of grade group.

Second, subjects who pass a particular level as their highest attainment should also pass the preceding lower levels.

Third, subjects who attain a concept at the formal level, in comparison with those who stop at the classificatory level, are predicted to use the concept more effectively in cognizing supraordinate-subordinate relationships, in understanding principles, and in problem solving.

Fourth, the first 11 items in the subtests were arranged according to an hypothesized level of difficulty. Therefore, the percentage correct within a grade group is expected to decrease across successive items, and the proportion of subjects correctly answering a given item should increase as a function of grade group.

Fifth, the CLD model implies that older subjects will perform better than younger subjects on the three uses--cognizing supraordinate-subordinate relationships, understanding cause-and-effect and other relationships, and solving problems. At the same time, no prediction is made concerning which of the three uses might be more or less difficult, since too little research has been done in this area to make predictions. Furthermore, difficulties were encountered in scoring the first two subtests because the three-dimensional blocks did not convey the concept equilateral triangle with mathematical accuracy. Therefore the results regarding uses must be interpreted in terms of this limitation.

Difficulty of the Four Attainment Levels

Table 3 presents the number and proportion of subjects in each grade group passing each of the four attainment levels. Within each grade group the percentage of subjects passing each of the four successive levels, concrete through formal, should decrease according to the CLD model. Similarly, a higher percentage of each grade group, preschool through tenth, should pass each level. Exceptions of five percent or more to these predictions are noted. Reading the proportions in the rows across Table 3, it is evident that at each grade level the proportions decrease across the four successive levels of attainment. Overall, the proportion of the 280 subjects who passed the various levels was .91 concrete, .82 identity, .40 classificatory, and .11 formal.

The second prediction was that the percentage of children passing each level of attainment would increase as a function of grade level. The information in the columns deals with this prediction. The only exception to this prediction involving a discrepancy of greater than five percent occurred at the classificatory level where a higher proportion of fourth-grade than sixth-grade children passed.

Chi-square tests were run to ascertain the significance of the proportion of individual grade groups from the proportion of all grade groups passing each of the four levels. The deviation was not significant for the concrete level; a very high percentage of all grade groups passed this level. The deviation for the identity level was significant at the .20 level; 70 percent or more of each grade group from kindergarten on passed this level. The deviations of each grade group from the overall proportion passing the classificatory level and the formal level were all significant beyond the .001 level.

Chi-square tests were also used between each set of grade groups to ascertain where

TABLE 3
NUMBER AND PROPORTION PASSING EACH OF THE FOUR ATTAINMENT LEVELS

Grade	Concrete	Identity	Classificatory	Formal
Pre				
Number	29	18	3	0
Proportion	.73	.45	.08	.00
K				
Number	30	28	7	0
Proportion	.75	.70	.18	.00
2nd				
Number	37	33	12	0
Proportion	.93	.83	.30	.00
4th				
Number	40	36	21	1
Proportion	1.00	.90	.52	.03
6th				
Number	40	38	18	0
Proportion	1.00	.95	.45	.00
8th				
Number	40	37	21	12
Proportion	1.00	.93	.52	.30
10th				
Number	39	39	29	18
Proportion	.98	.98	.73	.45
All Grades				
Number	255	229	111	31
Proportion	.91	.82	.40	.11

differences in attainment were significant at the .05 level or beyond. The results were as follows: preschool and fourth grade at the concrete level, preschool and second grade at the identity level, preschool and fourth grade at the classificatory level, and fourth grade and eighth grade at the formal level.

Passing Successive Levels of Attainment

Each successive level of attainment was hypothesized to require the use of one or more new cognitive operations. Items were written for the successive levels that would measure attainments requiring these operations. It was hypothesized that each subtest would be more difficult than the previous subtest because it entailed the use of an additional cognitive operation. Therefore, five patterns of passing and failing of the four successive levels of attainment (FFFF, PFFF, PPFF, PPPF, and PPPP) are consistent with the model.

Table 4 shows the number and proportion of each grade group that attained the successive levels according to the five patterns and also the number and proportion of each grade group that were exceptions to the five patterns. Information regarding those who performed according to the predicted patterns is given in the top five rows; the exceptions are given in the lower rows.

Two hundred fifty of the 280 subjects, or 89 percent, performed according to the predicted patterns; 30, or 11 percent, did not. More specifically, the following numbers conformed to each of the five predicted patterns: eight FFFF, 36 PFFF, 105 PPFF, 77 PPPF, and

TABLE 4
PASS-FAIL PATTERNS

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	Pre	K	2nd	4th	6th	8th	10th	All Grades
FFFF	6 .15	1 .025	1 .025	0 .00	0 .00	0 .00	0 .00	8 .02857
PFFF	15 .375	9 .225	5 .125	3 .075	2 .05	2 .05	0 .00	36 .12857
PPFF	12 .30	16 .40	21 .525	16 .40	20 .50	13 .325	7 .175	105 .375
PPPF	1 .025	3 .075	10 .25	19 .475	18 .45	13 .325	13 .325	77 .275
PPPP	0 .00	0 .00	0 .00	1 .025	0 .00	8 .20	15 .375	24 .08571
Subtotal conforming	34 .85	29 .73	37 .93	39 .98	40 1.00	36 .90	35 .88	250 .89
FFFP	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00
FFPF	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00
FFPP	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00
FPFF	4 .10	7 .175	1 .025	0 .00	0 .00	0 .00	1 .025	13 .04642
FPFP	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00
FPPF	1 .025	2 .05	1 .025	0 .00	0 .00	0 .00	0 .00	4 .01428
FPPP	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00
PFFP	0 .00	0 .00	0 .00	0 .00	0 .00	1 .025	0 .00	1 .00357
PFPF	1 .025	2 .05	1 .025	1 .025	0 .00	0 .00	1 .025	6 .02142
PFPP	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00
PPFP	0 .00	0 .00	0 .00	0 .00	0 .00	3 .075	3 .075	6 .02142
Subtotal not conforming	6 .15	11 .28	3 .08	1 .03	0 .00	4 .10	5 .13	30 .11

24 PPPP. The grade groups that conformed most closely to predicted patterns were second, fourth, sixth, and eighth.

No subject performed according to six of the 11 possible patterns of exceptions. Twenty-nine of the 30 exceptions were concentrated in four patterns as follows: 13 FPFF, four FPPF, six PFPF, and six PPFP. The grade groups represented by these exceptions were five preschool, nine kindergarten, two second, one fourth, three eighth, and five tenth. Unexpectedly, a large number of the preschool and kindergarten subjects, 14 of 80, failed the concrete level while subsequently passing either the identity level (11) or the identity and classificatory level (three). Understanding the instructions and following them may have been an important consideration here. The exceptions by the older age groups were represented heavily in the PPFP pattern with three eighth-grade and three tenth-grade students showing this pattern. The high percentage of subjects conforming to the predicted patterns is regarded as highly supportive of the major proposition of the CLD model, namely, that human beings attain concepts in a highly predictable pattern involving four successive levels--concrete, identity, classificatory, and formal.

Uses Passed by Attainers at the Classificatory and Formal Levels

Table 5 presents two sets of information: (a) the number and proportion of subjects who passed the classificatory level but not the formal level and who also passed the Supraordinate-Subordinate, Principle, or Problem Solving subtests and (b) the number and proportion who passed the formal level as well as each of the three uses. According to the CLD model, a higher percentage of those who pass the formal level should pass each use. Also, the percentage passing each use should increase as a function of grade level.

Table 5 shows that the total number of subjects passing the classificatory level but not the formal level was 87; another 31 subjects passed the formal level. Comparisons are now made related to the subtests for uses. Twenty-one percent of those who passed the classificatory level but not the formal level and 71 percent of those who passed the formal level also passed the Supraordinate-Subordinate subtest.

The Principle subtest was passed by 23 percent of the subjects whose top level of attainment was the classificatory level; 71 percent who passed the formal level also passed the Principle subtest.

Problem Solving was passed by 68 percent of the subjects whose highest attainment was the classificatory level. One hundred percent of the subjects who passed the formal level also passed the Problem Solving subtest.

The higher percentage of subjects who passed at the formal level and who also passed each of the various uses subtests, in comparison with those who passed only at the classificatory level, supports the prediction that attaining a concept at the formal level enables greater use of the concept than merely attaining it at the classificatory level. Six subjects who passed neither the classificatory level nor the formal level passed the Supraordinate-Subordinate subtest, and 14 subjects who passed neither the classificatory nor the formal level passed the Principle subtest. These outcomes do not fit the CLD model. However, these numbers are quite small in comparison with the numbers at the classificatory and formal levels who passed the two uses tests.

Difficulty of Successive Items Within the Levels

Table 6 gives the proportion of each grade group passing each item at the first three levels of attainment and also the first item at the formal level of attainment. As noted earlier in the chapter, the first item at the formal level called for giving the name equilateral triangle or an acceptable equivalent. This item had to be passed in order for a subject to pass at the formal level. No hypotheses were formulated with respect to the difficulty level of the items within the subtest at the formal level.

Several predictions were made concerning the difficulty of successive items. At the concrete level it was predicted that the items would become successively more difficult as the number of nonexamples increased and as the nonexamples had more attribute values in common with those of the target. At the identity level the prediction was that each additional change in the position of the target example would make the item more difficult. The three items at the identity level had the same number of nonexamples and the changes in position increased from one through three. At the classificatory level the prediction was that the second item would be more difficult for younger subjects and easier for older subjects inasmuch as older subjects should be able to use the information provided in the nonexamples of equilateral triangle.

In line with the predictions, the proportion of subjects answering a given item correctly

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TABLE 5
PATTERNS OF ATTAINMENT OF LEVELS AND USES SUBTESTS

Grade	Classificatory But Not Formal			Formal		
	Supraordinate-Subordinate	Principle	Problem Solving	Supraordinate-Subordinate	Principle	Problem Solving
Pre						
N Passing Level	3	*	*	0	*	*
N Passing Use	0			0		
Proportion	.00			.00		
K						
N Passing Level	7			0		
N Passing Use	0	1	0	0	0	0
Proportion	.00	.14	.00	.00	.00	.00
2nd						
N Passing Level	12			0		
N Passing Use	1	1	6	0	0	0
Proportion	.08	.08	.50	.00	.00	.00
4th						
N Passing Level	20			1		
N Passing Use	3	7	11	1	0	1
Proportion	.15	.35	.55	1.00	.00	1.00
6th						
N Passing Level	18			0		
N Passing Use	6	4	16	0	0	0
Proportion	.33	.22	.89	.00	.00	.00
8th						
N Passing Level	13			12		
N Passing Use	4	3	12	9	9	12
Proportion	.31	.23	.92	.75	.75	1.00
10th						
N Passing Level	14			18		
N Passing Use	4	4	14	12	13	18
Proportion	.29	.29	1.00	.67	.72	1.00
All Grades						
N Passing Level	87			31		
N Passing Use	18	20	59	22	22	31
Proportion	.21	.23	.68	.71	.71	1.00

*Not administered.

should increase as a function of grade level. At a given grade level the proportion of subjects answering correctly should decrease across successive items.

The successive grade groups did, in fact, perform better on each item with a few minor exceptions. Two exceptions are noted between the preschool and kindergarten groups where, on Co 3 and Id 8, the preschool subjects performed somewhat better than the kindergarten subjects. Four exceptions are noted for the fourth- and sixth-grade groups. The fourth graders did slightly better than the sixth graders on Items Co 5, Id 7, Id 8, and Cl 10. The final exception is on Id 6 where the sixth graders did better than the eighth graders.

TABLE 6
NUMBER AND PROPORTION PASSING EACH ITEM OF EACH ATTAINMENT LEVEL

Grade	Co 1	Co 2	Co 3	Co 4	Co 5	Id 6	Id 7	Id 8	Cl 9	Cl 10	Fo 11
Pre											
N	40	37	29	24	26	24	24	28	2	2	0
Pro	1.00	.93	.73	.60	.65	.60	.60	.70	.05	.05	.00
K											
N	39	40	24	33	27	31	31	25	5	5	0
Pro	.98	1.00	.58	.83	.68	.78	.78	.63	.13	.13	.00
2nd											
N	40	39	35	35	37	36	31	33	8	11	1
Pro	1.00	.98	.88	.88	.93	.90	.78	.83	.20	.28	.03
4th											
N	40	40	36	36	39	37	38	36	11	19	3
Pro	1.00	1.00	.90	.90	.98	.93	.95	.90	.28	.48	.08
6th											
N	40	40	39	39	36	40	33	31	10	16	6
Pro	1.00	1.00	.98	.98	.90	1.00	.83	.78	.25	.40	.15
8th											
N	40	40	39	39	36	37	40	36	11	19	24
Pro	1.00	1.00	.98	.98	.90	.93	1.00	.90	.28	.48	.60
10th											
N	40	40	37	39	37	39	40	38	19	24	28
Pro	1.00	1.00	.93	.98	.93	.98	1.00	.95	.48	.60	.70

These exceptions involved a relatively small number of subjects.

In general, the items at the concrete level were successively more difficult for each grade group. However, the first two items at the identity level were of about the same level of difficulty as the last two items at the concrete level.

The items at the classificatory level were more difficult than those at the identity level. In line with the prediction at the classificatory level, Cl 10 was easier than Cl 9 for the older subjects. On the other hand, Cl 10 was no more difficult for the younger subjects than was Cl 9.

It was stated earlier that acquiring the label for a concept may occur at any level but having the label is requisite for attainment at the formal level. No prior information was available that would permit predicting when this label might be acquired by the subject. However, the item is included for the purpose of indicating the extent to which subjects in the various grade groups had acquired the appropriate label.

As may be seen in Table 6, very few subjects had acquired the label equilateral triangle until the eighth grade. This phenomenon is apparently related to instruction in the school setting. Directed study of geometry, including equilateral triangle, was introduced into the curriculum during the seventh grade in the school system where the subjects were enrolled.

Difficulty of Three Uses

Interesting results were found regarding the difficulty of the three uses. Table 7 presents the number and proportion of subjects passing each of the three concept uses: supraordinate-subordinate, principle, and problem solving. The prediction here was that performance would increase as a function of grade. No exceptions to this were noted;

TABLE 7
NUMBER AND PROPORTION PASSING EACH CONCEPT USE

Grade	Supraordinate-Subordinate	Principle	Problem-Solving
Pre			
Number	0	*	*
Proportion	.00		
K			
Number	0	2	1
Proportion	.00	.05	.03
2nd			
Number	2	3	23
Proportion	.05	.08	.58
4th			
Number	4	8	25
Proportion	.10	.20	.63
6th			
Number	8	8	36
Proportion	.20	.20	.90
8th			
Number	13	15	37
Proportion	.32	.38	.93
10th			
Number	19	20	40
Proportion	.48	.50	1.00
All Grades			
Number	46	56	162
Proportion	.16	.23	.68

*Not administered.

however, the fourth graders did as well as the sixth graders on the Principle subtest.

A point of interest is that 16 percent of all subjects passed the Supraordinate-Subordinate subtest; 23 percent the Principle subtest; and 68 percent the Problem Solving subtest. These results must be interpreted in terms of the items used, the criteria established for passing the three subtests, and the proportion of subjects who passed each item.

Table 8 presents the proportion of each grade group that passed each item of each usage subtest, starting with Supraordinate-Subordinate. Items Su 2a and b were more difficult than items Su 1 and Su 2c. Su 1 and Su 2c required the subject to call equilateral-triangle blocks and right-triangle blocks "triangles," while Su 2a and b called for naming and discriminating between equilateral triangles and right triangles.

In the Principle subtest Item Pri 3 was easier than the other items, and Pri 1 and Pri 4 were quite difficult. Pri 1 and Pri 2 dealt with the key principle "equilateral triangles are similar in shape." Pri 3 and Pri 4 each dealt with one of two other principles. The passing criterion here was one of the first two and one of the last two items correct.

The items in the Problem Solving subtest were relatively easy. Pro 2 was quite easy for all the grade groups. Eighty-eight percent of the second graders passed it. Pro 4 was the most difficult across the grade groups. On the whole the Problem Solving items were

TABLE 8
NUMBER AND PROPORTION PASSING EACH ITEM OF EACH CONCEPT USE

Grade	Su 1	Su 2a	Su 2b	Su 2c	Pri 1	Pri 2	Pri 3	Pri 4	Pro 1	Pro 2	Pro 3	Pro 4
Pre												
Number	9	0	0	28	*	*	*	*	*	*	*	*
Proportion	.23	.00	.00	.70								
K												
Number	20	0	0	29	4	3	6	0	9	24	4	1
Proportion	.50	.00	.00	.73	.10	.08	.15	.00	.48	.60	.10	.03
2nd												
Number	24	0	1	33	1	3	13	0	35	32	23	1
Proportion	.60	.00	.03	.83	.03	.08	.33	.00	.88	.80	.58	.03
4th												
Number	25	0	4	33	3	7	29	0	37	30	34	15
Proportion	.63	.00	.10	.83	.08	.18	.73	.00	.93	.75	.85	.38
6th												
Number	32	0	11	35	3	8	32	1	40	35	35	31
Proportion	.80	.00	.28	.88	.08	.20	.80	.03	1.00	.88	.88	.78
8th												
Number	32	12	13	37	6	14	33	8	37	38	38	32
Proportion	.80	.30	.33	.93	.15	.35	.83	.20	.93	.95	.95	.80
10th												
Number	36	14	15	38	10	24	38	17	40	39	40	39
Proportion	.90	.35	.38	.95	.25	.40	.95	.43	1.00	.98	1.00	.98

*Not administered.

easier for younger subjects than anticipated. Apparently they perceived that the sides or the angles of the equilateral triangles, given as part of the item, were equal. They then performed a simple addition such as 2 + 2 + 2 or a division such as 180 ÷ 3. Most of the subjects below Grade 8 who solved these exercises did not have the concept of <u>equilateral triangle</u> at the formal level, and they did not understand or use any of the principles as stated.

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IV
Summary and Discussion

Short tests dealing with equilateral triangle were developed and administered to assess the performance of 40 subjects in each of seven grade groups--preschool, kindergarten, and Grades 2, 4, 6, 8, and 10. The tests were administered individually and in a random order to the subjects. The test materials included 36 three-dimensional blocks, the larger surfaces of which corresponded to equilateral triangles, right triangles, and squares. The test items were scored as correct or incorrect, and criteria for passing were established. The criteria were intended to be equally strict for each of the seven tests which assessed the four levels and the three uses.

Items were developed to measure attainment of each level, the levels being operationally defined by the CLD model. Based on the rationale that successive levels involve additional cognitive operations which emerge with learning and maturation, it was predicted that passing the items at each successive level would be more difficult than passing the items at the prior level. Two major hypotheses emerged from the reasoning: (a) the proportion of students passing a given level would increase as a function of age and (b) at a given grade level the proportion of students answering correctly would decrease across successive levels.

The results pertaining to grade groups, levels, and uses supported the major hypotheses and also others based on the CLD model. With few minor exceptions the percentage of students passing a given level of concept attainment increased across grade groups. Also, within each grade the percentage of subjects passing decreased across successive levels.

The CLD model also implies that subjects should manifest any of the following patterns of attainment: FFFF, PFFF, PPFF, PPPF, or PPPP. This prediction was confirmed to a greater degree than anticipated. The results were as follows: 89 percent of all the subjects conformed to the predicted patterns and only 11 percent did not. Furthermore, 14 of the exceptions showed either of two patterns, FPFF or FPPF. It is thought that the younger subjects may not have completely understood the instructions for the concrete level.

The CLD model also implies that a higher proportion of subjects who attain the formal level, in comparison with subjects who attain only the classificatory level, will pass the three uses. Here, the results for the classificatory and formal attainment levels are: .21 and .71 for supraordinate-subordinate relationships, .23 and .71 for principles, and .68 and 1.00 for problem solving. Subjects who can name the concept, define it, discriminate and name the defining attributes, and evaluate examples and nonexamples perform better than do those who can classify exhaustively but who fail on one or more of the preceding requirements for attainment of the formal level. Having appropriate language is the important requirement for passing the formal level and, therefore, is probably also important in using a concept.

It was predicted also that an increasing proportion of each successive grade group would be able to use their attained concepts. The proportions for understanding principles were: .05, kindergarten; .08, second grade; .20, fourth grade; .20, sixth grade; .38, eighth grade; and .50, tenth grade.

Predictions were made regarding the difficulty of items within the first three attainment levels. Increasing the number of the nonexamples presented with the target example and increasing the number of attribute values in common between the nonexamples and the target example tended to increase the difficulty of the items at the concrete level. Increasing from one to three the changes in orientation of the target block between the initial and test display tended to increase the difficulty of the items at the identity level. We found that the first two items of the identity level were about as easy as the last two at the concrete level. One may tentatively infer from this and other similar information that there is continuity among the levels and among the operations, rather than discontinuity.

While this cross-sectional study supports the major predictions based on the CLD model, two limitations are noted. The first deals with the test battery and the second with conclusions warranted from cross-sectional information.

The items and three-dimensional examples and nonexamples appeared to be most appropriate for assessing attainment at the concrete and identity levels. It is recognized, however, that line drawings are better representations of geometric figures than are three-dimensional blocks. The older subjects of this study may already have received instruction to the effect that three-dimensional figures are not plane figures.

Despite the fact that the three-dimensional blocks may have been highly appropriate at the concrete and identity levels, the instructions seemed difficult for the youngest subjects to comprehend. This may have partially accounted for the relatively large number of subjects who failed the concrete level but passed the identity level.

A final possible weakness of the test relates to the scoring of items. Certain items at the formal level and the uses called for the subject to give the label equilateral triangle or an acceptable equivalent. Questioning of the subject was used to elicit the label. The experimenter had reasonable assurance that a subject who did not give the label actually did not have the label in his spoken vocabulary; however, there was not complete certainty regarding this.

These preceding limitations have been dealt with in constructing a second version of the battery which uses line drawings instead of three-dimensional figures, changes the instructions somewhat, and employs multiple-choice items which give the correct label as one of five choices (Klausmeier, Ingison, Sipple, & Katzenmeyer, in preparation).

Another limitation of the study deals with the kind of conclusions that can be drawn from cross-sectional studies. One cannot infer from information gathered at a single point in time when the partial or full mastery of any developmental function may occur in individuals. Longitudinal study with successive measurements of the same individuals is required to do this. In a longitudinal study one is able to ascertain, for example, the year at which full attainment of each level first occurs. One can also identify at that same time the extent to which the individual can use the concept. Commonalities among individuals with respect to the first time of full mastery of the various levels may be observed. These, in turn, can be related to the uses.

The results of the present cross-sectional study appear to warrant a longitudinal study. A four-year longitudinal study that starts with 50 boys and 50 girls of each of four grade groups--kindergarten, third, sixth, and ninth--was initiated by Klausmeier and his associates at the Wisconsin R & D Center during the second semester of the 1972-73 school year.

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