ED 088 704 SE 017 445

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TITLE Mathematics Forum, May 1973.
INSTITUTION University of the South Pacific, Suva (Fiji). School of Education.
SPONS AGENCY United Nations Educational, Scientific, and Cultural Organization, Paris (France).
PUB DATE May 73
NOTE 88p.

EDRS PRICE MF-$0.75 HC-$4.20
DESCRIPTORS Cultural Factors; *Curriculum; *Geometric Concepts; *Instruction; *Mathematics Education; *Number Concepts; Number Systems

ABSTRACT

This is a magazine for teachers of mathematics in the South Pacific who teach at the Form I level or above. Materials which can be used in the classroom are included in most of the articles. Articles are on instructional strategies, curriculum developments, interesting problems, puzzles and math lab activities. Many articles are particularly relevant to the interests and situations of the South Pacific territory schools. In this issue one article addresses itself to the problems entailed with teaching mathematics in a second language different from the native language. Another article presents the linguistic structure of the natives' base ten numeration system. The syntactic structure reveals the manner in which number concepts are formulated with the culture of this area. A third article discusses the curriculum development project that has been instituted in the South Pacific Territory. (JP)

ED 088704

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Mathematics Forum is a magazine for all teachers of Mathematics in the South Pacific who teach at the Form I level or above.

It is a cooperative venture:

- The U.N.D.P. Curriculum Development Unit is financing the production.
- U.S.P. is financing the distribution to the various island territories.
- Departments of Education are handling the distribution to teachers within their own territories.

Mathematics Forum does not reflect the views or policies of any Education Department or other Educational body. Any opinions expressed are the opinions of the editors or contributors who are acting as individuals.

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EDITORIAL

Although we have a considerable amount of material for this edition of the magazine, it seems that its future is not assured.

We have not had many suggestions about its development nor have we received many articles recently. It appears that in many schools the magazine is not read. Perhaps if there was a charge the magazine would be valued more but then we have to produce all the apparatus necessary for collection of subscriptions etc. and this we don't want to do.

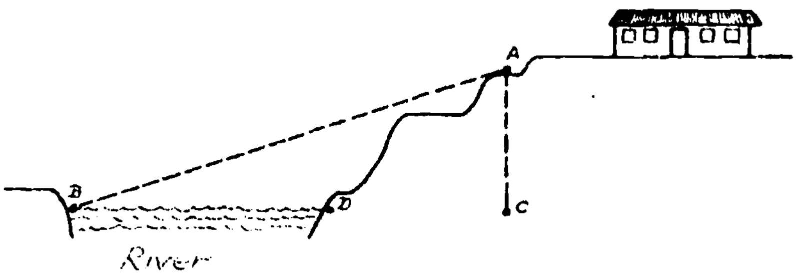
How can we get more teachers to read the magazine?

Please encourage any teachers of mathematics to read it and to help make it of more value to the South Pacific.

HOW HIGH THE RIVER

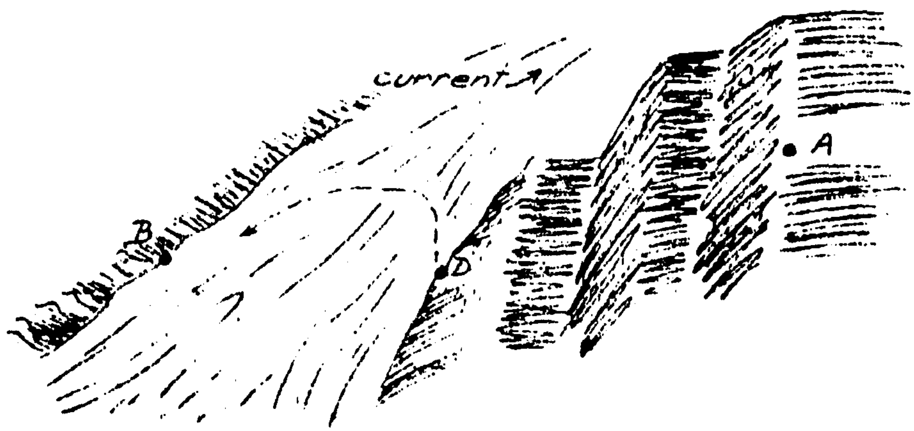
All over Fiji, rivers rose high at the time of Hurricane Bebe. The Sigatoka River was no exception. On Tuesday morning we watched the river rise from its dry season level to its disastrous flood level. For a long time after, many teachers, villagers, and people around the Mission asked just how high did the river rise?

This was Problem Number One. Some time later I put the question to my Class 6. Their estimates ranged from 10ft to 50ft, but most settled for around 25ft. So we decided to find out. The equipment available was several lengthy pieces of string all knotted together, a blackboard protractor, a stone and a battered chain tape.



The method seemed simple enough (though perhaps a bit sophisticated for the children at that time) - stretch the string from A to B, A the level of the water at time of flood, B the level of the water on day of calculation; measure the angle of elevation at B; find actual length of string, and from scale drawing measure AC.

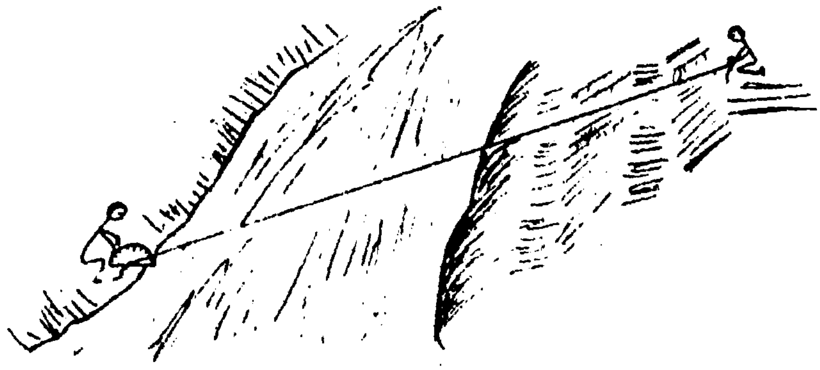
Problem Number Two: how to get string across the river. We decided that Merewai should stand on our bank of the river, while the boys took the other end of the string across the river in the canoe. They first poled up-stream to counter the very strong current, and so arrived at B. However the river was too strong and all 200ft of string meant to go across the 90ft wide river ended in it, and well down.



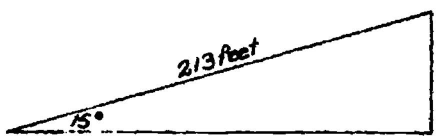
Niko, a fairly strong boy, decided he could swim across the river holding the string, and since this was a fairly common practice, we let him try. However, he too miscalculated and finished well down-stream, leaving the string behind him.

The third method proved more successful. Samuwela stood on the river bank, attached the string to a stone, and with an almighty heave, threw the stone and string across the river.

We then pulled the string up to point A and pulled it tight to minimize droop (all knots held) and measured the angle of elevation at B.



The string was later laid across the playing field, and by measuring it with a chain tape, it was 213ft. The angle measured 15°.



The next step was to make a scale drawing on graph paper. From that we figured the river rose 54ft. We than had to add another 4ft to take in the difference of levels prior to flood and at time of measurement. The river rose 58ft altogether.

I am not sure what to allow for error in measurement. A 2° error put the rise at about 45ft (+ 4ft) still making the rise around the 50ft mark. Little wonder that many people lost their food crops.

Well, it was an interesting morning's maths, but our only hope is that the problem doesn't rise again.

Laurie Williams
Bemana Mission School
1972

HOW DISCOVERY METHODS HELP US IN THE LEARNING OF MATHEMATICS

Editorial note: This is an abridged version of an essay produced by a third year Diploma student at U.S.P. in 1972.

One of the most important aims of mathematics teaching is to provide mathematical experiences which enable pupils to make observations, to discover patterns and relationships, to develop concepts, to draw logical conclusions, to express thoughts accurately, and to form generalisations. Secondly, they must have that understanding and ability to apply these principles to wider fields.

The question is "How can these aims be achieved?" What method should the mathematics teacher use? There is no single best approach to the teaching of any subject. There are so many combinations of approaches which can be used for effective teaching but this largely depends on the nature of the teacher.

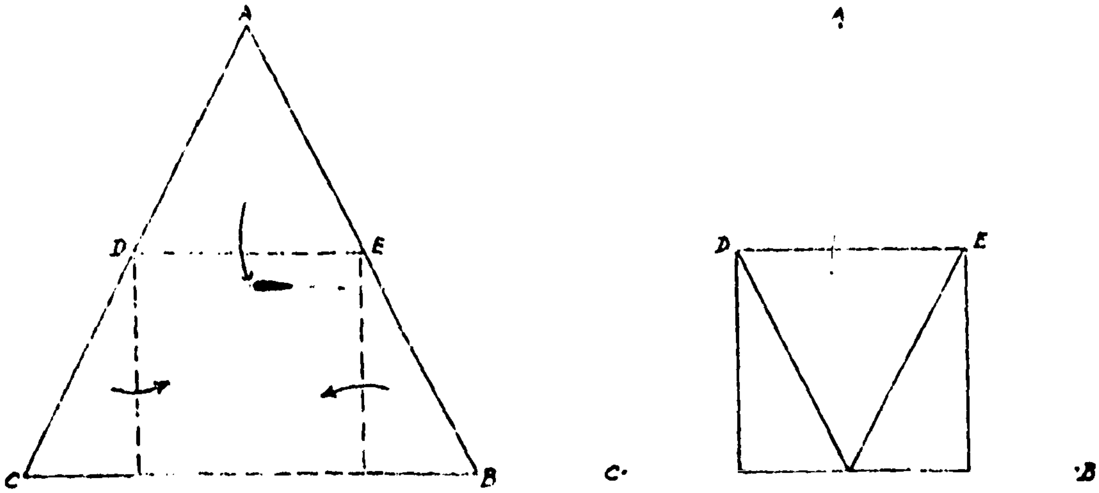
The best teaching is that which is intellectually stimulating to both students and teacher. If a subject is dull, one can immediately guess that the teacher is approaching it from the wrong angle. "Variety is the spice of life", and so it is in the hands of the teacher to make the lesson more interesting and lively for the students. The teacher must choose the content which will be at the standard understandable by the pupils. That is, the work should fit the capacity of the child. When a teacher stands in front of a class and starts talking, the students get the idea that whatever the teacher says is true and they tend to take in every word said by the teacher. But this is not the best thing to do. Moreover, a teacher must not be talking all the time or else he becomes a "jug" merely pouring knowledge into the "mug" - the child. One must not forget the fact that children are inquisitive. They are always curious to find out things for themselves and if their teacher uses "chalk-and-talk" method in all his lessons, then these pupils will surely become 'sloppy' thinkers.

However, if we consider a maths lesson, we notice that there are so many things which the children can do by themselves under the teacher's guidance. Since pupils learn, remember and are able to apply best those things that they have discovered for themselves, the teacher should put them in situations where the students can discover things for themselves. By doing this a child gets the opportunity to discover facts, principles and relationships for himself rather than the teacher telling him everything. This will lead them to invent their own methods for solving problems.

Suppose, in geometry, the teacher is dealing with theorems concerning properties of triangles. If he asks the class "to prove that the sum of the interior angles of a triangle is 180 degrees", then obviously there is nothing left for the class to discover. But if the problem is worded differently, such as "Can you discover anything special about the interior angles of a triangle?", the students will try their best to find an answer for themselves and compete with their classmates to see who is the first one to produce a good answer.

At this stage, I would mention something about visual aids. These visual aids and manipulative materials are very useful during the concept-building phase of each lesson. Children learn more from these aids than from the teacher's lecture on the same topic.

While we are still on this theorem, one of the very simple visual aids which came into my mind is as follows:

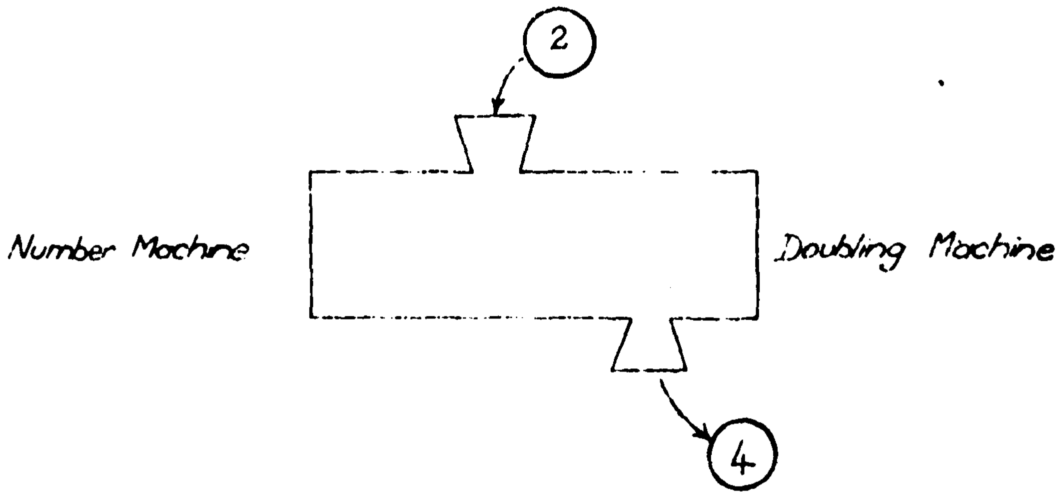


If we fold the three corners of the triangle, the three angles come together to form adjacent angles on a straight line. Since their sum is 180 degrees, then one can say that the interior angles of a triangle add up to 180 degrees. Also from the same piece of apparatus we can prove the mid-point theorem which says that the line joining the mid-points of two sides of a triangle (DE) must be parallel to the third side (BC) and also equal to one-half of its length.

When dealing with constructions, such as construction of triangles, angles of 30°, 60°, 90°, perpendicular bisectors, etc., I came across a teacher who told the class each step and did the actual construction on the blackboard. During this particular lesson, most of the students, especially at the back of the classroom, were not paying any attention to what he was saying. On the next day, he asked one of the pupils to construct an angle of 60° on the blackboard and explain to the class how to do it. I was not surprised when he said he could not do it. I personally feel that if the teacher had asked the children to find the construction by themselves, then surely most of them would have remembered the solution.

Problems, as the mathematicians say, are the "very flesh and blood of mathematics" and should appear at every stage of teaching this subject. The problems must be real and significant - they must arise from the interests and activities of the children. Puzzles are of great help. Children who know nothing about mathematics will tend to appreciate this subject through puzzles, magic squares, etc.

One can always start a lesson with a puzzle. For example, when teaching a topic like 'Relations and Functions', the teacher is in a difficult position because there is very little chance of making the lesson lively. However, the teacher can start off doing some work on number machines. Let us look at one example.



The above machine does the job of doubling any number that is put into the machine, i.e. when we put the number 2 inside the machine, the number which comes out is 4. So if we give this problem to the class, they can build up a set of numbers, i.e. $\{(2,4), (3,6), (1,2) \ldots\}$. The machine can be used in another way. The input number and the output number can be given, and the class can be asked to explain the function of the machine. This will lead them to find the relationship between numbers and seems a good introduction for the topic.

Geoboards are also very useful pieces of apparatus in the teaching of mathematics, especially 'AREAS'. The children can make various shapes by using rubber bands and they can compare their sizes by calculating their areas and so on.

There are many more methods of teaching this subject through the use of these simple visual aids and apparatus which help the students to find out facts for themselves.

A quiz is another way of finding out how much the students know. It can be held once a week or twice a month on one of the topics that they have been taught. This will enable them to keep up with their work and to do some research and in some cases, practical observations on a problem which may be one of the questions in the quiz. Most of the teachers think it is a waste of time and ignore them. But it is more interesting than a lecture.

I have explained how some visual aids can be used in a maths lesson and how the students can use apparatus to solve problems. When faced with a problem, a child at first will not know how to go about it. But if he draws a diagram to illustrate what the problem asks for or if he thinks that some apparatus are needed, then he should be in a position to solve the problem by manipulating these objects. Apparatus and booklets are being prepared by local teachers, lecturers, Education Department officials and the U.N.D.P. Curriculum Development Unit. The teachers who use these texts must be proud to say how the teaching of maths has been made easier with the help of these materials, especially at Form I and Form II levels.

Before I finish off this essay, I would like to include some puzzles which will be of interest to some students.

1. Magic Squares

Have each pupil draw four 3 by 3 squares as shown below. The 3 small squares in each row and in each column are to be filled with arrangements of the given set of 3 numerals such that the sum for each row and each column is 6. One typical solution is given for each set.

Given set: {1,2,3}

1	2	3
2	3	1
3	1	2

Given set: {1,1,4}

1	1	4

Given set: {0,1,5}

Given set: {0,2,4}

Fill in the rest of the squares. Can you use different sets of numerals such that the sum for each row and each column is 7, 8, 9, etc.?

2.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

(a) To solve 33 + 5 = [ ], find 33 in the table and move 5 spaces to the right.
(b) To solve 42 + 10 = [ ], find 42, and move down 1 space.
(c) To solve 34 + 13 = [ ], find 34, move down 1 space and move 3 spaces to the right.

Can you suggest a rule for subtraction of numbers? Verify your answers by using examples from the table.

3. Place 20 tokens (bottle caps, coins, sticks or other suitable objects) in a straight line. Explain that this is a "take away" game. Two players take alternate turns and play by the rule "at each turn you may take 1, 2 or 3 tokens". The player who takes the last token wins the game.

When introducing the game, the teacher should be one of the players, a pupil the other player, and the rest of the class should observe to see if they can discover how to win always. Let the pupil start. At each of your turns take enough tokens to make the total taken by both players equal to 4 for that turn. If the pupil takes 1, you take 3; if he takes 2, you take 2. You will always win provided you don't start and as long as the other player is unaware of the winning pattern.

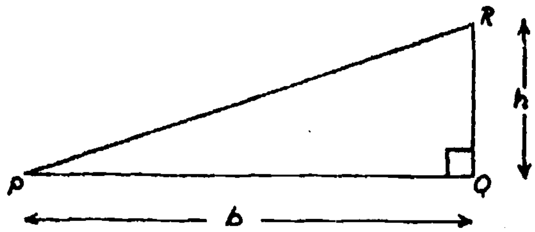
Once a pupil has discovered the winning pattern, he takes the teacher's place.

The total number of tokens can be any multiple of 4, not necessarily 20.

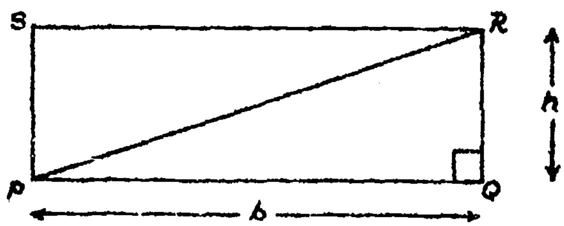
Chandra Mani

AREA OF A TRIANGLE
SOME PROOFS OF THE FORMULA $A = \frac{1}{2} bh$

Right angled triangle



This triangle PQR is a half of the rectangle PQRS.



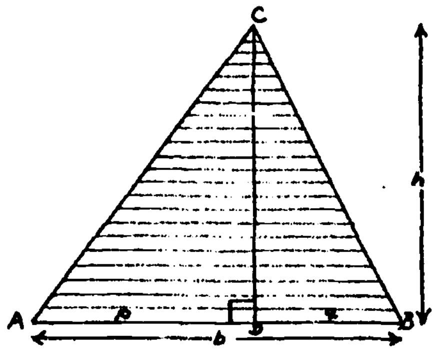
Area $\triangle$ PQR = ½ (area rectangle PQRS)

= ½ (bh units^2)

= ½ bh units^2

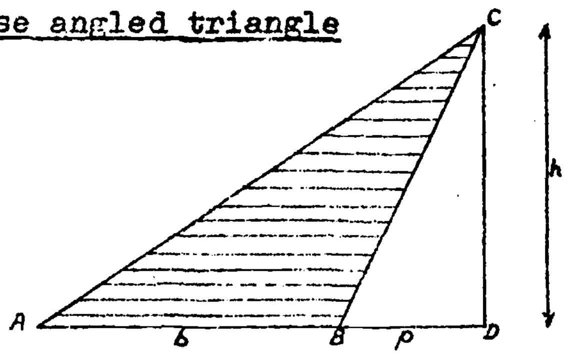
General triangle (algebraic proofs)

Acute angled triangle



$$
\begin{aligned}
\text{Area } \triangle \text{ ABC} &= \text{Area } \triangle \text{ ADC} + \text{Area } \triangle \text{ CDB} \\
&= \tfrac{1}{2}\text{ph} + \tfrac{1}{2}\text{qh units}^2 \\
&= \tfrac{1}{2}\text{h (p + q) units}^2 \\
&= \tfrac{1}{2}\text{hb units}^2 \\
&= \tfrac{1}{2}\text{bh units}^2
\end{aligned}
$$

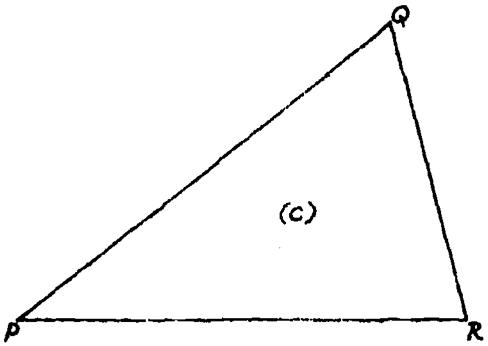
Obtuse angled triangle

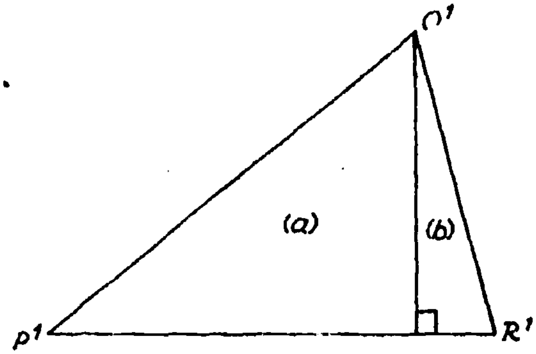


$$
\begin{aligned}
\text{Area } \triangle \text{ ABC} &= \text{Area } \triangle \text{ ADC} - \text{Area } \triangle \text{ BCD} \\
&= \tfrac{1}{2}\text{(b + p) h} - \tfrac{1}{2}\text{ph units}^2 \\
&= \tfrac{1}{2}\text{bh} + \tfrac{1}{2}\text{ph} - \tfrac{1}{2}\text{ph units}^2 \\
&= \tfrac{1}{2}\text{bh units}^2
\end{aligned}
$$

<u>General triangle</u> (geometrical proofs)

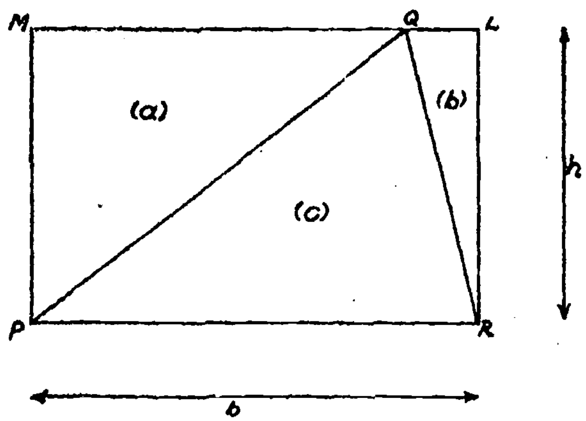
<u>Method 1</u>





Take two identical cardboard triangles PQR and $P^1Q^1R^1$.

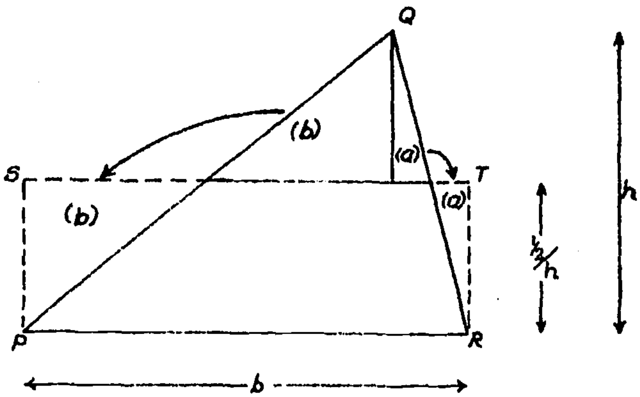
Cut the second triangle as shown and rearrange the pieces like this.



$$2 \times \text{Area } \triangle \text{ PQR} = \text{Area rectangle PRLM}$$

$$\text{Area } \triangle \text{ PQR} = \tfrac{1}{2}\text{bh units}^2$$

Method 2



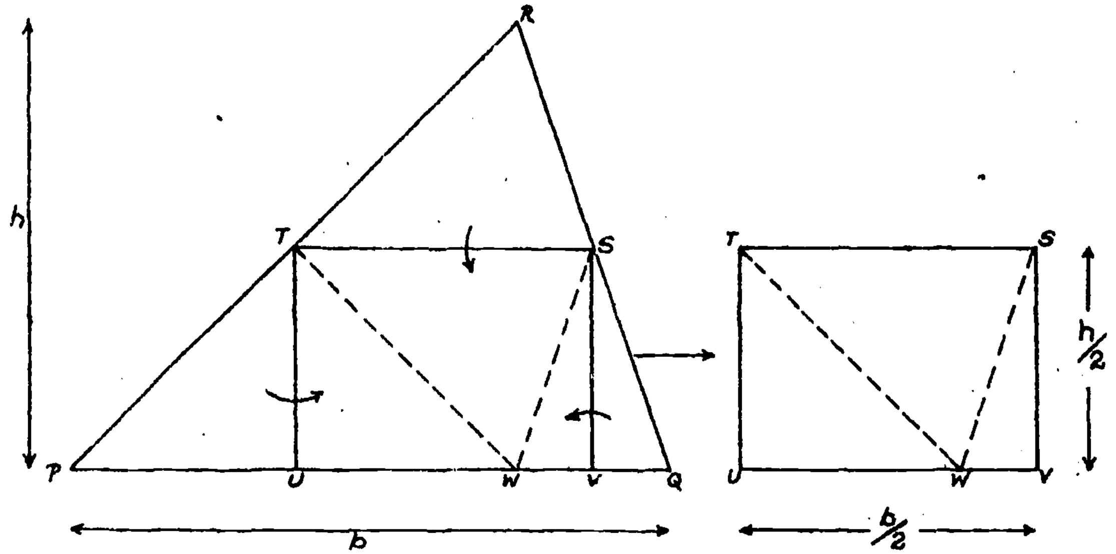
Cut the △ PQR and rearrange the pieces as shown.

Area △ PQR = Area rectangle PRTS

= $b \times \frac{h}{2}$ units^2

= ½ bh units^2

Method 2 (paper folding)



Folding the triangular pieces about ST, TU and SV we see that:

$$\text{Area } \triangle PQR = 2 \text{ Area rectangle UVST}$$
$$= 2 \times \frac{b}{2} \times \frac{h}{2} \text{ units}^2$$
$$= \tfrac{1}{2} bh \text{ units}^2$$

E. H. Leaton
U.N.D.P., U.S.P.

Editors: Do readers know of any other simple methods for proving mensuration results like these?

MATHS FOR CHILDREN WHO DROP OUT OF SCHOOL

The majority of children who enter Primary schools in Fiji drop out at the end of Primary school or at the end of the Form 4 year. Only a minority of our entrants continue their schooling beyond Form 4. I feel sure that the same sort of thing occurs in other countries in the South Pacific region.

How relevant are our mathematics syllabuses to the needs of these students (who are the majority of our students) when they leave school and go looking for a job? Our present syllabus is designed to cover the topics that are needed for the New Zealand School Certificate Examination, and it seems likely that when a South Pacific Examinations Council comes into existence in the not-too-distant future the syllabus will be designed around the requirements of their equivalent of the New Zealand examination. But the majority of our students do not sit the School Certificate examination, and it is difficult to see how the syllabus meets the needs of these students.

At present the syllabus covers the following major areas of mathematics:

algebra
geometry (either Euclidian or transformational)
statistics
trigonometry
computation (such as areas and volumes, and decimals).

How many of the students who drop out at the end of Form 4 have any use for algebra, geometry or trigonometry later in life? Not many. Most of them are only too happy to forget these subjects, and they manage to live the rest of their lives (successfully) without any further need for these subjects.

So far as these students are concerned, much of the mathematics we teach them is a waste of time. And, worse still, the students feel that they are wasting their time - how often does the teacher of a lower-stream mathematics class have to answer typical questions like

Why are we doing this topic?
What use is mathematics?
Why do we have to learn mathematics?

Questions like these are difficult to answer because much of the mathematics we now teach is irrelevant to these students.

What can be done about this?

One popular solution to this problem is to graft extra topics onto the syllabus such as 'banking', 'hire purchase' and 'social arithmetic', in the hope that these apparently everyday topics will give the syllabus a flavour of relevance. The hope is, I think, misplaced for two reasons. Firstly, topics like 'banking' and 'hire purchase' are arid in the sense that it is very difficult for the teacher to make them interesting to the students. And secondly, these topics are only marginally more relevant than topics such as algebra and trigonometry because the only contact most leavers will have with banks is when they go there to cash a cheque (which hardly needs explanation) or to raise a loan (banks employ experts to explain to customers the procedure for raising a loan), and if they go to a hire purchase firm for goods I am sure the salesman would be only too happy to explain to a potential customer the ins and outs of a hire purchase agreement.

Is there an alternative solution to the problem?

I suggest that we look at the problem from a different point of view and ask ourselves 'What kinds of attitudes should students who leave school at the end of Form 4, or earlier, have toward mathematics?' They should leave school

feeling that they know some worthwhile mathematics,
confident of their ability to solve mathematical problems,
flexible in their thinking about mathematics and other subjects.

This goal could be achieved using a combination of two approaches to the teaching of mathematics.

In the first place, mathematics should be taught in conjunction with other school subjects such as Science, Social Science and Technical work. (Too often mathematics is taught as an isolated exercise which has no application or significance in other subjects.) For example, a topic such as statistics could be taught from one point of view in the mathematics class, from another point of view in the science lab (e.g. designing an experiment gathering ecological data), from a third point of view in the Social Science room (e.g. the interpretation of trade figures), and from yet another point of view in the technical workshop. An approach like this obviously needs careful planning so that the teachers of the various subjects are working in close harmony and so that the children's understanding of the topic is reinforced by seeing the topic from various viewpoints. The topic would then immediately become more meaningful to the children because they can see its relevance. Furthermore, this kind of approach also obviates an educational problem that must concern teachers at all levels, namely the tendency of school children - particularly secondary school children - to think of the subjects they learn as existing in separate compartments, with no application outside the specific areas in which the subjects are taught.

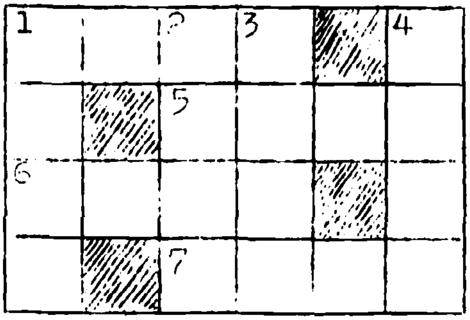
I think I would go so far as to suggest that, for a class of potential leavers, a mathematical topic that cannot be taught in the integrated manner sketched above should not be taught at all.

In the second place, our teaching approach should be designed to encourage students to do their own mathematics, and to explore topics that they find particularly interesting. Students who make their own mathematical discoveries are more likely to remember what they have learned, they are more likely to understand what they are doing because they are working at their own pace and their own level, and they are more likely to enjoy what they are doing because they are doing what they enjoy. They will also be more confident of their own ability to understand and cope with mathematics for the reason that the mathematics they have done is the product of <u>their own</u> ingenuity and skill. In this teaching situation the teacher's role becomes the role of a person who helps where necessary, encourages students who are in difficulties, and shepherds back students who have strayed from their goal.

Perhaps if we tried this kind of approach with our drop-outs we would at least produce leavers who had some understanding and appreciation of mathematics, and who did not regard mathematics as just one more of the subjects they found incomprehensibly difficult.

R. H. Metcalfe
USP

A MINI CROSS NUMBER PUZZLE



CLUES ACROSS

1. The cube of a whole number.
5. The number of square inches in a square yard.
6. The number of cubic inches in a cubic foot.
7. The number of millimetres in a metre.

CLUES DOWN

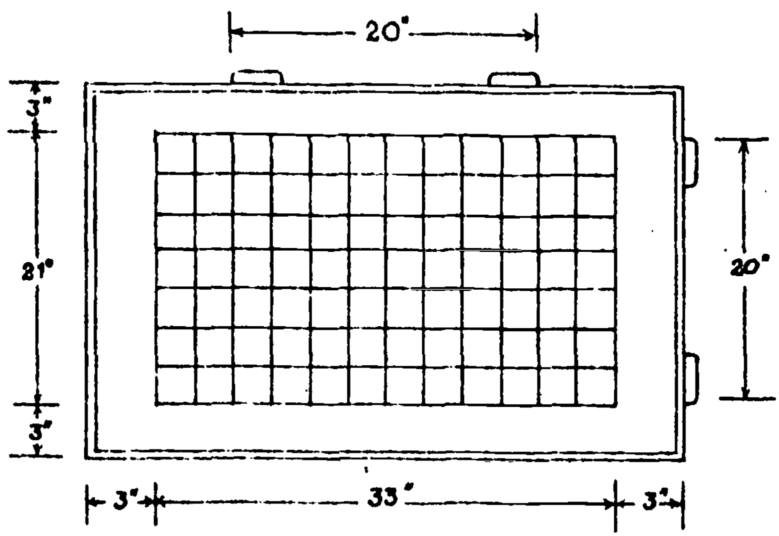
1. A number which is unchanged if the digits are reversed.
2. A prime number.
3. The number of feet in a mile.
4. The number of seconds in an hour.

CONSTRUCTION OF A BLACKBOARD GRAPH BOARD

(1) This should be of a similar shape to the graph paper used by students, but on a larger scale, say 3" squares, e.g. paper 11" x 7", graph board 33" x 21".

(2) The board can be used horizontally or vertically.

CONSTRUCTION PROCEDURE



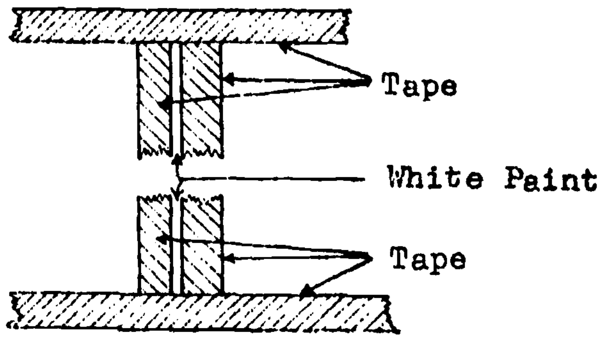
(1) Cut piece of old black board (or other board and paint it with blackboard paint) 27" x 39".

(2) Attach board to a frame for strength.

(3) Draw up graph grid, using 3" intervals, in pencil. Leave a side margin all around for writing labels on axis, etc.

(4) Place sticky tape (masking or cellotape) down each side of all vertical lines leaving a $\frac{1}{16}$" gap. Tape off both ends of the lines.

(5) Paint in all the vertical lines thickly with white paint.



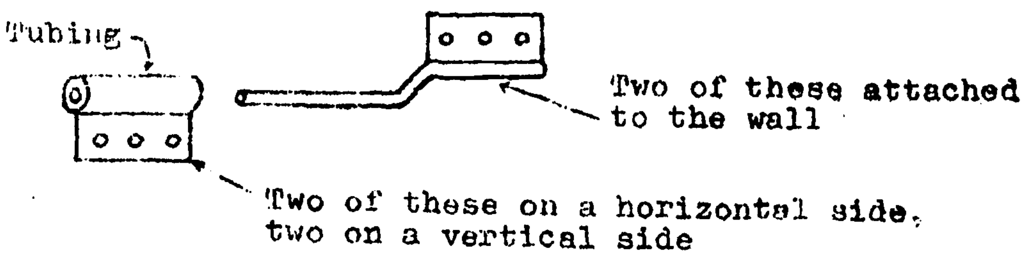
(6) Remove the tape while the paint is wet.

(7) Leave the paint to dry completely (1-2 days).

(8) Repeat steps 4 to 7 for the horizontal lines.

(9) Paint a white border also $\frac{3}{8}$" wide.

Detail of the hanging brackets:



(10) The board can be attached to a wall with the brackets illustrated above made of welded steel or hooks and eyelets could be used. If brackets or eyes are used place them the same distance apart.

From a Nauru contributor

TEACHING MATHEMATICS IN A SECOND LANGUAGE

I like to define language in two broad categories - 'basic functional language' and 'special language'.

LANGUAGE	
Basic Functional Language	Special Language
The basic language we need to communicate any ideas. The most important elements in this section are the structural items of the language.	The terms that belong to a specialist field and that have an explicit meaning when applied to this field.
Verbs	
Determiners with countable and uncountable nouns, Articles, Connectives, Prepositions, etc.	Such terms as commutative, associative, pie, angle, curve, intersection, data, etc., etc., in mathematics. Also the symbols used to express mathematical ideas. Geography, History, Economics, Electronics, Engineering, etc., all have their own special language.

The basic ideas I discuss below apply to teaching any specialist subject, but as I have been asked for comments on 'Teaching Mathematics in a Second Language', I shall confine myself to this subject.

HAVE YOUR CHILDREN THE UNDERSTANDING OF FUNCTIONAL LANGUAGE THAT WE WOULD EXPECT A NATIVE SPEAKER OF THE LANGUAGE TO HAVE?

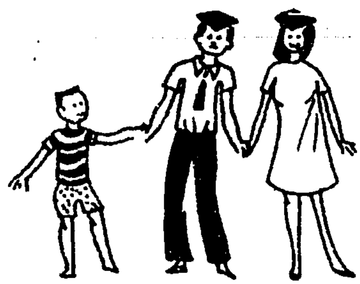
If your answer is 'Yes', then you have the relatively simple task of establishing the concepts conveyed by the special language of mathematics.

However, if you answer 'No', then your whole approach to mathematics must be a double one - to still teach the concepts conveyed by the special language of mathematics, but also to ensure that the children have the functional language that allows them to understand and to express these mathematical ideas.

Without doubt, we can assume that if we are working in a second language, our children will fall into the second category.

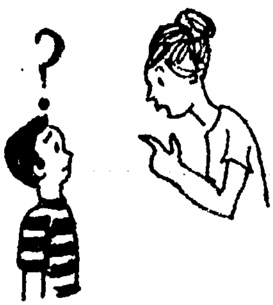
WHOSE TASK IS IT TO TEACH FUNCTIONAL LANGUAGE?

I think we can reasonably say that this is the job of the Language Specialist. In teaching a Second Language, he will be using a logically designed teaching plan and will systematically teach to the children the structure of the language, vocabulary and the social significance of the language. This task is necessarily slow, particularly in the early stages as the second language may bear no resemblance to the mother tongue of the learners; the learner must learn to automatically respond in the new language; language is so tied to the culture of native speakers of the language that cultural and social elements may need special emphasis in the teaching programme. The language specialist will always try to use the known to teach new items. The need for this is so obvious. To do otherwise would be like trying to teach the Pythagorean theorem before a child has an understanding of the idea of triangle. Yet, earlier I stated that the mathematics teacher, teaching via the medium of a second language, must teach the special language of his subject, but also ensure the children have the functional language to understand and convey mathematical ideas. Now having confused the issue by stating that the language specialist should teach the functional language, where do we stand as teachers of mathematics?



THIS or THAT?

I feel that the illustration above left portrays what is happening in many areas of the South West Pacific. (It may not be in yours; I congratulate you.) We have the language specialist battling away trying to establish in his students basic functional language, while on the other side we have the specialist in his subject trying, without due regard to the level of functional language, to pull on his side of the rope and desperately trying to teach his subject. Confusion on the part of the learner is all that can result. Surely the picture should be that portrayed on the right where the two specialists together to lead the child towards an understanding of both specialties. At the primary level the teacher is both specialists as he teaches all subjects, yet it is interesting to see the same teacher so dedicated to teaching the prescribed language work ruining his efforts by disregarding language control in other subject areas. At the secondary level we usually have two different teachers in conflict, each often secretly blaming the other for failing to produce results.



WHAT IS THE ANSWER?

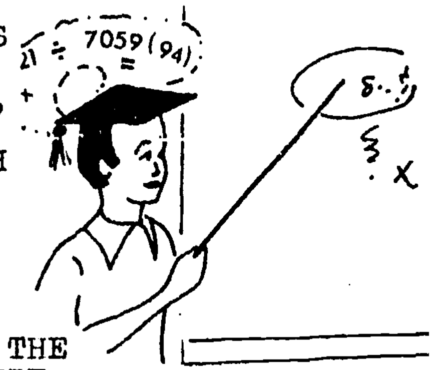
If you are a primary or secondary teacher, you must be aware of what language structures and vocabulary have been taught to the level at which you are teaching. Teachers throw up their hands in horror at this idea, yet at the lower school levels you have an excellent reference in Miss Tate's Handbook on Oral English. At the secondary level there must be liaison between the specialist teachers. Schemes of work need to be compared so that the language teacher knows what functional language is needed by the mathematics teacher. The mathematics teacher must know what has been taught and attempt to PHRASE QUESTIONS, PROBLEMS AND EXPLANATIONS, etc., using the known structures.

This brings us to a point where there are conflicting opinions.

- DO WE USE SPECIALIST SUBJECTS {MATHEMATICS, SOCIAL SCIENCE, ...} TO TEACH FUNCTIONAL LANGUAGE?

or

- DO WE ATTEMPT TO KEEP WITHIN THE KNOWN FUNCTIONAL LANGUAGE WHILE TEACHING OUR SPECIALIST SUBJECT?



We find keen supporters of both principles. The first group argue with merit that by using mathematics to teach functional language we are strengthening the children's language and at the same time helping the language specialist.

Their opponents argue strongly that there is enough special language to be taught in mathematics without confusing the children by also trying to teach functional language.

Let me elaborate with some examples. If we consider the terminology that has explicit meaning for the mathematician when discussing sets, we include element, member, braces, subset, superset, universal set, cardinality, ordinal, union, intersection, disjoint, null, ... When these terms are mixed with basic functional language which is limited, do you wonder why confusion over the meaning of terms arises? I agree that some of these terms cannot be replaced with simple phrases or words that convey the mathematical idea and that are part of basic functional language, but many of them can. For example, why talk about 'the cardinal number of a set' when we can say 'the number of elements (what is wrong with <u>things</u>) in a set'? Why talk about 'disjoint sets' when we n 'sets that do not have any elements that are the same'?

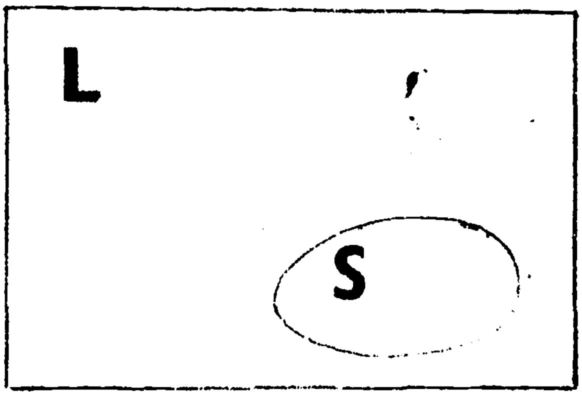
Why talk of the 'null set' when we can say 'the empty set'? Mathematicians, I can see you have angry thoughts ready to express, but let me finish. I do not say we should not teach the terminology of mathematics, but we do tend to teach it when the basic functional language of the child is not developed sufficiently to allow a great number of specialist terms to become established without causing confusion. I am quite happy that specialist language is taught, providing this is done in a way that follows the principles of teaching a second language, and that doesn't mean that the teacher says, 'We call this ________', and the children are expected to understand the term. Adequate time must be given for the learner to meet the idea in varying situations and also to use the language again and again and establish automatic response. Much of this can be done incidentally if the teacher uses functional language as well as the special terms. I argue that until the specialist teacher pays due regard to functional language the children understand, and to the principles of teaching a second language, language should be kept within the controls already established, and specialist language kept to a minimum when this is possible.

A WORD FOR THE COURSE WRITERS

To ask teachers to exercise control in their use of language is quite an imposition. They must study and collaborate with others to do this. It constitutes an extra burden for them. But I cannot see any excuse for the course writer who does not learn to use language controls in his writing. It is a little like the Englishman being expected to eat his roast beef without any Yorkshire pud. I must admit that our U.N.D.P. Mathematics Specialist does pay due regard to this matter, and despite the fact I should not deviate from the mathematics theme, I must take this opportunity to point out that in other specialist areas this is not always so. The teacher in the classroom has enough problems of his own without having to inherit yours.

SUMMARY

1.



L	=	{Language}
S	=	{Specialist Language}
L - S	=	{Functional Language}

2. KNOW WHAT FUNCTIONAL LANGUAGE HAS BEEN TAUGHT AT YOUR LEVEL.

Primary teachers - Study Miss TATE's HANDBOOK ON ORAL ENGLISH.

Secondary teachers - Study TATE and collaborate with your other specialists.

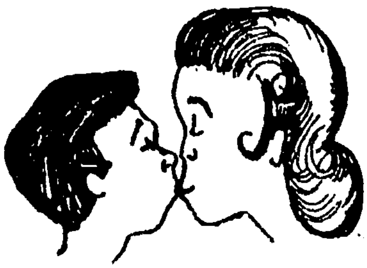
3. USE KNOWN FUNCTIONAL LANGUAGE IN YOUR TEACHING.

- Teach specialist terms when functional language is well established.
- When teaching specialist language pay due regard to the principles of second language teaching.

4. COURSE WRITERS

Don't hand problems caused by your inefficiency to teachers.

5. FINALLY, A WORD FOR THE DAY

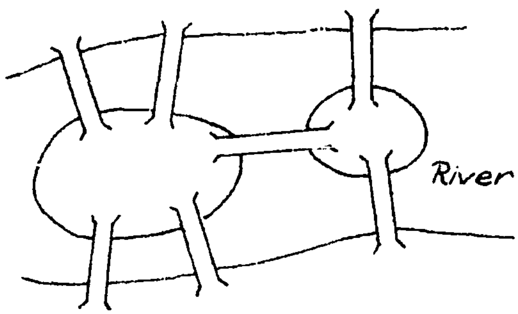


K ↕ Keep I ↕ It S ↕ Simple, S ↕ Sir!

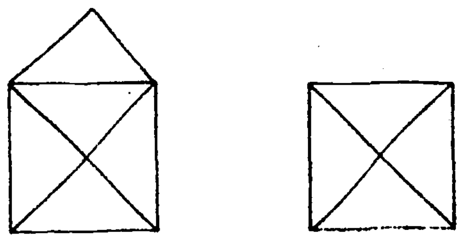
J. Keown
Department of Education
Rarotonga

TOPOLOGICAL PUZZLES

Editorial note: On pages 53-55 of the January edition of Mathematics Forum there were four apparently different puzzles. Laurie Williams [Stella Maris Primary School, Suva] has in this article shown how their structure and solutions are of the same type. Geometrical problems like these in which distance and angle are irrelevant come under the heading of TOPOLOGY.



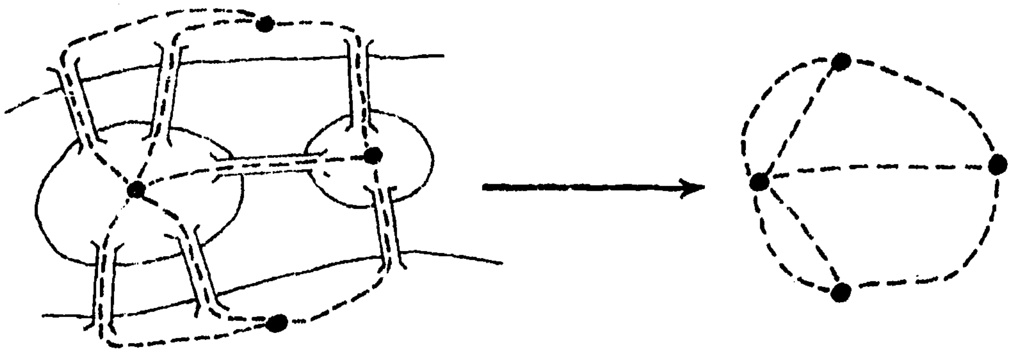
Is it possible to go over all the bridges once and once only?



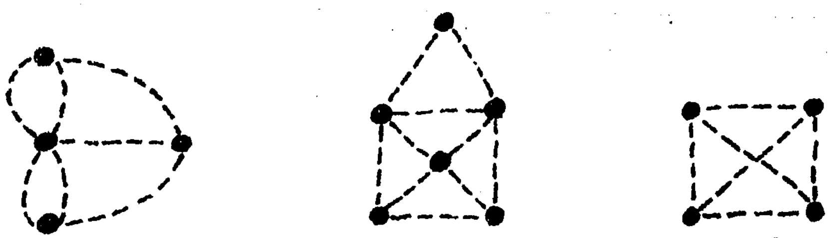
Is it possible to trace around these shapes going over each segment once and once only?

Many of us will have spent some time trying out these puzzles, tracing over the picture, using up masses of paper, and then forgetting whether we'd gone over this bridge, or that side, and ending up giving them to our children to solve. In the first puzzle, we may have tentatively said that it was not possible to cross all the bridges, but at the same time not being too sure that our trials may have been in error. The same may have been the case for the second and third puzzles, though our answers were perhaps a little more definite.

Looking at the bridges puzzle, it may be a help to draw a diagram showing all possible paths over the bridges.



Now we have a link with problem 2 - that of finding a path over all segments without going over any segment twice.



(a) (b) (c)

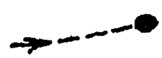
Can these puzzles be solved by means other than trial and error?

To do this let's call each large dot a 'node'. If there are 1, 3, 5, etc., segments leading from the node, we shall call it an 'odd node'.

Figure a has 4 odd nodes.
Figure b has 2 odd nodes.
Figure c has 4 odd nodes.

Why choose 'odd nodes'?

When there is one path leading to a node, to go over the segment, we must either start at that node, or finish there.



Where there are two segments, we can go to the node and away again.

Where there are three segments to the node, we can go in and out, and either start at the node or finish there.

Similarly, with a 5 node, we can go in and out twice, and either start or finish there.

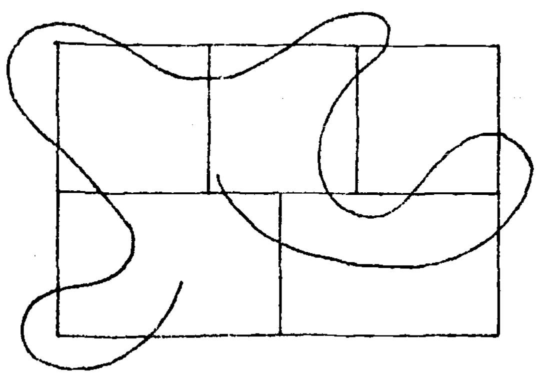
Now if a network has more than 2 odd nodes, can we start and finish at more than two different places?

This leads us to one of Euler's laws for Networks:

> 'A network can be drawn in one stroke if there are no more than two odd nodes. Such a network is said to be Unicursal.'

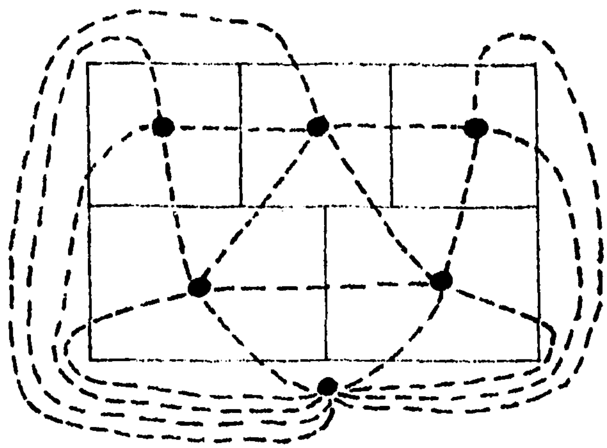
This means that figure a - with 4 odd nodes - and also figure c cannot be traversed, whereas figure b can.

Problem 3



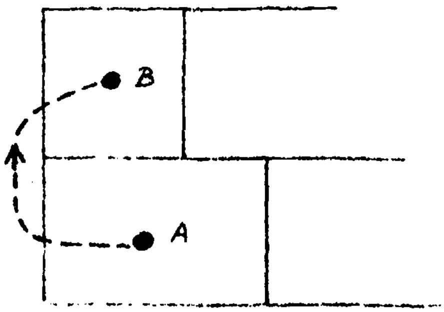
Can you draw a curved line that crosses every line segment once and once only?

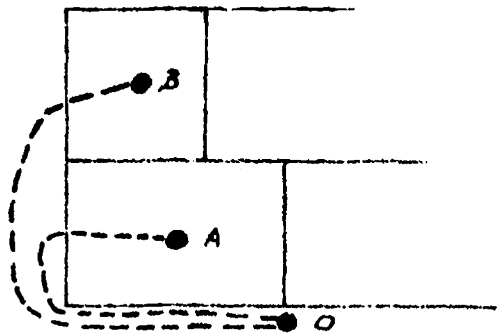
A figure can be drawn showing all the possible paths crossing the line segments into each region.



Regions can be thought of as being represented by the heavy dots.

Note: Going from A to B as shown is equivalent to going from A to O to B.





If this network can be traced around crossing each path once and once only, this means that we can draw a line crossing every line segment.

Looking at the diagram we see that there are more than 2 odd nodes (there are 4 odd nodes). Therefore we conclude that the network is not traversible, i.e. there is no line crossing all segments.

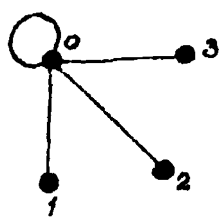
Problem 4   The Domino Puzzle

Is it possible to arrange the dominoes in one line?

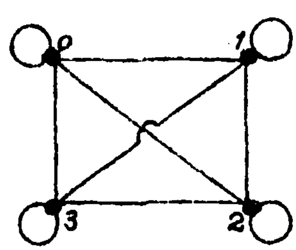
The complete set of dominoes is:

0 | 0	1 | 1	2 | 2	3 | 3
0 | 1	1 | 2	2 | 3	
0 | 2	1 | 3		
0 | 3			

We can show the dominoes with a '0' in them by the various segments below.



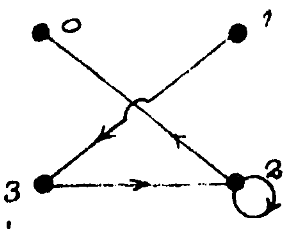
A diagram for the complete set of dominoes is like this:



Each continuous path represents a sequence of dominoes. Thus this sequence of dominoes

1	3		3	2		2	2		2	0

would be shown by this part of the network:



The problem of putting the complete sequence of dominoes out in a line is equivalent to traversing the network in one stroke of a pencil.

Can this be done?

By looking at the network we find that there are four odd nodes, and therefore there is no path - and consequently no continuous sequence using all the dominoes.

What if you have a whole set of dominoes with numbers from 0 to 6? Can you arrange them in one line? Try it with a network - but watch out for Diagram Dazzle!!! You will find that you can.

Note: A pamphlet on Topological Puzzles will be produced soon by the U.N.D.P. Curriculum Development Section.

Laurie Williams
Suva

CONJURING TRICKS USING THE BINARY SYSTEM

TRICK 1

8	12
9	13
10	14
11	15
A	

4	12
5	13
6	14
7	15
B	

2	10
3	11
6	14
7	15
C	

1	9
3	11
5	13
7	15
D	

- You ask someone to think of a number between 1 and 15.
- You then ask them to tell you which cards the number is on.
- You tell them immediately what their number is.

HOW IS IT DONE?

Suppose you are told the number is on cards A, C and D. You first produce the number in the Binary System like this:

A ✓	B ✗	C ✓	D ✓
↓	↓	↓	↓
1	0	1	1

You then change the Binary Number to Base Ten to get the answer.

$$1011_{(two)} = 8 + 2 + 1_{(ten)}$$
$$= 11_{(ten)}$$

Their number was ELEVEN.

EXPLANATION

BINARY NUMBER				BASE TEN NUMBER
A	B	C	D	
			1	1
		1	0	2
		1	1	3
	1	0	0	4
	1	0	1	5
	1	1	0	6
	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

On card A you put the base 10 numbers with a 1 in the A column.

On card B you put the base 10 numbers with a 1 in the B column.

And so on.

So when a person tells you which cards a number is on he in effect is telling you the binary form. This you convert to base 10. If we jumble up the numbers on the cards it makes it more difficult for anyone to see how the trick is done.

Children can be asked to extend this trick to 31 numbers or 63 numbers and so on.

TRICK 2

Someone writes down a sequence of 18 0's and 1's, perhaps like this:

101101111001110010

You look at the digits for a few seconds and then they are covered over.

You then repeat the digits in order.

EXPLANATION

First you divide the digits into groups of 3.

101 101 111 001 110 010

You then convert the group into base ten.

5 5 7 1 6 2

This you can remember easily as

557 ----- 162

To reproduce the original sequence all you do is to change back to the binary system.

This trick requires practice but once having had this you can extend it to 24 or more digits.

E. H. Leaton
U.N.D.P., U.S.P.

WHY DOES $a^0 = 1$?

A common proof that $a^0 = 1$ goes something like this.

Using the law of subtraction of indices

$$a^x \div a^x = a^0$$

But we know $a^x \div a^x = 1$ so a^0 must equal 1

We are saying in effect that if the law of subtraction of indices is to be valid in all cases we must define a^0 to be 1.

For some people this is a very roundabout way of proving something and consequently difficult to accept.

So I would like to suggest further evidence to support the contention that $a^0 = 1$.

Let us use square root tables and start with any value of a. Suppose a equals 10.

$$10^{\frac{1}{2}} = \sqrt{10} = 3.162$$

$$10^{\frac{1}{4}} = \sqrt{10^{\frac{1}{2}}} = 1.779$$

$$10^{\frac{1}{8}} = \sqrt{10^{\frac{1}{4}}} = 1.333$$

$$10^{\frac{1}{16}} = \sqrt{10^{\frac{1}{8}}} = 1.154$$

$$10^{\frac{1}{32}} = \sqrt{10^{\frac{1}{16}}} = 1.074$$

$$10^{\frac{1}{64}} = \sqrt{10^{\frac{1}{32}}} = 1.036$$

$$10^{\frac{1}{128}} = \sqrt{10^{\frac{1}{64}}} = 1.018$$

$$10^{\frac{1}{256}} = \sqrt{10^{\frac{1}{128}}} = 1.009$$

$$10^{\frac{1}{512}} = \sqrt{10^{\frac{1}{256}}} = 1.004$$

$$10^{\frac{1}{1024}} = \sqrt{10^{\frac{1}{512}}} = 1.002$$

What do we observe? The numbers in the right hand column tend to 1 and the indices in the left hand column tend to 0. This <u>suggests</u> that $10^0 = 1$.

Is it true, whatever value we give a, that $a^0 = 1$?

With a class of children we can

(i) divide them into groups,

(ii) get each group to choose a different positive value of a (some bigger and some smaller than 1),

(iii) use the square root tables to find $a^{1/2}$, $a^{1/4}$, $a^{1/8}$, etc.,

and (iv) then suggest the value of a^0.

They will find that for all values of a, their results suggest that $a^0 = 1$.

Students who have followed this approach using the square root tables will find the result of the rigorous proof that $a^0 = 1$ more convincing.

B.S. Prasad
USP

NUMBER SYSTEMS IN THE NORTHERN AND SOUTHERN COOK ISLANDS

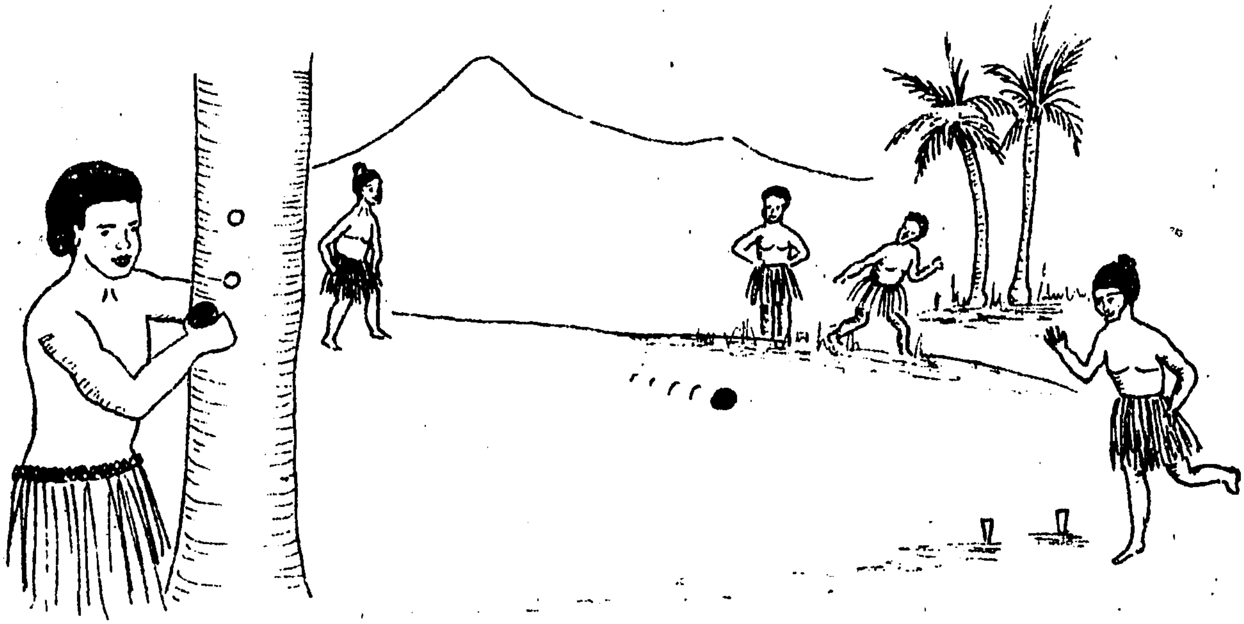
Editorial note: The January 1973 issue of Mathematics Forum contained an article on the Tongan numeration system. Below is a brief article on the numeration system still used in parts of the Cook Islands. In the next issue of the magazine we will publish an article on numeration systems in Fiji.

Prior to the advent of Christianity, there were well established number systems in the Cook Islands in the southern and northern Cooks. There were ways of counting all the things from the sea and different ways of counting all the things on the land. The system still exists in the northern Cook Islands - Manihiki, Rakahanga and Pukapuka.

Numbers were not recorded as numerals and real objects were required when counting. When big feasts were held and the left-over food was distributed, pebbles were collected equal in number to the total people at the feast. As the food was shared and given out to the individuals, a stone was removed from the heap representing the share given out.

Ti leaves (cordyline) were also used. Small strips were torn off the leaf and each strip represented one person.

Scoring of games was also done in a similar way except that each point was recorded by punching a hole in a tree trunk.



Holes were punched in the trunk of a tree to score winning discs when 'pua' (a disc game similar to bowls) was played.

Counting in the Maori language today is a translation of the Hindu-Arabic system.

Tai Ngauru	-	One Tens
Rua Ngauru	-	Two Tens
Tauatini	-	Thousand
Anere	-	Hundred

COUNTING THINGS FROM BOTH LAND AND WATER - Southern Cooks

English Number	Maori Number in Words	English Equivalent	Number Sentence
1	ta'i	one	1
2	rua	two	2
3	toru	three	3
4	a	four	4

5	rima	five	5
6	ono	six	6
7	itu	seven	7
8	varu	eight	8
9	iva	nine	9
10	ta'i ngauru	one ten	10
11	ta'i ngauru ma ta'i	one ten and one	10 + 1
12	ta'i ngauru ma rua	one ten and two	10 + 2
13	ta'i ngauru ma toru	one ten and three	10 + 3

The pattern continues up to 19.

20	ta'i takau	one score	1 x 20
21	ta'i takau ma ta'i	one score and one	1 x 20 + 1
22	ta'i takau ma rua	one score and two	1 x 20 + 2
23	ta'i takau ma toru	one score and three	1 x 20 + 3
24	ta'i takau ma a	one score and four	1 x 20 + 4

The pattern continues up to 29.

30	ta'i takau ma uru-ngauru	one score and ten	1 x 20 + 10
31	takau ma uru-ngauru ma ta'i	one score and ten and one	1 x 20 + 1 x 10 + 1
32	takau ma uru-ngauru ma rua	one score and ten and two	1 x 20 + 1 x 10 + 2

33	takau ma uru-ngauru ma toru	one score and ten and three	1 x 20 + 1 x 10 + 3

The pattern continues up to 39.

40	e rua takau	two score	2 x 20
50	e rua takau ma uru-ngauru	two score and ten	2 x 20 + 1 x 10
60	oko toru	three score	3 x 20
70	toru takau ma uru-ngauru	three score and ten	3 x 20 + 1 x 10
80	oko a or a takau	four score	4 x 20
90	a takau ma uru-ngauru	four score and ten	4 x 20 + 1 x 10
100	oko rima or rima takau	five score	5 x 20

The pattern continues up to 190.

200	e ta'i rau	one two hundred	1 x 200
300	e ta'i rau e rima takau	one two hundred and five score	1 x 200 + 5 x 20
400	e rua rau	two two hundred	2 x 200
500	rua rau e rima takau	two two hundred and five score	2 x 200 + 5 x 20

The pattern continues up to 900.

COUNTING THINGS ON LAND - Northern Cook Islands

English Number	Maori Number in Words	English Equivalent	Number Sentence
1	e tahi mea	one thing	1 x 1
2	e tahi fakahani	one pair	1 x 2
3	e teru mea	three things	3 x 1
4	e rua fakahani	two pairs	2 x 2
5	tapahi	five	5
6	e teru fakahani	three pairs	3 x 2
7	e hitu mea	seven things	7 x 1
8	e fa fakahani	four pairs	4 x 2
9	e iva mea	nine things	9 x 1
10	purupuru	ten	10
11	purupuru ma tahi mea	ten and one thing	10 + 1
12	purupuru ma tahi fakahani	ten and one pair	10 + 1 x 2
13	purupuru ma teru mea	ten and three things	10 + 3
14	purupuru ma rua fakahani	ten and two pairs	10 + 2 x 2
15	purupuru ma rima mea	ten and five things	10 + 5
16	purupuru ma teru fakahani	ten and three pairs	10 + 3 x 2
17	purupuru ma hitu mea	ten and seven things	10 + 7
18	fa fakahani	ten and four pairs	10 + 4 x 2

19	puruppuru ma iva mea	ten and nine things	10 + 9
20	e tahi takau	one score	1 x 20

The pattern continues up to 29.

30	e tahi takau ma purupuru	one score and ten	1 x 20 + 10
40	e rua kau	two scores	2 x 20
50	rua takau ma purupuru	two scores and ten	2 x 20 + 10
60	tei ta takau	three scores	3 x 20
70	tei ta takau ma purupuru	three scores and ten	3 x 20 + 10

The pattern continues up to 190.

200	e tahi ta rua	one two hundred	1 x 200

Note that this system actually catered for numbers into the millions.

Paiere Mokaroa
Demonstrator
Teachers' Training Centre
Rarotonga

SCRAMBLED WORDS

Unscramble each word. Each one is a word used in mathematics. Write the answers in the space at the side. Put the circled letters in the message box at the end.

TIPMAULLICNIOT

EARA

GOLYPNSO

BRATIUNSCTO

NEGALS

DEDVII

TERMIRPEE

STMAH

QATEINOSU

READOGOB

MACLSDEI

DDTNOIASI

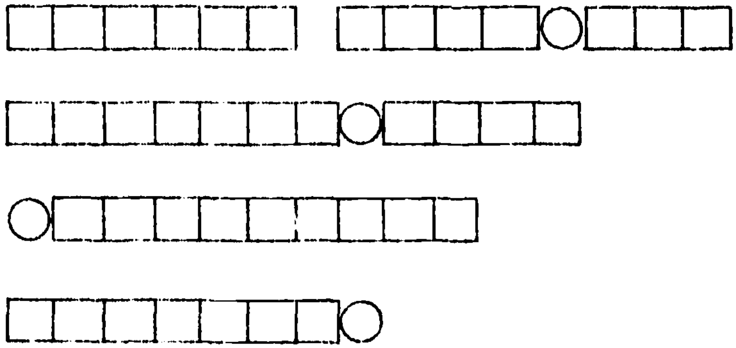
ORDNICOTASE

ENMURB ASEMHNCI

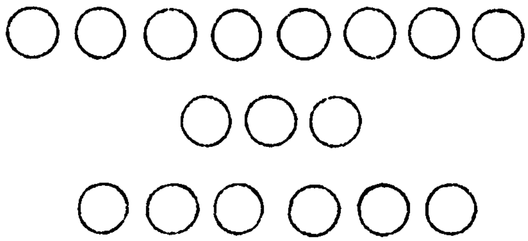
TRESNINTCEOI

KMILTOEERS

STMYYREM



FIND A MESSAGE



Maureen Young,
Pauline Chang,
Cynthia Cheer,
Alice Chow
of Fiji Chinese Primary School
Suva

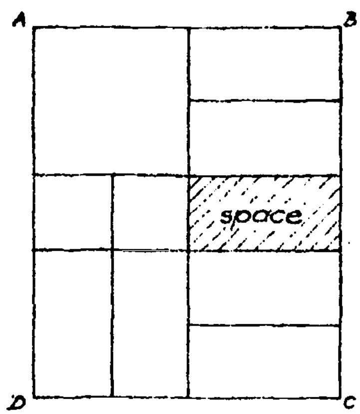
THE ELUSIVE SQUARE

Editorial note: This puzzle appeared on page 37 of the January edition. The square proved so elusive that at least one member of the editorial board was convinced that a solution did not exist. He then devoted his time to trying to prove the non-existence of the solution. The fact that he was not successful in this task is understandable having read the second part of Paul Schofield's article.

How often we see a puzzle and think, "I'll try that one day." Procrastination is an evil we see in our pupils, but . . . . .

Before reading through this solution DO TRY THE PROBLEM. Remember Lady Macbeth when she was urging her husband to stab Duncan?

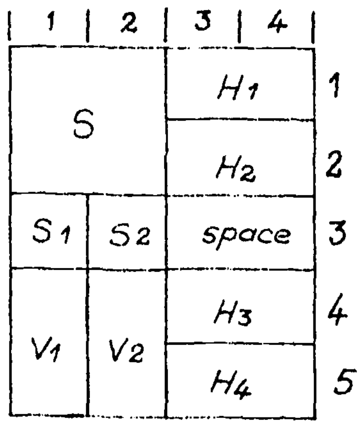
"We fail? But screw your courage to the sticking-place, And we'll not fail"!



<u>THE PROBLEM</u> is to move the large square from corner A to corner D. You are not allowed to remove any piece from the tray or to [illegible]te any piece.

SOLUTION

The tray may be thought of as twenty unit squares (4 along the top and 5 down the side).



The squares, horizontal and vertical rectangles have been labelled as shown, and the moves are described as follows:

(S_2, R_2) - move S_2 two units to the right

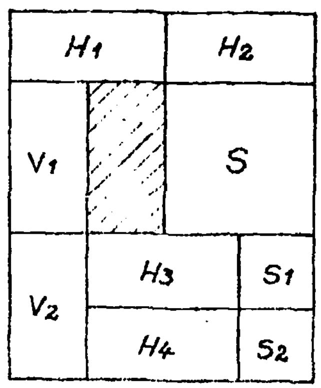
(S, D_1) - move S one unit down

(H_1, L_2) - move H_1 two units left

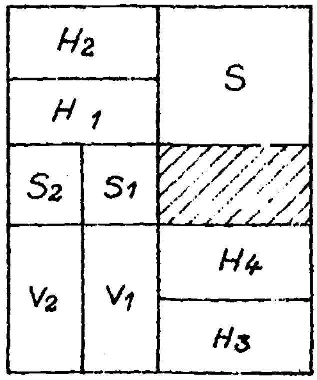
(H_2, U_1) - move H_2 one unit up

The diagrams show the various stages in the solution:

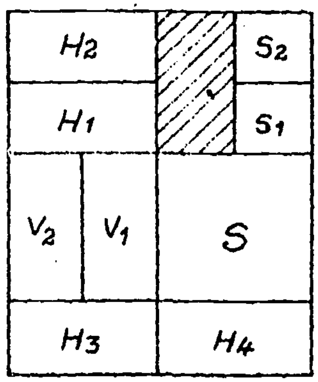
a) (S_2,R_2) , (S_1,R_2), (S , D_1), (H_1,L_2) , (H_2,U_1) , (S_1,U_1), (S_1,R_1) , (S , R_1) , (V_1,U_2), (V_2,L_1) , (H_3,L_1) , (H_4,L_1), (S_2,D_2) , (S_1,D_2) , (S , R_1).



b) (V_1,R_1), (V_2,U_2), (H_3,L_1),
(H_4,L_1), (S_2,L_1), (S_2,U_1),
(H_4,R_2), (H_3,D_1), (V_1,D_1),
(V_2,D_1), (H_1,D_1), (H_2,L_2),
(S,U_1), (S_2,U_1), (S_1,U_1),
(H_4,U_1), (H_3,R_2), (V_1,D_1),
(V_2,D_1), (S_2,L_2), (S_1,L_2). →



c) (S,D_1), (H_2,R_2), (H_1,U_1),
(S_2,U_1), (S_1,U_1), (V_1,U_1),
(V_2,U_1), (H_3,L_2), (H_4,D_1),
(S,D_1), (S_1,R_2), (S_2,R_2),
(H_1,D_1), (H_2,L_2), (S_2,U_1),
(S_2,R_1). →



d) (H_2,R_1), (H_1,R_1), (V_2,U_1),
(V_1,L_1), (S,L_1), (S_1,D_2),
(S_2,D_2), (H_2,R_1), (H_1,R_1),
(V_2,R_1), (V_1,U_2), (S,L_1),
(S_1,L_1), (S_1,U_1), (H_4,U_1),
(H_3,R_2), (S,D_1). →

If anyone has a shorter solution
I would be pleased to see it.

P. Schofield
Adi Cakobau School
Suva

WHAT'S NEW ABOUT NEW MATHEMATICS

Editorial note: In our January 1973 issue we published an article on 'new' mathematics by Mr. R. Nathan of Nauru. The following article by Mr. V. Raylu of Fiji is on the same topic, but offers a different point of view.

Today, in this technological age, the spirit of innovation is characteristic of mathematics education; new syllabuses continue to alter traditional content, classroom activities of pupils, and teacher's approach in mathematics teaching at all school levels. Many are talking about new mathematics, but only a few really know what it is. So the questions what new mathematics is and why it should be taught in schools are now matters of great concern to many educators as well as to the community at large in Fiji.

WHAT IS NEW MATHEMATICS?

The vastly divergent opinion about the new mathematics course comes at least partially from the fact that the term 'new mathematics' means so many different things to so many different people. So the origin of discussion surrounding new mathematics is a problem of semantics. With regard to this problem, the error here is in the use of the word 'new' in connection with mathematics at the post-primary level. My feeling is that to a layman this use of the word implies that the mathematics now being taught has just been discovered. This is not so. The term 'New Mathematics' or 'Modern Mathematics' does not necessarily imply the introduction of a completely revolutionary syllabus at the expense of a more traditional one, but more of the change in approach in the teaching of the subject and in the emphasis and selection of the topics.

WHY NEW MATHEMATICS?

It is a myth to regard mathematics as an ancient science dealing with static ideas. Mathematics in the past was regarded as a tripod of arithmetic, algebra and geometry which had little coherence and no structure. With the rapid explosion of mathematical knowledge, with the rapid development in the field of technology, with the advance of psychological and educational knowledge of child growth and development and of learning processes there is tremendous need for mathematics as a discipline to be kept under review to ensure progressive improvement in the quality of mathematical education for mathematics was and still is a dynamic subject. Today, there are new ideas about numbers, new ways of performing calculations, new theorems that are being proved in algebra and geometry and completely new fields of mathematics. Why then, should we not study new mathematics which had obvious pedagogical advantages and is more relevant to our needs? Surely because mathematics has been created by man, it should be enforced in this new form by man?

The new mathematics course does include some new subject-matter which is important for it has considerable new practical applications and contemporary usage. Scientists are utilizing new fields of mathematics such as topology, analysis, computers, vectors, statistics, etc., to fill in gaps in knowledge of biological, chemical and physical sciences. Mathematics is being used widely in the social sciences and by psychologists to study human learning and behaviour. Many economists today find that they must understand game theory - a branch of mathematics which did not really exist 25 years ago. In business, the mathematics of probability and linear programming is used to schedule production and distribution. The use of statistics in industry and many other fields has been increasing by leaps and bounds - usually in conjunction with computers. Today, anybody who does not have at least some knowledge of statistics cannot even read the newspaper intelligently. So the new programmes provide not only the fundamental processes in mathematics, but also the recognition of their social applications for the effective solutions of quantitative problems in daily life. In addition to providing intellectual flavour and intellectual challenge, a knowledge of the new subject-matter is essential to the ordinary citizen as well as being important for many vocations, and these then, have been given a new depth in the new mathematics programmes. To so many pupils Euclidian geometry was simply a repetition of a series of proofs that were of no abiding interest even to professional mathematicians. However, most of Euclidian geometry is now substituted by transformation geometry which is much more interesting and stimulating both in teaching and in learning.

It should be noted that to accommodate the new topics, the treatment of some of the old topics has been altered and the number of drill problems has been reduced. New mathematics offers more to a child both in content and value than did traditional mathematics. It is therefore a meaningful mathematics - meaningful to the teachers, to the pupils and even to the parents who have some understanding of the subject.

Modern high-speed computers have had a profound effect throughout the world. They are now not only used to carry out the mass of numerical calculations involved in solving problems in engineering and science but also in the field of general problem solving. The computer is connected to new mathematics in a number of ways. Some subject-matter in the new syllabuses, such as sets, binary numbers and symbolic logic, is utilized in computer programming. Mathematical ideas can be learned effectively through computer programming. Although the use of computers in Fiji is still in its infancy we can assume that its use will grow in the foreseeable future. So we must teach new mathematics to our children in order to develop their mathematical potential. We must give our youngsters adequate knowledge of mathematics so that they will be able to better direct their common task of building a single nation.

Probably the greatest drawback of traditional mathematics was that it did not explain the "why" of certain simple arithmetic operations. To a child, as I see it, the "why" as well as the "how" of arithmetic operations are important. How many pupils or teachers who have learnt or taught traditional mathematics can explain why the product of two negative numbers is a positive number, for example, $^{-}6 \times {}^{-}2 = {}^{+}12$? Perhaps very few. The teaching of earlier programmes of mathematics consisted largely of drills, memorization and computational skills at the expense of mathematical ideas which yield immediate enjoyment and satisfaction. Thus to some pupils mathematics became another of the bits of drudgery involved in growing up. However, the new mathematics programmes place primary emphasis on thinking, reasoning and understanding. I do not mean to say that the new programmes have neglected computational skills, but rather that they have introduced and emphasized computational skills only after concepts necessary for understanding the particular operation have been developed. Evidence suggests that pupils who understand a process before practising it learn it more efficiently and can use it more effectively. Thus the new programmes teach pupils to learn how to learn.

The new mathematics programmes emphasize the structure of mathematics rather than isolated topics. It integrates the ideas, materials and methods within each branch of mathematics and across the branches, for it is mathematics as a unified whole that is presented. This has led to the fact that there are certain important themes such as sets, functions, mathematical structure and nature of proof that pervade new mathematics today. It is largely the new content which integrates the branches into a unified discipline. For example, matrices integrates arithmetic, algebra and geometry and so does finite arithmetic. The associative and commutative laws, far from being limited to algebra, have important interpretations and applications in other branches of mathematics. Thus more attention is given to the understanding of mathematical concepts and structure and the relationship of mathematics to the child's environment as opposed to pure rote learning of arithmetic skills. The new mathematics courses reflect more adequately than did the traditional syllabuses the up-to-date nature and use of mathematics,

The new mathematics programmes have explored a modern approach to the teaching of mathematics - an approach that motivates pupils to learn and that emphasizes pupil activity of one sort or another. Often this method is described as the "discovery" approach. The discovery method is the method of choice because understanding is more likely to materialize if the learner plays an active part in developing ideas. Mathematical ideas which a student discovers make sense to him and he will be able to remember them longer. Discovery will lead him to feel that mathematics is a human and growing subject. The triumph of discovery nourishes curiosity for more learning. Discovery takes place through experiments, thinking, reasoning and the study of patterns. So, by using physical models in the classroom and various other experimental materials the child will have a better understanding of mathematics. The evidence of this can be seen in the Chinese proverb, "I hear and I forget, I see and I remember, I do and I understand". Thus using heuristic and inductive methods will stimulate pupils to be creative, develop their enthusiasm for mathematics, improve their intuitive thinking and enhance their learning and retention. Many modern textbooks use exploration exercises as a means of promoting discovery and non-verbalized awareness of mathematical principles and concepts. The object of new mathematics, therefore, is to give pupils a good understanding of basic ideas and principles so that he can make whatever applications that may be called for in the future.

The new mathematics, unlike traditional mathematics, is suitable for a range of pupils of varying abilities. It has been found that pupils who have been taught new mathematics exhibit an overall superior performance compared with pupils who have not been so taught. I do not dispute the fact that the top mark in new mathematics is not as high as the top mark in traditional mathematics. But surely we cannot make any value judgements of this nature at this stage for the teaching of traditional mathematics has reached its old age while the teaching of new mathematics is still in its infancy. However, one can say without any hesitation that the pupils who study new mathematics find it to be more interesting, stimulating, enjoyable and exciting.

V. Raylu
Senior Education Officer (Secondary)
Fiji Department of Education

FRACTIONS AND/OR DECIMALS

Editorial note: It seems as if the South Pacific Region will "go metric" by 1975. Quite apart from the direct implications that this will have on methods of teaching and on the content of courses there are some more subtle considerations.

One of them is "Do we need to spend so much time on the teaching of fractions once the metric system is used?"

What do readers **think**?

Here is an extract from a letter written by the U.N.D.P. mathematics adviser in answer to an enquiry about this subject.

. . . . . Now to come to your two points about fractions and decimals. Perhaps my first comment is to say I don't know what should be the relevant emphasis to give to the two topics. For the mathematicians to know about the rational number system would be reasonable but the same hardly applies to the majority of students.

Conversion from fractions to decimals is I think important. Then we could add, for example, $\frac{1}{3} + \frac{1}{2}$ like this:

$$\begin{aligned}\tfrac{1}{3} + \tfrac{1}{2} &= .33 + .5 \\ &= .83\end{aligned}$$

Subtractions, multiplications, divisions can all be handled in the same way.

At present however in Decimals I, II, III, we have used "fraction" ideas to introduce decimal procedures. For example:

$$.3 \times .4 = \frac{3}{10} \times \frac{4}{10} = \frac{12}{100} = .12$$

But we don't have to use fractions to produce this result. We could do something along these lines.

$$\begin{aligned} .3 \times .4 &= (3 \times .1) \times (4 \times .1) \\ &= (3 \times 4) \times (.1 \times .1) \\ &= (12 \times .01) \\ &= .12 \end{aligned}$$

Nevertheless in present circumstances the fraction method is simpler for teachers and children alike. Maybe in the future we shall decide the latter method is more appropriate.

I have been asked about the position on the Continent regarding school teaching. It appears that fractions and fraction operations are taught in the same way as they are in non-decimal areas.

With regard to shopping the situation varies on the Continent. Some people say ½ kilo, some 500 gms. Hectograms aren't used but hectolitres are (big drinkers!). So what happens is that either common fractions are used or the unit is changed to a smaller one. And surprisingly enough decimals aren't used in everyday speech. In the same way here we talk about a dollar 23 cents and write $1.23.

Fractions are used on the Continent in the case of "sharing". We may have a situation in which A gets $\frac{1}{3}$, B gets $\frac{1}{6}$, C gets $\frac{1}{2}$. Here we ought at least to be able to check that the total is one.

However I must admit I can't think of many situations where fractions are going to be of much value. So where do we go from here? It would certainly seem that the fractions could be played down. Also when we are a little clearer in our minds we could start revising some of the material. Perhaps we could put these ideas into Mathematics Forum and ask readers for examples of situations where fraction work is indispensible.

I don't think there is much to worry about in the sense that any work taught from a "problem" viewpoint develops ability and broadens experience. We have tried to use this approach in the units Fractions I and Fractions II and in fact in all the units. Moreover this is, as I see it, the fundamental task to which we should be devoting our energies.

I'll look forward to hearing your views about all this.

....

THE HISTORY OF MEASUREMENT

Editorial note: This is the second part of a chapter on the history of measurement in A Short History of Mechanical Engineering by W. F. Greaves and J. H. Carpenter, Longmans, and is reproduced with the permission of the authors. The first part of the chapter appeared in the Jan. 1973 edition of Mathematics Forum.

THE INTERNATIONAL PROTOTYPE METRE

An International Convention in 1875 proposed the construction of a new standard metre - to be called the International Prototype Metre. This was constructed from an alloy of platinum and iridium and took the form of a bar of cruciform section. The metre was defined as the distance between two lines engraved on the neutral plane of this standard when the temperature of the bar was nought degrees Centigrade. Copies of this standard are held by countries which were signatories to the Convention of 1875. The British copy, now called the U.K. primary standard of the metre is held by the Board of Trade, and is checked against the International Prototype, which is preserved at Sevres near Paris, at determined intervals.

STANDARDS OF WEIGHT

Weighing by means of the balance dates from about 4000 B.C., and apparently its earliest use was for the weighing of gold. The balance with two pans, one to hold the weights and the other to hold the commodity, was in use in the civilisations of the western Mediterranean by 2000 B.C. The shekel was the standard weight used, varying between seven and fourteen modern grammes, larger units were the mina and the talent which was approximately equivalent to a hundredweight. The steelyard, the type of balance still to be seen in butchers' shops, was introduced by the Romans.

In 1266 a statute of Henry III made legal a table of weights based on a grain of wheat taken from the middle of the ear. Thirty-two such grains were to be the weight of one silver penny, twenty pennyweights made one ounce, and there were twelve ounces to the pound. The pound was also to be the weight of twenty shillings. It will be seen that this system of weights bore a direct relationship to the monetary system of the period, and in fact coins were used as weights.

Our present avoirdupois system with its sixteen ounces to the pound was adopted early in the fourteenth century, and was used at first only for weighing bulky commodities. The two systems continued in use side by side, with the earlier system becoming known as Troy weight. The Weights and Measures Act of 1878 abolished the Troy pound, and the Troy ounce was reserved for use in the gem and precious metal trades.

In the reign of Elizabeth I a new set of standard weights was prepared, and the hundredweight was fixed at 112 pounds avoirdupois, with twenty hundredweights to the ton. The Pilgrim Fathers established the avoirdupois system in America, but the hundredweight became fixed at 100 pounds giving a ton of 2000 pounds. Thus the U.S.A. ton became known as the 'short' ton.

Although we have used the word weight so far in this commonly accepted sense it is more correct for engineers and scientists to refer to a mass of one pound. The weight of one pound being the force exerted on a one pound mass by the earth's gravitational field. This is more usually stated as that force which, when applied to a mass of one pound, produces an acceleration of 32.2 feet per second.

Weights, volumes, areas, and liquid measures, were all rationalised when the metric system was being created in France. A logical system was designed in which these quantities were expressed in terms of the metre. The standard unit of mass, the gramme, was taken to be the mass of one cubic centimetre of water at a temperature of 4°C. Larger and smaller units in multiples of ten were named with the same prefixes as those used for the linear units.

THE UNITED KINGDOM PRIMARY STANDARD OF THE POUND

The U.K. Primary Standard Pound, formerly the Imperial Standard Pound, was produced in 1856 and is defined by the Weights and Measures Act of 1963. It is a cylindrical solid of platinum with a small groove cut around its surface so that it may be lifted with an ivory fork. The arrangements for the disposal and testing of the Authorised Copies are the same as those for the Standard Yard. The Weights and Measures Act 1963 defines the pound in terms of the kilogram as being exactly 0.45359237 kg.

THE INTERNATIONAL PROTOTYPE KILOGRAMME

This was established in 1889 and consists of a cylindrical solid of an alloy of platinum and iridium with its diameter equal to its height. The arrangements for copies are the same as those for the Prototype Metre.

A REFLECTION ON REFLECTION

Have you ever taken a photograph of a scene and its reflection in still water? The pictures below show a photograph taken from a launch on the lagoon of a house with a tree behind it. The direct scene is given in Fig.(i) and two alternative reflections A and B are given in Fig.(ii). Which of these would appear in the photograph?

Fig.(i)

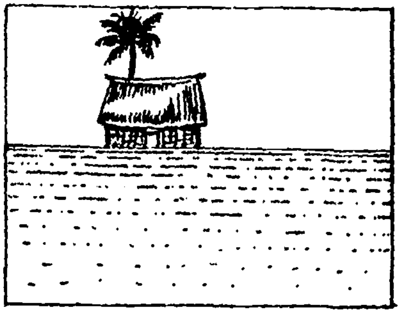
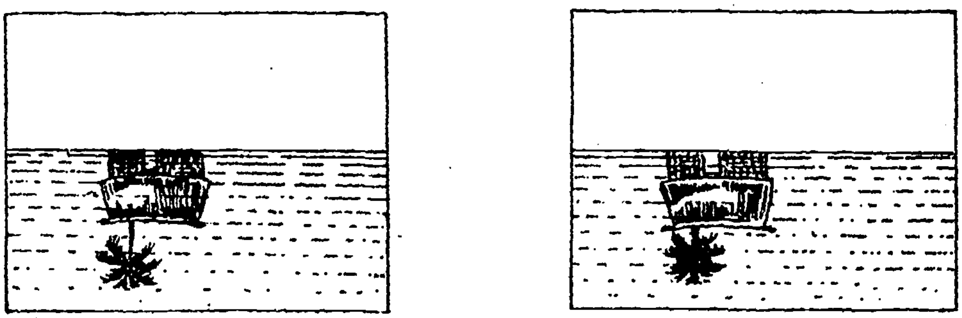


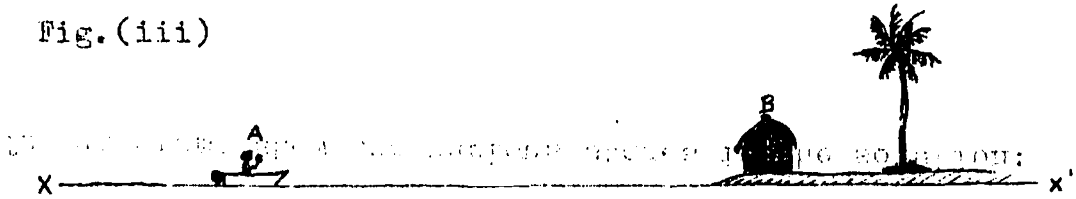
Fig.(ii)

(a) (b)



To consider this problem we must first look at the scene in cross section, and then for convenience reduce it to dia- rammatic form (Fig.(iv)).

Fig.(iii)



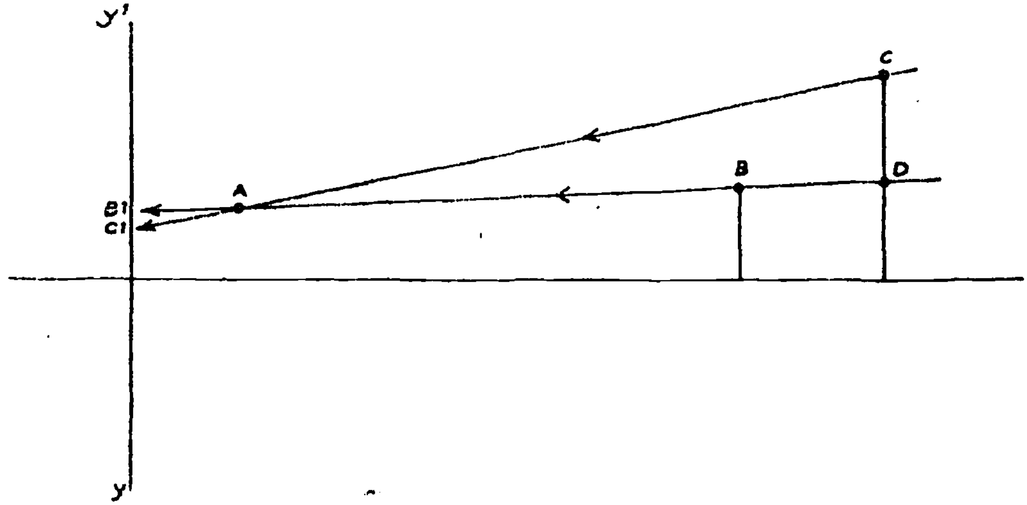


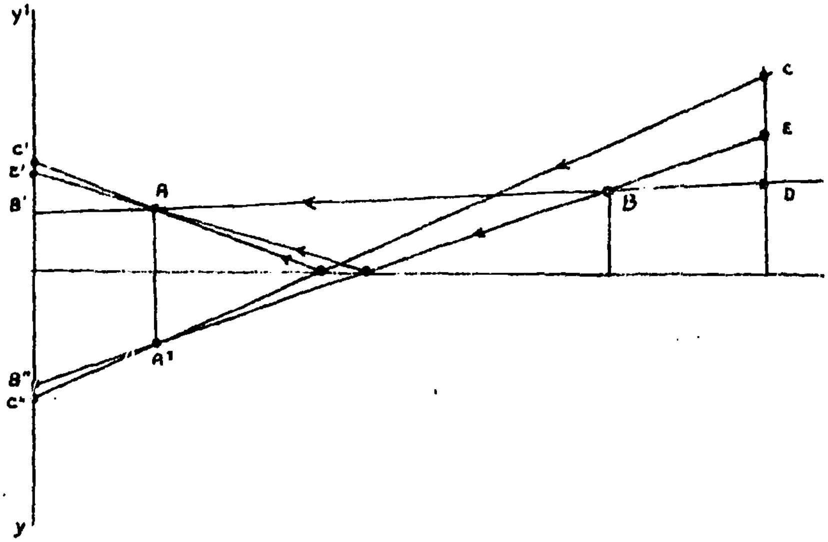
Fig.(iv)

When a photograph is taken each point if projected through A onto a vertical plane say (YY^1) at the back of the camera.

The image of C will appear at C^1 and that of B will appear at B^1. The image of the point D on the trunk of the tree will also appear at B^1 since A, B and D are in the same straight line.

From A we will only see the trunk of the tree above point D in the photograph, the rest of the trunk being hidden by the houso.

With the reflected scene, light from object points are reflected by the water and then go to point A.



To get an idea what the reflected scene looks like we can imagine A reflected in the water to A^1 and consider B^{11} and C^{11} the projections of B and C through A^1 onto the plane YY^1.

In this case it is point E which becomes coincident with point B in the reflected part of the photograph. Anything below can't be seen because of the house. In particular point D can't be seen but it could in the direct view.

Reflections become compressed as in Fig.(ii)B. This phenomenon can be seen in many pictures in magazines, calendars and books. Have you ever noticed it?

G. Longmore
British Secondary School
Vila

FINITE AND INFINITE SETS

At some stage in most new Mathematics courses work relating to sets is taught, and as part of this topic finite and infinite sets are mentioned.

Finite sets may be defined as countable (or numerable) sets, however large the number of elements may be. Infinite sets are those whose members are uncountable.

The cardinal number of a set means the number of elements the set contains. The cardinal number of a finite set is finite whilst the cardinality of infinite sets is infinite.

The finite set $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ has as its cardinality 8, whereas the set of whole numbers $W = \{0, 1, 2, 3, 4, ...\}$ has infinite cardinality.

One of the best tests to enable us to decide whether or not a set is finite or infinite is to take any proper subset of the set and try to match it with the set in a one-to-one correspondence.

With finite sets we cannot find a one-to-one matching whilst with infinite sets we are always able to match the set with a proper subset of itself.

Examples:

A. Finite sets

$$\text{Let } P = \{a, b, c, d\}$$

$$Q = \{1, 2, 3, 4\}$$

We can match each element of P with one element of Q like this.

$$\begin{array}{ccccccc} P & = & \{a, & b, & c, & d\} \\ & & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ Q & = & \{1, & 2, & 3, & 4\} \end{array}$$

The cardinal number of each set is 4. P and Q are called <u>equivalent sets</u>.

Let us now take P and a proper subset of P and try a similar matching.

$$P = \{a,\ b,\ c,\ d\}$$

$$\updownarrow \quad \updownarrow \quad \updownarrow \qquad\qquad R \subset P$$

$$R = \{a,\ b,\ c\ \ \}$$

This test shows that P must be finite.

B. <u>Infinite sets</u>

$$\text{Let } N = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \ldots\ldots\}$$

$$E = \{\ \ 2\ ,\ 4\ ,\ 6\ ,\ldots\ldots\}$$

so $E \subset N$ or E is part of N.

However we can match each element of E with a corresponding element of N like this.

$$N = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \ldots\ldots\}$$

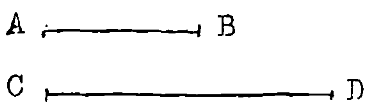
$$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$$

$$E = \{2,\ 4,\ 6,\ 8,\ 10,\ 12,\ \ldots\ldots\}$$

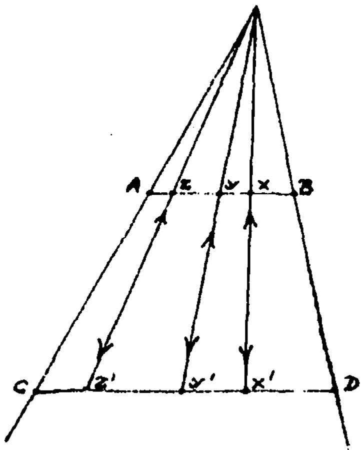
This may seem strange but remember each set has infinite cardinality.

Strange features about infinite sets are not limited to numbers.

C. Two unequal line segments.



Each point on $\overline{AB}$ can be matched with exactly one point on $\overline{CD}$ and vice versa. This can be shown like this.



$$\overline{AB} = \{X,\ Y,\ Z,\ \ldots\ldots\} \qquad \overline{AB} \subset \overline{CD}$$

$$\updownarrow \quad \updownarrow \quad \updownarrow$$

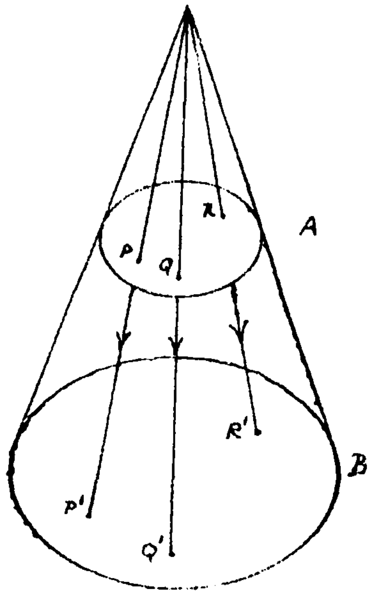
$$\overline{CD} = \{X,\ Y,\ Z,\ \ldots\ldots\}$$

Clearly $\overline{CD}$ has a greater measure of length than $\overline{AB}$ but each line contains the same cardinal number of points.

D. Two unequal discs.

A B

Each point in disc A can be matched with exactly one point in disc B and vice versa. This can be shown like this.



$$A = \{ P, \; Q, \; R, \; \ldots\ldots \}$$
$$\updownarrow \quad \updownarrow \quad \updownarrow$$
$$B = \{ P', \; Q', \; R', \; \ldots\ldots \}$$

Disc B has a greater area than disc A but each contains the same cardinal number of points.

It is interesting to note that the smallest possible circle contains only <u>one</u> point and so has finite cardinality, whilst any other circle which is slightly bigger than this smallest circle contains an infinite number of points.

Peter Etches
Tereora College
Rarotonga

Here are some interesting sets to think about.

(1) The set of all people on earth - Finite or Infinite?

(2) The set of all sand grains - Finite or Infinite?

(3) The set of all stars - Finite or Infinite?
(This depends on what theory of the universe you subscribe to.)

(4) The set of all molecules on the earth.

(5) Before we matched the set of natural numbers W with the set of even numbers E (or multiples of 2). Can we match W with the following subsets?

(a) Multiples of 10.

(b) Multiples of 100.

(c) Multiples of 10,000.

(d) Multiples of 1,000,000.

(e) Multiples of 1,000,000,000,000,000,000.

Are these subsets still infinite?

P. Etches
Tereora College
Cook Islands

U.N.D.P. CURRICULUM DEVELOPMENT WORKSHOP

Editorial note: During the long vacation a Curriculum Development Workshop was held at the University of the South Pacific. Participants from all the major islands in the South Pacific attended. The basic aim of the workshop was to give training in and opportunity for the writing of school material (pupils' pamphlets, teachers' guides, etc.). The subject areas covered were English, Social Science, Basic Science and Mathematics. To give readers some idea of what was a most useful workshop we include:

(i) A list of participants. (These people may be contacted for further information.)

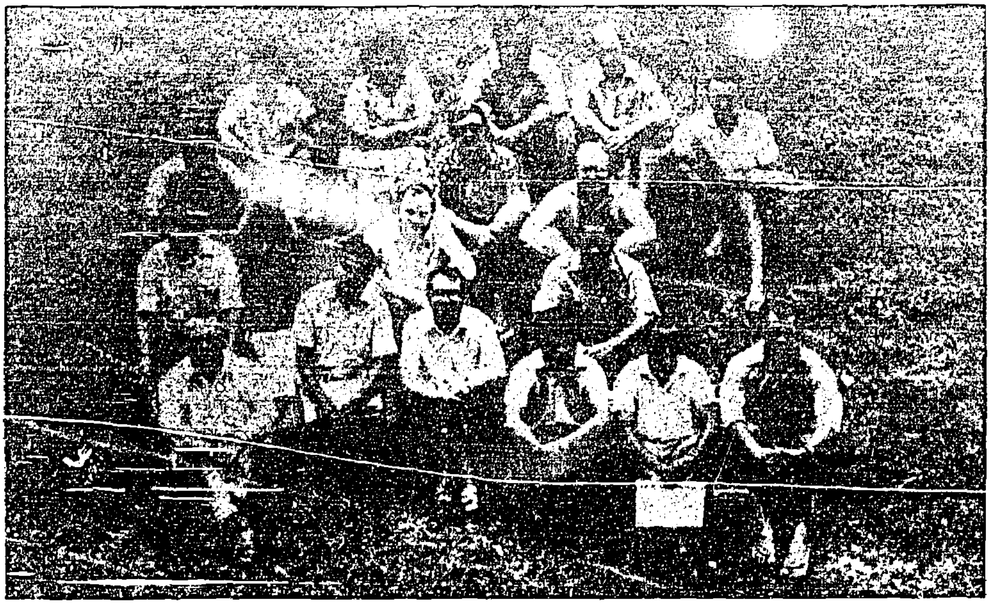
(ii) Some photographs.

(iii) The Mathematics Section Report.

MATHEMATICS PARTICIPANTS' NAMES AND ADDRESSES

	NAME	COUNTRY	ADDRESS
1.	GURDAYAL SINGH	Fiji	Curriculum Development Unit, Education Dept., Suva.
2.	KALI LAKSHMAN	Fiji	Curriculum Development Unit, Education Dept., Suva.
3.	LAURIE WILLIAMS	Fiji	Stella Maris Primary School, P.O.Box 97, Suva.
4.	SAMUELA R. DOMONI	Fiji	Nausori District School, P.O., Nausori.
5.	PIP LEATON	Fiji	U.N.D.P. Section, U.S.P., P.O.Box 336, Suva.
6.	SHANTILAL PATEL	Fiji	Shri Vivekananda High School, Nadi.

	NAME	COUNTRY	ADDRESS
7.	ANANDAN RAO	Fiji	Shri Vivekananda High School, Nadi.
8.	GANGA DHAR	Fiji	Central Fijian School, Nausori.
9.	'ATU[illegible]ISA HAVEA	Tonga	In-Service Training Centre, Nuku'alofa.
10.	LAFULOU TAULAHI	Tonga	Tonga High School, Nuku'alofa.
11.	MOSIKAKA KAVAPALU	Tonga	Tonga College, 'Atele.
12.	GINA TEKULU	B.S.I.P.	King George VI School, Honiara.
13.	NOEL McNAMARA	B.S.I.P.	King George VI School, Honiara.
14.	FRED JUNGWIRTH	New Hebrides	Onesua High School, Efate.
15.	JOHN TOURLAMAIN	New Hebrides	British Secondary School, Vila.
16.	PETER ELLERY	Cook Is.	Nugao Teachers' College, P.O.Box 117, Rarotonga.
17.	PETER ETCHES	Cook Is.	Tereora College, Rarotonga.
18.	DES BURGE	Australia	51 Wattle Road, Brookvale 2100.
19.	ROGER WHITING	G.E.I.C.	King George V School, Bikenibeu, Tarawa.
20.	GEOFF SYKES	G.E.I.C.	Hiram Bingham High School, Beru.
21.	MURRAY ROBERTSON	Niue	High School.
22.	LIVI TANUVASA	W. Samoa	Leififi Intermediate School, Apia.
23.	PERENISE SUFIA	W. Samoa	Education Department, Apia.



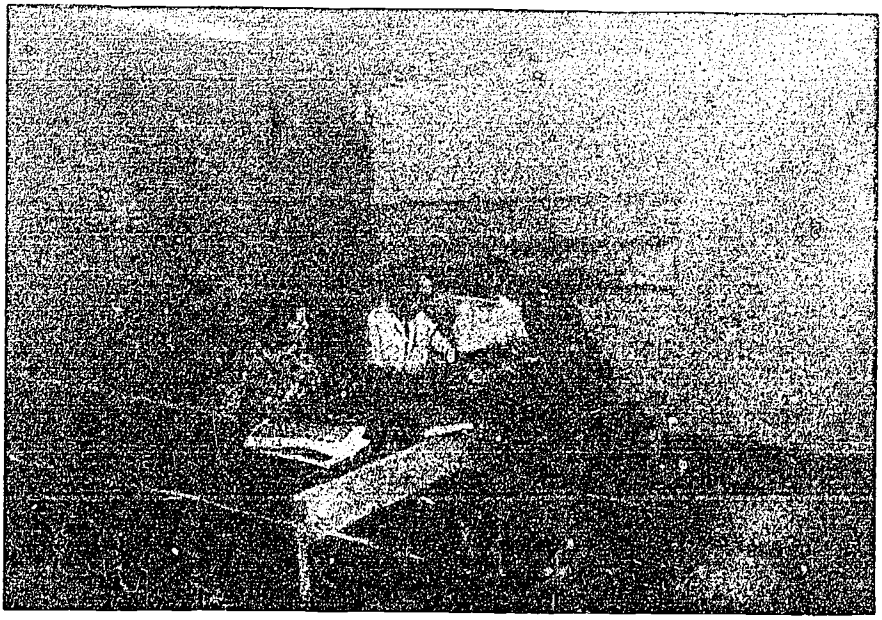
SOME OF THE MATHEMATICS SECTION PARTICIPANTS

MATHEMATICS SECTION REPORT

1. ORIGINAL AIMS

1.1 The basic aims of the workshop were to give training in and opportunity for the writing of mathematics material.

1.2 Although we felt that this had been accomplished, many participants expressed the opinion that the workshop had been of great value in other respects. Included among these were opportunity for



WORKING ON "NUMBER BASES"

PREPARING A UNIT ON "SETS"

- contact between people from different parts of the South Pacific region (this applies particularly to people from outlying islands),
- discussion of mathematical issues outside (as well as within) the main workshop framework,
- the promotion of co-operation and tolerance,
- the University to become more regional in nature.

2. METHODS OF WORKING

2.1 First we considered

- curriculum development procedures in the countries of the region,
- the way in which the U.N.D.P. Mathematics section has been functioning,
- the aims and objectives of mathematics teaching,
- the influence of these aims on
  - (a) methods of teaching,
  - (b) classroom organisation,
  - (c) the content of courses,
  - (d) textual material.

2.2 Following this, time was devoted to

- production of topic outlines,
- group (sometimes small, sometimes large) criticism and suggestions,
- draft writing and duplication,
- further group analysis of mathematical content and of English language,
- revision of draft,
- duplication of revised version.

2.3 Participants were able to take away all duplicated productions. Some of these were at first draft stage and some at the revised stage.

3. MATERIAL PRODUCED

3.1 Included were:

Teachers' Guide on "Methods of teaching",
Teachers' Guide on "The use of the newspaper",
Teachers' Guide on "The use of work cards",
Teachers' Guide on "The use of puzzles and problems",
Work card supplement to "Decimals I",
Work card supplement to "Angles",

Pupils' Pamphlets - "Indices",
"Logarithms",
"Trigonometry I",
"Trigonometry II",
"Trigonometry III",
"Calculating Devices",
"Air travel (Samoan version)",
"Reading and writing numbers",
"Circles",
"Shipping",
"Sets",
"Number bases I".

Teachers' Guides - "Money",
"Directed numbers",
"Air travel",
"Statistics",
"Area II".

New Pupils' Pamphlets which were started included:
"Solids and drawings",
"Networks",
"Geometrical transformations",
"Polygons",
"Matrices".

3.2 Continuation of production. After the present workshop we intend that

- the pattern of the workshop should continue (communication will be by post or, if possible, by satellite),
- country writing groups will be enlarged,
- the U.N.D.P. will circulate drafts, comments, etc.,
- commitments to further writing will be made by country writing groups.

We also envisage, besides producing new material, that the concept of a continual revision of existing material will be built into the curriculum development system we are creating.

4. RECOMMENDATIONS (These are general but not necessarily unanimous.)

4.1 Concerning workshops

- More time is needed to reach the production stage. A workshop of three weeks' duration is adequate if general lectures are given in the evening or made optional.
- Better facilities are needed for production.
- More notice should be given of the nature of a workshop.
- In a future workshop, it was felt desirable, for the sake of continuity, that perhaps half the people should be the same but that new people should also be given the opportunity of taking part.

4.2 Concerning communication

- Regional newsletters should be put into Mathematics Forum.
- This report of the Mathematics Group should be put into Mathematics Forum.

- The U.N.D.P. Unit generally should make more use of Extension Services representatives for imparting information.
- Country Broadcasting Commissions should be sent taped interviews, reports, etc.

4.3 Concerning U.N.D.P. assistance

It was recommended that this be continued after December 1973.

4.4 Concerning Regional Mathematics Development Committees

Where these did not exist, it was recommended that committees of local teachers be created. In other countries it was recommended that these committees be strengthened.

4.5 Concerning examinations

It was recommended that a written statement be sent to Directors of Education pointing out

- that New Zealand had agreed to examine South Pacific courses at School Certificate level,
- the extent to which these School Certificates are acceptable in countries overseas,
- the present stage in the formation of a South Pacific Examining Body.

CURRICULUM DEVELOPMENT WORKSHOP
SOME FEATURES AFFECTING CURRICULUM DEVELOPMENT

Editorial note: At the start of the workshop participants were asked to survey the curriculum development in Mathematics in their own country. Here are some of the points which were mentioned.

- Countries are at different stages of development. Some have an established Curriculum Development Committee, others none.

- In some areas the curriculum is dependent upon external examinations, in others on the particular wishes of teachers.

- Entrance examinations referred to are the New Zealand School Certificate and the Cambridge Overseas School Certificate.

- It is clear that teachers are not satisfied with these external examinations and very strongly desire a declaration concerning a South Pacific School Certificate.

- Because of the lack of such a declaration some countries are hesitant to join wholeheartedly in the U.N.D.P. project.

- It was felt that the U.N.D.P. materials could form a basic core for these overseas examinations. Even so special units would have to be written to cope with variations both in content and notation.

- While it was felt desirable that each country should develop its own production unit there were few in existence at present. Anxiety was expressed about the cost of booklets and the difficulty of production. One suggestion related to the formation of a Central Production Unit. It was agreed that the problem needed urgent consideration.

- The variations in standard and levels of English comprehension throughout the region were noted. It was accepted that the materials produced should cater for these differences.

- Primary curriculum development will soon have an effect on the starting points for secondary work. The present topic system should enable countries to cope with any changes which are necessary.

- Whilst accepting the idea that mathematics should be presented through problem situations it was agreed that sufficient attention should be given to the development of skills in computation.

Participants had these comments to make about the topic units on which the U.N.D.P. sponsored programme is based.

- It was felt desirable that at the back of each unit there should be a summary of the salient features.

- There was a suggestion that in some cases more exercises could be included to consolidate ideas. Also teachers would be encouraged to supplement the material with work cards of their own.

- Work cards were also of value for dealing with local environmental situations.

- There was an expression of opinion that there were many good traditional topics that should be included.

- Some participants felt very strongly the need for a unit on "Set Language", as this language provides for a unification of mathematics.

- Advantages of the pamphlet systems were outlined.
    - (i) Many people could join in with the writing.
    - (ii) The system was flexible.
        - Choices could be made by countries.
        - Alterations could easily be made in the light of changing situations or new ideas.
    - (iii) Children like the system.
    - (iv) Topic pamphlets do not have the inhibiting effect on development that a hard-covered textbook has.

- Difficulties of the pamphlet system were financial and administrative.