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ABSTRACT

A group of 161 kindergarten and first grade children were instructed in mathematics using exercises in linear measurement to increase understanding of unit-quantity relations. A comparison group made up from two adjacent and comparable middle-class schools was taught mathematics without intervention into the mathematics content. Treatment and comparison groups did not differ in general conservation performance throughout the study. However, an ancillary study demonstrated that linear measurement competence is preliminary to linear conservation. After five or six months of treatment, the measurement group was vastly superior in measuring competence, but slightly inferior in typical first grade mathematics. Appendices include sample lessons and test items as well as data tables. (Author/SET)

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FORMATION AND QUICK INTEGRATION OF MATHEMATICS CONCEPTS IN THE CHILD DURING THE FIRST SCHOOL YEAR

May 1973

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE

Office of Education

National Center for Educational Research and Development
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Abstract

Typically, children are introduced to mathematics by presenting them with an array of objects to be counted. The method is pedagogically, if not mathematically wrong, for the child will fail to gain a valid conception of number, and in consequence, his understanding of unit-quantity relations will be delayed. A different introduction to mathematics is possible which may avoid the above difficulties and, at the same time, provide unique learning opportunities. The child can be taught the physical operations of linear measurement so as to include practice in addition, subtraction, the tens number system, and other concepts without departing from linear measurement activity and with little or no paper-pencil arithmetic. Such measurement instruction was given to a sample of 161 children in the last half of kindergarten and in the first grade. A comparison group made up from two adjacent and comparable middle-class schools was taught mathematics without intervention into the mathematics content. Treatment and comparison groups did not differ in general conservation performance throughout the study. However, an ancillary study demonstrated that linear measurement competence is preliminary to linear conservation. After five or six months of treatment, the measurement group was vastly superior in measuring competence. This group was slightly inferior in "typical" first-grade mathematics.

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This research concerned the content of mathematics instruction for kindergarten and first-grade children. The rationale was complex, taking account of both what ought, and what ought not, to be a part of the first mathematics learned by the child. Exercises in linear measurement formed the basis of the treatment in order to establish in the learner basic quantitative concepts thought by the investigator to have their origin in such practice. Counting of objects as an element of didactic practice fails to provide the necessary quantitative experience.

The Problem

Immediate Goals

1. To accomplish in a treatment sample of subjects the concept of unit as a power, which requires that the learner demonstrate unit-quantity relations drawn from continuous variables.

2. To avoid the conception on the part of the learner that numbers assign only to counted objects.

3. To teach the treatment sample the operations of addition and subtraction of measured linear quantities as a generalization of the rule $n \pm 1$.

4. To accomplish the transfer of linear measurement operations so that the learner is able to perform measurements under a wide range of conditions.

5. To determine from tests of conservation whether children trained as above are superior in conserving and thereby give evidence of quantitative judgments, as opposed to qualitative ones in the solution of conservation problems.

A More Distant Goal

By continuous instruction over a period of nine to twelve months in linear measurement operations, it was the intention of the investigator to determine the relative mathematics competencies of the treatment sample and a comparison sample. This comparison was made on two bases. Two measures were constructed with the intention that one measure would cover the subject matter taught to the comparison group and the other the subject matter taught to the treatment group. Competence on both measures was determined for both groups of subjects.

The content of instruction for the comparison group was assumed to represent the range of subject matter commonly taught to children. As such, achievement in these subject matters was meant to represent an estimate of typical knowledge children acquire through the study of sets and counting operations.

The content of the treatment instruction was limited to linear distance measurement which will be given in greater detail in the method section of this report.

A Rationale for Making Measurement the Basis of Introductory Mathematics

The adult has used intuition to guide him when introducing mathematics to the small child, and he has chosen to teach the child to count. Numbers have been assigned to common objects as the child was taught to count boxes, apples, or pencils. The direct and easy way the child can be brought to use numbers in counting accounts for the almost universal acceptance of counting as the introductory method.

Pedagogists and mathematicians have not often paused to consider what should be the content of the child's first mathematical learnings. Recently, however, interest in the content of mathematics has increased and extended backward to early childhood as the well-known arithmetic gave way to modern mathematics. That mathematics is difficult to learn is generally acknowledged and the more inclusive content of the new mathematics has increased the learner's difficulties. This paper analyzes common practice and the difficulties children encounter and outlines an entré to mathematics learning which may make the child's task easier.

The Prime Concept

First consideration should be given to the concept of unit, for all numbers can be generated from it by the formula $n \pm 1$. The notion of unit can neither be explained to the child in relation to other numbers nor be based on other mathematical ideas, for such ideas do not pre-exist in the child. Nonetheless, the learner must correctly apprehend the unit concept as his subsequent learning will be based on it. For some years, the Russian educational psychologist, P. Ya. Gal'perin and his associates, have studied strategies to introduce neophyte learners to language, mathematics, and other subject matter (Reitman, 1962; Gal'perin, 1969; Gal'perin & Georgiev, 1969). It is the contention of these workers that much in mathematics education depends on what unit concept is learned at the very beginning of instruction. Gal'perin and Georgiev (1969) have called attention to the agreement methodologists unfortunately hold that unit is best explained by presenting groups of objects of which the individual object is designated "1." Teachers have directed their pupils to a group of things and required the learner to "bring me one ball (pencil, block, marble)." The practice has alternated between the above form and one requiring the designation of many as when the teacher asked, "How many books have I here?" Of the several properties possessed by the objects, the child must distinguish the property of "1." Since "oneness" cannot be separately explained to the learner, he is expected to grasp from these demonstrations that the separate object is one and that it gains that meaning from its individuality or separateness. Gal'perin and Georgiev asked rhetorically, "What exactly are we doing when we call an individual object one," and replied: "We are replacing one name with another." Then, too, they realized that unit belongs to one object no more than it belongs to aggregates that are given other numeral designations. The conceptual fault is that the unit cannot correctly be conceived in some discrete object the child calls "one."

The individuality or uniqueness of the object "one" refers to qualitative attributes and not to those that are mathematical or quantitative. Momentarily accepting such an interpretation, one can anticipate potential difficulties in separating meanings for the child that have the germane mathematical content from those that do not. As new information is presented to the learner, its meaning will depend to some degree upon the idea of unit as the child understands it. New learnings are likely to bear marks of distortion from whatever mislearning the child already has acquired.

A Correct Conception of Unit

Unit is appropriately conceived as the power of a set. It is an abstraction which applies to all members of the set but to none in particular. It can be applied to objects that are counted, but "unit" also applies in a more complete way to measuring. Before taking up the role which measurement plays in defining unit, consider what is theoretically necessary for a proper unit concept. Sinclair (1971) has discussed what the unit concept requires when applied to counting operations. Take the case of a finite assembly of objects. If these objects are used in teaching the learner an appropriate meaning for the concept "unit," he must divest the objects of all of their qualities so that they become identical and interchangeable. The objects can then be arranged so that one is included in the other in the serial order, $(1) < (1 + 1) < (1 + 1 + 1) \cdots)$.

In a practical sense, the objects must be distinguished so that a child can tell them apart and so they appear only once but are not missed altogether. The only way to keep them distinguishable is by their spatial or temporal order. Many adults intuitively sense the concept of unit, but the abstract ideas are explicitly understood by only the mathematically sophisticated. There is an obvious enigma in the above for anyone who aspires to get the small child started correctly in learning mathematics; namely, that the unit concept is basic and essential, but also that it is abstract and not easy to conceive. How, then, can it be taught to the child entering school with his limited repertoire? A set of tactics different from those now used will be required. The counting of objects is by no means the direct and adequate method it appears to be for starting the child toward mathematical understandings. The difficulties in separating idiosyncratic qualities from the unit are apparent when children are required to grasp the abstract concept of unit from some concrete quantity. Sinclair (1971) has documented the difficulties in children as they attempted to solve certain problems. Among the studies is one in which children counted poker chips (Greco, Inhelder, Matalon, & Piaget, 1963). In this study, the child was seated next to the experimenter. Before them were two piles of chips, red and blue, with the blue pile being much larger than the red. The experimenter proceeded by taking a blue chip from the large pile in front of him at the same time the child took a red one from the small pile. Experimenter and child repeated the draw four or five times, always drawing simultaneously.

Next, the experimenter asked, "Do we both have just as many counters? We each took our counters at the same time--you, a red one, and I, a blue one. Remember?" The child's response was not anticipated. A typical response was, "You have more than I have. Your pile is bigger than mine." The child recognized the red and blue attributes and behaved as if to say, "No matter what we do, there are more blues than reds." A difficulty is that the learner's attention has been directed toward the individuality of the objects (color), and he has not sensed the need for a unit common to both piles.

The meaning of quantity and the learner's dependence upon it. Quantity is best understood as a number based upon a given unit size. When a material representing the quantity is transformed, as it is in spreading out a small pile of rice so as to cover more space, the transformation does not disturb the number which represents the quantity. This is tacit to saying that the quantity has not changed. An interpretation of a stable quantity is not generally true of children below age six or seven. Young children do not ordinarily consider the quantity in relation to the unit. Equal amounts of material covering unequal amounts of space are judged by the small child to be unequal following his visual inspection. When a child is able to recognize the constancy of a quantity undergoing transformation, we say that the child "conserves" the quantity. Much attention has been given to conservation in the literature of mathematics learning (Copeland, 1970; Lovell, 1971; Rosskopf, Steffe, & Taback, 1971).

The failure in small children to conserve quantities may represent the absence of certain prerequisite concepts which integrate more or less rapidly at ages five and six, provided that the child has undergone certain experiences. An assumption underlying this paper is that the child must have at least an intuitive grasp of the relation of unit to quantity before he can conserve. By what other means can quantitative determinations be made? The more precise and complete is his understanding, the more certain that he will solve problems requiring quantitative ideas. The developing learner is not consistent; one time he may conserve by using the idea of unit, and the next time visually compare quantities and consequently fail to conserve. So it is in much of human learning; varied experience with different forms of a concept is required before one will master it and be dependably consistent (see Ellis, 1965, on transfer, and Dienes, 1959, 1960 on analytic and constructive thinking). When the child always resorts to the selection of a unit of measure and proceeds to correctly measure the quantity, whether the problem is volume, weight, or distance, he has a basis for further mathematics learning. Measurement has a special role in the rapid formation of number concepts. How this measurement differs from counting will now be considered.

Measurement of Continuous Quantities

Measurement operations vary according to the dimension one has chosen to measure. Some of these dimensions as weight, or distance, are specially favorable as the subjects of didactic exercise. According to Nagel (1960), "measurement is the correlation with numbers of entities which are not numbers." Without intention, this definition calls to mind expository advantages in measurement activity which are not present in counting.

Linear distance measurement is one form of measurement which can ideally represent the basic mathematical concepts. The advantage to the small child is that in measuring, he will manipulate a tangible analogue of the ideas we want him to apprehend and, in time, the learner will associate mathematical abstractions with the manipulanda. Then, too, the physical operations will provide a support for memory which one expects from a symbolic model well suited to the subject matter.

As measurement exercises are undertaken, the learner should first measure with some arbitrary but convenient unit, like a pencil or his shoe. The unit of measure should change frequently to help divest the physical measuring-object of unintended meaning that can arise from attention to its qualitative attributes. Further, the operations at first require that the measuring object be positioned, a distance marked, and the object transported forward for remarking while attention is given to accuracy of the measurement. Accuracy is an important new concept for the child--important because he has unfortunately practiced gross visual judgments of quantity which are now prepotent in his behavior, and because accuracy depends on the particular use that is made of a unit of measure. When continuous rather than discrete variables are expressed as quantities, accuracy is always an issue. One can count but not measure with complete accuracy. It is on the point of accuracy that a relation may be drawn between measuring and conservation of quantities. Quantities must come to be recognized in terms of unit and number. Measuring is a form of practice which sets the relation of number to quantity on every practice trial. When the unit-number relation is recognized, the child should be capable of conserving and he should hold the rudiments necessary for understanding number systems. Linear distance measurement is a more complete representation of quantitative concepts than counting, including, for instance, the concept of continuous quantity, and is probably comprehended by the child more readily than other measurement dimensions. There are a number of quantitative ideas that the learner must integrate before he can make appropriate use of subsequent mathematics instruction. Counting operations fall short of these requirements.

The meaning of quantity in relation to measurement. Not all number considerations are quantitative. Number may have only a nominal use as when data processing cards are punched "1" to represent "female," and "2" to represent "male." In this case, arithmetic operations cannot be carried out between the classes. It is correct to count only within the class. Certain scales of measurement embody quantitative attributes. Stevens (1960) has described four scales of measurement: nominal, ordinal, interval, and ratio. The importance of these scales to this discussion is that the scales are progressive, wherein the ratio scale includes all of the quantitative characteristics of the other three, mathematically less complete, scales. Attributes of the ratio scale of which linear distance is one, include transitivity of the numbers through a progressive order as is the case when one measures from unit to unit across the distance. The units are equal and interchangeable. There is a true zero point of origin for the numbers. One can form ratios or proportions within the scale, and the units are infinitely divisible.

Transfer of Learning and the Integration of Mathematical Concepts

Not all of these ideas of quantity can be comprehended by the small child, and not even all the easier ones at once. When instruction for the neophyte learner is centered on his measuring activities and measurement continues to be the basis of teaching and learning, there are repeated opportunities for the teacher to point out conceptual relations.

The general form of instruction and the manipulanda of instruction change only slightly from concept to concept when measuring is the instructional medium. The close relation between one set of measurement operations and the next is the basis for concept integration or transfer of learning.

Addition in measurement terms means addition of line segments, not paper-pencil operations. Subtraction, likewise, means the removal of a line segment and so with each additional mathematical idea--the referent is the same. Rapid formation of complex concepts is only possible when the individual associations the learner will make are abstractions from salient cues. Other things being equal, the cues will be strong if they are not continuously changed but rather are a part of a model of instruction that is used continuously for a lengthy period of time. There is reason to think that success in the widely-referenced Harlow (1949) study, "The Formation of Learning Sets," may have been due to the way the problems organized around the same set of manipulanda. Rhesus monkeys were confronted repeatedly with the same geometric objects, i.e., a cube, a sphere, as variations were brought about systematically among the discriminanda. The learning which took place was orderly and progressive.

Often the expository statements in mathematics textbooks are cryptic and the relations to other mathematical concepts are underdeveloped, or for that matter, not developed at all. Then, too, teachers usually depend on these textbooks to determine the sequence of subject matter to be learned and often permit a book to govern the amount of practice particular concepts will get. These circumstances do not favor mastery and consequent integration of subject matter. Chances are that the reader will be able to recount from his own observations, formal learning activities of children that were too brief and too devoid of related context provided by an immediately prior experience. The learning of highly-conceptual subject matter such as mathematics or language is dependent on a familiar context. The discriminations and generalizations these subjects require are many and they sometimes arise out of subtle similarities and differences in the stimuli of instruction. Ellis' (1965) analysis of the research and theory on transfer of learning provides evidence that: (a) the learner should have extensive experience with an original task if that task transfers to a next task; (b) greater pupil effort early in a series of tasks is better than later effort; and (c) a variety of examples should be given to the learner and practice should continue until he has mastered the governing principles.

The concrete operations of linear measurement can be made the basis of original learning in mathematics so that the learned elements associate together in the formation of numerous concepts, each of which is referenced to the same concretia. A number of observers have recognized measurement as an important aspect of mathematics learning and some have given it considerable emphasis (Dienes & Golding, 1966; Lovell, 1971; Wohlwill, 1964). That measurement can form the sole basis of mathematics learning for a time, with unusual transfer benefits to the learner, is the thesis of this paper.

Evidence that Learning to Measure Can Be the Core of Mathematics Instruction and that It Contributes to the Learning of Conservation of Quantities

Sequencing of subject matter is done in the hope that early topics will benefit the later ones and that some optimal sequence exists. A progression from less advanced subject matter to more advanced can, under appropriate conditions, fit what Gagné (1962, 1965, 1968) has called "learning hierarchies" or "hierarchical learning sets." To be hierarchically ordered, two levels of information, X and Y, must hold the following relations: (a) knowing X and being given a response cue, the learner must be able to perform Y; and (b) any learner already able to perform Y must necessarily be able, without further training, to perform X. Gagné (1962) has empirically validated a hierarchy hypothesis where elements of mathematics learning were the subject matters. The Gagné validation scheme is laborious and for that reason Phillips and Kane (1972) have studied alternative bases for determining hierarchical knowledge. These investigators used test data to compare seven different validation schemes for hierarchically ordered information presented to children in grades four through six. One of these seven schemes for ordering information was Gagné's (1962) a priori task analysis, which requires that subtasks be extrapolated from a description of the learner's terminal behavior. This task analysis is not to be confused with Gagné's method of validating a hierarchy. Phillips and Kane concluded that such a

> careful task analysis of instructional objectives can be a powerful tool in devising optimal instructional sequences. In fact, it may mean that in terms of overall cost, that careful analyses of instructional objectives to reveal

the prerequisite subtasks is an adequate procedure for developing a valid hierarchy.

Which comes first, measurement or conservation? Piaget, Inhelder, and Szeminska (1960) and Sawada and Nelson (1967) have asserted that the child should conserve before learning to measure. To these observers, it is a matter of the way the child has come to view the problem. If the learner thinks a quantity has changed when it is moved, as might be the case when rods of varying lengths are used to measure with, then he is not ready to learn to measure. Forcing the linear measurement task early will make of it a rote and mechanical operation. Gal'perin and his coworkers (Reitman, 1960; Gal'perin & Georgiev, 1969) quite obviously have taken a contrary view. They have argued that conservation results from learning the unit-quantity relation. Gal'perin and Georgiev (1969) taught 50 kindergarten children certain measuring exercises and required them to perform 15 different problems designed to assess mastery of the unit-quantity relation. Though these investigators did not use the term conservation, many of the problems will be recognized as conservation problems. Following measurement training, the children, trained by Gal'perin and Georgiev solved almost 100 percent of the problems. Comparison children, who were taught by the common means of counting and making visual comparisons, solved considerably fewer than half the problems.

Procedures

Subjects

The subjects were kindergarten children at three neighboring schools of comparable socio-economic status. The treatment group consisted of the entire kindergarten class at one school. The control group consisted of subjects from the other two schools who were matched with the treatment sample on the basis of age and performance on one of the two entering measures.

Determination of Entering and Leaving Behavior

Prior to the beginning of the mathematics instruction, the Goldschmid and Bentler, Concept Assessment Kit--Conservation, Forms A and C and the Metropolitan Readiness Test--Numbers were administered to the treatment and control groups. Forty-seven pairs of subjects were matched on the basis of age and the conservation test; 51 pairs were matched on age and the Metropolitan test.

After three and one-half months of instruction to the treatment group, the same two measures were administered again.

The Treatment

The regular classroom teachers and their aides were supplied with the instructional material. The lessons to be presented were discussed with the teachers in order to insure that they understood the objectives of each particular step in the sequence of instruction. Suggestions made by the teachers for more effective presentation were incorporated in the material, thus giving the teachers an active role in the development of the material as well as capitalizing on their interactions with the subjects.

The instructional sequence began with the development of the concept of a "unit of measure" in the context of linear measurement. The children were given a variety of experiences in comparing an assortment of everyday objects with an arbitrary "unit of measure" such as a pencil or blackboard eraser to determine if the object was longer than, shorter than, or the same as the unit of measure.

Once the concept of a unit of measure as a standard of comparison was established, it was used as the basis of measurement exercises by which the quantitative concept of number was developed. Making use of arbitrary units such as paper clips, lollipop sticks, the subjects measured objects such as tables, books, long rods, and their own arms to determine the number of units contained. This was done in one of two ways--using multiple copies of the unit or transporting a single unit across the distances to be measured.

The concept of one-to-one correspondence was developed at first by matching units in two different objects and then by representing each unit measured with some irregular fragment, as bits of styrofoam or scraps of paper to be counted to represent a total distance. The next natural step in the progression, then, was the symbolic representation by numbers of the units measured.

Simple addition of line segments whose total was five or less was also introduced.

Ancillary Conservation-Measurement Study

In conjunction with the kindergarten program, an investigation of the relation between linear measurement and ability to conserve was carried out. Martin (1972) attempted to establish which of two skills, linear measurement or conservation of length, is the higher order skill. She first completed a task analysis of length conservation through which she identified six measurement skills which appeared to be constituent subskills of conservation. For each subtask of the hierarchy, criterion mastery items were constructed and administered in random order to 42 kindergarten children, half of whom had recently had formal instruction in linear measurement. To a particular point in the hierarchy, 80 percent of the subjects passed all items and failed all after that point. These preliminary observations gave assurance that a systematic study of the hierarchical order of measurement and conservation could be undertaken.

In the next step, 55 children who could not conserve length were randomly assigned to three groups. One group first received training in measurement, then in conservation; a second group was trained in the inverse order, and a third group had no training.

Several days after the training sessions, a posttest that was primarily ar assessment of measurement skills, was administered to all of the groups. The results of this study will be discussed in the next section.

Continued study of the treatment sample into the first grade. As the sample of children entered first grade, the treatment group was expanded to include all the first-grade classes in the school where the treatment was administered. The treatment group now included approximately 120 children. The same subjects as in kindergarten constituted a comparison group.

The teaching staff involved in the first-grade program consisted of four regular classroom teachers, four teacher's aides, and several student teachers.

The subject matter in the first grade consisted of the following topics:

1. Review of kindergarten program.

2. Expansion of the concept of one-to-one correspondence.

3. Addition and subtraction of line segments up to 10.

4. Commutative and associative properties of numbers.

5. Introduction of the concept of place value.

6. Simple addition of 2-place numbers.

All of the above concepts were introduced and developed through measurement of line segments. Once the concepts were established, their representation was generalized to the traditional format, such as 2 + 7 = 9 and 9 - 4 = 5.

After three and one-half months of instruction, a measurement test, a test based on the content of a standard text used by the comparison group, and a conservation test, were administered. This latter test was the Goldschmid-Bentler Conservation Test which had been given to the original kindergarten sample.

Sample lesson plans and pages from the measurement test and the test based on the instruction given to the comparison group are included in the Appendix.

Results

Ancillary Conservation-Measurement Study

The task analysis conducted by Martin (1972) identified six measurement skills which appeared to be constituent subskills of conservation. The sequence of training was varied: conservation, then measurement training, or measurement, then conservation training. Subjects first learning the conservation task, an order not favored by the task analysis, averaged 19 trials and seven training errors to reach criterion. Those other subjects following the sequence favored by the task analysis, measurement, then conservation, averaged less than eight trials and one training error (trials $F = 9.44$, df 1/30, $p < .004$ and errors $F = 7.11$, df 1/30, $p < .01$).

Other evidence was obtained by Martin which also favors the interpretation that the hierarchical order of skills is measurement, then conservation. Significantly more subjects failed to meet the conservation criterion who began training on that topic ($\chi^2 = 4.36$, df 1, $p < .05$).

Kindergarten Programs

Before beginning the instructional program, the Goldschmid-Bentler Concept Assessment-Conservation Test and the Metropolitan Readiness Test--Numbers were given to subjects in the treatment and control groups. Pairs were matched on the basis of age and the score on either the conservation or the readiness test.

Conservation Test

Forty-seven pairs were matched on the conservation test with a mean of 6.68 and s.d. of 7.86 for the treatment group and a mean of 6.77 and s.d. of 7.90 for the control group. After approximately three and one-half months of instruction, a second conservation test was administered. The mean for the treatment group was 10.85 with an s.d. of 8.99. The mean for the control group was 10.74 with an s.d. of 8.23. An analysis of variance for the posttest revealed no significant difference between the two groups.

An analysis of covariance, using the pretest as the covariate, was also done to determine if there was a significant difference in the amount of change in the two groups. Again, no significance was obtained.

A third administration of the conservation test was given after the subjects had been in the first grade for approximately four months. The size of the sample was reduced due to subjects moving out of the district. The treatment group (N = 39) obtained a mean of 14.15 with an s.d. of 8.33. The mean for the control group (N = 32) was 13.78 with an s.d. of 7.68. However, there was a ceiling effect in these results with a number of subjects scoring at or near a perfect score of 24. The difference between the groups was not significant. The means and s.d.'s for all administrations appears in Table 1 in the Appendix, page 40.

Metropolitan Readiness Test. Fifty-one pairs were matched on the basis of the Numbers Subtest of the Metropolitan Readiness Test. The means and s.d. for both groups were 12.37 and 4.00 respectively. On the second administration, the treatment group had a mean of 14.69 and an s.d. of 3.25, while the control group had a mean of 15.59 and an s.d. of 4.52. There was no significant difference between the two groups of the posttest. An analysis of covariance using the pretest as the covariate also showed no significance. The results appear in Table 2 in the Appendix, page 40.

First-Grade Program

After approximately three and one-half months of instruction, two measures of achievement were developed and administered. One test was based on mathematics concepts derived from the measurement instruction. It had 67 items. Another test which had 48 items was tied closely to instruction carried out in the comparison group. The content of the two tests was governed by what the teachers indicated as material given the most practice and emphasis in instruction. The two measures were administered to both the treatment and comparison groups, but any one subject had instruction appropriate for only one of the tests and not the other. The means, standard deviations, and t-values are given in Table 3 in the Appendix, page 41. On the measurement-based test, the treatment group had a mean of 48.82, as compared to a mean of 14.41 for the comparison group ($p < .001$). On the comparison-based test, the control group had a mean of 36.36 while the treatment group had a mean of 32.54 ($p < .001$).

Omega square values were computed. These values revealed that 73 percent of the variance was accounted for by the treatment effect on the measurement-based test but only 6 percent of the variance was accounted for by the treatment effect on the comparison-based test.

Conclusions

The Martin (1972) study has provided evidence that the rationale is correct which claims that measurement knowledge aids the conservation of linear quantities. The evidence is contrary to Piaget's contention that a child must conserve before learning to measure. The reader should bear in mind that both the measurement task and the conservation task were limited to linear distance quantities. The significance of that realization is this: the Martin evidence cannot be used to support general transfer effects from linear measurement practice to other forms of conservation such as area, volume, or mass. The Goldschmid-Bentler measure of conservation includes items to cover conservation of number, area, volume, linear distance, etc. That test was repeatedly administered to the measurement-treatment and comparison groups without reliable differences appearing between the groups on conservation ability. If practice in linear distance measurement was to have a general, large-step transfer effect, such an effect would have been evidenced in a progressive separation between the groups in Goldschmid-Bentler conservation which favored the measurement practice group.

Such transfer is absent in research literature. Transfer of training studies as far back as those by Thorndike have shown that transfer occurs in small steps across closely related tasks. The present study sheds no light on the belief, some times expressed, that large quantum jumps in concept formation do occur. However, it was the hope of the investigators that long-term treatment would eventually result in rapid integration of mathematical ideas. Such an effect, if realized, could be called large-step transfer. No evidence was acquired which demonstrated such transfer. All that can be said is that perhaps after more treatment, some measure might be capable of showing unusually large transfer effects, namely those which show that a child can make broad application of one or more principles.

Concerning the achievement of treatment and comparison groups on the subject matters to which they were daily exposed, namely a standard mathematics content (Kelly, Dwight, Nelson, Schluet, & Anderson, 1970) and the measurement-based instruction, the following analysis seems reasonable. The superior performance on a measurement test of measurement-trained subjects is to be expected and the inferiority of this group on a standard subject matter test is also reasonable to expect. That the omega square values show these relative differences to be unequal, that is strongly favor the treatment group, is important. Performance on measurement activities part way into the first grade favored the treatment by more than three to one and .73 of the variance was a treatment effect. On the comparison mathematics, the treatment group was about 10 percent inferior and only .06 of the variance was a treatment effect. Because of the uniqueness of the measurement treatment, one will anticipate that its effect will endure for some time, and if continued, perhaps have long-term consequences in accord with the rationale of this study.

In the fall of 1973, a standardized achievement test will be administered to entering second-grade children. A comparison of the performances of the two groups will be made at that time. Since the content of standardized achievement tests tend to relate closely to what is usually taught in classrooms, one should expect the comparison group to do better. The investigators anticipate, on the contrary, that if there is a difference, it will favor the treatment group for reasons related to the rationale of the study reported here and because there are every day opportunities for the treatment sample to learn informally, some of what is taught in the typical first-grade mathematics.

References

Copeland, R. W. How children learn mathematics: Teaching implications of Piaget's research. New York: Collier-Macmillan, 1970.

Dienes, Z. P. The growth of mathematical concepts in children through experience. Educational Research, 1959, 2, 9-28.

Dienes, Z. P. Building up mathematics. London: Hutchinson, 1960.

Dienes, Z. P. & Golding, E. W. Exploration of space and practical measurement. New York: Herder and Herder, 1966.

Ellis, H. C. The transfer of learning. New York: Collier-Macmillan, 1965.

Gagné, R. M. The acquisition of knowledge. Psychological Review, 1962, 69, 355-365.

Gagné, R. M. The conditions of learning. New York: Holt, Rinehart and Winston, 1965.

Gagné, R. M. Learning hierarchies. Presidential Address, Division 15, The American Psychological Association, 1968.

Gal'perin, P. Y. Stages in the development of mental acts. In M. Cole & I. Maltzman (Eds.), A handbook of contemporary Soviet psychology. New York: Basic Books, 1969.

Gal'perin, P. Y. & Georgiev, L. S. The formation of elementary mathematical notions. In J. Kilpatrick & I. Wirszup (Eds.), Soviet studies in the psychology of learning and teaching mathematics. The University of Chicago, 1969.

Harlow, H. F. The formation of learning sets. Psychological Review, 1949, 56, 51-65.

Hayes, W. L. Statistics for psychologists. New York: Holt, Rinehart and Winston, 1963.

Keddy, M., Dwight, L., Nelson, C., Schluet, J., & Anderson, T. Exploring elementary mathematics, Grade I. New York: Holt, Rinehart, and Winston, 1970.

Lovell, K. The growth of basic mathematical and scientific concepts in children. New York: Philosophical Library, 1961.

Lovell, K. The growth of understanding in mathematics: Kindergarten through grade three. New York: Holt, Rinehart and Winston, 1971.

Martin, M. L. Sequential order in the learning of measurement skills and the conservation of length: Performance of preschool children. Unpublished master's thesis, Arizona State University, 1972.

Nagel, Ernest. Measurement. In A. Danto & S. Morgenbesser (Eds.), Philosophy of science. Cleveland: World Book Co., 1960.

Phillips, E. R. & Kane, R. B. Validating learning hierarchies for sequencing mathematical tasks. Paper presented at the meeting of the American Educational Research Association, Chicago, April 1972.

Piaget, J., Inhelder, B., & Szeminska, A. The child's conception of geometry. New York: Basic Books, 1960.

Reitman, W. R. Some Soviet investigations of thinking, problem solving, and related areas. In R. A. Bauer (Ed.), Some views on Soviet psychology. Washington, D.C.: The American Psychological Association, 1962.

Sawada, D. & Nelson, L. D. Conservation of length and the teaching of linear measurement: A methodological critique. The Arithmetic Teacher, May 1967, 345-348.

Sinclair, H. Number and measurement. In M. F. Rosskopf, L. P. Steffe, & S. Taback (Eds.), Piagetian cognitive-development research and mathematical education. Washington, D.C.: National Council of Teachers of Mathematics, 1971.

Stevens, S. S. On the theory of scales of measurement. In A. Danto & S. Morgenbesser (Eds.), Philosophy of science. Cleveland: The World Book Publishing Co., 1960.

Van Engen, H. Epistemology, research, and instruction. In M. F. Rosskopf, L. P. Steffe, & S. Taback (Eds.), Piagetian cognitive development research and mathematical education. Washington, D.C.: National Council of Teachers of Mathematics, 1971.

Wittrock, M. C. Three conceptual approaches to research on transfer of training. In R. M. Gagné & W. J. Gephart (Eds.), Learning research and school subjects. Itasca, Ill.: F. E. Peacock Publishers, 1968.

Wohlwill, J. F. Development and measurement. In R. Ripple & V. N. Rockcastle (Eds.), Piaget rediscovered: A report on cognitive studies and curriculum development. Ithaca, N.Y.: Cornell University, School of Education, 1964.

APPENDIX

Sample Lessons, Sample Test Items, and Tables

Introduction of Concept of Unit of Measure
(Single Copy)

OBJECTIVES

The child understands the concept of a single unit of measure when he:

(a) applies the principles of precise measurement;
(b) compares lengths of objects using varying sizes of single copies;
(c) measures lengths using different sizes of single copies
(d) manipulates a single unit to demonstrate the rules of precise measurement;
(e) solves problems using the rules of measurement.

VOCABULARY

measurement
rules of measurement

INTRODUCTION

Initially the teacher stresses the importance of precise measurement. The following rules of precise measurement may be introduced as the steps to measuring with single copies.

Rules of Precise Measurement

1. Place the copy directly on one end of the object to be measured.
2. Put a finger on the copy and mark the point at the end of the copy.
3. Pick up the copy and place it next to the mark . and continue marking and measuring.

The teacher may demonstrate the rules of precise measurement on some classroom examples. It is important to avoid any reference to standard units of measure, such as inch, foot, etc. Instead, emphasis should be placed on the idea of the unit and how it is used in measurement, The children will learn that they can use a single copy of the article that represents the unit of measure to determine the lengths of objects that are longer than the unit itself. During the measuring, the children need not be encouraged to count the units as they mark them off. It is essential that the children understand that the measured item may contain more than one unit of measure, yet it will remain a single item itself.

ACTIVITIES

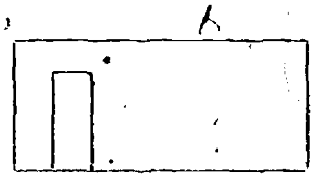
Group

A. The teacher demonstrates the use of a single copy by drawing a line on the blackboard and asks how to measure it with only one unit. Children volunteer answers. The teacher then demonstrates the careful placement of the unit each time it is moved.

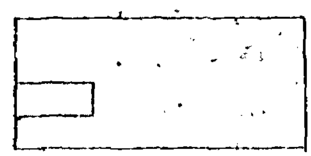
Review comparisons using the "greater than or longer than, shorter than or less than and same as concepts." Use review exercises and dittos.

B. Using a stick that is about a foot in length, make comparisons between it and several items in the classroom. Alternate asking which of the two is shorter, longer, and have the children express the comparison. To introduce the day's lesson, place the stick (1') on the narrow end of the piano bench, as shown, so that the bench exceeds the length of the stick by just a few inches. A small table can be used in place of a piano bench. When the children have answered that the bench is longer, place the stick on the bench as shown and ask again which is longer.

#1



#2



C. Then say "let's see how many sticks we'll need to go to the end of the bench." Place the stick on the bench and mark the end with a finger and pick up the stick and place it at the finger under the length of the bench that has been measured.

D. Place a piece of tape on each table and have the children measure using:

a. single copies
b. various size units

Have one child measure and one child mark the end of each unit--with a pencil mark and with a finger.

E. For 2 flannelboard

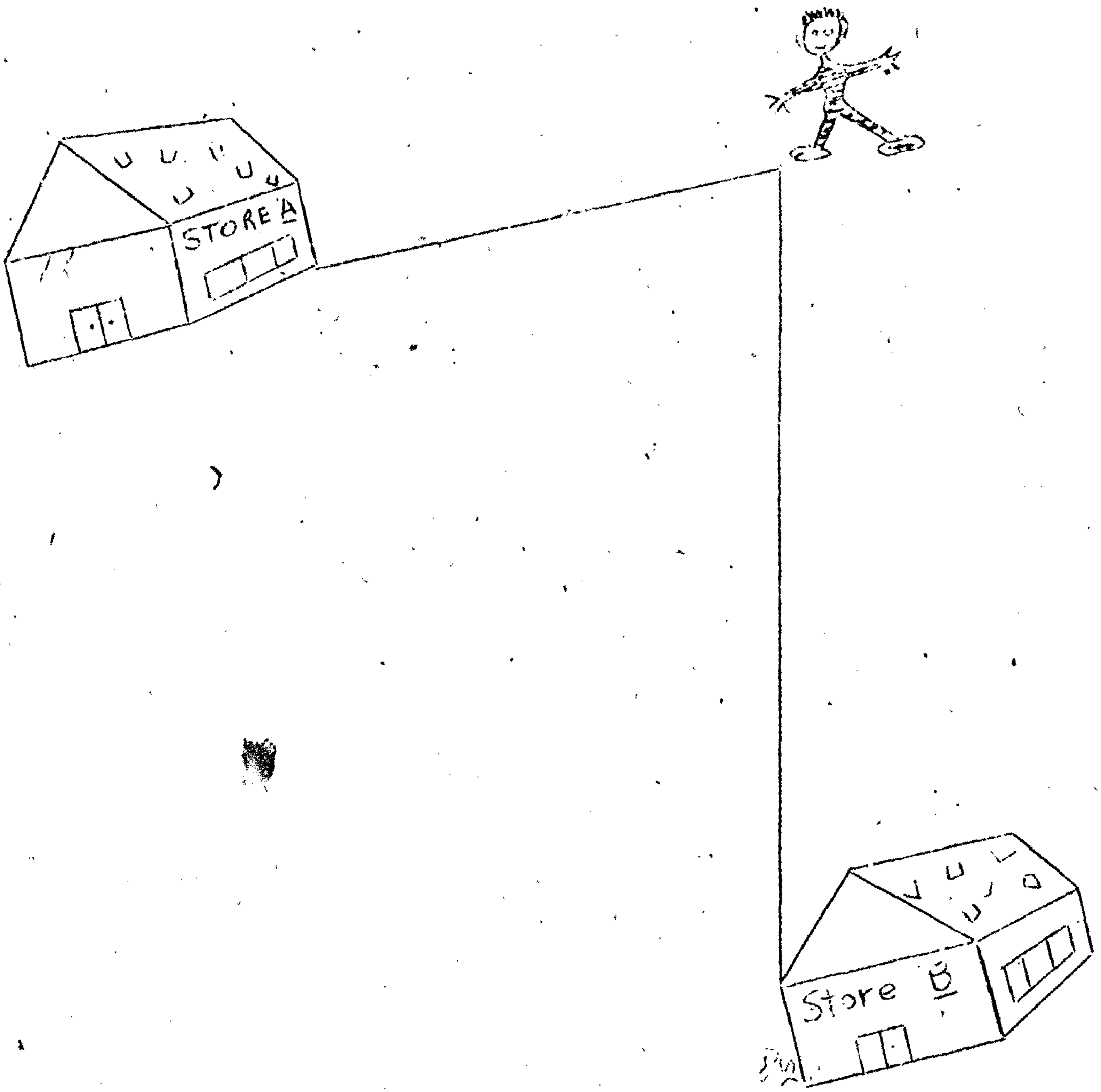
The children can tell that this shoe (the unit) is as long as the boy's step is. How many steps will it take him to go home? (The teacher can add other figures and ask who's closer to home, etc.)

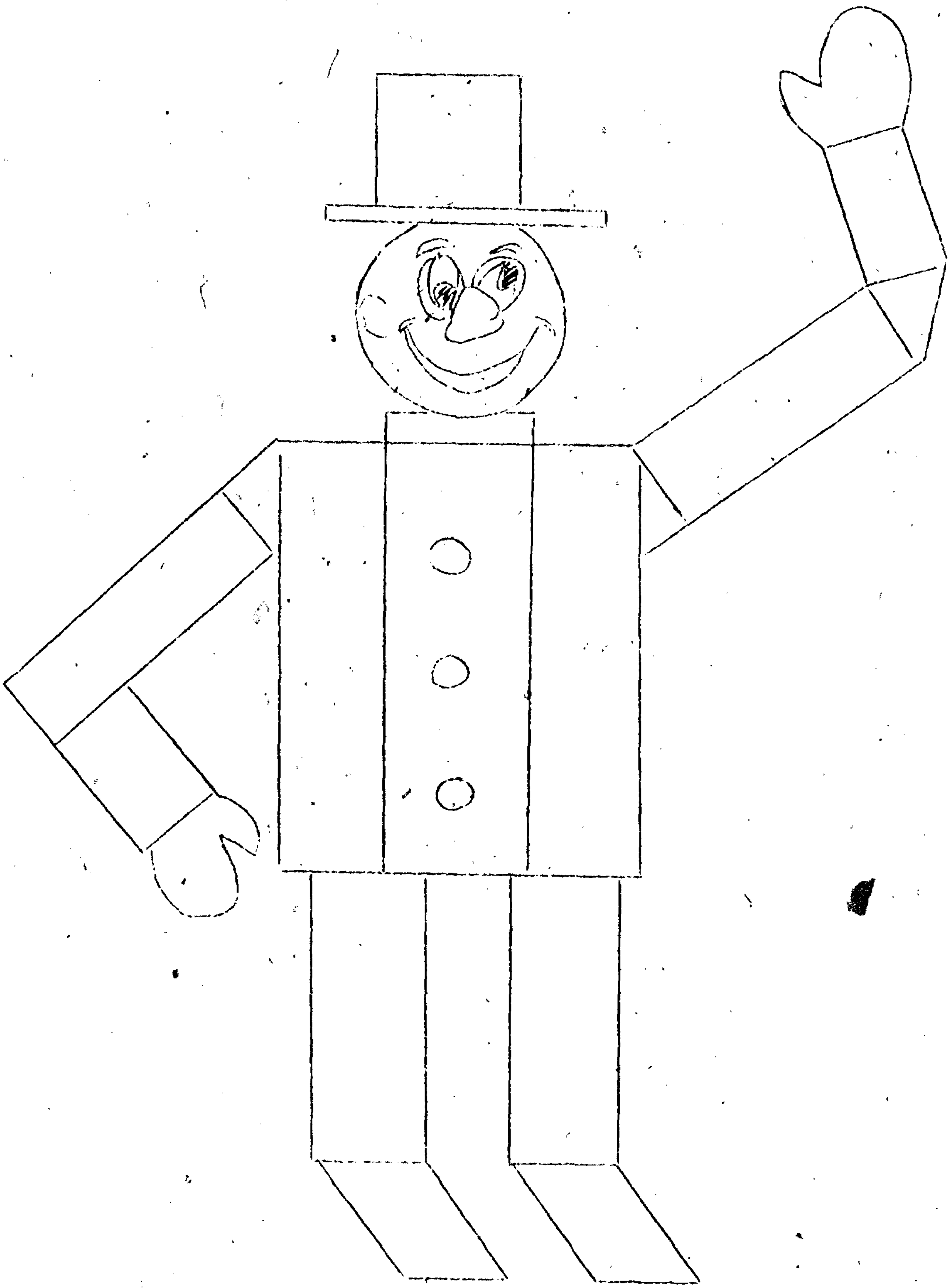
F. Give children a large unit, a small unit and a piece of paper. Ask the children, "would you rather have a piece of candy as long as four of these (small unit) or one as long as three of these (longer unit)." Let the children measure and mark on the paper and decide.

INDIVIDUAL

A. On the "map," the child is asked if the figure is closer to Store A or Store B. He is provided with a unit for measuring.

B. The child is given the clown and is told to color three units red, two units blue, and one unit yellow.


STORE A
Store B

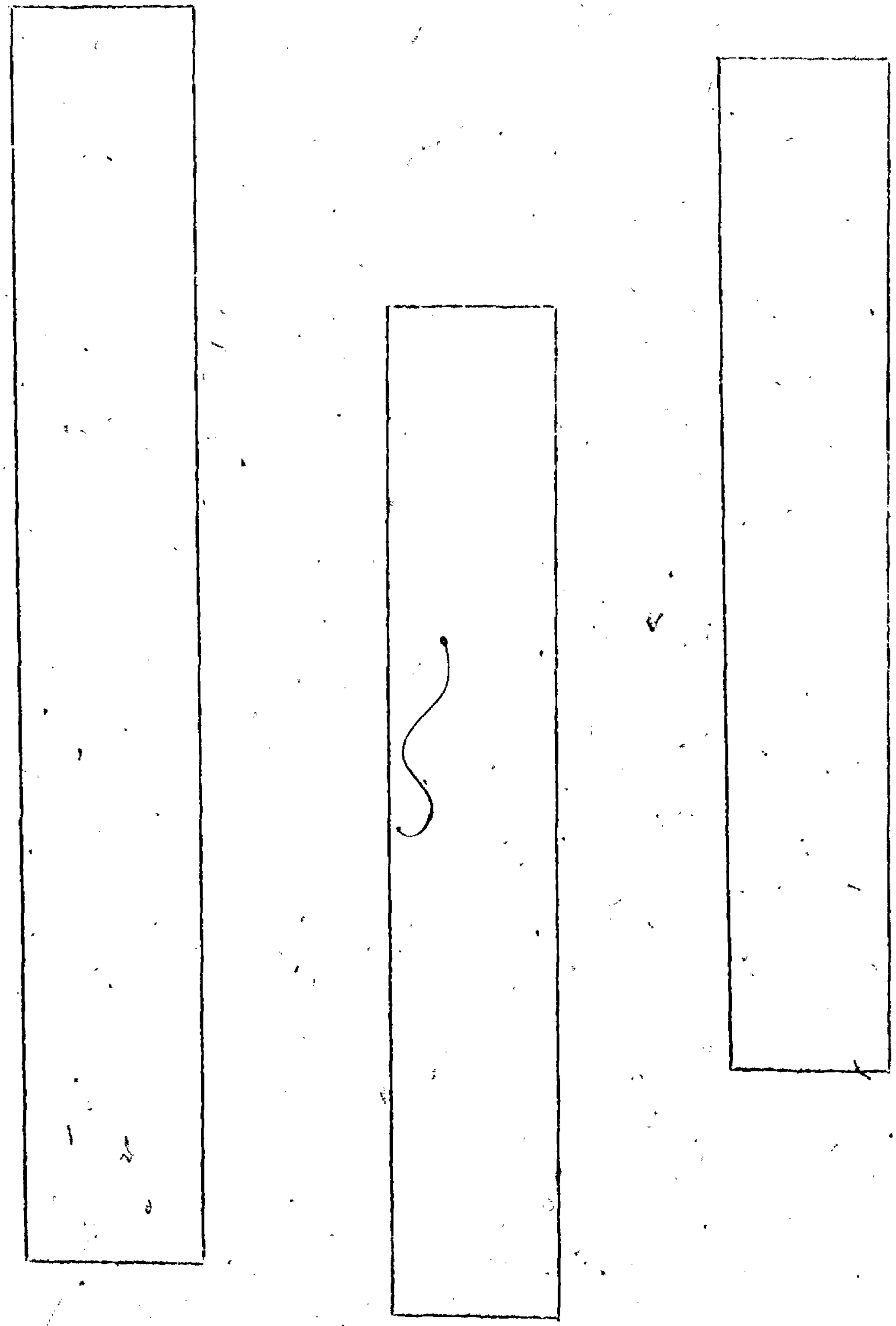


INSTRUCTION SHEET #5

One-to-One Correspondence

On the following worksheet:

1. Measure and mark each rectangle with the unit.
2. Next match the units by connecting them with a line.
3. Then circle the longest rectangle.
4. Use the 1½ inch measures.

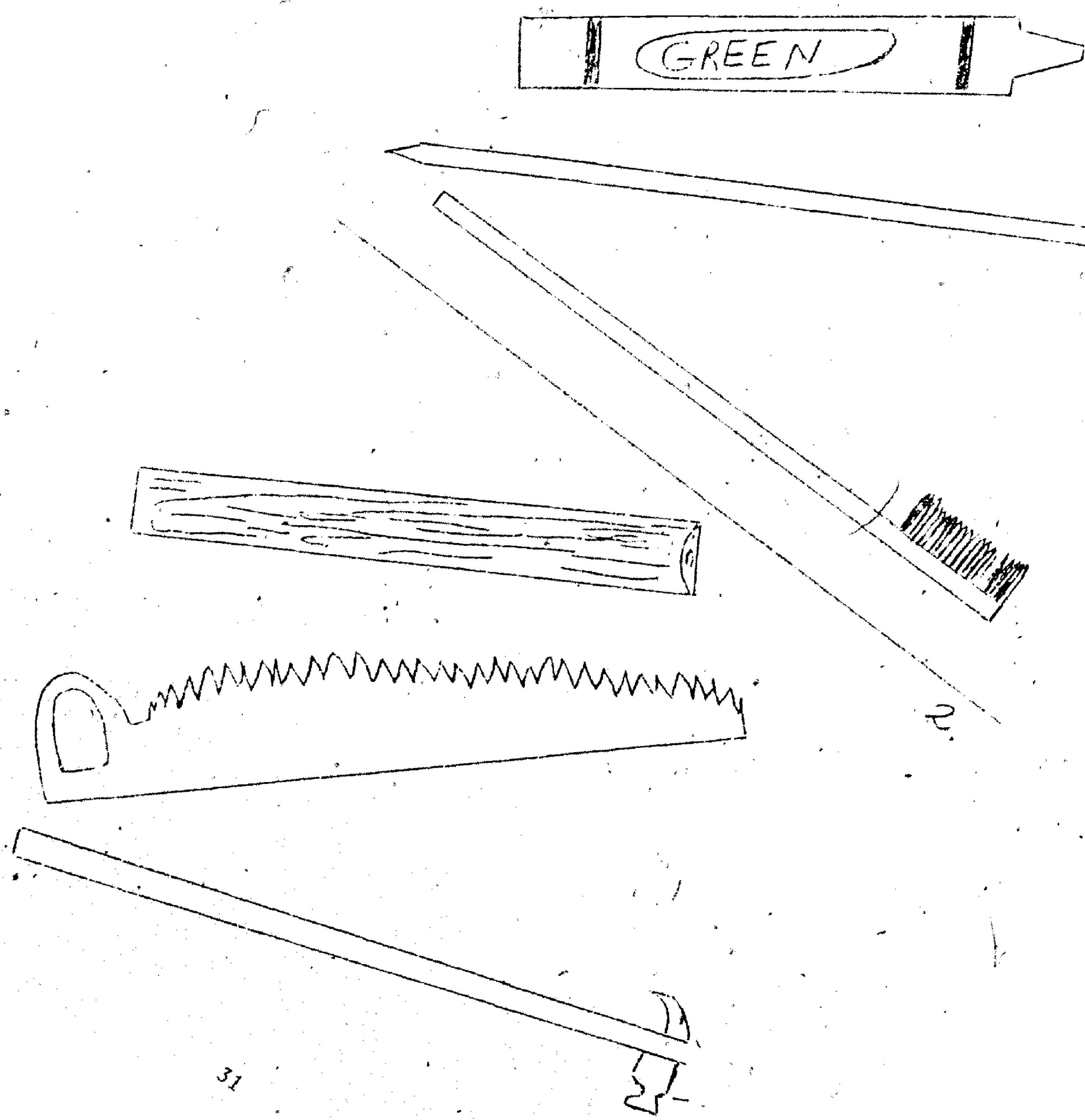


INSTRUCTION SHEET #4A

One-to-One Correspondence

On the following worksheet:

1. Measure and mark each object in problems 1 and 2.
2. Match the marked units by connecting them with a line.
3. For problem number 1, circle the shortest object.
4. For problem number 2, circle the two objects that are the same.
5. Use the 3/4 inch measures.


GREEN
2.

INSTRUCTION SHEET

Combination of Numbers

On the following worksheet:

1. Use yellow 3/4 inch units.
2. Measure each length.
3. Write the correct number under each unit.
4. Draw attention to the equation formed under each box. Use the term equation without any direct teaching of the term.
5. Point out that we are adding one more in each step. Try to bring out the relation that 2 is more than 1, 3 is 1 more than 2, etc.

Combination of Numbers

☐ and ☐ = ☐

___ ___ = ___

☐ and ☐ = ☐

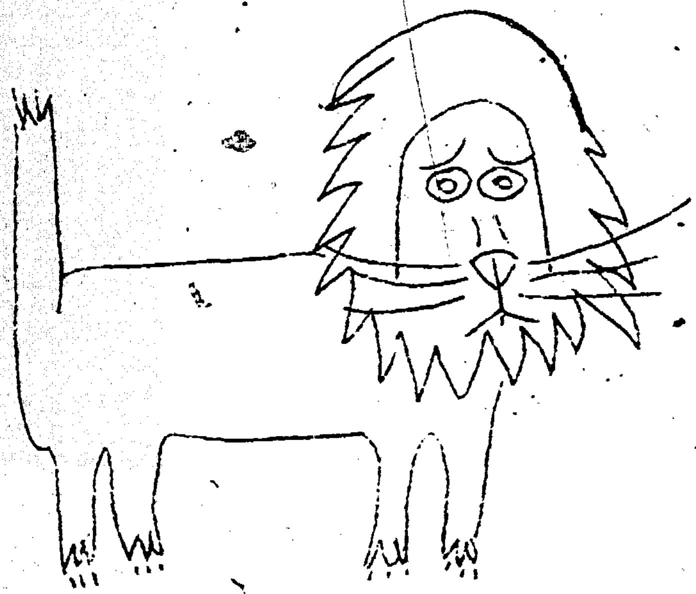
___ and ___ = ___

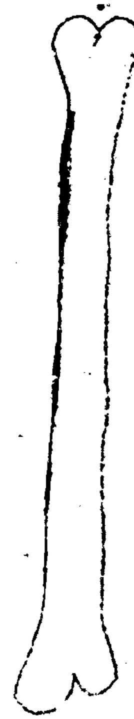
☐ and ☐ = ☐

___ and ___ = ___

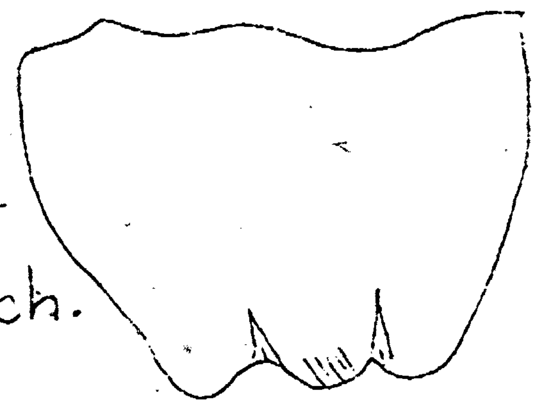
Marvin the Munching Monkey sat on a 5 unit branch which grew 1 more unit. How long is the branch? ________

The 7 unit tree that Marvin was holding onto grew 1 more unit. How tall is the tree? ________





Leonard the lazy lion
ate 3 units from a 5 unit
bone. How many units
of bone were left? ______
Later Leonard had 2 units
of a 5 unit apple for lunch.
How much apple was left?



______

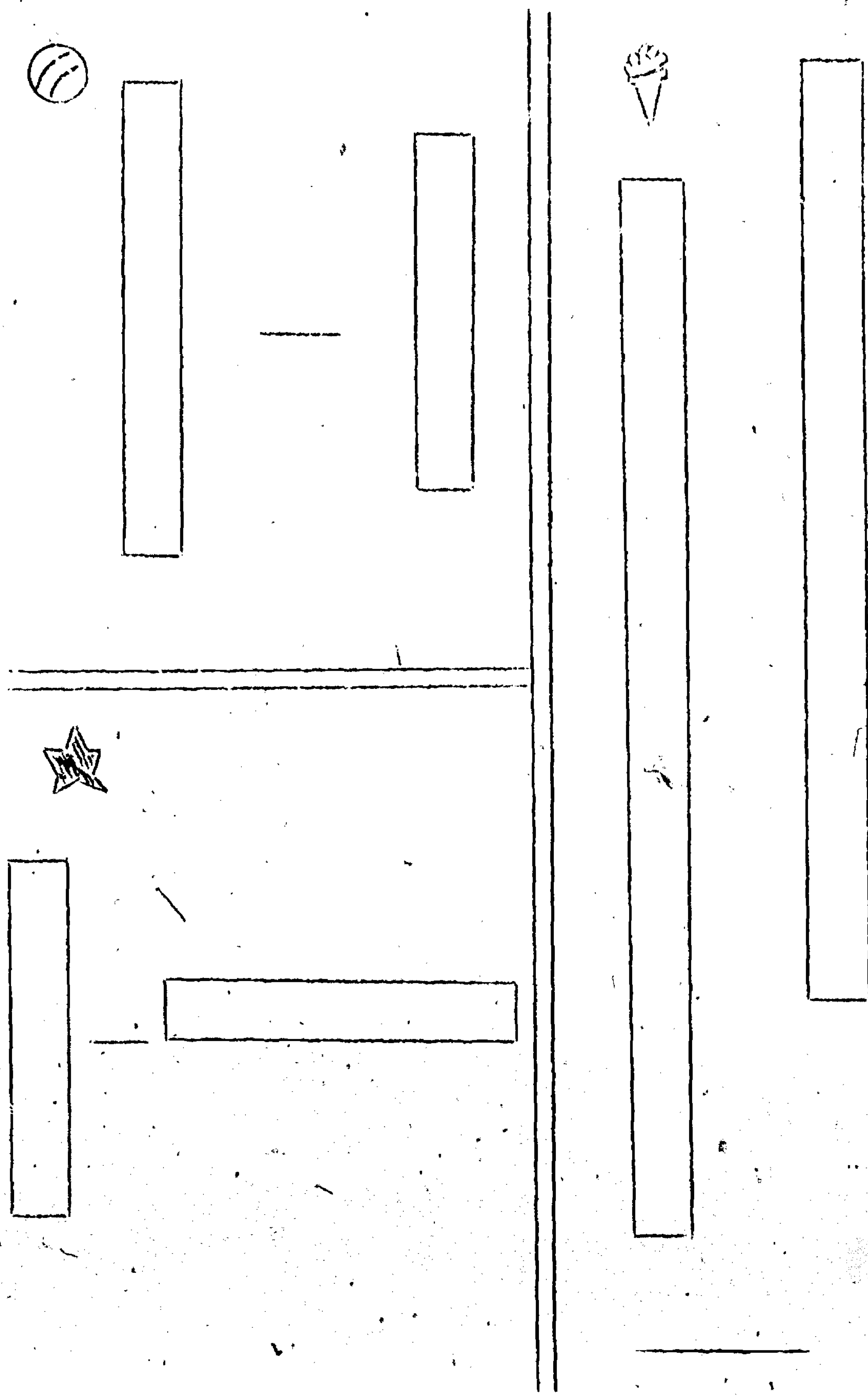
SAMPLE OF TEST ITEMS BASED ON MEASUREMENT INSTRUCTION

Materials: Worksheet with three pairs of rectangles, crayon, and 3/4 inch yellow unit.

Instructions: After materials are handed out, have the children measure the first pair of sticks saying: "Look at the two sticks in the box with the ball (demonstrate). Please measure each of these with the yellow unit. For each unit you measure draw a dot in the space below each stick. Now, next to the dots, write the number that tells how many dots you drew under each stick."

"Now look at the little line between the sticks. If the sticks are the same length put an equal sign on the line. If the first stick is longer put the sign that means 'greater than' on the little line."

Repeat these instructions for the pair below these (star) and then with the pair at the right (ice cream cone). Note on this last pair, the symbol is written between the fragment groups.



SAMPLE OF TEST ITEMS BASED ON COMPARISON INSTRUCTION

Instruction

Look at the fish in the corner of the paper. Please match each fish (apple, tree, butterfly) in this set with a boat (banana, candy cane, flower) from this set by drawing lines between them. ________

Now write the number in the square that tells how many fish (apples, trees, butterflies) are in that set. ________

In the circle write the number that tells how many boats (bananas, candy canes, flowers) are in that set. ________

If one set has more members, draw a circle around that set. If they are the same, do not draw any circles.

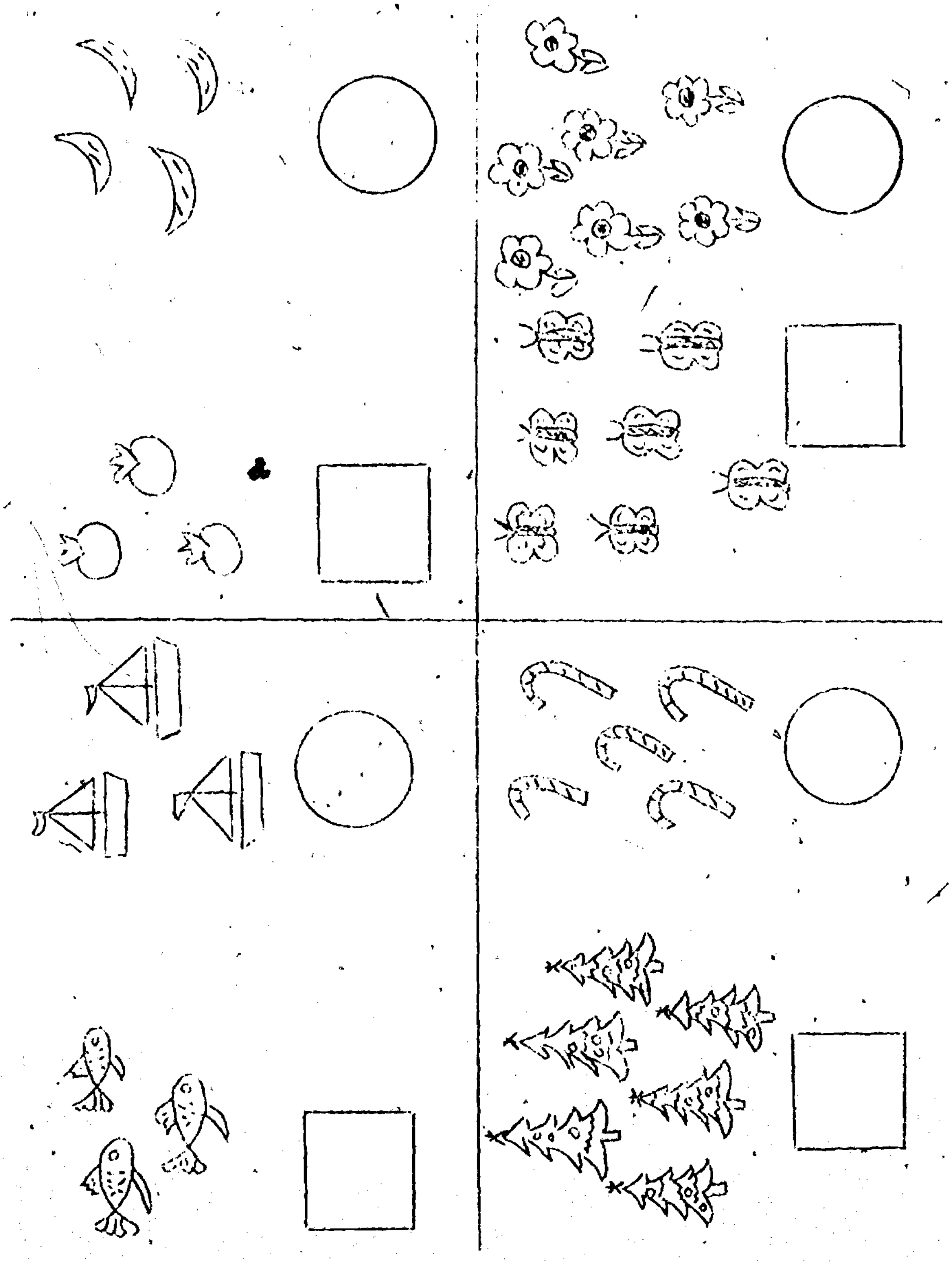


TABLE 1

Performance of Two Groups at Three Successive Administrations of the Goldschmid-Bentler Test: Concept Assessment--Conservation

Administration	Treatment					
	Measurement			Comparison		
	M	SD	N	M	SD	N
1-Pretreatment	6.68	7.86	47	6.77	7.90	47
2-During treatment	10.85	8.99	47	10.34	8.23	47
3-During treatment	14.15	8.33	39	13.78	7.68	32

TABLE 2

Performance of Two Groups at Two Successive Administrations of the Metropolitan Readiness Test--Numbers

Administration	Treatment					
	Measurement			Comparison		
	M	SD	N	M	SD	N
1-Pretreatment	12.37	4.00	51	12.37	4.00	51
2-Posttreatment	14.69	3.25	51	15.59	4.00	51

TABLE 3

Mean Achievement in Mathematics Determined from Tests of Measurement Related Concepts and Concepts Derived from Comparison Mathematics

Treatments	Competence Tests			
	Measurement		Comparison Mathematics	
	M	SD	M	SD
Measurement Instruction N = 115	48.82	10.55	32.54	6.34
Comparison Instruction N = 46	14.41	5.41	36.36	7.02
t-tests	21.14**		3.47**	
Omega Square Comparisons	.73		.06	

**p < .001