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ABSTRACT

This conference explored ways to improve mathematics education for inner-city schools. Five position papers, together with summaries of the discussions of these papers by conference participants, are contained in this report. Topics range over types of pre- and in-service teacher training, laboratories and materials for the inner-city school, and relevant instruction as well as other pedagogical considerations. One report reviews many of the projects that have been instituted by various agencies or institutions which attempt to upgrade mathematics education in the inner-city schools. In the summary of the small-group discussions, a list is given of the recommended ways in which SMSG could contribute to the improvement of inner-city mathematics education. (JP)

SCHOOL MATHEMATICS STUDY GROUP

Report of

A CONFERENCE ON MATHEMATICS EDUCATION IN THE INNER CITY SCHOOLS

March, 1970

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Financial support for the School Mathematics Study Group has been provided by the National Science Foundation.

Preface

From its inception in 1958, the concerns of the School Mathematics Study Group had been largely along the directions of curricular changes. However, it is clear that questions of teaching mode are vitally related to those of curriculum development. Equally, questions concerning the audience for whom the curriculum material is intended are related to those of teaching modes. Consequently, since the spring of 1964, the School Mathematics Study Group has been devoting increasing attention to mathematics education for the culturally disadvantaged population as well as for the "slow learner". With financial support of the Cooperative Research Branch of the U.S. Office of Education, the School Mathematics Study Group held a conference in Chicago, Illinois on April 10 and 11, 1964 to explore recommendations for experimentation and curriculum development with regard to students whose achievements in mathematics is below average.*

In the past few years, the impact of problems in the inner city has been brought to bear. In response to these sensitivities, the Advisory Board of the School Mathematics Study Group has recommended an exploratory conference similar to the one for below average achievers in 1964. Financial support was obtained from the National Science Foundation, and the Conference on Mathematics Education in the Inner City Schools was planned by a committee of Board members (William Chinn, Chairman; Karl Kalman; Dexter Magers) and was held on March 6 and 7, 1970 in Philadelphia.

Five position papers were invited: a keynote paper to relate the problems of mathematics education to the general problems of education in the inner city schools; a survey of existing projects which attempt to attend to inner city problems in mathematics education; attention to these problems at the level of the state department of education in a large state; and a panel, focusing on pedagogy and the laboratory approach as possible partial solutions to the problems. In addition to these papers, three reaction papers were invited responding to the report of projects, the state efforts, and the panel.

This report contains all the papers prepared for the conference, together with summaries of the discussions of these papers by the conference participants.

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*School Mathematics Study Group, Conference on Mathematics Education for Below Average Achievers, Stanford, 1964.

The program for the conference, reproduced on these pages, displays the order of events as they occurred. Rearrangement of this sequence for the purposes of this report is made simply to place the position paper and its reaction in juxtaposition.

Although the participants were grouped into four subgroups for the discussions, there were many common threads shared by all the groups. Therefore, the discussions of all four groups are summarized together. Much liberty has been taken in summarizing the comments; however, it is hoped that the spirit on the basis in which these discussions were made have been preserved.

William Chinn

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SCHOOL MATHEMATICS STUDY GROUP

Conference on Mathematics Education in the Inner City Schools

Benjamin Franklin Hotel - Philadelphia, Pennsylvania

Friday, March 6, 1970

Chairman: Mr. Dexter A. Magers, U.S. Office of Education

9:00 a.m. - 9:15 a.m. Professor Edward G. Begle
Director, School Mathematics Study Group
Stanford University
Opening Remarks

9:15 a.m. - 10:15 a.m. Professor Henry I. Willett
Virginia Commonwealth University
Problems of Mathematics Education in the Inner City Schools

10:15 a.m. - 11:15 a.m. Professor Lauren G. Woodby
Michigan State University
Survey of Projects

11:15 a.m. - 11:30 a.m. Break

11:30 a.m. - 12:15 p.m. Mr. Melvin Mendelsohn
Director, Computer Assisted Instruction
New York City Board of Education
One State Education Department's Activities Concerned with Improving Mathematics Education in the Inner City Schools

12:15 p.m. - 2:00 p.m. Lunch

2:00 p.m. - 2:45 p.m. Panel - Pedagogy, Laboratory

Moderator: Professor William C. Lowry
University of Virginia

Mrs. Elizabeth A. Collins
Director, Staff Development and Utilization
Dade County Public Schools
Pedagogy and Accountability in Teaching Society's Rejects

Mr. Charles Allen
Mathematics Consultant
Los Angeles City Unified School District
The Laboratory Approach to Teaching Mathematics in the Inner City

2:45 p.m. - 5:00 p.m. Group Discussions

#1 Mrs. Isabelle Rucker, discussion leader
Professor Paul A. White, recorder

#2 Mrs. Sarah Greenholz, discussion leader
Dr. Burton Colvin, recorder

#3 Professor Dora Skypek, discussion leader
Professor Frank L. Wolf, recorder

#4 Mr. Terry Shoemaker, discussion leader
Professor Peter A. Lappan, recorder

Saturday, March 7, 1970

Chairman: Professor Joseph Payne, University of Michigan

9:00 a.m. - 9:45 a.m. Professor Jack E. Forbes
Purdue University at Hammond
Reaction to Survey of Projects

9:45 a.m. - 10:30 a.m. Mrs. Emma M. Lewis
Supervisor of Mathematics
Public Schools of the District of Columbia
Reaction to One State Education Department's Activities

10:30 a.m. - 10:45 a.m. Break

10:45 a.m. - 11:30 a.m. Professor William M. Fitzgerald
Michigan State University
Reaction to Panel

11:30 a.m. - 12:45 p.m. Group Discussions

12:45 p.m. - 2:00 p.m. Lunch

2:00 p.m. - 3:00 p.m. General Session

Report from Group Discussions

Group 1: Mrs. Isabelle Rucker
Virginia Department of Education

Group 2: Mrs. Sarah Greenholz
Cincinnati Public Schools

Group 3: Professor Dora Skypek
Emory University

Group 4: Mr. Terry Shoemaker
Castle Rock High School

3:00 p.m. - 3:30 p.m. Open Discussions

Moderator: Professor Joseph Payne
University of Michigan

3:30 p.m. - 4:00 p.m. Professor Herbert J. Greenberg
University of Denver
Summary of Conference

PROBLEMS OF MATHEMATICS EDUCATION
IN THE INNER CITY SCHOOLS

Henry I. Willett
Virginia Commonwealth University

Introduction

An increasing number of articles and books are being written about the education of the disadvantaged, but no accepted definition is used to designate the disadvantaged. As I understand my assignment, it is to direct my comments to the problem of mathematics education for the inner city child. My instructions also indicated that you would be interested in the education of the slow learner. I am sure it is not necessary for me to point out to this distinguished group that it is very important that we not equate a child from the disadvantaged community with a slow learner. However, I am afraid that this attitude presents one of the problems that serves to retard the progress of children in the inner city. Consequently, I think it is important for us to make several basic assumptions:

1. The child from a disadvantaged community or home is not necessarily a slow learner.
2. Many of the problems experienced by the child from the disadvantaged area is the result of his inability to express himself verbally.
3. The child from the disadvantaged area is more likely to have language difficulties and a small vocabulary as a result of limited experience and meager opportunities for expression, particularly with adults.
4. There is a danger that all children from disadvantaged areas will be labeled with the same general characteristics, and will arouse the same expectations on the part of the teacher.
5. The mathematical concepts of some children in the disadvantaged areas may be on a higher-than-normal level because of experiences that involve mathematics and decision making. For example, some of the children in the inner city assume responsibility earlier and many develop mathematical concepts beyond that of children of an affluent neighborhood. A six- or seven-year-old child in the disadvantaged area in the inner city may be more likely to have such experiences as being given a $5 bill and told to go to the store

and purchase something to eat. This requires mathematical ability and decision-making ability that extends far beyond the experience of many children in the more affluent suburbs. Consequently, we must recognize that mathematics does not necessarily hold less significance for the child in a low income family.

Probably enough has been said to indicate that in this discussion, we are thinking of the term, disadvantaged, in two different settings, and we must be careful not to confuse the two. Disadvantaged refers to those children who, because of place and circumstances of birth, do not have those experiences that would tend to prepare a child for entry into school and to begin the formal learning process.

The other use of the word, disadvantaged, applied to slow learners who have less ability to achieve academically, not only as a result of environment, but also as a result of their own limitations in intellectual capacity.

In the case of the very young child, we have not found any ready or easy method of determining the immediate cause as to why a child at a given time appears to be disadvantaged. In this latter sense, we find various estimates of the percentage of total population included in this classification. Some authorities would place this number at fifteen percent of the population. Other authorities would place as high as twenty-five to thirty percent, those children who are rejected because they have been unable to acquire language facilities to compete in the mainstream of life, both in and out of school. There is a danger that false labeling on the part of the teacher in the earlier years of a child's experience at school will prove just as harmful and dangerous as the false labeling of drugs in a medicine cabinet. Perhaps a good starting point would be to accept the statement made by Barbara Biber in a pamphlet prepared for the Association for Childhood Education:

> "Fortunately we have had experience with education for children from disadvantaged areas and know that, while they need special understanding and adjustments to their particular needs and characteristics, they are like all other children. Fundamentally, they have the same potentialities, the same curiosity, the same basic human problems to face in life--except that life has given them some extra ones no children should really have. When we talk about education, we want to begin by talking about what our understanding is of good education for young children--all young children."

It is very important for persons in mathematics education to be concerned about the development of the whole child and to recognize that the child's mathematical ability or lack of it may be closely related to the development of verbal skills. This is a factor that applies not only to the disadvantaged child, but to children in general. In fact, it seems to persist even into adult life. I recall studies in high school algebra, many years ago, which indicated that pupils that could achieve on a score level of 10 when given the formula for a problem dropped to a skill score level of four when the same principles were expressed verbally, or when the formula had to be derived from the written statement of the problem.

I would now like to discuss several important areas that should be carefully considered by mathematics educators. Remember, these are the opinions of a generalist who makes no pretense of being an expert in the field of mathematics.

Take the Child Where He is

Treat him as an individual and apply the best that we know in producing an environment in which a child will want to learn and in which he can learn most effectively. This will require that the teacher separate fact from fancy in dealing with the disadvantaged child, and she should begin by providing experiences that have been absent or lacking in the life of the child. This means that the teacher must be concerned with language. Of course, mathematics itself is a language; however, in teaching mathematical concepts to the young child, there must be concern for language that gives meaning to objects and thoughts and ideas, along with symbols and computations. Whether or not this kind of approach can be made to the teaching of the disadvantaged child with the proper consideration for his mathematical concepts and understanding will depend very largely on the experience, training, understanding, attitude and commitment of the teacher.

The Training of Teachers for the Inner City

As a part of Project Aware, a study was made of 122 colleges and universities that incorporated work with the disadvantaged into the teacher-training curriculum. Sixty percent of these colleges and universities attempted to give this awareness and understanding in courses alone, but we now have enough experience to know that the proper training and orientation requires more than vicarious experience, that in order to prepare for teaching in the inner city, the teacher must have an understanding that extends beyond subject matter. This involves an understanding of how children learn, plus some direct experience in working with children in the inner city.

The Urban University program at Syracuse University affords a good example of this kind of program in that they have added a fifth year during which the teacher trainee actually spends time teaching under guidance and supervision in the inner city. Experiments over the country are applying this direct experiential approach that usually involves a cooperative program between the schools and colleges. It is a pretty traumatic experience for a young teacher, raised in a middle-class family, with no specific training and experience for the particular job, to begin teaching in an inner city situation with children from a different ethnic background. Not only is it essential for specific training to be given by the colleges, but the school systems also have the responsibility to provide a program of orientation supplemented by continuing in-service training.

Not all persons are so constituted mentally and emotionally that they can be successful teaching the inner city child. Along with the training and experience, there must be a strong commitment which, when present, can prove to be the most satisfying experience in teaching. A part of the training of the teacher in such a situation will involve specific training in making mathematics applicable to the needs and interests of the disadvantaged child. This question of the importance of teacher preparation is expressed very succinctly by McCloskey in the following statement:

> "Many teachers assigned to core city schools have had little or no preparation for working with impoverished children...most teachers are too immersed in middle-class outlooks to teach disadvantaged children successfully...difficulties are accentuated when teachers with the least experience are assigned to slum schools and, more so, when supervision is inadequate...in many schools, half of the teachers are substitutes...to make matters worse, teacher turnover in depressed area schools is exceptionally high. In some, it reaches sixty-one percent each year. Short tenure decreases teachers' opportunity to acquire an understanding of disadvantaged pupils. Adding to the disparities inherent in high turnover rates, some competent teachers refuse to work in "undesirable neighborhoods."

The Curriculum

Teachers still rely primarily on textbooks prepared for children from middle-class homes. Consequently, both the teacher and the child tend to be handicapped from the very start because the text deals with experiences and often describes objects and situations that are foreign to the child of the inner city. Experience and experimentation both indicate that in dealing with the disadvantaged child in developing mathematical concepts and understanding, it is important to deal with objects that can be seen, touched, handled, moved, counted. It is important that all children, and especially the disadvantaged, begin with concrete objects that can be identified verbally. It is generally accepted that considerable skill must be developed with concrete and verbal identification before the child can be expected to move to abstract thinking. This relationship has meaning for the child throughout his mathematical experiences.

The work of Piaget suggests experiences that enhance intellectual growth which can be distinguished in three stages:

> "In the first and pre-school stage the child's mental growth consists mainly in establishing relationships between experience and action. In this preoperational stage the child's perception of objects is paramount, and often only one aspect is considered and all others ignored. If the same amount of liquid is poured into two similar bottles and then the liquid from one is poured into a tall thin bottle, the child will concentrate on height only and say there is more in the tall bottle."
>
> "A second stage, usually associated with the early school years of the child, is identified by Piaget as the stage of concrete operations. In this stage the child is not only active, as in the previous stage, but operational. He manipulates physical objects to represent things and relations in his mind, and uses the data for solution of problems. In this manner cognitive growth is linked with concrete objects. In performing concrete operations the child deals directly with objects and learns to make classifications and seriations. Concepts of substance, weight, and volume are obtained in this stage. The child structures the immediate world."[1]

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[1] TIEDT, SIDNEY W., Teaching the Disadvantaged Child. Oxford University Press, 1968.

"But, Piaget observes, the child moves to this stage (formal operations) at about 11-12. Here, the child becomes capable of reasoning not only with objects and actions but with abstractions. The child uses mental data by means of symbols. Thought goes beyond the immediate real world. Formulating hypotheses and deducing all the consequences from them become part of the cognitive power of the youth. Possible variables and their relationships are explored. In this manner his problem-solving activities are forwarded by the power of abstract exploration.

"What has this to do with teaching mathematics in the public school classroom? Piaget has outlined a scheme of developmental patterns of intelligence. When aligned with developmental structures of mathematics, the teacher can, in turn, select and teach basic ideas; and do this in such a way as to pass from active exploration of concrete objects, to operation with concrete objects coupled with intuitive thought, to formal operation in the abstract for the same basic idea. The environment of the low-achieving child can be manipulated to follow, even force, the natural development of cognitive skills."[2]

Some interesting experiments are being carried on throughout the country that subscribe to this general philosophy of learning. For example, in the City of Richmond, the School System is experimenting with industrial arts in the primary grades as a means of giving meaning to mathematical concepts and helping children to operate with the concrete, to verbalize what they are doing, and, hopefully, by the middle or upper elementary grades to move into abstract thinking. A program of relating mathematics to economics is also proving effective since children are usually motivated by acquiring, handling, and spending money. We need to remember that mathematics education is not an entity in itself, closely related to verbal and written expression, but the basic purpose of mathematics is to help people to live better, to understand the world in which they live, and to become more proficient in making the right decisions. This means that the problems of mathematics teaching are related to the interests and abilities of the children that are being taught, and surely children cannot adjust to and succeed in a world of computers and trips to the moon without a growing understanding of the role and importance of mathematics in giving meaning to life.

[2] Ibid.

The implications of science and mathematics in approaching the solution to some of our social problems should not be overlooked. This thought is expressed by John W. Gardner, former Secretary of the U. S. Department of Health, Education and Welfare, when he made the following recommendation:

> "If we indoctrinate the young person in an elaborate set of fixed beliefs, we are insuring his early obsolescence. The alternative is to develop skills, attitudes, habits of mind, and the kinds of knowledge and understanding that will be the instruments of continuous change..."

Family Involvement

The expectation of the home is still by far the strongest motivating factor in the academic success of a child. This means that if education as a whole can have much chance of success, we must utilize the parents and the influence of the home in a two-pronged approach in order to achieve desirable goals. A number of cities, including Richmond, have some very far-reaching projects going on involving three-year-olds, their parents, and the schools. In fact, experience indicates that if a child doesn't get certain experiences as a three-year-old, it is difficult to compensate for this lack of experience when he is six. And, of course, the family as a unit provides the most ideal environment in which the child develops his first interest in numbers and their meaning in his communication with others.

Family planning of the use and value of money are important parts of mathematics education, and the school alone cannot adequately compensate for the lack of certain experiences in the home. The parents must be used as partners in the enterprise, for education extends beyond the school.

The Planning and Evaluating Process

Models of progress in other schools and communities can be very helpful, but each school and school system needs to develop its own program. Colleges and universities can and should be very helpful in this planning process.

There are a few suggestions that I would like to make that relate to this planning process as we attempt to set up a program whereby mathematics plays an important and coordinated, rather than fragmented, role.

1. A school or community needs to identify, isolate and define the problems, and then set goals.

2. Determine who should be involved in the planning of the program: teachers, principals, parents, pupils, university consultants, and so forth.

3. In setting up the program, identify preventative as well as corrective measures.

4. Outline and describe the program, courses of study, acitivities, experiences, objectives. Be sure that such a well-defined program for the child includes his total development. Insure that mathematical concepts, skills, and understandings are promoted where feasible throughout the curriculum. In developing the program for slow-learning children, it is most important to develop the proper balance between the mechanics of arithmetic and arithmetical applications and reasoning. This very often has to be determined on an individual basis.

5. Provide for built-in evaluation techniques for whatever program is set up.

Conclusion

I have attempted to emphasize the importance of mathematical education as a part of the total program with a close relationship to skills in language arts and communication. I have emphasized the importance of the teacher as a person, a human engineer who has some understanding of the pupils that she teaches as well as the subject matter. We discussed the kind of curriculum that would have meaning for the inner city child with materials, objects, activities, and experiences that have particular meaning for children at that stage of their growth, development, and background. It was noted that in developing the curriculum, it is particularly important to utilize the best that we know about how children learn and to recognize the desires, interests, and motivations of the inner city child. Parents and other educational forces play an important role in developing or retarding mathematical skills.

Finally, we talked about a process of planning whereby we identify the needs of children in a particular school, and involve parents, pupils, teachers, psychologists, consultants and other resources in the community that would be helpful in developing a program geared to the needs of the particular children to be served. And it was also stressed that in the process of planning, it is essential to weave in a program of continuing evaluation and reorientation in terms of the progress made toward the achievement of the agreed upon goal.

It is very important in working with a child of the inner city for both the child and the parent to understand not only what is being done, but also why it is being done. This serves as a part of the motivating force that can open new doors of opportunity to the inner city child.

Much of what has been said here seems to have been directed toward the child in the early years of his development, because, hopefully, if the proper programs can be developed, we would no longer be dealing with large numbers of children who are disadvantaged because of environmental inadquacies as they reach the higher levels of education. Perhaps the greatest challenge to all of us is the training of teachers who have some understanding of inner city children and who are motivated by a desire to help the inner city child create an image of himself that is consistent with his ability, but one that transcends the boundary of ghetto frustration because of the new avenues of opportunity that are available to him. In today's world, mathematics must play an important role in helping the disadvantaged child to find himself and to acquire the ability to read the maps that point the way to avenues of success.

SURVEY OF PROJECTS

Lauren G. Woodby
Michigan State University

Because of my intense interest, I welcomed the invitation to survey projects concerned with this problem in mathematics in inner city schools. It has been my privilege to be involved, to varying degrees, in many of these projects, and in fact, during this current year I am on leave from Michigan State University, working on the Tri-University Project at New York University. This project is concerned with the training of elementary teachers for inner city schools. We are working in P.S. 68 in Harlem, which is about as inner city as you can get.

Some historical perspective for this conference can be achieved by looking at three previous conferences that were concerned with some of the same problems. These conferences were

1. the Conference on the Low Achiever in Mathematics held in Washington, D.C. on March 25-27, 1964, sponsored jointly by the United States Office of Education and the National Council of Teachers of Mathematics,
2. the Conference on Mathematics Education for the Below Average Achievers, held in Chicago on April 10-11, 1964, sponsored by the School Mathematics Study Group and funded by USOE, and
3. the Conference for State Mathematics Supervisors on Programs in Mathematics for Low Achievers held in Charlottesville, Virginia, December 4-9, 1969, sponsored by the Association of State Supervisors of Mathematics (ASSM) and funded jointly by the National Science Foundation and USOE.

The reports of these conferences are relevant to this present conference. For example, the USOE report recommended the establishment of research and development centers, with one area of study to be the preparation of teachers for low achievers. More than half of the twelve pages of SMSG recommendations concerned city slum areas or segregated Negro schools. A general policy recommendation was priority for a long range approach beginning in pre-school or in primary years. Both action programs and research programs were recommended.

Major talks at the ASSM Conference included report of the COLAMDA (Committee on Low Achievers in Mathematics - Denver Area), The Madison Project (in Philadelphia), the UICSM Underachievers Project (in Philadelphia), the Low Achievers Program (in Duluth), the Project for Bilingual Pupils (in Texas), the Drill and Practice CAI (computer assisted instructions) Program (in Mississippi), and Behavioral Objectives and the Slow Learner (in Baltimore County). A report will be out soon.* Conversations with Russell Phelps from NSF and Dexter Magers from USOE indicate that this was a most useful conference because of the communication of ideas among the members of ASSM who are generally influential as far as mathematics curriculum decisions are concerned.

Before examining specific mathematics projects for inner city schools, I want to mention two legislative acts (besides the NSF) that have had considerable influence on these projects. The Elementary and Secondary Education Act (ESEA) of 1965 has had the most influence because of the amount of dollars allocated and also because of the intended benefactors--the disadvantaged. The second act is the Education Professions Development Act of 1968. Another act is California's Senate Bill 28, which provided support for projects in reading and mathematics for the disadvantaged. There are other legislative acts that exist, and there will be new ones in the future. But let's face this fact: current prospects are dim for the dollars that are needed.

Three Major Projects - NSF Supported

SMSG, UICSM, and the Madison Project are similar in that all three were initially concerned with mathematics curricula for children of average or high ability--generally the college-bound. All three of these projects are now concerned also with problems of learning and teaching mathematics to inner city children.

SMSG began its Special Curriculum Project--a pilot program on mathematics learning of culturally disadvantaged primary school children--to implement recommendations of the April, 1964 Conference. Disadvantaged children in kindergarten and first grade used existing SMSG materials in six large cities (Boston, Chicago, Detroit, Miami, Oakland, and Washington). Data were gathered from teachers' weekly reports and from individual tests given at the beginning, middle, and end of the year. In the second year, three new centers were added

*Professor William C. Lowry, Director of the ASSM Conference, provided each participant of the SMSG Inner City Conference in Philadelphia a copy of this report, <u>Programs in Mathematics for Low Achievers</u>.

(Austin, Charleston, and Chula Vista) and two centers were discontinued (Boston, Miami). An extensive testing program was carried out during this second year. (New textual materials were developed at the end of the first year and used beginning the second year.)

SMSG also did an exploratory study of very low achievers in a seventh grade class in 1965-1966, and followed with a two-year study of very low achievers in ten seventh grade classes in the San Francisco area. However, these children were in middle class or lower middle class areas--not slum areas. The purpose of the study was to investigate the use of "modern" materials, but at a slower pace for an experimental group of slow learners.

UICSM is currently involved in a massive program for underachieving junior high school students. The materials, Stretchers and Shrinkers, and Motion Geometry, were developed during the past six years and tried out extensively with underachievers especially from the ghetto areas. Goals are to improve attitudes as well as skills in problem solving capabilities. One feature of the UICSM effort is that teachers are given intensive training with the materials before try-out, and an orientation to pedagogical considerations of teaching inner city children. A major effort is now in progress to form a large number of two-person teams (consisting of a college person and a school system person) to provide inservice support for teachers in schools that implement the program.* Stretchers and Shrinkers is designed to give students an understanding of fractions, percent, and decimals. Rational numbers are thought of as operators on physical quantities. Motion Geometry develops the concepts through isometric motions. The four books are: (1) Slides, Flips, and Turns, (2) Congruence, (3) Symmetry, (4) Constructions, Areas, and Similarity.

In Philadelphia this year, 22 teachers are teaching this material to some 1700 students in 57 classes. I visited four of these classes. An analysis of test and other data will be made to determine progress in problem solving ability and computational skills of students involved. Attitude differences will also be studied.

The Madison Project is active in the inner cities of New York, Philadelphia, Chicago, St. Louis, and Los Angeles. The first inner city efforts of this project were in Chicago about 5 years ago. The pattern has been a

*This college/district team feature was also present in the SMSG Special Curriculum Project for inservice and consultancy.

summer workshop for teachers followed by inservice work with teachers during the year by an experienced Madison Project coordinator. The Madison Project attributes much of its success in New York and Philadelphia (in contrast with results in some of the other cities to the competency of the mathematics directors in New York and Philadelphia). Other positive factors mentioned included cooperation and sympathy with the aims of the Madison Project as well as success in identifying a corps of teachers who have had experience with the Project to train other teachers. Even so, the Project views these results as only moderate against the magnitude of the problem, stating, "New York is so large that even this magnificent effort is but a drop in the bucket and the vast majority of teachers in the city of New York has not been affected at all ..."

USOE Supported Projects

The Title III (ESEA) program is intended to assist school districts to develop model programs that demonstrate workable solutions to educational problems. Projects supported were to be innovative supplements to regular programs. Since this program has been in existence for 4 years, I spent considerable time trying to find some of these projects that are relevant to this conference although this Title is not aimed at inner city problems. In the morass of reports, two studies seemed to me to be most useful for this survey. The references are in the bibliography. The first is the dissertation by Norman Hearn on the continuation of programs after the termination of 3-year Title III grants. The most surprising information was that 92% of the 250 projects studied were continued after the grant was terminated. Innovative projects tended to be continued. Of the 28 projects that were rated "innovative", all were continued following termination of the grant. However, only one of these was a mathematics program, and it was not inner city. The other study is a report on Title III Projects in Elementary School Mathematics, prepared by Edwina Deans, and nearly ready for publication by the USOE. Projects involving the use of video-tape, individualized instructions, and CAI predominate in this report.

Two Title III projects in CAI are of special interest to us in this conference. One is the CAI program in McComb, Mississippi, which was also reported on at the State Supervisors Conference two months ago. The project began in the summer of 1967 when 20 teachers from McComb participated in a 4 week workshop at Stanford. They became familiar with the drill-and-practice program, learned to write behavioral objectives, and gained experience with lessons at the teletypewriter. In 1968-69, 81 additional teachers from McComb were trained. Preliminary results indicate that the CAI system in mathematics is effective in teaching skills as a supplement to classroom instruction. In addition, Mr. J.D. Prince, director of the project, points out in his cost analysis of the system that an effective cost of between 15 and 25 cents per student hour can be achieved. In his paper for the ASSM Conference, Mr. Prince states:

"This type of CAI program has particular relevance to current needs in providing instruction for the disadvantaged child. Gaps in achievement between populations of the disadvantaged...and other children can be narrowed, although the technique may not prove of value in all individual cases."

The other Title III project in CAI provides drill-and-practice lessons in mathematics for some 6000 children in 200 classrooms at 16 school sites in the five boroughs of New York City. The effectiveness of the program will be studied by a comparison of schools in the project with control schools on pre-test and post-test scores on the Metropolitan Achievement Test. You will have a detailed discussion of this project by the Director, Mr. Mendelsohn, in the next session.

The Central Iowa Low Achiever Mathematics Project (CILAMP), is a Title III project of the Des Moines Independent School District. The project began with a 3-week summer workshop at Central College in 1967 in which 30 teachers prepared several instructional units for low achievers in junior high school. Inservice training was continued through Drake University during the following year. In the summer of 1968, a group of low achieving junior high school students attended summer school along with the teachers in the project. Materials were developed and tried out in six classes. Inservice work continued during 1968-69, and in 1969, ten project teachers were selected to organize the material on Ratio and Proportion, Measurement, Whole Numbers, Decimals, Sets, Fractions, and Factoring. Sixteen units are available. A total of 58 teachers from 33 schools in Iowa were participants. About 275 teachers have participated in the dissemination workshops. Five outcomes are reported:

1. teachers have developed an interest in teaching low achievers;
2. teachers are using a variety of strategies;
3. student interest in mathematics has increased;
4. student achievement is at the expected level or higher;
5. most teachers are adapting available material (including CILAMP's) to fit the ability and interest of the child.

At this time, I would like to quote a statement appearing in the Bulletin of the National Association of Secondary School Principals (April, 1968):

> "Fear of mathematics and a distaste for any computation or for the kind of analytical thinking that typifies mathematics."

This characterization of the slow learner is from the article by Ruth Hoffman of the University of Denver. In this article, she comments on the Jefferson County program in senior high mathematics, the Los Angeles Special Projects, the University of Denver Workshop, Paul Rosenbloom's Concepts and Applications of Mathematics Program (CAMP), and some other projects for low achievers. She points out that a mathematics laboratory of some kind was an element common to programs for low achievers throughout the country. Other common elements she listed were:

1. the use of calculators to help the student find his pattern of error in computation and to enable him to get past computational blocks;
2. the use of many manipulative devices such as the abacus, Cuisenaire rods, and geoboards;
3. the use of games, puzzles, and other motivational techniques;
4. the use, where possible, of remote terminals tied into computers for computer-aided instruction.

Dr. Hoffman is director of a current project at the University of Denver, a special training program for returning Peace Corps workers for teaching the disadvantaged and low achievers in mathematics, Grades 7-9, in the Denver Metropolitan Area Schools.

Regional Educational Laboratories

The Regional Educational Laboratories which were established four years ago under the provisions of Title IV of ESEA, have had some influence on mathematics education in inner cities. For example, the Michigan-Ohio Regional Laboratory supported the Cleveland Mathematics Laboratory Project for Low Achievers in 1967. I shall describe this briefly, and from a biased point of view, because I was the Director. The project was a pilot exploratory effort to learn something about training junior high school teachers to use a laboratory-discovery approach with low achieving students in a ghetto school. Two teachers were released from their usual schedule, moved to a room where a wide variety of instructional aids and materials were available, and encouraged to use a laboratory approach in teaching two classes of 20 students each.

They were given extreme freedom to develop their own ideas and to make decisions. They were isolated from local supervision and furnished outside consultant support. Evaluation of the attitudinal changes of the two teachers and of the students was done by an outside agent. The outcomes were principally in terms of the changes that took place in the attitudes and behaviors of the two teachers, the insights gained by me and my colleagues, and two projects that resulted, in part, from that pilot project.

The first of these projects was the Grand Rapids Mathematics Laboratory Project, in which 30 junior high school teachers, mainly from inner city schools, learned how to use laboratory approaches to mathematics in a laboratory setting, trying out these techniques in their classes during 1967-68. The project was funded by NSF and sponsored by Michigan State University. The second project, which is related, is the Cleveland Remedial Mathematics Project, one of fourteen separate Title I (ESEA) projects in the Cleveland Public Schools. This remedial mathematics project is for children in Grades 3 to 6 in high poverty areas who are at least one year below grade level in mathematics. Thirty-one selected elementary schools have added a teacher, called a mathematics consultant, in each of these target schools. This teacher's responsibility is to teach some 60 low achievers in mathematics in small groups of about 10 each. The essential features of the program are:

1. the 31 teachers are much better than average; they asked for the assignment, they related to the children, and they have a special inservice program;
2. there is much individual and small group instruction;
3. much use is made of counters and other manipulative materials (e.g., Dienes blocks are available in each classroom); and
4. parents are involved through home visits by the consultants.

In the classes visited, the excitement of success by the children with mathematical activities was most noticeable. Evaluation by means of achievement test scores shows relatively high gains made by the children. The interesting question is, "What factors caused the gains in achievement?"

Mathematics for Inner City Teachers and Students

This project began at Michigan State University in the summer of 1969. It is funded jointly by three agencies: NSF, USOE, and the Center for Urban Affairs. Fifty-five teachers, mostly in junior high schools, were on campus for seven weeks, and 130 students from Grades 8 - 12 were on campus for six weeks. The teacher did three things:

1. studied mathematics in classes (5 were offered);
2. spent two hours per week in a mathematics laboratory; and
3. participated in a special seminar concerned with psychological, sociological, and historical aspects of inner city education.

The project is directed by Professor Irvin Vance in the mathematics department, and there is a heavy commitment on the part of the senior staff in the mathematics department at the Michigan State University. Twenty-four special classes were taught by mathematics professors for student participants in the 1969 summer program.

One feature of this project is the agreement with the schools for staff members of MSU to work with participating teachers in their classrooms during the current school year. This follow-up also often involves other teachers in the school. Nine school systems are involved: Jackson, Grand Rapids, Battle Creek, Lansing, Muskegan, Saginaw, Pontiac, Ecorse, and Inkster. Three graduate students are working full-time on this project, each assigned to three cities. Plans are for 180 students to be brought to the MSU campus during the summer of 1970, but for five weeks only, because of limited funding. Level of support is $200,000.

The Center for Urban Education (CUE), located in New York City, is one of the Regional Educational Laboratories. Since the primary focus of this laboratory is on urban problems, and since I am working with interns and teachers in P.S. 68 in Harlem, I visited CUE to look for mathematics projects. My own worm's eye view had led me to believe that practically nothing was being done to improve mathematics education in the inner city, and my visit to the Center for Urban Education confirmed this first impression.

One exception is an intense effort in one elementary school. The Center for Urban Education has contracted with Schools of the Future (Caleb Gattegno and his co-workers) for a two-year effort in P.S. 133 in Harlem to work with interested teachers and demonstrate pedagogical techniques as well as materials. Cuisenaire rods and geoboards are central to the mathematics program. I visited P.S. 133 and found that the acting principal and some teachers have high positive attitudes about the changes that are occurring in that school. I also visited Schools for the Future and discussed the program with the three full-time consultants in P.S. 133. Since participation by a teacher is voluntary, the fact that in 1969-70 there are 22 teachers participating (compared to

11 teachers a year ago) indicates acceptance by the teachers. In addition to a weekly seminar after school, there are six weekend seminars held at the Schools for the Future.

The Director of Mathematics for the New York Board of Education, George Grossman, recognizes the effectiveness of Gattegno's work with individual teachers and children, but doubts that there will be much spread to other teachers and to other inner city schools. His approach is to concentrate on the training of key teachers. Emphasis is on the problem, "How children learn". A major effort has been made in experimenting with mathematics laboratories, and there are signs of extremely effective, though sparse, pockets of excellence.

Project Beacon Training Program

This is a fellowship program at Yeshiva University that has been in operation for six years. Support was originally through NDEA, but this current year, by EPDA. I visited the Director, Doxey Wilkerson, and Dr. Julian Roberts, and also sat in on a seminar with the 18 interns.

The aim is to prepare the teachers to work with socially disadvantaged children. Premises are:

1. children in urban slums are characterized by a wide range of individual differences;
2. their academic retardation is a function of social conditioning, not of biological inheritence;
3. the academic handicap can be minimized through appropriate curricular experiences;
4. in order for teachers to develop "appropriate curricular experiences", the teachers need to be equipped with special theoretical insights, attitudes, and classroom skills relevant to the special learning problems;
5. the professional equipment needed can be developed by a program of relevant studies and field experiences;
6. on the pre-service level, liberal arts graduates who evidence genuine interest in working in depressed area schools are good prospects.

The program consists of three broad seminars (exclusively for the interns) in psychology of human development and learning; social organization and process; and curriculum and instruction. Special workshops and special seminars supplement these three seminars. The interns are assigned to slum schools for four weeks, then return to the university for four weeks.

Evaluation criteria are:

1. the extent to which graduates of the program obtain positions in the slum schools;

2. the quality of their performance in such schools;

3. the assessment of student performances by their classroom teachers and internship supervisors;

4. the appraisal of Beacon trainees
   a. during the year of training, and
   b. during the first year of employment.

Evaluation results are used to change procedures. For example, in 1966, the course "The Teaching of Reading" was considered inadequate and irrelevant.

There is a skillcenter approach with a heavy emphasis on reading and literacy skills. Briefly, the approach is to provide pupils the opportunity for children to engage in self-directing and self-correcting learning experiences which integrate reading with a highly motivational science experiment. The procedure involves three tasks prior to carrying out the experiment:

1. reading an introductory paragraph;

2. vocabulary development;

3. reading the directions.

After doing the experiment, another task is to write a descriptive paragraph of what he did. One example given was the making of a kaleidoscope. A second example is: "How can we measure very large distances?" Children do an experiment with indirect measurement using a simple static device.

Project Beacon is being evaluated by an outside group. The people I visited feel they are making a difference. Even though the project will not be funded by USOE next year, it will continue with University funding. My own assessment from reading the reports and talking with the participants, is that the teachers turned out are prepared to cope with the problems they will face in inner city schools. They have a Peace Corps type of personal commitment for working with people. I do not think that they are adequately prepared in mathematics. In my opinion, the effect of this input of some 15 teachers of this type is significant, even when we consider that there are 30,000 elementary teachers in the New York City schools.

Research for Better Schools

Research for Better Schools, a regional educational laboratory which is located here in Philadelphia, has for the past four years been engaged in a major effort of field-testing and dissemination of the program, Individually Prescribed Instruction (IPI). This program was developed at the University of Pittsburgh's Research and Development Center in the early 1960's. The six elements of IPI that distinguish this system from conventional school procedures are:

1. detailed specification of objectives;
2. organization of materials to attain these objectives;
3. determination of each pupil's present competence;
4. individual daily evaluation and prescription for each pupil;
5. provision for frequent monitoring of performance;
6. continued evaluation and strengthening of the instructional procedures.

If we assume that the tentative evaluation results are valid, we must consider the potential of IPI for application to some of the problems of mathematics instruction in the inner city. At the present time, very few inner city schools are involved. The growth of the IPI program has been very rapid--from 13 schools and 4000 pupils in 1966-67 to 175 schools and 50,000 pupils in 1969-70, IPI will likely expand to 264 schools and 75,000 next year.

Another individualized learning program, PLAN, was announced just a month ago. PLAN, created by Westinghouse Learning Corporation and American Institutes for Research, is computerized, and great claims are certain to be made for its effectiveness. This year, 63 schools and 9,000 students are participating, and the number will quadruple next fall.

Brief mention is made here of a group of projects supported by Senate Bill 28 in California. The projects were for disadvantaged (but not necessarily inner city) children, and were restricted to mathematics or reading. I visited with Professor Robert Heath at the Stanford Research and Development Center, read some of the reports of these projects, and discussed with him his evaluation of the projects. As an illustration, 57 seventh and eighth graders in the Richardson Bay School in Sausalito were given individual and small group instruction to overcome weakness in basic skills. One interesting feature was that about half of the junior high participants taught fifth or sixth grade classes to develop self-esteem.

In Los Angeles, the Mathematics Demonstration Center was built, and its use implemented with State funds.

Project SEED (Special Elementary Education for the Disadvantaged) under the direction of William F. Johntz, University of California at Berkeley, is unique in several ways. It is funded by a special act of the California State Legislature; University mathematicians and graduate assistants teach the children; the "culture-free" quality of mathematics is applied and somehow, the ghetto children "believe in" this idea and seems to thrive on abstract thinking with lots of symbolism. Johntz has the ability to get children to relate to him and his approach. The teachers also seem to develop a total commitment to the learning process as an exciting discovery experience that is different from most school activities.

Just three days ago, Bill Johntz gave a demonstration with a fifth grade class in P.S. 68 in Harlem. It was a pretty convincing demonstration of his theory that disadvantaged elementary school children can succeed with abstract conceptually-oriented algebra when presented to them by a person well-trained in mathematics, using the discovery method. He worked with the class in their own room for about a half hour, then worked with them on the stage in the school auditorium. His lesson was on operations, and the children were able to do 2 EXP 3, for example. The highlight of the demonstration, to me, was the response of a child that 9 EXP $\frac{1}{2}$ was a number. The class ended with a discussion of whether 2 EXP 0 was 1 or 0. They were using variables, for example, y EXP (r + z) = y EXP r · y EXP z.

Features of his project are:

1. the specialist teacher must be well-trained; he recruits mathematicians from universities (both faculty and graduate students) and from industry. Seven out of ten specialists are research mathematicians;
2. there is a full time commitment of day-to-day teaching at least four days per week;
3. the discovery approach is used;
4. project teachers meet weekly for two hours to discuss their teaching;
5. the regular classroom teacher is always in the room;
6. the Project teaching is a supplement to the regular mathematics work.

Johntz believes that mathematics is a "culture-free" mental discipline that has little or no relationship with socially-oriented subjects taught in the elementary school. It calls for pure intelligence, not social adjustment.

His fundamental goal is to raise the self-image of the disadvantaged student. He believes that the single most important cause of low achievement of the disadvantaged child is his lack of motivation due primarily to his negative self-image. Johntz believes that most compensatory education programs have two faults:

1. the "more and better of the same" syndrome in which the same teachers, with the same attitudes, and the same methods, teach the same content;
2. the remedial nature emphasizes the students' past failures and further negates his self-image.

At the University of California, SEED has top priority in the urban crisis package. The Graduate Community Teaching Fellowship program expressly permits a third category of graduate student support. Under this program, a student may be paid as a teaching assistant or a research assistant, but teach disadvantaged children in a local public school district.

There is no set curriculum, but topics usually included by the specialist teachers are: open sentences, operations defined on real numbers, ordered pairs, graphing, sequences, functions, intuitive geometry, mathematical systems.

Johntz believes in absolute freedom to think and learn. He insists on strict discipline so that every child can hear what is said. Control is essential because ghetto kids look upon permissiveness as undesirable. He thinks that the teacher gets through to the kids best in a firm but free atmosphere. They get the message "that you care" and "that you are not a fake".

The Innovation Team, Model School Division

The "Innovation Team" is a group of fifteen helping teachers organized into sub-teams to work in the Model School Division (MSD) in Washington, D.C. There are fourteen elementary schools, three high schools, one vocational school, and the Cardozo High School. However, financial support for the project is from the Educational Development Center, the Regional Education Laboratory in Massachusetts. This feature of building isolation from the D.C. schools may be one of the reasons for the unusual success of the Innovation Team. The innovation Team is a part of EDC's Pilot Community Program.

The Model School Division began in 1964 as a model sub-system of the D.C. schools to experiment across the board in an attempt to improve the quality of instruction. In order to report accurately the nature of this project, I obtained a copy of <u>An Evaluation of the Innovation Team Program</u>

in the D.C. Model School Division. This thorough study was done by an outside agency (the Washington School of Psychiatry) and published in September, 1969. Incidentally, this study would be a useful source for any agency (e.g., SMSG) contemplating or planning a project for inner city schools.

Briefly, the fifteen team members were selected from MSD. The two criteria for selection were: special training in at least one subject; and successful use of some trial unit in the classroom. The two functions specified were to coordinate programs and to support instructional programs. Notice that the role was not to initiate innovations. Its operational goal was to "make things possible for teachers". The Team was to be a facilitating unit: arrange workshops and provide support in the classroom. One important agreement was that the classroom teacher decided when and how she will make use of the Team member.

During the first year (1967-68), Team members had visited a majority of the teachers in the fourteen elementary schools at least ten times. These visits were primarily in response to varied teacher requests, to teach lessons, or to demonstrate the use of materials, or to deliver supplies. In the second year, there were seventeen mathematics workshops with an average of twenty-three teachers attending.

Some selected comments from the findings of the study are:

1. Staff development has aimed at improving instructional methods and procedures. Emphasis is on activity and discovery-oriented approaches to learning.
2. Classroom support included supply and delivery of a large variety of materials.
3. Although most principals would like to have control over and responsibility for the Team, the evaluation team's opinion is that the Team members can function better without being stationed in the schools they serve.

From the Innovation Team,

"Support is the development of positive teaching-learning methods. But it is more. Support is a warm feeling of communication--two-way, of course...Support is a form of on-the-job training. The whole process is one of internal growth and change which is brought about only when trust exists."

This situation in the MSD in the D.C. schools contrasts sharply with the situation in the schools in Harlem. There are literally so many different projects and programs going on at the same time, that the teachers simply don't know what the programs are. The teachers do not seek out assistance.

Colleges and universities are becoming directly involved with problems of inner city mathematics. For example, at the University of Illinois (Circle Campus, Chicago), there is a special Education Assistance Project. The mathematics department has developed a sequence of three special courses for disadvantaged freshment to prepare them to continue in mathematics. The philosophy is to try different things and keep those that work. Programmed instruction, TV, films, and large lectures were tried out and discarded because they did not work. At present, small sections of 15 to 20 students taught by a selected professor with a teaching assistant, seems to be working. This program has a high priority in the mathematics department.

A project which is now in the planning stage is likely to furnish useful information about teacher development. Professor Robert Hess of Stanford University is planning a survey of projects designed to train teachers of the disadvantaged. This research project will be supported by the Center for Research at the Stanford Research and Development Center.

I call your attention to the program for the 1970 annual meeting of the NCTM next month. In the planning session for this program, there was general agreement that teachers would like to know about special mathematics projects concerned with inner city problems, and a number of sections meetings has been scheduled with this in mind.

Summary

The more I learn about the problems of mathematics education in the inner city, the less certain I am about solutions. I have come to believe that curriculum efforts, while necessary, offer relatively little hope for changing the situation that exists. I am convinced that our money and our efforts should be concentrated on teacher development. My ultimate hope is that pre-service education will improve so that beginning teachers are equipped to go into ghetto schools with their eyes open. Meanwhile, we cannot wait; so in-service education efforts should have high priority. I agree with the statement of Paul Briggs, Superintendent of Cleveland Public Schools: "The plight of thousands of children in the inner city is the most serious domestic issue this nation has faced in the twentieth century. If the schools are to be relevant in the days ahead, they must adress themselves to this issue."

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REACTION TO SURVEY OF PROJECTS

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Professor Woodby reported on a wide variety of projects and programs for "inner city" students. Perhaps the most significant aspect of the report was the broad range of activities, approaches, philosophies, etc., represented among these projects. In terms of approach, they range from materials-oriented programs, to materials orientation with teacher training in the use of the materials, to teacher training programs which include training in materials selection. They include a program in which both teachers and students are involved in a summer residence program on a university campus.

Some are "basic" programs utilizing instruction by the (perhaps project-trained) classroom teacher. Others supplement basic instruction by the teacher, either laterally through computer-monitored drill and practice or vertically through presentation of enrichment topics by teachers from the project. At least one project requires a total change in teacher role, with materials providing the instruction while the teacher is involved in diagnosis and prescription.

In content, the programs are equally heterogeneous--from one described as "try harder with what they've failed at before, with the added factor of the genuine interest of a good teacher" to one with the philosophy, "no attempt at remediation will work--that simply emphasizes past failures".

Although the projects described differ in many ways, there are common themes which appear again and again:

Individualization of instruction
Diagnosis and prescription
Objectives-oriented programs
Success experiences for students
Student involvement in learning
Student's self-image
Teacher training and development

Thus, some "sameness" of strategies and tactics does emerge out of otherwise divergent programs. Furthermore, one begins to realize that the common themes in these programs have relevance for the education of <u>all</u> children. This, in turn, leads to a series of questions:

1. Is there a homogeneous group of students who are "inner city children", with educational goals and needs that are <u>essentially</u> different from those of non-inner city children?

2. Does the term "inner city children" connote slow learners? low achievers? culturally disadvantaged? culturally different?

If the answer to Question 1 is "No" and to Question 2 is "Perhaps any of these, and much more" as I believe is the case, then one must ask:

3. Were all of the projects reported in this paper directed at the same group of students or does "inner city" have different connotations in the titles of different projects?

It seems that most projects characterize "the inner city child" as one with rather severe learning problems in mathematics. These problems are variously ascribed to cultural deprivation, cultural difference, individual lack of ability, societal conflicts, etc. <u>If</u> it is the set of students who exhibit problems in learning mathematics in which we are interested, it is important to note that this neither includes nor is included in the set of all "inner city" students! Furthermore, it should be noted that the factors which we (perhaps tacitly) identify as "causes" will influence the choice of strategies used in dealing with the problems exhibited by the students. It is my conjecture that haziness in the areas of identification of the student to be aided and of the causes of his learning problems is the basis for the broad range of sometimes contradictory strategies exhibited within the various projects.

For the purposes of this discussion, let us make a very limited definition of "<u>the</u> inner city child". We assume he is <u>of</u> the inner city as well as <u>in</u> the inner city. That is, he reflects both the life style and value system of the area in which he lives. He is probably non-white. He is poor. His environment contains many threatening factors. He views all aspects of externally imposed authority as among these threatening factors. He is likely to view some group of peers as his "family"--the streets as his "home". Now, there are some obvious observations we can make about the education of <u>this</u> inner city child.

1. Methods used to evaluate ability, designed for use with children quite different from the inner city child are not valid for use with him.

2. Since his life style is less formal than that expected by the traditional school setting, this setting must be modified if it is to efficiently mediate his learning. If functioning successfully within the traditional setting is a goal of instruction, then it should be defined as a goal, not as a "technique of instruction", and activities designed to achieve this goal should be included in the overall instruction.

3. Since his environment puts a premium on action as opposed to verbalization, this must be recognized in planning his instruction. He is likely to be quite skilled at perception and to have extensive experience in solving "real life" problems.

4. It cannot be assumed that either the reward structure or the scheduling of rewards which are appropriate in traditional settings will work with him.

5. It is false to assume that his environment puts no premium on learning. However, the learning induced by the environment is not necessarily supportive of, and may be negative toward, traditional in-school learning. If the school cannot make its learning relevant to the out-of-school learning of the child, he will most surely opt for the out-of-school learning which facilitates immediate survival as opposed to the "long range" rewards (which he may have reason to doubt!) of in-school learning.

6. It is likely that his life style includes irregular attendance at school. Hence, there is no choice but to individualize (or rather recognize his individualization of) his instruction. Furthermore, there is no choice but to provide him with experiences which he perceives as successful, for otherwise he will not engage in the learning process. He has "better things to do outside" than to be continuously frustrated inside the school. These, in turn, imply the necessity for individual diagnosis and prescription, and this is possible only when based on well-defined objectives for the instruction. Thus, many of the recurrent themes mentioned in regard to the reported projects seem essential characteristics of programs for this child.

7. Teacher development is a key factor in any such program. However, content training is probably of little importance relative to training in modes of class conduct, diagnosis of learning needs, management of a variety of instructional materials and devices, and to establish

attitudes and behaviors which will allow the teacher to communicate with the child. It is quite likely that not all who cannot successfully teach <u>some</u> children can teach these children. Thus, study is needed in regard to the process of selecting teachers for inner city children.

In considering programs for the mathematical education of inner city children, we must be realistic. Funds to support such projects are, as Professor Woodby reported, limited, and this situation is not likely to improve. Furthermore, funds to implement demonstrably successful projects on a broad scale are, and will continue to be limited. Therefore, our emphasis must be on "reasonable" projects which, if successful, can be implemented in existing schools with existing teachers within existing budgets.

We must also consider the possibility that no program in mathematics alone can have more than limited success. Perhaps we must initiate programs in conjunction with other subject matter areas. Perhaps no manipulation of instructional variables in one or many content areas will succeeed without major changes in administrative personnel and practices within inner city schools. Finally, we must consider the possibility that we are "out of step" in attempting to improve the education of the inner city child. Perhaps a majority of society has a vested interest in providing limited educational opportunities for some easily identifiable subset of the population.

Even with the limited definition of "the inner city child" which we have used, the unanswered questions are many. With a more realistic definition, more questions arise. In finding answers to them, we will no doubt discover ways to improve mathematics education for all children.

ONE STATE EDUCATION DEPARTMENT'S ACTIVITIES CONCERNED WITH IMPROVING MATHEMATICS EDUCATION IN THE INNER CITY SCHOOLS

Melvin Mendelsohn
Board of Education of the City of New York

James E. Allen, Jr., U.S. Commissioner of Education, stated at a news conference on May 27, 1969: "The basic priority in U.S. education is to raise the level of those children who now are caught below minimum levels of competence. This is a critical area."

Mathematics educators have the responsibility to provide a compensatory mathematics learning program, kindergarten through grade 12, for that large group of students, throughout the country, working below grade level in basic skills.

Results of the mandated Arithmetic Tests for New York State Elementary Schools indicated a need for a re-evaluation of mathematics instructional programs and services offered in our schools for educationally disadvantaged students. The extent of the problem is reflected in the following table.

Percentage Of Pupils Below Minimum Competence* In Mathematics, Grades 3 and 6, In Selected Cities in New York State - 1968

	Percentage of Pupils Below Minimum Competence	
City	Grade 3	Grade 6
Albany	22	22
Buffalo	30	27
New York	51	49
Rochester	49	35
Syracuse	19	29

(*Minimum Competence - Below the 23rd percentile on the New York State Pupil Evaluation Program Tests)

The significant percentages of grade enrollments below minimum competence levels across the State directed the State Education Department to plan and implement federal and state aided programs in mathematics education. These programs would allow us to begin to cope with the serious problem of low achievement in mathematics, for large numbers of our educationally disadvantaged students.

On September 30, 1965, an additional position was added to the staff of the Bureau of Mathematics Education. The primary function of this position was to provide consultation and assistance to school systems in New York State in the formulation and operation of program proposals in mathematics education under various federal and state grant programs, make recommendations for approval or disapproval of submitted projects, and conduct evaluation of operational programs.

On March 17, 1967, the Bureau of Mathematics Education sponsored a conference, in Albany, with the theme: Low Achievers in Mathematics. The conference had four main objectives:

1. to identify the mathematics curricula being developed and implemented for mathematically disadvantaged students;
2. to consider approaches to the problem of in-service and pre-service education for mathematics teachers of low achievers;
3. to identify types of research studies being carried on in the field of low achievement in mathematics; and
4. to describe ongoing Title I, ESEA projects in mathematics.

On December 1, 1967, the Bureau of Mathematics Education sponsored a second conference in Albany, with the same theme: Low Achievers in Mathematics. It had three main objectives:

1. to describe the cognitive development and learning style of mathematically disadvantaged children;
2. to discuss the utilization of standardized tests in evaluation of mathematically disadvantaged students; and
3. to identify meaningful and concrete instructional techniques geared to low achievers in mathematics.

Invitations to both conferences were extended to leading mathematics educators from the city school districts of New York State.

Two major presentations have been delivered concerning mathematics education and the disadvantaged. Developing Mathematics Programs for the Educationally Disadvantaged was a presentation delivered in Syracuse, Massena, and Rochester in October 1965, April 1966, and May 1967, respectively. Following is a summary of the presentation.

As mathematics teachers and educators it is our mandate to provide a comprehensive mathematics program, K-12, for children at all levels of ability, and from various environmental strata. With federal and state funds as a tool we can now provide expanded and improved programs for the economically and educationally disadvantaged child. Our responsibility is to formulate ideas, and to develop, promote, and implement programs concerned with the mathematics education of such educationally deprived children.

After perusal and evaluation of approximately 500 mathematics projects submitted to the State Education Department for funding under various grant programs, a suggested format for such projects emerged.

The development of a comprehensive mathematics project is constructed with the thought of the project becoming functionally integrated into the overall school program.

Four general areas constitute a project format: evaluation, curriculum, teacher training, and organizational structure. Each of these areas is interrelated and developed in that context.

Pupil evaluation consists of three phases: identification, diagnosis, and achievement. The first step is identification of the mathematically disadvantaged. The U.S. Office of Education, in its pamphlet, School Programs for Educationally Disadvantaged Children, states it thusly: "Educationally deprived children are children whose educational achievement is below that normally expected of children of their age and grade, ..." Extending this statement into a working definition are the following criteria in the State of New York for mathematical disadvantage:

Primary grades (1-3) - below grade level achievement.
Intermediate grades (4-6) - below grade level achievement of one year or more.
Secondary grades (7-12) - below grade level achievement of two years or more.

The most commonly utilized standardized achievement tests in New York State at grades 1 through 6 are the Iowa Tests of Basic Skills (3-6), Metropolitan Achievement Tests, and Stanford Achievement Tests. In grades 7-12 the achievement tests used most often are either the Iowa Tests of Basic Skills (7-9), or the Iowa Tests of Educational Development (9-12). The popular aptitude tests in grades 7-12 are the California Tests of Mental Maturity, Differential Aptitude Tests (8,9), and Otis Mental Ability Test.

The New York State Pupil Evaluation Program (PEP) examinations, along with the standardized tests, service this phase as well as the other two phases of pupil evaluation. A list of the PEP mandated tests follows:

Grade 1 - New York State Readiness Tests (Arithmetic and Reading)

Grades 3 and 6 - Arithmetic Tests for New York State Elementary Schools

Grades 7 and 8 - New York State Test in Mathematics (also given at grade 9)

At the present time, the State Education Department is using an arbitrary minimum competence level, which is the 23rd percentile, as obtained in the fall 1966 Pupil Evaluation Program testing. This minimum competence level will remain the same for the next several years, so that school districts will be able to tell what progress has been made in reducing the number of disadvantaged pupils. Theoretically, it is possible that there will be no pupils below this point within a few years. This point in the distribution, the 23rd percentile, was selected primarily for administrative reasons. It is high enough to include those pupils with significant educational needs and low enough to insure an opportunity to provide for corrective measures. It was also selected because it is the point which separates the third and fourth stanines, or achievement levels as they are referred to in connection with New York State tests. With stanines, or achievement levels, the distribution of pupils' scores is divided into nine equal units. If a pupil obtains a score at level three or below he can be considered educationally disadvantaged.

After identification is diagnosis. The effectiveness of diagnosis increases in value as it passes from a mere locating of the places of difficulty to an analysis of the causes of difficulty. Test perusal and teacher observation are utilized in this phase. General areas of difficulty are discovered by investigating test results, while the specific nature and analysis of the student's difficulty is determined by teacher observation.

Teacher observation has few of the limitations of testing as to time and place. It imposes no unusual restrictions and exposes children to no unnatural tensions. Whereas tests are a description of a particular performance in a particular situation, informal observation can seize each instance of significant behavior as it occurs, along with the prior and accompanying behavior.

The final phase in pupil evaluation is the determination of the student's level of achievement. In a project, interest is focused upon the difference between achievement levels of the student at the beginning of the project year, and at the end of the project year. A significant difference between pre- and post-test scores for most students tends to indicate a successful project.

Curriculum materials is the next area to be considered. Desired objectives are decided upon before any other work in this area is contemplated. The general objective is improved performance in mathematics; and the three specific objectives that mathematics instruction strives for are improvement of: computational skills, mathematical understanding, and problem solving ability. After the objectives are decided upon, curriculum materials are developed.

Current thinking on curriculum emphasis for educationally disadvantaged students leans strongly toward academic content. Many renown educators advocate this approach. Kenneth Clark, a Regent of the University of the State of New York, in his book Dark Ghetto, says:

> "The evidence ... seems to indicate that a child who is expected by the school to learn, does so; the child of whom little is expected produces little. Stimulation and teaching based upon positive expectation seems to play (an) important role..."

Carl Hansen, ex-superintendent of schools of Washington, D.C., at a University of Detroit Symposium on the low achiever, said:

> "The primary responsibility of the formal educational systems of the country is to prepare for intelligent behavior, ... We are saying that every child -- the slow, the average, and the bright -- is capable of responding to growth in mental discipline, and that our responsibility is to discover the technique ... by which it will be possible to use ... the rational processes of his mind."

These two quotes focus in on exploring techniques and positive identification.

The materials provide for the development of mathematical understandings for vocational competence. Opportunity for success is a major aim in the design of materials. To achieve this success, the learning materials are graded in content to meet the differing rates of student learning. Units are short, and provide various approaches to the development of mathematical concepts.

Five topics need to be concentrated upon in the teacher training area:

1. Mathematics programs and contemporary instructional techniques.
2. Characteristics of the educationally disadvantaged.
3. Emphasis on instructional techniques for the low achiever.
4. Curriculum and textbook examination and study.
5. Purposes and procedures of evaluation.

The updated presentation of mathematics instruction is aimed at bringing to light all of the underlying structural properties, and, to show how certain mechanical algorithms will produce desired results. The object is to teach the algorithm, but only after leading the students, using the method of guided discovery, step by step through the background which makes the algorithm possible. Disadvantaged students require a modified, structural approach.

Characteristics of culturally deprived children are discussed candidly. Units such as home environment, language, cognition and learning, intelligence and aptitudes, personality and motivation, and school achievement are used as a basis for discussion.

In the area of instructional techniques, there is a need for clarity of structure, remediation of large deficits in common knowledge and skills, and for individualizing instruction. Teaching is directed toward simple, clear, achievable goals; and the means designed to accomplish them understood by the children. Immediate rewards for successful performance heightens motivation. The research on motivation suggests the need for developing school programs adapted to the motivational patterns of these children.

Tasks are challenging, but not discouraging. Even short-range training in perceptual skills, following directions, and other tasks has produced marked increases in intelligence test performance.

Disadvantaged children have shown difficulty in developing concepts of an abstract nature and in generalizing. Therefore, teachers provide a saturation approach for material to be retained.

An integral part of teacher training is the review and study of curriculum materials to be utilized in the classroom. Closely allied with this is the study of the textbook(s) to accompany the curriculum materials.

Purposes and procedures of evaluation provide an overview of their areas. The chief purposes of evaluation are:

1. diagnose class and individual ability;
2. inventory knowledge and abilities;
3. determine the extent of learning over a limited period;
4. measure learning over a relatively long period;
5. obtain rough measures for comparative purposes.

Four general classes of evaluation techniques considered are:

1. paper-and-pencil tests;

2. teacher observation;

3. individual interviews and conferences with pupils;

4. pupil reports and projects.

The last area is that of classroom organizational structure. This is where the other three areas: evaluation, curriculum development, and teacher training are put in operation. A full school year program is emphasized because of the eventual assimilation into the total school program. A possible summer session could be included for reinforcement of fundamentals and/or enrichment.

The environment of disadvantaged children causes depression of intellectual functioning. Provision for a more adequate environment can result in a considerable increase in aptitude, and in more learning and achievement. A child's full learning ability will be realized when a proper home environment is supported by good environmental conditions in the school. These conditions are fostered by various classroom organizational patterns. To identify a few: inter-class or intra-class ability grouping, ungraded classes, and small group instruction. In the intermediate grades particularly, the Dual Progress Plan (DPP) and the Joplin Plan are two types of classroom patterns. The mathematics part of the DPP has ungraded classes taught by elementary teachers specializing in mathematics instruction.

The particular structure decided upon, and the mathematics project developed should be an outgrowth of the local system's educational philosophy so as to fit into the overall school program.

Federal Funds for Mathematics Education was a presentation delivered in Oneonta, Greenlawn, Warsaw, Syracuse, Las Vegas, Nevada, and again in Syracuse in February 1966, March 1966, April 1966, May 1966, April 1967, and May 1968, respectively. Following is a summary of the presentation.

Three points are clear concerning federal funds for education. They are to be used for:

1. attacking special problems (represent categorical, not general aid);

2. developing new programs; and

3. supplement state and local revenues.

The Federal Government's effort appears to be more dramatic, only because it is more recent. Federal funds are approximatley nine percent of expenditures for public education in New York State.

The role of the three agencies should be thought of as Federal concern, State responsibility, and local control.

Exemplary projects under the following federal grant programs are then described:

1. NDEA, Title III - financial assistance for strenghtening instruction in science, mathematics, modern foreign languages, and other critical subjects;

2. ESEA, Title I - financial assistance to local educational agencies serving areas with concentrations of children from low-income families to expand and improve their educational programs by various means which contribute particularly to meeting the special educational needs of educationally deprived children;

3. ESEA, Title III - a program for making grants for supplementary educational centers and services, and to stimulate and assist in the development and establishment of exemplary elementary and secondary educationalprograms to serve as models for regular school programs; and

4. ESEA, Title IV - to make grants for research, surveys, and demonstrations in the field of education, and for the dissemination of information derived from educational research.

Guidelines for Developing A Title I, ESEA Mathematics Project was first published in the New York State Mathematics Teachers Journal, April 1966. The guidelines followed closely the presentation titled, Developing Mathematics Programs for the Educationally Disadvantaged, described previously.

25 Mathematics Education Programs, Volumes I and II are two publications distributed by the State Education Department. They were published by the Department in January and July, 1968. The compendiums provide program descriptions for 25 Title I, ESEA projects involved with mathematics education. The projects selected give a range of coverage in different instructional and service domains, including curriculum development, in-service training, and small group instruction.

Mathematics Education and the Educationally Disadvantaged is a publication that contains, basically, the major presentations of the two conferences held in 1967.

There are two programs that deserve exposure as vehicles addressed to the conference theme. The first is the BRIDGE project.

BRIDGE, (Building Resources of Instruction for Disadvantaged Groups in Education), was a project developed to find ways of effectively preparing teachers to work in culturally deprived neighborhoods. The problem was two-fold:

1. How to bridge the gap between essentially middle class oriented teachers and the lower class youth of varied ethnic backgrounds present in public junior high schools and,
2. How to modify the Queens College of the City University of New York curricula so as to meet the future teacher's needs and prepare the teachers for the unique problems of classroom instruction at the grass roots level in the low achievement schools.

The preparation of teachers is, of course, only one aspect of this problem. It has been true in the past that teachers generally prefer to teach in those schools where the best conditions of work are present and the responses of the pupils most stimulating. The consequence of this desire has been to relegate much of the education of the children in slum-area schools to inexperienced teachers, many of whom either leave teaching or escape to more desirable locations. The staff of the Education Department at Queens College was aware that their students tended to follow the general pattern, preferring either to teach in suburban communities or in those areas of the city where middle-class whites lived. They were also aware that many of the young graduates of the College who accepted teaching assignments in low socio-economic area schools reported encountering severe problems of bridging the gap between middle-class, academically oriented whites preparing to be teachers, and the lower-socio-economic white, Negro, Puerto Rican youth who form a large part of the pupil population in the slum.

The general goal of the BRIDGE Project was to discover what modification of, or addition to, the present program of teacher education would more effectively prepare teachers for work in secondary schools in culturally disadvantaged areas.

Three teachers, recent graduates of Queens College, were selected to teach English, mathematics, science, and social studies to the Project pupils for the three years of their junior high school education. One teacher taught both mathematics and science. Each teacher's full schedule was devoted to the instruction of the Project children. Pupils and teachers were together for approximately two-thirds of each school day throughout the three years.

A supervisor or coordinator was selected to discharge the duties of training these teachers on the job, giving them assistance and emotional support in their difficulties, organizing the meetings of the Project staff (both school and college) which were held in the school, and in supervising the collection of teacher records and reports which were necessary for the research.

The teachers were involved in the everyday duties of all teachers, no matter the school or the community. The unique aspects of their work was the every day planning aimed at meeting the unique group needs of the children. (Reading level average -- 2.2 years behind, range 3.0 - 10.0. Seven out of eight pupils were below reading level at the onset of the Project.) Throughout the teachers' work in this laboratory classroom they sought approaches to the teaching of mathematics. Planning was not a one-day seminar, or a two-day workshop, but an everyday item on the agenda for three full years. With the help of a master teacher-supervisor, they planned specific activities. The official mathematics texts were rarely used...semi-original and original teacher prepared materials were given to the students daily and combined by the students to form their own personal textbooks. (Texts were ordered as they discovered the needs of the three very different classes).

Evaluation of what took place in the classroom was done almost immediately and remedies, where necessary, could go into effect often at once. The advantage being, that they were teaching in a fish bowl and didn't have to wait several weeks for feedback. They were able to see the units of work take shape, and to participate weekly in planning sessions with a college mathematics consultant to evaluate what they tried.

The teachers on the project were able to share their common experiences, new materials, books and pertinent games and tricks. The staff as a group was able to focus attention on specific children (one per week in a case study conference) as to their reactions, and learning difficulties in all the subject areas. They soon found that as their knowledge of the children increased, that certain units of work were required in mathematics, namely basic mathematics and consumer education. (Consumer education included installment buying, loan sharks, discounts, tax packaging comparisons, etc.) They discovered that they could not teach any advanced mathematics concept without reviewing the basic algorithms, yet this was too dry for the children. They found that if they could create a desire or a need to learn a concept the basic computations needed for figuring out the answers became an easy thing to teach. However, isolated unrelated mathematics notions remained meaningless to students. They further discovered through trial and error, that because their students lacked the basic rudiments of mathematics, the children were gullible to business sharpshooters. Teachers tried to help them visualize mathematics in a realistic everyday context. They took trips to Macy's, watched the supermarket people train, and used a cash register and adding machine in their own classroom. The needs of the children dictated the activities.

In working on units in graphs, maps, and scales in general, the children had almost no concept of distance, e.g., distance from New York to Philadelphia ranged from 3 miles to 3000 miles...although many of the children moved their residences frequently, few had ever actually ventured out of their narrow neighborhoods, even to go as far as Times Square. So how could they know how far was Africa? Or where their brothers were fighting in Vietnam?

Those youngsters who needed skill work in the algorithms wanted to work alone with their problems because they hated to admit their shortcomings. Frustration was common to the children who hadn't learned their multiplication tables. Students' work was individualized. (Each child had a separate skills folder of his own and was tested only after he was ready to be tested.) This encouraged developing these vital skills as well as learning an elementary responsibility for handling of school materials. (Folders and pencils were distributed and collected by students.) Toward the end of the second year, the children were able to function on their own without direction from teachers.

Since the majority of the children were two or three years behind in reading at the onset of the project (grade 7), the teachers were obliged to teach reading, learning to do this on the job, as they needed these skills immediately. Generalities were soon translated into subject areas. Vocabulary in mathematics was developed where possible with words children had seen and learned before (e.g., fraction and fracture; percent and cent; equal and equality). Understanding what were once insolvable problems, were built through the teachers' knowledge of context clues and comprehension skills.

While the teachers and pupils were learning, the college staff was active. There was always a professor working with a group of youngsters or observing a lesson in session. The professors liked the laboratory atmosphere, and were observing at first hand, what works and what doesn't. Regular conferences in each area included the appropriate college teacher. Through this type of conference and observation and participation, the professors changed and exchanged ideas, and their curriculum and course content at Queens College changed.

The College learned from the BRIDGE experience that new teachers should not be sent into difficult areas as their first assignment. New teachers need a "period of adjustment": time to learn the ropes of the classroom, stage presence, administrative paper shuffling, and other routines. Nevertheless, there is the reality factor. Teachers are needed in difficult schools immediately. In New York City, the majority of newly appointed teachers have no choice. Therefore, what can be offered as concrete, realistic help?

A fifth year for the fledgling. A year to become a strong, confident teacher. A year in which to build materials file, unit plans, diagnostic devices, and lesson plans. A year in which being "alone" in the classroom is nonexistent. A year with skilled help and guidance as well as critical analysis of the work accomplished. A year to sit down and talk about difficult teaching problems with experts in the field. A year to think, and create. Time to meet the curriculum needs of the pupil by preparing new materials to be used in teaching. A year to understand the children in the classroom as individuals and try to meet their individual needs. No pre-service training in the college classroom can ever hope to provide the laboratory atmosphere BRDIGE created.

The second program to be described is the New York City CAI Program. CAI, Computer Assisted Instruction, provides an opportunity to give each student the individual teaching attention he needs to do his best work.

The computer, with its great speed, vast memory, and information given to it by a curriculum author, can drill 200 students working at student terminals, all at the same time. It asks each student questions hard enough to make him work, but not too hard for him to answer. Based on the student's previous performance, the computer selects the appropriate level of difficulty for each student and guides him on an individual path of learning. Each student receives daily lessons geared to his own progress and learning ability.

Teachers have more time to help students who need special attention, because the CAI system prepares, conducts, and grades daily drills, and provides every teacher with a complete report and analysis of class performance each day. Children using the system in New York City are being drilled in elementary mathematics, grades 2 through 6.

New York's CAI system has a total of 202 student terminals installed in 17 elementary schools in the Bronx, Brooklyn, and Manhattan. In all but one of these schools, 9 to 13 terminals are installed in a central classroom, where students go for their daily lessons. In the one remaining school, a terminal is installed in each classroom, so that students can take their CAI lessons without leaving the room. A computer terminal room is essential to the program.

All 202 student terminals, which are special teletypewriters, are connected by dedicated telephone lines to an RCA Spectra 70/45 computer installed in the central computer facility located at 42nd Street and Second Avenue. Any student who has been registered as a participant in the CAI program at least one day before may use any terminal for student lessons. A teacher may also use the terminal during this period to request any of the various features available to him. Both students and teachers use the Student Instructional Terminal to communicate with the computer. The computer sends messages and questions to the terminal which are displayed on a page printer. The student or teacher responds by pressing appropriate keys on the terminal keyboard. These characters are transmitted to the computer for evaluation and processing. They are also displayed on the page printer.

Every school day the system can give individual instruction to some 6,000 students during classroom hours. Late afternoons the system is used to help additional elementary school children, and in the evenings it is used for adult education, and other special instruction. Nights, weekends, and vacations it is used to prepare student progress reports for teachers, and for other information and administrative data processing functions. More of this kind of use will be made in the future for scheduling, test scoring, etc.

A student may begin taking daily lessons after he is registered. The curriculum material provided for mathematics lessons is organized as a series of concept blocks; approximately twenty-four at each grade level. A concept block is a set of material relating to a particular idea, or concept; for example, division of fractions. Within a concept block, the student receives questions and problems keyed to the level of difficulty for which he is currently best suited. The system provides material for five levels of difficulty. The blocks can be correlated with materials in most current textbooks being used in the classroom.

The pre-test, given on the first day of a concept block (there are 7 days of drill lessons in each concept block), establishes the level of difficulty for the second day's drill. If a student does very well on a drill, he is moved to the next higher level the next day. The student may also stay at the same level, or move down a level. The student's individual progress, as indicated by his daily score, dictates the level at which he is currently working. On the last session, the student is given a post-test covering the current concept block.

A student is also given review material in addition to drill material. Review material is selected from that previously drilled concept block on which the student made his lowest post-test score. This score establishes the level of difficulty for the next four days. The review test score is substituted for the previous post-test score for that concept block. A student may review a concept block up to four times, receiving different material each time.

During an average daily lesson in mathematics drill and practice lasting 10 minutes, the student would spend approximately seven minutes working on the current concept block, and an additional three minutes of review on his concept block with the lowest score.

The procedure used for drills, reviews, and tests is the same: a problem is displayed at the terminal and the student types his answer. When the student answers incorrectly or does not complete the answer within the time allowed (normally 10 seconds), the system redisplays the problem for another try. If the student is not successful on the second try, the answer is displayed and the problem is presented again for a last attempt. If the student is still unsuccessful, the system again displays the correct answer and goes on to the next problem. This is drill-and-practice and does not develop concepts. The teacher does this in the classroom.

The City University of New York prepared the CAI evaluation report for 1968-69. The purpose of the evaluation was to describe the outcomes of the program in terms of its effects upon pupils and teachers. More particularly, its purpose was to describe the effect of the Computer Assisted Instruction (CAI) arithmetic drill-and-practice program on pupil achievement, and on the opinions and attitudes of pupils, teachers, and administrators. A total of 3282 students (2930 CAI and 352 non-CAI) are the subjects of the report.

The Metropolitan Achievement Test (MAT) was selected for the study, because it is used extensively by the New York City Board of Education in evaluating pupil achievement. The data compared are the computation scores of this test.

The principal statistical method used in the study is the comparison of groups in terms of the means and standard deviations of the test scores; the significance of differences being tested by analysis of variance. In this report differences are considered significant if they are less than the .01 level of significance.

When all students' computation scores are examined, the mean raw score gains between the pre- and post-tests are higher in all grades for the CAI groups. The differences in gain scores are significant, at less than the .01 level, for grades 2, 3, 4, and 5 for all students. The CAI drill and practice program would be expected to have impact primarily on arithmetic computation skills. However, there was a high correlation ranging from .72 in grade 2 to .88 in grade 6 between computation scores and problem solving and concept scores. This is an indication that the program produces gains in the problem solving and concepts area as well as in the computation skills area.

A final concern with any program or project is its effectiveness. This is what is referred to as "accountability".

The educational community is being called upon to provide increased accountability to its many constituencies for the financial support received.

There are two aspects to accountability in education:

1. Have the funds been spent for the purposes intended? and,
2. what effective use has been made of them? These are the two fiscal and educational aspects. No one can protest that one should be held fiscally accountable for money received and spent. But educators, because they deal with a largely intangible product, are not quite as used to as others are, to providing a full reckoning for funds received. As Ewald Nyquist, New York State Commissioner of Education, in a recent speech said: "Education is too often thought of as in a class with the American flag, baseball, and motherhood -- they have a sanctity which should go unexamined."

Certainly one aspect of accountability is the significance of the improvement which has taken place in the pupils, but it is not the only aspect of accountability. There are other questions which should be raised in evaluating the effectiveness of a project.

1. Were there any undesirable outcomes of this project?
2. Could there or should there have been a greater improvement in pupil achievement?

3. Could this improvement have been made more efficiently or economically by some other means?

This is by no means a complete list of the questions that should be raised at the conclusion of a project. It is only intended to suggest what is meant by accountability.

Thus, there is a dual challenge to everyone in the educational community at both the state and local level. First, there is the challenge of helping disadvantaged children to find equality of educational opportunity. The second challenge is in doing this job with a sense of stewardship for the financial support received.

REACTION TO ONE STATE EDUCATION DEPARTMENT'S ACTIVITIES CONCERNED WITH IMPROVING MATHEMATICS EDUCATION IN THE INNER CITY SCHOOLS

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I am in agreement with the statement that "mathematics educators have the responsibility to provide a compensatory mathematics learning program, kindergarten through grade 12, for that large group of students, throughout the country, working below grade level in basic skills".

The procedures outlined to attack the problem were sound: namely; looking at mandated Arithmetic Tests for the State Elementary Schools; determining the percentage of pupils below minimum competence as defined by New York State for reasons mentioned in the presentation; getting the State Education Department to plan and implement federal and state aided programs in mathematics education; adding to the staff of the Bureau of Mathematics Education a person to provide consultation and assistance to school systems in the state in the formulation and operation of program proposed in mathematics education under various grant programs, to recommend for approval or disapproval the submitted projects and to conduct evaluation of operational programs; sponsoring conferences on "Low Achievers in Mathematics". The objectives of the two conferences were good; however no outcomes were stated. The only reference to a publication, <u>Mathematics Education and The Educationally Disadvantaged</u> stated that it contains basically the major presentations at the conferences.

After a listing of the objectives of the two conferences, the discussion moves into summaries of other presentations that have been delivered concerning mathematics education and the disadvantaged; a discussion of a project format, a listing of the most commonly utilized standardized achievement tests in New York State in grades 1-12, aptitude tests in grades 7-12, the New York State Pupil Evaluation Program examination; quotes from educators concerning curriculum emphasis for educationally disadvantaged students.

It became very difficult for a number of pages of reading generalizations or philosophy of education for disadvantaged children to pinpoint what activities were actually progressing in New York for these students. I could not relate all of this to specifics for the New York program.

I realize that much research, understanding of philosophy and reading is necessary before launching any new projects. However, I don't know from the paper exactly who in the New York programs were involved in this. Was this the kind of information given to teachers and curriculum developers for disadvantaged pupils?

The paper seems to break down into three parts: the state of mathematics education in New York State; a philosophy of education for educationally disadvantaged children; and specific steps taken by New York State to improve mathematics education for disadvantaged children, including a detailed discussion of two projects. About one-third of the paper is devoted to philosophy of education for disadvantaged children.

Two of the projects approved, operated, and evaluated are discussed here. There were the BRIDGE program (Building Resources of Instruction for Disadvantaged Groups in Education) conducted, in connection with Queens College of City University of New York City, for preparing teachers for inner-city work--and, the New York City CAI Program (Computer Assisted Instruction) which worked with the student directly by installing 202 student terminals in 17 elementary schools. These were connected to an RCA Spectra 70/45 computer installed in a central facility.

Evaluation of the computer program showed a statistically significant gain in achievement in mathematics on the Metropolitan Achievement Test by students. Evaluation of the BRIDGE program (which was not included in this paper but which I knew about) showed a significant number of teachers electing to stay in inner city work. In other words, both programs are good for New York City.

The weakness lies in the cost. As long as these programs are funded by grants, they can be operated. Most systems do not have the kind of money it would take to make these uniform throughout a system.

The College learned from the BRIDGE experience that new teachers should not be sent into difficult areas as their first year assignment. They recommend a fifth year for the fledgling -- a year to build a materials file, unit plans, diagnostic devices and lesson plans. (This is comparable to the Master of Arts in Teaching program that George Washington University and Catholic University have in Washington, D.C. I believe Trinity College, also in D.C., is moving in this direction.)

This detailed discussion of these projects was the real meat of the paper. However, there were many questions in my mind after reading and rereading. I have divided the questions into two categories, general and specific.

General Questions

How many teachers and pupils are involved in the project?

How many schools were involved in BRIDGE?

How were pupils grouped prior to being singled out for the study?

How are they grouped during the study? Where? Special schools or classes?

How are teachers assigned for the project? Are they specifically qualified? Is any incentive used? Do they feel stigmatized at having to work with disadvantaged children?

Who prepares teacher-pupil materials? Are guidelines given?

Is the content developmental? sequential? or merely corrective?

Are copies of course outlines, lessons, activity sheets (or anything) available?

Are there any special physical changes being tried? Special rooms?

Special arrangements of furniture or equipment?

Are there any special organizational changes being tried? Team-teaching? Flexible scheduling, etc.?

Since secondary (as well as elementary) seems involved in the project, are efforts being made to articulate these two instructional levels?

Are parents (or other community personnel) involved at any stage?

Are teaching problems shared? analyzed? revealed at all? How are they resolved?

Does the program use any professional consultants? social workers? sociologists? psychologists?

What tests are used? Do pupils use any of the same ones as before? Who selects texts?

Are any guidelines available for the selection of texts? (Reference was made to the fact that texts were ordered as teachers discovered the needs of the three very different classes.)

How are school administrators involved? What orientation? What feedback do they receive?

What consideration is given the children in the project? Is the project concerned solely with the sterile subject-matter and its pedagogical implications? What else is being offered to these pupils?

Have these programs had any impact on the other problems of inner city children?

Were basic skills emphasized over structure and broad understandings of mathematics?

Specific Questions

Did the conferences accomplish the objectives set forth?

What were the findings of the conferences? What was the later role of the "leading mathematics educators" in attendance at the conference as far as the projects were concerned?

What part did the aptitude test play in evaluating the pupils for these programs?

Are the children's interests, aptitudes, background, etc., considered as their curricula are developed? If so, to what degree?

Reference is made to the materials provided for the development of mathematical understandings for vocational competence. Is this stressed for all children? What are some of the "various approaches to the development of mathematical concepts" mentioned here?

Who are the teachers in the CAI Project? How are they selected? Who provides their training?

The title of the presentation was misleading for me. I had anticipated learning about mathematics programs for inner city children throughout New York State, then more particularly in the five cities singled out in the report on minimum competence in mathematics. It was difficult to believe that only two programs both in N.Y.C. were the only ones that deserved exposure as vehicles addressed to the conference theme, namely - Low Achievers in Mathematics. It had been mentioned earlier in the presentation that conference invitations were extended to leading mathematics educators from the city school districts of the state.

Various classroom organizational patterns that might foster good environmental conditions in the schools for inner city children were listed, but no reference was made concerning the use of any of these in the state.

I would be interested in knowing what programs in mathematics have been or are being tried in other inner city areas of New York State.

PEDAGOGY AND ACCOUNTABILITY IN
TEACHING SOCIETY'S REJECTS

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Introduction

The limited increases in achievement resulting from the millions of dollars expended during the sixties on compensatory education leads to the generalization that when sociological and physiological phenomena are manipulated, changes only occur in the sociology or physiology and no significant changes occur with respect to the nearness to or direction of movement towards school learnings. Few compensatory programs have resulted in growth in academic achievement.

Challenges regarding the appropriateness of standardized tests for evaluating the effectiveness of compensatory or any other special educational efforts for disadvantaged or inner city youth are "COP" outs. These are high intensity indications of still another hoax being perpetrated upon these youths. Are not such measures still the primary tool used for permitting entry into or for justifying exclusion from jobs, technical schools, or colleges? Is not part of the American Dream the promise that education is the first step in improving economic and social status? How then can such subterfuges be justified? Or are such excuses really affirmations that success in school is directly proportional to the expectations that educators have for the social class of the student's family?

Hopefully, the community of mathematics educators will not become active co-conspirators in the vast enterprise of continuing the process of swindling the many Blacks and persons of Spanish ancestry or lower class whites who populate America's inner cities.

In this paper, the details of the structure, experiences, and theoretical base of an informal ten-year study on teaching a segment of our school's population will be presetned. A desired concomitant of sharing these experiences is that they will contribute to the development of a theory of teaching and ultimately a theory of learning for society's rejects, many of whom have lost their identities because they wear the badge of identification of living in America's inner cities.

Toward a Theory of Teaching Society's Rejects

In the fall of 1961, this writer was teaching in an all Black secondary school (the largest in the country) for grades 7-12 in Dade County, Florida (Miami). The teaching assignment consisted of one section of seventh graders, two of eighth graders, and one section each of ninth grade Algebra I and eleventh grade Algebra II students. SMSG publications were selected for use. The following plan was then implemented:

1. Each student would be expected to exhibit at least those behaviors that are associated with persons having an I.Q. of 20 points more than the score that had been most recently recorded for that student.
2. Students would only be held responsible for those things in which they shared in making decisions. Therefore, since they were not involved in deciding either the what or how of teaching, the teacher bore sole responsibility for the effects of the teaching upon the student's learning. That is, the student was only responsible for those learnings over which he had control; all else was the responsibility of the teacher.
3. Only those rules that related directly to increasing the comfort of the learner would be enforced or established by the teacher.
4. Students in each grade or subject area would "cover" an amount of content equal to that which was projected by the organization of the text materials to be equivalent to at least the work projected for $1\frac{1}{2}$ years.
5. All seventh and eighth graders were to begin with the seventh grade SMSG text, the Algebra I students with Part II of the eighth grade text and the Algebra II students, Part II of the Algebra I text.
6. The natural language of the students would be the primary language for communication. This natural language would be systematically enriched with precise synonymous mathematical terms.
7. The teacher would "talk" through all new concepts and strategies that were to be considered in a set of "new" learnings, beginning with that which was known, moving to that which was ultimately expected. All new terms and symbols would be used in context with the only delineations being that each symbol or word would be prominently recorded by the teacher when used.

8. One day of each week would be used for drill and practice in those areas related to the diagnosed needs of the individual learner. Students not requiring such experiences could select any activity either initiated by the student or by the teacher, but directly related to mathematics.

9. No student was to ever complete more than five practice exercises before having them checked either by the teacher or another student whose efforts had been adjudged satisfactory by the teacher.

After the first six weeks of school, two other Black teachers of seventh and eighth graders joined in testing the efficacy of this plan. They, however, used Florida State-adopted texts written during the mid-fifties. Joint planning resulted in SMSG approaches being implemented by all. End of year evaluations indicated success in that:

The mean achievement for each grade or subject group increased an average of $2\frac{1}{2}$ years as measured by traditional standardized tests.

All students except one, purchased the SMSG texts, thereby providing evidence of involvement in affective dimensions.

Experiences thusly gained suggested that requisites for effective conceptual development are the availability of a wide range of stimuli, which are systematically ordered along a similarity-dissimilarity continuum. The initial "talk-through" of all new learnings, applying principles from Gestalt psychology, was the enabling mechanism for exerting control over the variety of stimuli and their relationship to each other.

It also became evident that teacher-pacing can interfere with conditioned preferences for position, distance, and speed, and that when such pacing is combined with defined teaching strategies, learners will demonstrate achievement gains compatible with the scope of concepts, skills, and understandings available to them.

This plan was again used in 1962-63 in the same school with the following changes:

1. This writer followed the seventh and eighth graders into eighth and ninth grades with both groups enrolled in Algebra I.

2. The instruction for these youths began the succeeding year without review--exactly where that student had ended the previous year.

3. Cumulative review and drill-and-practice sheets were designed for use during the "one-day-a-week" established for this purpose. A dialog format was created so that at least two students could be mutually supportive and would be provided extended support from the teacher in learning how to read, interpret, and translate mathematical ideas expressed in English and mathematical terms.

4. The top five seventh graders of 1962-63 were scheduled with the class of eighth graders taking Algebra I. Only one of these students had scored as high as the 85 percentile on either of the subtests of the standardized test that had been used. The other four scored between the 25th and 50th percentile on the same subtests.

This action-based study revealed the effects of quantity and quality of reinforcement upon performance. Investigations by Hull (1943), Crespi (1942), Hutt (1954), and Zeaman (1949), yielded results in agreement on the point that performance increases as a negatively accelerated function with increases in the amount of reinforcement. Guttman (1953) and Hutt (1954) manipulated both quantity and quality.

Hutt's investigation is of special interest because it is one in which amount of reinforcement was varied in two ways, by manipulating quantity and quality of reinforcement in a factorially designed experiment. Hutt used a semiliquid mixture of milk, flour, and water as a basic reinforcer. To produce three variations in quality of this basic mixture, Hutt added citric acid to it to make it sour and less acceptable for one group of rats. For another variation he added saccharin, producing a mixture which the rats preferred. The unadulterated basic mixture provided the third, intermediate, variation in quality. Different groups of rats were trained to press the bar in the Skinner box for each of these mixtures. Beyond this, three different subgroups within each quality group received different quantities of the mixture. The three quantities were: small, 3 mg.; medium, 12 mg.; and large, 50 mg.

The results of the experiment appear in the following table, where mean response rate appears for each of the 9 subgroups in the experiment on the fifth day of training. The mean rates entered in the table show that both variables had an effect--that of quantity being somewhat greater than that of quality. This, however, may only mean that a wider effective range of quantities than qualities was sampled. Close examination of the entries will show that there were no inversions. Increasing quantity led to an increasing response rate for each value on the quality dimension. Similarly, the order of the quality groups is citric, basic, saccharin for all quantity conditions.

MEAN RATES OF BAR-PRESSING UNDER DIFFERENT COMBINATIONS
OF AMOUNT AND QUALITY OF REINFORCEMENT (HUTT, 1954)

Quantity	Citric	Basic	Saccharin	Mean
Small	3.0	3.9	5.0	3.9
Medium	4.2	6.9	8.0	6.4
Large	8.2	11.1	13.8	11.0
Mean	5.1	7.3	8.9	

It is suspected that both quantity and quality of reinforcements were functioning as interrelated performance variables for the sampling of inner city youth previously described. For to assume that quantity had dominance over quality is equivalent to assuming that chocolate nuggets, judiciously distributed, would have achieved the same outcomes.

Reinforcement was also immediate in that no more than four practice exercises were to be completed before assessment was made of the S-R bonding and the scheduled one day of drill-and-practice. Several independent lines of evidence suggest that responses spatially or temporally near reinforcement are learned more quickly than responses remote from reinforcement. Tolman (1934) showed that rats, running a maze which required two black-white discriminations, one after the other, learned the second of two discriminations more easily than the first. It is well known that blinds in a complex maze tend to be mastered in a backward order (Montpellier, 1933). From the works of Yoshioka (1929) and Grice (1942), it has been established that rats tend to take a short path to a goal rather than a long one. Further, if during the extinction of a running response in a straight alley, rats are blocked at various distances from the goal, extinction is more rapid if the block is placed near the starting box than if it is near to the goal of the box (Lambert and Solomon, 1952). There is also a clear-cut speed of locomotion gradient in the behavior of a rat in a straight runway. The animal runs faster as it nears the goal (Hull, 1934c). All of these suggests that learning, or perhaps performance, varies directly with the immediacy with which reward follows response to be learned. The <u>goal gradient</u>, or <u>delay-of-reinforcement gradient</u>, represents one of the most important applications of conditioning principles to more complex behavioral situations. The goal-gradient hypothesis as Hull originally formulated it, is as follows:

"The mechanism...depended upon as an explanatory and integrating principle is that the goal reaction gets conditioned the most strongly to the stimuli preceding it, and the other reactions of the behavior sequence get conditioned to their stimuli progressively weaker as they are more remote (in time or space) from the goal reaction." (Hull, 1932, pp. 25-26.)

Delayed reinforcements are ineffectual with inner city youth. Societal conditions, legion in numbers, pervasive with skin and economic discrimination, raise question regarding the intent of proponents who advocate postponement of rewards until these youths enter the adult world. If they are to establish a pattern of success along a vertical continuum, then the learning climate and the controlling adult must engineer immediate applications of reward systems that are not contrived, but relate to further learnings.

This writer's direct influence over these seventh, eighth, and ninth grade youths ceased as of the spring of 1963. Case studies made, produced:

1. All of the students completed high school, even though at least five had been identified by counselors as potential dropouts as soon as they reach the age of sixteen.
2. Their interest and achievement rate in mathematics continued through the eleventh grade, with them earning an average grade of B, and with all seventh graders of 1962-63 transferred into an integrated senior high school upon reaching grade 10.
3. Two of the five accelerated seventh graders successfully completed the calculus, each winning academic scholarships to attend prestigious eastern colleges.

After the third year of implementing this plan, it was obvious that these youths would continue to have successful experiences in mathematics because they were now able to enter the excitement of mathematics normally reserved only for those who are permitted to engage in such experiences as suggested by the titles: geometry, trigonometry, probability, analytic geometry, analysis, calculus. How then could greater numbers of youths be provided similar opportunities? The question was also raised as to the probable contributing factors of either the organization or teaching methods used in elementary mathematics that uniformaly maintained low achievement for such youths? Black youths in Dade County consistently entered the junior high years with at least 60% of them achieving below the 25 percentile on subtests of standardized achievement tests.

From experiences teaching elementary mathematics gained as a demonstration teacher since 1965 and analysis of both "traditional" and "modern" text materials used, it was observed:

That the season of the year could be determined by the arithmetic skill with whole numbers being treated, diagnosed, or drilled.

That such a sequence of learning mandated successively ordered repetitions that are stultifying and stifling in that skills were not ordered in relative positioning of difficulty, but dependent upon perseverance with ever-increasing numbers of digits included in numerals. Why must $2012 - 789 = x$ precede $2 \times 3 = 6$ or $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ or $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \ldots$?

That the implications of the field properties for teaching and sequencing are only minimally abstracted, thereby establishing interactive contradictions between compatible theories of learning, teaching sequences, methods suggested by the field properties, and the present ordering of

addition to subtraction to multiplication to division

of

whole numbers to nonnegative rational numbers to integers to rationals to complex numbers.

That whereas theoretical approaches to geometry may be compatible with man-made deductive approaches, they are inappropriate and fraught with frustrations for young learners who from the instant of birth are required to operate in multidimensions.

That although the modern mathematics revolution enriched the mathematical language of elementary teachers and students at the same time, textbooks introduced more content into the curriculum, the profession has failed to make accessible to nonmathematically trained elementary teachers alternatives for teaching that are inherent in the discipline. Little evidence is available to indicate leadership from the profession in suggesting either sequences or approaches to teaching that are both mathematically and pedagogically sound.

That the present elementary mathematics sequence and emphasis upon singular algorithmic methods are devastatingly relevant for maintaining the prevailing social, economic, and educational conditions of society's rejects, many of whom live in America's inner cities. As soon as these youths begin to feel that school offers hope and begins to understand, for example, the number line as a tool for picturing addition, continued growth is impaired by a truncated number line being used. Similarly, as soon as they begin to translate $15 - 7 = x$ to $7 + \underline{?} = 15$ and experience success in applying this, they are forced to forget these relationships and discriminate between at least fifteen different stimuli, awkwardly executing:

How then are inner city youths enabled to deal with the negative and probabilities that are the substance of their life styles?

The Experiencing Mathematics Series is an attempt to make available to other students and teachers of junior high mathematics the features of the teaching model that has been described. These materials also reflect attempts to counteract the effects of conditioned positioning that are direct results of the sequence of skills in elementary mathematics. These materials indicate the position taken by its authors that:

1. The content would be mathematically correct, but approaches for teaching would be dependent upon simultaneous considerations of the discipline and compatible principles of learning.
2. Conditioned preferences for positioning with whole numbers would be systematically interfered with.
3. Two and three space geometric conditions and relationships would be used and only intuitively characterized with no attempt at conceptual development of deductive ideas until such time that an experience base has been established, thus grappling with the concept of readiness.
4. All important ideas would be presented through carefully designed dialogs, thereby maximizing the possibilities that students would be encouraged to read mathematics and supported "talking out" meanings to be inferred.
5. Support would be engineered to enable learners to discriminate among a field of stimuli to determine those that must initially be excluded and those which, although excluded, are related in some ordered fashion, and which must be at some point integrated together in some definite manner.
6. Pedagogy deemed vital would be detailed in materials for students with ~~interpretive data~~ provided teachers in a specially written Teachers' Commentary.
7. Student text materials would be thin so as to provide the status measure of success tied to "completing" a textbook, thereby enabling students to achieve closure in their accomplishments and facilitating pacing that is determined by either the student or the teacher.

8. Students would be provided the opportunity, as far as the text materials were concerned, to explore alternative means for arriving at solutions and selecting for perfection those methods that suited them best.

9. Upon completion of the series, students would be able to effectively function with mathematics instruction that is more open-ended than that which is now provided in General Mathematics sequences. This can be validated by profiles of achievement evaluated by both the teacher and the student. Evaluation would be based upon accomplishment of those objectives that are most frequently evaluated by "experts" who design standardized measures of mathematical achievement.

The position of the authors of the Experiencing Mathematics Series was not only influenced by data collected from observations and trial and error practice, but also from experimental results obtained from experiments conducted under controlled conditions by psychologists in guest of answers to how learning takes place. Significantly influencing these authors are the following principles:

generalization and the number of reinforcements;
stimulus intensity;
response generalization;
variables controlling the amount of stimulus generalization;
formation of learning sets;
principle of successive approximations;
patterns of reward;
observational learning;
transfer of relational responding.

Each of these points are amplified in the paragraphs to follow.

Generalization and the Number of Reinforcements

With continuing reinforcement, the range of stimulus generalization shows an initial increase. Beyond this point, it has sometimes been found to show a further increase (Margolious, 1955) and sometimes to show a decrease (Beritov, 1924, p. 123; Hovland, 1937d). On the basis of a summary of 67 Pavlovian experiments, Razran (1949) suggests that the initial increase may be followed by a decrease and then a second increase. The magnitudes of response strength providing the basis for this conclusion are small, however, and the reality of the details of this postulated function is, by no means, certain.

Hull (1952, p. 70) has proposed that changes in the form of the generalization gradient with practice depend upon the extent to which the response is conditioned to incidental stimuli in the learning situation. Such stimuli (apparatus noises, movements of the experimenter, sounds of the heating or cooling equipment in the room, distinctive odors, and so on) are often present at the time of reinforcement and should acquire a certain degree of potential for eliciting the conditional response early in learning. Later on, these stimuli would be expected to lose their power to evoke the CR, through extinction, since they are apt to be present between, as well as during, trials. Thus, early in learning, a generalization test should yield an increasing response strength as these nongeneralized incidental stimuli undergo conditioning.

Stimulus Intensity

Since increased stimulus intensity increases the vigor of at least some conditioned responses, the factor of stimulus intensity enters to complicate the generalization function for such responses. Intensities lower than the CS will lead to lessened response strength because of generalization decrement and the directly weakening effect of the lower intensity. With higher intensities, these two factors act against each other. Increasingly higher intensities will tend to strengthen responses through a dynamogenic effect, which Hull (1949) calls stimulus intensity dynamism, at the same time that generalization decrement tends to weaken the response.

The form of the generalization function produced by variations in intensity will depend upon the relative contributions of these two processes. At the very best, it is to be predicted that the generalization gradient for stimuli more intense than the CS will be flatter than the one obtained with weaker stimuli.

Response Generalization

The principle of stimulus generalization, stated somewhat abstractly, is that an organism which has learned to respond in a certain way to stimulus S, has thereby learned also to respond in a similar way to a different stimulus S'. The counterpart of the response side is as follows: if an organism has learned to react with response R to a stimulus S, it has also learned thereby to react with response R', which is unlike R, but in some respects equivalent to it. Wickens (1943, 1948) has described two experiments on response generalization involving finger withdrawal. In these studies, an extensor response was first conditioned by pairing a tone with a mild shock to the finger. During this original conditioning, the subject sat at the apparatus with the palm down, the middle finger resting on the shock-administering electrode. After conditioning, the subject turned his hand over so that the palm was now up. Under these circumstances, most of the subjects responded to the CS with a flexor response, demonstrating response generalization. In addition, one of these experiments (Wickens, 1948) successfully demonstrated stimulus generalization of the generalized response. That is, the flexor response transferred in reduced strength to tones other than the specific one used as a CS. Other studies of response generalization have been conducted by Antonitis (1951), Arnold (1945), and by Williams (1941).

Variables Controlling the Amount of Stimulus Generalization

At the most general level, there is good evidence that the breadth of the generalization gradient increases in the strength of the conditioned response. The evidence for this conclusion is to be found in the facts that:

1. increasing numbers of reinforcements broaden the generalization gradient (although a subsequent narrowing may occur),
2. the generalization gradient steepens with extinction unless training was under conditions of intermittent reinforcement,
3. increased motivation increases the range of the generalization gradient, and
4. the generalization gradient is flatter following intermittent reinforcement than following continuous reinforcement.

It has also been shown that the form of the generalization gradient can be influenced by the intensity of the test stimuli. If a response is conditioned to a particular stimulus and then tested with stimuli which are both weaker and stronger than the training stimulus, the amount of generalization will be greater for the stronger stimuli than it will be for the weaker. In experiments on the generalization of classically conditioned responses, this stimulus-intensity effect may be so powerful as to obscure the generalization gradient.

Finally, the extent of generalization depends upon the conditions of training. If a response is established with the procedures of discrimination learning, the generalization gradient is steeper than it is following training in which no discrimination is required.

The Formation of Learning Sets

In many areas of learning, practice on a series of learning tasks leads to an improvement of the organism's ability to deal with learning situations involved. This is a general statement which apparently applies at all levels of task complexity. Repeated conditioning and extinction increase the rapidity with which these processes occur (Bullock and Smith, 1953). The number of trials required to learn a list of nonsense syllables decreases by about 55 percent with practice on a series of a dozen lists (Meyers and Miles, 1953). And there is a remarkable improvement in the ability of organisms of several species to master discrimination problems with practice at making such discriminations. Harlow (1949, 1959) refers to such improvements as the formation of learning sets and explains learning sets in terms of error factor theory, which will be treated later.

The classic study of learning sets (Harlow, 1949) was one in which monkeys learned a series of over 300 discriminations in the Wisconsin General Test Apparatus, each presented for 6 trials. What is striking is the great improvement in performance over the series of problems. On the final block, the typical performance is chance (50 percent on the first trial and nearly perfect from then on). Occasional errors do occur, for reasons to be discussed later. But these are relatively infrequent and appear only once in about 20 trials, as is indicated by the leveling of the function for problems 201-312 at about 95 percent.

Principle of Successive Approximations

Skinner describes a simple experiment that illustrates the shaping up of response patterns through reinforcing successive approximations. In the Skinner experiment, a pigeon is placed in a box where its behavior may be observed through a one-way screen. The pigeon can be fed by using a small food tray that is operated electrically. The behavior to be developed is that of raising the head above a given height. A scale is pinned on the far wall of the box in order to observe the height of the pigeon's head. Initially, the experimenter observed the height at which the head of the pigeon is normally held. Then a line on the scale is selected that is reached only infrequently by the pigeon. Whenever the pigeon's head comes up to this height, it is rewarded with food. Almost immediately, there is a change in the frequency with which the head is raised up to the line. Also, the pigeon, on occasions, raises its head higher than the rewarded height. The experimenter next selects a second and higher height on the scale and reinforces the pigeon only when the head is raised up to this new height. Again there is a change in the frequency with which the head is raised up to the new line, and again the pigeon occasionally raises its head above the line.

Patterns of Reward

Under laboratory conditions it is possible to dispense reinforcers for every desired response or dispense them intermittently according to some schedule or plan. Generally speaking, continuous reinforcement results in the more rapid acquisition of responses, but once learned, the behavior is more stable and resistant to extinction if it has been acquired on an intermittent schedule.

With a fixed ration schedule, very stable rates of response are set up, with the speed of response varying positively with the frequency of the reinforcements. On a fixed interval schedule, the rate of response is low immediately after a reinforcement but increases rapidly as the time for the next reinforcement approaches.

In laboratory studies, the effects of both variable-ratio and variable-interval schedules have been investigated. Variable schedules result in very stable rates of response and generally speaking, in increased resistance to extinction.

Observational Learning

If someone were to ask how we acquired new response patterns, current stimulus-response theory has a reasonably pat answer: by reinforcement of variations in behavior that successively approximate the final form desired. In shaping a behavior sequence by successive approximations, we wait for and then reinforce some response that is at least grossly similar to the first element of the final pattern desired. Thereafter, reinforcement is provided only for variations of this first element that correspond more closely to the one desired. After the first element of a series is learned, we go on to teach the second element in a similar manner, and this is chained to the first element. After the two are learned, they are chained to the third element, and so on. Skinner was one of the first to describe the shaping method as a means for increasing an organism's repertoire. The procedure is often illustrated by training animals to perform tricks or complex behavioral sequences that make an interesting spectacle. Thus, pigeons can be trained to play competitive ping-pong, a raccoon to put a basketball through a hoop, a mynah bird to say strings of English words, and so forth. Shaping an animal to perform such a chain has by now become a standard laboratory exercise in most college courses in learning.

Shaping through differential reinforcement is indeed an important method for establishing new responses, but is it the only method? Examination of everyday learning by human beings suggests another method, and in almost all cases it is more efficient than the shaping method. This other method is simply to have the learner observe someone else performing the response that the learner is to acquire. By this means, the learner can often perform the novel responses sometime later without ever having performed them before or having been reinforced for them (since they have never occurred before). It seems obvious that a large portion of human learning is observational and, in one sense, imitative. It is obvious too, that many skills, (e.g., driving a car, pronouncing foreign words) are learned more readily by this method than they would be were the successive approximation method (without verbal instructions) used exclusively.

In several papers, Bandura (1962, 1965) has pointed out the ubiquity and efficiency of such observational learning in humans and has emphasized its unique features not found in the standard paradigms of shaping and instrumental conditioning. He has also carried out an admirable series of studies with young children that throw light upon the variables influencing such observational learning.

In the typical experiment, a kindergarten child (the subject) sits and watches some person (the model) perform a particular behavioral sequence. Later, the subject is tested under specified conditions to determine to what extent his behavior now mimics that displayed by the model. What he does is to imitate the model. A variety of factors can be varied in this situation, and many can be shown to affect the extent of imitative behavior performed by the subject. We list a few of those studied by Bandura:

A. Stimulus properties of the model

1. The model's age, sex, and status relative to that of the subject. High status models are more imitated.
2. Model's similarity to the subject: the model is another child in the same room, a child in a movie, an animal character in a movie cartoon, etc. Imitation induced in the subject decreases as the model is made more dissimilar to the real person.

B. Type of behavior exemplified by the model

1. Novel skills vs novel sequences of known responses. Presumably, the more complex the skills, the poorer the degree of imitation after one observation trial.

2. Hostile or aggressive responses. These are imitated to a high degree.

3. Standards of self-reward for good vs bad performances. The subject will adopt self-reward standards similar to those of model. Also, the subject will imitate the type of moral standards exhibited by an adult model.

C. Consequences of model's behavior

1. Whether model's behavior is rewarded, punished, or "ignored" (neither reinforced nor punished) by other agents in the drama. Rewarded behaviors of the model are more likely to be imitated.

D. Motivational set given to the subject

1. Instructions given the subject before he observes the model provide him with high or low motivation to pay attention to and learn the model's behavior. High motivation might be produced by telling the subject that he will be paid commensurate with how much of the model's behavior he can reproduce on a later test. Under minimal instructions, learning is classified mainly as "incidental".

2. Motivating instructions after the subject views the model and before he is tested. This aids in distinguishing learning from performance of imitative responses.

This listing of variables in the observational learning situation is not exhaustive; it is intended to show the range of possibilities. A wide range of behaviors can be transmitted under these conditions by the model and with the effect that the fidelity of the subject's mimicry (even under incidental learning conditions) is often remarkable.

Transfer of Relational Responding

All theoretical approaches to discrimination learning begin by trying to specify, either formally or intuitively, what it is that a subject has learned in his discrimination training. How are we to characterize the subject's knowledge gained by this educational procedure? For behaviorists, this question gets translated into one about stimulus control of responses: what is the effective stimulus CONTROLLING THE SUBJECT'S discriminative performance? At one level of analysis, practically all theories answer this general question in a similar manner: the effective stimulus variable that comes eventually to control discriminative performance is that feature (cue, attribute, etc.) or set of features present in S+ and absent (or different) in S-. Such features are called relevant cues because their variations correlate with presence or absence of reinforcement for responding. Cues not so correlated are termed irrelevant.

But let us consider a problem where the relevant cues consist of different values along some ordered stimulus continuum (such as size, brightness or heaviness, etc.). For example, suppose a monkey is trained to use size as a cue for securing a food reward. The setup may consist of simultaneously presenting two boxes between which the monkey is to choose; the one containing the reward has a top with an area of 160 square centimeters, whereas the one containing no reward has a smaller top, 100 square centimeters in area. The relational theory supposes that in this situation, the subject would learn the relation "the larger area is correct". The absolute theory supposes that the subject has learned specific stimulus-response connections; in particular, the rewarded stimulus (160) whereas the response is inhibited to the value of the nonrewarded stimulus (100).

Which mode of description of "what is learned" is better is more than a matter of taste, because transfer tests with new stimuli provide us with data for inferring what the subject has learned in the 160 vs 100 situation. If the subject learned a relation ("choose the larger one of the stimuli"), then he should in some degree be able to transfer his response to this situation to new stimulus pairs differing from those used in training. That is, the relation he has learned is one that transcends the specific stimulus pair used to exemplify the relation. Thus, if we test the animal with the new pair, 256 vs 160, he should still choose the larger stimulus in this pair--namely, 256--in preference to 160, despite the fact that 160 was rewarded in the prior training series. The usual experimental result is that animals do choose the 256 stimulus in preference to the 160 stimulus. That is, they transpose the relation "larger" along the size continuum. Such studies are thus called transposition experiments.

Conclusion

At best, the teaching model in the Experiencing Mathematics Series is a stopgap measure. The very fact that such a non-dead-end program was forced to come into existence validates the assumption that all is not well in the kindergarten to sixth grade program. The first teaching model demonstrated what was possible when energies are directed towards establishing irrelevant teaching and learning conditions, assuming that relevance infers appropriateness to or maintenance of the existing condition or state of being. It also demonstrates a strategy for enabling students to escape the doldrums of mathematics school experience at the secondary level which are equally as disenchanting as their mathematical learning experiences were at the elementary levels. Even inner city youth with television sets in every household know that more than 220,000 miles can be traversed in four days of travel, but their sequence of learning insists that this is too difficult for 4-8 year old minds to comprehend. Wherein is there excitement and relevance for future societies? Staid laboratory or quizzical games provide temporal excitement, but freeze upward mobilities.

It is imperative that teaching strategies, pedagogy, and the discipline of mathematics offer inner city youth experiences:

that are not as

- grey, dull, and monotonous as the physical structure in which these youths live.

or as

- barren and as depressing as the small playground areas surrounding their homes if such are available in their neighborhoods.

or as

- uneventful and unexciting as the many hours each day they while away the time, peering out of the windows of a concrete or brownstone prison called home, waiting for a guardian to return with a pittance of funds to purchase a few morsels of food for stomachs agonizing in pain.

Further, they should not be provided a curriculum that is as

- crowded and cluttered and as cold as the flats they inhabit with wall-to-wall people.

or as

- overwhelming, challenging, and defeating as their efforts are in attempting to do more than just survive.

Perhaps a few first steps is the examination of the following basic assumptions:

1. That inner city youths behave in those ways that are expected by the model adult figures.
2. That the nature of the discipline of mathematics makes at least limited inferences for abstracting appropriate teaching strategies and sequences.
3. That there exist some compatible theories and principles of learning which should be employed.
4. That if changes are to be effected in the amount, rate, or quality of mathem tics that youths are to learn, then such changes will only occur when the conditions for learning, the nature of teaching, and the relative positioning of concepts and skills in order of difficulty are manipulated.
5. That the nature and intensity of reinforcers may be functions of the learners' social background, the discipline, and the predisposition of the controlling adult figures.
6. That since social learnings are acquired through imitation, it therefore is vital that there exist in each classroom, teachers competent in subject matter, and who are also in-tuned to the precarious needs of youth and society. Such teachers must also be committed to facilitating delinations of identities and thereby capable of providing strength to rejected youths as these youths grope with executing control amid the dynamics of antagonistic elements, each yielding forces capable of rendering discontinuities in their very being.

Mathematics educators are morally and professionally obligated to explore in greater depth the implications of research and to commit those resources necessary for enabling this not necessarily college-bound population to acquire those competencies needed by fully functioning persons. What, then, is our commitment towards developing a theory of instruction for society's rejects? Remember, that the pedagogy that we design and implement today, returns in the tomorrows. We cannot escape accountability!

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THE LABORATORY APPROACH TO TEACHING MATHEMATICS IN THE INNER CITY

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A laboratory is defined as a building or room fitted up for conducting scientific experiments, analyses, or similar work. It is a department--as in a factory--for research, testing, and experimental technical work. Thus, a mathematics laboratory would be one fitted up for conducting mathematical experiments. The laboratory approach would then be one that provides the student with opportunity to conduct experiments, make analyses, research a particular concept, or test a hypothesis.

My experiments have convinced me that the laboratory approach--as any approach--will be as successful as the teacher using it. This approach is needed as badly in the outer city as in the inner city. Like all approaches, the laboratory will work as well with fast learners as it does with slow learners. This approach is heavily dependent upon pedagogy. It is greatly enhanced by the classroom atmosphere. The laboratory approach succeeds or fails regardless of the manipulative objects being used.

Today, we find many gimmicks being used in the mathematics classroom. Acceptance of the laboratory approach has opened the floodgates for all kinds and types of gimmicks. I want to caution against an overemphasis of the gimmick. While they are valuable as motivational devices, the gimmicks must be algorithmic. They must be symbolic as well as functional. To paraphrase Dr. Pólya, any gimmick used must be elegant. The elegance of the gimmick will vary directly as the number of ideas in it and will vary inversely as the effort it takes to see these ideas. I sincerely believe that any gimmick or medium used to teach mathematics should serve only as a cosmetic that enhances the beauty of the mathematical concept. Perhaps, like a cosmetic, the gimmick should be washed off at the end of each day.

Manipulative objects have proven very successful in the laboratory approach. They have a very high motivational value. The secret is to exploit this motivational value while it exists and to release it before the object has become a bore. The student must be introduced to the object and allowed time to free-play with it. He must be allowed to stretch the rubber bands and even to shoot them. He must be allowed to stack the poker chips, design with the rods and blocks, explore the calculator's keyboard with no particular purpose in mind.

Task cards have proven useful in introducing the student to the object, allowing him the free-play in a structured manner, and then sequentially leading him to discover a particular concept. With task cards, one must keep the reading level at least two grade levels below the mathematical level.

My experiences have also convinced me that the slow learner doesn't always have to touch the manipulative object. His weakness is in his inability to locate an idea in his mind and to bring it to the forefront. If the teacher holds the object up in front of the class, this is sufficient for the student to locate the object and call the object forward. Most slow learners are able to manipulate this object mentally. If the desire is to transfer the student to some abstract concept, then perhaps we should allow him only a mental contact with the object.

Calculators, computers, and terminals have proven very hot items in the mathematics laboratory. However, I feel that very little software has been developed to justify the expenditure for these items. They cannot be purchased just to make the student competent with the keyboards. Our objective should not be to teach only computer programming. The student must use these calculators and computers to learn mathematics. I also feel that the responsibility of developing software rests squarely on the shoulders of the producers of the hardware. They must, as it is evidently so, use classroom teachers to write this software material.

Games, competitions, and grouping have proven highly successful in the laboratory approach. Very few games have written directions that the slow learner can read and understand by himself. Quite often, the time needed to explain and introduce a game could well be used to teach the concept in a more traditional approach. Very few teachers have the ability to introduce games and maintain a high level of interest in the game. The inner city child competes in his everyday life. Why not allow him to compete in the mathematics classroom? Through grouping initially, the stigma of defeat is shared. How to meet a failure and overcome it is a very important lesson for the inner city child as well as the outer city child. I feel that most of our dropouts at Berkeley are the kids from suburbia who have never failed in their life until they get to Berkeley. Our success-oriented educational system has not taught them how to live with failure. It has, in fact, done a poor job of teaching them how to live with success.

The selection of the best games, the decision of when to inject competition, and the selection of groups involve an expertise that is not found in the typical classroom teacher. Producers of materials must assume the responsibility for including suggestions or giving directions for performing the tasks in a minimum amount of verbiage. They must also give some consideration to supplementing the typical teacher's personality with exciting and innovative student materials. Producers of manipulative objects must likewise assume a responsibility for software to accompany the objects. They must be practical and concern themselves with the amount of work involved in setting up a typical class with manipulative objects. They must give some consideration for in-service training of the teachers to use the objects successfully. It might be indicated here that teachers must be involved with the laboratory approach in the same manner that they are to involve their students with it.

Which direction, then, is the mathematics laboratory to follow? Is it to become merely a replica of the science laboratory? Does this type of laboratory produce the desired number of scientists? Does this type of laboratory work successfully with the slow learner? Has this type of laboratory found tremendous success in the inner city? I think not! The slow learner and most of the inner city students find it difficult reading the science text to obtain their theory and then applying this theory in the laboratory experiment.

Perhaps the mathematics laboratory is to develop along the lines of the "shop" classes. A student is given a job assignment or project to do, and then he is required to report on this job. Perhaps he is to stitch a curve, build a model, stack blocks, draw lines, measure distances, duplicate a tape on a calculator, or even fold paper. Once the job is accomplished, the student is then asked to complete some task cards that will lead him to discover the mathematical concept related to his job assignment. The reporting session could include many activities such as verbal explanation to the class, written data on a table or chart, charing of ideas about the job with a co-worker, or merely displaying the finished product for the critical eyes of the class.

A prototype of a laboratory assignment might suggest more specifically what I think a mathematics laboratory should be. The students arrive at the laboratory today to hear the Mission Impossible theme music being played on a tape recorder or record player. They are handed sealed envelopes with instruction printed on them, "DO NOT OPEN UNTIL TOLD TO DO SO." After all the students are seated, the tape recorder greets them and indicates that the head man cannot be there today, but the teacher will carry on in his absence. The agents are to pay close attention to the teacher as he is one of the sharpest agents in the organization. (Teacher smiles.)

Each student is instructed to open his envelope which contains six cubical counting blocks and a task card (4 × 6 index card). On the task card will be many activities such as the following:

1. You must be competent in measuring with these blocks. How long (in blocks) is your desk, book, the room, etc.?

2. You must learn to develop or reproduce arrangements rapidly. How many different arrangements of the six blocks can you make?
   How many arrangements are there if each block touches only two other blocks?
   How many arrangements of the blocks could be folded to make a cube?

3. You must become a master with five blocks.
   How many different patterns can you make with five blocks if all blocks must touch each other flush (side-by-side)?
   PENTAMINOES.

4. Three other agents have the same color blocks as you have.
   Get with them and await instructions for your team.
   How many different ways can your team stack the blocks into perfect stacks? (No blocks left over and a solid formation.)

5. Make a tinfoil cover for your team's favorite formation.

6. Draw and cut out a paper pattern for a container to hold your 24 blocks.

7. Make a cardboard container for the formation selected by your team.

8. You must measure the surface area of each of the team's containers.

9. Which arrangement has the maximum surface area? the minimum surface area?

10. Your team must complete all the questions on your task cards. Be prepared for any member of your team to report to the class!
    DO NOT TELL ANYONE WHAT YOU HAVE DONE ON THIS MISSION!
    RETURN YOUR BLOCKS TO THE ENVELOPE WITH YOUR TASK CARD AFTER EACH DAY'S ASSIGNMENT! REMEMBER YOUR ENVOLOPE NUMBER!

What are the implications for the types of text materials that will be produced if a mathematics laboratory is to become a reality? It will be more than one page in a text titled, "Math Lab". The laboratory approach must permeate throughout the entire text materials. The paperback books will be purely consumable! They will be changed frequently, as the approach is conducive to innovating and creating. The students will be asked, "Can you solve this?" rather than told to, "Solve this!" Guessing exercises will be provided for in the text. Home fun will replace homework. A concept will be introduced through an activity, project, or game.

The teacher will be given the concept and a few suggestions, then told to improvise. The student and teacher will have maximum room for changing the book's approach (many paths to the same mountain). Familiar gadgets will be used to introduce unfamiliar concepts and unfamiliar gadgets will get their introduction as a familiar concept is being presented. Humor and fun will replace the usual drab approaches in traditional texts. We must compete with Laugh-In! Audio-visual aids and multisensory media will be used as we compete with television commercials. The teacher will be able to rapidly complete the student exercises and thus obtain a more than adequate picture of what is being attempted. Teacher guides will be in the form of diaries that give one successful teacher's method of attack. Concepts will be presented in many different ways. The student will be conditioned to look many different ways at a problem.

Every facet of the materials must be geared toward developing learning as creative self-direction. Education will become not a preparation for living in the future; rather, a preparation by living in the present. Like the best materials and the best teacher, the laboratory approach will have high spots and low spots. No one will expect to remain on the mountain top! What is most important, everyone will know that there is a mountain top!

REACTION TO THE PANEL

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When reacting to the two papers of Mrs. Collins and Mr. Allen, I am impressed with the contrasts of the different points of view, each of which is valid when one is considering certain particular aspects of a mathematics classroom learning situation.

I would characterize Mrs. Collins' paper as convergent relying rather highly upon an S-R psychology. I feel she places too much faith in psychology being helpful in creating a healthy mathematics learning environment.

The characteristics of the classroom which is described by Mrs. Collins provides for us a good theoretical basis for this healthy learning environment. These characteristics are:

1. higher expectations;
2. student involvement in decision making;
3. increasing comfort of the learners;
4. use of natural language;
5. prior "talk through" of concepts (advance organizers);
6. special days for individual skill building;
7. immediate checking and correcting of problems.

The results seem highly desirable when children display two and one-half years of growth in one year, and when most of the children were motivated to buy their own textbooks.

We were not told what the "Experiencing Mathematics Series" was, how it was being used, nor who wrote it. The nine positions taken by the authors are general statements which would gain wide agreement, but without specific examples, they are not very helpful. To claim that such a series is based on the principles of S-R psychology as she describes them, seems to me to be irrelevant to mathematics instruction. The research cited refers to monkeys, rats, and pigeons learning cute little things or people learning to refrain from shocking themselves or to memorize nonsense syllables. It may be true that this kind of orientation toward mathematics instructions has, in fact, been the cause of many of our problems in mathematics teaching rather than requesting solutions. Isn't it that many poor teachers are "intensifying the stimuli" in order to obtain better successive approximation to the long division algorithm?

On the other hand, Mr. Allen's paper represents a rather divergent concept of the mathematics learning environment. My impression is that he feels that mathematics should be learned in a laboratory teaching situation, that the classrooms should provide a wide variety of experiences for children, that possibly we should throw away the text and that through the learning experience in the laboratory every child will achieve and be successful.

When one considers a mathematics classroom as a whole, and I mean here a healthy mathematics classroom, then one can appreciate the fact the classroom is an extremely complex phenomenon. And, in fact, a rather large collection of seemingly contradictory phenomenon. Let me cite some examples. First, in any healthy mathematics learning classroom there has to be both an element of cooperation and an element of competition. These two characteristics tend to be contradictory, but I think the best example of exploiting both of them in the classroom can be seen in the way Laymen and Robert Allen have devised techniques to introduce Equations games tournaments. Suppose one had a class of thirty students and wished to conduct an Equations game tournament for one period each week for six weeks. Begin by making a preliminary estimate of each child's ability to play Equations game and rank order the students from one to thirty. This could be done in a very subjective fashion because the procedure developed is self-correcting. Three-man teams are formed so that there are ten teams in the room. Each team consists of one of the brighter kids, one of the middle kids, and one of the slower kids in the room. For the first playing session, the children are asked to play games in groups of three, but not with their team members. Instead, there are ten tables in the room. These are ordered from top to bottom--one to ten. At table number one are the three best players in the room, at table number two are the three next best players and so forth, such that at table ten will be the three poorest players in the room.

At the end of the first period of play, and before the beginning of the second period of play during the next week, the winner at each table is moved to the table above and the loser of each table is moved to the table below. Therefore, through time, children tend to be playing with other children that are at their same skill level. Thus, the competition is fair. The cooperation comes in the team scoring and team effort. All of the students' scores are added into their total team score. The slower student usually contributes more points than the faster student to the team because the slower students play simpler and faster games. Thus, we have a good illustration of a classroom setting in which both the competitive and the cooperative aspects are used to the advantage of learning situation.

A second example of contradictory phenomenon is the role of success and failure in the classroom. Mr. Allen referred to the success-oriented curriculum in schools that has caused the drop-outs at Berkeley. While it certainly is true that many places in school don't tell the children the truth about their inadequacies, it's at least equally true that in many places in school we continually tell the children about their inadequacies or at least what we view as inadequacies. In other words, much of the school experience is failure-oriented experience for children. Children operate under the threat of failure as a basic motivator in many situations. Another phenomena is the relative role of work in groups as opposed to individual work. Both group work and individual work has a proper place in a successful mathematics learning study.

Still another contradiction is the relative emphasis upon conceptualization and understanding of mathematical ideas as opposed to practice in skill development. Another contradiction arises from the role of a classroom group discovery in an exciting learning situation as opposed to the idea that mathematics can be thought of as contemplative behavior. We have the contradiction of the cultural bias of standardized tests in mathematics versus the argument that mathematics provides culture-free content, and thus provides possibilities for children in deprived areas. We have documents such as the Cambridge Conference Report on the correlation of elementary science and mathematics and the SMSG Report No. 8 which shows no significant attitude changes when science is used as a motivator of mathematical ideas. We have the contradiction of attempting to predetermine what the mathematics curriculum shall be, and at the same time, to attempt to provide for individual learning characteristics of children. It is unfortunate that we were not able to have the paper on classroom atmosphere written as was planned for this conference* because, in some ways, the classroom atmosphere may be the most important characteristic of a mathematics classroom that we could consider.

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*Mr. Ogie Wilkerson of the St. Louis Public Schools was invited to present a paper on classroom atmosphere. He was unable to attend the conference due to illness.

Let me describe what I mean by healthy mathematics classroom atmosphere. A healthy classroom atmosphere would be one in which there is a degree of trust between the teacher and the student, and between student and student; one in which there is a spirit of inquiry both on the part of the teacher and on the part of the student. It is one in which the inquiry is aimed toward the solution of real problems--problems that are sensed by the children and problems which they feel need answering. The solution to these problems arises out of the cooperative efforts of the children and the teacher. There is the feeling of success in an achievement, and reports of the successful achievement to the children themselves, to the teachers, and to the parents. The healthy classroom has opportunity for the children to participate in a variety of activities at any time, and one in which there is an excitement for the learning taking place.

If the description above represents a classroom with an ideal atmosphere, what prohibits their existence so often? First, many classes that are unhealthy from an atmospheric point of view are permeated with mistrust. Why do teachers not trust students and why do students not trust teachers? To what extent is this because the teacher imposes the standard tasks and goals upon the students and actually establishes a competitive atmosphere between the students. How many times have we seen classrooms where the rule is that if you tell other children the answers, it will hurt you. In other words, we establish the setting in which imparting of knowledge is undesirable and we call this an educational system. Many classrooms do not reflect a spirit of inquiry. Might this be because teachers are saying to students, "Let me tell you what it is you are interested in." In many classrooms, children are not working on the problems that are real to them. If Johnny can mow 6 lawns in 3 hours, let him do it. In how many cases is the real problem the child is trying to solve, "How do I get this teacher off my back?" In how many cases is the main idea to get by with the minimum expectation? In how many classrooms is everyone expected to obtain the same concept or learn the same task at the same time? How often do we have a situation where the idea is to get through this drudgery so we can get to the errichment work? Why isn't all mathematics considered enrichment?

Recently I was in a fourth grade class in an inner city school in Michigan where the teacher had 6 children lined up at the front chalkboard. The rest were sitting quietly in their seats. Each child at the front of the board had a division problem with one-digit divisor and three-digit dividend. The children solved their problems and returned to their seats. Then, each child in turn was asked to go to the front and using a pointer, explain his problem. It would go something like this: "My problem was to divide 7 into 296. Seven won't go into 2. Seven into 29 goes 4 times. Four times seven is 28. I draw my line and subtract. Eight from nine is 1. Two from two is 0. I bring down the 6. Seven goes into 16 twice. Two times 7 is 14. I draw my line and subtract. Four from 6 is 2. My problem was to divide 296 by 7. The quotient is 42 and the remainder is 2."

Off to the side of this problem one could see a string of computations including

$$2 \times 7 = 14$$
$$3 \times 7 = 21$$
$$4 \times 7 = 28$$
$$5 \times 7 = 35.$$

When all six of the children had completed giving descriptions of their problems with precisely the same ritual with each description, and only the numbers being changed, the lesson was considered to be complete. Therefore, in a half hour time, the entire class of fourth grade children examined 6 identical division problems, had all 6 of them explained in minute detail. Represented in that half hour were some aspects of what I might call a healthy learning situation, but also many aspects which I would call unhealthy. Generally, the situation would have to be called stultifying. But because the classroom is so complex, it is difficult to name specific steps that could be taken to improve the learning environment. Clearly, the teacher could have behaved differently. There could have been a greater variety of materials being used in the classroom. Certainly more of the children should have been actively involved than were.

This brings brings me to a proposal which I would like to make to the Board of SMSG. I would propose that SMSG engage in a research project involving a rather careful study of school mathematics classrooms, and through the study develop a concise description of what one might mean by a healthy mathematics classroom. In such a study, one would describe all of the kinds of teacher behavior which might be desirable in certain circumstances, and those kinds of teacher behavior which might be undesirable in certain circumstances. Related to this, one could describe various kinds of student behavior, various kinds of material that are available, and how the materials are used. The study could observe precisely what mathematical ideas exist in the classroom at any given time and how to introduce mathematical ideas to a classroom. One could study the quantity and quality of questions that are asked by students and by teachers, and the quantity and quality of answers given to questions. One could study the various ways to provide for individual skill development, and to record individual skill profiles for each child. I would see after a year of study, that the development of diagnostic teams that would go into a classroom, make particular observations and measurements, and be able to describe to the teacher and to others how that particular classroom can be improved in what particular way. I think after a year of study, one could be able to describe what a healthy mathematics classroom, then spend two years learning how to change unhealthy classrooms into healthy classrooms. I feel that teachers in classrooms today are as ripe for help in learning how to create a healthy mathematics environment as they were ready for the content changes in mathematics that were provided by SMSG ten years ago.

SUMMARY OF SMALL-GROUP DISCUSSIONS

A summary of the discussions in the small groups is presented below. Although these comments emanated separately from four groups, many common threads of thought appeared from group to group. Occasionally, however, there were contradictory remarks and recommendations even within the same group. In the editing, we attempt to exercise no judgment on the merits of each. We classify "representative" recommendations and discussions according to each of 3 specific questions directed to the attention of the groups:

1. What can SMSG do to improve mathematics education in inner city schools?
2. What teacher training is necessary for success in inner city schools and what can SMSG do about it?
3. What curriculum materials need to be developed for inner city schools?

In addition to proposing answers to these questions, some of the participants raised other related questions in the attempt to clarify the situations. Such questions are listed under the category most closely typifying each.

General Remarks

In the course of the discussions, some general remarks emerged. We display these first since they seem to characterize the attitude of most of the participants.

There was near unanimity in all groups that priority in inner city education should be given to preschool, kindergarten, and the early primary years. Likewise, almost all participants felt that little of value could be accomplished unless the program includes great emphasis on developing appropriate attitudes, insights, and understanding among teachers in inner city schools.

What Can SMSG Do?

Comments responding to the question, "What can SMSG do to improve mathematics education in inner city schools?" refer to general and specific recommendations for SMSG different from what it might do in regard to teacher training and construction of curriculum materials. Recommendations in response to these latter two areas are listed separately under the questions on "Teacher Training" and on "Curriculum Materials".

1. SMSG might prepare packets of materials for classroom use consisting of a variety of articles such as booklets, worksheets, and toys along with its mathematics program.

2. SMSG might prepare parent-student packets of materials for use in the home. Such packets would include worksheets, toys, games, and directions to parents for directing children in learning activities at home.

3. SMSG might attempt to collect and disseminate nationwide, descriptions of outstanding practices and materials for inner city schools.

4. SMSG might join in efforts to produce more TV programs similar to Sesame Street for inner city children. It might offer assistance on a consultancy basis to such program series to ensure undistorted mathematical interpretations.

5. SMSG might develop tests for use in inner city schools in order to help evaluate accurately the contemporary program. Oral, nonverbal, and culture-free aspects should be considered carefully.

6. SMSG might develop liaison with some inner city school districts on a long-range basis. These locales would furnish a base for trial of materials and instructional techniques. The relationship should be of such a nature that SMSG would be responsible for the entire mathematics program in a K-12 system or subsystem.

   [In this connection, the following question was posed: "If entire school systems could be used as a basis, what factors must or could be manipulated to bring about increased learning in mathematics?]

7. SMSG might lend its prestige as a nationally recognized body to encourage experiments in the schools and by doing so, attract many better teachers to such experimental situations.

8. SMSG might interest itself in additional research in the following regards:

   (a) In an inner city setting, how does one differentiate between the ability-hindered slow learner and the low achiever whose lack of achievement is primarily caused by economic, cultural, and educational disadvantage?

   (b) What are the characteristics that an effective inner city teacher must have?

Teacher Training

In this category are responses to the question, "What teacher training is necessary for success in inner city schools and what can SMSG do about it?" General remarks, not necessarily suggestions and recommendations, are included in this summary as well as several questions raised in the midst of the discussions.

1. SMSG might develop or join in developing model inservice or preservice courses for teachers in inner city schools.

2. SMSG might set up teacher training centers similar to those in the Nuffield Project. Special training in methods and materials for use in inner city schools could be emphasized.

3. SMSG might develop training material to be used for voluntary tutors, paraprofessionals, and parents. These should be coupled with instructional materials these persons might use with children.

4. SMSG might identify different strategies of teaching mathematics to the inner city child.

5. SMSG might develop behavioral objectives for teacher inservice courses.

6. SMSG might provide a diary on how material was successfully presented to inner city children by a selected teacher.

7. SMSG might demonstrate micro-lessons on video tapes as guides for teachers, illustrating good techniques in inner city schools and special personality aspects of teaching for these schools.

Related questions proposed were: "What is a good classroom atmosphere? How can it be produced?" These questions were left unanswered.

In addition to the specific suggestions, a few general commentaries in connection with teacher preparation and teacher attitudes were offered:

(a) At the secondary level, the problem is that the teacher has preconceived notions of what the inner city students are like; for example, a prevalent assumption seems to be that the inner city students don't want to learn.

(b) There is considerable transiency in teaching staff of inner city schools. Consequently, any program of teacher training for these schools should recognize this situation and be geared to training new teachers each year.

Curriculum Materials

This part of the summary relates to the question, "What curriculum materials need to be developed for inner city schools?" Recommendations refer to textual materials for the general program as well as ancillary materials including use of multi-media. The discussions involved a variety of aspects of curriculum materials such as format, content, approach, and philosophy.

1. Materials that would promote individualization of instructions are needed. Current SMSG elementary text materials might be revised into short topical units.

2. A variety of short topical units should be developed to accommodate several approaches to the same topic without regard for mathematical structure. Explicit information regarding sequence and how each unit might be used in a classroom should be written for the teacher.

3. SMSG might develop materials suitable for heterogeneous groupings.

4. SMSG might pursue the same kind of program it had done for the disadvantaged elementary student. The trials of these materials which involved considerable teacher training were quite successful.

5. Geometry, probability, and statistics are relatively nonverbal areas of mathematics that might be exploited in individual learning packets for students.

6. The complexity, relevance, and sequence of computation must be carefully examined and taken into account when curriculum material is written.

7. A series of short tapes, films, and packages might be developed for use in inner city schools.

8. SMSG might use as examples, mathematics that are involved in games children play.

9. We need motivation; delayed rewards are not effective in the inner city school class.

10. SMSG should plan to use existing SMSG materials together with materials it might develop expressly for the inner city school child.

Ambivalence with Item 1, to ignore structure, is noted in a recommendation that SMSG should be concerned that materials for the inner city school child reflect mathematical structure just as much as in the standard materials.

Finally, in direct opposition to Item 4, a critical, philosophical, comment expostulated: "The mathematics community does not understand how to motivate with immediate rewards for this population."

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