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ABSTRACT

This paper presents the conceptual development and application of a new interactive approach for multicriterion optimization to the aggregate operating problem of an academic department. This approach provides a mechanism for assisting an administrator in determing resource allocation decisions and only requires local trade-off and preference information about his objectives and values. This interactive approach is described in the context of a specific mathematical programming algorithm (the Frank-Wolfe method). The mathematical model of the operations of an academic department is then detailed. A numerical example of the use of this model coupled with the interaction procedure is provided. This example is taken from actual experience with a pilot implementation of the approach for the Graduate School of Management at UCLA, where the authors are attempting to install it as a permanent decisionmaking model. The authors conclude that the approach used here will permit the successful treatment of many other problems in higher education not previously considered amenable to solution via mathematical programming due to the multiciplicity of criteria. (Author)

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# ACADEMIC DEPARTMENTAL MANAGEMENT: AN APPLICATION OF AN INTERACTIVE MULTI-CRITERION OPTIMIZATION APPROACH

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# PREFACE

This is one of a continuing series of reports of the Ford Foundation sponsored Research Program in University Administration at the University of California, Berkeley. The guiding purpose of this Program is to undertake quantitative research which will assist university administrators and other individuals seriously concerned with the management of university systems both to understand the basic functions of their complex systems and to utilize effectively the tools of modern management in the allocation of educational resources.

This paper presents the conceptual development and application of a new interactive approach for multi-criterion optimization to the aggregate operating problem of an academic department. This approach provides a mechanism for assisting an administrator in determining resource allocation decisions in an ever-improving sequence, and only requires local trade-off and preference information about his objectives and values.

# I. INTRODUCTION

One of the most common difficulties obstructing the successful application of mathematical programming techniques to the problems of higher education is the presence of multiple criteria. Incommensurate and often conflicting criteria arise at all levels of a system of higher education. The hierarchy of related problems of determining the allocation of a state's educational resources among types of institutions (e.g., universities, four-year colleges, and junior colleges), the allocation of university resources among several campuses, and the allocation of campus resources among academic departments each involve multiple decision-relevant criteria. This same condition exists in the problem we have chosen for study, the allocation of faculty resources to the various activities performed within a single academic department on a university campus. We believe that the approach taken in this paper has potential for application to many problems of higher education previously considered too "ill-structured" for treatment by mathematical programming.

Several prescriptions have been offered for the problem of allocating resources at the campus or departmental level. Among these are the deterministic simulation models of Judy [7] and of the National Center for Higher Education Management Systems at WICHE [11]. These models provide answers to "what if" questions posed by an administrator by translating particular resource allocations and policy decisions into a resulting set of operating characteristics (criteria). However, they provide no mechanism for assisting the administrator in determining decisions which <u>improve</u> the values of these criteria.

Mathematical programming has been suggested previously as a means of providing this improvement mechanism for problems in higher education (e.g., [1]). A comparative discussion of several such approaches is included in the survey by Weathersby and Weinstein [10]. However, the available approaches either tend to ignore or oversimplify the multi-criterion aspect of the problem, or deal with it in an ad hoc manner. This is not surprising, for until now the available methods for optimizing in the presence of multiple criteria have almost all been ad hoc [6,9].

One of the present authors recently proposed [2] a man-machine interactive mathematical programming approach which is applicable to a broad class of problems with multiple decision criteria. It assumes that a large-step gradient ascent algorithm would be applicable if the decision-maker were somehow able to specify an overall "preference function" to resolve the conflicts inherent in the given multiple criteria, but never actually requires this preference function to be identified explictly. Instead, the algorithm calls only for such local information about the preference function as is actually needed to carry out the optimizing calculations. In other words, the viewpoint taken is this: adopt a mathematical programming technique of known efficiency, but implement it so as to require minimal information from the decision-maker concerning his preferences over the criteria.

This interactive approach is described in the following section in the context of a specific mathematical programming algorithm (the Frank-Wolfe method). Section III then details our mathematical model of the operations of an academic department. A numerical example of the use of this model coupled with the interaction procedure is provided in Section IV. This example is taken from actual experience with a pilot implementation of the approach for the Graduate School of Management at UCLA, where we are attempting to install it as a permanent decision-making tool.

## II. AN INTERACTIVE MATHEMATICAL PROGRAMMING APPROACH TO MULTI-CRITERION OPTIMIZATION

This section briefly summarizes the interactive mathematical programming approach to the multi-criterion problem in the specific context of the well-known Frank-Wolfe algorithm (e.g., [13, pp. 158-162]). A more detailed discussion is presented in [3]. Although other algorithms could be rendered interactive in a similar manner, we have selected the Frank-Wolfe method because of its simplicity and appropriate theoretical properties [4,12].

The multi-criterion optimization problem is written as

$$\text{(1)} \qquad \text{Maximize } U[f_1(x),\ldots,f_r(x)] \quad \text{subject to } x \in X$$

where $f_1,\ldots,f_r$ are $r$ distinct criterion functions depending on the decision vector $x$ , $X$ is the set of feasible decisions, and $U$ is the decision-maker's preference function defined on the values of the criteria. We shall assume that $X \subseteq R^n$ is a compact, convex set,[1] and that the objective function of (1) is differentiable and concave on $X$ . The functions $f_i$ and the set $X$ must be explicitly known, but $U$ is <u>not</u>

[1] A set is convex if the line segment between any two points in the set is also in the set. A function defined on a convex set is said to be concave if linear interpolation never overestimates the value of the function. Two sufficient conditions for the concavity of the objective function of (1) are: (i) $U$ is concave and each $f_i$ is linear; (ii) $U$ is concave increasing and each $f_i$ is concave. The assumption of the concavity of $U$ seems reasonable since we expect the decision-maker to exhibit non-increasing marginal utility for additional units of each of the criteria (see [5, pp. 142-148]).

explicitly known (otherwise there would be no need for an interactive approach).

If $U$ were known, the Frank-Wolfe algorithm would be applied to (1) as follows:

Step 0. Choose an initial point $x_1 \in X$. Put $k = 1$.

Step 1. Determine an optimal solution $y_k$ of the direction finding problem[2]

$$\text{(2)} \qquad \underset{y \in X}{\text{Maximize}} \quad \nabla_x U[f_1(x_k),\ldots,f_r(x_k)] \cdot y \;.$$

Put $d_k = y_k - x_k$.

Step 2. Determine an optimal solution $t_k$ of the step-size problem

$$\text{(3)} \qquad \underset{0 \le t \le 1}{\text{Maximize}} \quad U[f_1(x_k+td_k),\ldots,f_r(x_k+td_k)] \;.$$

Put $x_{k+1} = x_k + t_k d_k$, $k = k+1$, and return to Step 1.

Since $U$ is not explicitly known, neither Step 1 nor Step 2 can be performed entirely by computer. However, as we now explain in more detail, the required information regarding $U$ may be obtained from the decision-maker as needed.

Step 1

The standard chain rule of calculus yields

$$\text{(4)} \qquad \nabla_x U[f_1(x_k),\ldots,f_r(x_k)] = \sum_{i=1}^{r} \left(\frac{\partial U}{\partial f_i}\right)^k \nabla_x f_i(x_k) \;,$$

where $(\partial U/\partial f_i)^k$ is the ith partial derivative of $U$ evaluated at the point $[f_1(x_k),\ldots,f_r(x_k)]$. Thus the (linear) objective function of (2) is incompletely known because the $(\partial U/\partial f_i)^k$'s are not known. Notice,

[2] We use $\nabla_x$ to denote the gradient with respect to $x$.

however, that the optimal solution $y_k$ of (2) is not affected by positive scaling of the objective function. Hence, one may divide the objective function by any positive coefficient $(\partial U/\partial f_i)^k$ . The criterion which plays this distinguished role will be called the reference criterion.[3] We may assume without loss of generality that the reference criterion is the first one, so that (2) is equivalent to

$$\text{(2')} \qquad \underset{y \in X}{\text{Maximize}} \sum_{i=1}^{r} w_i^k \nabla_x f_i(x_k) \cdot y$$

where we define

$$\text{(5)} \qquad w_i^k \triangleq (\partial U/\partial f_i)^k \,/\, (\partial U/\partial f_1)^k \;, \; i = 1,\ldots,r \;.$$

In the important case where all constraints are linear, (2') is a linear programming problem.

Each "weight" $w_i^k$ is the marginal rate of substitution or indifference tradeoff between each $f_i$ and $f_1$ at $x_k$ . The decision-maker can approximate these values by specifying what (small) change $\Delta_1$ in the first criterion "exactly compensates" for a change $\Delta_i$ in the ith criterion, with all other criteria remaining at their current values. Then we have approximately

$$\text{(6)} \qquad w_i^k = -\Delta_1 \,/\, \Delta_i$$

and the approximation becomes arbitrarily exact as the amounts of change approach 0 .

[3] Since the coefficients $(\partial U/\partial f_i)^k$ will usually be positive in practice, this is the case considered here. It is also possible to use a criterion with a negative coefficient as a reference criterion, in which case one would obviously divide through by the negative of this coefficient.

Alternatively, the decision-maker may be asked to identify the "ideal" marginal proportion of change for two criteria. With all criteria except the first and ith held constant, if U increases most rapidly in the incremental sense when criterion i changes by $\delta_i$ units for each $\delta_1$ change in criterion 1, then

(7) $$w_i^k = \delta_i \ / \ \delta_1 \ .$$

Both of these approaches to the determination of the tradeoff weights are justified in detail in [3], and will be illustrated in Section IV.

Step 2

Since U is not explicitly available, (3) must be solved directly by the decision-maker. This is not as difficult as it may appear because there is only one variable, t . Plots can be made of the values of all r criteria $f_i(x_k+td_k)$ as a function of t between 0 and 1 (this is done in the example presented in Section IV), or the computer can tabulate the values of the criteria at selected values of t .

We stress that what is required of the decision-maker is much easier than the comprehension of an arbitrary set of choices in r-space, which would indeed be a hopeless task. He need only comprehend a singly parameterized curve in r-space, an object which he can see directly in component-wise parametric form. Of course, this is not to deny that the task of carrying out Step 2 becomes more difficult as criteria become more numerous, but available evidence suggests that one can cope with at least 6-8 criteria without undue difficulty.

## The Interactive Frank-Wolfe Algorithm

Thus we see that the Frank-Wolfe algorithm for (1) can be executed interactively with the decision-maker as follows:

Step 0. The decision-maker chooses an initial point $x_1 \in X$. Put $k = 1$.

Step 1.

a. The decision-maker assesses his tradeoff weights $w_i^k$ by subjective analysis of the current trial point via the relations (6) or (7).

b. Compute an optimal solution $y_k$ of (2'). Put $d_k = y_k - x_k$.

Step 2. Plot or tabulate the functions $f_i(x_k+td_k)$ over the unit interval, and have the decision-maker subjectively determine an optimal solution $t_k$ to problem (3). Put $x_{k+1} = x_k + t_k d_k$, $k = k+1$, and return to Step 1.

It is worth noting that, except perhaps for Step 0, the entire procedure can be viewed by the decision-maker as taking place in criterion space rather than in decision-variable space (a space that is usually of much higher dimension). This assumes, of course, that the decision-maker relegates Step 1b and the mechanical portions of Step 2 to a technical assistant or an interactive computer program. The advantage of functioning entirely in criterion space is that it allows the decision-maker to concentrate directly on making tradeoff judgments, with no extraneous details to distract him.

# III. THE DEPARTMENTAL MODEL

We now describe the departmental model developed for implementation in the Graduate School of Management at UCLA. Since some of the terminology is unique to the University of California System, a Glossary of terms has been included at the end of the paper.

## Purpose and Criteria of the Model

The faculty of an academic department are viewed as engaging in three principal activities: formal teaching, departmental service duties (e.g., major administrative responsibilities and curriculum reform), and "other" tasks such as research, student counseling, committee work, and minor administrative duties. Formal teaching takes place at several graduate and undergraduate levels. We shall be concerned primarily with the allocation of faculty effort among these activities given an exogenous budget for personnel in terms of Full Time Equivalent (FTE) positions.

The use of nonacademic personnel or other resources, such as supplies and expenses, are not considered in this model. Neither do we address the more "micro" operating problems related to course scheduling or to the assignment of faculty and students to particular sections and classrooms.

The teaching load of a full-time lecturer in the Graduate School of Management, with no other assigned duties, is nine sections per academic year. Similarly, the time of a regular faculty member is considered as being divided into nine equal parts; normally five of these correspond to the teaching of five sections per academic year, and the remaining four are spent on "other " tasks such as those mentioned above. Consequently, it is natural to account for faculty effort in terms of "equivalent course sections." One faculty FTE therefore equals nine equivalent sections. Expressing all activities in related or equivalent units is by no means required by our procedure, but it does facilitate the assessment of trade-off weights.

The first four criteria of the model are the number of sections offered by the department at the advanced graduate, professional (graduate), upper division undergraduate, and lower division undergraduate levels. Advanced graduate courses are designed primarily for Masters of Science or Ph.D. students, while professional courses are for Masters of Business Administration students. Since the research emphasis and possible seminar format of the former dictates the need for smaller class sizes (and relatively more resources) than the latter, it is appropriate to differentiate among the offerings of the department at the graduate level. Criterion five is the number of equivalent sections of teaching assistant time used for the support of classroom instruction by the faculty, instead of for teaching additional sections.

Faculty members are compensated for departmental service duties by teaching releases. The sixth criterion, measured in terms of these releases, represents the allocation of the faculty to this activity. Finally, criterion seven is the regular faculty effort devoted to tasks other than teaching or formal departmental service, again measured in equivalent sections.

It should be noted that this model focuses on "inputs" to the educational process instead of "outputs." Although some work has been done to define the outputs of higher education and relate them to educational resources [8], we have restricted our initial efforts to this simpler formulation. When quantifiable output measures become available, they may be adopted as criteria within the present framework.

## Detailed Formulation of the Model

The determination of the appropriate level of detail is an important consideration in the development of a mathematical model. We have found that the use of four course levels, five student levels, and five types of faculty provides sufficient information for the aggregate operating plans of the Graduate School of Management. Course levels $j = 1, 2, 3, 4$ correspond to advanced graduate, professional, upper division undergraduate, and lower division undergraduate, respectively. Similarly, we let $\ell = 1, 2, 3, 4, 5$ correspond to Ph.D., Masters of Science, Masters of Business Administration, upper division undergraduate, and lower division undergraduate students. Finally, we have faculty of type $k$ ($k = 1, 2,3,4,5$), where types 1 and 2 represent tenured and non-tenured regular faculty, type 3 represents teaching assistants, and types 4 and 5 represent lecturers and senior lecturers.

The reader is again referred to the Glossary for definitions of specialized terms.

## Variables Under the Control of the Department

The departmental decision variables included in the model are the following:

$x_{1j}$ = the number of sections offered at course level $j$.

$x_{2k}$ = the number of regular FTE faculty of type $k$ hired beyond the department's contractual commitments.

$x_{3k}$ = the number of equivalent FTE of type $k$ released from teaching for departmental service duties ($x_{33}$ is the number of teaching assistant FTE used for the support of instruction by the regular faculty).

$x_4$ = the number of irregular academic FTE hired with unused FTE and releases.

All of these variables are required to be non-negative.

## Parameters Determined by the Campus Administration

The following departmental parameters are determined exogenously by the campus administration:

$y_{1k}$ = the academic FTE faculty of type $k$ allocated to the department, and

$y^i_{2\ell}$ = the enrollment at student level $\ell$ in department $i$ .

The actual limitations imposed by the campus administration do not differentiate between the two types of masters students ($\ell = 2$ and 3) , but the additional detail is useful for planning purposes. If enrollment levels are not set by the campus administration, they may be treated as additional decision variables and criterion functions in the model.

## Constraints

The constraints fall into two categories. Some are technical necessities (such as work balance equations for faculty time, or resource constraints), while others are discretionary reflections of departmental customs and policies. One of the important side benefits of the modeling process is that these policies and customs come under close scrutiny when they are explicitly formulated.

The total number of sections taught in the department per academic year is determined by the "work conservation" equation

$$(8)\qquad \sum_{j=1}^{4} x_{1j} = \sum_{k=1}^{3} [t_k(c_k+x_{2k}-a_k-s_k)-g_k(x_{3k}+r_k)] + \sum_{k=4}^{5} [t_k(c_k+x_{2k})]+g_4x_4$$

where

$t_k$ = the customary number of sections taught per academic year by a faculty member of type $k$ ,

$c_k$ = the estimated number of faculty FTE of type $k$ to whom the department is contractually committed for the planning period,

$a_k$ = the estimated number (in FTE) of leaves of absence without salary by faculty of type $k$ ,

$s_k$ = the estimated number (in FTE) of faculty of type $k$ taking sabbatical leave,

$r_k$ = the estimated faculty time (in FTE) of type $k$ released from teaching due to support from outside the department (e.g., extramural research support), and

$g_k$ = the number of equivalent sections corresponding to a full workload for an FTE faculty of type $k$ .

Relation (8) asserts that each faculty member teaches a full course load unless specifically released.[4]

The hiring of irregular faculty is limited by the available FTE generated by vacancies, leaves, and support from outside the department. The resulting constraint may be written

$$(9)\qquad x_4 \leq \sum_{k=1}^{5} (y_{1k}-c_k-x_{2k}+a_k+r_k)$$

---

[4]A simple modification can be made if desired to account for the fact that the actual number of sections offered may be slightly less due to last minute cancellations and other factors.

The constraints

$$(10) \qquad x_{2k} \leq y_{1k} - c_k \ , \ k=1,2,3,4,5$$

place an upper limit on the number of faculty hired;

$$(11) \qquad g_k(r_k + x_{3k}) \leq t_k(c_k + x_{2k} - a_k - s_k) \ , \ k=1,2,3$$

provide that teaching releases of regular faculty cannot exceed the total teaching obligations; and

$$(12) \qquad t_3(c_3 + x_{23} - x_{33}) \leq x_{14}$$

restricts the formal teaching by teaching assistants to lower division undergraduate classes.

Finally, there are the constraints which reflect policies, customs, and commitments of the department. We require that

$$(13) \qquad x_{1j} \geq m_{1j} \ , \ j=1,2,3,4$$

where $m_{1j}$ is the minimum number of sections which must be offered at course level $j$ in order to meet departmental commitments.
Also,

$$(14) \qquad \sum_{k=1}^{2} x_{3k} \geq m_2$$

where $m_2$ is the minimum number of releases required for essential departmental service duties. Another important policy decision is reflected by

$$(15) \qquad m_{3j}x_{1j} \leq \frac{\sum_{\ell=1}^{5} \sum_{i} y_{2\ell}^{i} \, p_{j\ell}^{i}}{h_j} \leq m_{4j}x_{1j} \ , \ j=1,2,3,4$$

which places limits on the average class size[5] at each course level, where

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[5]The average class size interpretation is evident upon dividing (15) by $x_{1j}$ .

$m_{3j}$ = desired lower bound on average class size at course level $j$ ,

$m_{4j}$ = desired upper bound on average class size at course level $j$ ,

$h_j$ = average number of student credit hours per student-section for courses given at course level $j$ , and

$p^i_{j\ell}$ = the preference by students enrolled in department $i$ at student level $\ell$ for courses offered by the department at course level $j$ , measured in student credit hours per student for an academic year (obtained by sampling from student records).

The second summation (over i) in (15) ranges over all departments in the university.

## Departmental Criterion Functions

The criterion functions used in the model are generally deemed individually desirable commodities from the departmental viewpoint, although this need not always be the case. The first four criterion functions,

(16) $$f_j = x_{1j} \ , \ j=1,2,3,4 \ ,$$

represent the number of sections offered by the department at the advanced graduate, professional, upper division undergraduate, and lower division undergraduate course levels, respectively. The fifth criterion

(17) $$f_5 = g_3 x_{33} \ ,$$

is the number of equivalent sections of teaching assistant time used for the support of instruction by other faculty. Criterion six,

(18) $$f_6 = \sum_{k=1}^{2} g_k x_{3k}$$

is the number of teaching releases granted to the regular faculty for departmental service duties. Finally, the seventh criterion,

(19) $$f_7 = \sum_{k=1}^{2} (c_k+x_{2k}-s_k-a_k)(g_k-t_k) ,$$

represents the balance of the regular faculty's time (measured in equivalent sections) devoted to areas other than classroom instruction exclusive of releases from formal teaching duties.

## IV. A NUMERICAL EXAMPLE

This example simulates the use of the model and the interactive optimization procedure to develop aggregate annual operating plans for the Department of Management at UCLA. Such planning may occur during the winter or spring quarter for the following academic year. Forecasts or estimates of the various coefficients included in the model must be provided. In addition, estimates must be made of the anticipated allocation of academic FTE from the central administration and of the restrictions on student enrollments. The model also requires certain policy parameters to be set. The results of the initial runs of the model may, of course, suggest desirable modifications of these policies.

### Initialization (Step 0)

The only requirement regarding the initial operating point is that it be feasible with respect to the constraints of the model. An obvious candidate for this initial point is the (perhaps adjusted) operating point of the Department for the current academic year. This strategy was used to obtain the result presented in Table 1.

TABLE 1

| Criterion | | Initial Value in (Equivalent) Sections |
|---|---|---|
| sections offered - advanced graduate | ($f_1$) | 206 |
| sections offered - professional | ($f_2$) | 103 |
| sections offered - upper division | ($f_3$) | 68 |
| sections offered - lower division | ($f_4$) | 15 |
| teaching assistant time used for support | ($f_5$) | 44 |
| releases for departmental service duties | ($f_6$) | 40 |
| additional activities of the regular faculty | ($f_7$) | 267 |

## Estimation of Tradeoffs (Step 1)

The next step requires the decision-maker to provide the tradeoff weights associated with each of the criteria. The numerical tradeoff weights and step sizes presented in this example were determined during the actual use of the procedure by an administrator, but the discussion below regarding their rationale is purely illustrative. We select $f_1$, the number of advanced graduate sections offered, as the reference criterion. By convention, its tradeoff weight $(w_1)$ is unity.

We now wish to obtain a tradeoff weight for $f_2$, the number of professional sections offered, versus $f_1$. This weight will be estimated by considering the "ideal" proportional change. The ratio of $f_2$ to $f_1$ at the current operating point is $103/206 = .5$. In other words, 1 professional section is offered for every 2 advanced graduate sections. Suppose the administrator feels that the professional offerings of the department should be given increased emphasis relative to the advanced graduate offerings. Then he may feel that an ideal proportional change from the current values for these two criteria is more like 6 professional sections for every 5 advanced graduate sections, or

$$w_2 = \delta_2/\delta_1 = 6/5 = 1.2 .$$

Similar reasoning relating $f_3$ and $f_4$ to $f_1$ led to $w_3 = 0.2$ and $w_4 = 0.4$.

To obtain $w_6$, the administrator must state his preference for changes in the number of advanced graduate sections versus changes in the number of course releases for departmental service duties. He realizes, of course, that the granting of one release reduces the total number of sections offered by one. However, he may feel that the loss of 3 releases for departmental service from the current number of 40 would be just offset, in terms of his preference, by a gain of 3.5 advanced graduate sections. In other words, an increase of 3.5 additional units of $f_1$ would exactly compensate for a loss of 3 units of $f_6$, so that

$$w_6 = -(\Delta_1/\Delta_6) = -(3.5/-3) = 1.17 .$$

These examples have illustrated both approaches to the estimation of tradeoff weights. The "ideal proportional change" approach [see (7)] was illustrated by the determination of $w_2$, and the "indifference" approach was illustrated by the estimation of $w_6$ [see (6)]. To obtain $w_5$, we shall illustrate a tactic useful when it is difficult to relate a criterion to the reference criterion, but relatively easy to relate it to some other criterion. In this case, $f_5$ is related to $f_7$, which in turn is related to $f_1$.

Suppose the administrator has difficulty in trading off teaching assistant support time $(f_5)$ against the number of advanced graduate sections $(f_1)$. However, he judges that teaching assistant support for 4 sections taught by regular faculty would be worth 1 equivalent course section of a regular faculty member's time devoted to "other" tasks. It follows that $-(\Delta_7/\Delta_5) = 1/4$ estimates $(\partial U/\partial f_5)/(\partial U/\partial f_7)$ (it would serve as the weight for $f_5$ <u>if</u> $f_7$ were the reference criterion). However, since $w_7 = (\partial U/\partial f_7)/(\partial U/\partial f_1)$, $w_5$ can be obtained from the relation

$$w_5 = \frac{\left(\frac{\partial U}{\partial f_5}\right)}{\left(\frac{\partial U}{\partial f_7}\right)} \cdot \frac{\left(\frac{\partial U}{\partial f_7}\right)}{\left(\frac{\partial U}{\partial f_1}\right)} = -(\Delta_7/\Delta_5)w_7 = 1/4\ w_7 .$$

Only $w_7$ remains to be estimated.

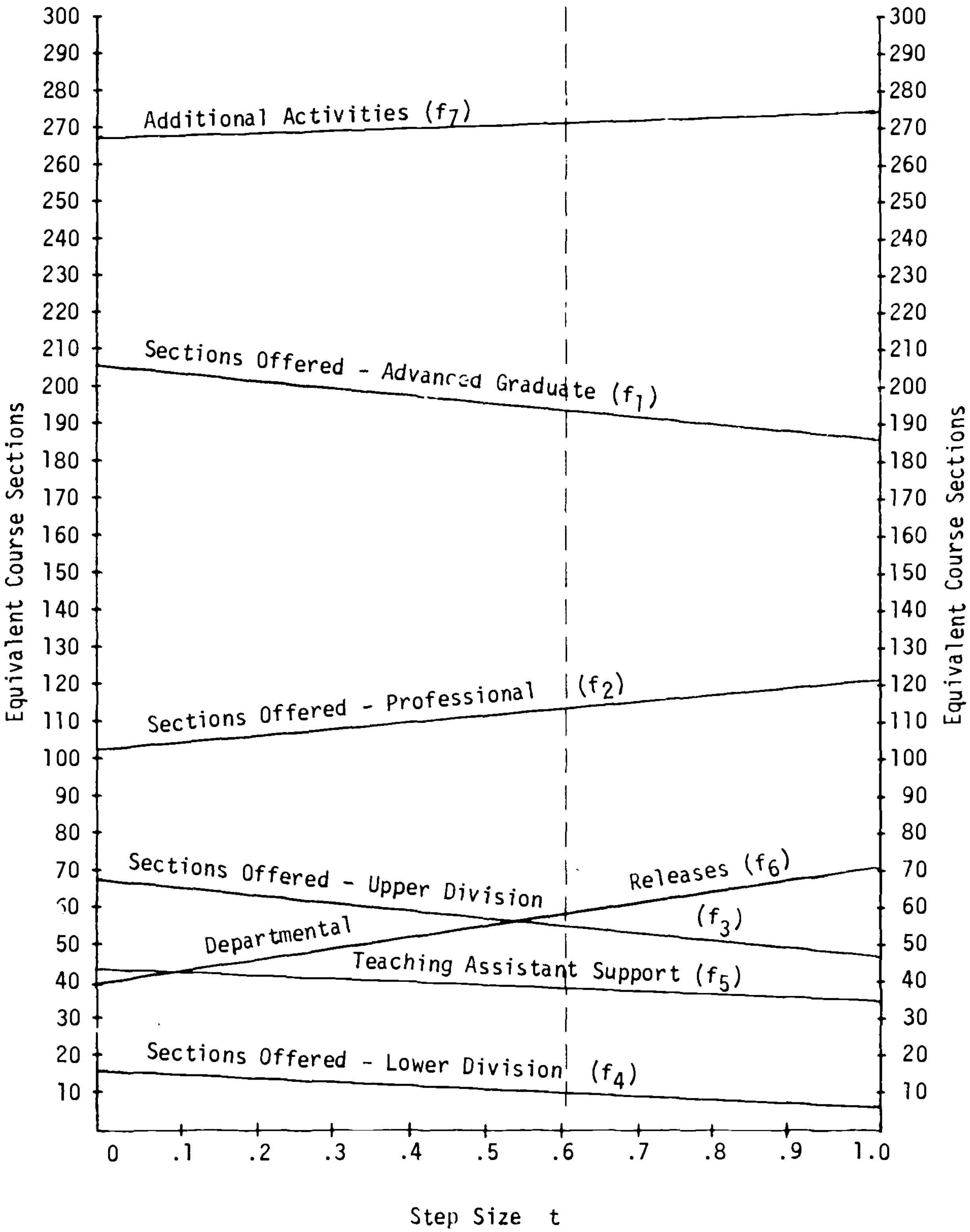
Suppose the administrator feels that time for "other" tasks (research, committee work, counseling, etc.) by the faculty is particularly important. He may feel that the ideal proportional change is an increase of 2 section equivalents in $f_7$ for every additional graduate section. Then, $w_7 = \delta_7/\delta_1 = 2/1 = 2.0$, and consequently $w_5 = (.25)(2.0) = 0.5$.

## Step-Size Determination (Step 2)

These tradeoff weights are used to compute a new feasible operating point by solving the optimization problem (2'). As described in Section II, we can now plot the values of all $r$ criteria as a function of $t$. Since all of our criteria are measured in similar units, it is natural to superimpose their plots on the same graph as shown in Figure 1. The values of the criteria at $t = 0$ are the initial operating point of Table 1, and the corresponding values for $t = 1$ are the solutions to (2'). Since all of our criteria happen to be linear (see Section III), the plots of their values are line segments.

To complete an iteration of the procedure, the decision-maker must determine a value of $t$ for which the corresponding values of the criteria are most preferred. A vertical line may be visualized as superimposed on Figure 1, intersecting the plots of the criteria and the $t$ axis; by shifting this line from left to right, the decision-maker may visualize all of the feasible solutions to this restricted one-dimensional optimization problem. In this particular example, the administrator selected $t = .6$ (the dashed line in Figure 1). The corresponding criteria values become his revised operating point, and he is ready to perform another iteration.

FIGURE 1. Step 2 of Iteration 1



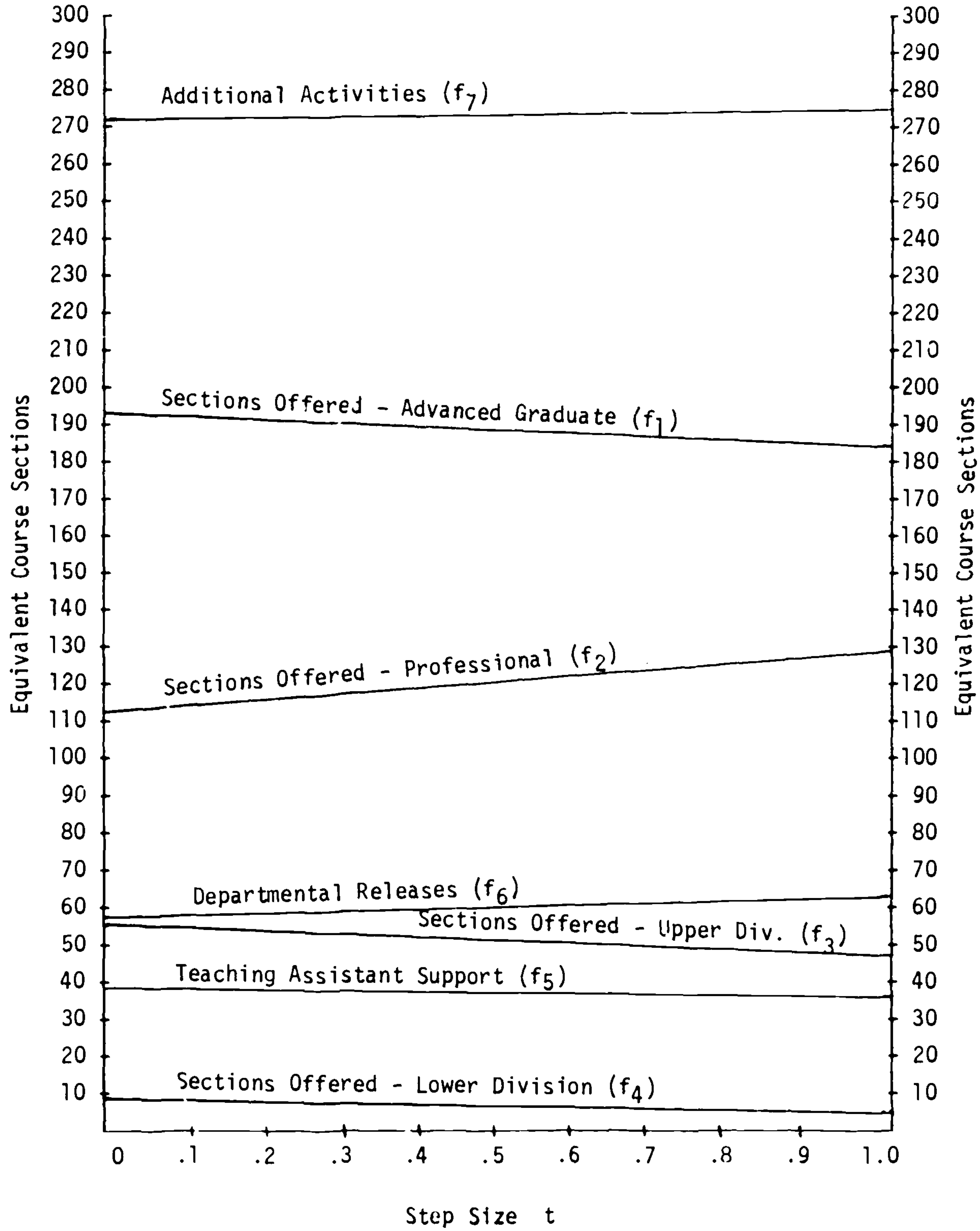
## Further Iterations of the Procedure

After considering the point selected in Step 2 of the first iteration, the administrator revised two of his tradeoff weights. He reduced $w_6$, the tradeoff weight for releases for departmental service duties, from 1.17 to .80, and decreased $w_2$, the tradeoff weight for sections of professional courses, from 1.2 to 1.15 . He felt that his relative preferences for the other criteria remained unchanged at this new point.

The revised weights and new operating point were used to compute the results presented in Figure 2. To complete the second iteration, the decision-maker selected $t = 1.0$ as his preferred solution to the one-dimensional optimization problem (Step 2).

For the third iteration, the administrator modified only $w_6$ from 0.80 to 0.90 . The new operating point selected at the conclusion of the second iteration is an extreme point solution to (2'). A post-optimality analysis of the direction-finding linear program indicated that the solution would be unchanged for the new value of $w_6$ . Since no improvement would occur in the third iteration, the procedure was terminated.

FIGURE 2. Step 2 of Iteration 2



## V. CONCLUSIONS

We have presented an example of the application of a new interactive approach for multi-criterion optimization to the aggregate operating problem of an academic department. In our efforts to install this procedure as a tool for the Graduate School of Management at UCLA, we have worked successfully with as many as seven levels of courses (five at the graduate level) and a total of ten criteria. In all cases, our experimental use of the procedure has indicated that decision-makers can provide the required information without significant difficulty.

We believe that the approach used here will permit the successful treatment of many other problems in higher education not previously considered amenable to solution via mathematical programming due to the multiplicity of criteria.

# GLOSSARY

Faculty - the various faculty types are broken down as follows:

Ladder Faculty - (hired for teaching, research, administration, and other duties)

| | |
|---|---|
| Professors<br>Associate Professors | (tenured; k = 1) |
| Assistant Professors<br>Instructors | (non-tenured; k = 2) |

Non-Ladder Faculty - (hired for teaching only; all non-tenured)

| | |
|---|---|
| Teaching Assistants[6] | (k = 3) |
| Lecturers[7] | (k = 4) |
| Senior Lecturers[7] | (k = 5) |

Full Time Equivalent (FTE) Faculty - a person or persons performing the academic duties of one full-time faculty member.

Regular vs. Irregular Faculty - a regular faculty member is employed on a continuing basis for one year or more. An irregular faculty member is a senior lecturer or lecturer employed on a short-term basis (often part-time) with unused faculty FTE positions.

Course Levels:

Advanced Graduate (j = 1) - graduate courses numbered 200-299 in the UCLA catalog. These classes generally have small average class sizes (10-25) and are taken by Ph.D. and Masters of Science students.

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[6]Teaching assistants are advanced graduate students employed to assist other faculty with instruction or to teach courses themselves.

[7]May be granted "security of employment," which is tantamount to tenure.

Professional ($j = 2$) - graduate courses numbered 400-499 in the UCLA catalog. The average class size generally ranges from 30 to 45, with the enrollment composed primarily of Masters of Business Administration students.

Upper Division Undergraduate ($j = 3$) - courses numbered 100-199 and 300-399 (teacher-training) in the UCLA catalog.

Lower Division Undergraduate ($j = 4$) - courses numbered 1-99 in the UCLA catalog.

Student Credit Hours -- the number of academic units which accrue to a student from the successful completion of a course.

Student Levels:

Ph.D. ($\ell = 1$) - graduate students enrolled in a Ph.D. program.

Masters of Science ($\ell = 2$) - graduate students enrolled in a Masters of Science program.

Masters of Business Administration ($\ell = 3$) - graduate students enrolled in the Masters of Business Administration program.[8]

Upper Division Undergraduate ($\ell = 4$) - undergraduate students with at least 84 quarter units completed.

Lower Division Undergraduate ($\ell = 5$) - undergraduate students with less than 84 quarter units completed.

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[8] No distinction is made among masters degree students from other departments.

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