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ABSTRACT

This study investigates the resource allocation problem of faculty hiring and promotion patterns using the techniques of optimal control theory. The mathematical structure of an academic faculty is described by a linear dynamic model whose parameters were estimated from actual data by two different techniques. The principal characteristics of the faculty system considered are: (1) linear system propagation; (2) a convex preference function to rank the relative values of varying the states of the system; and (3) four state variables and four control variables including the stocks and flows of (a) full professors, (b) associate professors, (c) assistant professors, and (d) instructors. The specific approach adopted for this investigation assumes that the promotion policies and attrition rates of faculty members are relatively fixed over the short run and the only variables left open to achieve a desired faculty structure are the institutional hiring policies. Under these conditions, the optimal open loop faculty hiring paths are calculated and their sensitivity is investigated. The study concludes by investigating and evaluating several solution procedures. (Author)

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A CONTROL THEORY SOLUTION TO OPTIMAL FACULTY STAFFING

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PREFACE

This is one of a continuing series of reports of the Ford Foundation sponsored Research Program in University Administration at the University of California, Berkeley. The guiding purpose of this Program is to undertake quantitative research which will assist university administrators and other individuals seriously concerned with the management of university systems both to understand the basic functions of their complex systems and to utilize effectively the tools of modern management in the allocation of educational resources.

This study investigates the resource allocation problem of faculty hiring and promotion patterns using the techniques of optimal control theory. If is both an extension and a synthesis of the conceptual analysis of faculty structure introduced in earlier papers in this series. The principal characteristics of the faculty system considered are: (1) linear system propagation; (2) a convex preference function to rank the relative values of varying the states of the system; and (3) four state variables and four control variables including the stocks and flows of (a) full professors, (b) associate professors, (c) assistant professors, and (d) instructors. The specific approach adopted for this investigation assumes that the promotion policies and attrition rates of faculty members are relatively fixed over the short run and the only variables left open to achieve a desired faculty structure are the institutional hiring policies. Under these conditions, the optimal open loop faculty hiring paths are calculated and their sensitivity is investigated. Finally, this study investigates and evaluates several solution procedures.

This paper was presented at the Eleventh American Meeting of The Institute of Management Science, October 19-21, 1970.

I. INTRODUCTION

The analytical techniques of optimal resource allocation have been applied to decision making in general and in institutions of higher education in particular.[1] One area that is both financially dominant in the public sector and academically essential is the educational administrator's need to decide on the most desirable pattern of faculty hiring and promotion. On a national basis, faculty salaries are the most expensive component of an institution's budget, accounting for $5.5 billion in 1968-69,[2] and the number of faculty positions is often fixed by law or slowly evolves under tenure restrictions. Furthermore, the quality of the faculty is essential because they set the tenor of the institution and participate in many of the academic and administrative decisions.

Some aspects of this problem were discussed in a recent paper concerning equilibrium faculty patterns resulting from current in   :utional appointment and promotion policies.[3] Using the Berkeley campus of the University of California as an example, it was shown that the logical extrapolation of the current data would result in no feasible equilibria for current policies.[4] More generally, present policies have resulted in faculties more heavily concentrated in tenure ranks than is often desired. Furthermore, under present policies this situation will tend to worsen over time because of the current youthful faculty age distribution reflecting

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[1] For an extensive discussion of this research area, see Weathersby [16].

[2] Howard R. Bowen, "Financial Needs of the Campus," in Robert H. Connery [3].

[3] Oliver [9]. See also: Bartholomew [1], Halpern [4], Reisman [10], and Reisman and Taft [11].

[4] In this context, feasibility was defined as those regions which fulfilled certain tenure/non-tenure proportional constraints as well as the enforced dynamics of the system under current appointment and promotion policies.

the rapid faculty expansion of the past decade. Therefore, one major problem of the educational administrator is to allocate his open faculty positions wisely by choosing hiring and promotion policies which fulfill his long-term goals of faculty rank distribution while meeting his budgetary or other resource constraints.

One approach to the analysis of this problem is for a campus or system level administrator to assume that in the short run the promotion policies and attrition rates of faculty members are both relatively fixed and beyond his immediate control. While it is true that over time an administrator can affect promotion and attrition rates by policy changes and financial incentives, often the only control variables available to the campus decision maker to help him achieve a desired faculty structure in the short run are the institutional hiring policies. The scarce resource constraint can be interpreted as either a limit on the total funds available for academic salaries or a limit on the total number of full time equivalent teaching positions allowed.

While there are clear political and bureaucratic costs associated with exceeding an administrator's available resources, there is also concern associated with the under-utilization of a resource in the public sector. Unused resources often have high opportunity costs associated with them and public fiducial responsibility requires the maximum productive use of public funds. Furthermore, unmet public needs can foster political discontent when funds are not used to their full availability. Finally, the future budget allocation of a scarce resource is frequently dependent upon the full use of that resource in the current period. Administrators are aware that under-utilization in one period may very well lead to a lesser budget for the following period.

In addition to the resource constraint, the decision maker must also consider institutional preferences for a well-balanced faculty, not only in terms of subject field but also in terms of professorial rank. While the definition of this factor may vary widely, the balance of senior and junior faculty is nevertheless important and relevant in the decision process.

The decision system described in this study is a multi-stage decision system, in which the results of current decisions are perceived in subsequent years and current decisions are constrained by past hiring decisions. The degree to which the decision maker fulfills his goals can be measured by a utility or scoring function which is defined for the particular decision maker in question.

This paper is concerned with the application of control theory to the solution of the optimal open-loop faculty hiring problem. Chapter II contains a description and estimation of the formal model of the faculty structure. Chapter III contains a mathematical formulation of the analysis. Chapter IV contains discussion of the numerical results derived from the application of the model, and Chapter V summarizes the conclusions drawn from this study, and discusses future research. The computer program used for these solutions was written in a generalized format so that it could be easily applied by other researchers to similarly defined allocation problems. Appendix B described how a copy of the user's manual or the program may be obtained from the authors.

II. THE CONCEPTUAL MODEL, DATA, AND ESTIMATION OF THE SYSTEM DYNAMICS

Conceptual Formulation

For the purposes of this analysis, the variables which characterize faculty structure can be divided into (a) those variables which the decision maker can directly control, called control variables and designated by the symbol u ; (b) those endogenous variables which cannot be controlled by the decision maker, called state variables and designated by the symbol x ; and (c) those exogenous variables impinging upon the system, which are designated by the symbol z . To avoid the possibility of the problem becoming degenerate, we assume that neither x nor u is empty.[5] For the purposes of this analysis, these variables are assumed to be related by the linear dynamic propagation equation:

$$x(i + 1) = Fx(i) + Gu(i) + Hz(i) \qquad i = 0, 1, \ldots, N - 1 \qquad (1)$$

where the initial $x(0)$ is given and where i is the planning period and N is the total number of planning periods considered. We either know or can measure the initial state of the system, $x(0)$. Also basic to the model is a preference function formulated by the decision maker to describe the relative values of the states of the system. The intensity of preference for a particular state can be written as a function of the current state variables, control variables and the decision period, and is denoted as $V(x, u, i)$. Because we are concerned with an N period decision pro-

[5] If u is empty, the decision maker has no control over the faculty structure and the decision problem is meaningless. If x is empty, the problem is directly analagous to the problem of consumer demand with the same solution. See Samuelson [12].

blem, we must further define a scalar summary measure for the future stream of preferences,[6] which we denote as J :

$$J = \alpha_N V\left(x(N),\ u^*(N),\ N\right) + \sum_{i=0}^{N-1} \alpha_i V\left(x(i),\ u(i),\ i\right) \quad . \tag{2}$$

The N^{th} term is separated from the first $N - 1$ terms to reflect the truncation of an infinite sequence after N periods. The symbol α_i denotes a weighting or discount factor introduced to reflect the time preference of the decision maker's utility. Using this notation, the multi-stage public sector resource allocation decision problem is to choose from the admissible set the control sequence $u(i)$, $i = 0, 1, 2, \ldots, N - 1$ to

$$\text{Max } \{J = \alpha_N V[x(N),\ u^*(N),\ N] + \sum_{i=0}^{N-1} \alpha_i V[x(i),\ u(i),\ i]\} \tag{3}$$

subject to

$$x(i + 1) = Fx(i) + Gu(i) + Hz(i) \qquad i = 0, 1, \ldots, N - 1 \tag{4}$$

$$x(0) \quad \text{given} ,$$

and any budgetary or physical constraints.

Application to the Faculty Structure Problem

To apply this resource allocation decision model to the optimal faculty structure problem, we define the system variables as follows. The state and control vectors, x and u , refer to the four academic instructional ranks of all disciplines: (1) full professor, (2) associate professor, (3) assistant professor, and (4) instructor, where $x(i)$ are the number of faculty of each rank continuing at the end of academic year i , and $u(i)$ are the number of faculty of each rank hired at the beginning of year i . We may write

[6] This assumes the intertemporal separability of preference to enable feasible assessment and tractable solution.

the dynamics of the faculty structure as

$$\begin{pmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \end{pmatrix} = \begin{pmatrix} f_{11} & f_{12} & 0 & 0 \\ 0 & f_{22} & f_{23} & 0 \\ 0 & 0 & f_{33} & f_{34} \\ 0 & 0 & 0 & f_{44} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix} + \begin{pmatrix} g_{11} & 0 & 0 & 0 \\ 0 & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & 0 \\ 0 & 0 & 0 & g_{44} \end{pmatrix} \cdot \begin{pmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{pmatrix}$$

In other words, the number of full professors in the system at the end of next year is a function of the number of full professors completing this year (persistence), the number of associate professors completing this year (promotion), and the number of full professors hired this year. We assume that (1) no faculty member is promoted more than one rank in each year, (2) no faculty member is demoted to a lower rank, and (3) no newly hired faculty member is promoted within the first year of his contract. There have been very few exceptions to these assumptions in the recent history of the University of California.

There are several logical restrictions that should be imposed on the elements of the F and G matrices. Each year some people leave the system because of death, retirement, or resignation; therefore,

$$0 \leq f_{ii} \leq 1.0 \quad \text{and}$$

$$0 \leq g_{ii} \leq 1.0 \quad \text{for } i = 1, 2, 3, 4, \text{ and}$$

$$\sum_{i=1}^{4} f_{ij} \leq 1.0 \quad \text{for } j = 2, 3, 4 \ .$$

The assumption of linear system equations allows one to use the techniques of multiple linear regression to estimate statistically the elements of the F and G matrices. Data were available for the total number of full time equivalent (FTE) faculty by rank for the academic years 1962-63, to 1967-68 and a summary of the data is given in Table 1 and a correlation matrix is given in Table 2. The number of new hires for each rank and year were subtracted out for use as an independent variable in the regression equations. The results of the ordinary least squares multiple regression on data for total University of California are shown in Table 3. While the coefficients of multiple determination (R^2) were very high (.96 to .99), five of the eleven coefficients violated the logical sign and magnitude restrictions. This is not surprising because the ordinary least squares regression did not recognize any inequality coefficient constraints.

The estimation of transition probabilities has been discussed extensively in the literature and a number of techniques have been developed[7] to avoid the difficulties of ordinary least squares. Most of these techniques are formulated for equations in which the transition probability is the dependent variable rather than the coefficient applied to an independent or predetermined variable. However, one approach that recognizes most of the logical coefficient restrictions is quadratic programming which minimizes $[x(t+1) - Fx(t) - Gu(t)]^t[x(t+1) - Fx(t) - Gu(t)]$. This quadratic programming estimation technique was used on the same data and produced the coefficient estimates given in Table 4. We observe that all sign and magnitude restrictions are met by these estimates.

It is interesting to compare the results derived by ordinary least squares with the estimates computed by quadratic programming. Table 5 shows that the sum of squared residuals and the standard error as a percent of the mean derived by quadratic programming estimation are very close to the corresponding quantities derived by ordinary least squares, except in the case

[7]See Lee, Judge and Cain [8] for a comparison of five alternative estimation techniques; Zellner and Lee [17] for a discussion of Logit, Probit, Gompit, weighted least squares and joint estimation techniques; and Theil and Rey [13] for a discussion of quadratic programming.

of instructors.

Finally, a set of subjectively assessed coefficients was used as a test for sensitivity and reasonableness. These values are given in Table 6 and indicate that annually 10% of the instructors are promoted to assistant professors, and 20% of both the assistant professors and the associate professors are promoted one rank. Furthermore, these judgmental values indicate that only 5% of the faculty leave the system each year. This is greater than the recent attrition experience of about 2% at the University of California.

TABLE 1: SUMMARY OF DATA USED

Variable	Mean	Standard Deviation
Full Professors	1576.9	211.15
Associate Professors	899.24	80.652
Assistant Professors	1299.4	246.15
Instructors	102.71	16.164
New Full Professors	76.333	25.617
New Associate Professors	56.833	25.816
New Assistant Professors	366.50	99.863
New Instructors	68.667	14.091

TABLE 2: DATA CORRELATION MATRIX

ariable		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
ull Professor	1	1.0	.85	.99	-.38	.93	.73	.83	-.20	.95	.85	.98	-.36	.92	.85	.96	-.19
ssociate Professor	2		1.0	.91	-.71	.87	.77	.93	.31	.88	.72	.84	-.09	.94	.97	.93	-.37
ssistant Professor	3			1.0	-.48	.91	.70	.84	-.06	.94	.89	.97	-.36	.94	.90	.99	-.16
nstructors	4				1.0	-.51	-.36	-.45	-.79	-.49	-.41	-.45	-.40	-.44	-.55	-.58	.38
ew Full Professor	5					1.0	.90	.88	-.05	.99	.64	.97	-.01	.91	.83	.87	-.54
ew Associate Professor	6						1.0	.90	-.03	.87	.29	.77	.19	.84	.76	.64	-.77
ew Assistant Professor	7							1.0	.08	.88	.55	.81	-.11	.98	.96	.82	-.48
ew Instructors	8								1.0	-.08	-.06	-.16	.51	.01	.20	.07	-.22
ull Prof. in t-1	9									1.0	.69	.98	-.07	.92	.84	.90	-.48
ssoc. Prof. in t-1	10										1.0	.80	-.61	.72	.73	.91	.28
sst. Prof. in t-1	11											1.0	-.21	.89	.81	.93	-.31
nstructor in t-1	12												1.0	-.26	-.25	-.33	-.77
ew Full Prof. in t-1	13													1.0	.97	.91	-.33
ew Assoc. Prof. in t-1	14														1.0	.91	-.27
ew Asst. Prof. in t-1	15															1.0	-.12
ew Instructor in t-1	16																1.0

TABLE 3: TRANSITION MATRICES DERIVED FROM UNCONSTRAINED MULTIPLE REGRESSIONS

F-Matrix

	Full	Associate	Assistant	Instructor
Full	0.6854	1.11	0.0	0.0
Associate	0.0	-0.0689	0.1088	0.0
Assistant	0.0	0.0	C.4956	-0.5818
Instructor	0.0	0.0	0.0	-0.2955

G Matrix

	Full	Associate	Assistant	Instructor
Full	0.9112	0.0	0.0	0.0
Associate	0.0	2.050	0.0	0.0
Assistant	0.0	0.0	C.8138	0.0
Instructor	0.0	0.0	0.0	0.2413

Z-Vector 'constants)

Full	Associate	Assistant	Instructor
-413.6	703.9	275.2	28.26

TABLE 4: TRANSITION MATRICES DERIVED FROM QUADRATIC PROGRAMMING ESTIMATION

F-Matrix

	Full	Associate	Assistant	Instructor
Full	0.7058	0.5242	0.0	0.0
Associate	0.0	0.9570	0.03	0.0
Assistant	0.0	0.0	0.960	0.450
Instructor	0.0	0.0	0.0	0.526

G-Matrix

	Full	Associate	Assistant	Instructor
Full	1.000	0.0	0.0	0.0
Associate	0.0	0.63	0.0	0.0
Assistant	0.0	0.0	0.23	0.0
Instructor	0.0	0.0	0.0	0.738

TABLE 5: COMPARISON OF EFFICIENCY OF ORDINARY LEAST SQUARES AND QUADRATIC PROGRAMMING ESTIMATION

	Full	Associate	Assistant	Instructor
Ordinary Least Squares				
Sum of Squared Residuals	4960	11590	19690	730
Standard Error of Estimate	49.8	76.1	99.2	15.6
Standard Error as Percentage of Mean	3.04	8.30	7.30	15.97
Quadratic Programming				
Sum of Squared Residuals	5045	11591	33541	1513
Standard Error of Estimate	50.2	76.1	129.5	22.5
Standard Error as Percentage of Mean	3.07	8.30	9.50	22.95

TABLE 6: JUDGMENTALLY ASSESSED TRANSITION MATRICES

F-Matrix

	Full	Associate	Assistant	Instructor
Full	0.95	0.20	0.0	0.0
Associate	0.0	0.75	0.20	0.0
Assistant	0.0	0.0	0.75	0.10
Instructor	0.0	0.0	0.0	0.0

G-Matrix

	Full	Associate	Assistant	Instructor
Full	0.90	0.0	0.0	0.0
Associate	0.0	0.90	0.0	0.0
Assistant	0.0	0.0	0.90	0.0
Instructor	0.0	0.0	0.0	1.0

III. FORMULATION OF THE ANALYSIS

Chapter II presented the general framework of public sector resource allocation decision analysis in terms of the decision maker's utility function V defined over the state and control variables. However, it is often more convenient to transform the analysis from one of maximizing utility to one of minimizing loss, with loss defined around the most desired targets. Near the targets, the loss structure is approximately quadratic independent of the form of the utility function.[8]

Several criterion functions were used and most combined considerations of the relative composition of faculty by rank and either a monetary or absolute constraint on the total number of faculty positions. The relative composition of the faculty was measured by the ratio of the number in each rank to the number of full professors. The desired distribution is then a target or goal sought in some or all periods. The various criterion functions are summarized in Table 7.

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[8] If $V(x, u)$ has a maximum at x^*, u^* (desired targets), then the second order Taylor series expansion about x^*, u^* is

$$V(x, u) = V(x^*,u^*) + \frac{\partial V}{\partial x}\Delta x + \frac{\partial V}{\partial u}\Delta u + 1/2\Delta x^t \frac{\partial^2 V}{\partial x^2}\Delta x + 1/2\Delta u^t \frac{\partial^2 V}{\partial u^2} \Delta u + \Delta x^t \frac{\partial^2 V}{\partial x \partial u}\Delta u + HOT .$$

Near the optimum

$$\left.\frac{\partial V}{\partial x}\right|_{x^*,u^*} = \left.\frac{\partial V}{\partial u}\right|_{x^*,u^*} = 0 \qquad \text{and}$$

$$V(x, u) = V(x^*,u^*) + 1/2[\Delta x^t \Delta u^t] \begin{pmatrix} V_{xx} & V_{xu} \\ V_{ux} & V_{uu} \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta u \end{pmatrix} + HOT$$

Therefore, the maximization of $V(x, u)$ is equivalent to minimization of the quadratic term on the right hand side.

TABLE 7

SUMMARY OF CRITERION FUNCTIONS USED IN THIS ANALYSIS

Criterion Function	Targets Met in	Constrained by	Weights Used
1	all periods	budget	various
2		budget	various
3	all periods	budget	Kalman
4	last period	budget	various
5	last period	budget	Kalman
6	all periods	positions	various
7	last period	positions	various

Denoting $p_j(i)$ as the average annual salary for faculty of rank j in period i, $B(i)$ as the total academic salary budget for period i, r_j as the target ratios sought, and k_j and β as weights indicating relative loss, we may write the criterion function used as

$$V(i) = \sum_{j=2}^{4} k_j \left(\frac{x_j(i)}{x_1(i)} - r_j \right)^2 + \beta \left[\sum_{j=1}^{4} p_j(i)\left(x_j(i) + u_j(i)\right) - B(i) \right]^2$$

for $i = 0, 1, \ldots, N-1$ (1)

and

$$V(N) = \sum_{j=2}^{4} k_j \left(\frac{x_j(N)}{x_1(N)} - r_j \right)^2 \quad . \tag{2}$$

The weights k_j describe the relative loss of deviating from the distributional targets of each rank. Several methods were used to derive these weights including (1) the average salary of each rank relative to the average salary of full professors expressing the relative monetary loss of a unit deviation from the desired targets, (2) the Kalman variance in each period of each state variable relative to the variance in that period of the number of full professors expressing the relative uncertainty in each state variable, and (3) weights chosen roughly proportional to the first derivative of $V(i)$ to insure rapid convergence. With the exception of the Kalman variance weights, both types (1) and (3) were used in most numerical experiments.

There are other important objectives served in the management of a faculty structure. In some cases, the total number of faculty positions is fixed in each period and all composition adjustments must occur within these ceilings. This case can be accommodated within our framework by setting all the annual faculty salaries equal to 1.0 and the total academic salary budget equal to the chosen ceilings. Two other objectives not included in this analysis are (1) the maintenance of a steady flow of promotions to avoid faculty ossification and discouragement and (2) the elimination of no longer productive faculty through early retirement or some other means. These last two examples show that this analysis cannot solve all of the problems in the management of an academic faculty structure. On the contrary, this study illustrates that some faculty management decisions can be analyzed in a cogent and sophisticated manner.

Stochastic Considerations

In addition to the deterministic model just described, we may also introduce a stochastic element in the academic structural equations in the following fashion. We consider an additive random error in a linear dynamic system:

$$x(i + 1) = Fx(i) + Gu(i) + Hz(i) + \tilde{\varepsilon}(i) \qquad (3)$$

where $\tilde{\varepsilon}(i)$ is assumed to be independently normally distributed with zero mean and variance-covariance matrix Q , which is assumed constant over time.[9] The decision maker can observe the state of his system at the beginning of period i by the linear scheme

$$y(i) = S(i)\, x(i) + \tilde{v}(i) \qquad (4)$$

where $S(i)$ is the current sampling scheme matrix, $\tilde{v}(i)$ is the sampling error with variance-covariance matrix $R(i)$, and $y(i)$ the observation vector. It may now be derived[10] through use of the Kalman filter that, while the prior

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[9]Estimates of the elements of the variance-covariance matrix Q can be derived from the results of either the linear regressions or the quadratic programming estimation performed to develop the F and G matrices. Where e_{ij} is the i^{th} residual and $\bar{e}_j$ the mean residual of the j^{th} prediction equation (i.e., for state $j = 1,2, \ldots, NX$), and N is the number of data points used in each calculation, we have that

$$Var(e_{ij}) = \frac{\sum_{i=1}^{N} (e_{ij} - \bar{e}_j)^2}{N - 1} \quad \text{for } j = 1, 2, \ldots, NX$$

and

$$Cov(e_{ij}, e_{ik}) = \frac{\sum_{i=1}^{N} (e_{ij} - \bar{e}_j)(e_{ik} - \bar{e}_k)}{N - 1}$$

for $j = 1, 2, \ldots, NX \; ; \; k = 1, 2, \ldots, NX \; ; \; j \neq k$.

[10]Bryson and Ho [2], Chapter 12.

variance-covariance matrix of x before measurement, M, is

$$M(i + 1) = FP(i)F^t + Q \quad , \tag{5}$$

the posterior variance-covariance matrix of x after measurement, P, is

$$P(i) = M(i) - M(i)S^t(i)[S(i)M(i)S^t(i) + R(i)]^{-1}S(i)M(i) \quad . \tag{6}$$

In this way, given $M(0)$, Q, $H(i)$, and $R(i)$ for $i = 0, 1, . . ., N$, we may precompute the posterior variance-covariance matrix, P, before attacking the deterministic optimization problem. While this stochastic element is not considered in the optimization process *per se*, its use in the formulation of the preference function can be quite relevant. In the special case of a quadratic criterion function and linear system dynamics perturbed by additive Gaussian noise, a "certainty equivalence" principle is applicable which allows one to separate the estimation and optimizaton procedures without affecting the final optimal solution.[11] This is due to the fact that for the quadratic preference function, the expected value of the preference function, $V(i)$, can be separated into its mean and variance. This is also true of negative-exponential and linear preference functions.

[11] Joseph and Tou [6].

IV. NUMERICAL RESULTS

This chapter is devoted to presenting numerical examples of the results derived from the implementation of the model discussed in the previous chapters. All of the examples described here were designed to investigate the behavior of the model under various conditions. Basic to this investigation were the following characteristics:

1. The criterion functions used were those described in Section III.
2. All the faculty ratios are in proportion to the number of full professors.
3. Five or ten planning periods were used.
4. The data employed were for the total University of California. The initial state variable assignments were adapted from the actual FTE faculty appointment schedule[12] for the 1967-68 academic year: full professors (1807), associate professors (822), assistant professors (1189), and instructors (13).[13]
5. No discounting was used on the preference function (i.e., all $\alpha_i = 1.0$).
6. All prices and the total number of positions are assumed to increase 5.0% per year while the budget constraint increases at the historical 12.0% per year.
7. The control assignments of the initial iteration were all zeroes.
8. For practical purposes, the computations were terminated when neither the control set nor the criterion function value showed any pronounced variation. This was usually 50-100 iterations.[14]

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[12] University of California, 1967-68 Statistical Summary - Students and Staff [14].

[13] The unusually low figure for instructors was due to the one-year contract for faculty of this rank. Practically all instructors are new hires.

[14] The inaccuracies caused by any pre-mature terminations were thought to be of minimal importance for this particular study due to the cancellations of error upon comparison with other similarly handled runs.

In addition to examining several criterion functions, a number of variations in the basic formulation were investigated by changing the relative size of the parameters of the criterion function. Three specific variations were in the: (1) price and budget vectors, (2) relative penalty weights, and (3) faculty ratios. In addition to these preference function variations, three different sets of F and G transition matrices were used as were several different solution algorithms including both first and second order methods. Examples of these variations along with their derived results will now be described.

Analysis with Constrained Least Square Transition Matrices and Second Order Optimization Technique

Budget Constrained

The first investigation focused on criterion functions 1 and 4 which represent an institutional decision maker striving to achieve faculty distributional targets in each period or only in the final period while meeting the budget constraints. In this case, five planning periods were used. The results of this analysis are given in Table 8 and Figures 1 and 2.

In all the analyses performed, the distribution of faculty chosen as the desired target was either the distribution included in the Regents Budget of the University of California, the existing pattern in 1967-68, or an arbitrary target. (See below)

Table 8

Source	Distributional Ratios		
	Assoc. Prof. / Full Prof.	Asst. Prof. / Full Prof.	Instr. / Full Prof.
Actual 1967-68	.459	.666	.007
Budget 1967-68	.544	1.192	.200
Arbitrary	.900	1.500	.050

Table 9 shows that even under optimal control the arbitrary faculty distributional targets were unachievable in any period because of resource and system constraints and there was very little difference in performance between the two criterion functions. Furthermore, as in virtually all of the cases investigated, the budget constraint was very closely approximated.

Figure 1 shows the values of the control variables that are optimal for each of the two criterion functions under open loop control. While both hiring patterns average roughly 300 new faculty of each rank in each year, the patterns have striking differences. The large number of new assistant professors hired under the first criterion function is necessary to maintain the requisite balance in the face of the severe estimated first year attrition of new assistant professors (see Table 4). Meanwhile, the fourth criterion function includes faculty distribution in the last period only and, consequently, the last decisions diverge considerably from the previous pattern in an effort to achieve the desired targets.

From Figure 2 we observe that the total number of faculty of each rank also display quite different growth patterns. Under the first criterion function, the number of full professors increases very slowly at first and more rapidly at the end of the planning period, while under the fourth criterion function the number of full professors follows exactly the opposite growth pattern. The number of instructurs is another extreme case which varies from a smooth increase (#1) to an initial jump of 700 followed by a gradual decline of 200 positions (#4). It is interesting to note that with the same resources available and striving for the same targets an institution managed by criterion function 1 acquires more of the tenured ranks while an institution managed by criterion function 4 acquires proportionately more junior ranks.

TABLE 9: COMPARISON OF PREFERENCE FUNCTIONS 1 AND 4

A decision maker with preference function number 1 seeks to achieve targets in each planning period with a budget constraint.

	assoc/full	asst/full	inst/full
weights	100.0	50.0	25.0
targets	0.9	1.5	0.05

Ratios Generated

Period	assoc/full	asst/full	inst/full
1	0.455	0.658	0.007
2	0.644	0.861	0.075
3	0.675	0.819	0.096
4	0.677	0.811	0.113
5	0.623	0.735	0.104
6	0.659	0.739	0.104

Budget Weight = 1.0×10^{-6}

Period	Salaries Generated	Salaries Budgeted
1	87150.063	88516.000
2	98443.875	99138.000
3	110348.500	111034.000
4	123424.875	124358.000
5	129106.688	139281.000

Preference Function Values

Period	
1	57.18
2	27.52
3	28.76
4	29.69
5	37.07
6	34.83

Criterion Function Value = 215.1

A decision maker with preference function number 4 seeks to achieve targets in the last period only with a budget constraint.

	assoc/full	asst/full	inst/full
weights	100.0	50.0	25.0
targets	0.9	1.5	0.05

Ratios Generated in Period 6

assoc/full	asst/full	inst/full
0.643	0.909	0.177

Budget Weight = 5×10^{-7}

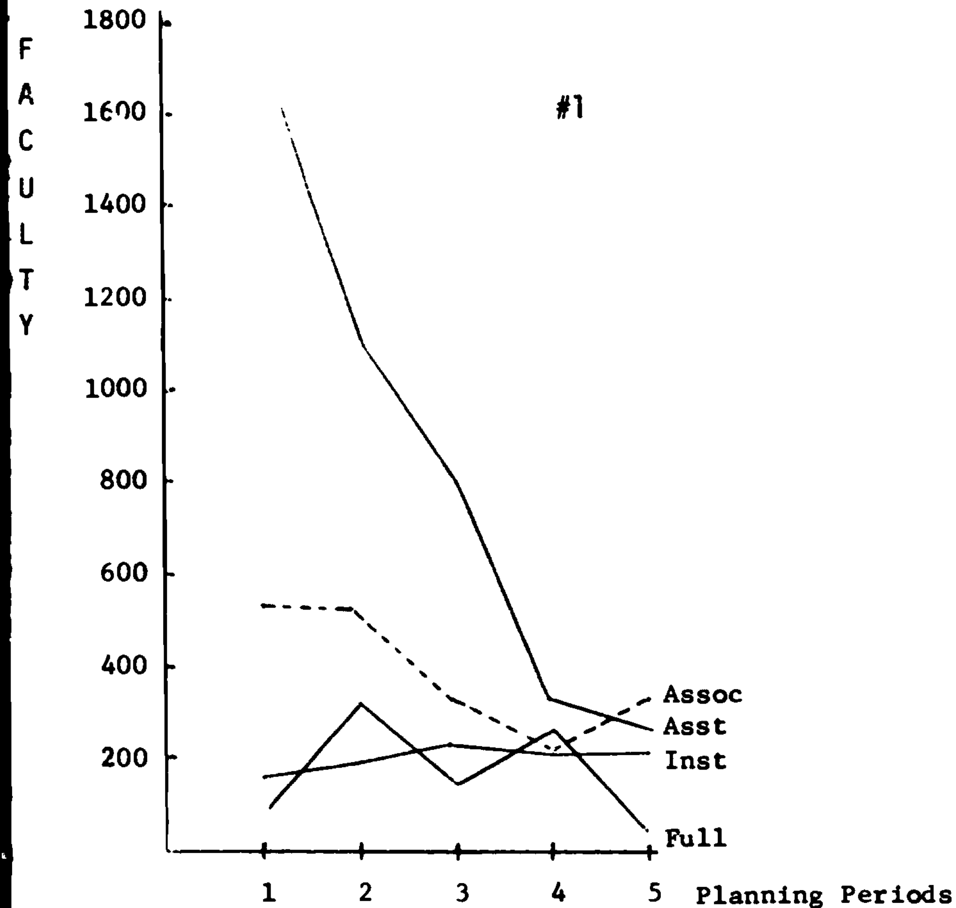
Period	Salaries Generated	Salaries Budgeted
1	89213.375	88516.000
2	100449.625	99138.000
3	110982.687	111034.000
4	123769.875	124358.000
5	137987.937	139281.000

Preference Function Values

Period	
1	.2432
2	.8602
3	.001316
4	.1729
5	.8360
6	24.47

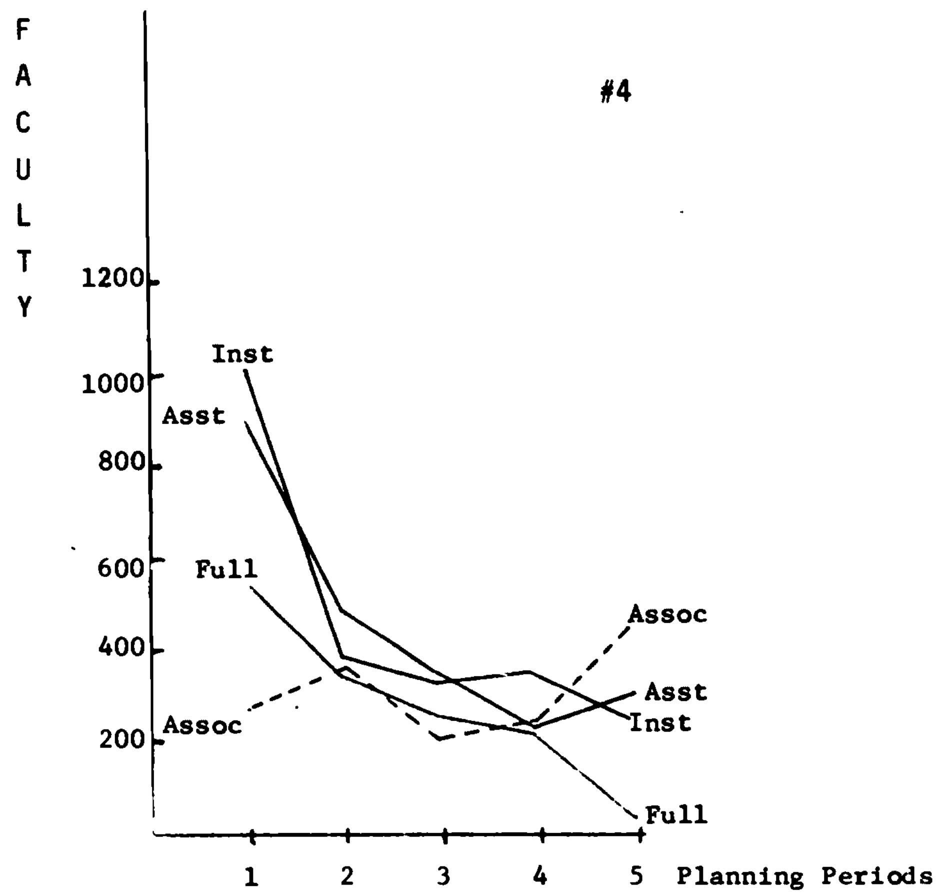
Criterion Value = 26.58

DECISION MAKER SEEKS TARGETS IN EACH PERIOD; BUDGET CONSTRAINT



Per	Full	Assoc	Asst	Inst
1.	99.4	539.5	1767.	173.7
2.	322.5	524.3	1107.	191.2
3.	147.3	321.5	803.	229.7
4.	270.9	222.5	324.	208.9
5.	42.7	329.5	277.	211.7

DECISION MAKER SEEKS TARGETS IN LAST PERIOD: BUDGET CONSTRAINT



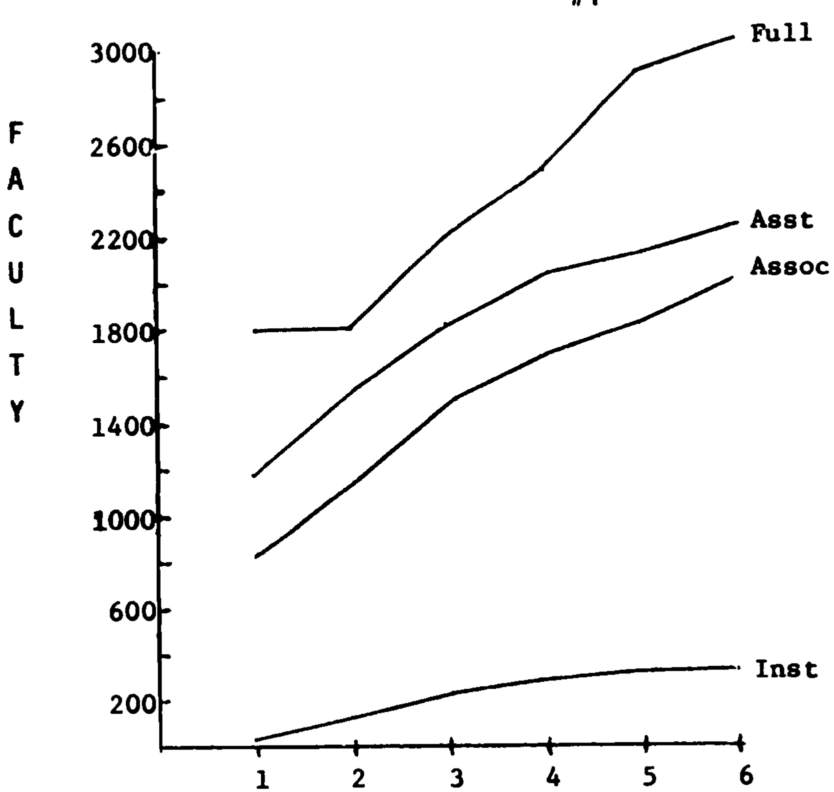
Per	Full	Assoc	Asst	Inst
1.	542.7	275.8	892.4	996.5
2.	345.4	359.8	488.8	371.8
3.	258.8	204.2	349.2	336.1
4.	209.3	241.5	225.7	356.2
5.	37.3	445.6	299.9	254.7

CONTROL VARIABLES

FIGURE 1

DECISION MAKER SEEKS TARGETS IN
EACH PERIOD: BUDGET CONSTRAINT

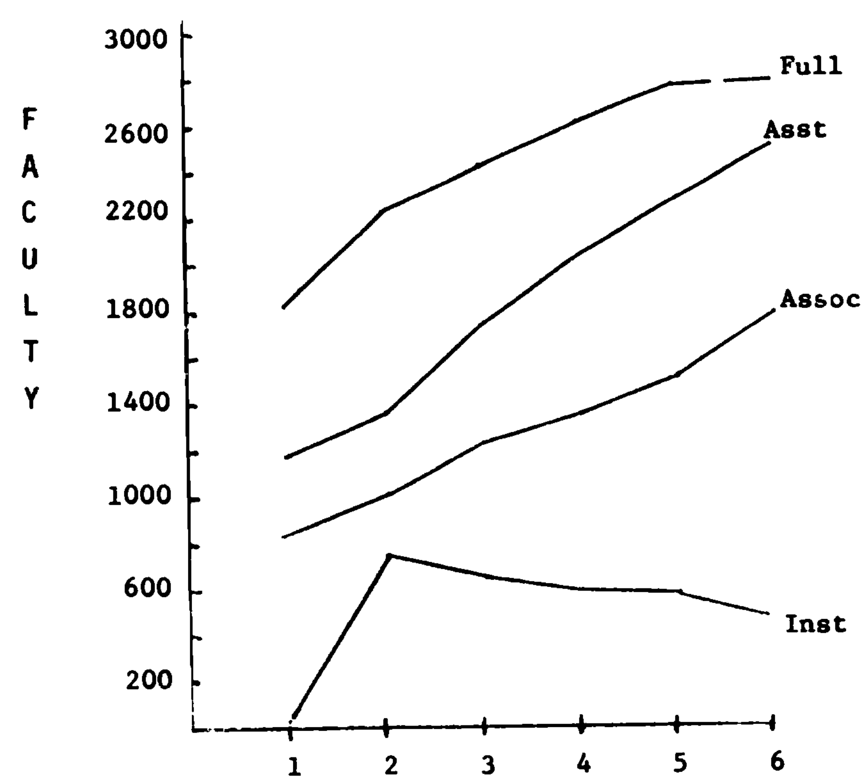
#1



Per	Full	Assoc	Asst	Inst
1.	1807.	822.	1189.	13.2
2.	1806.	1162.	1554.	135.1
3.	2206.	1489.	1807.	212.2
4.	2845.	1682.	2015.	281.2
5.	2906.	1810.	2135.	302.1
6.	3043.	2004.	2250.	315.1

DECISION MAKER SEEKS TARGETS IN
LAST PERIOD ONLY: BUDGET CONSTRAINT

#4



Per	Full	Assoc	Asst	Inst
1.	1807.	822.	1189.	13.2
2.	2249.	996.	1353.	742.4
3.	2455.	1220.	1745.	664.9
4.	2631.	1349.	2055.	597.8
5.	2773.	1505.	2293.	577.3
6.	2783.	1789.	2530.	491.7

STATE VARIABLES

FIGURE 2

Position Constrained

The second series of analyses focuses on a similar set of decision problems with total position constraints instead of total budget constraints. The faculty distributional targets in this case are the budgeted ratios of 1967-68. Once again, the decision maker could seek to achieve these targets in every decision period (#6) or only in the final decision period (#7). The results of this analysis are given in Table 10 and Figures 3 and 4.

The evidence of Table 10 indicates that the budgeted faculty distributional targets are also infeasible with the system description and position constraints used in this analysis. While the numbers of associate professors exceed its target, both assistant professors and instructors have significantly lower ratios. Meanwhile, both formulations closely adhere to the forecasted budgets.

The initial presence of full professors in excess of the proportion indicated by the budgeted targets leads the analysis to recommend firing several hundred full professors as shown in Figure 3. While this is institutionally infeasible and non-negativity constraints could be imposed on the new hires, these results are included to show the logical consequences of the budgeted faculty distributional ratios. The average new hires of about 100 per rank per year is significantly less than the approximately 300 new hires of each rank in each year found under the budget constraint because the number of positions increases at an assumed 5.0% per year as opposed to an assumed net 7.0% per year budget increase. The patterns of total faculty at each position are shown in Figure 4 which again reveals abrupt alterations in the last decision period for the case of distributional targets in the last period only.

TABLE 10: COMPARISON OF PREFERENCE FUNCTIONS 6 AND 7

A decision maker with preference function number 6 seeks to achieve targets in each planning period with a total positions constraint.

	assoc/full	asst/full	inst/full
weights	1000	1000	1000
targets	.544	1.192	.2

Ratios Generated

Period	assoc/full	asst/full	inst/full
1	0.455	0.658	0.007
2	0.579	0.813	0.038
3	0.579	0.773	0.135
4	0.581	0.779	0.135
5	0.579	0.784	0.116
6	0.598	0.788	0.113

Budget Weight = .01

Period	Positions Generated	Positions Budgeted
1	3799.30	3831.02
2	4007.34	4022.57
3	4218.06	4223.20
4	4428.82	4434.89
5	4662.63	4656.63

Preference Function Values

Period	
1	340.3
2	175.5
3	186.6
4	181.7
5	180.0
6	177.9

Criterion Function Value = 1242

A decision maker with preference function number 7 seeks to achieve targets in the last period only with a total positions constraint.

	assoc/full	asst/full	inst/full
weights	1000	1000	1000
targets	.544	1.192	.2

Ratios Generated in Period 6

assoc/full	asst/full	inst/full
.616	.722	.131

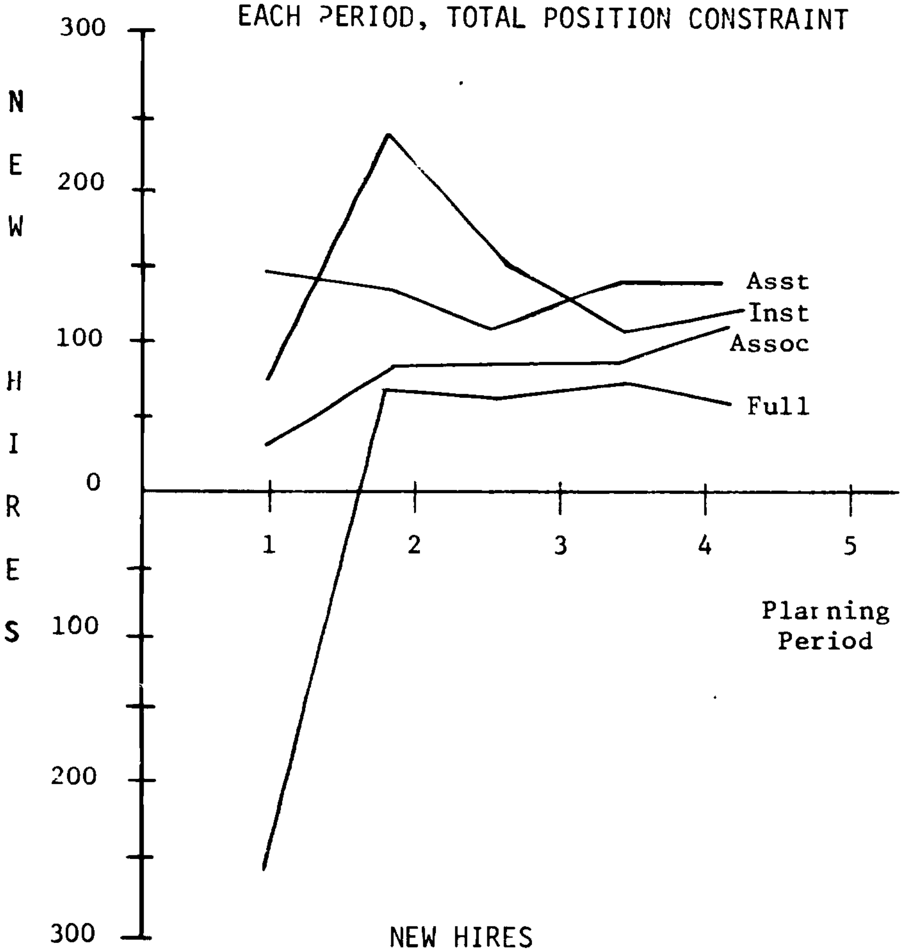
Budget Weight = .01

Period	Positions Generated	Positions Budgeted
1	3829.55	3831.02
2	4027.61	4022.57
3	4220.62	4023.70
4	4397.78	4434.89
5	4607.38	4656.63

Preference Function Values

Period	
1	.0215
2	.2538
3	.0950
4	13.77
5	24.27
6	231.3

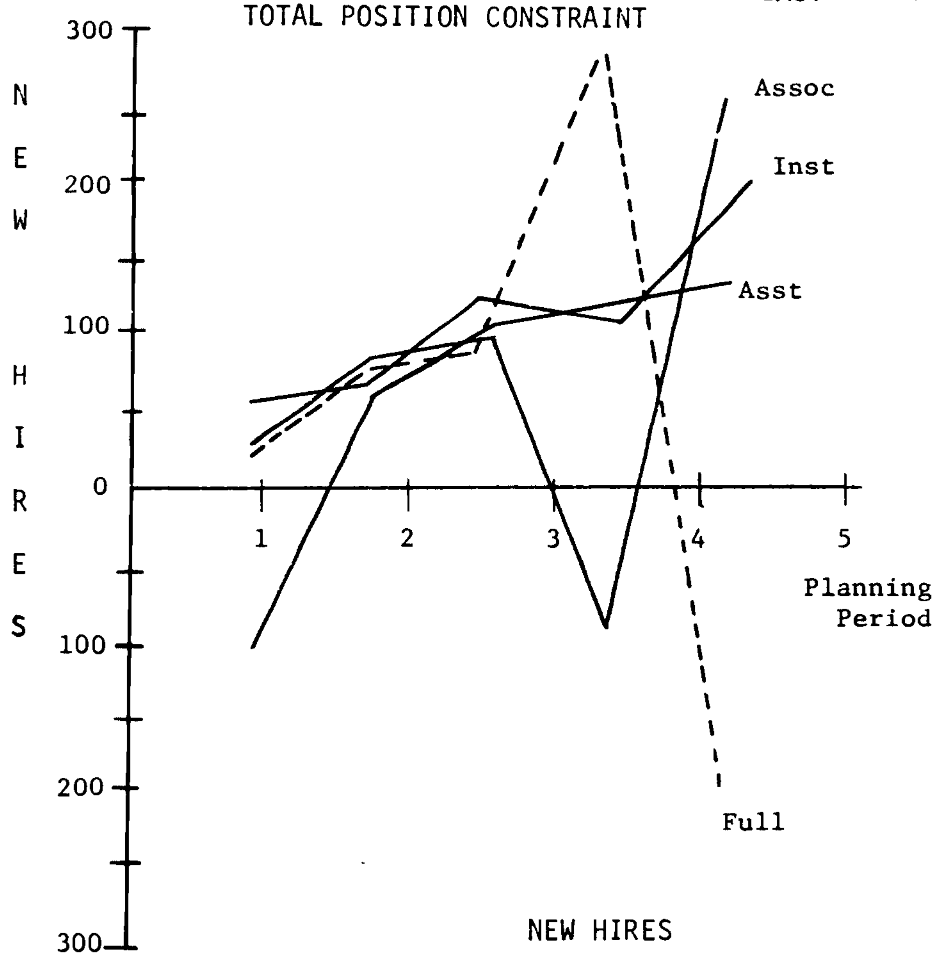
DECISION MAKER SEEKS TARGETS IN EACH PERIOD, TOTAL POSITION CONSTRAINT



NEW HIRES

Per	Full	Assoc	Asst	Inst
1.	-254.20	30.37	142.0	65.1
2.	69.93	78.03	130.8	241.6
3.	66.23	83.77	113.2	148.7
4.	74.71	82.42	137.5	113.2
5.	57.37	124.0	127.3	131.2

DECISION MAKER SEEKS TARGETS IN LAST PERIOD, TOTAL POSITION CONSTRAINT



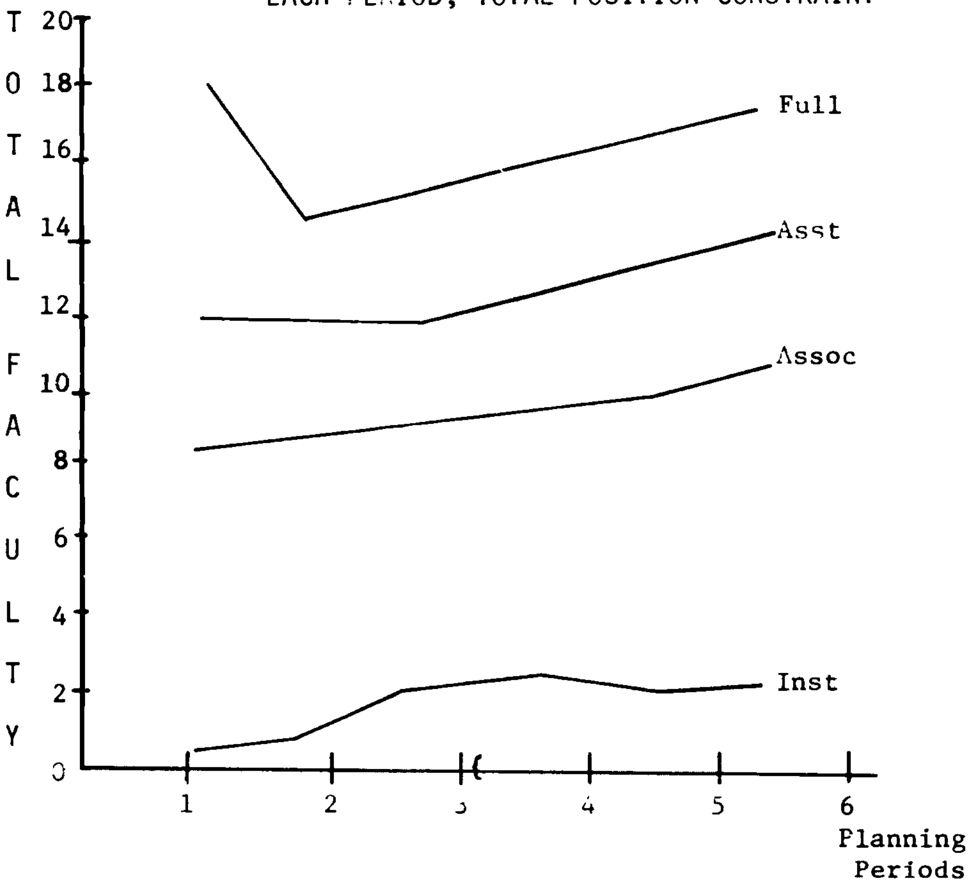
NEW HIRES

Per	Full	Assoc	Asst	Inst
1.	20.66	30.77	-109.8	56.9
2.	76.44	85.71	61.3	64.7
3.	88.63	92.57	104.0	119.1
4.	270.30	-93.89	122.4	118.0
5.	-200.40	259.70	133.4	190.6

CONTROL VARIABLES

FIGURE 3

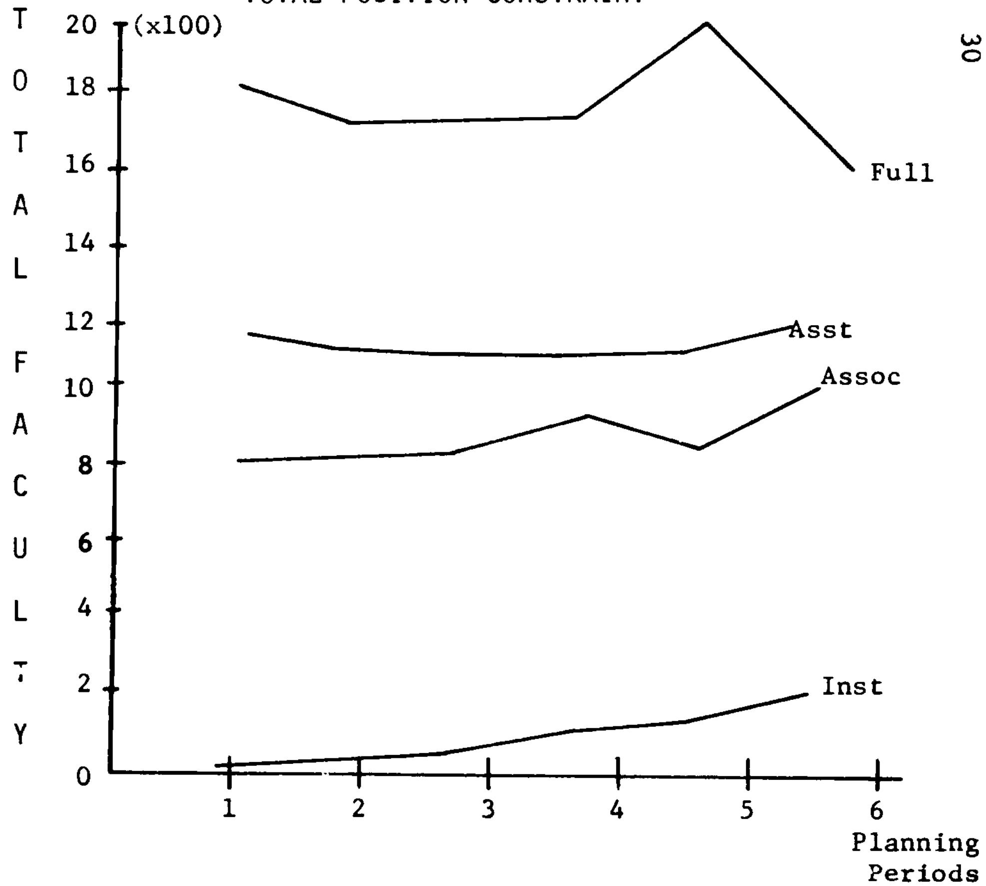
DECISION MAKER SEEKS TARGETS IN
EACH PERIOD, TOTAL POSITION CONSTRAINT



TOTAL FACULTY

Per	Full	Assoc	Asst	Inst
1.	1807.	821.8	1189.	13.2
2.	1452.	841.3	1180.	55.0
3.	1536.	889.7	1188.	207.2
4.	1616.	939.8	1259.	218.7
5.	1708.	989.1	1339.	198.6
6.	1782.	1065.0	1404.	201.3

DECISION MAKER SEEKS TARGETS IN LAST PERIOD,
TOTAL POSITION CONSTRAINT



TOTAL FACULTY

Per	Full	Assoc	Asst	Inst
1.	1807.	821.8	1189.	13.2
2.	1727.	841.5	1122.	49.0
3.	1736.	893.0	1113.	73.6
4.	1782.	946.3	1126.	126.6
5.	2024.	880.0	1166.	153.7
6.	1690.	1041.	1219.	221.5

STATE VARIABLES

FIGURE 4

Use of Varianc s as Penalty Weights

The weights used in the previous criterion functions were chosen either to balance the convergence of the solution algorithm or to reflect the relative cost of a one unit deviation from the targets. Another approach is to incorporate in the criterion function the current uncertainty in the estimate of the future magnitude of the state variables. In other words, a decision maker may choose to exert stricter control over the most uncertain components of his system. As discussed in Section III, the propagation of variance in a linear dynamic system is independent of the control chosen and, therefore, may be computed in advance and used as a set of fixed weights in the optimization. Furthermore, the relative magnitudes of the variance of the state variables will differ if the decision maker chooses to sample in the future. The results calculated using equation (III-6) for variance propagation are given in Table 11 for the case in which faculty distributional targets are sought in every decision period and in Table 12 for the case of only final period target ratios. The corresponding hiring and total faculty sequences are shown in Figures 5, 6, 7, and 8.

While the previous example penalized deviations from target ratios equally, the relative variances differ by about 50-100 to 1 with much less weight given the instructor target. As a consequence, the optimal number of newly hired instructors goes negative at some point in 3 out of 4 of the cases shown in Figures 5 and 7. This suggests that an educational decision maker would readily eliminate instructors to maintain balance in his other ranks - a result observed in practice (see Table 8). Otherwise, the hiring patterns for the top three ranks with the variance weightings closely resemble the previous arbitrarily weighted results shown in Figure 1.

As before, the terminal target only case exhibits more extreme behavior in the last decision period.

TABLE 11: COMPARISON OF TWO USES OF THE ERROR PROPAGATION ROUTINE TO OBTAIN WEIGHTS FOR A PREFERENCE FUNCTION OF A DECISION MAKER WHO SEEKS TO ATTAIN TARGET RATIOS IN EACH PLANNING PERIOD

Matrix of weights derived from EPROP with Sampling

Period	assoc/full	asst/full	inst/full
1	0.67178	1.07022	0.13651
2	2.31164	6.75631	0.20176
3	2.31257	6.75275	0.20158
4	2.31259	6.75234	0.20157
5	2.31278	6.75445	0.20162
6	2.31278	6.75445	0.20162

Target Ratios

	0.544	1.192	0.200

Ratios Generated

1	0.455	0.658	0.007
2	0.480	0.629	0.186
3	0.507	0.660	0.193
4	0.529	0.679	0.080
5	0.542	0.669	0.113
6	0.551	0.662	0.115

	Salaries Generated	Salaries Budgeted
1	88452.1875	88516.00
2	99461.5625	99138.00
3	110743.8125	111034.00
4	124190.5625	124358.00
5	139333.5625	139281.00

Budget Weight: 0.0000005

Preference Function Values

1	0.3176	4	1.794
2	2.205	5	1.852
3	1.960	6	1.901

Criterion Function Value: 10.03

Matrix of weights derived from EPROP without Sampling

Period	assoc/full	asst/full	inst/full
1	0.44444	0.66667	0.11111
2	2.29023	6.62333	0.19931
3	3.18549	9.12204	0.18084
4	2.53874	7.18544	0.10396
5	1.76997	4.94075	0.05630
6	1.76997	4.94075	0.05630

Target Ratios

	0.544	1.192	0.200

Ratios Generated

1	0.455	0.658	0.007
2	0.458	0.598	0.138
3	0.491	0.607	0.166
4	0.518	0.636	0.050
5	0.535	0.617	0.089
6	0.547	0.610	0.106

	Salaries Generated	Salaries Budgeted
1	88089.6250	88516.00
2	99625.0625	99138.00
3	111061.9375	111034.00
4	123931.1875	124358.00
5	139288.6250	139281.00

Budget Weight: 0.0000005

Preference Function Values

1	0.2887	4	2.316
2	2.473	5	1.635
3	3.127	6	1.677

Criterion Function Value: 11.52

TABLE 12

COMPARISON OF TWO USES OF THE ERROR PROPAGATION ROUTINE TO OBTAIN WEIGHTS FOR PREFERENCE FUNCTIONS OF A DECISION MAKER WHO SEEKS TO ACHIEVE TARGETS IN LAST PLANNING PERIOD ONLY

Weights derived from EPROP with Sampling

	assoc/full	asst/full	inst/full
Weight:	2.313	6.754	0.202

Target Ratios

assoc/full	asst/full	inst/full
0.544	1.192	0.200

Ratios Generated

Period	assoc/full	asst/full	inst/full
1	0.548	0.484	-0.59

Period	Salaries Generated	Salaries Budgeted
1	88446.375	88516.00
2	99384.625	99138.00
3	110989.500	111034.00
4	123914.625	124358.00
5	139321.000	139281.00

Budget Weight = 0.0000005

Period	Preference Function Values
1	0.002424
2	0.03041
3	0.0009901
4	0.09829
5	0.00080
6	3.401

Criterion Function Value = 3.534

Weights derived in EPROP without Sampling

	assoc/full	asst/full	inst/full
Weight:	1.770	4.941	0.056

Target Ratios

assoc/full	asst/full	inst/full
0.544	1.192	0.200

Ratios Generated

Period	assoc/full	asst/full	inst/full
1	0.555	0.705	0.138

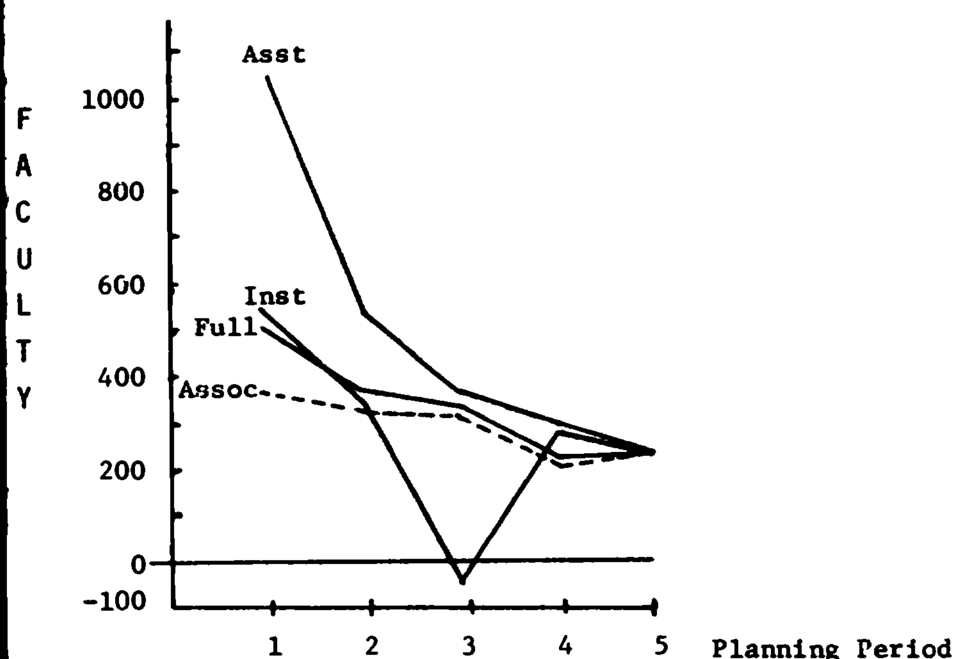
Period	Salaries Generated	Salaries Budgeted
1	88560.688	88516.00
2	99220.563	99138.00
3	111680.000	111034.00
4	123780.063	124358.00
5	139383.000	139281.00

Budget Weight = 0.0000005

Period	Preference Function Values
1	0.0009985
2	0.003408
3	0.2087
4	0.1670
5	0.005202
6	1.171

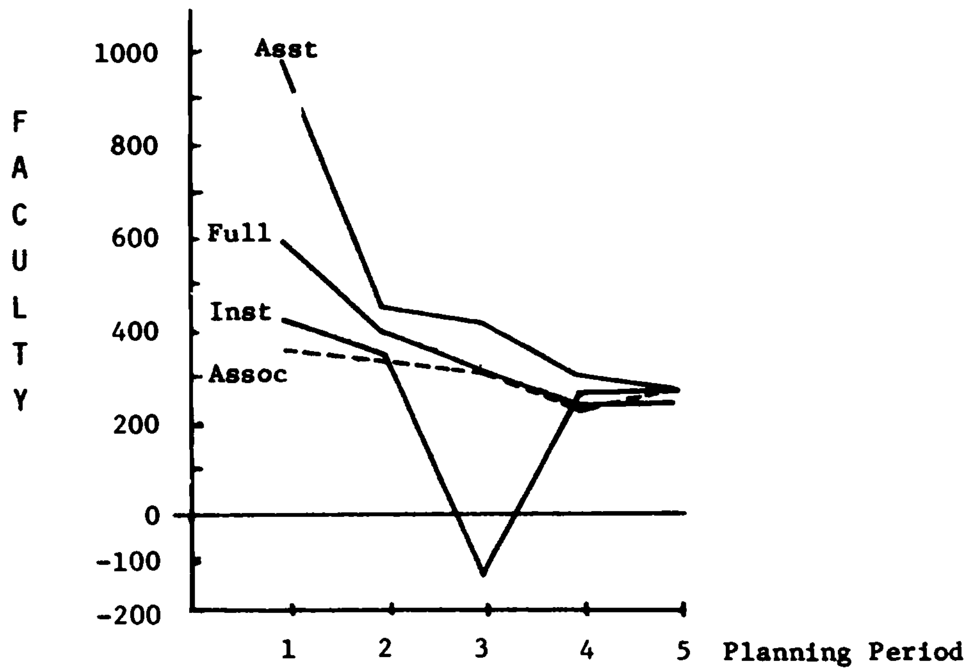
Criterion Function Value = 1.556

PF #3 SAMPLING



Per	Full	Assoc	Asst	Inst
1.	509.1	381.6	1068.0	549.3
2.	381.0	331.5	551.8	360.4
3.	345.4	327.2	363.9	-42.9
4.	231.8	224.5	306.3	294.5
5.	258.5	257.3	255.6	257.3

PF #3 NO SAMPLING

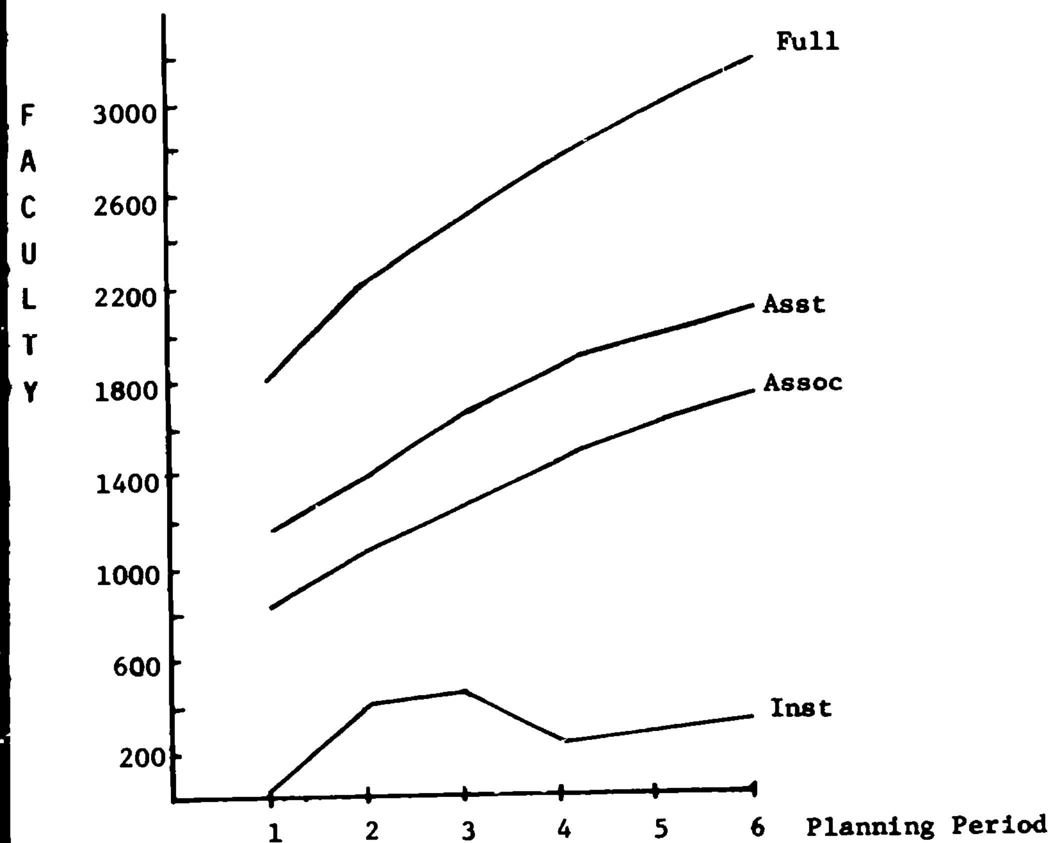


Per	Full	Assoc	Asst	Inst
1.	593.5	365.1	991.0	422.1
2.	405.7	346.6	450.4	353.1
3.	339.6	326.1	433.3	-114.2
4.	251.5	246.3	304.4	263.0
5.	260.5	272.2	282.0	272.3

CONTROL VARIABLES

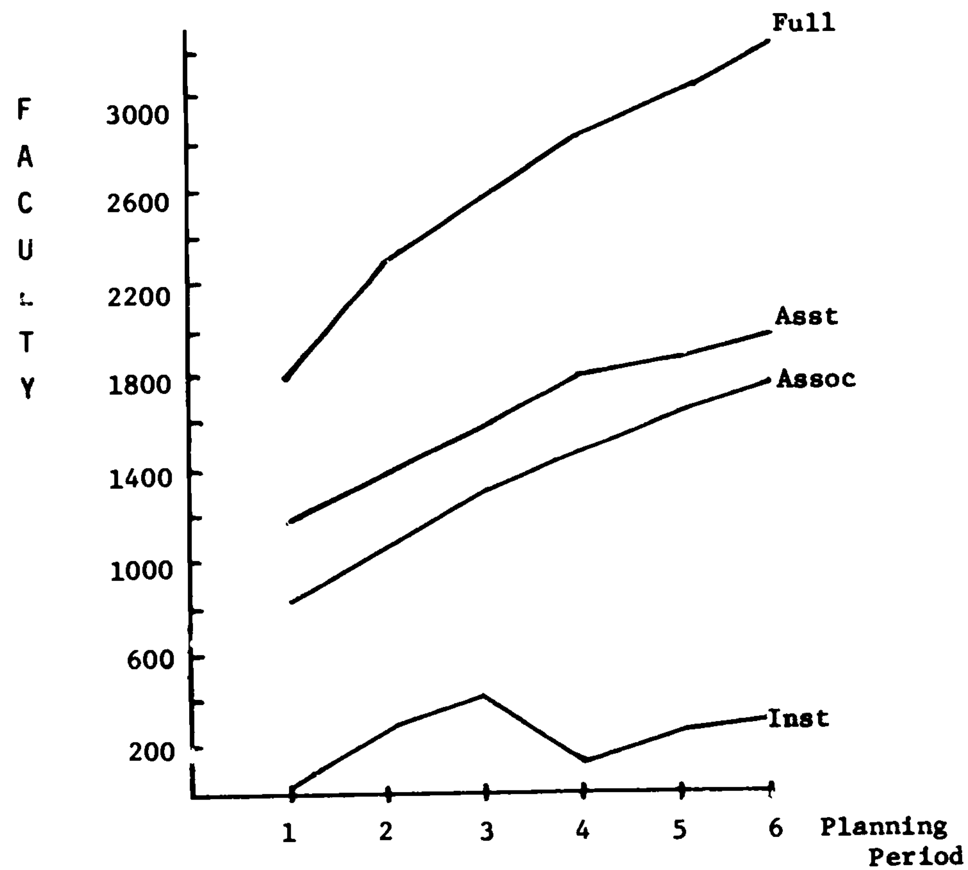
FIGURE 5

PF #3 SAMPLING



Per	Full	Assoc	Asst	Inst
1.	1807.	822.	1189.	13.2
2.	2215.	1063.	1393.	412.3
3.	2501.	1267.	1650.	482.8
4.	2775.	1469.	1885.	222.3
5.	2961.	1603.	1980.	334.2
6.	3189.	1756.	2110.	365.7

PF #3 NO SAMPLING

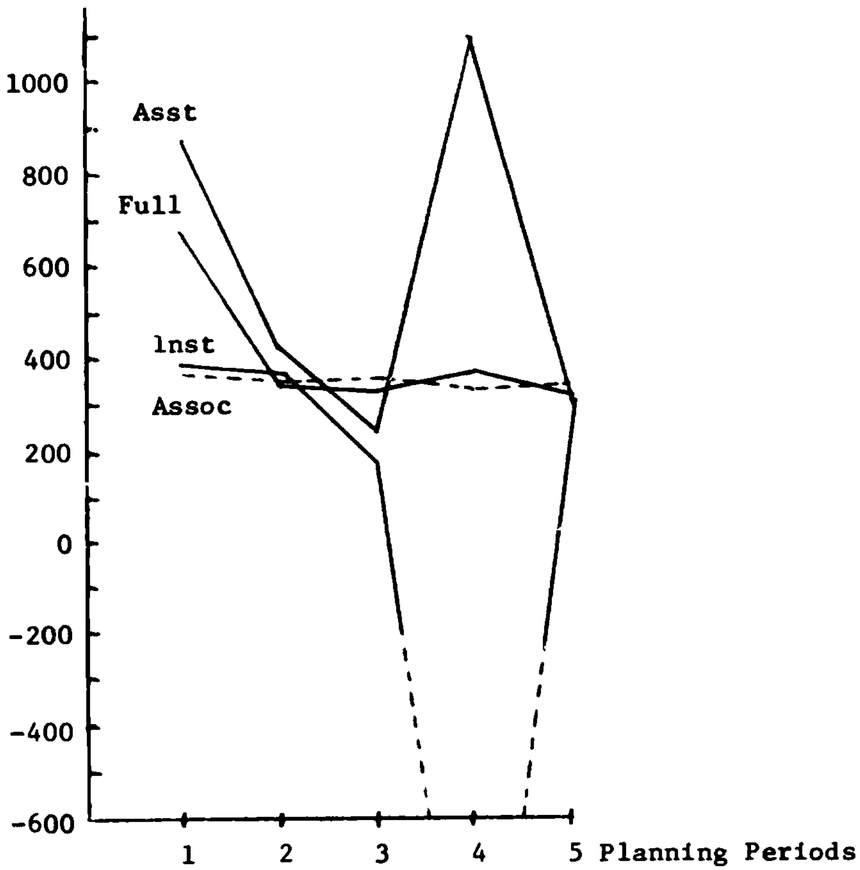


Per	Full	Assoc	Asst	Inst
1.	1807.	822.	1189.	13.2
2.	2300.	1052.	1375.	318.4
3.	2580.	1266.	1567.	428.1
4.	2825.	1464.	1797.	140.9
5.	3013.	1611.	1858.	268.2
6.	3231.	1769.	1970.	342.0

STATE VARIABLES

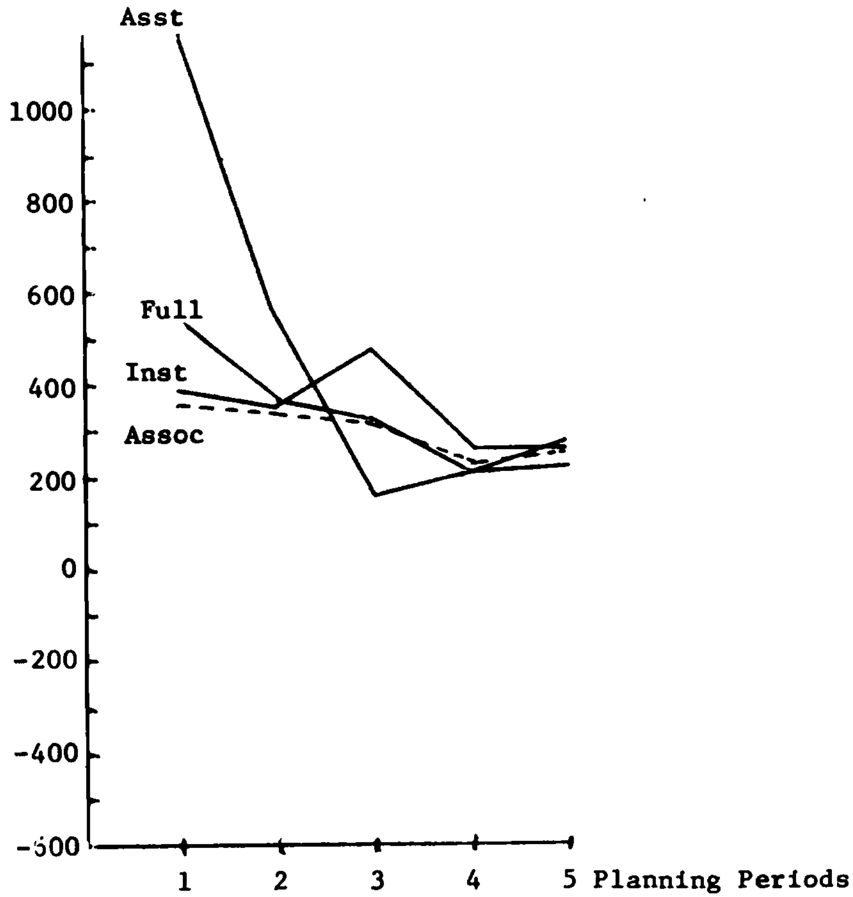
FIGURE 6

PF #5: SAMPLING



Per	Full	Assoc	Asst	Inst
1.	684.4	365.6	874.3	397.9
2.	341.9	346.0	414.2	358.9
3.	329.3	353.3	242.0	183.0
4.	376.7	331.9	1107.0	-1419.0
5.	335.5	340.1	309.5	339.3

PF #5: NO SAMPLING

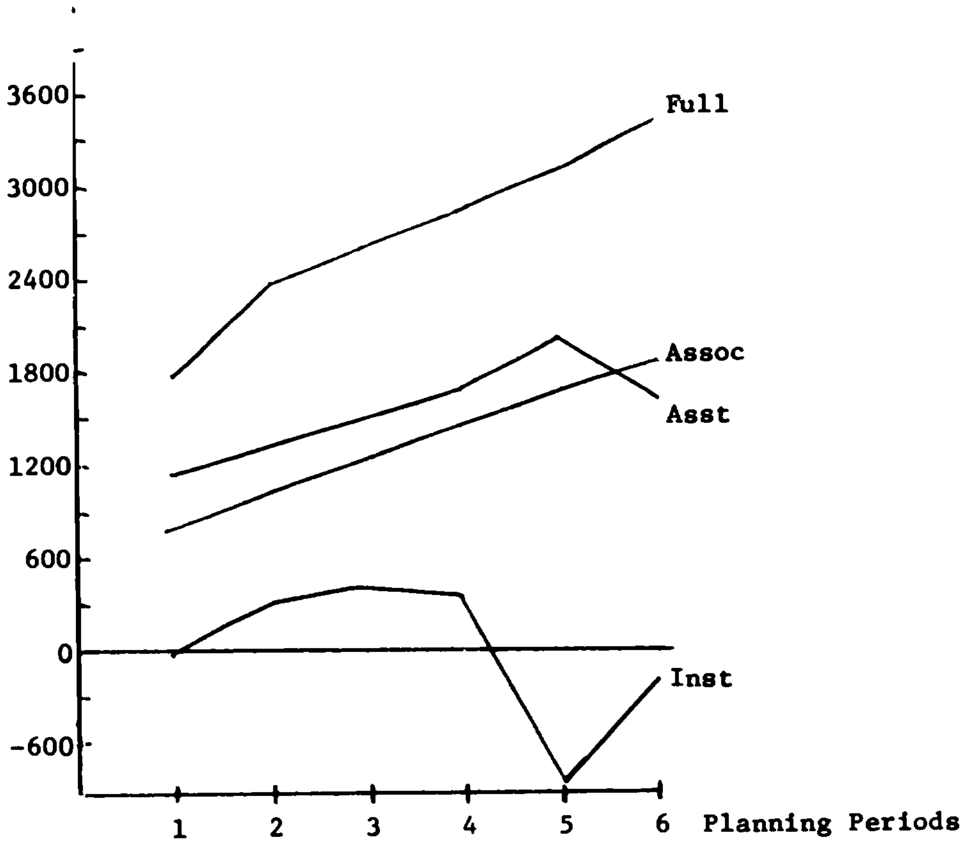


Per	Full	Assoc	Asst	Inst
1.	528.7	365.8	1174.0	394.8
2.	378.4	347.2	549.8	359.0
3.	325.8	316.6	177.4	479.3
4.	216.6	232.2	205.1	243.1
5.	233.1	242.1	246.5	242.2

CONTROL VARIABLES

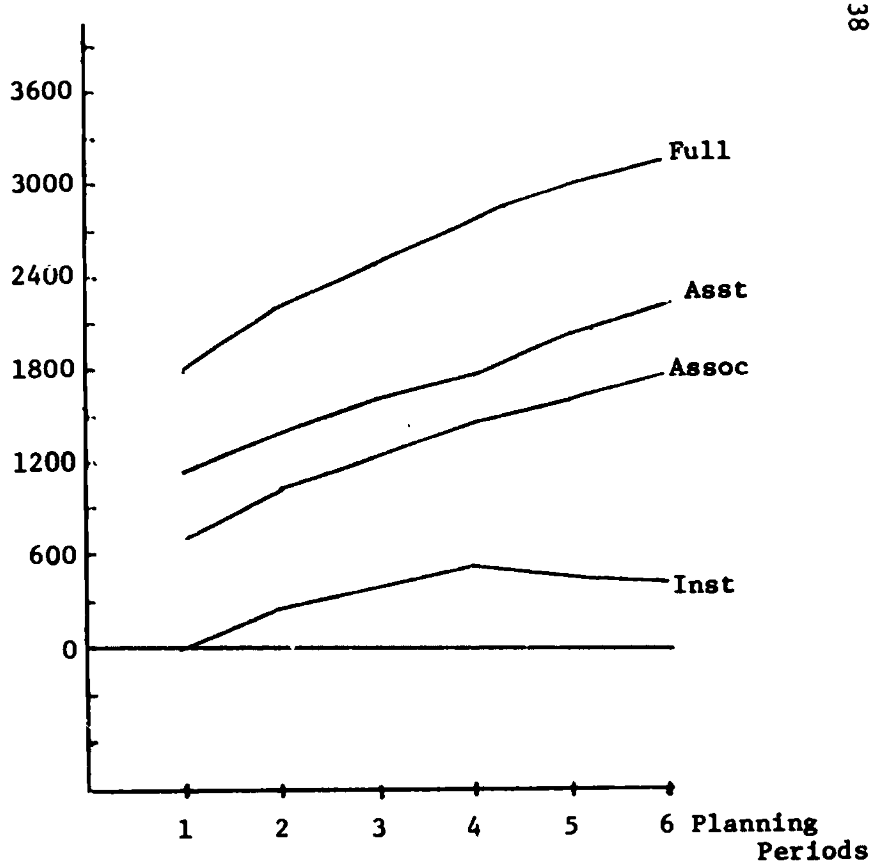
FIGURE 7

PF #5: SAMPLING



Per	Full	Assoc	Asst	Inst
1.	1807.	822.	1189.	13.2
2.	2391.	1052.	1348.	300.6
3.	2581.	1266.	1525.	423.0
4.	2814.	1480.	1710.	357.5
5.	3139.	1676.	2057.	-859.5
6.	3429.	1880.	1659.	-201.7

PF #5: NO SAMPLING



Per	Full	Assoc	Asst	Inst
1.	1807.	822.	1189.	13.2
2.	2235.	1053.	1418.	298.3
3.	2508.	1269.	1622.	421.9
4.	2761.	1462.	1787.	575.7
5.	2932.	1599.	2022.	482.2
6.	3141.	1744.	2215.	432.4

STATE VARIABLES

FIGURE 8

TABLE 13: SUMMARY OF THE EFFECTS OF CHANGING THE PLANNING HORIZON

A decision maker with a preference function that seeks to achieve faculty ratios in last period only.

	assoc/full	asst/full	inst/full	budget
weights:	1000.0	500.0	250.0	0.0000005

Period	Salaries Generated; 10 Yr.	Salaries Budgeted	Salaries Generated; 5 Yr.
1	88446.0625	88516.00	88361.875
2	99073.875	99138.00	99161.0625
3	111155.875	111034.00	111467.500
4	124507.4375	124358.00	124746.125
5	139624.5000	139281.00	139421.4375
6	155578.1875	155995.00	
7	175045.6250	174714.00	
8	195762.8750	195680.00	
9	218870.4750	219161.00	
10	244766.5625	245461.00	

Ratios

	assoc/full	asst/full	inst/full
Targets	0.459	0.666	0.007
Generated (10 Yr.)	0.519	0.617	0.101
Generated (5 Yr.)	0.475	0.661	0.064

Preference Function Values

10 Year Horizon:

1) 0.002446 2) 0.002056 3) 0.007427 4) 0.01117 5) 0.05900

6) 0.08687 7) 0.05499 8) 0.003434 9) 0.04223 10) 0.2411 11) 7.005

Criterion Function Value: 1.251

5 Year Horizon:

1) 0.01188 2) 0.0002659 3) 0.09396 4) 0.07532 5) 0.009861

6) 1.060

Criterion Function Value: 1.251

TABLE 13: SUMMARY OF THE EFFECTS OF CHANGING THE PLANNING HORIZON

A decision maker with a preference function that seeks to achieve faculty ratios in last period only.

	assoc/full	asst/full	inst/full	budget
weights:	1000.0	500.0	250.0	0.0000005

Period	Salaries Generated; 10 Yr.	Salaries Budgeted	Salaries Generated; 5 Yr.
1	88446.0625	88516.00	88361.875
2	99073.875	99138.00	99161.0625
3	111155.875	111034.00	111467.500
4	124507.4375	124358.00	124746.125
5	139624.5000	139281.00	139421.4375
6	155578.1875	155995.00	
7	175045.6250	174714.00	
8	195762.8750	195680.00	
9	218870.4750	219161.00	
10	244766.5625	245461.00	

Ratios

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Criterion Function Value: 1.251

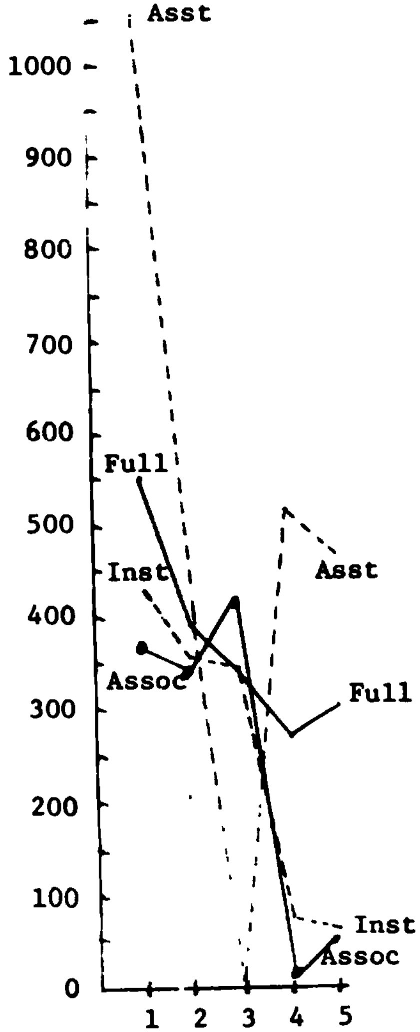
5 Year Horizon:

1) 0.01188 2) 0.0002659 3) 0.09396 4) 0.07532 5) 0.009861

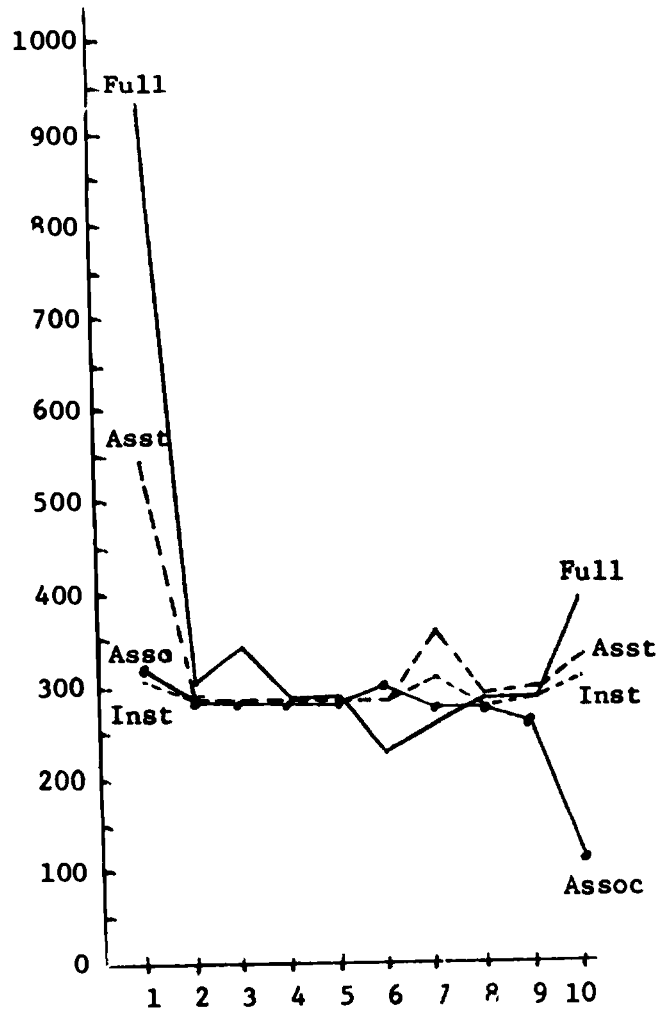
6) 1.060

Criterion Function Value: 1.251

FIVE YEAR HORIZON



TEN YEAR HORIZON



TEN YEAR HORIZON

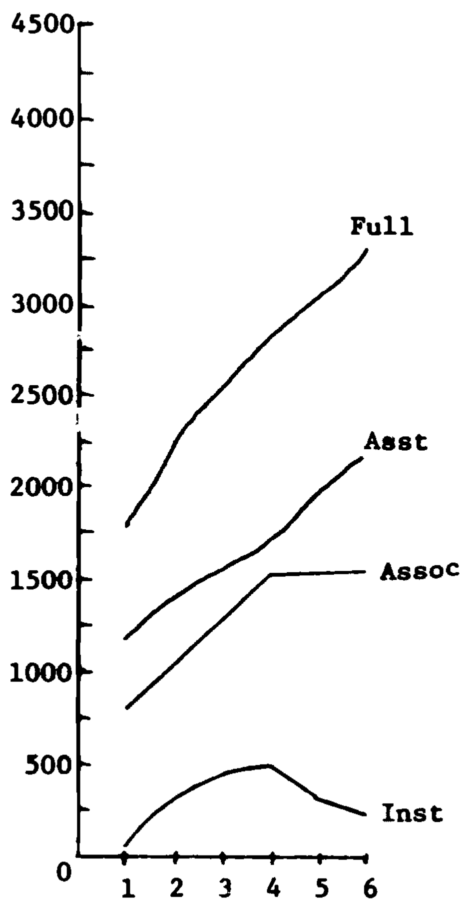
Per	Full	Assoc	Asst	Inst
1.	930.6	317.3	547.8	307.8
2.	305.8	286.8	288.4	287.8
3.	348.0	286.6	290.6	287.8
4.	290.5	286.3	289.8	287.2
5.	286.7	287.1	288.0	286.4
6.	224.9	299.6	287.3	287.2
7.	261.9	274.2	357.6	308.4
8.	283.4	277.3	289.3	282.0
9.	285.7	252.6	296.5	285.7
10.	391.2	108.3	333.3	308.7

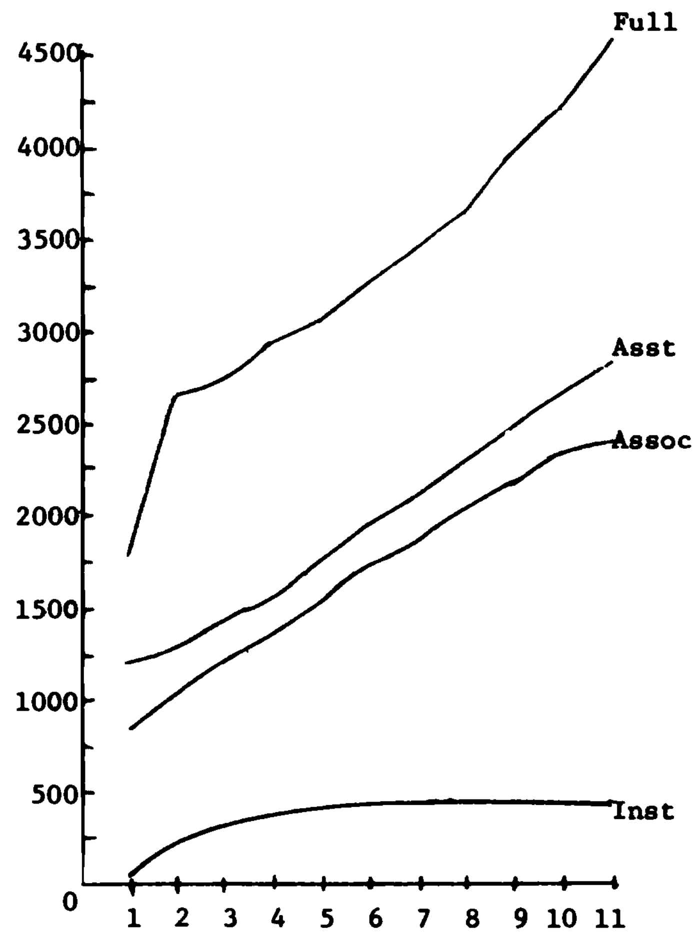
FIVE YEAR HORIZON

Per	Full	Assoc	Asst	Inst
1.	562.6	372.0	1056.0	434.7
2.	397.3	347.4	443.3	365.2
3.	346.9	425.1	10.0	349.8
4.	276.2	7.3	522.7	74.9
5.	311.5	57.9	473.1	61.5

CONTROL VARIABLES

FIGURE 9





TEN YEAR HORIZON

Per	Full	Assoc	Asst	Inst
1.	1807.	822.	1189.	13.2
2.	2637.	1022.	1273.	234.1
3.	2703.	1197.	1394.	335.5
4.	2883.	1368.	1556.	388.9
5.	3042.	1536.	1736.	416.5
6.	3239.	1703.	1920.	430.4
7.	3404.	1876.	2103.	438.4
8.	3648.	2031.	2298.	458.2
9.	3923.	2188.	2479.	449.2
10.	4201.	2327.	2650.	447.1
11.	4576.	2375.	2822.	463.0

FIVE YEAR HORIZON

Per	Full	Assoc	Asst	Inst
1.	1807.	822.	1189.	13.2
2.	2269.	1057.	1390.	327.8
3.	2552.	1272.	1584.	441.9
4.	2815.	1532.	1722.	490.6
5.	3066.	1523.	1994.	313.3
6.	3274.	1554.	2164.	210.2

STATE VARIABLES

FIGURE 10

Analysis of Sensitivity to Alternative Transition Matrices - First Order Optimization Technique

Basic to the propagation of the state of the system over time is the structure and composition of the F and G transition matrices. The resulting effect of these matrices on the optimum allocation pattern is vividly shown in Figure 11. These curves show the results of using two different sets of transition matrices on otherwise identically defined allocation problems. Example (a) uses matrices derived from unconstrained multiple regressions on actual data (see Table 3). Example (b) uses matrices assessed by subjective reasoning (see Table 6).

The total faculty curves for example (a) show some system instability as opposed to the smooth growth pattern characteristic of example (b). This is due to the multiplicative effect of the coefficients present in the transition matrices estimated by regression analysis which contained problems of realism, as earlier discussed. These results should be compared with Figure 2 (#1) which used the constrained least squares estimate.

This comparison accentuates the importance of developing valid and reasonable coefficients to describe the propagation of the system, without which one cannot hope to produce reasonable results. It appears that constrained least squares regression is the best approach for determining these coefficients provided enough valid data are available. However, care must be taken in using the results of the regressions to evaluate their accuracy and reasonableness.

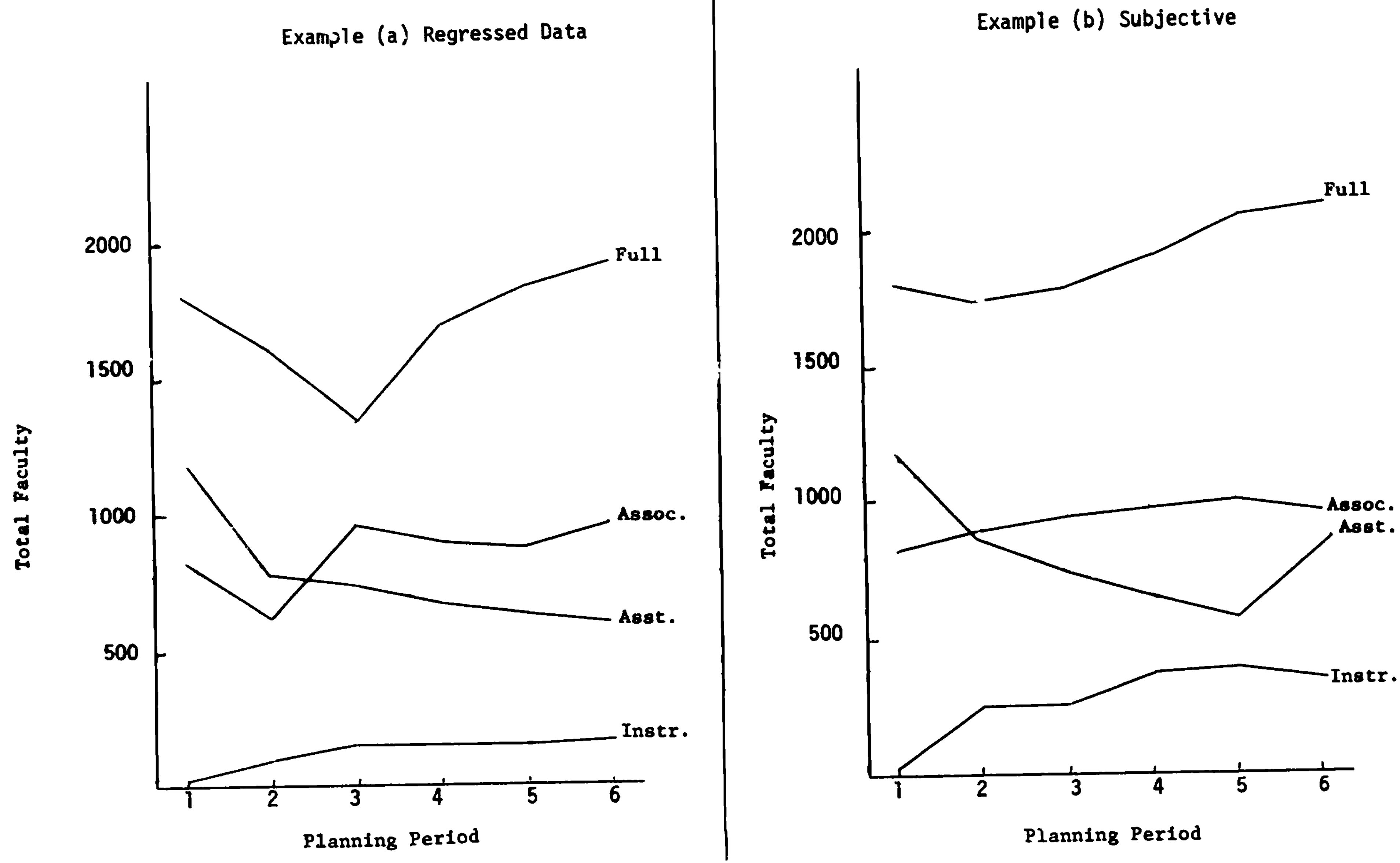


FIGURE 11: SENSITIVITY TO TRANSITION MATRICES

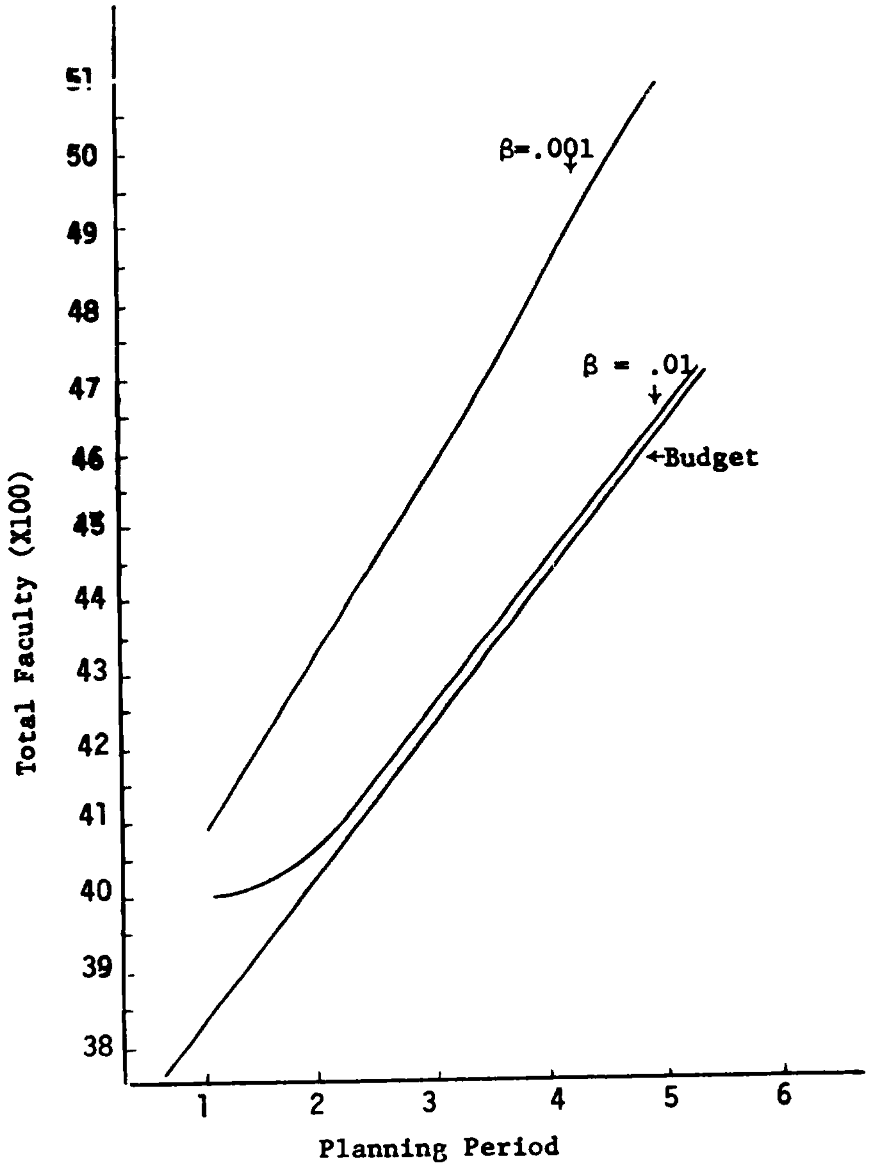
Variation of Weight on Resource Constraint with First Order Solution Technique

As described in Section III, the preference function can be divided into two parts: (1) the faculty distribution constraint and (2) the total resource constraint. Each of these terms is weighted by coefficients which determine how strictly the constraint is to be enforced. A large weighting coefficient will increase the penalty resulting from the deviation; similarly, a small weighting coefficient will decrease the penalty. This section investigates the effects of the weighting coefficient, β , on the resource constraint.

Again we compare the results of decision problems which are identical in every respect except in the single characteristic of the budget weighting coefficient, β . These runs were made using the ordinary least squares regression results for transition matrices. Figure 12 shows the results of a moderate variation in β . The resource constraint is still operative when β is decreased from 0.01 to 0.001, although it is obviously not as effective. Notice that there is little change in the degree to which the system continues to fulfill the faculty ratio constraint.

In Figure 13, we see a much greater effect resulting from a variation in β . In this case, reducing β from 0.001 to 0.0001 makes the resource constraint completely inoperative. There is now a negative correlation between the budgeted total faculty and the calculated faculty. Furthermore, we note that there is now a marked increase in the system's ability to fulfill the faculty ratio constraints.

In other words, the relative size of β regulates the balance between the resource constraint and the faculty ratio constraints. Consequently, β must be chosen carefully to simulate correctly the goals of the decision maker.



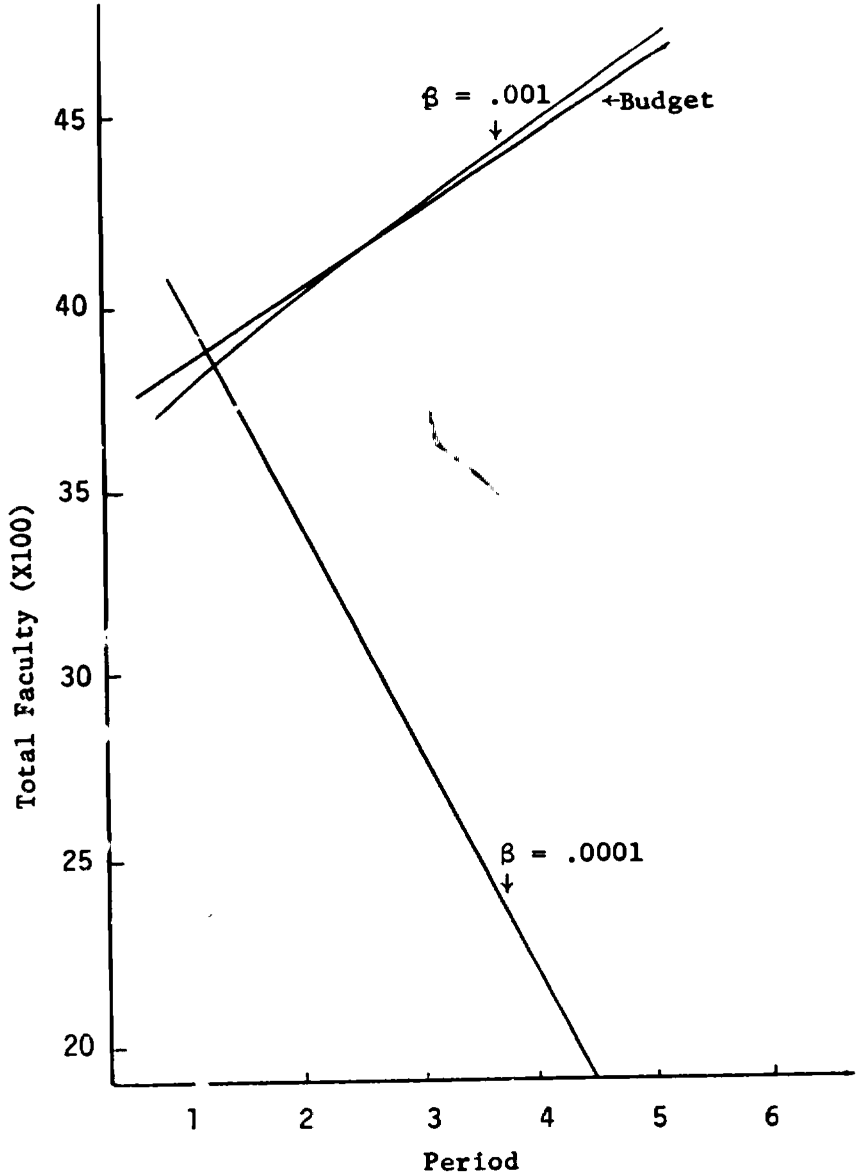
Total Faculty

Period	β = .001	β = .01	Budget
1	4096	4010	3821
2	4345	4065	4023
3	4555	4261	4224
4	4851	4465	4434
5	5090	4645	4657

Average Faculty Ratios

Ratio	β = .001	β = .01	Target	Weight
Full/Instr.	134.6	135.4	136.0	18.4
Assoc./Instr.	63.4	62.4	62.0	12.8
Asst./Instr.	88.8	89.0	90.0	10.0

FIGURE 12: VARIATION OF WEIGHT ON RESOURCE CONSTRAINT



Total Faculty

Period	β = .0001	β = .001	Budget
1	4017	3753	3821
2	3407	4020	4023
3	2850	4252	4224
4	2235	4471	4434
5	1631	4643	4657

Average Faculty Ratios

Ratio	β = .0001	β = .001	Target	Weight
Full/Instr.	5.10	5.07	5.00	18.4
Assoc/Instr.	2.78	3.32	2.72	12.8
Asst./Instr.	5.98	4.87	5.96	10.0

FIGURE 13: VARIATION OF WEIGHT ON RESOURCE CONSTRAINT

Variation in Method for Calculating the Step-size in First Order Solution Procedure

When investigating the convergence properties of the solution to this formulation, two different methods for calculating the incremental step-size, ε , were investigated: (1) a single step-size, and (2) a multiple step-size. With the single-ε method, the same step-size is applied to the entire control set through the relationship

$$u_{new} = u_{old} \pm \varepsilon \frac{\partial H}{\partial u}^{15} . \qquad (1)$$

In this way, the size of the increment is proportional to the associated gradient.

The multi-ε method uses an individually computed step-size, $\varepsilon_j(i)$, for each control element and is applied using the relationship,

$$u_j(i)_{new} = u_j(i)_{old} \pm \varepsilon_j(i) . \qquad (2)$$

Under the multi-ε method, adaptive increments are computed by bisecting the previous iteration's step-size when the associated gradient changes sign.

Although the single-ε method was the most straightforward and most easily implemented of the two procedures, its convergence peoperties often proved to be less satisfactory than the convergence of the multi-ε method. In almost all cases, the speed of convergence dropped off considerably as the optimum point was approached. This was caused by a large decrease in the size

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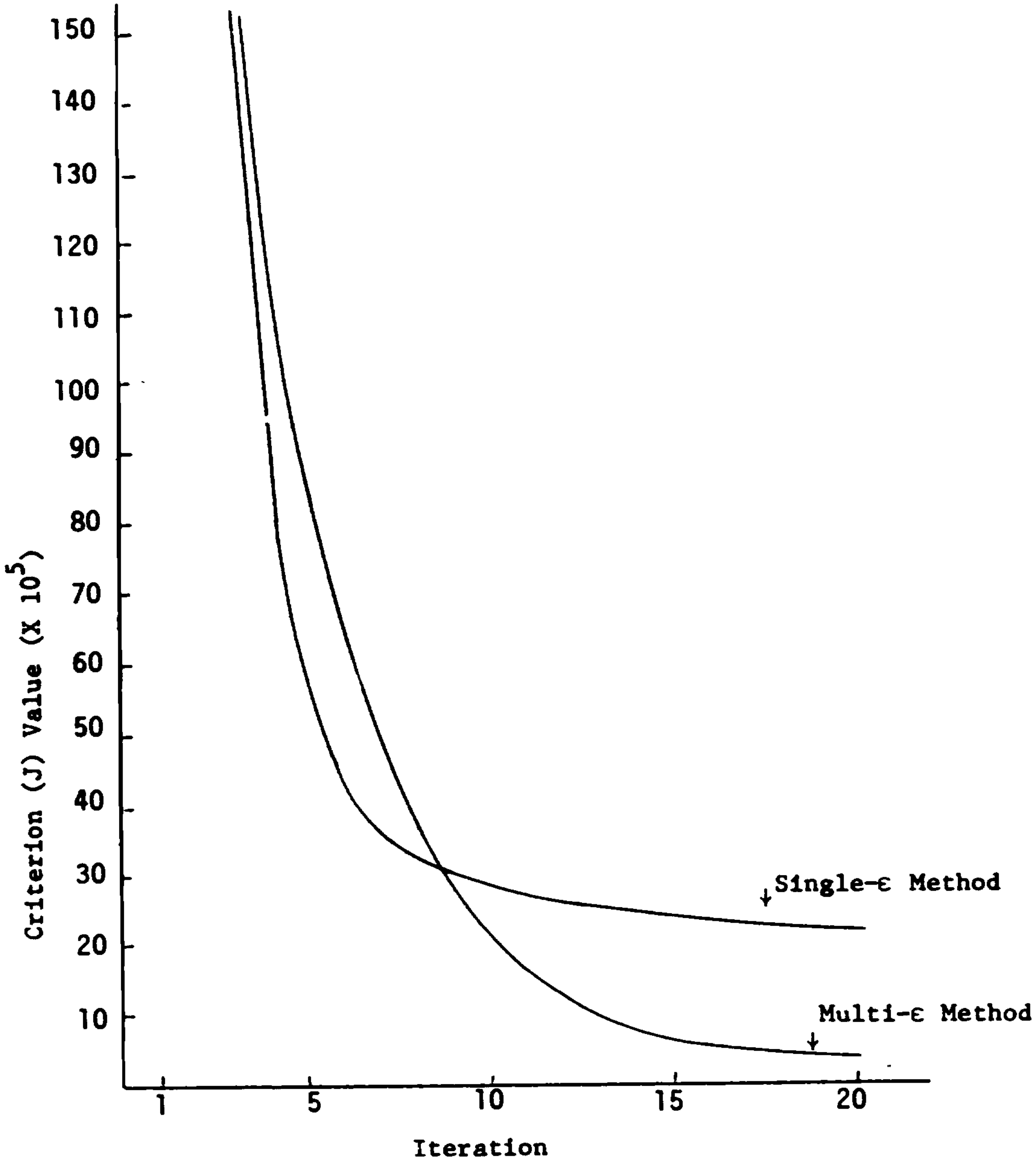
[15] $H \equiv V(x,u,t) + \lambda^t(t+1)[Fx(t) + Gu(t)]$.

of the gradient which then resulted in too small an incremental step-size. Moreover, attempts to correct this by starting with a large initial ε-value resulted in divergence of the solution algorithm. These problems with the single-ε method led to the development of the multi-ε method. The result was a much quicker overall convergence.

An example of the difference in convergence properties for the two methods is shown in Figure 14. There is a break-even point between the speed of convergence for the two methods; consequently, a preferable mixed strategy would be to change from the single-ε method to the multi-ε method when the slopes of the two curves are equal. However, there is no method available at this time for determining where this point occurs.



Cycle	Single-ε Method	Multi-ε Method
1	604	604
2	292	340
3	151	160
4	86	120
5	56	90
6	42	67
7	35	50
8	32	38
9	29	28
10	28	21
11	27	15
12	26	12
13	25	9
14	25	7
15	24	6
16	23	5
17	22	4
18	22	4
19	21	3
20	21	3

FIGURE 14: SINGLE-ε METHOD VS. MULTI-ε METHOD

Comparison of Three Solution Algorithms

Finally, we present a comparison of the first-order gradient and multi-ϵ methods and the second order gradient method. Table 14 compares similar solutions to one of the decision formulations and shows that the second order method is much better at meeting the chosen targets than either of the other methods, while maintaining approximately a 1% tolerance on the budget target. For this reason, the second order method was primarily used on this analysis.

TABLE 14: SUMMARY OF SOLUTION ALGORITHMS

	Multi-ε	Gradient	Second Order
Budget Weight	.0001	.0000001	.000001
Faculty Weights	100,50.25	1000,800,500	1000,800,500
F and G matrices		UNCONSTRAINED MULTIPLE REGRESSION	

After 30 iterations with initial controls = 0

Ratios	Targets	Achieved	Achieved	Achieved
assoc/full	0.9	0.363	0.582	0.897
asst/full	1.5	0.245	0.426	1.480
inst/full	0.05	0.024	0.016	0.035

Salaries	Budget		% Diff.		% Diff.		% Diff.
1	88516.	88507.	.01	59102.	32.23	89568.	1.19
2	99138.	99149.	.01	57702.	41.8	98171.	.08
3	111034.	111039.	.01	5,397.	48.31	110989.	.05
4	124358.	124373.	.01	56401.	54.65	119774.	3.69
5	139281.	139276.	.01	55845.	59.91	139181.	.08
J-value		107.7		2728.		25.32	

V. CONCLUSIONS AND FUTURE RESEARCH

This analysis has described the mathematical structure of an academic faculty by a linear dynamic model whose parameters were estimated from actual data by two different techniques. The new faculty hiring decision problem was formulated as an optimal control problem defined in terms of desired faculty distributional targets. Finally, either total budget or total position constraints were imposed upon the system through a penalty function. This multi-stage optimization problem subject to dynamic constraints was then solved by three alternate numerical methods.

While many interesting and illuminating results were obtained and discussed in this paper, a number of unresolved problems remain. The proper estimation of meaningful transition matrices is difficult without a long history of individual faculty flows. The transition probabilities used in this analysis were based on aggregate data available for an entire institution and consequently obscured individual faculty time paths through the rank structure. If this kind of model is to be used for institutional decision making, a more complete and disaggregated data base would be needed.

The criterion functions used in this analysis focused exclusively on the number and mix of faculty independent of student enrollments, available facilities, support staffs, or other relevant institutional parameters. This deliberate abstraction enabled a first step of demonstrable computational feasibility; however, the scope of the model should be expanded in future formulations to include at least the student body, the physical plant and the financial parameters of the system.

Uncertainty is explicitly included in the system dynamics in the form of a random additive error term. This presumes that the transition probability matrices are known and fixed, which certainly is not the case. A full analysis of uncertainty would include random transition matrices as well as random errors and would recognize the improved information available through sampling in the future - information which would provide new, updated estimates of transition probabilities.

The conceptual formulation used in this analysis is most appropriate for a campus or system executive officer and provides little guidance to the chairman of a department. Presumably, the department chairman can keep most of the relevant parameters in mind while he considers possible new faculty members. However, a college chancellor or president must balance many competing pressures calling for more faculty and would probably desire a more disaggregate model than the one discussed in this paper. It would be feasible to extend the current model to the discipline or departmental level and to recognize additional categories of faculty including visiting and irregular faculty and teaching assistants. This would necessitate a larger computer program and a calculation time longer than the current 1-2 minutes on an IBM 360/65; one hundred organizational units or groupings and ten categories of faculty could easily be accommodated.

This analysis is only a beginning. It shows that a decision and control formulation, which is more comprehensive and informative than simulation analysis, is economically feasible and potentially very useful in the analysis of faculty hiring decisions. The problems and difficulties of this approach deserve the attention of quantitative analysts. The promise and potential of this approach deserve the serious consideration of academic administrators.

APPENDIX A: SOLUTION METHOD

The solution of this new faculty allocation problem may be approached in a number of ways, one of which is the use of the techniques of modern control theory. Since the mathematical background of these methods is adequately described elsewhere,[1] only a brief discussion of the specific technique used will be presented here.

We begin the solution method by adjoining the linear propogation constraint to the criterion function with the use of a Lagrange multiplier, thereby reducing the problem to an unconstrained optimization problem.[2] Letting

$$V(i) = V[x(i), u(i), i] \tag{1}$$

and

$$f(i) = Fx(i) + Gu(i) + Hz(i) \tag{2}$$

we may write the augmented criterion function, $\bar{J}$, as

$$\bar{J} = V(N) + \sum_{i=0}^{N-1} \{V(i) + \lambda^t(i + 1)[f(i) - x(i + 1)]\} \quad . \tag{3}$$

We now define the Hamiltonian, $H(i)$, as

$$H(i) = V(i) + \lambda^t(i + 1)f(i) \tag{4}$$

[1] Bryson, A. E., Jr., and Yu-Chi Ho, Applied Optimal Control: Optimization, Estimation and Control, (Waltham, Massachusetts: Blaisdell Publishing Co.), 1969.

[2] For convex criterion functions with linear constraints the stationary solution of the adjoined criterion function is the same as the stationary solution of the original criterion function. In other cases, the properties of the criterion function and constraint set need to be investigated carefully.

and may subsequently write the augmented criterion function as

$$\bar{J} = V(N) + \sum_{i=0}^{N-1} [H(i) - \lambda^t(i + 1)x(i + 1)] \quad . \tag{5}$$

By changing the index of summation, we obtain

$$\bar{J} = V(N) - \lambda^t(N)x(N) + \sum_{i=0}^{N-1} [H(i) - \lambda^t(i)x(i)] + H(0) \quad . \tag{6}$$

For a stationary point, we require that the total differential of J be equal to zero along the entire sequence of allocations.[3] Therefore,

$$dJ = 0 = d\bar{J} = \left[\frac{\partial V(N)}{\partial x(N)} - \lambda^t(N)\right]dx(N)$$

$$+ \sum_{i=0}^{N-1}\left\{\left[\frac{\partial H(i)}{\partial x(i)} - \lambda^t(i)\right]dx(i) + \left[\frac{\partial H(i)}{\partial u(i)}\right]du(i)\right\}$$

$$+ \left[\frac{\partial H(0)}{\partial x(0)}\right] dx(0) + \left[\frac{\partial H(0)}{\partial u(0)}\right] du(0) \quad . \tag{7}$$

Since the initial state of the system, $x(0)$, is assumed given and fixed, $dx(0) = 0$. In order to force dJ to zero we require that

$$\lambda^t(i) = \frac{\partial H(i)}{\partial x(i)} \qquad i = 0, 1, 2, \ldots, N - 1 \tag{8}$$

subject to the boundary condition that

$$\lambda^t(N) = \frac{\partial V(N)}{\partial x(N)} \tag{9}$$

and also that

$$\frac{\partial H(i)}{\partial u(i)} = 0 \qquad i = 0, 1, \ldots, N - 1 \quad . \tag{10}$$

This provides us with the following first-order conditions necessary for an optimal allocation pattern:

[3] The requisite second order conditions necessary for a maximum or a minimum are given in Bryson and Ho, op. cit., Chapter 6.

(1) $x(i + 1) = Fx(i) + Gu(i) + Hz(i) \qquad i = 0, 1, \ldots, N - 1$ (11)

$x(0)$ given;

(2) $$\lambda^t(i) = \frac{\partial H(i)}{\partial x(i)} \qquad i = 0, 1, \ldots, N - 1$$

$$= \frac{\partial V(i)}{\partial x(i)} + \lambda^t(i + 1)F \tag{12}$$

$$\lambda^t(N) = \frac{\partial V(N)}{\partial x(N)} \tag{13}$$

and

(3) $$0 = \frac{\partial H(i)}{\partial u(i)} \qquad i = 0, 1, \ldots, N - 1$$

$$= \frac{\partial V(i)}{\partial u(i)} + \lambda^t(i + 1)G \quad .$$

It should be noted that nowhere in the previous discussion are there any specific limitations on the format of the preference function, $V[x, u, i]$, other than certain convexity requirements[4] and the necessary inclusion of at least two non-empty variables sets: $u(i)$ and $x(i)$. In other words, the formulation of the preference function is left completely up to the user.

[4] See footnote 2.

APPENDIX B: OPCON: A USER'S MANUAL

A copy of both the user's manual and the program are available upon request and may be obtained by writing

Dr. George B. Weathersby
Ford Foundation Research Program
Office of the Vice President -
Planning and Analysis
247 University Hall
University of California
Berkeley, California 94720.

A nominal fee will be charged for duplicating and loading the program. The manual is available at no charge. Requests for further information should be sent to the same address.

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