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ABSTRACT

In this paper the authors argue that the decision structures of educational institutions are multi-level, multi-decision-maker hierarchies which can be described and analyzed in decision theoretic terms and that these multi-levels, multi-decision-maker hierarchies can be reduced to equivalent one-level, one-decision-maker formulations, which can be solved either analytically or numerically by the techniques presented. Illustrative examples are given which identify and then solve for a set of optional resource allocation and policy decisions. The computer program used for the problem and the input data specifications are included in an appendix. A 24-item bibliography is included. (Author)

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OPTIMALITY IN COLLEGE PLANNING:
A CONTROL THEORETIC APPROACH

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PREFACE

This is one of a continuing series of reports of the Ford Foundation sponsored Research Program in University Administration at the University of California, Berkeley. The guiding purpose of this Program is to undertake quantitative research which will assist university administrators and other individuals seriously concerned with the management of university systems both to understand the basic functions of their complex systems and to utilize effectively the tools of modern management in the allocation of educational resources.

In this paper we argue that the decision structures of educational institutions are multi-level, multi-decision-maker hierarchies which can be described and analyzed in decision theoretic terms. Furthermore, we show that these multi-level, multi-decision-maker hierarchies can be reduced to equivalent one-level, one-decision-maker formulations, which can be solved either analytically or numerically by the techniques presented in this paper. An illustrative example is given which first identifies and then solves for a set of optimal resource allocation and policy decisions. A listing of the computer program used in this problem and the input data specifications are included in the Appendix.

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INTRODUCTION

Institutions of higher education currently face a number of major policy choices which will largely determine their character for the next twenty-five years. The tremendous expansion of American higher education in the last twenty-five years was driven by burgeoning enrollment growth and by massive federal commitment to doctoral production in the sciences and technologies. Both of these forces are abating rapidly. Nationally, enrollments in higher education are forecasted to peak in 1980, then decline until the late 1980's and not approach the 1980 level until after 1995. Many schools are now experiencing enrollment levels below their previous expectations. This is not a short-run phenomenon; rather, current enrollment shortfalls are harbingers of the next twenty-five years. Colleges and universities must learn to survive and to prosper with a decreasing demand for their services.

It is far less likely that in the future the federal government will rescue the expectations of higher education as they have done in the post-Sputnik era. The United States will probably have a surplus of highly trained scientific and technical manpower for at least the next decade without major additional federal expenditures [Brode (1971)]. The reduced rate of undergraduate enrollment expansion will drastically reduce the number of new teachers needed in colleges and universities, thereby reducing the future demand for additional Ph.D.'s and, therefore, the need for large doctoral programs. Furthermore, the federal priorities have shifted from scientific manpower to equality of student access and the quality of educational experience. Both of these major federal objectives impact undergraduate education far more than graduate programs and they move counter to many institutions' prestige and elitist orientations.

While the demands for educational services by students and governments will probably be decreasing in the coming decades (first in rate of growth and then in absolute number), the costs of educational institutions continue to rise. As Cheit [1970] has pointed out, a significant number of America's colleges and universities are headed for financial difficulties and current institutional rigidities preclude those cost adjustments necessary to maintain fiscal viability.[1] Furthermore, the technology of education has changed very little in the last three or four decades; indeed, some would argue that educational technology has changed very little since Socrates. In essence, there has been no observable productivity increase in American post-secondary education in the last four decades [O'Neill (1971)].

If it were not so painful, we might examine with considerable intellectual interest the experience of public and private eleemosynary institutions beset by diminishing demands for services and rising costs, increasing institutional rigidities and no productivity increases.[2] Unpleasant as it may be, educational administrators are having to ask the

[1] The "varieties of the financial crisis" are explored by Balderston [1971] in a recent paper prepared for the American Council on Education.

[2] This description closely resembles the experience of the American railroads. One inciteful observation on the decline of the railroad companies was that unfortunately railroad managers viewed their industry as "railroads" versus "transportation." At the time of burgeoning new modes of transportation, the railroad companies were in an excellent position to diversify and expand--but that was neither their tradition nor their self-concept. Are our schools in the "formal instruction" industry or the "education" industry?

tough questions: What are our objectives? How would we know if we achieved them? How can we reallocate resources to be more productive? What activities are really essential for an educational institution? Who should make these decisions; and many others?

The purpose of this paper is to look at institutional resource management decisions in the context of institutional goals and objectives. After describing one view of institutional decision making, we present a simple yet comprehensive mathematical model which explicates the interrelationships of major institutional variables. Sample data are then used to derive resource allocations which would be optimal for the institution. The use of this model in educational policy analysis is then discussed before presenting our conclusions.

INSTITUTIONAL DECISION MAKING

The decision making structures of educational institutions are as diverse as the institutions themselves. Some colleges and universities are highly authoritarian while others are highly egalitarian; some institutions are ruled by presidents and others by committees. Some educational systems have many layers of administrative superstructure while others do not. There is often little resemblance between the organizational structure and the decision or power structure of an institution. Often individuals with no delegated authority have great influence on decisions.

While these complex interactions have been analyzed from many perspectives,[3] we have chosen to analyze institutional decision making from a decision theoretic basis. Initially we distinguish between the values used in arriving at policy decisions and the authority structure in which the decisions are made. Focusing first upon the structure of decisions, we observe that the decision structures of most educational institutions are hierarchical, with students, individual faculty members, department chairmen, deans, provosts, and presidents playing different, but important, decision making roles.

These roles are distinguished primarily by the variables each level can control. For example, students decide which of the available courses they will take; faculty decide how to allocate their time between formal

[3] Wildavsky [1964] looks at the resource allocation process in government from a political theory perspective; Downs [1967] and Braybrooke and Lindblom [1963] view bureaucratic decision making as a behavioral and organizational process; Glenny [1969] and Palola [1970] approach educational decision making from the perspective of governance.

instruction, preparation, informal meetings with students, research, committees, community service, professional advancement, and leisure; department chairmen decide, with consultation to be sure, the course and committee assignments of faculty, the allocation of support services, recommendations on salaries and promotions, and curriculum proposals; deans allocate new faculty positions to departments, increasingly will reallocate faculty positions between departments, determine salaries and promotions, establish departmental budgets, endorse curriculum changes, and approve research programs; provosts or presidents in turn allocate faculty positions and budgets between schools, review or approve salaries and promotions, recruit deans, approve curricula and academic programs. Table 1 summarizes some of these distinctions.

Another characteristic of the hierarchical structure of educational decision making is the direct interrelationships of the various decision making levels. As illustrated in Table 1, the control variables at one level often become constraints at the next lower level. For example, the president can allocate faculty positions to the various schools in his institution to the limit of his budget. In turn, deans can allocate faculty to departments up to the limit permitted by the president's budget. What was initially a decision to the president later becomes a constraint to the dean.

Another component of our analysis of institutional decision making is the distinction between the implementation structure and the decision structure. The implementation structure is usually reflected in the institution's organization chart; it is the array of deans, department chairmen, accounting officers, purchasing agents, budget officers, admissions officers, registrars, librarians, and all the other functional

TABLE 1

Examples of Interrelationships of Institutional Decision Makers

Decision Makers	Control Variables	Constraints
President/Provost	Budgets	Income or Appropriations
	Faculty Positions	
	Program Approval	
Dean	Departmental Budgets	President's Budget
	Faculty Positions	President's Budget
	Program Approval	President's Approval
Department Chairmen	Faculty Assignments	Dean's Budget
	Support Services	Dean's Budget
	Salaries and Promotions	Dean's Budget
	Curriculum	Dean's Approval

specialists who keep an institution running effectively. On the other hand, the decision structure is rarely reflected in a school's organization chart. At issue here is who is responsible for what decisions and how are the recommendations for these decisions made.

The operations cycle of an institution is illustrated in Figure 1. Once made, a decision is communicated to the implementation structure where functional specialists establish the operating policies and procedures which actually move the organization in the desired direction. These implementation managers need operating data for their effective functioning. For example, the accounting officer needs payroll information to process checks and charge the appropriate accounts. On the other side of this circle, those who are charged with recommending decisions need institutional data to evaluate past decisions and as input to future decisions. In addition, external data on student demand, manpower supply and demand, community needs, attractive research areas and a variety of other issues are needed for decision recommendations. The process of synthesizing these data and institutional objectives into a coherent, consistent strategy for future action is nebulous if not nonexistent at most institutions. Yet, if educational leaders are going to be able to deal effectively with the serious social and economic challenges confronting their institutions, much more attention will have to be devoted to their decision structures. The approach and the mathematical model presented in this paper is one small step in this direction.

In addition to the process of decision are the values upon which the decision is based. One of the functions of each decision maker is to choose the values appropriate for the decisions at hand. This is another way of raising the question of governance: Who will

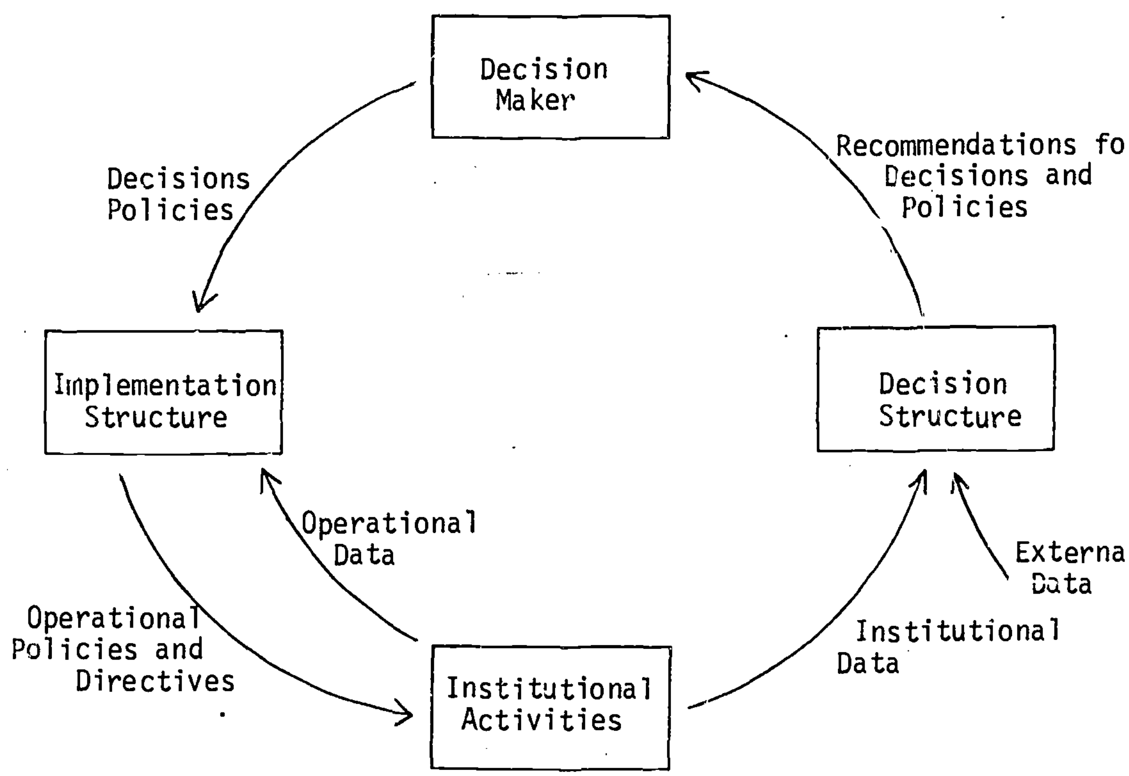

FIGURE 1

Operations Cycle of an Institution:
Decision - Execution - Evaluation - Decision

decide and whose values will he use when he decides? Furthermore, what attributes of the educational system are important to the decision maker, what does he consider to be the outputs of his system?[4] How important are more undergraduates versus more graduate students, more researchers versus more instructors, more computing power versus more library services, more faculty versus more facilities, and a thousand-and-one other possible tradeoffs? Taken together, all of these choices and tradeoffs comprise a decision maker's value system.

These value systems also serve to connect the hierarchical decision systems which were discussed earlier. In many cases, the president of an institution is deeply concerned about the classroom environment and the interaction of students and faculty, even though he cannot directly control any of the operative variables. However, the president often makes his budgetary and faculty allocations with their educational consequences in mind and adjusts his allocations to correspond to his assessment of the educational use to which these resources are put. In other words, the consequences of decisions at a lower level are important to decision makers at higher levels.

There is a circular flow of information in a hierarchical decision system: decisions are passed downward and value signals are passed upward. These in turn affect the decisions which are passed downward in a subsequent cycle, as shown in Figure 2. It is this two-way flow of information that makes delegated authority operational and renders a decentralized or hierarchical system controllable; this notion of decentralized control will be explored in more detail shortly.

[4]The outputs of higher education have received increasing attention in the last few years; see Lawrence, Weathersby and Patterson [1970], Breneman and Weathersby [1970] and Huff [1971].

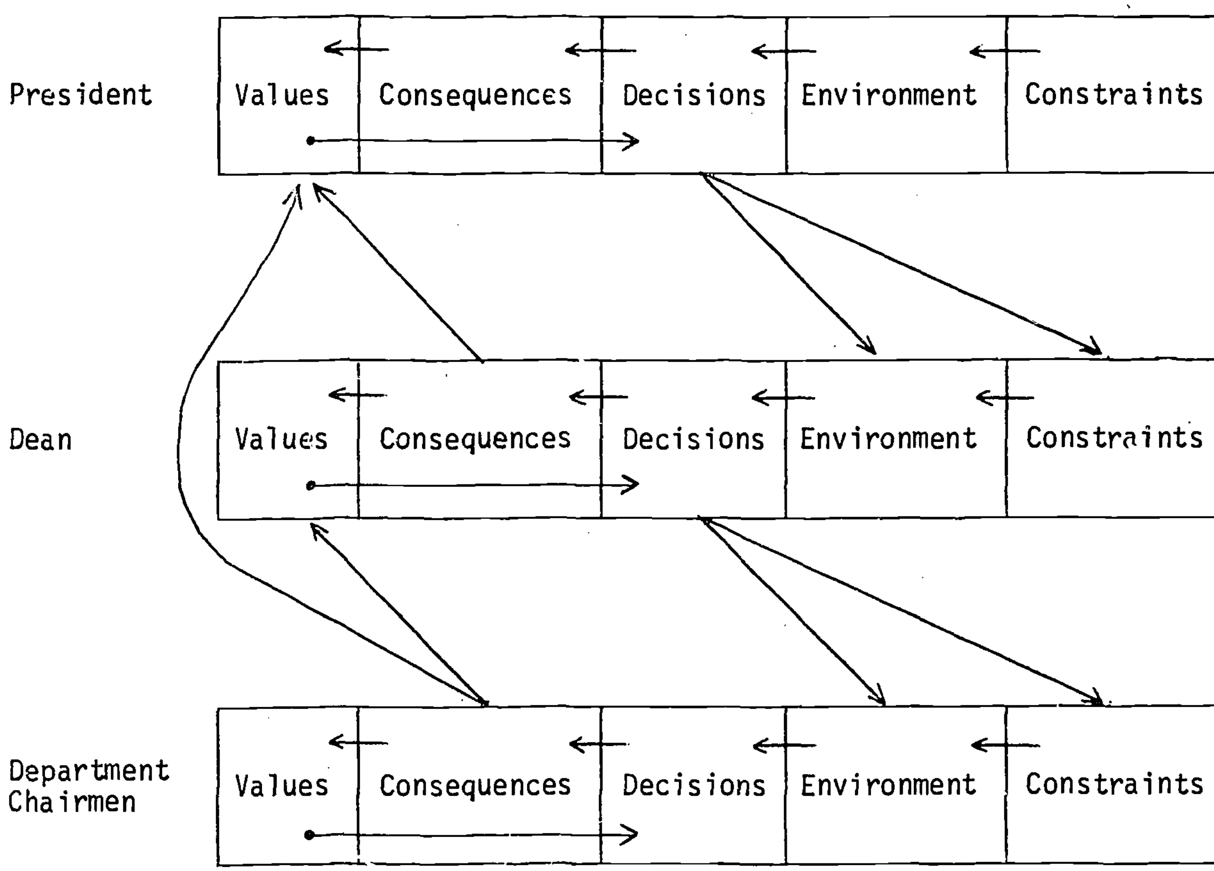

FIGURE 2

Interrelationship of Decisions and Values in a Hierarchical Decision Structure

Analytical Description of Institutional Decision Making

This conceptual analysis of educational decision making can be made more precise by describing the decision interrelationships in mathematical terms. To begin with, we need some definitions:[5]

$u_i(t)$ = the vector of decision variables available to decision maker (DM) i in period t ;

$z_i(t)$ = the vector of predetermined variables impinging upon the system relevant to DM i in period t ;

$x_i(t)$ = the vector of consequences (or state variables) in period t resulting from the decisions of DM i and relevant exogenous influences.

The relationship of consequences to decisions (or output to input) is often called the production function:

$$x_i(t+1) = f_i(x_i(t),\ u_i(t),\ z_i(t),\ t). \qquad (1)$$

Finally, the value to DM_i of making a decision $u_i(t)$ when confronted with the predetermined variables $z_i(t)$ is written

$$V_i(x_i(t),\ u_i(t),\ z_i(t),\ t). \qquad (2)$$

Expressions (1) and (2) describe the horizontal flows shown in Figure 2 at each decision making level.

The decision problem faced by each administrator is to maximize his own values subject to his constraints of authority and resources and subject to the responsiveness of his system to the application of policy or resource decisions. Furthermore, a decision maker often looks several

[5]For a complete exposition of this approach and the motivation for these definitions, see Weathersby [1969a, 1969b].

years in advance and wants to maximize his values over a planning horizon of N periods. We may write this decision problem as:[6]

$$\max_{u_i(o),\ldots,u_i(N-1)} \left\{ J = \sum_{t=o}^{N} V_i\left[x_i(t),\ u_i(t),\ z_i(t),\ t\right] \right\} \qquad (3)$$

subject to:

$$x_i(t+1) = f_i\left[x_i(t),\ u_i(t),\ z_i(t),\ t\right] \quad t=o,1\ldots,n-1 \qquad (1)$$

$x_i(o)$ known

and

$$C\begin{bmatrix} x_i(t) \\ u_i(t) \end{bmatrix} \leq b_i(t) \qquad t=o,1\ldots,n-1 . \qquad (4)$$

In this formulation, C is the constraint function and $b_i(t)$ are the resource and other constraints relevant to DM_i in period t .

In general, there is a solution $u_i^*(t)$, t=0,1,...,n-1 which maximizes the overall value function, J , provided the necessary and sufficient conditions are satisfied.[7] Furthermore, an optimal solution is in general a function of all preceding variables.[8]

$$u_i^*(t) = g_i\left[u_i^*(o),\ldots,u_i^*(t-1),\ x_i(o),\ldots,x_i(t),\ z_i(o),\ldots x_i(t),\right.$$
$$\left. b_i(o),\ldots b_i(t),\ t\right] . \qquad (5)$$

---

[6] The general form of the N period value function is $V_i\left[x_i(o), x_i(1),\ldots,x_i(N),\ u_i(o),\ u_i(1),\ldots,u_i(N-1),\ z_i(o),\ldots,z_i(N-1)\right]$ which can be separated into $\sum_{t=o}^{N-1} V_i\left[x_i(t),\ u_i(t),\ z_i(t),\ t\right] + V_i\left[x_i(N),\ N\right]$ by the assumption of weak separability (Weathersby [1969a]). Notice that present value discounting is a special case of the time function of $V(x,u,z,t)$.

[7] The solution procedure will be described in more detail later and the algorithm used in this study is described in the Appendix.

[8] See Aoki [1969] for a discussion of general recursive solution.

The consequences which result from an optimal decision sequence can be calculated from equation (1);

$$x_i^*(t+1) = f_i\left\{x_i(t),\ u_i^*(t),\ z_i(t),\ t\right\}$$

$$= h_i^*\left\{x_i(o),\ldots,x_i(t),\ z_i(o),\ldots,z_i(t),\ b_i(o),\ldots,b_i(t),t\right\} \quad (6)$$

after equation (5) is substituted for $u_i^*(t)$. In other words, when the parameters of the decision problem are known, i.e., equations (1), (3), (4), and (5), one can replace the decision problem by equation (6) which describes the consequences of an optimally controlled system.

In a strict hierarchical decision structure, there is one such decision problem for each decision maker. In the three level hierarchy shown in Figure 3, n deans report to the president and m_i department chairmen report to the i^{th} dean. The interrelationships are:

1. At levels 1 and 2, the predetermined variables $z(t)$ and resource constraints $b(t)$ can be controlled or influenced at the next higher level. For example,

$$x_{1,1}(t+1) = f_{1,1}\left\{x_{1,1}(t),\ u_{1,1}(t),\ z_{1,1}(t),\ u_{2,1}(t),\ z_{2,1}(t),\ u_{3,1}(t),\ z_{3,1}(t)\right\}.$$

2. The values associated with the consequences of decisions by decision-makers at levels 2 and 3 can include the decisions and consequences of lower level decision-makers. For example:

$$V_{3,1} = V_{3,1}\left\{x_{3,1}(t),\ u_{3,1}(t),\ z_{3,1}(t),\ x_{2,1}(t),\ x_{2,2}(t),\ldots,x_{2,n}(t),\ u_{2,1}(t),\ldots,u_{2,n}(t),\ t\right\}.$$

The first interrelationship describes the downward flow of decisions either directly or indirectly while the second form of interrelationship describes the upward flow of accountability or value. Both of these interrelationships

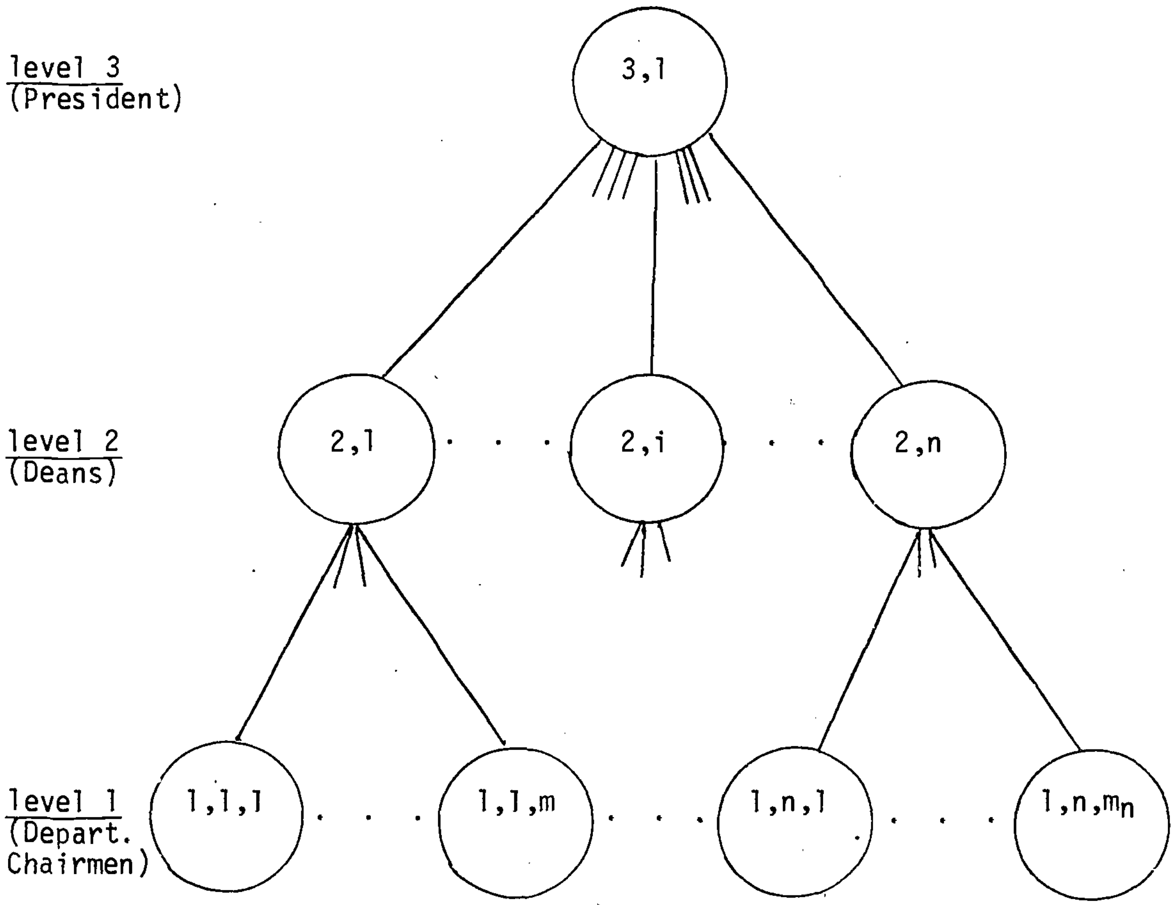

FIGURE 3

Array of Decision Makers in Three Level Hierarchy

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are necessary for a controllable hierarchical system; however, they are not sufficient conditions for total systems controllability.

The basic strategy of solution of a strict hierarchical decision structure (i.e., interrelationships only between adjoining levels) is to reduce the structure down to a one decision maker problem by folding up from the bottom.[9] This approach would replace each decision making node at level 1 in Figure 3 by his corresponding optimal decision function $g^*_{1,i,j}$ and the corresponding optimal production function $h^*_{1,i,j}$. For each level 2 decision maker, the m_i departmental chairmen's decision

[9] For a complete presentation of this approach, see Weathersby [1969b].

problems are replaced by $2m_i$ vector equations for the g^* and h^* functions. These $2m_i$ vector equations then are effectively production functions to the i^{th} dean which augment his own production function. Now we can fold the dean's decision problems up to the presidents' level by the same technique. Thus, we can collapse a multi-layer, multi-decision maker hierarchy to an equivalent one-decision-maker problem. Correspondingly, if we can solve the one-decision-maker decision problem, we can conceptually solve the multi-layer, multi-decision maker problem. Therefore, the remainder of this study will focus on the single decision maker problem.

The President's Model: Single Decision Maker Paradigm

The basic decision problem of the president is to maximize the achievement of his own values, or the values he chooses to operate with as president, subject to resource limitations and the responsiveness of his institution. The formal statement of this problem was given previously in equations (3), (1), and (4). The three major components of the problem are: (1) the president's value function; (2) the institutional response or production function; and (3) the resource, legal and other constraints.

One obvious difficulty with the decision theoretic formulation is that generally presidents, and other administrators, cannot articulate their value function. Most of us are not trained to think in terms of multi-attributed utility functions and, therefore, any approach which requires a mathematical description of a decision maker's value function is destined to grave difficulty if not failure.

There have been two major techniques for circumventing the assessment difficulties associated with a full description of the value function.

Geoffrion and Dyer [1970, 1971] have shown that one need only assess the local gradient of the value function at the current operating point. In their work, they ask a dean or department chairman to select one of his variables as a numeraire and then assess the pairwise tradeoffs of all the other policy relevant variables with respect to the chosen numeraire. This is the local gradient which shows an improving direction along which the decision maker selects a new and improved operating point. At this new point, however, the local gradient must be assessed again because it is generally different at every point on the utility surface. In other words, the Geoffrion and Dyer approach replaced a global assessment of the multidimensional value surface with a series of local assessments of the tangent plane, which is a much easier task. This requires interaction between the decision maker and the mathematical programming algorithm because the path along the value surface is unpredictable a priori.

A second approach to the reduction of the dimensionality of the value assessments is to express the decision-maker's objectives in terms of targets. This is the approach used in the study reported in this paper. For simplicity of exposition, consider a president's value function that is defined over only the consequences of state vector x, i.e., $V(x)$, and that the president wants to achieve a most desirable level of x, say x^*. In other words, the president believes that the optimal state of his institution would be a student enrollment of 10,000 with 1,000 faculty members, 600 of whom would be tenured, and so forth.

The key to the target approach is that if the institution is initially reasonably near the desired targets, then the general utility maximization problem can be expressed by an approximately equivalent loss minimization problem where the loss function is quadratic, independent of the form of the utility function as long as the utility function is twice differentiable, i.e., smooth and continuous. Although we may know nothing more about the president's utility function than that it ought to be concave and smooth,[10] we do know that in the neighborhood of his targets his loss function is quadratic to second order.

By choosing targets x^* , a decision maker indicates that

$$V(x^*) > V(x) \quad \text{for all } x \,. \tag{7}$$

For x near x^* , we can expand $V(x)$ about x^* by Taylor series

$$V(x) = V(x^*) + \nabla V\big|_{x^*}(x-x^*) + 1/2(x-x^*)^t\, \nabla^2 V\big|_{x^*}(x-x^*) + \text{Higher Order Terms.} \tag{8}$$

If $V(x^*)$ is a maximum, as indicated by expression (7), then the local gradient must be zero at x^* and the second right hand side term in equation (8) must be zero. Furthermore, the second derivative of V must be negative definitive for x^* to be a strict maximal point. Therefore, the third right hand side term in equation (8) must be negative for all x . This argument proves that, to second order,

$$\max_x V(x) \doteq \min_x 1/2\left\{(x-x^*)^t \nabla^2 V\big|_{x^*}(x-x^*)\right\} \,. \tag{9}$$

One point of indeterminancy remains in equation (9); in general, the matrix of second partial derivatives of V is not known. Two approaches may be used here. One can ask the decision maker to choose a numeraire and assess the relative pairwise comparison losses that he would exper-

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[10] We can assess the relative relationship between the first and second derivatives of a decision-maker's utility function by a discussion of his risk aversion (Pratt [1964]).

ience at x^* and use this one set of assessments in place of $\nabla^2 V$. Alternatively, one could recognize that the magnitudes of $\nabla^2 V$ change the relative shape of the quadratic loss structure but not its minimum, which is x^*. Near the minimum, the solution to (9) is often insensitive to the global shape of $\nabla^2 V$ and a much simpler procedure is possible: namely choose an arbitrary weighting matrix K in place of $\nabla^2 V$ such that the magnitudes of loss of one unit variation in every dimension are identical. Both of these approaches require minimal assessment.

In summary, we have argued that the decision structures of educational institutions are multi-level, multi-decision-maker hierarchies which can be described and analyzed in decision theoretic terms. Furthermore, we have argued that these multi-level, multi-decision-maker hierarchies can be reduced to equivalent one level, one decision-maker formulations. In turn, these single decision-maker problems can be solved either analytically or numerically by the techniques discussed in this and the following section. We now proceed with the formulation and solution of a specific decision model and discuss its implications.

SMALL CAMPUS PLANNING MODEL

The concepts of the previous section are illustrated in this section in a specific analytical modeling context. For the purposes of exposition, we have focused on the instructional program of an institution partly because this seems to be an area of great interest to most colleges and partly because instructional activities have far more in common among institutions than the various research and public service programs. The paradigm of this model is the liberal arts undergraduate institution or that component of a major university.

The Small Campus Planning Model (SCPM) is designed to provide a control theoretic solution to the problem of finding an optimal sequence of new student admissions, new faculty hires, and new physical construction over an N-year planning horizon. It assumes that the flows of students, faculty, construction, and money can be characterized by linear dynamic equations and that the campus administrator's preferences for student and faculty mix, for space, and for solvency are sufficiently close to the institution's current experience that actual deviations from targets can be adequately expressed in terms of quadratic penalty functions.

The model is still in the investigatory stages and will undoubtedly undergo further revision before it is considered a finished product. Ultimately it is hoped that SCPM, because of its minimal data requirements and low implementation and calculation costs, may serve as a useful planning device for college administrators who have neither the funds nor the data base to support implementation of other, more complex, models.[11]

[11]For a structural comparison of other recent analytical models for university planning, see Weathersby and Weinstein [1970].

SYSTEM DYNAMICS OF SCPM

The generalized form of SCPM's dynamics may be characterized by

$$x(t+1) = F_t\, x(t) + G_t\, u(t) + H_t\, z(t) \qquad (1)$$

where:

$x(t)$ = n-vector of state variables at time t
$u(t)$ = m-vector of control variables at time t
$z(t)$ = n-vector of predetermined variables at time t
F_t = n x n matrix of transition coefficients for period t
G_t = n x m matrix of transition coefficients for period t
H_t = n x n matrix of transition coefficients for period t .

It is assumed in SCPM that $F_t = F$, $G_t = G$, and $H_t = H$ for all t , i.e., that the transition matrices are not time dependent. This is not necessary for solution, but facilitates estimation and reduces substantially the data requirements. Furthermore, these matrices are not Markovian, i.e., the row sums do not total 1.0, because the absorption states for students, faculty, space and money are excluded. Under the stationarity assumption, equation (1) becomes

$$x(t+1) = F\, x(t) + G\, u(t) + H\, z(t) \quad . \qquad (2)$$

The state, control, and predetermined variables as defined in SCPM are divided naturally into four groups: students, faculty, space, and money. We consider each separately for purposes of exposition.

Faculty

We define the variables $x_1(t)$ to $x_4(t)$ to be the number of full-time equivalent full professors, associate professors, assistant professors and instructors who were in the institution last year (at time t-1) and who remained in the system at the start of this year (period t). Similarly, we define the variables $u_1(t)$ to $u_4(t)$ to be the number of faculty who are hired at corresponding ranks at the start of period t. Then, expressing the matrices F and G by their elements f_{ij} and g_{ij}, we have

$$
\begin{aligned}
x_1(t+1) &= f_{11}x_1(t) + f_{12}x_2(t) + g_{11}u_1(t) + g_{12}u_2(t) \\
&\vdots \\
x_4(t+1) &= f_{44}x_4(t) \qquad\qquad + g_{44}u_4(t) \ . \qquad (3)
\end{aligned}
$$

Here, f_{ij} is the promotion rate of faculty from level j in period t to level i in period $t+1$, f_{ii} is the continuation rate for faculty at the same rank, and g_{ij} is the promotion and continuation rates of new faculty who were hired in period t at level j and who are at level i in period $t+1$.

Students

We define the variables $x_5(t)$ to $x_8(t)$ to be the number of continuing freshman, sophomore, junior and senior students at the start of period t. Similarly, we define the variables $u_5(t)$ to $u_8(t)$ to be the number of students admitted to the corresponding student levels at the start of period t. Once again, we may write the scalar equations

$$
\begin{aligned}
x_5(t+1) &= f_{55}x_5(t) \qquad\qquad + g_{55}u_5(t) \\
&\vdots \\
x_8(t+1) &= f_{87}x_7(t) + f_{88}x_8(t) + g_{87}u_7(t) + g_{88}u_8(t) \ . \qquad (4)
\end{aligned}
$$

The coefficients f_{ij} and g_{ij} have the same advancement and retention interpretations as before. Attrition of faculty and students is accounted for by omission of state variables corresponding to the "out" state. For manpower planning or other purposes one could define two additional states of successful degree completion and "stopping out." This would provide specific degree output information and render the student system Markovian.

Space

We assume that physical construction takes an average of four years to complete once it has begun. A conscious simplification at this stage is the assumption of fully interchangeable space types and uses. Equation (5) could be repeated for each space type if the additional detail would be worth the additional cost. SCPM also assumes a constant depreciation rate of $\alpha = [1-f_{12,12}]$. Accordingly we define

$u_9(t)$ = amount of new construction measured in Assignable Square Feet (ASF) which begins in period t

$x_9(t)$ = ASF begun in period $t-1$

$x_{10}(t)$ = ASF begun in period $t-2$

$x_{11}(t)$ = ASF begun in period $t-3$

$x_{12}(t)$ = ASF which is available and usable at the start of period t .

Thus,

$$
\begin{aligned}
x_9(t+1) &= u_9(t) && \text{i.e., } g_{9,9} = 1.0 \\
x_{10}(t+1) &= x_9(t) && \text{i.e., } f_{10,9} = 1.0 \\
x_{11}(t+1) &= x_{10}(t) && \text{i.e., } f_{11,10} = 1.0 \\
x_{12}(t+1) &= x_{11}(t) + f_{12,12}\, x_{12}(t) && \text{i.e., } f_{12,11} = 1.0 \ .
\end{aligned}
\tag{5}
$$

Money

Finally, we assume that there are two kinds of funds which adequately describe the administrator's financial concerns: restricted funds (endowment) and unrestricted funds (operating plus capital funds). Once again, these fund categories could be expanded if needed. It is further assumed that interest earned on endowment funds may be allocated arbitrarily between funds, but that the income and capital gains use policy is fixed in advance. If $f_{14,j}$ is the value (cost if negative) of one unit of x_j and $g_{14,j}$ is the value (cost) of one unit of u_j , and performing all calculations in constant dollars, we can define

$z_{13}(t)$ = restricted gifts in period t (estimated or assumed known)
$z_{14}(t)$ = unrestricted gifts in period t (estimated or assumed known)

and write:

$$x_{13}(t+1) = f_{13,13}x_{13}(t) + h_{13,13}z_{13}(t)$$
$$x_{14}(t+1) = \sum_j f_{14,j}x_j(t) + \sum_j g_{14,j}u_j(t) + h_{14,13}z_{13}(t) + h_{14,14}z_{14}(t) \quad (6)$$

While equation (6) looks quite complicated, each of its components is very simple and traditional.

$f_{14,1}$ to $f_{14,4}$ = the average faculty salary by rank including direct support costs.

$f_{14,5}$ to $f_{14,8}$ = the net institutional cost per student by level (excluding faculty salaries and direct faculty support costs and including student fees and tuition).

$f_{14,9}$ to $f_{14,12}$ = the average cash flow cost per ASF in each year of new construction.

$f_{14,13}$ = the average rate of return on endowment that is available for operating expenses.

$f_{14,14}$ = the proportion of last year's net cash balance available in the current year (usually 1.0).

The numerical values of these and the other coefficients used in the computational example are given in Table 3. Note that equation (2) is just the aggregate of equations (3) through (6). If the state vector $x(1)$ is known and the gift funds $z(t)$, $t = 1,\dots,N-1$ are predicted, then the control sequence $u(t)$, $t = 1,\dots,N-1$ with (2) determines $x(t)$ for all future periods $t = 1,\dots,N$. After making various assumptions about the likely levels of state aid and gifts in the future, SCPM determines optimal enrollment, hiring, and construction policies for a given set of institutional objectives, which are described in the next section.

Student tuition could be included as a control variable instead of a predetermined factor. This would more accurately reflect the decisions of most private institutions and a growing number of public institutions. However, two major problems have to be dealt with to include tuition as a unconstrained control variable. A conceptual problem is the effect of additional tuition on student demand for attendance and on the quality of students able to pay the higher tuition.[12] A minor technical problem is the non-linearity of the money dynamics introduced by controllable tuition. The solution algorithm given in the Appendix will accommodate both linear and nonlinear dynamic systems.

However, this does raise the issue of the validity of the linearity assumption embodied in the SCPM systems dynamics. While in any specific implementation, the functional form of the dynamics would be an empirical question, there are several justifications for the use of linear dynamics in our example: (1) the ease of interpretation of coefficients in terms of transition probabilities, depreciation factors, faculty salaries, etc.; (2) the experimental ease of formulation and modification; and (3) the lack of any information of a more generally useful and accurate formulation.

---

[12] See Miller [1971] for a discussion of recent attempts to estimate student demand functions and Jewett [1971] for a presentation of a national student ability - willingness to pay model and analysis.

CRITERION FUNCTION

The general form of the criterion function used by SCPM is

$$\min \left\{ J = P\left(x(N)\right) + \sum_{t=1}^{N-1} V_t\left(x(t),u(t),z(t)\right) \right\} \qquad (7)$$

where P and V_t are the relative quadratic loss functions derived in a previous section. In this study, P is the sum of four quadratic terms and V_t (V_t=V for all t) is a summation of nine quadratic terms. For purposes of exposition we separate V into five sets of terms relating to the administrator's objectives expressed in terms of the student/faculty ratio, faculty mix, space requirements, student mix, and financial stability. P will be discussed separately.

Student/Faculty Ratio

One proxy measure of the amount and quality of student/faculty interaction at an institution is the ratio of students to its (FTE) faculty. SCPM enables a campus administrator to specify a targeted ratio and then seeks a set of controls which minimizes the deviation of the actual student/faculty ratio from the target. If we define

r_1 = target student/faculty ratio,

k_1 = some scalar weight,

$TF_t = \sum_{i=1}^{4}\left[x_i(t) + u_i(t)\right]$, which is the total faculty at time t, and

$TS_t = \sum_{i=5}^{8}\left[x_i(t) + u_i(t)\right]$, which is the total students at time t,

then the first term of V_t may be written

$$k_1(r_1 TF_t - TS_t)^2 \quad . \qquad (i)$$

If an administrator wished, this term could be expanded to reflect students by level and faculty by rank to describe, for example, the exposure of lower division students to tenured faculty. Similar expansions are possible in most of the terms of the criterion function but were not reported here because they do not alter the solution procedure or the basic utility and results of the model.

Faculty Mix

Another proxy criterion for assessing the quality or prestige of a college is the mix of its faculty by rank. [See Rowe, Wagner and Weathersby (1970).] In the case of community colleges, or any other cases for which there are no ranks but rather salary schedules, we may interpret the four (or fewer) levels of faculty purely in terms of salary. In any case, we assume that the administrator has preferences over different mixes of faculty by level and we allow him to specify target ratios which describe the desired mix.

If we define

$F_i(t) = x_i(t) + u_i(t)$, $i = 1,2,3,4$, which is the number of FTE faculty at level i in year t,

r_i = target ratios of each rank relative to the number of full professors, $i = 2,3,4$, and

k_i = scalar weights $i = 2,3,4$,

then the next three terms of V_t are

$$k_i\left(r_i\, F_1(t) - F_i(t)\right)^2, \quad i = 2,3,4 \quad . \qquad \text{(ii)}$$

Space

Typically space needs are largely determined by either student enrollment or faculty size or a combination of the two. SCPM makes the simplifying assumptions that space is interchangeable and available in continuously variable amounts. For a small, homogenous college, space interchangeability may not be a devastating assumption because in liberal arts subjects rooms of each type can be used by most disciplines even though one cannot easily interchange lecture halls and offices. The assumption of continuously variable space is a weakness of the model, because new construction occurs by project or building and, therefore, occurs in quantum jumps. However, we do include the time lag of construction from start to completion. Recalling that $x_{12}(t)$ is the available space at t, we let

k_5 be a scalar weight and

c_1, c_2, c_3 be space standard coefficients determining space needs as a linear combination of total faculty (TF_t) and total students (TS_t).

The fifth term of V_t is then

$$k_5\left(c_1 + c_2 TS_t + c_3\, TF_t - x_{12}(t)\right)^2 . \qquad \text{(iii)}$$

Student Mix

Fiscal planning can be much more effective if student enrollments can be forecasted several years into the future. SCPM does not attempt to describe enrollments by discipline, although it could by defining additional state variables and equations; instead, SCPM focuses on student levels. Furthermore, for small colleges it was felt that average costs would not vary significantly across disciplines because these small colleges rarely can afford massive commitments of dollars to facilities and faculties in the hard sciences and engineering, traditionally the most expensive disciplines. (Schools such as Cal Tech and MIT are clearly exceptions to this ..le and they would need to recognize student discipline and level.) The next three terms of V_t are constructed from the same pattern as for faculty.

$$k_i\left(r_i\ S_5(t) - S_i(t)\right)^2\ ,\ i = 6,7,8\ ,\ \text{and}\ S_i = \text{students of level } i\ . \qquad \text{(iv)}$$

Monetary Balance

The last term of V_t is a balance equation, expressed as a quadratic penalty function, which forces the annual net cash balance at the end of each period towards zero. Campus administrators are assumed to seek policies so that the cash inflow, e.g., transfers from endowment, gifts, and student revenues, is equal to the outflow, e.g., transfers to endowment, faculty salaries, construction costs, maintenance costs, and other operating costs. Otherwise, too much is withdrawn from income producing investments or, conversely, not enough is invested—both of which have an opportunity cost to the institution and should be avoided. The net cash balance is given by $x_{14}(t+1)$; however it is included in V_t as a function of $x(t)$, $u(t)$, $z(t)$. With

k_9 = a scalar weight and

$f_{14,j}$, $g_{14,j}$ defined as before, the final expression in V_t is

$$k_9\left(\sum_j f_{14,j}x_j(t) + \sum_j g_{14,j}u_j(t) + h_{14,13}z_{13}(t) + z_{14}(t)\right)^2\ . \qquad \text{(v)}$$

Terminal Conditions

Because this optimal decision problem is formulated as a finite horizon differential dynamic programming problem, it is necessary to introduce "artificial" targets in the last planning period to correct for the truncated horizon. To prevent SCPM from "selling off" uncompleted space in the last three periods (space which is not usable during the model's lifetime, and therefore, space of no value to the institution) we drive $x_9(N), x_{10}(N), x_{11}(N)$ to zero by including in $P(N)$

$$k_{10}x_i(N)^2 \text{ , } i = 9,10,11 \text{ , where } k_{10} \text{ is a scalar weight.} \qquad \text{(vi)}$$

The last term of $P(N)$ is the net cash balance equation expressed as a function of $x(N)$, and write

$$k_9x_{14}(N)^2 \text{ , } k_9 \text{ a scalar weight.} \qquad \text{(vii)}$$

When put in the form of equation (7), expressions (i) - (vii) constitute the criterion function currently being investigated.

As discussed previously, the scalar weights were chosen to equalize unit losses and thereby avoid inducing artificial minima. Table 3 shows the values for the scalar weights used in this study.

Although the target ratios were chosen to be constant over time, it would be possible to choose target paths showing the evolution of the variables over time and SCPM would then solve for the optimal decisions for this path. Another formulation of the criterion function would enable the campus planner to determine the shortest time in which the desired targets could be achieved. However, the numerical results reported in the next section employed equations (i) through (vii).

SUMMARY OF RESULTS

To assess the realism of the model, a reference set of input data was developed which represents the operations of an "average" small liberal arts college. Actually, these data were modelled after data in the University of Santa Clara [1970]. Changes in this reference set (denoted Data Set 1) were made to reflect different operating policy decisions which might be made by campus administrators and to reflect different assumptions about future levels of external financial support. The optimal decisions for these new environments were then compared to the base case results. (See Tables 2 and 3 for a summary of Data Set 1).

Additionally, the college's optimal decision problem was solved for only non-negative values for the control variables. The resulting values thereby achieve sub-optimal solutions which trade optimally for more realistic results. It was shown that these solutions achieve the targeted ratios and provide adequate space as specified by the objective function but that they result in a much greater variation in the net cash balances at the end of each operating period. (See Table 5 for a comparison of net cash balances.)

Student admissions, faculty hiring, and new construction decisions are the normal outputs of the model. In addition, the model can answer questions about the operational effects of alternative funding methods which are of considerable interest to external funding agencies such as state and local governments. To examine these questions, three additional data sets were analyzed from this policy perspective.

TABLE 2

Accounting Summary for SCPM Example*
(Enrollment Level: 800)

EXPENSE		% of Total	INCOME		% of Total
Instruction	$ 104,400	6.39	Tuition & Fees	$ 970,400	58.07
Student Aid	119,200	7.31	Transfers from Endowment	115,200	6.89
Student Services	266,800	16.33	Gifts & Grants	364,000	21.78
Total Student Related	$ 490,400	30.03	Other (gov't. aid)	221,600	13.26
Salaries: Teaching Faculty	418,000	25.60	TOTAL	$1,671,200	100.00
Salaries: Admin./Other Fac.	222,000	13.60			
Total Fac./Admin. Salaries	$ 640,000	39.20			
Plant M&O	208,000	12.73			
Plant Additions	294,450	18.04			
Total Plant	$ 502,450	30.77			
TOTAL	$1,632,850	100.00			

*This accounting summary was derived from "An Introduction to Program Planning Budgeting and Evaluation for Colleges and Universities" - July 1970 - University of Santa Clara--Office of Institutional Planning.

Additional Assumptions:

1) Eighty percent of instructional costs are faculty salaries;

2) There are $3,840,000 of restricted funds yielding 3% per annum;

3) Plant M&O costs $2/ASF; there are 130 ASF/student;

4) Student/faculty ratio is 16/1; faculty includes teaching, research and administrative staff exclusive of support.

TABLE 3

Data Set 1

F-MATRIX

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.93	0.05												
2		0.85	0.05											
3			0.80	0.50										
4				0.10										
5														
6					0.95	0.02								
7						0.70	0.03							
8							0.70	0.04						
9														
10									1.00					
11										1.00				
12											1.00	0.98		
13													1.00	
14	-1.7	-1.4	-1.1	-0.8	0.06	0.06	0.06	0.06	-4.5	-4.5	-2.0	-0.2	0.03	1.00

G-MATRIX

	1	2	3	4	5	6	7	8	9
1	0.70								
2		0.70							
3			0.95	0.30					
4				0.10					
5					0.03				
6					0.60	0.02			
7						0.70	0.03		
8							0.70	0.04	
9									1.00
10									
11									
12									
13									
14	-1.7	-1.4	-1.1	-0.8	0.06	0.06	0.06	0.06	-1.5

TABLE 3 (continued)

Vector of Initial x-Values

x(1)	x(2)	x(3)	x(4)	x(5)	x(6)	x(7)	x(8)	x(9)	x(10)	x(11)	x(12)	x(13)	x(14)
10.	12.	15.	1.0	10.	170.	140.	130.	2.08	2.08	2.08	104.	384.	0.0

Vector of Predetermined Funds ($10,000's)

Per.	Amount	Per.	Amount	Per.	Amount	Per.	Amount
1)	58.56	6)	61.55	11)	64.69	16)	67.99
2)	59.15	7)	62.17	12)	65.34	17)	68.67
3)	59.74	8)	62.79	13)	65.99	18)	69.36
4)	60.34	9)	63.42	14)	66.65	19)	70.05
5)	60.94	10)	64.05	15)	67.32		

TARGETS AND WEIGHTS

Item	Target	Weight	Item	Target	Weight
Assoc/Full:	1.5	1.0	Soph/Frosh:	0.7	1.0
Asst/Full:	2.0	1.0	Fr./Frosh:	0.65	1.0
Inst/Full:	0.5	1.0	Sr./Frosh:	0.5	1.0
Stu/Faculty:	16.0	0.01	ASF/Student:	130.	50.0
Net Cash Bal:	0.0	0.5			

Comparison of Optimal versus Sub-Optimal Results

The general form of the optimal control problem is to

$$\min_{u(t)} \left\{ J = P\big(x(N)\big) + \sum_{t=0}^{N-1} V\big(x(t),u(t),z(t),t\big) \right\}$$

subject to

$$x(0) : \text{known}$$
$$x(t+1) = f\big(x(t),u(t),z(t),t\big) \ .$$

This formulation does not constrain the sign of either the state or control variables. This is the form for which the solution algorithm given in the Appendix was designed. While firing full professors and selling newly constructed space may appeal to some interests, negative values for control variables are in general not meaningful. Furthermore, our study revealed that unconstrained solutions to the optimal policy problem usually contained several such negative decision values, which, while small in magnitude, were nevertheless inappropriate and unrealistic.

Rather than impose inequality constraints which would require a reformulation of the model or attach penalty functions to the criterion function to facilitate the use of a sequential optimization algorithm, we initially included a switch in the computer program which set negative values of the computed control variables to zero within the iteration sequence.[13] Since the algorithm computes the improved $u(t)$ values based on small variations in $x(t)$ and since those variations are not substantially altered by zeroing out negative values of $u(t-1)$, the

[13] One can solve the general formulation with inequality constraints at considerable additional complexity and expense, see Jacobson [1969], but we concluded that at this stage of development these refinements were not worth the additional cost because the constrained and unconstrained results were so similar.

resulting sub-optimal path should provide acceptable solutions. A comparison of the two sets of results confirms this assertion.

Three sets of comparison runs were made to determine the loss from sub-optimization. These runs were based on the original Data Set 1, Data Set 2 in which exogenous funds grow at 2% per year, and Data Set 3 in which per-student income increased by $100 per year. Tables 4 and 5 show the differences in control variables and in the yearly net cash balances for Data Set 1. Similar results obtained for Data Sets 2 and 3. The differences between the two sets of control variable solutions for Data Set 1 are summarized below.

Control Variable	Mean Value Unconstrained	Mean Value Constrained	Mean of The Absolute Differences
Full Professors	0.12	0.15	0.25
Assoc. Professors	1.86	1.85	0.63
Asst. Professors	2.26	2.35	0.73
Instructors	4.74	4.88	0.30
Freshmen	286.39	287.52	5.03
Sophomores	23.37	23.75	5.39
Juniors	42.22	42.62	4.73
Seniors	8.12	8.36	4.01
New Construction (000's ASF)	2.92	2.94	3.41

It is clear from an examination of Table 6 that in terms of the targeted values specified by the objective function, there is very little loss of utility associated with using the constrained formulation. Because the negative values for the control variables generated by the unconstrained solutions are unrealistic, we shall concentrate on the constrained, sub-optimal results for the balance of this discussion.

TABLE 4

Comparison of Control Variables
Data Set 1
Unconstrained vs Non-Negative Modes

	NEW FACULTY							
	Unconstrained				Non-Negative			
Per.	Prof.	Assoc.	Asst.	Inst.	Prof.	Assoc.	Asst.	Inst.
1	0.10	3.20	5.11	3.82	0.10	3.20	5.11	3.82
2	-0.03	1.68	1.43	4.63	0.00	1.68	1.43	4.63
3	0.12	1.75	2.25	4.43	0.00	1.48	2.10	4.66
4	0.02	1.54	2.03	4.53	0.16	1.89	2.19	4.37
5	0.39	2.12	2.78	4.67	0.31	1.94	2.67	4.77
6	-0.21	1.23	1.36	4.47	0.00	1.14	1.50	4.85
7	0.43	2.20	3.13	4.73	0.12	2.08	2.45	4.38
8	-0.06	1.46	1.62	4.63	0.32	2.00	2.78	4.93
9	0.22	1.92	2.60	4.66	0.00	0.38	1.08	5.68
10	0.10	1.71	2.21	4.80	0.00	1.38	1.77	5.24
11	0.18	1.90	2.36	4.69	0.97	4.01	4.18	4.49
12	0.05	1.66	2.18	4.79	0.00	1.14	1.33	5.14
13	0.25	1.98	2.62	4.90	0.00	0.62	1.53	5.50
14	0.12	1.84	2.14	4.72	0.00	1.73	2.17	5.18
15	0.00	1.56	2.28	5.02	0.53	3.25	3.64	4.95
16	0.53	2.53	3.07	4.89	0.00	1.05	1.36	5.47
17	-0.42	0.96	1.13	4.83	0.00	1.70	2.05	4.88
18	0.77	2.81	4.12	5.33	0.39	2.67	3.40	5.04
19	-0.27	1.37	0.87	4.56	0.02	1.83	1.89	4.77

	NEW STUDENTS							
	Unconstrained				Non-Negative			
Per.	Frosh	Soph.	Jr.	Sr.	Frosh	Soph.	Jr.	Sr.
1	270.8	26.5	42.5	10.4	270.7	26.5	42.5	10.4
2	272.8	20.7	39.5	7.1	272.7	20.8	39.6	7.1
3	272.8	21.4	39.5	7.1	272.8	21.4	39.5	7.1
4	272.8	21.3	39.5	7.0	272.8	21.3	39.5	7.0
5	282.1	27.9	45.6	11.7	282.4	28.0	45.7	11.8
6	271.9	15.2	34.4	2.3	276.3	18.1	37.1	4.4
7	285.5	30.5	48.0	13.7	277.6	22.3	40.7	7.7
8	279.4	18.4	37.6	4.5	289.1	29.8	47.7	12.9
9	284.3	25.1	43.5	9.5	282.8	18.5	37.9	4.5
10	286.3	23.8	42.5	8.4	278.9	19.2	38.3	5.2
11	287.9	23.5	42.5	8.2	301.4	37.4	54.9	18.4
12	288.1	22.7	41.8	7.5	294.5	19.4	39.5	4.6
13	294.1	26.7	45.5	10.4	288.6	18.7	38.7	4.6
14	292.2	21.8	41.4	6.7	289.1	22.7	42.0	7.6
15	294.5	24.4	43.8	8.7	307.0	35.2	53.5	16.4
16	304.2	29.8	48.9	12.5	300.2	19.7	40.3	4.7
17	289.5	13.8	34.6	0.6	294.2	19.1	39.5	4.6
18	315.3	40.2	58.4	20.1	309.3	33.4	52.3	15.1
19	294.0	10.4	32.2	-2.2	302.5	19.8	40.6	4.8

TABLE 5

Comparison of Unrestricted Funds Balances

Unconstrained vs Non-Negative Modes
(Data Set 1)

Per.	Unconstrained	Non-Negative
1	$ 15,240.	$ 15,220.
2	-24,096.	- 47,390.
3	13,195.	16,260.
4	- 5,572.	26,660.
5	856.	54,500.
6	787.	14,030.
7	678.	45,840.
8	- 4,560.	-139,320.
9	9,166.	- 97,000.
10	-11,996.	146,450.
11	10,543.	109,970.
12	- 3,486.	-128,660.
13	- 8,290.	- 52,060.
14	20,927.	101,710.
15	-28,550.	23,120.
16	25,150.	- 11,670.
17	- 7,584.	16,080.
18	-21,526.	7,077.
19	25,923.	14,157.
NET	$ 6,833.	$ 5,974.

Comparison of Control Variables

Unconstrained vs Non-negative Modes
(Data Set 1)

New Construction

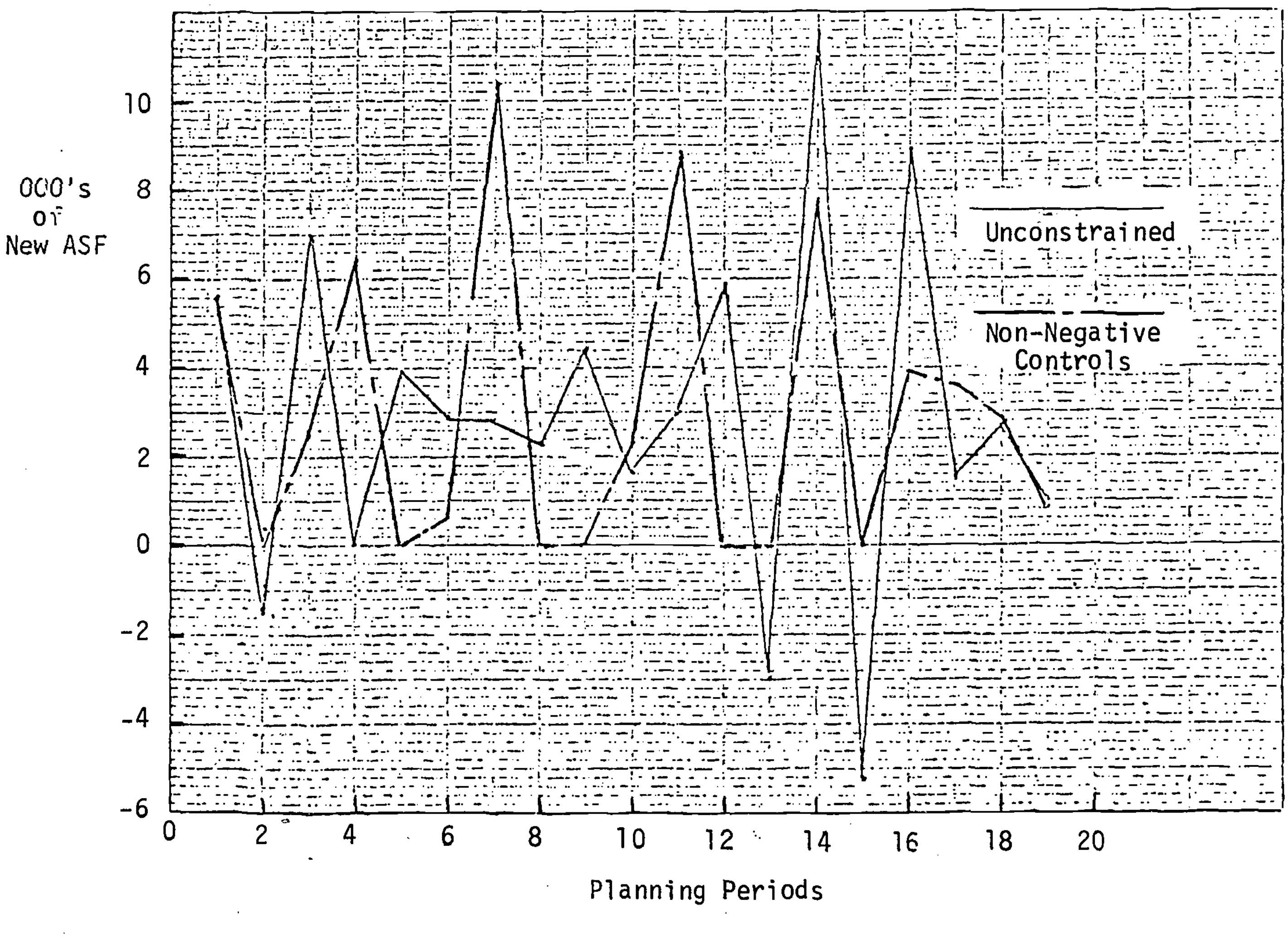

FIGURE 4

TABLE 6

Comparison of Unconstrained (A) vs Non-Negative (B) Modes
Preference Function Targets
(Data Set 1)

Period	Associates Professors		Assistants Professors		Instructors Professors		Sophomores Freshmen		Juniors Freshmen		Seniors Freshmen		Students Faculty		ASF Required ASF Built	
	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
Target	1.500	1.500	2.000	2.000	0.500	0.500	0.700	0.700	0.650	0.650	0.500	0.500	16.00	16.00	1.000	1.000
1	1.506	1.506	1.992	1.992	0.478	0.478	0.700	0.700	0.650	0.650	0.500	0.500	15.93	15.93	1.000	1.000
2	1.496	1.492	2.005	1.999	0.514	0.513	0.700	0.700	0.650	0.650	0.500	0.500	16.06	16.05	1.001	1.001
3	1.502	1.490	1.997	2.003	0.493	0.521	0.700	0.700	0.650	0.650	0.500	0.500	15.99	16.08	1.001	1.001
4	1.499	1.504	2.001	1.995	0.503	0.485	0.700	0.700	0.650	0.650	0.500	0.500	16.03	15.97	1.001	1.001
5	1.500	1.497	2.000	2.004	0.500	0.511	0.700	0.700	0.650	0.650	0.500	0.500	16.00	16.05	1.000	1.001
6	1.500	1.460	2.000	1.973	0.499	0.528	0.700	0.700	0.650	0.650	0.500	0.500	16.00	16.08	1.000	1.001
7	1.500	1.505	2.000	1.992	0.500	0.478	0.700	0.700	0.650	0.650	0.500	0.500	16.01	15.95	1.001	1.001
8	1.499	1.496	2.001	2.006	0.503	0.516	0.700	0.700	0.650	0.650	0.500	0.500	16.03	16.07	1.001	1.001
9	1.501	1.383	1.998	1.934	0.495	0.599	0.700	0.700	0.650	0.650	0.500	0.500	15.98	16.26	1.000	1.001
10	1.498	1.398	2.003	1.929	0.508	0.565	0.700	0.700	0.650	0.650	0.500	0.500	16.04	16.16	1.001	1.001
11	1.502	1.512	1.998	1.982	0.494	0.451	0.700	0.700	0.650	0.650	0.500	0.500	15.98	15.85	1.000	1.001
12	1.499	1.460	2.001	1.967	0.503	0.518	0.700	0.700	0.650	0.650	0.500	0.500	16.01	16.05	1.000	1.001
13	1.499	1.378	2.002	1.902	0.505	0.558	0.700	0.700	0.650	0.650	0.500	0.500	16.02	16.11	1.000	1.001
14	1.503	1.415	1.996	1.928	0.488	0.534	0.700	0.700	0.650	0.650	0.500	0.500	15.97	16.06	1.000	1.001
15	1.495	1.503	2.007	1.996	0.518	0.490	0.700	0.700	0.650	0.650	0.500	0.500	16.08	15.98	1.001	1.000
16	1.504	1.440	1.994	1.962	0.485	0.544	0.700	0.700	0.650	0.650	0.500	0.500	15.96	16.12	1.001	1.001
17	1.498	1.459	2.003	1.952	0.507	0.497	0.700	0.700	0.650	0.650	0.500	0.500	16.03	15.97	1.000	1.001
18	1.497	1.502	2.004	1.997	0.511	0.492	0.700	0.700	0.650	0.650	0.500	0.500	16.05	15.99	1.001	1.001
19	1.507	1.506	1.989	1.990	0.470	0.474	0.700	0.700	0.650	0.650	0.500	0.500	15.92	15.93	1.001	1.001

Comparison of Results from Various Policy Alternatives

Perhaps the most striking feature of the solutions generated by the model is that they do not show a smooth expansion path, either in terms of total enrollments or in terms of any of the control variables. This is not surprising mathematically, but it may surprise administrators unfamiliar with controllable, dynamic systems with different response times.

Intuitively, what has happened to the reference data set for the constrained case is the following. The preference function desires the ratio of available ASF per student to be fixed at 130. Enrollments are therefore constrained during the initial four years by the amount of physical space under construction at the beginning of the planning period, (x_9,x_{10}, and x_{11} at t = 1). For the reference data set, these are set at a level which exactly counteracts the depreciation of the existing capital stock. For the first four years therefore, the physical space available remains constant. Because faculty is linked to enrollments through the Student/Faculty Ratio, it is unnecessary to spend operating dollar balances resulting from the 1% growth in outs''a funds, to increase the size of the faculty during this period. As a result, the model spends any "excess funds" on new capital construction. This becomes available after the fourth year at which time enrollments begin to increase.

Although students yield a net dollar gain from the tuition level, they induce costs in the form of faculty salaries, capital needs, etc., so that it is impossible to pay for additional space by simpl, adding students to the rolls. When enrollments increase, therefore, the amount of new construction must decline relatively. This will eventually cause some decline in enrollment levels from peak periods, and thus induce a cyclical pattern of physical expansion and student growth.

Although the system dynamics and the solution process are considerably more complex, the foregoing is the predominant reason for the cyclical variability in the expansion path. Another reason is the different time constants or response times for the various state variables. Full professors spend a longer average time in the system than instructors, freshmen more than entering seniors, and so forth. The time behavior of aggregates is built up from many of these overlapping transients and, therefore, the aggregates show a cyclical time behavior.

Data Sets 1 and 2 differ only in the assumptions concerning the levels of external aid, with a 1% yearly increase in gifts, grants and government assistance reflected in Data Set 1 and a 2% yearly increase contained in Data Set 2. Both employ the same levels of aid for the initial year. At enrollment levels of 800 students, these funds account for 35% of current income in the initial year and 36.02% and 37.91% respectively for the total planning horizon. A comparison of the enrollment patterns generated is shown in Table 7. As can be seen from the table, there is no substantial increase in enrollments until the 10th period despite the fact that an additional $225,700 has been received by the institution through the 9th period. Since the 19-year "marginal-cost" of increased enrollments is approximately $5,500, one might intuitively expect that a smooth expansion path would be generated which should have enrolled an additional 41 students through the first 10 years. In fact, the school has only enrolled an additional 3 students.

Another way of looking at the data is to examine the increase in external funds in relation to the total income over the 19-year period. Here it can be seen that the additional $1.1876 million generated by Data

TABLE 7

Comparison of Data Set 1 and Data Set 2
Enrollments and Costs

Period	Data Set 1 Enrollment	Data Set 2 Enrollment	DS2 -- DS1 Difference	Data Set 1 Exog.Funds*	Data Set 2 Exog.Funds	DS2 -- DS1 Difference	% Change Enroll.	% Change* Exog.Funds
1	800	800	0	585,600	585,600	0	0.00	0.00
2	801	800	-1	591,500	597,300	5,800	-0.12	0.98
3	801	800	-1	597,400	609,300	11,900	-0.12	1.99
4	801	800	-1	603,400	621,400	18,000	-0.12	2.98
5	828	828	0	609,400	633,900	24,500	0.00	4.02
6	812	812	0	615,500	646,500	31,000	0.00	5.03
7	815	820	5	621,700	659,500	37,800	0.61	6.08
8	848	848	0	627,900	672,700	44,800	0.00	7.13
9	831	832	1	634,200	686,100	51,900	0.12	8.18
10	819	842	23	640,500	699,800	59,300	2.80	9.25
11	883	876	-7	646,900	713,800	66,900	-0.79	10.34
12	865	858	-7	653,400	728,100	74,700	-0.80	11.43
13	848	860	12	659,900	742,700	82,800	1.41	12.54
14	848	916	68	666,500	757,500	91,000	8.01	13.65
15	900	897	-3	673,200	772,700	99,500	-0.33	14.78
16	882	880	-2	679,900	788,100	108,200	-0.22	15.96
17	864	938	74	686,700	803,900	117,200	8.56	17.06
18	906	921	15	693,600	820,000	126,400	1.65	18.22
19	889	930	41	700,500	836,400	135,900	4.61	19.40
Total	16,041	16,258	217	12,187,700	13,375,300	1,187,600	1.35	9.74

*Exogenous funds constitute approximately 36% of current income, so that a 9.74% increase in exogenous funds represents a 3.5% increase in current income.

Set 2 is 3.51% of the total income generated via Data Set 1. This 3.5% increase in total income thus would result in a 1.35% increase in total enrollments if it were to come in the form of additional gifts, grants, and government aid according to this exponentially increasing function of time.

Suppose instead the increase in income had come from tuition charges. Data Set 3 simulates the effect of a $100 increase in tuition charges (or equivalently, an additional subsidy of $100 per student) by changing the net return from students from $600 to $700. (Recall that the coefficients $f_{14,j}$ and $g_{14,j}$, j=5-8 , are the difference between tuition and average student-related costs such as admissions, counseling, student-aid and health facilities.)

If the institution were to increase tuition and fees by $100, neglecting per-student student aid increases, enrollments would increase 3.57% over the 19-year period. The increase in total income represented by this policy change is $2,356,449 or approximately 7%. Table 8 shows a comparison of enrollments and income under the assumptions of Data Sets 1 and 3.

Looking at these results from the point of view of a potential fundor, such as the State or Federal government, it would appear at first blush that the most productive means of funding the institution would be the per-student institutional subsidy. In order to investigate the question more thoroughly, Data Set 4 was developed. This data set increases the external inputs by an amount equal to the $100/student-additional income generated by the increase in tuition at optimal enrollment levels. Table 9 shows the enrollments generated by Data Sets 2, 3 and 4 and the marginal costs of enrollments. A comparison of these costs shows that for the purpose of increasing enrollments, it is more effective to fund the institution directly

TABLE 8

Comparison of Data Set 1 and Data Set 3
Enrollments and Costs

Period	Data Set 1 Enrollment	Data Set 3 Enrollment	DS3 -- DS1 Difference	Additional Income Generated by Tuition Increase	% Change Enrollments	% Change* Tuition Income
1	800	800	0	80,000	0.00	0.00
2	801	800	-1	78,787	-0.12	8.10
3	801	800	-1	78,787	-0.12	8.10
4	801	801	0	80,100	0.00	8.24
5	828	858	30	122,190	3.62	12.16
6	812	841	29	119,277	3.57	12.10
7	815	824	9	93,317	1.10	9.43
8	848	870	22	113,686	2.59	11.05
9	831	860	29	121,177	3.48	12.02
10	819	861	42	137,046	5.12	13.79
11	883	904	21	115,873	2.37	10.81
12	865	886	21	114,073	2.42	10.87
13	848	885	37	133,381	4.36	12.96
14	848	943	95	209,535	11.20	20.37
15	900	924	24	121,512	2.66	11.13
16	882	905	23	118,399	2.60	11.06
17	864	961	97	213,761	11.22	20.39
18	906	943	37	139,181	4.08	12.66
19	889	948	59	166,367	6.63	15.42
Total	16,041	16,614	573	2,356,449	3.57	12.11

*Tuition and fees account for approximately 58% of current income, i.e., a 12% change in tuition income is approximately a 7% change in total current income.

TABLE 9

Comparison of Data Set 1, Data Set 2, Data Set 3, and Data Set 4
Enrollments and Marginal Costs

	Data Set 1	Data Set 2			Data Set 3			Data Set 4		
Period	Enrollment	Enrollment	Change in Exog.Funds	Marginal Cost	Enrollment	Subsidy= $100/Stu.	Marginal Cost	Enrollment	Change in Exog.Funds	Marginal Cost
1	800	800	0	0	800	80,000	∞	800	80,000	∞
2	801	800	5,800	- 5,800	800	80,000	-160,000	801	80,000	∞
3	801	800	11,900	- 8,850	800	80,000	-120,000	801	80,000	∞
4	801	800	18,000	-11,900	801	80,100	-160,000	801	80,100	∞
5	828	828	24,500	-20,067	858	85,800	14,496	858	85,800	13,530
6	812	812	31,000	-30,400	841	84,100	8,596	841	84,100	8,305
7	815	820	37,800	64,500	824	82,400	8,673	824	82,400	8,418
8	848	848	44,800	86,900	870	87,000	7,493	870	87,000	7,327
9	831	832	51,900	75,233	860	86,000	6,371	863	86,000	6,110
10	819	842	59,300	10,961	861	86,100	5,230	859	86,100	5,133
11	883	876	66,900	18,521	904	90,400	5,122	908	90,400	4,930
12	865	858	74,700	35,550	886	88,600	5,027	890	88,600	4,767
13	848	860	82,800	21,225	885	88,500	4,618	875	88,500	4,598
14	848	916	91,000	6,526	943	94,300	3,583	959	94,300	3,409
15	900	897	99,500	7,864	924	92,400	3,601	940	92,400	3,297
16	882	880	108,200	9,288	905	90,500	3,622	921	90,500	3,208
17	864	938	117,200	5,747	961	96,100	3,087	915	96,100	3,067
18	906	921	126,400	5,976	943	94,300	3,048	999	94,300	2,734
19	889	930	135,900	5,473	948	94,800	2,899	979	94,800	2,506
Total	16,041	16,258	1,187,600	5,473	16,614	1,661,400	2,899	16,704	1,661,400	2,506

rather than paying a per-student amount. The same subsidy, in terms of both total dollars and timing, has been assumed by Data Sets 3 and 4, yet the solution to the planning proble. generates greater enrollments when the subsidy is given as a flat grant.[14]

Conclusion

These examples show that realistic and relevant results can be obtained relatively easily and inexpensively by SCPM. The control theoretic approach both incorporates the multi-level, multi-decision maker hierarchical structures of higher education and enables educational planners to derive improved institutional plans and to evaluate many alternative operating policies. The robustness and flexibility of SCPM suggest that it could make a major contribution to improved educational planning.

[14]This counterintuitive result does not hold for the unconstrained case, if the optimal value of the criterion function is zero. If it is possible to achieve all the targets exactly in all planning periods, then MIN $J = 0$ and the optimal enrollments for Data Sets 3 and 4 would be identical. An alternative view of student tuition models and the effects of government subsidies is given in Weathersby [1970]. The above conclusion is supported in this supply and demand analysis.

APPENDIX

Solution Procedure

The linear-quadratic minimization problem is solved in one step using an adaptation of an algorithm devised by David Mayne [1966]. The following computer program is based on this algorithm.

A non-optimal trajectory is generated using a nominal control sequence. The effect on the criterion (penalty) function of small variations of the control sequence is determined. This enables an improved sequence to be chosen. In the case of a quadratic criterion and linear system dynamics, the first improved sequence is optimal.

The advantage of Mayne's approach over conventional dynamic programming approaches lies in immense reduction of core requirements. In place of the optimal return function V° of Bellman, Mayne uses ∇V°, the optimal variation in the non-optimal return function due to variation of the state variable. ∇V° is expanded in a power series (to second order) of the variation in x, and difference equations are derived for the coefficients of the series. It is the identification of these coefficients which provide, in analytic form, the optimal change in $x(t)$, and hence, by working backward in time, of $u(t - 1)$.

The program, in its present form handling 25 variables and 20 planning periods, requires approximately 50K bytes of core and 1.5 minutes CPU time on IBM's 360-65 O.S. On the University of California Administrative Data Processing System this costs approximately $10.00 for an adequate numerical solution for one case. It is hoped that future versions will reduce the size and cost of running this program.

COMPUTER PROGRAM

```
FORTRAN IV G LEVEL  18                    MAIN              DATE = 71300           10/27/59

                C
                C     THIS PROGRAM EMPLOYS AN ITERATIVE PROCEDURE TO FIND A SEQUENCE
                C  OF CONTROL VECTORS WHICH MINIMIZES A NON-LINEAR N-PERIOD OBJECTIVE
                C  FUNCTION SUBJECT TO LINEAR-DYNAMIC CONSTRAINTS.  THE PROGRAM IS
                C  BASED ON AN ALGORITHM DEVELOPED BY DAVID MAYNE, PUBLISHED BY
                C  INT. JOURNAL OF CONTROL (1966).
                C
 0001                 COMMON X(14,20),U(09,19),NX,NU,NPP,VX(14),   VXX(14,14),VUX(9,14),
                     1VUU(9,9),VXX2(14,14),VU(9),LPP,VX2(14),V(20),F(14,14),G(14,9),
                     2IIN,IOUT,NUMRUN
 0002                 REAL JJ,OPT
 0003                 DIMENSION A(14,14),B(9,14),ALPHA(9,19),BETA(9,14,19),HU(9),
                     1H(14,14),Z(14,19),XNEW(14,20),DELU(9),W(14,14),
                     2C(9,9),CINV(9,9),CC(45)
 0004                 DOUBLE PRECISION C,CINV    ,CC
 0005                 IIN=1
 0006                 IOUT=6
                C
                C  READ BASIC DATA
                C
 0007               1 READ(IIN,900) NUMIT,LPRNT,NU,NX,NPP,NUMRUN,OPT
 0008                 IF (NUMIT.GT.99) STOP
 0009                 LPP=NPP+1
 0010                 IF(NUMRUN.GT.1) GO TO 150
 0011               2 DO 100 I=1,NX
 0012             100 READ(IIN,901)(F(I,J),J=1,NX)
 0013                 IF(NUMRUN.GT.1) GO TO 150
 0014               3 DO 101 I=1,NX
 0015             101 READ(IIN,901)(G(I,J),J=1,NU)
 0016                 IF(NUMRUN.GT.1) GO TO 150
 0017               4 DO 102 I=1,NX
 0018             102 READ(IIN,901)(H(I,J),J=1,NX)
 0019                 IF(NUMRUN.GT.1) GO TO 150
 0020               5 DO 103 I=1,NX
 0021             103 READ(IIN,901)(Z(I,J),J=1,NPP)
 0022                 IF(NUMRUN.GT.1) GO TO 150
                C
                C  INITIALIZE SYSTEM
                C
 0023               6 READ(IIN,901)(X(I,1),I=1,NX)
 0024                 IF(NUMRUN.GT.1) GO TO 150
 0025               7 DO 110 I=1,NU
 0026             110 READ(IIN,901)(U(I,J),J=1,NPP)
 0027                 IF(NUMRUN.EQ.1) GO TO 1000
 0028             150 READ(IIN,922) ICHNGE
 0029                 GO TO (1000,2,3,4,5,6,7),ICHNGE
                C
                C  CALCULATE INITIAL STATE VARIABLES
                C
 0030            1000 NCYCLE=0
 0031                 DO 120 L=1,NPP
 0032                 M=L+1
 0033                 DO 121 I=1,NX
 0034             121 X(I,M)=0.0
 0035                 DO 120 I=1,NX
 0036                 DO 122 J=1,NX
 0037             122 X(I,M)=X(I,M)+F(I,J)*X(J,L)+H(I,J)*Z(J,L)
 0038                 DO 120 J=1,NU
```

```
0039          120 X(I,M)=X(I,M)+G(I,J)*U(J,L)
          C
          C  ECHO CHECK INITIAL DATA
          C
0040              WRITE(IOUT,902) NUMRUN
0041              WRITE(IOUT,903) (I,I=1,NX)
0042              DO 200 I=1,NX
0043          200 WRITE(IOUT,904) I,(F(I,J),J=1,NX)
0044              WRITE(IOUT,905)
0045              WRITE(IOUT,903) (I,I=1,NU)
0046              DO 201 I=1,NX
0047          201 WRITE(IOUT,904) I,(G(I,J),J=1,NU)
0048              WRITE(IOUT,906)
0049              WRITE(IOUT,903)(I,I=1,NX)
0050              DO 202 I=1,NX
0051          202 WRITE(IOUT,904) I,(H(I,J),J=1,NX)
0052              WRITE(IOUT,907)
0053              WRITE(IOUT,903) (I,I=1,NX)
0054              DO 203 J=1,NPP
0055          203 WRITE(IOUT,904) J,(Z(I,J),I=1,NX)
          C
          C  BEGIN ITERATIVE PROCEDURE
          C
0056              CALL CALCV(0)
0057          300 WRITE(IOUT,908) NCYCLE
0058              WRITE(IOUT,909) (I,I=1,NU)
0059              DO 301 J=1,NPP
0060          301 WRITE(IOUT,910) J,(U(I,J),I=1,NU)
0061              WRITE(IOUT,911)
0062              WRITE(IOUT,909) (J,J=1,LPP)
0063              DO 302 I=1,NX
0064          302 WRITE(IOUT,912) I,(X(I,J),J=1,LPP)
0065              CALL CALCV(LPP)
          C
          C  CALCULATE J-VALUE
          C
0066              JJ=0.0
0067              DO 310 I=1,LPP
0068          310 JJ=JJ+V(I)
0069              WRITE(IOUT,913)(I,I=1,LPP)
0070              WRITE(IOUT,914)(V(I),I=1,LPP)
0071              WRITE(IOUT,915)JJ
          C
          C  CHECK TO SEE IF CURRENT CONTROLS ARE OPTIMAL
          C
0072              CHECK=ABS(JJ-OPT)
0073              IF(CHECK.GT.0.02) GO TO 320
0074              WRITE(IOUT,916)
0075              GO TO 1
0076          320 IF(NUMIT-NCYCLE) 321,322,322
0077          321 WRITE(IOUT,917)
0078              GO TO 1
          C
          C  BEGIN MAJOR LOOP
          C
0079          322 AA=0.0
0080              DO 400 II=1,NPP
0081              IT=LPP-II
```

```
0082            CALL CALCV(IT)
          C
          C  CALCULATE MATRIX A = VXX + F' VXX(T+1) F
          C
0083            DO 401 J=1,NX
0084            DO 401 I=1,NX
0085            A(I,J)=0.0
0086            DO 401 K=1,NX
0087        401 A(I,J)=A(I,J)+F(K,I)*VXX2(K,J)
0088            DO 402 J=1,NX
0089            DO 402 I=1,NX
0090            W(I,J)=0.0
0091            DO 402 K=1,NX
0092        402 W(I,J)=W(I,J)+A(I,K)*F(K,J)
0093            DO 403 J=1,NX
0094            DO 403 I=1,NX
0095        403 A(I,J)=W(I,J)+VXX(I,J)
          C
          C  CALCULATE MATRIX B = VUX + G' VXX(T+1) F
          C
0096            DO 404 I=1,NU
0097            DO 404 J=1,NX
0098            B(I,J)=0.0
0099            DO 404 K=1,NX
0100        404 B(I,J)=B(I,J)+G(K,I)*VXX2(K,J)
0101            DO 405 I=1,NU
0102            DO 405 J=1,NX
0103            W(I,J)=0.0
0104            DO 405 K=1,NX
0105        405 W(I,J)=W(I,J)+B(I,K)*F(K,J)
0106            DO 406 I=1,NU
0107            DO 406 J=1,NX
0108        406 B(I,J)=W(I,J)+VUX(I,J)
          C
          C  CALCULATE MATRIX C = VUU + G' VXX(T+1) G
          C
0109            DO 407 I=1,NU
0110            DO 407 J=1,NX
0111            W(I,J)=0.0
0112            DO 407 K=1,NX
0113        407 W(I,J)=W(I,J)+G(K,I)*VXX2(K,J)
0114            DO 408 I=1,NU
0115            DO 408 J=1,NU
0116            C(I,J)=0.D0
0117            DO 408 K=1,NX
0118        408 C(I,J)=C(I,J)+W(I,K)*G(K,J)
0119            DO 409 I=1,NU
0120            DO 409 J=1,NU
0121        409 C(I,J)=C(I,J)+VUU(I,J)
          C
          C  PLACE UPPER TRIANGLE OF C INTO CC BY COLS.
          C
0122            K=1
0123            DO 419 J=1,NU
0124            DO 418 I=1,J
0125            IF(C(I,J)) 415,416,416
0126        415 CC(K)=DMAX1(C(I,J),C(J,I))
0127            GO TO 418
```

```
      416 CC(K)=DMIN1(C(I,J),C(J,I))
      418 K=K+1
      419 CONTINUE
          IER=0
          EPS=.000001
          CALL DSINV(CC,NU,EPS,IER)
C
C  PLACE CC IN CINV
C
          L=1
          DO 485 J=1,NU
          DO 480 I=1,J
          CINV(I,J)=CC(L)
          IF(I.NE.J) CINV(J,I)=CC(L)
      480 L=L+1
      485 CONTINUE
          IF(IER.EQ.-1) GO TO 1
          DO 410 J=1,NU
          HU(J)=0.0
          DO 411 I=1,NX
      411 HU(J)=HU(J)+VX2(I)*G(I,J)
      410 HU(J)=HU(J)+VU(J)
C
C  CALCULATE ALPHA(IT),BETA(IT)
C
          DO 420 I=1,NU
          ALPHA(I,IT)=0.0
          DO 421 J=1,NU
      421 ALPHA(I,IT)=ALPHA(I,IT)+CINV(I,J)*HU(J)
      420 ALPHA(I,IT)=-ALPHA(I,IT)
          DO 430 I=1,NU
          DO 430 J=1,NX
          BETA(I,J,IT)=0.0
          DO 431 K=1,NU
      431 BETA(I,J,IT)=BETA(I,J,IT)+CINV(I,K)*B(K,J)
      430 BETA(I,J,IT)=-BETA(I,J,IT)
C
C  CALCULATE NEW VXX2 AND NEW VX2
C
          DO 440 I=1,NX
          W(I,1)=0.0
          DO 440 J=1,NU
      440 W(I,1)=W(I,1)+BETA(J,I,IT)*HU(J)
          DO 441 I=1,NX
          W(I,2)=0.0
          DO 441 J=1,NX
      441 W(I,2)=W(I,2)+VX2(J)*F(J,I)
          DO 442 I=1,NX
      442 VX2(I)=VX(I)+W(I,1)+W(I,2)
          DO 450 I=1,NU
          DO 450 J=1,NX
          W(I,J)=0.0
          DO 450 K=1,NU
      450 W(I,J)=W(I,J)+CINV(I,K)*B(K,J)
          DO 451 I=1,NX
          DO 451 J=1,NX
          VXX2(I,J)=0.0
          DO 452 K=1,NU
```

```
      452 VXX2(I,J)=VXX2(I,J)+B(K,I)*W(K,J)
      451 VXX2(I,J)=A(I,J)-VXX2(I,J)
C
C  CALCULATE AA TO INDICATE EXPECTED IMPROVEMENT IN J-VALUE
C
          W(1,1)=0.0
          DO 460 I=1,NU
      460 W(1,1)=W(1,1)+HU(I)*ALPHA(I,IT)
      400 AA=AA+0.5*W(1,1)
C
C  CALCULATE NEW CONTROLS AND NEW STATE VARIABLES
C
          DO 500 I=1,NU
          U(I,1)=U(I,1)+ALPHA(I,1)
          IF(U(I,1).LT.0.) U(I,1)=0.
      500 CONTINUE
          DO 501 I=1,NX
          XNEW(I,2)=0.0
          DO 502 J=1,NX
      502 XNEW(I,2)=XNEW(I,2)+F(I,J)*X(J,1)+H(I,J)*Z(J,1)
          DO 501 J=1,NU
      501 XNEW(I,2)=XNEW(I,2)+G(I,J)*U(J,1)
          DO 510 L=2,NPP
          M=L+1
          DO 511 I=1,NU
          DELU(I)=0.0
          DO 512 J=1,NX
      512 DELU(I) =     BETA(I,J,L)*(XNEW(J,L)-X(J,L))+DELU(I)
          U(I,L)= DELU(I)+ALPHA(I,L)+U(I,L)
          IF(U(I,L).LT.0.) U(I,L)=0.
      511 CONTINUE
          DO 513 I=1,NX
          XNEW(I,M)=0.0
          DO 514 J=1,NX
      514 XNEW(I,M)=XNEW(I,M)+F(I,J)*XNEW(J,L)+H(I,J)*Z(J,L)
          DO 513 J=1,NU
      513 XNEW(I,M)=XNEW(I,M)+G(I,J)*U(J,L)
      510 CONTINUE
          DO 520 I=1,NX
          DO 520 J=2,LPP
      520 X(I,J)=XNEW(I,J)
          NCYCLE=NCYCLE+1
          WRITE(IOUT,918)AA
          GO TO 300
      900 FORMAT(6I3,F8.0)
      901 FORMAT(10F8.0)
      902 FORMAT(1H1,20X,'DATA FOR RUN NUMBER',I3//20X,'F-MATRIX')
      903 FORMAT(/3X,14I9)
      904 FORMAT(1X,I2,14F9.2)
      905 FORMAT(//20X,'G-MATRIX')
      906 FORMAT(1H1,20X,'H-MATRIX')
      907 FORMAT(//20X,'Z-TRANSPOSE')
      908 FORMAT(1H1,10X,'STATISTICS FOR CYCLE NUMBER',I4//20X,'CONTROL VARI
         1ABLES')
      909 FORMAT(/(3X,10I12))
      910 FORMAT(1X,I2,5X,10G12.4)
      911 FORMAT(1H1,20X,'STATE VARIABLES')
      912 FORMAT(/2X,'X(',I2,') ',10G12.4/(8X,10G12.4/))
```

```
      C ************************************************************************
      C
      C     PURPOSE
      C      INVERT A GIVEN SYMMETRIC POSITIVE DEFINITE MATRIX
      C
      C     USAGE
      C      CALL DSINV(A,N,EPS,IER)
      C
      C     DESCRIPTION OF PARAMETERS
      C      A     DOUBLE PRECISION UPPER TRIANGULAR PART OF GIVEN SYMMETRIC
      C            POSITIVE DEFINITE N BY N MATRIX.
      C            ON RETURN A CONTAINS THE RESULTANT UPPERTRIANGULAR MATRIX
      C            IN DOUBLE PRECISION.
      C      N     ORDER OF THE GIVEN MATRIX
      C      EPS   SINGLE PRECISION INPUT PARAMETER WHICH IS USED AS RELATIVE
      C            TOLERANCE FOR TEST ON LOSS OF SIGNIFICANCE.
      C      IER   RESULTING ERROR PARAMETER CODED AS FOLLOWS
      C            IER=0      NO ERROR
      C            IER=-1     NO RESULT BECAUSE OF WRONG INPUT PARAMETER N OR
      C                       BECAUSE SOME RADICAND IS NON-POSITIVE (MATRIX A IS
      C                       IS NOT POSITIVE DEFINITE, POSSIBLY DUE TO LOSS OF
      C                       SIGNIFICANCE,)
      C            IER=5      WARNING WHICH INDICATES LOSS OF SIGNIFICANCE.
      C
      C     REMARKS
      C      THE UPPER TRIANGULAR PART OF GIVEN MATRIX IS ASSUMED TO BE STORED
      C      COLUMNWISE IN N*(N+1)/2 SUCCESSIVE STORAGE LOCATIONS.  IN THE
      C      SAME STORAGE LOCATIONS THE RESULTING UPPER TRIANGULAR MATRIX IS
      C      STORED COLUMNWISE TOO.
      C      THE PROCEDURE GIVES RESULTS IF N IS GREATER THAN 0 AND ALL
      C      CALCULATED RADICANDS ARE POSITIVE.
      C
      C     SUBROUTINE REQUIRED - DMFSD
      C
      C ************************************************************************
0001         SUBROUTINE DSINV(A,N,EPS,IER)
      C
0002         DIMENSION A(1)
0003         DOUBLE PRECISION A,DIN,WORK
      C
      C         FACTORIZE GIVEN MATRIX BY MEANS OF SUBROUTINE DMFSD
      C         A=TRANSPOSE(T) * T
0004         CALL DMFSD(A,N,EPS,IER)
0005         IF(IER) 9,1,1
      C
      C         INVERT UPPER TRIANGULAR MATRIX T
      C         PREPARE INVERSION-LOOP
0006       1 IPIV=N*(N+1)/2
0007         IND=IPIV
      C
      C         INITIALIZE INVERSION-LOOP
0008         DO 6 I=1,N
0009         DIN=1.D0/A(IPIV)
0010         A(IPIV)=DIN
0011         MIN=N
0012         KEND=I-1
0013         LANF=N-KEND
0014         IF(KEND)5,5,2
```

```
FORTRAN IV G LEVEL  18                 DSINV              DATE = 71300          10/27/59

 0015                 2 J=IND
              C
              C           INITIALIZE ROW-LOOP
 0016                   DO 4 K=1,KEND
 0017                   WORK=0.D0
 0018                   MIN=MIN-1
 0019                   LHOR=IPIV
 0020                   LVER=J
              C
              C           START INNER LOOP
 0021                   DO 3 L=LANF,MIN
 0022                   LVER=LVER+1
 0023                   LHOR=LHOR+L
 0024                 3 WORK=WORK+A(LVER)*A(LHOR)
              C           END OL INNER LOOP
              C
 0025                   A(J)=-WORK*DIN
 0026                 4 J=J-MIN
              C           END OF ROW-LOOP
              C
 0027                 5 IPIV=IPIV-MIN
 0028                 6 IND=IND-1
              C           END OF INVERSION-LOOP
              C
              C           CALCULATE INVERSE(A) BY MEANS OF INVERSE(T)
              C           INVERSE(A)=INVERSE(T)*TRANSPOSE(INVERSE(T))
              C           INITIALIZE MULTIPLICATION-LOOP
 0029                   DO 8 I=1,N
 0030                   IPIV=IPIV+I
 0031                   J=IPIV
              C
              C           INITIALIZE ROW-LOOP
 0032                   DO 8 K=I,N
 0033                   WORK=0.D0
 0034                   LHOR=J
              C
              C           START INNER LOOP
 0035                   DO 7 L=K,N
 0036                   LVER=LHOR+K-I
 0037                   WORK=WORK+A(LHOR)*A(LVER)
 0038                 7 LHOR=LHOR+L
              C           END OF INNER LOOP
              C
 0039                   A(J)=WORK
 0040                 8 J=J+K
              C           END OR ROW- AND MULTIPLICATION-LOOP
              C
 0041                 9 RETURN
 0042                   END
```

```
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              C ************************************************************************
              C
              C     PURPOSE
              C      FACTOR A GIVEN SYMMETRIC POSITIVE DEFINITE MATRIX
              C
              C     USAGE
              C      CALL DMFSD(A,N,EPS,IER)
              C
              C     DESCRIPTION OF PARAMETERS
              C      A      DOUBLE PRECISION UPPER TRIANGULAR PART OF GIVEN
              C             SYMMETRIC POSITIVE DEFINITE N BY N MATRIX
              C      N      THE NUMBER OF ROWS(COLUMNS) IN GIVEN MATRIX
              C             ON RETURN A CONTAINS THE RESULTANT UPPER
              C             TRIANGULAR MATRIX IN DOUBLE PRECISION
              C      EPS    SINGLE PRECISION INPUT CONSTANT WHICH IS USED
              C             AS RELATIVE TOLERANCE FOR TEST ON LOSS OF
              C             SIGNIFICANCE
              C      IER    RESULTING ERRORPARAMETER CODED AS FOLLOWS
              C             IER=0  - NO ERROR
              C             IER=-1 - NO RESULT BECAUSE OF WRONG INPUT PARAMETER
              C                      N OR BECAUSE SOME RADICAND IS NON-POSITIVE
              C                      (MATRIX A IS NOT POSITIVEDEFINITE -
              C                      POSSIBLY DUE TO LOSS OF SIGNIFICANCE)
              C             IER=K  - WARNING WHICH INDICATES LOSS OF SIGNIFICANCE
              C                      THE RADICAND FORMED AT FACTORIZATION
              C                      STEP K+1 WAS STILL POSITIVE BUT NO LONGER
              C                      GREATER THAN ABS(EPS*A(K+1,K+1))
              C     REMARKS
              C      THE UPPER TRIANGULAR PART OF GIVEN MATRIX IS ASSUMED TO BE
              C      STORED COLUMNWISE IN N*(N+1)/2 SUCCESSIVE STORAGE LOCATIONS.
              C      IN THE SAME STORAGE LOCATIONS THE RESULTING UPPER TRIANGULAR
              C      MATRIX IS STORED COLUMNWISE TOO.
              C      THE PROCEDURE GIVES RESULTS IF N IS GREATER THAN 0 AND ALL
              C      CALCULATED RADICANDS ARE POSITIVE.
              C      THE PRODUCT OF RETURNED DIAGONAL TERMS IS EQUAL TO THE
              C      SQUARE-ROOT OF THE DETERMINANT OF THE GIVEN MATRIX.
              C
              C     METHOD
              C      SOLUTION IS DONE USING THE SQUARE-ROOT METHOD OF CHOLFSKY.
              C      THE GIVEN MATRIX IS REPRESENTED AS PRODUCT OF TWO TRIANGULAR
              C      MATRICES. THE LEFT HAND FACTOR IS THE TRANSPOSE OF THE
              C      THE RETURNED RIGHT HAND FACTOR.
              C
              C ************************************************************************
0001                 SUBROUTINE DMFSD(A,N,EPS,IER)
              C
              C
0002                 DIMENSION A(1)
0003                 DOUBLE PRECISION DPIV,DSUM,A
              C
              C         TEST ON WRONG INPUT PARAMETER N
0004                 IF(N-1)12,1,1
0005               1 IER=0
              C
              C         INITIALIZE DIAGNONAL-LOOP
0006                 KPIV=0
0007                 DO 11 K=1,N
0008                 KPIV=KPIV+K
```

```
      IND=KPIV
      LEND=K-1
C
C        CALCULATE TOLERANCE
      TOL=ABS(EPS*SNGL(A(KPIV)))
C
C        START FACTORIZATION-LOOP OVER K-TH ROW
      DO 11 I=K,N
      DSUM=0.D0
      IF(LEND)2,4,2
C
C        START INNER LOOP.
    2 DO 3 L=1,LEND
      LANF=KPIV-L
      LIND=IND-L
    3 DSUM=DSUM+A(LANF)*A(LIND)
C        END OF INNER LOOP
C
C        TRANSFORM ELEMENT A(IND)
    4 DSUM=A(IND)-DSUM
      IF(I-K) 10,5,10
C
C        TEST FOR NEGATIVE PIVOT ELEMENT AND FOR LOSS OF SIGNIFICANCE
    5 IF(SNGL(DSUM)-TOL) 6,6,9
    6 IF(DSUM) 12,12,7
    7 IF(IER) 8,8,9
    8 IER=K-1
C
C        COMPUTE PIVOT ELEMENT
    9 DPIV=DSQRT(DSUM)
      A(KPIV)=DPIV
      DPIV=1.D0/DPIV
      GO TO 11
C
C        CALCULATE TERMS IN ROW
   10 A(IND)=DSUM*DPIV
   11 IND=IND+I
C        END OF DIAGONAL LOOP
C
      RETURN
   12 IER=-1
      RETURN
      END
```

```
0001            SUBROUTINE CALCV(IPER)
          C  IN WHICH IS CALCULATED VX,VU,VXX,VUX,VUU,AND V
          C     THERE ARE THREE SECTIONS TO THE ROUTINE
          C   1 PARAMETERS ARE READ IN,SECOND DERIVATIVE MATRICES SET TO 0.0
          C   2 V IS CALCULATED USING UPDATED X AND U,ALSO VX2(LPP) AND UXX2(LPP)
          C     SUMMARY STATISTICS ARE PRINTED OUT
          C   3 VX,VU,VXX,VUX,VUU, ARE CALCULATED FOR PERIOD IT AS DEFINED IN MAIN
          C
0002            COMMON X(14,20),U(09,19),NX,NU,NPP,VX(14),   VXX(14,14),VUX(9,14),
               1VUU(9,9),VXX2(14,14),VU(9),LPP,VX2(14),V(20),F(14,14),G(14,9),
               2IIN,IOUT,NUMRUN
0003            DIMENSION R(8),P(8)
0004            REAL K(9)
0005            IF(IPER.EQ.0) GO TO 100
0006            IF(IPER.EQ.LPP) GO TO 200
0007            GO TO 300
          C
          C  READ BASIC DATA
          C
0008        100 IF(NUMRUN.EQ.1) GO TO 103
0009            READ(IIN,904) ICHNGE
0010            IF(ICHNGE.EQ.0) GO TO 101
0011        103 READ(IIN,900) K,R,C1,C2,C3
0012        101 WRITE(IOUT,901) K,R,C1,C2,C3
          C
          C CALCULATE VXX,VUX,VUU (WHICH IS INVARIANT)
          C
0013            DO 102 J=1,14
0014            DO 102 I=1,J
0015        102 VXX(I,J)=0.0
0016            D=K(1)*R(1)**2 +K(5)*C3**2
0017            VXX(1,1)=D+K(2)*R(2)**2+K(3)*R(3)**2+K(4)*R(4)**2
0018            VXX(1,2)=D-K(2)*R(2)
0019            VXX(1,3)=D-K(3)*R(3)
0020            VXX(1,4)=D-K(4)*R(4)
0021            VXX(2,2)=D+K(2)
0022            VXX(2,3)=D
0023            VXX(2,4)=D
0024            VXX(3,3)=D+K(3)
0025            VXX(3,4)=D
0026            VXX(4,4)=D+K(4)
0027            DO 110 I=1,4
0028            DO 110 J=5,8
0029        110 VXX(I,J)=-K(1)*R(1)+K(5)*C2*C3
0030            DO 111 I=1,4
0031        111 VXX(I,12) = -K(5)*C3
0032            D=K(1)+K(5)*C2**2
0033            VXX(5,5)=D+K(6)*R(6)**2+K(7)*R(7)**2+K(8)*R(8)**2
0034            VXX(5,6)=D-K(6)*R(6)
0035            VXX(5,7)=D-K(7)*R(7)
0036            VXX(5,8)=D-K(8)*R(8)
0037            VXX(6,6)=D+K(6)
0038            VXX(6,7)=D
0039            VXX(6,8)=D
0040            VXX(7,7)=D+K(7)
0041            VXX(7,8)=D
0042            VXX(8,8)=D+K(8)
0043            DO 112 I=5,8
```

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0044          112 VXX(I,12)=-K(5)*C2
0045              VXX(12,12)=K(5)
0046              DO 121 J=1,14
0047              DO 121 I=1,J
0048          121 VXX(I,J)=K(9)*F(14,I)*F(14,J) +VXX(I,J)
0049              DO 120 I=2,14
0050              IM1=I-1
0051              DO 120 J=1,IM1
0052          120 VXX(I,J)=VXX(J,I)
0053              DO 130 I=1,8
0054              DO 130 J=1,8
0055              VUU(I,J)=VXX(I,J)
0056          130 VUX(I,J)=VXX(I,J)
0057              DO 131 I=1,8
0058              DO 131 J=9,14
0059          131 VUX(I,J)=VXX(I,J)
0060              DO 132 J=1,14
0061          132 VUX(9,J)=K(9)*F(14,J)*G(14,9)
0062              DO 133 J=1,8
0063              VUU(9,J)=VUX(9,J)
0064          133 VUU(J,9)=VUU(9,J)
0065              VUU(9,9)=K(9)*G(14,9)**2
0066              GO TO 800
        C
        C  CALCULATE PREFERENCE FUNCTION AND VXX2(LPP),VX2(LPP)
        C
0067          200 WRITE(IOUT,902)
0068              DO 210 J=1,NPP
0069              DO 201 I=1,8
0070          201 P(I)=X(I,J)+U(I,J)
0071              TS=0.0
0072              TF=0.0
0073              DO 202 I=1,4
0074              TF=TF+P(I)
0075          202 TS=TS+P(I+4)
0076              P(2)=P(2)/P(1)
0077              P(3)=P(3)/P(1)
0078              P(4)=P(4)/P(1)
0079              P(1)=TS/TF
0080              A1=P(6)/P(5)
0081              A2=P(7)/P(5)
0082              A3=P(8)/P(5)
0083              A4=C1+C2*TS+C3*TF
0084              WRITE(IOUT,903)J,TS,TF,(P(I),I=1,4),A1,A2,A3,A4,X(12,J)
0085              A1=R(1)*TF-TS
0086              A2=R(2)*(X(1,J)+U(1,J))-(X(2,J)+U(2,J))
0087              A3=R(3)*(X(1,J)+U(1,J))-(X(3,J)+U(3,J))
0088              A4=R(4)*(X(1,J)+U(1,J))-(X(4,J)+U(4,J))
0089              A5=C1+C2*TS+C3*TF-X(12,J)
0090              A6=R(6)*(X(5,J)+U(5,J))-(X(6,J)+U(6,J))
0091              A7=R(7)*(X(5,J)+U(5,J))-(X(7,J)+U(7,J))
0092              A8=R(8)*(X(5,J)+U(5,J))-(X(8,J)+U(8,J))
0093              V(J)=K(1)*A1**2+K(2)*A2**2+K(3)*A3**2+K(4)*A4**2+K(5)*A5**2
                 1+K(6)*A6**2+K(7)*A7**2+K(8)*A8**2+K(9)*X(14,J+1)**2
0094          210 V(J)=V(J)/2.0
0095              V(LPP)=0.5*K(9)*X(14,LPP)**2
                 1 + 0.5*R(5)*(X(9,LPP)**2+X(10,LPP)**2+X(11,LPP)**2)
0096              DO 220 I=1,13
```

```
  220 VX2(I)=0.0
      DO 221 I=1,14
      DO 221 J=1,14
  221 VXX2(I,J)=0.0
      DO 222 I=9,11
      VX2(I)=R(5)*X(I,LPP)
  222 VXX2(I,I)=R(5)
      VX2(14)=    K(9)*X(14,LPP)
      VXX2(14,14)=K(9)
      GO TO 800
C
C  CALCULATE VX(IPER)
C
  300 TF=0.0
      DO 301 I=1,4
  301 TF=TF+X(I,IPER)+U(I,IPER)
      LD = X(5,IPER) + X(6,IPER)  +U(5,IPER)+U(6,IPER)
      UD = X(7,IPER) + X(8,IPER)  +U(7,IPER)+U(8,IPER)
      TS = LD + UD
      DO 302 I=1,14
  302 VX(I)=0.0
      J=IPER
      A1=R(1)*TF-TS
      A2=R(2)*(X(1,J)+U(1,J))-(X(2,J)+U(2,J))
      A3=R(3)*(X(1,J)+U(1,J))-(X(3,J)+U(3,J))
      A4=R(4)*(X(1,J)+U(1,J))-(X(4,J)+U(4,J))
      A5=C1+C2*TS+C3*TF-X(12,J)
      A6=R(6)*(X(5,J)+U(5,J))-(X(6,J)+U(6,J))
      A7=R(7)*(X(5,J)+U(5,J))-(X(7,J)+U(7,J))
      A8=R(8)*(X(5,J)+U(5,J))-(X(8,J)+U(8,J))
      D=K(1)*A1*R(1)+K(5)*A5*C3
      VX(1)=D+K(2)*A2*R(2)+K(3)*A3*R(3)+K(4)*A4*R(4)
      VX(2)=D-K(2)*A2
      VX(3)=D-K(3)*A3
      VX(4)=D-K(4)*A4
      D=-K(1)*A1+K(5)*A5*C2
      VX(5)=D+K(6)*A6*R(6)+K(7)*A7*R(7)+K(8)*A8*R(8)
      VX(6)=D-K(6)*A6
      VX(7)=D-K(7)*A7
      VX(8)=D-K(8)*A8
      VX(12)=-K(5)*A5
      DO 303 I=1,14
  303 VX(I)=VX(I)+K(9)*X(14,J+1)*F(14,I)
      DO 310 I=1,8
  310 VU(I)=VX(I)
      VU(9)=K(9)*X(14,J+1)*G(14,9)
  800 CONTINUE
      RETURN
C
  900 FORMAT(9F8.0/8F8.0/3F8.0)
  901 FORMAT(1H1,20X'PREFERENCE FUNCTION WEIGHTS'//1X'K1-K9',9F12.6//
     11X'R1-R8',8F12.6//1X,'C1-C3',3F12.6)
  902 FORMAT(1H1,50X,'SUMMARY STATISTICS'//1X,'PER',4X,'TS',9X,'TF',8X,
     1'TS/TF',6X,'F2/F1',6X,'F3/F1',6X,'F4/F1',6X,'SO/FR',6X,'JR/FR',
     26X,'SR/FR',5X,'ASF REQ',6X,'ASF BUILT'/)
  903 FORMAT(1X,I3,2(3X,F8.1),7(3X,F8.3),2(3X,F9.2))
  904 FORMAT(I1)
      END
```

DATA INPUTS FOR MULTIPLE RUNS

To make multiple comparison runs without respecifying all the data inputs, coded data-change cards are used. Changes may be made in either the data inputs to the main routine or to CALCV or both.

Changes in the data for MAIN must be preceded by a RUN card and followed by a card with "1" in column 1. Changes made in CALCV data must be preceded by a card with "2" in column 1. If no changes are made in MAIN data, a card with "1" in column 1 must follow the RUN card. If no changes are made in CALCV data, a card with "Ø" (zero) must follow the last card in the data set which changes MAIN, i.e., the card with "1" in column 1. The END card which follows the data in single runs is to be made the last card of the change-data sets. (There is only one END card for the program.)

FORM OF INPUT DATA

I. DATA FOR MAIN ROUTINE

Item	Var. Name	Value	# Cols.	# Rows	Format
1) RUN card:					(I3,3x,4I3,F8.0)
a) # Iterations this run	NUMIT				
b) # Control Variables	NU				
c) # State Variables	NX				
d) # Planning Periods	NPP				
e) Run Number	NUMRUN	001			
f) Est. Optimal Value of p.f.	OPT				
2) State Variable Transition Matrix (read in by rows)	F		NX	NX	(10 F8.0)
3) Control Variable Transition Matrix (read in by rows)	G		NX	NU	(10 F8.0)
4) Exogenous Variable Transition Matrix (read in by rows)	H		NX	NX	(10 F8.0)
5) Exogenous Variables for all Periods (read in by rows)	Z		NPP	NX	(10 F8.0)
6) Initial State Variables	X			1	(10 F8.0)
7) Initial Control Variables for all Periods (read in by rows)	U		NPP	NU	(10 F8.0)

II. DATA FOR CALCV SUBROUTINE

1) Vector of p.f. Weights	K		9	1	(9 F8.0)
2) Vector of p.f. Ratios (Note R(5) is wt for LPP x_9, x_{10}, x_{11})	R		8	1	(8 F8.0)
3) Vector of Coefficients for ASF Needs	C		3	1	(3 F8.0)
END CARD		100			(I3)

DATA STRUCTURE FOR MULTIPLE RUNS

Item	Var.Name	Value	# Cols.	# Rows	Format
1) RUN card (as before) 2) MAIN data (as before) 3) CALCV data (as before)					
4) RUN card (as before except run number) e)	NUMRUN	>002			
5) Change code	ICHNGE	1-7			(I1)
New F-Matrix follows		2			
New G-Matrix follows		3			
New H-Matrix follows		4			
New Z-Matrix follows		5			
New Initial-state Variables follow		6			
New Initial-control Variables follow		7			
No (further) changes in MAIN data		1			
Following each change coded card, the appropriate variables are specified as before.					
6a) If there are no changes in CALCV data	ICHNGE	0			(I1)
6b) If there are changes in CALCV data Complete set of CALCV data as before	ICHNGE	2			(I1)
[Repeat 4 - 6 as needed]					
7) END CARD		100			(I3)

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