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AUTHOR	Moore, Charles N.
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ABSTRACT

A computer-based Statistical Program to Assist in Teaching Statistics (SPATS) has been successfully developed to aid the teaching of statistics to undergraduates with business and economics majors. SPATS simplifies the problem of experimentally creating and analyzing a variety of populations and of selecting and analyzing different kinds of random samples, together with individual data and summary statistics. SPATS is also used by students to carry out statistical experiments to prove theorems, similar to experiments used in chemistry to prove Boyle's and Charles' laws. Although experience with this application of computers is limited, results to date indicate that computer assisted statistical experiments will be a major breakthrough in the process of providing students with a real understanding of basic statistical principles and theorems. Research is continuing on the problem of sharpening and testing the tools of SPATS so as to make them more feasible for undergraduate classroom use. (Author/PB)

COMPUTER ASSISTED LABORATORY PROBLEMS FOR TEACHING BUSINESS AND ECONOMIC STATISTICS

Charles M. Moore
Mississippi State University
State College, Mississippi 39762
(601) 325-5904

Students taking courses in statistics (theory and/or methods) typically have difficulty learning the details of the methods for describing and analyzing quantitative data. They have more difficulty understanding the theory behind the methods.

Unfortunately this condition leads to the use of rote memory and generates little enthusiasm for the potential of statistical methods to supply useful information and facilitate decision making. In the pre-computer era, we used a variety of visual aids in the classroom to generate enthusiasm and interest among the students. For example, "2 X 2" or 35mm projector slides and VU-Graph transparencies were created to show the student some of the basic ideas in time series analysis, correlation and regression, and probability. In an attempt to clarify the problems of variation, sampling errors, and inference, populations of various colored beads and wooden sampling paddles were used by instructors for classroom demonstration. With some ingenuity a teacher could use the bead population and samples for shedding further light on the cause of the basic problem in statistics--variation(within and among populations, within and among samples, and between sample statistics and population parameters). For example, the cigar boxes of beads and the paddle were used to generate real time sampling data from known populations. The data were used for problems involving chi-square, analysis of variance, and correlation and regression. Although better than using purely static data, these methods were obviously inflexible (e.g., size and parameters of population, and size of sample were not easily changed). In addition, it was not wise to permit hundreds of undergraduate students to "play" with the beads of controlled populations because of the likelihood of the populations getting out of control. Another major limitation of pre-computer teaching aids was that they provided no relief from the frequently difficult, monotonous, and time consuming task of data analysis. Fish bowls, cigar boxes of beads, and wooden paddles could help in generating dynamic type data, but the necessity for manual processing of data greatly limited the opportunity to experiment with and analyze different kinds and sizes of populations and different sizes and types of samples.

When electronic computers became available on the campus the writer began to experiment with the problem of creating populations of various kinds, selecting random samples (with or without replacement), and analyzing and summarizing the results of thousands of samples. A basic "tool" needed is a random number generator, (uniformly distributed from zero to one)[1] from which a variety of different kinds of random numbers (continuous and discrete) can be generated. The great potential of the electronic computer for dynamic creation and analysis of data from populations and samples became apparent.

In 1960 the writer had the opportunity to use a medium size electronic computer (the UNIVAC SS80 at the University of Alabama) for classroom instruction. At that time he was teaching both undergraduate courses in business statistics and a course in statistical quality control. In the quality control course, the computer was used to simulate the classical fish bowl experiments of Walter Shewhart[2]. With the aid of the computer and a random number generator, all of Shewhart's finite populations (normal, triangular, and rectangular) were created and thousands of random samples of various sizes were selected and analyzed to verify the validity of the quality control charts (and the underlying central limit theorem). This was the real beginning of the writer's interest in having students use the electronic computer for conducting experiments to verify concepts, to prove theorems, and to explore relationships not previously explained by theorems.

SPATS - statistical program to assist in teaching statistics

In 1967 the writer had the opportunity to develop a master computer program to be used easily by students for generating and analyzing various kinds of populations and samples. This program is called SPATS (Statistical Program to Assist in Teaching Statistics) and has been used by both graduate and undergraduate students. Briefly, SPATS consists of a series of FORTRAN subroutines which permit the user to create, analyze, organize, and present the data for a variety of different kinds of populations (continuous and discrete) and random samples (with or without replacement). The number and size of populations one can create and store is dependent upon the specific hardware used; originally SPATS was written in FORTRAN

IV for the IBM 360/40, and provided for the creation and storage of up to 10 different populations of 1000 items or less, and up to 6 different random samples of 500 items or less. The analyst can see a printout of the individual population or sample items by indicating that choice in the argument list. One of the most powerful features of SPATS is that numerous random samples (with or without replacement) can easily be selected, analyzed, summarized, and presented for inspection.

SPATS - Shewhart experiment

To illustrate the simplicity of using SPATS for creating various kinds of populations and selecting and analyzing random samples of various sizes from each of these populations, the following short FORTRAN program is submitted. This program is sufficient to re-create (and improve upon) [3] the classical experiments of Shewhart in which he created two populations (normal, and rectangular) and selected 1000 random samples (with replacement) of four items each. The power of this sort of program for producing information is indicated by the extent of the analysis obtained from statements number 30 and 40. In addition, with a few modifications the Shewhart experiment can be greatly expanded to include many more different kinds and sizes of populations and samples.

STATEMENT NUMBER	FORTRAN STATEMENT
10	CALL MVPOPJ (3, 0., 1., 998, 1)
20	CALL MBPOPJ (1, -3., 3., 998, 2)
30	CALL SAMPLE (1, 998, 4, 2, 1, 1000)
40	CALL SAMPLE (2, 998, 4, 2, 1, 1000)
50	STOP
60	END

Notes:

1. Statement number 10 creates and stores a normal population of 998 items with a mean of 0. and a standard deviation of 1.

2. Statement number 20 creates a rectangular population of 998 items ranging from -3. to +3.

3. Statement number 30 selects 1000 random samples of size 4, with replacement, from the normal population and analyzes the population and sample data by computing the following measurements:

   a. From population

      1) Mean
      2) Variance
      3) Standard Deviation
      4) Standard error of sample mean

   b. From each sample

      1) Sample mean, variance, standard deviation, standard error of sample mean, confidence limits for population mean;
      2) Unbiased estimates of variance, standard deviation, and standard error of estimate
      3) List of each item in random sample (optional)

   c. From summary of all samples

      1) Average sample mean, average variance, average standard deviation, average standard error of sample means;
      2) Unbiased estimates of each of the above averages
      3) Measures of dispersion from the experimental distributions of sample statistics such as:

         a) Variance and standard deviation of sample means
         b) Variance and standard deviation of sample variances
         c) Variance of standard errors of estimate

EXHIBIT I

Laboratory Problem No. 2

Experimental Study of Samples, Sample Statistics, and Sample Distributions

A. Introduction to problem

The experiment in laboratory problem no. 1 was concerned with populations--the problem of designing, creating, analyzing, and studying different kinds of populations.

In this laboratory problem the focal point is on samples, and the experiment is designed to permit the student to design and create random samples, and to study experimentally the characteristics of sample statistics and sample distributions in the light of known characteristics.

This kind of exploration (made feasible by SPATS and the electronic computer) should provide a better understanding and appreciation of sampling theory, and the basic concepts of random behavior.

B. Perform the following operations:

1. Design, create, and store two different kinds of populations. You may use the parameters in laboratory problem no. 1, if you desire, but increase the size of each of the populations to 1000.

2. (Optional). Print the data from the two populations.

3. a) Compute the first four moments for each population. Construct a table summarizing for each population the following measurements; mean, variance, coefficient of skewness, and coefficient of kurtosis.

   b) Construct a frequency distribution for each population. Use arguments for FRED as suggested in (5) of lab problem No. 1.

4. Take four random samples of the indicated size of each of the two populations. (Use printing code, IP = 2, in SAMPLE)

Sample Size	Sampling Procedure
5	with replacement
5	without replacement
50	with replacement
50	without replacement

5. Take 100 random samples of size 50 each from your populations. Print only the individual statistics and the final averages (IP = 1)

6. Take 1000 random samples of size 5 each from your normal population. Print only the final averages and measures of dispersion (IP = 0). Repeat using sample size = 50.

Note: As a by-product of this use of the subprogram SAMPLE, the following sample statistics have been computed and stored.

Sample Statistics	Column in XX Matrix
mean	7
variance	8
standard deviation	9
standard error of estimate	10

Thus, you will create at this point in the experiment--distributions of sample statistics or "sampling distributions" [4, p.142]. These distributions will be in the form of arrays which can then be summarized and analyzed as desired.

7. It is now desired to make an additional analysis of selected statistics.

   a) Compute the first four moments for each of the four sampling distributions listed in the note in item 6.

   b) Construct a frequency distribution from each array of sampling distributions. Use class limits of your choice.

8. Repeat steps 6 and 7 using your second population (the non-normal one) in place of the normal population. Use samples of size 5 items or 50 items.

C. Now, based on your experiments in Part B, answer the following questions:

(Chapter 8, Spiegel, is a good reference)

1) Comment on your observations from the summary table in B 3(a). (Are the measurements in line with the characteristics you expected based on your work in lab problem no. 1?)

2) Comment on your observations from the frequency distributions constructed in B 3(b). (Is the distribution in each case about what you expected?)

3) What effect does sampling with or without replacement have on the computation of sample statistics? Give an example from your data.

4) a) Formulate a definition of "68% and 95% confidence limits", Test this definition with your experimental data from B (5).

   b) Formulate a definition of biased and unbiased estimates. Give two examples of each kind of estimate from your experimental data obtained in B (5).

   c) Examine the results of B (6) and comment on the observed averages of biased and unbiased statistics. How do they compare with the corresponding population parameters?

5) Formulate a definition of the central limit theorem (see Spiegel, p. 142 and/or other references). Test the validity of this theorem with the results of your computations in B, parts (6), (7), and (8). (Hint: Examine the distributions of sample means from each of the two populations for both small and large samples; examine the theoretical and experimental standard errors of sample means).

   Does your experimental data support the theorem? Defend your answer.

5) Record the following statistics from your calculations in B (6) and test these experimental results against the theoretical values [4, p.144].

   a) Standard deviation of sample means

   b) Standard deviation of sample variances

   c) Standard deviation of sample standard deviations.

Comment on your observations.

Note: Unless specifically indicated, samples may be selected with or without replacement.

EXHIBIT II

FORTRAN IV Main Program for Lab Problem No. 2, Using SPATS

	STATEMENT	REMARKS
	CALL MVPOPJ(2, 70., 10., 1000, 2)	Creates normal pop.; μ=70, σ=10
	CALL MVPOPJ(1, 40., 100., 1000, 1)	Creates uniform pop.; A=40, B=100
	PRINT (2, 1000, 2)	Prints the 2 populations.
	DO 3 J=1, 2 CALL MOMPS(4, 1000, 100., 2, J) BOT = XBAR - 3.*SDEV TOP = XBAR + 3.*SDEV CI = SDEV/3.	Computes the first four moments for each of the populations and constructs a frequency distribution for each pop. using μ and σ for constructing classes
3	CALL FRED (BOT, TOP, CI, 1000, J)	
4	CALL SAMPLE (2, 1000, 5, 2, 0, 1) CALL SAMPLE (2, 1000, 5, 2, 1, 1,)	Select random sample of size 5 from normal pop., without and with replacement.
	CALL SAMPLE (2, 1000, 50, 2, 0, 1) CALL SAMPLE (2, 1000, 50, 2, 1, 1)	same as above except sample size = 50.
5	CALL SAMPLE (2, 1000, 50, 1, 1, 100) CALL SAMPLE (1, 1000, 50, 1, 1, 100)	Select and analyze 100 random samples from normal pop. and from uniform population.
	DO 8 NS = 5, 50, 45 DO 8 NP = 1, 2	
6	CALL SAMPLE*(NP, 1000, NS, 0, 1, 500)	Select and analyze 500 random samples of size NS from pop. (Kind = NP).
	DO 7 J = 7, 10 CALL MOMPS(4, 500, 100., 2, J)	Compute 1st 4 moments from Jth pop.
	BOT = XBAR - 3.*SDEV TOP = XBAR + 3.*SDEV CI = SDEV/3.	Compute arguments for FRED using mean (XBAR) and σ (SDEV) from last MOMPS.
7	CALL FRED(BOT, TOP, CI, 2, 500, J)	Construct freq. distribution for Jth pop., J=7, 8, 9, 10; Note: Pop. here are actually the newly created <u>sampling distributions</u> created from statement no. 6; see lab prob. no. 2.
8	CONTINUE STOP END	

Note: For a re-run using a different random start, add the following statement at the beginning of this program:

NSTART = 613247 [Or use any other <u>odd</u> number with 8 or less digits]

<u>Using SPATS in the Classroom</u>

To illustrate how SPATS may be used in the classroom, an example of a laboratory problem is presented in Exhibit I. The SPATS program for performing the required operations appears in Exhibit II. The problem included is the second in a set of four which were used in a three semester hour course offered at Mississippi State University entitled, Business Statistics Using the Computer (required of all undergradutae majors in business statistics and data processing). The prerequisite is six semester hours of business statistics and three semester hours of data processing (including FORTRAN).

The idea of laboratory experimentation with populations and samples being somewhat foreign, the students were slow to catch on to the idea when first explained. However, when they began to see some tangible examples of quantities which previously were somewhat nebulous, such as discrete and continuous populations, random samples, confidence limits, biased and unbiased estimates, and sampling distributions, their enthusiasm increased. Some students stated after laboratory problem no. 2 that for the first time they understood some concepts and relationships that they had previously derived in mathematical statistics courses. For example, "95% confidence limits," and biased and unbiased estimates become more meaningful when supported by empirical evidence.

Proving Theorems with SPATS

One further extension of SPATS which has great potential is that of proving theorems experimentally. This has not been attempted with undergraduates but I have tried this approach with a small group of graduate students in business. The intent was to have the students make a parallel proof of each theorem experimentally and analytically, and to evaluate the merits of each approach. The analytical proofs had to be omitted due to lack of time and the students' insufficient mathematical backgrounds. Since the experimental approach was new to them, as was the case with the undergraduates in the other course, it was more of a challenge than I realized. Instead of having them perform all of the experiments on the assignment sheet (Exhibit III), it was necessary to settle for no more than six lessons. One drawback is that SPATS was not created originally to prove theorems directly. Therefore, it is necessary to use only limited subsets of SPATS and a great deal of FORTRAN to carry out the experiments. One of the graduate students, however, is now working on a dissertation which is concerned with the problem of developing easy to use computing "apparatus" and "materials" (analogous to that used in chemistry experiments). It is hoped that these new materials will greatly simplify the problem of designing and carrying out a large number of experiments for testing statistical theorems, and even exploring relationships not supported by mathematical proof.

EXHIBIT III

A Dozen Additional Statistical Experiments Feasible with SPATS

1. Prove experimentally that $\sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2}$

2. Test experimentally the theory of the mathematician D'Alembert who reasoned that the three possible outcomes of a toss of two coins are equally likely.

3. Design an experiment similar to Walter Shewhart's classical experiment in quality control concerning the distribution of sample means where the samples are of various sizes taken from both non-normal and normal populations.

4. Design an experiment to test the frequency concept of probability.

5. Perform an experiment to show the influence of experimental data on subjectively assigned prior probabilities concerning various hypotheses (or events) which are mutually exclusive and completely exhaustive. This would amount to an experimental test of Bayes theorem (see Mosteller, Rourke, and Thomas[5], p. 146).

6. Prove experimentally that the variance of the sum of n independent random variable is the sum of their variances.

7. Let C = a constant, and X = a random variable. Prove experimentally the following:

$$\sigma^2(C \cdot X) = C^2 \cdot \sigma^2(X)$$

$$\sigma^2(C+X) = \sigma^2(X)$$

$$\sigma(C+X) = \sigma(X)$$

$$\sigma^2(a+bX) = b^2 \cdot \sigma^2(X)$$

8. Chebyshev's theorem states that the probability is at least $(1 - (1/h^2))$ that a random variable from a population falls within h standard deviations from the population mean. Test this theorem experimentally using several different kinds of random variates.

9. Chebyshev's theorem applies to the probability of items in a sample as well as to the random variates from a population. Test experimentally that at least $(1 - (1/h^2))$ of

the items in a random sample are within h sample standard deviations from the sample mean.

10. Prove experimentally the DeMoivre-LaPlace theorem. Let X_i, (i = 1, n) be the number of successes in a binomial experiment with n trials each with a probability of success = p.

Prove that $P(Z_n = ((X_n - np)/\sqrt{npq}) \geq z)$ approaches the area to the right of z for a standard normal distribution.

11. Experimentally construct a chi-square distribution with mean = n and variance = 2n.

12. Experimentally check the validity of F. N. David's Tables of Correlation Coefficients (Cambridge Press, 1938) using population correlation coefficients (rho) = 0., .1, .2, .9; and using sample size n = 3 to 25 (increments of 1), and large samples where n = 50, 100, 200, and 400.

Summary and Conclusions

SPATS is a master statistical program developed originally for the purpose of simplifying the problem of experimentally creating and analyzing a variety of populations (discrete and continuous), and simplifying the task of selecting and analyzing a variety of different kinds of random samples, together with individual data and summary statistics. It has been used in the classroom and does give students a better understanding of some of the basic concepts and relationships pertaining to populations and samples.

In the process of developing and using SPATS, the idea occurred that some of the basic components of SPATS could be used by the students for carrying out statistical experiments to prove theorems similar to those experiments used in chemistry to prove Boyle's and Charles' laws. Although experience with this application of computers has not been as extensive as other more traditional applications, results to date give support to the belief that computer assisted statistical experiments will be a major breakthrough in the process of providing students with a real understanding of basic statistical principles and theorems. Research is continuing on the problem of sharpening and testing the tools of SPATS so as to make them more feasible for undergraduate classroom use.

NOTES AND REFERENCES

1. Originally the mid-square method was used but more recently some form of the multiplicative congruential method has been found to be more satisfactory.

2. W. A. Shewhart, Economic Control of Quality of Manufactured Product. Princeton, N. J.: D. Van Nostrand Co., 1931.

3. Although both Shewhart's and SPATS normal and rectangular populations do not exactly duplicate the theoretical distributions, the SPATS populations are more continuous (less discrete); also the rectangular population in SPATS contains 998 items whereas Shewhart's contains only 122.

4. Murray R. Spiegel, Theory and Problems of Statistics. New York: Schaum Publishing Company, 1961.

5. Mosteller, Rourke, and Thomas, Probability and Statistics Addison-Wesley Publishing Co., 1961.