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ABSTRACT

This teacher's manual accompanying the ninth-grade general mathematics workbook, Drop-In Mathematics, states objectives for each of the topics covered, suggests teaching methods, lists resource materials, and provides an answer key for problems in the text. Enrichment activities are included in the appendix. For student materials, see SE 016 406.. (DT)

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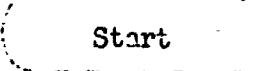
UNIT 1

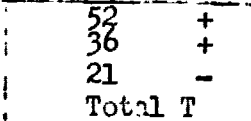
FLOWCHARTS


I. Objective: To introduce use of calculator; to introduce symbols and techniques used in making flowcharts


II. Suggestions:


A Introduce symbols used in flowcharting and tell how each symbol is used.


 Start Symbol used to begin flowchart.

 Information on left and operation on right.

 Used for decisions. The question is written inside the symbol.

 Recorded subtotal or partial answer.

 Used to record final result.

 Used to indicate the end of the flowchart.

B. Advantages of Flowcharts:

1. Prevents wasted time, by knowing what is to be done and pre-planning how to do it.
2. Changes complex mathematical problems to a set of simple steps.
3. Provides a clear means of communication between persons working on the same problem
4. A flow chart sections the job to be done. If it is necessary to stop at a particular point, reference to the flowchart tells what has been done and what remains to be done. The printed tape makes it easy to check with a flowchart.
5. Checks understanding. If a problem can be flowcharted, it is more likely to be understood.

Flowcharting may be used to advantage on all problems involving computation. Properly instructed, the machine cannot make an error. If the flowchart is correct and is followed accurately, the machine will be properly instructed and you must obtain the correct result.

C. Presentation: When

Flowcharting can be used without the calculator. Flowcharting is most helpful as a device for teaching the operation of the calculator.

The calculators can be introduced several different ways:

1. At the beginning of the course as an introductory unit to the 4 basic operations.
2. With the units as the operations addition, subtraction, etc. are studied.
3. With some of the local business problems.

No matter how the calculators and/or flowcharting are introduced it is important to keep the lessons varied in content, time, and methods so that the students will remain interested.

D. Presentation: How

One way to develop student interest is to give them a basic skills test and then show how much more quickly and easily it can be done with the calculator.

Another method is to use flowcharts with a difficult problem showing how much it can be simplified.

Calculators can take the drudgery out of problems.

Remember that flowcharts should be simple, logical steps to the solution of a problem. You can make your own with little difficulty after you have gone over a few formal charts.

Most calculator companies can give you plenty of examples for their specific calculator.

Note: On flowcharts on page 3 the operation symbols have been left out. The teacher should help the students place the symbols on their machine in the appropriate place.

ANSWERS: Page (2)

- (1) 3015; (2) 3643; (3) 34,656; (4) 30,357; (5) 13,056; (6) 32,559;
- (7) 36,368; (8) 37,789; (9) 42,568; (10) 428,687; (11) 4,507; (12) 6,767;
- (13) 799,684; (14) 81,005; (15) 91,337; (16) 1,019,800,242;
- (17) 8,981,939,620; (18) 965,000; (19) 9,753,086,421.

TM-1-5

ANSWERS: Page 4.

- (1) 2,007,167; (2) 2,223,840; (3) 20,862; (4) 33,612,460,758;
(5) 517,332; (6) 7,506,140; (7) 9 610,700; (8) 1,959,999,363;
(9) 84,113,863,726; (10) 560,764,958,000; (11) 424; (12) 17; (13) 84;
(14) 225; (15) 541; (16) 854; (17) 6.7; (18) 156; (19) 4,440; (20) 2,200.

Page 5.

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UNIT II

SETS

I. Objective: To introduce the basic set theory as a language for future use to clarify some concepts.

II Suggestions and Answers:

- A. Explain the idea of sets and elements by example.
May use bulletin board display, film strips and transparencies.
Answers will vary.
- B. Stress the relationship between the description (Sometimes called Rule) and listing of the elements. Some answers will vary.

ANSWERS: Page 7

- 4 Set D {Set of three smallest coins}
5 " E {Set of symbols for Roman Numerals}
7 " G {This will be a way for students to become acquainted}
2 " K {1, 3, 5, 7, 9, . . . , 97, 99}
3 " L {a, b, c, d, . . . , v, z}
4 " M {Multiples of 3 from 3 through 33}
5 " N {Odd counting numbers 1 through 27 inclusive}
6 " O {12, 14, 16 . . . , 994, 996, 998}

- C Point out that there are sets which may have elements we cannot list (i.e., the leaves on any one tree, the people of any given nation) by pattern but only by description. Also, there are sets which do not contain any elements. Point out there are finite sets so large that it would be difficult to count the elements (set of counting numbers from 1 to 1000, etc.,) and there are infinite sets which have no final element. Such sets can be listed by pattern.

ANSWERS: Page 8

- 1 Set $Y \Leftrightarrow 2, 4, 6, 8, \dots$
2 " $X \Leftrightarrow 3, 6, 9, 12, \dots$
3 " $W \Leftrightarrow$ Counting multiples of 4
4 " $V \Leftrightarrow$ Squares of all counting numbers beginning with 2
1. $\emptyset, 2, \text{none}, \{ \}$ may be used for \emptyset

- D. Point out subsets may contain one element, many elements, all elements or no elements of the universal set.

ANSWERS: Page 9

1. a. {Sun} {Sat.} {Fri} d. None
b. {Sun., Sat., Fri} {Sun., Fri} {Sat., Fri} e. Yes
c. {Sun., Sat., Fri} f. 8

ANSWERS: Page 10.

2. a. $\{ \}$; $\{ \text{Joe} \}$; $\{ \text{Jane} \}$; $\{ \text{Mary} \}$; $\{ \text{Joe, Jane} \}$
 $\{ \text{Joe, Mary} \}$; $\{ \text{Jane, Mary} \}$; $\{ \text{Joe, Jane, Mary} \}$
- b. Finite.
3. a. $\{ 3 \}$; $\{ 7 \}$; $\{ 6 \}$; $\{ 1 \}$
- b. $\{ 3, 7 \}$; $\{ 3, 6 \}$; $\{ 3, 1 \}$; $\{ 7, 6 \}$; $\{ 7, 1 \}$; $\{ 6, 1 \}$
- c. $\{ 3, 7, 6 \}$; $\{ 3, 7, 1 \}$; $\{ 3, 6, 1 \}$; $\{ 7, 6, 1 \}$
- d. $\{ 3, 7, 6, 1 \}$
- e. Set S can be a subset of any larger set which contains all elements of set S; i.e., the complex number system, counting numbers, such sets as $\{ 3, 7, 6, 1, 4 \}$, etc.
4. a. 3 element set has 2^3 subsets or 8
- b. 4 element set has 2^4 subsets or 16

A. Venn Diagrams

Since Venn diagrams can become very complex, we are leaving the amount covered to the individual teachers. Following are some possibilities:

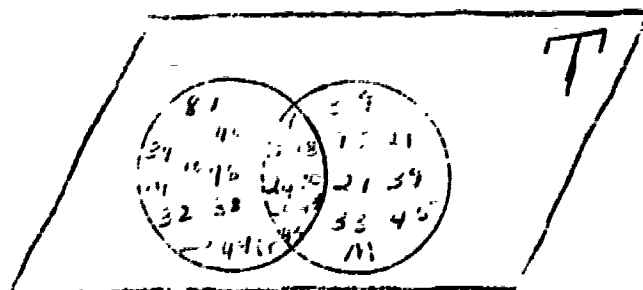
B. Be sure students understand which sets are to be shown as subsets.

ANSWERS: Page 11.

1.



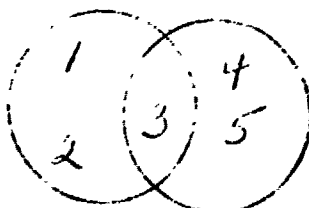
2.



A. Intersection of Sets

ANSWERS: Page 12

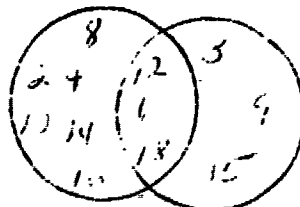
1. $\{3\}$



2. Set $K \leftrightarrow \{2, 4, 6, 8, \dots, 18, 20\}$

" $J \leftrightarrow \{3, 6, 9, \dots, 15, 18\}$

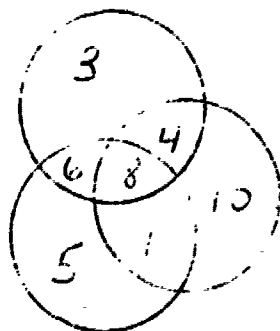
" $D \leftrightarrow \{6, 12, 18\}$



To challenge the students, you might present the following problem to be solved using Venn diagrams.

Of the members of 3 athletic teams in a certain school, 21 are on the basketball team, 26 on the baseball team, and 29 on the football team; 6 play basketball and baseball, 7 play baseball and football, 4 play basketball and football, and 8 play on 3 teams. How many members are there altogether?

Basketball



Baseball

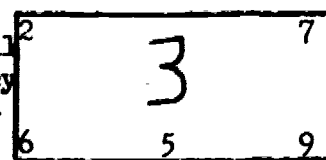
43 members in the athletic department.

RENAMING NUMBERS

I Objective: To introduce the student to the idea that a number is only a concept, but has many names

II. Suggestions:

Have the class think of a new pet such as a puppy. Have them offer suggestions for naming it. Of course, many likes and dislikes of a name will result. Now follow up with an explanation, that number cannot be written on paper or board, that number is only an idea in a person's mind concerning quantity whereas numeral is the symbol itself representing the number. Symbols, like pets, people, or things can bear many names according to the wishes of the person naming the symbol. Put the word "milk" on the board and ask the class what they see on the board. This will serve as a beginning motivator. Some will say they see "milk" on the board, whereas others may say they only see a name or symbol for the liquid milk itself. Use other stimulating statements on the board as "George Washington slept here" and have class discuss its veracity. Turn next to a box with several symbols and ask what is the largest number they see. Really they can't see number, but '3' will be the largest numeral they see. If they think "quantity", then '9' would be correct, but the number '9' cannot be seen.



Next ask students to suggest names for the number seven. If response is slow, suggest names like these:

VII	8-1	5+2
$\frac{14}{2}$	$\sqrt{49}$	$3^2 - 2$
$2^2 + 3$		$10 - 3$
$(3 \times 2) + 1$	$-1 +$	$\sqrt{64}$

(Students are not expected to understand all these names used) Can this list be made longer? How much? Point out that the symbol 7 is the simplest way to express the idea of seven. Have students write a numeral at the top of the page and give them 3 to 5 minutes to write as many other names as possibly. This can be in the form of a game and used at any time.

III. Uses:

- (a) The renaming of numbers can be used to introduce number bases other than ten.

- (b) Renaming numbers can be used to introduce exponents
- (c) Renaming numbers are used in applying the distributive principle or factoring
- (d) Finding common denominators of fractions—reduction of fractions

IV Resources:

Encyclopedia Britannica Press, Discovering Modern Mathematics—A Program for Parents, "A Number Has Many Names"

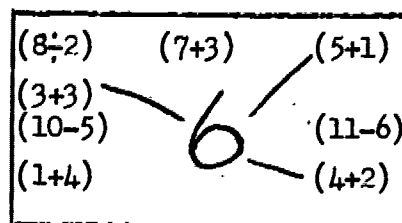
Introduction to Secondary Mathematics, Vol. I, Haag and Dudley.

Mathematics, A Modern Approach, AVE Filmstrip, Wilcox and Yarnell (if available)

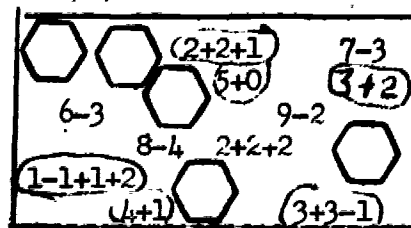
V Answers: Page 14

1. $11-1$, $9+1$, x , $100/10$, 20_5 , ten, $2(5)$, $3^2 + 1$, $70/7$, $4(2 \frac{1}{2})$

2



3



UNIT III.

COUNTING NUMBERS

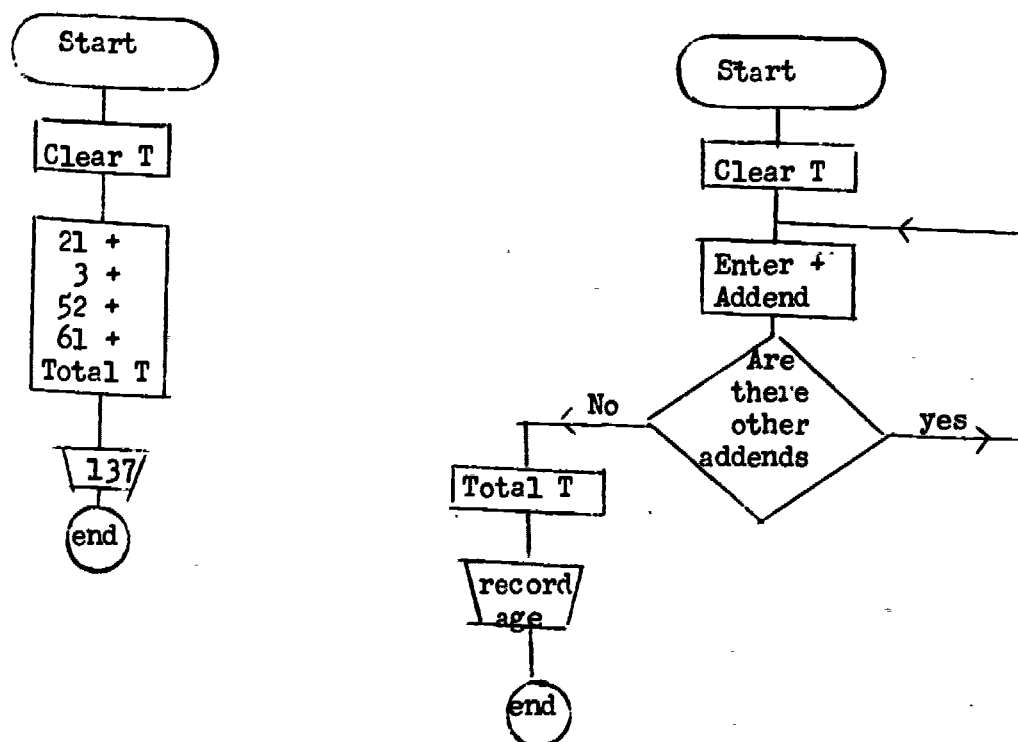
- I Objective: To teach the meaning of the set of counting numbers as used in daily life
- II Suggestions:
 - A Use stories from the history of development of numbers to illustrate the use of counting numbers and to create interest
 - 1 Counting sheep, notched sticks, tally marks, pebbles, counting books in certain section of library or classroom, etc
 - B Go to street corner, Count cars by using tally marks
 - C Use arrangement for tallying number of manually calculated problems which prove to be correct when student checks his work on calculator An abacus for each student could be used.
 - D Characteristics of Counting Numbers
 - 1 Least counting number; greatest counting number
 - 2 How to get the next counting number after the first (By adding the number one to any given counting number the next counting number can be obtained)
 - E Show use of counting numbers in making inventory
 - F Good place to introduce definition of operation Use weird operations for optional exercises, page 241
- III Resource Material:
 - A Sticks, pebbles, movie or filmstrip, cubes, empty boxes (for counting materials)

IV. Addition Flowcharts

Examples:

(for Problem 1, Student Page 17)

General form for addition



 Answers: Page 15 7) 100; (8) 441; (9) 4.

Page 16 (1) 1; (2) 7; (3) 123; (4) 1; (5,a) $1 + 1 + 1$; (b) $2 + 1$; (6,a) $1 + 1 + 1 + 1$; (b) $1 + 3$; (c) $3 + 1$; (7) By adding the number one; (8,a) $1 + 1 + 1 + 1$; $3 + 1$; $1 + 3$; (b) Order principle of counting numbers (commutative principle) (c) $2 + 2$.

Page 17 Machine Practice Sheet (answers)

(1) 137 (2) 64 (3) 2171 (4) 923 (5) 767 (6) 9117
(7) 5393 (8) 1798 (9) 8246 (10) 471,213 (11) 58,473

Page 18 Addition practice on calculator. Answers:

(1) 1212 (2) 1988 (3) 2628 (4) 1267 (5) 1769
(6) 1582 (7) 1731 (8) 1193 (10) 1773 (9) 2195
11) 915 (12) 1893 (13) 1965 (14) 1589 (15) 1388
(16) 1773 (17) 1786 (18) 1771

Page 19. 8.07

NUMBER LINE

- I. Objective: To introduce the number line using counting numbers.
- II. Suggestions:
 - A. Use rope and clothes pins.
 - B. Use cigar box with the lid as a flannel board.
 - C. Walking across floor.
 - D. Number line of counting numbers leaves many points without a name.
- III. Resource Material

See your elementary school friends.

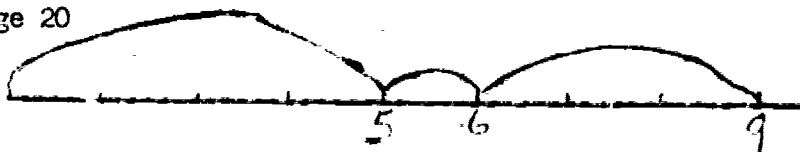
Books: Arithmetic, An Introduction to Mathematics, by Law, Macmillan.

Audio Visual:

The Number Line: Colonial Films.
The Number Line: SVE Filmstrip.

ANSWERS: Page 20

4. Start



Page 21.

- (7) c. 14, d. 5, 9, 14, e. 14, f. Addition
- (8) c. 9, d. 2, 3, 4, 9, e. 9, F. Addition
- (9) b. zero, c. 49 d. 49
- (10) c. 11 d. 5, 3, 1, 2, 11 e. 11 f. Addition

CLOSURE FOR ADDITION OF COUNTING NUMBERS

I. Objective: To introduce and teach the concept of closure of counting numbers in addition.

.. Suggestions: The student page is designed to direct students' thinking toward the definition of closure under addition.

Definition: When adding two or more counting numbers, the sum is always a counting number.

III. Directions for Plant Population: Students will need to find out the distance between rows and distance between plants in the rows for the various crops. This is a good outside assignment for students associated with farms or gardens. Other crops may be substituted for those listed depending on local interest. Obviously there will be differences in information.

IV. Resource Materials:

A. Baskets or boxes, apples, oranges, empty food cans with prices on one end, empty sugar sacks with price attached, blocks.

B. Books

C. Todd and McDermott: Programmed In-Service Mathematics, Arkansas State Department of Education, 1965.

C. Audio Visual

The Closure, Commutative and Associative Properties of Real Numbers. SVE Filmstrip.

ANSWERS: Page 24,

- (1) 5, (2) 4, (3) 9, (4) Yes, (5) vary
 (6) counting number (7) 37; 45; 21; 383; 239.
 (8) No; (9) counting number (10) 16, 53, 21, 21, 57, 74.

ANSWERS: Page 25,

	<u>No. of Seed</u>	<u>Total</u>		<u>No of Seed</u>	<u>Total</u>
Cotton	52,269	5,226,900	Beans	104,538	15,680,700
Corn	8,711	487,816	Cantaloupe	4,840	58,080
Watermellon	2,987	74,675	Potatoes	52,269	261,345

COMMUTATIVE PROPERTY OF COUNTING NUMBERS

I. Objective: To teach the commutative property in addition of the counting numbers. Also, introduce the calculator to the student as a device that will help him with his work.

II. Suggestions:

- A. Teacher and pupils working together make a flow chart for addition on machine.
- B. Explain how to add on machine by referring to Student Manual provided by manufacturer.
 1. Have students insert into the machine the counting numbers 17 and 36. Add.
 2. Now insert in order 36 and 17. Add.
 3. Ask class what seems to be true of the results found on the tapes.
 4. Have them insert 29, 45, 37, and 13 into calculator and add.
 5. Now insert 13, 37, 45, 29 and add.
 6. Have students check results on tape.
 7. Ask, "What do you find when comparing these results?" (Same)
 8. Can there be other arrangements that will give the same results?
 9. What conclusion can you make concerning addition of the counting numbers? (Order of addition not important)
- C. Have the class add any eight different counting numbers. Rearrange and add. Are the results the same?
- D. Place several grocery items on counter near calculators. Let each check. Compare order with neighbors.

III. Resource Materials:

- A. Student manuals accompanying calculators.
- B. Items (blocks, marbles, books, etc.) that students may hold exchange, and compare.

ANSWERS: Page 27,

- (1). 53; (2). 23; (3). 124; (4) Yes; (5) a. 29, b. Yes, c. 6, d. (2, 15, 12) (2, 12, 15) (15, 2, 12) (15, 12, 2) (12, 2, 15) (12, 15, 2).
- (6). No, Commutative Property; (7). 377

NEED FOR A NUMBER

I. Objective: By way of student discovery introduce the need for zero, which is the identity element for addition of counting numbers, and lead into the whole numbers.

II. Suggestions:

A. Definition: The identity for addition is a number that can be added to any given number and the sum is that given number.

B. Go over questions 1-6 on student page with class.

C. Leading questions:

1. What number would you add to 3 to get 3 (0)
2. $5 + ? = 5$ Notice that the sum is "same as" the given addend. What number makes this statement true? (Zero)
3. A synonym for "same as" might be a hint.
(answer: identical)
4. From this response, teacher might suggest the correct word "identity" for the additive identity element.
5. In Problem 10 is a good place to start practical experience in counting out change. (Use play money).

III. Resource Material:

Books: Haag-Dudley, Introduction to Sec. Math.

ANSWERS: Page 29-30.

(8). 537, 737, 100, 74, 110	(10) \$13.17, \$6.83
(11). \$202.81, less, \$7.19	(12) \$8.56, \$1.44

Page 32.

(1) 1,265.59	(2) 1,241.436	(3) 26,475.03	(4) 2,000,338.37
(5) 41,001,773	(6) 858.4	(7) 44428	(8) 48,229 (9) 427,542

Page 33-34.

a. DP-64	Soda 50	b. 561
ctns. 39	NG 52	c. 60
KSDP 68	SD 55	d. 61, 32, 66, 41, 34, 37, 44, 50
KSDP ctns. 50	TC 54	54, 51, 31
Diebetic 45	GC 35	e. 501
ctns. 49		

Total Net Sales will vary.

UNIT IV--WHOLE NUMBERS

Place Value

I. Objective: To present an interesting and clear understanding of the place value of numbers with respect to base ten.

II. Suggestions:

- A. To have the students put toothpicks into sets of ten, then bundle the sets of ten into piles of ten. Always have an odd number of toothpicks left over.
- B. You may use thumb tacks, buttons, cards, blocks, and many other things for this experiment.
- C. Use a film or filmstrip if available.
- D. Introduce number systems such as the Egyptian, Roman or Babylonian which do not have place value.
- E. Have students make up own numeration systems.

III. Resource Material:

A. Books:

1. Haag and Dudley, Introduction to Secondary Mathematics, Vol. 1 and 2, Heath.
2. Lay, An Introduction to Mathematics, Macmillan
3. SRA

B. Audio Visual:

1. State Department of Education: Film, Place Value: Ones, Tens, Hundreds.
2. Filmstrip: Expanded Notation, Colonial Films, Series 3 New Elementary Math.

ANSWERS: Page 35.

1. 3 digits, 2 hundreds, 3 tens, 8 ones.
2. 4 ten thousands, 5 thousands, 6 hundreds, 5 tens, 3 ones.
3. Each group is 10 times as large.

Page 36,

1. $5(10 \times 10) + 6(10) + 4$ 2. $7(10 \times 10 \times 10) + 3(10) + 2$
 $5(10^2) + 6(10) + 4$ $7(10^3) + 3(10) + 2$
3. $2(10 \times 10 \times 10 \times 10 \times 10) + 3(10 \times 10 \times 10 \times 10) + 1(10 \times 10) + 5(10) + 6$
 $2(10^5) + 3(10^4) + 1(10^2) + 5(10) + 6$

PUZZLE TERMS: Inverse, null, multiply, whole, closed, subtract, subset, set, infinite, finite, rule, divide, add, identity, inverse, number square, integer, flowchart, line, calculator. (Note: "inverse" is on the puzzle twice.)

ANSWERS: Page 37,

ACROSS: (1) 112 (4) 132 (7) 1932 (8) 6006 (10) 10 (11) 200 (13) 26
 (14) 1012 (16) 3456 (18) 0 (19) 0 (20) 1809 (23) 2054 (26) 21
 (27) 900 (29) 44 (30) 1799 (32) 1234 (34) 999 (35) 202
DOWN: (1) 1900 (2) 13 (3) 2222 (4) 1603 (5) 30 (6) 2025 (7) 111
 (9) 666 (12) 0 (15) 100 (17) 400 (20) 121 (21) 8179 (22) 9999
 (23) 2012 (24) 5432 (25) 444 (28) 0 (31) 99 (33) 20

ASSOCIATIVE PROPERTY OF ADDITION

I. Objective: To introduce the meaning of the associative principle in addition of whole numbers and use of sub-total on calculator.

II. Suggestions:

- A. Using calculator show how you can sub-total when combining several terms.
- B. Show, also, that addition is binary (subtotal on machine gives this binary result).
- C. By commutative principle change order in entering in machine or by use of grand total, show associativity.
- D. Flow chart on machine operation.
- E. Drill material on calculator.

III. Resource Material:

- A. Use of blocks, marbles, people, etc. placed in small groups and then combine to make larger groups.
- B. Drills from manual or other sources.

ANSWERS: Page 38,

1. (a) and (b) = 84
2. (a) and (b) = 1088
3. (a) and (b) = 5693
4. (a) $(13 + 45) + 32 = 90$
(b) $13 + (45 + 32) = 90$

CLOSURE - MULTIPLICATION OF WHOLE NUMBERS

- I. Objective: To establish the property of closure in multiplication of whole numbers. Introduce use of multiplication on calculator.

II. Suggestions:

- A. By use of simple examples show that multiplication of whole numbers will give a whole number.
- B. Number of desks in a row times the number of rows having that number of desks.
- C. Number of tiles in a row in wall (bricks, concrete) times number of rows.
- D. Introduce method of multiplication on a calculator by flowcharting.

III. Resource Material:

- A. More drill can be used from other books or calculator material.
- B. Napiers Bones

ANSWERS: Page 39,

1. (a) 192 (b) 1728 (c) 7,629,336 (d) 504 (e) 53,264,400
(f) 39 (g) 4606 (h) 321,843,289,584
2. Whole number 3. No
4. (a) 63 (b) 96 (c) 68 (d) 378 (e) 5312 (f) 6399 (g) 10,458
(h) 25,976 (i) 38,766 (j) 329,958

Page 40,

(1) 226,128	3,006,168	2,470,545	30,960
(2) 381,874	6,589,024	436,277,268	5,310,972
(3) 376,875	37,278	7,431,149,166	432,092,552
(4) 18,371,605	15,946,812	18,444,577,959	
(5) 2,982,316,177,848	536,308,938,624	27,572,832	
(6) 40,940	3,170,054	42,431,368	34,650
(7) 316,731,656	6,625,667,919	785,174,423,532	

COMMUTATIVE

I. Objective: To show that the commutative and the associative properties hold for multiplication of whole numbers.

II. Suggestions:

- A. Work with students with such problems as 3×2 and 2×3 to show commutativity.
- B. $(3 \times 2) \times 7 = 3 \times (2 \times 7)$ associativity. May or may not use name.
- C. Review. (cost pants + cost shirts) + cost shoes = cost pants + (cost shoes + cost shirt).
- D. Can use transparency with $3 \times 2 \times 7$ cover 3×2 with transparent rectangle $3 \times 2 \times 7$. Write "6" over 3×2 making problem $6 \times 7 = 42$ with another transparent rectangle.

Ask if it can be placed on the problem in a different place. (2×7) so that the problem becomes $3 \times 14 = 42$.

III. Resource Material:

- A. SVE Filmstrip
- B. Colored geometric shapes with appropriate numerals printed thereon.
- C. Todd and McDermott

ANSWERS: Page 41,

Example:

1. 3552; 2. 4176; 3. 45018; 4. (a) 214,656

Page 42,

1. (a) 8,100; (b) 35,392; (c) 60,696; (d) 42,355
2. (a) 138,306; (b) 288,360; (c) 288,360; (d) 152,148
3. (a) 397,548 (b) 9,629,496; (c) 14,684,466; (d) 14,684,466
4. (a) 48,346,755; (b) 27,212,886; (c) 32,115,451
5. (a) 16,068,000; (b) 2,120,212; (c) 64,821,212
6. (a) 51,812,259; (b) 54,722,316; (c) 261,011,376;
(d) 249,119,415.

Page 41

1	3	4	2
4	2	1	3
2	4	3	1
3	1	2	4

WHOLE NUMBERS - SUBTRACTION

- I. Objective: To introduce subtraction of whole numbers by use of the number line.

To introduce subtraction on the calculation machine.

- II. Suggestions:

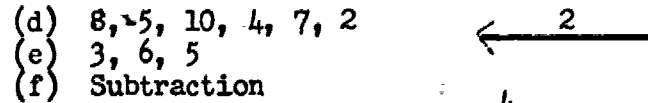
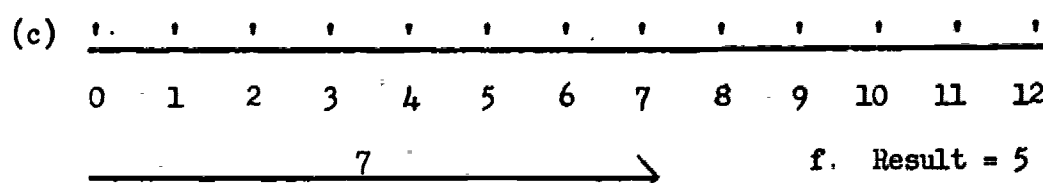
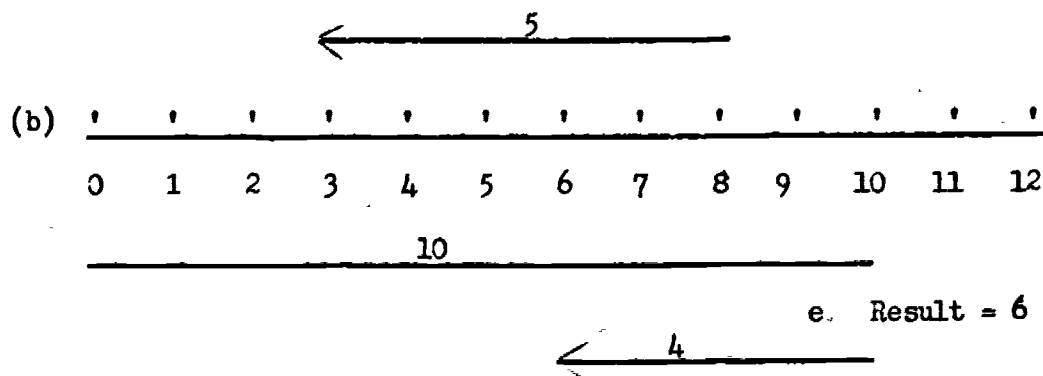
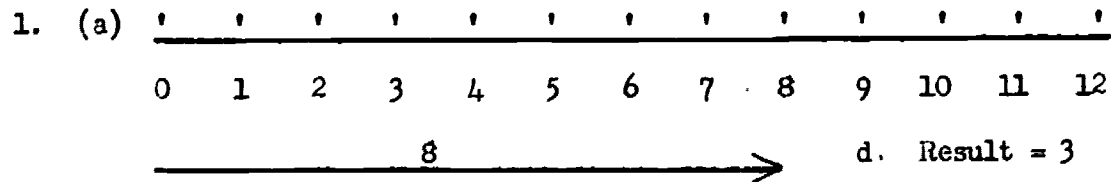
- A. Having the students take 10 steps forward, then take 7 steps backward.
- B. Repeat this operation 5 times more: if needed
- C. Temperature (rise and fall)
- D. Use calculator manual on subtraction

- III. Resource material:

- A. River falls
- B. Stock market
- C. Weather reports
- D. Hospital admittances and dismissals
- E. Calculator manual

- IV. ANSWERS: (next Page)

ANSWERS: Page 43-44



4.

2. (a) 174 lbs.
(b) Whole numbers
3. (a) \$110
(b) Whole number

Page 45,

1. 7483 (2) 15,848 (3) 39,971
(4) 8889 (5) 23,701 (6) 10,343
(7) 16,439 (8) 60,655 (9) 22,715
(10) 16,398 (11) 12,525 (12) 14,216
(13) 12,847 (14) 25,626 (15) 1759
(16) 21,614 (17) 68,668 (18) 28,261

Little goodie twice the present year.

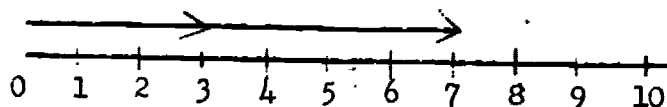
a	b		c	d	e
2	4		3	8	2
f	7	8	g	3	h
				2	6
i	6		j	3	k
				3	5
	l	4	9	m	n
				7	2
o	6	2	8	p	7
					q
r	1	7		s	6
				t	9
					2

DISTRIBUTIVE PROPERTY

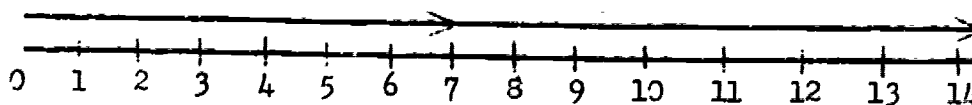
- I. Objective: To introduce the distributive principle for multiplication of whole numbers and show that multiplication is distributive over addition but addition is not distributive over multiplication

II. Suggestions:

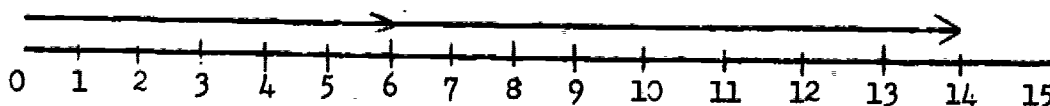
By use of a number line, start by showing how one would graph $2 \times (3 + 4)$.



1. (a) Show the addition of $3 + 4$ on line by beginning at 0 and going out to 3 with first arrow, then from 3 go 4 units farther out the number line with a second arrow. You are now at 7 and this represents the sum of 3 and 4.
- (b) On a new number line we can picture the product 2×7 which would result from above step as 2 "7's".

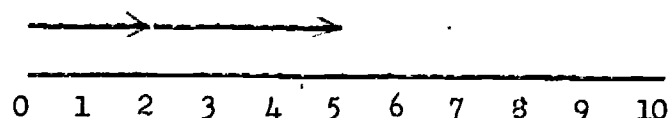


- (c) From 0 proceed 7 units to the mark "7". Then from "7" go seven more spaces beyond with a second arrow. You are now at 14 — the product of 2×7 .
- (d) Now consider $(2 \times 3) + (2 \times 4)$ on the number line. First we note the product of 2×3 is 6 and product of 2×4 is 8. Hence you need only show on the number line the sum of 6 and 8.



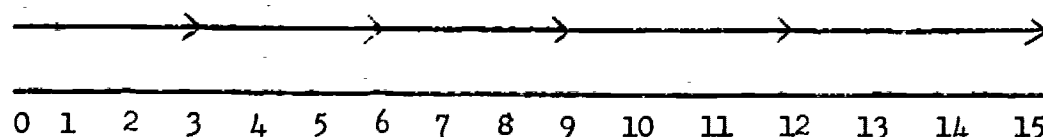
From 0 proceed 6 units out to "6" on the line with first arrow. Then with a second arrow begin at 6 and count 8 spaces farther out. Note-you are now at 14 the same place as you were by part (b). This shows multiplication is distributive over addition.

2. (a) Now consider $2 + (3 \times 1)$. Using number line show that addition is not distributive over multiplication.

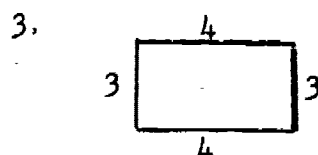


From 0 go right 2 units with first arrow. The product of 3×1 being 3, with a second arrow starting at 2, proceed 3 units farther right. Note- you are now at 5.

- (b) Graph $(2 + 3) \times (2 + 1)$



Now that the sum of $2 + 3$ is 5 and $2 + 1$ is 3 you note that the problem resolves into 5×3 which can be shown by 5 successive arrows of 3 units each. You stop at 15 which is not where you stopped in part 2a. Hence this can be used for visual proof that addition is not distributive over multiplication.



Another way to introduce the distributive principle would be by asking class to write the perimeter of a rectangle such as one shown. One student might write $3 + 4 + 3 + 4$ while another might write $(2 \cdot 3) + (2 \cdot 4)$ while a third might add 3 and 4 and then multiply by 2 such as $2(3 + 4)$. Note—all 3 obtain the same result but the last two have found different mathematical shortcuts for expressing the same concept. Then you show $2(3 + 4) = (2 \cdot 3) + (2 \cdot 4)$.

4. Every time a two digit number is used as a multiplicand one is really using the distributive principle.

$$\text{Thus } 4 \times 12 = 4 \times (10 + 2) = (4 \times 10) + (4 \times 2) = 48$$

$$4 \times 12 = 48 \quad \text{also} \quad 12$$

$\frac{4}{8}$	(4×2) Dist. prin.	$\frac{3242}{x 4}$	(4×2)
$\frac{40}{48}$	(4×10) Dist. Prin.	$\frac{160}{800}$	(4×40)
		$\frac{12000}{12968}$	(4×200)
			(4×3000)

5. Note: Problem 5 SM is intended to be multiplied as shown in problem 4 above

III. Resource Materials:

- A. SVE Filmstrip
- B. Haag-Dudley Introduction to Secondary Math
- C. SMSG, Studies in Mathematics, Vol. VI
- D. Brown, Simson, Snader, General Math, Book I

ANSWERS: Page 46

1 (a) $\xrightarrow{8}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

$\xrightarrow{5}$

$\xrightarrow{3}$
1 1

$\xrightarrow{16}$

(d) 4×2

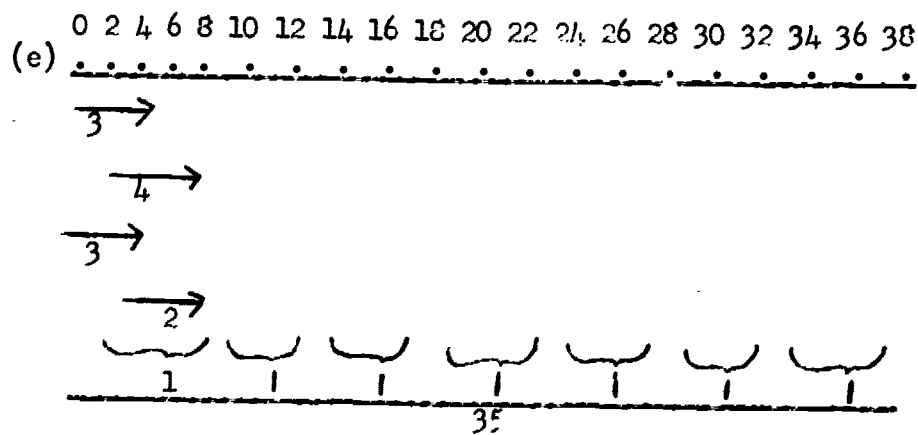
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

$\xrightarrow{1}$ $\xrightarrow{1}$ $\xrightarrow{1}$ $\xrightarrow{1}$

$\xrightarrow{3} +$

$\xrightarrow{8}$

$\xrightarrow{11}$



2. a. 2
b. 40
c. 9
d. x
e. x, +

Page 47,

3. a. $(3 \times 13) + (3 \times 28)$
b. $6 \times (7 + 9)$
c. $(24 \times 9) + (18 \times 9) + (31 \times 9)$
d. $(37 + 13) \times 18$
4. a. 6966
b. 363
c. 2695
5. a. 612
b. 399
c. 1300
d. 12183
e. 7306

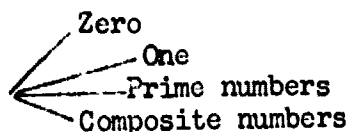
PRIMES

I. Objective:

- To teach — meaning and use of prime numbers
 — meaning of composite numbers
 — review rules of divisibility (2, 3, 5)

II. Suggestions:

A. Set of whole numbers



If a whole number is not zero or one it must be a prime or a composite. If it is not zero, one, or prime it must be composite and can be written as a product of primes.

B. Define:

1. A prime number is a counting number other than one with no more than 2 factors. (itself and one).
 2. Composite number is a counting number with at least three factors. (itself, one, and one other).
- C. One way of teaching primes is to have the students take a list of counting numbers beginning with 2 and extend it as far as you wish. Eliminate the multiples along this list in the following order (multiples of 2, multiples of 3, multiples of 5, . . .) i.e. (2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~). You might use a different colored pencil for marking out multiples of each succeeding prime to get over the idea of composite numbers and multiple factors of counting numbers. On the student sheet there is a chart 1 to 100. You may develop primes in any way you choose. This is the Sieve of Eratosthenes (MSG Vol. VI, page 74-75).
- D. Using the different colors as indicators could lead to the prime factors of composite numbers. Ex: $2 \cdot 3 = 6$
 $2 \cdot 3 \cdot 2 = 12$ etc.

Review or teach divisibility rules if desired. What numbers can be divided by 2 without remainders; by 3? by 5?

III. Resource Material:

A. SMSG, Volume VI

B. Mathematics, A Modern Approach - Wilcox, Yarnelle- Addison -
Wesley

C. Introduction to Secondary Mathematics, Vol. 1, Haag-Dudley,
D.C. Heath.

ANSWERS: Page 48-50

1. Prime numbers between 1 & 100

2. 25

3. 2

4. 2, 3

5. 97

6. 2

7. 4

8. No

9. 3

10. $2 \times 2 \times 2$

11. 2×5

12. 9×10

$3 \times 3 \times 2 \times 5$

13. 30×3

$5 \times 3 \times 2 \times 3$

14. 15×6

$3 \times 5 \times 3 \times 2$

15. $2 \times 3 \times 3 \times 5$

16. Yes; 45×2 or 18×5

17. One

18. One and only one

19. No.

20. a. $18 = 9 \times 2$
 $= 3 \times 3 \times 2$

b. $72 = 9 \times 8$
 $= 3 \times 3 \times 4 \times 2$
 $= 3 \times 3 \times 2 \times 2 \times 2$ (others possible)

21. Found prime factors by short division.

22. $2 \overline{)108}$

$2 \overline{)54}$ (other orders
possible)

$3 \overline{)27}$

$3 \overline{)9}$

3

23. (a) $17 \times 3 \times 3$

(b) $11 \times 5 \times 5$

(c) $2 \times 3 \times 3 \times 3 \times 5 \times 7$

PRIME FACTORS

I. Objectives:

- A. To teach addition and multiplication of odd and even numbers.
 B. Using prime numbers
- (1) renaming using exponents
 - (2) reducing fractions
 - (3) finding common denominators (least common multiple)

II. Suggestions:

- A. Using clock plate 1 make a transparency (or a large chart for front of room) by placing even and odd numbers in center as addend and multipliers, establish patterns of the following: odd x odd, odd x even, and even x even.
 B. Using o for odd and e for even

+	o	e
o	e	o
e	o	e

x	o	e
o	o	e
e	e	e

Note: None of above on student pages.

- C. Discuss common factors (may be a review)
 D. Directions for THINK BEFORE YOU ACT. Page 52
- (↓ ADD 10) (↘ ADD 11) (↑ SUBTRACT 10) (↗ SUBTRACT 9)
 (→ ADD 1) (↙ ADD 9) (← SUBTRACT 1) (↖ SUBTRACT 11)

ANSWERS: Page 51

(2) 2 (3) 5, 3 (4) 2, 2, 15 (5) 2, 2 (6) 6, 10
 (7) 2, 2, 3, 3, 3, 1 (8) 2, 3, 3, 3, 54 (9) 2, 2, 4, 27
 (10) 3, 2, 2, 3 (11) 2, 3, 3, 6, 18 (12) 2, 2, 3, 12

ANSWERS: Page 52

(13) 2 (14) 3 (15) 27, 3 (16) answers will vary

THINK BEFORE

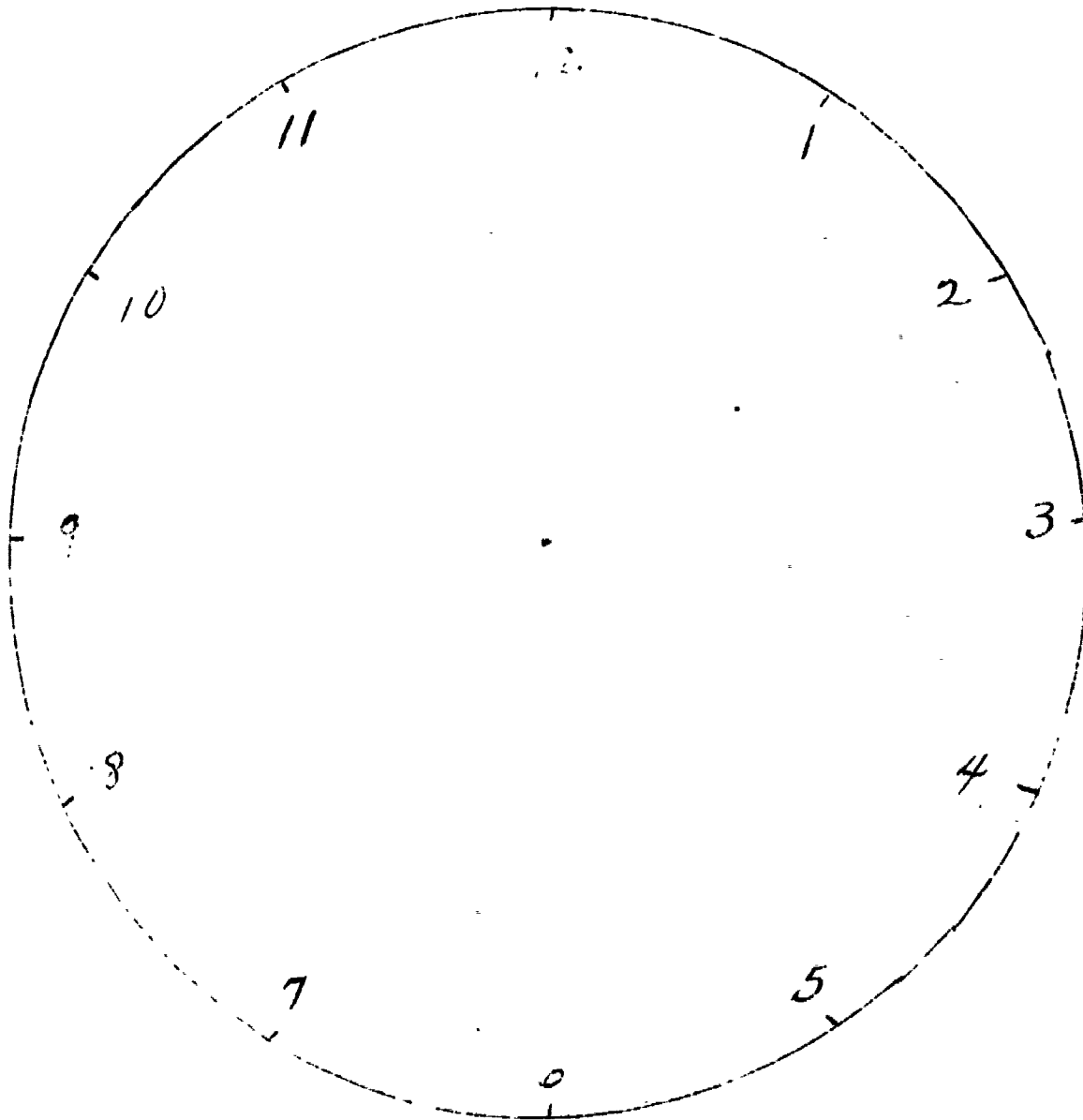
YOU ACT (1) 8 (2) 54 (3) 3 (4) 5 (5) 9 (6) 19
 (7) 19 (8) 58 (9) 22 (10) 0 (11) 18 (12) 6

ANSWERS: Page 54

(1) 7 (2) 11² (3) 7, 23 (4) 2, 3, 5² (5) 2², 7, 11
 (6) $\frac{2 \times 2 \times 2 \times 2 \times 3 \times 3}{2 \times 2 \times 47}$ 2 x 2

TM -51-54
2

CLOCK PLATE



DIVISION OF WHOLE NUMBERS

I. Objective: To provide practice in the operation of division.

II. Suggestions:

- A. Introduce division on machine.
- B. Divide numbers which result in no remainder. (By machine and by hand).
- C. Practical problems (few) (No remainders)
- D. Choose easier problems to have students do manually. More difficult by machine.

III. Resource Material:

- A. Averaging grades or scores.
- B. Find the price of one item or group when the group's price is given.
- C. Flowchart for division.

IV. Division: (ANSWERS) Page 55-56

(1) 25 (2) 144 (3) 213 (4) 121 (5) 163 (6) 3 (7) 3

(8) 2 (9) 9 (10) 28 (11) 25 (12) 128 (13) 561

(14) 352 (15) 153 (16) 789 (17) 1123 (18) \$1125 per share

(19) \$325 per acre (20) 83 - average grade.

(21) a.) 13 e.) 6 i.) 16 m.) 15
 b.) 3 f.) 28 j.) 12 n.) 7
 c.) 17 g.) 6 k.) 4 o.) 23
 d.) 17 h.) 10 l.) 18 p.) 5

Each row - 50

Each column - 50

Each diagonal - 50

UNIT V

INTEGERS

CLOSURE -- SUBTRACTION

I. Objective: To extend the whole numbers in order to have a set so that the closure property holds for subtraction. This set is called the integers.

II. Suggestions:

1. Distance Problems
2. Temperature problems
3. Debt
4. Stock market (rise and fall)
5. Football games

III. Resource Material:

1. Wall Street Journal
2. Temperature charts, newspaper
3. Cost and selling price, Profit and Loss Statement.
4. Audio Visual

Introduction to Signed Numbers, SVE, Algebra Series.

IV. ANSWERS: Page 58-59 Subtraction

- | | |
|-----------------|-----------------|
| (1) 0 | (13) -2,228 |
| (2) -1 | (14) -1,646,469 |
| (3) -7125 | (15) +79 |
| (4) -192 | (16) +50 |
| (5) -5,928,947 | (17) +17 |
| (6) -40,000,000 | (18) +86,007 |
| (7) -100,000 | (19) +63,266 |
| (8) -73 | (20) +8 |
| (9) -54,195 | (21) +9 |
| (10) -14,280 | (22) +50 |
| (11) -519,903 | (23) +17 |
| (12) -38,133 | (24) +45 |

8/12	9/12	4/12
3/12	7/12	11/12
10/12	5/12	6/12

INTEGERS - PROPERTIES OF ADDITION

- I. Objective: To show the properties of addition on the set of integers.
- II. Suggestions: Discuss.
 - A. Closure (established by previous examples on addition of integers)
 - B. Commutative: Use many examples on calculator and manually.
 - C. Associative law: Use subtotals: Give examples.
- III. Resource Material:

Preliminary Mathematics - Dressler - Amsco School Pub ; Inc.

ANSWERS: Page 60

B. (1) 0, yes; (2) -600, yes; (3) 199, yes; (4.a) 9212
 (4.b) 9212 (5.a) -74 (5.b) -74 (6.a) -4084 (6.b) -4084

Answers same, commutative property

(7) 111 (8) 111 (9) -657 (10) -657 (11.a) 1699
 (11.b) 1699 (12.a) 100 (12.b) 100

answers in 11 (a & b) same

answers in 12 (a & b) same

associative property

NEGATIVE INTEGERS

I. Objective: To acquaint the student with the use of negative integers in practical problems.

II. Suggestions:

A. Give many problems involving addition.









1. Drill problems (with machine)
2. Practical problems
 - (a) A count of bottles
 - (b) Day to day count in number of a specific article on the grocery shelf
 - (c) Follow a particular stock for a period of time (make a graph of results)
 - (d) Profit and loss problems
 - (e) Hospital admittances and releases for a week.
 - (f) River stages for a week

B. To introduce number line.

III. Resource Material:

- A. Newspaper: Stock Market report - River report.
- B. Report on hospital admittances and releases.
- C. Profit and loss report of particular company or business.

IV. ANSWERS Page 61

a	8	b	9		c	1	d	9	e	0
f	1	1	1	1	7	8				
h	2	4		i	9	5	j	4		
		k	3		l	6	4			
m	2	n	3	0	7		p	6		
q	2	1	4		r	3	8			

Students are to find
the squares to be
shaded.

INVERSES

I. Objective: To introduce the idea of inverse.

II. Suggestions:

1. Define inverse (in terms of any operation): If the operation on two numbers produces the identity element (of that operation), then one number is the inverse of the other.

2. Examples: $5 + -5 = 0$

$$-5 + 5 = 0$$

3. If possible, show film or filmstrip.

4. Use application problems

III. Resource Material:

1. Use temperature

2. Use stock market

ANSWERS Page 62

(1) -6 (2) 0 (3) $+3$ (4) $+547$ (5) -140 (6) 0 (7) 0

(8) 0 (9) $+31$ (10) -31

DIVISION OF INTEGERS

I. Objective: Show a need for the rational numbers. For our purpose division will be limited to the integers.

II. Suggestions:

1. Show the integers are not closed under division:
Example: $6 \div 4 \neq$ integer. In order to divide any two integers, we must extend the set of integers. The integers plus these new numbers we call the rational numbers.

2. Use grocery advertisement to find the price per item where the answer needs to be rounded off.
Example: 3 for \$1.00.

III. ANSWERS Page 63

1. 6	Yes	4. 2564	Yes
2. 3	Yes	5. 429	Yes
3. 25	Yes	6. 1111 rem.	1111 No

TM 64 - 65

TM 64 - 65

Answers: Page 64

Summer rates

1. 1232
2. 4536
3. 14,449
4. 16,797
5. 5768

Winter rates

1. 1557.5
2. 6370.5
3. 20557.5
4. 23,932.5
5. 7928

Answers: Page 65

- | | |
|-------------|----------------|
| 1. Closed | 11. Rule |
| 2. Set | 12. Flowchart |
| 3. Subset | 13. Null |
| 4. Identity | 14. Finite |
| 5. Whole | 15. Infinite |
| 6. Number | 16. Calculator |
| 7. Add | 17. Line |
| 8. Subtract | 18. Integer |
| 9. Multiply | 19. Inverse |
| 10. Divide | 20. Square |

Answer - Fill the 5, pour into 3, leaves 2 in the 5 gal.,
Pour out 3, pour the two into 3 gallon, fill 5,
finish filling 3 gallons from 5 gallon. Leaves
4 gallons in 5 gallon bucket.

UNIT VI
RATIONAL NUMBERS
MULTIPLICATION OF RATIONALS

I. Objectives:

- To introduce multiplication of rational numbers (in fractional form).
- To investigate the properties of multiplication on the set of rational numbers.

II. Suggestions:

- Provide practice in multiplying fractions.
- To emphasize these properties:
 - Closure
 - Commutative
 - Identity
 - Inverse
 - Associative

III. ANSWERS Page 66-67

- (a) $\frac{4}{15}$ (b) $\frac{25}{43}$ (c) $\frac{12}{35}$ (d) $\frac{63}{23}$ (e) yes (f) there are none (g) yes
- (a) $\frac{45}{43}$ (b) $\frac{16}{65}$ (c) $\frac{45}{43}$ (d) $\frac{16}{65}$ (e) same results (f) commutative
- (a) $\frac{2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{63}{13}$ (d) $\frac{63}{16}$ (e) same (f) same
- (a) $\frac{13}{7}$ (b) $\frac{8}{9}$ (c) 1 (d) 1 (e) $\frac{13}{14}$ (f) number, :1
- (a) 1 (b) 1 (c) 1 (d) $\frac{8}{7}$ (e) $\frac{91}{24}$ (f) $\frac{17}{16}$ (g) $\frac{12}{7}$
(h) $\frac{1}{3}$ (i) $\frac{1}{9}$ (j) Divide 1 by the number or find a number which multiplied by given number produces 1
(k) $\frac{8}{7}$ (l) $\frac{1}{12}$ (m) $\frac{3}{7}$

ANSWERS: Page 63 (Across)

- $\frac{4}{3}$
- $\frac{3}{4}$
- $\frac{25}{12}$
- $\frac{1}{9}$
- $\frac{1}{6}$
- $\frac{1}{16}$
- $\frac{5}{4}$
- $\frac{1}{20}$
- 1
- $\frac{352}{245}$
- $\frac{27}{2}$
- $\frac{1}{2}$
- 3
- $\frac{7}{3}$

+	+	=
x	÷	+
x	÷	=
+	÷	-
		=
-	-	-
÷	x	=

REDUCING RATIONALS USING PRIMES

I. Objective: To teach reduction of fractions by using prime numbers.

II. Suggestions:

A. Recall renaming numbers, prime factors, numerator and denominator - other names for the number one, divisibility of a number.

ANSWERS: Page 69

$$(1) \frac{35}{21} = \frac{5 \times 7}{3 \times 7} = \frac{5}{3} \times \frac{7}{7} = \frac{5}{3} \times 1 = \frac{5}{3}$$

$$(2) (a) \frac{15}{25} = \frac{3 \times 5}{5 \times 5} \quad (b) \frac{81}{18} = \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 2} \quad (c) \frac{12}{18} = \frac{2 \times 2 \times 3}{2 \times 3 \times 3}$$

$$(d) \frac{75}{21} = \frac{3 \times 5 \times 5}{3 \times 7} \quad (e) \frac{39}{65} = \frac{3 \times 13}{5 \times 13} \quad (f) \frac{4620}{231} = \frac{2 \times 2 \times 3 \times 5 \times 7 \times 11}{3 \times 11 \times 7}$$

$$(3) (a) \frac{15}{25} = \frac{3}{5} \times \frac{5}{5} \quad (b) \frac{81}{18} = \frac{9}{2} \times \frac{9}{9} \text{ or } \frac{9}{2} \times \frac{3}{3} \times \frac{3}{3} \text{ or } \frac{27}{66} \times \frac{3}{3}$$

$$(c) \frac{12}{18} = \frac{2}{3} \cdot \frac{6}{6} \text{ or } \frac{4}{6} \cdot \frac{3}{3} \text{ or } \frac{2}{2} \cdot \frac{3}{3} \cdot \frac{2}{3} \quad (d) \frac{75}{21} = \frac{25}{7} \times \frac{3}{3}$$

$$(e) \frac{39}{65} = \frac{3}{5} \times \frac{13}{13} \quad (f) \frac{4620}{231} = 2 \times 2 \times 5 \times \frac{3}{3} \times \frac{7}{7} \times \frac{11}{11} \text{ or}$$

$$2 \times 2 \times 5 \times \frac{21}{21} \times \frac{11}{11}$$

(also other possibilities)

$$(4) (a) \frac{3}{5} \quad (d) \frac{25}{7}$$

$$(b) \frac{9}{2} \quad (e) \frac{3}{5}$$

$$(c) \frac{2}{3} \quad (f) 20$$

ANSWERS: Page 70

$$(5) \frac{153}{102} = \frac{3 \times 3 \times 17}{3 \times 2 \times 17} = \frac{3}{2}$$

- | | |
|--------------------|------------------|
| (6) a. 2 | g. $\frac{7}{8}$ |
| b. $\frac{10}{9}$ | h. $\frac{1}{9}$ |
| c. $\frac{25}{11}$ | i. $\frac{3}{4}$ |
| d. $\frac{16}{3}$ | j. $\frac{1}{8}$ |
| e. $\frac{7}{6}$ | k. $\frac{1}{8}$ |
| f. $\frac{6}{7}$ | l. $\frac{1}{6}$ |

ANSWERS: Page 71

(1) $\frac{1}{4}$ acre unit or $\frac{1}{32}$ royalty interest

$$\text{Royalty} = \frac{1}{32} \times 40 \times 2.60 \times 30 = \$97.50$$

(2)

Brown	\$448.406
Black	373.671
Lowe	1793.625
R. Rogers	523.141
C. Rogers	298.938
B. Franks	896.813
Young	448.406

MULTIPLICATION OF DECIMALS

I. Objective: To multiply rational numbers in decimal form.

II. Suggestions:

- A. Give explanation of placing decimal in product.
- B. Provide practice in multiplying decimals.

EXAMPLES:
$$\begin{array}{r} .25 \\ \times .33 \\ \hline \end{array}$$

$$\begin{array}{r} 1.34 \\ \times 1.56 \\ \hline \end{array}$$

- C. Use calculators to multiply these numbers without the decimal, then have students locate decimal in product.

III. Resource Material:

- A. Preliminary Mathematics - Dressler AMSCO School Pub., Inc.
- B. Mathematics a Modern Approach - Wilcox, Yarnelle Addison-Wesley
- C. Mathematics in Daily Use - Hart Schult, Irvin. D.C. Heath and Co.

ANSWERS: Page 72

- | | |
|------------------|--------------|
| (5) (a) 18.68496 | (e) 1645.00 |
| (b) 1316.601 | (f) 3302.387 |
| (c) 86.3656 | (g) 1561.804 |
| (d) 4338.00 | (h) 0.020402 |

ANSWERS: Page 73

Corsage - \$22.50; fruit and vegetables, asst. - \$9.10; Total - \$173.45

ANSWERS: Page 74

- (1) 184.898; 1,896,405,052.62; 18,800.50
- (2) 2,732,6725.9; 4,290.6632; 68.022798
- (3) 781.360; 39,530.1546; 500,488.812
- (4) 38.32; 68,740.0356; 106.7394
- (5) 847,462.85940; 1,319,608.22640; 27,459,151,833.75
- (6) 43,244.8194555; 4,507.26727825; 736,850.55436
- (7) a. 1.471065 b. 95.46 c. .9776 d. 7.456733

TM-75-76

ANSWERS: Page 75

\$45.20
28.80
13.44
76.95
18.72
19.44
1.80
4.48
2.24
6.20
3.20
1.70
1.70
3.80
2.80
3.74
3.96
3.80
7.40
5.60
8.88
6.00
3.60
7.92
9.30
\$293.67

Total

Page 76

Sales Tax \$ 2.79
Total 99.70
Total payment 89.70
Carrying Charge 6.64
Time Balance 96.34
10 Months

DIVISION OF FRACTIONS

I. OBJECTIVES:

- A. To introduce the division of rationals (in fractional form)
- B. To investigate the properties of division over the rational numbers.

II. Suggestions:

- A. Show closure.
- B. Show division is not commutative.
- C. Show division is not associative.
- D. Provide practice to emphasize suggestions A, B, and C.
- E. Emphasize the use of the multiplicative property of 1 in division. Example: On page (77)

III. Resource Material:

- A. S.R.A. Materials
- B. S.M.S.G. Vol. 4

ANSWERS: Page 77 Exercises

(1) (a) $\frac{6}{7}$ (b) $\frac{25}{3}$ (c) $\frac{15}{6}$ (d) $\frac{12}{15}$ (e) $\frac{6}{8}$ (f) Yes (g) No

(h) Yes

(2) No. (3) No

ANSWERS: Page 78

(1) $\frac{3}{5}$ (2) 12 (3) $\frac{84}{169}$ (4) $\frac{5}{2}$ (5) $\frac{1}{3}$ (6) $\frac{5}{4}$ (7) $\frac{17}{13}$

(8) $\frac{8}{9}$ (9) $\frac{9}{5}$ (10) 1 (11) $\frac{3}{7}$ (12) $\frac{8}{9}$ (13) $\frac{19}{15}$ (14) $\frac{3}{4}$

(15) $\frac{10}{13}$ (16) $\frac{8}{17}$

Compound Savings: \$5,368,709.12

ANSWERS: Page 79

- E. (1) eight and six tenths
 (2) one hundred and forty-three hundredths
 (3) fifty thousand sixty-two hundred thousandths
 (4) nine hundred thousandths
 (5) five
 (6) two hundred twenty-five and eighty-nine thousandths
 (7) four and one thousand five hundred seventy-five ten thousandths
- F. (1) .80
 (2) 05
 (3) .008
 (4) .0001
 (5) 128.2
 (6) 206.26
 (7) 3000.08

ROUNDING OFF

I. Objectives:

- A. To introduce approximation.
- B To practice rounding off quotients.

II. Suggestions:

- A. Show why rounding off is necessary by choosing examples such as:
 - 1. Distance to nearest town would be rounded off to miles
 - 2. Total federal debt might be rounded off to nearest billion dollars.
 - 3. Height of person given usually to the nearest inch.
 - 4. Money transactions are always rounded off to the nearest cent.
 - 5. Averaging involving people usually to nearest whole person.

III Resource Material;

- A. Average daily attendance
- B. Let them take some activity and keep a record. Then find their daily average.
- C Flowchart for dividing rationals.

ANSWERS: Page 81

Page 82

		<u>Hundreds:</u>	<u>Thousands:</u>
(1) a. 7253.48	(2) a. 147		
b. 7253.5	b. 96		
c. 7253.	c. 4723	413,718,800	1,213,000
d. 7250.	d. 94	124,900,400	685,000
e. 7300.	e. 95	288,818,400	5,191,000
f. 7000.	f. 90		11,997,000
	g. 26	<u>Hundred Th:</u>	201,000
		1,000,000	601,000
		3,700,000	77,303,000
		4,700,000	
		<u>Tens:</u>	<u>Millions:</u>
		5,706,690	2,000,000
		68,200	8,000,000
		215,390	10,000,000
		20,925,100	380,000,000
		16,798,150	
		9,051,520	
		4,651,410	

DIVISION OF DECIMALS

I. Objective:

- A. To show how to express a fraction as a decimal.
- B. To show how to locate the decimal point in the quotient.
- C. To provide practice in division of decimals.

II. Suggestions:

- A. It will be necessary to show students how to locate a decimal point in the quotient before teaching them to express a fraction as a decimal.
- B. Provide practice in changing from fractional form to decimal form. (Problems should not require rounding off)
- C. Have the student divide 12 sticks into 4 equal stacks, into 3 equal stacks, into 6 equal stacks, into $1/2$ equal stacks. This is to emphasize why the divisor should be a whole number. Then take up topic of dividing decimals by decimals
- D. Provide practice in dividing decimals. (Problems should not require rounding off).

III. ANSWERS: Page 83

(1) .8	(7) .55
(2) .875	(8) .9375
(3) .25	(9) .125
(4) .5	(10) 1.6
(5) .2	(11) .18
(6) 52	(12) .246

Page 84

.75; .5; .6; $\overline{.571428} - -$;
 .15625; .86; .5; $\overline{.285714} - -$;
 $\overline{.166} - -$; 125; $\overline{.0833} - -$; 3;
 .35; $\overline{.1388} - -$; .003; .8
 .375; .25; $\overline{.714285} - -$; $\overline{.166} - -$;

Page 84

Brain Teaser:

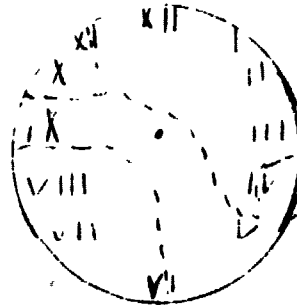
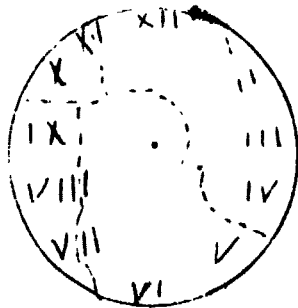
$$(-20+30+50=100)$$

First friend $\frac{20}{100}$ or $\frac{1}{5}$, \$125.20

Second friend $\frac{30}{100}$ or $\frac{3}{10}$, \$187.80

Third friend $\frac{50}{100}$ or $\frac{1}{2}$, \$313.00

ANSWERS: Page 86 "Broken Clock"



There are at least 2 other solutions. Can you find them?

ANSWERS: Page 87

(1)	.00424	(12)	33.3
(2)	1.56	(13)	88.223
(3)	.647	(14)	22.
(4)	65.3	(15)	33.22
(5)	8.63	(16)	.977
(6)	.4538	(17)	.031
(7)	.17	(18)	225.
(8)	84.	(19)	85.4
(9)	3.2	(20)	23.
(10)	.752	(21)	54.1
(11)	.021	(22)	4.44

Page 88

JAYCEE SUPPLY

Total	\$59.85
Tax	1.80
Net Total	61.65
	\$1.995
	1.990
	3.744

Page 89

CROSS COUNTY HOSPITAL

- (1) 40.77
- (2) .057 or \$0.06
- (3) Medical Records \$2.377 or \$2.38

Page 90

- | | |
|------------------------------------|----------------|
| (1) 10-24-71 | (7) 144 |
| (2) 10-29-71 | (8) 12 |
| (3) Truck | (9) 4 |
| (4) Mallard Pen & Pencil Co. | (10) 243. lbs. |
| (5) Yes | (11) 33 1/3% |
| (6) 2% EOM Net 60 | (12) \$0.10 |
| (7) 2% if paid by end of month. | (13) \$1.20 |
| Net amount due 60 days after date. | (14) 4 1/2 |
| Amount of discount \$3.21 | |

USES OF RENAMING NUMBERS

- I. Objective: To introduce instances where renamed numbers can be used. Also, give students some drill in renaming numbers or making number expressions.
- II. Suggestions: Numbers can be renamed just like people or places. A woman changes her name upon marriage but remains the same person.
- A. Begin by showing 1 can be renamed 1 or $1 = 1$. Then show 1 can be renamed as the ratio of any number (except 0) to itself, thus $1 = \frac{8}{8}$ or $1 = \frac{10}{10}$ or $1 = \frac{\frac{4}{3}}{\frac{4}{3}}$, or $1 = \frac{.8}{.8}$
- B. Numbers can be renamed for use in reducing fractions or finding prime factors of a number. E.g.
- $$\frac{15}{25} = \frac{5 \cdot 3}{5 \cdot 5} = \frac{5}{5} \cdot \frac{3}{5} = 1 \cdot \frac{3}{5} = \frac{3}{5} \text{ i.e. } \frac{5}{5} = 1.$$
- It should be pointed out to the class that "=" means "same as"; hence any sequence such as example given here is nothing more than a sequence or series of different names applied to the same quantity concept.
- C. Explain how 2 can be written or renamed $2/1$ or $7 = \frac{7}{1}$.
- Any whole number may be written over 1 since the whole numbers are a subset of the rationals. Then by multiplying the number by an appropriate form of one, equivalent fractions can be introduced. E.g. $\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$, $7 = \frac{7}{1} \times \frac{3}{3} = \frac{21}{3}$,
- or $1 = \frac{1}{1} \times \frac{8}{8} = \frac{8}{8}$. This is necessary for adding or subtracting rationals. E.g. $\frac{2}{3} + \frac{3}{4}$: Lowest common denominator is 12: $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$ and $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$. Hence the renamed problem is $\frac{8}{12} + \frac{9}{12}$.

- D. Introduce the idea that $8\frac{1}{3}$ can be renamed $8 + \frac{1}{3}$. This is needed for use with the distributive principle, factoring, or converting the so-called "mixed number" to fractional form. Thus $8 = \frac{8}{1} \times \frac{3}{3} = \frac{24}{3}$, $\frac{24}{3} + \frac{1}{3} = \frac{25}{3} = 8\frac{1}{3}$. On occasion $7\frac{3}{4}$ might be renamed $8 - \frac{1}{4}$ for short cut advantage in solving a certain problem, i.e. $(8\frac{1}{4} \times 7\frac{3}{4}) = (8 + \frac{1}{4})(8 - \frac{1}{4}) = 64 - \frac{1}{16} = 63\frac{15}{16}$ and vice versa. Thus $\frac{4}{5} = \frac{8}{10} = .8$, $\frac{1}{4} = \frac{25}{100} = .25$, or $.375 = \frac{375}{1000} = \frac{3}{8}$. An advantage in renaming numbers in this manner would be that decimals can be entered into the calculator whereas fractions as such cannot.
- E. Numbers such as 9 can be renamed 3^2 or $25 = 5^2$ or $8 = 2^3$. This can be very beneficial in very large or very small numbers used in scientific notation.
- F. In measurement, it is often convenient to rename denominate numbers. E.g. 3 ft. = 1 yd, 12 in. = 1 ft. = $\frac{1}{3}$ yd.
- G. In changing from one base system to another base numeration system, renaming is useful from the standpoint of positional values. E.g. In the quantity 423 base 10, the numeral 2 denotes 2 tens or 2×10 , whereas in base 5 the 2 denotes 2 fives or 2×5 .

ANSWERS: Page 92-93

- I. (1) 1,1,7,8,1 (2) a. $\frac{3}{3}$, b. $\frac{7}{7}$, c. $\frac{4}{4}$, d. $\frac{4}{4}$, e. $\frac{3}{3}$
 (3) a. 1, b. (15 + 12), c. 6, d. (3 + 4), ($\frac{6}{6} + \frac{6}{6}$) (4) a. $\frac{2}{3}$,
 b. (16 + 1), c. $\frac{54}{6}$, d. (7 + 7) (5) a. .4 b. .875
 c. 1.2 (6) a. 4 b. 27 c. 16
 (7) a. 7^2 b. 5^2 c. 9^2 or 3^4 (8) a. $\frac{1}{4}$ b. 32 c. 30"
 d. 5 e. 2 (optional) (9) a. 122_{eight} b. 17_{ten}

LEAST COMMON MULTIPLE

I. Objective:

- A. Teach multiple of a number.
- B. Teach common multiple.
- C. Teach least common multiple.

II. Suggestions:

- A. Multiple-cylindered engine 4, 8, 12; gear ratios, speeds of tape recorder.
- B. Show connection of lowest common denominator and lowest common multiple.

III. ANSWERS: Page 94-95

1. $R = \begin{Bmatrix} 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, \\ 65, 70, 75, 80, 85, 90, 95, 100, \dots \end{Bmatrix}$
2. $Q = \begin{Bmatrix} 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, \\ 78, 84, 90, 96, \dots \end{Bmatrix}$ (a) infinite (b) infinite
3. $E = \{30, 60, 90, \dots\}$
4. 30, yes
5. $\begin{Bmatrix} (a) \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, \dots\} \\ (b) \{12, 24, 36, 48, 60, 72, 84, 96, \dots\} \\ (c) \{24, 72, 96, \dots\} \\ (d) \{24\} \end{Bmatrix}$
6. $\begin{Bmatrix} (a) \begin{Bmatrix} 12, 24, 36, 48, 60, 72, 84, 96, \dots \end{Bmatrix} \\ \begin{Bmatrix} 15, 30, 45, 60, 75, 90, 105, \dots \end{Bmatrix} \\ \begin{Bmatrix} 60, 120, 180, \dots \end{Bmatrix} \\ \{60\} \end{Bmatrix}$
 $\begin{Bmatrix} (b) \begin{Bmatrix} 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, \\ 78, 84, 90, 96, \dots \end{Bmatrix} \\ \begin{Bmatrix} 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, \dots \end{Bmatrix} \\ \begin{Bmatrix} 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, \dots \end{Bmatrix} \\ \begin{Bmatrix} 72, 144, 216, \dots \end{Bmatrix} \\ \{72\} \end{Bmatrix}$
(c) Can be found in 6 (a) and 6 (b) lcm = 72
12 found in 6 (a)
 $\begin{Bmatrix} 14, 28, 42, 56, 70, 84, 98, \dots \end{Bmatrix}$
 $\begin{Bmatrix} 15, 30, 45, 60, 75, 90, \dots \end{Bmatrix}$
lcm = 420
7. $\begin{Bmatrix} (a) 30 \\ (b) 45 \\ (c) 30 \end{Bmatrix} \quad \begin{Bmatrix} (d) 14 \\ (e) 27 \\ (f) 42 \end{Bmatrix} \quad \begin{Bmatrix} (g) 6 \\ (h) 234 \\ (i) 630 \end{Bmatrix} \quad \begin{Bmatrix} (j) 60 \\ (k) 45 \\ (l) 1131 \end{Bmatrix}$

ADDITION OF FRACTIONS

I. Objectives:

- A. To introduce addition of rationals in fractional form.
- B. Establish closure.

II. Suggestions:

- A. To provide practice in the addition of fractions which will emphasize the properties of closure.
- B. Discuss addition of fractions containing whole numbers.

ANSWERS: Page 96

- | | | |
|----------------------|-----------------------|---------------------|
| K. 1. $\frac{19}{3}$ | 7. $2\frac{1}{4}$ | 13. $\frac{104}{9}$ |
| 2. $\frac{7}{3}$ | 8. $\frac{22}{4}$ | 14. $\frac{29}{3}$ |
| 3. $\frac{10}{3}$ | 9. $\frac{30}{4}$ | 15. $\frac{319}{6}$ |
| 4. $\frac{7}{2}$ | 10. $\frac{1116}{21}$ | 16. $\frac{11}{4}$ |
| 5. $\frac{9}{2}$ | 11. $\frac{128}{41}$ | 17. $\frac{66}{7}$ |
| 6. $\frac{11}{2}$ | 12. $\frac{21}{6}$ | 18. $\frac{31}{10}$ |

How old am I? 55 or 56

ANSWERS: Page 97

- L. (1) $\frac{2}{3}$ (2) $\frac{6}{7}$ (3) $\frac{11}{13}$
 (4) $\frac{9}{8}$ (5) $\frac{9}{12}$ (6) $\frac{9}{2}$
 (7) $\frac{7}{12}$ (8) $\frac{19}{8}$ (9) Yes
 (10) Yes

Page 98- 10a.

1 $\frac{4}{10}$	2 $\frac{12}{18}$	3 $\frac{7}{6}$	4 $\frac{7}{5}$	5 $\frac{19}{30}$
6 $\frac{9}{10}$	7 $\frac{26}{26}$	8 $\frac{24}{20}$	9 $\frac{12}{12}$	10 $4\frac{1}{10}$
11 $\frac{19}{12}$	12 $\frac{21}{21}$	13 $\frac{28}{30}$	14 $\frac{11}{6}$	15 $5\frac{7}{20}$
16 $\frac{16}{16}$	17 $\frac{17}{12}$	18 $\frac{10}{6}$	19 $\frac{16}{18}$	20 $5\frac{1}{12}$
21 $3\frac{53}{60}$	22 $4\frac{1}{12}$	23 $4\frac{29}{30}$	24 $5\frac{7}{30}$	25

TRICK NUMBERS

- (1) 13,212
 (2) $\{ 1+2+3+4+5+6+7+8+9=100$
 $\quad 74+25+\frac{3}{6}+\frac{9}{18}=100$
 (3) $9+9+\frac{9}{9}=19$

ADDITION PROPERTIES OF RATIONALS COMMUTATIVE

- I. Objectives:
 - A. To investigate the properties of addition on the rational numbers.
 - B. Provide practice.
- II. Suggestions:
 - A. By example show:
 1. Commutative
 2. Associative
 3. Identity
 4. Inverse
 - B. Explanations of Student Sheet
 - C. Fraction-O-Gram.

III. Resource Material:

- A. Refresher Arithmetic - Stein, Allyn & Bacon.

ANSWERS: Page 99-100

- | | | |
|--|---|---|
| <p>M. (1) a. $\frac{4}{6}$
 b. $\frac{4}{5}$
 c. $\frac{3}{10}$
 d. $\frac{2}{3}$
 Commutative</p> | <p>(2) a. $\frac{1}{5} + \frac{2}{5}, \frac{4}{5}, \frac{7}{5}$
 b. $\frac{2}{5} + \frac{4}{5}, \frac{1}{5}, \text{same}$
 c. $\frac{3}{4} + \frac{2}{3}$, rename them
 using lowest common denominator, $\frac{4}{5}$, yes $\frac{133}{60}$
 d. $\frac{3}{4} + (\frac{2}{3} + \frac{4}{5})$, same, associative</p> | <p>(3) a. 0 same
 b. $\frac{1}{10}$ yes
 c. $\frac{5}{6}$ 0</p> |
| <p>(4) a. 0
 b. $\frac{4}{5}$
 c. $\frac{2}{3}$
 d. 0
 e. $\frac{9}{7}$
 f. $-\frac{5}{9}$
 Opposites
 Same, 0
 Inverse
 Inverse</p> | <p>(5) a. $12 \frac{1}{20}$
 b. $\frac{179}{72}$ or $2 \frac{35}{72}$
 c. $1 \frac{1}{17}$
 d. $\frac{939}{560}$ or $1 \frac{379}{560}$
 e. $10 \frac{17}{40}$
 f. $2 \frac{3}{8}$
 g. $2 \frac{1}{4}$
 h. $1 \frac{9}{10}$</p> | |

Page 101

- | | | |
|---|---|---|
| <p>N. (1) $\frac{1}{3}, \frac{8}{10}$
 $\frac{3}{5}, \frac{3}{14}$
 $\frac{5}{8}, \frac{7}{9}$</p> | <p>(2) a. Increases
 b. Increases</p> | <p>(3) $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{23}{24}$</p> |
| <p>(4) $\frac{5}{6}, \frac{3}{4}, \frac{7}{10}, \frac{5}{12}, \frac{3}{8}$</p> | | |

ADDITION OF DECIMALS

- I. Objective;; To provide practice in the addition of decimal fractions.
- II. Suggestions:
 1. Stress place value to illustrate the proper position of the decimals.
 2. Use money problems to stress importance of proper placement of decimal.
 3. Work a few problems manually but many problems on calculator.
 4. Emphasize the use of 0's as place holders. (0's must be entered in machine.)
 5. Practical problems.

III. Resource Problems:

Calculator manual
 Millage Problems (see county tax assessor)

ANSWERS: Page 102		Page 103 Junior Achievement	
(1) 158.277	(6) 3,939.4325	(1). Total income, Rent, Net Income before taxes.	
(2) 2162.354	(7) 416.625		
(3) 680.3546	(8) 121.253		
(4) 10306.8833	(9) \$198.950	(2) \$928.34, \$53.47, \$281.42	
(5) 604.654	(10) 767.017		

NOTE: Problems on student page - number 3, 6, and 10 have a negative number.

Page 104

NOTE: Find extensions and total all items marked. C means price per 100. Students may apply special terms to find discount and write check for payments. Total \$136.33.

Page 105

Total \$43.89	Total \$19.86
Tax 1.32	Tax .60
Net Total 45.21	Net Total 20.46

SUBTRACTION OF RATIONALS

I. Objectives:

- A. To introduce the subtraction of rationals (in fraction form).
- B. To investigate the properties of rational numbers under subtraction.

II. Suggestions:

- A. Show closure.
- B. Show subtraction is not commutative.
- C. Show subtraction is not associative.
- D. Provide practice.

III. Resource Material:

Review text, Preliminary Mathematics, Dressler: AMSCO School publications, Inc.

IV. ANSWERS: Page 106

- P. (1) $\frac{2}{7}$ (2) $\frac{6}{15}$ (3) $\frac{7}{10}$ (4) $\frac{19}{40}$ (5) $\frac{25}{72}$ (6) $-\frac{7}{12}$
- (7) Yes (8) $1\frac{2}{3}$ (9) $6\frac{9}{16}$ (10) $2\frac{1}{8}$ (11) $8\frac{7}{12}$
- (12) $1\frac{5}{12}$ (13) $11\frac{8}{15}$ (14) $1\frac{11}{16}$ (15) $3\frac{34}{45}$

WHAT TIME IS IT? Page 108-109

- (1) 12:07 or 12:08
- (2) :45
- (3) 2:47 or 2:48
- (4) 11:17 or 11:18
- (5) 2:40
- (6) 4:02 or 4:03
- (7) 10:32 or 10:33

ANSWERS: Page 107

- Q. (1) a. $\frac{1}{2}$ b. $\frac{2}{5}$ c. $-\frac{17}{18}$
- (2) a. $\frac{1}{12}$ b. $-\frac{1}{12}$ Commutative
No
No
- (3) a. $\frac{7}{12}$ b. $-\frac{5}{12}$

Same fractions

Yes

Subtraction is not associative

Page 110-111

- (1) $\frac{3}{1}$ $\frac{3}{3}$
- (2) $\frac{6}{4}$ $\frac{6}{6}$
- (3) $\frac{0}{5}$ $\frac{2}{5}$
- (4) $\frac{3}{2}$ $\frac{5}{2}$
- (5) $\frac{4}{8}$ $\frac{4}{9}$
- (6) $\frac{2}{6}$ $\frac{0}{5}$
- (7) $\frac{3}{6}$ $\frac{5}{6}$
- (8) $\frac{7}{4}$ $\frac{8}{5}$
- (9) $\frac{5}{5}$ $\frac{5}{6}$
- (10) $\frac{0}{4}$ $\frac{0}{6}$
- (11) $\frac{3}{7}$ $\frac{4}{7}$
- (12) $\frac{5}{1}$ $\frac{5}{3}$
- (13) $\frac{3}{4}$ $\frac{3}{6}$
- (14) $\frac{0}{4}$ $\frac{2}{4}$
- (15) $\frac{3}{4}$ $\frac{5}{4}$

- (16) $\frac{5}{4}$ $\frac{5}{5}$
- (17) $\frac{6}{6}$ $\frac{7}{6}$
- (18) $\frac{7}{6}$ $\frac{8}{5}$
- (19) $\frac{5}{7}$ $\frac{4}{7}$
- (20) $\frac{2}{4}$ $\frac{2}{5}$
- (21) $\frac{1}{4}$ $\frac{7}{4}$
- (22) $\frac{4}{7}$ $\frac{4}{8}$

	0	1	2	3	4	5	6	7	8	9
0
1
2
3
4
5
6
7
8
9

SUBTRACTION OF DECIMALS

I. Objective: To provide practice in the subtraction of rationals in decimal form.

II. Suggestions:

1. Emphasize again (1) the importance of keeping the decimal point in line, and (2) using zeros as placeholders. (especially when entering the number on the machine)

III. Resource Material:

1. All types of money problems.

How much they had yesterday, how much they spent, and how much they retained.

2. Sale ad's from grocery store--How much more or less in each store is one item?

IV. ANSWERS: Page 112 Page 113

- | | |
|---------------|--|
| 1. 89.913 | (1) 449.13; 6908.333; 340.802 |
| 2. 145.500 | (2) 4444; 3329; 2222; 2000; 309 |
| 3. 56.873 | (3) 953,067; 8,308,687; 7,003,881 |
| 4. 489.05 | (4) 810,978.12; 721,229.0293 |
| 5. 770.112 | (5) 297,098,189.90; 79,208,497.13 |
| 6. .3089 | (6) 5019.0920; 304,319.9913; -20.22 |
| 7. .97231 | (7) 949,980,806 42; -32,255,110,889 |
| 8. .1222 | Page 114 |
| 9. 142.407 | (1) .06; .34778; .5683; .77; .88; .9 |
| 10. 165.45 | (2) 1.009; 1.08; 1.35; 1.50; 1.70; 1.77; |
| 11. \$784.16 | 1.8 (in reverse order) |
| 12. \$1069.24 | (3) .8875; 2.975; 5.1; 6.005; 11.663; |
| | 14.50998; 70.466 |
| | (4) (a) .07 (b) .009 (c) 2.005 |

ORDER OF OPERATIONS

I. Objective: To show students that order must come to mathematical operations performed in series or sequence or else uncertainty and confusion will prevail. To teach this order once the need is shown.

II. Suggestions:

Just as there is order in putting on one's socks and shoes, the days of the week, or driving on the highway on the right side, there must be an orderly way to arrive at the answer to problems in math like $5 + 2 \cdot 3$. Changing the order of putting on socks and shoes would at least be humorous if not embarrassing. If the order were changed in the days of the week, it would be confusing. To drive on the left would be disastrous if not deadly. To argue whether $5 + 2 \cdot 3 = 11$ or $5 + 2 \cdot 3 = 21$ would be wasted time and answer nothing. To avoid these circumstances, math must have a rule for order of performing operations. Mathematicians have agreed that proceedings from left to right we first go thru the problem performing multiplications and divisions, then the additions and subtractions in order from left to right. Thus $5 + 2 \cdot 3 = 11$. If parentheses () are used to group numbers, this indicated the enclosed operation is to be done first. Thus $(5 + 2) \cdot 3 = 21$. Here addition is done before multiplying by 3.

In a longer problem other grouping symbols may be used to indicate subsets. Brackets [], Braces { } and vinculum or bar — may be used. Thus in the problem $[3 + \{2 + 4 - (3 + 1) + 2\} - 3]$, one would begin by removing the innermost symbol first. In the example one would add $3 + 1$ in the parentheses first, giving the result $[3 + \{2 + 4 - 4 + 2\} - 3]$. Then the bar is removed giving the result $[3 + \{2 + 2\} - 3]$. Next remove braces and you have $[3 + 4 - 3]$. Finally the brackets and you have $3 + 4 - 3$ which is a name for 4. Caution: never try removing group symbols two or more at a time or from left to right as you read.

Since order of operations is essentially grouping (associativity), one could introduce flow charting, which is "planning your solution." This planning of solution must be an orderly, as well as ordered process.

IV. ANSWERS: Page 115

- (1) a. $(4.3) - 5$
 b. $8 + (7.2)$
 c. $8 - (4 + 4)$
 d. $(6.3) - (2 \cdot 4)$
 e. $12 + (3 \cdot 5)$

- (2) a. 33
 b. $\frac{30}{15} = 2$
 c. 53
 d. 10
 e. 33

Page 116

- f. 4
 g. 8
 h. 21
 i. 29
 j. 48

- (3) a. 21; b. 7; c. 16; d. 22; e. 160; f. 69; g. 4; h. 13

I. Objective: Stimulate interest in a review of rational numbers.

II. Suggestions:

- A. These pages may be used through the unit as interest builders or at the end as review.
- B. Dot-to-dot page when completed, forms an Arkansas Razorback.
- C. Ten Mathematical errors might be used as a test question.

III. ANSWERS: Dot-to-dot Page 117

(1) 4.73	(11) 3.144	(21) .071	(31) 5	(41) 90
(2) 3.38	(12) 91.3	(22) 7.00	(32) 50%	(42) .0078
(3) 51.2	(13) 1.000	(23) 1.5625	(33) 1.0125	(43) 57.34
(4) .48	(14) 41.863	(24) 13	(34) 25%	(44) .3
(5) .090	(15) 49	(25) 44.68	(35) 438.75	(45) .0024
(6) 60	(16) 199.2	(26) 392.42	(36) .68%	(46) 8.6
(7) 9.89	(17) 6.24	(27) .586	(37) 40.80	(47) 30%
(8) 305.3	(18) .03	(28) 167.65	(38) 242.29	(48) 1.1
(9) .27	(19) 738.89	(29) .2496	(39) 1.07	
(10) 47.3	(20) 6.86	(30) .9	(40) 27.405	

Page 118 Mathematical Errors:

- | | |
|------------------------|--|
| (1) 3 tons = 6000 lbs. | (6) $L = 3.14 \times 10' \times 40$ |
| (2) $11 + 4 = 15$ | (7) $36.7 + 5.50 + .784 + 6320 = 6362.984$ |
| (3) $3 \times 3 = 9$ | (8) $L = 50.24 \text{ sq. in.}$ |
| (4) $P = 24"$ | (9) Length of bow 2" |
| (5) $1/4 + 1/3 = 3/4$ | (10) $5/4 \cdot 1/12 = 5/48$ |

Page 119-120 Pizza Party:

- | | | | |
|-------------|---------------|-----------------|---------------|
| (1) Start | (6) Zero | (11) Opposites | (16) Closure |
| (2) Null | (7) Square | (12) Place | (17) Fraction |
| (3) Sets | (8) Prime | (13) Odd | (18) Property |
| (4) Renamed | (9) Whole | (14) Associate | (19) Integer |
| (5) Symbols | (10) Counting | (15) Operations | (20) Numbers |
| | | | (21) Products |

UNIT VII

PER CENT

- I. Objective: To define and introduce per cents.
- II. Suggestions: Define per cent (%) as $\frac{1}{100}$. In place of this sign (%) you may substitute $\frac{1}{100}$ and in place of $\frac{1}{100}$ you may substitute %.
- III. Resource Material::

Practical Problems

ANSWERS: Page 122

- (1). 20% (2) 50% (3) 83% (4) 60% (5) 60% (6) 5%
- (7) 4% (8) .4% (9) 2% (10) 75% (11) 7.5% (12) $8\frac{1}{3}\%$
- (13) 68% (14) 40% (15) .8% (16) .40, $\frac{2}{5}$ (17) .75 $\frac{3}{4}$
- (18) .125, $\frac{1}{8}$ (19) .098, $\frac{49}{500}$ (20) .001, $\frac{1}{1000}$ (21) .05, $\frac{1}{20}$

ANSWERS: Page 123

- | | |
|----------|----------|
| (1) 287 | (2) 2.34 |
| (3) 1500 | (4) 183 |
| (5) 36 | (6) 630 |
| (7) 3.25 | (8) 125 |
| (9) 33 | (10) .25 |

- | | |
|-------------------------|----------|
| (1) 66 $2\frac{1}{3}\%$ | (2) 15% |
| (3) 33 $1\frac{1}{3}\%$ | (4) 225% |
| (5) 250% | (6) 25% |
| (7) 36% | (8) 200% |
| (9) 27% | (10) 6% |

- | | |
|----------|----------|
| (1) 25 | (2) 800 |
| (3) 1500 | (4) 3600 |
| (5) 78 | (6) 70 |
| (7) 1000 | (8) 2500 |
| (9) 2500 | (10) 100 |

Page 126-128

Note on Big Game

Let 2 students work together on one sheet as opponent teams. They may flip coin for goal or start. Let each use a different colored pencil. Each student works the problems when he has the ball and draws his line of travel. Watch the downs.

Page 124 - Matching

- | | |
|-------|----------------------|
| (1) o | (1) 59.22 |
| (2) h | (2) 138.18 |
| (3) g | (3) 301.64 |
| (4) a | (4) 46.06 |
| (5) u | (5) 32.90 |
| (6) d | Total 100% Total 658 |

- | |
|--------|
| (7) t |
| (8) k |
| (9) l |
| (10) q |

Page 125

- | | |
|-----------------|------------------|
| (1) 23% | (1) 33; 66 |
| (2) 6% | (2) 49.28; 24.64 |
| (3) 25% | (3) 38; 9.5 |
| (4) 11% | (4) .750; 3 |
| (5) 18% | |
| (6) 17% | |
| Total 789, 100% | |

INTEREST

- I. Objective:; To introduce and define interest
- II Suggestions:
- Define interest as $I = PRT$. Then by practical problems lead the students to find the many ways interest enters into his everyday life
- III Resource Material:
- Filmstrips

IV	ANSWERS: Page 130-131		<u>Dot-To-Dot Puzzle</u>	
(1)	\$1250	(7)	8%	(13) 1
(2)	\$1600	(8)	6%	(14) 3
(3)	\$172.50	(9)	7%	(15) 2
(4)	\$7.44	(10)	8%	(16) 90
(5)	\$6.25	(11)	5 $\frac{1}{2}$ %	(17) 45
(6)	\$3375	(12)	8%	(18) 30
				(19) \$1820
				(20) \$300
				(21) \$198
				(22) \$1998
				(23) \$838
				(24) \$1000

- V ANSWERS: Page 132, Simmons First National Bank
- | | |
|---------------|----------------------------------|
| (1) a. \$900 | (2) a. \$11,500 |
| b. \$2.50 | b. \$31.94(44) |
| c. 165 | c. \$2875.00 |
| d. 9 - 5 - 71 | d. Annual \$11.50 90 days \$2.88 |
| | e. 12, 4, 71 |

COMMISSION

I Objective: To study commission by use of practical problems.

II Suggestions:

A Use Rul-e-a to introduce commission rule.

B Discuss meaning of commission. Ask students who receives a commission and for what?

Examples: Insurance salesmen, real estate agents, auctioneers, newsboys, car salesmen.

C If a boy has paper route, substitute his figures in the illustrative example. (S.M.)

III Resource Material:

Rul-e-a

ANSWERS: Page 133

Ex: \$2 93 (Rounded off) Amount of commission

Practice Problems:

1 Mr Graves' Commission: \$375

2 Don's Commission: \$367.20

3 Total sales: \$8475
Total commission: \$339

ANSWERS Page 134

Pool table \$ 6.49

TV 25.99

Refrigerator 21.99

Total \$54.47

DISCOUNT

- I Objective: To introduce discount as an example of percentage problems.
- II Suggestions: Where do we find examples of discount?
(Retail stores, purchases in quantity, prompt payment, employees of companies.)
- III Resource Material: Practical problems.

ANSWERS: Page 135

Amount of discount:	\$1.79	Judy's price:	\$7.16
1 Amount of discount:	\$38.02	Phil's price:	\$342.23
2 Amount of discount:	\$637.50	Jim's price:	\$3612.50
3 Amount of discount:	\$17.94 or \$17.90	Phyllis' price:	\$41.86 or \$41.90

ANSWERS: Page 136

Page 137

Rate	Rate of Discount		Amount of Disc.	Sale Price
	Amt of Discount			
(1) 20%	\$480.00	(1)	\$.13	\$.37
(2) 4%	.75	(2)	297.00	603.00
(3) 25%	3.75	(3)	3.44	5.15
(4) 4 1/2%	4.50	(4)	37.91	12.64
(5) 4%	32.00	(5)	4.56	14.44
(6) 4 1/2 2/3%	.60	(6)	15.83	8.15
(7) 6 1/4%	1.00	(7)	3.49	31.39
(8) 6 2/3%	5.00	(8)	29.22	116.88
(9) 12 1/2%	4.00	(9)	2.30	4.60
(10) 15%	12.00	(10)	59.66	139.22
(11) 20%	35.00	(11)	3.24	9.71
(12) 11 1/9%	.50	(12)	1.76	9.99

ANSWERS: Page 138 N.S. Athletic Department

(1) Totals:	\$136.25	\$102.70		
(2) Amount	Rate	3. List	Team	
A. \$7.55	30.8%	A. \$ 784.00	\$ 542.40	
B. 8.55	25.5%	B. 1072.00	798.40	
C. 5.05	16.8%	C. 960.00	798.40	
D. 7.55	26.5%	D. 912.00	670.40	
E. 4.55	24.6%	E. 592.00	446.40	
F. 0.30	24.0%	F. 40.00	30.40	
		G. 4360.00	3286.40	

ANSWERS:: Page 139

Page 141

(1) Sterling Stores Co., Inc.

Barton Lumber Co.

Towels	\$1.19	\$2.38
Records	2.59	10.36
Candy	.59	1.77
Total		14.51
Tax		.44
Save	\$7.60	

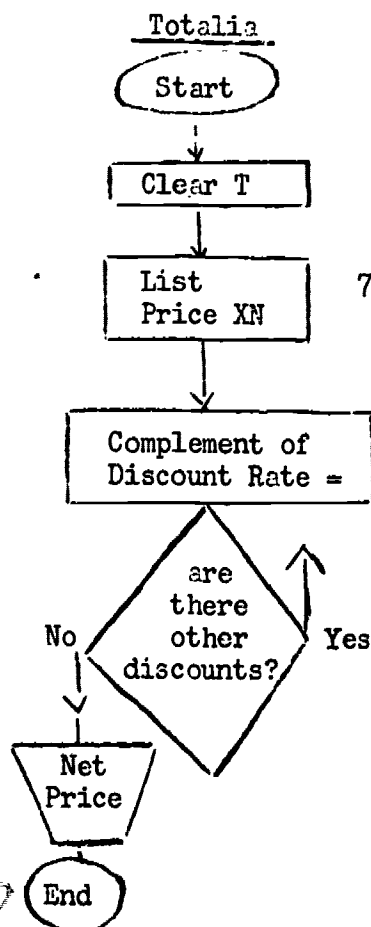
Balance

Paint	\$572.38
Roofing	396.24
Molding	52.39
Total	\$1021.01

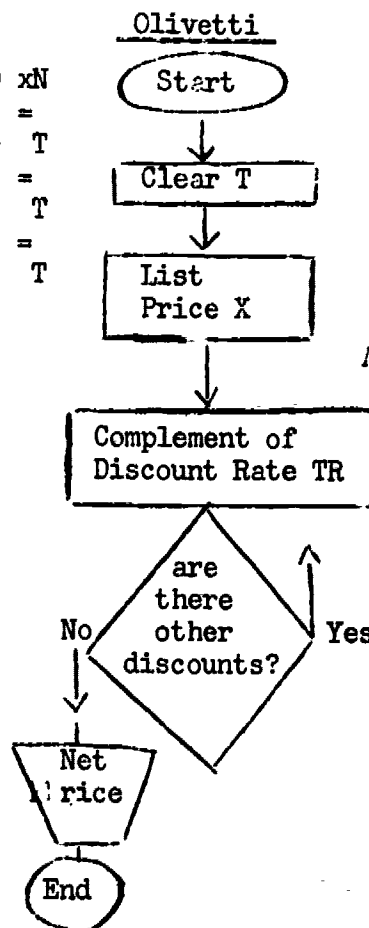
(2) Save \$15.07
Rate of discount 37.7%

Page 140

Flowcharts for Successive discounts: Example: \$16.20 with discounts of 30%, 25%, and 15%.



1620 xN
70 =
113400 T
75 =
8505000 T
85 =
722925000 T



1620 x <
70 =
113400 xT
75 =
8505000 xT
85 =
722925000 xT

ANSWERS: PAGE 140

(1)
a. \$52.50
b. \$50.63

(2)
a. \$15.82
b. 411.89
c. 14.46
d. 13.80
e. 75.58

(3)
a. \$324.00
b. 320.63

(4)
a. \$118.13
b. 112.50

TAXES

I. Objective: To introduce taxes.

II. Suggestions:

- A. This should include sales tax, school millage; gas taxes, cigarette taxes, personal or property taxes.
- B. Each tax will be introduced by examples. Let students investigate local tax rates.

III. Resource Material:

Practical Problems

ANSWERS: Page 142-143

- E. 1. (a) \$0.53 (b) 1.02 (c) 8.85 (d) .27 (e) .02
(f) .03 (g) 44.64 (h) .07
2. (a) \$31.85 (b) \$3.83 (c) \$47.00 (d) \$87.72 (e) \$10.40
- 3.
- | | <u>Total</u> | <u>Federal Tax</u> | <u>State Tax</u> |
|-----|--------------|--------------------|------------------|
| (a) | \$4.42 | \$0.57 | \$1.07 |
| | 1.65 | .20 | .38 |
| | 6.48 | .79 | 1.48 |
| | 3.98 | .46 | .86 |
| | 5.26 | .66 | 1.24 |
4. (a) \$2.17 $\frac{1}{2}$ or \$2.18; 42 $\frac{1}{2}$ ¢ or 43¢
5. (a) Subtotal \$18.03 Note: Jonesboro city tax equals
Fed. Tax 1.80 2.79% of monthly service
St. Tax .31 charge.
Total 20.14
- (b) \$6.56
- (a) City tax \$ 0.16
Total 20.30
- (b) \$6.72
6. (a) Total \$18,725. (g) \$107.10
Assessed Value \$3,745. (h) \$366.44
(b) Assessed value \$1,700. (i) \$266.81
(c) Total assessed value \$5,445 (j) \$5.45
(d) 69.25 mills (k) \$18.73
(e) 63 mills
(f) \$259.34

Page 144: Cooksey's Flower Shop

Amount	\$12.98
Tax	.39
Total	\$13.37

MARK-UP

I. Objective: To introduce Mark-Up

II. Suggestions: Show that mark-up is overhead + profit. Take different articles in a particular business and compare the percentage of mark-up by graph or chart. Discuss why wide variations occur, if they do.

III. Resource Material:

Practical Problems.

ANSWERS: Page 145

F. (1) a.	11¢	b.	17.7%	c.	15%
(2) a.	15¢		25%		33.3%
b.	21¢		20%		25%
c.	25¢		9.1%		10%
d.	75¢		16.7%		20%
e.	\$3.00		13%		15%

PERCENT OF INCREASE OR DECREASE

I. Objective: To define and introduce per cent of increase and decrease

II. Suggestions:

Work with class to develop a flowchart to show steps involved in finding the percent of increase and decrease.

III. Resource Material:

ANSWERS: Page 146-147

G. (1) Amount of increase: \$100
Per cent of increase: $33 \frac{1}{3}\%$ or
decimal 33.33%

(2) Amount of decrease: 9¢
Per cent of decrease: 21.5%

(3) Price last year: \$620
Per cent of decrease:
approx. 23.4%

(4) Amount of increase: \$500
Per cent of increase: 14.7%

(5) (b)
$$\begin{array}{r} 237 \\ 621 \\ \hline 147177 \end{array}$$

$$\begin{array}{r} 3 \\ \times 0 \\ \hline 0 \end{array}$$

(c)
$$\begin{array}{r} 725 \\ 527 \\ \hline 382075 \end{array}$$

$$\begin{array}{r} 5 \\ 5 \\ \hline 25 \end{array}$$

$$1 + 4 + 7 + 1 + 7 + 7 = 27$$

$$27 \div 9 = 3 \quad 0 \text{ remainder}$$

$$2 + 7 + 3 + 8 + 5 = 25$$

$$7 \text{ remainder}$$

I. Objective: To familiarize the students with the procedure involved in buying on the installment plan.

II. Suggestions:

- A. Discuss advantages and disadvantages of time payments.
- B. Discuss items involved such as insurance; service charge, if any; taxes; down payment; number of payments; balance to be financed, etc.
- C. Use accompanying retail installment contract blank to make out installment contract.
- D. Secure additional blanks from local merchants in quantity for classroom use.

III. ANSWERS: Page 148

Sales Tax	\$ 11.70	Cash price	\$401.65
Price (inc. Tax)	401.65	Unpaid bal.	350.00
		Total other chgs.	19.76
		Amt. financed	369.76
		Total of payments	418.92
		Deferred pay price	470.57
		Total payments	30
		1st payment date 1 month from	
		date of contract.	
		Amount of payment	13.97
		Final payment	13.79

UNIT VII

MEASUREMENT: LENGTH

- I. Objective: To interest students in measurement of length.
- II. Suggestions:
 - A. Discuss standard units of measure.
 - B. Give students copy of ruler work page, to work problems on reading ruler.
 - C. Give one student a ruler, one student a yardstick, and one student a steel tape and have each measure the length and width of classroom. Compare answers. Who will probably give the most accurate answer? Students will have to change answers to same units of measure before comparing. (denominate numbers)
 - D. Measure desks first in inches.
 - E. Measure them again in feet. What part of a yard is this?
 - F. Measure the teacher's desk in inches, feet, and yards.
 - G. Have each student choose the student they think is tallest. Then measure to see who was correct.
 - H. One lap around a track is 440 yards: How many laps would be necessary for the mile race? (Hint: How many yards in a mile?)
 - I. For additional problems measuring segments and adding lengths of segments. See enrichment problem no. 6 and 7.
- III. Resource Material: Rulers, Steel tapes, yardsticks, and string.

ANSWERS Page 149 Geometric Problem

- A. (1) $a = 1 \frac{3}{16}"$, $b = 1 \frac{1}{4}"$, $c = 1 \frac{3}{4}"$, Total = $4 \frac{3}{16}"$
 (2) $a = 1"$, $b = 1"$, $c = 1 \frac{1}{16}"$, $d = 1 \frac{1}{2}"$, Total = $4 \frac{9}{16}"$
 (3) $a = 1 \frac{1}{8}"$, $b = 2"$, $c = 1 \frac{1}{16}"$, $d = 1 \frac{1}{4}"$, $e = 1 \frac{5}{16}"$,
 Total = $6 \frac{3}{4}"$.

Page 150

- (1) 2, 4
- (2) 8, 4
- (3) 2, 4, 8, 16
- (4) 8, 3, 4, 2

Page 151

Example 1. 51¢
 Example 2. 5 oz.
 Line is approximately 62"

MEASUREMENT - LENGTH
PERIMETER

- I. Objective: To extend measurement of length to perimeter.
- II. Suggestions:
 - A. Show by example the meaning of perimeter.
 - a. How many "dubs" does it take to go around edge of desk top?
 - b. How many inches in (a)?
 - B. Measure geometric blocks (rectangle, triangle, square, trapezoid, many-sided polygons). Emphasize the names of these figures. Point out to students the relation between the sides of such figures as square, triangle, and rectangle.
 - C. Find the distance around the blackboard.
- III. Resource Material.
 - A. Blocks for unit of measure.
 - B. Geometric blocks and figures to measure.
 - C. Geometric board.
 - D. Peg board and plastic golf tees.
- IV. ANSWERS: Page 152
 - (1). 260'
 - (2). (a) Square (b) $1\frac{1}{4}"$ (c) A square has four equal sides.
(d) 36'
 - (3). (a) Triangle (b) A triangle has three sides, but the sides do not have to be equal. (d) Any three values whose sum is 135 where one value is less than the sum of the other two.
 - (4). (a) Rectangle (b) The two lengths the same and the two widths the same. (c) each side $1\frac{1}{2}"$, each end 1" (d) end 20', each side 50'.

MEASUREMENT
LENGTH
CIRCUMFERENCE

I. Objective: To introduce circumference as the perimeter of a circle.

II. Suggestions:

- A. SVE film strip with record would be very good here.
- B. Try to measure distance around a circle with the "dub" (invented unit of measure). If this is not successful, let students suggest a method of measuring. (Use string, then find the length of string in dubs and/or inches).
- C. Measure the equator on a globe.
- D. Send or take students to measure circumference of some trees on school ground.
- E. Remove label from a can of food and measure the length. Measure the distance through the center of the top of the can. See if the students know a name for this distance; also, half this distance. (1. diameter 2. radius).
- F. Have cylinders with diameters 1 in., 2 in., 4 in.... Measure circumference. Make a chart recording diameters and circumferences. Have the students make a comparison. Try to lead them to discover that the circumference is always about 3 times the diameter. Show that this comparison can be written as ratio $\frac{c}{d}$. Let them find the value of this ratio in all the cases above and take the average. Then ask the students if they know the name for this ratio. (π .)

Resource Material:

- A. Tape Measure.
- B. Block as a unit of measure.
- C. Circles with different diameters.

ANSWERS: Page 153

(1).	$\frac{C}{131.88}$	$\frac{D}{42}$	$\frac{\pi}{3.14}$
	$\frac{87.92}{28}$		$\frac{3.14}{3.14}$

(2) $\pi d = C$ — answers will differ.

Page 141 continued,

- (3). a. 25.12 inches
b. 197.82 miles
c. 27.475 inches
d. 7.85 yards
- (4). a. 1 inch
b. $\frac{1}{2}$ inch
c. $\frac{1}{2}$ of diameter
d. radius, $\frac{1}{2}$ diameter
e. $C = 2 \pi r$
- (5). a. 56.52 feet
b. 87.92 inches
c. 23.55 miles
d. 14.65 yards
e. 14.65 inches
- (6). 21.98 feet
- (7). 1.75 feet

Page 154-155

ANSWERS:

Sedum	\$22.80
Lzaleas	<u>18.27</u>
Total	\$41.07

Optional: 299 square feet - 1 bag - \$2.25

Page 156,

- (1). 13.29" (2). 293.60"

- I. Objective: To provide practice in measurement of segments and angles,
To provide materials for identification of triangles.
- II. Suggestions:

Page 157 can be used to provide practice in:

- (1). Define angle.
- (2). Locating and naming angles (how many) (naming as $\angle B$ or $\angle BFG$)
- (3). Teach right, acute, and obtuse angles.
- (4). Locating and naming triangles (how many)
- (5). Locating and naming segments (how many)
- (6). Use overhead to teach use of protractor.
- (7). Measure selected angles such as $\angle B$, $\angle DAH$, $\angle HGJ$
- (8). Angle sums
 - a. Complements
 - b. Supplements
 - c. In a triangle
- (9). Measure segments
 - a. Which are congruent?
 - b. Compare those that are not congruent
- (10). Perimeters
 - a. Triangles
 - b. rectangles
- (11). USE YOUR IMAGINATION ON THIS FIGURE.

ANSWERS: Page 157,

- B. (1). $(33 \frac{1}{2}^\circ)$ (2). $(28 \frac{1}{2}^\circ)$ (3). 84° (4). 100°
 (5). 128° (6). 90° (7). 45° , 135° , 45° , 135° (8). 52°
 Opposite angles are equal

Page 158,

- (1). 16 (2). Compare answers 19 $\frac{9}{16}$ (3). Compare answers 3 $\frac{7}{16}$

RATIO

I. Objective: To introduce, define, practice, and strengthen students' understanding of Ratio.

II. Suggestions:

- A. Define Ratio as a comparison of two numbers by division, and show the different ways a ratio may be written ($5/12$, 5 to 12, 5:12). (Ratio compares first number to second number—always).
- B. Refer to the circle chart and compare by division the shaded portion to the entire drawing, shaded portion to white portion, white to shaded, and white to whole
- C. Bring in denominate numbers to emphasize that numbers must be in the same units to write as a ratio.

III. Resource Material:

Circle Chart, Page 161

- (1). a. $\frac{1}{6}$ f. $\frac{1}{7}$
 b. $\frac{18}{1}$ g. $\frac{1}{12}$
 c. $\frac{3}{2}$ h. $\frac{3}{20}$
 d. $\frac{1}{9}$ i. $\frac{1}{24}$
 e. $\frac{1}{60}$ j. $\frac{1}{13}$
- (2). 22, 75, 33; 60, 102, 110
- (3). a. $\frac{6}{10}$ b. $\frac{10}{7}$ c. $\frac{7}{23}$
 d. $\frac{17}{23}$ e. $\frac{17}{6}$
- (4). Answers not included

Page 161,

- (1). Badminton Set $\frac{488}{688}$ (2) $\frac{200}{688} = 29.07\%$
 Insulated Bag $\frac{88}{129}$ $\frac{41}{129} = 31.78\%$
- (3). \$2.41
- (4). Bag.

SERIES OF TESTS FOR CIRCLE CHART

Test 1

Using the shaded parts only:

- 1). Find the sum of column 1. _____
- 2). Find the sum of column 2. _____
- 3). Find the sum of column 3. _____
- 4). Find the sum of column 4. _____
- 5). Col. 1 + $\odot_1 > 1 \frac{1}{2}$ T or F
- 6). Col. 2 + $\odot_4 > 1 \frac{1}{2}$ T or F
- 7). Col. 3 + $\odot_1 > 2 \frac{1}{2}$ T or F
- 8). Col. 4 + $\odot_8 > 3$ T or F
- 9). Col. 1 $>$ Col. 2. T or F
- 10). Col. 2 $>$ Col. 3 T or F

Test 2

Using the shaded parts only:

- 1). Find the sum of Row A. _____
- 2). Find the sum of Row B. _____
- 3). Find the sum of Row C. _____
- 4). Row A $>$ Row B T or F
- 5). Row B $>$ Row C T or F
- 6). Row C $>$ Row A T or F
- 7). Row A + $\odot_1 \neq \odot_2 + \odot_3$ T or F
- 8). Row A + \odot_2 Row C T or F
- 9). Row A + $\odot_9 \neq 2 \frac{1}{2}$ T or F
- 10). Row A + $\odot_1 < 1 \frac{3}{4}$ T or F

SERIES OF TESTS FOR CIRCLE CHART

Test 3

- (1). Find the sum of column 1. _____
- (2). Col 1 $>$ Row A T or F
- (3). Find the sum of column 2 _____
- (4). Col. 2 $<$ Row B T or F
- (5). Find the sum of column 3. _____
- (6). Col. 3 $>$ Row C T or F
- (7). Col. 1 + Col 2 $>$ Col. 1 + Col. 1 T or F
- (8) Col. 2 + Col. 3 $>$ Col. 1 + Col. 1 T or F
- (9). Col. 1 + Row A = ? _____
- (10). Col. 2 + Row B = ? _____

Test 4

- (1). Find the sum of Row A. _____
- (2). Row A $>$ Col. 4 T or F
- (3). Find the sum of Row B. _____
- (4). Row B $>$ Col. 3 T or F
- (5). Find the sum of Row C. _____
- (6). Row C $>$ Col. 2 T or F
- (7). Row A + Row B $>$ Row A + Row A T or F
- (8). Row B + Row C $>$ Row A + Row A T or F
- (9) Col. 2 + Row A = ? _____
- (10). Col. 3 + Row B = ? _____

SERIES OF TESTS FOR CIRCLE CHART

Test 5

- (1). Col. 1 $>$ Col. 3 T or F
- (2). Col. 1 \div Row A = ? _____
- (3). Row A $<$ Col. 4 T or F
- (4). Row A \times Col. 3 = ? _____
- (5). Col. 2 \neq Col. 3 T or F
- (6). THE SUM OF THE ODD NUMBERED COLUMNS = ? _____
- (7). $\odot_1 > (\odot_3 + \odot_5)$ T or F
- (8). $\odot_4 < (\odot_1 + \odot_2)$ T or F
- (9). Col. 1 + Col. 2 = ? _____
- (10). Col. 3 + Col. 4 = ? _____

Test 6

- (1). Col. 1 - Col. 2 = ? _____
- (2). Col. 3 - Col. 4 = ? _____
- (3). Row A + Row B = ? _____
- (4). Row C - Row A = ? _____
- (5). $(\text{Row A} + \odot_7) < \text{Row B}$ T or F
- (6). $(\text{Row B} + \odot_4) < 3$ T or F
- (7). $(\text{Col. 1} + \odot_1) > \text{Row A}$ T or F
- (8). $(\text{Row A} + \odot_5) \neq 2$ T or F
- (9). $(\text{Col. 3} + \odot_{10}) \neq 3$ T or F
- (10). $(\text{Col. 4} + \odot_1) < 2$ T or F

SERIES OF TESTS FOR CIRCLE CHART

Test 7

- (1). Col. 4 - \odot_4 = ? _____
- (2). Col. 4 + \odot_4 > 2 1/2 T or F
- (3). Col. 3 + \odot_1 = ? _____
- (4). Col. 3 + \odot_4 < 2 1/2 T or F
- (5). (Row A + 7) > 8 T or F
- (6). (1/2 Row A + 5) > 6 T or F
- (7). Col. 3 > Row A T or F
- (8). Col. 1 + Row A = ? _____
- (9). Col. 1 + Col. 4 = ? _____
- (10). $\odot_5 + \odot_6$ > \odot_9 T or F

Test 8

- (1). $\odot_9 : \odot_1 = \odot_2 : \odot_?$ _____
- (2). Row A : $\odot_9 = \odot_3 : \odot_?$ _____
- (3). 15 : $\odot_1 = 45 : \odot_?$ _____
- (4). 22 : $\odot_9 = 11 : \odot_?$ _____
- (5). Row A + Row B = ? _____
- (6). Row A : Row B = 3 : ? _____
- (7). Row A > 3 T or F
 \odot_{12}
- (8). $\odot_3 : \odot_9 = \odot_6 : \odot_9$ T or F
- (9). $\odot_9 : \odot_1 = \odot_{11} : \odot_6$ T or F
- (10). $\odot_{10} \div \odot_9$ > 1 T or F

SERIES OF TESTS FOR CIRCLE CHART

Test 9

- (1) $\odot_7 : \odot_{10} = \odot_1 : \odot_2$ _____
- (2) $\odot_5 : \odot_1 = ? : \odot_3$ _____
- (3) Col 1 : $\odot_3 = \odot_{10}$: Row B T or F
- (4) $20 : \odot_1 = 60 : \odot_?$ _____
- (5) Col. 2 + Col. 3 = ? _____
- (6) $\odot_{11} + \odot_{12} > 1.10$ T or F
- (7) $(\odot_3 + \odot_7) < (9 + 5)$ T or F
- (8) $\frac{\text{Col. 1}}{2} > 0.75$ T or F
- (9) Row C > 2.18 T or F
- (10) Col. 3 < 2.19 T or F

Test 10

- (1) Col. 1 + $\odot_{10} = ?$ _____
- (2) $\odot_1 : \odot_9 = \odot_4 : (\odot_8 + \odot_8)$ T or F
- (3) $\odot_{11} : \odot_4 = \odot_9 : (\odot_1 + \odot_9)$ T or F
- (4) $\odot_6 : \odot_3 = \odot_7 : \odot_4$ T or F
- (5) THE SUM OF ALL ODD NUMBERED CIRCLES > 3.5 T or F
- (6) THE SUM OF ALL EVEN NUMBERED CIRCLES < 2.5 T or F
- (7) THE SUM OF ODD NO. \odot 's $>$ THE SUM OF EVEN NO. \odot 's T or F
- (8) $(\odot_{11} + \odot_9) > (\odot_7 + \odot_4)$ T or F
- (9) THE SUM OF CIRCLES WITH A DENOM. FACTOR OF 4 > 1.5 T or F
- (10) THE SUM OF CIRCLES WITH A DENOM. FACTOR OF 3 < 1.75 T or F

ANSWERS TO CIRCLE CHART TESTS

<u>TEST 1</u>	<u>TEST 3</u>	<u>TEST 5</u>	<u>TEST 7</u>	<u>TEST 9</u>
(1). $\frac{27}{20}$	(1). $\frac{27}{20}$	(1). F	(1). $\frac{20}{15}$	(1). $\frac{3}{10}$
(2). $\frac{43}{40}$	(2). F	(2). $\frac{27}{30}$	(2). T	(2). $\frac{216}{120}$ or $\frac{9}{5}$
(3). $\frac{44}{20}$	(3). $\frac{43}{40}$	(3). F	(3). $\frac{42}{20}$	(3). F
(4). $\frac{26}{15}$	(4). T	(4). $\frac{1320}{400}$	(4). F	(4). Circle 3 or 11
(5). T	(5). $\frac{44}{20}$	(5). T	(5). T	(5). $\frac{131}{40}$
(6). F	(6). T	(6). $\frac{71}{20}$	(6). F	(6). F
(7). F	(7). F	(7). F	(7). T	(7). F
(8). F	(8). T	(8). F	(8). $\frac{57}{20}$	(8). F
(9). T	(9). $\frac{57}{20}$	(9). $\frac{97}{40}$	(9). $\frac{185}{60}$	(9). T
(10). F	(10). $\frac{150}{40}$	(10). $\frac{236}{60}$	(10). F	(10). T

<u>TEST 2</u>	<u>TEST 4</u>	<u>TEST 6</u>	<u>TEST 8</u>	<u>TEST 10</u>
(1). $\frac{30}{20}$	(1). $\frac{30}{20}$	(1). $\frac{11}{40}$	(1). Circle 9	(1). $\frac{78}{40}$
(2). $\frac{107}{40}$	(2). F	(2). $\frac{28}{60}$	(2). Circle 9	(2). T
(3). $\frac{131}{60}$	(3). $\frac{107}{40}$	(3). $\frac{167}{40}$	(3). Circle 3 or 11	(3). T
(4). F	(4). T	(4). $\frac{41}{60}$	(4). Circle 9	(4). T
(5). T	(5). $\frac{131}{60}$	(5). T	(5). $\frac{167}{40}$	(5). T
(6). T	(6). T	(6). F	(6). 5.35	(6). T
(7). F	(7). T	(7). T	(7). T	(7). T
(8). F	(8). T	(8). F	(8). T	(8). T
(9). F	(9). $\frac{103}{40}$	(9). T	(9). T	(9). F
(10). F	(10). $\frac{195}{40}$	(10). T	(10). T	(10). T

ANSWERS TO CIRCLE CHART TESTS

<u>TEST 1</u>	<u>TEST 3</u>	<u>TEST 5</u>	<u>TEST 7</u>	<u>TEST 9</u>
(1). $\frac{27}{20}$	(1). $\frac{27}{20}$	(1). F	(1). $\frac{20}{15}$	(1). $\frac{3}{10}$
(2). $\frac{43}{40}$	(2). F	(2). $\frac{27}{30}$	(2). T	(2). $\frac{216}{120}$ or $\frac{9}{5}$
(3). $\frac{44}{20}$	(3). $\frac{43}{40}$	(3). F	(3). $\frac{49}{20}$	(3). F
(4). $\frac{26}{15}$	(4). T	(4). $\frac{1320}{400}$	(4). F	(4). Circle 3 or 11
(5). T	(5). $\frac{44}{20}$	(5). T	(5). T	(5). $\frac{131}{40}$
(6). T	(6). T	(6). $\frac{71}{20}$	(6). F	(6). F
(7). F	(7). F	(7). F	(7). T	(7). F
(8). F	(8). T	(8). F	(8). $\frac{57}{20}$	(8). F
(9). T	(9). $\frac{57}{20}$	(9). $\frac{97}{40}$	(9). $\frac{185}{60}$	(9). T
(10). F	(10). $\frac{150}{40}$	(10). $\frac{236}{60}$	(10). F	(10). T

<u>TEST 2</u>	<u>TEST 4</u>	<u>TEST 6</u>	<u>TEST 8</u>	<u>TEST 10</u>
(1). $\frac{30}{20}$	(1). $\frac{30}{20}$	(1). $\frac{11}{40}$	(1). circle 9	(1). $\frac{78}{40}$
(2). $\frac{107}{40}$	(2). F	(2). $\frac{28}{60}$	(2). Circle 9	(2). T
(3). $\frac{131}{60}$	(3). $\frac{107}{40}$	(3). $\frac{167}{40}$	(3). Circle 3 or 11	(3). T
(4). F	(4). T	(4). $\frac{41}{60}$	(4). Circle 9	(4). T
(5). T	(5). $\frac{131}{60}$	(5). T	(5). $\frac{167}{40}$	(5). T
(6). T	(6). T	(6). F	(6). 5.35	(6). T
(7). T	(7). T	(7). T	(7). T	(7). T
(8). F	(8). T	(8). F	(8). T	(8). T
(9). F	(9). $\frac{103}{40}$	(9). T	(9). T	(9). F
(10). F	(10). $\frac{195}{40}$	(10). T	(10). T	(10). T

PROPORTION

- I. Objective: To introduce, define and practice working with proportions
- II. Suggestions:
- A. Use with unit on similar triangles, also, with equations if introduced
 - B. A proportion is a statement that one ratio is equal to another ratio. (Show ways of writing a proportion Examples:

$$\frac{2}{3} = \frac{8}{12}, 2:3 = 8:12, \text{ or } 2:3::8:12$$

Read "2 is to 3 as 8 is to 12". Explain the position of the terms such as 2 is first term, 3 is the second term, 8 is the third term, and 12 is the fourth term)

III Resource Material:

Unit on similar triangles

Unit on fractions

Unit on Ratio

Unit on Measurement

ANSWERS: Page 163

(1) a. $x = 12$

b. $x = 7$

c. $x = 35$

d. $x = 5$

e. $x = 5$

(2). $x = \$.94$

(3). $x = 720 \text{ lbs}$

(4) $x = \$4 \text{ } 17$

(5). $x = \$1 \text{ } 63$

MEASUREMENT: SCALE DRAWINGS

- I. Objective: To introduce scale drawings
- II. Suggestions: Have road maps for each student. Discuss how far it is to the next town. Let students measure in inches how far apart the towns are on the map. Ask the students how this distance in inches was determined. Lead them to say scale. Some student may find the scale on the top of the map. Discuss this scale with the students to bring out meaning of scale. Find a picture in dictionary drawn a fraction of actual size. What does fraction mean? (Ratio of picture to actual size). Refer to drawing, page 119-A

Take up and keep students' scale drawings for future use, such as finding area of court and center circle, etc

III Resource Material:

Road maps, rulers, dictionary.

ANSWERS: Page 164,

- (1). 1 in., 3 in., 6 in., 6 in., 18 in.
- (2). 3 in., 9 in., 4 1/2 in., 1 yd
- (3). Example 1" = 2' or 1" = 10'
- (4). 1200 rods.

MEASUREMENT - SIMILAR TRIANGLES

- I. Objective: Present indirect measure.
- II. Suggestions:
 - A. Give pairs of similar triangles, and explore characteristics (corresponding sides in ratio). Then give similar triangles and find missing parts.
 - B. Possibly go to school ground and observe shadow of tall boy in comparison to flag shadow. How high is the flag pole?
 - C. Introduce right triangle and its parts and characteristics.
- III. Resource Material.
 - A. Steel tape
 - B. Models of similar triangles.
 - C. Page 157-158

ANSWERS: Pages 165-166

1. corresponding sides have same ratio.
2. (a) 10 (b) 4 (c) $7\frac{1}{2}$ (d) $6\frac{3}{4}$
3. (a) right (b) 75'
4. 6'
5. 42'
6. 80'
7. \$478.

MEASUREMENT - PYTHAGOREAN THEOREM
(INDIRECT MEASUREMENT)

I. Objective: To introduce the Pythagorean Theorem in relation to indirect measurement.

II. Suggestions:

- A. Ask students to find distance they must throw the ball from 3rd base to 1st base. Discuss shape of baseball field. If we drew the line the ball traveled from 3rd to 1st, what kind of triangles would be formed? Do students know a name for this line or this part of the triangle? Follow this by a discussion of the right triangle. (Parts: hypotenuse, sides, right angles). Talk about length of sides. Draw squares on the sides. Tell the students Pythagoras developed a way of finding this length. Give special examples....

Then go back and find distance from 3rd base to 1st base.
(Read answer from square root table).

III. B. Probably need to spend some time here to teach students to read a square root table.

III. Resource Material:

Square root table for each student..

ANSWERS: Page 167

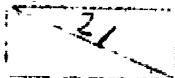
1. 10

2. 9

3. 20

4. 13

5. answers will differ

6. 12  - 16"

7. 500 feet—one wire—1500 for 3 wires

MEASUREMENT - AREA - SQUARE - RECTANGLE

I. Objective: To introduce area of rectangle, square.

II. Suggestions:

- A. Present imaginary problem such as needing to paint walls of room. Painters must know how many square feet need painting to know how much paint will be needed.
- B. Discuss area of rectangle, square. Ask student the dimensions of his room. Let him figure how much it would cost to paint or wall paper his room.
- C. Go through problem of putting tile or carpeting on classroom floor. Other problems.
- D. Mention (base x height) as being (length x width) in order to connect to parallelogram. The angles of rectangle as being right angles.

III. Resource Materials.

- A. Geo Board would be useful here.
- B. Problems of tile, linoleum, carpeting could be used here.
- C. SVE filmstrips.

ANSWERS: Page 168

- (1). \$130.00
- (2). \$8.64
- (3). 100 square feet
- (4). 8100 square feet
- (5). 30.4 square feet

Page 169

- (1). 19 square yds., \$151.05
- (2). 21 square yds., \$136.50
(rounded)
- (3). 20 square yds., \$179.00
- (4). 448 tiles, Cost \$ 67.20

S.P. 103.04



Profit \$ 35.84

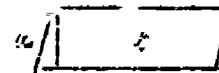
MEASUREMENT-AREA-TRIANGLE-PARALLELOGRAM

I. Objectives:





- A. To introduce method for finding area of parallelogram and then introduce the area of a triangle as being half the area of a parallelogram.
- B. OR To introduce the area of a parallelogram and introduce the area of a triangle as $\frac{1}{2}$ rectangle.

II. Suggestions:

- A. The parallelogram may not be a familiar geometric figure to the students. Relate it to the rectangle. Explain the base and height of . Ask the students if they can make a rectangle by moving one part of  divided in this way.



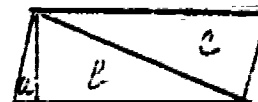
Lead them to see that if you moved part (a) to the right side of part (b) you will have a rectangle. Then discuss area as being $b \times h$.

- B. On the same  draw a diagonal . Lead them to see that it forms two equal triangles. Then that area of the  would be $\frac{1}{2}$ area of . Thus developing $\frac{1}{2} b \times h$.

- OR Draw diagonal of rectangle. Lead them to see that it forms two equal triangles. Then the area of a triangle would be $\frac{1}{2}$ the area of a rectangle.

III. Resource Material:

- A. Use Geo. Board.
- B. A model of parallelogram of some material which has these parts so students can actually see.



ANSWERS: Page 170

- (1). a. 48,400 square yards. Total square yards = 198,440
10 acres
Total acres = 41
b. 150,400 square yards
31 acres

(2).

1 440	2 2250	3 340	4 250
5 930	6 180	7 4130	8 380
9 1220	10 2440	11 170	12 910
13 690	14 750	15 100	16 2000

(3). Triangle

Rectangle or parallelogram

Triangle

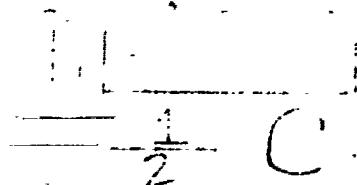
Rectangle or parallelogram

MEASUREMENT-AREA-CIRCLE

I. Objective: Teach students to find the area of a circle.

II.1 Suggestions:

- A. Have students draw a circle as large as possible with compass. Mark off circle in 10° sections. Cut circle in half. Start at center and cut almost to edge on all lines on both halves. Then dovetail them together to form approximate rectangle. Find area of rectangle.



$$A = r \cdot \frac{1}{2} C$$

but $C = 2\pi r$
 So $A = 2\pi r \cdot \frac{1}{2} r$
 $A = \pi r^2$

- B. Then give students practice using the formula found.

III. Resource Material:

- A. Compass, paper, scissors, protractor.
 B. dimensions of basketball court, center circle, free throw circle, etc.

ANSWERS: Page 171-172

- (1). 144 sq. ft. —/rea of square
113.04 sq. ft. —/rea of circle
 30.96 sq. ft. —/rea of shaded portion
- (2). 254.34 sq. ft. —/rea of larger circle
113.04 sq. ft. —/rea of smaller circle
 141.30 sq. ft. —/rea of patio
- (3). 14.13 quarts of paint needed
 \$35.1837 (cost) —\$35.18
- (3). 675 sq. ft. —/rea of park
 144 sq. ft. —/rea of square flower garden
 56.52 sq. ft. —/rea of two circular flower beds.
 474.48 sq. ft. —/rea of park grass covered
- (4). Answers will differ according to demensions you use with your court.

The Repetitious Number.

The product of 7, 11, and 13 is 1001.
 1001 times any number repeats the digits.

MEASUREMENT SURFACE AREA OF CUBE

I. Objective: To introduce surface area of cubes.

II. Suggestions:

- (1). Make a cube by first drawing a pattern of a cube.
- (2). Then see how many different patterns they can design consisting of only one piece.
- (3). This idea could be extended to other solid figures.

III. Resource Material:

Ruler, protractor, paper and tape.

IV. Page 173:

CUT OUT - FOLD IT - NAME IT (Geometric Paper Folding)

Octahedron

Cube

Tetrahedron

Have students cut out along dotted lines and fold along solid lines. If students cannot name the solid suggest they search the library for names.

These solids fit within each other, the octahedron within the tetrahedron and the tetrahedron within the cube.

MEASUREMENT
SURFACE AREA OF SOLIDS
RECTANGLES

I Objective: To introduce surface area of rectangular boxes.

II. Suggestions:

1. Make a rectangular box and measure all sides.

(1). Are all sides the same?

(2). What did you discover?

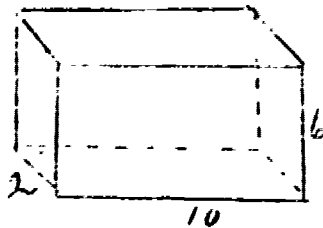
III. Resource Material:

Ruler, protractor, paper, and tape.

ANSWERS:- Page 175

(1). Answer will differ.

(2).



Total area: 120 sq. "
24 sq. "
40 sq. "
184 sq. "

(3). a. 88 sq. ft.

b. \$10.56

"Time Killer"

2^3	$\sqrt{64}$	$9-1$	$10-3$	$5+3$
$(2-3)+2$	$15+3$	VIII	eight	
$12-5$	$8+0$	$8+0$	$6/3 \times 2$	
$4+1+1+2$	$16/2$	$6+2$		

MEASUREMENT
SURFACE AREA OF SOLIDS
CYLINDER

I. Objective: To introduce surface area of solids

II. Suggestions:

1. Make cylinder of rectangular piece of paper to show surface area is same as circumference times height. Then measure diameter and find area of top and bottom. Find total area of cylinder.

III. Resource Material:

Ruler, protractor, paper, and tape.

ANSWERS: Page 176

- (2). a. 133.45 sq. in. or 133.46
- b. 207.24 sq. in.
- c. 49.0625 sq. in. - total surface area of can.

MEASUREMENT - VOLUME

I. Objective: To present volume in a more understanding way.

II. Suggestions:

A. Get a gallon can and punch a hole about $\frac{3}{4}$ of the way up from the bottom. Insert a small piece of rubber tubing to make a spout. Fill the can and let overflow until water is level with spout. Choose a smaller container to catch overflow. Submerge 1 inch cube in water and catch overflow. Mark graduation on small container. Refill gallon can and repeat submerging cube and catching overflow and marking small container until you have a sufficient scale to measure many volumes. Submerge other models. Read volume from scale on small cup.

B. Then give students formulas for finding volumes of cube, rectangle, and cylinder. Be sure to discuss what each letter or symbol used in formula means to be sure they can read and understand formula. Find volumes of models by experiment and formula and compare.

III. Resource Materials:

Gallon can, smaller can inner, 1 inch cube, small rubber tube for spout.

ANSWERS: Page 177

(Good to use page 178 for extra credit)

	No of sides		No. of inch
	Red	Green	Cubes
(1). 3 in., 4 in., 2 in.	0	0	648
(2). 125 cubic inches	0	1	153
(3). 972 pounds of green beans	0	2	9
(4). 2520 cubes	0	3	0
(5). 225 hours	0	4	0
(6). (a). 2970 pounds	1	0	153
(b). approx. 45.7 bu.	1	1	26
	1	2	1
	1	3	0
	2	0	9
	2	1	1
	2	2	0
	3	0	0
	3	1	0
	4	0	0

Page 179,

H.S. Athletics

A. 24	E. 228,000
B. 2400	F. \$48.00
C. 9600	G. \$4800.00
D. 96000	H. \$.50

Page 180, Volume of Cylinder

- (1). 1695.6 cu. in.
- (2). approx. 10097 gal
- (3). 39.25 cu. yds.

Total 1000

Page 181.

(1). Redi-Pool

Area = 92.10 ft.

Volume = 20.93 ft.

313.95 or 314 gal.

(2). File cabinet- Area = 1832 sq. in.

12 13
18 sq.ft..

Make a guess.

Horizontal Vol. = 132.6 cu. in.

Vertical Vol. = 169.6 cu. in.

More

MEASUREMENT

WEIGHT - LIQUID - DRY

I. Objective: To review the topic of liquid and dry measure.

II. Suggestions: Introduce this with practical problems.

Visit a produce market that buys from local people and sells retail.

Ask students how often they measure something. What unit of measure did they use?

III. Resource Material:

Have on hand examples of different containers for the measures.

ANSWERS: Page 182

- | | |
|-------|--------|
| (1) g | (9) s |
| (2) c | (10) p |
| (3) b | (11) u |
| (4) d | (12) k |
| (5) t | (13) h |
| (6) r | (14) a |
| (7) l | (15) j |
| (8) e | |

ANSWERS: Pages 183-184

- (1) 18' 3"
- (2) 7 yds. 2 ft.
- (3) 15 lbs. 11 oz.
- (4) 12 tons 1625 lbs.
- (5) 16 tbles.

- (6) 13 bu. 1 pk.
- (7) 10 bu. 2 pks. 5 qts.
- (8) 15 gal. 1 pt.
- (9) 8 qts 1 pt. 1 cup
- (10) 270 29' 34"
- (11) 295 56' 56"

Page 185

- (1) 8 ft. 7 in.
- (2) 2 yds. 1 ft. 9 in.
- (3) 3 mi. 1580 yds.
- (4) 3 lbs. 13 oz.
- (5) 12 lb. 13 oz.
- (6) 3 T 225 lbs.
- (7) 4 tbles. 2 teas.

- (6) 6 bu. 2 pks.
- (9) 1 pk 6 qts.
- (10) 12 gal. 2 qts.
- (11) 5 qts, 1 pt, 1 cup
- (12) 35 38' 43"
- (13) 51 40' 47"

Page 186

- (1) 25 ft. 8 in.
- (2) 52 yds. 1 ft. 11 in.
- (3) 13 mi. 109 yds.
- (4) 43 lbs. 5 oz.
- (5) 82 lbs. 2 oz.
- (6) 25 T 607 lbs.
- (7) 81 tbles. 2 teas.

- (8) 100 bu. 2 pks.
- (9) 23 bu. 2 pks. 3 qts.
- (10) 55 gal. 1 pt.
- (11) 23 qts.
- (12) 684' 7' 39"
- (13) 220' 22'

Page 187

- (1) 2 ft. 3 in.
- (2) 1 mi. 900 yds.
- (3) 2 ft. 10 in.

- (7) 1 Tbles. 2 teas.
- (8) 3 pks.
- (9) 2 pks, 4 qts.

- (4) 1 lb. 2 oz.
- (5) 15 oz.
- (6) 900 lbs.

- (10) 1 gal. 3 qts. 1 pt.
- (11) 2 qt. 1 cup
- (12) 16 14' 18"
- (13) 14 15' 3"

MEASUREMENT - TIME

I. Objectives:

- A. To teach telling differences in time of day and time of years.
- B. Add, subtract, multiply, and divide time

II. Suggestions:

- A. What is basis of time? How long is a day and why? Day is not really 24 hours, but 23 hours 59 minutes. Talk about leap year, etc
- B. How was clock devised? Why 12 hours? (Originally designed to measure hours of night. Day was measured by sun).
- C. Suppose there were no clocks. How would your day be more disorganized? (For example)
- D. (1) How would you know when to turn on the TV to watch Batman?
(2) How would you know when to catch the bus?
- D. Ask bus student what time he catches the school bus? What time does he get to school? How long is he on the bus each morning? Each day? Each week?

ANSWERS: Page 183

- (1) 2 hours, 45 minutes
- (2) variable
- (3) 16 days
- (4) 174 days
- (5) (a) May 9 (b) July 17 (c) Jan. 29 (d) Sept. 28
- (6) (a) 13 years 10 months (b) 9 weeks (c) 16 hrs 56 min. 10 sec.
- (7) (a) 2 days 15 hours (b) 15 min. 28 sec.
(c) 1 hr. 53 min. 46 seconds
- (8) (a) 45 days 18 hours
(b) 21 hours 47 minutes 50 seconds
- (9) (a) 3 years 2 months
(b) 7 min. 15 seconds

MEASUREMENT - TIME

I. Objective:

To extend the unit on measurement of time to include the use in wage problems..

II. Suggestions:

A. Talk about workers being paid on hourly basis.

1. Without time card
2. With time card.

B. Refer to practical problem on wage-time card.

ANSWERS: Page 189

- (1) a. 24.80
b. \$124.00

(2) \$103.50

(3) \$5.88

(4) Mon 7 hrs.

Tues. 7 hrs.

Wed. 6 1/2 hrs.

Thurs

Thurs. 7 hrs.

Fri. 7 hrs.

34 1/2 hrs. \times 1.80 = \$62.10

Total wage

Page 191

(1) 135 days

- (2) (a) 97 days
(b) 27 days
(c) 264 days
(d) 274 days
(e) 39 days
(f) 87 days

- (3) (a) 133
(b) 224
(c) 619
(d) 111

- (4) July 2
(5) Ans. will vary
(6) Ans. will vary
(7) July 12
(8) 694
(9) Ans. will vary
(10) (a) Feb. 28
(b) Dec. 25

ANSWERS: Page 192-193

Little Joo

- | | |
|-------------------|------------------|
| (1) Mathematics | (19) Payment |
| (2) Angle | (20) Installment |
| (3) Measuring | (21) Replace |
| (4) Estimate | (22) Points |
| (5) Unequal | (23) Counting |
| (6) Square | (24) Number |
| (7) Quadrilateral | (25) Triangle |
| (8) Circle | (26) pi |
| (9) Replace | (27) Chords |
| (10) Set | (28) Increased |
| (11) Diameter | (29) Rate |
| (12) Inches | (30) Per cent |
| (13) Volume | (31) Ruler |
| (14) Calculation | (32) Compass |
| (15) Discount | (33) Protractor |
| (16) Ratio | (34) Average |
| (17) Tax | (35) Curves |
| (18) Down | (36) Height |

Suggestions: Page 194

- (1) Measure line segments
- (2) Naming angles
- (3) Naming line segments
- (4) Sum of segments

Page 195 Definition of Board Feet: 1 in x 1 ft. x 1 ft. = 1 B.F.

- | | | | |
|------------|-------------|-------------|------------|
| (1) 10 | (7) 13 1/2 | (13) 8 | (19) 24 |
| (2) 12 | (8) 16 | (14) 1 | (20) 24 |
| (3) 8 | (9) 2 | (15) 42 2/3 | (21) 8 |
| (4) 4 | (10) 24 | (16) 5 1/3 | (22) 5 1/3 |
| (5) 4 | (11) 3 1/3 | (17) 2 2/3 | |
| (6) 21 1/3 | (12) 13 1/3 | (18) 85 1/3 | |

ORDER RELATIONS

I. Objective: To introduce order relation between numbers by using inequalities.

II. Suggestions:

- A. Introduce symbols: $=$, \neq , $>$, $<$, (or \geq), (or \leq)
Review number line.
Emphasize that on the number line the number on right is greater.
- B. Use only whole numbers at first. D. Use many illustrations such as:
- C. Also use positive fractions. 5 "is greater than" 3: 5 3
or 3 "is less than" 5: 3 5
- E. Still using the number line, extend it to the left of zero.
- F. Still emphasize that the number to the right on the number line is greater.

JUDY'S PROBLEM - Page 200

- G. Also use negative fractions.
- H. Use illustrations such as:
1 "is less than" 2
2 "is greater than" 1

ANSWERS: Page 196

- | | |
|-----------------------|------------------------|
| (1) (answer may vary) | (7) $>$ |
| (2) $>$ | (8) $<$ |
| (3) $<$ | (9) $<$ |
| (4) $<$ | (10) $<$ |
| (5) (answer may vary) | (11) $>$ |
| (6) (answer may vary) | (12) (answer may vary) |

ANSWERS: Page 197

- | | | | | |
|----------|----------|----------|----------|----------|
| (1) $<$ | (2) $>$ | (3) $<$ | (4) $<$ | (5) $>$ |
| (6) $>$ | (7) $<$ | (8) $<$ | (9) $>$ | (10) $>$ |
| (11) $>$ | (12) $<$ | (13) $>$ | (14) $<$ | (15) $>$ |
| (15) $<$ | (16) $=$ | | | |

ANSWERS: Page 198

- (1) addend, dividend, subtrahend (2) even, seven, eleven
- (3) pint, point, midpoint (4) line, nine, determine
- (5) year, clear, linear, collinear (6) rate, ratio, rating, rational

ANSWERS: Page 199

- (7) vector, bisector, trisector, intersector, (8) segment element, tangent adjacent, congruent, equivalent, concurrent complement (9) meter, center, roster, greater, incenter, perimeter, diameter, centimeter (10) eight, right, weight, height, straight.

UNIT VIII

HOW TO SPEND YOUR TIME

- I. Objective: To develop an understanding of time and a need for budgeting time so we may accomplish a desired goal.
- II. Suggestions: This should be two or three days work.
 - A. Attempt to bring students to a realization of time.
 1. Ask questions.
 - a. What do they plan to do after school today?
 - b. What do they plan to do Saturday, Sunday, etc?
 - c. What is their favorite T. V. program? Do you plan to see the next program?
 2. Students complete top of student page 158.
 3. Use machine to find what % of day is used for each activity.
 - B. Attempt to get students to talk about something they would like to own which they do not have.
 1. Ask questions.
 - a. Why do they want it?
 - b. Why do they not have it?
 - c. How badly do they want it?
 - d. Will they have time to work that they might buy it?
 - e. How much does it cost?
 2. Students complete bottom of page 158. (Students should keep this page for later reference).
 - C. Discussion of purchase by savings or by installment.
 1. Teachers could reproduce flowcharts (transparency) TM-2, 3 for class.
 2. Acquire information from Employment office about the following:
 - a. Laws concerning children
 - (1) Ages that can be employed.
 - (2) Hours that can be worked by different ages.
 - b. Minimum wage
 - c. Overtime

III. Resource Materials.

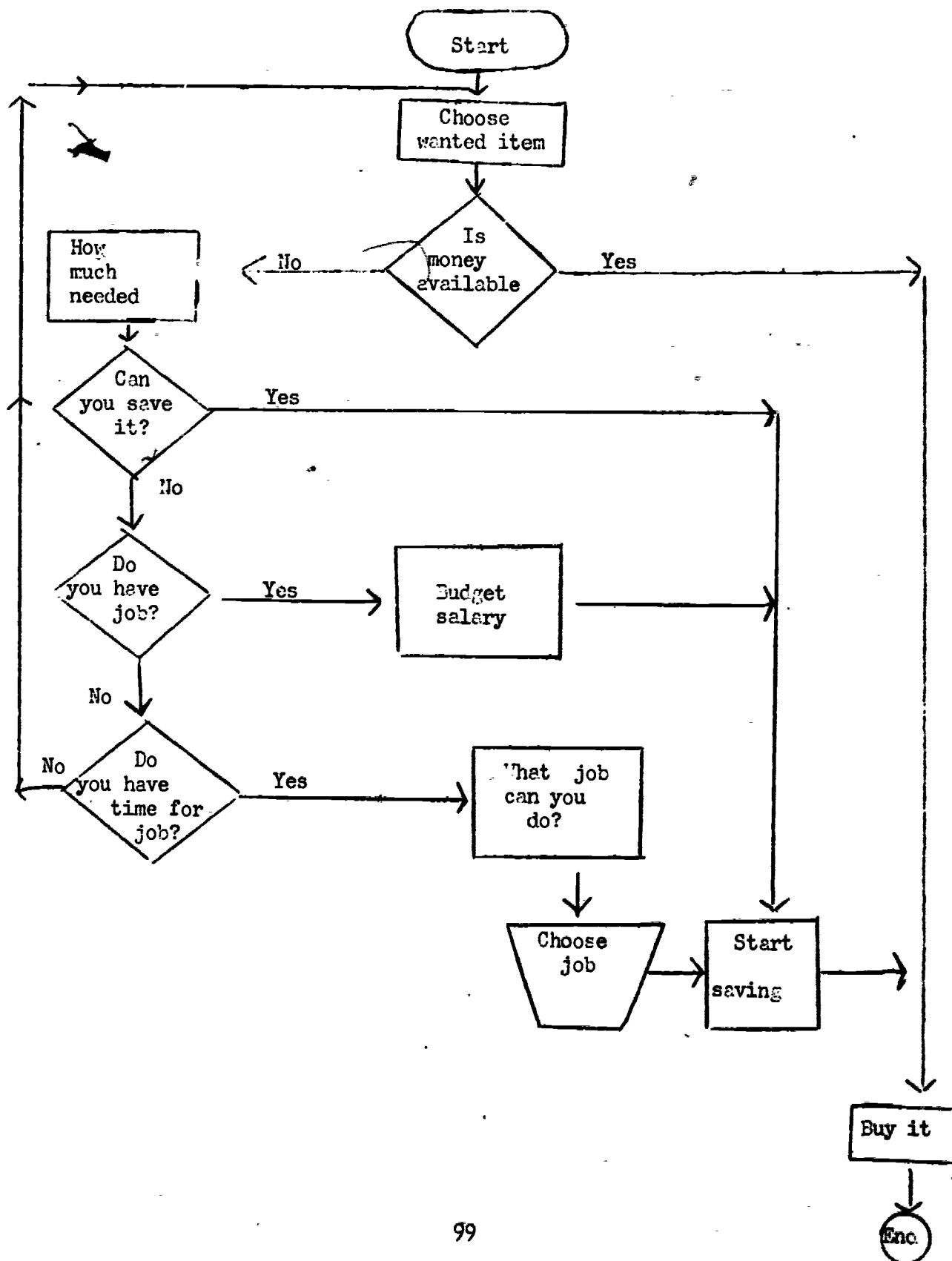
- A. Newspapers - ad section.
 - B. Checks.
 - C. Deposit slips.
 - D. Information from Employment office.
 - E. Income Tax tables
 - F. Social Security Tables.
-

IV. Answers: Page 202 Tool and Die Maker

- (1) \$113.20; \$23.48; \$89.72
- (2) \$113.20; 6; \$25.47; \$138.67 \$28.64; \$110.03
- (3) State 1.4% 1.2%; Fed. 15.1%; 15.2; Soc. Sec. 4.2%
and 4.2%

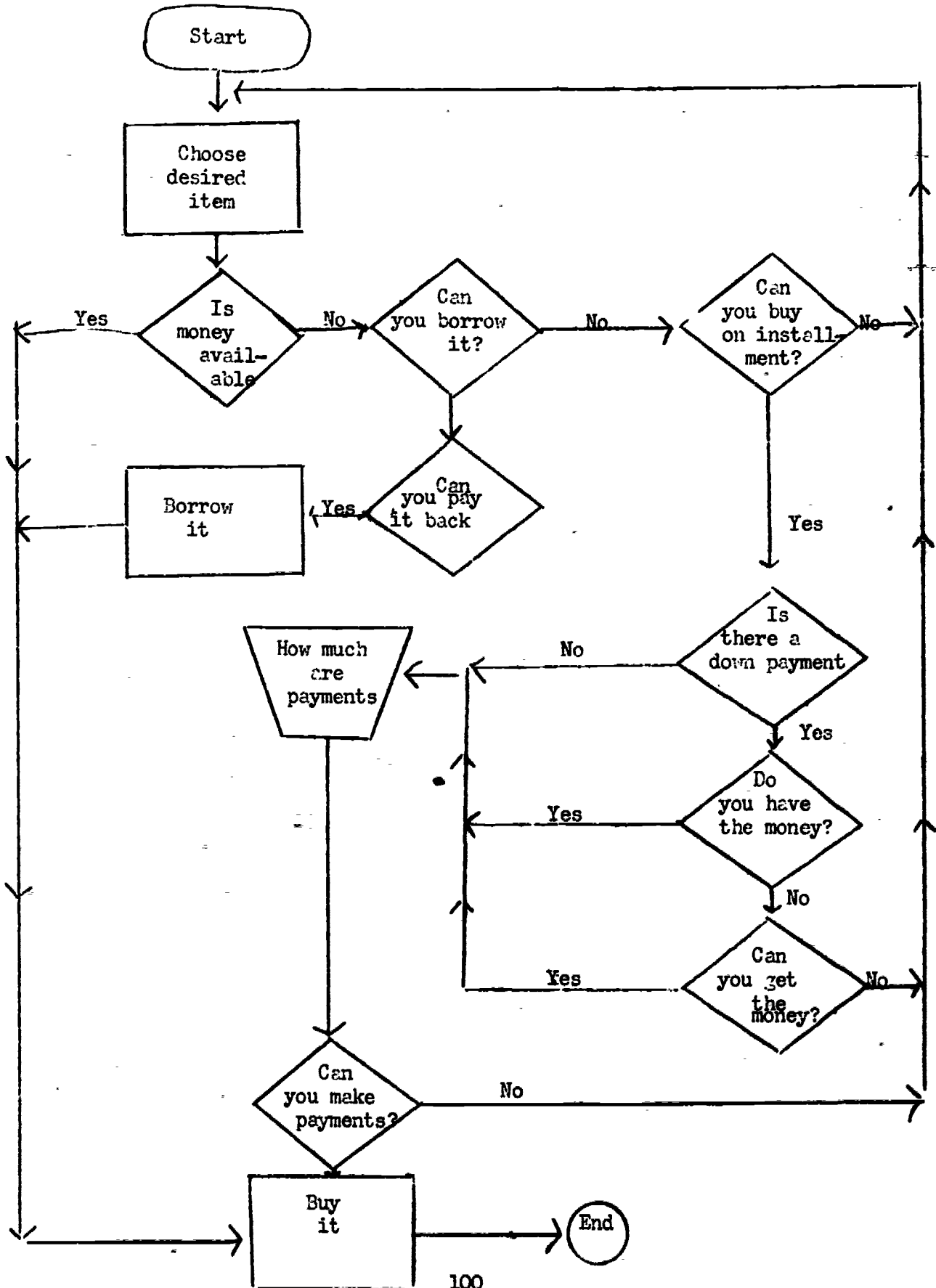
(A)

PURCHASE BY SAVING



(D)

PURCHASE BY INSTALLMENT



ORDERING FROM A CATALOG

I. Objectives: To teach students how to order from catalogs.

II. Suggestions:

- A. This should be a good interest developing situation.
- B. Be sure to have available some type of catalog to use.
- C. After student has completed this lesson then let him make up other orders. This will give some other good work besides math. Stress the importance of neatness.
- D. You might even bring in check writing in that they could write a check to pay for the order. Read the Shipping Information on the back of a Sears order blank so that students will be able to make shipping decisions.

III. Resource Material:

- A. Several catalogs, may be any company which uses a catalog ordering service; possible even school supplies
- B. Duplicates of each catalog order blank

IV. Answers: Page 162

- | | |
|--|------------------------------------|
| (1) \$17.59, 7.98, 8.67, 3.78 | (9) Depends on method of shipping. |
| (2) \$36.02 | (10) Vary |
| (3) \$1.14 | (11) Vary |
| (4) Parcel Post | (12) Will vary |
| (5) 8 pounds 29 ounces | (13) None |
| (6) 10 pounds | (14) Check |
| (7) will vary | |
| (8) Depends on method of shipping. | |
| (15) Collect on delivery. | |
| (16) Cash order is cheaper. There is a higher rate charged by the post office for a C.O.D. order than there is for parcel post because of the responsibility of the post office to collect on the C.O.D. | |

BANKING UNIT

I Objective: To teach the services of a bank and the proper procedures in the uses of these sources

II. Suggestions:

- A. Speaker from bank.
- B. Opening an account.
- C. Making deposits (pages included are for banks using computers for processing).
- D. Writing checks.
- E. Keeping a check record.
- F. Checking running balance on bank statements.
- G. Filling in the running balance on a bank statement.
- H. Visiting a bank.

III. Resource Material:

- A. Deposit slips.
- B. Checks
- C. List of banks in town.
- D. Bulletin board using bank terms.

Answers: Page 206-209

D Check register final balance: \$364 26

How much money? \$2.00 (Hint: work backward)

Page 210

Statement of Account:

\$371.96 \$361.20 \$340.98 \$300.98 \$280.64 \$141.64

\$392.85 \$381.62 \$144.83 \$ 66.49 \$ 62.03 \$ 20.66 \$261.04

Page 211

Balance your bank statement:

Balance on statement--\$261 04; Deposits--0

Outstanding checks--\$166 04; Balance--\$95 00

IV. Answers: Page 212

1. 3 months
2. \$150.00
3. April 1, 1971
4. (a) Years & months — 1 yr. 10 months; 1 yr. 7 months;
1 yr. 4 months; 1 yr. 1 month; 10 months, 7 mo.,
4 months, 1 month
- (b) 19.95, 19.95, 19.51, 19.51, 19.05, 19.05, 18.75,
18.75
5. \$4.52

Savings and Loan, Page 213-214.

- (1) $4\frac{3}{4}\%$ (2) \$0.096956 or 10¢ (3) 52¢ (4) \$1.42
- (5) \$2.35 (6) \$3.29 (7) \$44.27 (8) \$83.19
- (9) \$123.04 (10) \$157.58

*Depending on method of multiplying time (decimal or fraction) the answer may vary. Answers were found by multiplying by months and dividing by 12.

- (11) \$7.58
- (12) Savings account earns more than savings bonds
for 2-year period.
- (13) \$4.12

OPERATING AN AUTOMOBILE

I. Objective: To teach the cost of owning and operating an automobile.

II. Suggestions:

A. Invite car salesman to speak to class.

1. Discuss manufacturer's suggested retail price. Page 164
2. Discuss purchase of new compared to used car.
3. Discuss security agreement.
 - a. Who may sign? (under age)
 - b. How is amount of payment figured?

B. Invite insurance agent to speak to class.

1. Explain different types of coverages (meaning of 10-20-5)
2. What does the state law require?
3. Cost of insurance for boys - girls
 - a. Age limits - under 21
 - b. Benefits from driver education
 - c. Accident rate
4. How does age of car affect insurance rate?

C. Cost of operating a car.

1. Second hand Page 215
2. New Car Page 216

III. Resource Material:

A. Copies of security agreements

B. Copies of suggested retail prices

C. Insurance policies

D. Registration papers (including assessment and dealer's certificates if new, or title and pink slip if new)

E. Cost of automobile license

1. State
2. City

Answers:Page 215

1. Expenses:

Used Car

Gas	321.03
Oil	23.65
Lub.	13.50
Oil Filters	5.70
Air Filters	3.65
Head Lamp	1.76
Muffler	9.88
Tail Pipe	5.25
Plugs	8.24
Points	2.10
Tires	42.90
Battery	18.95
Cables	2.00
Hubcaps	31.80
Mirrors	6.50

Sub total	496.91
License	26.00
Depreciation	450.00
Interest	55.00
Insurance	208.00
Repair	98.47
Flats	8.75
Total cost	\$1343.13

Av. cost/mile \$0.113

Page 218

2. Expenses:

New Car

Gas	\$ 488.16
Oil	27.00
Lub.	13.50
Oil Filter	5.40
Air Filter	4.10
License	96.00
Depreciation	850.00
Interest	145.00
Insurance	233.40
Total	\$1662.50

Av cost/mile \$3.099

Note: You might have students check how accurate these cost are currently.

COST PER UNIT

I. Objective: Recognizing values by comparative analysis and to recognize values in quantity buying.

II. Suggestions:

- A. Discuss quantity buying - cost per unit page 219.
- B. Discuss better buy page 221 and 222.
- C. Field trip to grocery store during inventory (Krogers very interesting)
- D. Assign students to bring cost, weight, and volume of one item.
- E. Talk about cost of feeding a family - each child's family
 1. Have each child estimate cost of food for one week for him.
 2. Have each child estimate cost of feeding entire family for one week.
 3. Have each child figure cost of feeding entire family for one week, page 173.
- F. Assign children to bring a grocery bill to class for exercises 2 and 3, page 174.

III. Resource Material:

- A. Home economist
- B. Complete page ads from newspaper

IV. Answers: Welch's Grape Juice, Page 219

9 items: Meat pies 12 1/2¢ (13¢); corn 22 1/4¢ (23¢); Sausage 22 1/4¢ (23¢); Grape Juice 33 1/3¢ (34¢); Treet 4 1/12¢; Okra 1 2/3¢; Peanut Butter 3 1/4¢; Orange Juice, 3 for 49¢ is 1/3¢ per can less, but one can of either would be 17¢.

IGA, Page 220

Safeway, Page 221-222

Item	Mark-up	%	
10 lb. sugar	.011	1.8%	16-oz. cokes
5 lb. sugar	.206	21%	1 gallon milk
Pet Milk	.014	8.7%	6 oz. coffee (sale item)
Soap Powder	.05	7.5%	16 oz. orange juice
Coffee	.006	.3%	5 lb. canned ham
Biscuits	.012	16.2%	10 lb. sugar
Milk	.06	11.7%	1 lb. 13 oz. peaches
Baby food	.015	11.5%	8 oz. biscuits
Flour	.03	13.5%	2 lb. crackers
Cake Mix	.02	25%	1 lb. weiners
Can Cola	.13	21.6%	1 lb. pork and beans
One-way Pepsi	.10	22.2%	Highway Corn
			Mrs. Wright's Breed
			Hills-of Home Corn
			Wrigley's Gum

TM 219-224
2

TM 219-224
2

The answers for problems 1, 2, and 3 will vary. Page 223-224

RECORD PARTY

Cokes	cost	\$2.58
Spudnuts	Cost	\$2.66
		<u>\$5.24</u>
	tax	.16
		<u>\$5.40</u>

$$\$5.40 \div 18 = 30\text{¢}$$

UNIT X
CONSTRUCTIONS

I. Objective:

- A. To teach construction of an angle congruent to a given angle; to review acute, right and obtuse angles.
- B. Construct the bisector of an angle.

II. Suggestions:

- A.
 - 1. Use only straight-edge and compass.
 - 2. Teach meaning of "congruent"
 - 3. Is the measure of the constructed angle the same as the given angle?
 - 4. Such terms as adjacent, complementary and supplementary angles could be introduced if desired at this point.
 - 5. This could be used as a change-of-pace unit at any time of the year.
- B.
 - 1. Discuss meaning of bisector. Page 226
 - 2. Draw an angle on the board and construct its bisector. Each student should follow the construction on his paper.
 - 3. Refer to figure EASY on student page. What angle is formed by the angle bisectors of angles AYE and AYS? (A right angle)
 - 4. Draw any triangle and have students practice construction of an angle bisector by bisecting each angle of the triangle to find the incenter of the triangle (point of intersection of 3 angle bisectors) with this point as center and a radius equal to the shortest distance to a side draw the circle inscribed in the triangle.

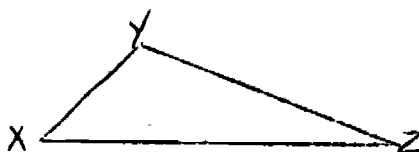
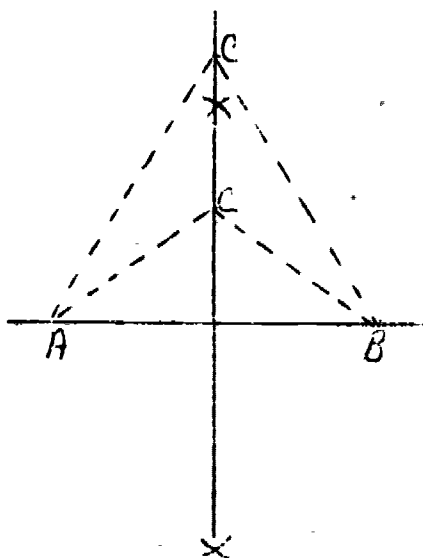
III. Resource Material:

- A. Any good geometry book.
- B. Geometric constructions --Webster

I. Objective: To teach construction of a perpendicular bisector of a line segment.

II. Suggestions:

- A. 1. Put a line segment on board and construct its perpendicular bisector.
2. Have each student construct a perpendicular bisector of a line segment.
3. Connect a point on the bisector to the end points of the line segment. Have students use compass and measure AC and BC. Are they congruent? What specific polygon have we formed? What name is given to this specific kind of triangle? Is $AC \cong BC$? Could we alter this so that $AB \cong BC \cong AC$? What name is given to this triangle?
4. Draw triangle as figure XYZ, have students measure sides. Are any 2 sides congruent? What is the name of this triangle?
5. Have students practice construction of bisector of line segments.



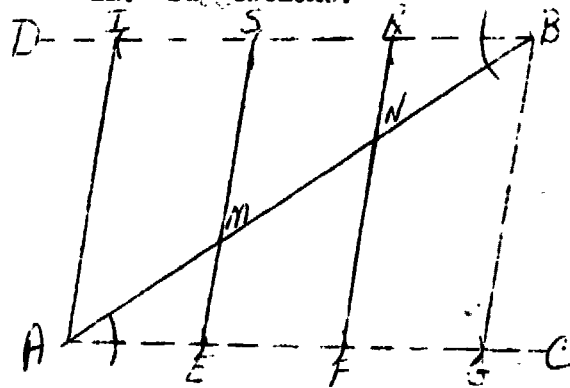
- B. Extend construction to include perpendicular bisectors of sides of a triangle to get circumcenter and draw the circle circumscribing the triangle.

III. Resource Material:

Geometry text.

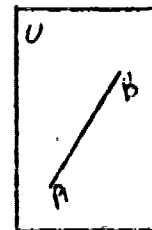
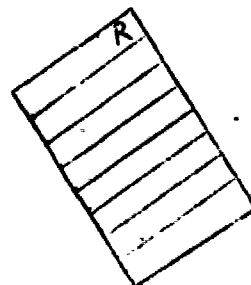
- I. Objective: To teach how to divide any line segment into a given number of segments.

III. Suggestions:



- A. Given segment \overline{AB}
- B. Draw any convenient angle $\angle BAC$.
- C. Construct angle $\angle ABD$ congruent angle $\angle BAC$.
- D. Using any convenient radius, mark off congruent segments on \overline{AC} , beginning at A. Letter these points E, F, G

- E. Using the same radius, mark off congruent segments on \overline{BD} beginning at B. Letter these points R, S, T.
- F. Draw \overline{EG} and \overline{AT}
- G. Draw \overline{RF} and \overline{SE} .
- H. Then \overline{AM} congruent \overline{MN} congruent \overline{NB} .



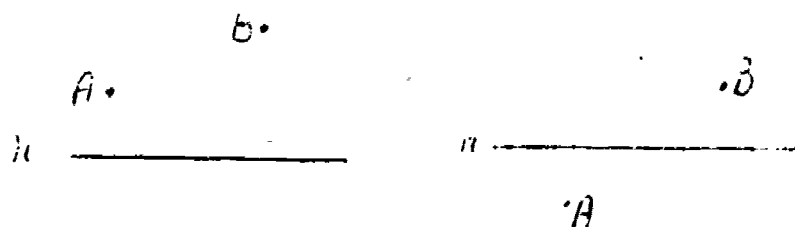
To divide segment \overline{AB} into five congruent parts place page U containing segment \overline{AB} over page R which contains parallel lines. Place A on line 1 and B on line 5. Mark on \overline{AB} the points where the rulings intersect it. The line will be divided into 5 congruent parts.

III. Resource Material:

- I. Objective. To teach the construction of: (1) a \perp to a line from a point outside the line; (2) a \perp to a line from a point on the line.

II. Suggestions.

- A. Put a line segment and a point Outside the line on the blackboard. Go through the steps and construct a line through the point.
- B. Repeat the construction and have all students do the step with you.
- C. Given line h and 2 points, A and B, (see figure). Construct lines through the points perpendicular to h . Point out that 2 lines constructed perpendicular to the same line are parallel.



- D. Put a line segment and a point on the line the blackboard or overhead projector. Go through steps and construct a \perp line through the point.
- E. Repeat construction having each student do the steps on his paper as you do them.
- F. If 2 points appear on same line, will perpendicular through each point be parallel?

I. Objective: Teach simple construction of parallel lines

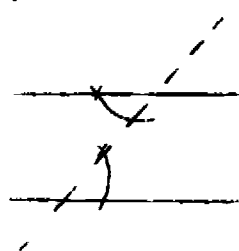
by:

- (1) making a pair of alternate interior angles equal.
- (2) Making a pair of corresponding angles equal.
- (3) making a perpendicular to a perpendicular.

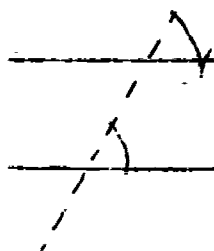
II. Suggestions:

- A. Discuss parallel lines and angles formed by a transversal cutting the parallel lines. Define alternate interior angles. Then by constructing equal corresponding angles and also by constructing a perpendicular to a perpendicular or a certain distance apart.

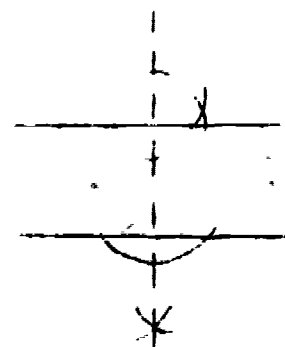
1.



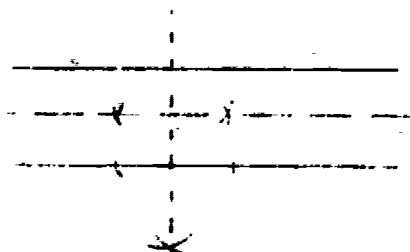
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3.



4.



B. Optional Problems:

1. A fence located a given number of feet on each side of the highway. (Locate fence from center of highway)
2. Construct any equilateral triangle. Then using each side of original triangle as a base construct another equilateral triangle. Cut out, fold along sides of original triangle and name it.

C Then have students do a practice page.

UNIT IX

PROBABILITY CHARTS - GRAPHS

I. Objective: To introduce students to the idea of probability.

II. Suggestions:

- A. Make two circular discs of slightly different sizes. Paint them blue on one side and red on the other. Drop them on table top; what is the chance they will both land red up? (1 to 4)
- B. Drop 3 circular discs red on one side, blue on the other (3 different sizes).
 1. List all the different ways they can fall (8).
 2. List all the different ways you can get 2 red and one blue. (answer 3). Probability of 2 reds and 1 blue is $\frac{3}{8}$. What are the chances of not getting 2 reds and one blue? ($\frac{5}{8}$).
- C. Number of discs determine number of different outcomes. Rule is 2^n , where n = number of discs.
- D. Take 26 cards each containing a different letter. What is the chance of drawing an M at random? ($\frac{1}{26}$) What chance of drawing a letter after T? ($\frac{6}{26}$ or $\frac{3}{13}$). Note: Probability is determined by actual outcomes divided by number of possible outcomes. $P = \frac{A}{N}$. What is the possibility of not drawing a letter after T? $\frac{20}{26}$ or $\frac{10}{13}$. Notice if you are adding $\frac{20}{26} + \frac{6}{26}$ you get 1. \therefore 1 minus the probability of drawing a card after T = probability of not drawing one after T. Hence $1 - P = \text{not } P$. (That is: One minus the probability of the event occurring is equal to the probability of the event not occurring.)
- E. Suppose there are 10 numbered slips of paper in a pile.. What chances are there of drawing a 5 or a 6? Probability of a 5, written $P(5) = \frac{1}{10}$. $P(6) = \frac{1}{10}$. Since it matters not which, but either can be drawn, then $\frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$. When either of two outcomes are desired you add separate probabilities. In one draw, you can't pick both at the same time. Rule would therefore be $P(A \text{ or } B) = P(A) + P(B)$.
- F. In problem 5, had your desire been to have drawn the 5 on the first draw, replace it, and then draw the 6 on the second draw, the probability of both would be $P(5 \text{ and } 6) = P(5) \cdot P(6)$.

or $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$. i.e. one chance in one hundred of drawing the two you want in 2 successive draws.

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

- G This unit might be most successful if used as experimental work.

III. Resource Material:

- A. Wilcox and Yarnelle, Mathematics, A Modern Approach, Addison Wesley, 1963.
- B Haag and Dudley, Introduction to Secondary Mathematics, Vol. 1, Heath, 1964.

Answers: Page 233

1. (a)

R R R R	R B R R	B R R R	B B R R
R R R B	R R R B	B R R B	B B R B
R R B R	R B B R	B R B R	B B B R
R R B B	R B B B	B R B B	B B B B

- (b)

R R R B
R R B R
R B R R
B R R R

- (c) $\frac{4}{16}$ or $\frac{1}{4}$

2. Number of discs

3. 2^n where n is number of discs

4. (a) $\frac{1}{16}$ (b) $\frac{6}{16}$ or $\frac{3}{8}$ (c) $\frac{4}{16}$ or $\frac{1}{4}$ (d) $\frac{1}{16}$

- (e) $\frac{1}{16} : \frac{3}{8} : \frac{1}{4} : \frac{1}{16}$

5. (a) $\frac{14}{40}$ or $\frac{7}{20}$; (b) $\frac{10}{40}$ or $\frac{1}{4}$; (c) $\frac{1}{2}$; (d) $\frac{1}{2}$

- (e) $\frac{3}{40}$ or $\frac{1}{5}$; (f) $\frac{6}{40}$ or $\frac{3}{20}$; (g) $\frac{1}{2}$

- (h) $\frac{4}{40}$ or $\frac{1}{10}$ (i) $\frac{6}{40}$ or $\frac{3}{20}$; (j) $\frac{4}{40}$ or $\frac{1}{10}$

- (k) $\frac{2}{40}$ or $\frac{1}{20}$ (l) $\frac{4}{40}$ or $\frac{1}{10}$ (m) $\frac{3}{40}$ (n) $\frac{12}{40}$ or $\frac{3}{10}$

PROBABILITIES

- I. Objective: To extend probability to two outcomes in succession.
- II. Suggestions:

A. With Replacement.

- 1.. Suppose you toss a coin twice, what is the probability it will show heads both times? We find all possible outcomes. (4) The probability of (HH) is $\frac{1}{4}$. Another way of finding this is $P(H)$ on 1st toss $\frac{1}{2}$; $P(H)$ on 2nd toss $\frac{1}{2}$.

If we form product $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$ which is the probability that the same outcome will occur twice in succession

The two probabilities do not have to be the same. For example, probability of 1st outcome is $\left(\frac{3}{4}\right)$, probability of second outcome $\left(\frac{1}{2}\right)$ thus probability of 2 outcomes will occur in succession is $\left(\frac{3}{4}\right)\left(\frac{1}{2}\right) = \frac{3}{8}$.

2. A jar contains three marbles, one blue, one red, one white. Draw one, observe color, replace it and mix with others. Then draw another. What is the probability that you drew the red marble each time? $\left(\frac{1}{9}\right)$. List

All possible outcomes for the two successive drawings.

First drawing	R	R	R	W	W	W	B	B	B
Second drawing	R	W	B	R	W	B	R	W	B

Note: 9 possible outcomes and only one is RR.

3. Suppose you have 8 black marbles and 2 white marbles in a cup. Draw one and replace it. Draw a second. What is the probability both will be white? $\left(\frac{4}{100}\right)$. First white has the probability of $\frac{2}{10}$. Second white has the probability of $\frac{2}{10}$. Thus $\left(\frac{2}{10}\right)\left(\frac{2}{10}\right) = \frac{4}{100} = \frac{1}{25}$

B. Without Replacement

4. Suppose there are 6 black marbles and 4 white marbles in a jar. What chance have you of drawing a white on the first draw? $\left(\frac{4}{10} = \frac{2}{5}\right)$. Without replacing a white one drawn there are left 6 black and 3 white and for the one

drawn on the second try to be white, the probability is now $3/9$ or $1/3$.

The chances of getting both white is $\frac{2}{7} \cdot \frac{1}{3} = \frac{2}{15}$.

5. In problem 4, had a white been desired on the first draw and a black on the second draw without replacing the first, the chance would be $2/5$ for white, and

$$\frac{6}{9} = \frac{2}{3} \text{ for a black. Thus}$$

$$P(W_1 \text{ and } B_2) = P(W_1) \cdot P(B_2) \text{ or } \frac{2}{5} \cdot \frac{2}{3} = \frac{4}{15}.$$

III. Resource Material:

- A. Wilcox and Yarnelle, Mathematics, A Modern Approach, Addison-Wesley, 1963.
- B. Haas-Dudley, Introduction to Secondary Mathematics, Vol 1, Heath, 1964.

IV. Answers: Page 234

1. (a) $\frac{15}{27} \cdot \frac{15}{27}$ or $\frac{5}{9} \cdot \frac{5}{9}$ or $\frac{25}{81}$ (d) $1 - \frac{40}{81}$ or $\frac{41}{81}$
 (b) $\frac{12}{27} \cdot \frac{12}{27}$ or $\frac{4}{9} \cdot \frac{4}{9}$ or $\frac{16}{81}$ (e) $\frac{15}{27} \cdot \frac{12}{27}$ or $\frac{5}{9} \cdot \frac{4}{9}$ or $\frac{20}{81}$
 (c) $2\left(\frac{15}{27} \cdot \frac{12}{27}\right)$ or $2\left(\frac{5}{9} \cdot \frac{4}{9}\right)$ or $\frac{40}{81}$ (f) $\frac{12}{27} \cdot \frac{15}{27}$ or $\frac{4}{9} \cdot \frac{5}{9}$ or $\frac{20}{81}$
 (g) $\frac{12}{27} + \frac{12}{27} - \frac{16}{81} = \frac{56}{81}$
2. (a) $1/4$ (b) $2/3$ (c) $2/3 \cdot 3/4$ or $6/12$ or $1/2$
 (d) $1/3 \cdot 1/4$ or $1/12$ or $1/4$ (e) $2/3 \cdot 1/4$ or $2/12$ or $1/6$
 (f) $1/3 \cdot 3/4$ or $3/12$ or $1/4$
3. (a) $6/16$ or $3/8$
 (b) $6/16 : 5/15$ or $1/8$ if you keep first apple.
 (c) $6/16 \cdot 10/16$ or $15/64$
 (d) $6/16 \cdot 10/15 = 1/4$

GRAPHS - CHARTS

- I. Objective: To get a better understanding of graphs and charts.
- II. Suggestions:
 - A. Have students bring 5 small cubes
 - B. Color sides
 - (a) 2 sides red
 - (b) 1 side green
 - (c) 1 side orange
 - (d) 2 sides blue
 - C. Count cars in parking lot.
- III. Resource Material:

Filmstrip, cubes

Refer to food buying lesson for charts. Also use of income tables under personal finance.

UNIT XI

Bases (Optional)

BASE 8

- I. Objective: To introduce number systems with bases other than ten for the intended purpose of strengthening student's understanding of place value.. To learn to count in base 8. Specifically base 8 (electronic computers are programmed in base eight—though they function on base two) to be followed by an introduction of base two and base five.

II. Suggestions:

- A. In counting, base 8, one might begin by counting the books in the library. For each book counted, put down a tally mark. For each eighth book draw a diagonal tally through first seven tallies, as shown. For the 9th book begin a new set.

Can you represent with tally marks a count of 16 books? Continue until 8 sets of 8 elements (64 books) have been counted. Draw a square around these 8 sets and put them aside to left until you build up another set of 8 subsets of 8 elements each. Draw square and repeat. Thus 111 would mean one square containing 64 tallies plus one array containing 7 vertical tallies and 1 diagonal tally plus 1 single tally left over. i.e. 1 1 1 or 73 books were counted..

$$8^2 + 8 + 1$$

Have students count from 1 to 50 in base eight recording the names: Thus:

1 = 1	12 = 14
2 = 2	13 = 15
3 = 3	14 = 16
4 = 4	15 = 17
5 = 5	16 = 20 (read two, zero)
6 = 6	17 = 21 (read two, one)
7 = 7	50 = 62 (read six, two)
8 = 10 (read "one, zero" — never "ten")	
9 = 11 (read "one, one" — never "eleven")	
11 = 13	

Note: Never use the numeral 8 for eight in base eight.

- B. For notation 62 means 62 in base eight and should be read "six, two, base (8)," and not "sixty-two".
- C. Have students locate as many sets of eight elements each as possible in floor tiles, squares on a sheet of quadrille or graph paper, ceiling board, flooring boards.
- D. Then ask for students to find sets of 64 elements each, or a set of 8 subsets of 8 elements each = 8×8 .
- E. Next ask them to find a set containing 512 elements or $8 \times 8 \times 8$ if possible.
- F. Direct their attention to how many subsets of eight elements in a set of 64 elements, in a set of 512 elements.
- G. How many subsets consisting of 64 elements in a set of 512 elements?
- H. As we can count by ones, tens, hundreds, and thousands, show the student with a little practice that it is just as easy to count by ones, eights, sixty-fours, and five hundred twelves. Thus 475 would be in base 4
- $$(8 \times 8) + 7(8 \times 1) + 5(1) = 4(64) + 7(8) + 5 =$$
- $$256 + 56 + 5 = 317.$$
- Have student make several conversions from base 10 to base eight and vice versa.

III. Resource Material:

Wilcox-Yarnelle. "Mathematics, A Modern Approach".

IV. Answers: Page 240

- | | |
|----------------------------|---------------------------|
| 2. (a) 45_{eight} | 3. (a) 124_{ten} |
| (b) 117_{eight} | (b) 61_{ten} |
| (c) 217_{eight} | (c) 35_{ten} |
| (d) 721_{eight} | (d) 131_{ten} |
| (e) 2430_{eight} | (e) 511_{ten} |

BASE EIGHT - ADDITION AND SUBTRACTION

I. Objective: To enhance the student's understanding of place value by introducing addition, subtraction in base eight system.

II. Suggestions:

- A. Begin by reviewing notations used such as 325^{eight}. This is read "three, two, five, base eight" and not "three hundred twenty-five, base eight". 325^{eight} means $3(8 \times 8) + 2(8 \times 1) + 5$. This names the same quantity as 213 in the decimal (base ten) system.
- B (1) Addition: Make an addition table for base eight with help of class.

	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

(2) By use of table ask simple problems such as

$$\begin{array}{r} 6 \\ +7 \\ \hline \end{array}$$

eight

$$\begin{array}{r} 4 \\ +2 \\ \hline \end{array}$$

eight

$$\begin{array}{r} 7 \\ +4 \\ \hline \end{array}$$

eight

The answer to the first example is one, five, base eight which means one group of eight plus five 1's
Next two are: 6, 13.

- (3) Then try adding larger numbers such as
$$\begin{array}{r} 276_{\text{eight}} \\ +754_{\text{eight}} \\ \hline \end{array}$$

First step would be
$$\begin{array}{r} (1) \quad 276_{\text{eight}} \\ +754_{\text{eight}} \\ \hline 2_{\text{eight}} \end{array}$$

Second step would be:
$$\begin{array}{r} (1) (1) \quad 276_{\text{eight}} \\ +754_{\text{eight}} \\ \hline 52_{\text{eight}} \end{array}$$

Third step would be:
$$\begin{array}{r} (1) (1) \quad 276_{\text{eight}} \\ +754_{\text{eight}} \\ \hline 1252_{\text{eight}} \end{array}$$

Final answer is read one, two, five, two, base eight.

Have a student rename addends in base ten and add again as a check.

- (4) As a third example such as
$$\begin{array}{r} 503_{\text{eight}} \\ +677_{\text{eight}} \\ \hline \end{array}$$
 have class tell

You what to write. When sum is complete, check by adding in base ten. Then assign extra problems.

- (5) By looking at table, can you find an identity element for addition? (0)
- (6) Is addition commutative? (yes)

C. Subtraction:

Example 1:
$$\begin{array}{r} 21_{\text{eight}} \\ -15_{\text{eight}} \\ \hline \end{array}$$

Before subtracting 5 from 1, borrow 1 eight from the 2 eights and rename it eight ones, then add to the 1 in the problem. Think 9 ones - 5 ones + 4 ones.

Thus
$$\begin{array}{r} 21_{\text{eight}} \\ -15_{\text{eight}} \\ \hline 4_{\text{eight}} \end{array}$$

Then rename the subtrahend and minuend in the decimals and subtract for a check on understanding.

Example 2:
$$\begin{array}{r} 415_{\text{eight}} \\ -237_{\text{eight}} \\ \hline 156 \end{array}$$

Check:
$$\begin{array}{r} 269_{\text{eight}} \quad 156_{\text{eight}} \\ -159_{\text{eight}} \\ \hline 110_{\text{ten}} \end{array}$$

TM 241
3

TM 241
3

III. Resource Material:

Wilcox and Yarnelle, Mathematics, A Modern Approach,
Addison-Wesley, 1963

IV. Answers, Page 241

1. Add: $\begin{array}{r} 116 \\ 531 \\ \hline \end{array}$ _{eight}

check

$$\begin{array}{r} 29 + 49 = 78 \\ 86 + 259 = 345 \end{array}$$
 _{ten}

$$\begin{array}{r} 1400 \\ \hline \end{array}$$
 _{eight}

$$468 + 300 = 768$$
 _{ten}

$$\begin{array}{r} 707 \\ \hline \end{array}$$
 _{eight}

$$312 + 143 = 455$$
 _{ten}

$$\begin{array}{r} 5051 \\ \hline \end{array}$$
 _{eight}

$$1667 + 934 = 2601$$
 _{ten}

2. Subtract.

$$\begin{array}{r} 42 \\ \hline \end{array}$$
 _{eight}

$$\begin{array}{r} 24 \\ \hline \end{array}$$
 _{eight}

$$\begin{array}{r} 117 \\ \hline \end{array}$$
 _{eight}

$$\begin{array}{r} 342 \\ \hline \end{array}$$
 _{eight}

$$\begin{array}{r} 405 \\ \hline \end{array}$$
 _{eight}

$$\begin{array}{r} 62 \\ 28 \\ \hline 34 \end{array}$$
 _{ten}

$$\begin{array}{r} 43 \\ 23 \\ \hline 20 \end{array}$$
 _{ten}

$$\begin{array}{r} 132 \\ 53 \\ \hline 79 \end{array}$$
 _{ten}

$$\begin{array}{r} 400 \\ 174 \\ \hline 226 \end{array}$$
 _{ten}

$$\begin{array}{r} 458 \\ 197 \\ \hline 261 \end{array}$$
 _{ten}

OPTIONAL:

(1) 0

(6) 25

(11) 2

(2) 51

(7) 6

(12) 1

(3) 5

(8) 25

(13) 2

(4) 34

(9) 9

(14) 68

(5) 2

(10) 3

(15) 0

OPTIONAL BASE EIGHT (OPERATIONS)

I. Objective: To continue work with base eight by introducing student to multiplication for purpose of further strengthening the concept of place value.

II. Suggestions:

A. Begin by having class help you construct a multiplication table for base eight.

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61

F. Start with
$$\begin{array}{r} 34_{\text{eight}} \\ \times 27_{\text{eight}} \\ \hline \end{array}$$

eight

First step would be
$$\begin{array}{r} (3) \\ 34_{\text{eight}} \\ \times 27_{\text{eight}} \\ \hline 4_{\text{eight}} \end{array}$$

Second step
$$\begin{array}{r} (3) \\ 34_{\text{eight}} \\ \times 27_{\text{eight}} \\ \hline 304_{\text{eight}} \end{array}$$

third step
$$\begin{array}{r} 34_{\text{eight}} \\ \times 27_{\text{eight}} \\ \hline 304_{\text{eight}} \\ 70_{\text{eight}} \\ \hline \end{array}$$

Fourth step
$$\begin{array}{r} 34_{\text{eight}} \\ \times 27_{\text{eight}} \\ \hline 304_{\text{eight}} \\ 70_{\text{eight}} \\ \hline 1204_{\text{eight}} \end{array}$$

TM 242
2

TM 242
2

Check:

$$34_{\text{eight}} = 28_{\text{ten}}$$

$$27_{\text{eight}} = 23_{\text{ten}}$$

$$\begin{array}{r} 28_{\text{ten}} \\ \times 23_{\text{ten}} \\ \hline 84 \\ \hline 644_{\text{ten}} \end{array}$$

Example 2: $\begin{matrix} (1) & (1) \\ (1) & (2) \\ (3) & (4) \end{matrix}$ Numbers carried

$$\begin{array}{r} 346_{\text{eight}} \\ \times 236_{\text{eight}} \\ \hline 2544 \\ 1262 \\ 714 \\ \hline 106764_{\text{eight}} \end{array}$$

$$\begin{array}{r} 230_{\text{ten}} \\ 158_{\text{ten}} \\ \hline 1840 \\ 1150 \\ 230 \\ \hline 36340_{\text{ten}} \end{array}$$

$$106764_{\text{eight}} = 1 \cdot 8^5 + 0 \cdot 8^4 + 6 \cdot 8^3 + 7 \cdot 8^2 + 6 \cdot 8^1 + 4 = 32768 + 0 + 3072 + 448 + 48 + 4 = 36340_{\text{ten}}$$

- C. By inspection of table, can you find an identity element for multiplication in base eight?
- D. Is multiplication in base eight commutative?
- E. Division may be undertaken if the teacher so desires and time permits.

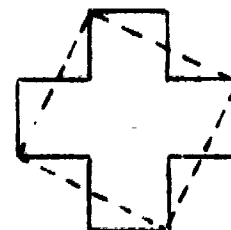
III. Resource Material:

Wilcox - Yarnelle

IV. Answers 242

- 1. 52_{eight} 2. 614_{eight} 3. 3040_{eight} 4. 3705_{eight}
- 5. 14772_{eight} 6. 61163_{eight} 7. 46737_{eight} 8. 5116_{eight}

Solution "Squaring the Cross"



OPTIONAL: BASE TWO

I. Objective: To introduce the binary (base two) number system to further strengthen the notion of place value.

II. Suggestions:

A. Explain that the use of the binary number system is almost Unlimited as far as electronics are concerned since an ordinary switch has only two positions that can be matched with the two symbols 0 and 1. The binary system can be used to represent any type of "Yes-No" response. In any base number system the number of symbols used is the same as the base. Thus base two has only two symbols 0 and 1, hence the name binary since bi means "two".

B Counting: $1 = 1_{\text{two}}$
 $2 = 10_{\text{two}}$ (read one, zero)
 $3 = 11_{\text{two}}$
 $4 = 100_{\text{two}}$
 $5 = 101_{\text{two}}$
 $6 = 110_{\text{two}}$
 $7 = 111_{\text{two}}$
 $8 = 1000_{\text{two}}$
 $9 = 1001_{\text{two}}$
 $10 = 1010_{\text{two}}$

The positional value that a zero or one has is determined in much the same way as explained in base eight previously. i.e. if we have 111_{two} , this means we have $1 \cdot 2^2 + 1 \cdot 2^1 + 1$ which in decimal notation would be $4 + 2 + 1 = 7$. Instead of grouping by eights, we now group by two's.

C. In base-two notation the place value of any digit is two times greater than the value of the digit to the right.

D. Have the student count beyond 10, using binary notation, and then have him write in expanded notation the meaning of several binary numbers such as $11011 = 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 = 1 \cdot 16 + 1 \cdot 8 + 0 \cdot 4 + 1 \cdot 2 + 1 = 27$ in decimal notation. Read the previous example as one 16 plus one eight plus no four plus one two plus 1 gives 27_{ten} .

- E A short cut for obtaining binary notation could be illustrated by the following short division process.

$$\begin{array}{r} 2 \overline{) 37} \\ 2 \overline{) 18} \\ 2 \overline{) 9} \\ 2 \overline{) 4} \\ 2 \overline{) 2} \\ 2 \overline{) 1} \\ 2 \overline{) 0} \end{array}$$

Remainder

1
0
1
0
0
1

Now read the remainders from bottom up and they will be the binary notation for 37.

Thus $37_{\text{ten}} = 100101_{\text{two}}$

Let students try $75_{\text{ten}} = \boxed{}_{\text{two}}$

- F. ADDITION: Can be performed in much the usual way with any other base.

e.g.

$$\begin{array}{r} 101_{\text{two}} \\ + 11_{\text{two}} \\ \hline 1000_{\text{two}} \end{array} \begin{array}{l} \longrightarrow 4 + 0 + 1 \\ \longrightarrow 2 + 1 \\ \longrightarrow 8 + 0 + 0 + 0 \end{array} \begin{array}{l} \longrightarrow 5_{\text{ten}} \\ \longrightarrow 3_{\text{ten}} \\ \longrightarrow 8_{\text{ten}} \end{array}$$

$$\begin{array}{r} 11001_{\text{two}} \\ + 1011_{\text{two}} \\ \hline 100100_{\text{two}} \end{array} \begin{array}{l} \longrightarrow 16 + 0 + 0 + 0 + 1 \\ \longrightarrow 8 + 0 + 2 + 1 \\ \longrightarrow 32 + 0 + 0 + 4 + 0 + 0 \end{array} \begin{array}{l} \longrightarrow 25_{\text{ten}} \\ \longrightarrow 11_{\text{ten}} \\ \longrightarrow 36_{\text{ten}} \end{array}$$

- G. SUBTRACTION:

$$\begin{array}{r} 1110_{\text{two}} \\ - 101_{\text{two}} \\ \hline 1001_{\text{two}} \end{array} \begin{array}{l} \longrightarrow 8 + 4 + 2 + 0 \\ \longrightarrow 4 + 0 + 1 \\ \longrightarrow 8 + 0 + 0 + 1 \end{array} \begin{array}{l} \longrightarrow 14_{\text{ten}} \\ \longrightarrow 5_{\text{ten}} \\ \longrightarrow 9_{\text{ten}} \end{array}$$

"Borrowing" may be necessary in subtraction. In the above example borrow the 1 two and think two 1's and subtract one 1 leaving one 1. Then no two's minus no two's leaves no two's.

- H. MULTIPLICATION: Multiplication is even easier than addition and no additional work is needed but a sharp eye must be kept on position.

Check

Thus:

$$\begin{array}{r} 1101_{\text{two}} \\ 101_{\text{two}} \\ \hline 1101 \\ 0000 \\ 1101 \\ \hline 100001 \end{array} \begin{array}{l} \longrightarrow 8 + 4 + 0 + 1 \\ \longrightarrow 4 + 0 + 1 \\ \longrightarrow 64 + 0 + 0 + 0 + 0 + 1 \end{array} \begin{array}{l} \longrightarrow 13_{\text{ten}} \\ \longrightarrow 5_{\text{ten}} \\ \longrightarrow 65_{\text{ten}} \end{array}$$

III. Resource Material:

- A. Wilcox - Yarnelle Mathematics - A Modern Approach
- B. Brown-Simon-Snader General Math Book I.
- C. SVE filmstrip if available.

IV. Answers: Page 243

1. 1010_{two} , 1011_{two} , 1100_{two} , 1101_{two} , 1110_{two} ,
 1111_{two} , 10000_{two} , 10001_{two} , 10010_{two} , 10011_{two} ,
 10100_{two} , 10101_{two} , 10110_{two} , 10111_{two} , 11000_{two} ,
 11001_{two} , 11010_{two} , 11011_{two} , 11100_{two} , 11101_{two} ,
 11110_{two} .
2. (a) 13 (e) 62 (i) 59
(b) 11 (f) 59 (j) 61
(c) 35 (g) 25
(d) 24 (h) 23
3. (a) 1010011_{two} (b) 1101001_{two} (c) 101001_{two}
(d) 1101010_{two} (e) 111011_{two}
4. (a) 101000_{two} (b) 1001000_{two} (c) 111100_{two}
(d) 101011_{two} (e) 100100_{two}
5. (a) 1110_{two} (b) 110110_{two} (c) 1010_{two}
(d) 1101_{two} (e) 10010_{two}
6. (a) 10101_{two} (b) 101101_{two} (c) 1010100_{two}
(d) 1001011101_{two} (e) 101111001_{two}

OPTIONAL: BASE FIVE

I. Objective: To introduce base five as a third numeration system in which place value can be fortified.

II. Suggestions:

A. Man used base ten to count because he had ten fingers on which to record his counting. But suppose by accident he lost a hand. Now he would have to learn to count using only one hand. Hence a need for base five (Quincy System)

B. In base five, begin by counting a collection of marks or tallies:

1111 1111 1111 1111 1111 1111 111

Here are shown six sets of five tallies each plus 3 tallies. If a loop is drawn around five of those sets of five elements each, there is now one set consisting of five subsets of five elements each, plus 1 set of five elements plus 3 elements, i.e. one group of twenty-five tallies, 1 group of five tallies, plus 3 tallies. This is $1 \cdot 5^2 + 1 \cdot 5 + 3$, Thus $113_{\text{five}} = 33_{\text{ten}}$.

C. Have students count from one to 100_{ten} in base five.

	base ten	=	base five	
Thus	1	=	1	5 = 10 (one, zero, base five)
	2	=	2	6 = 11 (one, one, base five)
	3	=	3	7 = 12
	4	=	4	

D. Make an addition chart

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

Adding in base five is very much like other bases already introduced.

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2

$$\begin{array}{r} \text{e.g. } 23_{\text{five}} \\ + 42_{\text{five}} \\ \hline 0_{\text{five}} \end{array}$$

second step

$$\begin{array}{r} 23_{\text{five}} \\ + 42_{\text{five}} \\ \hline 120_{\text{five}} \end{array} \quad \begin{array}{l} 10 + 3 = 13_{\text{ten}} \\ 20 + 2 = 22_{\text{ten}} \\ 25 + 10 = 35_{\text{ten}} \end{array}$$

E. Subtraction:

Subtraction is carried out in a manner similar to other base subtraction. In an example such as

$$\begin{array}{r} 31_{\text{five}} \\ - 14_{\text{five}} \\ \hline \end{array}$$

five taken from 1, so borrow 1 five from the 3 fives and rename the borrowed five ones and add to the given one. Think 6 ones when subtracting 4 ones. Thus the answer to example would be 12_{five} .

$$\begin{array}{r} 31_{\text{five}} \\ - 14_{\text{five}} \\ \hline 12_{\text{five}} \end{array} \quad \begin{array}{l} 15 + 1 = 16_{\text{ten}} \\ 5 + 4 = 9_{\text{ten}} \\ 5 + 2 = 7_{\text{ten}} \end{array}$$

Is addition commutative? Can you find an identity?

F. Multiplication:

X	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

e.g. (1)

$$\begin{array}{r} 24_{\text{five}} \\ \times 32_{\text{five}} \\ \hline 3_{\text{five}} \end{array}$$

second step

$$\begin{array}{r} 24_{\text{five}} \\ \times 32_{\text{five}} \\ \hline 103_{\text{five}} \end{array}$$

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3

third step

$$\begin{array}{rcl}
 24_{\text{five}} & \longrightarrow & 10 + 4 \longrightarrow 14_{\text{ten}} \\
 \times 32_{\text{five}} & \longrightarrow & 15 + 2 \longrightarrow 17_{\text{ten}} \\
 \hline
 103_{\text{five}} & & 98_{\text{ten}} \\
 \\
 132 & & 14 \\
 \hline
 1423_{\text{five}} & \longrightarrow & 125 + 100 + 10 + 3 \longrightarrow 238_{\text{ten}}
 \end{array}$$

Is multiplication commutative? Can you find from the table an identity element?

III. Resource Material:

A Brown, Simon, Snader, General Mathematics, Book 1

IV. Answers Page 244

(1)	200	220	240	310	330	400
	201	221	241	311	331	
	202	222	242	312	332	
	203	223	243	313	333	
	204	224	244	314	334	
	210	230	300	320	340	
	211	231	301	321	341	
	212	232	302	322	342	
	213	233	303	323	343	
	214	234	304	324	344	

(2) 16_{ten} , 14_{ten} , 110_{ten} , 7_{ten} , 28_{ten} , 293_{ten}

(3) 243_{five} , 444_{five} , 2341_{five} , 142_{five} , 10000_{five} ,
 1120_{five}

(4) Add:

104_{five} , 110_{five} , 1100_{five} , 1144_{five} , 1014_{five}

(5) Subtract:

4_{five} , 21_{five} , 42_{five} , 213_{five} , 2243_{five}

(6) Multiply:

2110_{five} , 1103_{five} , 14344_{five} , 33141_{five} , 143143_{five}

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1

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1

UNIT XIII

TM

Since there are times you may need material to give to a substitute, we are including this general review unit on miscellaneous assignments for this purpose.

It may be used as a review lesson or a test.

Answers: Page 245

Whole numbers and fractions

1. (a) $2, 2, \frac{3}{4}$ (b) $2, 3, 3, \frac{1}{2}$ (c) $2, 3, \frac{4}{7}$ (d) $3, 3, 3, \frac{2}{5}$
(e) $2, 2, 2, 2, 2, \frac{4}{7}$ (f) $5, 7, \frac{2}{7}$
2. (a) $\frac{9}{2}$ (b) $\frac{2}{5}$ (c) $\frac{5}{8}$ (d) $\frac{77}{10}$
3. (a) $28 \frac{11}{12}$ (b) $3 \frac{2}{5}$ (c) $72 \frac{11}{72}$ (d) $15 \frac{11}{12}$ (e) $49 \frac{1}{4}$
(f) $2 \frac{1}{3}$
4. $470 \frac{7}{12}$ yards
5. (a) $11 \frac{5}{8}$ (b) $6 \frac{7}{10}$ (c) $4 \frac{37}{40}$
6. (a) $\frac{1}{3}$ (b) \$2.50
7. \$.41625 or 42¢
8. (a) 4 (b) $\frac{1}{6}$ (c) $\frac{69}{12}$ or $5 \frac{3}{4}$ (d) $\frac{605}{4}$ or $151 \frac{1}{4}$
(e) $\frac{189}{8} = 23 \frac{5}{8}$ (f) $\frac{1}{4}$
9. 33 miles per hour
10. (a) 2 (b) $\frac{240}{65}$ or $3 \frac{9}{13}$ (c) $\frac{16}{25}$
11. (a) $\frac{1}{12}$ (b) $\frac{2}{5}$ (c) $\frac{6}{1}$ (d) $\frac{4}{5}$ (e) $\frac{3}{10}$
12. $3 \frac{1}{30}$

Decimals — Page 247-248

- 1 (a) .000156 (b) 0019 (c) 45.65
- 2 (a) $g = 1/10$ (b) $i = .75$ (c) $e = .875$ (d) $b = 5/8$
 (e) $h = 1/8$ (f) $e = .333$ (g) $d = 1/6$ (h) $c = .4$
 (i) $f = 1/2$
- 3 (a) $\frac{34}{100} = \frac{17}{50}$ (b) $\frac{235}{1000} = \frac{57}{200}$ (c) $\frac{5}{1000} = \frac{1}{200}$
 (d) $\frac{375}{1000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$ (e) $\frac{13}{10,000}$
- 4 (a) .444 (b) .292 (c) .553 (d) .133
 (e) .094
- 5 (a) 35.64 (b) 58.1 (c) .007
- 6 (a) 1288.80 (b) 91.243 (c) 268 69 (d) .2484
 (e) 5.812
- 7 (a) 21,287.70 (b) 3246.88 (c) 69 282 (d) 32.7836
- 8 (a) .35 (b) 72 (c) .30 (d) .88
9. 171.750

NUMBER SEQUENCE

I. Objectives:

- A. To look for patterns in number sequences.
- B. To extend sequence of numbers to progression of numbers.

Answers: Page 249

- | | |
|---------------|-------------------------------------|
| 1. (a) 11, 13 | (f) 28, 36 |
| (b) 12, 14 | (g) $\frac{1}{7}, \frac{1}{8}$ |
| (c) 105, 110 | (h) $\frac{15}{16}, \frac{16}{17}$ |
| (d) 19, 18 | (i) $\frac{13}{12}, \frac{10}{14}$ |
| (e) 63, 60 | (j) $\frac{96}{19}, \frac{192}{22}$ |
| 2. (a) 36 | (e) 405 |
| (b) 42 | (f) 56 |
| (c) 405 | (g) a, b, d, e, |
| (d) 129 | |

Page 250 Geometric Progression

- | | |
|--|---|
| 1. (a) 128, 256 | 3. (a) $\frac{16}{3}, \frac{32}{3}$ |
| (b) 243, 729 | (b) $\frac{16}{162}, \frac{32}{486}$ |
| 2. (a) 3, 6, 96, 192 | (c) $\frac{32}{243}, \frac{64}{729}$ |
| (b) $\frac{1}{64}, \frac{1}{16}, 16, 64$ | (d) $\frac{243}{256}, \frac{729}{1024}$ |
| (c) 125, 25, $\frac{1}{25}, \frac{1}{125}$ | |
| (d) 12, 6, $\frac{3}{8}, \frac{3}{16}$ | |

PERCENTAGE ANSWERS

(Page 251-253)

1. (a) 3% (b) 20% (c) 25% or $1/4\%$ (d) 8.25% or $8\frac{1}{4}\%$
 (e) $.4\%$ or $2/5\%$ (f) 7.6% or $7\frac{3}{5}\%$ (g) 12.5% or $12\frac{1}{2}\%$
 (h) 24.68% (i) 5% or $1/2\%$
2. (a) $75/1000$ or $3/40$ (e) $25/10,000$ or $1/400$
 (b) $125/100$ or $1\frac{1}{4}$ (f) $309/100$
 (c) $5/100$ or $1/20$ (g) $20/100$ or $1/5$
 (d) $81/1000$ (h) $52/700$ or $26/350$ or $13/175$
3. 25%
4. John 80%
5. (a) 10.64 (f) \$1120
 (b) 13.50 (g) .747
 (c) 13.00 (h) 6.176
 (d) 1000 (i) 2
 (e) 1580. (j) 9/11
6. (a) 75%
 (b) $66\frac{2}{3}\%$
 (c) 55%
 (d) $18\frac{3}{10}\%$
 (e) $7\frac{3}{7}\%$
7. (a) 72
 (b) 460
 (c) 120
 (d) 90
 (e) 1000
8. (a) 50% (f) \$855
 (b) 30 lbs (g) 20%
 (c) \$392.04 (h) \$603.75
 (d) \$165.71 (i) \$22.00
 (e) 120 (j) 107.7%
9. (a) 5 (f) 2
 (b) 4 (g) 1
 (c) 2 (h) 3
 (d) 1 (i) 4
 (e) 5 (j) 1

MI 354-256

MI 354-256

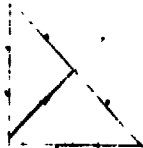
I. Objective: To furnish extra work for students who finish work early.

II. Suggestions:

1. Cut these and any other challenging problems into small strips. Fold them and put them in the fish bowl. Have students pick one at a time. They must solve it before taking another.
2. Extra credit could be given for each one solved.

III. Answers:

2.



4. $\begin{matrix} 111 & 111 & 111 \\ 222 & 222 & 222 \\ 333 & 444 & 444 \\ \text{etc.} \end{matrix}$

5. Pattern sum is square of middle digit.

6. (a) Answers are same three digits in different order.
(b) Sum is same as middle multiplier.

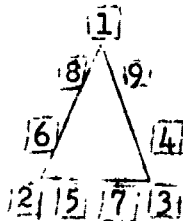
7. $10 + 5 - 12 = 3$

8. $50 - 2 = 48 - 13 = 35$

9. $11 + 9 = 20 \div 5 = 4 + 6 = 10$

11.

There are other solutions.



TM (App. #1)
1

TM (App. #1)
1

APPENDIX I

Enrichment Activities

Objective: To provide special activities which are not included as part of the regular outlined materials,

Plan: The activity is outlined so that it can be followed step by step

FAMOUS MEN OF MATH

Purpose: To acquaint the students with people who have made important contributions to mathematics.

Plan:

1. Show several filmstrips on biographies of mathematicians. After these are shown in the classroom they should be placed in the library for individual viewing.
2. Have students listen to tape-recordings placed in the library on lives of mathematicians. The tapes could be made by advanced students who report on this same topic.
3. Students are now to select one person and read additional material concerning his life. An oral report of less than five minutes should be given to the class.
4. The report should include the following:
 - (a) Time in which the mathematician lived or the number of years ago. (Give rounded off - not date of birth and death)
 - (b) Events from his childhood
 - (c) An interesting or amusing event in his life.
 - (d) A major contribution to mathematics. (This does not have to be familiar or understood by the student.)
 - (e) Any other information about his life.
5. After a student has given the oral report, a teacher demonstration or students working in small groups should explore an idea presented in the report.

Example 1: A report on Archimedes would most likely include buoyancy. Show by a demonstration the principle of buoyancy.

Example 2: A report on Galileo would most likely include the pendulum. Students working in pairs could make a pendulum that has a period of 2 seconds - then a period of 1 second.

STUDENT VISITATIONS

Purpose: To acquaint students with local businesses and the mathematics used in their operation.

To give the student experience with appointments, punctuality, and appropriate dress.

To give experience in acquiring information by interview and organizing this information for reporting to the class.

To involve students in acquiring mathematics problems.

Plan:

1. Group or allow students to group into units of 2 to 5 students each.
2. Each group selects a business to visit. Selection is to be approved by teacher to avoid duplications.
3. Each group must have an appointment with a person connected with the business - may be after school, on Saturday, or during study halls.
4. Visitation made at appointed time.
5. Group reports to class allowing time for class questions and discussions.
6. The following two pages are suggested forms which might be used in connection with these visitations

TM (App. #1)
4

TM (App. #1)
4

STUDENT VISITATIONS

What establishment do you wish to visit?

Where is it located?

What arrangements have been made for this visit?

Date _____ Time _____

With whom is your appointment? _____

Transportation? _____

Committee members:

Teacher approval _____

Principal _____

Host's Signature _____

TM (App. 1)
5

TM (App. 1)
5

BUSINESS VISITATION WORKSHEET

Student's Name _____

A. General Information

1. Name of Business _____
2. Type of Business _____
3. Owner or Manager _____
4. Name of person in the business who was most helpful
to you _____

B. Questions to consider

1. Is there any math associated with the business? _____
Explain _____

2. Number of calculators _____
3. Are the personnel interested in and seem to enjoy their
work? _____
Why do you think so? _____

4. Would you like this type of work? _____
Why or why not? _____

5. What was the most interesting part of the business?

What was least interesting to you? _____

C: Notes to use in talking about this business to the class:

D: Attach one recent math problem that occurred in this
business:

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TM (App. #1)
6

USE OF MATH BUILDER OR CONTROLLED READER

Purpose: To increase span of attention and furnish drill.

Plan:

1. Use controlled reader and film number I of the General Mathematics - Business mathematics course, EDL arithmetic skills program.
2. Use only drill section.
3. Set controlled reader on a slow speed and increase until class working level is reached.
4. Have students answer in rotation and keep check on each other's answers.
5. Limit time to not more than 15 minutes.

Suggestions:

1. Blank film is available in kits to make films needed for a specific use.
2. Two especially good films are: General Mathematics - Business Mathematics Course, and Arithmetic Number Facts, EDL Arithmetic Skills Program.

MATH GAMES AND INTEREST DEVICES

Objective:

- (a) To provide mathematical experiences through a fun situation.
- (b) To occupy a student's time who has finished an assignment before the class period ends.
- (c) To improve the students attitude toward mathematics classes.

Plan:

- 1 Secure the games to be used before school starts
Many of the games can be made (see following pages)
- 2 Present only one game to the class at a time Explain the directions for playing the game.
- 3 Place the game on a table, preferably in the back of the room, so it will be accessible to the students.
- 4 Repeat steps 2 and 3 for each game.
- 5 Remove a game from the table when students lose interest.

TM App. 1
8

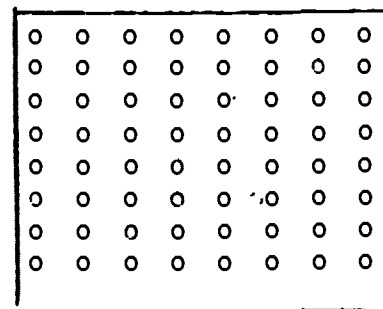
GAMES EASILY BUILT

1. 16 Pegs

Cut from peg board. Cut off golf tee's to use as pegs.

RULES: Place the 16 pegs so no more than two pegs are on the same line vertically, horizontally, or diagonally.

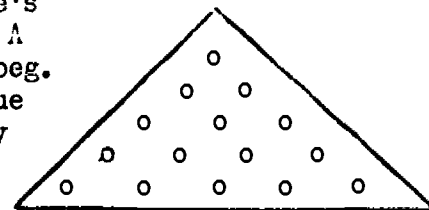
TM App. 1
8



2. Pascal's Triangle

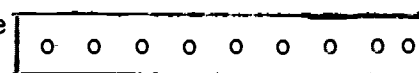
Cut from any peg board. Cut off golf tee's

RULES: Place pegs in all but one hole. A move can be made by jumping one peg. Remove the peg jumped and continue until no moves are possible. Try to remove all but one peg.



3. Eight Men in a Boat:

Cut from peg board. Place 4 pegs on one end and 4 pegs of a different color on the other end.

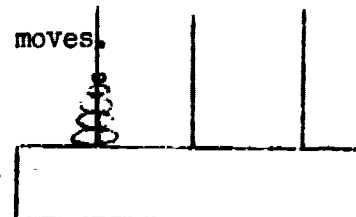


RULES: A move consists of jumping one man or moving one space. The object is for the men to change ends of the boat with the fewest moves possible.

(24) Using other than eight men the formula is $(\frac{1}{2}n)^2 + n = \text{fewest moves}$.

4. Hanoi's Towers

(a) Place 5 washers of different sizes on one tower (arrange from largest to smallest)



(b) Place all 5 on a different tower by moving only one washer at a time. A larger washer cannot be placed over a smaller one.

(c) Fewest moves possible are 31. For other than 5 washers, fewest moves are $2^n - 1$.

USING CALCULATORS IN GENERAL MATHEMATICS

After using calculators in general mathematics in 14 pilot classes during the 1966-67 year the teachers involved have drawn the following conclusions:

- (1). The machines are a strong motivational device and can be used effectively as such but cannot be justified solely for this reason.
- (2). The ideal situation would be one machine for each student but an effective program can be carried on with one for each two students.
- (3). The machines should be used as a tool just like the chalkboard, pencil and paper.
- (4). The program operates better if the machines are on tables in the back of the room or shelves along the walls around the room. With the machines located in this manner the students should be given the privilege to use the machines at will.
- (5). If the program is being operated with the minimum number of machines, the student's use of machines can be organized in one of two ways: One group of students can work on the machines while another group works at the desk, or two students could work together on each machine.
- (6). The machines are not a cure-all and they can be used too much. It is not necessary and is not advisable to have assignments on the machines every day.
- (7). In many occasions the students should be guided toward discovering how to make the machine work for them.
- (8). The teacher should try to have three or four people to visit in the class and demonstrate how they used the machine in their occupation..
- (9). Students can learn the skillful operation of calculators and have a salable skill. However, this is only a side value not related to the teaching of mathematics.
- (10). The ideal class would be 15 to 20 students - because of need to give the individual student personal attention. The maximum size of this class should not be over 25.