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ABSTRACT

Two groups of kindergarten children had activities involving the transitive property of matching relations and length relations, respectively. Both groups had activities involving transitivity of weight relations. A control group had instruction only on relations. Pretests of Matching Relations, Length Relations, Matching Relations Conservation, Length Relations Conservation, Matching Relations Transitivity and Length Relations Transitivity were given. Posttests were given on the latter four. Analysis of covariance on each posttest, using the six pretest as covariables, showed significant difference (p .05) for treatment on Matching Relations Transitivity. Further analysis revealed that both treatment groups outperformed the control group, but did not perform differentially. (Author)

THE EFFECTS OF SELECTED EXPERIENCES ON THE ABILITY OF KINDERGARTEN CHILDREN TO USE CONSERVATION AND THE TRANSITIVE PROPERTY OF SIX RELATIONS

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One purpose of the study was to determine the effectiveness of each of two specified sets of activities which were designed to teach the transitive property of selected relations to a group of kindergarten children. In particular, one set of activities was prepared to teach the children to use the transitive property of matching relations, and the other set of activities was designed to teach the transitive property of length relations. A second purpose of the study was to determine the effect of the learning activities involving matching relations on the ability of the children to use length relations and to determine the effect of the experiences involving length relations on the ability of the children to use the transitive property of matching relations. A third purpose of the study was to determine the effect of the experiences in using the transitive property on the ability of the children to conserve the relation once the physical comparison was destroyed by a spatial transformation.

To establish a matching relation between two sets A and B, a child forms pairs of elements, where one member of each pair is chosen from set A and the other is chosen from set B, until one or both sets are exhausted. Whenever both sets are exhausted, there are the same number of a's as b's. If set B is exhausted and set A is not, there are more a's than b's (and fewer b's than a's). For a definition of the length relations, consider two segments A and B. A is the same length as B, if whenever (transformations of) A and B lie on a line such that two end points (right or left) coincide, the remaining two end points coincide. A is longer than B and B is shorter than A if the remaining end point of B coincides with a point between the end points of A. Note that a child is not required to associate a number with a set nor with the length of a segment in order to establish these relations.

In this study, the matching relations were operationally defined on such finite sets of physical objects as checkers and tiles. Sticks and straws provided physical representations of segments and were used in the operational definitions of the length relations.

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All of the matching relations and length relations are transitive. That is, if R denotes any one of these relations, then R has the following property: If aRb, and bRc, then aRc. In this study, the instructional settings were designed so that the transitive property could be empirically observed by the children. Once a child had established the relations aRb and bRc, it was suggested that he compare a and c to observe aRc. No activities were included specifically to emphasize transitivity of the relations "fewer than" or "shorter than." Presumably, if a child could establish both relations, an observation of the transitive property of "more than" provides experience with the transitive property of "fewer than."

It appears from Piaget's theory of underlying cognitive structure, that if a child can use the transitive property in one category of relations, then he can use the transitive property in any category of transitive relations regardless of the physical embodiment. Piaget (1952, p. 204) has indicated, on the contrary, that a formal structure of transitivity is not acquired all at once, but it must be reacquired every time a new embodiment is encountered. In a previous study by the investigator (Owens, 1972), performance was improved on transitivity of matching relations by activities involving conservation and the transitive property of matching relations, but no transfer occurred to length relations. The present study provided additional data on this previous point and allowed for a test of transfer from the transitive property of length relations to transitivity of the matching relations. Also in this study, activities were included on the transitive property in two relational categories. After experiences in either the transitive property of matching relations or length relations, the treatment included activities on the transitive property of weight relations, "same weight as" and "heavier than." This was based on the conjecture that observation of the transitive property in two settings might foster generalization to the third relational category.

In Piaget's (1952) classical conservation of number tasks, a child is asked to establish the equivalence of two sets. Then one set is taken through a physical transformation and the child is then asked, "Is there the same number or does one have more?" Van Engen (1971, p. 43) has argued that this task may measure whether or not the child conserves the one-to-one correspondence rather than conservation of number. In this study a task similar to the

above is considered to be a measure of conservation of the relation "same number as." Conservation need not be limited to cases of equivalence. Tasks for conservation of "more than," "fewer than," and the three length relations are included.

Smedslund (1963) has argued that from a logical point of view, conservation precedes transitivity in the child's development. Consider the example in which it is established that set A contains more objects than set B. This relation must be conserved while set B is moved to a new location and compared with set C. This view, that conservation precedes transitivity, raises the question of whether conservation ability might be enhanced by activities aimed primarily at improving the ability to use transitivity.

Method

Sample

The 36 children in the sample were chosen from the 44 children in a morning and afternoon kindergarten having the same teacher. The private kindergarten located in Richmond, British Columbia, was in a school district which had no public kindergartens at the time of the study. The study was conducted in the month of June, so the subjects were between the ages of 66 months and 78 months.

Tests

Tests were constructed to determine a child's abilities to establish relations, conserve relations and use the transitive property of relations in length and matching relational categories. All tests were designed to be administered on a one-to-one basis.

The purpose of the Matching Relations (MR) Test was to measure the ability of a child to establish the matching relations "same number as," "more than," and "fewer than." The Conservation of Matching Relations (CMR) Test was designed to measure the ability of a child to conserve a matching relation, provided that he could establish the relation. In one item, for example, the child was presented seven blue discs in a row attached to a piece of cardboard. He was also given six red discs and instructed to pair the red discs and the blue discs. After the pairing the examiner asked two questions, "Is there the same number of red discs as blue discs?" and "Are there fewer red discs than blue discs?" After the child responded, the

examiner rearranged the red discs into a row the same length as the row of blue discs and repeated the two questions. In each case the correct answer to one question was "yes" and the other was "no." In each item, the rearrangement was perceptually biased in favor of the incorrect conclusion. The first two questions about each situation were scored as an item of the MR Test and all four questions were considered in scoring the CMR item. Each of the MR and CMR Tests was comprised of six items--two items for each matching relation.

The Length Relations (LR) Test was constructed to measure the ability of a child to establish the length relations "same length as," "longer than," and "shorter than," and the purpose of the Conservation of Length Relations (CLR) Test was to assess the ability of a child to conserve these relations once they were established. Two items were included for each of the length relations for a total of six items in the LR and CLR Tests. In an item of the LR Test the child was asked to establish a relation between the lengths of two straws (or sticks) by answering two questions. For example, "Is the red straw the same length as the green straw?" and "Is the red straw longer than the green straw?" Afterward, for completion of the CLR item, the examiner slid one straw along or made a "T" arrangement so that the new configuration presented a perceptual bias against the correct solution. The two questions which had been asked earlier were repeated. All four questions were considered in the CMR Test item.

The purpose of the Transitivity of Matching Relations (TMR) Test was to measure a child's ability to use the transitive property of matching relations. In a TMR item, a child was presented a piece of cardboard on which two rows of objects were attached in such a way that if the child focused on the lengths of rows rather than on the transitive property, he would reach an incorrect conclusion. Each item involved a child's pairing a third set of objects with each of the two sets attached, observing the relations, and making an inference about the relation between the two attached sets. In the example shown in Figure 1, eight tiles and six checkers were attached in rows of equal length to a piece of cardboard. Seven jacks were available and the child was instructed to pair the tiles and the jacks. The examiner asked, "Are there more tiles than jacks?" After the response, the child was instructed to pair the jacks and the checkers. The question, "Are there more jacks than checkers?" followed. The examiner then removed the jacks,

asked two questions, and allowed time for responses. The questions were, "Is there the same number of tiles as checkers?" and "Are there more tiles than checkers?"

Insert Figure 1 about here

Three items exhibiting each of the matching relations were included in the TMR Test for a total of nine items. Six items had question formats similar to the example above. In one item for each relation the question format was "Is there the same number of tiles as checkers, or does one have more (fewer)? Which one?" All items followed the materials format of Figure 1 with such additional materials as colored wooden discs, cutout stars, and bottle caps.

The Transitivity of Length Relations (TLR) Test was constructed to measure a child's ability to use the length relations of this study. In the example shown in Figure 2 a red stick and a green stick each 8 inches long were attached to a piece of cardboard and a blue stick was available. The examiner had the child to place the blue stick beside the red stick, and then asked, "Is the red stick the same length as the blue stick?" After the response, the examiner suggested that the child place the blue stick beside the green stick and asked, "Is the blue stick the same length as the green stick?" The examiner then removed the blue stick and asked, "Is the red stick shorter than the green stick?" and "Is the red stick the same length as the green stick?"

Insert Figure 2 about here

Each item of the TLR Test had a red and a green stick, straw, or pencil attached as in Figure 2 and a blue object of the same kind available. If the objects were not the same length, the longer one occupied the position of the red straw in Figure 2, and the red and green ones differed by one-half inch. A transitive inference was possible in each case. As in the TMR Test there were three items for each of the three length relations for a total of nine items. The question format of one item was, "Is the red pencil the same length as the green pencil or is one longer (shorter)? Which one?"

The purpose of the Transitivity of Weight Relations (TWR) Test was to determine if a child could use the transitive property of the relations "same weight as" and "heavier than." The child was asked to find if a plastic bottle weighed the same as a styrofoam cup without placing them both on the balance at the same time. The two were "loaded" to the same weight as a third object which was available.

In the other item the child was asked to find if the orange (styrofoam) cup and the purple (styrofoam) cup weighed the same or if one was heavier without placing them both on the balance at the same time. A yellow styrofoam cup, the same size and shape as the other two, was available. The weight of the yellow cup was intermediate to the other two, but the three weights could not be distinguished by handling.

Scoring Tests

An item was scored "pass" provided that a child answered correctly all the questions contained in the item and "fail" otherwise. The number of items scored "pass" by a child on each test was considered to be the child's score on the test.

Units of Instruction

Six units of instruction were designed to improve the abilities of the children to establish certain relations and use the transitive property of those relations. Each lesson of the units was written for a single session of 20-30 minutes.

Unit I: Length Relations and Matching Relations. The purpose of this unit was to insure that a child, whenever presented appropriate stimuli, could determine that certain relations hold and that other relations do not hold. The first three lessons involved the length relations "same length as," "longer than," and "shorter than." For example, in one activity, each child was given a green stick 6 3/4 inches long and a 7-inch red stick. The teacher then asked for responses to, "Are the two sticks the same length?" (Children respond.) "Is the green stick longer than the blue stick? Is the green stick shorter than the blue stick?" The children responded to each question in turn and the teacher explained if an explanation appeared to be necessary. Other materials such as boards, ropes, and straws were used for comparison. A distinction was made between "longer than" and "higher than."

The next three lessons in Unit I dealt with the matching relations "same number as," "more than," and "fewer than." Small toys, tiles, checkers, colored wooden discs, and colored plastic tags were the materials used. The children paired two sets of objects and the teacher introduced appropriate relational terminology. Later in the instructional sequence the teacher asked questions analogous to those mentioned above, and gave explanations when deemed necessary.

Unit II: Length Relations and Matching Relations Continued. This unit contained five lessons and provided practice for the children in establishing length or matching relations.

Unit III: Transitivity of Length Relations. The purpose of the five lessons in Unit III was to provide for the children experiences in using the transitive property of the relations "same length as" and "longer than." For an example of an activity in this unit, the children compared two straws and found that the blue straw was longer than the green straw. They were then to place the blue straw in a paper bag and determine that the green straw was longer than the red straw. The children were then asked to make a conjecture about the relation between the blue straw and the red straw. After conjectures, the children were encouraged to verify their conclusion by direct comparison of the blue straw and the red straw.

Unit IV: Transitivity of Matching Relations. Unit IV was composed of five lessons. Experience involving the transitive property of the relations "same number as" and "more than" were included. Consider, for example, the activity in which each child was given six jacks and six tiles. After it had been established that there was the same number of jacks as tiles, each child was given a bag containing six checkers. It was suggested that the checkers be poured from the bag and left in a random arrangement. The tiles were to be "paired" with the checkers. After the children had established that there was the same number of tiles as checkers, they were asked to make a conjecture about the jacks and checkers. A direct comparison of the jacks and checkers followed the conjecture.

In each of the transitivity units, Unit III and Unit IV, the sequence began with arrangements of materials which were supportive of the correct inference. For example, three sticks were laying rather close together so that, in fact, a direct comparison was possible. The supportive case was

followed by a neutral arrangement such as the one described above in which the checkers were left just as they fell from the bag. Next in sequence following the neutral arrangement were activities involving screened stimuli. For example, whenever the straws or counters were placed in a bag or box before the children were asked to make an inference, the stimuli were said to be screened. Finally, each transitivity unit contained activities in which the physical situation presented a perceptual bias against the correct solution. For example, a longer row contained fewer objects.

Each transitivity unit contained activities in which each child had his own set of materials and also group activities in which one child or the teacher manipulated and responses were elicited from different children in turn. Examples of group activities were those in which the flannel board and felt cutouts were used for matching and boards were used for length relational activities.

Unit V: Weight Relations. The three lessons in this unit were prepared to give the children experiences in establishing the relations "heavier than" and "same weight as." The children could perceive by handling the materials that a rock was heavier than a piece of foam rubber about the same size. Similar materials with distinguishable weight differences were used to introduce the beam balance and its behavior when the objects were placed on the opposite pans. The children were led to conjecture that the beam would be level if the two objects were the "same weight." In some comparisons the two objects were congruent (e.g. two styrofoam cups with lids), but one was heavier than the other. In other comparisons the two objects were not of the same size, but they were of the same weight or the smaller object was heavier.

Unit VI: Weight Relations and Transitivity. The weight relations "heavier than" and "same weight as" were introduced using a balance in a manner similar to that of Unit V. Very little practice had been given in establishing the relations when the transitive property was introduced. Of course, relations had to be established in each problem before the transitive inference was possible. Materials identical to those used in Unit V were used in the three lessons of Unit VI.

In one transitivity activity, the children were given congruent brass and aluminum cylinders and instructed to use the balance to determine which one was heavier. They were then to compare the weight of the aluminum cylinder with

a third wooden cylinder congruent to the other two. Finally, the teacher said, "Tell me about the wooden cylinder and the yellow (brass) cylinder." Following responses the teacher suggested that the children compare the two directly. Care was taken throughout the unit to insure that volume could not be considered an indicator of a weight relation.

Procedure

All 44 children had the six lessons of Unit I on establishing length relations and matching relations. Following instruction on relations, 40 children (four were absent) were given pretests. The children's abilities to establish matching relations and length relations were assessed by the MR and LR Tests described above. Conservation ability was measured by the CMR and CLR Tests. As a measure of transitivity of matching (length) relations, a six-item subset of the TMR (TLR) Test was used. As indicated in the test descriptions, a relations test was administered with a conservation test of those same relations. A transitivity pretest in the same relational category followed in the same testing session. However, for a given child, tests in the two relational categories were in separate testing sessions.

Following the pretests, four children who failed to score 50 percent on each relations pretest, MR and LR, were dropped from the study. The remaining 36 children were randomly ordered, and every third child was assigned to one of three treatment groups. Let the treatment groups be denoted by M, L, and C.

During the treatment period Group L was given the experiences of Unit III followed by Unit VI. Thus, the treatment for Group L consisted of five lessons on the transitive property of the length relations, and three lessons on the transitive property of weight relations. Note that this group did not have experiences in the transitive property of matching relations.

The treatment for Group M consisted of Unit IV and Unit VI. Thus, Group L had five lessons on the transitive property of matching relations followed by three lessons of experiences in transitivity of weight relations. No experiences on the transitive property of length relations were included for Group M.

Group C was considered the control group. After the five additional lessons of Unit II on length and matching relations Group C then had activities of Unit V intended to define weight relations. Thus, this group had experiences only in three kinds of relations but no activities involving transi-

tivity were included.

The investigator, the regular teacher of the kindergarten, and a graduate student in mathematics education* served as instructors. In a morning or afternoon session of kindergarten, the three treatment groups were instructed at the same time. Thus, about six or seven children were in an instructional group. The three instructors rotated from one treatment group to another on a daily basis to prevent confounding of any instructor effects with treatment effects.

Following the activities of the differential treatments, the posttests were administered on a one-to-one basis. The two relations tests, MR and LR, and the two conservation tests, CMR and CLR, were administered in the same session. The longer nine-item transitivity tests, TMR and TLR, were administered in separate sessions. The order in which the items of each of these tests were given was randomized for each child independently of other children. The two items for transitivity of the weight relations were contained in a fourth testing session.

Analysis of the Tests

Means, standard deviations, and KR-20 reliabilities were computed for the six posttests (excluding weight relations). These are presented in Table 1. The relations tests were the easiest, the conservation tests were more difficult, and the transitivity tests were the most difficult. The test standard deviations were smallest for the relations tests and greatest for the transitivity tests. All six KR-20 reliabilities were between .76 and .85.

Insert Table 1 about here

Data Analysis

A one-way analysis of covariance was performed for each of the following six variables defined by the posttest measures: Matching Relations (MR), Conservation of Matching Relations (CMR), Transitivity of Matching Relations (TMR), Length Relations (LR), Conservation of Length Relations (CLR), and

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*The investigator is indebted to Miss Ann Johnson and Mr. Robert Verner for their assistance.

Transitivity of Length Relations (TLR). There were six covariables, each defined by a pretest measure corresponding to one of the posttests. Let the covariables be denoted by PMR, PMRC, PMRT, PLR, PLRC, and PLRT, respectively. The three levels of treatment were: (1) Group M, the group which had lessons on the transitive property of matching relations; (2) Group L, the group which had activities on the transitive property of length relations; (3) Group C, the control group which had experiences with relations only. Of particular interest, also, will be the contrasts, Group M--Group C and Group L--Group C.

Data on weight relations are presented but no analyses were done.

Results

Analyses of Covariance

The six analyses of covariance are reported in Table 2. The only variable on which the F-ration was significant was TMR and $F = 4.50$ ($p < .02$). Thus, the null hypothesis of all three group means equal, may be rejected in favor of an alternative that at least one pair of means is different for the variable TMR.

Insert Table 2 about here

Table 3 contains the adjusted group means of the three groups for all six variables. For transitivity of matching relations the adjusted means are 5.01, 4.87, and 2.95, for the M, L, and C Groups, respectively. In order to determine which pairs of these means were significantly different, Scheffe's Test was used. Using Scheffe's Test, the adjusted means of 5.01 and 4.87 are not significantly different. However, 4.87 and 2.95 are significantly different ($p < .05$) by Scheffe's method. This implies that 2.95 and 5.01 are significantly different. Thus, it appears from using Scheffe's Test that both treatment groups outperformed the control group on TMR.

Insert Table 3 about here

Two contrasts for the variable TMR were performed directly by analysis of covariance. Of interest to the investigator were the comparison of each treatment group to the control group. The results of these analyses of covariance are reported in Table 4. The F-ratio of 5.90 for the Group L versus

control contrast was significant ($p < .02$). However, the F-value for the Group M versus control was not significant. This result appears to conflict with the result of Scheffe's Test.

Insert Table 4 about here

Correlations

The adjusted correlations between pairs of variables are presented in Table 5. Generally speaking, these correlations are small. Only ones which give a clear indication of a relationship are $r = .49$ and $r = .61$. These show a relationship of the variable, MR with the variables CLR and LR, respectively. There is no logical reason which will explain why these relationships exist while others do not.

Insert Table 5 about here

Weight Relations Transitivity

No analyses were performed on the variable Transitivity of Weight Relations (TWR). However, since instruction was given on this transitive property with the hope of fostering generalization, it is of interest to note the results. The data are presented in terms of group totals in Table 6. While no statistical tests were made, it may be observed that each treatment group made more transitive inferences than the control group. In this case both treatment groups M and L had the same instruction on the transitive property while the control group had experiences only with the weight relations.

Insert Table 6 about here

Conclusion

It appears that Group L, the group which had the instruction on the transitive property of length relations outperformed the control group on the transitive property of matching relations. A somewhat more tentative result indicates that Group M, which had instruction on transitivity of

matching relations, outperformed the control group on the same property. In light of the purposes of this study, the second result indicates that the activities on transitivity of matching relations were successful to some degree in improving the children's ability to use the property. This is consistent with the results of a previous study by the investigator (Owens, 1972). In the previous study, children who had the lessons on transitivity of matching relations outperformed a control group on a measure of the property.

In consideration of the second purpose of the inquiry, transfer did not occur across relational categories. While the group which had the lessons in the length category improved in the matching category, this is not considered transfer. In order for transfer to occur, there must be learning of the material on which the instruction was given. Evidence is not provided here that Group L achieved at any higher level than the control group on the property on which instruction was given to Group L.

With regard to the third question of the study, no group performed at a higher level than any other group on a conservation measure. No evidence is provided that instruction in transitivity is facilitating to conservation performance.

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Table 1

Number of Items, Means, Standard Deviations, and

KR-20 Reliability Coefficients for Six Tests

Test	Number of Items	Mean	Standard Deviation	KR-20
Matching Relations (MR)	6	5.38	1.21	.76
MR Conservation	6	3.79	2.17	.85
MR Transitivity	9	3.88	2.78	.84
Length Relations (LR)	6	5.52	1.10	.77
LR Conservation	6	3.86	2.16	.85
LR Transitivity	9	5.38	2.54	.76

Table 2

Univariate Analyses of Covariance with Six Covariates

Variable	M. S. Treatment	M. S. Error	F	p<
Matching Relations (MR)	1.80	.83	2.16	.13
Conservation of MR (CMR)	.29	2.06	<1	
Transitivity of MR (TMR)	13.76	3.06	4.50	.02
Length Relations (LR)	.42	.35	1.20	.32
Conservation of LR (CLR)	3.73	2.35	1.58	.22
Transitivity of LR (TLR)	8.08	3.80	2.13	.14

Note: Treatment has 2 df; Error 27.

Table 3

Group Means on Each of Twelve Variables,

Adjusted for Six Covariables

Variable	Treatment Group		
	M	L	C
MR	5.18	6.07	5.42
CMR	3.93	4.17	3.82
TMR	5.01	4.87	2.95
LR	5.69	5.86	5.45
CLR	3.65	4.57	3.36
TLR	5.63	6.76	4.95

Table 4

Univariate Analysis of Covariance for Variable

TMR with Six Covariables--Contrasts

Contrast	Source	df	Mean Square	F	p<
Group M versus C	Treatment	1	7.01	2.29	.14
Group L versus C	Treatment	1	18.04	5.90	.02
	Error	27	3.06		

Table 5

Matrix of Correlations Between the Variables

with Covariates Eliminated

	MR	CMR	TMR	LR	CLR
CMR	.27				
TMR	-.10	.20			
LR	.61	.32	.01		
CLR	.49	.29	.27	.34	
TLR	.00	.00	.27	-.07	-.02

Table 6

Group Totals for Transitivity of the Weight Relations

Treatment Group	n for which data available	Transitivity of "same weight"	Transitivity of "heavier than"
M	11	11	9
L	12	10	12
C	11	7	6

Figure 1

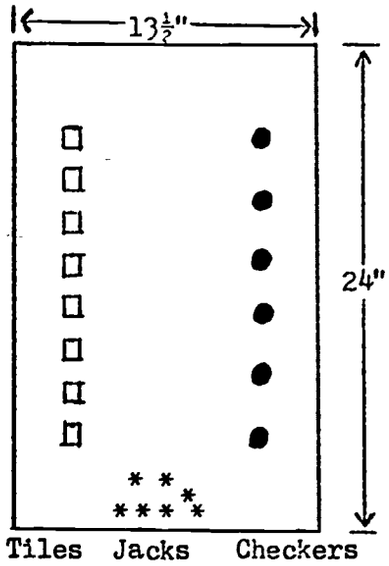

Figure 2