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ABSTRACT

The second review segment of the Self-Paced Physics Course is presented in this volume and arranged to match study segments 19 through 40. The segment is divided into five subsegments, each of which is composed of a set of problems and solutions. A study guide is provided for each subsegment. The problem set is designed in a back-referencing system, and the scrambling method is used in solution presentation. Directions for reaching solutions are revealed through the use of latent image study guides. The purpose of the review segment is to help students in isolating and organizing essential physics concepts which are common to problem situations. (Related documents are SE 016 065 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

Set #5

UNITED STATES NAVAL ACADEMY

Self-Paced PHYSICS

SEGMENTS 41A-41E

DEVELOPED AND PRODUCED UNDER THE
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NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY

P	STEP	NAME	P	STEP	SECTION SEGMENT 41-A
1		______ (ans)	9		______ (ans)
2		______ (ans)	10		______ (ans)
3		a) ______ nt/coul (ans)	11		______ (ans)
		b) ______ nt (ans)	12		______ (ans)
4		______ (ans)	13		______ (ans)
5		______ (ans)	14		______ (ans)
6		______ (ans)	15		______ (ans)
7		______ (ans)	16		______ (ans)
8		______ (ans)			

STUDY GUIDE SELF-PACED PHYSICS

P	STEP	NAME	P	STEP	SECTION SEGMENT 41-B
1		_______ (ans)	10		_______ (ans)
2		_______ (ans)	11		_______ (ans)
3		_______ (ans)	12		_______ (ans)
4		_______ (ans)	13		_______ (ans)
5		_______ (ans)	14		_______ (ans)
6		_______ (ans)			
7		_______ (ans)			
8		_______ (ans)			
9		_______ (ans)			

P	STEP	NAME	P	STEP	SECTION SEGMENT 41-C
1		(ans)	10		(ans)
2		(ans)	11		(ans)
3		(ans)	12		(ans)
4		(ans)	13		(ans)
5		(ans)	14		(ans)
6		(ans)	15		(ans)
7		(ans)	16		(ans)
8		(ans)			
9		(ans)			

P	STEP	NAME	P	STEP	SECTION SEGMENT 41-D
1		(ans)			(ans)
2		(ans)	9		(ans)
3		(ans)	10		(ans)
4		(ans)			(ans)
5		(ans)	11		(ans)
6		(ans)	12		(ans)
7		(ans)			
8		(ans)			
		(ans)			

STUDY GUIDE SELF-PACED PHYSICS

P	STEP	NAME	P	STEP	SECTION SEGMENT 41-E
1		(ans)	10		(ans)
2		(ans)	11		(ans)
3		(ans)	12		(ans)
4		(ans)	13		(ans)
5		(ans)			SEG 41-E
6		(ans)			
7		(ans)			
8		(ans)			
9		(ans)			

PREFACE TO REVIEW SEGMENTS

This volume of your Self-Paced Physics course contains five Review Segments which have been carefully arranged to match corresponding sections of your PROBLEMS AND SOLUTIONS. You may find the grouping below helpful in organizing your review time.

Review Segment	Covers Topics in Study Segments
41-A	19 through 23
41-B	24 through 27
41-C	28 through 31
41-D	32 through 36
41-E	37 through 40

The review problems are numbered in sequence in each Segment. Previous study problem(s) to which each review problem relates is shown in parentheses following the review problem number.

For example, review problem 3 in Segment 41-A identifies the related material as

3 (20-9, 20-5, 19-15)

which means that the topical substance of review problem 3 contains concepts and operations which are also involved in Segment 20, Problem 9; Segment 20, Problem 5; and Segment 19, Problem 15.

We recommend that you establish a pattern of review which will make full use of this back-referencing system. Despite the fact that there is almost an infinite number of ways to state a problem in a given field, the number of relevant concepts and operations are limited. By referring to the original problems before or after solving the review problem, you will be able to view the essential operations and procedural sequences from several different vantage points. It will enable you to isolate and organize the essential concepts that are common to so many problem situations.

next page

continued

Each Segment is accompanied by its own individual STUDY GUIDE. And, as in the learning Segments, the problems are in numerical order but the solutions are scrambled so that the latent image STUDY GUIDE must be used to locate the solution in which you are interested. You will observe, however, that the STUDY GUIDES for the Review Segments differ from those you have used previously in that there are no references to Information Panels, Audiovisuals, assigned reading, or homework. All review problems are to be solved by everyone; there are no alternative paths. Finally, no provision is made for true-false follow-up questions or answers in the solutions or STUDY GUIDE.

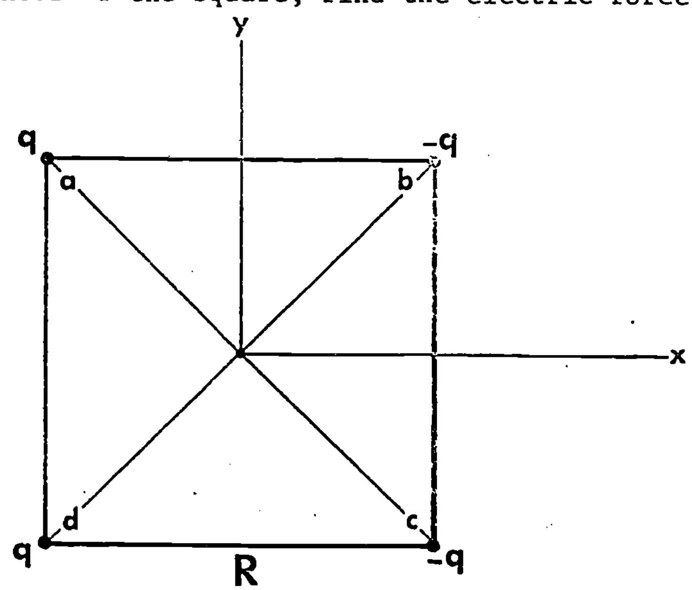
Completed STUDY GUIDES should be submitted for evaluation to your instructor in accordance with previously established procedures.

Good luck.

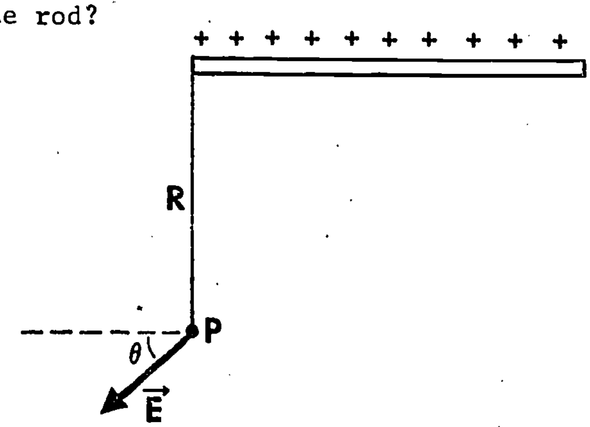
1 (19-11). A charge $Q = 6.0$ coul is to be divided into two parts and placed a distance $R = 10$ m apart. Find the maximum force of repulsion.

2 (20-1). A charge q is placed in an electric field $\vec{E}$. Write an expression for the force acting on the charge.

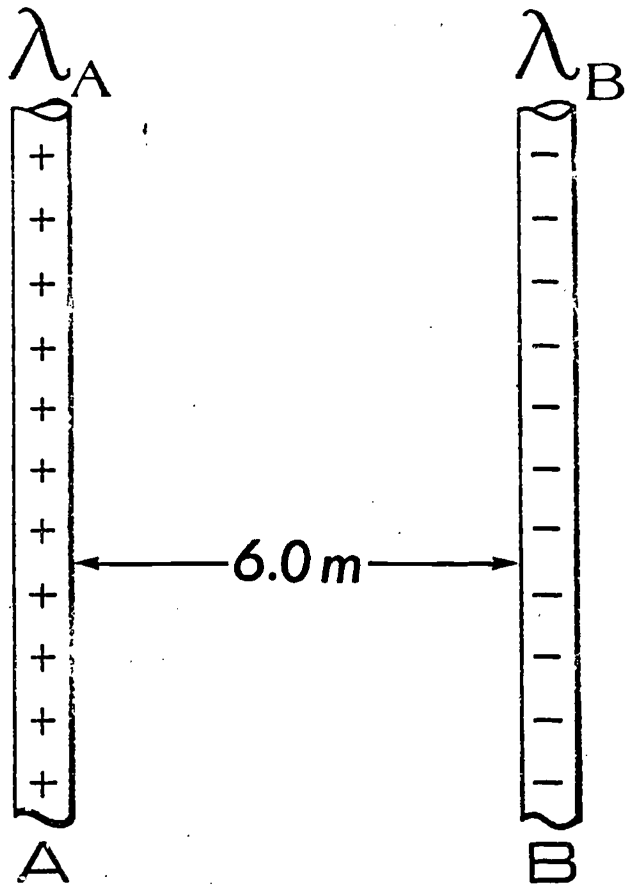
3 (20-9, 20-5, 19-15). (a) Find the magnitude and direction of the electric field $\vec{E}$ at the center of the square due to the charge distribution shown in the diagram. Assume that $q = 1.0 \times 10^{-8}$ coul and $R = 5.0$ cm. (b) If an additional charge $+Q = 5.0 \times 10^{-8}$ coul is placed at the center of the square, find the electric force $\vec{F}$ acting on the charge Q.



4 (20-13). A semi-infinite insulating rod carries a constant charge per unit length λ. What is the angle θ that the electric field at P makes with the rod?

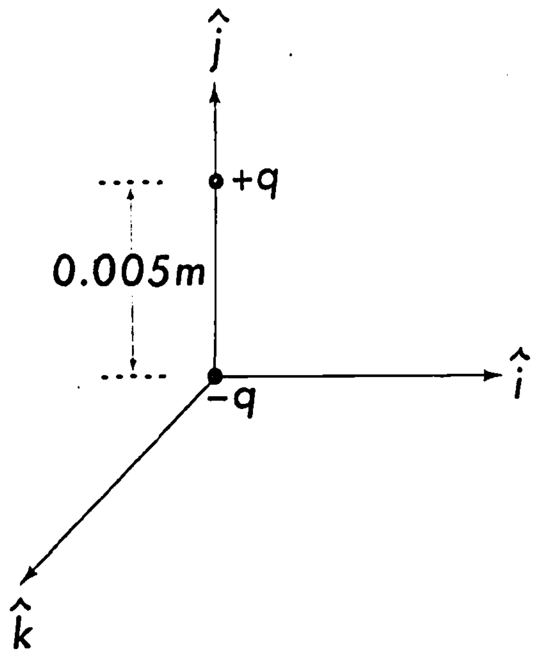


5 (21-1). Two infinitely long wires A and B have uniform charge densities of $\lambda_A = 3.0 \times 10^{-6}$ coul/m and $\lambda_B = +6.0 \times 10^{-6}$ coul/m respectively. The wires A and B are 6.0 m apart. Find the distance from wire A between wires A and B such that the electric field at the point is zero.



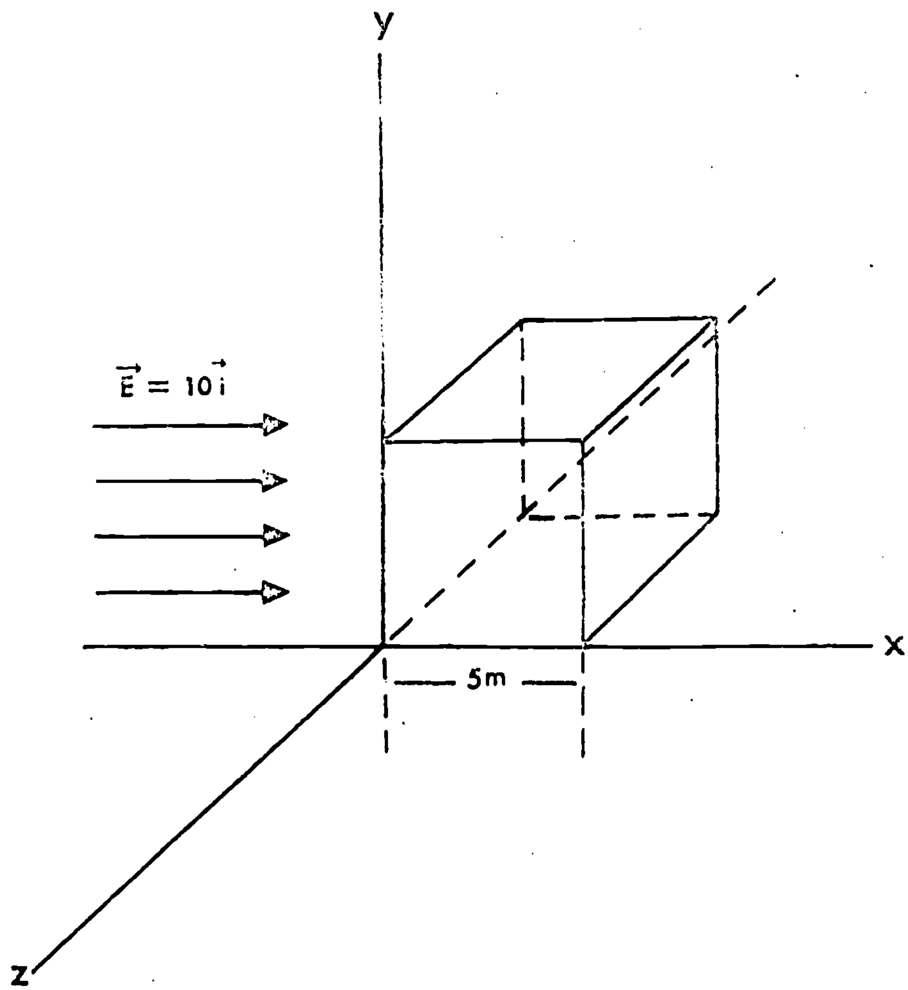
6 (21-14, 21-10, 19-1, 19-2). A constant electric field $\vec{E}$ is applied to a charge q with mass m. Find an expression for the acceleration of the charge in terms of $\vec{E}$, q, and m.

7 (22-4, 22-1).

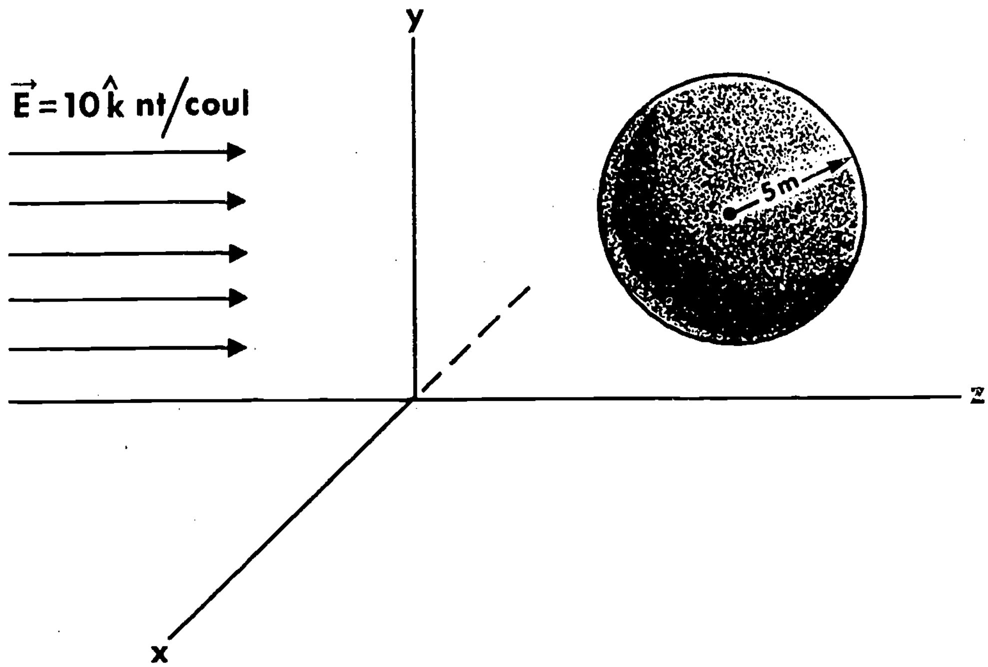


An electric dipole, consisting of opposite charges each of magnitude 3.2×10^{-19} coul, is arranged as shown in the above diagram. What is the magnitude and direction of the dipole moment?

8 (22-9, 22-5). Study the An open cubical surface 5
on edge is placed in a uni ield of 10 newtons per coul
The missing surface is in the ne. Find the electric flux Φ_E
the surface.



9 (22-14). A spherical surface of radius 5 meters is shown in the diagram. What is the value of the electric flux through the spherical surface?

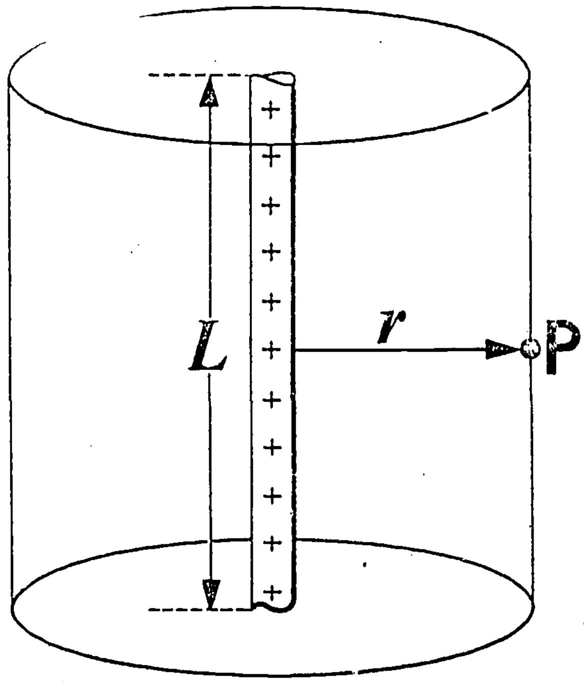


10 (23-5, 19-6). A hollow conductor initially carries a charge of +3 coul. Then an object carrying a charge of -2 coul is introduced into the interior of the conductor. Finally another object of charge -4 coul is brought close to the outside of the conductor. Find the net charge on the outer surface of the conductor.

11 (23-9). Write the relationship between electric flux ϕ_E through a closed surface and the charges enclosed within that surface, $q_1, q_2, \ldots, q_n$.

12 (23-17, 19-5). A long, nonconducting solid cylinder is uniformly charged with charge density ρ. Find the electric field E *inside* the cylinder as a function of r, the distance from the cylinder axis.

13 (23-21). The figure below shows a portion of an infinitely long wire with a uniform charge λ = 2 coul/m. Apply Gauss's law to determine the electric field at point P which is a distance of 2 m from the wire.



14 (23-23, 23-22, 21-5). Two concentric conducting spherical shells have radii R_1 = .2 m and R_2 = .3 m. The inner sphere carries a charge -6×10^{-2} coul. An electron escapes from the inner sphere with negligible speed. What is the magnitude of the electron's acceleration at the instant it escapes into the region between the two spheres?

15 (23-24, 23-1, 20-18). A hollow sphere and a solid sphere each of radius R are both uniformly charged. Is it possible to identify each simply by measuring their respective electric fields at various points inside and outside the spheres?

16 (23-25). Two coaxial hollow metal cylinders of length L with radii a and b (b > a) carry charges +q and −q respectively. Find the magnitude of the electric field at a point r > b, measured from the common axis.

[a] CORRECT ANSWER: $250 \frac{\text{nt-m}^2}{\text{coul}}$

We could easily consider the problem in six steps by saying

$$[illegible] + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6$$

where

ϕ_1 = flux through the side facing the E field,

ϕ_2, ϕ_3, ϕ_4, ϕ_5 = flux through the sides with edges parallel to the x-axis,

and

ϕ_6 = flux through the side on the back of the cube from the E field

The area vector $d\vec{S}$ is pointing outward from all the surfaces and consequently we can see the surfaces 2, 3, 4, and 5 all have area vectors perpendicular to the electric field. Therefore,

$$\phi_2 = \phi_3 = \phi_4 = \phi_5 = 0$$

Through side one S = 0, so

$$\phi_1 = 0$$

Through side six,

$$\phi_6 = ES \cos 0°$$

or

$$\phi_6 = ES$$

Finally,

$$\phi_E = 0 + ES$$

or

$$\phi_E = ES = 10\ (5 \times 5) = 250\ \text{nt-m}^2/\text{coul}$$

[a] CORRECT ANSWER: Yes

Choosing a spherical Gaussian surface of radius r, we find that for either sphere

$$\phi_E = \frac{Q}{\varepsilon_0} = ES$$

where Q is the total charge of the sphere.

We know that for a sphere

$$S = 4\pi r^2$$

Therefore,

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

We conclude that an electric field due to a spherical charge distribution acts as if all the charge were located at the center, that is, when r is greater than the radius of the distribution.

This result holds for all spherically symmetric distributions, so we cannot know if a sphere is hollow or solid just by measuring the electric field outside of the sphere. However, the electric field inside the hollow sphere is zero since there is no net charge inside a Gaussian surface of radius $r < R$. On the other hand, for a solid sphere we may write, using Gauss's law,

$$\varepsilon_0 E \, 4\pi r^2 = \rho \frac{4\pi}{3} r^3 \qquad r < R$$

or

$$E = \frac{\rho r}{3\varepsilon_0}$$

where ρ is the constant charge density.

[b] CORRECT ANSWER: $\vec{F} = q\vec{E}$

The electric field $\vec{E}$ is defined by the relation

$$\vec{F} = q\vec{E}$$

where $\vec{F}$ is the electrostatic force on the charge.

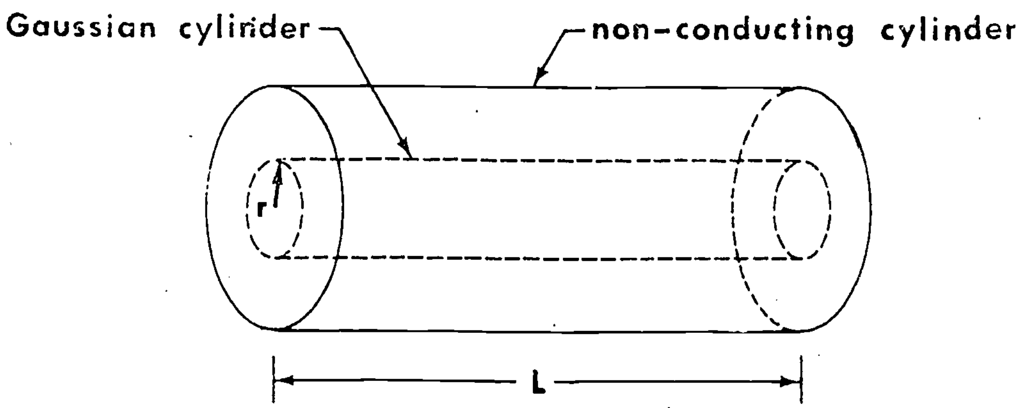
[a] CORRECT ANSWER: $\frac{\rho r}{2\varepsilon_0}$

A Gaussian cylinder inside the nonconducting cylinder yields

$$ES = \frac{q}{\varepsilon_0}$$

where q represents the *net* charge enclosed by the Gaussian cylinder. If ρ is the charge density (coul/m^3) within the nonconducting cylinder, then

$$q = (\pi r^2 L)\rho$$



Notice that $\pi r^2 L$ is the *volume* of the Gaussian cylinder. Therefore,

$$ES = \frac{\pi r^2 L\rho}{\varepsilon_0}$$

If we substitute for S the relationship for the *area* of the Gaussian cylinder, we obtain

$$E(2\pi rL) = \frac{\pi r^2 L\rho}{\varepsilon_0}$$

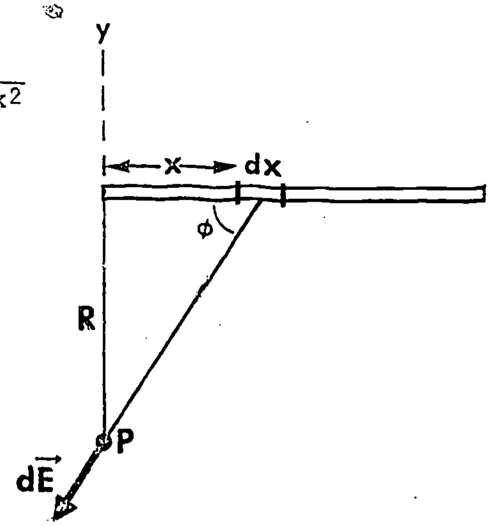
This expression can be reduced to

$$E = \frac{\rho r}{2\varepsilon_0}$$

[a] CORRECT ANSWER: 45°

The electric field at P due to a charge element λdx is given by the expression

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{R^2 + x^2}$$



The x and y components of this field are

$$dE_x = -dE \cos\phi = -\frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda dx}{R^2 + x^2}\frac{R}{\sqrt{R^2 + R^2}}\right)$$

and

$$dE_y = -dE \sin\phi = -\frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda dx}{R^2 + x^2}\frac{R}{\sqrt{R^2 + x^2}}\right)$$

The x and y components of the field at P is obtained by integrating dE_x and dE_y over x from 0 to ∞:

$$E_x = \int_0^\infty dE_x = -\frac{1}{4\pi\varepsilon_0}\int_0^\infty \frac{\lambda x dx}{(R^2 + x^2)^{3/2}} = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{R}$$

$$E_y = \int_0^\infty dE_y = -\frac{1}{4\pi\varepsilon_0}\int_0^\infty \frac{\lambda R dx}{(R^2 + x^2)^{3/2}} = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{R}$$

Since $E_x = E_y$, it is clear that the angle θ is 45°.

[b] CORRECT ANSWER: Zero

In the absence of sources or sinks, the flux through any *closed* surface is zero. This really says that for continuous flow "what goes in the closed surface must come out."

[a] CORRECT ANSWER: 8.1×10^8 nt

Let the charges be q and (Q - q). The magnitude of the coulomb force of repulsion $\vec{F}$ due to charges q and (Q - q) distance R apart is

$$F = \frac{q(Q - q)}{4\pi\varepsilon_0 R^2} \qquad (1)$$

F is a function of q, and in order to maximize F, we defferentiate F with respect to q and equate to zero. Thus

$$\frac{dF}{dq} = \frac{Q}{4\pi\varepsilon_0 R^2} - \frac{2q}{4\pi\varepsilon_0 R^2} = 0$$

Therefore,

$$q = \frac{Q}{2}$$

for maximum electrostatic repulsion.

The maximum force of repulsion F_{max} is obtained by substituting $q = \frac{Q}{2}$ in equation (1). Thus

$$F_{max} = \frac{Q^2/4}{4\pi\varepsilon_0 R^2} = 8.1 \times 10^8 \text{ nt}$$

[b] CORRECT ANSWER: 1 coul

To solve this problem, we should first determine how much charge is going to be on the inner surface of the conductor. For this purpose, we can apply Gauss's law by choosing a Gaussian surface which lies within the conductor. Since we know that E = 0 within any conductor, the net charge within the Gaussian surface must be zero. This means that the inner conductor surface must have a charge of +2 coul since there is a charge of -2 coul inside the conductor.

Therefore, with +2 coul on the inner surface, the outer surface must have a charge of +1 coul.

When another charge of -4 coul is brought near the conductor, nothing changes insofar as the net charges are concerned. Of course, the external charge will influence the distribution of charges over the outer surface but the net charge remains unaffected.

[a] CORRECT ANSWER: 1.8×10^{10} nt/coul

Gauss's law gives

$$\epsilon_0 \oint \vec{E}\cdot d\vec{S} = q$$

From symmetry, $\vec{E}$ due to a uniform linear charge can only be radially directed. As a Gaussian surface, we choose a circular cylinder of radius r and length L, closed at each end by plane caps normal to the axis. E is constant over the cylindrical surface and the area of the surface is $2\pi rL$. There is no flux through the circular caps because $\vec{E}$ here lies in the surface at every point. The charge enclosed by the Gaussian surface is λL. Gauss's law

$$\epsilon_0 \oint \vec{E}\cdot d\vec{S} = q$$

then becomes

$$\epsilon_0 E\, 2\pi rL = \lambda L$$

whence

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Substituting numerical values, we obtain

$$E = \frac{2}{2\pi\epsilon_0(2)} = \frac{2}{4\pi\epsilon_0} = 2 \times 9 \times 10^9 = 1.8 \times 10^{10} \text{ nt/coul}$$

[b] CORRECT ANSWER: $\vec{a} = \frac{q}{m}\vec{E}$

The force on q due to $\vec{E}$ is

$$\vec{F} = q\vec{E}$$

and, from Newton's second Law,

$$\vec{F} = m\vec{a}$$

or

$$q\vec{E} = m\vec{a}$$

Finally,

$$\vec{a} = \frac{q}{m}\vec{E}$$

[a] CORRECT ANSWER: a) $2.0 \times 10^5\ \hat{i}$ nt/coul b) $1.0 \times 10^{-2}\ \hat{i}$ nt

a) The magnitude of electric field due to a charge q at a distance r from the charge is

$$E = \frac{q}{4\pi\varepsilon_o r^2}$$

The direction of $\vec{E}$ is radially outward from q when q is positive, and inward when q is negative. Therefore, the field at the center due to charges placed at corners a and c is

$$E_{ac} = \frac{q}{4\pi\varepsilon_o\left(\frac{R}{\sqrt{2}}\right)^2} + \frac{q}{4\pi\varepsilon_o\left(\frac{R}{\sqrt{2}}\right)^2} = \frac{q}{\pi\varepsilon_o R^2}$$

and is towards the corner c. Similarly

$$E_{bd} = \frac{q}{\pi\varepsilon_o R^2}$$

and is towards the corner d, and

$$\vec{E} = \vec{E}_{ac} + \vec{E}_{bd}$$

$$= \frac{2q}{\pi\varepsilon_o R^2}\cos 45°\ \hat{i}$$

Therefore

$$E = 2.0 \times 10^5 \text{ nt/coul}$$

and is towards the positive x-axis.

b) The force on a charge Q placed at a point where the field is $\vec{E}$ may be computed from

$$\vec{F} = \vec{E}Q$$

Therefore

$$F = 1.0 \times 10^{-2} \text{ nt}$$

and is directed towards the positive x-axis. The force may also be calculated using Coulomb's law and is left as an exercise for the student.

[a] CORRECT ANSWER: $\phi_E = \frac{q_1 + q_2 + \dots q_n}{\varepsilon_o}$

The relationship for flux, inrespective of the size or shape of the closed surface, is

$$\phi_E = \frac{\Sigma q}{\varepsilon_o}$$

where Σq is the algebraic sum of all charges *inside* the *closed* surface.

In this case, $q_1, q_2, \dots q_n$ are interior to the surface, so

$$\phi_E = \frac{q_1 + q_2 + \dots q_n}{\varepsilon_o}$$

[b] CORRECT ANSWER: Zero

The magnitude of the electric field may be obtained by using Gauss's law:

$$\varepsilon_o \oint \vec{E} \cdot d\vec{S} = q$$

We may choose our Gaussian surface to be a coaxial cylindrical surface of length L and radius r > b. Since the total charge contained inside the Gaussian surface is zero and E on this surface is constant, we obtain

$$\varepsilon_o \, E \, 2\pi r L = 0$$

Therefore

$$E = 0$$

[c] CORRECT ANSWER: $1.6 \times 10^{-21} \, \hat{j}$ coul-m

The dipole moment $\vec{p}$ due to the given pair of charges of magnitude q and separated by a distance ℓ is given by

$$\vec{p} = \ell q \, \hat{j}$$

Since the dipole moment vector is directed from (-) toward the (+) charge along the dipole axis. Therefore,

$$\vec{p} = 1.6 \times 10^{-21} \, \hat{j} \text{ coul-m}$$

[a] CORRECT ANSWER: 2.0 m

The magnitude of the electric field due to an infinite line charge a distance r is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

and is away from the charge for positive λ and towards the charge for negative λ. We are asked to find a point between A and B such that the electric field is zero. Therefore,

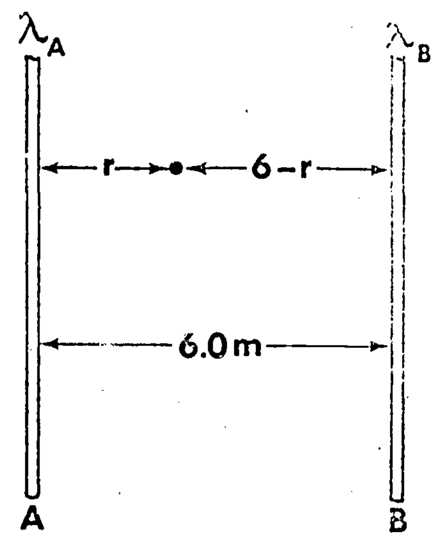
$$0 = \frac{\lambda_A}{2\pi\varepsilon_0 r} - \frac{\lambda_B}{2\pi\varepsilon_0 (6 - r)}$$

where r is the distance in meters from point A, or

$$0 = \frac{3}{2\pi\varepsilon_0}\left(\frac{1}{r} - \frac{2}{6 - r}\right)$$

Therefore,

$$2\,r = 6 - r \qquad \text{or} \qquad r = 2.0 \text{ m}$$



[b] CORRECT ANSWER: 2.4×10^{21} m/sec^2

The electric field in the region between two shells may be obtained from Gauss's law:

$$\varepsilon_0 \int \vec{E}\cdot d\vec{S} = q$$

Thus the magnitude of the electric field at the surface of the inner sphere is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R_1^2} = \frac{9 \times 10^9 \times 6 \times 10^{-2}}{(.2)^2} = 1.4 \times 10^{10} \text{ nt/coul}$$

The acceleration on the electron at the surface of the inner sphere is

$$a = \frac{eE}{m} = 1.76 \times 10^{11} \times 1.4 \times 10^{10} = 2.4 \times 10^{21} \text{ m/sec}^2$$

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

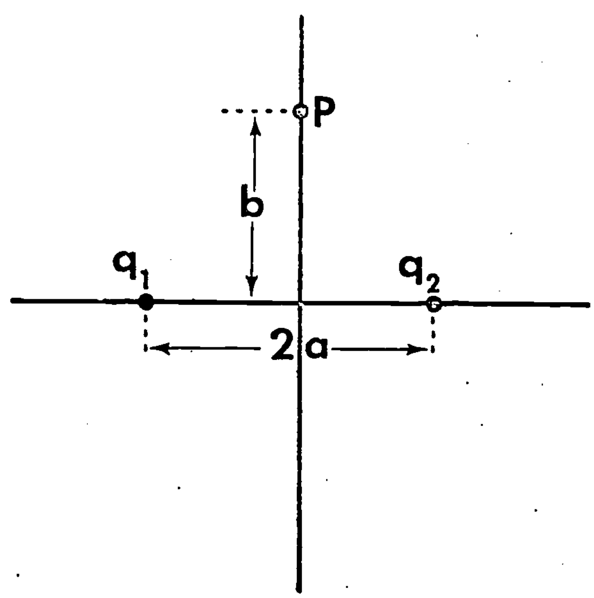
1 (24-1). A particular electric field can be described by the following equation:

$$\vec{E} = 4x^2 \ \hat{i}$$

How much work must be performed to move a charge $q = 3.0$ coul from $x = 10$ m to $x = 5.0$ m?

2 (24-11). What is the electric potential at a distance 3×10^{-3} m from a charge of 3×10^{-5} coul?

3 (24-15). Two charges, q_1 and q_2, are a distance 2 a apart. Find the potential due to the system of charges at point P shown below (q_1 and q_2 are equal distances from the vertical line).



4 (25-1). An electric potential at a point (x, y, z) in rectangular coordinates is given as

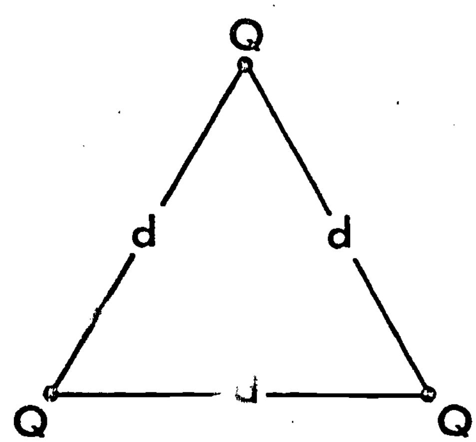
$$V = C(x + y + z)$$

where C is a constant. Find an expression for the force exerted on a charge q at point (x, y, z).

5 (25-6, 24-6). What is the charge density on the surface of a conducting sphere of radius 0.10 m whose potential is 500 volts?

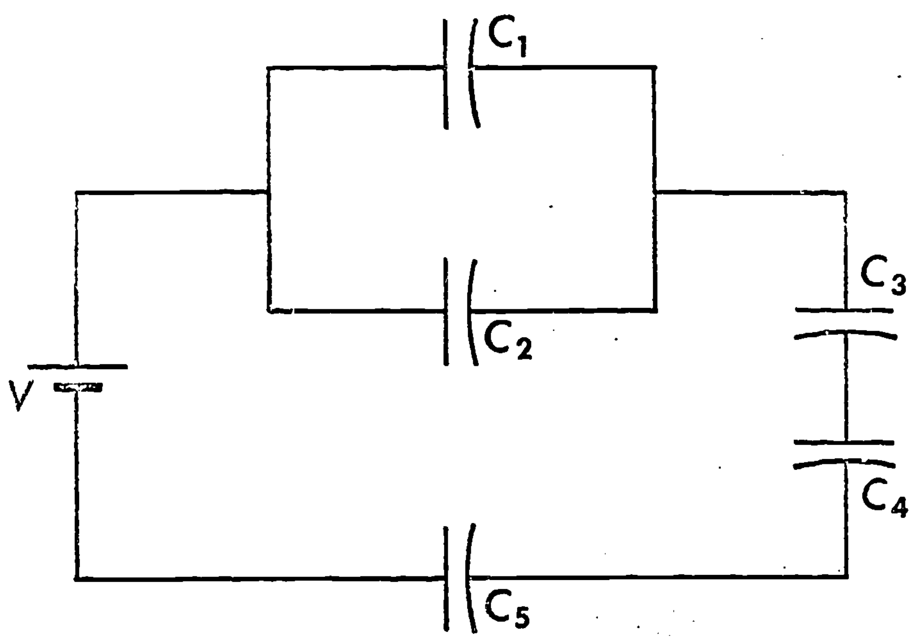
6 (25-10). A circular metal ring has a radius a and charge per unit length λ uniformly distributed over the ring. What is the electric potential at a point on the axis at distance y from the plane of the ring?

7 (25-14). Calculate the work required to assemble the three charges shown in the following diagram, starting with the charges at infinity.



8 (26-6, 26-1). What is the capacitance of a spherical capacitor consisting of two concentric spherical shells of radii 2.0 m and 3.0 m?

9 (26-10). For the circuit shown below, what is the equivalent capacitance?



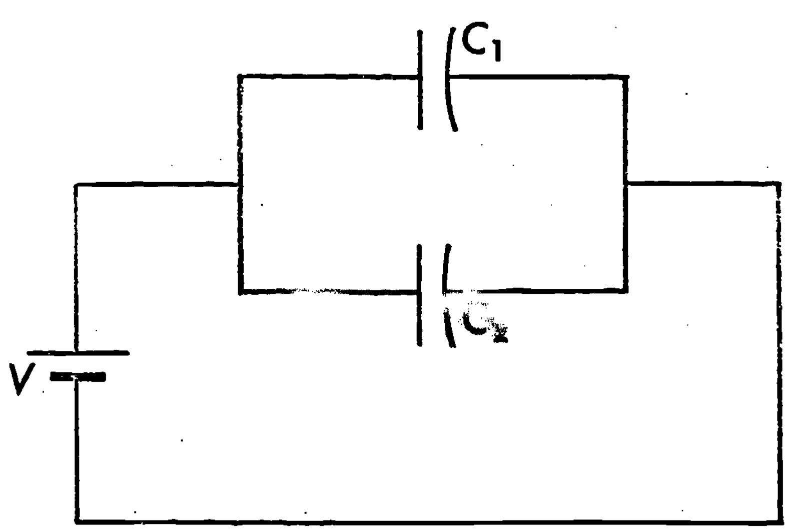
$V = 12$ volts

$C_1 = C_3 = 2.0\ \mu f$

$C_2 = 1.0\ \mu f$

$C_4 = C_5 = 3.0\ \mu f$

10 (26-15). In the diagram below, what fraction of the total charge will be on one plate of the capacitor C_2?



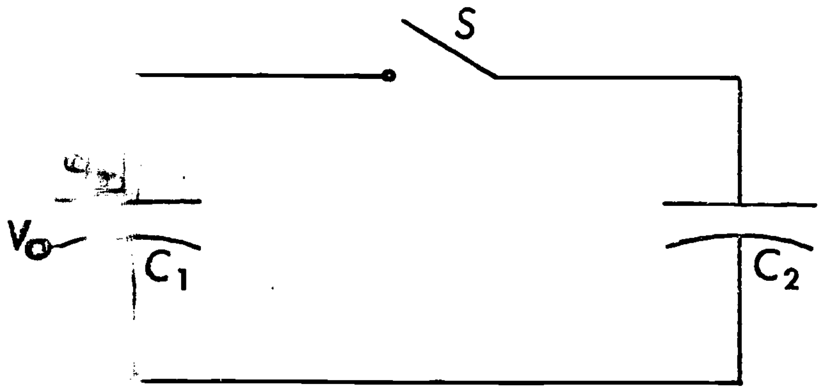
$C_1 = 1.0\ \mu f$

$C_2 = 10\ \mu f$

V = unknown

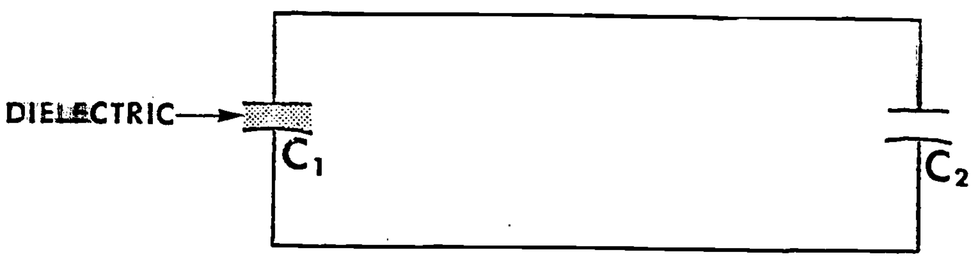
11 (27-1) ... the work required to move an infinitesimal charge dq from one ... a capacitor to the other in terms of dq, the charge on the capac... nd the capacitance C.

12 (27-6). ...acitor C_1 is initially charged to a potential difference V_o. The c... battery is then removed and the capacitor is connected to an unch... capacitor C_2. What is the difference in the stored energy before and ... the switch is thrown?



13.(27-10). Two identical capacitors have their plates initially separated by a layer of air. A sheet of mica (κ = 8) is then inserted into the air space of the second capacitor, completely filling it. What is the ratio of the capacitance without dielectric to that of the other; namely, C_a/C_d?

(2 6). An air capac tor having a capacitance $C_1 = 1.0$ μf is connected
-volt battery. After the capacito fully charged, it is
ected from the battery and filled wit dielectric material of
tric constant $\kappa = 3.0$. If the capacit with the dielectric is now
nnected to another uncharged capacitor C_2 .0 μf as shown in the
m, find the energy stored in the final system.



[a] CORRECT ANSWER: $\frac{3}{4\pi\varepsilon_o}\frac{Q^2}{d}$

The work required to assemble a charge configuration is equal to the potential energy of the configuration. In computing the potential energy we must take every possible pair into account. The potential energy between neighboring charges is (for each pair)

$$U_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q^2}{d}$$

and there are three such pairs. Consequently,

$$U = 3U_1 = \frac{3}{4\pi\varepsilon_o}\frac{Q^2}{d}$$

[b] CORRECT ANSWER: 1/8

The capacitance of a parallel plate capacitor is given by $C = \kappa\varepsilon_o A/d$ where A is the area and d is the separation of the plates. Thus, the ratio of the capacitances is:

$$\frac{C_a}{C_d} = \frac{\varepsilon_o A/d}{\kappa\varepsilon_o A/d} = \frac{1}{\kappa} = \frac{1}{8}$$

where the subscript d and a refer to dielectric and air respectively. (We have taken κ for air to be equal to 1.)

[c] CORRECT ANSWER: 9×10^7 volts

The potential due to a point charge q at a distance r from it is

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

Thus,

$$V = \frac{1}{4\pi\varepsilon_o}\frac{3 \times 10^{-5}}{3 \times 10^{-3}} = 9 \times 10^9 \times 10^{-2} = 9 \times 10^7 \text{ volts}$$

[a] CORRECT ANSWER: $-Cq(\hat{i} + \hat{j} + \hat{k})$

The force on a charge q in an electric field $\vec{E}$ is given by

$$\vec{F} = q\vec{E}$$

$\vec{E}$ may be found from the electric potential,

$$\vec{E} = -\vec{\nabla}V \equiv -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

In this case $V = C(x + y + z)$, and

$$E_x = -\frac{\partial V}{\partial x} = -C$$

$$E_y = -\frac{\partial V}{\partial y} = -C$$

$$E_z = -\frac{\partial V}{\partial z} = -C$$

so

$$\vec{F} = q(-C\hat{i} - C\hat{j} - C\hat{k})$$

$$= -Cq(\hat{i} + \hat{j} + \hat{k})$$

[b] CORRECT ANSWER: 10/11

Since t[illegible]ntial difference across C_1 and C_2 will be the same, we have

$$q_1 = C_1V \qquad (1)$$

and

$$q_2 = C_2V \qquad (2)$$

The total charge q on the combination is

$$q = q_1 + q_2 = (C_1 + C_2)\,V \qquad (3)$$

Thus, the fraction of the total charge on C_2 is

$$\frac{q_2}{q_1 + q_2} = \frac{C_2}{C_1 + C_2} = \frac{10}{11}$$

[a] CORRECT ANSWER: $\dfrac{C_1 C_2 V_o^2}{2(C_1 + C_2)}$

The initial stored energy is

$$U_o = \frac{1}{2} C_1 V_o^2 \qquad (1)$$

The final stored energy is

$$U = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 \qquad (2)$$

It is necessary to express V in terms of the initial potential difference V_o. After the switch is closed, the original charge q_o is shared by the two capacitors. Thus,

$$q_o = q_1 + q_2$$

Applying the relation q = CV, we obtain

$$C_1 V_o = C_1 V + C_2 V$$

or

$$V = V_o \frac{C_1}{C_1 + C_2} \qquad (3)$$

Substituting (3) into (2), we have

$$U = \frac{1}{2} (C_1 V^2 + C_2 V^2) = \frac{1}{2} (C_1 + C_2) \left(\frac{V_o C_1}{C_1 + C_2}\right)^2 = \frac{C_1}{C_1 + C_2} U_o$$

Thus the difference in the stored energy is

$$U_o - U = \frac{C_2}{C_1 + C_2} U_o = \frac{1}{2} \left(\frac{C_1 C_2}{C_1 + C_2}\right) V_o^2$$

Note $U < U_o$. This difference in the energy appears as heat in the connecting wires.

[a] CORRECT ANSWER: 6.7 × 10^{-10} f

The capacitance of a capacitor is given by $C = \frac{Q}{V}$, where V is the magnitude of the potential difference between the plates, and Q is the magnitude of charge on one plate. The potential difference V between two concentric spheres is

$$V = -\int_a^b \vec{E}\cdot d\vec{\ell}$$

The electric field intensity between the two spherical shells is

$$E = \frac{Q}{4\pi\varepsilon_o}\frac{1}{r^2}$$

$$V = -\int_a^b \vec{E}\cdot d\vec{\ell} = -\frac{Q}{4\pi\varepsilon_o}\int_a^b \frac{dr}{r^2} = -\frac{Q}{4\pi\varepsilon_o}\left(-\frac{1}{r}\right)_a^b$$

$$\text{magnitude } V = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{a} - \frac{1}{b}\right) \quad b > a$$

Substituting this expression of V into the expression for capacitance C, we obtain

$$C = 4\pi\varepsilon_o \frac{ab}{b - a} = \frac{1}{9 \times 10^9}\left(\frac{6.0}{1.0}\right) = 6.7 \times 10^{-10} \text{ f}$$

[b] CORRECT ANSWER: $\frac{q}{C}$ dq

The work required to move a charge dq through a potential difference V is

$$dW = V\,dq$$

Using the definition of capacitance, C - q/V, we find the potential difference across a capacitor with capacitance C carrying a charge q

$$V = \frac{q}{C}$$

Therefore,

$$dW = \frac{q}{C}\,dq$$

[a] CORRECT ANSWER: 4.4 10^{-8} coul/m^2

The potential on the conductor is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad (1)$$

where R is the radius of the sphere. The charge may be considered to be concentrated at the center of the sphere.

In this problem, the potential and the radius of the sphere are given. Thus from (1), we may write

$$q = 4\pi\varepsilon_0 VR \qquad (2)$$

The charge density on the surface of the sphere is given by

$$q = 4\pi R^2\sigma \qquad (3)$$

Combining equations (2) and (3), we obtain

$$\sigma = \frac{4\pi\varepsilon_0}{4\pi R} V = 4.4 \times 10^{-8} \text{ coul/m}^2$$

[b] CORRECT ANSWER: 0.67 μf

For the capacitors C_1 and C_2, the equivalent capacitance is simply the sum of them:

$$C_0 = C_1 + C_2$$

For the series combination of C_0, C_3, C_4 and C_5 the equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C_0} + \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5}$$

Substituting numerical values, we obtain

$$\frac{1}{C} = \frac{1}{3.0} + \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{3.0}$$

$$= \frac{1.0}{2.0} + 1.0 = \frac{3.0}{2.0}$$

or

$$C = \frac{2.0}{3.0}\ \mu f = .67\ \mu f$$

[a] CORRECT ANSWER: $\dfrac{q_1 + q_2}{4\pi\varepsilon_o \sqrt{a^2 + b^2}}$

The expression for the potential due to the two charges is

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

r_1 is the distance from q_1 to P and r_2 is the distance from q_2 to P. By the Pythagorean theorem,

$$r_1^{\,2} = a^2 + b^2$$

$$r_2^{\,2} = a^2 + b^2$$

and V becomes

$$V = \frac{1}{4\pi\varepsilon_o}\,\frac{q_1 + q_2}{\sqrt{a^2 + b^2}}$$

[b] CORRECT ANSWER: 6.3×10^{-4} j

The total charge in the capacitor system remains unaltered since C_2 is originally uncharged. Thus, the net charge on each plate of an equivalent capacitor C is

$$Q = C_1 V \qquad (1)$$

The equivalent capacitance of the final system is

$$C = \kappa C_1 + C_2 \qquad (2)$$

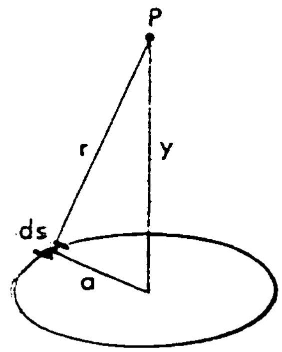
and the final stored energy of the system may be obtained from

$$E_f = \frac{1}{2}\,\frac{Q^2}{C}$$

or

$$E_f = \frac{1}{2}\,\frac{C_1^{\,2}\,V^2}{(\kappa C_1 + C_2)} = 6.3 \times 10^{-4} \text{ j}$$

[a] CORRECT ANSWER: $\frac{\lambda a}{2\varepsilon_o(y^2 + a^2)^{1/2}}$



$$dV = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_o}\frac{\lambda ds}{(y^2 + a^2)^{1/2}}$$

The total contribution to the potential due to the ring is obtained by integrating over the entire ring,

$$V = \frac{1}{4\pi\varepsilon_o}\int_0^{2\pi a}\frac{\lambda dS}{(a^2 + y^2)^{1/2}} = \frac{1}{4\pi\varepsilon_o}\frac{\lambda}{(y^2 + a^2)^{1/2}}\int_0^{2\pi a} ds$$

$$= \frac{\lambda}{4\pi\varepsilon_o}\frac{2\pi a}{(y^2 + a^2)^{1/2}} = \frac{\lambda a}{2\varepsilon_o(y^2 + a^2)^{1/2}}$$

[b] CORRECT ANSWER: 3.5×10^3 j

The work done by an external agent in moving a charge q through a displacement $d\vec{x}$ is

$$dW = \vec{F}\cdot d\vec{x}$$

Where $\vec{F}$ is the force, the external agent must apply to keep the charge q from accelerating and is exactly equal to $-q\vec{E}$. Thus

$$dW = -q\vec{E}\cdot d\vec{x}$$

or

$$dW = -q4x^2dx(\hat{i}\cdot\hat{i})$$

$$= -q4x^2dx$$

Therefore the total work W is

$$W = \int_{10}^{5} -4qx^2dx$$

$$= -4q\left[\frac{x^3}{3}\right]_{10}^{5}$$

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

1 (28-6). Current enters a cylindrical wire of radius $r_1 = 1/4$ cm. The wire eventually tapers down to a radius $r_2 = 1/8$ cm. Find the ratio of the current density j_1/j_2. j_1 and j_2 are the current densities in the portions of the wire where the radii are r_1 and r_2 respectively.

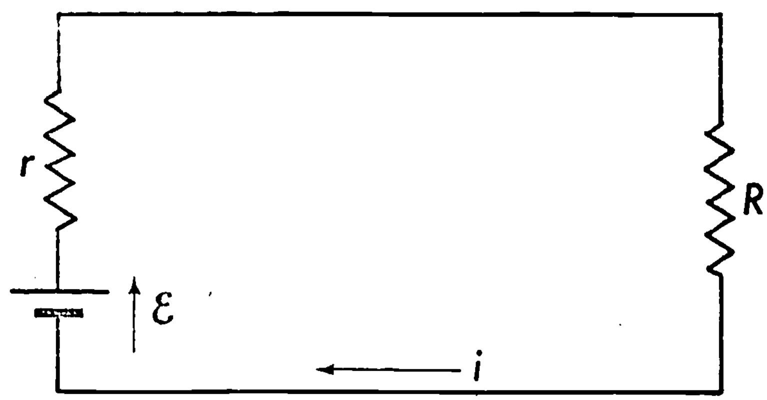
2 (28-15, 28-10). What is the current through a wire of length ℓ, cross-sectional area A, and resistivity ρ, through which power P is being dissipated?

3 (29-1, 28-1). Two electrodes are placed in a solution that obeys Ohm's law. With a potential difference of 100 V across the electrodes a current of 5 amps flows in the circuit. What is the resistance of the solution?

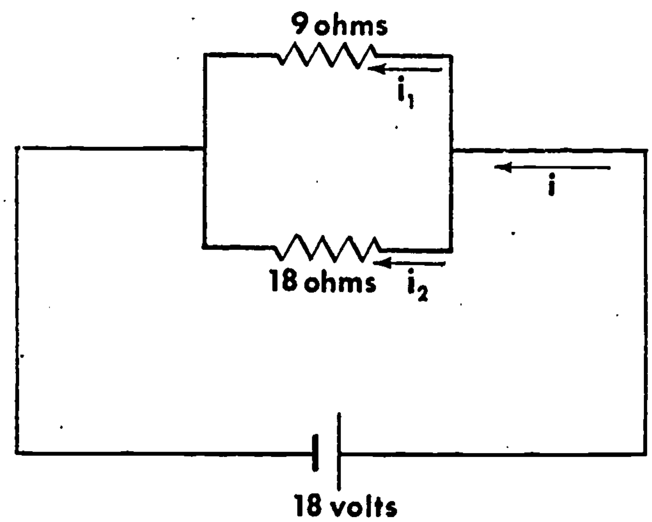
4 (29-5). A small commercial hot water heater generates 500 watts of heat when it is connected to 120 volts source. The original heating unit is now replaced by the one whose resistance is 80% of the original one. How much heat does the replaced heater generate when it is connected to the same source?

5 (29-9). How can the emf be defined in terms of the work dW done by a seat of emf in moving charge dq from a lower potential to a higher potential?

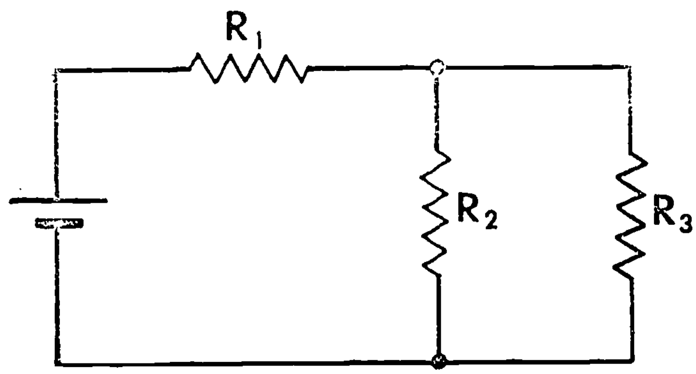
6 (29-15). In the figure shown below, the following values are given: ε = 10 volts, r = 1.5 ohms, R = 3.5 ohms. How many joules of electrical energy are converted into heat by resistor R in every second?



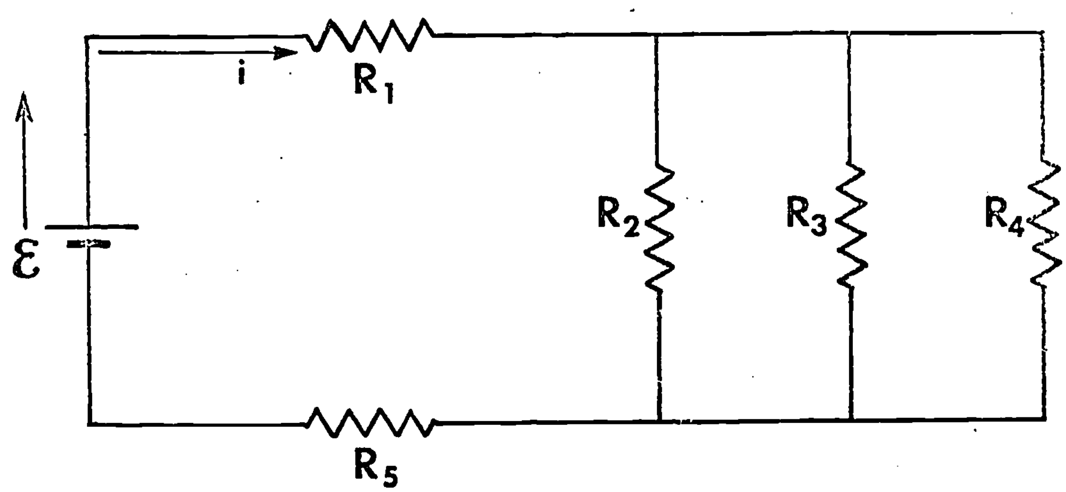
7 (29-21). In the circuit shown below, find the currents i_1, i_2, and i.



8 (30-5, 1). What is the equivalent resistance of the circuit below when each of the resistors has resistance r?



9 (30-10). What is the current i in the circuit shown below?



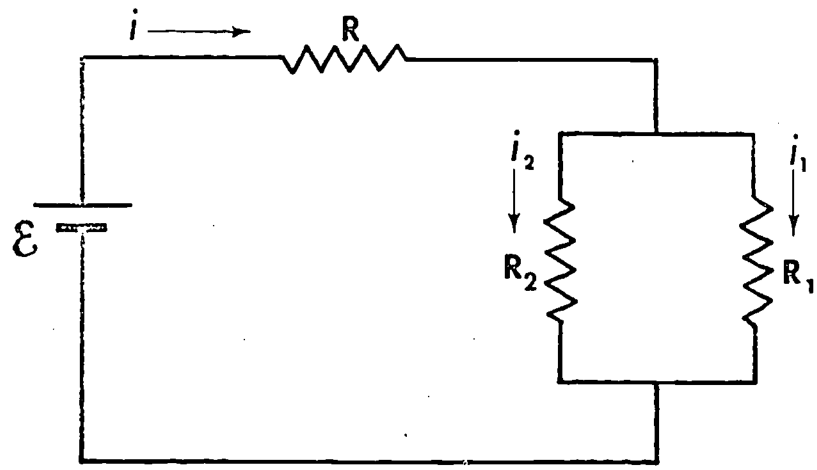
R_1 = 20 ohms

$R_2 = R_3 = R_4$ = 30 ohms

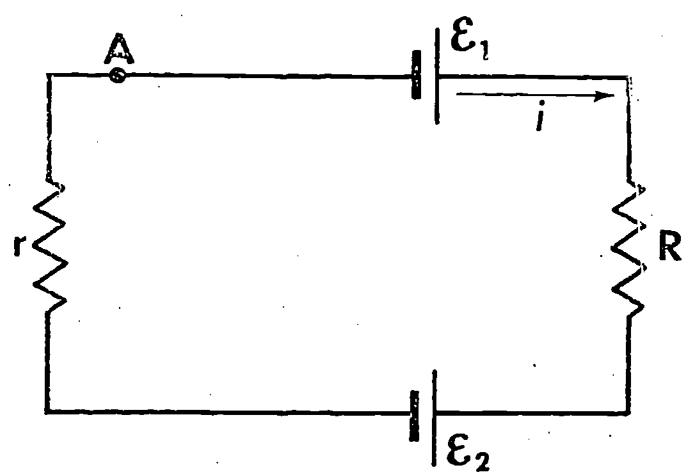
R_5 = 40 ohms

ε = 7 volts

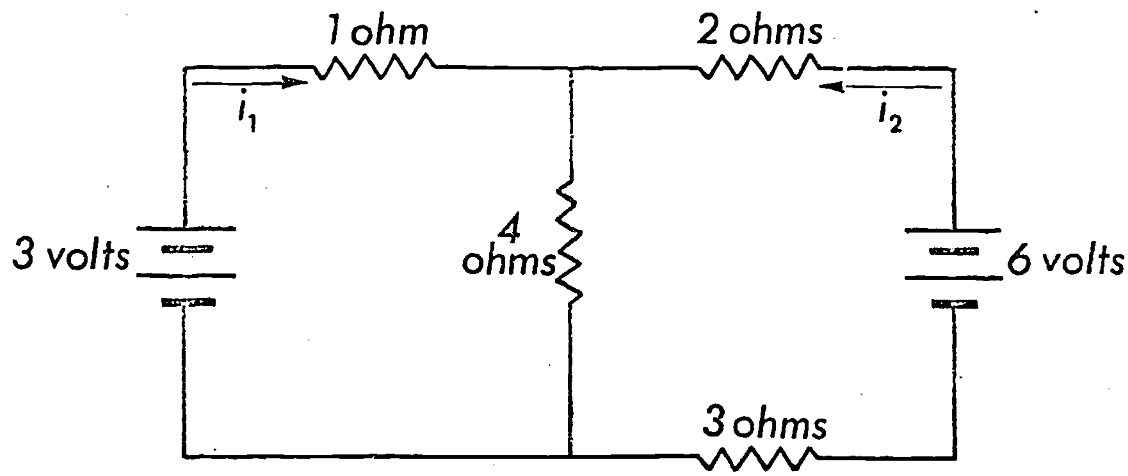
10 (30-16). Write one relation between currents i, i_1, and i_2 for the circuit below.



11 (30-19). Write the loop equation for the circuit shown below. Assume that current i flows clockwise through the circuit.

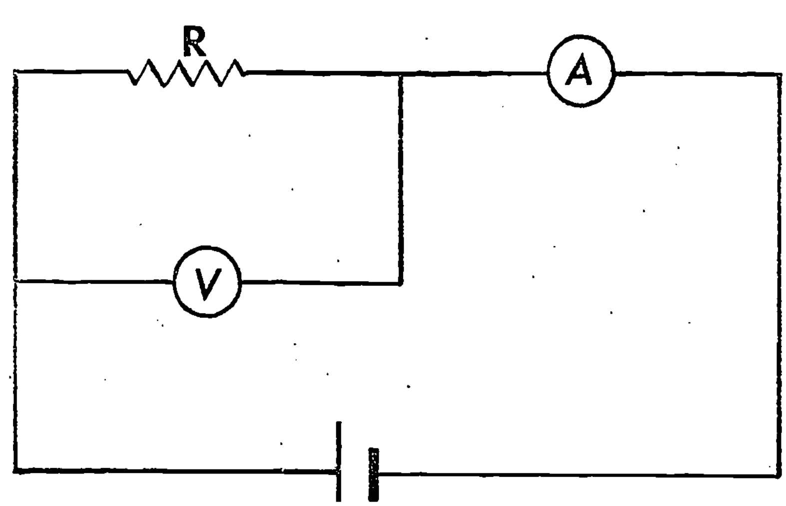


12 (30-23). Find the current in the 1-ohm resistor in the circuit shown below.

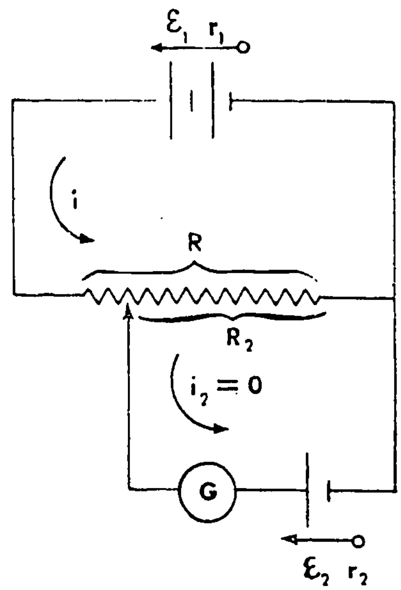


13 (31-1). A milliammeter which has a maximum deflection for $i = 10.0 \times 10^{-3}$ amp is converted to read in the range of 0 to 1.00 amp by means of a 5.00-ohm shunt. What is the internal resistance of the meter?

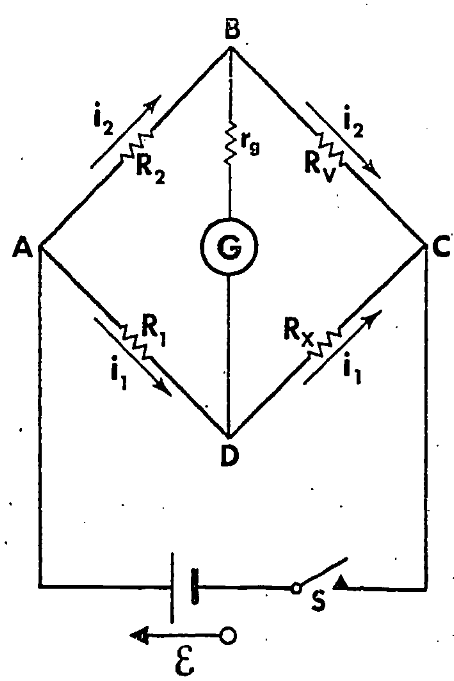
14 (31-7). A voltmeter and an ammeter are used to determine an unknown resistance R as shown in the following diagram. The voltmeter resistance is 400 ohms and the ammeter resistance is assumed to be negligible. The ammeter reads 0.4 amp and the voltmeter 25.0 volts. Compute the real value of R.



15 (31-11) The potentiometer depicted below is measuring voltage ε_2. Find ε_2 in terms of ε_1, r_1, R_2, and R.



16 (31-15). In the Wheatstone bridge shown below, R_1 = 5 ohms, ε = 10 volts, and i_1 = 0.2 amp. Find R_x.



[a] CORRECT ANSWER: 3r/2

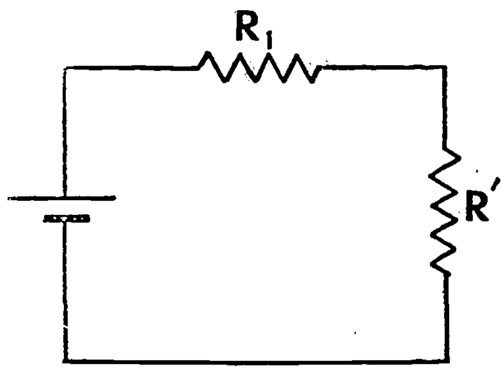
Resistors R_2 and R_3 are connected in parallel. Therefore, their equivalent resistance R' is given by

$$\frac{1}{R'} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{r} + \frac{1}{r} = \frac{2}{r}$$

or

$$R' = \frac{1}{2} r$$

The circuit can now be redrawn to include R', and

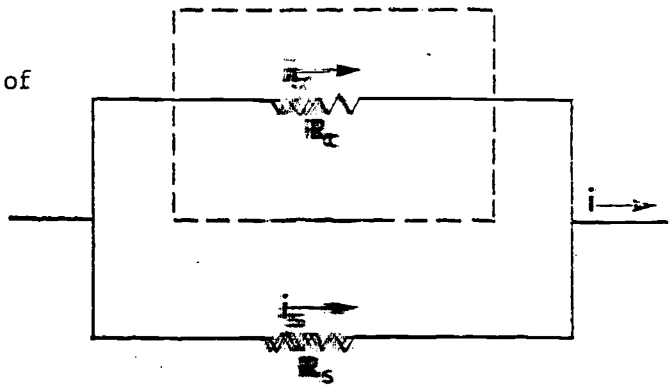


we see that R_1 and R' are connected in series, so the equivalent resistance of the entire circuit, R, is

$$\begin{aligned} R &= R_1 + R' \\ &= r + \frac{1}{2} r \\ &= 3r/2 \end{aligned}$$

[a] CORRECT ANSWER: 495 ohms

We wish to find R_c, the resistance of the ammeter, given



$$i = 1.00 \text{ amp}$$

$$i_c = 10.0 \times 10^{-3} \text{ amp}$$

and $R_s = 5.00$ ohms

Using loop and junction equations we obtain

$$i_s + i_c = i$$

$$i_c R_c = i_s R_s$$

Solving for R_c, we obtain

$$R_c = \frac{i_s R_s}{i_c} = \frac{(i - i_c)\, R_s}{i_c}$$

$$= 495 \text{ ohms}$$

[b] CORRECT ANSWER: 625 watts

The power generated by a heater is given by the expression $P = i^2 R$. Since both heaters are connected to the same source,

$$i_1 R_1 = i_2 R_2$$

and the ratio of the power generated is

$$\frac{i_1{}^2 R_1}{i_2{}^2 R_2} = \frac{(i_1 R_1)}{(i_2 R_2)} \cdot \frac{i_1}{i_2} = \frac{i_1}{i_2} = \frac{R_2}{R_1} = 0.8$$

Thus

$$i_2{}^2 R_2 = \frac{i_1{}^2 R_1}{0.8} = \frac{500}{0.8} = 625 \text{ watts}$$

[a] CORRECT ANSWER: $\varepsilon_1 - iR - \varepsilon_2 - ir = 0$

An organized method for solving these circuit problems is as follows:

(1) Select the direction in which you will traverse the ... This is arbitrary, either clockwise or counterclockw. .

(2) Write the ε's as positive if they are in the direction you have chosen; negative otherwise.

(3) Write iR's (the resistive potential drops) as negative if you go through the resistor in the direction of the current; positive otherwise.

For the circuit in this problem, starting at point A and going around in a clockwise sense,

$$\varepsilon_1 - iR - \varepsilon_2 - ir = 0$$

[b] CORRECT ANSWER: 14 watts

The current in this circuit is obtained by applying the loop theorem:

$$\varepsilon - ir - iR = 0$$

or

$$i = \frac{\varepsilon}{r + R}$$

Substituting numerical values, we obtain

$$i = \frac{10}{1.5 + 3.5} = 2 \text{ amps}$$

The rate of converting electrical energy into heat by R is given by the expression

$$\frac{dH}{dt} = i^2R$$

and is

$$\frac{dH}{dt} = 2^2 \times 3.5 = 14 \text{ watts}$$

[a] CORRECT ANSWER: $\frac{\varepsilon_1 R_2}{r_1 + R}$

The potentiometer is a null instrument; i.e., it makes measurement by giving a zero reading. In this case, ε_2 is measured by adjusting the variable resistance until $i_2 = 0$, as measured by the galvanometer. By writing down Kirchhoff's voltage rule for the upper and lower loops, we obtain

$$ir_1 + iR = \varepsilon_1$$

and

$$iR_2 =$$

Dividing the bottom equation by the top yields

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{iR_2}{i(r_1 + R)}$$

or

$$\varepsilon_2 = \frac{\varepsilon_1 R_2}{r_1 + R}$$

[b] CORRECT ANSWER: 0.1 amp

The equivalent resistance for resistances R_2, R_3 and R_4 connected in parallel is

$$\frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{3}{30}$$

or

$$R = 10 \text{ ohms}$$

Thus the current in the circuit is

$$i = \frac{\varepsilon}{R_1 + R + R_5} = \frac{7}{20 + 10 + 40} = 0.1 \text{ amp}$$

[a] CORRECT ANSWER: $(PA/\rho\ell)^{1/2}$

The resistance of a wire, in terms of the resistivity ρ, its length ℓ and its cross-sectional area A, is given by

$$R = \rho \frac{\ell}{A} \tag{1}$$

The power is given by

$$P = i^2 R$$

so

$$i = (P/R)^{1/2}$$

and, from (1),

$$i = (PA/\rho\ell)^{1/2}$$

[b] CORRECT ANSWER: 45 ohms

We need not concern ourselves with the upper path of the bridge. We know that the potential difference between points A and C is $\varepsilon = 10$ volts. Since resistors R_1 and R_x are connected in series, we obtain

$$i_1 (R_1 + R_x) = \varepsilon = 10 \text{ volts}$$

Thus,

$$R_x = \frac{\varepsilon - i_1 R_1}{i_1} = \frac{10 - 1}{0.2} = 45 \text{ ohms}$$

[a] CORRECT ANSWER: $i_1 = 2$ amps; $i_2 = 1$ amp; $i = 3$ amps

There are several approaches to this problem, but perhaps the most direct is the application of Kirchhoff's rules.

Three unknowns require three equations for their solution. By the first Kirchhoff rule, the current entering a branch point must equal the current leaving that point,

$$i = i_1 + i_2 \qquad (1)$$

By the second rule, the sum of all the emf changes around a loop must be zero. Traversing the outer loop counterclockwise, we have

$$18 - i_1 9 = 0 \qquad (2)$$

Similarly, for the inner loop

$$18 - i_2 18 = 0 \qquad (3)$$

Solving (1), (2), and (3) gives

$$i_1 = 2 \text{ amps}$$

$$i_2 = 1 \text{ amp}$$

$$i = 3 \text{ amps}$$

[b] CORRECT ANSWER: $i = i_1 + i_2$

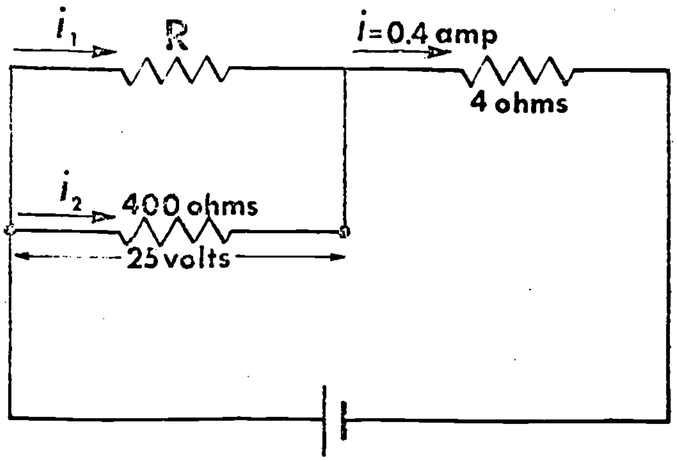
Kirchhoff's first rule states that the current entering a branch point is equal to the current leaving that branch point. That is,

$$i = i_1 + i_2$$

No other relations between these currents are attempted because we are not given any particulars about the resistances or emf.

[a] CORRECT ANSWER: 74.1 ohms

The circuit is essentially given in the following diagram and we may write down the following equations



$$i_1 R = i_2\, 400 = 25 \quad (1)$$

$$i_1 + i_2 = 0.4 \quad (2)$$

From (1) we get

$$i_2 = \frac{25}{400} = 0.0625 \text{ amp}$$

and from (2)

$$i_1 = 0.3375 \text{ amp}$$

Using the value of i_1, we obtain from (1)

$$R = \frac{25}{0.338} = 74.1 \text{ ohms}$$

[b] CORRECT ANSWER: 20 ohms

Using Ohm's law,

$$V = iR$$

$$R = \frac{V}{i}$$

$$= \frac{100 \text{ volts}}{5 \text{ amps}} = 20 \text{ ohms}$$

[a] CORRECT ANSWER: 1/4

The current density j is defined as

$$j = \frac{\text{Current}}{\text{Cross-Sectional Area}} \quad \frac{i}{A}$$

Therefore,

$$\frac{j_1}{j_2} = \frac{i}{A_1} \; \frac{A_2}{i} = \frac{A_2}{A_1}$$

or

$$\frac{j_1}{j_2} = \frac{\pi r_2^{\,2}}{\pi r_1^{\,2}}$$

$$= 1/4$$

[b] CORRECT ANSWER: $i_1 = 0.1$ amp

Let us assume a clockwise current in the left loop, i_1; and a counter-clockwise current i_2 in the right loop. The loop equation for the first loop is

$$3 - i_1 - 4(i_1 + i_2) = 0 \qquad (1)$$

For the second loop we have

$$6 - 2i_2 - 4(i_1 + i_2) - 3i_2 = 0 \qquad (2)$$

Solving equations (1) and (2) simultaneously, we obtain

$$i_1 = \frac{3}{29} \text{ amp} = 0.1 \text{ amp}$$

[c] CORRECT ANSWER: $\varepsilon = \frac{dW}{dq}$

The emf must do an amount of work dW on the (positive) charge dq to make it move to the point of higher potential. It is defined by the expression

$$\varepsilon = \frac{dW}{dq}$$

and has the units J/coul = volt.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

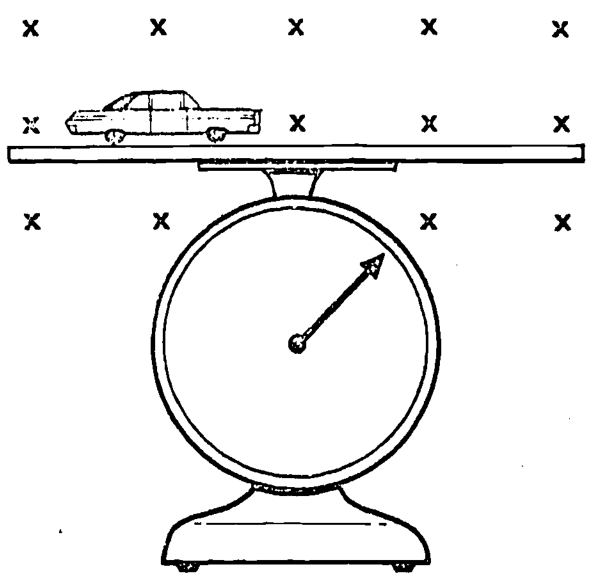
SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

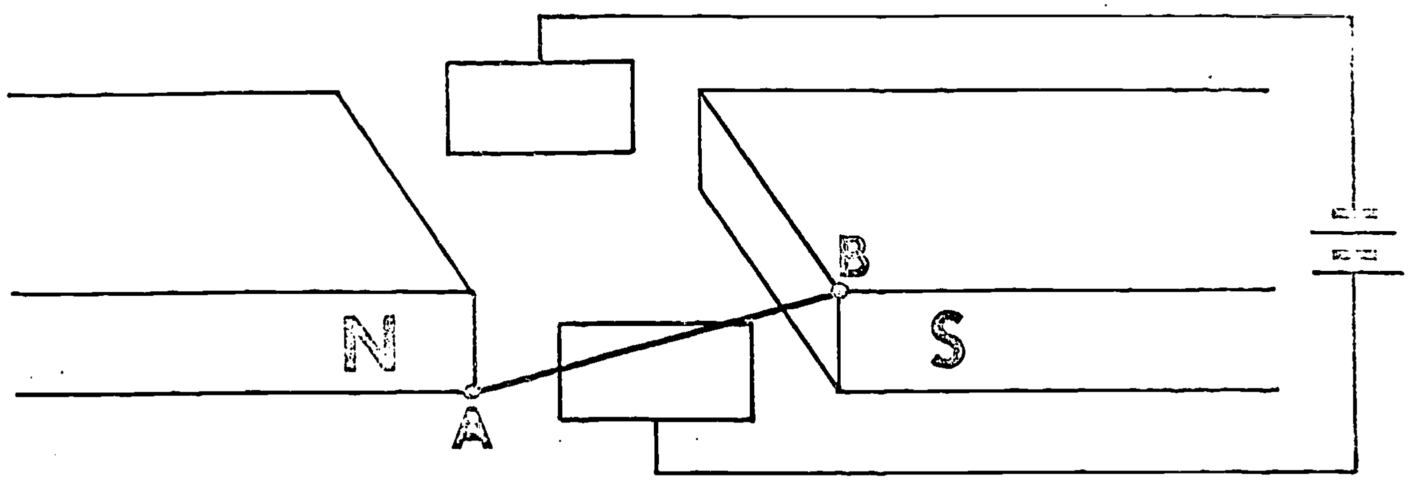
1 (32-1). A toy automobile carries an excess positive charge and rolls to the right along the platform of a scale as shown in the figure.

A magnetic induction field directed into the page surrounds the apparatus. The weight of the car is 2 ounces; will the scale read exactly 2 ounces, more than two ounces, or less than 2 ounces?



2((32-9, 4). A particle with mass 10^{-25} kg and charge 2e (3.2×10^{-19} coul) moves perpendicularly to a magnetic induction field of 1.0×10^{-4} T in circular orbit of radius .050 m. Find the tangential velocity of the particle.

3 (32-9, 32-4).

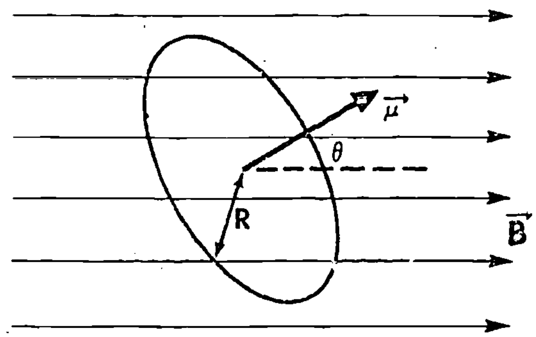


The above figure shows crossed electric and magnetic fields. The electric field intensity is 100 nt/coul, the magnetic induction is 2 T. A particle enters the field from below and travels along a line parallel to the line AB. This path makes an angle of 30° with the horizontal. With what speed must the particle move to remain undeflected in its path?

4 (33-1). A straight rod of length ℓ carries current i. A uniform magnetic induction field causes a force of magnitude F to be exerted on the rod. Find the component of magnetic induction which is perpendicular to the rod.

5 (33-9). A 10 cm by 10 cm square loop carrying a current of $i = 2$ amperes is in a uniform $\vec{B}$ field of magnitude $B = 3 \times 10^{-5}$ T at an angle of 30° with the normal to the plane of the loop. Find the magnitude of the torque experienced by the loop.

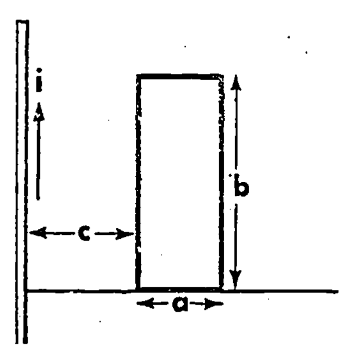
6 (34-1). A single loop of radius R, carrying a current i is placed in a uniform magnetic field $\vec{B}$ as shown in the diagram. Calculate the average torque acting on the loop as θ varies from 0 to π.



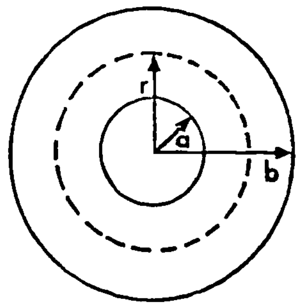
7 (34-5, 33-14). In the Bohr model of the hydrogen atom, an electron revolves around a nucleus in a circular orbit of radius $r = 5.00 \times 10^{-11}$ m with a speed $v = 2.25 \times 10^6$ m/sec. If the hydrogen atom is placed in a uniform magnetic field $\vec{B} = 5.00\, \hat{k}$ T, the magnetic dipole moment vector is found to make an angle of 60° with the z-direction. If, at a later time, the magnetic dipole moment vector lines up with the z-direction (i.e., $\mu\hat{k}$) find the change in the potential energy of the system.

8 (34-13). The magnitude of the Earth's magnetic induction at a place in the northern hemisphere is B = 58 μT. The inclination and declination are 73° N and 15° W respectively. What are the eastward (B_E), northward (B_N) and upward (or vertical B_V) components of B there?

9 (35-1, 34-9). A rectangular loop as shown in the diagram has dimensions a and b and its nearer side is at distance c from an infinitely long wire carrying a current i. What is the total magnetic flux through the loop?

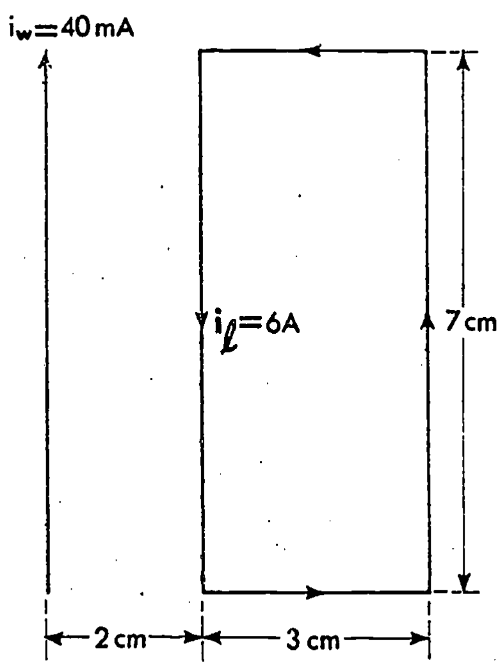


10 (35-6). A conducting cylindrical shell with inner and outer radii a and b respectively carries a current of 2 amp uniformly distributed over the cross section of the shell. For which values of r is the magnetic induction equal to zero?



11 (36-1). A solenoid which measures 20.0 cm in length and 5.00 cm in diameter is wound with 4 layers of 250 turns each. How much current i_0 must the windings carry in order to produce a magnetic field of 1.00×10^{-4} tesla inside the solenoid?

12 (36-15, 35-10, 33-15, 33-5). A rectangular loop 3 cm × 7 cm is oriented with its longer sides parallel to an infinitely long wire which is 2 cm from the near side of the loop. The currents in the wire and loop are respectively 40 milliamperes and 6 amp, and the currents in the wire and the *far* side of the loop are parallel. What is the force on the loop?



[a] CORRECT ANSWER: -2.25×10^{-23} j

The potential energy for a magnetic dipole moment in a magnetic field is:

$$U = -\vec{\mu}\cdot\vec{B}$$

However, the magnitude of μ is

$$\mu = iA \qquad (1)$$

where i and A are the current and area of the equivalent current loop and are given by

$$i = q_e \dot{i} = q_e \frac{w}{2\pi} = q_e \frac{v}{2\pi r} \qquad (2)$$

and

$$A = \pi r^2 \qquad (3)$$

Therefore

$$\mu = \frac{q_e v r}{2}$$

The change in the potential energy is

$$\Delta U = U_f - U_i = (-\mu B) - (-\mu B \cos 60°)$$

$$= -\frac{\mu B}{2} \qquad (4)$$

Substituting numerical values in equation (4), we obtain

$$\Delta U = -2.25 \times 10^{-23} \text{ j}$$

[b] CORRECT ANSWER: 1.59×10^{-2} amp

In the formula for the magnetic induction inside an ideal solenoid, $B = \mu_o i_o n$, n is the number of turns per unit length. For a solenoid wound with 4 layers of 250 turns each and of length 20 cm,

$$n = \frac{4 \times 250}{0.2} = 5000 \text{ m}^{-1}$$

Therefore, the current required to produce a field of 10^{-4} T is

$$i_o = \frac{B}{\mu_o n} = \frac{10^{-4}}{4\pi \times 10^{-7} \times 5 \times 10^3} = 1.59 \times 10^{-2} \text{ amp}$$

[a] CORRECT ANSWER: 16 m/sec

A charge q having mass m and velocity $\vec{v}$ in a magnetic induction field $\vec{B}$ experiences a force

$$\vec{F} = q\vec{v} \times \vec{B}$$

If the charge moves in a circle of radius r, this force must equal the centripetal force. In addition, it is given that $\vec{v}$ is perpendicular to $\vec{B}$:

$$\frac{mv^2}{r} = qvB$$

Solving for v and setting q equal to 2e, we have

$$v = \frac{2eBR}{m} = 16 \text{ m/sec}$$

[b] CORRECT ANSWER: $2iR^2B$

The torque acting on a current loop is

$$\bar{\tau} = \vec{\mu} \times \vec{B}$$

where

$$\vec{\mu} = i\vec{A}$$

The magnitude of the torque, when $\vec{\mu}$ makes an angle θ with the direction of $\vec{B}$ field is

$$\tau = iAB \sin\theta$$

or

$$\tau = i\pi R^2B \sin\theta$$

The average value of the torque $\bar{\tau}$ may be found from

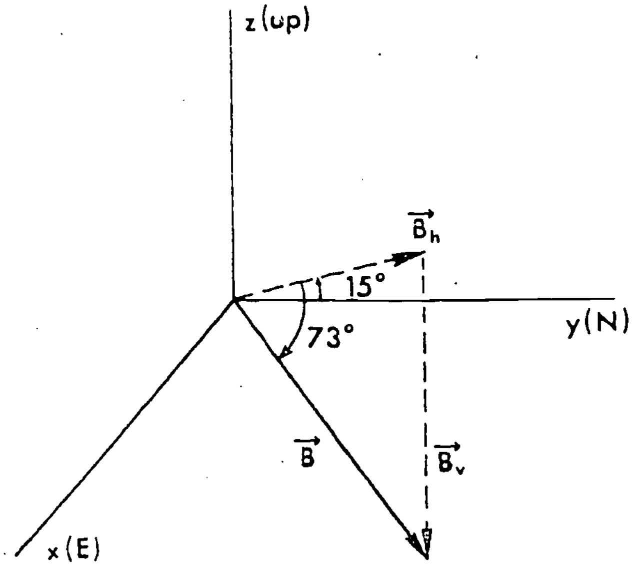
$$\bar{\tau} = \frac{\int_0^\pi r\,d\theta}{\int_0^\pi d\theta}$$

Therefore

$$\bar{\tau} = \frac{i\pi R^2B \int_0^\pi \sin\theta\, d\theta}{\int_0^\pi d\theta} = 2iR^2B$$

[a] CORRECT ANSWER: $B_E = -4.4\ \mu T$ $B_N = 16.4\ \mu T$ $B_v = -55.5\ \mu T$

We choose a right-handed coordinate system with origin at the point in question.



The vertical and horizontal components of $\vec{B}$ are given by

$$B_v = -B \sin 73° = -(58\ \mu T) \times 0.957 = -55.5\ \mu T$$

and

$$B_h = B \cos 73° = (58\ \mu T) \times 0.292 = 17\ \mu T$$

where the negative sign for B_v denotes that it is downward. We now resolve $\vec{B}_h$ into components along east and north.

$$B_E = -B_h \sin 15° = -(17\ \mu T) \times 0.259 = -4.4\ \mu T$$

and

$$B_N = B_h \cos 15° = (17\ \mu T) \times 0.966 = 16.4\ \mu T$$

[a] CORRECT ANSWER: $\frac{F}{i\ell}$

The force on the rod is

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

from which we can write

$$F = i\ell B_\perp$$

or

$$B_\perp = \frac{F}{i\ell}$$

where $B_\perp$ stands for the component of $\vec{B}$ normal to the rod.

[b] CORRECT ANSWER: Less than 2 ounces

A charge q moving with velocity $\vec{v}$ in a magnetic induction field $\vec{B}$ is acted upon by a force $\vec{F}$:

$$\vec{F} = q\vec{v} \times \vec{B}$$

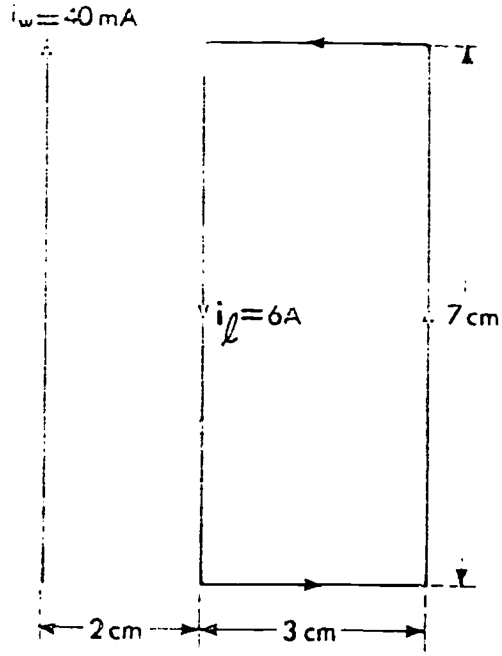
In this case, $\vec{v}$ is a vector pointing to the right, $\vec{B}$ is a vector pointing into the page, and the cross product $\vec{v} \times \vec{B}$ is therefore directed vertically upward.

The magnetic force is opposite to the direction of the weight; consequently, the resultant vertical force on the car (directed downward) is less than its weight.

[c] CORRECT ANSWER: $r \leq a$ and $r \to \infty$

Because of the cylindrical symmetry, Ampere's law gives $B = \mu_0 i/2\pi r$, where i is the current enclosed by a circle of radius r. The cylindrical shell has an inner radius a, and so $i = 0$ for $r \leq a$. Therefore, $B = 0$ for $r \leq a$. Furthermore, the current enclosed by an infinitely large circle is finite (namely 2 amp), but since $B \propto 1/r$, $B \to 0$ as $r \to \infty$.

[a] CORRECT ANSWER: 10^{-7} nt



We utilize the symmetry of the configuration to reason that the forces on the two short sides of the loop are equal and opposite and will add to zero. The force on the near side of the loop is repulsive (anti-parallel currents). Its magnitude is

$$F_1 = \frac{\mu_o i_w i_\ell}{2\pi d}\ \ell$$

where ℓ and d are 7 cm and 2 cm, respectively. Similarly, the force on the far side of the loop is attractive and has a magnitude of

$$F_2 = \frac{\mu_o i_w i_\ell}{2\pi d'}\ \ell$$

where d' = 2 cm + 3 cm = 5 cm. Obviously $F_1 > F_2$ so the net force is repulsive and has a magnitude of

$$F = F_1 - F_2 = \frac{\mu_o i_w i_\ell \ell}{2\pi}\left(\frac{1}{d} - \frac{1}{d'}\right)$$

$$= \frac{4\pi \times 10^{-7} \times 4 \times 10^{-2} \times 6 \times 7 \times 10^{-2}}{2\pi}\left(\frac{1}{0.02} - \frac{1}{0.05}\right)$$

$$= 3.36 \times 10^{-9} \times (50 - 20) = 1.01 \times 10^{-7} \text{ nt} \approx 10^{-7} \text{ nt}$$

next page

continued

We note that in addition to the above computed forces, each long side experiences forces resulting from the field set up by the current in the other long side and the current in the short sides as well. The net force on each side due to the current in the loop, however, is equal in magnitude and opposite in direction to the net force on the opposite side; as a result, these forces do not enter in the net force *on the loop.*

[a] CORRECT ANSWER: 3×10^{-7} nt-m

The relation between torque τ on a loop and field $\vec{B}$ is given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

where $\vec{\mu} = i\,\vec{A}$ is the magnetic dipole moment of the loop and A is the area of the loop. Thus

$$\tau = i\,A\,B\,\sin 30° = 3 \times 10^{-7} \text{ nt-m}$$

[b] CORRECT ANSWER: $\dfrac{\mu_o i b}{2\pi}\,\ell n\left(\dfrac{c + a}{c}\right)$

The magnetic induction at the distance x from the wire may be obtained from Ampere's law:

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_o i$$

and

$$B = \frac{\mu_o i}{2\pi x}$$

The flux is given by the expression

$$\phi = \int \vec{B}\cdot d\vec{S}$$

and since $\vec{B}$ and $\vec{S}$ are parallel to one another, we obtain the total flux by integrating over the area of the loop:

$$\phi = \int B\,dS = \int_c^{c+a} \frac{\mu_o i}{2\pi x}\,b\,dx = \frac{\mu_o i b}{2\pi}\,\ell n\left(\frac{c + a}{c}\right)$$

[a] CORRECT ANSWER: 100 m/sec

In order for the particle to remain undeflected, the electric and magnetic forces must be equal and oppositely directed. This will yield a net force of zero on the particle. Hence the force equation is written as

$$0 = q\vec{E} + q\vec{v} \times \vec{B}$$

or

$$0 = E - vB \sin\theta$$

where the minus sign is introduced to account explicitly for oppositely directed forces.

Solving for v yields

$$v = \frac{E}{B \sin\theta} = \frac{100}{2 \sin 30^\circ} = 100 \text{ m/sec}$$

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Self-Paced

Physics

SEGMENT SEPARATOR

note

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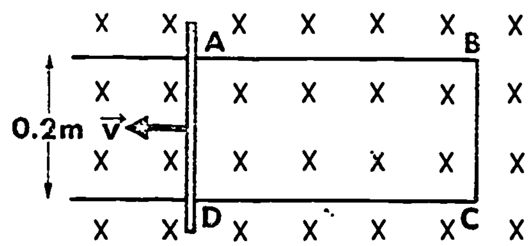
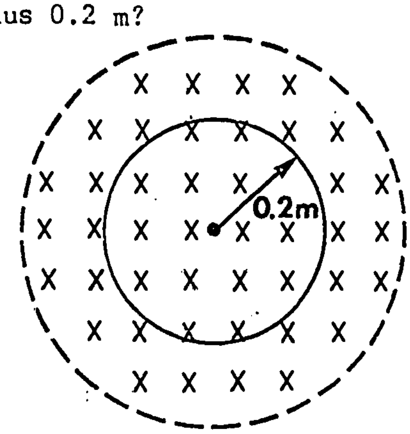
1 (37-1). A coil of 2 turns in a uniformly changing flux develops an emf of 10 volts. If this 2 turn coil is replaced by a 100 turn solenoid, find the emf developed across the ends of the solenoid.

2 (37-10, 6). A conducting rod AD makes contact with the metal rails AB and DC which are 0.2 m apart in a uniform magnetic field of 2.0 T perpendicular to the plane as shown in the diagram. The total resistance of the circuit ABCD is 0.8 ohms. What is the magnitude and direction of current induced in the rod at the instant it is moved to the left with a velocity of 100 m/sec?



3 (37-14). In the preceding problem, what force (magnitude and direction) is required to keep the rod in motion?

4 (38-1). The magnetic field at all points within the dotted circle of the diagram below equals 0.5 T. It is directed into the plane of the diagram and is decreasing at the rate of 2 T/sec. What are the magnitude and direction of the induced electric field at any point of the circular conducting ring of radius 0.2 m?



5 (38-11, 6). Two parallel wires of equal radius a whose centers are a distance d apart carry equal currents i in opposite directions. Find the inductance of a length ℓ of these wires neglecting the flux within the wires themselves.

6 (38-18). An inductor with inductance L = 15 millihenrys is connected in a series circuit with an open switch. When the switch is closed, the current in the circuit builds up from zero to a steady state current of 4 amp. Calculate the energy stored in the inductor.

7 (38-22). A long cylindrical conductor with radius R carries a current of uniform density. If the total current carried by the wire is i, find the magnetic energy per unit length stored within the conductor in terms of i and R.

8 (39-1). A series circuit consists of a resistor of resistance R = 2.0 megaohm, a capacitor of capacitance C = 2.0 microfarad, a seat of emf of ε = 6.0 volts, and an open switch. Find the current in the circuit 4.0 sec after the switch is closed.

9 (39-10, 39-7). A 60-ohm resistor and a 3-microfarad capacitor are connected in a single-loop circuit with a seat of emf equal to 6 volts. After one minute, the seat of emf is removed and the capacitor is allowed to discharge. What is the magnitude of the current just after the capacitor starts to discharge?

10 (39-15). An uncharged 50.0-microfarad capacitor is charged by a constant emf through a 100-ohm resistor to a potential difference of 50.0 volts. What is the total work done?

11 (40-7). A series circuit consists of an inductor of inductance L = 6.0 millihenrys, a seat of emf of ε = 6.0 volts, total series resistance R = 6.0 ohms and an open switch. How long must one wait after the switch is closed before the current is 50% of it's equilibrium value?

12 (40-11, 40-1). A 40-volt potential difference is applied to a coil with L = 50 millihenrys and R = 200 ohms for one minute. The applied voltage is suddenly removed from the circuit. What is the current in the circuit after one time constant from the time the applied voltage is removed?

13 (40-15). A coil with an inductance of 2.0 henrys and a resistance of 10 ohms is suddenly connected to a resistanceless battery with ε = 50 volts. How much energy is stored in the magnetic field when the equilibrium current exists in the coil?

[a] CORRECT ANSWER: $\frac{\mu_o i^2}{16\pi}$

The magnetic field within the conductor can be obtained from Ampere's law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o i$$

which leads to

$$B\ 2\pi r = \mu_o i \left(\frac{\pi r^2}{\pi R^2}\right)$$

or

$$B = \frac{\mu_o i r}{2\pi R^2}$$

where

$$r < R$$

The energy density for points inside the conductor is therefore

$$U = \frac{1}{2\mu_o} B^2 = \frac{\mu_o i^2 r^2}{8\pi^2 R^4}$$

Consider a volume element dV consisting of a cylindrical shell whose radii are r and r + dr and its length ℓ. The energy dU contained in it is

$$dU = udV = \frac{\mu_o i^2 r^2}{8\pi^2 R^4} (2\pi r \ell)\ dr$$

$$= \frac{\mu_o i^2 r^3 \ell}{4\pi R^4} dr$$

Integrating over r from 0 to R, we obtain the total magnetic energy contained in the conductor of length ℓ, radius R.

$$U = \int_0^R \frac{\mu_o i^2 \ell}{4\pi R^4} r^3 dr = \frac{\mu_o i^2}{16\pi} \ell$$

The magnetic energy per unit length stored within the conductor is obtained by dividing U by ℓ:

$$\frac{U}{\ell} = \frac{\mu_o i^2}{16\pi}$$

Note that this energy per unit length is independent of the radius of the conductor.

[a] CORRECT ANSWER: 6.9×10^{-4} sec

The expression for the current in an L-R circuit is

$$i = \frac{\varepsilon}{R}(1 - e^{-Rt/L}) \tag{1}$$

The equilibrium value of the current i_∞ is given by

$$i_\infty = \frac{\varepsilon}{R} \tag{2}$$

The time T when $i = .5\, i_\infty$ is obtained by substituting the values of i and $\frac{\varepsilon}{R}$ in equation (1). Thus

$$.5\, i_\infty = i_\infty(1 - e^{-RT/L}) \tag{3}$$

or

$$.5 = e^{-RT/L}$$

Taking the natural logarithm of both sides, we find

$$\ell n\ 2 = \frac{RT}{L}$$

or

$$T = \frac{L}{R}\ \ell n\ 2 = \frac{6 \times 10^{-3}}{6}(.693) = 6.9 \times 10^{-4} \text{ sec}$$

[b] CORRECT ANSWER: 50 amps, counterclockwise

The induced emf can be calculated from Faraday's law:

$$\varepsilon = -\frac{d\Phi}{dt}$$

During the time interval dt, the increment in flux $d\Phi$, is

$$d\Phi = -B\ell\, v\, dt$$

Where $\ell\, v\, dt$ is the differential area covered in the interval dt. We have

$$\varepsilon = -\frac{d\Phi}{dt} = B\, \ell\, v$$

The induced current is

$$i = \frac{\varepsilon}{R} = \frac{B\ell v}{R} = \frac{2 \times 0.2 \times 100}{0.8} = 50 \text{ amps}$$

The direction of the induced current may be obtained from Lenz's law and it must be counterclockwise as it opposes the change (in this case increase in Φ) by setting up a field that is anti-parallel to the external field within the loop ABCD.

[a] CORRECT ANSWER: 0.1 amp

Since one minute is much larger than the capacitive time constant $RC = 1.8 \times 10^{-4}$ sec of the circuit, the charge on the capacitor has attained its equilibrium value after one minute of charging. Thus the charge on the capacitor after one minute of charging is

$$q = C\varepsilon = 3 \times 10^{-6} \times 6 = 1.8 \times 10^{-5} \text{ coul}$$

This amount of charge is on the capacitor at the start of discharging. The discharging current in an RC circuit is given by

$$i = -(q_o/RC)\ e^{-t/RC}$$

Thus, at the beginning of the discharge, the current is

$$i_o = -q_o/RC$$

where

$$q_o = C\varepsilon = 1.8 \times 10^{-5} \text{ coul}$$

Substituting the numerical values, we obtain

$$i_o = -1.8 \times 10^{-5}/(60 \times 3 \times 10^{-6}) = -0.1 \text{ amp}$$

[b] CORRECT ANSWER: 25 J

The energy stored in the magnetic field is

$$U_B = \frac{1}{2} L i^2$$

In this case, the current i refers to the equilibrium current in the coil which is given by the expression $i_\infty = \varepsilon/R$.

Thus

$$U_B = \frac{1}{2} L \frac{\varepsilon^2}{R^2} = \frac{1}{2} \times 2.0 \times (50/10)^2 = 25 \text{ J}$$

[a] CORRECT ANSWER: 0.2 volt/m, clockwise

The magnetic flux through the ring is

$$\Phi = \pi r^2 B$$

where r is the radius of the ring

From Faraday's law

$$\varepsilon = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi}{dt}$$

or

$$E 2\pi r = -\frac{d}{dt}(\pi r^2 B) = -\pi r^2 \frac{dB}{dt}$$

Solving for E yields

$$E = -\frac{r}{2}\frac{dB}{dt}$$

The minus suggests that the induced electric field $\vec{E}$ acts to oppose the change of the magnetic field. In this case, the flux Φ is decreasing so the induced current in the ring will tend to oppose this change by setting up a magnetic field of its own that points into the plane of the loop. Thus, the induced current i must be clockwise and the direction of electric field $\vec{E}$ must also be clockwise. The magnitude of the electric field is

$$E = \frac{1}{2} r \frac{dB}{dt} = \frac{1}{2} \times 0.2 \times 2 = 0.2 \text{ volt/m}$$

[b] CORRECT ANSWER: 1.1×10^{-6} amp

The equation for charging an RC circuit is

$$q = C\varepsilon(1 - e^{-t/RC}) \qquad (1)$$

The equation for the current is obtained by differentiating equation (1) with respect to time, thus

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$$

Substituting numerical values, we obtain

$$i = 3 \times 10^{-6} e^{-1}$$

$$= 1.1 \times 10^{-6} \text{ amp}$$

[a] CORRECT ANSWER: 1.2×10^{-1} J

The energy is equal to work done in raising the current from zero to 4 amp through the inductor. During this process, when the current in the inductor is i, the work done in duration dt is found from

$$dW = \varepsilon\, dq = \varepsilon \frac{dq}{dt} dt$$

$$= \varepsilon\, i\, dt \qquad (1)$$

where ε, the applied emf to the inductor, is

$$\varepsilon = L \frac{di}{dt} \qquad (2)$$

Substituting this expression for ε in equation (1) and integrating, we obtain

$$W = \int_0^4 Li\,di = \frac{1}{2} Li^2 \Big|_0^4 = 1.2 \times 10^{-1} \text{ J}$$

[b] CORRECT ANSWER: 7.4×10^{-2} amp

Since the time constant for the circuit, L/R, is

$$L/R = 50 \times 10^{-3}/200 = 2.5 \times 10^{-4} \text{ sec}$$

which is much less than one minute, the current reaches its equilibrium value after one minute of charging. The equilibrium value of the current is $i_\infty = \varepsilon/R = 0.2$ amps.

After the applied voltage is removed, the current decays exponentially, i.e.,

$$i = (\varepsilon/R)\, e^{-Rt/L}$$

Thus, after one time constant, the current in the circuit is

$$i = (\varepsilon/R)\, e^{-1} = 0.074 \text{ amp}$$

[a] CORRECT ANSWER: 500 volts

Faraday's law may be written as

$$\varepsilon = -\frac{d(N\Phi)}{dt}$$

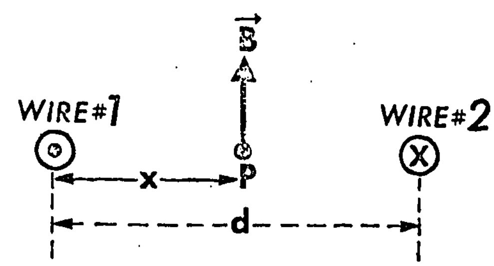
where $N\Phi$ are the number of flux linkages in a coil. The rate of change of flux $d\Phi/dt$ will be the same for both the coil and the solenoid; thus ε will be proportional to the number of turns of wire. We may then write the proportionality

$$\frac{10 \text{ volts}}{2 \text{ turns}} = \frac{x \text{ volts}}{100 \text{ turns}}$$

$$x = 500 \text{ volts}$$

[b] CORRECT ANSWER: $\frac{\mu_0 \ell}{\pi} \ln\left(\frac{d-a}{a}\right)$

The magnetic fields at P due to the two wires point in the same direction.



Using Ampere's law,

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i$$

we obtain the total field at P:

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} + \frac{1}{d-x}\right) \qquad (1)$$

where x is to be measured from wire 1. The self inductance of this pair is obtained from the relation

$$Li = \phi = \int \vec{B}\cdot d\vec{S} \qquad (2)$$

Substituting (1) into (2) and integrating over the area which is perpendicular to $\vec{B}$ between the two wires, we obtain

$$Li = \int_a^{d-a} \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} + \frac{1}{d-x}\right)\ell \, dx$$

$$= \frac{\mu_0 i \ell}{\pi} \ln \frac{d-a}{a}$$

so that

$$L = \frac{\mu_0 \ell}{\pi} \ln\left(\frac{d-a}{a}\right)$$

where the flux within the wires themselves has been neglected.

[a] CORRECT ANSWER: 0.125 J

The work done per unit time by an emf is given by either side of the equation

$$\varepsilon i = \frac{q}{C} i + i^2 R$$

Integration over t will yield the total work done. It is probably simplest to calculate from the left-hand side. Since the current is given by the expression $i = \frac{\varepsilon}{R} e^{-t/RC}$, the integral of the left side may be written as

$$W = \int_0^\infty \varepsilon i dt = \frac{\varepsilon^2}{R} \int_0^\infty e^{-t/RC} dt = \varepsilon^2 C$$

Substituting the given numerical values, we obtain

$$W = \varepsilon^2 C = (50)^2 \times 50 \times 10^{-6} = 0.125 \text{ J}$$

[b] CORRECT ANSWER: 20 nt to the left

The force on rod AD due to the magnetic field is given by

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

The magnitude of $\vec{F}$ in this case is

$$F = i\ell B$$

$$= (50)\ (0.2)\ (2.0) = 20 \text{ nt}$$

and the direction of $\vec{\ell} \times \vec{B}$ is toward the right. Consequently, in order to help the rod in motion, one must apply a force of 20 nt to the left.