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ABSTRACT

Four study segments of the Self-Paced Physics Course materials are presented in this eighth problems and solutions book used as a part of course assignments. The content is related to magnetic induction, Faraday's law, induced currents, Lenz's law, induced electromotive forces, time-varying magnetic fields, self-inductance, inductors, resistor-capacitor circuits, resistor-inductor circuits, and current decay problems. Contained in each segment are an information panel, core problems enclosed in a box, core-primed questions, scrambled problem solutions, and true-false questions. Study guides are provided and used to reveal directions for reaching solutions. When the core problem is answered incorrectly, the study guide requires students to follow the remedial loop, leading to the solutions of core-primed questions. Also included is a sheet of problem numbers with corresponding page numbers which locate correct answers. (Related documents are SE 016 065 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

UNITED STATES

Self-Paced
PHYSICS

U. S.
NAVAL ACADEMY
EX SCIENTIA TRIDENS

NAVAL ACADEMY

SEGMENTS 37-40

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NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY

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Segment 1 [illegible]		Segment #2 [illegible]		Segment #3 [illegible]		[illegible]	
No.	Page	No.	Page	No.	Page	No.	Page
1	[illegible]	1	19a	1	[illegible]	[illegible]	[illegible]
2	[illegible]	2	23a	2	15a	[illegible]	[illegible]
3	19b	3	24b	3	14a	[illegible]	[illegible]
4	21b	4	[illegible]	4	[illegible]	[illegible]	[illegible]
5	18a	5	26a	5	21a	[illegible]	[illegible]
6	21a	6	20a	6	15a	[illegible]	[illegible]
7	26a	7	26b	7	15a	[illegible]	[illegible]
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10	30a	10	17b	10	16a	[illegible]	[illegible]
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12	24a	12	23b	12	15a	[illegible]	[illegible]
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14	19a	14	28a	14	20b	[illegible]	[illegible]
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16	29a	16	29a	16	14b	[illegible]	[illegible]
17	20b	17	24a	17	19a	[illegible]	[illegible]
18	26b	18	17a	18	16a	[illegible]	[illegible]
		19	25b				
		20	21a				
		21	28b				
		22	25a				
		23	18a				
		24	27a				
		25	24b				
		26	22a				

Reading:

*HR 35-1/35-4;
35-6
SW *33-1,33-3
33-4

Reading:

HR *35-3, 35-6
*36-1,*36-2
*36-4,*36-5
SW 37-4/37-9

Reading:

*HR 32-8
SW 32-5

Reading:

*HR [illegible]-9
SW 37-7

P	STEP	NAME
	0.1	Reading: *HR 35-1/35-3; 35-4 SZ *33-1; 33-3, 33-4
	0.2	Audiovisual, "FARADAY'S LAW OF INDUCTION"
	0.3	Information Panel, "Faraday's Law of Induction"
1		T F (ans)
	1.1	If correct, advance to 5.1; if not, continue sequence.
2		A B C D
3		A B C D
4		A B C D
5		T F (ans)
	5.1	Information Panel, "Induced Current"
6		A B C D T F
	6.1	If first choice was correct, advance to 9.1; if not, continue sequence.

P	STEP	SECTION SEGMENT 37
7		A B C D
8		A B C D
9		A B C D
	9.1	Information Panel, "Lenz's Law"
10		A B C D T F
	10.1	If first choice was correct, advance to 13.1; if not, continue sequence.
11		A B C D
12		A B C D
13		A B C D T F
	13.1	Information Panel, "Direction of Induced Electromotive Force--Vector Approach"

P	STEP	NAME	P	STEP	SECTION SEGMENT 37
14		A B C D T F			
	14.1	If first choice was correct, advance to 18.1; if not, continue sequence.			
15		A B C D			
16		A B C D			
17		A B C D			
18		A B C D T F			
	18.1	Homework: HR 35-15			

P	STEP	NAME
	0.1	Reading: HR *35-5, 35-6 *36-1, *36-2, *36-4, *36-5 SW 37-4/37-7
	0.2	Information Panel, "Time-Varying Magnetic Fields"
1		T F _______________(ans)
	1.1	If correct, advance to 5.1; if not, continue sequence.
2		A B C D
3		A B C D
4		_______________(ans)
5		T F _______________(ans)
	5.1	Information Panel, "Self-Inductance"
6		A B C D T F
	6.1	If first choice was correct, advance to 10.1; if not, continue sequence.

P	STEP	SECTION SEGMENT 38
7		A B C D
8		A B C D
9		A B C D
10		A B C D T F
	10.1	Information Panel, "Self-Induced Electromotive Force"
11		A B C D T F
	11.1	If first choice was correct, advance to 17.1; if not, continue sequence.
12		A B C D
13		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 38
14		(ans)		21.1	Information Panel, "Energy Density"
15		(ans)	22		A B C D T F
16		A B C D		22.1	If first choice was correct, advance to 26.1; if not, continue sequence.
17		A B C D T F	23		A B C D
	17.1	Information Panel, "Energy Stored in an Inductor"	24		(ans)
18		T F (ans)	25		A B C D
	18.1	If correct, advance to 21.1; if not, continue sequence.	26		A B C D T F
19		A B C D		26.1	Homework: HR 36-7
20		(ans)			
21		A B C D T F			

P	STEP	NAME
	0.1	Reading: *HR 32-8 SW 32-5
	0.2	Information Panel, "Time-Varying Current and Charge in an RC Charging Circuit"
1		T F _____ (ans)
	1.1	If corrent, advance to 6.1; if not, continue sequence.
2		A B C D
3		A B C D
4		A B C D
5		A B C D
6		A B C D T F
	6.1	Information Panel, "Graphical Approach to the RC Charging Circuit"
	6.2	Audiovisual, RC TRANSIENTS

P	STEP	SECTION SEGMENT 39
7		A B C D T F
	7.1	If first choice was correct, advance to 9.1; if not, continue sequence.
8		_____ (ans)
9		A B C D T F
	9.1	Information Panel, "The Discharge Process"
10		T F _____ (ans)
	10.1	If correct, advance to 14.1; if not, continue sequence.
11		A B C D
12		A B C D
13		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 39
14		T F _____ (ans)			
	14.1	Information Panel, "Energy Associated with the Elements of an RC Circuit"			
15		T F _____ (ans)			
	15.1	If correct, advance to 15.1; if not, continue sequence.			
16		_____ (ans)			
17		_____ (ans)			
18		T F _____ (ans)			
	18.1	Homework: HR 32-32			

P	STEP	NAME
	0.1	Reading: *HR 36-3 SW 37-7
	0.2	Audiovisual, LR TRANSIENTS
	0.3	Information Panel, "The LR Time Constant"
1		T F (ans)
	1.1	If correct, advance to 6.1; if not, continue sequence.
2		A B C D
3		A B C D
4		(ans)
5		(ans)
6		T F (ans)
	6.1	Information Panel, "Current Growth in an LR Circuit"

P	STEP	SECTION SEGMENT 40
7		T F (ans)
	7.1	If correct, advance to 10.1; if not, continue sequence.
8		(ans)
9		A B C D
10		T F (ans)
	10.1	Information Panel, "Current Decay in an LR Circuit"
11		T F (ans)
	11.1	If correct, advance to 14.1; if not, continue sequence.
12		A B C D
13		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 40
14		T F _____ (ans)			
	14.1	Information Panel, "Energy Associated with the Elements of an LR Circuit"			
15		T F _____ (ans)			
	15.1	If correct, advance to 18.1; if not, continue sequence.			
16		A B C D			
17		_____ (ans)			
18		T F _____ (ans)			
	18.1	Homework: HR 36-14			

INFORMATION PANEL Faraday's Law of Induction

OBJECTIVE

To state and analyze Faraday's law of induction.

The descriptive results of Faraday's experiments as well as the quantitative analytical statement of the law which finally emerged from these experiments are quite informative and definitely worth reviewing.

Of great significance is the experimental fact that an emf appears across the ends of a conductor *when there is relative motion between the conductor and a magnetic field in which the conductor is included* provided that the conductor does not move parallel to the field. Such an emf is termed "induced" and will give rise to an induced current if the circuit is complete across the ends of the conductor. As long as the relative motion has a component at right angles to the direction of the magnetic induction, an induced emf will appear.

If one visualizes the conductor as *cutting through* lines of magnetic induction as a result of the relative motion, it is not difficult to show that the magnitude of the emf thus induced is *proportional to the rate of cutting*. Starting with a single conductor in a given magnetic field, experiment discloses that the magnitude of the emf may be increased by

(a) increasing the magnetic induction through which the conductor is moving, that is, strengthening the magnetic field;

(b) increasing the speed with which the relative motion occurs;

(c) making the angle of relative motion closer to 90°;

(d) increasing the number of conductors (connected in series) that are moving through the field.

Relative motion between a conductor (or a series of conductors in the form of a coil) and a magnetic field may be realized by actual physical motion of the conductor and/or a permanent magnet, or by placing the conductor in the vicinity of another conductor in which a varying current exists. In the latter case, *the magnitude of the induced emf depends upon the rate at which the current is changing and not upon the magnitude of the current.*

next page

continued

Quantitatively, the relationship between these factors may be expressed by Faraday's law as follows:

$$\varepsilon = -\frac{d\phi_B}{dt}$$

In this statement, ε = the induced emf, ϕ_B = the magnetic flux, and t = time. Thus, the law may be verbally given as:

The induced emf ε in a circuit is equal to the negative of the rate at which the magnetic flux through the circuit is changing.

If the rate of change of the flux is expressed in webers per second, the induced emf ε will then be in volts.

The negative sign is related to the direction of the induced emf, a matter to be discussed in the subsequent Information Panels.

When the conductors to be moved relative to a field are wound in the form of a coil of, say, N turns an emf will be induced across each turn and, since the turns are effectively in series, the net emf induced across the coil will be the sum of the individual loop emf's. For very closely wound coils, and for ideal solenoids and toroids, the flux through each turn may be considered equal. Hence, Faraday's law for such coils is:

$$\varepsilon = -\frac{d\phi_B}{dt} N$$

or

$$\varepsilon = -\frac{d\left(N\phi_B\right)}{dt}$$

The product $N\phi_B$ is referred to as the number of *flux linkages* in the device.

In approaching the core problem in the following group, it will be helpful if you remember that

$$\phi_B = \int \vec{B}\cdot d\vec{S}$$

and if $\vec{B}$ is constant and perpendicular to the area of a conducting loop (or coil) then

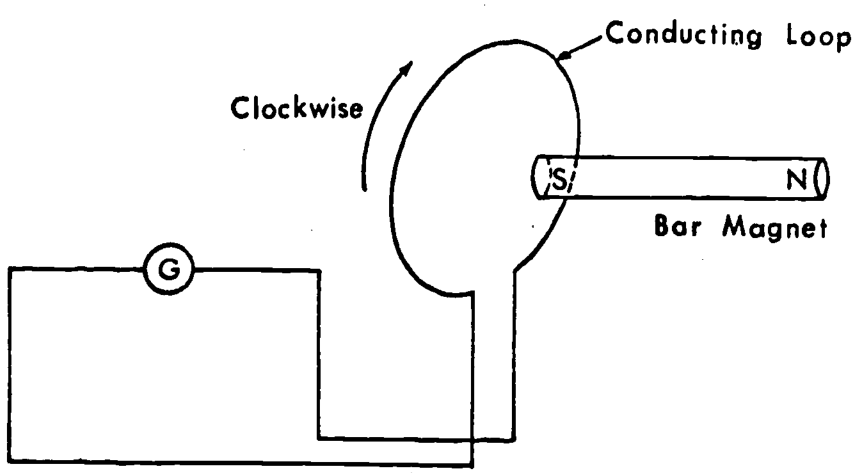
$$\phi_B = B\int dS = BA$$

1. A flat coil of 50 turns is placed perpendicularly to a uniform magnetic field B = 2.0 T. The coil is collapsed so that the area is reduced with a constant rate of 0.1 m^2/sec. What is the emf developed in the coil?

2. If the unit of magnetic flux is the weber and unit of time is the second, then the unit of induced emf, ε, is the

A. coulomb
B. newton
C. farad
D. volt

3. Suppose both the magnet and the loop as shown below are both moving to the right with speed v. The current in the loop is



A. clockwise

B. counterclockwise

C. zero

D. increasing in a clockwise direction

4. Suppose that the magnet in the preceding diagram approaches the loop and then stops moving as its south pole reaches the plane of the loop. What will be the current in the loop some time after that?

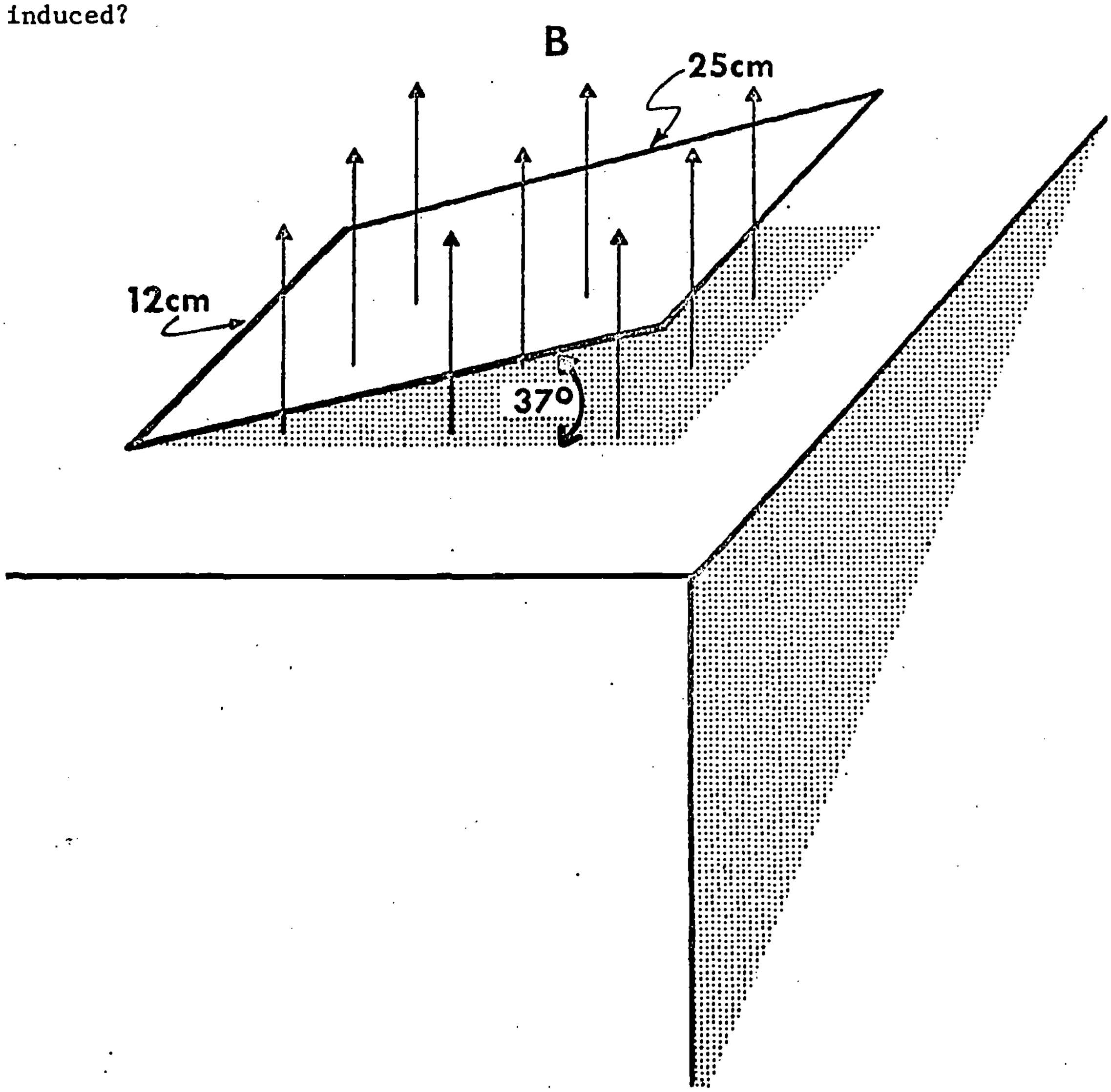
A. clockwise

B. zero

C. counterclockwise

D. increasing in the counterclockwise direction

5. A closely wound rectangular 50-turn coil has dimensions of 12 cm × 25 cm. It is located in a uniform magnetic field of B = 2.0 T, oriented as shown in the diagram. If the loop is brought from its position as indicated to the horizontal position in 0.10 sec, what is the magnitude of the average emf induced?



INFORMATION PANEL Induced Current

OBJECTIVE

To solve problems involving currents induced in conductors and coils by electromagnetic methods.

The magnitude of the emf induced in a closely-wound coil, ideal solenoid, or ideal toroid is given by Faraday's law:

$$\varepsilon = \frac{d\left(N\Phi_B\right)}{dt} \qquad (1)$$

When the conductor or coil in which the emf is induced is part of a complete circuit, the current that appears in the circuit is readily determined from:

$$i = \frac{\varepsilon}{r} \qquad (2)$$

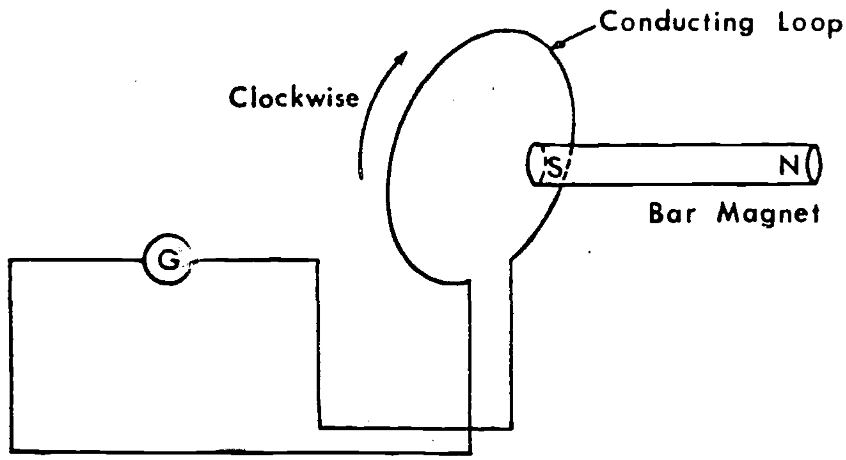
in which r = the resistance of the coil.

When a pair of loops or coils are placed adjacent to one another in such a way that flux linkage is possible, an emf will be induced in one loop if a varying current exists in the other. If the second loop circuit is complete, a current due to the induced emf will be present, its magnitude being given by equation (2) above. There will be no emf, and consequently no current induced by a steady-state current in the first loop.

The induced current in a closed loop produced by a permanent magnet in motion along the loop axis will have a magnitude that depends upon the magnitude of the induced emf and the resistance of the loop. As given previously, the magnitude of the induced emf depends upon the strength of the permanent magnet, the speed of the relative motion between loop and magnet, the angle of the relative motion, and the number of turns in the coil, if one is used instead of a single loop.

The information provided above should be helpful in the problem work in this section.

6. As shown in the diagram, the loop is moved away from the magnet with a speed v. Next, the loop is replaced by a coil of N turns of identical wire and wound closely so that it occupies approximately the same space as the original loop. If this coil is moved away from the magnet exactly in the same manner as the single loop and with the same speed v, the *current* in the N-turn coil as compared to that in the single loop will be



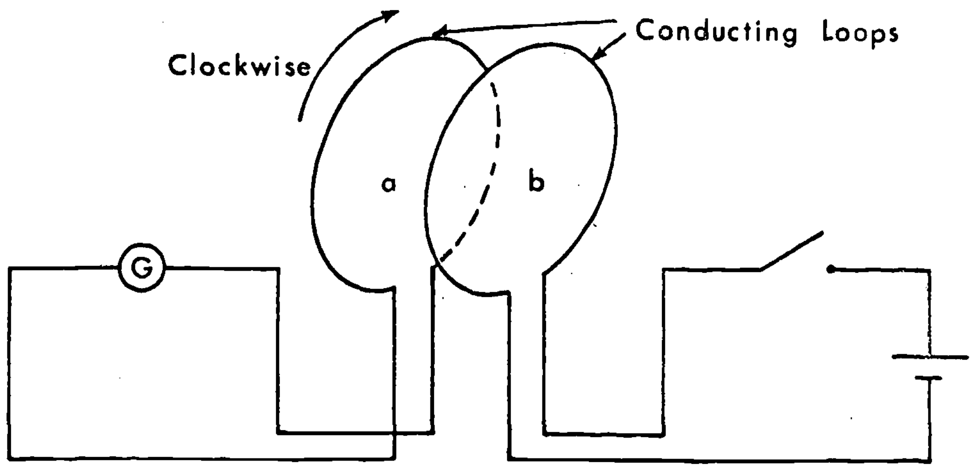
A. unchanged

B. N times as large

C. N times less

D. N^2 times as large

7. After the switch in the figure below has been closed long enough so that a steady current has been established in loop b, the current in loop a is



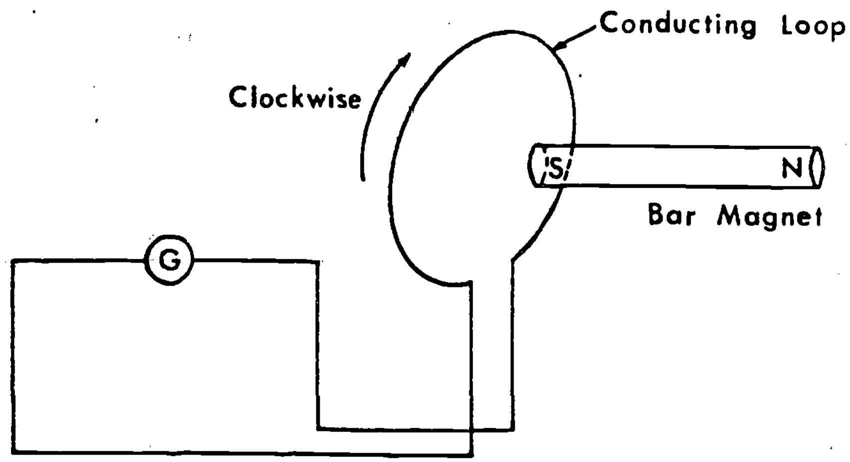
A. clockwise

B. counterclockwise

C. zero

D. increasing in a clockwise direction

8. The bar magnet as shown in the diagram is moved toward the loop covering a distance d in a given time. Then, the same procedure is repeated but with the same distance d covered in half the time. The current in the loop the second time as compared to the first time is



A. unchanged

B. reversed in direction

C. decreased

D. increased

9. The loop in the preceding question is approached in exactly the same manner first by one bar magnet and then by a second bar magnet that is stronger than the first. For these cases, the first current as compared to the second current produced in the loop is

A. unchanged

B. reversed in direction

C. larger

D. smaller

INFORMATION PANEL Lenz's Law

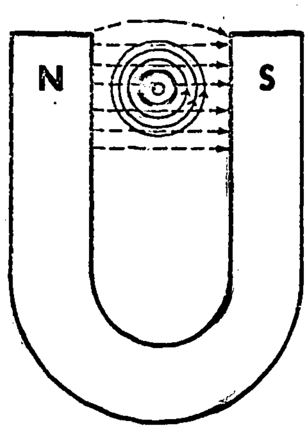
OBJECTIVE

To solve problems in which the direction of the induced emf in an electromagnetic system is to be determined or plays a key role.

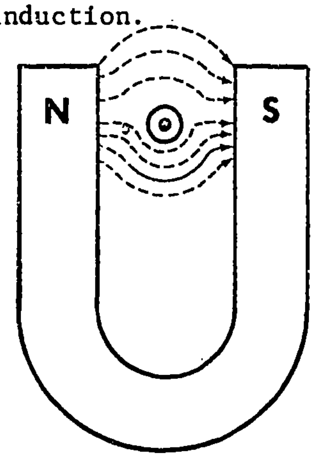
Lenz's law is an expression of the principle of conservation of energy in electromagnetic terms. Succinctly stated, the law reads:

> *A current induced in a conductor will have a direction such that the magnetic field it produces will oppose the change that gave rise to the induced emf.*

Fundamentally, Lenz's law is based on conservation principles because it mandates that the agent that causes the relative motion required for induction meet with *opposition* to its motion. Thus, the agent is required to do work in order to generate the electrical energy of the induced current. Assuming no mechanical losses occur, all of the work done by this agent on the system is converted directly into Joule heat when the circuit is complete. On open circuit, there can be no induced current, hence no Joule heating and, therefore, no work required of the agent. But it must be remembered that even on open circuit, an induced emf will appear across the ends of the conductor being cut by lines of magnetic induction.



(A)



(B)

Consider a straight conductor that is part of a complete circuit being moved downward through the magnetic field illustrated in the drawings above. The cross section of the conductor seen as a small circle in the field shows the induced current emerging from the plane of the paper toward the reader. (Drawing A.) The lines of magnetic induction due to the permanent magnet poles are shown as straight lines from N to S, while the circular field of the induced current is described by the concentric circles, the arrows indicating the direction of the field.

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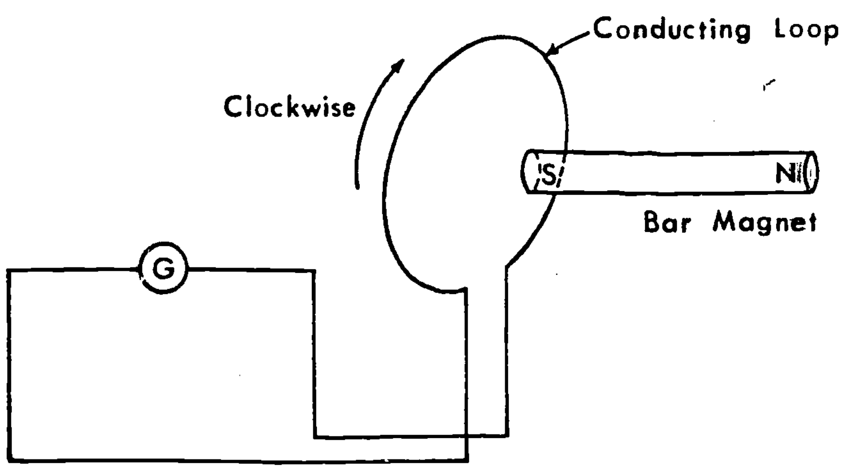
continued

In Drawing B, the *resultant field* has been illustrated, this resultant being obtained by the vector addition of the field due to the magnet and the field due to the induced current. The crowding of the lines under the moving conductor shows a field of high density; the few lines above the conductor suggests a field of low density. The conductor experiences an opposition, then, in moving from a region of low to high density as it does in this case. Should the direction of motion of the conductor be reversed, the high density region would then appear above the conductor because the current would reverse in sense. Hence, the moving wire would again be opposed by the resultant field and work would again be required of the agent moving the wire.

10. If the south pole of the magnet in the diagram is moving toward the loop (toward the left), the current in the loop is (the magnet is parallel to the axis of the loop)



A. clockwise

B. counterclockwise

C. zero

D. decreasing in the counterclockwise direction

11. An induced emf is *always* such that it

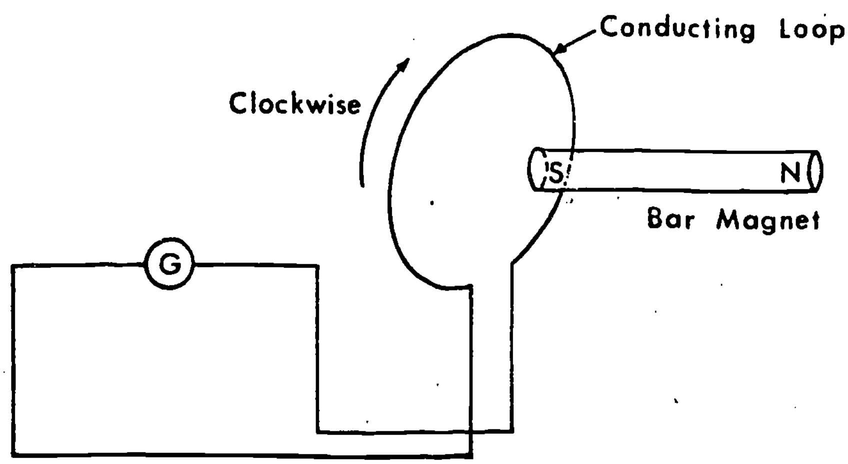
A. aids the current producing it

B. opposes the change of the current producing it

C. aids the change of the current producing it

D. first aids, then opposes, the change of current producing it

12. In the diagram, the south pole of the magnet is removed from its position at the center of the loop by moving the magnet toward the right. The current in the loop as indicated by the galvanometer is



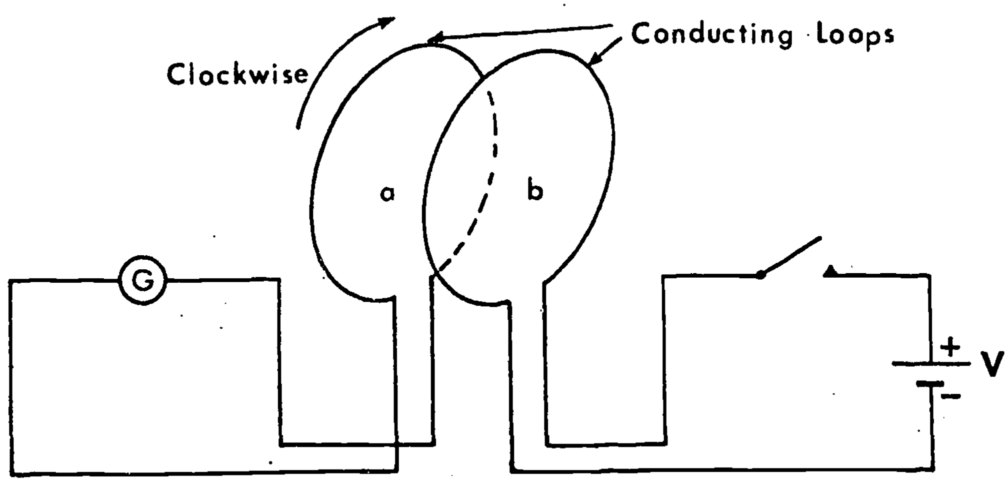
A. counterclockwise

B. clockwise

C. zero

D. decreasing in the clockwise direction

13. Immediately after the switch of circuit b is closed, the current in circuit a is



A. clockwise

B. counterclockwise

C. zero

D. decreasing

INFORMATION PANEL

Direction of Induced Electromotive Force--Vector Approach

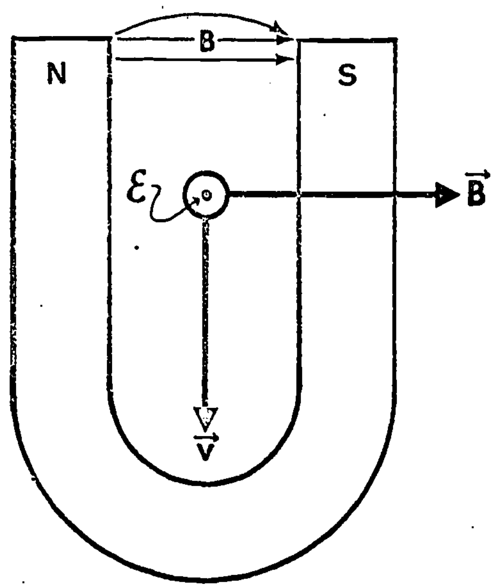
OBJECTIVE

To solve problems in which the direction of the induced emf in a circuit moving through a magnetic field is a key factor.

When a conductor of length $d\vec{\ell}$ moves through a magnetic field of induction $\vec{B}$ with a velocity $\vec{v}$, the general expression for the induced emf $d\varepsilon$ is:

$$d\varepsilon = (\vec{v} \times \vec{B})\cdot d\vec{\ell} \qquad (1)$$

When the length dimension of the conductor, its velocity $\vec{v}$, and the magnetic induction $\vec{B}$ are all mutually perpendicular (see accompanying figure), then equation (1) may be written in scalar form as:



$$\varepsilon = vB\ell \qquad (2)$$

The direction of the induced emf ε is that of

$$(\vec{v} \times \vec{B})$$

Referring to the conditions as depicted in the diagram where the conductor is moving downward ($\vec{v}$), and the sense of the field is to the right ($\vec{B}$), it should be clear that if vector $\vec{v}$ is rotated into vector $\vec{B}$, the induced emf will be directed out of the paper toward the reader in the direction of advance of a right-handed screw. This result corresponds (as it must) with the direction of the induced *current* obtained by applying Lenz's law.

Thus, it follows that the induced current will have the same direction as the induced emf.

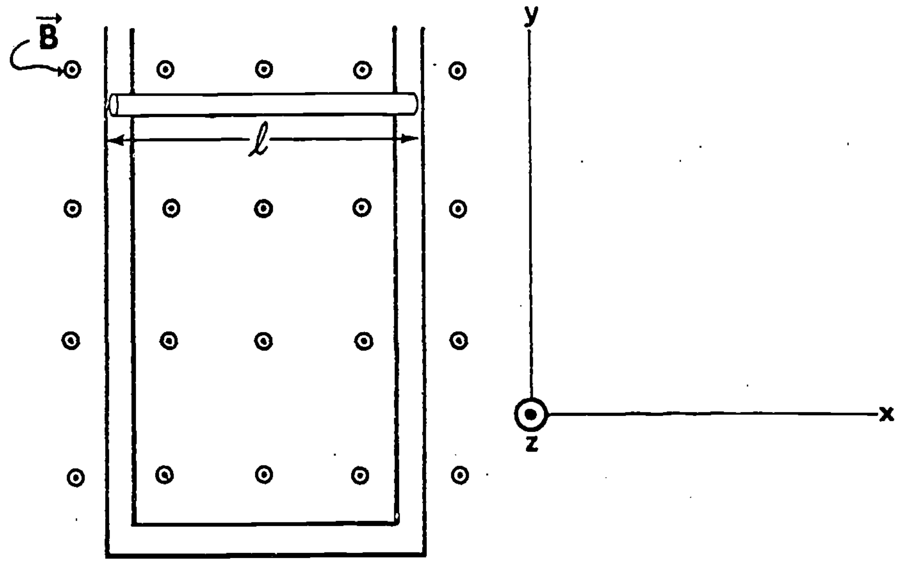
14. A wire of length ℓ, mass m and resistance R slides without friction vertically downward along parallel conducting rails of negligible resistance as shown in the diagram. The rails are connected to each other at the bottom by a conductor of negligible resistance. The wire and the rails form a closed rectangular conducting loop. A uniform magnetic field $\vec{B}$ pointing in the +Z direction (out of the plane of paper) exists throughout the region. The steady state speed of the wire is

A. zero

B. $\frac{mg\ell}{R^2B^2}$

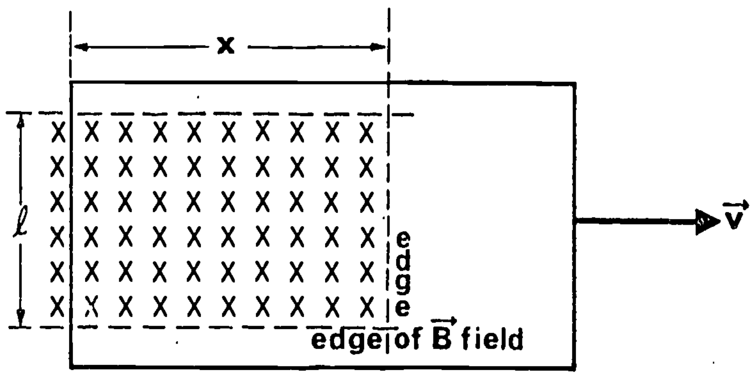
C. $\frac{mg\ell}{2R^2B^2}$

D. $\frac{mgR}{\ell^2B^2}$



Introductory Note

The following four questions refer to the diagram shown below and are interrelated.



15. A closed conducting loop as shown in the diagram is moved to the right at a constant speed v in a region where a magnetic field $\vec{B}$ exists. The resistance of the loop is R. If the field is uniform and normal to the plane of the loop the magnetic flux through the loop at the instant shown is given by (let the direction out of the paper be the positive direction)

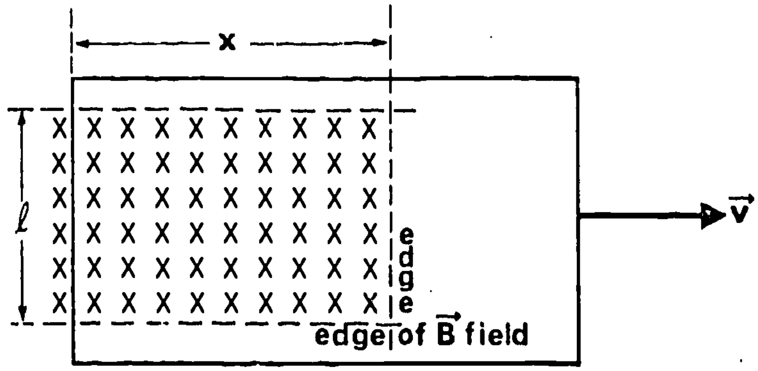
A. $-B\ell x$

B. $|\vec{B} \times \vec{v}|$

C. $-B\ell v$

D. $|\vec{\ell} \times \vec{B}|$

16. The closed conducting loop shown in the diagram is being moved to the right at a constant speed v. The induced emf in the circuit is



A. $\varepsilon = -Bv$

B. $\varepsilon = Bv$

C. $\varepsilon = -B \ell v$

D. $\varepsilon = B \ell v$

17. If the loop in the preceding question has a total resistance R, then the current in the loop is given by

A. $i = B \ell vR$, clockwise

B. $i = B \ell v/R$, clockwise

C. $i = B \ell v/R$, counterclockwise

D. $i = B \ell vR$, counterclockwise

18. What power is required to maintain the speed of the loop in the preceding questions?

A. $B^2\ell^2v^2/R^3$

B. $B^2\ell^2v^2/R$

C. $B^2\ell^2v^2$

D. $B^2\ell^2v^2R$

[a] CORRECT ANSWER: 6.0 volts

We are trying to find the av ot the instantaneous emf induced in the coil. Thus, we may write y's law of induction as

$$\bar{\varepsilon} = -N \frac{\Delta\Phi_B}{\Delta t} \qquad (1)$$

The only unknown, other than 1) is $\Delta\Phi_B$. To find this we must compute the initial and final flux t the coil, using the definition of flux

$$\Phi_B = \int \vec{B}\cdot d\vec{S} \quad \ldots s\theta \; dS \qquad (2)$$

where θ is the angle between $\vec{B}$ the normal to the plane surface S.

The field $\vec{B}$ is constant so (2) be written as

$$\Phi_B = BS \cos\theta \qquad (3)$$

and

$$\Delta\Phi_B = \Phi_{Bf} - \Phi_{Bi} = BS(\cos\theta_f - \cos\theta_i) \qquad (4)$$

The initial angle θ_i is 37° and the final is 0°, so from (4) we obtain

$$\Delta\Phi_B = (2.0 \ldots \left[(0.12 \text{ m}) \times (0.25 \text{ m})\right] \times (1.0 - 0.80)$$

$$= 2.0 \times 3.0 \times 10^{-2} \times 0.20 \text{ T-m}^2 = 1.2 \times 10^{-2} \text{ weber}$$

Finally substituting numerical values in (1), we obtain

$$|\bar{\varepsilon}| = \left|-50 \times \frac{1.2 \times 10^{-2}}{10^{-1}}\right| = 6.0 \text{ volts} \qquad (5)$$

TRUE OR FALSE? In equation (5), the quantity

$$\frac{1.2 \times 10^{-2}}{10^{-1}}$$

is the rate of change of th

[a CORRECT ANSWER

The wire will [illegible] a constant speed when the sum of the [illegible]rces acting on the wire is zero. There are two forces acting on the [illegible], (1) $\vec{F}_g$ downwards ($-\vec{F}_g$) due to gravity, and (2) $\vec{F}_B$ due to magnet[illegible] field $\vec{B}$. Force $\vec{F}_B$ acting [illegible] the wire of length ℓ due to the magneti[illegible] field $\vec{B}$ is given by

$$\vec{F}_B = (\vec{\ell} \times \vec{B}) \tag{1}$$

where the current [illegible] is due to the induced emf ε in the circuit of resistance R, that is

$$\vec{i}R = \ell(\vec{v} \times \vec{B})$$

$$\vec{i}R = \ell(-v\hat{j} \times B\hat{k})$$

$$\vec{i}R = -\ell vB\hat{i} \tag{2}$$

and where v is the instantaneous speed of the wire under the influence of the forces. Note that the wire starts from rest and accelerates downwards due to the resultant force and we are asked to determine the steady state speed attained where the resultant force is zero. Substituting the expression for $\vec{i}$ from equation (2) into equation (1) we find

$$\vec{F}_B = \ell\left(-\frac{\ell vB}{R}\hat{i} \times B\hat{k}\right)$$

$$\vec{F}_B = \frac{\ell^2 vB^2}{R}\hat{j} \tag{3}$$

Thus we find $\vec{F}_B$ is opposite to $\vec{F}_g$ and hence the wire will attain a constant speed v_c which is given by

$$\frac{\ell^2 v_c B^2}{R}\hat{j} - mg\,\hat{j} = 0$$

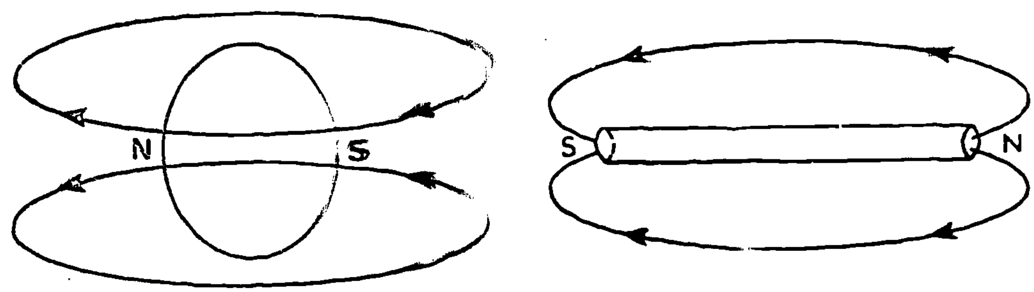
or

$$v_c = \frac{mgR}{\ell^2 B^2}$$

TRUE OR FALSE? At the instant the sliding wire is released to start its downward motion, the resultant force on it is $\vec{F}_g$.

[a] CORRECT ANSWER: [illegible]

We can apply Lenz's law. The induced current will have a direction which opposes the change that produced it. In this case, since the south pole of the magnet approaches the loop the current in it will be such as to create a "south pole" to oppose the magnet pole, as shown below.



Using the right-hand rule we see that our fingers must curl around the loop clockwise in order for our thumb to point toward the loop's north pole (toward left). Hence, the direction of the current will be clockwise.

TRUE OR FALSE? The same current direction would result if a north pole were to move away from the loop toward the right.

[b] CORRECT ANSWER: B

The magnitude of the induced emf ε, as derived in the preceding question is

$$\varepsilon = B\ell v$$

Thus,

$$i = \varepsilon/R = B\ell v/R$$

and the direction of the induced ε is the direction of $\vec{v} \times \vec{B}$, which is clockwise.

Notice that we could have worked with magnitudes and used Lenz's law to establish the direction of the current. The flux is into the paper and is decreasing as the loop moves to the right. A clockwise current is needed to produce a field into the paper which will tend to counteract the decrease in flux.

[a] CORRECT ANSWER: A

The emf induced in the N-turn coil will be N times as large as that induced in the single loop, as can be seen from Faraday's law of induction

$$\varepsilon = N d\Phi_B/dt$$

Since, however, the N-turn coil is made up of a continuous wire, and since the resistance is proportional to length, the resistance of the coil will also be N times as high as that of the single loop. The current is given by

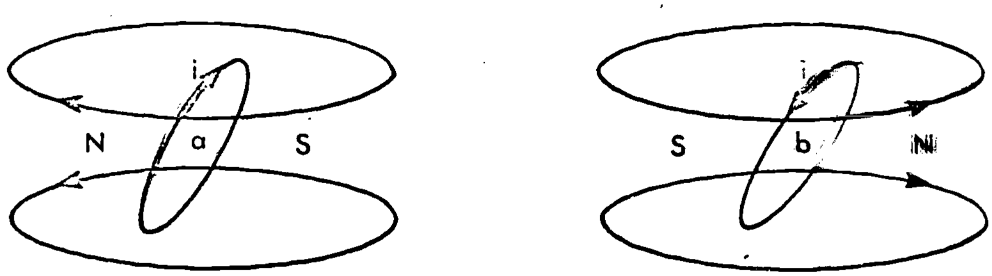
$$i = \varepsilon/R$$

so we see that if both ε and R are multiplied by the same number (N), the current remains unchanged.

TRUE OR FALSE? The above solution shows that the induced current is directly proportional to the number of turns in the coil.

[b] CORRECT ANSWER: A

As soon as the switch is closed a counterclockwise current



will start in loop b. This will generate a (changing) magnetic field in the direction shown above. The induced current in loop a must be such as to oppose this changing field. As can be seen above a clockwise current in a will produce this effect.

TRUE OR FALSE? To arrive at the above solution, one must apply Faraday's law of induction.

[a] CORRECT ANSWER: 10 volts

For an N-turn coil the induced emf is given by

$$\varepsilon = -N \frac{d\Phi_B}{dt}$$

The flux Φ_B is given by the expression

$$\Phi_B = \int \vec{B} \cdot d\vec{s}$$

In this case, $\vec{B}$ is constant and perpendicular to the area $\vec{A}$ s hat

$$\Phi_B = B \int ds = BA$$

We have been told that

$$\frac{dA}{dt} = -0.1 \ m^2/sec$$

so

$$-\frac{d\Phi_B}{dt} = -B \frac{dA}{dt} = (2.0)(0.1) \text{ volts}$$

Finally,

$$\varepsilon = -N \frac{d\Phi_B}{dt} = 50 \ (0.2 \text{ volts}) = 10 \text{ volts}$$

TRUE OR FALSE? The rate of change of flux varies inversely as the rate of change of the coil area.

[b] CORRECT ANSWER: B

There is no source of emf in the circuit of the loop, and once the magnet stops moving the induced emf becomes zero.

[a] CORRECT ANSWER: A

From the definition of magnetic flux

$$\Phi_B = \int \vec{B}\cdot d\vec{S} = \int B \cos\theta \; dS$$

and in view of the fact that $\vec{B}$ is anti-parallel to $\vec{S}$ ($\theta = 180°$), since $\vec{S}$ points out of the plane of the paper, then

$$\Phi_B = -BS$$

where S represents the part of the loop that is influenced by the magnetic field, that is

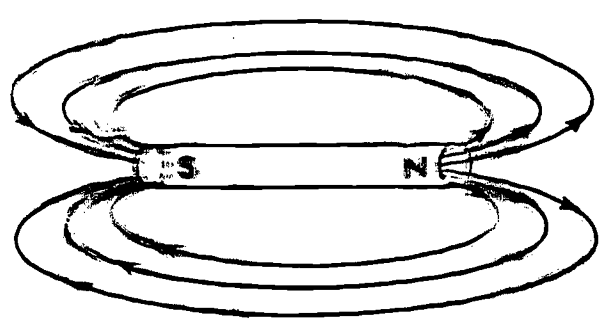
$$S = \ell x$$

Thus,

$$\Phi_B = -B\ell x$$

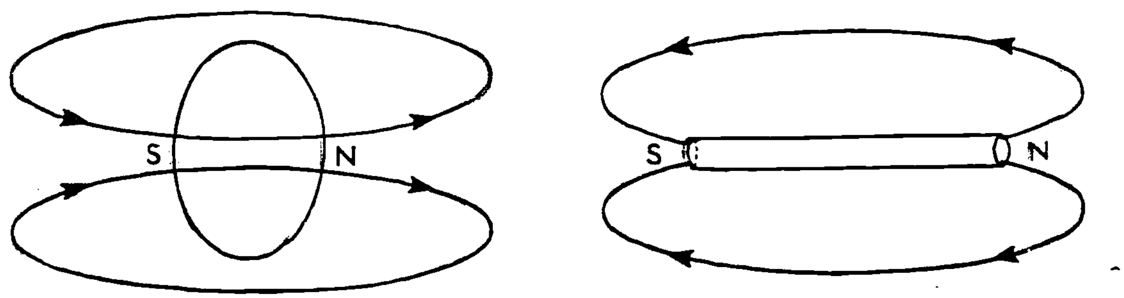
[b] CORRECT ANSWER: D

The density of the lines of induction increases as one approaches the poles of a magnet. When a faster moving magnet approaches a loop, the rate at which additional lines of induction cut through the loop will be higher than it would be for a slower moving magnet. Hence, the second time the current in the loop will be greater.



[a] CORRECT ANSWER: A

In this case, the direction of the induced current must be such that the current will form a "north pole" on the right side of the coil. The coil will then oppose (by attraction) the movement of the south pole to the right.



We notice this time that our fingers must curl around the loop in a counterclockwise sense in order for our thumb to point toward the right (where we want the "north pole" of the magnetic dipole produced by the induced current to be).

[b] CORRECT ANSWER: C

Both the loop and the magnet have velocity $\bar{v} = -v\hat{i}$. Hence their relative velocity is zero, and the magnetic flux Φ_B through the loop is constant. But if Φ_B = constant then $d\Phi_B/dt = 0$ and $\varepsilon = 0$ so there will be no current in the loop.

[c] CORRECT ANSWER: C

A current is induced only when the flux through the loop is changing. A steady state current will not produce a changing flux, hence, the induced current will be zero.

[a] CORRECT ANSWER: C

In the preceding question we established the flux through the loop to be

$$\Phi_B = -B\ell x \tag{1}$$

where the minus sign indicates that the flux is into the paper (recall that we took the direction out of the paper as the positive direction). In (1) B and ℓ remain constant as the loop moves to the right. Thus, Faraday's law of induction gives

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(-B\ell\,\frac{dx}{dt}\right) = B\ell\,\frac{dx}{dt} \tag{2}$$

The relationship between v and dx/dt is

$$\frac{dx}{dt} = -v \tag{3}$$

since x, as defined, decreases as the loop is being moved to the right. Thus, (2) becomes

$$\varepsilon = -B\ell v \tag{4}$$

[b] CORRECT ANSWER: D

The density of the lines of induction increases with the strength of the magnet. Since both magnets are moved toward the loop with the same speed, the current induced by the first (weaker) magnet will be smaller because its rate of cutting lines of induction is smaller.

TRUE OR FALSE? The total number of lines of induction cutting through the coil is the same for both cases.

[c] CORRECT ANSWER: B

This is a statement of Lenz's law. This law in conjunction with Faraday's law of induction enables us to determine both the magnitude and the direction of an induced emf.

[a] CORRECT ANSWER: D

We certainly do not need to know the units of flux and time to deduce that the unit of emf is the volt. We first introduced this unit in connection with electrical potential. Later we saw that emf has the same unit.

It is interesting to note, however, that Faraday's law,

$$\varepsilon = -d\Phi_B/dt$$

gives the relation

$$1 \text{ volt} = 1 \frac{\text{weber}}{\text{sec}}$$

so

$$1 \text{ weber} = 1 \text{ volt-sec}$$

[b] CORRECT ANSWER: B

Neglecting any frictional losses due to air resistance, we conclude that all the mechanical energy required in maintaining the motion of the wire at speed v is eventually converted into heat via Joule heating. The rate at which energy is dissipated in a current-carrying circuit (power) is given by

$$P = i^2R = \varepsilon^2/R$$

Using the result of the preceding question we obtain

$$P = [B\ell v/R]^2/R = B^2\ell^2v^2/R$$

Alternatively, we can recall that

$$P = F\cdot v \quad \text{for constant velocity and force.}$$

Then

$$F = i\ell B = \frac{B\ell v}{R}\ \ell B, \text{ and}$$

$$\text{Power} = Fv = \frac{B^2\ell^2v^2}{R}, \text{ the identical result.}$$

TRUE OR FALSE? If air resistance is *not* ignored, the P would have a larger value than $B^2\ell^2v^2/R$.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Time-Varying Magnetic Fields

OBJECTIVE

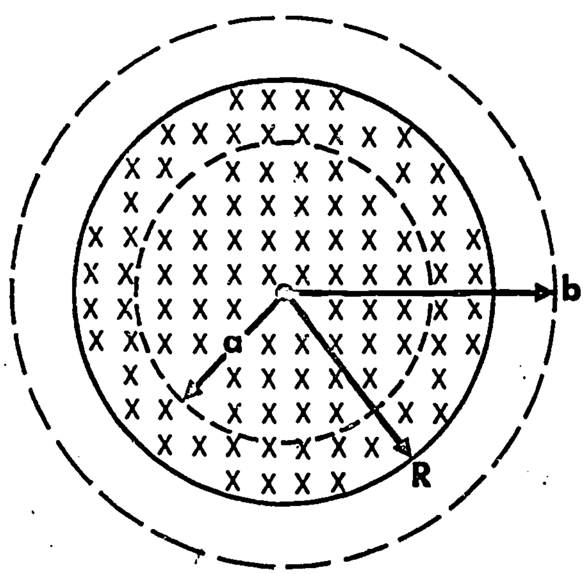
To solve problems involving the relationship between time-varying magnetic fields and the electric fields which result from this variation.

The basic theme of this portion of your study is that a time-varying magnetic field sets up an electric field that can act upon charges to produce acceleration, hence induced currents. When we deal with such time-varying fields we assume that there is no motion of the physical objects involved (i.e., there is no moving magnet nor is there a moving conductor.) As shown in your source material, Faraday's law for a time-varying magnetic field may be written as:

$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\phi_B}{dt} \tag{1}$$

in which E = the induced electric field at specific points in the magnetic field and the other symbols have their usual meanings. Thus, the time-rate of change of the magnetic flux is expressed by $d\phi_B/dt$ and the path of integration by $d\ell$. The time-rate of change of the magnetic field is generally given in teslas per second (T/sec).

In many of the problems you encounter, the magnetic field is confined to a region of cylindrical volume and you are asked to determine the effect of the induced electric field on given charges either inside or outside this space. If a is the radius of any loop *inside* the region of the uniform magnetic field (see drawing) and b is the radius of any loop outside the same region, the magnitude of the electric field for each of these cases is:



Loop radius a, *where* a < R

$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\phi}{dt}, \quad E\,2\pi a = -\pi a^2 \frac{dB}{dt},$$

$$E = -\frac{1}{2}\,a\,\frac{dB}{dt} \tag{2}$$

Loop radius b, *where* b > R

$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\phi}{dt}, \quad E(2\pi b) = -\frac{d(BA)}{dt},$$

$$E(2\pi b) = -\pi R^2 \frac{dB}{dt},$$

$$E = -\frac{1}{2}\frac{R^2}{b}\frac{dB}{dt} \tag{3}$$

next page

continued

You are asked to bear in mind that the induced emf ε resulting from a time-varying magnetic field may be expressed by:

$$\varepsilon = \oint \vec{E}\cdot d\vec{\ell} \qquad (4)$$

since

$$\varepsilon = -\frac{d\phi_B}{dt} \qquad (5)$$

Note that the negative sign should be retained to remind you that the induced emf always *opposes* the changing flux.

The problems in this section call upon you to

(a) find the instantaneous acceleration of a charged particle placed outside the region of the uniform magnetic field;

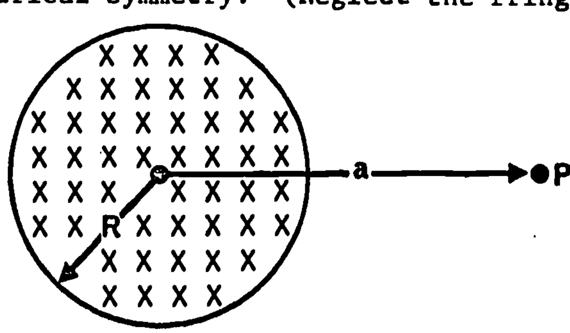
(b) find the instantaneous acceleration of a charged particle placed inside the region of the uniform magnetic field;

(c) apply Joule's law to determine the heat generated in a loop due to an induced current in the loop when placed in a magnetic field;

(d) apply equations (2) and (3) of this Information Panel to specific problems applicable to them.

PROBLEMS

1. The figure below shows a uniform magnetic field $\vec{B}$ confined in a region of cylindrical volume of radius $R = 10$ cm. The $\vec{B}$ field is decreasing in magnitude at a constant rate of 2×10^{-2} T/sec. Find the magnitude of the instantaneous acceleration in meters per second per second of an electron placed at point P a distance $a = 20$ cm from the center of the cylindrical symmetry. (Neglect the fringing effect of the $\vec{B}$ field beyond R.)



2. Let us consider a circular loop of radius r in a magnetic field which is varying with time. The magnetic flux enclosed in the loop is ϕ_B, changing with time. An emf is induced around the loop, and is given by

$$\varepsilon = -\frac{d\phi_B}{dt}$$

The electric field $\vec{E}$ induced at various points of the loop is alorg the tangent to the loop at that point. The expression for emf ε in terms of $\vec{E}$ is given by

$$\varepsilon = \oint \vec{E}\cdot d\vec{\ell} = -\frac{d\phi_B}{dt}$$

Use the above relation to consider the following question: Assume R is the radius of the cylindrical region in which a magnetic field $\vec{B}$ exists. What is the magnitude of the electric field $\vec{E}$ at any radius r where r < R?

A. $E = -\frac{1}{2r^2}\frac{dB}{dt}$

B. $E = -\frac{R^2}{2r}\frac{dB}{dt}$

C. $E = -\frac{1}{2}r^2\frac{dB}{dt}$

D. $E = -\frac{1}{2}r\frac{dB}{dt}$

3. In the preceding problem, what is the magnitude of the electric field where r > R?

A. $E = -\frac{1}{2}r\frac{dB}{dt}$

B. $E = -\frac{1}{2}\frac{r}{R^2}\frac{dB}{dt}$

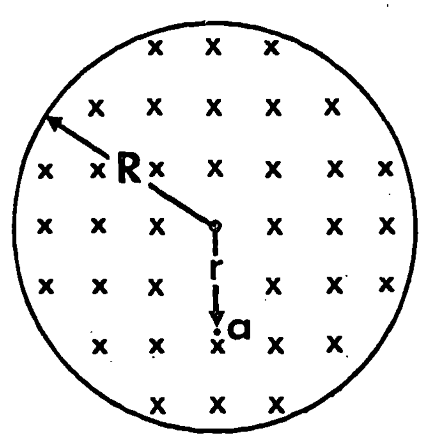
C. $E = -\frac{1}{2}R^2\frac{dB}{dt}$

D. $E = -\frac{1}{2}\frac{R^2}{r}\frac{dB}{dt}$

4. A circular conducting loop of radius 20 cm and with total resistance of 0.50 ohms is placed with its plane perpendicular to a time-varying magnetic field. If the whole area of the loop is inside the field, and the rate at which the field varies is constant and equal to $10/\pi$ T/sec, how much heat in joules is generated in the loop in 1.0 sec?

5. The figure below shows a uniform field of induction $\vec{B}$ confined in a region of cylindrical volume of radius R. $\vec{B}$ is decreasing in magnitude at a constant rate of 2.0×10^{-2} T/sec. What is the magnitude of the instantaneous acceleration of an electron in meters per second per second placed at point a? (Neglect the fringing effect of the field beyond R.)

$r = 5.0 \times 10^{-2}$ m



INFORMATION PANEL | Self-Inductance

OBJECTIVE

To define self-inductance (or simply *inductance*) as a relationship between flux linkages in a coil and the current; to use this relationship for solving appropriate problems.

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continued

An induced emf appears in a given coil if the current in the coil varies to produce a time-varying magnetic field. The process is called *self-induction*. The phrase "flux linkages" signifies the product of the actual flux ϕ and the total number of turns in the coil so that:

$$\text{Number of flux linkages} = N\phi_B$$

where N = the number of turns. If the coil is isolated from magnetic materials such as iron, cobalt, nickel or magnetic alloys, it is found that the number of flux linkages for a given coil is proportional to the current i in the coil. That is:

$$N\phi_B = Li$$

in which L, the constant of proportionality, is known as the *inductance* of the particular coil.

It is important to regard inductance L as a property of the *coil* rather than as a property of the *circuit* in which the coil is placed. The inductance of a coil depends upon its geometry, the number of turns, the way the layers are wound, etc. Although inductance *is* often affected by the current in the coil in practical circuits, for theoretical purposes L is considered fixed and constant once the coil has been built. The inductance of a coil is, however, sharply changed by inserting a core of ferrous material. For this section, the coil is to be taken as isolated from such materials. The relationship above is most exactly applicable to very closely-wound coils ("close-packed"), toroids, and at the center of a reasonably long solenoid.

Before starting the problems that follow, it is advisable to review the relationships between magnetic flux and magnetic induction, as, for example,

$$\phi_B = \int \vec{B}\cdot d\vec{S}$$

and

$$B = n\,\mu_0 i$$

for an ideal solenoid where n = the number of turns per unit length.

The core problem in this set is based upon a closely-wound toroid with a given inner radius and outer radius, a given current, and a given number of turns for which your are to determine the inductance L. In doing this problem, you should be able to show that

$$L = NBA/i$$

where A is the cross sectional area of the toroid.

6. A coreless, closely wound toroidal coil carries current i and has an outside radius b, inner radius a, and N turns. Assuming that the magnetic field B inside the coil is $\mu_o Ni/(\pi a + \pi b)$, find the self-inductance.

A. $(1/4)\ \mu_o N\ (b - a)^2/(b + a)$

B. $(1/4)\ \mu_o N^2\ (b - a)^2/(b + a)$

C. $\mu_o Nb^2/(b + a)$

D. $\mu_o N^2 b^2/(b + a)$

7. How is inductance L related to flux linkage $N\phi_B$ and the current i which causes the flux?

A. $(N\phi_B)^2 = Li^2$

B. $N\phi_B = Li$

C. $N\phi_B i = L$

D. $N\phi_B iL = \text{constant}$

8. A closely wound coil with a self-inductance of 5.1 millihenrys carries a current of 0.01 amp. The instantaneous total self flux linked by the coil is

A. 5.1×10^{-2} weber

B. 5.1×10^{-5} weber

C. 2.5×10^{-5} weber

D. 2.5×10^{-2} weber

9. The following problem is the first of two steps to find the inductance of an ideal solenoid.

Recall that the magnetic field in an ideal solenoid is $n\mu_0 i$, where n is the number of turns per unit length. Find the magnetic flux ϕ_B through a cross section of the solenoid (cross section area A, and length ℓ). Neglect edge effects

A. $\mu_0 i A \ell$

B. $n\mu_0 i A \ell$

C. $n\mu_0 i A$

D. $n\mu_0 i \ell^2$

10. Find the inductance of a long solenoid having n turns per unit length, length ℓ, and cross-sectional area A.

A. $L = \mu_0 n^2 \ell A$

B. $L = \mu_0 n \ell A$

C. $L = \mu_0 n^2 \ell^2 A$

D. $L = \mu_0 n \ell A^2$

INFORMATION PANEL Self-Induced Electromotive Force

OBJECTIVE

To analyze the defining equation for inductance and to solve problems in which it is used.

Faraday's law for an ideal coil in which the flux linkage may be given as $N\phi$ is written:

$$\varepsilon = -\frac{d(N\phi_B)}{dt} \qquad (1)$$

next page

continued

As shown in the previous set, the number of flux linkages is equal to the product of the inductance L and the current in the coil i. Thus:

$$N\phi_B = Li \tag{2}$$

Equations (1) and (2) may be combined to give equation :

$$\varepsilon = -L\frac{di}{dt} \tag{3}$$

And equation (3) may be written in this form:

$$L = -\frac{\varepsilon}{di/dt} \tag{4}$$

Equation (4) is considered to be the defining equation for the inductance of any device in which an induced emf appears as a result of a time-varying magnetic field. As the defining equation shows, the unit of inductance is the volt per ampere per second, or simply the volt-second per ampere. This unit is called the *henry* and may therefore be defined:

> *A coil (or any other device in which an induced emf appears as a result of the presence of a time-varying magnetic field) has an inductance of 1 henry if a current through it varying at the rate of one ampere per second causes an induced emf of one volt to appear across its terminals.*

The henry may be abbreviated "h". Its common subunits are the millihenry (mh) and the microhenry (μh).

The direction of an induced emf is found from Lenz's law; this emf always opposes the direction of di/dt. If the current is falling, the induced emf takes a direction such that it tends to maintain the current; if the current is rising, the direction of the induced emf tends to prevent it from rising.

When an emf is applied to a coil, the rate at which the current rises is given by:

$$\varepsilon_{app} = L\frac{di}{dt} \tag{5}$$

in which it must be noted that the sign of di/dt is the *same* as that of the *applied* emf. This is in contradistinction to the signs when one refers to the *induced* emf.

next page

continued

The inductance of a coil may, therefore, be measured by applying a known emf, measuring the rate of change of the current and using the relation:

$$L = \frac{\varepsilon_{app}}{di/dt} \qquad (6)$$

The summary above will be helpful in your work on the problems in this set.

11. An emf is applied to a device with a self inductance L and a resistance R causing the current to increase. The power delivered, εi, is equal to

A. $i^2R - Li\ di/dt$

B. $Li\ di/dt$

C. $i^2R + Li\ di/dt$

D. $-Li\ di/dt$

12. How is inductance L defined in terms of the electromotive force ε produced by a time-varying current i?

A. $L = \frac{-\varepsilon}{(di/dt)}$

B. $L = -\varepsilon\ (di/dt)$

C. $L = \frac{(di/dt)}{\varepsilon}$

D. $L = -i(d\varepsilon/dt)$

13. Analyzing the defining equation for inductance, the MKS unit of inductance (called the henry) is equivalent to

A. $\dfrac{\text{emf - sec}}{\text{amp}}$

B. $\dfrac{\text{volt - sec}}{\text{coul}}$

C. $\dfrac{\text{volt - amp}}{\text{sec}}$

D. $\dfrac{\text{volt - sec}}{\text{amp}}$

14. In a closely wound coil with a self-inductance of 5.1 millihenrys, the current is increasing at the rate of 0.0050 amp/sec. Find the self-induced emf in volts for a fixed geometry.

15. An emf of 12.5 volts is applied across an induction coil. If the current through the coil is changing at the rate of 0.0800 amp/sec, what must be the inductance of the coil in henrys.

16. Use Ohm's law to deduce which one of the following expressions corresponds to electrical power delivered by an emf ε. The power expended by current i in a resistor R is i^2R.

A. εi^2

B. ε/i^2

C. ε/i

D. εi

17. The applied power required to cause a rate of current rise di/dt in a coil of self-inductance L is:

A. $i dL/dt$

B. $i^2 L\, di/dt$

C. $i L\, di/dt$

D. $-i dL/dt$

INFORMATION PANEL

Energy Stored in an Inductor

OBJECTIVE

To solve problems in which the energy stored in an inductor is a key factor.

Work must be done to increase the current in an inductor from zero or some other value to a higher steady-state value. When the current has reached its steady-state value and is no longer changing, its magnitude is determined solely by the *resistance* of the inductor. To maintain the steady current, the emf required is calculated from Ohm's law. However, when the steady-state circuit is opened, the magnetic field around the coil collapses and induces a counter emf which opposes the collapsing field. Thus the work done initially in raising the current to its steady-state value against the counter emf is returned to the circuit upon collapse of the field.

This suggests that energy is stored in the magnetic field of the coil while current is present. To determine the quantity of stored energy, we first observe that the work done in raising the current from some initial value i by an amount di in time dt is given by:

$$dW = i\varepsilon dt \qquad (1)$$

Note that the product $i\varepsilon$ is the instantaneous power so that the product on the right side of the equation is power x time, or work. In equation (1), ε, the emf *applied* to the inductor, is given by:

$$\varepsilon = L\frac{di}{dt} \qquad (2)$$

next page

continued

Substituting the expression for ε from equation (1) into equation (2) we obtain:

$$dW = i\left(L\,\frac{di}{dt}\right)dt \tag{3}$$

and integrating to obtain the total work done in raising i from zero to I, we have:

$$W = \int_0^I i\;L\,\frac{di}{dt}\,dt$$

or

$$W = \frac{1}{2}\,LI^2 \tag{4}$$

Thus, the energy stored in the magnetic field of the inductor is:

$$U_B = \frac{1}{2}\,LI^2 \tag{5}$$

When the inductance of the coil is expressed in henrys and the current in amperes, then the energy comes out in joules as shown below:

$$\frac{\text{volt-second}}{\text{amp}} \times \text{amp}^2 = \text{volt-sec-amp}$$

But a volt is a joule/coulomb and an ampere is a coulomb/sec, hence

$$\frac{\text{joule}}{\text{coul}} \times \text{sec} \times \frac{\text{coul}}{\text{sec}} = \text{joule}$$

18. An inductor with inductance L = 5 millihenrys is connected in a series circuit with an open switch. When the switch is closed, the current in the circuit builds up from zero to a steady state current of 2 amp. Calculate the energy in joules stored in the inductor.

19. Which of the following best explains why energy must be stored in the magnetic field of a current carrying inductance?

A. An emf is required to drive a current through an inductance.

B. When the current is interrupted, field builds up and induces an emf which can supply power to a closed circuit.

C. When the current is interrupted, the collapsing field induces an emf which can supply power to a closed circuit.

D. None of the above.

20. A coil with a self inductance of 4 millihenrys carries a current of 0.5 amps. Find the energy stored in the magnetic field, in joules.

21. An initial steady-state current i in an inductor with inductance L is allowed to decay to zero. What is the total energy lost to heat, sound, and radiation?

A. Li^2

B. 0

C. $\frac{1}{2} Li$

D. $\frac{Li^2}{2}$

INFORMATION PANEL Energy Density

OBJECTIVE

To solve problems involving the energy density of a magnetic field.

The concept of *energy density* provides a useful tool for some types of problems in electromagnetic induction. We have already seen that the total energy stored in the magnetic field of an inductor is:

$$U_B = \frac{1}{2} Li^2 \quad (1)$$

Energy density is defined as *energy per unit volume* or, if u symbolizes energy density then:

$$u = \frac{U_B}{V} \quad (2)$$

in which V = the volume of the space occupied by the magnetic field in question. Here again, we are assuming that the magnetic field has a uniform magnitude distribution.

Your source reading demonstrates that the following equation can be derived for u:

$$u = \frac{B^2}{2\mu_o} \quad (3)$$

in which μ_o is the familiar *permeability of a vacuum.* Attention is called to the fact that, although the derivation of equation (3) is obtained by referring to the characteristics of a solenoid, it is valid for all magnetic field configurations.

The core question in this set asks that you compute the energy density of a circular loop of wire with given radius and current. To do this you must first obtain the value of B (magnitude) using the Biot-Savart law for this special case, and then substitute this value into equation (3).

22. A long coaxial cable consists of two concentric cylinders with radii a and b. Its central conductor carries a steady current i, the outer conductor providing the return path. What is the energy stored in the magnetic field for a length ℓ of such a cable? You may assume that the energy is stored in the space between the conductors.

A. $\dfrac{\mu_o i^2}{8\pi^2 r^2}$

B. $\dfrac{\mu_o i^2 \ell}{4\pi} \ln (b/a)$

C. $\dfrac{\mu_o i^2 \ell}{4\pi} \left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

D. $\dfrac{\mu_o i^2 \ell}{4\pi} (b - a)$

23. The total energy stored in a solenoid of inductance L = 4.0 millihenrys is 5.0×10^{-4} j. If the solenoid is 50 cm long with 10^5 turns, the energy *per unit volume* stored in the magnetic field is

(Note: $\mu_o = 4\pi \times 10^{-7}$ mks units)

A. 3.1 joules/m^3

B. 5.0 joules/m^3

C. 6.3×10^3 joules/m^3

D. Zero

24. A circular loop of wire 10.0 cm in radius carries a current of 100 amp. Calculate the energy density at the center of the loop in joules/m^3.

25. A long wire of radius R carries a current of uniform density. Let i be the total current in the wire. What is the magnetic energy per unit length stored within the wire?

A. $\frac{\mu_o i^2}{8\pi^2 R^2}$ joules/m

B. $\frac{\mu_o i^2}{8\pi}$ joules/m

C. $\frac{\mu_o i^2}{16\pi}$ joules/m

D. 0

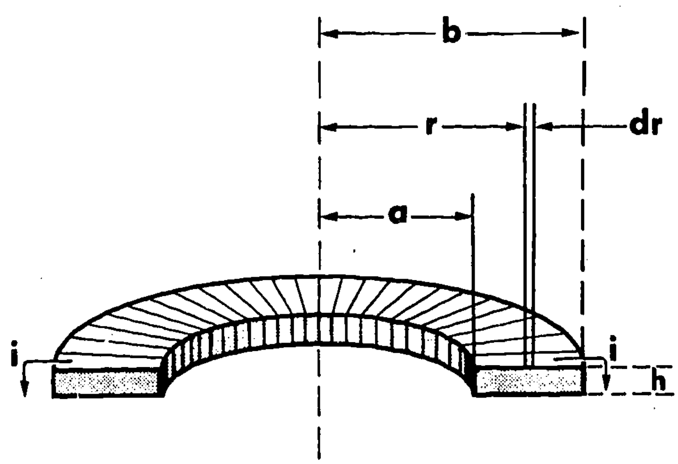
26. A rectangular toroid of N turns carries a current i as shown in the diagram. The inner and outer radii are a and b, and the height is h. The energy stored in the toroid is:

A. $\frac{\mu_o N^2 i^2 h}{8\pi^2 r^2}$

B. $\frac{\mu_c N^2 i^2 h}{4\pi r}$ (b - a)

C. $\frac{\mu_o N^2 i^2 h}{4\pi}$ ℓn (b/a)

D. $\frac{\mu_o N^2 i^2 h}{4\pi} \left(\frac{1}{a} - \frac{1}{b}\right)$



[a] CORRECT ANSWER: 1×10^{-2}

The energy is stored in the magnetic field associated with the inductor. The energy is equal to work done in raising the current from zero to $I = 2$ amp through the inductor. During this process, when the current in the inductor is i, the work done in increasing the current by di in time dt is

$$dW = i\,\varepsilon\,dt \tag{1}$$

where ε, the emf applied to the inductor, is

$$\varepsilon = L\,\frac{di}{dt} \tag{2}$$

Substituting the expression for ε in equation (1) and integrating, we obtain:

$$W = \int_0^I i\,L\,\frac{di}{dt}\,dt$$

$$= \frac{1}{2}\,LI^2 = 1 \times 10^{-2}\ \mathrm{j}$$

TRUE OR FALSE? The stored energy is equal to the total work done only for an ideal inductor with zero resistance.

[b] CORRECT ANSWER: A

For a closely packed coil with N turns,

$$N\phi_B = Li$$

thus

$$L = N\phi_B/i$$

In the previous problem, you found

$$\phi_B = BA = n\mu_o iA$$

where n is the number of turns per unit length. Hence,

$$L = N\mu_o inA/i = N\mu_o nA$$

Finally, we note that $N = n\ell$. Substituting in the expression for L gives

$$L = \mu_o n^2 \ell A$$

TRUE OF FALSE? In accord with the symbols used here, $n = N/\ell$.

[a] CORRECT ANSWER: C

We are given that the total energy stored is 5.0×10^{-4} joules. We must, therefore, compute the volume from the data. We know the field is confined to the core region of the solenoid and that

$$L = 4.0 \times 10^{-3} \text{ henry} = \frac{N^2 \mu_o A}{\ell}$$

$$\frac{(10^5)^2 \ (4\pi \times 10^{-7})A}{0.50}$$

giving

$$A = \frac{1}{2\pi} \times 10^{-6} \text{ m}^2$$

Therefore

$$V = A\ell = \frac{1}{4\pi} \times 10^{-6} \text{ m}^3$$

and the ratio gives the solution:

$$\frac{5.0 \times 10^{-4}}{V} = 6.3 \times 10^3 \text{ joules/m}^3$$

Note that this solenoid would be very difficult to construct, with 100,000 turns and a cross-sectional area of 1.6×10^{-7} m^2.

[b] CORRECT ANSWER: 0.32

Since the loop is entirely immersed in the magnetic field, the flux through it is given by

$$\phi_B = \pi r^2 B \qquad (1)$$

The field is varying, so an emf will be induced in the loop given by Faraday's law of induction,

$$\varepsilon = -\frac{d\phi_B}{dt} = -\pi r^2 \frac{dB}{dt} \qquad (2)$$

The sign in (2) is immaterial since we are not interested in the direction of emf. We may use Joule's law to find the power at which heat is being generated in the loop which, of course, is the amount of heat generated in one second. Thus,

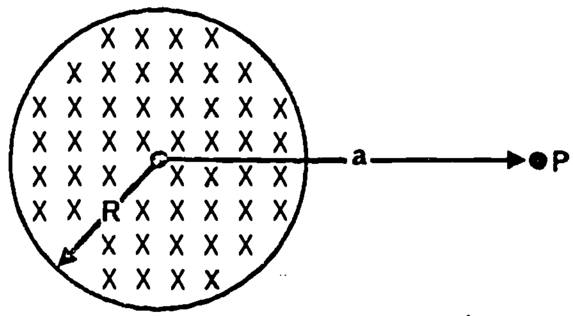
$$P = \frac{\varepsilon^2}{R} = \frac{[\pi r^2 (dB/dt)]^2}{R}$$

Substituting numerical values we obtain

$$\text{Heat generated per second} = P \times (1.0 \text{ sec}) = \frac{[\pi \times (2.0 \times 10^{-1})^2 \times (10/\pi)]^2}{0.50}$$

$$= \frac{(4.0 \times 10^{-1})^2}{0.50} = 0.32 \text{ joules}$$

[a] CORRECT ANSWER: 8.8×10^7



The electric field $\vec{E}$ at point P due to the changing magnetic field may be found from

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_B}{dt} \qquad (1)$$

Let us choose a circular path of integration of radius a. Due to symmetry, the induced electric field lines are concentric circles. Further $\vec{E}$ is tangent to the path of integration. Therefore,

$$\oint \vec{E} \cdot d\vec{\ell} = E\ 2\pi a \qquad (2)$$

and

$$\phi_B = \int \vec{B} \cdot d\vec{S} = B\pi R^2 \qquad (3)$$

because the $\vec{B} \cdot d\vec{S}$ is zero for all points that lie outside the effective boundary of the magnetic field. Substituting expressions (2) and (3) in (1), we find

$$E\ 2\pi a = -\pi R^2 \frac{dB}{dt} \qquad (4)$$

or

$$E = -\frac{R^2}{2a}\frac{dB}{dt} \qquad (5)$$

Therefore, the magnitude of the instantaneous acceleration $\frac{dv}{dt}$ experienced by the electron is

$$\frac{dv}{dt} = \frac{|e|\ |E|}{m_e} \qquad (6)$$

$$= \frac{e}{m_e}\frac{R^2}{2a}\frac{dB}{dt}$$

$$= 8.8 \times 10^7 \text{ m/sec}^2$$

TRUE OR FALSE? The product $|e|\ |E|$ in equation (6) gives the magnitude of the instantaneous force acting on the electron in the varying magnetic field.

[a] CORRECT ANSWER: B

This problem is a direct application of the defining equation for inductance L,

$$N\phi_B = Li$$

where ϕ_B is the magnetic flux $\int \vec{B}\cdot d\vec{S}$ through the cross section of the coil. B is a given constant, so $\phi_B = \int \vec{B}\cdot d\vec{S} = BA$, where A is the area of the cross section. The diameter of the ring portion of the doughnut is b - a, and the area is then

$$A = \pi \left(\frac{b - a}{2}\right)^2$$

Substituting A and the given value for B into

$$L = NBA/i$$

we obtain

$$L = N \frac{\mu_o Ni}{\pi(a + b)} \cdot \pi \left(\frac{b - a}{2}\right)^2 \Big/ i$$

$$= \frac{1}{4} \mu_o N^2 \frac{(b - a)^2}{(b + a)}$$

TRUE OR FALSE? The magnetic flux through the cross section of the toroid is given by

$$\pi B \left(\frac{b - a}{2}\right)^2$$

[b] CORRECT ANSWER: D

The MKS unit of inductance can be found from the definition in terms of the induced emf,

$$L = - \frac{\varepsilon}{di/dt}$$

Simply inserting the units for ε (volts), i (amps), and t (secs) gives the unit for L. Note that emf is not a unit.

[a] CORRECT ANSWER: 5×10^{-4}

The work done in increasing the current from i to $i + di$ in time dt is

$$dW = i\,\varepsilon\,dt \tag{1}$$

where ε, the applied emf to the inductor, is

$$\varepsilon = L\frac{di}{dt} \tag{2}$$

Substituting the expression for ε in (1) and integrating, we obtain

$$W = \int_0^L Li\frac{di}{dt}\,dt = \frac{1}{2}LI^2$$

so the work done is

$$W = LI^2/2 = L\frac{(0.5)^2}{2} = 4 \times 10^{-3}\frac{(0.25)}{2}$$

$$= 5 \times 10^{-4}\ \text{j}$$

[b] CORRECT ANSWER: 156

For the *applied* voltage ε,

$$\varepsilon_{app} = L\frac{di}{dt}$$

and solving for inductance

$$L = \varepsilon_{app}\Big/\frac{di}{dt} = (12.5\ \text{volts})/(.0800\ \text{amp/sec}) = 156\ \text{henrys}$$

Notice that the sign of di/dt is opposite for *applied* emf to that for *induced* emf.

[a] CORRECT ANSWER: C

The $\vec{B}$ field lines inside a toroid are concentric circles. Applying Ampere's law to a circular path of radius r, we obtain

$$\oint \vec{B}\cdot d\vec{\ell} = B\ 2\pi r = \mu_o N i$$

or

$$B = \frac{\mu_o N i}{2\pi r}$$

The energy density for points inside the toroid is

$$u = \frac{B^2}{2\mu_o} = \frac{\mu_o N^2 i^2}{8\pi^2 r^2} \tag{1}$$

The total energy stored in the toroid is obtained by integrating expression (1) over the volume of the toroid. Thus, the energy

$$U = \int_V u\ dV \tag{2}$$

where $dV = 2\pi rh\ dr$. Therefore,

$$U = \int_a^b \frac{\mu_o N^2 i^2\ 2\pi rh\ dr}{8\pi^2 r^2}$$

$$= \frac{\mu_o N^2 i^2 h}{4\pi}\int_a^b \frac{dr}{r}$$

$$= \frac{\mu_o N^2 i^2 h}{4\pi}\ \ell n\ (b/a)$$

TRUE OR FALSE? The volume of the toroid discussed in this solution is $V = 2\pi rh$.

[b] CORRECT ANSWER: B

Flux linkage $N\phi_B$ is related to inductance L and current i according to

$$N\phi_B = Li$$

Thus, we obtain

$$N\phi_B = (5.1 \times 10^{-3}\ \text{henrys})\ (0.01\ \text{amp}) = 5.1 \times 10^{-5}\ \text{weber}$$

[a] CORRECT ANSWER: D

Faraday's law can be rewritten as

$$\varepsilon = \oint \vec{E}\cdot d\vec{\ell} = E\ 2\pi r = -\frac{d\phi_B}{dt}$$

and

$$\phi_B = \pi r^2 B$$

Therefore, we may write

$$E = -\frac{r}{2}\frac{dB}{dt}$$

The minus sign signifies the fact that the induced electric field $\vec{E}$ acts to oppose the change in the magnetic field. Note also that the line integral

$$\oint \vec{E}\cdot d\vec{\ell}$$

here is not equal to zero. Consequently, the electric field associated with a changing magnetic field is non-conservative. In contrast to this the electric field produced by stationary charges is conservative, and in that case

$$\oint \vec{E}\cdot d\vec{\ell} = 0$$

[b] CORRECT ANSWER: A

From Faraday's law of induction we see that the induced emf depends upon the flux linkages $N\phi_B$,

$$\varepsilon = -\frac{d(N\phi_B)}{dt} \qquad (1)$$

The flux linkages are proportional to the current according to

$$N\phi_B = Li \qquad (2)$$

where L is a proportionality constant called inductance. Substituting eq. (2) into (1) gives the correct relation between ε, L, i, and t. Therefore,

$$L = -\varepsilon \Big/ \left(\frac{di}{dt}\right)$$

[a] CORRECT ANSWER: C

The applied emf ε_{ap} is given by

$$\varepsilon_{app} = L\,\frac{di}{dt}$$

Knowing that power applied would be found by the product of the applied emf, ε_{ap}, and the current i, your answer then follows:

$$\varepsilon_{app} i = iL\,\frac{di}{dt}$$

Notice that the sign of the applied emf is opposite that of induced emf ε.

TRUE OR FALSE? In this expression: $\varepsilon = -L\,\frac{di}{dt}$, ε is the applied rather then the induced emf.

[b] CORRECT ANSWER: C

Let us apply Ampere's law to find the magnetic field inside the wire. At a distance r from the center of the wire we have

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_o i\,\frac{\pi r^2}{\pi R^2}$$

or

$$B\,2\pi r = \mu_o i\,\frac{r^2}{R^2}$$

and

$$B = \frac{\mu_o i r}{2\pi R^2}$$

The energy density at r is therefore

$$u = \frac{B^2}{2\mu_o} = \frac{\mu_o i^2 r^2}{8\pi^2 R^4}$$

The total energy is obtained by integrating over the entire cross section of the wire

$$U = \int u\,dV = \int_o^R \frac{\mu_o i^2 r^2}{8\pi^2 R^4}\,2\pi r dr\,\ell = \frac{\mu_o i^2}{16\pi}\,\ell$$

where $dv = 2\pi r dr\,\ell$. Therefore, the energy per unit length is

$$U/\ell = \frac{\mu_o i^2}{16\pi}$$

[a] CORRECT ANSWER: B

In the space between the two conductors, Ampere's law

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_o i$$

leads to

$$B\, 2\pi r = \mu_o i$$

or

$$B = \frac{\mu_o i}{2\pi r}$$

The energy density for points between the conductors is

$$u = \frac{1}{2\mu_o} B^2 = \frac{\mu_o i^2}{8\pi^2 r^2}$$

The total magnetic energy is obtained by integrating over the space enclosed between the conductors. Thus

$$U = \int u \, dV$$

and

$$dV = (2\pi r \ell)\, dr$$

Therefore,

$$U = \int \frac{\mu_o i^2}{8\pi^2 r^2} (2\pi r \ell)\, dr$$

$$= \frac{\mu_o i^2 \ell}{4\pi} \int_a^b \frac{dr}{r} = \frac{\mu_o i^2 \ell}{4\pi} \ln (b/a)$$

TRUE OR FALSE? The energy density in the space between conductors is directly proportional to the magnitude of the magnetic induction.

[b] CORRECT ANSWER: C

It is helpful to think of the inductance as a device which tends to keep the current at its uninterrupted or steady value. When we try to reduce the current, the magnetic field associated with the current collapses. This changing magnetic field induces an emf in the coil which tends to maintain the current.

[a] CORRECT ANSWER: 8.8×10^7

The electric field due to this changing magnetic field is given by

$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(B\pi r^2) \tag{1}$$

which may be reduced to

$$E\ 2\pi r = -\pi r^2 \frac{dB}{dt} \tag{2}$$

or

$$E = -\frac{1}{2} r \frac{dB}{dt} \tag{3}$$

Therefore, the magnitude of the acceleration experienced by the electron is

$$a = \frac{|e|\ |E|}{m_e} \tag{4}$$

Substitution of the numerical values gives

$$E = 1/2 \times 5.0 \times 10^{-2} \times 2.0 \times 10^{-2} = 5.0 \times 10^{-4}\ \text{nt/coul} \tag{5}$$

Putting the values for $|e|$ and m_e in the expression for acceleration, we get

$$a = \frac{1.6 \times 10^{-19} \times 5.0 \times 10^{-4}}{9.1 \times 10^{-31}} = 8.8 \times 10^7\ \text{m/sec}^2 \tag{6}$$

TRUE OR FALSE? Referring to step (5) in the above solution, this result could be written as 5.0×10^{-4} volts/m^2.

[b] CORRECT ANSWER: B

Flux linkage $N\phi_B$ is proportional to current i, and L is the proportionality constant. By convention, L is placed with i to convert the proportionality

$$N\phi_B \propto i$$

to an equality

$$N\phi_B = Li$$

[a] CORRECT ANSWER: 0.157

The magnetic field at the center of the current carrying loop is obtained from the Biot-Savart law:

$$B = \int \frac{\mu_o i}{4\pi} \frac{|d\vec{\ell} \times \vec{r}|}{r^3} = \frac{\mu_o i}{4\pi R^2} \int d\ell = \frac{\mu_o i}{4\pi R^2} (2\pi R)$$

$$= \frac{\mu_o i}{2R}$$

The energy density is thus

$$u = \frac{1}{2} \frac{B^2}{\mu_o} = \frac{\mu_o i^2}{8R^2}$$

The substitution of the numerical values yields

$$u = 0.157 \text{ joules/m}^3$$

[b] CORRECT ANSWER: D

As in the preceding question, the flux through the loop is

$$\phi_B = \int \vec{B} \cdot d\vec{S} = B\pi R^2$$

because $\vec{B} \cdot d\vec{S}$ is zero for all points that lie outside the effective boundary of the magnetic field.

Thus, from Faraday's law,

$$\oint \vec{E} \cdot d\vec{\ell} = E\, 2\pi r = -\frac{d\phi_B}{dt} = -\pi R^2 \frac{dB}{dt}$$

or

$$E = -\frac{1}{2} \frac{R^2}{r} \frac{dB}{dt}$$

[a] CORRECT ANSWER: 2.5×10^{-5}

This problem is a straightforward application of the relation between inductance L, changing current, and induced emf ε,

$$\varepsilon = -L\ (di/dt)$$

An important point to notice is that current i does not have any effect on ε. Only the rate of change of current is relevant here. The information about the number of turns is also superfluous.

Inserting

$$L = 5.1 \times 10^{-3} \text{ henry and } di/dt = .0050 \text{ amp/sec}$$

gives

$$\varepsilon = -5.1 \times 10^{-3} \times .0050 \simeq 2.5 \times 10^{-5} \text{ volts}$$

[b] CORRECT ANSWER: D

The energy stored originally in the inductor is (1/2) Li^2, and finally the energy is zero (because $i = 0$). All the energy is lost, therefore, to heat, sound, and/or radiation. That is, the total energy lost by the inductor is (1/2) Li^2. Just how the new forms of energy will share in the dissipation of the original energy depends on the nature of the unspecified external circuit.

TRUE OR FALSE? The energy interchanges which occur as an inductor discharges never violate the law of energy conservation.

[c] CORRECT ANSWER: C

The power delivered is partly dissipated by the resistance in amount i^2R and partly stored as energy in the magnetic field at a rate $Li\ di/dt$ since the current (hence field) is increasing.

TRUE OR FALSE? The energy stored in the magnetic field is $Li\ di/dt - i^2R$.

[a] CORRECT ANSWER: D

Ohm's law is

$$\varepsilon = iR$$

and the power dw/dt lost to joule heating is given by

$$\frac{dw}{dt} = i^2R$$

Multiply Ohm's law by i and compare the result with dw/dt.

Comparing the right side of

$$\varepsilon i = i^2R$$

with the right side of $dw/dt = i^2R$, we note that things equal to the same thing are equal to each other and

$$\frac{dw}{dt} = \varepsilon i$$

[b] CORRECT ANSWER: C

The definition of magnetic flux is $\phi_B = \int \vec{B}\cdot d\vec{S}$. In this case, $d\vec{S}$ is an element of cross-sectional area and the area is perpendicular to the magnetic field $\vec{B}$,

$$\phi_B = \int BdS$$

The magnetic field is constant so $\phi_B = \int Bds = BA = n\mu_o iA$

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

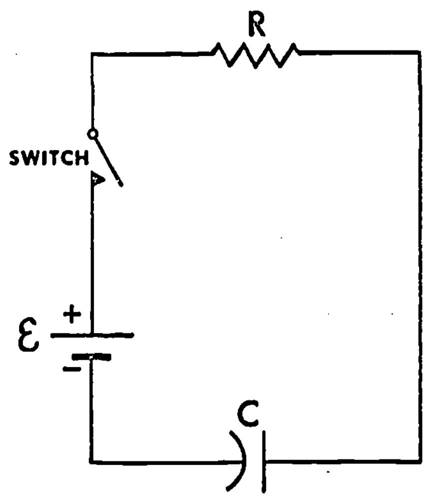
INFORMATION PANEL

Time-Varying Current and Charge in an RC Charging Circuit

OBJECTIVE

To answer questions and solve problems relating to the time-varying current in an RC charging circuit; to the time-varying charge that appears across the capacitor.

A resistor and capacitor (R and C in the diagram) connected in series across a source of emf ε form a single-loop RC circuit. To appreciate the physical significance of the events that occur after the switch has been closed, consider the electrical condition of the configuration just before this is done: the current in the circuit is zero and the capacitor charge is zero; there is no potential difference across either of the two circuit components.



At the instant of switch closure, a charging current begins to build up the charge on the capacitor. At this instant ($t = 0$), the magnitude of the current is the same as it would be if the capacitor were short-circuited, i.e., if the resistor alone were connected across the source of emf; when $t = 0$, the potential difference across the capacitor is zero since it has not yet had time to acquire a charge.

As time passes, the capacitor acquires an increasing charge and develops a potential difference equal to q/C. At any instant during the charging interval, the sum of the potential differences across resistor and capacitor must equal ε (loop theorem). Thus, at some time t after charging has begun,

$$\varepsilon - iR - q/C = 0 \qquad (1)$$

in which iR is the potential difference across the resistor and q/C is the potential difference across the capacitor. Compare this with the initial condition immediately after the switch is closed ($t = 0$)

$$\varepsilon - iR - 0 = 0 \qquad (2)$$

next page

continued

which shows that the potential difference across the resistor is equal to the source emf while the potential difference across the capacitor is still zero. This comparison indicates that after any finite time, the *potential difference across the resistor is decreased by an amount equal to the potential difference that has built up across the capacitor.* This means that the charging current must also decrease as the charge builds up across the capacitor.

The reason for the decreasing current may be approached in a different way. As the capacitor charges, the potential difference across its terminals grows *in opposition to the applied emf* so that the potential difference across the resistor is the difference between the two. This may be shown by solving equation (1) for iR:

$$iR = \varepsilon - q/C \qquad (3)$$

hence the current in the loop at any time is given by:

$$i = \frac{\varepsilon - q/C}{R} \qquad (4)$$

Referring to equation (4), it is immediately apparent that when the capacitor becomes fully charged and the potential difference across it becomes equal to the source emf ($q/C = \varepsilon$), the loop current becomes zero. In summary, then, we have,

Initial condition ($t = 0$): $iR = \varepsilon$; i = maximum; $q = 0$; and $q/C = 0$

Condition at full charge: $iR = 0$; i = zero; q = maximum; and $q/C = \varepsilon$

For your convenience, the relationships that you will need for answering questions and solving problems involving the charging process are given below:

CHARGE ON CAPACITOR AT ANY TIME t: $q = C\varepsilon(1 - e^{-t/RC})$ (5)

CURRENT IN THE RC LOOP AT ANY TIME t: $i = \frac{\varepsilon}{R} e^{-t/RC}$ (6)

RELATIONSHIP AMONG APPLIED EMF, RESISTOR POTENTIAL DIFFERENCE, AND CAPACITOR POTENTIAL DIFFERENCE AT ANY TIME: $\varepsilon = iR + q/C$ (7)

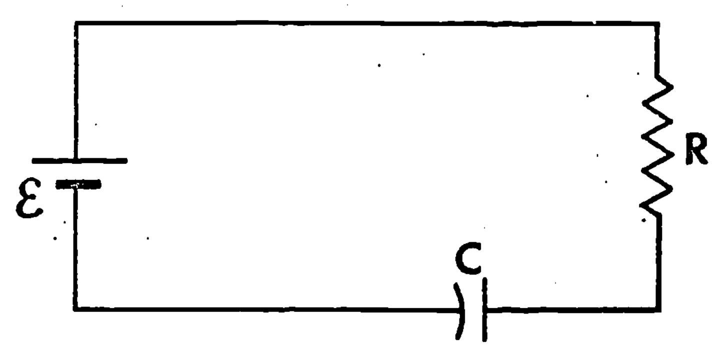
CHARGE ON CAPACITOR WHEN $t = RC$: $q = C\varepsilon(1 - e^{-1}) = 0.63\ C\varepsilon$ (8)

The problems in this section are based upon the interpretation and application of the equations summarized above. The student is strongly urged to review their derivations in the assigned reading before starting to work on the questions and problems.

PROBLEMS

1. A 3.0 megohm resistor and a 1.0 microfarad capacitor are connected in series with a seat of emf of $\varepsilon = 6.0$ volts. At 3.0 sec after the connection is made, what is the rate at which the charge on the capacitor is increasing (in amps)?

2. Let us consider the circuit given in the diagram below. A charge dq moves through any cross section of the circuit during a time interval dt. This charge enters the seat of emf ε at its low-potential end and leaves at its high-potential end. The emf must do an ammount of work dw on the positive charge carriers to force them to go to the point of higher potential. This work must equal the energy that appears as Joule heat in the resistor and the increase in the amount of energy U stored in the capacitor in accordance with the conservation of energy principle. Which one of the statements below best expresses this principle in this specific case?



A. $\varepsilon\, dq = i^2 R\, dt + d(q^2/2C)$

B. $\varepsilon\, dq = i\, R^2\, dt + d(q^2/2C)$

C. $\varepsilon\, dq = i^2 R\, dt + d(C^2/2q)$

D. $\varepsilon\, dq = i^2 R\, dt + d(C^2/2q^2)$

3. We have established previously that the equation for the RC circuit (the circuit with resistor and capacitor) is

$$\varepsilon = iR + q/C = R\frac{dq}{dt} + q/C$$

This is a differential equation with q as the dependent variable and t as the independent variable. The solution of this equation is

$$q = C\varepsilon(1 - e^{-t/RC})$$

This means that when the solution, i.e., q as a function of t, is substituted in the equation, the equation will be an identity. This equation describes how the charge on a capacitor increases with time, and hence is given the name "charging equation" for RC circuits. In an R-C circuit the charge on a capacitor will approach the equilibrium value $C\varepsilon$ when

A. either $t \to \infty$ or $R \to \infty$

B. either $t \to \infty$ or $R \to 0$

C. both t and $R \to 0$

D. both t and $R \to \infty$

4. The relation between q with t during the charging process in an RC circuit is given by expression

$$q = C\varepsilon(1 - e^{-t/RC})$$

The instantaneous current in the circuit is

A. $i = \frac{\varepsilon}{R} e^{-t/RC}$

B. $i = -C\varepsilon\ e^{-t/RC}$

C. $i = C\varepsilon\ e^{-t/RC}$

D. $i = 0$

5. From the equation for the instantaneous current in an RC charging circuit, determine the value of the initial current.

A. $i_o = 0$

B. $i_o = \varepsilon/R$

C. $i_o = (\varepsilon/R)e^{-1}$

D. $i_o = (\varepsilon/R)e$

6. At the end of a time interval equal to one capacitive time constant, the charge on the capacitor in an RC circuit has increased to

A. 50% of its equilibrium value

B. 37% of its equilibrium value

C. 63% of its equilibrium value

D. 100% of its equilibrium value

INFORMATION PANEL — Graphical Approach to the RC Charging Circuit

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OBJECTIVE

To answer questions and solve problems relating to the RC charging circuit by using the information obtainable from graphs of the relevant equations.

In the RC time-varying charge equation

$$q = C\varepsilon\,(1 - e^{-t/RC}) \qquad (1)$$

next page

continued

the quantity RC is called the *time constant* of the circuit. The fact that RC must have the dimensions of time is evident when one considers that the exponent $-t/RC$ must be dimensionless. In a specific circuit, the value of the time constant is especially significant since it represents the time interval required to bring the charge on the capacitor up from its initial value of zero to 63% of full charge (equilibrium value). This is readily shown by putting $t = RC$ in equation (1)

$$q = C\varepsilon(1 - e^{-1}) \tag{2}$$

The quantity e is the base of the Naperian system of logarithms and has a value of 2.718 to four significant digits. Thus:

$$e^{-1} = 1/e = 0.3678$$

hence

$$1 - e^{-1} = 0.6322$$

so that the charge on a capacitor in an RC circuit after an interval of one time constant is:

$$q = 0.6322\ C\varepsilon$$

or roughly 63% of the value the charge will have after it reaches its equilibrium condition. A capacitor is considered fully charged (very nearly) after a charging time of 5 time constants.

A great deal of information may be obtained from a graph of the time-varying charging current versus the time interval over which the capacitor charges. The graph given in Figure 1 has been drawn for $R = 2000$ ohms, $C = 1.0$ microfarad, and an applied emf of $\varepsilon = 10$ volts.

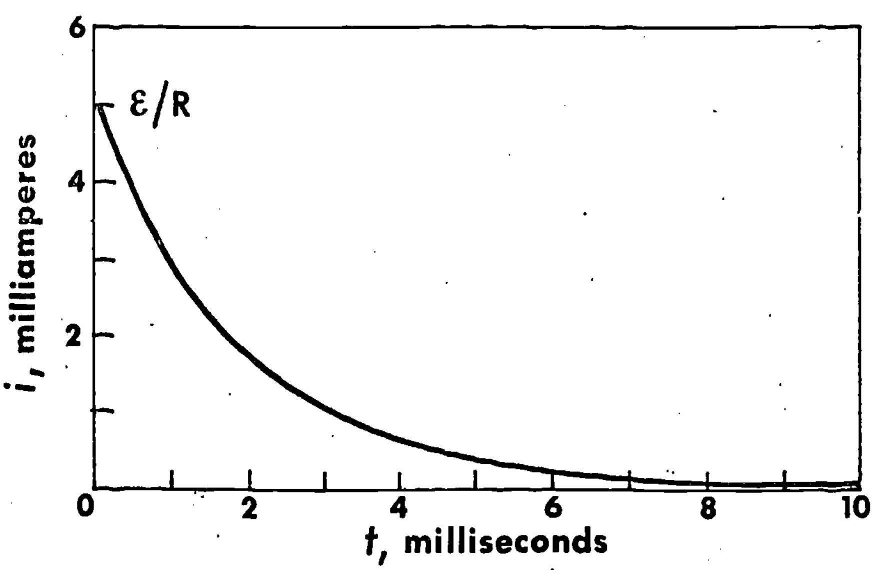


Figure 1

next page

continued

The graph shows that at $t = 0$, the current $i = 5 \times 10^{-3}$ amp since, for this initial condition, the entire source of emf appears across the resistor (capacitor charge = 0) so that $i = \varepsilon/R = 10$ volts/2000 ohms = 5×10^{-3} amp. The time constant for this particular circuit is:

$$RC = (2 \times 10^{3} \text{ ohms})(1.0 \times 10^{-6} \text{ farad})$$

$$= 2 \times 10^{-3} \text{ sec}$$

To determine the time constant from the graph, it is necessary to locate the point on the curve where the instantaneous current is 37% of its initial value, or 1.85 ma, and project this point on to the time axis to find the value of the time constant.

A similar graph, this time showing the way *charge* varies with time, is presented in Figure 2. The values of R, C, and ε are the same for this curve as for the curve in Figure 1, hence the time constant is 2.0×10^{-3} sec as before.

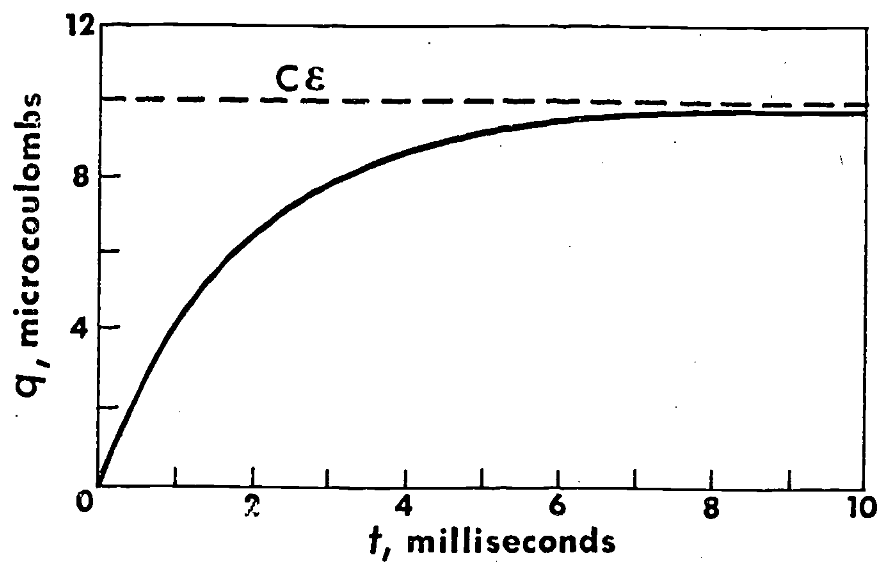


Figure 2

A number of important conclusions can be drawn directly from this graph. Again note that the charge on the capacitor after one time constant period is 6.3 microcoulombs which is 63% of the value that the charge will attain after equilibrium is reached. Observe that after 5 TC periods (10 milliseconds), the capacitor has charged to very nearly 10 microcoulombs which is its equilibrium charge. This verifies a previous statement that a capacitor can be considered fully charged after 5 time constants. To determine the potential difference that the capacitor has acquired after any given number of time constants, merely find the charge that has built up for this interval using equation (1) or the graph itself and then calculate the voltage using: $V = q/C$

7. In Figure 1 of the preceding Information Panel, the current in an RC circuit is plotted against the time. Using this graph, determine the approximate value of the RC time constant.

A. 1 millisecond

B. 2 milliseconds

C. 5 milliseconds

D. 10 milliseconds

8. In an RC circuit, the capacitive time constant is 2 milliseconds. Knowing that $R = 2000$ ohms, find the value of the circuit capacitance.

9. From the curve given in Figure 2 of the preceding Information Panel determine the voltage on the capacitor after two time constants ($RC = 2 \times 10^{-3}$ sec). Take $R = 2000$ ohms and $e^{-1} = 0.37$.

A. 3.7 volts

B. 6.3 volts

C. 1.0 volts

D. 8.6 volts

INFORMATION PANEL — The RC Discharge Process

OBJECTIVE

To answer questions and solve problems in which a capacitor previously charged by a source of emf then discharges through a known resistance.

next page

continued

When analyzed by methods similar to those used in the charging process, the following relationships are obtained:

RESIDUAL CHARGE ON THE CAPACITOR AT ANY TIME t AFTER DISCHARGE HAS BEGUN:

$$q = q_o e^{-t/RC} \qquad (1)$$

in which q_o is the initial charge on the capacitor. If the capacitor has been fully charged initially, then $q_o = C\varepsilon$.

CURRENT AT ANY TIME t AFTER THE DISCHARGE HAS BEGUN:

$$i = -\frac{\varepsilon}{R} e^{-t/RC} \qquad (2)$$

where the negative sign preceding the right member indicates that the direction of the current now is opposite that of its direction during the charging process.

Note that equation (1) may be used to determine the residual charge on a capacitor after it has been allowed to discharge for a given time. If the discharge time is equal to one time constant ($t = RC$), then the residual charge is easily shown to be 37% of the charge it had initially at $t = 0$.

10. A 60-ohm resistor and a 2.1-microfarad capacitor are connected in series with an emf equal to 5.3 volts. After 1 minute, the emf is removed and the capacitor is allowed to discharge. What is the magnitude of the current immediately after the capacitor starts to discharge?

11. A capacitor is first fully charged by a seat of emf. The seat of emf is then removed and the capacitor begins to discharge through resistor R. Using the loop theorem to help you, select the correct discharge equation.

A. $iR + q/C = 0$

B. $iR + q^2/C = 0$

C. $iR - q/C = 0$

D. $iR + q^2C = 0$

12. In the preceding problem, we derived the equation for a discharging RC circuit. The solution to that equation is

$$q = q_o e^{-t/RC}$$

where q_o is the initial charge on the capacitor.

Now the capacitor is allowed to discharge for an interval t = RC. What percentage of the original charge still remains on the capacitor?

A. 33%

B. 37%

C. 50%

D. 63%

13. In a discharging RC circuit, the charge on a capacitor at any time t is

$$q = q_o e^{-t/RC}$$

where q_o is the charge on the capacitor at t = 0.

What is the expression for the current during discharging?

A. $i = -(q_o/RC)\ e^{-t/RC} = -\ (\varepsilon/R)\ e^{-t/RC}$

B. $i = -q_o RC\ e^{-t/RC} = -RC^2\varepsilon\ e^{-t/RC}$

C. $i = -q_o e^{-t/RC} = -C\varepsilon\ e^{-t/RC}$

D. $i = -C\varepsilon\ e^{-t/RC}$

14. A 60.0-ohm resistor and a 2.10-microfarad capacitor are connected in series with a seat of emf equal to 5.30 volts. After one minute, the seat of emf is removed and the capacitor is allowed to discharge. What is the magnitude of the discharging current in amperes after 1.26×10^{-4} sec?

INFORMATION PANEL — Energy Associated with the Elements of an RC Circuit

OBJECTIVE

To calculate the rate at which energy is stored in the capacitor and the rate at which Joule heat is produced in the resistor when the two elements are connected to form a charging RC circuit.

In a previous segment it was shown that the energy U stored in a capacitor is given by the expression:

$$U = \frac{q^2}{2C} \tag{1}$$

The rate at which energy is stored as a function of charge q is then:

$$\frac{dU}{dq} = \frac{q}{C} \quad \text{or} \quad dU = \frac{q}{C}\, dq \tag{2}$$

Thus, the rate at which energy is stored as a function of time is:

$$\frac{dU}{dt} = \frac{q}{C}\frac{dq}{dt} = \frac{q}{C}\, i \tag{3}$$

since $dq/dt = i$.

However, the rate at which energy is delivered to a resistor is i^2R, hence if a resistor and a capacitor are connected as an RC circuit across a seat of emf, the instantaneous power delivered to the circuit by the seat of emf is the sum of the above two terms so that:

$$\varepsilon i = \frac{q}{C}\, i + i^2R \tag{4}$$

To calculate the total work done in charging a given capacitor to a given potential difference through a given resistor, it is then necessary to integrate equation (4) over the time t from zero to infinity. (You must remember that the theoretical time required for a capacitor to reach full charge is infinity.)

Since the charging current in an RC circuit is:

$$i = \frac{\varepsilon}{R}\, e^{-t/RC} \tag{5}$$

next page

continued

then the left side of equation (4) may be converted to:

$$\frac{\varepsilon^2}{R} e^{-t/RC} \qquad (6)$$

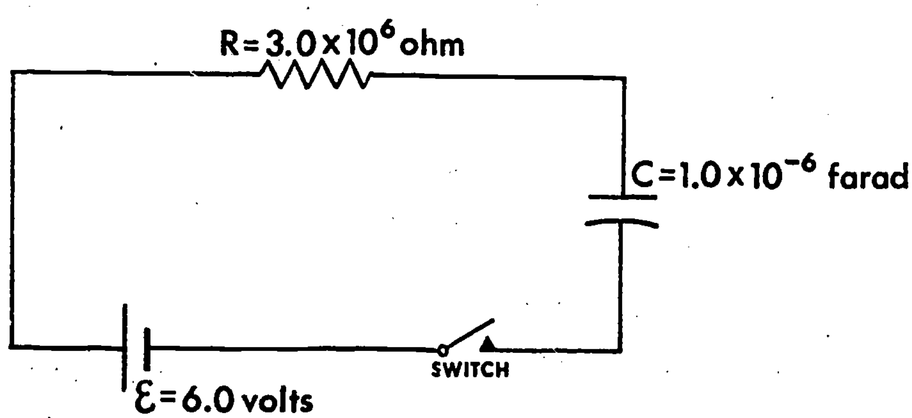
and integrating, we have:

$$W = \frac{\varepsilon^2}{R}\int_0^\infty e^{-t/RC}\, dt = \varepsilon^2 C \qquad (7)$$

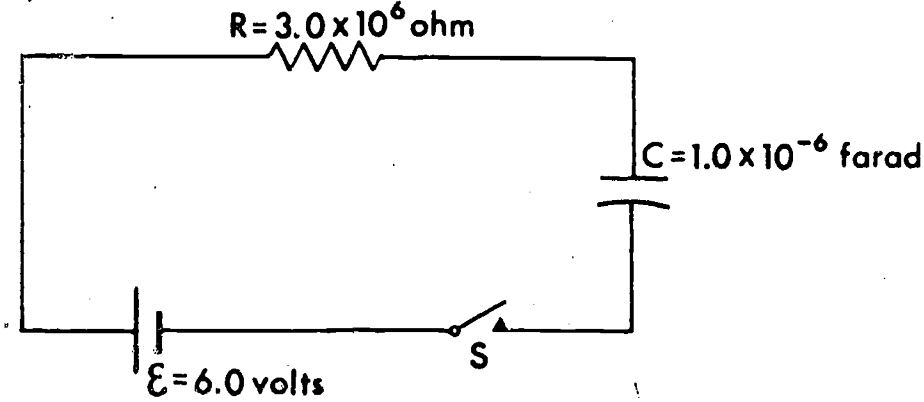
A very interesting aspect of this result is that the total work done in charging a capacitor through a resistor is *independent of the magnitude of the resistor,* since R does not appear in the final term.

15. An uncharged 10-microfarad capacitor is charged by a constant emf through a 100-ohm resistor to a potential difference of 50 volts. What is the total work done?

16. In the circuit shown, what is the rate at which Joule heat is appearing in the resistor 3.0 sec after the switch is closed? (Answer in watts.)



17. In the circuit shown, what is the rate at which energy is being stored in the capacitor 3.0 sec after the switch S is closed? (Answer in watts.)



18. In the circuit of the preceding problem, what is the rate at which energy is being delivered by the seat of emf 3.0 sec after the connection is made? (Answer in watts.)

[a] CORRECT ANSWER: 0.088 amp

Since one minute is much larger than the capacitive time constant, ($RC = 1.2 \times 10^{-4}$ sec), after one minute of charging the capacitor is in equilibrium. Therefore, after one minute of charging, the charge on the capacitor is equal to

$$q_o = C\varepsilon$$

The symbol q_o has been used because this charge is on the capacitor at the start of discharging. We know that the discharging current in an RC circuit is

$$i = -(q_o/RC)\ e^{-t/RC}$$

so that

$$i = -(\varepsilon/R)\ e^{-t/RC}$$

Since we are seeking the current at the beginning of the discharge, time is set equal to zero with the result

$$i_o = -\varepsilon/R = -.088 \text{ amp}$$

The minus sign signifies that the circuit is discharging.

TRUE OR FALSE? The capacitor in the solution above is considered to be in equilibrium after 3 time-constant intervals.

[b] CORRECT ANSWER: 1.6×10^{-6} watt

The rate at which the Joule heat is appearing in the resistor is i^2R.

The expression for the current is

$$i = (\varepsilon/R)\ e^{-t/RC}$$

and at $t = 3$ sec, we have

$$i = (\varepsilon/R)\ e^{-t/RC} = (6/3 \times 10^6)\ e^{-1} = 0.74 \times 10^{-6} \text{ amp}$$

Thus,

$$i^2R = (0.74 \times 10^{-6})^2 \times 3 \times 10^6 = 1.6 \times 10^{-6} \text{ watt}$$

[a] CORRECT ANSWER: A

The expression for the Joule heat which will appear in resistor R during the time interval dt is i^2 R dt and the amount of the electric energy which will be stored in the capacitor during the same length of time is $d(q^2/2C)$. Thus we have

$$\varepsilon dq = i^2 R\, dt + d(q^2/2C)$$

The equation can be reduced to

$$\varepsilon dq = i^2 R\, dt + (q/C)\, dq$$

Dividing by dt, and identifying $\frac{dq}{dt}$ as i we obtain

$$\varepsilon = iR + q/C$$

This equation can be readily obtained from the loop theorem.

[b] CORRECT ANSWER: B

The current is given by the equation

$$i = \frac{\varepsilon}{R} e^{-t/RC}$$

At t = 0, we find $\frac{\varepsilon}{R} = 5 \times 10^{-3}$ amp. At t = RC,

$$i = 5 \times 10^{-3} \times e^{-1} = 5 \times 10^{-3} \times .37 = 1.85 \times 10^{-3} \text{ amp}$$

From the figure we find that the current is approximately 1.85×10^{-3} amp at $t = 2 \times 10^{-3}$ sec. Thus, $RC = 2 \times 10^{-3}$ sec.

TRUE OR FALSE? The quantity e raised to the minus one power is 0.37.

[c] CORRECT ANSWER: B

Substitution of t = RC in the given solution of the equation for a discharging RC circuit will yield $q = q_o e^{-1}$ and $e^{-1} = .37$, thus $q = .37\, q_o$.

[a] CORRECT ANSWER: 4.4×10^{-6} watt

The rate at which energy is being delivered by the seat of emf is equal to the sum of the rates at which electric energy is being stored in the capacitor (qi/C) and Joule heat (i^2R) is appearing in the resistor. Thus

$$\varepsilon i = qi/C + i^2R$$

You may calculate either side of the above equation to obtain the answer. If you choose to calculate the left side, we have at $t = 3$ sec

$$i = (\varepsilon/R)\ e^{-t/RC} = (6.0/3 \times 10^6)\ e^{-1} = .74 \times 10^{-6} \text{ amp}$$

and

$$\varepsilon i = 6.0 \times (.74 \times 10^{-6}) = 4.4 \times 10^{-6} \text{ watts}$$

If you calculate the right side, using the values for i^2R and qi/C obtained in preceding problems, you should obtain the same result.

TRUE OR FALSE? The unit for qi/C is the watt.

[b] CORRECT ANSWER: A

Since $i = \frac{dq}{dt}$, we take the derivative of q with respect to t, and obtain

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R}\ e^{-t/RC}$$

This is the expression for the instantaneous current in a charging circuit.

[c] CORRECT ANSWER: 1.0 microfarad

The capacitive time constant is given as $RC = 2 \times 10^{-3}$ sec. R is given as 2000 ohms. The capacitance of an RC circuit may be computed from $t = RC$. Thus,

$$C = \frac{2.0 \times 10^{-3}}{2.0 \times 10^3} = 1.0 \times 10^{-6} \text{ f} = 1.0 \text{ microfarad}$$

[a] CORRECT ANSWER: 0.74×10^{-6} amp

The rate of change of charge on the capacitor is equivalent to the current in the RC growth circuit. Thus, from the equation for charge

$$q = C\varepsilon(1 - e^{-t/RC})$$

we get

$$\frac{dq}{dt} = i = (\varepsilon/R)\ e^{-t/RC}$$

Substitution of numerical values yields

$$i = 0.74 \times 10^{-6}\ \text{amp}$$

TRUE OR FALSE? In the equations above, the quantity e is the base of natural logarithms.

[b] CORRECT ANSWER: 2.5×10^{-2} j

The energy stored in a capacitor is

$$U = q^2/2C$$

Therefore,

$$\frac{dU}{dt} = qi/C$$

The power delivered to the resistor R is i^2R. Therefore, the instantaneous power delivered by the seat of emf ε is

$$\varepsilon i = \frac{q}{C}\,i + i^2R \qquad (1)$$

In order to obtain the total work done, we integrate eq. (1) over t from 0 to ∞ (i.e., until the steady state is reached).

Since $i = \frac{\varepsilon}{R}\,e^{-t/RC}$, the integral of the left hand side of (1) may be written as

$$\int_0^\infty \varepsilon i\ dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-t/RC}\ dt = \varepsilon^2 C = 2.5 \times 10^{-2}\ \text{j}$$

TRUE OR FALSE? The same result (total work) can be obtained from the right side of equation (1).

[a] CORRECT ANSWER: C

The equation for charging an RC circuit is

$$q = C\varepsilon(1 - e^{-t/RC})$$

The capacitive time constant is t = RC. At t = RC, the equation for charge becomes

$$q = C\varepsilon(1 - e^{-1}) = .63\ C\varepsilon$$

Since $C\varepsilon$ is the equilibrium charge on the capacitor, at t = RC, the amount of charge on the capacitor is equal to 63% of its equilibrium value.

TRUE OR FALSE? If, in a given RC circuit, the equilibrium charge on the capacitor is 63 volts, the charge on the capacitor after one time-constant interval is 10 volts.

[b] CORRECT ANSWER: 3.26×10^{-2} amp

In the preceding problem, we obtained $i = -\frac{\varepsilon}{R} e^{-t/RC}$. At t = 0, $i_o = \frac{\varepsilon}{R}$. Therefore

$$i = -i_o\ e^{-t/RC}$$

Here RC = 1.26×10^{-4} sec.

Thus at t = 1.26×10^{-4} sec

$$i = -.088 \times e^{-1} = -.088 \times .37$$

$$= -3.26 \times 10^{-2} \text{ amp}$$

TRUE OR FALSE? At t = 0, the capacitor behaves as though it is short-circuited.

[c] CORRECT ANSWER: B

When t becomes very large, the term $e^{-t/RC}$ approaches zero. When R becomes very small, the ratio t/RC becomes very large and again $e^{-t/RC}$ approaches zero. Therefore, the charge q attains an equilibrium value of $C\varepsilon$ for either of these events.

[a] CORRECT ANSWER: 2.8×10^{-6} watt

The energy U in a capacitor is given by

$$U = \frac{q^2}{2C}$$

Therefore the rate $\frac{dU}{dt}$ is

$$\frac{dU}{dt} = \frac{q}{C}\frac{dq}{dt} = \frac{q}{C} i$$

and the expression for q and i are

$$q = C\varepsilon(1 - e^{-t/RC})$$

and

$$i = \frac{\varepsilon}{R} e^{-t/RC}$$

Thus

$$\frac{dU}{dt} = \frac{\varepsilon^2}{R}(1 - e^{-t/RC})\ e^{-t/RC} = \frac{36}{3 \times 10^6}\ (1 - e^{-1})e^{-1}$$

$$= (12 \times .63 \times .36) \times 10^{-6}$$

$$= 2.8 \times 10^{-6} \text{ watt}$$

[b] CORRECT ANSWER: A

Since $i = \frac{dq}{dt}$, we can obtain i by differentiating q with respect to t. Hence from $q - q_o e^{-t/RC}$ we get

$$i = \frac{dq}{dt} = -(q_o/RC)\ e^{-t/RC}$$

Since q_o here is the equilibrium value of the charge on the capacitor in a charging RC circuit, may write $q_o \approx C\varepsilon$. Thus, we have

$$i = -(q_o/RC)\ e^{-t/RC} \approx -(\varepsilon/R)\ e^{-t/RC}$$

[a] CORRECT ANSWER: A

The general equation for an RC circuit is

$$iR + q/C = \varepsilon$$

Now if we put $\varepsilon = 0$ in the above equation, it becomes

$$iR + q/C = 0$$

This is the equation for the discharging circuit. Writing current as the time rate of change of charge, this becomes

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

It is easy to check that the solution to this equation is

$$q = q_o e^{-t/RC}$$

where q_o is the initial capacitor charge.

[b] CORRECT ANSWER: D

From the curve we have

$$C\varepsilon = 10 \times 10^{-6} \text{ coul}$$

At $t = 2$ RC either from the curve or from the equation

$$q = C\varepsilon(1 - e^{-t/RC})$$

we have

$$q = 10 \times 10^{-6} (1 - e^{-2}) = 8.6 \times 10^{-6} \text{ coul}$$

Since we know $RC = 2 \times 10^{-3}$ sec and $R = 2000$ ohms, we obtain

$$C = 1.0 \times 10^{-6} \text{ f}$$

Therefore, the voltage on the capacitor after two time constants is

$$V = \frac{q}{C} = 8.6 \text{ volts}$$

TRUE OR FALSE? In this problem, an interval of two time constants is 2.0 milliseconds.

[a] CORRECT ANSWER: B

In order to obtain the initial current, we put t = 0 in the equation

$$i = \frac{\varepsilon}{R} e^{-t/RC}$$

Since $e^{\circ} = 1$, the initial current is

$$i_0 = \frac{\varepsilon}{R}$$

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note

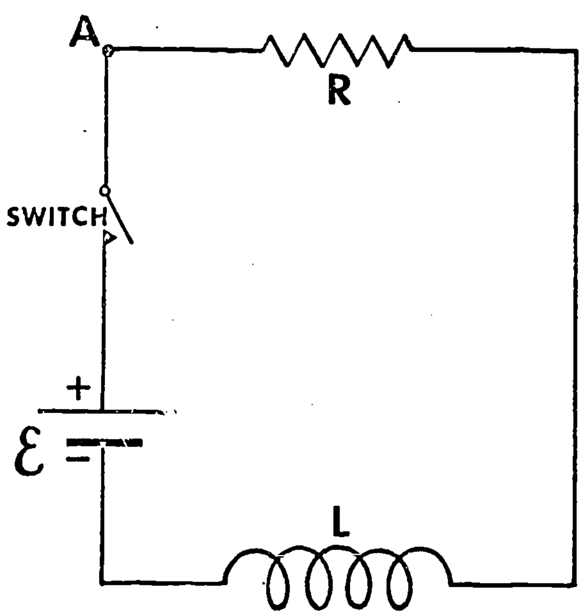
ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL The LR Time Constant

OBJECTIVE

To define and calculate the LR time constant for typical inductive-resistive circuits.

An LR circuit contains a resistor and an inductor in series; if a seat of emf is included in the series configuration as shown in the accompanying diagram, the growth of current occurs at a predictable rate when the switch is closed. At the instant of closure ($t = 0$), the current begins to rise. If the inductor were replaced by a simple conductor, the current would grow rapidly to reach a steady value of ε/R. With the inductor present, however, a self-induced emf appears in the circuit. In accordance with Lenz's law, this emf opposes the applied emf in direction. The resistor, therefore, is acted on by the *difference* between the two emfs. The emf due to the battery ε is a constant one while the emf due to self-induction is variable and is equal to $-L\, di/dt$.



The fundamental relationships for an LR circuit are obtained by following a sequence of steps very similar to those used in developing the equations for the RC circuit. From the loop theorem, we can immediately write: (starting at point A above and traversing the circuit clockwise)

$$-iR - L\frac{di}{dt} + \varepsilon = 0 \qquad (1)$$

and transposing terms:

$$L\frac{di}{dt} + iR = \varepsilon \qquad (2)$$

next page

continued

The solution to the linear differential equation given in (2) is:

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right) \tag{3}$$

which, you will observe, is very similar in form to the equivalent expression obtained for the RC circuit discussed in the previous segment.

Equation (3) may be rewritten in the form of equation (4):

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) \tag{4}$$

in which the symbol τ has replaced the quotient L/R or

$$\tau_L = L/R \tag{5}$$

which serves to define the *inductive time constant*. That is, the inductive time constant τ_L is the ratio of the inductance of the inductor to the total resistance in the circuit. Analysis indicates that τ_L has the dimension of time and may be measured in seconds.

When τ_L is made equal to the time t in equation (4), the latter becomes:

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1}\right) \tag{6}$$

and since e^{-1} is equal to 0.37, we have finally that:

$$i = 0.63\,\frac{\varepsilon}{R} \tag{7}$$

which states that the current in the LR circuit is 63% of its final equilibrium value if the growth time t is equal to one time-constant interval τ_L.

PROBLEMS

1. It is found that the time constant for the decay of current through a certain coil is halved when a 10-ohm resistor is added in series with the coil. Furthermore, when a pure inductance of 30 millihenrys is added in series with the original coil and the series resistor, the time constant is the same as that for the coil alone. What is the coil's internal resistance?

2. For a circuit having a resistance and an inductance in series, the current is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$

This current will approach the final equilibrium value ε/R when

A. $t \to \infty$

B. $L >> Rt$

C. $t \to 0$

D. $R/L \to 0$

3. The quotient L/R in the equation

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$

is known as

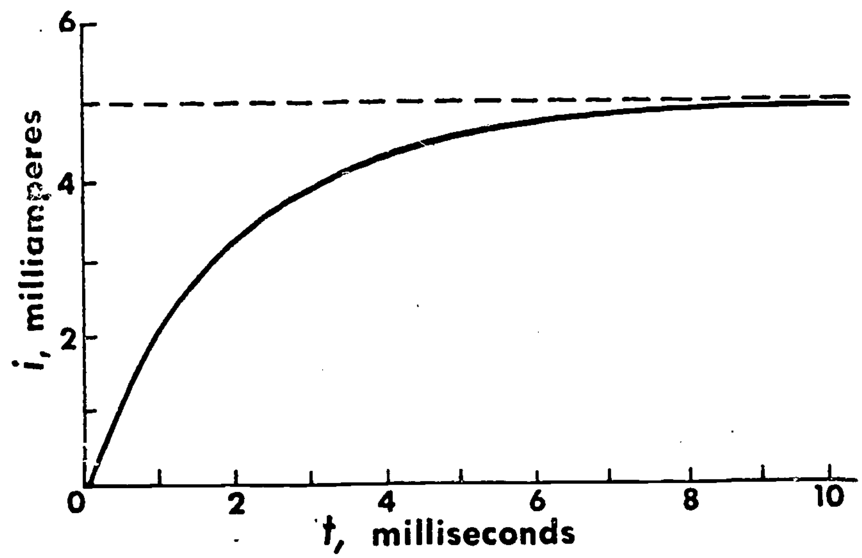
A. the capacitive time constant

B. the resistive time constant

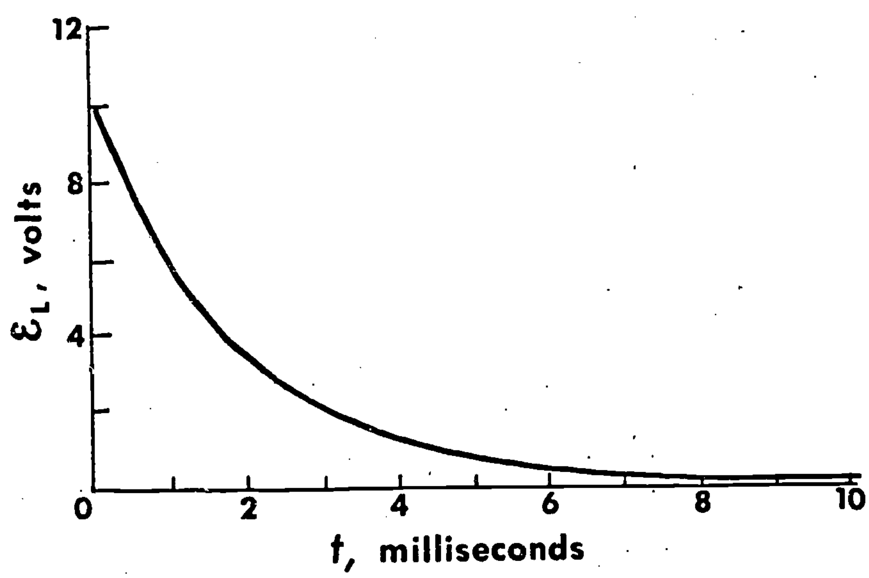
C. the inductive time constant

D. the inductive-resistive time constant

4. Using the curve shown below, determine the inductive time constant of the RL circuit in milliseconds.



5. The curve given in the preceding problem and the graph below both apply to the same RL circuit. Find the value of L for this circuit.



6. The inductance and the internal resistance of a certain coil are unknown. Experiment shows that current in the inductor decays to e^{-1} of its initial value in 0.6 millisecond when a 4.0-ohm resistor is placed in series with the inductor. If the additional series resistor is removed, the inductor's initial current falls to e^{-1} of its initial value in 0.8 milliseconds. What is the inductors inductance L in henrys?

INFORMATION PANEL — Current Growth in an LR Circuit

OBJECTIVE

To answer questions and solve problems relating to the times associated with current changes in the LR circuit.

The core problem in the group that follows is typical of a number of practical situations that arise in electrical engineering. Fundamentally, the problem requires that one determine the time required for the current in a given LR circuit containing a given applied emf to reach some pre-selected percentage of the final equilibrium value.

To solve problems of this type, one must have a clear understanding of the way in which current grows in an LR circuit. Equation (1) below is basic to this understanding:

$$i = \frac{\varepsilon}{R}\ 1 - e^{-Rt/L} \qquad (1)$$

If the growth time t is allowed to be infinite (in practice, 5 time-constant intervals are usually considered adequate for design purposes), then equation (1) reduces to equation (2):

$$i_\infty = \frac{\varepsilon}{R} \qquad (2)$$

in which i_∞ is the equilibrium current and is equal to the applied emf divided by the total resistance in the RL circuit; this includes the coil's resistance plus any external resistances that may be present.

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Let us run through the solution of a typical problem of this type:

> *A resistor of 12 ohms and a coil of 5.0 millihenrys are connected in series with a battery of 24 volts. If the ohmic resistance of the coil is 3.0 ohms, what time will be required after closing the circuit switch for the current to reach half of its ultimate equilibrium value?*

To start, we note that we want to determine the time t when the instantaneous current [i in equation (1)] is half the value of the ultimate equilibrium current. That is to say

$$i = 0.5\ i_\infty = 0.5\ \frac{\varepsilon}{R} \qquad (3)$$

Thus, we can rewrite equation (1) in this way:

$$0.5\ \frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right) \qquad (4)$$

As shown in this equation, the emf does not enter into the solution since the quantity ε/R appears on both sides of the equation and therefore drops out. Simplifying

$$0.5 = 1 - e^{-Rt/L}$$

or

$$e^{-Rt/L} = 0.5 \qquad (5)$$

The exponent of e, that is, the quantity $-Rt/L$ is the natural logarithm of 0.5, so we have:

$$\frac{1}{e^{Rt/L}} = 0.5$$

so

$$\ln 2 = Rt/L \qquad (6)$$

and then solving for t yields:

$$t = \frac{L}{R}\ \ln 2 \qquad (7)$$

The given quantities are:

$$R = 12\text{ ohms} + 3.0\text{ ohms} = 15\text{ ohms}$$

and

$$L = 5.0\text{ millihenrys} = 5.0 \times 10^{-3}\text{ henry}$$

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continued

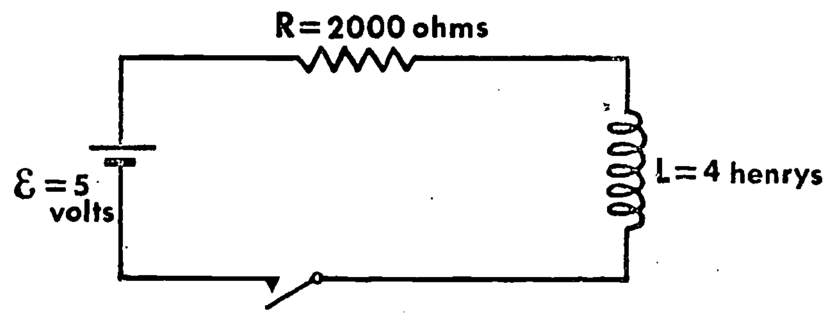
so that upon substituting in equation (7) we have:

$$t = \frac{5.0 \times 10^{-3}}{15}(0.693) = 2.3 \times 10^{-4} \text{ sec} \qquad (8)$$

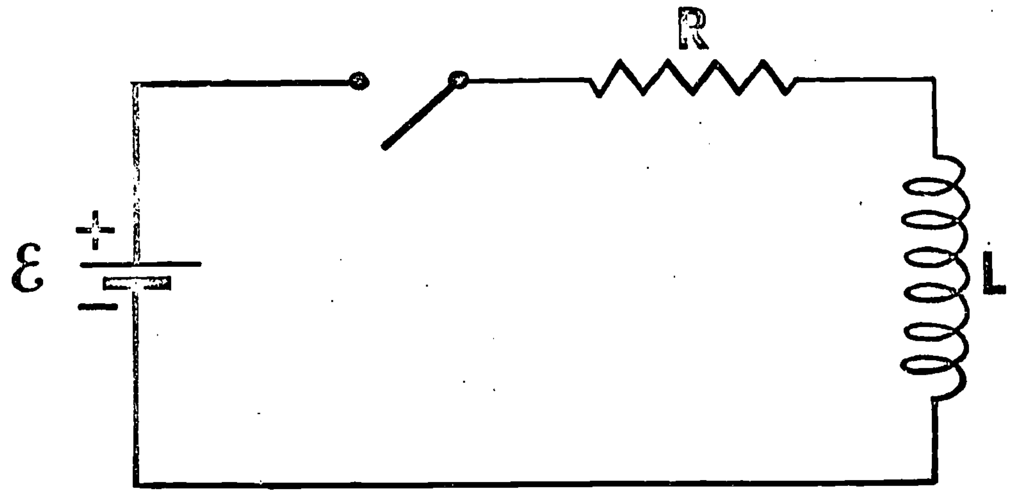
from which we can see that the time required for the current to reach 50% of its final equilibrium value is 2.3×10^{-4} second.

7. A coil having an inductance of 4 millihenrys and a resistance of 10 ohms is connected to a battery with an emf of 12 volts and internal resistance of 2 ohms. How long must one wait after the switch is closed before the current is 90% of its equilibrium value?

8. In the circuit below, what is the current (in milliamperes) two time constants after the switch is closed?



9. In the circuit below, what is the magnitude and direction of the voltage across the inductor at the instant the switch is closed?



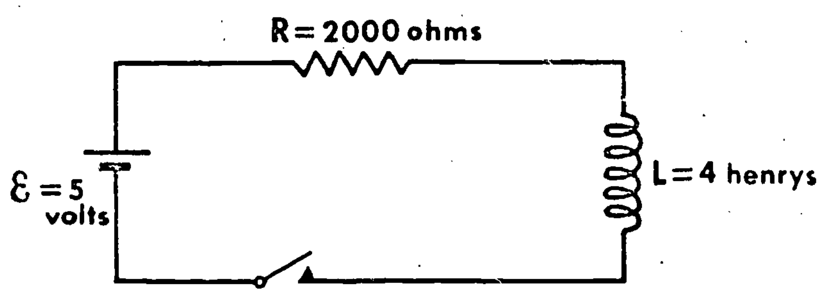
A. 0

B. ε, aiding the battery

C. ε, opposing the battery

D. $\varepsilon - iR$, opposing the battery

10. In the circuit given below, what is the magnitude of the induced emf across the inductance two time constants after the switch is closed?



INFORMATION PANEL

Current Decay in an LR Circuit

OBJECTIVE

To answer questions and solve problems relating to the decay time of the current in an LR circuit.

The problems in this section are similar to those you were required to solve in the previous section. In the case of current decay in an LR circuit, the relevant equations are:

$$i = i_o e^{-Rt/L} \qquad (1)$$

where i_o is the equilibrium current achieved previously when the switch had been closed for more than 5 time-constant intervals. As before, the equilibrium current is given by:

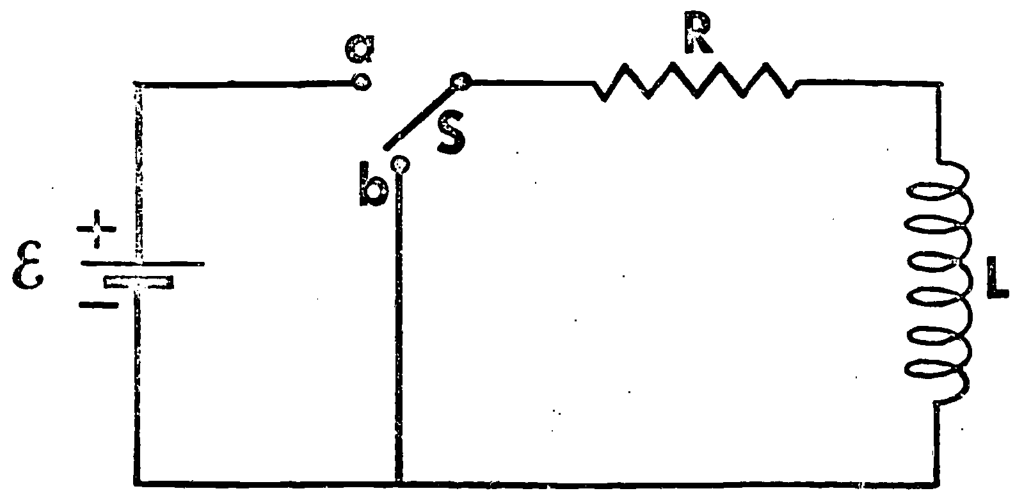
$$i_o = \frac{\varepsilon}{R} \qquad (2)$$

An identical approach may be used in solving decay problems as was employed for growth problems.

11. A 20-ohm resistor and a 2-henry inductor are connected in series with a seat of emf equal to 5 volts. After equilibrium is reached, the seat of emf is removed and the inductor is allowed to discharge its stored energy through the resistor. Find the time when the current through the circuit is 50 percent of the equilibrium current.

12. If the switch in the figure below, having been left in position a long enough for the equilibrium current ε/R to be established, is thrown to b, the effect is that of removing the battery from the circuit. The equation that describes the subsequent decay of the current in the circuit is

$$L\frac{di}{dt} + iR = 0$$



The solution to this equation is

A. $i = \frac{\varepsilon}{R}\left(1 - e^{-tR/L}\right)$

B. $i = \frac{\varepsilon}{R} e^{-tR/L}$

C. $i = \frac{\varepsilon}{R}$

D. $i = -\frac{L}{R}$

13. In the current-decay equation for an LR circuit, what percent of the initial current remains after a period of one time constant from the time when the battery or applied voltage is suddenly reduced to zero?

A. 63%

B. 37%

C. 50%

D. 33%

14. A 20-ohm resistor and a 2-henry inductor are connected in series with a seat of emf of 5 volts. After equilibrium is reached the seat of emf is removed and the inductor is allowed to discharge its energy through the resistor. What is the magnitude of the discharging current in amperes after 0.2 sec?

INFORMATION PANEL

Energy Associated with the Elements of an LR Circuit

OBJECTIVE

To answer questions and solve problems in which the energy stored in an inductor in an LR circuit at a given time after switch closure is a key factor.

When the work required to set up a magnetic field in a coil is calculated, the expression obtained gives the energy stored in the field. For example, after many time constant periods after switch operation occurs, the current in the coil has reached its equilibrium value and the energy stored therein is given by:

$$U_B = \frac{1}{2} Li^2 \qquad (1)$$

We are often concerned, however, with the magnitude of the energy stored in the field at some time *before* equilibrium is attained; or, in some cases, we want to know the time required to store up a given percentage of the equilibrium value of the energy. In these cases we must apply the relationships already studied, that is,

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right) \qquad (2)$$

where ε/R is the equilibrium current.

For example, in the core problem in this section you are asked to determine the time required for the energy stored in a given inductor in series with a given resistor and seat of emf to reach some percentage of its equilibrium value.

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One approach to this problem is to

(a) first calculate the equilibrium energy from equation (1);

(b) next determine the current required for a coil energy that would be present at the given percentage of equilibrium value;

(c) as a third step, equate the current just calculated to the right-hand member of equation (1) above and then solve for the growth time.

Another of the problems in this set calls for you to find the rate in watts that energy is being stored in the magnetic field of an inductor after a given number of time-constant intervals have elapsed since activation. Recognizing that the rate of energy storage is equivalent to the power at the instant in question, you might organize your solution this way:

(a) Write: rate of energy storage = $i\varepsilon_L = dU_B/dt$

(b) Write: $\varepsilon_L = L\,\dfrac{di}{dt}$

(c) Combine these statements into the equation:

$$dU_B/dt = iL\,\frac{di}{dt}$$

(d) Recall the general expressions for i and di/dt and write:

$$i = \varepsilon/R\left(1 - e^{-Rt/L}\right)$$

and

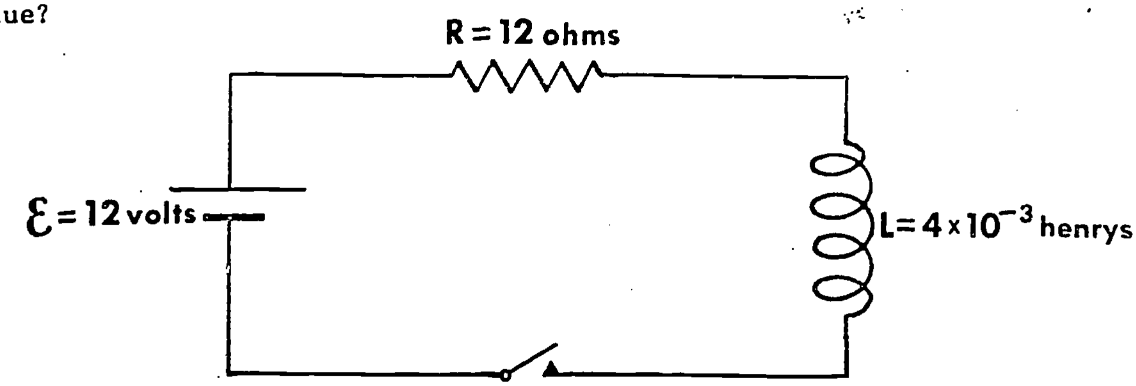
$$\frac{di}{dt} = \varepsilon/L\left(e^{-Rt/L}\right)$$

(e) Substitute these expressions for i and di/dt in the combined equation in (c).

(f) Substitute the given quantities of L and R into the definition of the time constant to determine the actual time in seconds that elapses after activation.

(g) Finally, substitute the required known values into the equation obtained in (e) and solve for the power.

15. In the circuit shown below, how long must one wait after the switch is closed before the energy stored in the inductor is 90% of its equilibrium value?



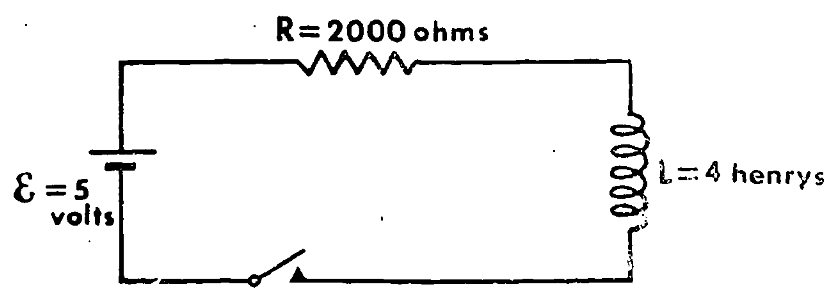
16. In the circuit given below, at what rate does the energy appear as joule heat in the resistor two time constants after the switch is closed?

A. 5×10^{-2} W

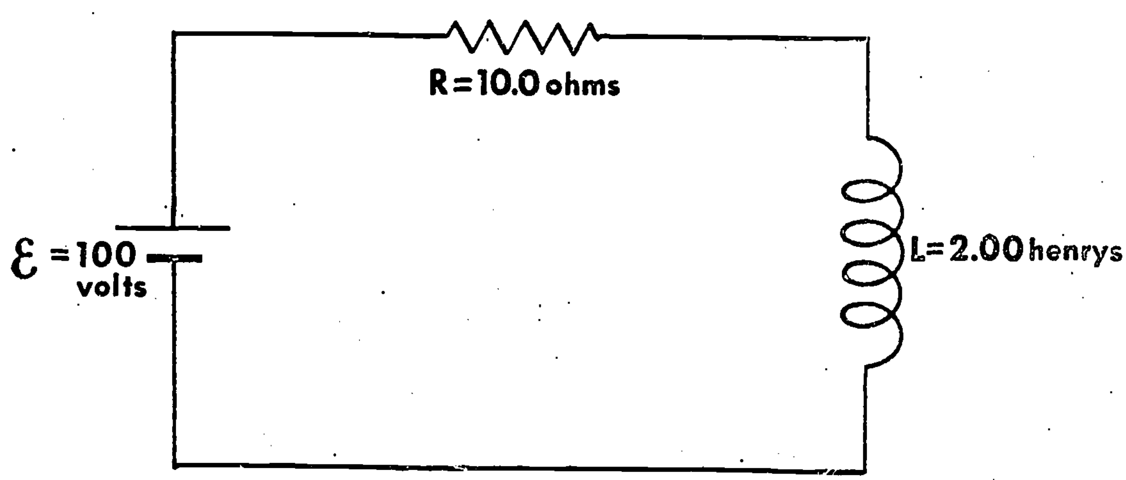
B. 5.12×10^{-3} W

C. 9.3×10^{-3} W

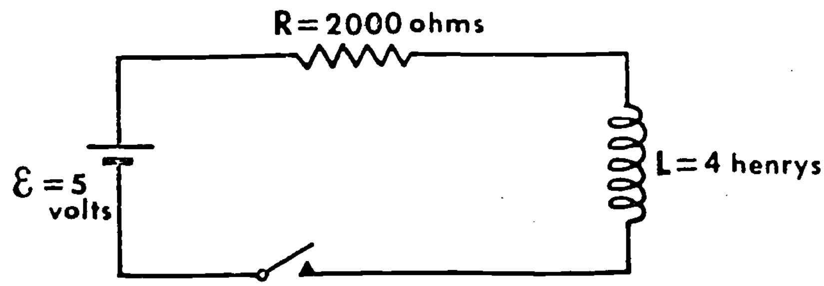
D. 1.25×10^{-2} W



17. In the circuit shown, how much energy is stored in the magnetic field when the equilibrium current exists in the coil?



18. In the circuit given below, at what rate in watts is the energy being stored in the magnetic field two time constants after the switch is closed?



[a] CORRECT ANSWER: 0.7 volts

The magnitude of the induced emf is given by the expression

$$\varepsilon_L = L \frac{di}{dt}$$

Knowing that, and using the expression for current i in an LR circuit, we find that

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$

so that

$$L \frac{di}{dt} = \varepsilon\ e^{-Rt/L}$$

Substitution of $t = 2\ L/R$ gives the induced emf at two time constants. The numerical value in this case is

$$\varepsilon_L = 5.0 \times e^{-2} = 5.0 \times (.37)^2 = 0.7 \text{ volts}$$

TRUE OR FALSE? The induced emf in the solution above may be given as $\varepsilon_e{}^{-Rt/L}$.

[b] CORRECT ANSWER: 9.6×10^{-3}

From the definition of the inductive time constant, we get the following two equations:

$$L/r = 0.8 \times 10^{-3} \text{ sec} \qquad (1)$$

where r is the internal resistance.

Also

$$L/(4 + r) = 0.6 \times 10^{-3} \text{ sec} \qquad (2)$$

From equation (1), we get

$$r = L/(0.8 \times 10^{-3}) \text{ ohms} \qquad (3)$$

Substituting this into equation (2), we obtain

$$L = 9.6 \times 10^{-3} \text{ henrys}$$

TRUE OR FALSE? In equation (3) above, r is the external resistance.

[a] CORRECT ANSWER: B

The current in the circuit after the emf is removed is given by

$$i = \frac{\varepsilon}{R} e^{-tR/L}$$

Hence, at $t = L/R$ we have

$$i = \frac{\varepsilon}{R} e^{-1} = .37 \frac{\varepsilon}{R}$$

Since ε/R is the initial current in the circuit, the current after one time constant is 37% of the initial current.

[b] CORRECT ANSWER: 1.5×10^{-3}

The rate at which the energy is being stored in the magnetic field is given by $i\varepsilon_L$ where

$$\varepsilon_L = L \frac{di}{dt} \qquad (1)$$

The general expressions for i and di/dt are

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right) \qquad (2)$$

and

$$\frac{di}{dt} = \frac{\varepsilon}{L} e^{-Rt/L} \qquad (3)$$

Thus

$$\frac{dU_B}{dt} = i\varepsilon_L = \frac{\varepsilon^2}{R} e^{-Rt/L}\left(1 - e^{-Rt/L}\right) \qquad (4)$$

At $t = 2\ L/R$

$$\frac{dU_B}{dt} = \frac{\varepsilon^2}{R} e^{-2}\left(1 - e^{-2}\right) = 1.5 \times 10^{-3} \text{ watts}$$

TRUE OR FALSE? In the above solution, if equation (3) is integrated from t = 0 to 5 = ∞, equation (2) would be obtained.

[a] CORRECT ANSWER: 2.16

The general expression for current in an LR circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$

Substitution of t = 2 L/R in the above expression yields

$$i = \frac{\varepsilon}{R}\left(1 - e^{-2}\right) = \frac{5}{2000}\left[1 - (.37)^2\right]$$

$$= 2.16 \text{ milliamperes}$$

[b] CORRECT ANSWER: 1.0×10^{-3} sec

Since the equilibrium current is 1 amp, we get the stored energy at equilibrium to be

$$U_\infty = \frac{1}{2} L\left(i_\infty\right)^2 = 2 \times 10^{-3} \text{ j} \qquad (1)$$

Thus, in this problem

$$.9\ U_\infty = \frac{1}{2} L i_1{}^2 = 2 \times 10^{-3}\, i_1{}^2 \qquad (2)$$

where i_1 corresponds to the current in the inductor when the energy stored in the inductor is 90% of the equilibrium value. Solving for i_1 from Equation (2) we obtain

$$i_1{}^2 = .9$$

or

$$i_1 = .95 \qquad (3)$$

Substituting (3) into the general equation for current, we have

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right) = i_\infty\left(1 - e^{-Rt/L}\right)$$

So that

$$.95 = 1 - e^{-3 \times 10^3\, t}$$

Finally, the required time t is found to be

$$t = 1.0 \times 10^{-3} \text{ sec}$$

TRUE OR FALSE? In the above solution, the equilibrium current is consistently symbolized as i_∞.

[a] CORRECT ANSWER: 2

After one time constant the exponential term $e^{-RT/L}$ becomes e^{-1}. So the expression

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$

reduces to

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1}\right) = \frac{\varepsilon}{R}\left(1 - .37\right) = .63\,\frac{\varepsilon}{R} = 3.2 \text{ milliamperes}$$

Corresponding to $i = 3.2$ milliamperes, the value of L/R can be read off the graph as 2 milliseconds.

[b] CORRECT ANSWER: B

From the given circuit equation, we can write

$$L\,di = -\,iR\,dt$$

or

$$\frac{di}{i} = \frac{-R}{L}\,dt$$

Therefore,

$$\int \frac{di}{i} = -\int \frac{R}{L}\,dt$$

or

$$\ln i = \frac{-R}{L}\,t + A \qquad (1)$$

The constant A can be determined from the initial condition, i.e., at $t = 0$, $i = i_o$ thus $A = \ln i_o$. We may, therefore, rewrite equation (1) as

$$i = i_o\, e^{-Rt/L}$$

where i_o is the equilibrium current $i_o = \varepsilon/R$.

[a] CORRECT ANSWER: 10

From the definition of the inductive time constant, we have the following equation:

$$L/(r + 10) = L/2r \tag{1}$$

where r is the internal resistance.

Dividing equation (1) by L, we get

$$\frac{1}{r + 10} = \frac{1}{2r}$$

or

$$r + 10 = 2r$$

which leads to r = 10 ohms.

We also have the following equation

$$(L + 30 \times 10^{-3})/(r + 10) = L/r$$

which can be solved for L to obtain

$$L = 30 \times 10^{-3} \text{ henrys.}$$

TRUE OR FALSE? In this solution, L turns out to be 30 millihenrys.

[b] CORRECT ANSWER: 3.4×10^{-2}

The current at time t in an LR decay circuit is

$$i = i_o e^{-Rt/L}$$

where i_o is the equilibrium current for the circuit. Thus,

$$i_o = \varepsilon/R$$

and at t = 0.2 sec, the current is

$$i = \frac{\varepsilon}{R} e^{-Rt/L} = \frac{\varepsilon}{R} e^{-2} = 3.4 \times 10^{-2} \text{ amp}$$

TRUE OR FALSE? In this solution, it is assumed that the resistance of the inductor is equal to that of the resistor.

[a] CORRECT ANSWER: 7.7×10^{-4} sec

The expression for the current in an LR circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right) \qquad (1)$$

After the current reaches the equilibrium value ($t \to \infty$), equation (1) reduces to

$$i_\infty = \frac{\varepsilon}{R} \qquad (2)$$

We are looking for time t when the instantaneous current $i = 0.9\, i_\infty = .9\,\frac{\varepsilon}{R}$

Substituting into equation (1) we obtain

$$.9\,\frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-12t/4 \times 10^{-3}}\right) \qquad (3)$$

where R = 10 + 2 = 12 ohms is the total resistance (two resistors connected in series) of the circuit. Thus, from (3)

$$e^{-3 \times 10^3 t} = 0.1$$

Therefore, $t = 7.7 \times 10^{-4}$ sec

TRUE OR FALSE? The total resistance in this LR circuit is 10 ohms.

[b] CORRECT ANSWER: C

Joule heat is the rate of energy dissipation in a resistor. By definition, this rate is power. Therefore, joule heat

$$P = i^2 R$$

The time constant of the circuit is $L/R = 2 \times 10^{-3}$ sec. Therefore, at two time constants the value of the current is

$$i_1 = \frac{\varepsilon}{R}\left(1 - e^{-2}\right) = 2.16 \times 10^{-3} \text{ amps}$$

Thus,

$$P = \left(2.16 \times 10^{-3}\right)^2 \times 2000 = 9.3 \times 10^{-3} \text{ W}$$

[a] CORRECT ANSWER: .07 sec

The current at time t in an LR decay circuit is

$$i = i_o e^{-Rt/L} \qquad (1)$$

where i_o is the equilibrium current for the charging LR circuit and is given by

$$i_o = \frac{\varepsilon}{R}$$

The time T when the current $i = i_o/2$ is obtained by substituting these values in equation (1). Thus,

$$\frac{1}{2} i_o = i_o e^{-RT/L}$$

Therefore,

$$\ell n\ 2 = RT/L$$

or

$$T = \frac{L}{R} \ell n\ 2 = \frac{2}{20} (.693)$$

$$= .07 \text{ sec}$$

TRUE OR FALSE? The magnitude of current i can never rise above the current i_o.

[b] CORRECT ANSWER: A

When $t \to \infty$, $e^{-Rt/L} \to 0$, and consequently $i \to \varepsilon/R$. In practice, if $t >> L/R$, then $i = \varepsilon/R$. This situation for an RL circuit is very similar to the situation in an RC circuit when $t >> RC$.

[a] CORRECT ANSWER: 100 j

The total energy stored in an inductor L carrying a current i is given by

$$U_B = \frac{1}{2} L i^2$$

At equilibrium

$$U_\infty = \frac{1}{2} L i_\infty^2$$

where

$$i_\infty = \frac{\varepsilon}{R}$$

Thus,

$$U_\infty = \frac{1}{2} L \left(\frac{\varepsilon}{R}\right)^2$$

or

$$U_\infty = \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ j}$$

[b] CORRECT ANSWER: 4 henrys

Since we know $\varepsilon/R = 5 \times 10^{-3}$ amp and $\varepsilon = 10$ volts from the curves, we can get

$$R = 2000 \text{ ohms}$$

From the solution of the preceding problem

$$L/R = 2 \times 10^{-3} \text{ sec}$$

Therefore,

$$L = R\left(2 \times 10^{-3}\right) = 2000 \times \left(2 \times 10^{-3}\right) = 4 \text{ henrys}$$

[a] CORRECT ANSWER: C

The equation for current in an LR circuit is $i = (\varepsilon/R)\left(1 - e^{-Rt/L}\right)$. The self-induced emf ε_L across the inductor is

$$\varepsilon_L = -L\frac{di}{dt} = -L\frac{d}{dt}\left[\frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)\right]$$
$$= -\varepsilon\, e^{-Rt/L}$$

The self-induced emf at t = 0 is readily obtained from the above expression by substituting t = 0 in the right-hand side. Thus,

$$\left(\varepsilon_L\right)_{t\,=\,0} = -\varepsilon$$

The minus sign signifies that the induced emf ε_L opposes the battery emf.

[b] CORRECT ANSWER: C

The constant L/R has the dimensions of time, and is called the "inductive" time constant.