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ABSTRACT

Five study segments of the Self-Paced Physics Course materials are presented in this seventh problems and solutions book used as a part of student course work. The content is related to magnetic fields, magnetic moments, forces on charged particles in magnetic fields, electron volts, cyclotron, electronic charge to mass ratio, current-carrying loops, torques, electric motors, magnetic flux, earth's magnetic fields, Ampere's law, solenoids, toroids, and Biot-Savart law. Contained in each segment are an information panel, core problems enclosed in a box, core-primed questions, scrambled problem solutions, and true-false questions. Study guides are provided and used to answer the true-false questions and to reveal directions for reaching solutions. When the core problem is answered incorrectly, the study guide requires students to follow the remedial or enabling loop, leading to the solutions of core-primed questions. Also included is a sheet of problem numbers with corresponding page numbers which locate correct answers. (Related documents are SE 016 065 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

UNITED STATES NAVAL ACADEMY

Self-Paced PHYSICS

SEGMENTS 32-36

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Spring Semester, 1971

PROBLEMS -- AND THEIR CORRECT-ANSWER PAGES

Segments 32, 33, 34, 35 and 36

Segment 32		Segment 33		Segment 34		Segment 35		Segment 36	
P #	Solution Page	P #	Solution Page	P #	Solution Page	P #	Solution Page	P #	Solution Page
1	23a	1	27a	1	20a	1	22a	1	17a
2	28a	2	20b	2	17b	2	15b	2	15a
3	31b	3	18a	3	24b	3	24b	3	19a
4	24a	4	26a	4	22c	4	18a	4	23a
5	27b	5	24a	5	17a	5	21a	5	21a
6	26a	6	20a	6	22a	6	23a	6	12a
7	31a	7	28a	7	19a	7	17b	7	24a
8	28b	8	17a	8	24a	8	19b	8	19b
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13	31c	13	25a	13	25a	13	18b	13	11a
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16	32a	16	28b	16	19c	16	20b	16	20a
17	30b	17	21a	17	25b	17	22b		
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20	33a								
21	29a								
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Segment 32 — Reading

HR 33-1,33-2;
33-6/33-8
SZ 30-5,30-6;
30-11
SW 34-9

Segment 33 — Reading

*HR 33-1/33-4
SZ 30-1;30-3;
30-4;31-1;
31-3;31-5;
31-6;31-8
SW 34-1;34-2;
34-4,34-5
AB 4-7;38-2;
39-1;39-2

Segment 34 — Reading

*HR 33-4
*SW 34-3;34-4
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Segment 35 — Reading

*HR 34-1/34-4
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Segment 36 — Reading

*HR 34-5,34-6
SZ 32-1,32-2
32-4;32-6

P	STEP	NAME
	0.1	Reading: HR 33-1, 33-2; 33-6/ 33-8 SZ 30-5, 30-6; 30-11 SW 34-9
	0.2	Information Panel, "Force on a Charged Particle in a Magnetic Field"
	0.3	Audiovisual, "DEFINITION OF A B-FIELD"
1		A B C D T F
	1.1	If your first choice was correct, advance to P 4; if not, continue sequence.
2		A B C D
3		A B C D T F
4		A B C D T F
	4.1	If your first choice was correct, advance to 8.1; if not, continue sequence.
5		A B C D
6		A B C D

P	STEP	SECTION SEGMENT 32
7		A B C D
8		A B C D T F
	8.1	Information Panel, "The Cyclotron"
9		A B C D T F
	9.1	If your first choice was correct, advance to 15.1; if not, continue sequence.
10		______ (ans)
11		A B C D
12		______ (ans)
13		A B C D
14		______ (ans)

P	STEP	NAME	P	STEP	SECTION SEGMENT 32
15		[ ] T [ ] F [ ] ________________ (ans)	22		A [ ] B [ ] C [ ] D [ ] T [ ] F [ ]
	15.1	Audiovisual, MOTION OF AN ELECTRON IN COMBINED E AND B FIELDS		22.1	Homework: HR 33-31
	15.1	Information Panel, "Measurement of Electronic Charge to Mass Ratio (e/m)			
16		[ ] T [ ] F [ ] ________________ (ans)			
	16.1	If correct, advance to 22.1; if not, continue sequence.			
17		A [ ] B [ ] C [ ] D [ ]			
18		A [ ] B [ ] C [ ] D [ ]			
19		A [ ] B [ ] C [ ] D [ ]			
20		A [ ] B [ ] C [ ] D [ ]			
21		A [ ] B [ ] C [ ] D [ ]			

P	STEP	NAME
	0.1	Audiovisual, "DEFINITION OF A $\vec{B}$-FIELD"
	0.2	Reading: *HR 33-1/33-4 SZ 30-1; 30-3; 30-4, 31-1; 31-3; 31-5; 31-6; 31-8 SW 34-1; 34-2; 34-4, 34-5 AB 4-7; 38-2; 39-1, 39-2
	0.3	Information Panel, "Units for Magnetic Fields"
1		T F (ans)
	1.1	If correct, advance to P 5; if not, continue sequence.
2		A B C D
3		A B C D
4		T F (ans)
5		A B C D T F
	5.1	If your first choice was correct, advance to 8.1; if not, continue sequence.

P	STEP	SECTION SEGMENT 33
6		A B C D
7		(ans)
8		T F (ans)
	8.1	Information Panel, "Torque on a Current-Carrying Loop"
9		T F (ans)
	9.1	If correct, advance to 13.1; if not, continue sequence.
10		(ans)
11		(ans)
12		A B C D
13		T F (ans)

P	STEP	NAME	P	STEP	SECTION SEGMENT 33
	13.1	Information Panel, "Magnetic Moment of a Current Loop"			
14		[ ] T [ ] F [ ] ______ (ans)			
	14.1	If correct, advance to 17.1; if not, continue sequence.			
15		[ ] ______ (ans)			
16		A [ ] B [ ] C [ ] D [ ]			
17		[ ] T [ ] F [ ] ______ (ans)			
	17.1	Homework: HR 33-6			

P	STEP	NAME
	0.1	Reading: *HR 33-4 SW 34-3*; 34-4 SZ 31-3; 34-10
	0.2	Information Panel, "Average Value of Torque on a Current-Carrying Loop"
1		A B C D T F
	1.1	If your first choice was correct, advance to 4.1; if not, continue sequence.
2		A B C D
3		A B C D
4		A B C D T F
	4.1	Information Panel, "Work and Energy Considerations for Current-Carrying Loops in Magnetic Fields"
5		T F _____ (ans)
	5.1	If correct, advance to 8.1; if not, continue sequence.

P	STEP	SECTION SEGMENT 34
6		A B C D
7		A B C D
8		T F _____ (ans)
	8.1	Information Panel, "Magnetic Flux"
9		T F _____ (ans)
	9.1	If correct, advance to 12.1, if not, continue sequence.
10		_____ (ans)
11		_____ (ans)
12		T F _____ (ans)
	12.1	Information Panel, "The Magnetic Field of the Earth"

P	STEP	NAME	P	STEP	SECTION SEGMENT 34
13		A B C D T F			
	13.1	If your first choice was correct, advance to 19.1; if not, continue sequence.			
14		A B C D			
15		A B C D			
16		A B C D			
17		A B C D			
18		A B C D			
19		A B C D T F			
	19.1	Homework: SW 34-3			

P	STEP	NAME
	0.1	Reading: *HR 34-1/34-4 SZ 32-6
	0.2	Information Panel, "Ampere's Law"
	0.3	Audiovisual, AMPERE'S LAW APPLIED TO A LONG STRAIGHT CONDUCTOR
1		T F (ans)
	1.1	If correct, advance to 5.1; if not, continue sequence.
2		A B C D
3		A B C D
4		A B C D
5		T F (ans)
	5.1	Information Panel, "Applying Ampere's Law"
6		A B C D T F

P	STEP	SECTION SEGMENT 35
	6.1	If your first choice was correct, advance to 9.1; if not, continue sequence.
7		(ans)
8		A B C D
9		T F (ans)
	9.1	Information Panel, "Forces Between Current-Carrying Conductors"
	9.2	Audiovisual, FORCE BETWEEN PARALLEL CURRENT-CARRYING CONDUCTORS
10		A B C D T F
	10.1	If your first choice was correct, advance to P 15; if not, continue sequence.
11		A B C D
12		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 35
13		A B C D			
14		A B C D T F			
15		T F _______________(ans)			
	15.1	If correct, advance to 19.1; if not, continue sequence.			
16		A B C D			
17		A B C D			
18		A B C D			
19		A B C D T F			
	19.1	Homework: HR 34-11			

P	STEP	NAME
	0.1	Reading: *HR 34-5, 34-6 SZ 32-1, 32-2 32-4; 32-6
	0.2	Infromation Panel, "The Magnetic Field of a Solenoid and Toroid"
1		T F (ans)
	1.1	If correct, advance to 5.1; if not, continue sequence.
2		(ans)
3		(ans)
4		(ans)
5		A B C D T F
	5.1	Information Panel, "The Biot-Savart Law"
6		A B C D T F
	6.1	If first choice was correct, advance to 9.1; if not, continue sequence.

P	STEP	SECTION SEGMENT 36
7		A B C D
8		A B C D
9		T F (ans)
	9.1	Audiovisual, "THE LAW OF BIOT-SAVART"
	9.2	The following problems are optional and provide further practice in the use of Biot-Savart.
10		A B C D T F
	10.1	If first choice was correct, advance to P 15; if not, continue sequence.
11		(ans)
12		A B C D
13		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 36
14		T F			
15		A B C D T F			
16		A B C D T F			
	16.1	Homework: HR 34-20			

INFORMATION PANEL — Force on a Charged Particle in a Magnetic Field

OBJECTIVE

To determine the path of charged particles in magnetic fields by finding the magnitude and direction of the force acting on them.

Several definitions and criteria should be emphasized before you begin work on this Segment. The first of these answers the question, "How does one know when a magnetic field exists in a given region of space?"

> *A magnetic field is said to exist at a point if a force over and above any electrostatic force is exerted on a moving charge at that point.*

The second item deals with the *direction* of the magnetic field and may be stated as follows:

> *The direction of a magnetic field is that direction in which a charged particle may move through it without experiencing any magnetic force.*

Thus, an electron, a proton, a deuteron, or an alpha particle moving *parallel* to the magnetic lines of force of a field at any instant is not acted upon by a force due to the presence of the magnetic field.

A third important consideration suitable for inclusion here is:

> *When the velocity vector $\vec{v}$ of a moving charge is perpendicular to the magnetic field, the force acting on the charge is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.*

A fourth item essential to a proper approach to this subject is:

> *The magnitude of the force acting on a charged particle q moving with velocity $\vec{v}$ perpendicular to a magnetic field $\vec{B}$ is given by the relation $F = Bqv$.*

next page

continued

In the second statemen· ›ove, we have given a method of deciding the direction of a magne·· · ·ld on the basis of the absence of force acting on a charge part moving through the field. In order to specify the *sense* o· th· ·eld, however, it is necessary to state the relationships between the force $\vec{F}$, the charge q, the velocity $\vec{v}$ and the magnetic intensity $\vec{B}$ vectorially:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Applying the rule for cross-products, the magnitude of the magnetic force for any direction of motion of the charged particle through the field is given by the equation:

$$F = qvB \sin\theta$$

where θ is the angle between the velocity vector $\vec{v}$ and the magnetic induction vector $\vec{B}$.

In a given situation, the direction and sense of the magnetic force may be determined from any one of the "rules" developed thus far in this course or from the "palm rule" described below. In general, for a positive particle in motion through a magnetic field, as $\vec{v}$ is rotated into $\vec{B}$ through the smaller angle between them, the direction of $\vec{F}$ is at right angles to the plane containing $\vec{v}$ and $\vec{B}$ and its sense is that of the advance of a right-handed screw undergoing this rotation.

To use the palm rule, extend the thumb of the right hand outward, point the fingers in the direction of the magnetic field and the thumb in the direction of motion of the positive charge, and the sense of the thrust of the palm will then indicate the sense of the force.

When the moving particle is negative, the sense of the force is opposite that obtained for a positive particle.

A charged particle entering a magnetic field perpendicularly with uniform velocity will follow a circular path while in the influence of the field. The magnetic force is centripetal in direction and sense in this case so that we may write:

$$F_B = qvB = \frac{mv^2}{r}$$

in which m is the mass of the particle, v is its velocity perpendicular to the magnetic field, and r is the radius of the arc it describes.

next page

continued

The radius of the path is then given by:

[illegible] qB or substituting momentum p for mv

$$r = \frac{p}{qB}$$

This equation expresses the radius of the particle's circular path in terms of its momentum and charge in a given field. Another useful expression in which the radius is given in terms of the kinetic energy of the particle rather than its momentum is obtained this way:

Since kinetic energy $K = mv^2/2$

then $K = (p)v/2$

but v may be expressed as p/m so that we can write

$$K = \frac{(p)(p/m)}{2}$$

hence $p = \sqrt{2mK}$

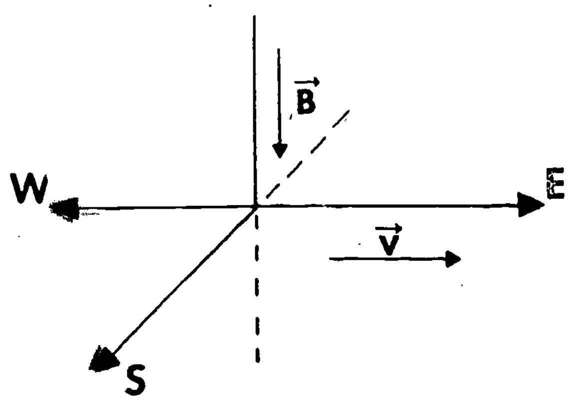
Thus, the radius of the circular path can be expressed as

$$r = \frac{\sqrt{2mK}}{qB}$$

The unit of magnetic field intensity is the Tesla, symbol T, and corresponds to the units weber/m^2 found in older texts.

PROBLEMS

1. An electron in a television picture tube has a speed of 6×10^5 m/sec. The tube is oriented so that the electrons move horizontally from west to east. The vertical component of the Earth's magnetic field points downward and has an intensity of $B = 5 \times 10^{-5}$ T. What is the force exerted on the electron? (Recall that $q_e = -e = -1.6 \times 10^{-19}$ coul.)



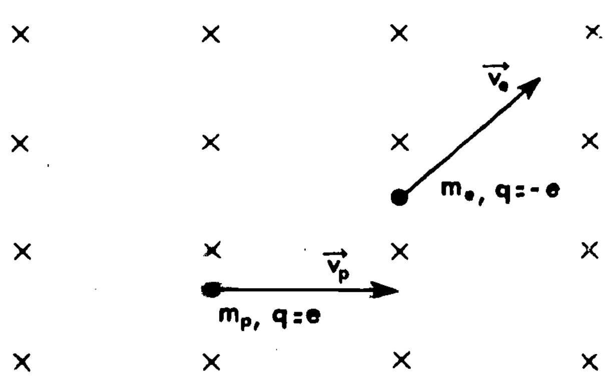
A. 9.6×10^{-14} nt; north

B. 4.8×10^{-18} nt; north

C. 9.6×10^{-14} nt; south

D. 4.8×10^{-18} nt; south

2. The proton is positively charged and 1836 times as massive as the negatively-charged electron. Each is released with its velocity in the plane of the paper, there being a uniform magnetic field directed perpendicularly into the plane of the paper. Which of the following statements correctly describes the motion of the particles?



A. The electron spirals clockwise, the proton counterclockwise, along the direction of $\vec{B}$.

B. The electron spirals counterclockwise, the proton clockwise, along the direction of $\vec{B}$.

C. The electron rotates clockwise, the proton counterclockwise, in a circular path parallel to the plane of the paper.

D. The electron rotates counterclockwise, the proton clockwise, in a circular path parallel to the plane of the paper.

3. In the absence of gravitational and electric fields, if we fire a test charge q_o with a velocity $\vec{v}$ through a point P in space and observe no change in the test charge's velocity, then we can conclude that

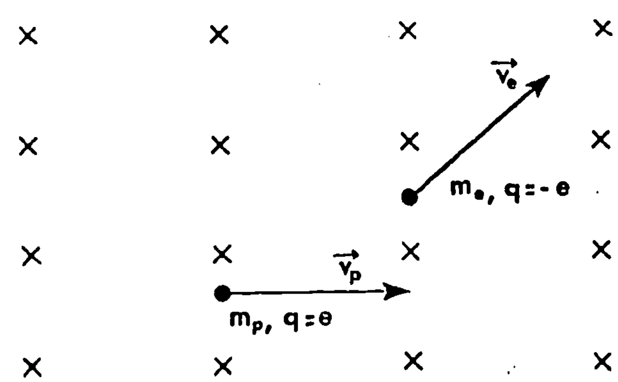
A. if there is a magnetic field at P, it must be uniform

B. if there is a magnetic field at P, it must be directed parallel to $\vec{v}$

C. if there is a magnetic field at P, it must be directed perpendicular to $\vec{v}$

D. there is no magnetic field at P.

4. The proton is positively charged and 1836 times as massive as the negatively-charged electron. Each is released with its velocity in the plane of the paper, there being a uniform magnetic field directed perpendicularly into the plane of the paper. If the proton and the electron are released with equal kinetic energies, the electron's orbit is



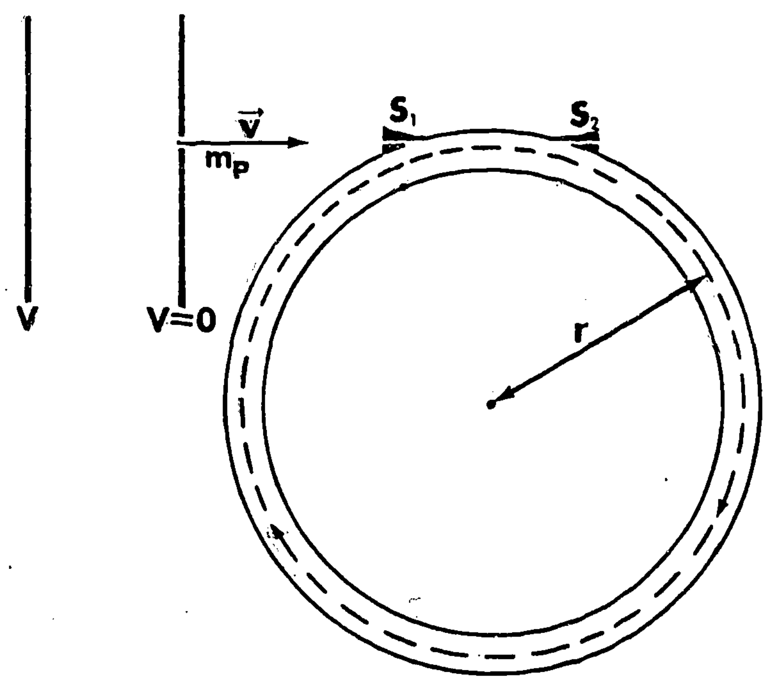
A. larger than the proton's orbit

B. smaller than the proton's orbit

C. the same size as the proton's orbit

D. no conclusion can be drawn about the relative sizes of the orbits

5. A proton is accelerated from rest through a potential difference V, between the two plates shown in the diagram. It leaves the plate at the right with a velocity $\vec{v}$, and then enters a circular region where a magnetic field exists. What must be the direction of the magnetic field if the proton must follow the circular path shown in the figure?



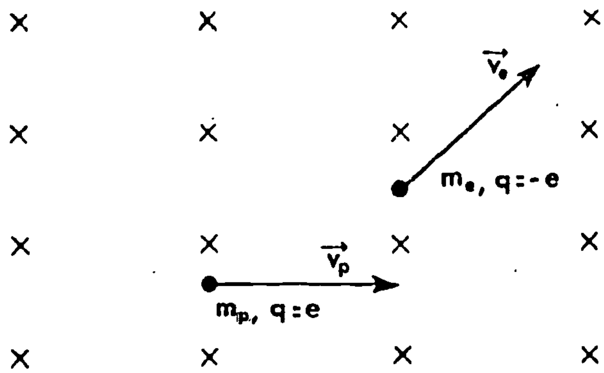
A. same at the direction of $\vec{v}$

B. opposite to the direction of $\vec{v}$

C. out of the page

D. into the page

6. The proton is positively charged and 1836 times as massive as the oppositely-charged electron. Each is released with its velocity in the plane of the paper, there being a uniform magnetic field directed perpendicularly into the plane of the paper. If the electron is released with a speed three times the speed of the proton, then the radius of the electron's orbit will be, approximately



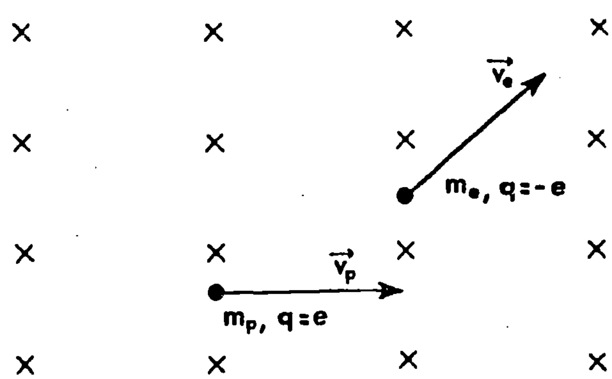
A. three times larger than the radius of the proton's orbit

B. nine times larger than the radius of the proton's orbit

C. the same as the radius of the proton's orbit

D. six hundred times smaller than the radius of the proton's orbit

7. If the proton and electron of the previous problem have the same speed, then the frequency with which the electron revolves in its orbit is



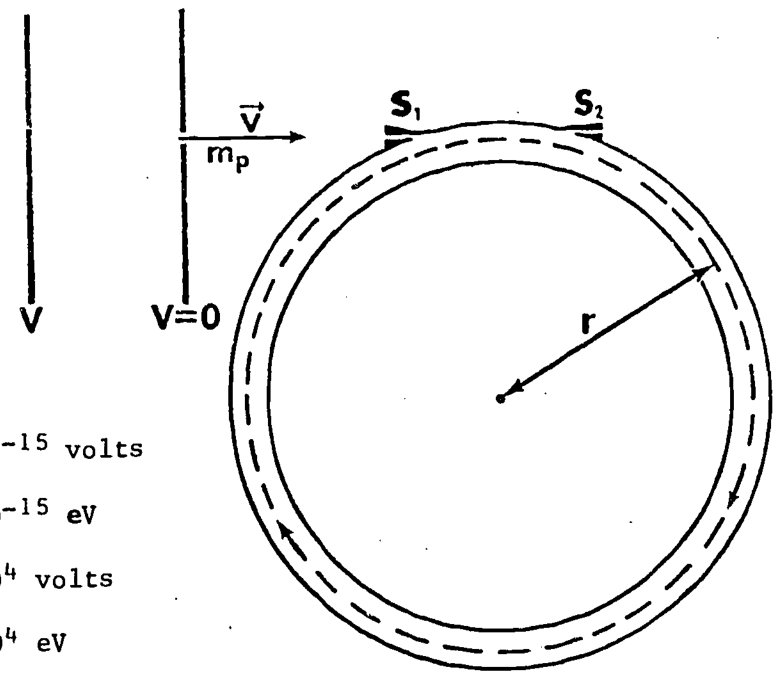
A. greater than the proton's frequency

B. less than the proton's frequency

C. the same as the proton's frequency

D. no conclusion can be drawn about the relative frequencies of the particles

8. The radius of the proton's circular path in the accompanying diagram is 5 meters and its period is 20 microseconds. What was the potential difference V through which it was accelerated before entering the magnetic field region?



A. 2.06×10^{-15} volts

B. 2.06×10^{-15} eV

C. 1.29×10^{4} volts

D. 1.29×10^{4} eV

INFORMATION PANEL — The Cyclotron

OBJECTIVE

To study the mechanical and electrical relationships needed to solve various problems dealing with the cyclotron.

Before starting this section of your work, it is imperative that you study carefully all of the material in all of the referenced reading indicated in the Study Guide for this Segment. A discussion of the cyclotron cannot be undertaken here, except to pinpoint a few bits of information which you will need in solving the problems.

next page

continued

The *electron volt* is an energy unit used extensively in connection with accelerators like the cyclotron. It is abbreviated eV and is frequently encountered in more convenient multiples:

$$1 \text{ MeV} = 10^6 \text{ eV}$$

$$1 \text{ BeV} = 10^9 \text{ eV}$$

$$1 \text{ GeV} = 10^{12} \text{ eV}$$

The definition of the electron volt is most conveniently obtained by considering that the kinetic energy K of a charged particle q that has moved from one point to another between which a potential difference V exists is given by:

$$K = qV$$

where K is in joules, q is in coulombs, and V is in volts. For an electron the charge is e so that:

$$K = eV$$

Thus, from this equation we have:

1 electron volt = (1 unit of electronic charge) (1 volt)

and it is seen that the electron volt is the amount of energy gained by an electron as it falls through a potential difference of 1 volt. Since the electronic charge is 1.60×10^{-19} coulomb, then the magnitude of one electron volt is 1.60×10^{-19} joule.

It will also be helpful to remember the relative masses of a few of the charged particles other than the electron used in accelerators. Taking the mass of the proton as unity for purposes of comparison, the relative masses of the other particles are:

electron: $m_e = \frac{m_p}{1836}$

deuteron: $m_D = 2m_p$

α-particle: $m_\alpha = 4m_p$

next page

continued

As developed in the previous Information Panel, you will recall that the radius r of the circular path followed by a charged particle q of mass m moving with velocity v perpendicularly to a field of magnetic induction B is expressed by:

$$r = \frac{mv}{qB}$$

The angular velocity ω of the circling particle is given by:

$$\omega = \frac{v}{r}$$

hence ω is then:

$$\omega = \frac{qB}{m}$$

and this equation can be given in terms of frequency f (say in revolutions per second) as

$$f = \frac{qB}{2\pi m}$$

wherein it should be noted that the frequency does not depend on the speed of the particle.

For the oscillator of the cyclotron to add energy to the particle at the proper point in each revolution, its frequency f_o must be *in resonance* with the frequency of revolution so that the oscillator frequency is (for resonance)

$$f_o = \frac{qB}{2\pi m}$$

9. If the oscillator frequency of a cyclotron is fixed at 15.3 MHz but the magnitude of the magnetic induction can be changed from zero to 1 T and its direction can be reversed, for which of the following particles, other than the proton, can this cyclotron be used?

A. only the electron

B. only the electron and deuteron

C. only the deuteron and the α-particle

D. all three (electron, deuteron and α-particle)

10. If a cyclotron which employs an accelerating potential of 1.0×10^5 volts has a dee separation of 5 cm, what is the magnitude of the electric field set up between the dees? (In volts/meter.)

11. What is the ratio of the magnitudes of the magnetic and the electric forces on the proton, during its passage through the gap?

A. $\frac{vB}{E}$

B. $\frac{vE}{B}$

C. $\frac{BE}{v}$

D. $\frac{B}{vE}$

12. What is the work per cycle done on the proton by the magnetic field when a proton is being accelerated in a cyclotron? Here are some data you may need: $v_p = 5 \times 10^6$ m/sec, B = 1 T and R = 45 cm

13. If a cyclotron which has an accelerating potential of 1.0×10^5 volts is used to accelerate protons, deuterons, or α-particles to an energy of 10 MeV, how many passages through the gap must each particle make?

A. proton, 100; deuteron, 200; α-particle, 400

B. proton, 100; deuteron, 100; α-particle, 50

C. proton, 100; deuteron, 50; α-particle, 50

D. proton, 100; deuteron, 100; α-particle, 100

14. The mass and charge of the proton are 1.67×10^{-27} kg and 1.60×10^{-19} coul respectively. What must be the minimum dee radius to accelerate a proton to 10 MeV if the magnetic induction is 1 T?

15. What must be the frequency of the oscillator in MHz if the cyclotron is to be used to accelerate protons to an energy of 10.0 MeV using a magnetic induction of 1.00 T?

INFORMATION PANEL

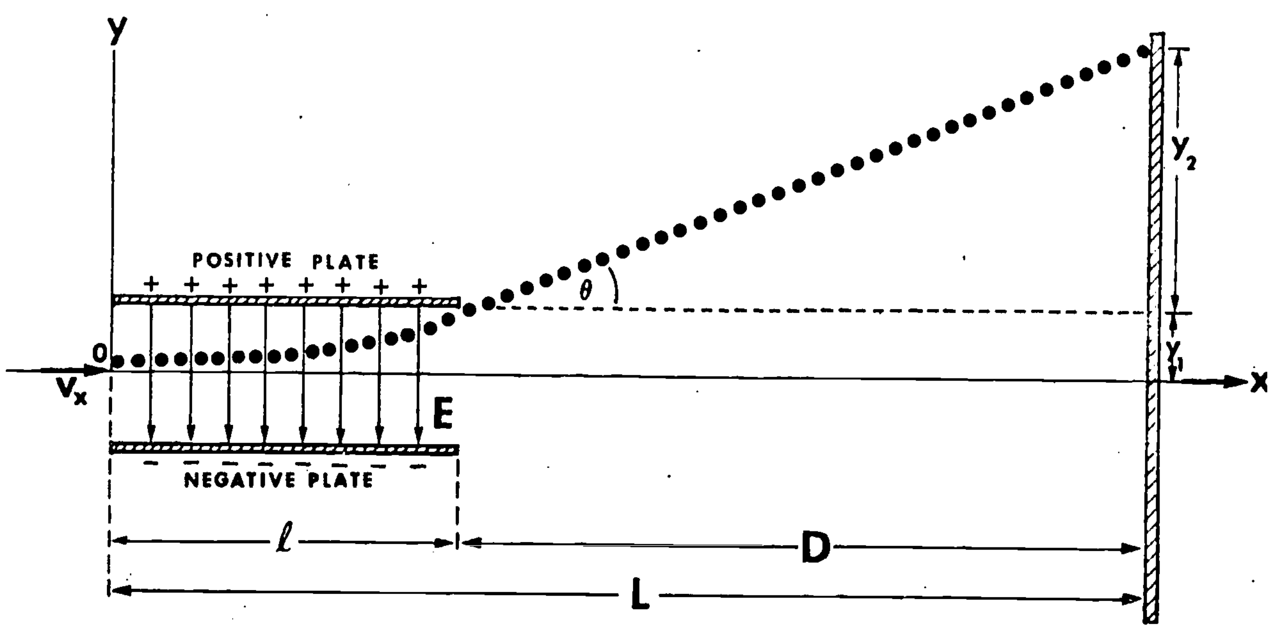
Measurement of Electronic Charge to Mass Ratio (e/m).

OBJECTIVE

To study the mechanism of the Thomson e/m experiment; to use the equations applicable to this experiment in solving relevant problems.

The reading assigned for this Segment contains extensive discussions of the mechanics and mathematics of the Thomson e/m experiment and will not be repeated here. However, to save you the effort of digging out the derivation of one of the key relationships involved in the Thomson equipment, we shall present a concise development here.

A beam of electrons moving with uniform velocity is made to enter a unifrom electric field between two charged, parallel metal plates. As the particles continue to move through the field, they experience a deflecting force which produces a deviation of the path. They ultimately strike a fluorescent screen, producing a spot of light which is displaced from the position it would have occupied had the electric field been absent. Refer to the accompanying diagram.



The electrons enter the region between plates at the origin 0 with a velocity v_x. Since the x-component of the force acting on the particles is zero, then $a_x = 0$ and v_x is therefore constant. Thus, in time t

$$x = v_x t \qquad (1)$$

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continued

The y-component of the force is given by the expression:

$$F_y = eE \tag{2}$$

where e is the charge on the electron. Since the acceleration is the force divided by the mass, or

$$a_y = \frac{eE}{m} \quad \text{(constant)} \tag{3}$$

then in time t

$$y = y_o + (v_y)_o t + \frac{1}{2} a_y t^2 \tag{4}$$

But $v_o = 0$ and also $(v_y)_o = 0$, then

$$y = \frac{1}{2} a_y t^2 \tag{5}$$

Substituting eE/m from equation (3) in place of a_y in equation (5):

$$y = \frac{eEt^2}{2m} \tag{6}$$

Eliminating t between equations (1) and (6) permits us to write:

$$y = \frac{eE}{2mv_x^2} \cdot x^2 \tag{7}$$

We shall define the quantity y_1 as the value of y when $x = \ell$. As the electrons pass beyond the deflecting plates, the trajectory becomes a straight line because they are now moving in a region where there is no electric field. The value of y_2 is D $\tan\theta$ where D and θ are defined as shown in the drawing.

The slope of this straight line is the ratio of the y-component of the velocity of an electron to the x-component of the velocity when the electron is about to leave the [illegible] where $x = \ell$. These components are v_x (constant) and

$$v_y = a_y t \tag{8}$$

Refer again to equation (1). This tells us that when $x = \ell$, then $t = \ell/v_x$. Combining this with the value of a_y from equation (3), we obtain:

$$v_y = \frac{eE\ell}{mv_x} \tag{9}$$

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Now since $\tan\theta = v_y/v_x$, we can substitute and get:

$$\tan\theta = \frac{eE\ell}{mv_x^2} \qquad (10)$$

Thus, we know the value of y_1 to be as shown below from equation (7):

$$y_1 = \frac{eE\ell^2}{2mv_x^2} \qquad (11)$$

and the value of y_2 now to be:

$$y_2 = \frac{DeE\ell}{mv_x^2} \qquad (12)$$

Since the total deflection $y_E = y_1 + y_2$, then

$$y_E = \frac{eE\ell}{2mv_x^2} + \frac{DeE\ell}{mv_x^2}$$

or

$$y_E = \frac{eE\ell}{mv_x^2}\left(\frac{\ell}{2} + D\right) \qquad (13)$$

Instead of using D, the distance from the plate ends to the screen, it is frequently more convenient to use L, the distance from the origin to the screen. When L is used, equation (13) then becomes

$$y_E = \frac{eE\ell}{mv_x^2}\left(L - \frac{\ell}{2}\right) \qquad (14)$$

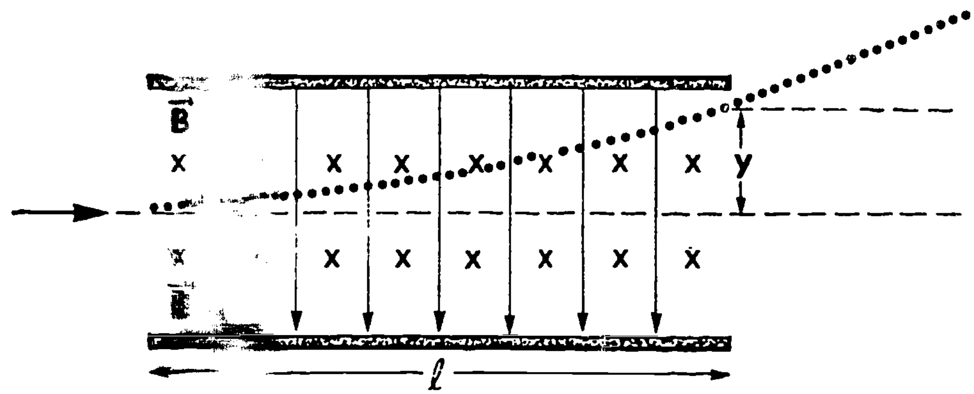
In the Thomson experiment, the electron beam was also acted on by a uniform magnetic field. A similar approach leads to the analogous equation for magnetic displacement y_M:

$$y_M = \frac{eB}{mv_x}\left(L - \frac{\ell}{2}\right) \qquad (15)$$

For balanced fields, where the beam deflection is zero, we have finally:

$$y_E - y_M = 0 \qquad (16)$$

16. A beam ... rons enters a region where it is acted upon by an electric and ... ic field simultaneously. The initial velocity, the direction of ... tric field and the direction of the magnetic field are mutually ... ular to each other. The electrons are found to leave the region ... ength ℓ = 10 cm undeflected if E = 50 nt/coul and B = 1.0 × 10 ... the $\vec{B}$ field is turned off, the electrons are found to be deflect... stance y = 1.7 mm; find the ratio e/m for the electrons in coul



17. An ele... has an initial velocity $\vec{v}$ = 3.25 × 10^7 m/sec in the horizontal ... ion. What is the deflection of the electron due to the gravitational force between x = 0 and x = 0.31 m? (Take the upward as the positive direction.)

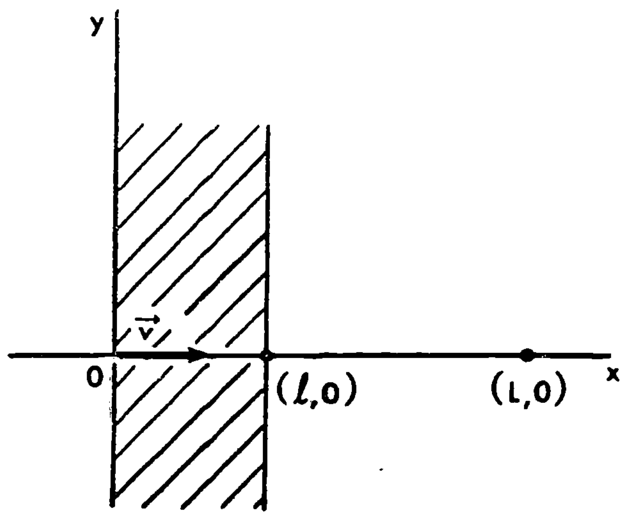
A. -1.9×10^{-18} m

B. -5.6×10^{-17} m

C. -4.5×10^{-16} m

D. 0.1 m

18. A particle of mass m enters the shaded area shown in the diagram at the origin and with a velocity $\vec{v}_0 = v_0\hat{i}$. Between $x = 0$ and $x = \ell$, it experiences a constant acceleration $\vec{a} = a\hat{j}$.



Derive an expression for the particle's y-coordinate when it leaves the shaded region $(x = \ell)$.

A. $a\ell^2/2v_0^2$

B. v_0t

C. $a\ell/v_0$

D. ℓ/v_0

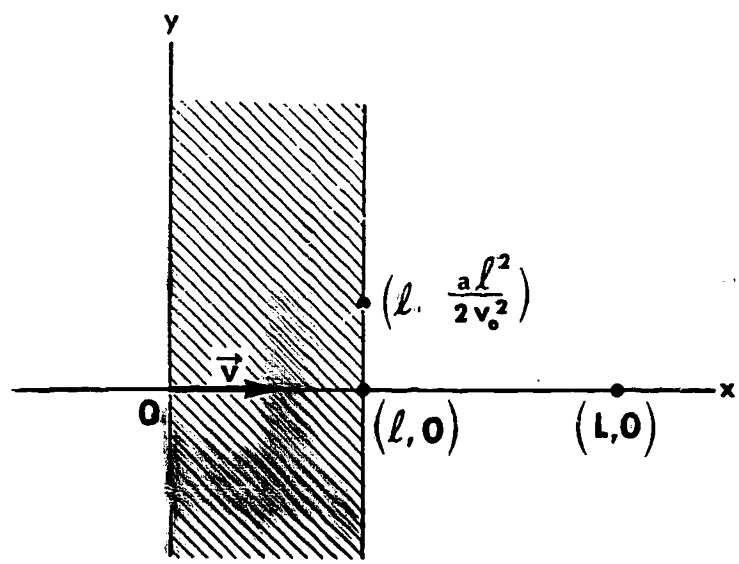
19. A particle of mass m enters the shaded area at the origin with velocity $\vec{v}_0 = v_0\hat{i}$. Between $x = 0$ and $x = \ell$, it experiences a constant acceleration $\vec{a} = a\hat{j}$. After passing the point $(\ell, a\ell^2/2v_0^2)$, the particle experiences no further acceleration. What is its y-coordinate when $x = L$? Neglect gravitational acceleration.

A. $\dfrac{aL^2}{2v_0^2}$

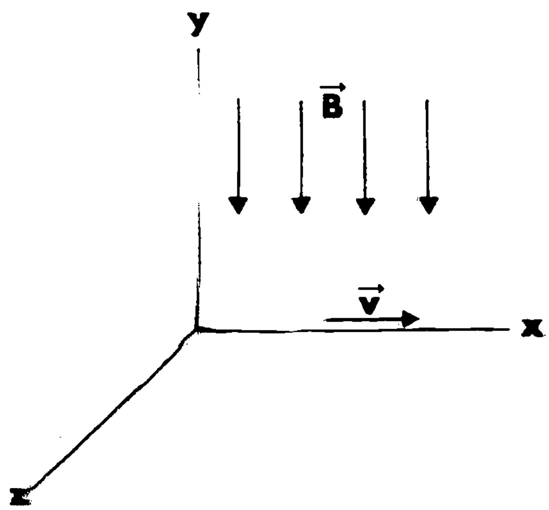
B. $\dfrac{a\ell}{v_0^2}(L - \ell)$

C. $\dfrac{a\ell}{v_0^2}\left(L - \dfrac{\ell}{2}\right)$

D. $\dfrac{a\ell^2}{2v_0^2}$



20. A beam of charged particles moves with constant velocity $\vec{v} = 1.34 \times 10^7\ \hat{i}$ m/sec through a region which contains uniform magnetic and electric fields. If $\vec{B} = -2.76 \times 10^{-3}\ \hat{j}$ T, what must the electric field be?



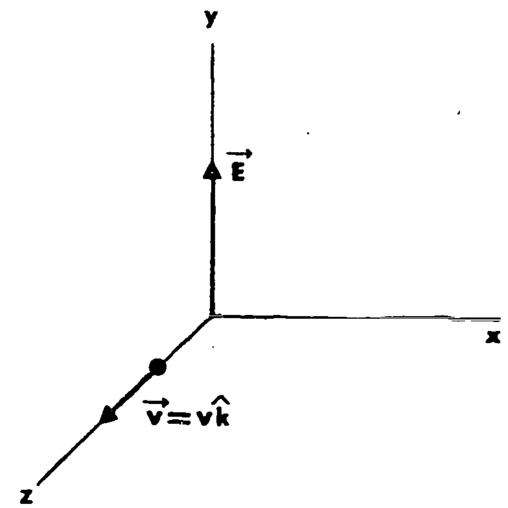
A. $\vec{E} = [illegible].70 \times 10^4\ \hat{k}$ nt/coul

B. $\vec{E} = -[illegible].70 \times 10^4\ \hat{k}$ nt/coul

C. $\vec{E} = 5.91 \times 10^{-15}\ \hat{i}$ nt/coul

D. $\vec{E} = 5.91 \times 10^{-15}\ \hat{j}$ nt/coul

21. An electric field $\vec{E}$ is directed along the positive y-axis. A charge q is fired along the positive z-axis with a speed v.



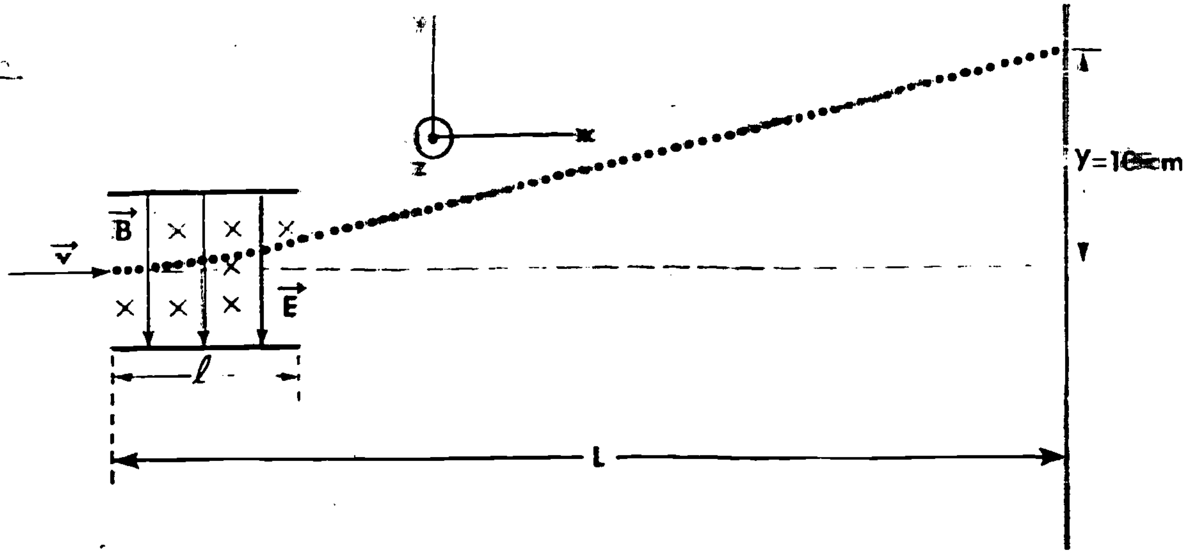
What is the minimum value of $\vec{B}$ and its corresponding direction such that the velocity of the particle does not change?

A. $\vec{B} = 0$

B. $B = \sqrt{2}\,\frac{E}{v}$, $\vec{B}$ in xz-plane making an angle of 45° with the x-axis

C. $B = \frac{E}{v}$, $\vec{B}$ anywhere in the xz-plane

D. $B = \frac{E}{v}$, $\vec{B}$ along the negative x-direction [illegible].



The dimensions of the above apparatus to measure the electron's charge to mass ratio, e/m, are ℓ = 2.00 cm, L = 31.0 cm. When an electric field of magnitude E = 1.00 × 10^5 nt/coul is applied, the deflection of the electron beam is measured on the screen to be y = [illegible].0 cm

$$\left[y = \frac{eE\ell}{mv^2}\left(L - \frac{\ell}{2}\right)\right].$$

A uniform magnetic field is applied between the deflecting plates along the negative z-axis, $\vec{B} = -B\hat{k}$ (into the paper). The magnitude of the magnetic induction is increased until the electron beam returns to its undeflected position. The field required is measured to be B = 3.08 × 10^{-3} T. What is the corresponding value of e/m?

A. (1.758796) × 10^{11} coul/kg

B. 3.25 × 10^7 coul/kg

C. 1.77 × 10^{11} coul/kg

D. 5.43 × 10^3 coul/kg

[a] CORRECT ANSWER: A

For a cyclotron, the force equation is given by

$$\frac{mv^2}{R} = m\omega^2 R = qvB$$

Therefore, the frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

for q/m, which is the quantity that varies as we change particles. We find

$$\frac{q}{m} = \frac{2\pi\nu}{B}$$

The frequency ν is fixed, so that as we vary B from zero to its maximum value (1 T, which puts the cyclotron in resonance for protons), q/m varies from infinity to

$$\frac{q}{m} = \frac{2\pi\nu}{B_{max}} = \frac{2\pi \times 15.3 \times 10^6}{1}$$

$$= 9.6 \times 10^7 \text{ coul/kg}$$

$$= \frac{q_p}{m_p} \text{ (proton)}$$

In other words, this cyclotron can be used for any particles whose q/m ratio is equal to, or greater than 9.6×10^7 coul/kg.

The q/m ratio for the proton is

$$\frac{q_p}{m_p} = \frac{1.6 \times 10^{-19}}{1.6 \times 10^{-27}} = 9.6 \times 10^7 \text{ coul/kg}$$

The q/m ratio of the three particles in question are (relative to the proton's):

a) Electron: $\left|\frac{q_e}{m_e}\right| = \left|\frac{q_p}{m_p/1840}\right| = 1840\left(\frac{q_p}{m_p}\right)$

b) Deuteron: $\frac{q_d}{m_d} = \frac{q_p}{2m_p} = \frac{1}{2}\left(\frac{q_p}{m_p}\right)$

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c) α-particle: $\frac{q_\alpha}{m_\alpha} = \frac{2q_p}{4m_p} = \frac{1}{2}\left(\frac{q_p}{m_p}\right)$

Hence, we note that only the electron has a q/m ratio larger than that of the proton.

TRUE OR FALSE? The q/m ratio of an α-particle is half as great as the same ratio for an electron.

[a] CORRECT ANSWER: A

This is a review question on kinematics. Using the equations

$$x = x_o + v_{ox}t + \frac{1}{2} a_x t^2 \quad (1)$$

with $a_x = 0$, $x_o = 0$ and $v_{ox} = v_o$ we obtain

$$x = v_o t \quad (2)$$

Thus, the particle reaches the line $x = \ell$ at time $t = \ell/v_o$

For the motion in the y-direction, we have $y_o = 0$, $v_{oy} = 0$ and $a_y = a$, so

$$y = \frac{1}{2} at^2 \quad (3)$$

Substituting (2) into (3) we obtain

$$y = \frac{1}{2} a\left(\frac{\ell}{v_o}\right)^2 = \frac{a\ell^2}{2v_o^2} \quad (4)$$

[a] CORRECT ANSWER: D

The magnetic force is given by $\vec{F} = q\vec{v} \times \vec{B}$. The vectors $\vec{v}$ and $\vec{B}$ here are mutually perpendicular and $\vec{v} \times \vec{B}$ points to the north. However, the direction of the force on the negatively charged particle is in the direction of $-\vec{v} \times \vec{B}$, or south. Finally, the magnitude of $\vec{F}$ is

$$F = qvB = (1.6 \times 10^{-19}\ \text{coul}) \times (6 \times 10^{5}\ \text{m/sec}) \times (5 \times 10^{-5}\ \text{T})$$

$$= 4.8 \times 10^{-18}\ \text{nt}$$

TRUE OR FALSE? The direction of the magnetic force on an electron is opposite that on a proton moving in the same direction.

[b] CORRECT ANSWER: C

Since the electron beam is undeflected, we know that the electric and magnetic forces are equal in magnitude and opposite in direction. Now

$$\vec{F}_B = q\vec{v} \times \vec{B} = (-e)vB\hat{i} \times (-\hat{k}) = -evB\hat{j}$$

and

$$\vec{F}_E = q\vec{E} = (-e)\ (-E\hat{j}) = eE\hat{j}$$

Setting $\vec{F}_B = -\vec{F}_E$, we find

$$eE = evB$$

or

$$v = \frac{E}{B} = \frac{1.00 \times 10^{5}}{3.08 \times 10^{-3}} = 3.25 \times 10^{7}\ \text{m/sec}$$

Since

$$y = \frac{eE\ell}{mv^2}\left(L - \frac{\ell}{2}\right) = .100$$

we may solve for e/m

$$\frac{e}{m} = \frac{yv^2}{E\ell(L - \ell/2)} = 1.77 \times 10^{11}\ \text{coul/kg}$$

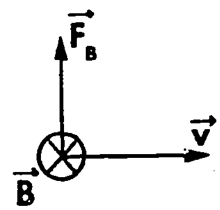
Note that choice A has too many significant figures for the data given.

TRUE OR FALSE? If the length ℓ of the deflecting plates is increased, the magnitude of y will decrease.

[a] CORRECT ANSWER: B

The force on a moving charged particle in a magnetic field is given by

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad (1)$$



From this it is clear that $\vec{v}$ and $\vec{F}_B$ are always perpendicular to one another since, in this case, $\vec{v}$ is perpendicular to $\vec{B}$. Therefore, the charged particle will describe a circular path of radius r given by the equation

$$F_B = qvB = \frac{mv^2}{r} \qquad (2)$$

Thus,

$$r = \frac{mv}{qB} = \frac{p}{qB} \qquad (3)$$

where p = mv and represents the particle's momentum.

Using the relation between momentum and kinetic energy

$$p = \sqrt{2mK} \qquad (4)$$

we can rewrite (3) as

$$r = \frac{\sqrt{2mK}}{qB} \qquad (5)$$

From (5) we see that for equal q, B, and K, the radius increases with mass. Therefore, the radius of the electron's orbit is smaller than that of the proton by a factor of $\sqrt{1836}$.

TRUE OR FALSE? In the solution above, K is a constant of proportionality.

[a] CORRECT ANSWER: A

The magnitudes of the magnetic and electrical forces are respectively

$$F_B = |q\vec{v} \times \vec{B}| = qvB \quad \text{(since } \vec{v} \text{ is normal to } \vec{B}\text{)}$$

and

$$F_E = |q\vec{E}| = qE$$

The ratio of the two is therefore

$$\frac{F_B}{F_E} = \frac{qvB}{qE} = \frac{vB}{E}$$

[b] CORRECT ANSWER: 15.3

For a cyclotron, the force equation is

$$\frac{mv^2}{R} = m\omega^2 R = qvB \qquad (1)$$

The oscillator must be in "resonance" with the proton's orbit. The frequency from (1) is

$$\nu = \frac{qB}{2\pi m} \qquad (2)$$

After one-half of a revolution of the proton, the oscillator must have gone through one-half of a cycle; namely, from maximum magnitude of $\vec{E}$ in one direction to maximum magnitude of $\vec{E}$ in the opposite direction in order to accelerate the proton as it re-enters the gap. Thus,

$$\nu_o = \nu = \frac{qB}{2\pi m} = \frac{(1.6 \times 10^{-19} \text{ coul}) \times (1 \text{ T})}{2\pi \times (1.67 \times 10^{-27} \text{ kg})} = 15.3 \text{ MHz}$$

TRUE OR FALSE? For resonance to occur, the frequency of the proton's revolutions must be equal to the oscillator frequency.

[a] CORRECT ANSWER: D

The magnetic force on any moving charged particle is given by

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad (1)$$

From this it is clear that $\vec{v}$ and $\vec{F}_B$ are always perpendicular to each other. The charged particle will describe a circular path of radius r given by the equation

$$F_B = qvB = \frac{mv^2}{r}$$

or

$$r = \frac{mv}{qB}$$

Thus, the electron's and proton's orbital radii are given respectively by

$$r_e = \frac{m_e v_e}{eB}$$

and

$$r_p = \frac{m_p v_p}{eB} = \frac{(1836\ m_e) \times (v_e/3)}{eBeB} = \frac{612\ m_e v_e}{eB}$$

We have used the fact that the magnitudes of the two particles' charges are equal and that the electron has a speed three times that of the proton. Dividing r_e by r_p, we obtain

$$\frac{r_e}{r_p} = \frac{m_e v_e/eB}{612\ m_e v_e/eB} = \frac{1}{612} \simeq \frac{1}{600}$$

That is, the radius of the electron's orbit is only 1/600 that of the proton's orbit.

[b] CORRECT ANSWER: zero

A magnetic field never does work! From

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

it follows that the magnetic force is perpendicular to the velocity and therefore to the displacement. Hence,

$$W_B = \int \vec{F}_B \cdot d\vec{s} = 0$$

[a] CORRECT ANSWER: C

Since to the right of the line $\ell = 0$ the particle experiences no acceleration, we can write

$$x = x_o + v_{ox} (t - t_o) \qquad (1)$$

and

$$y = y_o + v_{oy} (t - t_o) \qquad (2)$$

for $t \geq t_o = \ell/v$.

We already know that $x_o = \ell$, $y_o = a\ell^2/2v_o^2$ and $v_{ox} = v_o$. To find v_{oy} we use the fact that for $t < t_o$, $v_y = at$, so at $t_o = \ell/v$, $v_{oy} = a\ell/v$. Equations (1) and (2) now become

$$x = \ell + v_o (t - t_o) \qquad (3)$$

and

$$y = \frac{a\ell^2}{2v_o^2} + \frac{a\ell}{v_o} (t - t_o) \qquad (4)$$

We can solve for the time at which the particle reaches the line $x = L$. From equation (3), we find

$$t = t_o + (L - \ell)/v_o = L/v_o$$

Using this time into (4) we obtain

$$y \text{ (at } x = L) = \frac{a\ell^2}{2v_o^2} + \frac{a\ell}{v_o}\left(\frac{L}{v_o} - \frac{\ell}{v_o}\right)$$

$$= \frac{a\ell}{v_o^2}\left(\frac{\ell}{2} + L - \ell\right) = \frac{a\ell}{v_o^2}\left(L - \frac{\ell}{2}\right) \qquad (5)$$

[b] CORRECT ANSWER: C

The magnetic force on the proton is given by:

$$\vec{F} = q\vec{v} \times \vec{B}$$

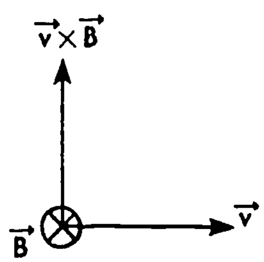
Since q is positive, the direction of $\vec{v} \times \vec{B}$ will be the same as that of the centripetal force. Using the right-hand rule, if we curl our fingers from the direction of $\vec{v}$ into the page, our thumb points upward. If we curl them out of the page, on the other hand, the thumb points downward. Therefore, $\vec{B}$ must be directed out of the paper.

] CORRECT ANSWER: C

The fact that the particles move in planar orbits follows from the fact that both the initial velocity and the acceleration,

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{v} \times \vec{B}}{m}$$

are in a plane normal to $\vec{B}$. The orbit is a circle because $\vec{a}$ is normal to $\vec{v}$, which means that it will not affect the magnitude of $\vec{v}$, only its direction. Thus, the speed will be uniform, and as we know, this type of acceleration results in a circular motion. The sense of rotation is obtained by assuming a direction for $\vec{v}$ and realizing that the center of the circle is in the direction in which the force, $\vec{F} = q\vec{v} \times \vec{B}$, points. For positive q, $\vec{F}$ is upward in the diagram (counterclockwise rotation). For negative q, $\vec{F}$ is downward (clockwise rotation).



] CORRECT ANSWER: C

The proton was accelerated from rest by a potential difference V, so using the definition of potential and the work energy theorem, we obtain

$$qV = W = \Delta K = \frac{1}{2} mv^2 \qquad (1)$$

For circular motion, the velocity is related to the radius and the period by

$$v = 2\pi r/T \qquad (2)$$

Solving (1) for the potential difference and using (2), we obtain

$$V = \frac{mv^2}{2q} = \frac{m}{2q}\left(\frac{2\pi r}{T}\right)^2 = \frac{2m}{q}\left(\frac{\pi r}{T}\right)^2$$

Now we substitute in the numerical values.

$$V = \frac{2 \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19}}\left(\frac{\pi \times 5}{2 \times 10^{-5}}\right)^2 = 1.29 \times 10^4 \text{ volts}$$

TRUE OR FALSE? In uniform circular motion, the velocity of the circling particle is directly proportional to the period of rotation.

[a] CORRECT ANSWER: D

The velocity of the particle will not change if the total force on it is zero.

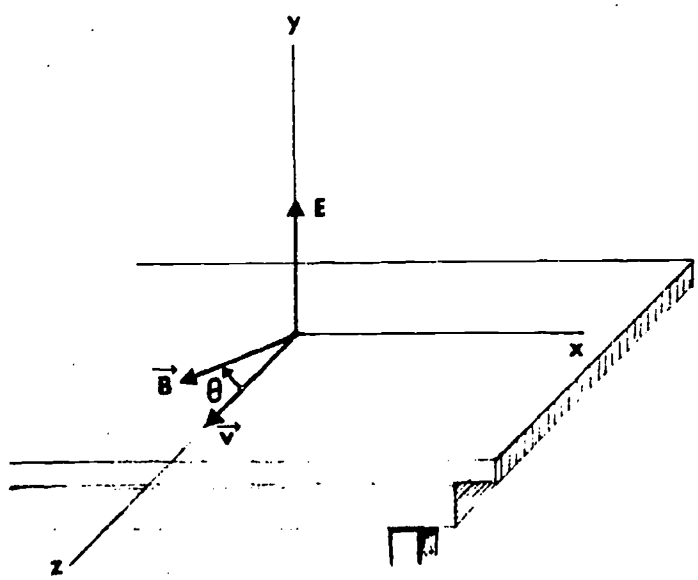
Thus,

$$0 = \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \qquad (1)$$

or

$$\not{q}\vec{v} \times \vec{B} = -\not{q}\vec{E} \qquad (2)$$

Since $\vec{E}$ is directed along the positive y-axis, this tells us that $\vec{v} \times \vec{B}$ must be along the negative y-axis. Now $\vec{v} \times \vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{B}$, so B must lie in the xz-plane, and so oriented relative to $\vec{v}$ that the product $\vec{v} \times \vec{B}$ is opposite to $\vec{E}$.



Letting θ be the angle between $\vec{v}$ and $\vec{B}$, we may write equation (2) as

$$vB \sin\theta = E$$

or

$$B = \frac{E}{v \sin\theta} \qquad (3)$$

Now E and v are fixed. However, the values of B and sinθ in (3) can be adjusted (within limits). It is to our advantage to use the full influence of $\vec{B}$; that is to apply the required force with a minimum B. For minimum B in (3) we must have a maximum value for sinθ; namely, sinθ = 1 or θ = 90°. Thus,

$$\vec{B} = \frac{-E}{v} \hat{i}$$

[a] CORRECT ANSWER: 0.46 m

The kinetic energy of a particle in circular motion with radius R in a uniform magnetic field is

$$K = \frac{q^2B^2R^2}{2\ m}$$

Thus, the dee radius necessary to accelerate a proton to an energy of 10 MeV is

$$R = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{(2 \times 1.67 \times 10^{-27}\ \text{kg}) \times (10\ \text{MeV}) \times (1.6 \times 10^{-13}\ \text{j/MeV})}}{(1.6 \times 10^{-19}\ \text{coul}) \times (1\ \text{T})}$$

$$= 0.46\ \text{m} = 46\ \text{cm}$$

[b] CORRECT ANSWER: C

The deflection is given by

$$y = -\frac{1}{2}gt^2$$

because $v_{oy} = 0$. However,

$$t = \frac{L}{v}$$

where $L = \Delta x$

Thus,

$$y = -\frac{gL^2}{2v^2} = -\frac{9.8 \times (.31)^2}{2 \times (3.25 \times 10^7)^2} = -4.5 \times 10^{-16}\ \text{m}$$

[c] CORRECT ANSWER: 2×10^6

The equation for the potential difference, V, between two points separated by a displacement $\vec{d}$ in a uniform electric field $\vec{E}$ is

$$V = \vec{E}\cdot\vec{d}$$

For $\vec{d}$ along $\vec{E}$ and the values given, we obtain

$$E = \frac{V}{d} = \frac{10^5\ \text{V}}{5 \times 10^{-2}\ \text{m}} = 2 \times 10^6\ \frac{\text{volts}}{\text{meter}}$$

[a] CORRECT ANSWER: A

In the preceding problem we derived the equation

$$r = \frac{mv}{qB} \qquad (1)$$

The frequency of revolution of the particle is given by

$$\nu = \frac{\omega}{2\pi} = \frac{v}{2\pi r} = \frac{v}{2\pi mv/qB} = \frac{qB}{2\pi m} \qquad (2)$$

For the proton and electron q and B are the same but the mass, m, is not. From (2) we see that the frequency is inversely proportional to the mass of the particle, so the frequency of revolution of the less massive particle (the electron) will be higher than that of the proton.

[b] CORRECT ANSWER: B

From the definition of the magnetic induction $\vec{B}$,

$$|\vec{F}| = q_o|\vec{v} \times \vec{B}| = q_o\, vB \sin\theta$$

we see that for $v \neq 0$ and $B \neq 0$, $\vec{F}$ becomes zero only if $\sin\theta = 0$. This is satisfied when $\theta = 0°$ or $180°$; that is when $\vec{v}$ and $\vec{B}$ are parallel or anti-parallel.

TRUE OR FALSE? The presence of even a small component of $\vec{B}$ perpendicular to $\vec{v}$ would cause a change in the velocity of the test charge.

[c] CORRECT ANSWER: B

The work done on a particle of charge q in passing through a potential difference V is equal to its change in kinetic energy and is given by $W = \Delta K = qV$, independently of the mass of the particle. (The *velocity* with which the particle emerges depends upon its mass.) For $\Delta K = 10$ MeV, and $V = n \times 100$ KV (and since 1 eV = 1.6×10^{-19} joule) we have

$$10^7 \times 1.6 \times 10^{-19} \text{ j} = qn \times 10^5 \text{ volts}$$

or

$$n = \frac{1.6 \times 10^{-19} \times 10^7}{q \times 10^5} = 100\,\frac{e}{q}$$

For q = e and 2e we obtain n = 100 and 50, respectively.

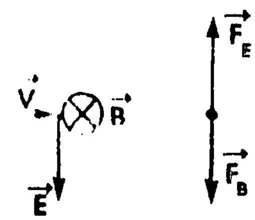
[a] CORRECT ANSWER: 1.7×10^{11}

Initially the resultant force on the electron is zero. Therefore

$$\vec{F}_E + \vec{F}_B = 0$$

or

$$e\vec{E} + e(\vec{V} \times \vec{B}) = 0 \qquad (1)$$



where v is the initial speed of the electrons. Since $\vec{E}$, $\vec{v}$, and $\vec{B}$ are mutually perpendicular to each other, equation (1) yields

$$eE = evB$$

or

$$v = \frac{E}{B} \qquad (2)$$

When the $\vec{B}$ field is turned off, the electrons are acted upon only by the $\vec{F}_E$ force which causes the electrons to accelerate in the upward direction (Note electrons are negatively charged particles). The acceleration is constant, directed upwards and its magnitude is

$$a = \frac{eE}{m} \qquad (3)$$

where m is the mass of an electron. The deflection y is given by

$$y = \frac{1}{2} at^2 \qquad (4)$$

where t is the time the electrons spent in traversing region ℓ, i. e.,

$$t = \frac{\ell}{v} \qquad (5)$$

Note: The electrons have no initial speed in the y-direction. Substituting the expression for t, a, and v in equation (4), we obtain

$$y = \frac{1}{2} \frac{eE}{m} \frac{\ell^2}{v^2}$$

or

$$y = \frac{1}{2} \frac{eE}{m} \frac{\ell^2 B^2}{E^2}$$

Solving for e/m we obtain

$$\frac{e}{m} = \frac{2yE}{\ell^2 B^2} = \frac{2 \times 1.7 \times 10^{-3} \times 50}{(.10)^2 \times (1.0 \times 10^{-5})^2}$$

$$= 1.7 \times 10^{11} \text{ coul/kg}$$

TRUE OR FALSE? The equation v = E/B is valid only if $\vec{E}$, $\vec{v}$, and $\vec{B}$ are mutually perpendicular to each other.

[a] CORRECT ANSWER: A

Since the velocity of the beam is constant, we conclude that the resultant force on the beam is zero. Thus

$$\vec{F} = q\vec{v} \times \vec{B} + q\vec{E} = 0$$

Solving for $\vec{E}$, we obtain

$$\vec{E} = -\vec{v} \times \vec{B} = -(1.34 \times 10^7\ \hat{i}) \times (-2.76 \times 10^{-3}\ \hat{j})$$

$$= (3.70 \times 10^4)(\hat{i} \times \hat{j}) = 3.70 \times 10^4\ \hat{k}\ \text{nt/coul}$$

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL — Units for Magnetic Fields

OBJECTIVE

To define the units to be used in working with magnetic fields; to solve problems involving the force that acts on electric charges moving through a magnetic field.

A number of definitions of quantities used in the study of magnetism parallels those developed in the analysis of electric fields. A logical place to start the work that now lies before us is the definition of *magnetic induction* $\vec{B}$. A magnetic field may be visualized as a region permeated by lines of magnetic force. If we designate the magnetic induction B as the number of field lines that cross perpendicularly through a unit area, and the magnetic flux as the total number of lines perpendicular to the area A, then the relation:

$$B = \frac{\phi}{A}$$

follows directly. In the MKS system, the unit of flux ϕ is the *weber* and, since the unit of area is the square meter, magnetic induction $\vec{B}$ is measured in webers per square meter.

By international agreement, the weber per square meter is to be called the *tesla* (abb. T). In basic MKS units, the tesla is equivalent to:

$$1\ T = 1\ \frac{nt}{amp\text{-}m}$$

This equivalence can be derived from the relationship between the force $\vec{F}$ acting on a wire carrying a current i in a magnetic field of intensity $\vec{B}$:

$$d\vec{F} = i(d\vec{\ell} \times \vec{B}) \qquad (1)$$

wherein $d\vec{\ell}$ is a differential element of length and $d\vec{F}$ is the force acting on $d\vec{\ell}$. Proper integration of this expression will yield the force acting on linear or nonlinear conductors.

In the special case of a current-carrying straight wire at right angles to $\vec{B}$, equation (1) reduces to the scalar form:

$$F = i\ell B$$

or for a straight wire at any other angle to the direction of $\vec{B}$

$$\vec{F} = i(d\vec{\ell} \times \vec{B})$$

next page

continued

The force $\vec{F}$ is directed at right angles to the plane formed by the line of the current-carrying wire and the direction of $\vec{B}$. The direction of the force may be determined by the usual method involving the advance of a right-handed screw as one vector is rotated into the other, or by using the *palm rule* described in your supplementary reading.

The CGS unit of magnetic induction is sometimes encountered in problem solving and should be converted into teslas. This unit is called the *gauss* and is related to the tesla as follows:

$$1 \text{ gauss} = 10^{-4} \text{ T}$$

In the problems to be solved in this section you will want to remember that the symbol e represents the absolute value of the electronic charge. Thus, for the electron

$$q_e = -e$$

and for the proton

$$q_p = +e$$

where $e = 1.69 \times 10^{-19}$ coul.

The problems that follow require that you

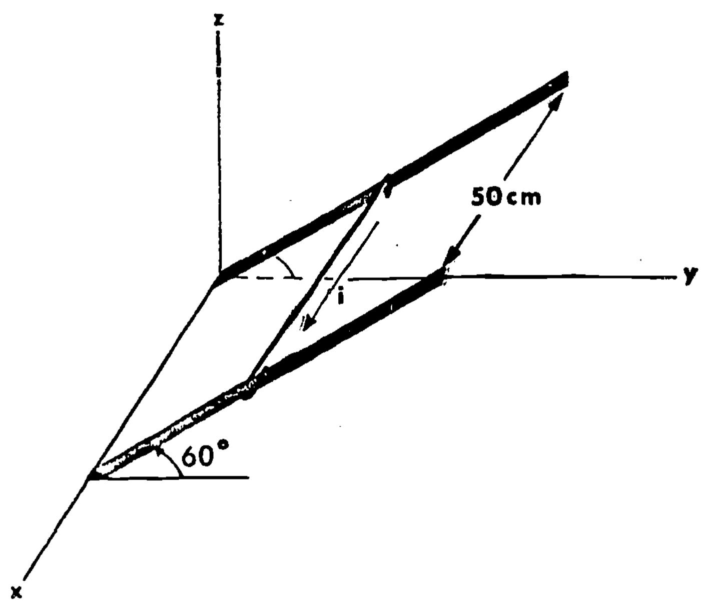
(a) calculate the force acting on a straight wire of given dimensions carrying a given current in a given magnetic field, or any variation of this general relationship;

(b) determine the direction of the force on a current-carrying wire from a descriptive diagram;

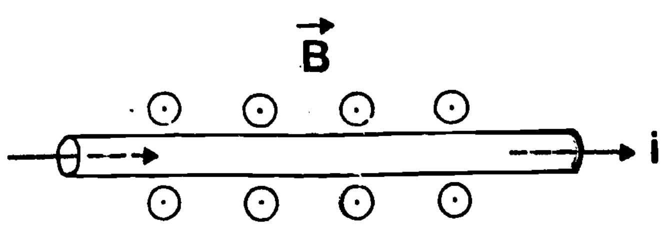
(c) decide what will happen to the force on a current-carrying wire if the cross-sectional area of the wire changes, all other factors remaining the same.

PROBLEMS

1. A metal wire of length 50 cm and mass 20 gm carries a current of 0.1 amp. It rests on a pair of frictionless rails inclined at an angle of 60° to the horizontal (the xy-plane is the horizontal plane and the wire is parallel to the x-axis). A horizontal uniform magnetic field exists in the region. What must be the magnitude of the field in teslas and its direction if the wire is not to slide up or down the incline?



2. In the situation shown in the diagram below, what is the direction of the force on the wire?



A. Into the paper

B. Out of the paper

C. Upward

D. Downward

3. The magnetic force on a wire of length ℓ which carries a current i that is perpendicular to a megnetic field $\vec{B}$ has magnitude $F = i\ell B$. If the cross-sectional area of the wire is doubled, but the current remains the same, the magnitude of the force

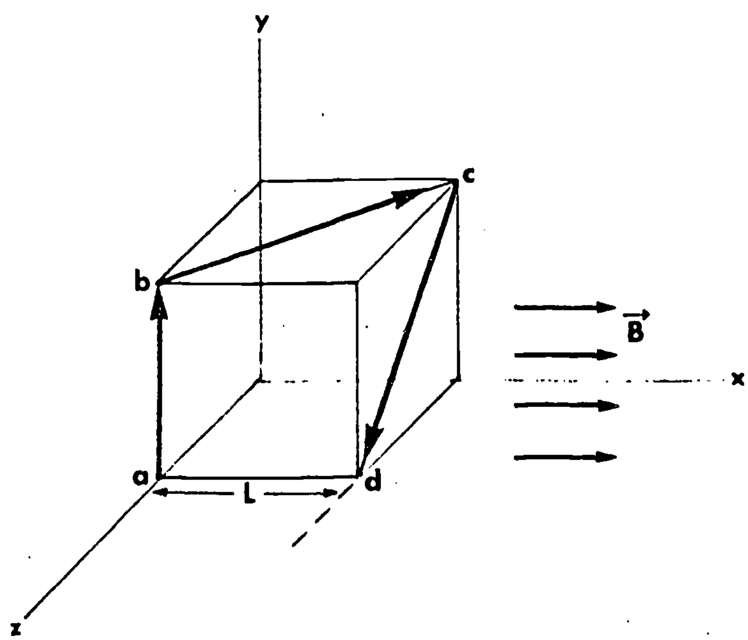
A. remains the same

B. is doubled

C. is multiplied by 4

D. is halved

4. A cube of side L = 2 m is in a uniform magnetic field B = .5 T, parallel to the x-axis. A wire abcd carries a current i = 3 amp in the direction as shown in figure below. Find the force acting on the wire abcd.



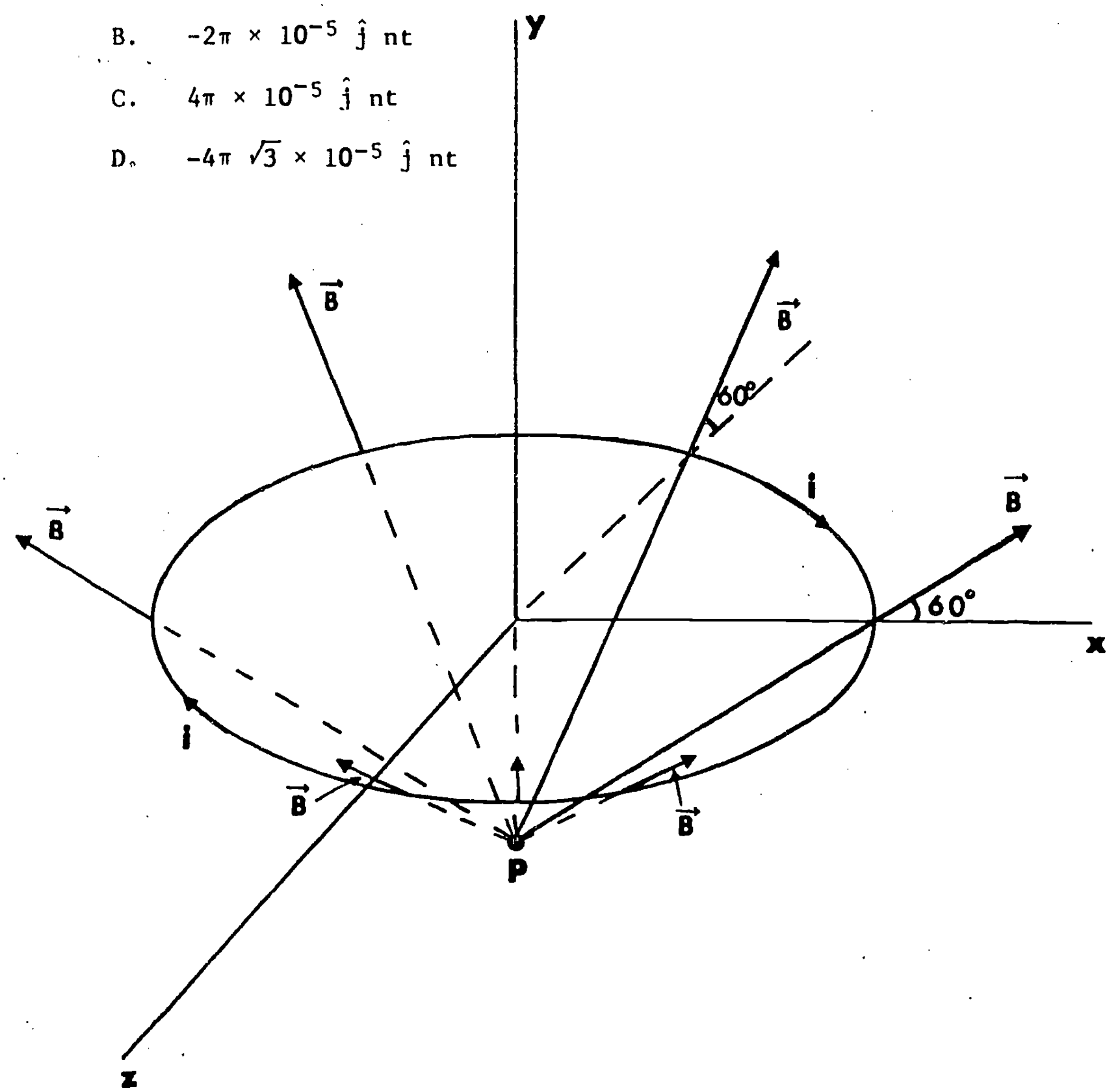
5. A circular loop of radius 40 cm carries a current of 1 milliampere in the sense shown in the diagram. The loop is placed in a symmetrically diverging magnetic field such that $\vec{B}$ is everywhere perpendicular to the loop itself and makes an angle of 60° with the plane of the loop (the plane of the loop is the xz-plane, and the magnetic field lines meet at a point P on the negative y-axis. The magnitude of $\vec{B}$ at the site of the loop is 0.1 T. What is the net force on the loop?

A. $2\pi \times 10^{-5}\ \hat{j}$ nt

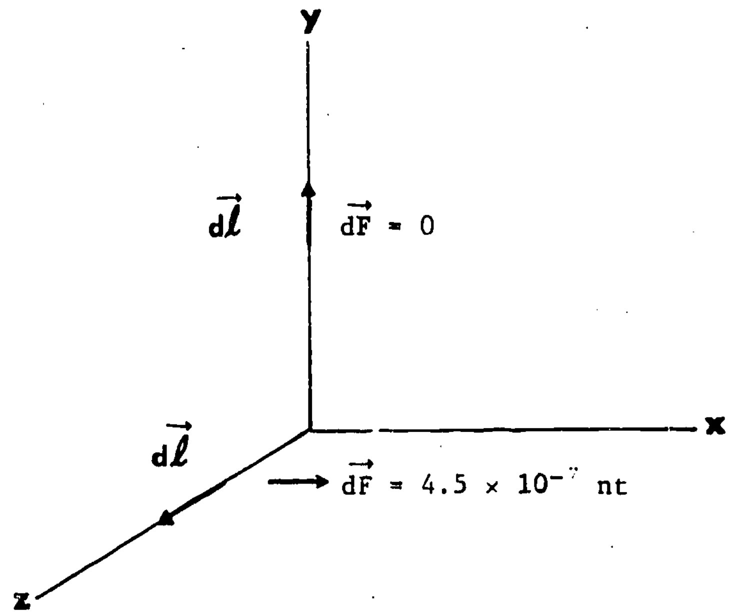
B. $-2\pi \times 10^{-5}\ \hat{j}$ nt

C. $4\pi \times 10^{-5}\ \hat{j}$ nt

D. $-4\pi\sqrt{3} \times 10^{-5}\ \hat{j}$ nt



6.



Consider a current element of length 1.0 cm and carrying a current of 0.25 amp. When oriented along the positive y-axis, $d\vec{\ell} = d\ell\ \hat{j}$, the element experiences no force. But when it is oriented along the positive z-axis, $d\vec{\ell} = d\ell\ \hat{k}$, it experiences a force in the positive x direction, $d\vec{F} = 4.5 \times 10^{-7}\ \hat{i}$ nt. Assuming there is a uniform magnetic field throughout the region, find its magnitude and direction.

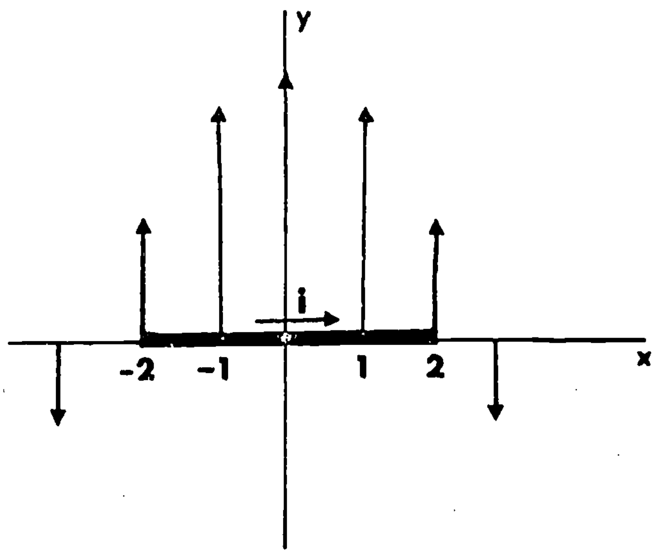
A. $\vec{B} = -1.8 \times 10^{-6}\ \hat{j}$ T

B. $\vec{B} = 1.8 \times 10^{-7}\ \hat{j}$ T

C. $\vec{B} = 1.8 \times 10^{-4}\ \hat{j}$ T

D. $\vec{B} = -1.8 \times 10^{-4}\ \hat{j}$ T

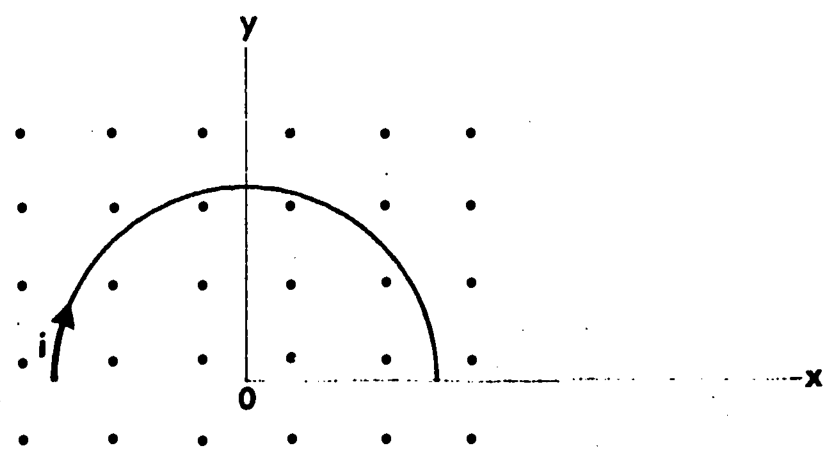
7. A rigid conducting wire carries a current of 0.50 amp in the i direction. The y-component of the magnetic field near the origin is as sketched in the diagram; namely,



$$B_y = (20 - 3x^2) \text{ T}$$

The z-component of the field is zero. If the wire is 4.0 m long and is centered at the origin, what is the magnetic force on the wire?

8. A wire bent in the form of a semicircle with radius 10 cm carries current i = 2 amp and is placed in a uniform magnetic field of magnitude 2 T as shown in the diagram. What is the total force on the wire?



INFORMATION PANEL — Torque on a Current-Carrying Loop

OBJECTIVE

To calculate the torque acting on a current-carrying loop immersed in a magnetic field.

Students beginning their study of the forces and torque acting on a current-carrying loop in a magnetic field are often confused in their interpretation of the schematic diagrams that illustrate the orientation of the loop and its current in the magnetic field. Perhaps the simplest way to represent the assembly is to show it in two steps. In Figure 1, the loop is displayed in a broadside view showing the direction of the current (light arrows) and the direction of the uniform magnetic field (into the plane of the paper as indicated by the "x's.") Figure 2 is obtained by rotating the whole diagram *clockwise as seen by looking at the loop in the direction indicated by the heavy arrow.* After a 90-degree clockwise rotation, the loop would appear as shown in figure 2.

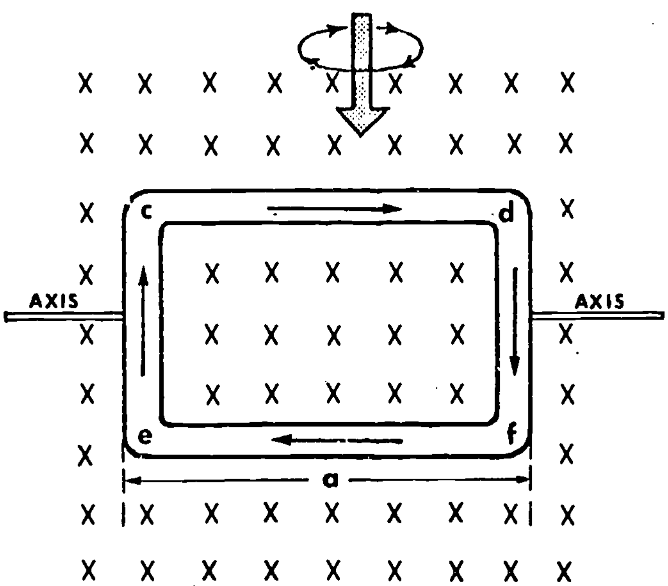


Figure 1

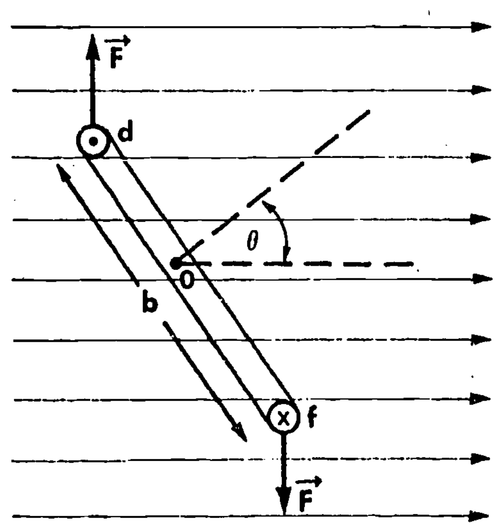


Figure 2

To investigate the forces acting on the loop, we'll start with side df which, in Figure 2, is nearest the reader. The current in this side is directed downward at the angle of the loop so that the force acting on this wire is outward, toward the reader. (Check this with the palm rule.)

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continued

However, side ce, which is directly behind df in Figure 2 and therefore not visible to the reader, experiences an *equal force into the plane of the paper;* this is evident from the symmetry of the assembly. Since these forces have the same line of action, the resultant force and the torque are both zero. Thus, these sides may be ignored in the remainder of the analysis.

Now, let's look at sides cd and ef. Both of these sides are perpendicular to the field, hence equal and opposite forces are exerted on them, but the lines of action of these forces are *not* the same. Since, as seen in Figure 1, the length of cd is designated as a, the force on side cd is:

$$F = Bia$$

The force on side ef is exactly the same in magnitude but is oppositely directed. The moment arm of each force is the perpendicular distance to the axis O, or

$$\text{moment arm} = \frac{b}{2}\sin\theta$$

Thus, the torque due to the force on *both* cd and ef is given by:

$$\tau = (2)\ (Bia)\left(\frac{b}{2}\sin\theta\right)$$

$$\tau = Bi\ (ab)\sin\theta$$

$$\tau = BiA\sin\theta$$

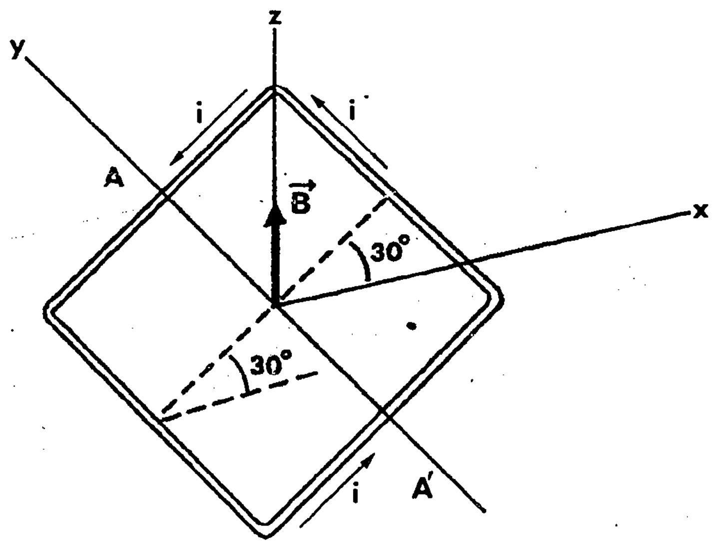
in which A is the area of the loop given by the product ab.

Since the same torque acts on every turn of a multiturn loop, if the loop is replaced by a coil having N turns, the torque would then be

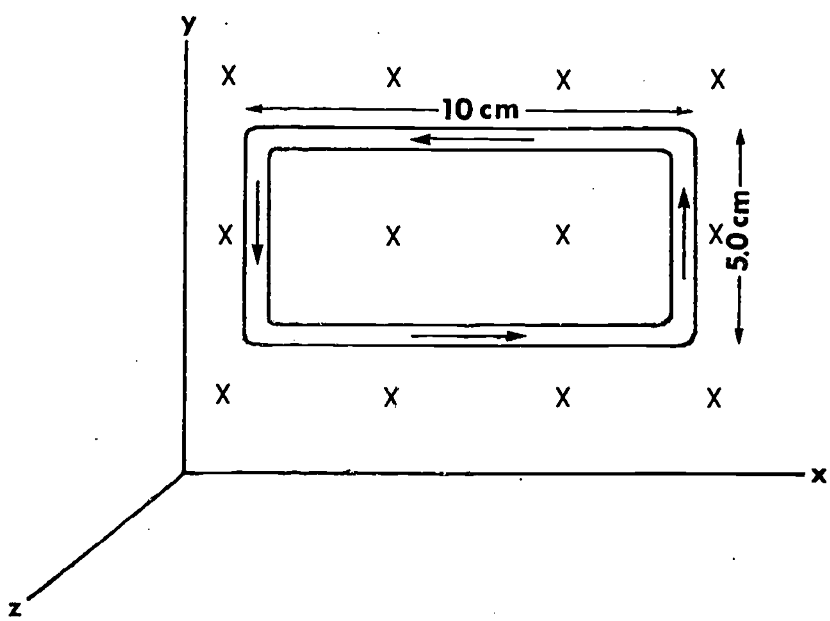
$$\tau = NBiA\sin\theta$$

Please note carefully when working on the problems in this group, that the angle θ is measured from the *normal to the plane of the loop* to the line of action of the magnetic induction vector.

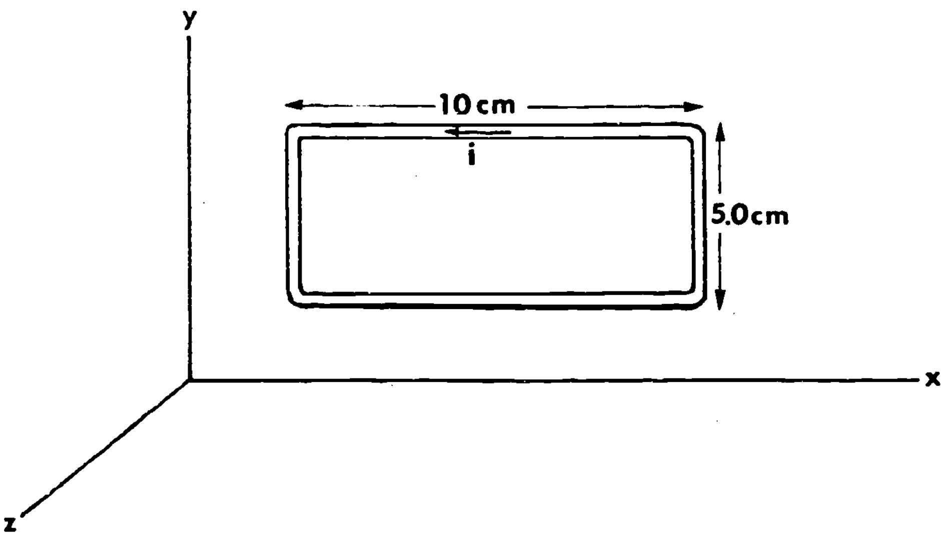
9. A rectangular loop of sides 5 cm and 6 cm carrying a current i = 2 amp, is placed in a uniform magnetic field B = 2 T directed along the z-axis as shown. The normal to the plane of the loop makes a 30° angle with the direction of $\vec{B}$. What is the torque in nt - m on the loop about axis AA'?



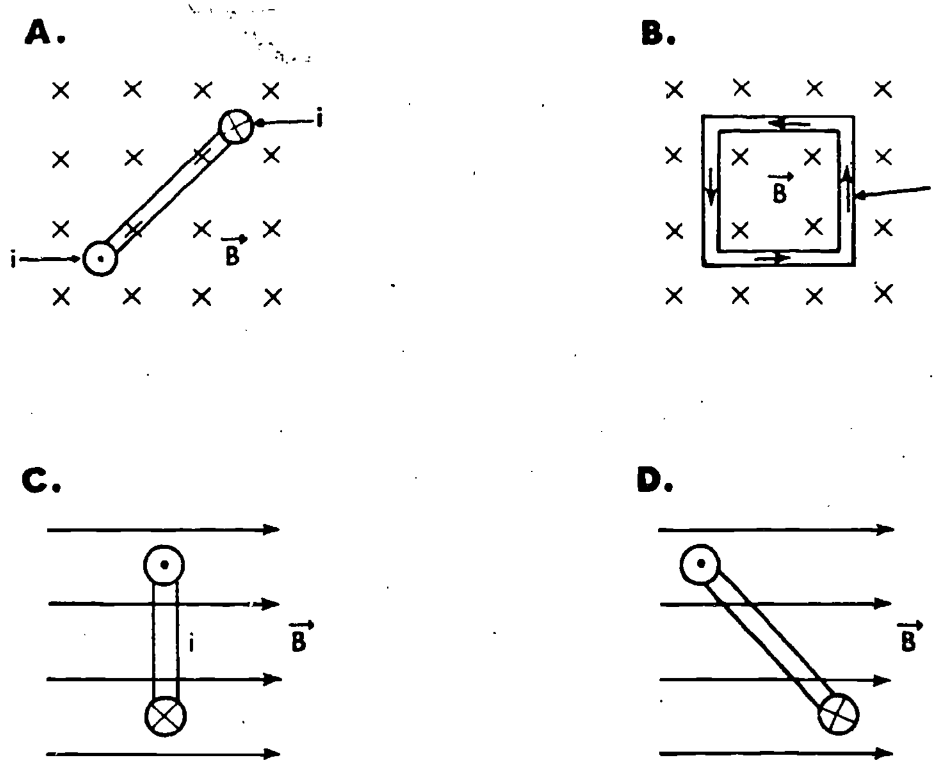
10. A rectangular loop of wire, 10 cm by 5.0 cm carrying a current i = 2.0 amp lies in the x-y plane as shown in the diagram. If a uniform magnetic field $\vec{B} = -0.5\,\hat{j}$ T crosses the plane of the entire loop, find the net force acting on the loop.



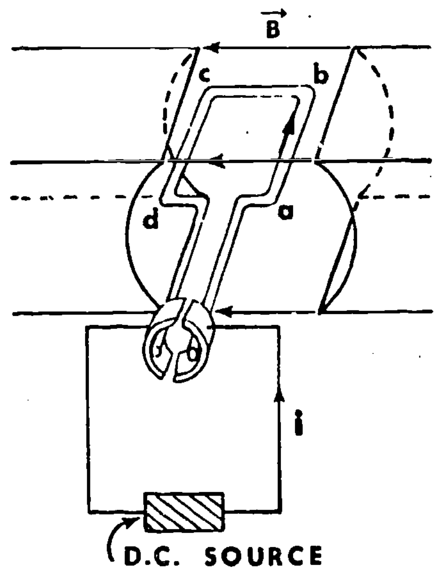
11. A rectangular loop of wire, 10 cm by 5.0 cm carrying a current i = 2.0 amp lies in the x-y plane as shown below. If a uniform magnetic field $\vec{B} = -0.5\ \hat{k}$ T crosses the plane of the loop, find the torque acting on the loop.



12. In which of the cases below is the torque a maximum? (In all cases the loops are identical, the currents are the same, and the magnetic fields have the same magnitude.)



13. In the simple motor shown in the diagram, the coil has n = 50 turns. The rectangular coil is 10 cm by 5.0 cm and carries a current of 0.50 amp. The magnitude of $\vec{B}$ field is 0.50 T. Calculate the instantaneous magnitude of the torque in nt - m acting on the loop.



INFORMATION PANEL

Magnetic Moment of a Current Loop

OBJECTIVE

To define and exemplify the magnetic dipole moment of a current loop; to use the magnetic moment in solving certain types of problems.

When an ordinary magnetic compass is placed in a magnetic field at some angle to the $\vec{B}$ vector, a torque acts upon it and causes it to rotate so

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that its north-seeking (N) and south-seeking (S) poles line up along the $\vec{B}$ lines. (Figure 1). In this action, the axis of rotation of the

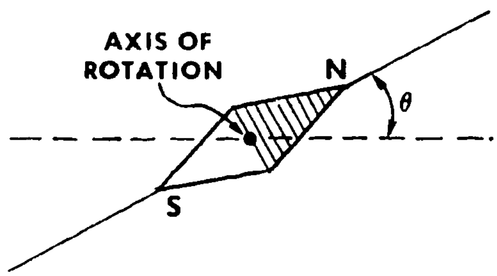


Figure 1 (a)

BEFORE IMMERSION IN $\vec{B}$-FIELD

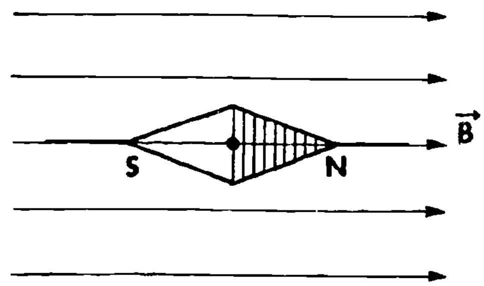


Figure 1 (b)

AFTER IMMERSION IN $\vec{B}$-FIELD

compass is to be visualized as perpendicular to the paper, hence perpendicular to the $\vec{B}$ field, too.

A current loop with its axis of rotation similarly oriented, and the direction of current flow as shown in Figure 2 (a) experiences a torque in exactly the same direction as the magnetic compass in the example above. The loop will then rotate so that the axis perpendicular to its faces lines itself up with the $\vec{B}$ field just as did the compass.

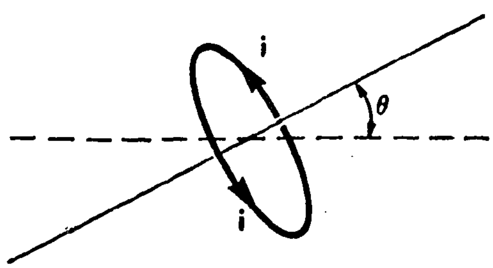


Figure 2 (a)

BEFORE IMMERSION IN $\vec{B}$-FIELD

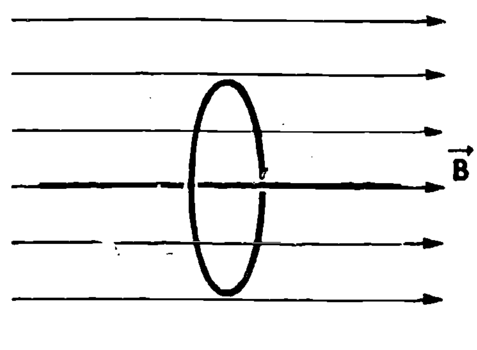


Figure 2 (b)

AFTER IMMERSION IN $\vec{B}$-FIELD

A compass is a *magnetic dipole;* it is quite analogous to the electric dipole previously studied in the subject of electric fields. The behavior of the current loop suggests that this may also be considered as a magnetic dipole. Furthermore, the torque acting on it can be analyzed in a manner similar to that used for the electric dipole.

next page

continued

In the previous Information Panel, it was shown that the torque τ acting on a current loop carrying a current i in a magnetic field of $\vec{B}$ magnitude is:

$$\tau = BiA \sin\theta \qquad (1)$$

where A is the area of the loop. If the loop has N turns, then

$$\tau = NBiA \sin\theta \qquad (2)$$

The magnitude of the torque acting on an *electric* dipole in an $\vec{E}$ field is given by:

$$\tau = pE \sin\theta \qquad (3)$$

where p is the electric dipole moment. The similarity between equation (1) and equation (3) is apparent: they both contain field intensity terms ($\vec{E}$ in one case, $\vec{B}$ in the other), and in both cases the intensity of the torque is related to the sine of the angle between the field lines and the dipole axis. This strong analogy enables us to designate iA for a single turn loop, or NiA for a loop of N turns, as the magnetic dipole moment just as p in equation (3) is called the electric dipole moment. If iA (or Nia) is symbolized by μ, then the torque on a current loop may be written as:

$$\tau = \mu B \sin\theta \qquad \text{(magnitude)} \qquad (4)$$

or in vector form as:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad (5)$$

It is clear that the magnetic dipole moment $\vec{\mu}$ is a vector; it is directed along the axis of the loop (perpendicular to the loop faces) and its direction is such that it can always be determined by applying the so-called right-hand rule for loops:

> *With the thumb of the right hand extended, encircle the loop in the direction of the current. The extended thumb will then point in the direction of the magnetic moment vector,* $\vec{\mu}$

The relation:

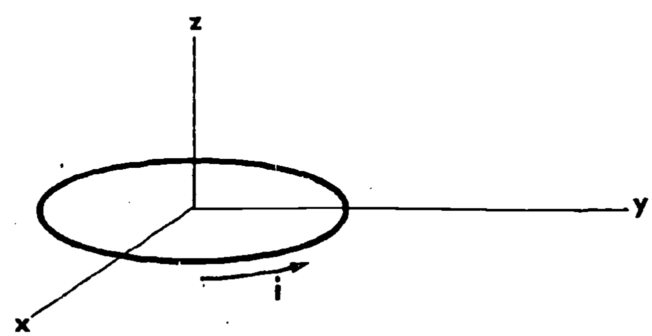
$$\vec{\mu} = NiA$$

will be used in some of the problems that follow. As a further note, it should be remembered that an electron circling a nucleus in an atom is an elementary current loop to which the concept of magnetic dipole moment may be applied.

14. In the Bohr model of the hydrogen atom, an electron revolves around a nucleus in a circular orbit of radius $r = 5.00 \times 10^{-11}$ m. If the electron has a speed $v = 2.25 \times 10^{6}$ m/sec, find the magnitude of the magnetic moment (in amp - m^2) of the electron (orbital). Assume the circulating charge to be equivalent to a tiny current loop of radius r.

15. In the Bohr model of the hydrogen atom, the electron makes 6.0×10^{15} rev/sec around the nucleus. Find the average current in amperes at a point on the orbit of the electron.

16. A circular loop of radius 2.0 cm carries a current of 2.0×10^{-3} amp. What is the magnitude and direction of the magnetic dipole moment of the loop?



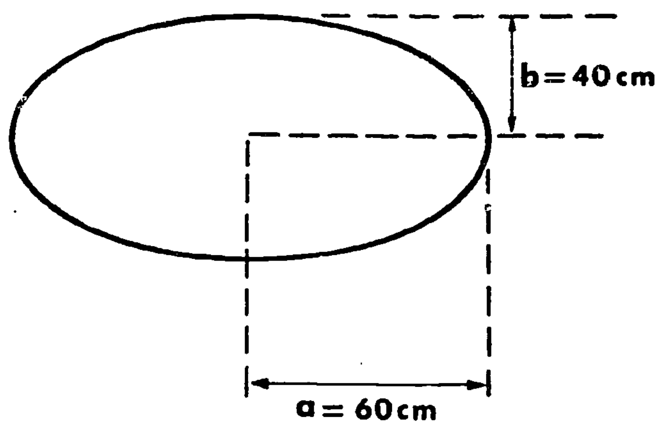
A. 2.5×10^{-6} amp - m^2 in the negative z-direction

B. 2.5×10^{-6} amp - m^2 in the positive z-direction

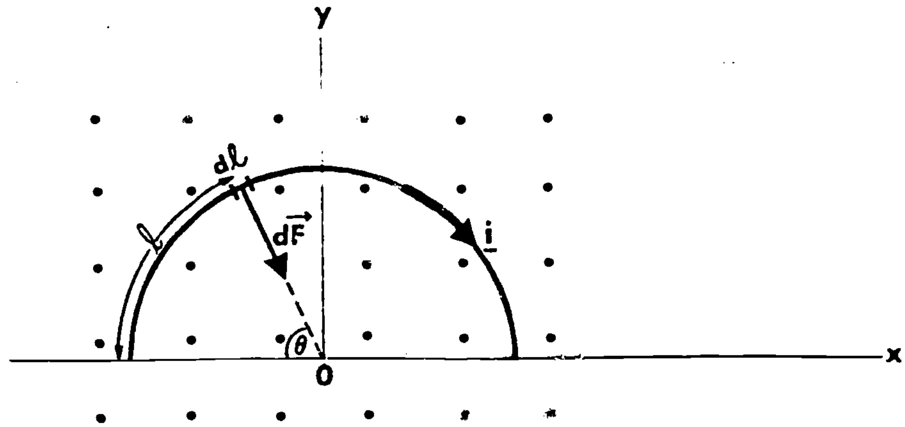
C. 4.0×10^{-6} amp - m^2 in the positive z-direction

D. 4.0×10^{-6} amp - m^2 in the negative z-direction

17. An elliptical current loop has 3 turns, each carrying a current of 2.0×10^{-3} amp. The semi-major and semi-minor axes of the ellipse are $a = 60$ cm and $b = 40$ cm. What is the magnitude of the magnetic dipole moment μ of the loops in amp - m^2? (The area of an ellipse is πab.)



[a] CORRECT ANSWER: $-0.8\ \hat{j}$ nt



The segment of wire of length $d\vec{\ell}$ on the arc has a force $d\vec{F}$ on it. Its magnitude is

$$dF = iBd\ell$$

and its direction is radially toward 0, the center of the arc. Since the horizontal component of this force is being canceled by an oppositely directed component associated with a corresponding arc segment on the other side of 0, only the negative y-component of the force will remain. Thus the total force on the wire is pointing downward and the magnitude is

$$F_y = \int_0^\pi dF_y = \int_0^\pi dF \sin\theta = \int_0^\pi iB \sin\theta\, d\ell$$

Since $\ell = R\theta$, then $d\ell = Rd\theta$ and the integral becomes

$$F_y = \int_0^\pi iB \sin\theta\, d\ell = \int_0^\pi iB \sin\theta\, Rd\theta = 2iBR$$

Substituting numerical values, we obtain

$$F_y = 2iBR = 0.8 \text{ nt}$$

TRUE OR FALSE? Each negative y-component of the magnetic force has a balancing y-component on the other side of the wire arc.

[a] CORRECT ANSWER: A

Looking at the expression giving the magnetic force on a current-carrying wire,

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

we note that this force does not depend on the cross-sectional area of the wire. Increasing the area, therefore, will not alter the force as long as i, $\vec{\ell}$ and $\vec{B}$ remain the same.

What follows is a derivation of the above expression starting from a microscopic point of view. It is given for review purposes and constitutes optional reading.

From the relation $i = jS = nev_dS$, the drift speed must halve when S doubles in order that i remain the same. (The number of conduction electrons per unit volume is fixed for a given conducting material and the electronic charge is a constant.) But the force on the wire is equal to the number of electrons times the force on each electron, or

$$F = (nS\ell) \times (ev_dB) = (nev_dS)\ell B = i\ell B$$

Thus the force remains the same. It is a function of i, ℓ, and B alone, just as the expression

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

says.

[b] CORRECT ANSWER: 9.6×10^{-4}

The average current i is the net charge, Q, passing a point on the orbit in time T. Therefore,

$$i = \frac{Q}{T} \qquad (1)$$

Let us find the current averaged over a period of *one* sec. Hence, Q is the net charge passing the point in *one* sec. For every revolution, the amount of charge passing through the point is q_e, the charge of electron. Therefore,

$$Q = q_e f \qquad (2)$$

where f is the frequency. Thus

$$i = q_e f$$
$$= 1.6 \times 10^{-19} \times 6.0 \times 10^{15}$$
$$= 9.6 \times 10^{-4} \text{ amp}$$

[a] CORRECT ANSWER: A

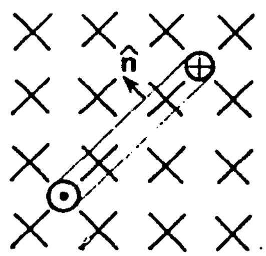
The expression giving the torque on a loop is, as we saw in the preceding questions as

$$\tau = i|\vec{A} \times \vec{B}| = iAB \sin\theta$$

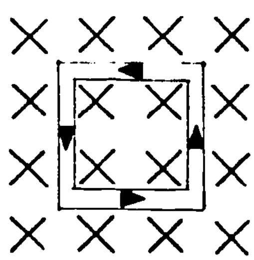
where i is the current, A is the area of the loop, B is the magnetic field, and θ is the angle between the normal to the plane of the loop and the magnetic field induction vector. The normal to the loop is taken along the direction that the right-hand thumb points when the fingers curl around the loop in the direction of the current.

Let us examine the various cases presented in the diagrams:

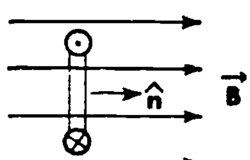
A. We redraw the diagram here. From the indicated direction of the current we find the normal to the loop (along unit vector $\hat{n}$). We note that $\hat{n}$ is parallel to the page, while $\vec{B}$ points into the page. Thus $\theta = 90°$ and $\sin\theta$ has its maximum value. Hence the torque has its maximum value in this case.



B. From the direction of the current we note that the ("positive") normal to the loop is pointing out of the page, while the field $\vec{B}$ is pointing into the page. Thus, $\theta = 180°$ and $\sin\theta = 0$, which makes $\tau = 0$.



C. In this case the normal, $\hat{n}$, is pointing parallel to the field $\vec{B}$. Thus, $\theta = 0°$ and $\sin\theta = 0$, which again makes $\tau = 0$.



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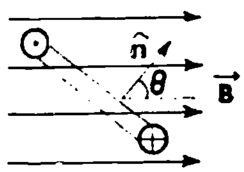
continued

D. The direction of the normal is shown below. It makes an angle θ with B. Thus, the torque is not zero. Nevertheless, τ does not have its maximum value which occures when $\theta = 90°$.



[a] CORRECT ANSWER: C

The force on the small element is given by

$$d\vec{F} = i\, d\vec{\ell} \times \vec{B} \qquad (1)$$

From the fact that $d\vec{F} = 0$ when $d\vec{\ell} = d\ell\, \hat{j}$, we conclude that the angle between $\hat{j}$ and $\vec{B}$ is either 0° or 180°; i.e.,

$$\vec{B} = \pm B\, \hat{j} \qquad (2)$$

Using this field for the differential length $d\vec{\ell} = d\ell\, \hat{k}$, we obtain

$$d\vec{F} = i\, d\vec{\ell} \times \vec{B} = i\, d\ell\, \hat{k} \times (\pm B\hat{j}) = \mp\, i\, d\ell\, B\, \hat{i} \qquad (3)$$

We were given, however, that this force is along the positive x-direction $(+\hat{i})$. Therefore, the lower (+) is the appropriate sign in (3), and from (2) we see that $\vec{B} = -B\hat{j}$. Now we use the fact that in this situation $dF = 4.5 \times 10^{-7}$ nt to solve for B. Thus,

$$B = \frac{dF}{i\, d\ell} = \frac{4.5 \times 10^{-7}}{0.25 \times 10^{-2}} = 1.8 \times 10^{-4}\ \text{T}$$

[b] CORRECT ANSWER: D

The magnetic induction $\vec{B}$ is coming out of the paper and the vector $\vec{\ell}$ is in the direction of the current; i.e., to the right. Turning $\vec{\ell}$ into $\vec{B}$, we see that a right-handed screw would advance downward. This is the direction of the force

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

[a] CORRECT ANSWER: 4.5×10^{-3}

The magnitude of the magnetic dipole moment of a current loop is

$$\mu = N i A$$

where A is the area of the loop, i is the current and N is the number of loops. This expression holds for any plane loop, irrespective of its shape. The area of an ellipse is given by

$$A = \pi ab$$

so

$$\mu = N i A$$

$$= 4.5 \times 10^{-3} \text{ amp - m}^2$$

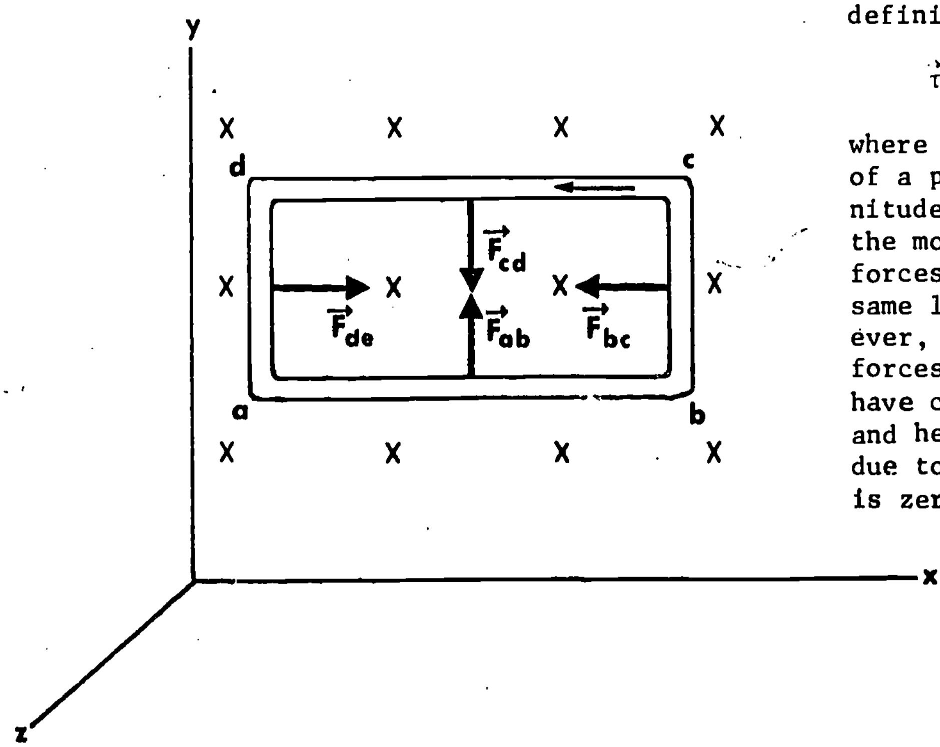
TRUE OR FALSE? If this problem had dealt with a 30-loop arrangement, the v^lue of μ would have been 0.045 amp - m^2.

[b] CORRECT ANSWER: 0

The forces as calculated in the preceding problem are shown in the diagram. The torque $\vec{\tau}$ by definition is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where F is the magnitude of a pair of "common magnitude" forces and r is the moment arm when the forces do not have the same line of action. However, both the pairs of forces in this problem have common lines of action and hence, the net torque due to each pair of forces is zero.



[a] CORRECT ANSWER: 9.00×10^{-24}

The magnitude of the magnetic moment is

$$\mu = iA \qquad (1)$$

where A is the area of the equivalent current loop, i.e.,

$$A = \pi r^2 \qquad (2)$$

and i, the current of the equivalent loop, is the rate at which the charge passes any given point. Therefore,

$$i = q_e f \qquad (3)$$

where f is the frequency of the revolving electron. The frequency f in terms of radius r and speed v may be obtained from

$$2\pi f = w = \frac{v}{r}$$

or

$$f = \frac{v}{2\pi r} \qquad (4)$$

Substituting the expressions for A, i, and f from equations (2), (3), and (4) respectively into equation (1), we find

$$\mu = \pi r^2 \frac{q_e v}{2\pi r}$$

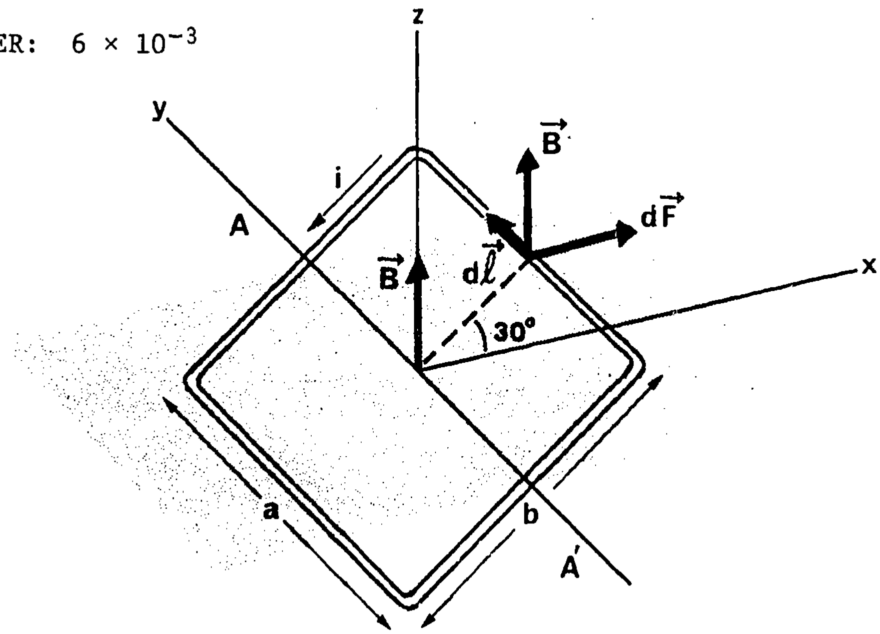
$$= \frac{r\, q_e v}{2}$$

$$= \frac{(5 \times 10^{-11}\ \text{m})\ (1.6 \times 10^{-19}\ \text{coul})\ (2.25 \times 10^{6}\ \text{m/sec})}{2}$$

$$= 9.00 \times 10^{-24}\ \text{amp} - \text{m}^2$$

TRUE OR FALSE? The quantity represented by the symbol f is dimensionless.

[a] CORRECT ANSWER: 6×10^{-3}



The force $d\vec{F}$ on the element $d\ell$ equals $i\ d\vec{\ell} \times \vec{B}$, and its direction is parallel to the x-axis toward the right. The magnitude of total force on side of length a is

$$F = i\ aB$$

A force of the same magnitude but in the opposite direction acts on the opposite side of the loop. The forces on the sides b are of magnitude i bB and are oppositely directed, but along the same line of action; that is, along the y-axis.

The total force on the loop is clearly zero, but the forces on the sides of length a do not have the same line of action and have torque τ about axis AA':

$$\tau = |\vec{r} \times \vec{F}| = \frac{1}{2} b \sin 30° F + \frac{1}{2} b \sin 30° F$$

$$= b \sin 30° F = i\ abB \sin 30° = 6 \times 10^{-3} \text{ nt - m}$$

You should note that this torque may be written as

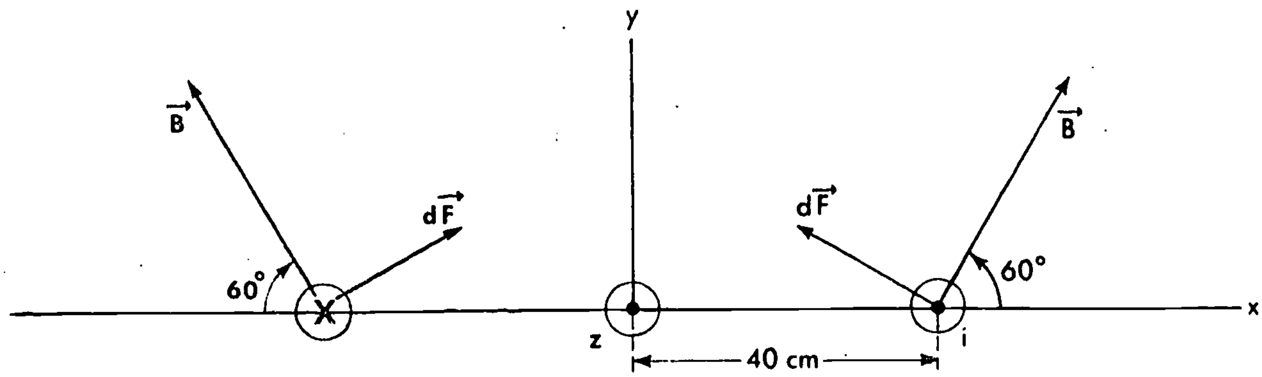
$$\vec{\tau} = i(\vec{A} \times \vec{B})$$

where $\vec{A}$ represents the area and the direction is normal to the plane of the area.

TRUE OR FALSE? The loop is in translational but not rotational equilibrium.

[a] CORRECT ANSWER: C

Let us draw the cross-section of the loop as viewed down the z-axis.



It is obvious from the symmetry of the problem that all points along the loop are equivalent. Let us analyze the point where the loop crosses the positive x-axis. The current, and therefore $d\vec{\ell}$, comes out of the page, $d\vec{\ell} = \hat{k}\, d\ell$. The magnetic induction vector lies in the xy-plane and can be written as

$$\vec{B} = \hat{i}\ B \cos 60° + \hat{j}\ B \sin 60°$$

Therefore, the force exerted on this element of the loop is

$$d\vec{F} = i\ d\vec{\ell} \times \vec{B} = i\ d\ell\ B\ \hat{k} \times (\hat{i} \cos 60° + \hat{j} \sin 60°)$$

$$= i\ d\ell\ B\ (\hat{j} \cos 60° - \hat{i} \sin 60°)$$

This force has a component upward (along $\hat{j}$) and one radially inward toward the center of the circle (at this point, along $-\hat{i}$). It is clear, from symmetry considerations, that this latter component will sum to zero as we integrate around the loop, so we conclude that $\vec{F}$ is along $\hat{j}$ and has the magnitude

$$F = \int d\ell\ i\ B \cos 60° = i\ \ell\ B \cos 60° = 2\pi r\ i\ B \cos 60°$$

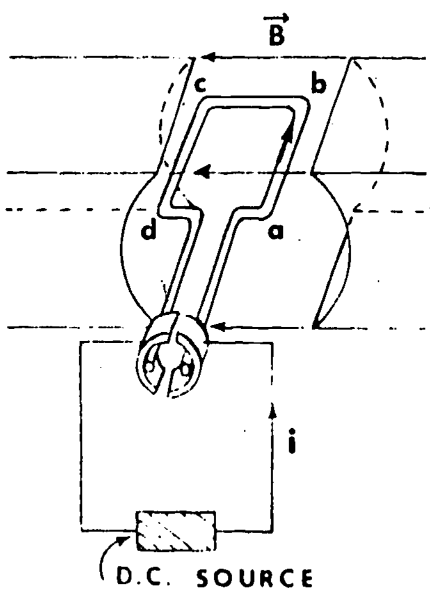
$$= 2\pi \times (0.4\ \text{m}) \times (10^{-3}\ \text{amp}) \times (0.1\ \text{T}) \times \frac{1}{2}$$

$$= 4\pi \times 10^{-5}\ \text{nt}$$

TRUE OR FALSE? The radial component of the magnetic force in this example has a net value equal to the component along $\hat{j}$.

[a] CORRECT ANSWER: 6.25×10^{-2}

Let the length and width of the loop be ℓ and w, respectively. The force acting on a current-carrying conductor $d\vec{\ell}$ in a uniform magnetic field $\vec{B}$ is



$$d\vec{F} = i(d\vec{\ell} \times \vec{B})$$

Therefore, the forces acting on sides bc and da are both zero. The forces acting on sides ab and cd are

$$F_{ab} = ni\ell B$$

and is directed out of the plane of the paper and

$$F_{cd} = ni\ell B$$

and is directed into the plane of the paper.

The total force on the loop is clearly zero, but the forces $\vec{F}_{ab}$ and $\vec{F}_{cd}$ do not have a common line of action and constitute a torque $\vec{\tau}$ which is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where F is the magnitude of the force $F_{ab} = F_{cd}$ and r is the moment arm. Therefore, the magnitude of the torque acting on the loop is

$$\tau = wni\ell B$$

$$= 6.25 \times 10^{-2} \text{ nt - m}$$

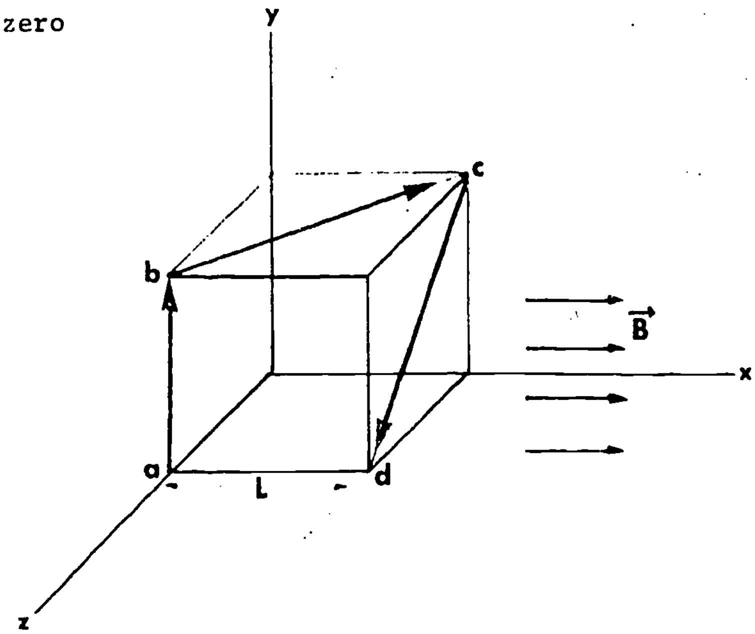
Note that the coil will rotate in counter-clockwise direction; note also that the torque may be written as

$$\vec{\tau} = ni(\vec{A} \times \vec{B})$$

where $\vec{A}$ is the vector area of the loop.

TRUE OR FALSE? Only two of the four sides of the loop contribute to the torque.

[a] CORRECT ANSWER: zero



The force on a straight conductor $\vec{\ell}$ carrying a current i in the presence of a uniform magnetic field $\vec{B}$ is

$$\vec{F} = i[\vec{\ell} \times \vec{B}]$$

In computing the force on each segment of the conductor it is convenient to resolve the vector $\vec{\ell}$ into its components. Thus the force $\vec{F}_{ab}$ acting on segment ab is

$$\vec{F}_{\ell} = i[L\,\hat{j} \times B\,\hat{i}]$$

$$= -iLB\,\hat{k}$$

(NOTE: $\hat{j} \times \hat{i} = -\hat{k}$)

$$\vec{F}_{bc} = i[(L\,\hat{i} - L\,\hat{k}) \times B\,\hat{i}]$$

$$= -iLB\,\hat{j}$$

and

$$\vec{F}_{cd} = i[(L\,\hat{k} - L\,\hat{j}) \times B\,\hat{i}]$$

$$= iLB\,\hat{j} + iLB\,\hat{k}$$

Hence

$$\vec{F}_{ab} + \vec{F}_{bc} + \vec{F}_{cd} = -iLB\,\hat{k} - iLB\,\hat{j} + iLB\,\hat{j} + iLB\,\hat{k}$$

$$= 0$$

TRUE OR FALSE? This solution specifically indicates that $\hat{j} \times \hat{i} = \hat{k}$.

[a] CORRECT ANSWER: 3.92 ĵ

The magnetic force on a current-carrying wire is given by

$$\vec{F} = i\vec{\ell} \times \vec{B} \qquad (1)$$

Since $\vec{B}$ is horizontal and the current flows along the positive x-direction, $\vec{\ell} = +\ell\,\hat{i}$, the direction of $\vec{B}$ that will yield maximum force (so that a minimum B is required) must be perpendicular to $\vec{\ell}$ (recall that $|\vec{\ell} \times \vec{B}| = \ell B \sin\theta$). Thus, the two possible directions of $\vec{B}$ are $\hat{j}$ and $-\hat{j}$. If $\vec{B} = B\,\hat{j}$, then

$$\vec{F}_{+} = i\ell B[(+\hat{i}) \times \hat{j}] = i\ell B\,\hat{k} \qquad (2)$$

On the other hand if $\vec{B} = -B\hat{j}$, then

$$\vec{F}_{-} = i\ell B[(+\hat{i}) \times (-\hat{j})] = -i\ell B\,\hat{k} \qquad (3)$$

There is an additional force acting on the wire; namely, the gravitational force (the weight) given by

$$\vec{w} = -mg\,\hat{k} \qquad (4)$$

For the wire to remain stationary

$$\vec{F} + \vec{w} = 0$$

or

$$\vec{F} = -\vec{w} \qquad (5)$$

Thus

$$\vec{F} = \vec{F}_{+} = i\ell B\,\hat{k}$$

The magnitude of B may be obtained from (5)

$$i\ell B = mg$$

Thus

$$B = \frac{mg}{i\ell} = \frac{.02 \times 9.8}{0.1 \times .50} = 3.92\,\hat{j}\ \text{T}$$

TRUE OR FALSE? The magnetic force on the wire must be exactly balanced by the gravitational force.

[a] CORRECT ANSWER: 32 k nt

The force on an infinitesimal segment of the wire is

$$d\vec{F} = i\, d\vec{\ell} \times \vec{B} = (i\, dx\, \hat{i}) \times (B_x\, \hat{i} + B_y\, \hat{j} + B_z\, \hat{k})$$

$$= i\, dx(0 + B_y\, \hat{k} - B_z\, \hat{j})$$

But $B_z = 0$, so the force is in the z-direction ($d\vec{F} = dF_z\, \hat{k}$), and $dF_z = i\, B_y\, dx$

We can now integrate this expression from x = -2 m to x = 2 m to obtain

$$F_z = \int_{-2}^{2} i\, B_y\, dx = i \int_{-2}^{2} (20 - 3x^2)\, dx$$

$$= i \left[20x - \frac{3x^3}{3}\right]\Bigg|_{-2}^{2} = i\, [40 - 8 - (-40 + 8)]$$

$$= 64\, i$$

Using i = 0.5 amp, we obtain

$$F_z = 32 \text{ nt}$$

or

$$\vec{F} = 32\, \hat{k} \text{ nt}$$

CORRECT ANSWER: B

The magnitude of a magnetic dipole moment of a current loop is given by the expression

$$\mu = Ni\, A$$

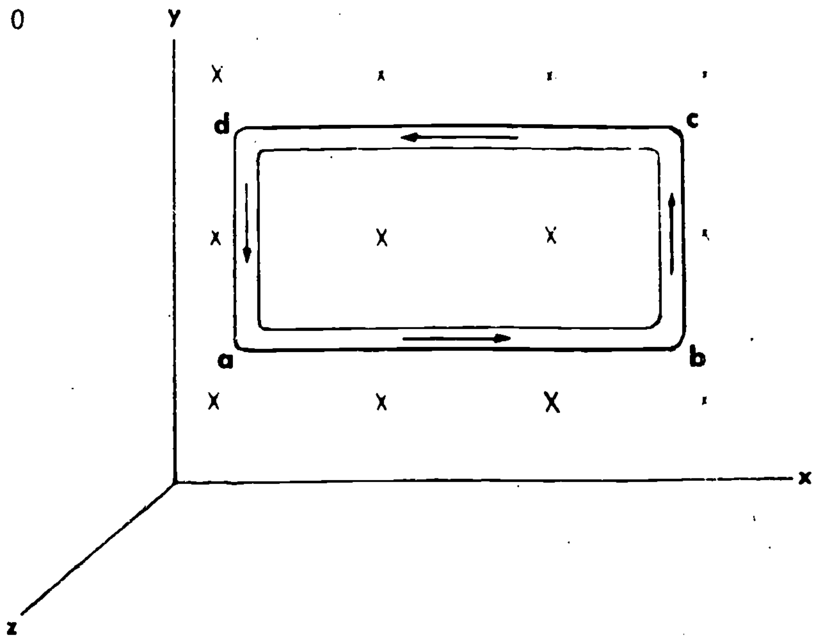
where N is the number of turns of the loop and A is the area of the loop. The direction is given by the rule that if you let your right-hand fingers curl around the loop in the direction of the current, the direction of the extended right thumb will be that of $\vec{\mu}$.

Thus is our case, the magnitude of μ is

$$\mu = i\, \pi r^2 = 2.5 \times 10^{-6} \text{ amp - m}^2$$

directed along the positive z-axis

[a] CORRECT ANSWER: 0



Let the length and width of the loop be ℓ and w, respectively.

The force acting on a current-carrying conductor $d\vec{\ell}$ in a uniform magnetic field $\vec{B}$ is

$$d\vec{F} = i\,(d\vec{\ell} \times \vec{B})$$

Hence, the forces acting on sides ab, bc, cd, and da of the loop are, respectively,

$$\vec{F}_{ab} = i\,\ell B\,\hat{j}$$

$$\vec{F}_{bc} = -i\,wB\,\hat{i}$$

$$\vec{F}_{cd} = -i\,\ell B\,\hat{j}$$

$$\vec{F}_{da} = i\,wB\,\hat{i}$$

Therefore, the net force $\vec{F}$ is

$$\vec{F} = \vec{F}_{ab} + \vec{F}_{bc} + \vec{F}_{cd} + \vec{F}_{da} = 0$$

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL

Average Value of Torque on a Current-Carrying Loop

OBJECTIVE

To discuss and evaluate the average torque that acts on a current-carrying loop in a magnetic field.

It has been shown that the magnitude of the torque τ_1 on a single turn (loop) of wire in a magnetic field is given by the expression:

$$\tau_1 = BiA \sin\theta \quad (1)$$

in which A is the area of the loop and θ is the instantaneous angle between the normal to the loop area and the magnetic induction.

In building practic ` dc motors, it is advantageous to arrange conditions so that the torque remains reasonably constant throughout each rotation of the armature. In practice, this condition is closely approximated by using a large number of loops wound around a soft-iron core, each loop oriented at a slightly different angle to the magnetic field at any given instant. Current is led into and out of these loops through graphite brushes that make contact with a cylinder on the shaft called the *commutator*. The commutator is an automatic switching device which maintains the currents in the loops in the proper direction to produce the desired torque at all times as the system rotates.

The average value of the torque for N loops in such an arrangement is equal to the average value of the torque for one loop ($\overline{\tau}_1$) multiplied by N. It can be shown that the average value for *one loop* is given by:

$$\overline{\tau}_1 = \frac{\int_0^{\pi} \tau_1 \, d\theta}{\int_0^{\pi} d\theta} \quad (2)$$

Combining equations (1) and (2), we can then write:

$$\overline{\tau}_1 = \frac{BiA}{\pi} \int_0^{\pi} \sin\theta \, d\theta \quad (3)$$

and by integrating this expression, obtain an equation for average torque into which numerical values may be directly substituted for evaluation of a specific example. As stated, the average torque for N loops may then be obtained merely by multiplying equation (3) by N.

next page

continued

The core question in this portion of the segment is based upon the relationships given above. The enabling questions review the application of magnetic dipole moment to the solution of problems requiring the calculation of the torque on a current loop in a magnetic field.

PROBLEMS

1. In order to develop a fairly constant torque in a dc motor, it is customary to wrap a large number, N, of rectangular current loops around a cylinder (the armature), which necessitates a correspondingly more complicated commutator. In the limit of very large N, the torque is constant and equal to its average value. Derive an expression for this average value of τ for N loops. The loop area is A.

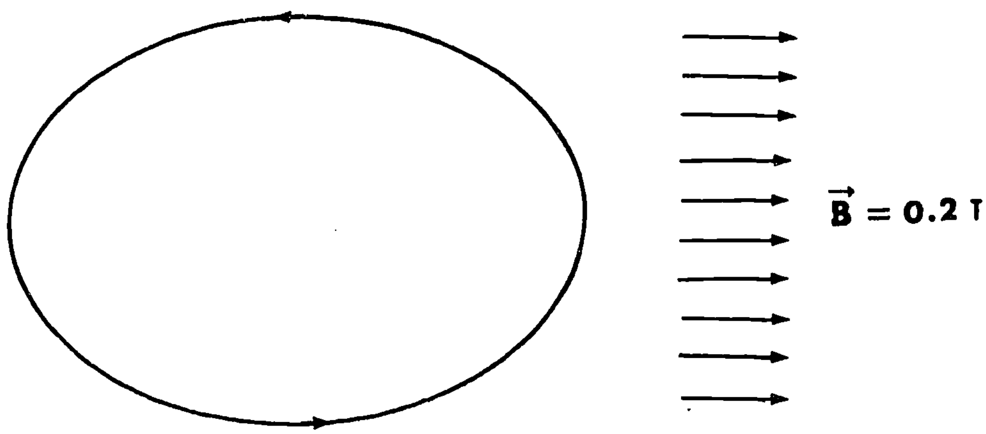
A. $\tau = NiAB$

B. $\tau = 2NiAB$

C. $\tau = NiAB/\pi$

D. $\tau = 2NiAB/\pi$

2. A current loop has a magnetic moment $\vec{\mu}$ of magnitude 4.5×10^{-3} amp-m^2. When it is placed in the plane of the paper in a uniform magnetic field as shown, the torque on the coil then is



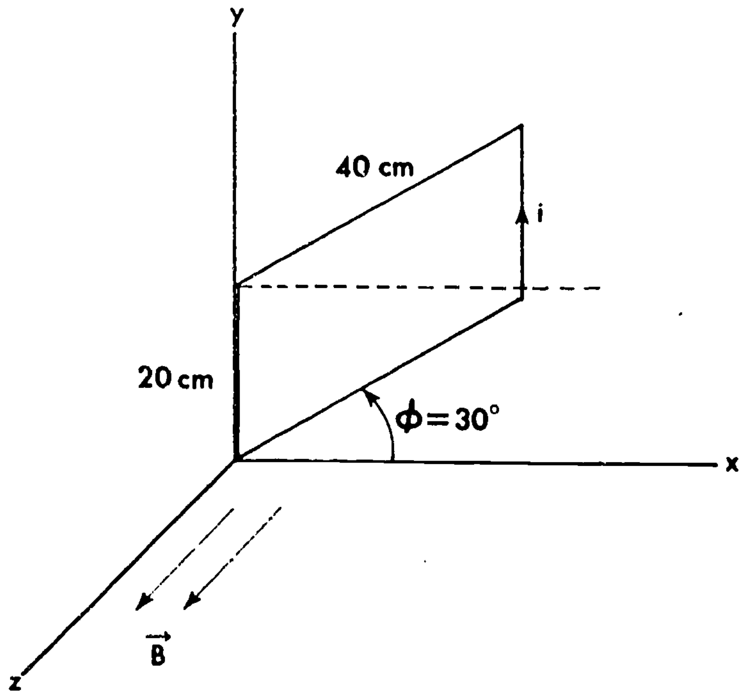
A. zero

B. 9×10^{-4} nt-m upward

C. 9×10^{-4} nt-m downward

D. 9×10^{-4} nt-m out of the page

3. A rectangular coil has 50 turns and carries a current of 10 amp. It is hinged so that it is free to rotate about the y-axis (see diagram).



There is a uniform magnetic field in the region given by $\vec{B} = 3.8 \times 10^{-3}\ \hat{k}$ T. What is the torque on the coil at the instant the angle between the plane of the coil and the xy-plane is $\phi = 30°$ (toward the negative z-axis)?

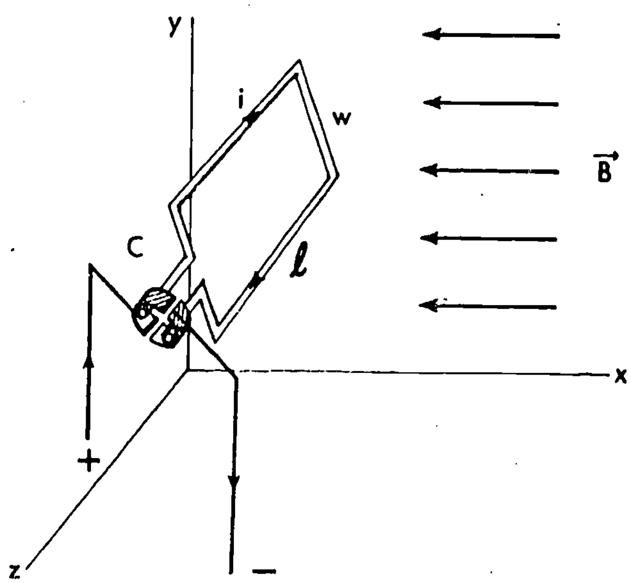
A. $-76 \times 10^{-3}\ \hat{j}$ nt-m

B. 76×10^{-3} nt-m in the xz-plate at 30° to z-axis and 60° to x-axis

C. 132×10^{-3} nt-m in the xz-plate at 30° to z-axis and 60° to x-axis

D. $-132 \times 10^{-3}\ \hat{j}$ nt-m

4. The diagram shows a prototype of the dc electric motor. The current loop is free to rotate about the z-axis; the magnetic field is uniform and along $-\hat{i}$. The commutator, C, is a device for switching the direction of the current in the loop so that the current in a given side of the loop is always toward the commutator when it is on the right and away from the commutator when it is on the left.



Which of the following statements about the torque exerted by the magnetic field on the loop is correct?

A. The torque always has magnitude $i\ell wB$ and is directed along $-\hat{k}$.

B. The torque always has magnitude $i\ell wB$, but half of the time the torque is directed along $\hat{k}$, and half of the time along $-\hat{k}$.

C. The magnitude of the torque varies between zero and $i\ell wB$, but the torque is always directed along $-\hat{k}$.

D. The magnitude of the torque varies between zero and $i\ell wB$ and half the time the torque is directed along $\hat{k}$ and half the time along $-\hat{k}$.

INFORMATION PANEL

Work and Energy Considerations for Current-Carrying Loops in Magnetic Fields

OBJECTIVE

To determine the work done in rotating a loop through a given angle in a magnetic field.

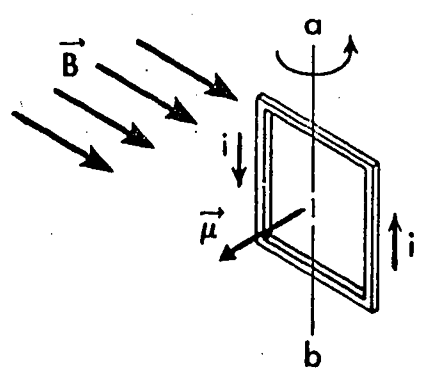
Reviewing briefly: the torque on a current loop immersed in a magnetic field may be expressed in vector form as:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{1}$$

in which μ is the magnetic dipole moment and $\vec{B}$ is, as usual, the magnetic induction. The magnetic dipole moment lies along the axis of the loop and its direction is given by the right-hand rule:

When the fingers of the right hand curl around the loop in the direction of the current, the extended thumb then points in the direction of the magnetic dipole moment.

Thus, in the accompanying diagram, the magnetic dipole moment points at right angles to the rectangular current loop. The current loop shown, if free to rotate about axis ab, would do so as indicated in the diagram, i.e., in a counterclockwise direction as viewed from above. Since torque acts on a current loop, it follows that work must be done by an external agent if this agent is to change the orientation of the loop in the field. Whether this work is to be considered positive or negative depends upon whether the potential energy of the loop increases or decreases. The zero-potential energy position is arbitrarily taken as the position of the loop when μ and $\vec{B}$ are at right angles to one another, or when $\theta = 90°$.



Thus, the magnetic potential energy possessed by the loop in any position θ can be defined as the work that the external agent must do to rotate the loop from its zero-energy position where $\theta = 90°$ to the new position θ. As shown in your assigned reading, the potential energy U is thus expressed:

$$U = -\vec{\mu} \cdot \vec{B} \tag{2}$$

next page

continued

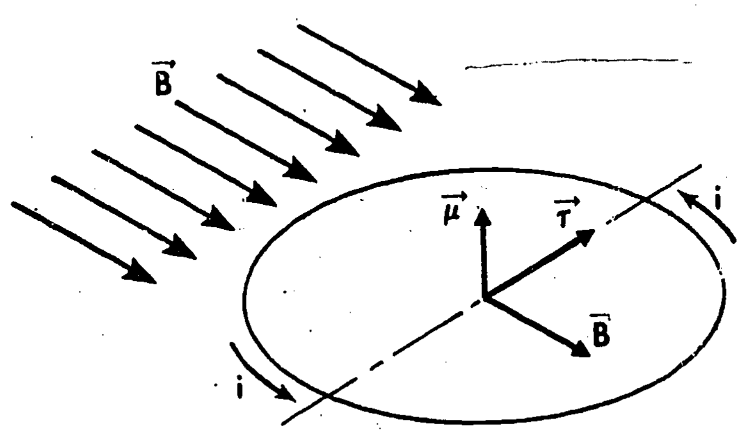
To find the work done by an external agent on the loop (positive or negative), it is therefore necessary to find the *change* in potential energy that occurs as the dipole is rotated through the given angle. It follows that:

$$W = U_f - U_i$$

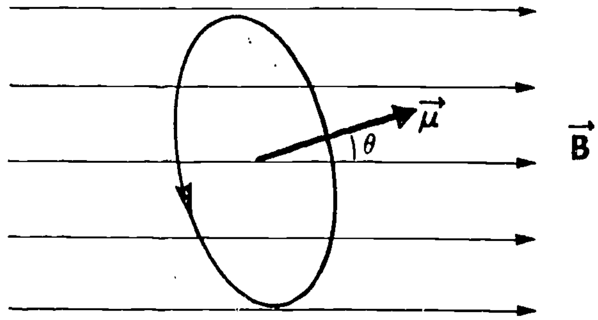
where the subscripts represent the final and initial potential energies, respectively.

The core problem in this section of the segment deals with the situation described above. The enabling questions are concerned primarily with a review of the methods used to calculate the force acting on a current-carrying conductor in a magnetic field.

5. If a current loop of magnetic moment $\mu = 4.5 \times 10^{-3}$ amp-m^2 is free to rotate about its minor axis in the $\vec{B}$ field of 0.2 T magnitude as shown, it will do so according to the right-hand rule; i.e., if the thumb of your right hand points in the direction of the torque, the loop will accelerate in the sense your fingers curl. How much work in joules is done by the magnetic field in turning the loop through one quarter of a revolution from the rest position shown?



6. A single loop of radius R, carrying a current i is placed in a uniform magnetic field $\vec{B}$ as shown in the diagram. What is the expression for the potential energy of the loop in the magnetic field?



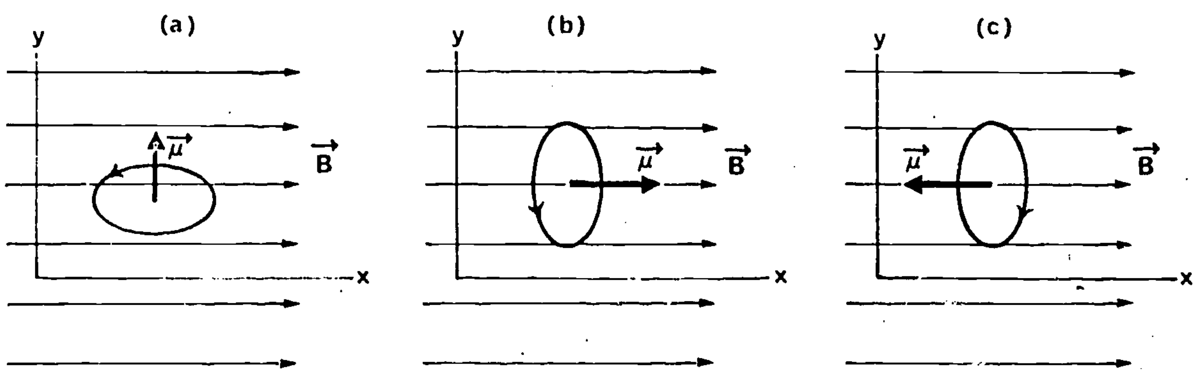
A. $U = -\vec{\mu} \cdot \vec{B}$

B. $U = -\vec{\mu} \times \vec{B}$

C. $U = \vec{\mu} \cdot \vec{B}$

D. $U = \vec{\mu} \times \vec{B}$

7. A small current loop of magnetic moment $\vec{\mu}$ is placed in a uniform magnetic field $\vec{B}$. The potential energy U of the system as shown in diagram (a), (b), and (c) are:



A. $U_a = 0$, $U_b = 0$ and $U_c = 0$ respectively

B. $U_a = \mu B$, $U_b = 0$ and $U_c = 0$ respectively

C. $U_a = 0$, $U_b = \mu B$ and $U_c = \mu B$ respectively

D. $U_a = 0$, $U_b = -\mu B$ and $U_c = \mu B$ respectively

8. A circular coil with radius 10 cm is carrying a current 2.0 amp. The magnetic moment vector $\vec{\mu}$ of the coil makes an angle of $\theta = 60°$ with a uniform magnetic field $\vec{B}$ of magnitude 4 T. How much work is required to turn the coil from its initial position to a position such that the vector $\vec{\mu}$ makes an angle of 120° with the $\vec{B}$ field.

INFORMATION PANEL — Magnetic Flux

OBJECTIVE

To calculate the magnetic flux through given surfaces in a field of known magnetic induction.

The flux associated with a magnetic field is defined as the surface integral of the magnetic induction or

$$\Phi = \oint \vec{B}\cdot d\vec{S} \qquad (1)$$

The integral is taken over the entire surface, closed or open, through which the flux passes. For the special case where the magnetic induction $\vec{B}$ is uniform and normal to a finite surface S, the flux across the area is:

$$\Phi = BS \qquad (2)$$

When B is measured in teslas (T) and the area in square meters, the flux is expressed in *webers* (Wb). The weber may therefore be defined as

> *the magnetic flux through a plane surface of 1 square meter area normal to the magnetic field of intensity 1 T.*

next page

continued

Expressed in scalar form, equation (1) is usually written:

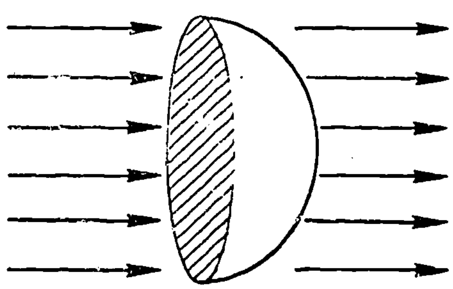
$$\Phi = \int B \cos\theta \, dS \tag{3}$$

in which θ is the angle between the magnetic induction vector $\vec{B}$ and the element of area $d\vec{S}$.

> *Since magnetic lines have no beginning or end but form closed curves in space, the net magnetic flux passing outward through any arbitrary closed surface is zero.*

The problems in this section involve calculations of flux in circumstances for which the information provided above is essential.

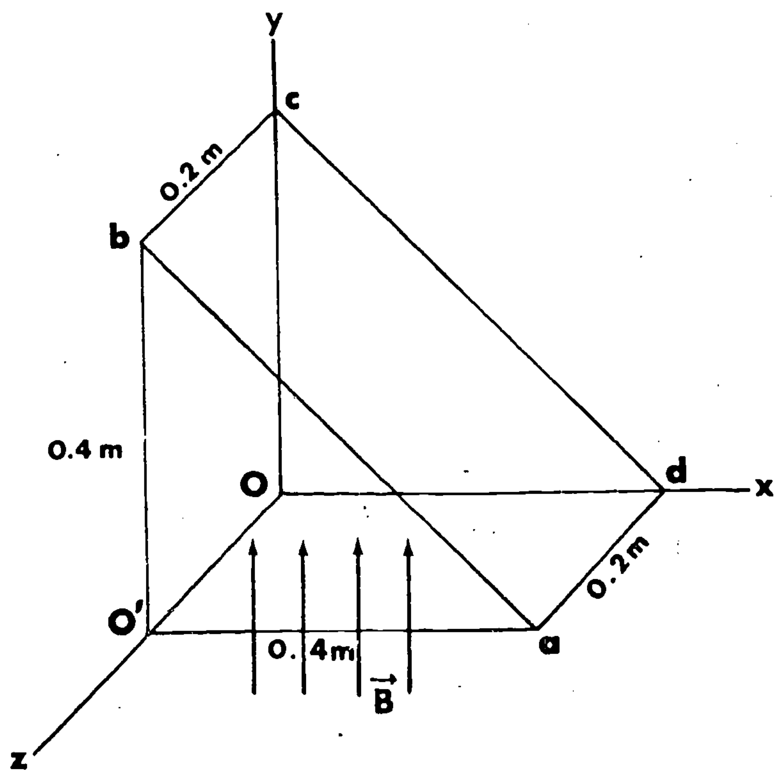
9. A hollow hemispherical bowl of radius 15 cm is placed in a uniform magnetic field of magnitude 2.0 T. The open (flat) end of the bowl is normal to the field. Calculate the magnetic flux through the bowl.



10. A closed spherical surface 5 cm in radius is placed in a uniform magnetic field of magnitude 0.5 T. Calculate the magnetic flux through the surface.

11. A uniform magnetic field of magnitude 5.0 T makes an angle of 20° with a plane surface S of area 3.0 m^2. Calculate the flux Φ_B through S.

12. The magnetic induction (in T) in the region shown is $\vec{B} = 3.0\ \hat{j}$. What is the magnetic flux through the surface abcd?



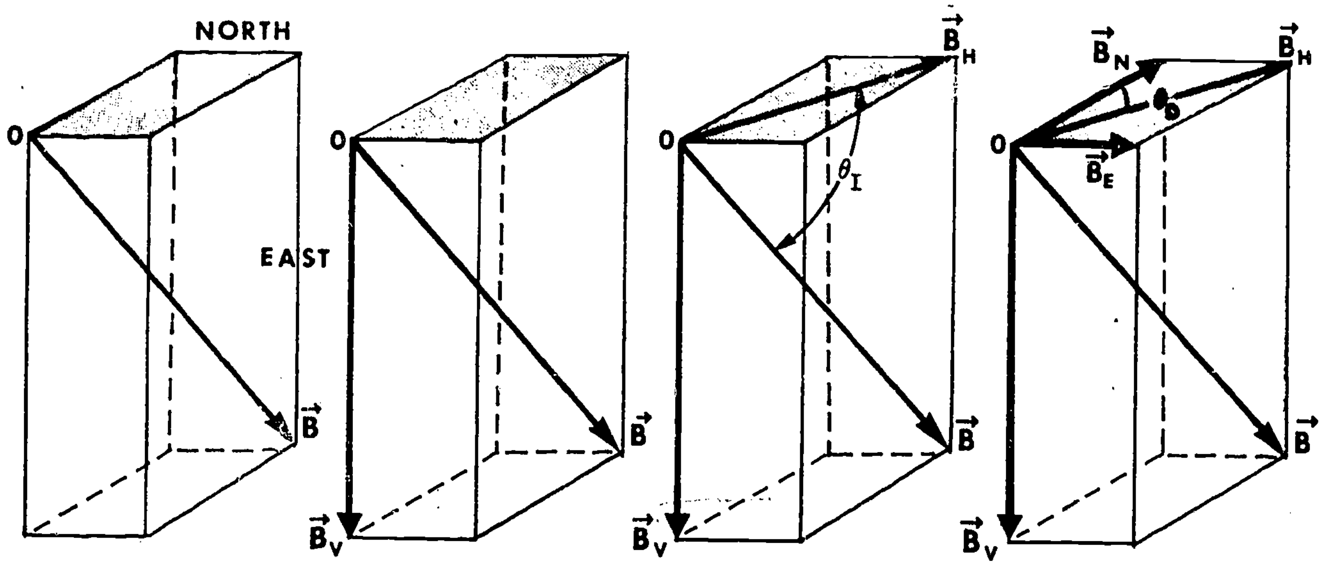
INFORMATION PANEL

The Magnetic Field on the Earth

OBJECTIVE

To answer questions and solve problems involving inclination, declination, and other characteristics of the Earth's magnetic field.

The accompanying diagram, adapted from one of your source texts, should be very helpful to you in visualizing the vectors into which the magnetic



field of the Earth may be resolved. The sequence of drawings will be of most assistance to you if you scan them from left to right as the discussion proceeds.

(A) The shaded area at the top of the block of "Earth" is to be taken as the surface with a point of observation at O. The actual magnetic vector of the $\vec{B}$-field of the Earth has been drawn in pointing downward and slightly to the east. This vector is identified as $\vec{B}$.

(B) The magnetic vector of the Earth's field $\vec{B}$ has been resolved to show its vertical component, labeled $\vec{B}_V$. A dipping needle at the point of observation would point along the vector $\vec{B}$. Theoretically, $\vec{B}$ and $\vec{B}_V$ would coincide only at the north magnetic pole of the Earth.

(C) In this drawing, the vector $\vec{B}$ has been projected on the surface to show its horizontal component $\vec{B}_H$. An ordinary magnetic compass, held horizontally, points along this component. θ_I is the *dip* or *inclination* angle.

next page

continued

(D) Finally, the horizontal component has been resolved into its northerly component $\vec{B}_N$ and its easterly component $\vec{B}_E$. The angle θ_D is the angle of *declination*. The angle of declination measures the deviation of the horizontal component from true north.

The relationships among the various components and the actual field vector may be listed as follows:

$$\vec{B}_H = \vec{B}\cos\theta_I$$

$$\vec{B}_V = -\vec{B}\sin\theta_I$$

$$\vec{B}_N = \vec{B}_H\cos\theta_D$$

$$\vec{B}_E = -\vec{B}_H\sin\theta_D$$

You will find the relationships presented above helpful in solving the forthcoming problems in this section of your work.

13. The magnitude of the Earth's magnetic induction at Cambridge, Massachusetts is B = 58 μT. The inclination and declination are 73° north and 15° west, respectively. What are the eastward (B_E), northward (B_N) and upward (or vertical B_V) components of B there?

A. B_E = 17 μT ; B_N = 17 μT ; B_V = 55 μT

B. B_E = 0 ; B_N = 17 μT ; B_V = 55 μT

C. B_E = -14 μT ; B_N = 54 μT ; B_V = -17 μT

D. B_E = -4.4 μT ; B_N = 16 μT ; B_V = -55 μT

14. The horizontal component of the Earth's magnetic field is generally directed

A. northward in the northern hemisphere and southward in the southern hemisphere

B. southward in the northern hemisphere and northward in the southern hemisphere

C. northward in both hemispheres

D. southward in both hemispheres

15. The vertical component of the Earth's magnetic field is *generally* directed

A. downward in the northern hemisphere and upward in the southern hemisphere

B. upward in the northern hemisphere and downward in the southern hemisphere

C. downward in both hemispheres

D. upward in both hemispheres

16. Select the statement which correctly describes the variation of the magnitudes of the horizontal and vertical components of the Earth's magnetic field $\vec{B}$ as you travel from the magnetic equator to the magnetic poles.

A. B_h decreases from 35 μT to 0; B_v decreases from 70 μT to 0.

B. B_h increases from 0 to 35 μT; B_v increases from 0 to 70 μT.

C. B_h decreases from 35 μT to 0; B_v increases from 0 to 70 μT.

D. B_h increases from 0 to 35 μT; B_v decreases from 70 μT to 0.

17. The declination (or variation) at Cambridge, Massachusetts is 15° west. This means that

A. the north end of a compass needle points 15° west of north

B. the north end of a compass needle points 15° north of west

C. the north end of a dipping needle aligned toward the west dips 15° below the horizontal

D. the north end of a dipping needle aligned 15° west of north will dip below the horizontal

18. The inclination (or angle of dip) at Cambridge, Massachusetts is 73° north. (The declination there is 15° west.) This means that

A. the north end of a dipping needle, aligned 15° south of wes dips 73° below the horizontal

B. the north end of a dipping needle, aligned in the south to north direction, points 73° above downward

C. the north end of a dipping needle, aligned 15° west of north, dips 73° below the horizontal

D. the north end of a dipping needle, aligned 15° west of north, points 73° above downward

19. How should a straight current-carrying wire be oriented so as to achieve the maximum *upward* force on it by the Earth's magnetic field at Cambridge, Mass.? The inclination and declination at Cambridge are 73° north and 15° west, respectively.

A. horizontal with the current in a direction 15° north of east

B. horizontal with the current in a direction 15° west of north

C. 17° above horizontal with the current in a direction 15° north of east

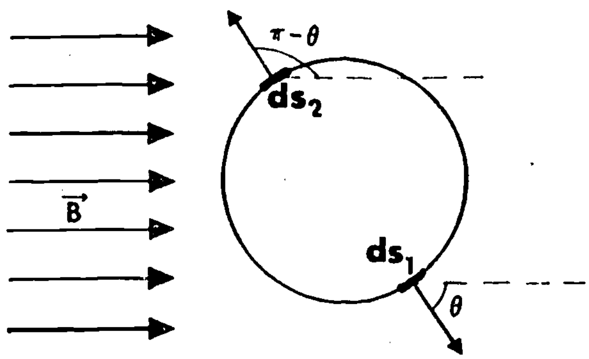
D. 17° above horizontal with the current in a direction 15° west of north

[a] CORRECT ANSWER: zero

The definition of flux is

$$\Phi_B = \int \vec{B} \cdot d\vec{S} = \int B \cos\theta \, dS \qquad (1)$$

where θ is the angle between $\vec{B}$ and the element of area $d\vec{S}$. Looking at the diagram, we note that for every element of area, $d\vec{S}_1$, making an angle θ with the uniform magnetic field $\vec{B}$, there is a diametrically opposite element of area, $d\vec{S}_2$, which makes an angle $\pi-\theta$ with $\vec{B}$. Since



$$\cos\theta = -\cos(\pi-\theta)$$

the contributions of these two elements of area to the integral in (1) cancel each other. Thus when the integration is carried out over the whole surface of the sphere it yields zero. Hence, the flux is zero.

This result in fact is independent of the shape of the surface. The flux through any closed surface placed in a magnetic field is zero. You may, of course, recall the same result for closed surfaces placed in a uniform electric field.

[b] CORRECT ANSWER: A

The lines of magnetic induction have the same direction as the *north* end of a dipping needle. In general, the north end dips in the northern hemisphere and the south end dips in the southern hemisphere. Therefore the vertical component of the Earth's magnetic field is downward (into the ground) in the northern hemisphere and upward (out of the ground) in the southern hemisphere. Due to the fact that the geographic and magnetic equators do not coincide, there are some positions on the Earth's surface where the vertical components have directions opposite to the above, but this is not the general case.

[a] CORRECT ANSWER: 9×10^{-4}

For a current loop in a magnetic field, the potential energy is given by

$$U = -\vec{\mu}\cdot\vec{B}$$

Thus, initially

$$U_i = -\mu B \cos 90° = 0$$

whereas, after one-quarter revolution

$$U_f = -\mu B \cos 0° = -\mu B$$

$$= -4.5 \times 10^{-3} \text{ amp-m}^2 \times 0.2 \text{ T} = -9 \times 10^{-4} \text{ J}$$

The work done by the magnetic field on the loop is

$$W = -(U_f - U_i) = 9 \times 10^{-4} \text{ J}$$

Note the work done by the loop is $U_f - U_i$, which is the negative of that done by the field.

TRUE OR FALSE? The magnitude of the work done by the loop on the field is the same as the magnitude of the magnetic induction.

[b] CORRECT ANSWER: B

By the right-hand rule, the direction of the magnetic dipole moment of the loop is out of the page. Rotating this into B would advance a right-hand screw upwards. Hence, the torque

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

is directed upward. Since $\vec{\mu}$ and $\vec{B}$ are perpendicular, the angle between them, θ, is 90°. Using the given value of μ we obtain

$$\tau = \mu B \sin\theta = (4.5 \times 10^{-3} \text{ amp-m}^2) \times (0.2 \text{ T}) \times (\sin 90°) = 9 \times 10^{-4} \text{ nt-m}$$

[a] CORRECT ANSWER: A

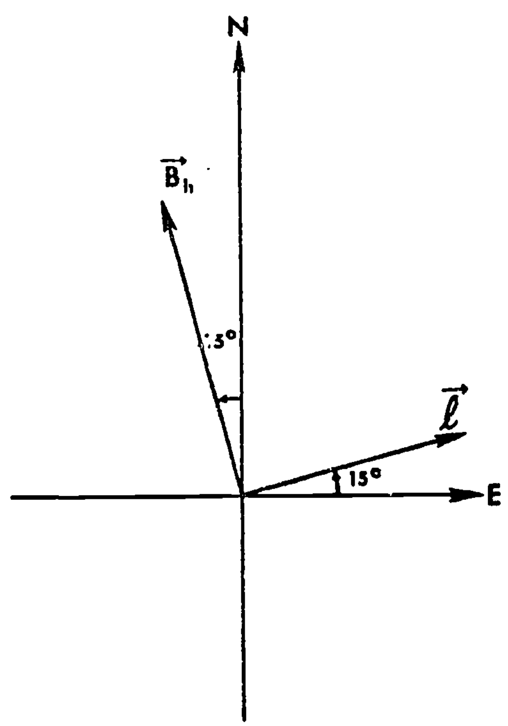
The force on a wire-carrying current

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

is perpendicular to both $\vec{\ell}$ and $\vec{B}$. Therefore, the vertical component of $\vec{B}$ cannot contribute to the vertical component of $\vec{F}$. The horizontal component of $\vec{B}$ can, however, contribute to the horizontal component of $\vec{F}$. (The reason is that by "horizontal" we mean some direction in a plane, whereas by "vertical" we mean a single direction). Whereas

$$\vec{F} = i\vec{\ell} \times \vec{B}_h$$

is a maximum for all orientations of $\vec{\ell}$ perpendicular to $\vec{B}_h$, it is maximum and vertical only when $\vec{\ell}$ is perpendicular to $\vec{B}_h$ and, at the same time horizontal. Now $\vec{B}_h$ is 15° west of north, so $\vec{\ell}$ must be (see diagram) 15° north of each in order for it to be normal to $\vec{B}_h$ and at the same time $\vec{\ell} \times \vec{B}$ be pointing up. Note that although the vertical component of the force is now a maximum, the total force is not vertical. Its horizontal component is given by



$$\vec{F}_h = i\vec{\ell} \times \vec{B}_v$$

which is, in fact, greater than the vertical component,

$$\vec{F}_v = i\vec{\ell} \times \vec{B}_h$$

since $\vec{\ell}$ is perpendicular to both $\vec{B}_v$. and B_h and $B_v > B_h$.

TRUE OR FALSE? Since $\vec{\ell}$ is perpendicular to both $\vec{B}_v$ and $\vec{B}_h$, the force vector on the wire due to the magnetic field of the Earth lies within the B_v - B_h plane.

[a] CORRECT ANSWER: D

The potential energy of a small current loop of magnetic moment $\vec{\mu}$ when placed in a uniform magnetic field $\vec{B}$ is

$$U = -(\vec{\mu} \cdot \vec{B})$$

Therefore.

$$U_a = -(\mu\,\hat{j} \cdot B\,\hat{i})$$
$$= 0$$
$$U_b = -(\mu\,\hat{i} \cdot B\,\hat{i})$$
$$= -B$$
$$U_c = -(-\mu\,\hat{i} \cdot B\,\hat{i})$$
$$= \mu B$$

[b] CORRECT ANSWER: C

The field lines run *generally* from the south to the north magnetic pole. At various positions on the Earth's surface there is an east-west component, and due to the fact that the magnetic and geographic poles do not coincide exactly, the lines are even sometimes directed southward, but this is not the general case.

[c] CORRECT ANSWER: C

From pictorial representations of the Earth's magnetic field shown in your textbooks, it is readily seen that the field is horizontal at the magnetic equator and vertical at the magnetic poles. Closer examination of the figures also reveals that the field lines are denser at the poles than at the equator.

[a] CORRECT ANSWER: D

The average value of τ for N loops is equal to N times the average value of τ for one loop. The magnitude of the torque τ_1 for one loop is

$$\tau_1 = iAB \sin\theta$$

where A is the area of the loop and θ is the instantaneous angle between the normal to the area and the $\vec{B}$ field. An average value for one loop is given by

$$\bar{\tau}_1 = \frac{\int_0^\pi \tau_1 \, d\theta}{\int_0^\pi d\theta}$$

Therefore,

$$\bar{\tau}_1 = iAB \frac{1}{\pi} \int_0^\pi \sin\theta \, d\theta = iAB \frac{1}{\pi} (-\cos\theta) \Big|_0^\pi$$

$$= iAB \frac{1}{\pi} (1 + 1) = \frac{2iAB}{\pi}$$

The average torque for N loops is thus

$$\bar{\tau}_N = N \bar{\tau}_1 = \frac{2NiAB}{\pi}$$

TRUE OR FALSE? In the above solution, θ is the instantaneous angle between the plane of the loop and the magnetic field vector.

[b] CORRECT ANSWER: 5.1 Wb

For a uniform magnetic field over a surface of area S

$$\Phi_B = BS \cos\theta$$

where θ is the angle between $\vec{B}$ and the normal to S. If θ' is the angle which $\vec{B}$ makes with the surface S, then $\theta' + \theta = 90°$, so $\cos\theta = \sin\theta'$. In our case, $\theta' = 20°$, so

$$\Phi_B = 15 \sin 20° \text{ Wb}$$

$$= 5.1 \text{ Wb}$$

[a] CORRECT ANSWER: 0.14 Wb

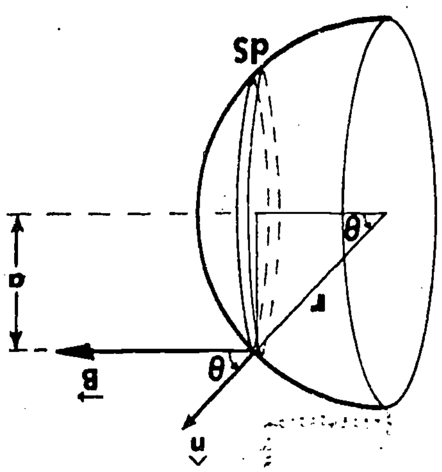
We can use the definition of magnetic flux

$$\Phi_B = \oint \vec{B} \cdot d\vec{S} \qquad (1)$$

to solve the problem. The given surface is open, of course, so that the result cannot be zero. We can simplify the problem considerably by noticing that the projection of the hemispherical surface on a plane normal to B is the "equatorial" plane of the bowl which is a circle of radius r. The area of this circle is πr^2, so the flux is

$$\Phi_B = \pi r^2 B = \pi \times (1.5 \times 10^{-1}\ \mathrm{m})^2 \times (2.0\ \mathrm{T}) = 0.14\ \mathrm{Wb} \qquad (2)$$

What follows is a calculation of the flux by integrating (1). For our element of area we choose a strip of the spherical surface. The mean radius of this "circular" strip is



$$a = r \sin\theta \qquad (3)$$

and its width is

$$dw = r d\theta \qquad (4)$$

Thus, the differential of area becomes

$$dS = 2\pi a dw = 2\pi r \sin\theta\ r d\theta = 2\pi r^2 \sin\theta\ d\theta \qquad (5)$$

The direction of $d\vec{S}$, $\hat{n}$, is not constant but it makes a constant angle with $\vec{B}$ all around the strip, so

$$\vec{B} \cdot d\vec{S} = 2\pi r^2 \sin\theta\ d\theta\ (B \cos\theta) = \pi r^2 B \sin 2\theta\ d\theta \qquad (6)$$

Integrating (6) from $\theta = 0°$ to $\theta = \pi/2$, we obtain

$$\Phi_B = \pi r^2 B \int_0^{\pi/2} \sin 2\theta\ d\theta = \pi r^2 B \left(- \frac{\cos 2\theta}{2} \right) \Bigg|_0^{\pi/2}$$

$$= \pi r^2 B - \frac{1}{2} (-1 - 1) = \pi r^2 B$$

Compare this with the expression evaluated in (2).

TRUE OR FALSE? In the latter part of the solution above, it is shown that the flux through the entire inner surface of the hemispherical bowl is equal in magnitude to the flux through the "equatorial" plane.

[a] CORRECT ANSWER: A

The torque on a current loop is

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

where

$$\vec{\mu} = i\vec{A}$$

The magnetic potential energy at an angle θ is defined as the work that an external agent must do to turn the current loop from its zero-energy position (θ = 90°) to the given position. Thus

$$U = \int_{90°}^{\theta} \tau \, d\theta = \int_{90°}^{\theta} iAB \sin\theta \, d\theta = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$$

[b] CORRECT ANSWER: C

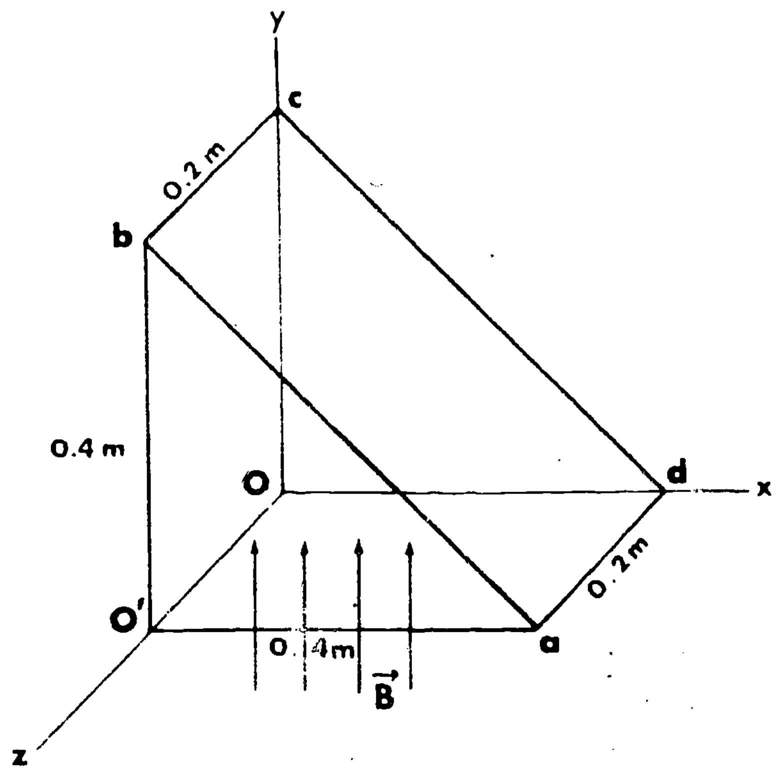
The dipping needle is aligned 15° west of north; i.e., in the direction of the horizontal component of the Earth's field, so that it is free to rotate into the direction of the resultant field, which is necessarily in the plane formed by the vertical and horizontal components of the Earth's field. The resultant here is 15° west of north and 73° below of horizontal.

[c] CORRECT ANSWER: C

The torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$, so its magnitude is $\tau = \mu B \sin\theta$, where θ is the angle between $\vec{\mu}$ and $\vec{B}$. $\vec{\mu}$ is not always normal to $\vec{B}$. Instead, sinθ oscillates between 0 and 1 as the loop rotates. The angle is actually restricted to lie between 0° and 180°, because $\vec{\mu}$ changes direction when the direction of the current switches; so the angle which would have been 181° becomes 1°. The purpose of the commutator is to arrange that the torque is always in the same direction; in this case, along $-\hat{k}$.

TRUE OR FALSE? One function of the commutator is to cause the current in the loop to change direction when θ passes through 180°.

[a] CORRECT ANSWER: 0.24 Wb



The equation for the magnetic flux, Φ_B, passing through a surface S is

$$\Phi_B = \int \vec{B} \cdot d\vec{S} \quad (1)$$

the integral being over the surface S.

For a uniform magnetic field, let θ be the angle between $\vec{B}$ and the normal to S. Then

$$\Phi_B = \int B \cos\theta \, dS = BS \cos\theta \quad (2)$$

Now we use the fact that the triangle abO' (or dcO) is a right isosceles triangle so that the length ab is $0.4\sqrt{2}$ m. The area abcd is thus $0.08\sqrt{2}$ m^2.

$$\Phi_B = (3 \text{ T}) \times (0.08\sqrt{2} \text{ m}^2) \cos 45°$$

$$= 0.24 \text{ Wb}$$

Note that S cosθ is the projection of area on the xz-plane which is OO'ad in the diagram. Therefore

$$\Phi_B = 3 \times 0.2 \times 0.4 = 0.24 \text{ Wb}$$

TRUE OR FALSE? The flux through the base of the right triangular prism is twice the flux through its largest face.

[a] CORRECT ANSWER: 0.25 j

The work required is the difference in potential energy between the two positions. The potential energy may be written as:

$$U = -\vec{\mu} \cdot \vec{B}$$

where

$$\mu = iA$$

Thus the work is

$$W = U_{\theta} = 120° - U_{\theta} = 60°$$

$$= (-\mu B \cos 120°) - (-\mu B \cos 60°)$$

$$= 2\, iAB \cos 60° = 0.25 \text{ j}$$

TRUE OR FALSE? It is quite possible for the resultant force on a current-carrying loop in a magnetic field to be zero yet have a torque acting on it.

[b] CORRECT ANSWER: A

We wish to calculate the torque from

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

so we must first calculate $\vec{\mu}$. The magnitude of the magnetic moment is

$$\mu = NiS = 50 \times 10 \times (.20 \times .40) = 40 \text{ amp-m}^2$$

Its direction is given by the direction in which a screw advances when rotated in the sense of the current. For $\phi = 0°$, $\vec{\mu}$ is along $\hat{k}$. Therefore, for $\phi = 30°$.

$$\tau = |\vec{\mu} \times \vec{B}| = \mu B \sin\phi = (40 \text{ amp-m}^2) \times (3.8 \times 10^{-3} \text{ T}) \times \frac{1}{2}$$

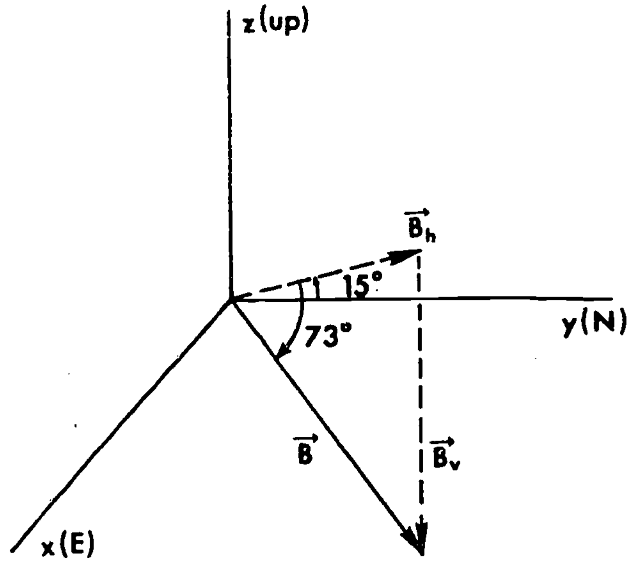
$$= 76 \times 10^{-3} \text{ nt-m}$$

The direction of $\vec{\tau}$ is that in which a screw advances when rotated from $\vec{\mu}$ into $\vec{B}$; i.e., along $-\hat{j}$. Therefore,

$$\vec{\tau} = -76 \times 10^{-3}\ \hat{j} \text{ nt-m}$$

[a] CORRECT ANSWER: D

We choose a right-handed coordinate system with origin located at Cambridge.



The vertical and horizontal components of $\vec{B}$ are given by

$$B_v = -B \sin 73° = -(58\ \mu T) \times 0.957 = -55\ \mu T$$

and

$$B_h = B \cos 73° = (58\ \mu T) \times 0.292 = 17\ \mu T$$

where the negative sign for B_v denotes that it is downward. We now resolve $\vec{B}_h$ into components along east and north.

$$B_E = -B_h \sin 15° = -(17\ \mu T) \times 0.259 = -4.4\ \mu T$$

and

$$B_N = B_h \cos 15° = (17\ \mu T) \times 0.966 = 16\ \mu T$$

TRUE OR FALSE? The inclination of the Earth's field at Cambridge is represented by the direction of B_v in the figure.

[b] CORRECT ANSWER: A

A declination of 15° west means that the north end of a compass needle points 15° west of north. "Magnetic north" is 15° west of geographic (true) north at this location. That is why declination is also known as variation; it gives the magnitude (here 15°) and direction (here west) of the variation of magnetic compass needle from true north.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Ampere's Law

OBJECTIVE

To define and relate the quantities that appear in the statement of Ampere's Law.

This Information Panel is intended to serve as a supplement to your reading on the subject of Ampere's law. It is concerned specifically with the definitions of the terms used in the statement of the law, and with interpretations of the relationships among these quantities.

Ampere's law is a quantitative relationship which gives the connection between an electric current and the magnetic field produced by this current. In its most basic form, it may be stated descriptively as follows:

The line integral of the magnetic induction $\vec{B}$ around any closed path is directly proportional to the net current through the area bounded by the path.

Symbolically,

$$\oint \vec{B}\cdot d\vec{\ell} = k \sum i \qquad (1)$$

in which $\sum i$ is the *net* current; this is further explained below.

The proportionality constant is specifically symbolized by μ_o as in

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_o \sum i \qquad (2)$$

where μ_o is known as the *permeability constant* and is assigned a value of

$$\mu_o = 4\pi \times 10^{-7} \text{ webers/amp-m} \qquad (3)$$

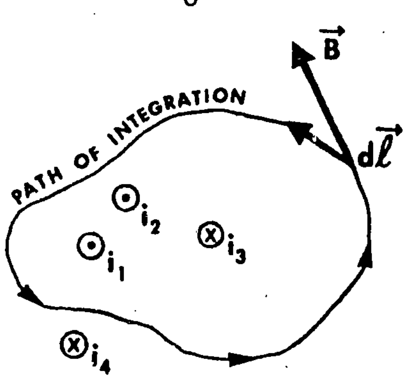


Figure 1

In explaining the significance of the terms, reference is made to Figure 1. The closed path of integration is so labeled. Inside the area bounded by this path are, say, three conductors carrying currents while, outside the area is another current-carrying conductor. The currents in two of the three conductors inside the area (centered dots) are directed out of the paper;

next page

continued

these are i_1 and i_2. The remaining two conductors carry current into the plane of the paper. As stated in Ampere's law, the line integral of the magnetic induction is proportional to the *net* current *through* the area bounded by the path of integration. Thus, current i_4 which is *outside* the path makes no contribution to the magnetic induction whatever. The net current is therefore given by

$$i = i_1 + i_2 - i_3 \qquad (4)$$

Note the algebraic signs and their meanings. If, for example, the magnitudes of these currents were, respectively, 2 amperes, 3 amperes, and 5 amperes in that order, the net current through the bounded area would be *zero*. Hence the magnitude of the $\vec{B}$ field would also be zero.

You will note in your reading that the formalized quantitative statement of Ampere's law as given in equation (2) is derived in your texts for the special case of the field around a long, straight current-carrying conductor. Experiment shows, however, that this equation is valid for any magnetic field shape, any combination of current-carrying conductors, and any path of integration.

Except for configurations of high symmetry, Ampere's law is seldom applied for the purpose of determining magnetic flux density. However, for situations in which symmetry considerations make it possible, the law is extremely useful. Some examples follow:

(1) SOLENOID. The field near the center of a solenoid whose length is much greater than its diameter is given by

$$B = \mu_0 n i$$

in which n = the number of turns per unit length; and i is the current in the solenoid wire.

(2) PARALLEL PLATES. The field in the region between parallel plates and not too close to the edges is given by

$$B = \mu_0 i/w$$

in which i = the current in either plate; and w = the width of either plate. This applies to parallel plates that are very much longer than they are wide.

(3) LONG WIRE. The field near a long wire is given by

$$B = \mu_0 i/2\pi r$$

in which r = the distance perpendicularly from the wire at which the magnetic induction is measured.

PROBLEMS

1. An infinitely long, thin copper wire carries a 50-amp current. What is the magnitude of magnetic field $\vec{B}$ at a distance of 0.50 m from the wire?

2. The magnetic field lines around a long, straight current-carrying conductor are

A. circular in a plane perpendicular to the wire

B. elliptical in a plane perpendicular to the wire

C. parallel to the wire and in the direction of the current

D. parallel to the wire and directed opposite to the current

3. Which of the following rules can help you determine the sense of the magnetic field lines generated by a current?

A. With the thumb of the left hand pointing in the direction of the current, the fingers will curl in the same sense as the magnetic field lines.

B. With the thumb of the right hand pointing in the direction of the current, the fingers will curl in the same sense as the magnetic field lines.

C. With the thumb of the right hand pointing in the direction of the electron flow, the fingers curl in the same sense as the magnetic field lines.

D. Both statements A and C above.

4. In Ampere's law, $\oint \vec{B} \cdot d\vec{\ell} = \mu_o i$, the symbol i represents the net current

A. along the path of integration

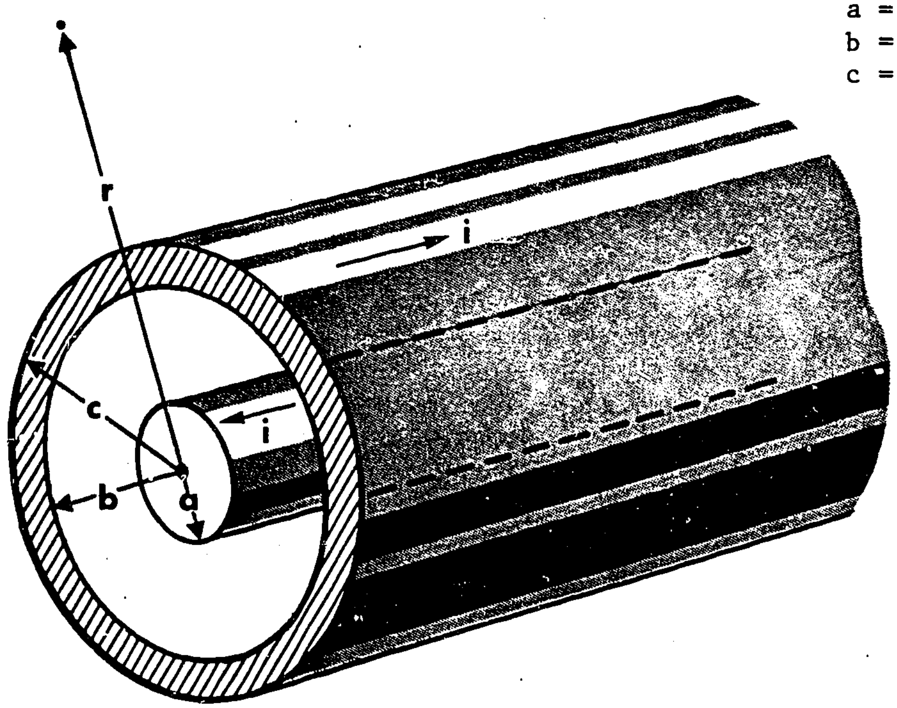
B. enclosed by the contour of integration

C. generating the field $\vec{B}$

D. generated by field $\vec{B}$

5. An infinitely long coaxial cable consists of two concentric conductors as shown in the diagram. There are equal and opposite currents i = 2 amp in the conductors. Find the magnitude of the $\vec{B}$ field outside the cable at r = 3.0 m.

a = 0.50 cm
b = 1.0 cm
c = 1.5 cm



INFORMATION PANEL

Applying Ampere's Law

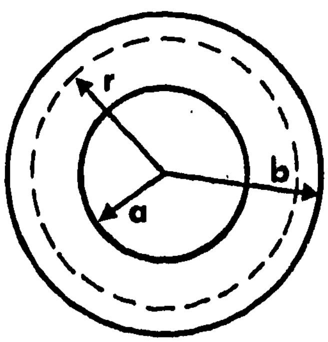
OBJECTIVE

To apply Ampere's law to the solutions of problems involving configurations of high symmetry.

Ampere's law

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i \qquad (1)$$

may be applied and relatively easily evaluated when applied to the configuration illustrated in the accompanying diagram. A long, cylindrical conductor in the form of a hollow sheath has cylindrical symmetry and yields to a direct application of the law. As shown, the radius of the hollow section is a, the radius of the outer sheath surface is b, and we are interested in determining the magnitude of the induction at some distance r from the center. Noting that r is larger than a but smaller than b, we can immediately see that conducting area enclosed in the circle of radius r is given by:



$$s = \pi(r^2 - a^2) \qquad (2)$$

If the total current in the conducting material is I, then the current density j may be expressed as:

$$j = \frac{I}{\pi(b^2 - a^2)} \qquad (3)$$

But only a fraction of this total current is present within the area enclosed by the path of integration; that is, the conducting area inside r. This fraction, symbolized by i, is the net current required for use in Ampere's law and is clearly:

$$i = \frac{(r^2 - a^2)}{(b^2 - a^2)} I \qquad (4)$$

For this case, equation (1) can be made specific by using equation (4) to determine the net current i.

6. What is the magnitude of $\vec{B}$ at [illegible] r from the axis of a current-carrying cylindrical shell in which the current density is uniform? The inner radius is a, the outer radius is b, and b > r > a.

A. zero

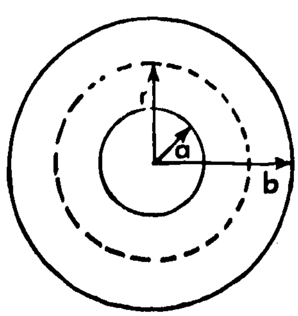
B. $\frac{\mu_0 (r^2 - a^2)}{2\pi (b^2 - a^2)} \frac{I}{r}$

C. $\frac{\mu_0 a^2}{2\pi\ b^2 - r^2} \frac{I}{r}$

D. $\frac{\mu_0 b^2}{2\pi\ r^2 - a^2} \frac{I}{r}$

7. An infinitely long cylindrical wire of diameter 1.0 cm carries a current of 6.0 milliamperes uniformly distributed over its cross section. Use Ampere's law to calculate the magnitude of the magnetic induction at a distance of 5.0 cm from the center of the wire.

8. A conducting cylindrical shell with respective inner and outer radii a and b carries a current I uniformly distributed over the cross section of the shell. For which values of r is the magnetic induction equal to zero?



A. only at $r = \infty$

B. only at $r = 0$

C. for $r \leq a$

D. for $r < b$

9. Use Ampere's law to calculate the magnitude of the magnetic induction at a distance of 2.0 mm from the center of an infinitely long cylindrical wire of radius R = 0.50 cm which carries a current i_R = 6.0 milliamperes uniformly distributed over its cross section.

INFORMATION PANEL — Forces Between Current-Carrying Conductors

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OBJECTIVE

To solve problems in which the force exerted by one current-carrying conductor on another current-carrying conductor is to be determined.

When two current-carrying conductors are placed parallel to one another, each one exerts a force on the other. This force is attractive if the current directions in the two wires are the same and repulsive if the currents are opposite in direction. To evaluate the magnitude of this force, we apply Ampere's law:

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_o i \qquad (1)$$

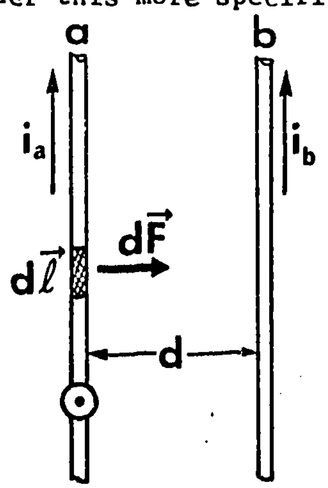
in which, for this case,

$$\oint \vec{B}\cdot d\vec{\ell} = B(2\pi d) \qquad (2)$$

Thus:

$$B = \frac{\mu_o i}{2\pi d} \qquad (3)$$

To render this more specific, we rewrite it in the form shown in equation (4) which *then expresses the magnetic induction at the location of wire a due to the current in wire b.*



$$B_b = \frac{\mu_o i_b}{2\pi d} \qquad (4)$$

Applying the right-hand rule, we let the extended thumb of the right hand point in the direction of the current in wire b and observe that the fingers encircle wire b in such a manner as to point out of the plane of the paper at the site of wire a. For this reason, we have placed a small circle and a central dot on wire a to show the direction of the magnetic field due to the current in wire b at this point.

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continued

From your previous work, you know that the force acting on an element of length $d\ell$ of wire a carrying a current i_a is given by:

$$d\vec{F} = i_a d\vec{\ell} \times \vec{B}_a \tag{5}$$

The magnitude of the force on wire a is then:

$$F_a = i_a \ell B_b \tag{6}$$

where ℓ is the full length of wire a.

Combining equations (4) and (6), we can write for the force on wire a:

$$F_a = i_a \ell \frac{\mu_o i_b}{2\pi d} \tag{7}$$

In most problems, this relation is more useful when expressed in terms of *force per unit length.* Thus,

$$\frac{F_a}{\ell} = \mu_o \frac{i_a i_b}{2\pi d} \tag{8}$$

It should be observed that, although this derivation was based on considerations involving determining the force on wire a due to the field produced by the current in wire b, exactly the same relationship also gives the force on wire b due to the field produced by the current in wire a. Aside from the fact that the commutative relationship of $i_a i_b$ permits one to write $i_b i_a$ with equal validity, Newton's third law indicates that the force on wire a must be equal to the force on wire b but opposite in direction.

10. Two long wires carrying parallel currents of 2.7 and 5.0 amp, respectively, in the same direction are separated by a distance of 3.0 cm. What is the force per unit length of each wire on the other?

A. 9.0×10^{-5} nt/m, attractive

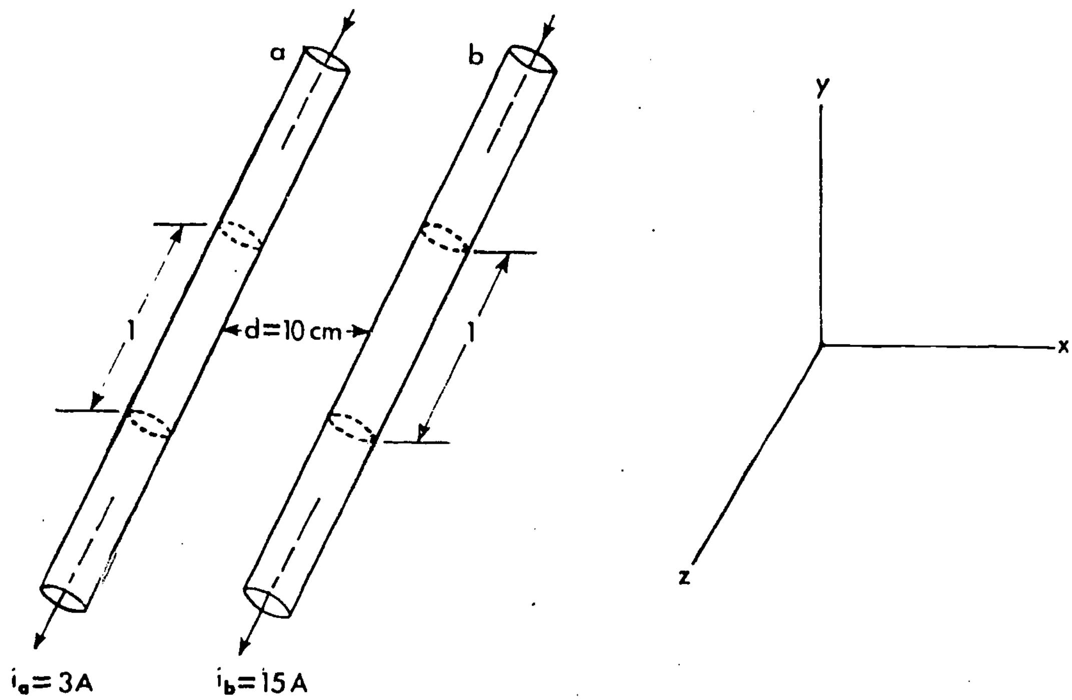
B. 9.0×10^{-5} nt/m, repulsive

C. 9.0×10^{-7} nt/m, attractive

D. 9.0×10^{-7} nt/m, repulsive

<u>Introductory Notes</u>

The following four questions are related to the two parallel current-carrying conductors shown below.



11. Two long parallel conductors separated by a distance d = 10 cm carry parallel currents of i_a = 3 amps and i_b = 15 amps. The directions of the currents and coordinate system are as shown in the diagram. Both conductors are parallel to the z axis and lie in the x-z plane. At the position of conductor a, the magnetic field set up by conductor b is

A. $\vec{B}_b = 3 \times 10^{-5}\ \hat{j}$ T

B. $\vec{B}_b = -3 \times 10^{-5}\ \hat{j}$ T

C. $\vec{B}_b = 6 \times 10^{-6}\ \hat{j}$ T

D. $\vec{B}_b = -6 \times 10^{-6}\ \hat{j}$ T

12. Calculate the magnetic force on a 2 meter length of conductor a in the previous question.

A. $\vec{F}_a = 9 \times 10^{-5}\ \hat{i}$ nt

B. $\vec{F}_a = -9 \times 10^{-5}\ \hat{i}$ nt

C. $\vec{F}_a = 1.8 \times 10^{-4}\ \hat{i}$ nt

D. $\vec{F}_a = -1.8 \times 10^{-4}\ \hat{i}$ nt

13. For the conductors shown in the diagram of question 11, what is the force exerted on a 2 meter length of conductor b due to current i_a?

A. $\vec{F}_b = 1.8 \times 10^{-4}\ \hat{i}$ nt

B. $\vec{F}_b = -9.0 \times 10^{-4}\ \hat{i}$ nt

C. $\vec{F}_b = 6.0 \times 10^{-5}\ \hat{i}$ nt

D. $\vec{F}_b = -1.8 \times 10^{-4}\ \hat{i}$ nt

14. If the direction of the current in conductor a, of the preceding question, $i_a = 3$ amp, is reversed, which of the following statements about the directions of the forces on a and b is correct?

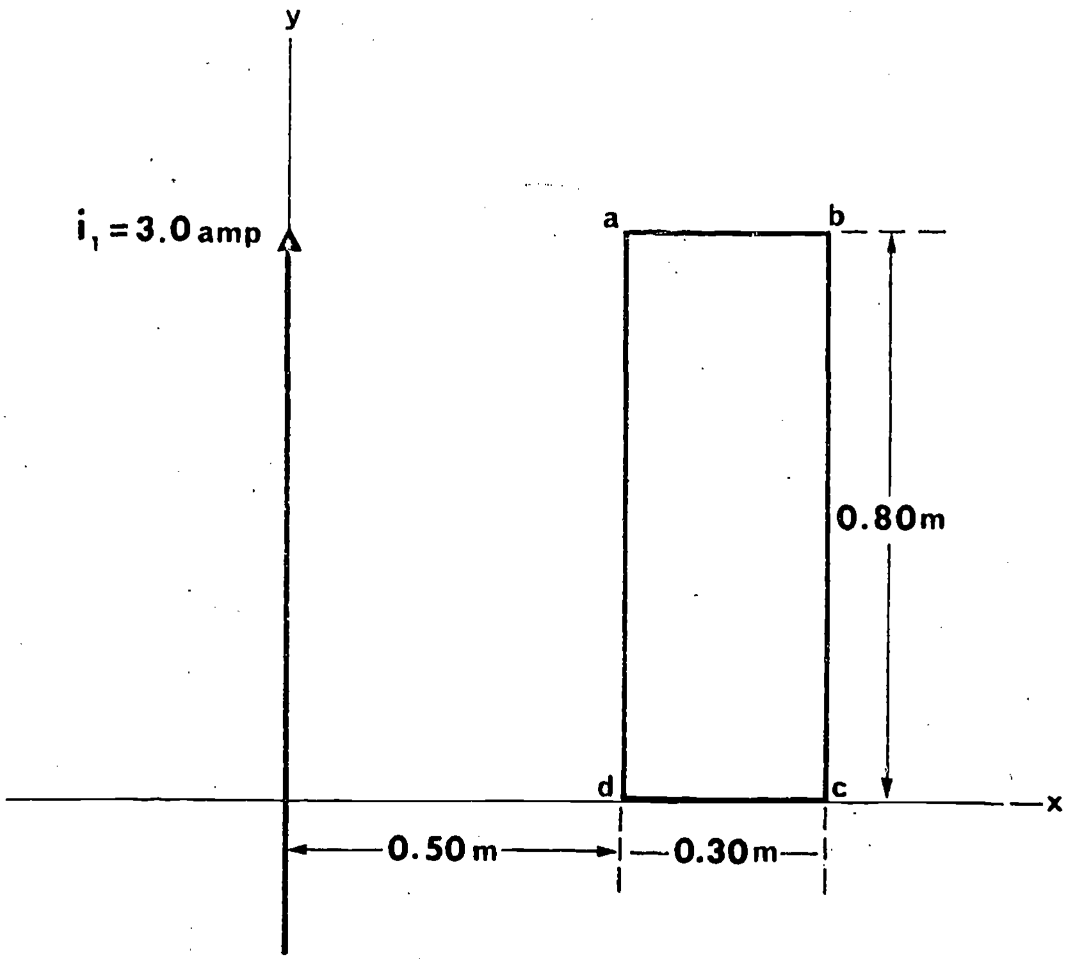
A. The directions of both forces remain unchanged

B. The direction of the force on conductor a only is reversed

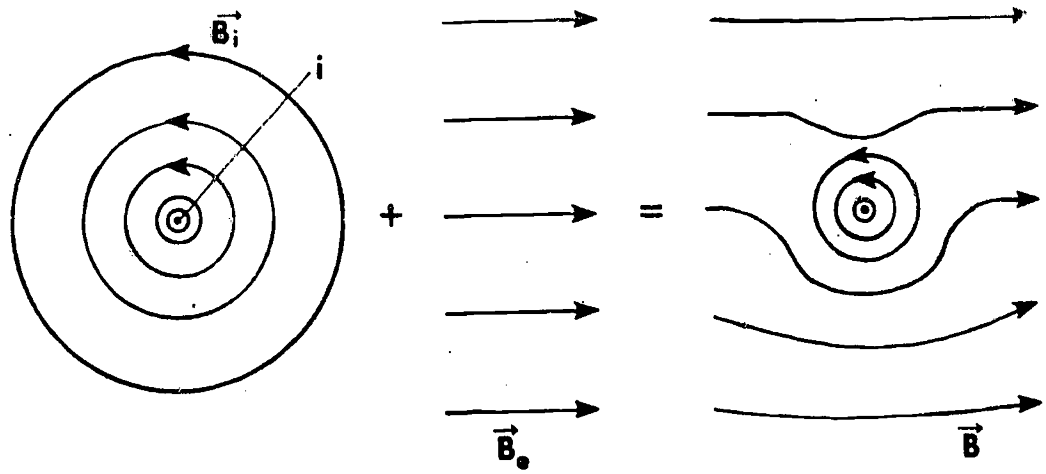
C. The direction of the force on conductor b only is reversed

D. The direction of both forces is reversed.

15. A clockwise current i_2 = 2.0 amp is set up in the rectangular loop in the accompanying diagram. What is the net force on the loop due to the magnetic field produced by i_1?



16. A current-carrying wire is immersed in a uniform external magnetic field $\vec{B}_e$ as shown below



The resultant field is now $\vec{B} = \vec{B}_i + \vec{B}_e$, where

$$B_i = \frac{\mu_o i}{2\pi r}\,\hat{\theta}$$

$\hat{\theta}$ being the unit vector tangent to the circle of radius r. The force on the wire is now

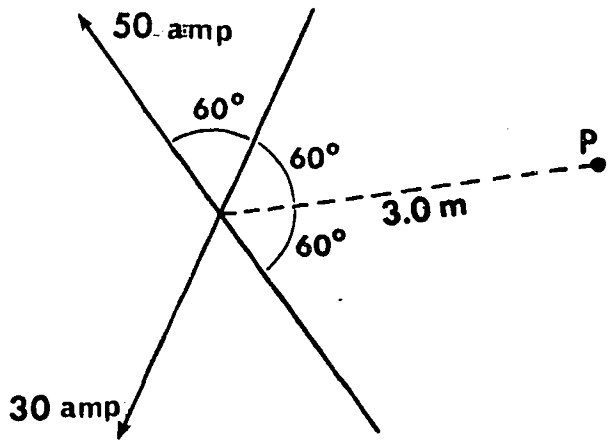
A. zero

B. $\vec{F} = i\vec{\ell} \times \vec{B}_i$

C. $\vec{F} = i\vec{\ell} \times \vec{B}_e$

D. $\vec{F} = i\vec{\ell} \times \vec{B}$

17. Two infinitely long thin straight wires carry currents of 30 amp and 50 amp, and cross each other as shown in the diagram. Calculate the magnetic induction at point P, which is in the plane of the wires.



A. 1.3 μT into the paper

B. 1.5 μT into the paper

C. 5.3 μT into the paper

D. 6.2 μT into the paper

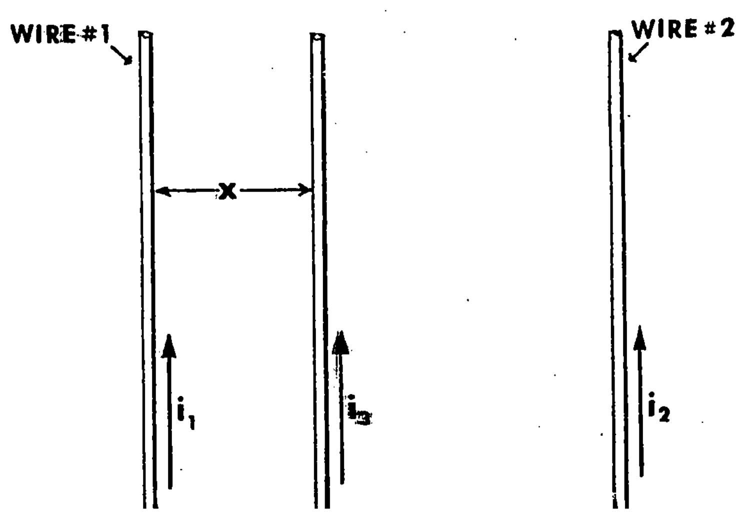
18. Two long parallel wires are separated by a distance of 60 cm. They carry currents i_1 = 5.0 amp and i_2 = 10 amp, both in the same direction. A third 2.0 m long wire is placed between the two, and parallel to them in such a location that when a current i_3 is set up in this wire it experiences no net force. How far from wire #1 is this third wire located?

A. 60 cm

B. 40 cm

C. 30 cm

D. 20 cm



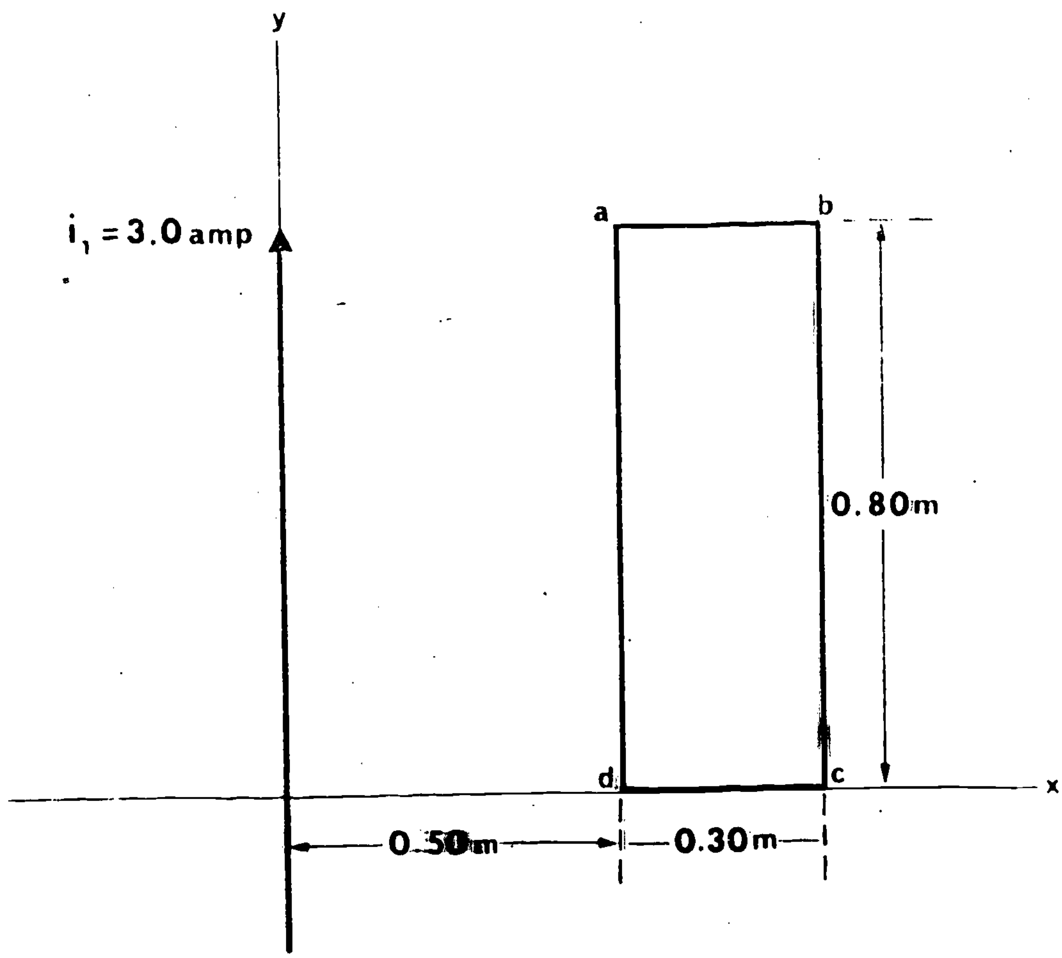
19. What current i_2 must be set up in the loop below in order that the force exerted by the current i_1 on the element ab of the loop be $5.6 \times 10^{-7}\ \hat{j}$ nt?

A. 1.4 amp clockwise

B. 1.8 amp counterclockwise

C. 2.0 amp clockwise

D. 2.2 amp counterclockwise



[a] CORRECT ANSWER: 9.6×10^{-8} T

Cylindrical symmetry once again enables us to use Ampere's law to calculate B. But here we choose a circle of radius 2.0 mm as our integration path, and so the current enclosed by the contour is not 6.0 milliamperes. It is, instead, the portion of the current enclosed by the path of integration. Since the current is uniformly distributed throughout the wire, the current density (current per unit area) is constant. Therefore,

$$j = \frac{i_r}{\pi r^2} = \frac{i_R}{\pi R^2} = \text{constant}$$

and

$$i = i_r = (6.0 \text{ milliamperes}) \times \frac{r^2}{R^2} = (6.0 \text{ milliamperes}) \times \left[\frac{2.0 \text{ mm}}{5.0 \text{ mm}}\right]^2$$

$$= 0.96 \text{ milliamperes}$$

Thus,

$$0.96 \text{ milliamperes} = i = \frac{1}{\mu_o}\int \vec{B}\cdot d\vec{\ell} = \frac{B \times (2\pi r)}{\mu_o}$$

with r = 2.0 mm, and

$$B = \frac{\mu_o i}{2\pi r} = \frac{4\pi \times 10^{-7} \times 0.96 \times 10^{-3}}{2\pi \times 2.0 \times 10^{-3}} = 9.6 \times 10^{-8} \text{ T}$$

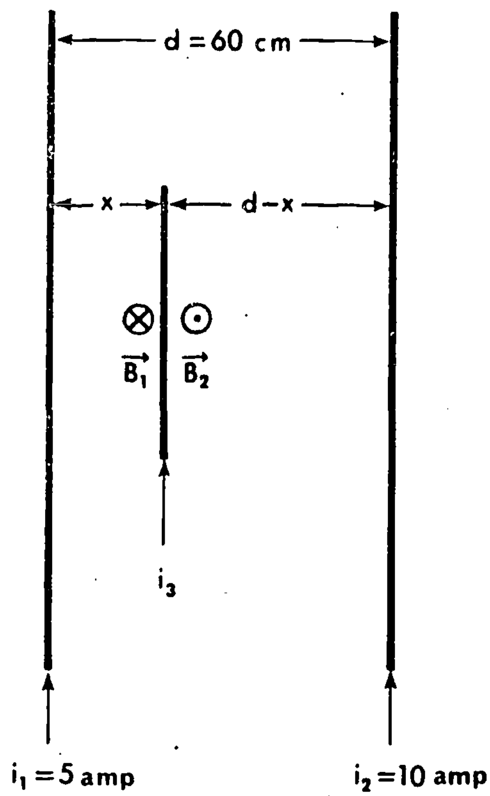
TRUE OR FALSE? The portion of the current enclosed by the path of integration is less than the total current in the wire.

[b] CORRECT ANSWER: A

The fact that the magnetic field lines around a long, straight current-carrying conductor are circular in a plane perpendicular to the conductor may be seen most graphically by performing a simple experiment. Iron filings, strewn onto a glass plate through which a current-carrying conductor passes perpendicularly, align themselves in the direction of the magnetic field when a current is passed through the conductor. The concentric, circular pattern is readily visible.

[a] CORRECT ANSWER: D

A current-carrying wire placed in a magnetic field will experience a force given by



$$\vec{F} = i\vec{\ell} \times \vec{B} \qquad (1)$$

It should be apparent from the cross product that the force can be zero if (a) the wire is parallel to B or (b) if B is equal to zero. The first possibility is ruled out since $\vec{B}_1$ is directed into the paper and $\vec{B}_2$ is directed out of the paper while the wire is parallel to the plane of the paper. This means that

$$\vec{B}_1 + \vec{B}_2 = 0$$

Now $\vec{B}_1$ and $\vec{B}_2$ are oppositely directed, so we must find the position at which $B_1 = B_2$. Let that position be a distance x from wire number 1. Then we must have

$$\frac{\mu_o i_1}{2\pi x} = \frac{\mu_o i_2}{2\pi(d - x)} \qquad (2)$$

or

$$i_1(d - x) = i_2 x$$

and

$$x = \frac{i_2}{i_1 + i_2} d \qquad (3)$$

Substituting the given values in (3), we obtain

$$x = \frac{5.0 \text{ amp}}{(5.0 + 10) \text{ amp}} \times (60 \text{ cm}) = 20 \text{ cm}$$

[a] CORRECT ANSWER: D

When the direction of i_a changes, the *field* at a remains unchanged ($\vec{B}_b' = \vec{B}_b$), while the direction of $\vec{\ell}$ changes ($\vec{\ell}_a' = -\vec{\ell}_a$). Therefore,

$$\vec{F}_a' = i_a\vec{\ell}_a' \times \vec{B}_b' = -i_a\vec{\ell}_a \times \vec{B}_b = -\vec{F}_a$$

Similarly, the direction of the field at b due to i_a reverses ($\vec{B}_a' = -\vec{B}_a$) but the direction of $\vec{\ell}$ remains unchanges ($\vec{\ell}_a' = \vec{\ell}_a$). Thus

$$\vec{F}_b' = i_b\vec{\ell}_b' \times \vec{B}_a' = i_b\vec{\ell}_b \times \vec{B}_a = -\vec{F}_b'$$

Hence, the directions of both forces are reversed.

TRUE OR FALSE? When the direction of i_a changes, the field at a does *not* change since this field is due to the current in wire b.

[b] CORRECT ANSWER: 2.4×10^{-8} T

Cylindrical symmetry insures that the magnetic induction has a constant magnitude along, and is tangential to, any circular path centered on the wire. Thus, by Ampere's law,

$$\mu_o i = \oint \vec{B}\cdot d\vec{\ell} = B(2\pi r)$$

where i is the current enclosed by a circle of radius r and B is the magnitude of the magnetic induction along the circle. A circle of radius r = 5.0 cm encloses the entire wire, and so i = 6.0 milliamperes. Therefore,

$$B = \frac{\mu_o i}{2\pi r} = \frac{4\pi \times 10^{-7} \times 6.0 \times 10^{-3}}{2\pi \times 0.050} = 2.4 \times 10^{-8}\ T$$

Note: For points that lie outside of the conductor, the $\vec{B}$ field is independent of the radius of conductor.

[a] CORRECT ANSWER: B

Ampere's law for magnetism is the analog of Gauss's law for electricity. Note the similarity:

Gauss's law	Ampere's law
$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_o}$	$\oint \vec{B}\cdot d\vec{\ell} = \mu_o i$
Integrate over a closed surface.	Integrate around a closed path.
q is the net charge enclosed by the surface.	i is the net current enclosed by the path.
Enables one to find $\vec{E}$ for configurations of high symmetry.	Enables one to find $\vec{B}$ for configurations of high symmetry.

[b] CORRECT ANSWER: D

Following the procedure (and notation) used in the previous question, we find

$$\vec{F}_b = i_b\vec{\ell} \times \vec{B}_a = i_b \ell\ \hat{k} \times \left(\frac{\mu_o\ i_a}{2\pi\ d}\ \hat{j}\right)$$

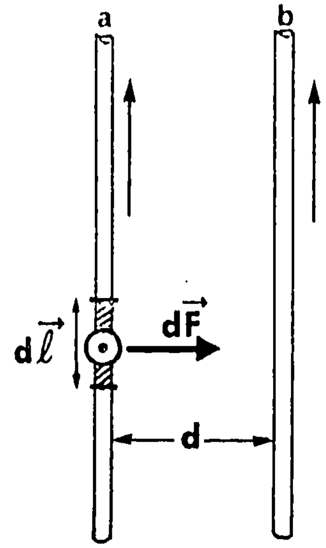
$$= -\frac{\mu_o\ i_a\ i_b}{2\pi\ d}\ \ell\ \hat{i} = -\frac{4\pi \times 10^{-7} \times 3 \times 15}{2\pi \times 10^{-1}} \times 2\ \hat{i}$$

$$= -1.8 \times 10^{-4}\ \hat{i}\ \text{nt}$$

We might have come to the same conclusion by making use of Newton's third law of motion which holds regardless of the way in which a force is applied on a body by another body; i.e., by direct contact or via a field as in the present case. Since the force on conductor a was found to be $1.8 \times 10^{-4}\ \hat{i}$ nt, the force on b is opposite to that; i.e., $-1.8 \times 10^{-4}\ \hat{i}$ nt.

[a] CORRECT ANSWER: A

The force is attractive because the currents have the same direction.



Let us calculate the magnetic field due to the current-carrying wire b at a point on wire a. Using Ampere's law, we find

$$\oint \vec{B}\cdot d\vec{\ell} = B\ 2\pi\ d = \mu_o i_b$$

or

$$B_a = \frac{\mu_o i_b}{2\pi\ d}$$

The direction of the magnetic field is out of the plane of paper.

The force acting on an element $d\vec{\ell}$ of wire a carrying current i_a is

$$d\vec{F} = i_a\ d\vec{\ell} \times \vec{B}$$

Therefore, the magnitude of the force on a unit length is

$$F = i_a \frac{\mu_o i_b}{2\pi\ d} = \frac{\mu_o i_a i_b}{2\pi\ d} = 9.0 \times 10^{-5}\ \text{nt/m}$$

The direction of $d\vec{F}$ is toward wire b; i.e., attractive.

You may similarly calculate the force on a unit length of wire b due to the field generated by wire a.

TRUE OR FALSE? The force on a unit length of wire b due to the current in wire a is also given by $\mu_o \frac{i_a i_b}{2\pi\ d}$, but this force is repulsive.

[b] CORRECT ANSWER: C

Because of the cylindrical symmetry, Ampere's law gives $B = \mu_o i/2\pi r$, where i is the current enclosed by a circle of radius r. The cylindrical shell has an inner radius a, and so $i = 0$ for $r \leq a$. Therefore $B = 0$ for $r \leq a$.

[a] CORRECT ANSWER: C

The force on an infinitesimal element $d\vec{\ell}$ of the horizontal sides of the loop is

$$d\vec{F} = i_2\, d\vec{\ell} \times \vec{B}$$

where

$$B = \mu_0 i_1 / 2\pi x$$

Now $d\vec{\ell} = \pm\, \hat{i}\, dx$ ("+" if the current is toward right and "-" if the current is toward left), and $\vec{B} = -B\,\hat{k}$, so

$$d\vec{F} = i_2(\pm\,\hat{i}\,dx) \times (-B\,\hat{k}) = \mp\, i_2\, B\, dx(\hat{i} \times \hat{k}) = \pm\, i_2\, B\, dx\, \hat{j}$$

For the element ab of the loop the upper sign stands for clockwise, and the lower sign for counterclockwise current. For $\vec{F}$ to be along $\hat{j}$, the current i_2 must be clockwise. In order to determine the magnitude of the force, we must integrate $d\vec{F}$ from x = 0.50 m to x = 0.80 m. Thus

$$F = \int_{0.50}^{0.80} i_2\, B\, dx = \frac{\mu_0\, i_1 i_2}{2\pi} \int_{0.50}^{0.80} \frac{dx}{x} = \frac{\mu_0\, i_1 i_2}{2\pi}\, \ell n\left(\frac{0.80}{0.50}\right)$$

$$= \frac{4\pi \times 10^{-7} \times 3.0 \times i_2}{2\pi}\, \ell n\, (1.6) = 6.0 \times 10^{-7}\, i_2 \times 0.47$$

$$= 2.8 \times 10^{-7}\, i_2$$

In order that F be 5.6×10^{-7} nt, i_2 must be 2.0 amp; i.e.,

$$i_2 = \frac{F}{2.8 \times 10^{-7}} = \frac{5.6 \times 10^{-7}}{2.8 \times 10^{-7}} = 2.0 \text{ amp}$$

TRUE OR FALSE? The requirement that must be met in this problem is that the vector direction of the force exerted by current i_1 on element ab be along $-\hat{j}$.

[b] CORRECT ANSWER: C

The meaning of the symbol $\vec{B}$ in $\vec{F} = i\vec{\ell} \times \vec{B}$ is just $\vec{B}_e$; namely, the external magnetic field which exists at the location of the wire.

[a] CORRECT ANSWER: 0

The magnitude of the B field may be calculated using Ampere's law

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \sum i \qquad (1)$$

From symmetry, the magnitude of the $\vec{B}$ field is constant for every point in a circular path of radius r centered on the inner conductor. If we take the path of integration to be a circle of radius r, then $d\vec{\ell}$ and $\vec{B}$ are always tangent to the path of integration. Therefore,

$$\oint \vec{B}\cdot d\vec{\ell} = B\ 2\pi r \qquad (2)$$

where B is the magnitude of the magnetic field at a distance r from the center. However $\sum i$ enclosed by the path of integration is

$$\sum i = i - i = 0 \qquad (3)$$

Substituting the results of equations (2) and (3) into equation (1), we find

$$B\ 2\pi r = 0$$

or

$$B = 0$$

TRUE OR FALSE? If the currents in both conductors had the same direction, the magnitude of $\vec{B}$ at a distance r = 3.0 m would still be zero.

[b] CORRECT ANSWER: C

The force on a length ℓ of conductor a is

$$\vec{F}_a = i_a \vec{\ell} \times \vec{B}_b$$

where B_b is the field set up at the position of conductor a by current i_b (B_b was calculated in preceding question.) Substituting the numerical values, we obtain

$$\vec{F}_a = 3 \times 2\ \hat{k} \times (-3 \times 10^{-5}\ \hat{j})$$

$$= -1.8 \times 10^{-4}\ (\hat{k} \times \hat{j}) = 1.8 \times 10^{-4}\ \hat{i}\ \text{nt}$$

[a] CORRECT ANSWER 10^{-5} T

The magnitude ... ic field $\vec{B}$ may be calculated from Ampere's law:

$$\oint \vec{B} \cdot d\vec{\ell}$$

The integral is ... evaluated for a path consisting of a circle of radius r centered on ... For all points on this circle, the magnetic induction has the s... nitude, B, and $\vec{B}$ and $d\vec{\ell}$ have the same direction; namely, tangen... ne path of integration. Thus, the integral becomes

$$\oint \vec{B}\cdot d ... \oint d\ell = B\ 2\pi r = \mu_0 i$$

or

$$B = ...$$

Substituting num... al values, we obtain

$$B = \frac{\mu_0}{2\pi} ... = \frac{4\pi \times 10^{-7}}{2\pi} \cdot \frac{50}{0.5} = 2.0 \times 10^{-5}\ \mathrm{T}$$

TRUE OR FALSE? In ... is solution, the symbol μ_0 represents the dipole moment of the ... of integration.

[b] CORRECT ANSWER: B

The field of a long current wire has been calculated by Ampere's law. We can use this result, $B = \mu_0 i/2\pi r$, for each of the wires and add the two contributions vectorially. The perpendicular distance of each wire from P (this is what "r" stands for) is

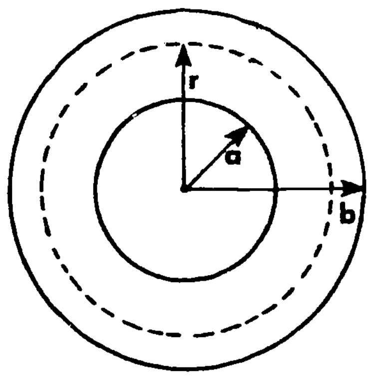
$$r = (3.0\ \mathrm{m}) \times \sin 60° = 2.6\ \mathrm{m}$$

The field of the 50 amp current is into the paper; that of the 30 amp current is out of the paper. The resultant field is therefore into the paper and has a magnitude

$$B = ... (50 - 30) = \frac{4\pi \times 10^{-7} \times 20}{2\pi \times 2.6} = 1.5 \times 10^{-6}\ \mathrm{T}$$

[a] CORRECT ANSWER: B

Since we have cylindrical symmetry, we can use Ampere's law to find



$$B = \mu_0 i/2\pi r \qquad (1)$$

where i is the current enclosed in a circle of radius r. The current density is uniform, so

$$j = i/S = \text{constant} \qquad (2)$$

The cross-sectional area of the part of the shell enclosed in a circle of radius $r \geq a$ is given by

$$S = \pi(r^2 - a^2) \qquad (3)$$

However, j is given by the expression

$$j = \frac{I}{\pi(b^2 - a^2)} \qquad (4)$$

Substituting (3) and (4) into (2), we obtain i:

$$i = I\,\frac{(r^2 - a^2)}{(b^2 - a^2)} \qquad (5)$$

Thus, we can find B using (5) and (1)

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 I(r^2 - a^2)}{2\pi(b^2 - a^2)r}$$

TRUE OR FALSE? I and i both symbolize current, but I stands for the total current in the conducting material.

[a] CORRECT ANSWER: 3

The magnetic field of current i_b is (from Ampere's law and cylindrical symmetry) $B_b = \mu_0 i_b/2\pi r$, where r is the distance from b. Thus at the position of conductor a, r = d and we have

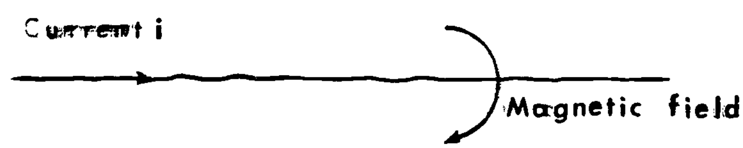
$$B_b = [illegible] = \frac{4\pi \times 10^{-7} \times 15}{2\pi \times 10^{-1}} = 3 \times 10^{-5} \text{ T}$$

The magnetic field vector points in the negative y direction. Thus, the correct answer is

$$\vec{B}_b = -3 \times 10^{-5} \hat{j} \text{ T}$$

[b] CORRECT ANSWER: [illegible]

Two points involved in this statement are worth noting. First, we have followed the convention about current used throughout the course; namely, consider the direction that positive charges would move as the direction of the current. Second, we have used the right-hand rule, which has been used throughout the course (right-handed coordinate systems, cross product, etc.)



[a] CORRECT ANSWER: $-7.2 \times 10^{-7}\ \hat{i}$ nt

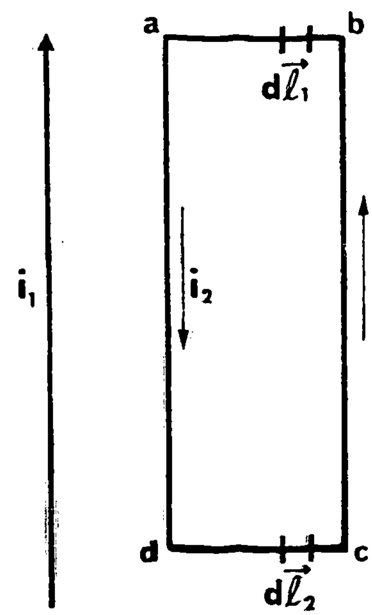
The force on current-carrying element $d\vec{\ell}$ is

$$d\vec{F} = i\, d\vec{\ell} \times \vec{B} \qquad (1)$$

The magnetic field $\vec{B}$ is the same for the two symmetric elements $d\vec{\ell}_1$ and $d\vec{\ell}_2$. However, the currents in $d\vec{\ell}_1$ and $d\vec{\ell}_2$ are opposite to each other. Hence, from equation (1), the forces on the upper and lower segments of the loop are equal in magnitude (corresponding elements of each segment are at equal distances from the wire) and oppositely directed (along +j for the upper, and -j for the lower segment). The force on each vertical side is

$$\vec{F} = i_2\, \vec{\ell} \times \vec{B}$$

where $\vec{B}$ is constant along each side, but has different value at each side.



For the near side, $\vec{\ell} \times \vec{B}_n$ is along $-\hat{i}$ and

$$\vec{F}_n = -i_2\, \ell\, \frac{\mu_o}{2\pi} \frac{i_1}{x_n}\, \hat{i}$$

with x_n = 0.50 m. Similarly, the force on the far side is

$$\vec{F}_f = i_2\, \ell\, \frac{\mu_o}{2\pi} \frac{i_1}{x_f}\, \hat{i}$$

with x_f = 0.80 m.

Therefore, the net force on the loop is

$$\vec{F} = \vec{F}_n + \vec{F}_f = i_2\, \ell\, \frac{\mu_o}{2\pi}\, i_1 \left(-\frac{1}{x_1} + \frac{1}{x_o}\right) \hat{i}$$

$$= 2.0 \times 0.80 \times \frac{4\pi \times 10^{-7}}{2\pi} \times 3.0 \left(-\frac{1}{0.50} + \frac{1}{0.80}\right) \hat{i}$$

$$= 9.6 \times 10^{-7} \times (-0.75) = -7.2 \times 10^{-7}\ \hat{i} \text{ nt}$$

TRUE OR FALSE? The forces acting on the upper and lower segments of the loop cancel one another.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE TEXT.

INFORMATION PANEL — The Magnetic Field of a Solenoid and Toroid

OBJECTIVE

To calculate the magnetic field characteristics of several typical solenoids; of several typical toroids.

The derivation of the equation which gives the magnetic induction for an infinitely long, ideal solenoid appears in your assigned reading and also in the first of the enabling-problem solutions that follow. It will not be repeated here. Several individual points of information are worth reemphasizing, however.

(1) The relationship is

$$B = \mu_0 i_0 n$$

The current symbol is given a subscript to distinguish it from the *net current* that passes through the area bounded by the path of integration. The current i_0 is the current in the wire of the solenoid, i.e., the current that would be measured by an ammeter in series with the winding. The symbol n is used to represent the *number of turns per unit length* contained in the solenoid winding.

(2) Despite the fact that the equation was derived for an ideal solenoid, it works quite well for the magnetic induction for internal points near the axis of the solenoid.

(3) For practical solenoids, the magnitude of the induction does not depend on the diameter or length; it may also be considered *uniform across the cross section* of the solenoid winding.

(4) The number of turns per unit length is given by the product of the total number of turns and the number of layers, divided by the total length of the winding. Care must be taken to keep the units consistent.

For a toroid, the magnitude of $\vec{B}$ is obtained from

$$B = \frac{\mu_0 i_0 N}{2\pi r}$$

in which N is the *total* number of turns and r is the distance from the center of the "doughnut" to the point inside the winding for which B is to be determined. Two important facts should be reiterated for a toroid:

next page

continued

(1) Unlike a solenoid, a toroid develops an internal field that is *not* uniform in cross section. The magnetic induction for points inside the winding decreases in magnitude as the distance from the center is increased.

(2) The field outside the toroid structure is negligible and may be taken as zero in problem work.

In this section, you will encounter problems in which you are to

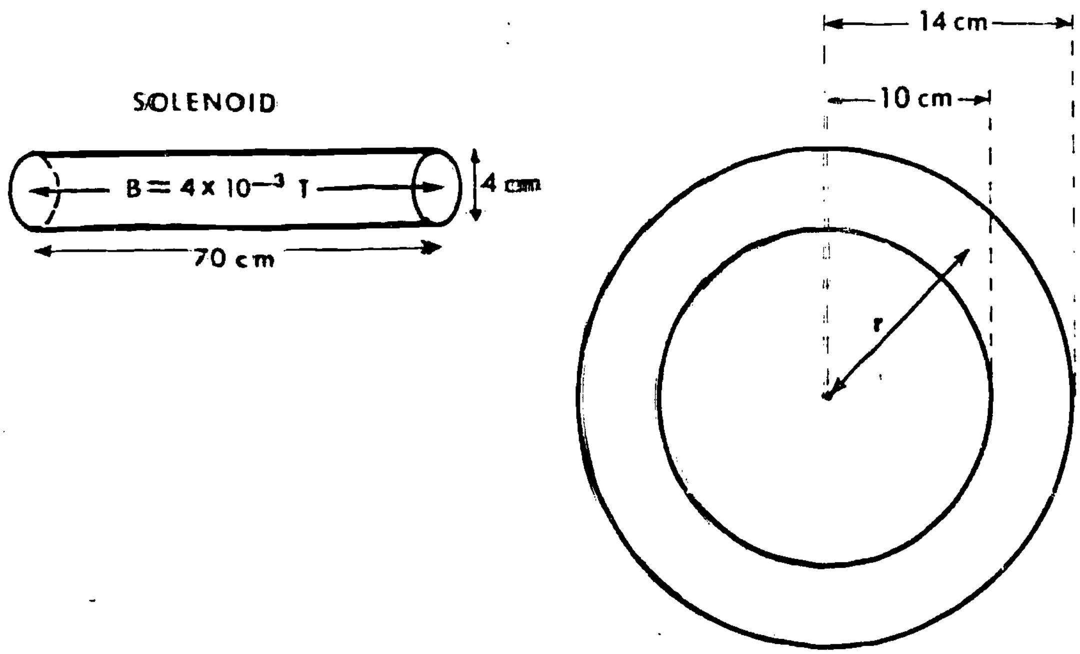
(a) calculate the value of B inside a typical toroid;

(b) determine the value of B at the center of a typical solenoid;

(c) prove that the value of B outside a typical toroid is zero.

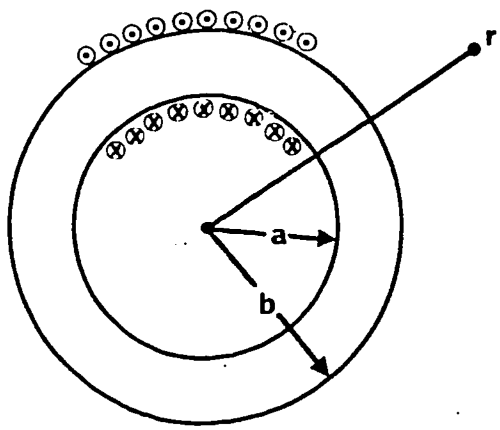
PROBLEMS

1. A flexible solenoid of length 70 cm and diameter 4 cm is bent into a toroid (the shape of a doughnut) which has inner and outer radii of 10 cm and 14 cm respectively. If the solenoid produces a uniform magnetic field of $B = 4 \times 10^{-3}$ T, what is the value of B inside the toroid at a distance r = 11 cm as shown in the diagram?



2. A solenoid is 2.0 m long and 5.0 cm in diameter. It has 6 layers of windings of 1000 turns each and carries a current of 2.0 amp. What is the magnitude of $\vec{B}$ at its center?

3. A toroid of inner radius a and outer radius b has N turns. The direction of current i_o is shown in the diagram. Find the magnitude of the $\vec{B}$ field at a point r, where r > b.



4. A solenoid which measures 20 cm in length and 5.0 cm in diameter is wound with 4 layers of 250 turns each. How much current i_o must the windings carry in order to produce a magnetic field of 10^{-4} T inside the solenoid?

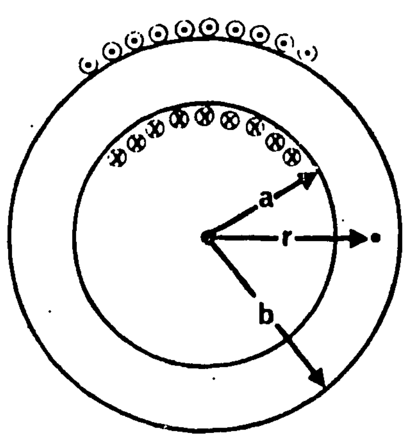
5. A toroid of inner radius a and outer radius b has N turns. The direction of current i_o is shown in the diagram. Find the magnitude of the $\vec{B}$ field at a point r, where a < r < b.

A. $B = \frac{\mu_o N i_o}{4\pi r}$

B. $B = \frac{\mu_o N i_o}{4\pi r^2}$

C. $B = \frac{\mu_o N i_o}{2\pi r}$

D. $B = \mu_o N i_o 2\pi r$



INFORMATION PANEL — The Biot-Savart Law

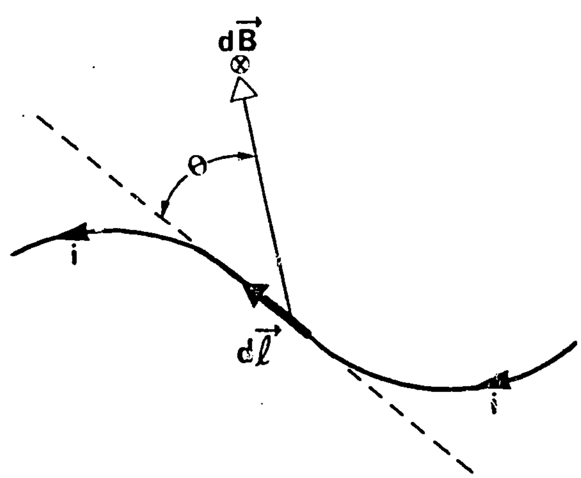
OBJECTIVE

To utilize the Biot-Savart law for solving basic problems involving the magnetic induction produced by a current in a conductor in configurations of low symmetry.

The Biot-Savart law replaces Ampere's law in configurations where the latter is too difficult to apply. The Biot-Savart law may be written in scalar form as

$$dB = \frac{\mu_o i\ d\ell\ \sin\theta}{4\pi r^2}$$

where r is a displacement vector from a current element to the point P at which the induction is to be determined; θ is the angle between r and the current element as indicated in the accompanying diagram. The direction of $d\vec{B}$ is that of the vector $d\vec{\ell} \times \vec{r}$. In vector form, the Biot-Savart law may be written as

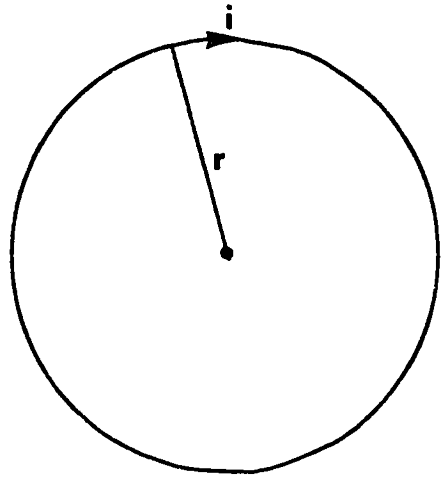


$$d\vec{B} = \frac{\mu_o i\ d\vec{\ell} \times \vec{r}}{4\pi r^3}$$

The resultant field at P is found by calculating the *vector* integral of $d\vec{B}$.

In this portion of your work, you will be asked to solve problems in which the magnitude of the magnetic induction is to be determined for configurations like a long, straight wire, a circular loop, and a hydrogen atom with its single revolving electron.

6. A wire in the form of circle of radius r carries a current i as shown in the diagram. The expression for the magnitude of the magnetic field at its center is



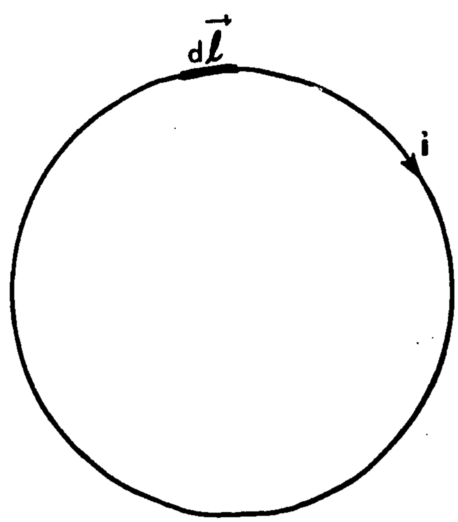
A. $B = \frac{\mu_o i}{2\pi r}$

B. $B = \frac{\mu_o i}{2r}$

C. $B = \frac{\mu_o i}{2r^2}$

D. $B = \frac{\mu_o i}{4\pi r^2}$

7. The diagram shows a circular loop which carries a current i in the clockwise sense. Find the contribution $d\vec{B}$, of the infinitesimal loop element, $d\vec{\ell}$, to the magnetic induction at the center of the loop.



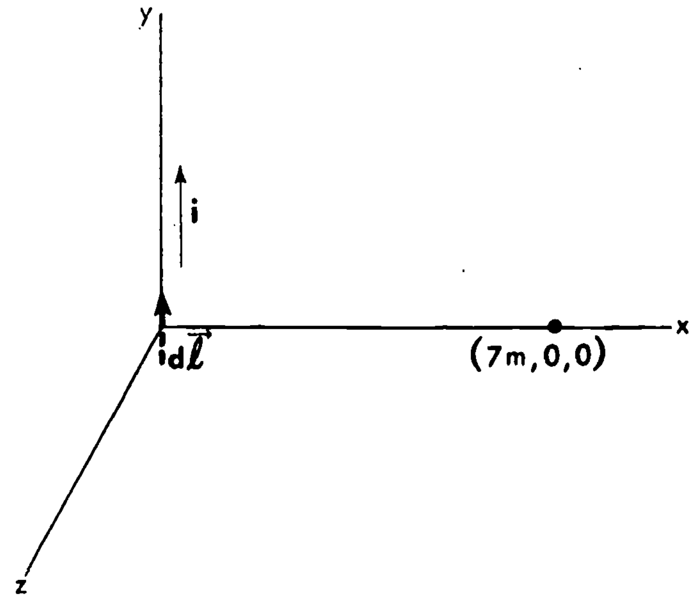
A. $dB = \frac{\mu_o i}{4\pi} \frac{d\ell}{r^3}$ out of the page

B. $dB = \frac{\mu_o i}{4\pi} \frac{d\ell}{r^3}$ into the page

C. $dB = \frac{\mu_o i}{4\pi} \frac{d\ell}{r^2}$ out of the page

D. $dB = \frac{\mu_o i}{4\pi} \frac{d\ell}{r^2}$ into the page

8. Find the magnetic induction at point (7.00 m, 0, 0) due to a current element 0.300 mm long which is oriented along the y-axis centered at the origin and carries a current $i = 4.00 \times 10^{-3}\ \hat{j}$ amp.



A. $-2.45 \times 10^{-15}\ \hat{k}$ T

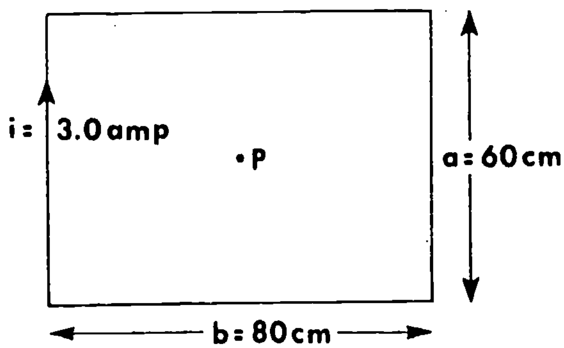
B. $2.45 \times 10^{-15}\ \hat{k}$ T

C. $-1.63 \times 10^{-11}\ \hat{k}$ T

D. $1.63 \times 10^{-11}\ \hat{k}$ T

9. In the Bohr model of the hydrogen atom the electron circulates around the nucleus in a path of radius 5.1×10^{-11} m at a frequency $f = 6.8 \times 10^{15}$ rev/sec. What value of $\vec{B}$ is set up at the center of orbit?

10. A rectangular loop having dimensions 60 cm × 80 cm carries a current of 3.0 amp in the clockwise sense. Find the magnetic induction at the center of the loop.



A. Zero

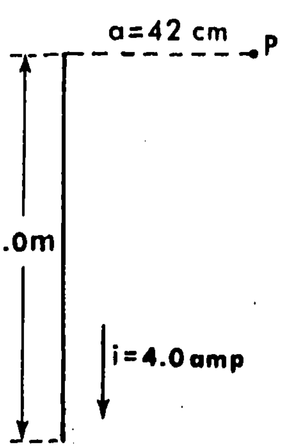
B. 9.0×10^{-7} T into the paper

C. 1.6×10^{-6} T into the paper

D. 5.0×10^{-6} T into the paper

11. An infinitely long, thin copper wire carries a current i = 25 amp. Using the Biot-Savart law, find the magnitude of the magnetic field $\vec{B}$ at a distance a = 2.0 m from the wire.

12. A conductor of length 6.0 m carries a current of 4.0 amp in the direction shown. Find the magnetic induction at point P, 42 cm along a line perpendicular to the wire at its upper end.



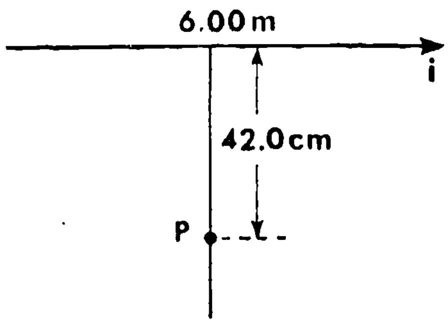
A. 6.7×10^{-8} T out of the paper

B. 9.5×10^{-7} T out of the paper

C. 1.9×10^{-6} T out of the paper

D. 1.4×10^{-5} T out of the paper

13. A conductor of length 6.00 m carries a current of 4.00 amp in the direction shown. Find the magnetic induction at point P, 42.0 cm along a line perpendicular to the wire at its midpoint.



A. 9.40×10^{-8} T into the paper

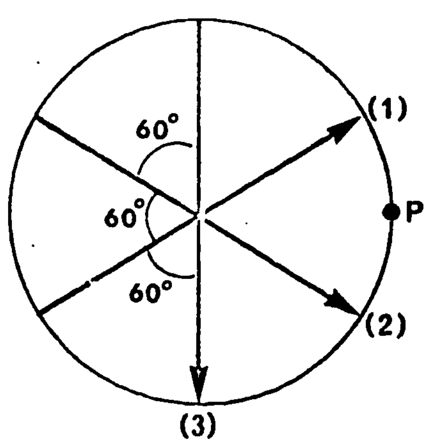
B. 2.65×10^{-8} T into the paper

C. 1.88×10^{-6} T into the paper

D. 1.90×10^{-6} T into the paper

14. A rigid wire bent into the form of a regular 18-sided polygon inscribed in a circle of radius 20.0 cm carries a current of 3.00 amp. Calculate the magnitude of the magnetic induction at the center of the polygon.

15. Three 10-m insulated wires, each carrying a current of 2.0 amp intersect at their midpoints making angles of 60° with respect to each other as shown in the diagram. Find the $\vec{B}$ field at point P due to the three conductors.



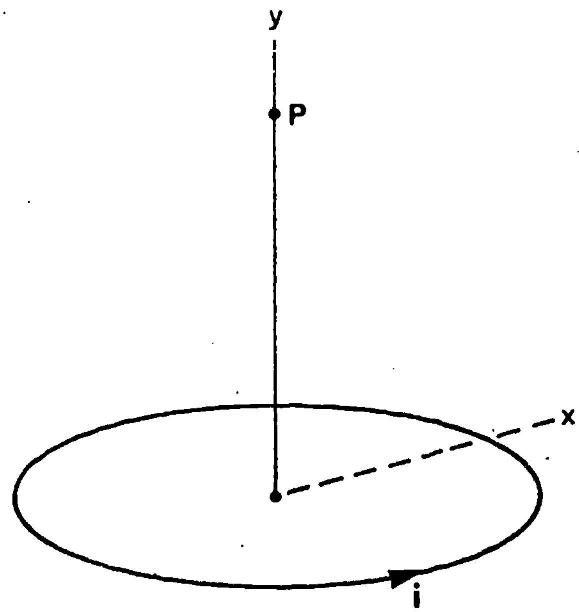
A. 2.8×10^{-8} T into plane of paper

B. 5.6×10^{-8} T into plane of paper

C. 2.8×10^{-8} T out of plane of paper

D. 5.6×10^{-8} T out of plane of paper

16. A circular loop of radius a is carrying a current i. What is the magnetic field $\vec{B}$ for points on the axis?



A. $\dfrac{\mu_0 i a^2}{2(a^2 + y^2)^{3/2}}\,\hat{j}$

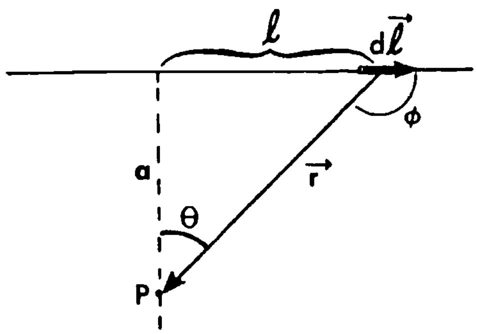
B. $\dfrac{\mu_0 i}{2(a^2 + y^2)}\,\hat{j}$

C. $-\dfrac{\mu_0 i}{2(a^2 + y^2)}\,\hat{j}$

D. $\dfrac{\mu_0 i}{2y}\,\hat{j}$

[a] CORRECT ANSWER: C

We use the Biot-Savart law, which gives the field at P due to the infinitesimal segment $d\vec{\ell}$ as



$$d\vec{B} = \frac{\mu_o i}{4\pi}\frac{d\vec{\ell} \times \vec{r}}{r^3} \quad (1)$$

The direction of $d\vec{B}$ is into the page at P and the magnitude is

$$dB = \frac{\mu_o i}{4\pi}\frac{d\ell}{r^2}\sin\phi = \frac{\mu_o i}{4\pi r^2}\cos\theta\, d\ell \quad (2)$$

We express $d\ell$ and r as functions of θ, where

$$\ell = a\tan\theta \qquad d\ell = a\sec^2\theta\, d\theta \qquad r = a/\cos\theta = a\sec\theta$$

Substituting these expressions into (2) we obtain

$$B = \frac{\mu_o i}{4\pi}\int \frac{a\sec^2\theta\cos\theta\, d\theta}{a^2\sec^2\theta} = \frac{\mu_o i}{4\pi a}\int \cos\theta\, d\theta \quad (3)$$

The upper and lower limits on θ are given respectively by

$$\tan^{-1}(\pm\ell/a) = \tan^{-1}[(\pm3.00\text{ m})/(0.420\text{ m})] = \tan^{-1}(\pm7.14) \quad (4)$$

$$= \tan^{-1}(7.14) \simeq \pm82°$$

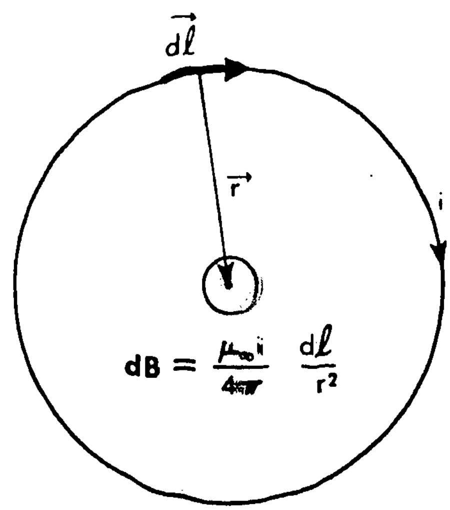
The integral of $\cos\theta$ is $\sin\theta$, so

$$B = \frac{\mu_o i}{4\pi a}\sin\theta\,\Big|_{-82°}^{82°} = \frac{\mu_o i}{4\pi a}[\sin82° - \sin(-82°)] = \frac{\mu_o i}{2\pi a}\sin82°$$

$$= \frac{4\pi \times 10^{-7} \times 4.00}{2\pi \times 0.420} \times 0.990 = 1.88 \times 10^{-6}\text{ T} \quad (5)$$

We notice that as the length of the wire increases, $B \rightarrow \mu_o i/2\pi a$ as it should.

[a] CORRECT ANSWER: B



The field due to a current carrying element $i d\vec{\ell}$ at the distance r is given by the Biot-Savart law:

$$d\vec{B} = \frac{\mu_o i}{4\pi} \cdot \frac{d\vec{\ell} \times \vec{r}}{r^3} \qquad (1)$$

Since $d\vec{\ell}$ and $\vec{r}$ are perpendicular to each other and $d\vec{B}$ is directed into the plane of the paper, we have

$$dB = \frac{\mu_o i}{4\pi} \frac{d\ell}{r^2} \qquad (2)$$

Therefore

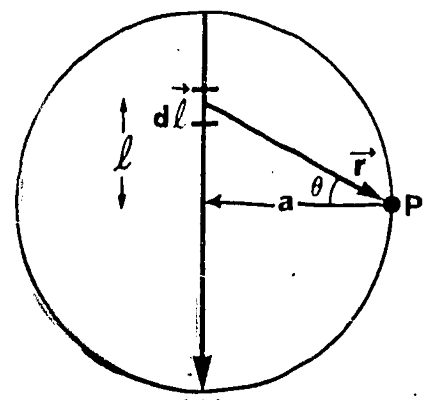
$$B = \int_0^{2\pi r} \frac{\mu_o i \, d\ell}{4\pi r^2} = \frac{\mu_o i}{4\pi r^2} \int_0^{2\pi r} d\ell = \frac{\mu_o i}{2r} \qquad (3)$$

TRUE OR FALSE? If the current direction had been counterclockwise, equation (1) would not have led directly to equation (2) in the solution.

[a] CORRECT ANSWER: D

According to the Biot-Savart law the magnetic field $d\vec{B}$ due to current carrying element $d\vec{\ell}$ at a distance r is given by

$$d\vec{B} = \frac{\mu_o i}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3} \qquad (1)$$



From symmetry, the contribution to the magnetic field at P from conductors (1) and (2) is zero. The direction of the $\vec{B}$ field due the remaining conductor, i.e. (3), is out of the plane of the paper. Therefore

$$dB = \frac{\mu_o i}{4\pi} \frac{d\ell \; r \cos\theta}{r^3}$$

$$= \frac{\mu_o i}{4\pi} \frac{\cos\theta \; d\ell}{r^2}$$

The magnitude of the $\vec{B}$ field due to the 10-m wire is

$$B = \int_{-5\text{ m}}^{5\text{ m}} \frac{\mu_o i}{4\pi} \frac{\cos\theta \; d\ell}{r^2}$$

From geometry we have

$$r = a \sec\theta \qquad \ell = a \tan\theta \qquad \text{and} \qquad d\ell = a \sec^2\theta \; d\theta$$

where

$$a = 5.0 \text{ m}$$

The limits of integration become $-\pi/4$ to $\pi/4$.

Hence equation (1) becomes

$$B = \frac{\mu_o i}{4\pi} \int_{-\pi/4}^{\pi/4} \frac{\cos\theta \; a \sec^2\theta \; d\theta}{a^2 \sec^2\theta}$$

next page

continued

or

$$B = \frac{\mu_o i}{4\pi a} \int_{-\pi/4}^{\pi/4} \cos\theta \, d\theta$$

Therefore

$$B = \frac{\mu_o i}{4\pi a} (2 \sin 45°)$$

$$= \frac{\mu_o i}{2\pi a} \frac{\sqrt{2}}{2}$$

$$= 5.6 \times 10^{-8} \text{ T}$$

and is out of the plane of the paper.

TRUE OR FALSE? If point P were not at the midpoint of the arc formed by conductors (1) and (2), this solution would not have been valid.

[a] CORRECT ANSWER: 13.4 T

The current is the rate at which charge passes any point on the orbit and is given by

$$i = ef = (1.6 \times 10^{-19} \text{ coul})(6.8 \times 10^{15} \text{ sec}^{-1}) = 1.09 \times 10^{-3} \text{ amp}$$

The magnetic field B at the center of the orbit may be calculated from the Biot-Savart law:

$$B = \int \frac{i\mu_o}{4\pi} \frac{d\ell}{R^2} = \frac{\mu_o i}{2R} = \frac{(4\pi \times 10^{-7})(1.09 \times 10^{-3})}{(2)(5.1) \times 10^{-11})} = 13.4 \text{ T}$$

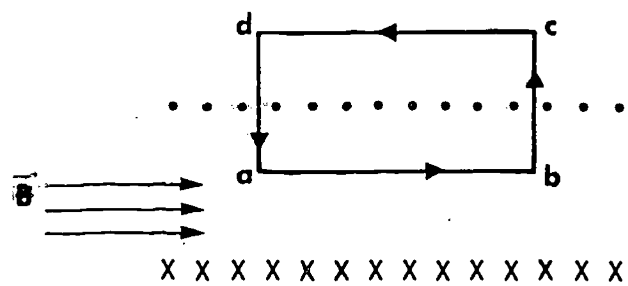
TRUE OR FALSE? If the frequency of revolution were half the value given in this problem, then the magnitude of $\vec{B}$ would be twice as great.

[a] CORRECT ANSWER: 7.5×10^{-3} T

Let us first calculate the magnitude of the magnetic field of an infinitely long solenoid by applying Ampere's law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o i$$

The integration is to be evaluated along the rectangular path abcd shown below



The integral may be written as

$$\oint \vec{B} \cdot d\vec{\ell} = \int_a^b \vec{B} \cdot d\vec{\ell} + \int_b^c \vec{B} \cdot d\vec{\ell} + \int_c^d \vec{B} \cdot d\vec{\ell} + \int_d^a \vec{B} \cdot d\vec{\ell}$$

The first integral on the right is Bh where B is the the magnitude of the magnetic field inside the solenoid and h is the arbitrary length of the path from a to b. The second and fourth integrals are zero because $\vec{B}$ is perpendicular to every path elements $d\vec{\ell}$. The third integral which includes the part of the rectangle that lies outside the solenoid is zero for all external points on an ideal solenoid. Thus, the integral is

$$\oint \vec{B} \cdot d\vec{\ell} = Bh = \mu_o i$$

where i is equal to i_o nh and n is the number of turns per unit length.

Thus, Ampere's law becomes

$$Bh = \mu_o i_o nh$$

or

$$B = \mu_o i_o n$$

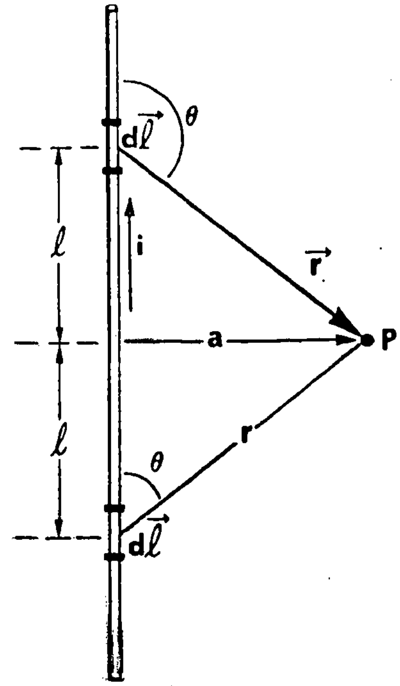
It is interesting to note that B does not depend on diameter for an infinitely long, ideal solenoid.

Substituting the given data, we obtain

$$B = \mu_o i_o n = 4\pi \times 10^{-7} \times 2 \times 6 \times 1000/2$$

$$= 7.5 \times 10^{-3} \text{ T}$$

[a] CORRECT ANSWER: 2.5×10^{-6} T



According to the Biot-Savart law, the magnetic field $d\vec{B}$ due to current carrying element $d\vec{\ell}$ at a distance r is given by

$$d\vec{B} = \frac{\mu_o i}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$$

If we assume the direction of the current as shown in the diagram, then the $\vec{B}$ field due to every element of the wire is into the plane of the paper on the right side.

Therefore

$$dB = \frac{\mu_o i}{4\pi} \frac{d\ell \ r \sin\theta}{r^3}$$

or

$$dB = \frac{\mu_o i}{4\pi} \frac{\sin\theta}{r^2} d\ell$$

The magnitude of the $\vec{B}$ field due to the infinite wire is

$$B = \int_{\ell = -\infty}^{\ell = \infty} \frac{\mu_o i}{4\pi} \frac{\sin\theta \ d\ell}{r^2} \qquad (1)$$

From geometry we have

$$r = a \csc\theta \qquad \ell = -a \cot\theta \qquad \text{and} \qquad d\ell = a \csc^2\theta \ d\theta$$

Hence equation (1) becomes

$$B = \frac{\mu_o i}{4\pi} \int_0^{\pi} \frac{\sin\theta \ a \csc^2\theta \ d\theta}{a^2 \csc^2\theta}$$

or

$$B = \frac{\mu_o i}{4\pi a} \int_0^{\pi} \sin\theta \ d\theta$$

Therefore,

$$B = \frac{\mu_o i}{2\pi a} \qquad (2)$$

$$B = 2.5 \times 10^{-6} \text{ T}$$

Note: The result could have been obtained using Ampere's law without integration due to symmetry of the configuration. However, the Biot-Savart law is useful in obtaining the $\vec{B}$ field for any curren distribution.

[a] CORRECT ANSWER: 4×10^{-3} T

From symmetry, we know that the magnetic field inside the toroid takes the form of concentric circles, each circle being uniform along its length. Applying Ampere's law to the circular path of integration of radius r, the magnetic field $\vec{B}_1$ inside the toroid is

$$\oint \vec{B}_1 \, d\vec{\ell} = \mu_0 i$$

or

$$B_1 \, 2\pi r = \mu_0 i_0 N$$

Solving for B_1 we obtain

$$B_1 = \frac{\mu_0 i_0 N}{2\pi r} \qquad (1)$$

In equation (1) the product $\mu_0 i_0 N$ is unknown but may be found from an expression for B inside a *solenoid*. Using Ampere's law, it can be readily shown that, inside a solenoid

$$B = \frac{\mu_0 i_0 N}{\ell} \qquad (2)$$

Evaluating equation (2) numerically we have

$$\mu_0 i_0 N = B\ell = (4 \times 10^3 \text{ T}) \times (0.7 \text{ m}) = 2.8 \times 10^3 \text{ T-m}$$

so that the field inside the toroid B_1 from equation (1) is

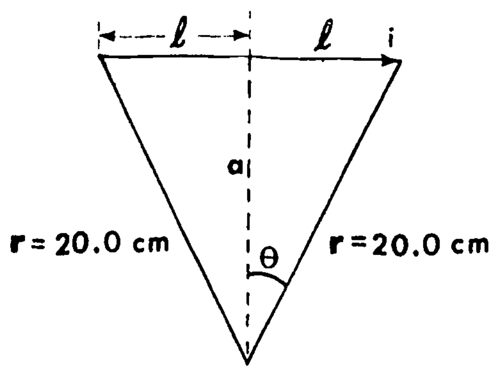
$$B_1 = \frac{\mu_0 i_0 N}{2\pi r} = 4 \times 10^{-3} \text{ T}$$

Note that the field inside a toroid depends on the distance from the center whereas in a solenoid the field is uniform.

TRUE OR FALSE? In a toroid, the value of $\vec{B}$ is the same for all values of r.

[a] CORRECT ANSWER: 9.51×10^{-6} T

We use the result of the preceding question for each side of the polygon. The contribution to $\vec{B}$ of each side of the polygon is directed into the page (for the choice of current sense shown) and has a magnitude



$$B_1 = \frac{\mu_0 i}{2\pi a} \sin\theta$$

where $a = r \cos\theta$ and

$$\theta = 2\pi/2n = \pi/n = \pi/18$$

Thus,

$$B_1 = \frac{\mu_0 i}{2\pi a} \tan\left(\frac{\pi}{18}\right)$$

The contributions of the 18 sides add algebraically, so

$$B = 18\,B_1 = \frac{\mu_0 i}{2\pi r}\,18\,\tan\left(\frac{\pi}{18}\right) = \frac{4\pi \times 10^{-7} \times 3.00 \times 18 \times 0.176}{2\pi \times 0.200}$$

$$= 9.51 \times 10^{-6}\ \text{T}$$

Note that

$$\lim_{N \to \infty} N \tan\left(\frac{\pi}{N}\right) = \lim_{N \to \infty} N \left(\frac{\pi}{N}\right) = \pi$$

and so $B \to \mu_0 i/2r$ as the number of sides of the polygon increases and it approaches a circular current loop of radius $r = 20.0$ cm. (For $N = 18$, $N \tan(\pi/n) = 3.17 \approx 3.14 = \pi$.) This agrees with the calculation for the field at the center of a circular current loop.

TRUE OR FALSE? For a polygon of 180 sides (instead of 18), we would have $N = 180$, $N \tan(\pi/n) \approx \pi$.

[a] CORRECT ANSWER: Zero

From symmetry, the $\vec{B}$ field lines form concentric circles outside the toroid. The magnitude of the $\vec{B}$ field at the point r may be calculated using Ampere's law. Let us choose a circle of radius r as the path of integration. Therefore,

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o i$$

$$= \mu_o (N i_o - N i_o)$$

or

$$\oint \vec{B} \cdot d\vec{\ell} = 0 \qquad (1)$$

The magnitude of $\vec{B}$ (if any) is constant at every point on the circle and further $\vec{B}$ and $d\vec{\ell}$ are always tangent to the path of integration. Therefore,

$$\oint \vec{B} \cdot d\vec{\ell} = B\ 2\pi r \qquad (2)$$

Equating equations (1) and (2) we find

$$B = 0$$

[b] CORRECT ANSWER: A

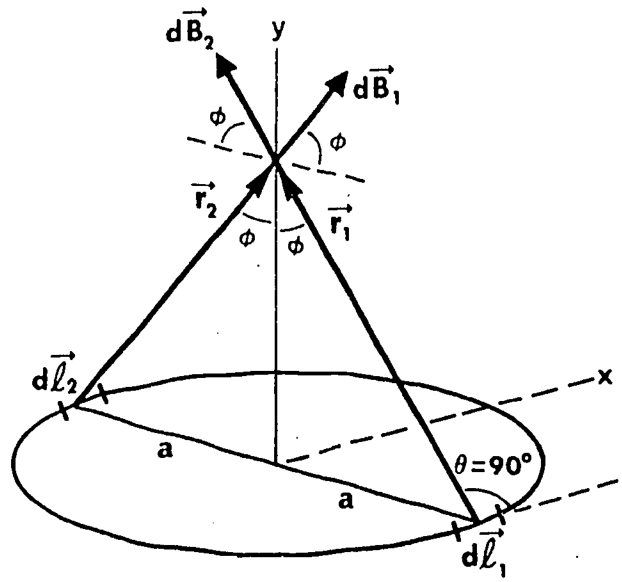
The current element is very short so we approximate $d\vec{\ell}$ by the element length 0.300 mm. Applying the Biot-Savart law directly, we obtain

$$d\vec{B} = \frac{\mu_o i}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3} = \frac{\mu_o i}{4\pi} \frac{d\ell\ \hat{j} \times \hat{i}}{x^2}$$

$$= -\frac{\mu_o i\ d\ell}{4\pi\ x^2} \hat{k} = -\frac{4\pi \times 10^{-7} \times 4.00 \times 10^{-3} \times 3.00 \times 10^{-4}}{4\pi\ (7.00)^2} \hat{k}$$

$$= -2.45 \times 10^{-15}\ \hat{k}\ T$$

[a] CORRECT ANSWER: A



The magnetic field due to an element $d\vec{\ell}_1$ at P is given by the Biot-Savart law as

$$d\vec{B}_1 = \frac{\mu_o i}{4\pi} \frac{d\vec{\ell}_1 \times \vec{r}_1}{r_1^{\,3}}$$

where

$$r_1 = \sqrt{y^2 + a^2}$$

and y is the distance from the plane of the loop. The angle between $d\vec{\ell}_1$ and $\vec{r}_1$ is 90° and the direction of $d\vec{B}_1$ is as shown in the diagram. The magnitude of dB_1 is thus

$$dB_1 = \frac{\mu_o i}{4\pi} \frac{d\ell_1}{r_1^{\,2}}$$

Let us now take another line element $d\vec{\ell}_2$ which is diametrically opposite to $d\vec{\ell}_1$. Using again the Biot-Savart law we obtain

$$dB_2 = \frac{\mu_o i}{4\pi} \frac{d\ell_2}{r_2^{\,2}}$$

and its direction is shown in the diagram. It is clear from the diagram that the horizontal components of $d\vec{B}_1$ and $d\vec{B}_2$ are of equal magnitudes but are pointing in opposite directions.

Thus the horizontal components will cancel out in pairs and only the axial components will contribute to $\vec{B}$ at P. Thus,

$$B = \int dB_{axial}$$

where

$$dB_{axial} = \frac{\mu_o i}{4\pi} \frac{d\ell}{r^2} \sin\phi = \frac{\mu_o i}{4\pi} \frac{d\ell}{r^2} \cdot \frac{a}{r}$$

next page

continued

and the integration over $d\ell$ yields B:

$$B = \int dB_{axial} = \frac{\mu_o i}{4\pi} \frac{a}{r^3} \int d\ell = \frac{\mu_o i\ a^2}{2r^3}$$

Substituting

$$r = \sqrt{y^2 + a^2}$$

we obtain finally

$$B = \frac{\mu_o i\ a^2}{2(y^2 + a^2)^{3/2}}$$

pointing in the +y direction.

TRUE OR FALSE? The axial components of $d\vec{B}_1$ and $d\vec{B}_2$ are equal in magnitude and opposite in direction.

[a] CORRECT ANSWER: C

From symmetry considerations, the $\vec{B}$ field lines form concentric circles inside the toroid. The magnitude of $\vec{B}$ at point r may be calculated using Ampere's law. Let us choose a circle of radius r as the path of integration. Therefore,

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o i$$

$$= \mu_o N i_o \qquad (1)$$

The magnitude of $\vec{B}$ is constant at every point on the circle and further $\vec{B}$ and $d\vec{\ell}$ are always tangent to the path of integration. Therefore,

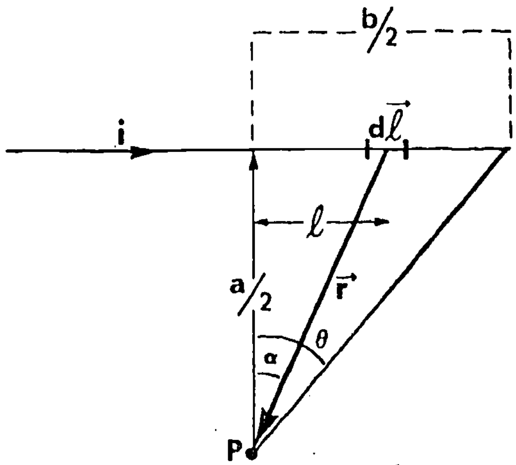
$$\oint \vec{B} \cdot d\vec{\ell} = B\ 2\pi r \qquad (2)$$

Equating equations (1) and (2) we find

$$B = \frac{\mu_o N i_o}{2\pi r}$$

TRUE OR FALSE? For a given circle of integration in the toroid, the magnitude of $\vec{B}$ is constant along its entire length.

[a] CORRECT ANSWER: D



According to the Biot-Savart law, the magnetic field $d\vec{B}$ due to a current carrying element $d\vec{\ell}$ at a position r is given by

$$d\vec{B} = \frac{\mu_o i}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^2}$$

or

$$dB = \frac{\mu_o i}{4\pi} \frac{\cos\alpha \; d\ell}{r^2}$$

and is directed into the plane of the paper.

Therefore, the contribution to the magnetic field due to the top side is

$$B_t = \frac{2\mu_o i}{4\pi} \int_0^\theta \frac{\cos\alpha \; d\ell}{r^2} \qquad (1)$$

However,

$$\ell = (a/2)\tan\alpha \qquad d\ell = (a/2)\sec^2\alpha d\alpha \qquad r = (a/2)\sec\alpha$$

Substituting these expressions into equation (1) we find

$$B_t = \frac{2\mu_o i}{4\pi} \int_0^\theta \frac{\cos\alpha \; a \sec^2\alpha d\alpha}{2(a^2/4)\sec^2\alpha}$$

$$B_t = \frac{\mu_o i}{\pi a} \sin\theta$$

A similar calculation applies to each side. We note that

$$\sin\theta = \frac{b/2}{\sqrt{(b/2)^2 + (a/2)^2}} = \frac{b}{\sqrt{a^2 + b^2}}$$

so

$$B_t = \frac{\mu_o i}{\pi} \frac{b}{\sqrt{a^2 + b^2}}$$

next page

continued

The bottom side has the same geometry as the top side; therefore, $B_b = B_t$. And for the left and right sides, we need only interchange b and a to obtain

$$B_\ell = B_r = \frac{\mu_o i}{\pi} \frac{a/b}{\sqrt{a^2 + b^2}}$$

The direction of the field produced by each side is into the paper for a clockwise current, and the contributions add arithmetically. The total magnetic induction at the center of the loop is

$$B = \frac{\mu_o i}{\pi} \frac{2}{\sqrt{a^2 + b^2}} \left(\frac{b}{a} + \frac{a}{b}\right) = \frac{\mu_o i}{\pi} \frac{2\sqrt{a^2 + b^2}}{ab}$$

Finally, substituting the given numerical values, we find

$$B = \frac{4\pi \times 10^{-7} \times 3}{\pi} \times \frac{2\sqrt{0.36 \times 0.64}}{0.6 \times 0.8} = 5.0 \times 10^{-6}\ \mathrm{T}$$

TRUE OR FALSE? In the above solution, the angle α is assigned a value such that $\alpha = \theta/2$.

[a] CORRECT ANSWER: 1.6×10^{-2} amp

In the formula derived in the preceding problem for the magnetic induction inside an ideal solenoid, $B = \mu_o i_o n$, n is the number of turns per unit length. For a solenoid wound with 4 layers of 250 turns each and of length 20 cm,

$$n = \frac{4 \times 250}{0.2} = 5000\ \mathrm{m}^{-1}$$

Therefore, the current required to produce a field of 10^{-4} T is

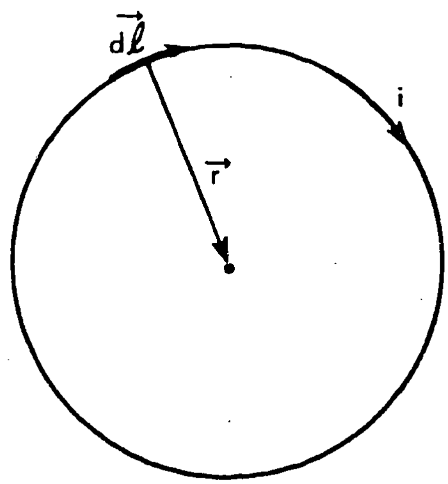
$$i_o = \frac{B}{\mu_o n} = \frac{10^{-4}}{4\pi \times 10^{-7} \times 5 \times 10^3} = 1.6 \times 10^{-2}\ \text{amp}$$

[a] CORRECT ANSWER: D

The Biot-Savart law states that

$$d\vec{B} = \frac{\mu_o i}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$$

Let us redraw the diagram and label $d\vec{\ell}$ and $\vec{r}$.

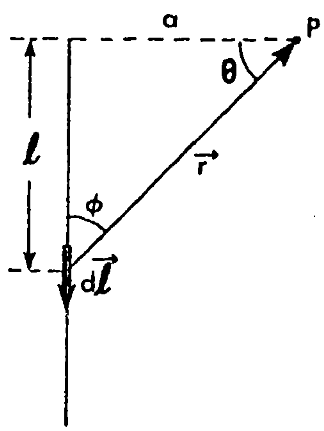


The vector $d\vec{\ell}$ is in the direction of i; $\vec{r}$ is the vector from the element to the field point. The direction of $d\vec{B}$ is that of $d\vec{\ell} \times \vec{r}$; i.e., into the page. Since $d\vec{\ell}$ and $\vec{r}$ are perpendicular, the magnitude of $d\vec{B}$ is

$$dB = \frac{\mu_o i}{4\pi} \frac{|d\vec{\ell} \times \vec{r}|}{r^3} = \frac{\mu_o i}{4\pi} \frac{d\ell}{r^2}$$

[a] CORRECT ANSWER: B

We use the Biot-Savart law. The field at P due to the infinitesimal segment of the wire $d\ell$ is



$$d\vec{B} = \frac{\mu_o i}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$$

Now $d\vec{\ell} \times \vec{r}$ is out of the page and from the diagram we see that

$$|d\vec{\ell} \times \vec{r}| = r\, d\ell \sin\phi$$
$$= r\, d\ell \cos\theta$$

Thus,

$$B = \frac{\mu_o i}{4\pi} \int \frac{d\ell\ r \cos\theta}{r^3} = \frac{\mu_o i}{4\pi} \int \frac{d\ell \cos\theta}{r^2}$$

Let us express the variables ℓ and r as functions of θ.

$$\ell = a \tan\theta$$

$$d\ell = a \sec^2\theta\, d\theta$$

$$r = a/\cos\theta = a \sec\theta$$

Therefore,

$$B = \frac{\mu_o i}{4\pi} \int \frac{a \sec^2\theta \cos\theta\, d\theta}{a^2 \sec^2\theta} = \frac{\mu_o i}{4\pi a} \int_{0°}^{86°} \cos\theta\, d\theta$$

where we found the upper limit of θ from

$$\tan^{-1}\left(\frac{\ell}{a}\right) = \tan^{-1}\left(\frac{6\ m}{0.42\ m}\right) = \tan^{-1}(14.3) = 86°$$

The integral of $\cos\theta$ is $\sin\theta$, so

$$B = \frac{\mu_o i}{4\pi a} \sin\theta \Big|_{0°}^{86°} = \frac{\mu_o i}{4\pi a} \sin 86° = \frac{\mu_o i}{4\pi a} \times 0.9976$$

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continued

Notice that as ℓ increases,

$$B \to \frac{\mu_o i}{4\pi a}$$

which is just one-half the value of the field due to an infinitely long conductor. This is reasonable, since this wire is "half" of a very long conductor. The numerical value of B at a = 0.42 m is

$$B = \frac{\mu_o i}{4\pi a} \times 0.9976 = \frac{4\pi \times 10^{-7} \times 4 \times 0.9976}{4\pi \times 0.42} = 9.5 \times 10^{-7}\ \text{T}$$