DOCUMENT RESUME

ED 075 245 | 24 | SE 016 074

TITLE	Self-Paced Physics, Segments 28-31.
INSTITUTION	Naval Academy, Annapolis, Md.; New York Inst. of Tech., Old Westbury.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO	BR-8-0446
PUB DATE	70
CONTRACT	N00600-68-C-0749
NOTE	126p.
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	College Science; Electricity; *Instructional Materials; *Physics; *Problem Sets; Problem Solving; *Remedial Programs; Science Education; *Self Help Programs
IDENTIFIERS	Self Paced Instruction

ABSTRACT

Four study segments of the Self-Paced Physics Course materials are presented in this sixth problems and solutions book used as a part of student course work. The subject matter is related to electric currents, current densities, resistances, Ohm's law, voltages, Joule heating, electromotive forces, single loop circuits, series and parallel circuits, Kirchhoff's rules, ammeters, galvanometers, potentiometers, and Wheatstone Bridges. Contained in each segment are an information panel, core problems enclosed in a box, core-primed questions, scrambled problem solutions, and true-false questions. Study guides are provided to answer the true-false questions and to reveal directions for reaching solutions. When the core problem is answered incorrectly, the study guide requires students to follow the remedial or enabling loop, leading to the solutions of core-primed questions. Also included is a sheet of problem numbers with corresponding page numbers which locate correct answers. (Related documents are SE 016 065 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

UNITED STATES NAVAL ACADEMY

Self-Paced PHYSICS

SEGMENTS 28-31

DEVELOPED AND PRODUCED UNDER THE
U.S. OFFICE OF EDUCATION, BUREAU OF RESEARCH,
PROJECT #8-0446, FOR THE U.S. NAVAL ACADEMY
AT ANNAPOLIS. CONTRACT #N00600-68C-0749.

NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY

P	STEP	NAME
	0.1	Reading: *HR 31-1/31-3 SZ 28-1/28-3; 25-5 AB 40-1/40-3
	0.2	Information Panel, "Introduction to the Study of the Electric Current"
1		A B C D T F
	1.1	If your first choice was correct, advance to 5.1; if not, continue sequence.
2		A B C D
3		A B C D
4		A B C D
5		A B C D T F
	5.1	Information Panel, "Current and Current Density"
6		T F (ans)
	6.1	If correct, advance to 9.1; if not, continue sequence.

P	STEP	SECTION SEGMENT 28
7		A B C D
8		A B C D
9		T F (ans)
	9.1	Information Panel, "Resistance and Resistivity"
10		T F (ans)
	10.1	If correct, advance to 14.1; if not, continue sequence.
11		A B C D
12		A B C D
13		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 28
14		T F (ans)			
	14.1	Information Panel, "Applications of Ohm's Law"			
15		T F (ans)			
	15.1	If correct, advance to 19.1; if not, continue sequence.			
16		A B C D			
17		(ans)			
18		(ans)			
19		T F (ans)			
	19.1	Homework: HR 31-8			

P	STEP	NAME
	0.1	Reading: *HR 31-5; 32-1/32-4 *SZ 28-7
	0.2	Information Panel, "Energy Transfers in an Electric Circuit"
1		T F (ans)
	1.1	If correct, advance to 4.1; if not, continue sequence.
2		A B C D
3		A B C D
4		T F (ans)
	4.1	Information Panel, "Problems in Joule Heating"
5		T F (ans)
	5.1	If correct, advance to 8.1; if not, continue sequence.
6		(ans)

P	STEP	SECTION SEGMENT 29
7		(ans)
8		T F (ans)
	8.1	Information Panel, "Electromotive Force"
9		A B C D T F
	9.1	If your first choice was correct, advance to 14.1; if not, continue sequence.
10		A B C D
11		A B C D
12		A B C D
13		A B C D

P	STEP	NAME
14		A B C D T F
	14.1	Information Panel, "Single Loop Circuits - One Seat of Emf"
15		T F (ans)
	15.1	If correct, advance to 20.1; if not, continue sequence.
16		A B C D
17		A B C D
18		(ans)
19		(ans)
20		T F (ans)
	20.1	Information Panel, "Single Loop Circuits - Two or More Seats of Emf"

P	STEP	SECTION SEGMENT 29
21		T F (ans)
	21.1	If correct, advance to 24.1; if not, continue sequence.
22		A B C D
23		A B C D
24		T F (ans)
	24.1	Homework: HR 32-6

P	STEP	NAME
	0.1	Reading: *HR 32-3/32-5 *SZ 29-1, 29-2 SW 32-1/32-3
	0.2	Information Panel, "Basic Characteristics of Series and Parallel Circuits"
1		A B C D T F
	1.1	If your first choice was correct, advance to 4.1; if not, continue sequence.
2		A B C D
3		A B C D
4		A B C D T F
	4.1	Information Panel, "Equivalent Resistance"
5		T F (ans)
	5.1	If correct, advance to 9.1; if not, continue sequence.
6		(ans)

P	STEP	SECTION	SEGMENT 30
7		(ans)	
8		(ans)	
9		T F (ans)	
	9.1	Information Panel, "Problems Involving Equivalent Resistance"	
10		A B C D T F	
	10.1	If your first choice was correct, advance to P 16; if not, continue sequence.	
11		(ans)	
12		(ans)	
13		(ans)	

P	STEP	NAME
14		A B C D
15		T F (ans)
16		T F (ans)
	16.1	If correct, advance to 18.1; if not, continue sequence.
17		(ans)
18		A B C D T F
	18.1	Information Panel, "Using Kirchhoff's Rules"
	18.2	Audiovisual, KIRCHHOFF'S RULES
19		A B C D T F
	19.1	If your first choice was correct, advance to P 23; if not, continue sequence.
20		A B C D

P	STEP	SECTION SEGMENT 30
21		A B C D
22		A B C D T F
23		T F (ans)
	23.1	If correct, advance to 26.1; if not, continue sequence.
24		(ans)
25		(ans)
26		T F (ans)
	26.1	Homework: HR 32-22

P	STEP	NAME
	0.1	Reading: *HR 32-6, 32-7 *SZ 29-3/29-6 SW 32-4
	0.2	Information Panel, "The Ammeter"
1		T F (ans)
	1.1	If correct, advance to 6.1; if not, continue sequence.
2		A B C D
3		A B C D
4		A B C D
5		(ans)
6		T F (ans)
	6.1	Information Panel, "The Voltmeter"

P	STEP	SECTION SEGMENT 31
7		T F (ans)
	7.1	If correct, advance to 10.1; if not, continue sequence.
8		A B C D
9		(ans)
10		T F (ans)
	10.1	Information Panel, "The Potentiometer"
11		T F (ans)
	11.1	If correct, advance to 14.1; if not, continue sequence.
12		A B C D
13		(ans)

P	STEP	NAME	P	STEP	SECTION SEGMENT 31
14		T F (ans)			
	14.1	Information Panel, "The Wheatstone Bridge"			
15		T F (ans)			
	15.1	If correct, advance to 18.1; if not, continue sequence.			
16		(ans)			
17		A B C D			
18		T F (ans)			
	18.1	Homework: SZ 29-18			

INFORMATION PANEL Introduction to the Study of the Electric Current

OBJECTIVE

To recognize the conditions required for an electric current, and relate such a current to the electric field or potential gradient in the conductor.

When electric charges are at rest in an object they are referred to as static charges. If the total charge comprises equal numbers of negative and positive charges, then the body is said to be neutral. A charge is detectable by means of an electroscope or a similar instrument only if there is a net positive or negative charge produced by an excess or deficiency of one of the charge types.

An isolated electrical conductor contains a large number of electrons that are so loosely bound to atomic nuclei that they may be called "free" electrons. Such electrons are in constant random motion so that the total number passing through a given cross section of the conductor in one direction is, on the average, equal to the total number passing through it in the opposite direction at any instant. This is analogous to the condition of electrostatic neutrality mentioned above since there is no net flow of electrons in the conductor, hence, no detectable effect due to charge motion. In short, in order to observe a flow of charge, it is necessary that the total number of charges per unit time moving in one direction be greater than the total number per unit time moving in the opposite direction through the same cross section.

When a conductor that is insulated from its surroundings is placed in an electric field, the charges in the conductor do undergo a net motion in the process of rearranging themselves so as to make the inside of the conductor a field-free region. When this process is completed, the charges again come to rest showing that there is no difference of potential between various points in the interior of the conductor. Thus, during the interval when the charges are shifting their positions, there is a short-duration electric current called a *transient*. However, this current ceases when the electric field inside the conductor becomes zero and the potential becomes constant.

To maintain a steady flow of charge or a *continuous* current in a metal wire, the external source of the electrostatic field must be capable of preventing the field within the conductor from dropping to zero. That is, the source must maintain a potential difference between two given points in the wire; this is tantamount to maintaining a potential gradient throughout the length of conductor between the points in question.

next page

continued

When an electric field is maintained in the manner described, this field $\vec{E}$ will give rise to a force that acts on the free electrons, giving them a resultant motion in the direction of $-\vec{E}$. This establishes an electric current i which is related to the net charge q passing through any cross section in a given time t by the expression:

$$i = q/t$$

assuming a constant current. Thus, an electric current may be defined as the *rate of flow of charge past any cross-sectional plane in a conductor.*

In this section you will encounter problems which require that you

(a) recognize that a continuous current can be established if there is an electric field or a potential gradient in the interior of the conductor;

(b) recognize that the electric field inside an isolated conductor is zero;

(c) recognize that the net motion of charges in an isolated conductor is zero.

PROBLEMS

1. A continuous current will be present in a metallic conductor if

   A. a continuous field or potential gradient is maintained within it

   B. the conductor has a connection to ground

   C. the conductor has an induced charge on its surface

   D. charges in the conductor are free to move

2. Which of the following is a true statement about electrons in an isolated, uncharged metallic conduc

A. The electrons are always o the surface of the conductor.

B. Their net effect is such that they can be considered to be at the center of the conductor.

C. They are in random motion, so that the net motion in any direction is zero.

D. The electrons are permanently attached to the positive ions comprising the lattice until an electric field is applied.

3. The electric field intensity inside a solid, isolated, negatively-charged conductor is:

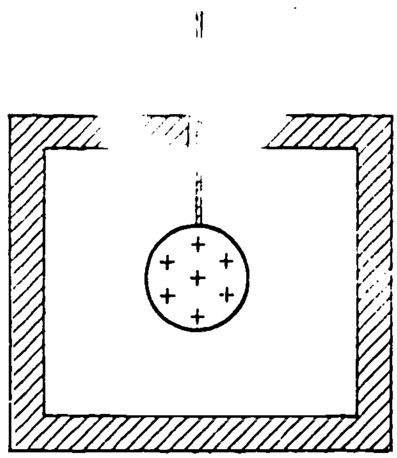
A. constant in magnitude and pointing away from the center of the conductor

B. constant in magnitude and pointing towards the center of the conductor

C. pointing towards the center and decreasing in magnitude as one goes away from the surface, the center having zero magnitude

D. zero everywhere

4. The figure depicts a positively charged metal sphere hanging from a non-conducting string. The string passes through a small hole in the lid of a metal container. What will happen when the charged sphere touches the inside walls of the container?



A. All the charge will leave the sphere and will distribute itself uniformly on the exterior surface of the container.

B. All the charge will leave the sphere and distribute itself uniformly throughout the walls of the container.

C. Half of the charge will leave the sphere and will distribute itself uniformly on the exterior surface of the container.

D. All the charge will remain on the sphere.

5. A continuous field or potential gradient is maintained in a conductor when properly connected to

A. a resistor
B. a capacitor
C. a battery or a generator
D. ground

INFORMATION PANEL — Current and Current Density

OBJECTIVE

To relate the magnitude of an electric current to the rate of transfer of charge; to define current density.

For a constant current,

$$i = q/t$$

but when the current varies with time, its instantaneous value is given by the differential limit of this expression or,

$$i = dq/dt$$

The MKS unit of electric current, *one coulomb per second*, is called the ampere and is abbreviated amp. Small currents may be more conveniently given in terms of the

milliampere (ma) where 1 ma = $10^{-[illegible]}$ [illegible]p

or

microampere (μa) where 1 μa = $10^{-[illegible]}$ amp

In the literature describing *very* small currents, you will encounter the nanoampere (10^{-9} amp) and the picoampere (10^{-12} amp). Current is a scalar quantity.

Current density j for a current that is uniformly distributed throughout the body of the conductor is given by the following relation (for a given cross section):

$$j = i/A$$

so that current density is essentially current magnitude per unit cross-sectional area. Current density is a *vector* quantity whose magnitude is given by the last equation. The direction of $\vec{j}$ is the direction in which a positive charge would move at that point, hence, an electron at the same point would move in the direction $-\vec{j}$.

next page

continued

Despite the fact that we recognize the electron as the charge carrier in metals, we shall continue to consider all charge carriers as *positive* in order not to conflict with definitions given for the direction of the electric field and potential gradient. Thus, the arrow showing the direction of an electric current in a wire is drawn in the direction that a positive charge would move in that field.

The problems in this section entail understandings associated with the definition of the electric current, charge motion, current density, and the units used to measure charge and current in the MKS system.

6. Current enters a cylindrical wire of diameter 1/4 in., the current density being 80 amp/m^2. The wire eventually tapers down to a diameter of 1/16 in. What is the current density in this thinner portion of the wire, in amp/m^2?

7. What is the relationship between current (i) and the rate of transfer of charge dq/dt?

A. $i = C(dq/dt)$ where C is the capacitance

B. $i = \rho(dq/dt)$ where ρ is the resistivity

C. $i = R(dq/dt)$ where R is the resistance

D. $i = dq/dt$

8. The unit of electric current is the ampere. One ampere is the equivalent of:

A. one coulomb of charge

B. the difference between the number of coulombs/second entering a conductor and the number of coulombs/second leaving the conductor

C. the flow of charge through any cross section of a conductor at the rate of 6.242×10^{18} electronic charges per second

D. the flow of charge through a conductor at the rate of one coulomb per second passing through a cross-sectional area of one square centimeter

9. A wire of 10 cm^2 cross section has 100 amperes of current in it. What is the magnitude of the current density in the wire?

INFORMATION PANEL — Resistance and Resistivity

OBJECTIVE

To define and relate the quantities resistance and resistivity.

Consider a specimen of substance A having a unit length (say 1 cm) and unit cross-sectional area (say 1 cm^2) placed in an electric field of intensity E. As a result, a definite magnitude of current density will appear in the conductor as long as the field is maintained in it. Now hypothetically replace substance A with substance B, the specimen having the same dimensions as the previous one, in a field of the same intensity. Suppose in this case that the resulting current density was found to be one-tenth that of the previous value. It would then be reasonable to conclude that substance B offered an *opposition* to the flow of electric charge that was effectively ten times greater than the opposition set up by substance A.

next page

continued

The opposition to the flow of charge set up by a given substance is a *characteristic of the substance itself*, all other factors remaining constant. It is given the name *resistivity* and is operationally defined as the ratio of the electric intensity to the current density:

$$\text{resistivity} = \rho = E/j$$

Thus, resistivity is electric intensity per unit current density. It follows from this that a greater intensity is needed to establish a specified current density in a subtance of high resistivity than in one of low resistivity. Saying this another way: a good conductor such as copper or silver has a very low resistivity since it is easy to establish a relatively large current density with a small electric field intensity. Similarly, a poor conductor like glass or bakelite has a high resistivity. For example, common glass has a resistivity that is at least 20 orders of magnitude greater than that of silver!

> *Thus, resistivity ρ is a characteristic attributed to a substance or material rather than a specific sample of material.*

Next, consider a certain cylindrical rod of given length, thickness, and temperature; a potential difference V is applied across its ends and the current i through it measured. The magnitude of the current will be governed, at least partially, by the resistivity of the material of which the rod is fabricated, but since it will be found that the other factors such as length and thickness will play a part in establishing the current level, it is convenient to define another term: resistance R.

> *The resistance R of a given sample of material at a given temperature is defined as the ratio of the potential difference across the ends of the sample to the current thereby established.*

$$R = V/i$$

The unit of resistance, equivalent to the volt per ampere, is called the *ohm*. To establish a mental picture of the magnitude of one ohm let us state that the resistance of 1,000 ft of number 10 gauge soft drawn copper wire at 20° C is very nearly 1 ohm.

As you have seen in your required reading, resistance and resistivity are related by:

$$R = \rho \frac{\ell}{A}$$

next page

continued

in which ℓ is the length of the conductor and A its cross-sectional area. To obtain the unit for resistivity, solve this equation for ρ and substitute the units for the other terms.

$$\rho = \frac{RA}{\ell}$$

$$= \frac{\text{ohm-meters}^2}{\text{meters}} = \textit{ohm-m}$$

Clearly, at a given temperature, the resistance R of a specimen is directly proportional to the length ℓ of the specimen and inversely proportional to its cross-sectional area A.

This section includes questions and problems in which you will be expected to

(a) relate a change of resistance to a change of length and cross-sectional area;

(b) define resistivity correctly;

(c) define and use the relation $R = \rho \frac{\ell}{A}$

(d) recognize that $R = V/i$ is valid for either linear or nonlinear conductors.

10. A wire with a resistance of 9.0 ohms is drawn out so that its new length is three times its original length. Find the new value of its resistance, assuming that the resistivity and the density of the material are not changed during the drawing process.

11. The equation defining the resistivity (ρ) of a conducting, electrically isotropic material is

A. $\rho = \frac{E\ell}{j}$ where: E is the electric field strength, j is the current density, and ℓ is the length of the conductor

B. $\rho = \frac{j}{E}$ where: E is the electric field strength and j is the current density.

C. $\rho = \frac{Ej}{\ell}$ where: E is the electric field strength, j is the current density, and ℓ is the length of the conductor.

D. $\rho = \frac{E}{j}$ where: E is the electric field strength and j is the current density.

12. Knowing that $E = V/\ell$ and $j = i/A$, derive a formula relating resistance R to resistivity ρ, where ℓ is the length of the conductor and A is the cross-sectional area of the conductor.

A. $R = \rho \frac{\ell}{A}$

B. $R = \rho \frac{A}{\ell}$

C. $R = \frac{A\ell}{\rho}$

D. $R = \frac{A}{\rho\ell}$

13. Which of the following is true of both ohmic (linear) and nonohmic (nonlinear) conductors?

A. $R = \frac{V}{i}$

B. $V \propto i$

C. Resistance is a constant at a given temperature

D. $\frac{V}{i}$ = constant

14. A copper wire and iron wire of the same length have the same potential difference applied to them. What must be the ratio of their radii (copper to iron) if the current is to be the same at 20°C? The resistivity of copper and iron are 1.7×10^{-8} ohm-m and 1.0×10^{-7} ohm-m, respectively, at 20°C and you may assume the wire to be a cylinder.

INFORMATION PANEL — Applications of Ohm's Law

OBJECTIVE

To solve basic problems in which Ohm's law is used to relate various electrical parameters.

Unlike Newton's laws of motion, or the conservation principles, Ohm's law is not a law of nature. Ohm's law describes a certain special property of some materials, but cannot be applied to many others.

In many conductors, notably metals, the resistivity remains constant despite changes in potential difference or current as long as the temperature is held constant. For such conductors, the current is directly proportional to the potential difference with the resistance serving as the proportionality constant in accordance with:

$$i = V/R$$

next page

continued

Thus, if current is plotted against applied potential difference (voltage), the graph is a straight line with positive slope passing through the origin of the axes. Such conductors are know as *ohmic* or *linear* conductors; materials that do not obey Ohm's law are referred to as *nonohmic* or *nonlinear*.

The problems and questions in this section make use of the relationships that exist between current, current density, the factors that govern the resistance of a specimen of a given material, potential difference, and the fact that current is the rate of flow of charge. You will be asked to

(a) calculate current density for a wire of given length, cross-sectional area, and current density;

(b) determine how the current in a conductor changes when specified changes are made in its length, cross-sectional area, and the potential difference applied across its ends;

(c) find the potential difference applied to a conductor when its resistance is known, and the total charge flowing through it in a given time is also specified;

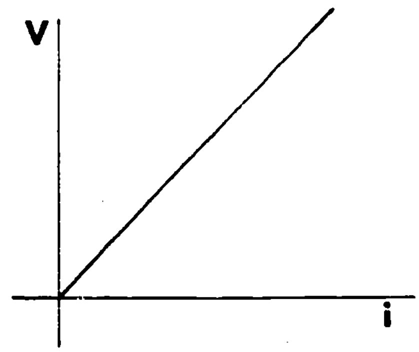
(d) recognize the graph which applies to a linear conductor.

15. A current of 2 amp exists in a wire 2 m long and 2 mm in diameter, when a 12-volt battery is connected across it. What will be the current through a wire 4 m long and 4 mm in diameter, made up of exactly the same material (same ρ), if a 6-volt battery is connected across it?

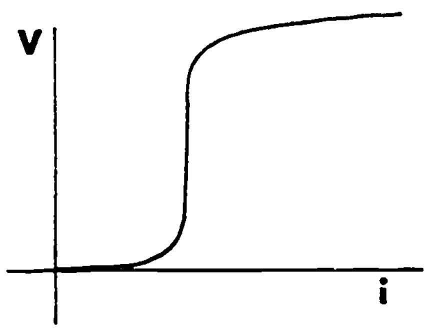
16. Which of the following curves of voltage versus current represents a conductor which obeys Ohm's law?

A.

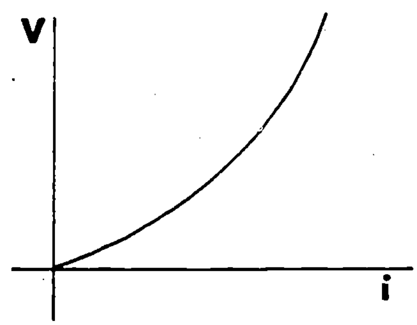
C.



B.



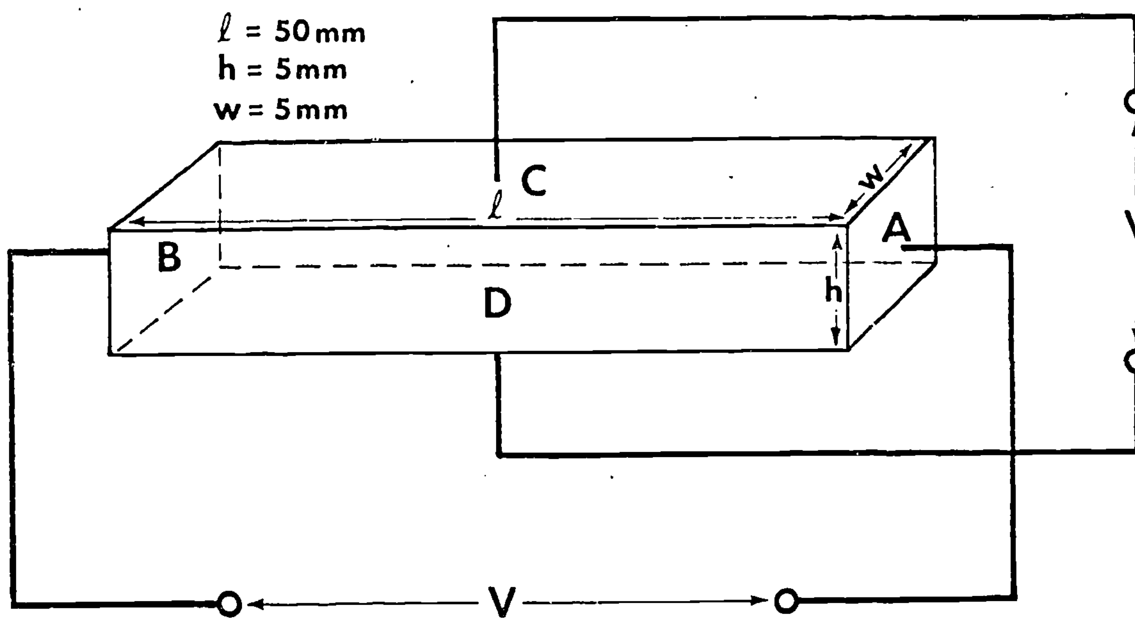
D.



17. A potential difference of 2 V is applied across a cylindrical conductor 50 cm long and 1 mm in diameter. The resistivity of the conductor is 10^{-6} Ω-m. Calculate the magnitude of the current density in the conductor in amp/m^2.

18. How much voltage must be applied across a 2-ohm resistor if 120 coulombs are to flow through the resistor in one minute?

19. A rectangular (solid) copper bar of resistivity ρ has dimensions of 50 mm × 5 mm × 5 mm. Find the ratio of electric current when a potential difference of V is maintained across surfaces A and B, to that when the same potential difference is maintained across surfaces C and D.



[a] CORRECT ANSWER: 0.41

The resistivity of a material is defined as

$$\rho = \frac{E}{j} \tag{1}$$

and E and j may be rewritten as

$$E = \frac{V}{\ell} \qquad \text{and} \qquad j = \frac{i}{A} \tag{2}$$

Thus

$$\rho = \frac{E}{j} = \frac{\frac{V}{\ell}}{\frac{i}{A}} \tag{3}$$

For a cylindrical wire, the cross section A is simply πr^2. Thus, taking the ratios of resistivity of copper and iron, we have

$$\frac{\rho_1}{\rho_2} = \frac{\frac{V}{\ell} \Big/ \frac{i}{A_1}}{\frac{V}{\ell} \Big/ \frac{i}{A_2}} = \frac{A_1}{A_2} = \frac{r_1{}^2}{r_2{}^2} \tag{4}$$

since V, ℓ and i are the same.

Thus,

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_1}{\rho_2}} \tag{5}$$

Substituting numerical data, we obtain

$$\frac{r_{copper}}{r_{iron}} = \sqrt{0.17} = 0.41$$

TRUE OR FALSE? For a wire of square cross section, step 4 in the above solution would have to be modified.

[a] CORRECT ANSWER: D

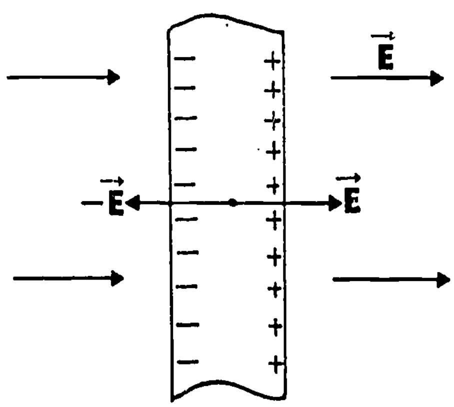
In a conductor electrons are free to move. Thus, if a potential difference is set up inside the conductor, the electrons will move in such a way as to tend to eliminate this potential difference.

Unless this potential difference is maintained by an external agent (an electromotive force--emf), it will eventually vanish, making the potential inside the conductor constant, and the electric field zero. What follows is proof of the last statement; i.e., that the electric field is zero when the potential is constant. You may omit it if you so desire.

Recall that the electric field strength is equal to the negative gradient of the electric potential; i.e.,

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

For a constant potential V, the partial derivatives are all equal to zero. Thus, the electric field inside a conductor, whether charged or uncharged, will be zero everywhere.



As shown in the diagram, if a conductor is placed in an external field $\vec{E}$, the charges inside the conductor will arrange themselves in such a way as to produce a field $-\vec{E}$, making the net field inside the conductor equal to zero.

[b] CORRECT ANSWER: 1280

The solution follows: Th the diameters is (1/16)/(1/4) = 1/4; thus, the ratio of areas 1/16. Since the current densities are inversely proportional to th reas, their ratio will be 16/1 or 1280/80. (Recognize that the cu ent in the conductor does not ange from point to point.)

TRUE OR FALSE? Current density is directly proportional to the square of the wire radius.

[a] CORRECT ANSWER: 4×10^6

The expression relating the magnitude of current density to the quantities given in this problem is

$$j = \frac{V}{\rho \ell} \qquad (1)$$

Substituting the given numerical values in (1) we obtain

$$j = \frac{2}{10^{-6} \times 0.5} = 4 \times 10^6 \text{ amp/m}^2$$

As a review we present the steps that are taken in the derivation of equation (1).

We use the definition of resistivity,

$$\rho = E/j$$

along with the relationship $E = V/\ell$ to arrive at equation (1).

[b] CORRECT ANSWER: A

For both linear and nonlinear conductors, resistance R is always defined by V/i. For linear conductors, R is a constant and does not change when the current in it changes (provided we keep the temperature constant).

For nonlinear conductors, however, the ratio V/i is dependent on the magnitude of the current. But if we know the resistance for a current i, we can still determine the voltage drop across the resistance from $V = iR$. We need only remember that a new value of 'R' is needed for every new 'i.'

The question may arise, "How can one know the value R corresponding to each current i?" The answer is quite simple. For important nonohmic "resistors" there are engineering charts or graphs proving the value of 'R' for various current values. Examples are presented in your reading.

[a] CORRECT A

The ratio ... es the resistivity of a conductor. The resistivity is a prope ... material from which the conductor is made, and does not depend ... ape of the conductor. In order that the resistivity be useful, ... a constant for each material. To assure this the restrictio ... ic material" was added to the definition of ... Electrical ... pic materials are those materials in which electrical properties ... epend on direction.

Materials ... the properties depend on direction are called anisotropic. In ... materials the ratio E/j still defines a resistivity which, howe ... not constant. In these materials, there are two, and sometimes th ... nstant resistivities along certain directions of the solid conduc ... The resistivity in directions other than these is usually a compli ... function of these two (or three) resistivities.

[b] CORRECT ANSW ... amp

From the given ... tage and current ($V = 12$ volts; $i = 2$ amp), we can find the initia ... sistance

$$R_i = \ldots_i = 12/2 = 6\ \Omega$$

Using the ... ta ($\ell_i = 2$ m, $\ell_f = 4$ m, $d_i = 2$ mm and $d_f = 4$ mm) and recalling ... d^2, we compute the new resistance.

$$R_f = \frac{\ell_f}{\ell_i}\frac{A_i}{A_f}R_i = \frac{\ell_f}{\ell_i}\frac{d_i^2}{d_f^2}R_i = \frac{4}{2}\times\frac{4}{16}\times 6 = 3\ \Omega$$

The new voltage is $V_f = 6$ volts; so, the current becomes

$$i_f = \frac{V_f}{R_f} = \frac{6}{3} = 2 \text{ amp}$$

TRUE OR FALSE? If a wire has a resistance R, then a second wire (of the same material and at the same temperature) that is twice as long and has twice the diameter will have a resistance R/2.

CORRECT ANSWER: . 00

Let the current be i_{AB} when the potential difference of V is maintained across surfaces A and B. Using Ohm's law, we obtain

$$i_{AB} = \frac{V}{R_{AB}} \qquad (1)$$

where

$$R_{AB} = \frac{\rho \times \text{length}}{\text{Cross-sectional Area}}$$

$$= \frac{\rho \ell}{hw} \qquad (2)$$

Similarly, the current i_{CD} is

$$i_{CD} = \frac{V}{R_{CD}} \qquad (3)$$

where

$$R_{CD} = \frac{\rho \times \text{length}}{\text{Cross-sectional Area}}$$

$$= \frac{\rho}{\ell}\frac{h}{w} \qquad (4)$$

Therefore, the ratio

$$\frac{i_{AB}}{i_{CD}} = \frac{V}{R_{AB}} \cdot \frac{R_{CD}}{V}$$

$$= \frac{R_{CD}}{R_{AB}} \qquad (5)$$

Substituting values of R_{AB} and R_{CD} from equations (2) and (4) respectively in equation (5) we obtain

$$\frac{i_{AB}}{i_{CD}} = \frac{\rho}{\ell}\frac{h}{w} \cdot \frac{h}{\rho}\frac{w}{\ell}$$

$$= \frac{h^2}{\ell^2} = \frac{1}{100}$$

TRUE OR FALSE? The current ratio is the inverse of the respective resistance ratio.

[a] CORRECT ANSWER: C

From the definition of current, $i = dq/dt$ the relationship

1 ampere = 1 coulomb/1 second

is easily . . .

Now, an electron carries a charge of magnitude $1.602 \cdot 10^{-19}$ coulombs. so one coulomb is equal to $1/1.602 \times 10^{-19} = 6.242 \times 10^{18}$ electronic charges. Consequently, one ampere is equivalent to the net flow through the conductor of 6.242×10^{18} electrons per second.

[b] CORRECT ANSWER: 4 volts

Often a problem is given in such a manner that the information is fragmented. In all cases you must have some insight. Notice that charge/time defines current. Therefore

$$i = \frac{120 \text{ coulombs}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ sec}} = 2 \frac{\text{coul}}{\text{s}} = 2 \text{ amp}$$

Now,

$$V = iR = (2 \text{ amp}) \times (2 \text{ ohms}) = 4 \text{ volts}$$

[c] CORRECT ANSWER: A

Current is the movement of electric charge in a conductor. Charges move when placed in an electric field. Therefore, in order to maintain a continuous current, there must exist a continuous field which, in turn, gives rise to a potential gradient.

TRUE OR FALSE? In order for a field to be continuous, it must also be constant in intensity.

[a] CORRENT ANSWER: 81 ohms

Since the density (mass per unit volume) remained unchanged, so did the volume since the mass, of course, could not have changed. Using the subscript f for final values and i for initial, obtain (V here stands for volume)

$$V_i = V_f \quad \text{or} \quad \ell_i A_i = \ell_f A_f$$

Thus,

$$\frac{A_i}{A_f} = \frac{\ell_f}{\ell_i} = 3$$

Thus, the final cross-sectional area is 1/3 the initial area while the length is 3 times as great. Since resistance R varies directly as length and inversely as cross-sectional area

$$R_f = R_i \frac{3}{1/3} = 9\ R_i = 81 \text{ ohms}$$

TRUE OR FALSE? The resistance of a conductor varies inversely as the square of its diameter, all other factors remaining the same.

[b] CORRECT ANSWER: C

Batteries and electric generators are able to maintain a potential difference between two points to which they are connected. These devices are sources of electrical energy.

TRUE OR FALSE? For a uniform conductor. the potential gradient can be calculated from the potential difference produced by the battery and the length of wire between the points of connection to the battery.

[a] CORRECT ANSWER: C

For an Ohmic resistor (usually metallic), the voltage drop is always directly proportional to the current (linear conductor). This makes most of our calculations easy since we know that the resistance R will be a constant. A nonlinear (or nonohmic) conductor is one for which the voltage is not directly proportional to the current.

Even for linear conductors, of course, the resistance is also a function of temperature. To assure the flawless operation of very sensitive electronic devices (like computers), we must make sure that the temperature remains constant.

[b] CORRECT ANSWER: 10^5 amp/m^2

The unit of current is the ampere which is abbreviated "amp." The general relationship for the current density is

$$i = \int \vec{j} \cdot d\vec{S} = \int j \cos\theta \, dS$$

where $d\vec{S}$ is an element of surface and the integral is taken over the surface in question.

When $\vec{j}$ is normal to the cross section of the conductor, which is the case here, then $\cos\theta = 1$ and

$$j = i/(\text{area of cross section})$$

Thus,

$$j = \frac{100 \text{ amp}}{10 \text{ cm}^2} \times 10^4 \frac{\text{cm}^2}{\text{m}^2} = 10^5 \text{ amp/m}^2$$

TRUE OR FALSE? In this solution, j symbolizes the current density at right angles to the cross section of the conductor.

[a] CORRECT ANSWER:

Starting from the definition of resistivity, we obtain

$$\rho = \frac{E}{j} = \frac{\ \ }{A} = \frac{VA}{i\ell} \qquad (1)$$

We know, however, that

$$R = \frac{V}{i} \qquad V = iR \qquad (2)$$

Substitution of (2) into (1) yields

$$\rho = \frac{(iR)\ A}{i\ell} = \frac{RA}{\ell}$$

and finally

$$R = \frac{\rho\ell}{A}$$

[b] CORRECT ANSWER: C

The charge carriers in a metallic conductor are electrons which are loosely bound in their atoms. When no electric field is present, the electrons are free to move at random. When an electric field is applied to a conductor, the electrons move in a direction opposite to the field at a rate which is proportional to the magnitude of the applied field. The proportionality constant depends on the composition of the conductor, its temperature and other parameters.

[c] CORRECT ANSWER: A

A detailed explanation of the distribution of charge for the case [illegible] point appears in your reading. Specifically you are referred to SZ 25-[illegible] for this.

[a] CORRECT ANSWER: [illegible]

An electric current is established in a conductor if a net charge q passes through any cross section of the conductor in time t. The current, if constant, has the value

$$i = q/t$$

If the current varies with time, the rate of flow of charge is then given by the differential limit of the equation above, or

$$i = dq/dt$$

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL — Energy Transfers in an Electric Circuit

OBJECTIVE

To develop and use the relationships involving energy transfers in an electric circuit; to define and use the concept of electric power; to solve problems involving *Joule heating*.

A useful term common in engineering is the word "load." A load may be defined as any device through which electric charges can move, the potential energy of the electrical system being converted into some other form of energy in the process.

When a source of electric potential difference (V_{ab}) such as a battery is connected across points a and b of a load, a charge dq may be assumed to move through the load from a to b. Thus, point a is considered to be at a higher potential than point b. As this motion occurs, this element of charge will undergo a decrease of potential energy given by:

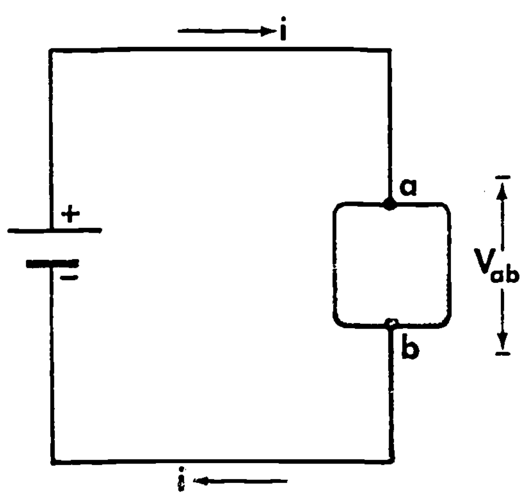


Figure 1

$$dU = V_{ab}dq \qquad (1)$$

since work per unit charge (V_{ab}) times charge (dq) equals work done on the charge. The power in an electric circuit is defined as the *rate of energy transfer*, or rate at which work is being done, hence

$$P = dU/dt = V_{ab}dq/dt$$

$$P = iV_{ab} \qquad (2)$$

Equation (2) applies to energy transfer of *any* type. If the load is a motor, the electrical energy will be largely transformed into mechanical energy; if the load is a resistor, the electrical energy will be transformed into heat. The heat thus produced is called *Joule heating*, an irreversible thermodynamic effect. The rate at which Joule heat is produced is given by the expression:

$$dH/dt = i^2R \qquad (3)$$

next page

continued

obtained by substituting $R = V/i$ into equation (2). Equation (3) applies only to resistors while, as previously mentioned, equation (2) is universally true for any kind of electrical load. Clearly, equation (3) is an expression of the power dissipated in the resistor and so may be rewritten:

$$P = i^2R \qquad (4)$$

The unit of electrical power is the *watt*; this unit is equivalent to the volt-ampere and also to the joule per second.

To summarize: electrical power P is the equivalent of (a) the rate at which work is done in moving a charge from one potential to another; (b) the rate at which energy is transferred through a load; (c) the rate at which Joule heat is developed in a resistor.

Equation (4) may be expressed in another useful form by utilizing the fact that, in a resistor, $i = V/R$. Substituting in equation (4) then gives:

$$P = V^2/R \qquad (5)$$

It must be remembered that equation (5) applies only to resistors.

In this section, the problems are based on the following ideas:

(a) To recognize and use the fact that Joule heating is given by i^2R and is an irreversible action;

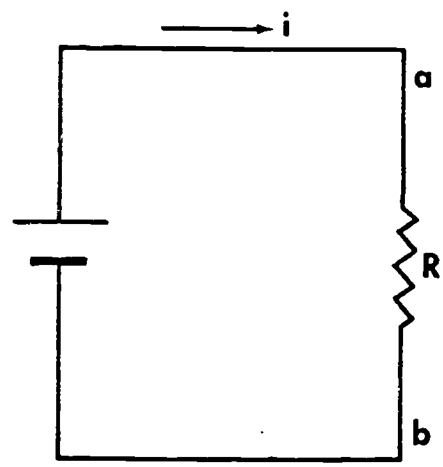
(b) To recognize and use the fact that $dU = V_{ab}dq$;

(c) To recognize and use the fact that P and dU/dt are equivalents;

(d) To find the value of a resistance, given the current through it and the rate at which energy is being consumed in it.

PROBLEMS

1. In the circuit shown in the accompanying diagram, the power developed in the resistor may be given as $P = iV_{ab}$. Derive from this the equation which expresses the rate at which heat is developed in the resistor R in terms of i and R.



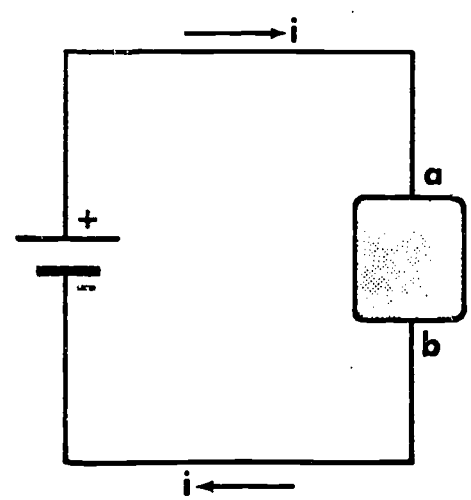
2. The figure below shows a device across which there is a potential difference V_{ab}. In transferring an amount of charge dq from point a to point b, the amount of energy that is transformed from one form to another is:

A. $dU = (V_{ab})dq$

B. $dU = \frac{1}{2} (V_{ab})^2 dq$

C. $dU = \int (V_{ab})dq$

D. $dU = i^2 R$



3. In the figure of the preceding problem, you found that the energy transferred is $dU = (V_{ab})dq$. From this expression, which of the following relations can be readily derived?

A. $P = iV_{ab}$ where P is the power delivered by the device

B. $U = (dq/dt)V_{ab}$

C. $V_{ab} = iR$

D. None of the above

4. By measuring the increase in temperature of water surrounding a resistor in which there exists a current of 2 amperes, it is found that 8 joules are converted into heat by the resistor in every second. What is its resistance in ohms?

INFORMATION PANEL — Problems in Joule Heating

OBJECTIVE

To solve some typical problems involving Joule heating.

As a refresher, look over the following equations for Joule heating before you begin the problems in this section:

(1) Given the voltage and resistance, and knowing the time during which charges are in motion:

$$H = \frac{V^2}{R}t \quad \text{(joules in MKS system)} \qquad (1)$$

or

$$P = V^2/R \quad \text{(watts in MKS system)} \qquad (2)$$

next page

continued

(2) Given the current and resistance, and knowing the time:

$$H = i^2Rt \tag{3}$$

or

$$P = i^2R \tag{4}$$

(3) Given the voltage and current, and knowing the time:

$$H = iVt \tag{5}$$

or

$$P = iV \tag{6}$$

The problems that follow require that you determine

(a) the percentage drop in heat output for the same load when you are given the power rating of a resistor at some initial applied voltage and are then told that the voltage drops to some new given figure;

(b) the power dissipated by a wire whose length, diameter, and resistivity are given together with the current in the wires;

(c) the current in a circuit when the voltage, heat in joules, and time of current are given;

(d) the ratio of the power dissipated in two resistances that are related to each other in a specific way.

5. A resistor dissipates 100 watts when it is connected to a 100-volt supply. If this voltage drops to 90 volts, what will be the percentage drop in heat output, provided the resistance remains the same?

6. A wire 100 m long and 2.0 mm in diameter has a resistivity of 4.8×10^{-8} ohm-m. If a current of 1.0 ampere flows through this wire, how much power is dissipated in it?

7. When a constant voltage of 6.0 volts is maintained across a resistor, 3600 joules of heat are generated in 5 minutes. What is the constant current in the resistor?

8. A heater coil of resistance 40 ohms consists of 20 ft of wire. When connected to a 220 volt source, this coil dissipates power P_1. The coil is now cut into two equal halves, each of 10 ft of wire and both coils are connected to the same source. The total power dissipated now is P_2. Find the ratio $P_2:P_1$.

INFORMATION PANEL Electromotive Force

OBJECTIVE

To define electromotive force; to answer questions dealing with a seat of emf as a source of energy; energy conversions involving a seat of emf; the effect of internal resistance in a seat of emf.

In our work thus far, we have been emphasizing potential difference in terms of energy transformations in which potential energy lost by a moving charge is converted into some other form of energy such as heat, kinetic energy, and so forth.

next page

continued

To keep a charge moving continuously through a load, the potential energy lost in the energy transformation in the load must be restored by some external device such as a battery or generator. This external device is called a *seat of emf* (electromotive force). In restoring the energy to the circuit, the seat of emf must be capable of doing the required work on the positive charge carriers to raise them from a lower potential to a higher potential so that the next traversal of the circuit by the charges will occur again just as it did before. The emf of a seat is therefore defined as:

$$\varepsilon = dW/dq$$

so that electromotive force is *work per unit charge* and is measured in joules per coulomb, or *volts*. (NOTE: The symbol used in printed texts is $\mathcal{E}$. We will use the lower case ε.) The name "electromotive force" is not a good one because emf is not a force at all; common usage, however, has made it acceptable and we shall continue to use it in this course.

A seat of emf is any device in which any form of energy is converted into electrical energy. A battery uses chemical energy in this process; a rotary generator, mechanical energy; a thermoelectric junction, thermal energy; and a photoelectric cell, light energy.

Alternatively, one may describe a seat of emf as a device capable of maintaining a fixed potential difference between two points between which there is a continuous flow of charge (through a load of some kind). An *ideal* seat of emf would have no internal resistance of its own, and if one assumes that the wires connecting the seat of emf to the load are also resistanceless, then one can write:

$$i = \varepsilon/R$$

Thus, an ideal seat of emf produces a potential difference across the terminals of the load equal to the emf itself. Since all real seats of emf do have some resistance, however, the actual current is given by

$$i = \frac{\varepsilon}{R + r}$$

where R is the net resistance outside the seat of emf, and r is the internal resistance inside the seat of emf.

When a current is present in such a real circuit, the voltage appearing at the terminals of the load is *less* than that produced by the seat of emf because of the fall of potential within the seat due to its internal resistance. The so-called "terminal voltage" is then

next page

continued

$$V_{ab} = \varepsilon - ir$$

where the produce ir is the internal voltage drop of the seat of emf.

You should have no difficulty with the questions and problems in this section if the preceding material is thoroughly understood.

9. Which of the following correctly defines emf in terms of the work done by a seat of emf in moving a charge dq from a lower potential to a higher potential?

A. $\varepsilon = -qdW$

B. $\varepsilon = \frac{dW}{dq}$

C. $\varepsilon = -\frac{dW}{dq}$

D. $\varepsilon = \frac{dq}{dW}$

where: dW is the work done by the source of emf on a charge dq, in moving this charge from a lower to a higher potential

10. A source of emf is defined as a system which

A. always converts chemical to electrical energy

B. is capable of maintaining a potential difference between two points to which it is connected

C. stores electrical energy

D. is capable of converting electrical to chemical energy

11. Which of the following energy conversions is incorrectly paired with the device which makes the conversion?

A. Chemical into electromagnetic energy: a discharging battery

B. Electrical into mechanical energy: a motor

C. Electrical into chemical energy: a charging battery

D. Thermal into electrical energy: a resistor

12. Which one of the following devices converts energy in a theoretically reversible process?

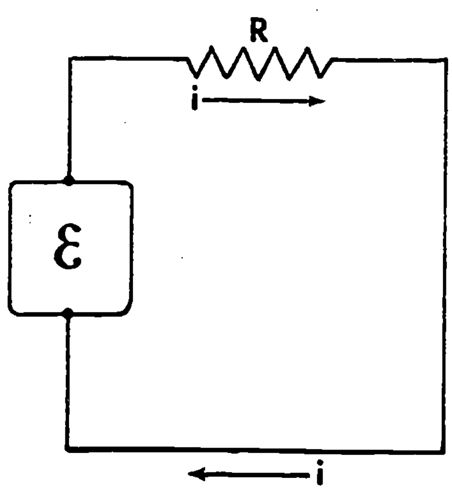
A. Any ideal seat of emf

B. A battery with an internal resistance

C. Any energy-converting mechanism as long as it is not a seat of emf

D. Only a source of emf which does not store its energy chemically, such as a generator.

13. Let the seat of emf, ε, in the circuit shown below be an ideal one; i.e., have zero internal resistance. The current flowing through the resistor, R, is given by



A. $i = \varepsilon/R$

B. $i = \varepsilon R$

C. $i = \varepsilon^2/R$

D. $i = V/R$

14. If an electrical device loses potential energy,

    A. all of this energy will be transformed into kinetic energy

    B. all of this energy will be transformed into thermal energy (heat)

    C. some of this energy may be lost and not accounted for

    D. all of this energy will be transformed into one or more forms of energy such as thermal, kinetic, etc.

INFORMATION PANEL — Single Loop Circuits - One Seat of Emf

OBJECTIVE

To develop a system for tracing the loop of a circuit; to apply the loop theorem to circuit problem solving.

A circuit such as that shown in Figure 1 will be referred to as a single loop circuit containing one seat of emf. The seat of emf develops an electromotive force ε; its internal resistance is r and the external resistance (assuming resistanceless wires) is lumped in the resistor R. The arrow adjacent to the seat of emf indicates the direction in which positive charge carriers would be driven through the seat, or simply the direction of the emf. The current, considered to consist of moving positive charge carriers, has the same direction as the emf, and is symbolized by i. We select a point anywhere in the circuit (point a) and propose two significant questions about it: (a) what is the potential of point a and (b) can point a have more than one potential?

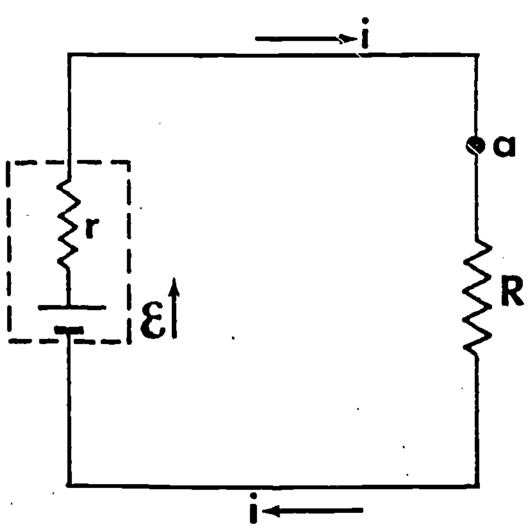


Figure 1

Answering question (a) first, we have already defined the potential of a point as the *difference of potential* between this point and some other arbitrarily chosen point to which we assign a potential of zero. It doesn't matter where the zero reference point is taken just as long as we use it consistently in a given circuit.

next page

continued

As to question (b), it seems self-evident that point a can have *one and only one potential* with respect to the preselected zero reference level. For the situation to be otherwise, it would be like assigning two different altitudes with respect to the Earth's surface to a single flag on a mountain. Thus, the potential of point a with respect to the arbitrary zero point, wherever it may be, is V_a and nothing else.

Viewing the circuit as a loop to be traversed by the moving charges, it is evident that each charge will experience a loss of potential as it pssses through a resistor--call this a *voltage drop*--and a rise of potential as it passes through the seat of emf--a voltage rise. However, if a charge starts at point a, traverses the loop and returns to point a, whatever has occurred in the form of voltage drops and rises during the traversal must leave the charge with the same potential energy as it had when it started *because point a can have only one potential.*

It is then clear that the totality of the voltage drops and voltage rises have come to naught with respect to any change in potential they might have caused at point a. Stated formally:

> The algebraic sum of the changes in potential encountered in a complete traversal of a circuit is equal to zero.

In the simple circuit of Figure 1, the current direction is readily established by the direction of the emf. Calling a voltage drop negative and a voltage rise positive, let's traverse the circuit clockwise (in the direction of the current) starting from point a, recording drops and rises as we go. Thus

$$-iR + \varepsilon - ir = 0$$

Note that a negative sign precedes voltage drops and a positive sign precedes a voltage rise. If this equation is solved for the current i we obtain

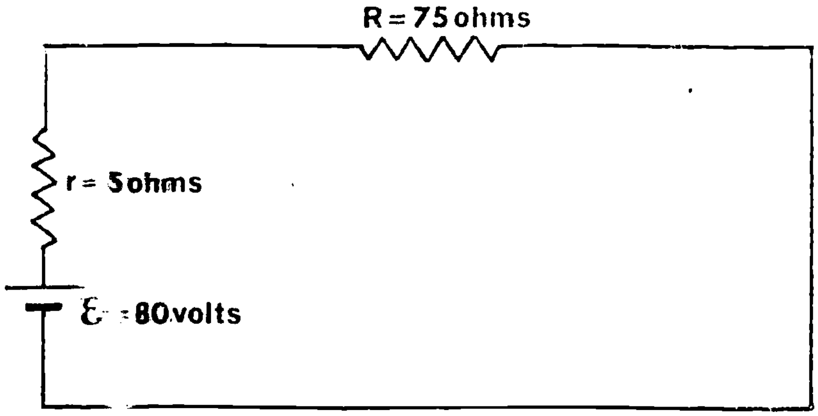
$$iR + ir = \varepsilon$$

$$i = \frac{\varepsilon}{R + r}$$

which is identical with the equation obtained in the last Information Panel for a battery with an internal resistance r working into a circuit with a lumped resistance of R.

The problems in this section are generated around this central idea of single loop systems containing a single seat of emf. They introduce the relationships between the currents, voltages and resistances in such loops and the power and heat developed in their various parts. In addition, there are a number of questions which test your understanding of the so-called loop theorem as stated above.

15. For the data given in the circuit below, what is the rate at which heat is being generated in the 75-ohm resistor?



16. The algebraic sum of the changes in potential encountered in going once around a single loop circuit is

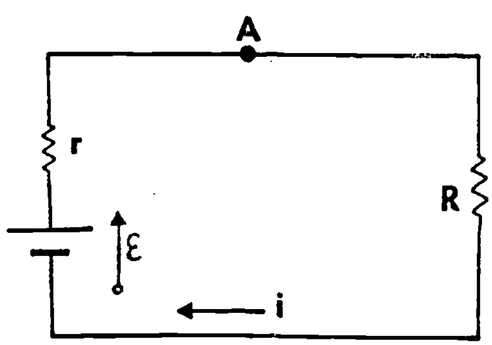
A. dependent on the direction in which the loop is traversed

B. dependent only on the magnitude of the circuit resistances and the direction of current through them

C. dependent only on the emf's present in the circuit

D. zero

17. The loop equation for the circuit shown in the figure is:



A. $\varepsilon + ir + iR = 0$

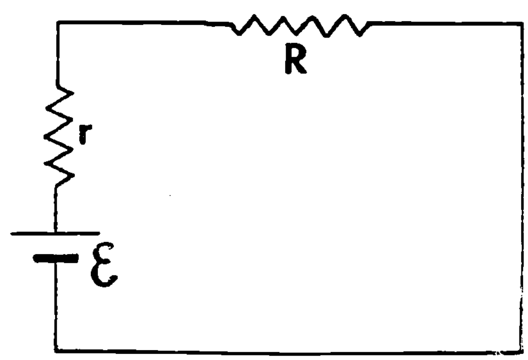
B. $-\varepsilon + ir - iR = 0$

C. $\varepsilon - ir - iR = 0$

D. $-\varepsilon - ir - iR = 0$

18. Let R = 100 ohms and r = 10 ohms in the circuit diagram of the previous question. If a potential difference of 200 volts is established across R, what is emf ε?

19. For the circuit shown below, let ε be the emf of a 12 volt battery. For R = 5 ohms, a current of 2 amps exists in the circuit. What is the internal resistance r of the battery?



20. In the circuit shown, r = 5 ohms and R = 75 ohms. What must be the emf ε if the source is to supply energy at the rate of 80 j/sec?

INFORMATION PANEL — Single Loop Circuits - Two or More Seats of Emf

OBJECTIVE

To apply the loop theorem to single loop circuits containing two or more seats of emf; to introduce the notation $V_b - V_a$ for expressing the difference of potential between points b and a.

When a second or third source of emf is introduced into a single loop circuit, the only additional procedure of significance relates to the direction assumed for the current i. Referring to Figure 1, you will see that the current may be assumed to have either a clockwise or a counterclockwise direction around the loop, depending upon which of the two seats of emf produces the greater voltage rise. These seats are connected so that their emf's oppose one another; thus, if ε_1 is larger than ε_2, the current will be clockwise and if the reverse is true, the current will be counterclockwise.

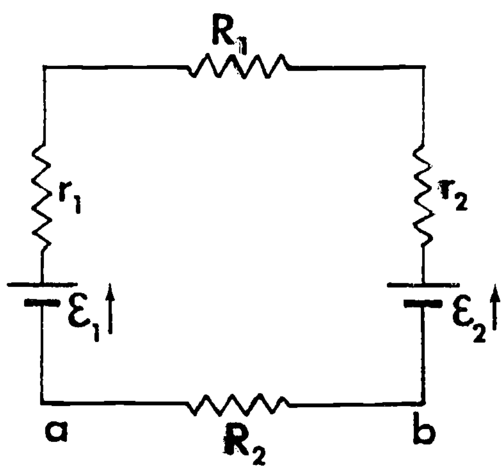


Figure 1

Suppose we wanted to find the potential difference between points a and b. We shall use a very explicit notation for this; instead of referring to this potential difference as V_{ab} or V_{ba}, we shall designate the potential at point a as V_a (referred to some unspecified zero reference) and the potential at point b, V_b. This potential is referred to the same zero reference, of course. Hence, the potential difference between these points will be expressed as $V_b - V_a$. If it happens that point b is at a higher potential than point a, this difference will be positive; if a is at a higher potential than b, the difference will be negative.

In most circuits, there is no way to tell which way the current will actually traverse the loop; this is of no importance because, when the sign conventions are correctly followed, they will insure the correct direction at the end of the problem. If the emf's are all known, of course, the logical direction for the current is that of the *net* emf but if one or more are unknown, one guess is as good as another.

next page

continued

One additional point is important in tracing the loop of this kind of circuit: if the direction of traversal through a seat of emf is in the same direction as the emf, this is to be taken as a voltage rise; if the traversal opposes the direction of the emf it is considered to be a voltage drop since work is being done *against* the emf.

To see how this works out for the circuit of Figure 1, let's assume a clockwise current direction and take as our objective the determination of $V_b - V_a$, the potential difference from point b to point a. Starting at point a and going clockwise, we would then write:

$$\varepsilon_1 - ir_1 - iR_1 - ir_2 - \varepsilon_2 - iR_2 = 0$$

Note that all terms are negative *except* the first emf. Rearranging and simplifying:

$$\varepsilon_1 - \varepsilon_2 - i(r_1 + R_1 + r_2 + R_2) = 0$$

Solving for the current i:

$$i = \frac{\varepsilon_1 - \varepsilon_2}{r_1 + R_1 + r_2 + R_2}$$

At this point you would substitute numerical values for the factors on the right. If ε_1 had been larger than ε_2, you would obtain a positive answer, indicating that the assumed current direction had been correct; if the reverse had been true, then the answer would come out negative, showing that the current has a true direction opposite that assumed. Finally, once the current magnitude and direction have both been determined, the potential difference $V_b - V_a$ is then readily obtained:

$$V_b - V_a = iR_2$$

Here again, if the current had a negative value, then $V_b - V_a$ turns out to be negative so that you would know that point a had a higher potential than point b.

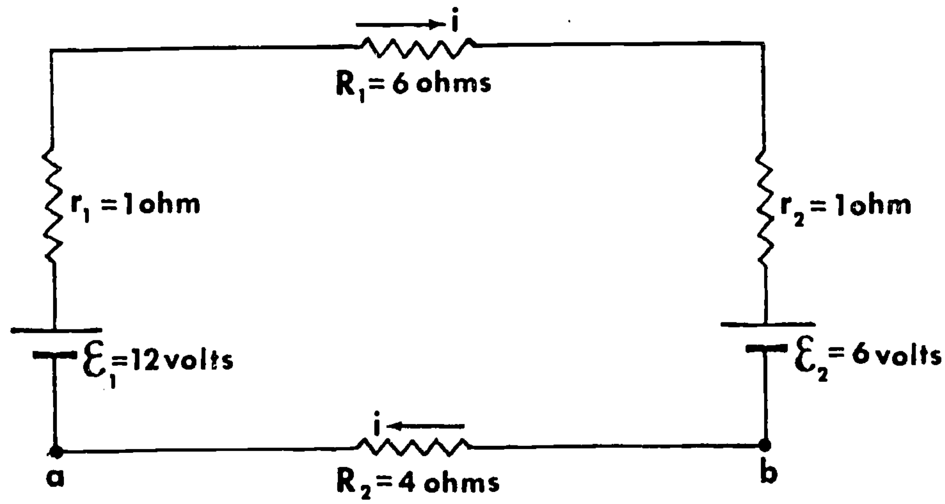
You will be aided by studying and following the set of rules thus far developed:

1. Choose a logical current direction, if possible; if not, assume *any* current direction.

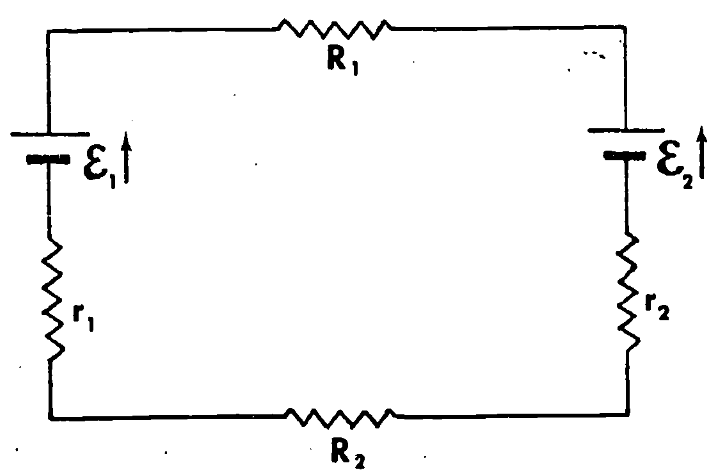
2. Traverse the loop in the same direction as the assumed current; in this way, all the "iR" and "ir" terms are negative--consistently.

3. In going through a seat of emf, if the traversal is in the same direction as the emf, it is a voltage rise (+); if in the opposite direction from the direction of the emf, it is a voltage drop (-).

21. Find the potential difference $V_b - V_a$ in the circuit shown below.



22. The loop equation for the circuit shown below is



A. $\varepsilon_1 - iR_1 - \varepsilon_2 - ir_2 - iR_2 - ir_1 = 0$

B. $\varepsilon_1 - iR_1 + \varepsilon_2 - ir_2 - iR_2 - ir_1 = 0$

C. $\varepsilon_1 + iR_1 + \varepsilon_2 + ir_2 + iR_2 + ir_1 = 0$

D. $\varepsilon_1 + iR_1 - \varepsilon_2 - ir_2 + iR_2 + ir_1 = 0$

23. If in the preceding question ε_1 = 20 volts and ε_2 = 30 volts, then

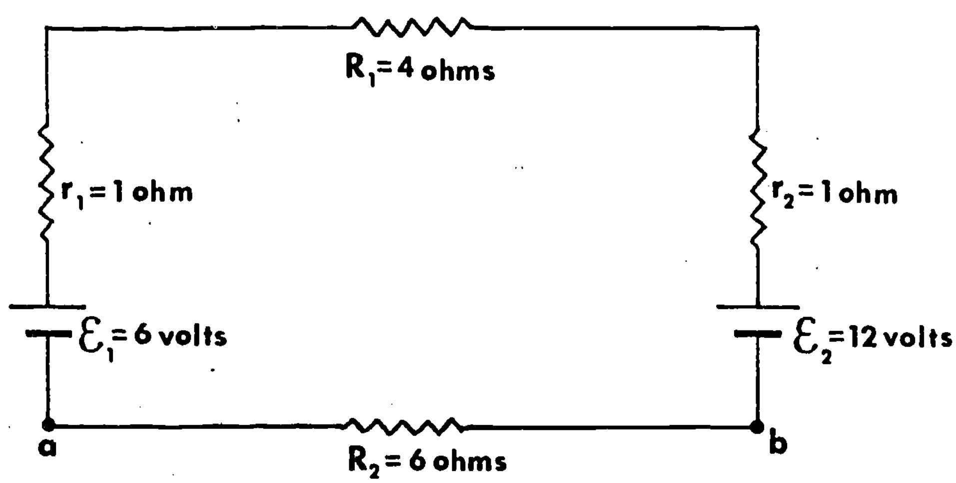
A. the current will be clockwise

B. the circuit will not function since the equation derived for the loop will then give negative current

C. more current will go through R_1 than through R_2

D. the current will be counterclockwise

24. Find the potential difference $V_b - V_a$ in the circuit shown.



[a] CORRECT ANSWER: 4

The power P_1 developed across a single coil of 20 ft-length is

$$P_1 = \frac{V^2}{R} = \frac{(220)^2}{40} = 2100 \text{ watts}$$

The power developed across a single coil of 10 ft-length is

$$P_1 = \frac{(220)^2}{20} = 4200 \text{ watts}$$

and, therefore, the total power developed with both coils operating simultaneously, is

$$P = 2P = 8400 \text{ watts}$$

Thus, the ratio of the total power developed is

$$\frac{P_2}{P_1} = \frac{8400}{2100} = 4$$

TRUE OR FALSE? The sequence of events described in the problem really involves (1) a series connection of the two 10 ft lengths, then (2) a parallel connection of the 10 ft lengths.

[b] CORRECT ANSWER: A

We note that regardless of the direction we assign to the current, and of the sense in which we go around the loop the two seats of emf are connected in such a way that one of the two will always be traversed in a direction opposite that of the other. Hence, we know that the emf's will enter the loop equation having opposite signs. Choosing the clockwise as the direction of the current i and also starting from the lower potential side of ε_1 and going around the loop clockwise, we obtain for the potential changes

$$\text{rise } \varepsilon_1 + \text{drop in } R_1 + \text{drop in } \varepsilon_2 + \text{drop in } r_2$$
$$+ \text{drop in } R_2 + \text{drop in } r_1 = 0$$

or

$$\varepsilon_1 - iR_1 - \varepsilon_2 - ir_2 - iR_2 - ir_1 = 0$$

[a] CORRECT ANSWER: C

The method for solving these circuit problems consists of the following steps:

(1) Select the direction in which you will traverse the loop. This is arbitrary, either clockwise or counterclockwise.

(2) Write the ε's as positive if they are in the direction you have chosen; negative otherwise.

(3) Write iR's (the resistive potential drops) as negative if you go through the resistor in the direction of the current; positive otherwise.

Let's try it together, starting at Point A and going around in a clockwise sense,

$$V_A - iR + \varepsilon - ir = V_A$$

or

$$V_A - V_A = 0 = \varepsilon - iR - ir$$

[b] CORRECT ANSWER: B

A source of electromotive force (emf) is capable of converting a specific form of energy for which it is designed into electrical energy. A battery uses chemical energy for this purpose; a rotary generator converts mechanical into electrical energy; a photocell does it with light energy. The essential point is that the seat of emf must be capable of maintaining a potential difference between two points to which it is connected even when a flow of charge occurs continuously between these points.

[c] CORRECT ANSWER: A

The potential V_{ab} is the work required per unit charge to move a positive charge from _b_ to _a_. Therefore,

$$dU = V_{ab}dq$$

[a] CORRECT ANSWER: -3 volts

Choosing clockwise as the direction of the current i and starting from point a in clockwise direction, the circuit equation is

$$\Sigma\varepsilon - i\Sigma R = 0$$

or

$$\varepsilon_1 - \varepsilon_2 - i(r_1 + R_1 + r_2 + R_2) = 0$$

Solving for i, we obtain

$$i = \frac{\varepsilon_1 - \varepsilon_2}{r_1 + R_1 + r_2 + R_2}$$

$$= \frac{6 - 12}{1 + 4 + 1 + 6}$$

$$= -0.5 \text{ amps}$$

Note that the value of the current i is negative, therefore, the current is opposite to the chosen clockwise direction, that is in the counter-clockwise direction. Since the direction of current is always from higher potential to the point of lower potential, $V_b - V_a$ is negative, that is

$$V_b - V_a = - |iR_2|$$

$$= -3 \text{ volts}$$

TRUE OR FALSE? The solution shows that point a is at a higher potential than point b.

[b] CORRECT ANSWER: A

In the absence of an internal resistance all the potential difference that the emf can maintain in the circuit, namely ε, appears across the resistor R. Now the potential difference (or drop) across the resistor is given by iR. Therefore, $\varepsilon = iR$ or $i = \varepsilon/R$.

[a] CORRECT ANSWER: 2

The unit of power in the MKS system (the watt) is defined as one joule/second. Therefore, the power dissipated by the given resistor is P = 8 W. From

$$P = i^2 R$$

we obtain

$$R = \frac{P}{i^2} = \frac{8\ \text{W}}{4\ \text{amp}^2} = 2\ \text{ohms}$$

TRUE OR FALSE? As shown above, an ohm is the equivalent of a watt per ampere.

[b] CORRECT ANSWER: 2 volts

Choosing clockwise as the direction of the current i and starting from point a in a clockwise direction, the circuit equation is

$$\Sigma\varepsilon - i\Sigma R = 0$$

or

$$\varepsilon_1 - \varepsilon_2 - i(r_1 + R_1 + r_2 + R_2) = 0$$

Solving for i we obtain

$$i = \frac{\varepsilon_1 - \varepsilon_2}{r_1 + R_1 + r_2 + R_2}$$

$$= \frac{12 - 6}{1 + 6 + 1 + 4}$$

$$= 0.5\ \text{amp}$$

Note that the value of current i is positive, therefore, the current direction was properly chosen (clockwise direction). Therefore,

$$V_b - V_a = iR_2$$

$$= .5 \times 4 = 2\ \text{volts}$$

TRUE OR FALSE? In problems of this type, the final algebraic sign of the current i specifies whether or not the choice of current direction was initially correct.

[a] CORRECT ANSWER: $dH/dt = i^2R$

The current through the resistor is given by $i = V_{ab}/R$. Thus $V_{ab} = iR$, and the expression for the power becomes

$$P = iV_{ab} = i\ (iR) = i^2R$$

thus,

$$\frac{dH}{dt} = i^2R$$

Notice that power is the rate of energy transfer, or in this case, the rate at which electrical energy is converted into heat. This process is called *Joule heating* and is an irreversible process.

TRUE OR FALSE? This solution assumes that all the electrical energy transferred through the circuit is converted into heat in resistor R.

[b] CORRECT ANSWER: 2 amp

Power is defined as the energy dissipated per unit time; i.e., per second. The time here is 5 min, or 300 sec. Thus,

$$P = \frac{3600\ j}{300\ sec} = 12\ watts$$

Now Joule's law can be writen as

$$P = i^2R = \frac{V^2}{R} = iV$$

so,

$$i = \frac{P}{V} = \frac{12\ watts}{6\ volts} = 2\ amp$$

[c] CORRECT ANSWER: D

A resistor is always a one-directional energy conversion device. That is, the resistor removes electrical energy from the circuit in the form of heat. It, therefore, converts electrical energy into heat.

[a] CORRECT ANSWER: 80 volts

For a simple circuit, the power supplied by the seat of emf may be written as

$$P = \frac{\varepsilon^2}{\Sigma R} \qquad (1)$$

Solving for ε, we obtain

$$\varepsilon^2 = \sqrt{P\ \Sigma R} = \sqrt{80 \times (5 + 75)} = 80 \text{ volts}$$

Alternate solution:

Power supplied by emf ε may also be written as

$$P = i\varepsilon \qquad (2)$$

However, ε is also given by the expression

$$\varepsilon = i(\Sigma R) = i(R + r) \qquad (3)$$

Therefore,

$$i = \frac{\varepsilon}{\Sigma R} \qquad (4)$$

Substituting (4) into (2), we obtain

$$P = \frac{\varepsilon^2}{\Sigma R}$$

which is the same as equation (1).

TRUE OR FALSE? Either solution shows that power P varies directly with emf ε.

[b] CORRECT ANSWER: D

Energy is conserved in electrical circuits as it is in any physical interaction. However, electrical energy may be converted partially into kinetic energy, partially into thermal energy, partially into light energy--or any combination of these depending on the nature of the device and the load into which it works.

TRUE OR FALSE? Even though various combinations are possible, thermal energy always predominates over the others as the final form of energy produced by electricity.

[a] CORRECT ANSWER: 1.5 watts

The resistance of a wire, in terms of the resistivity ρ, its length ℓ and its cross-sectional area A, is given by

$$R = \rho \frac{\ell}{A} \qquad (1)$$

The area of a circle of diameter d is given by

$$A = \pi \frac{d^2}{4} \qquad (2)$$

Using the given data in equations (1) and (2) yields

$$R = 4.8 \times 10^{-8} \text{ (ohm-m)} \times \frac{10^2 \text{ m}}{\pi(4 \times 10^{-6})/4 \text{ m}^2} = \frac{4.8}{\pi} \text{ ohms}$$

The power is given by

$$P = i^2R = \left[1 \text{ amp}^2\right] \times \left[\frac{4.8}{\pi} \Omega\right] = 1.5 \text{ watts}$$

[b] CORRECT ANSWER: D

We must realize that electric potential is a single-valued quantity. Even though the absolute value of the potential at a point can be chosen arbitrarily, once a reference has been chosen and the potential at other points is measured relative to the same reference these points will have the fixed potential unless there is a change in the environment. Thus, unless we make a change in a circuit (remove a battery, add a resistor, etc.), the potential of a certain point will always remain the same, regardless of whether the circuit consists of one loop or is a multiloop circuit. We may, therefore, take any possible path in going around a circuit and when we return to the starting point the potential will be the same as the one we started with. Hence, the net change in potential; i.e., the algebraic sum of the potential changes will be zero.

[a] CORRECT ANSWER: 1 ohm

The loop equation for the circuit is

$$\varepsilon - iR - ir = 0$$

Thus,

$$ir = \varepsilon - iR$$

or

$$r = \frac{\varepsilon - iR}{i} = \frac{12 - (2 \times 5)}{2} = 1 \text{ ohm}$$

[b] CORRECT ANSWER: A

From the definition of current,

$$i = \frac{dq}{dt} \qquad (1)$$

we can write

$$dq = i\ dt$$

The energy transferred is

$$dU = dq\ V_{ab} \qquad (2)$$

Substituting (1) into (2) yields

$$dU = i\ dt\ V_{ab} \qquad (3)$$

and upon division by dt

$$\frac{dU}{dt} = iV_{ab} \qquad (4)$$

Recalling that dU/dt is the definition of the power P, we see that equation (4) agrees with selection A.

[a] CORRECT ANSWER: D

We can answer this question intuitively. ε_2 alone would generate a counterclockwise current. On the other hand, ε_1 alone would generate a clockwise current. When the two seats of emf are connected opposing each other the "stronger" one will "win." Thus, the current must be in the counterclockwise direction.

The problem can also be solved formally. If we substitute the values of ε_1 and ε_2 in the equation for this loop derived previously we obtain

$$i(R_1 + r_2 + R_2 + r_1) = \varepsilon_1 - \varepsilon_2 = 20 - 30 = -10 \text{ V}$$

Thus

$$i = -\frac{10}{(R_1 + r_2 + R_2 + r_1)} < 0$$

Recall that we had arbitrarily chosen the clockwise direction as the direction of the current (at that time, of course, we did not know ε_1 and ε_2). All a negative current means then, is that the choice was wrong and that the actual direction of the current is counterclockwise.

[b] CORRECT ANSWER: 19%

The percent change in power can be expressed as follows,

$$\left|\frac{\Delta P}{P}\right| \times 100 = \frac{P_{\text{final}} - P_{\text{initial}}}{P_{\text{initial}}} \times 100 = \frac{V_f^2/R - V_i^2/R}{V_i^2/R} \times 100$$

$$= \frac{V_f^2 - V_i^2}{V_i^2} \times 100 = \frac{8{,}100 - 10{,}000}{10{,}000} \times 100 = 19\%$$

TRUE OR FALSE? Electrical power and Joule heat are dimensionally equivalent.

[a] CORRECT ANSWER: 75 watts

Using Joule's law we obtain

$$P = i^2R \tag{1}$$

The current, i , may be obtained from the circuit equation

$$\varepsilon = i\ \Sigma R = i(r + R)$$

Therefore

$$i = \frac{\varepsilon}{r + R} \tag{2}$$

Substituting equation (2) into equation (1), we obtain

$$P = \frac{\varepsilon^2 R}{(r + R)^2} = \frac{(80)^2 \times 75}{(5 + 75)^2} = 75 \text{ watts}$$

TRUE OR FALSE? The symbol "r" represents the internal resistance of the seat of emf.

[b] CORRECT ANSWER: B

The emf must do an amount of work dW on the (positive) charge dq to make it move to the point of higher potential. It is defined by the expression

$$\varepsilon = \frac{dW}{dq}$$

and has the units j/coul = volt.

TRUE OR FALSE? A seat of emf is not needed to move a charge from a point of higher potential to a point of lower potential.

[a] CORRECT ANSWER: 220 volts

From the given potential drop across the res[illegible] one finds the current in the loop

$$i = \frac{V}{R} = \frac{200\ V}{100\ ohms} = 2\ amp$$

The loop equation, $\varepsilon - iR - ir = 0$, gives

$$\varepsilon = i(R + r),$$

which upon substitution yields

$$\varepsilon = 2(100 + 10) = 220\ volts$$

[b] CORRECT ANSWER: A

You should recall that a reversible process is one that passes through equilibrium states and can be reversed by making an infinitesimal change in the environment of the system. Joule heating is an electrical energy conversion that is *not* reversible. Hence, the word "ideal" was added since all real seats of emf (batteries, motors, etc.) have non-zero internal resistance.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL

Basic Chargacteristics of Series and Parallel Circuits

OBJECTIVE

To recognize and use the basic characteristics of series and parallel circuits containing resistors and a seat of emf.

The essential information relating to *parallel* circuits such as that of Figure 1 may be summarized as follows:

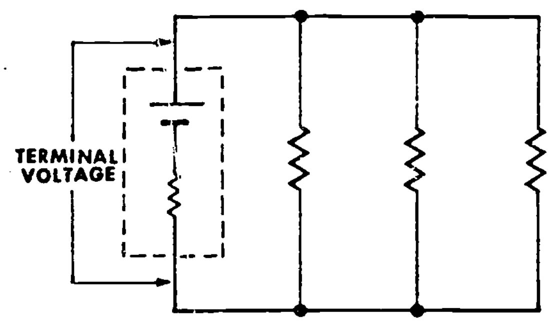


Figure 1

(1) The potential difference across each external resistor is equal to that across every other external resistor, and is equal to the terminal voltage of the source of emf.

(2) The sum of the currents in the external resistors is equal to the total circuit current supplied by the source of emf.

(3) The current in each individual resistor is inversely proportional to the individual resistance.

For *series* circuits such as that shown in Figure 2, the essential characteristics are:

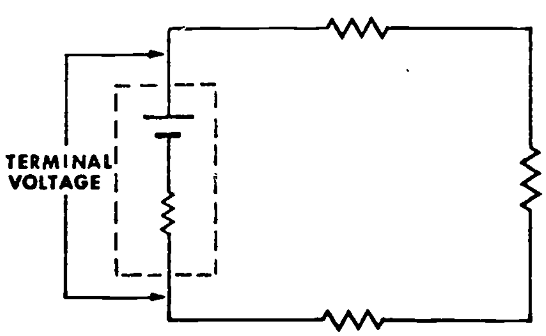


Figure 2

(4) The current is everywhere the same throughout the entire circuit.

(5) The sum of the voltage drops across each individual resistor is equal to the terminal voltage of the source of emf.

(6) The voltage drop across any individual resistor is directly proportional to the individual resistance.

The questions in this section are all descriptive and are easily answered by applying the above characteristics properly. Note carefully that, if the internal resistance of a seat of emf is not given in the statement of the problem data, it is to be ignored.

PROBLEMS

1. A circuit consists of three resistors, $R_1 = 1$ ohm, $R_2 = 2$ ohms, and $R_3 = 3$ ohms. The current in each resistor is found to be inversely proportional to its resistance. This means that

A. all three resistors are connected in series

B. all three resistors are connected in parallel

C. the first two resistors are connected in parallel and the combination is connected in series with the third resistor

D. this is always true regardless of the way these resistors are connected

2. When resistors are connected in series

A. the current through each resistor is the same

B. the voltage across each resistor is identical

C. the voltage across each of the resistors is inversely proportional to the current

D. the current through each of the resistors is proportional to its resistance

3. When resistors are connected in parallel

A. the current through each resistor is the same

B. the current through each of the resistors is proportional to its resistance

C. the voltage across each resistor is identical

D. the voltage across each of the resistors is proportional to its resistance

4. A circuit consists of three resistors, R_1 = 1 ohm, R_2 = 2 ohms, and R_3 = 3 ohms. It is found that the potential differences across the resistors are directly proportional to their resistances, the proportionality constant being the same for all three resistors. This means that

A. all three resistors are connected in series

B. all three resistors are connected in parallel

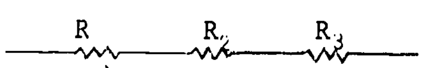
C. this statement is always true; it is a statement of Ohm's law

D. there is no possible combination that can accomplish this

INFORMATION PANEL — Equivalent Resistance

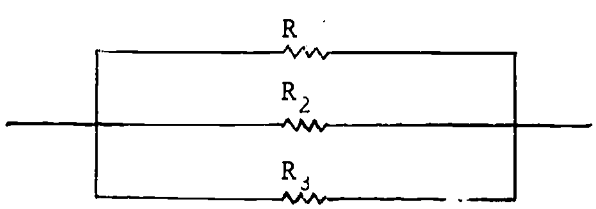
OBJECTIVE

To calculate the equivalent resistance of a series circuit; of a parallel circuit; of a series-parallel combination.



The equivalent resistance of a series circuit is the simple sum of its individual resistances. Thus, in the circuit shown at the left:

$$R = R_1 + R_2 + R_3$$



The equivalent resistance of a parallel circuit is the reciprocal of the sum of the reciprocals of the individual resistances. In the circuit at the left:

$$1/R = 1/R_1 + 1/R_2 + 1/R_3 .$$

next page

continued

Clearly, equivalent resistance may be defined as the resistance into which the seat of emf works; stated otherwise, the equivalent resistance of any simple or complex circuit is that single resistance which could replace the entire circuit without changing the current delivered to the external components by the seat of emf.

In breaking down a complex circuit for the purpose of determining its equivalent resistance, one of two patterns is generally followed: (1) if the complex circuit looks like a series circuit containing paralleled groups, as Figure 1, each parallel group should be resolved into its equivalent resistance to leave a simple series arrangement as in Figure 2.

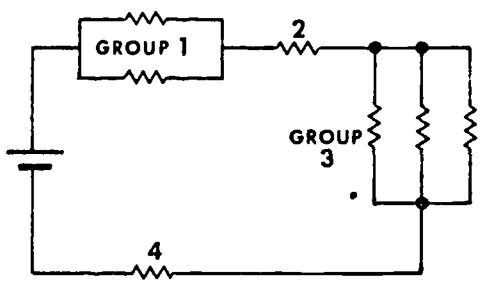


Figure 1

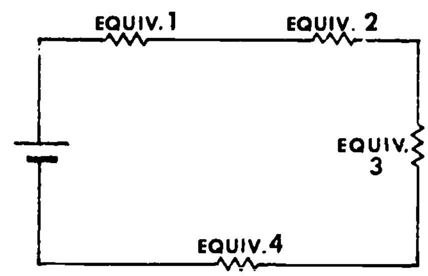


Figure 2

(2) if the complex circuit looks like a parallel circuit containing series or parallel groups, each group should be resolved into a single equivalent resistance to leave a simple parallel arrangement. This is shown in Figures 3 and 4.

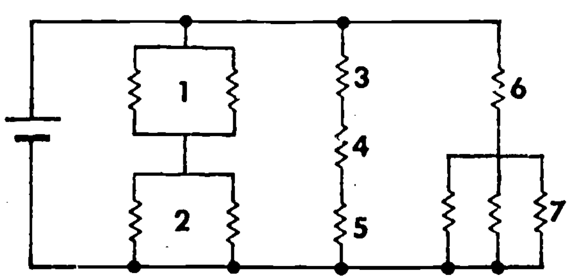


Figure 3

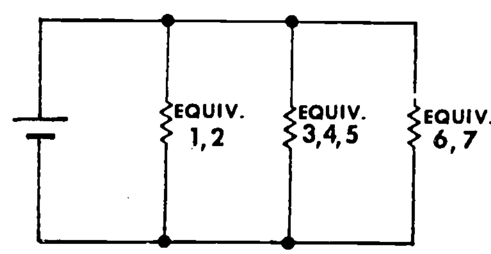


Figure 4

Circuits are sometimes obscurely drawn from the point of view of recognizing the general species involved. When you meet a circuit of this kind, try to redraw it so that its basic form can be recognized and appropriately handled. For example, if you study and analyze the peculiar looking circuit of Figure 5, you should be able to recognize it as a simple parallel circuit, redrawn as shown in Figure 6.

next page

continued

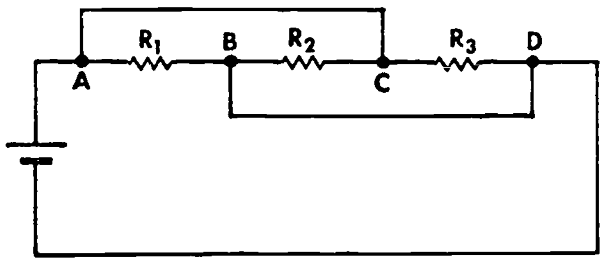


Figure 5

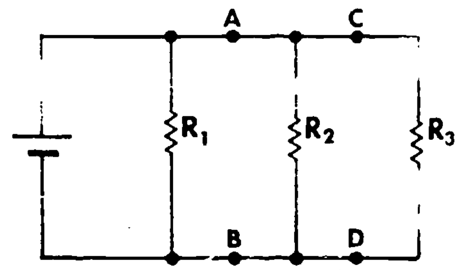
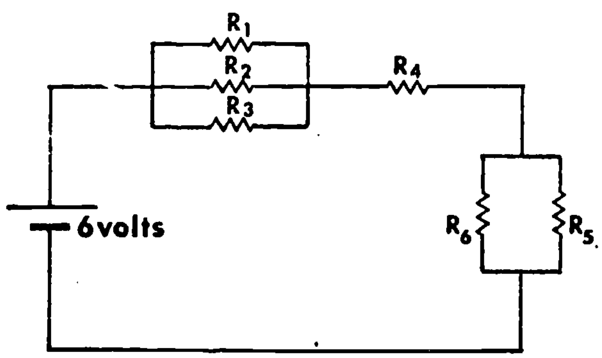


Figure 6

The problems that follow anticipate that you will be able to obtain the equivalent resistance of any type of resistive circuit, simple or complex.

5. What is the equivalent resistance of the circuit shown below?



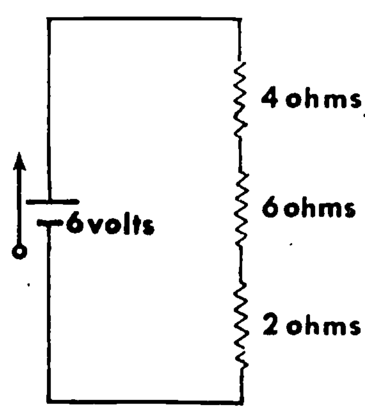
$R_1 = R_2 = R_3 = 15$ ohms

$R_4 = 10$ ohms

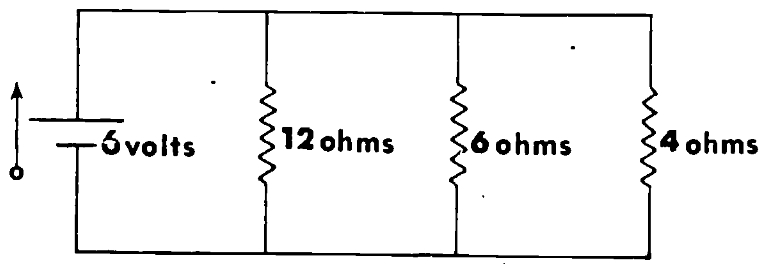
$R_5 = 10$ ohms

$R_6 = 5$ ohms

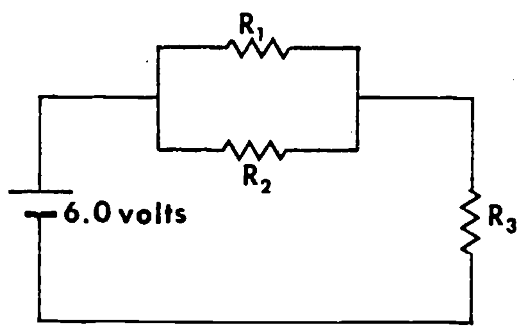
6. What is the equivalent resistance of the circuit shown below?



. What is the equivalent resistance of the circuit shown below?



What is the equivalent resistance of the circuit shown below?

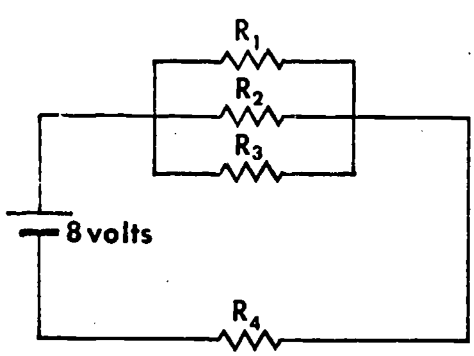


R_1 = 6.0 ohms

R_2 = 12 ohms

R_3 = 10 ohms

What is the equivalent resistance of the circuit shown below?



R_1 = 5 ohms

R_2 = 10 ohms

R_3 = 15 ohms

R_4 = 10 ohms

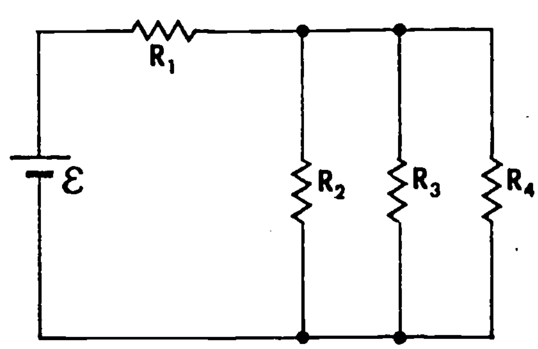
INFORMATION PANEL — Problems Involving Equivalent Resistance

OBJECTIVE

To solve problems involving the effect of various resistor combinations on the currents and voltages in electric circuits.

The circuit designer has at his disposal a multitude of ways to combine resistors. As a matter of fact, there are so many possible permutations that it would be fruitless to attempt to develop a step-by-step approach to the solution of problems involving resistor combinations. The best one can do in this respect is to indicate some general rules of operation and anticipate that the student will set up the necessary sequence of steps in a logical way.

Consider a typical circuit as given in the accompanying figure. Suppose that numerical data has been given for the values of the emf, and each of the individual resistors, and that you are asked to determine:



(a) the voltage drop across R_1;

(b) the current in R_3;

(c) the voltage drop across R_2.

There is a variety of ways to approach this problem. The method we shall suggest below is just that: suggestion, not a hard and fixed procedural mandate. But you should be aware that you may very well hit upon another approach entirely which is just as direct and logical as ours.

(a) The voltage drop across R_1: To determine this, you will first need to know the current in R_1; this requires that you calculate the equivalent resistance of the branched circuit containing R_2, R_3, and R_4 which you then add to the resistance of R_1 to obtain the *total* circuit resistance. Once this is established, you can then calculate the total circuit current with the aid of Ohm's law. This current is then used in the expression:

$$V_1 = iR_1$$

continued

(b) The current in R_3: The voltage drop across R_1 was found in step (a); the emf ε has been given in the data; therefore the voltage drop across each of the resistors in the parallel group is

$$V = \varepsilon - V_1$$

Since the voltage across each resistor in the parallel group is V, then the current in R_3 is obtained from:

$$i_3 = V/R_3$$

(c) The voltage drop across R_2: This has already been determined in step (b) since the voltage across each resistor in the parallel group is V: hence, V is the answer to this question.

The preceding sequence of reasoned steps is typical and springs from approaching the problem, first from a broad or "gestalt" point of view to establish the general function of each component of the circuit and its effect on the circuit as a whole, and then from the standpoint of specifics--setting up each step as a logical consequence of the preceding one, always with the broad aspect of the problem in mind.

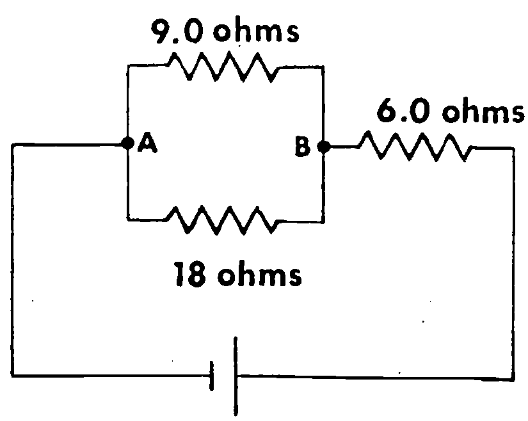
10. In this circuit, the voltage drop across the 6.0-ohm resistor is

A. equal to V_{AB}

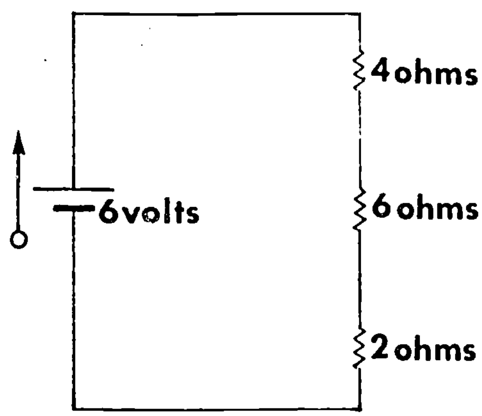
B. greater than V_{AB}

C. smaller than V_{AB}

D. zero

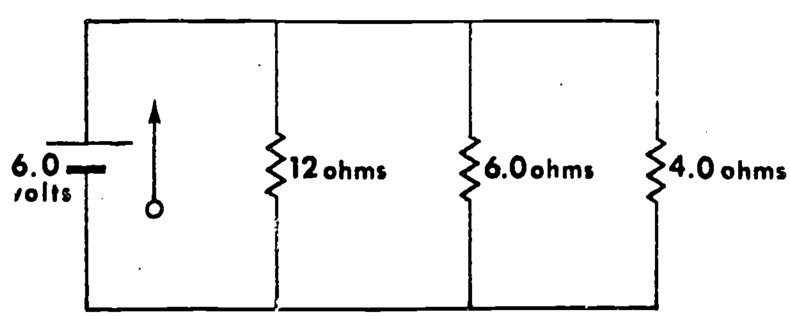


11. In the circuit shown below, what is the current in the 2-ohm resistor?

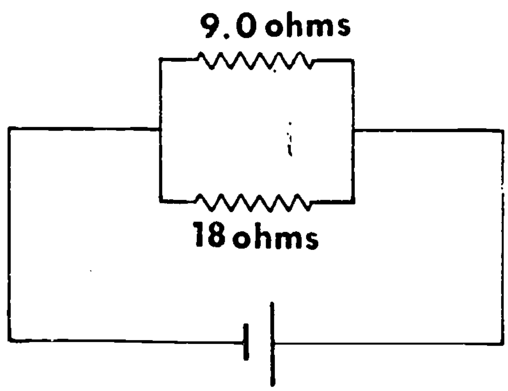


12. For the circuit of the preceding question, what is the potential difference across the 4-ohm resistor?

13. For the circuit shown below, what is the current in the 4.0-ohm resistor?



14. For the circuit below, one can say that the current in the 9.0-ohm resistor is ______ that in the 18-ohm resistor.



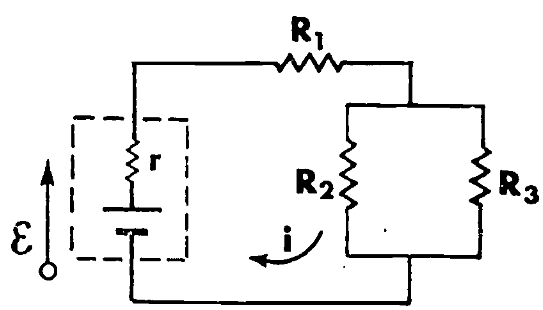
A. equal to

B. twice

C. 1/2

D. 1/3

15. For the circuit in the figure, the values of the circuit elements are



ε = 6.0 volts

r = 0.30 ohms

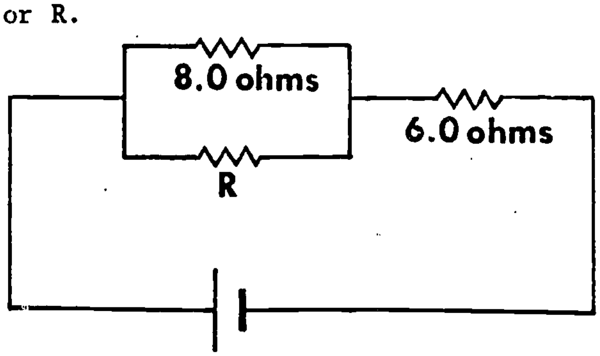
R_1 = 5.0 ohms

R_2 = 3.0 ohms

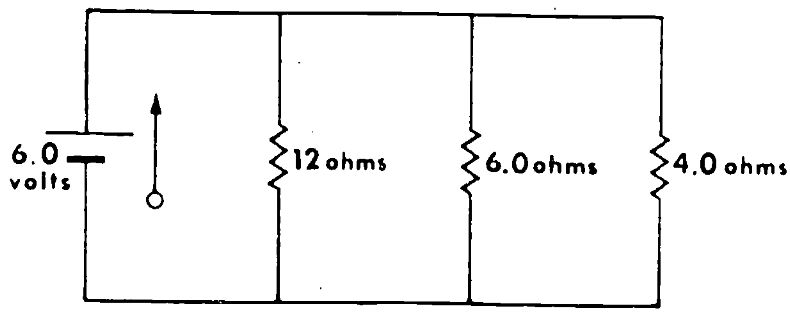
R_3 = 2.0 ohms

What is the current in R_1?

16. For the circuit shown in the figure, find the value of the resistance R such that the current in the 6-ohm resistor is *three* times the current in the resistor R.



17. Calculate the power delivered by the battery in the circuit shown below.



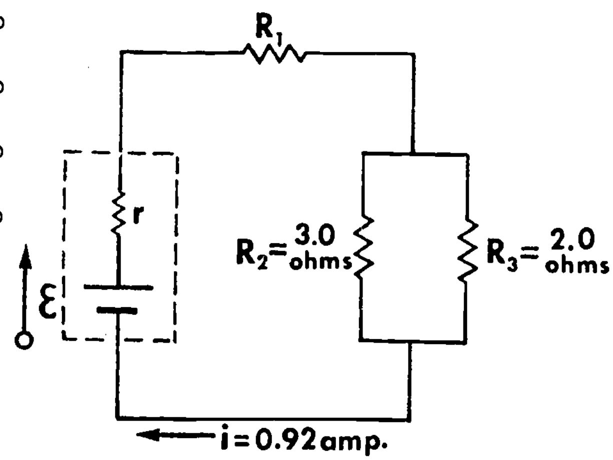
18. In the accompanying circuit, what are the currents i_2 and i_3 in resistors R_2 and R_3, respectively?

A. i_2 = 0.37 amp; i_3 = 0.55 amp

B. i_2 = 0.55 amp; i_3 = 0.37 amp

C. i_2 = 0.46 amp; i_3 = 0.46 amp

D. i_2 = 0.92 amp; i_3 = 0.92 amp



INFORMATION PANEL

Using Kirchhoff's Rules

OBJECTIVE

To apply Kirchhoff's rules to the solution of network problems.

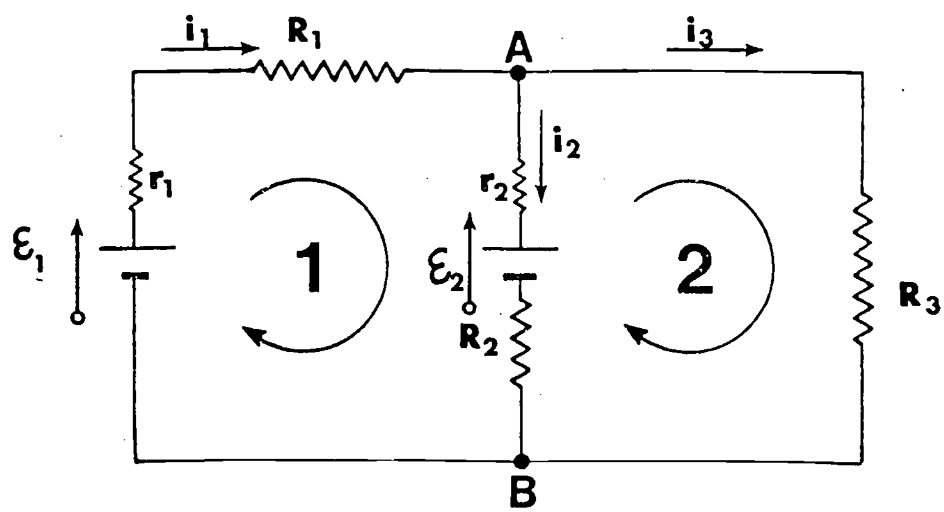
next page

continued

Various physics and engineering texts present conventions for the use of Kirchhoff's rules which differ from each other, usually to a minor extent. The fact that there do exist variations, however, may often lead the student astray as a result of inconsistency in the utilization of one and only one set of conventions. Although personal preferences do exist among workers in the field of electricity, we should like to recommend that you follow the procedure outlined below, adhering to the conventions we are going to use.

Upon presentation of the circuit and the problem:

(1) *Establish the number of independent loops in the circuit.* For example, the circuit given in the diagram below consists of only two independent loops identified as 1 and 2. You may feel that there is a third loop containing r_1, R_1, R_2, and ε_1 but you should recognize



that this loop is *not* independent of the other two; it is merely their sum. If we wished, however, we could use either one of the two identified loops and this third "sum" loop: this is a matter of choice depending entirely on the individual solving the problem.

(2) *Count the number of individual branch points.* In this circuit there are two branch points, A and B, which are not independent, however. At branch point A, the current i_1 splits into two parts, i_2 and i_3. At B, these two currents recombine to yield i_1 once again. The equations involving Kirchhoff's current rule are set up for these branch points, then, they will not be independent of one another. Thus, in this example there is only one *independent* equation possible. In general,

> *if a network has N branch points, only N - 1 independent branch points are available, and only N - 1 independent current equations may be set up.*

next page

continued

(3) The next step calls for *setting up assigned current directions in each loop*. In selecting the current direction, try to be logical about it if the data given to you includes applicable information relative to the comparative values and directions of the emfs. If this data is not available, or if it is obscure in any fashion, don't worry about it. Assign *any* direction to each loop current. If your selection turns out to be incorrect, the problem solution will show this. It will yield a negative sign for the current, thus indicating the wrong assigned direction initially.

(4) *Applying Kirchhoff's rules*. Write the equations for the chosen branch points using the first rule. The convention we use is that a positive sign precedes a current entering a branch point, and a negative sign precedes a current leaving a branch point. Selecting A as the branch point, we may write:

$$i_1 - i_2 - i_3 = 0 \qquad (1)$$

As we have shown, only one independent current equation can be set down for the sample circuit.

Next write the loop equations using Kirchhoff's second rule. The convention used here is that a positive sign precedes a voltage rise and a negative sign precedes a voltage drop. These are recognized as follows:

Voltage rise:	traversing a seat of emf in the direction of the emf; and traversing a resistor in the direction *opposite* that of the current.
Voltage drop:	traversing a seat of emf in the direction *opposite* that of the emf; and traversing a resistor in the direction of the current.

For the example at hand, the loop equations (identified loops 1 and 2) are:

$$\text{LOOP 1:} \quad \varepsilon_1 - i_1 r_1 - i_1 R_1 - i_2 r_2 - \varepsilon_2 - i_2 R_2 = 0 \qquad (2)$$

$$\text{LOOP 2:} \quad \varepsilon_2 + i_2 r_2 - i_3 R_3 + i_2 R_2 = 0 \qquad (3)$$

(5) *Solving the simultaneous equations*. This is the final step. In step (4) we have written three independent equations and can therefore solve for the three unknowns i_1, i_2, and i_3.

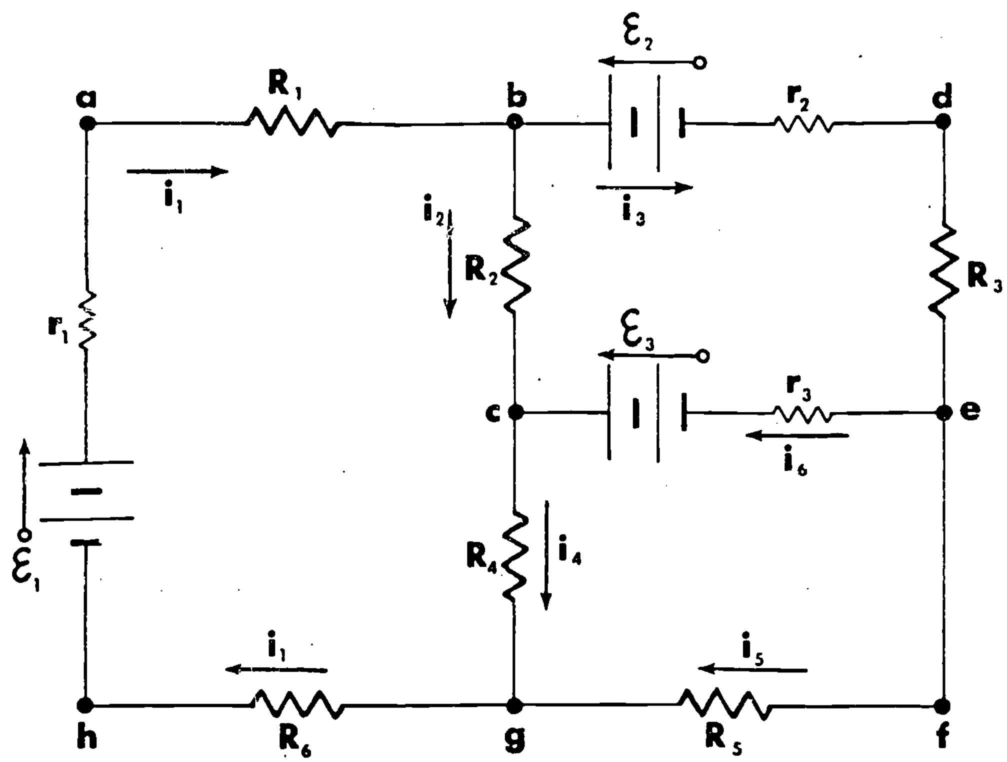
19. The current equations for the three branch points b, c, g in the accompanying circuit are, respectively

A. $i_1 - i_2 - i_3 = 0$; $i_2 - i_4 + i_6 = 0$; $-i_1 + i_4 + i_5 = 0$

B. $i_1 + i_2 + i_3 = 0$; $i_2 + i_4 + i_6 = 0$; $i_1 + i_4 + i_5 = 0$

C. $i_1 - i_2 - i_3 = 0$; $i_2 + i_4 - i_6 = 0$; $i_1 - i_4 - i_5 = 0$

D. $i_1 - i_2 - i_3 = 0$; $i_2 - i_4 + i_6 = 0$; $-i_1 + i_2 + i_4 + i_5 = 0$



20. What general conservation law leads directly to Kirchhoff's first rule (current rule)?

A. Conservation of energy

B. Conservation of current

C. Conservation of charge

D. Conservation of momentum

21. If the solution of a network problem yields a negative current, then most probably

A. the answer is wrong

B. the problem is insoluble

C. this is indicative of the fact that electrons (negative charges) are responsible for the current

D. the current direction initially assumed is incorrect

22. For the currents' directions as chosed in the diagram on the opposite page, write the loop equation for loop *abgha* starting from point a and going around the loop in a clockwise sense.

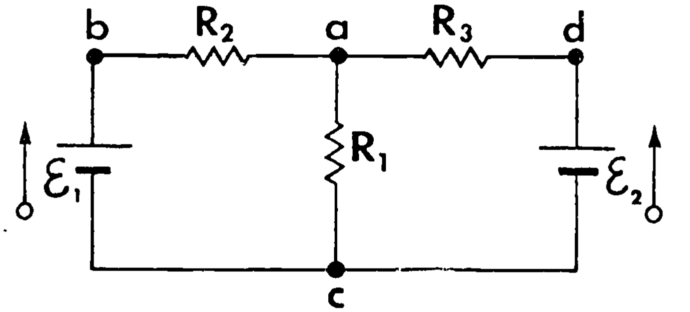
A. $i_1R_1 + i_2R_2 + i_4R_4 + i_1R_6 - \varepsilon_1 + i_1r_1 = 0$

B. $-i_1R_1 - i_2R_2 - i_4R_4 - i_1R_6 + \varepsilon_1 - i_1r_1 = 0$

C. $-i_1R_1 - i_2R_2 - i_4R_4 - i_6R_6 + \varepsilon_1 - i_1r_1 = 0$

D. $-i_1R_1 - i_2R_2 - i_4R_4 - i_1R_6 - \varepsilon_1 - i_1r_1 = 0$

23.



R_1 = 5.0 ohms

R_2 = 10 ohms

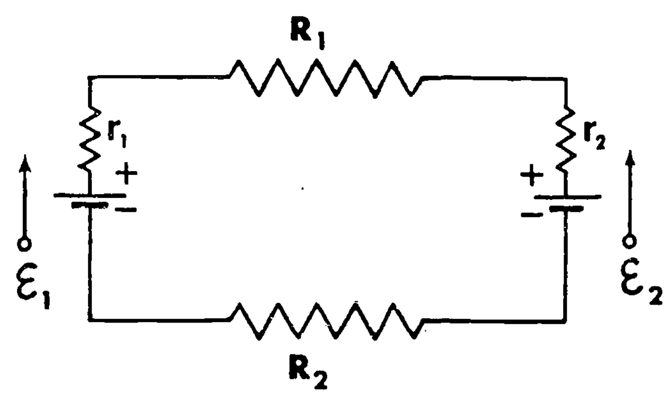
R_3 = 15 ohms

ε_1 = 12 volts

ε_2 = 6.0 volts

For the circuit shown in the figure, find the magnitude of the current through resistor R_1.

24. The values of the components of the circuit shown below are



R_1 = 9.0 ohms

R_2 = 3.0 ohms

ε_1 = 6.0 volts

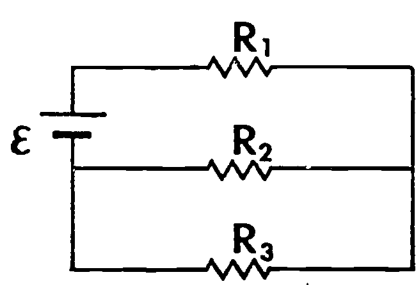
ε_2 = 12 volts

r_1 = 0.60 ohms

r_2 = 0.40 ohms

where r represents the internal resistance of a source of emf. Calculate the current i in the circuit.

25. For the circuit shown here, what is the potential difference across R_3?



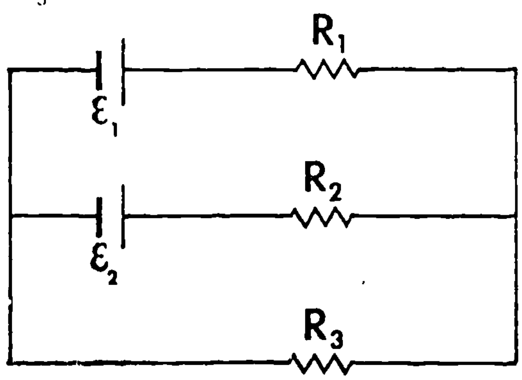
ε_1 = 4.0 volts

R_1 = 10 ohms

R_2 = 15 ohms

R_3 = 20 ohms

26. For the circuit in the diagram, what is the magnitude of the current in R_3?



ε_1 = 4.0 volts

ε_2 = 6.0 volts

R_1 = 10 ohms

R_2 = 15 ohms

R_3 = 20 ohms

[a] CORRECT ANSWER: 0.5 amp

The current in each series resistor is identical. Furthermore, we know that, by definition, the equivalent resistance of a circuit is the resistance of a resistor that can replace all the components of the circuit without changing the circuit's characteristics; i.e., without changing the current delivered by the source of emf.

The equivalent resistance of the given circuit is

$$R = R_1 + R_2 = R_3 = 12 \text{ ohms}$$

Thus, the current delivered by the source of emf is

$$i = V/R = 6/12 = 0.5 \text{ amp}$$

This is the current in each of the resistors.

[b] CORRECT ANSWER: 18 watts

The power can be computed using one of the following three expressions

$$P = i^2R = V^2/R = iV$$

It is immaterial, of course, which one we use. The information available to us determines this. For the present problem we were given V, the potential difference across the terminals of the battery; the equivalent resistance of the circuit is 2.0 ohms. Thus the power delivered by the battery is

$$P = \frac{V^2}{R} \quad \frac{6^2}{2} \quad \frac{36}{2} = 18 \text{ watts}$$

Note the equivalent resistance for this circuit is

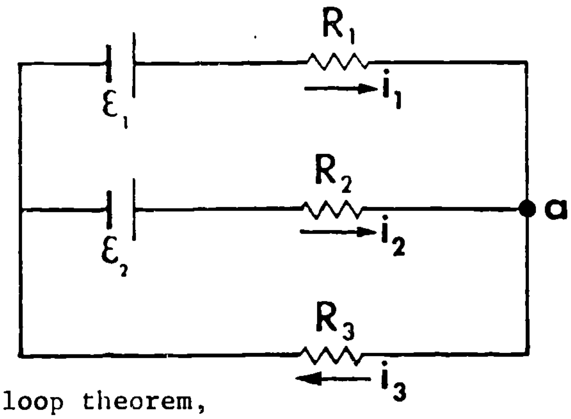
$$\frac{1}{R} = \frac{1}{12} + \frac{1}{6.0} + \frac{1}{4.0}$$

or

$$R = 2.0 \text{ ohms}$$

[a] CORRECT ANSWER: 0.18 amp

If we designate the current in each resistor as indicated in the drawing



we obtain, using the loop theorem,

$$\varepsilon_1 - i_1R_1 + i_2R_2 - \varepsilon_2 = 0 \quad (1)$$

$$\varepsilon_2 - i_2R_2 - i_3R_3 = 0 \quad (2)$$

and at the junction a, we have

$$i_1 + i_2 - i_3 = 0 \quad (3)$$

Since we are looking for i_3, we may rewrite (3) as

$$i_1 = i_3 - i_2 \quad (4)$$

Substituting (4) into (1), we obtain

$$\varepsilon_1 - \varepsilon_2 - (i_3 - i_2)R_1 + i_2R_2 = \varepsilon_1 - \varepsilon_2 - i_3R_1 + i_2(R_1 + R_2) = 0 \quad (5)$$

Finally, from (2) and (5), we obtain

$$i_3 = \frac{\varepsilon_1R_2 + \varepsilon_2R_1}{R_1R_2 + R_2R_3 + R_3R_1} = 0.18 \text{ amp} \quad (6)$$

TRUE OR FALSE? In this situation, with the directions of the currents as shown, i_3 must equal the sum of i_1 and i_2.

[b] CORRECT ANSWER: A

Since charge is neither destroyed nor created within a conductor, the same charge per unit time (current) must pass through any two points of a series circuit. Thus, the same current must be present in each of the resistors connected in series.

[a] CORRECT ANSWER: 18.3 ohms

In the circuit R_1, R_2, and R_3 and R_5 and R_6 are connected in parallel. Let us calculate the equivalent resistances for these first.

The equivalent resistance for combination of R_1, R_2, and R_3 is

$$\frac{1}{R_7} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15}$$

or

$$R_7 = 5 \text{ ohms}$$

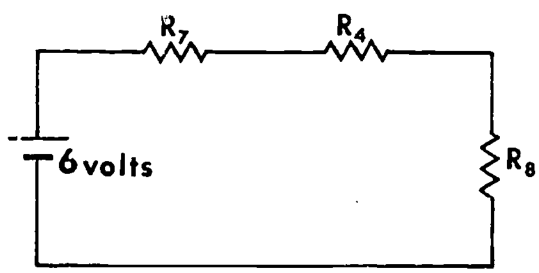
Similarly for the combination of R_5 and R_6, the equivalent resistance is

$$\frac{1}{R_8} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{10} + \frac{1}{5} = \frac{3}{10}$$

or

$$R_8 = 3.3 \text{ ohms}$$

Thus the original circuit may be replaced by the circuit shown here.



The equivalent resistance of this series circuit is:

$$R = R_7 + R_4 + R_8$$

$$R = 18.3 \text{ ohms}$$

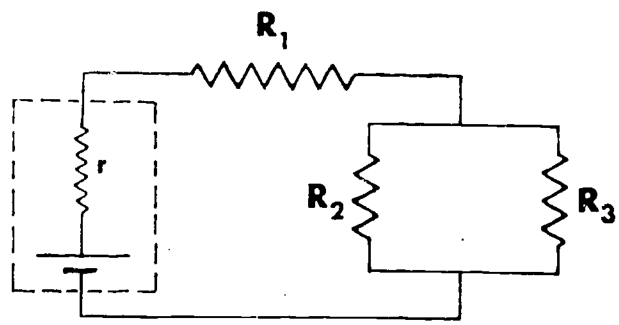
TRUE OR FALSE? The current in the circuit shown would not exceed 1/3 amp.

[b] CORRECT ANSWER: D

Obtaining a negative current in the solution of a network problem simply means that we "guessed" the direction of the current incorrectly (provided, of course, our algebraic and arithmetic operations have been carried out correctly). Since there are only two possible directions for the current, however, this poses no problem. We still have a solution to the problem, simply keeping in mind that the current has the obtained magnitude and a direction opposite to the one assigned.

[a] CORRECT ANSWER: 0.92 amp

The current in R_1 is the total current flowing in the circuit, which, by Ohm's law, is



$$i = \frac{\varepsilon}{R}$$

where R is the total resistance of the circuit. The emf ε is 6.0 volts and the resistances are

R_1 = 5.0 ohms, R_2 = 3.0 ohms, R_3 = 2.0 ohms, and r = 0.30 ohms

The equivalent resistance, R_p, of R_2 and R_3 is given by

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

or R_p = 6/5 ohm. The total resistance R of the circuit is then

$$R = R_1 + R_p + r = 5 + 1.2 + 0.3 = 6.5 \text{ ohms}$$

Therefore, the total current (the current in R_1) is

$$i = \frac{\varepsilon}{R} = \frac{6}{6.5} = 0.92 \text{ amp}$$

TRUE OR FALSE? In the final equivalent circuit, all the R's are treated as a series configuration.

[b] CORRECT ANSWER: 2 ohms

In the given circuit the three resistors are connected in parallel. Therefore, their equivalent resistance is given by

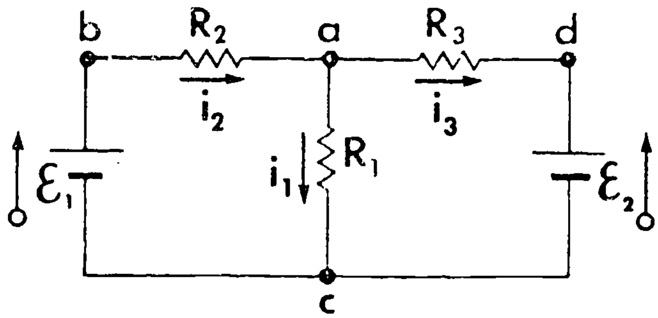
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{6}{12}$$

or

$$R = 2 \text{ ohms}$$

[a] CORRECT ANSWER: 0.87 amp

Let the currents through R_1, R_2, and R_3 be i_1, i_2, and i_3, respectively. Further, let us assume the arbitrary directions of the currents as shown in the diagram



The current equation for the branch point a is

$$i_2 - i_1 - i_3 = 0 \qquad (1)$$

Traversing clockwise, the loop equation for the loop abc is

$$\varepsilon_1 - i_2R_2 - i_1R_1 = 0 \qquad (2)$$

Similarly, the loop equation for the loop adc is

$$-\varepsilon_2 - i_3R_3 + i_1R_1 = 0 \qquad (3)$$

Substituting given values and rearranging the equations (1), (2), and (3) become

$$-i_1 + i_2 - i_3 = 0 \qquad (4)$$

$$5.0\ i_1 + 10\ i_2 + 0 = 12 \qquad (5)$$

$$5.0\ i_1 + 0 - 15\ i_3 = 6.0 \qquad (6)$$

Multiplying equation (4) by -15 and adding to equation (6) we obtain

$$20\ i_1 - 15\ i_2 = 6.0 \qquad (7)$$

Multiplying equation (5) by 3 and equation (7) by 2 and adding, we obtain

$$55\ i_1 = 48$$

or

$$i_1 = 0.87 \text{ amp}$$

TRUE OR FALSE? Referring to equation 3, the negative sign preceding ε_2 indicates that the traversal occurs in a direction opposite that of the emf in the seat.

[a] CORRECT ANSWER: 12.7 ohms

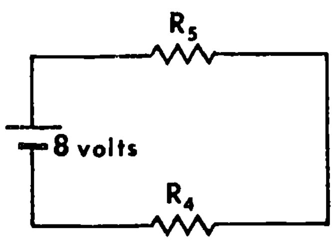
Since R_1, R_2, and R_3 are connected in parallel, the equivalent resistance of this group is

$$\frac{1}{R_5} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{5} + \frac{1}{10} + \frac{1}{15} = \frac{11}{30}$$

or

$$R_5 = 2.7 \text{ ohms}$$

Thus the original circuit may be replaced by the circuit given below



The ~~total~~ equivalent resistance is thus

$$R = R_5 + R_4 = 12.7 \text{ ohms}$$

TRUE OR FALSE? If the seat of emf were changed to 12 volts, the total equivalent resistance would then be greater than 12.7 ohms.

[b] CORRECT ANSWER: B

The circuit under consideration is the one appearing in the problem. Writing Ohm's law for the current through and the voltage across each resistor, we obtain

$$V_1 = i_1 R_1$$

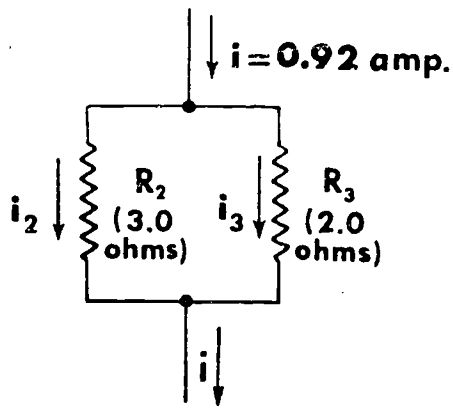
and

$$V_2 = i_2 R_2$$

Since $V_1 = V_2$

$$\frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{18}{9} = 2$$

[a] CORRECT ANSWER: A



The total current is given as 0.92 amp.

$$i_2 = \frac{R_3}{R_2 + R_3}\, i = \frac{2}{5} \times 0.92 = 0.37 \text{ amp}$$

and

$$i_3 = \frac{R_2}{R_2 + R_3} = \frac{3}{5} \times 0.92 = 0.55 \text{ amp}$$

Another way to view this problem is to use the equivalent resistance of the $R_2 - R_3$ combination; i.e.,

$$\frac{R_2 R_3}{R_2 + R_3} = R_p = \frac{6}{5} \text{ ohms}$$

to calculate the potential difference across the combination; i.e.,

$$V_p = i\, R_p = 0.92 \times \frac{6}{5} \doteq 1.1 \text{ volts}$$

Thus,

$$i_2 = \frac{V_p}{R_2} = \frac{1.1}{3} = 0.37 \text{ amp}$$

and

$$i_3 = \frac{V_p}{R_3} = \frac{1.1}{2} = 0.55 \text{ amp}$$

TRUE OR FALSE? The current i *leaving* the $R_2 - R_3$ group is less than 0.92 amp.

[a] CORRECT ANSWER: A

The mathematical expression of Ohm's law is

$$V = iR \quad (1)$$

Therefore the ratio of voltage to resistance is

$$\frac{V}{R} = i \quad (2)$$

This ratio, according to the data, is the same for all three resistors, i.e.,

$$\frac{V_i}{R_i} = i = \text{constant} \quad (3)$$

This means the resistors are connected in series since, in a series circuit, the current is everywhere the same.

TRUE OR FALSE? When corresponding values of two variables form a constant ratio, the variables are directly proportional to one another.

[b] CORRECT ANSWER: 1.5 amp

In the given circuit, the three resistors are connected in parallel. Thus, the potential difference across each one of them is the same as that across the battery (source of emf); namely 6.0 volts. For the 4.0 ohm resistor we use Ohm's law to obtain

$$i_4 = (6.0 \text{ volts})/(4.0 \text{ ohms}) = 1.5 \text{ amp}$$

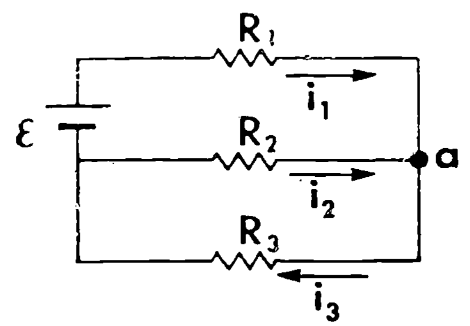
[c] CORRECT ANSWER: A

We follow this convention: currents flowing toward a branch point enter the equation with a plus sign and current flowing away from the branch point enter the equation with a minus sign. Note how this convention inevitably leads to the three equations (for b, c, and g) given in answer A.

TRUE OR FALSE? The current equation for point e is: $i_3 - i_6 - i_5 = 0$

[a] CORRECT ANSWER: 1.8 volts

If we designate the current in each resistor as shown in the diagram,



we obtain the following equation by using the loop theorem:

$$\varepsilon_1 - i_1R_1 + i_2R_2 = 0 \quad (1)$$

$$-i_2R_2 - i_3R_3 = 0 \quad (2)$$

and at the junction a we have

$$i_1 + i_2 - i_3 = 0 \quad (3)$$

Since we are looking for the potential drop across R_3 we have to solve for i_3 first. Eliminating i_1 from (1) and (3) we obtain

$$i_3 = \frac{R_2\varepsilon_1}{R_1R_2 + R_2R_3 + R_3R_1} \quad (4)$$

Thus, the potential drop across R_3 is

$$i_3R_3 = \frac{R_2R_3\varepsilon_1}{R_1R_2 + R_2R_3 + R_3R_1} \quad (5)$$

Substituting numerical values, we obtain

$$i_3R_3 = \frac{R_2R_3\varepsilon_1}{R_1R_2 + R_2R_3 + R_3R_1} = 1.8 \text{ volts}$$

Note that this problem may be solved by first replacing R_2 and R_3 with an equivalent resistance R, and then solving for the current in this modified circuit. Then we may calculate the current i_3 in R_3 and finally obtain the potential difference across R_3.

[a] CORRECT ANSWER: 14 ohms

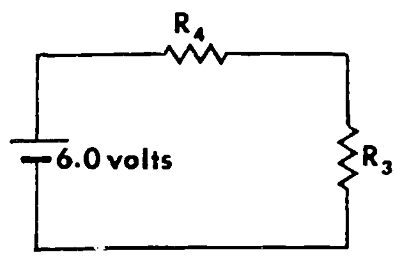
Since R_1 and R_2 are connected in parallel, the equivalent resistance of this group is

$$\frac{1}{R_4} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6.0} + \frac{1}{12}$$

or

$$R_4 = 4.0 \text{ ohms}$$

Now we may replace the original circuit by the following circuit.



The total equivalent resistance is

$$R = R_4 + R_3 = 14 \text{ ohms}$$

[b] CORRECT ANSWER: B

The statement is true regardless of the resistances of the resistors involved; i.e., the currents through resistors connected in parallel are inversely proportional to their resistance. We can show this using the fact that resistors in parallel have the same potential difference across them and the mathematical statement of Ohm's law; namely

$$V = iR$$

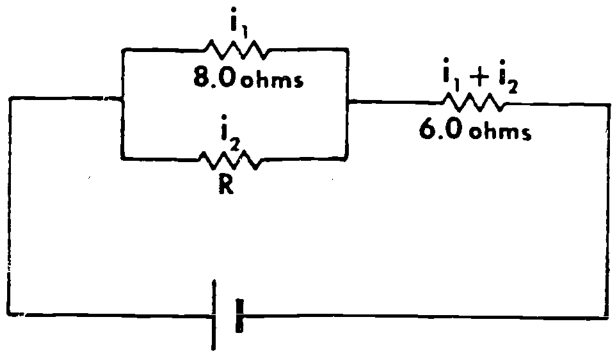
If V is common for all resistors connected in parallel then their respective currents are given by

$$i_i = V/R_i$$

i.e., the currents are inversely proportional to the resistances.

TRUE OR FALSE? For resistors of R_1 = 2 ohms, R_2 = 4 ohms, and R_3 = 14 ohms the current in each resistor would no longer be inversely proportional to its resistance in the parallel circuit.

[a] CORRECT ANSWER: 16 ohms



Let the currents through the 8.0-ohm resistor and R be i_1 and i_2 respectively. Therefore, the current through the 6.0-ohm resistor is $i_1 + i_2$. However, the 8.0-ohm resistor and R are connected in parallel; therefore, the iR drops across the 8.0-ohm resistor and R are the same, that is

$$R\, i_2 = 8.0\, i_1 \qquad (1)$$

and we are given

$$\frac{i_1 + i}{i_2} ,$$

or

$$2\, i_2 = i_1 \qquad (2)$$

Dividing equation (1) by equation (2) we obtain

$$\frac{R\, i_2}{2\, i_2} = \frac{8.0\, i_1}{i_1}$$

or

$$R = 16 \text{ ohms}$$

TRUE OR FALSE? The current in R is twice the current in the 8.0-ohm resistor.

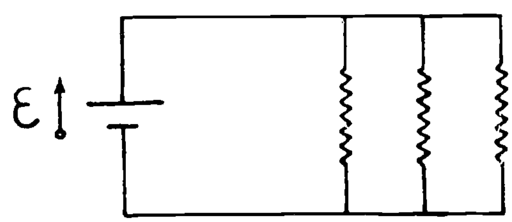
[b] CORRECT ANSWER: 12 ohms

In the given circuit the three resistors are connected in series. The equivalent resistance of any number of resistors connected in series is equal to the sum of their resistances. Hence

$$R = R_1 + R_2 + R_3 = 4 + 6 + 2 = 12 \text{ ohms}$$

[a] CORRECT ANSWER: C

The diagram shows three resistors connected in parallel. We must realize that in diagrams of this sort all lines representing conductors are taken as perfect conductors (zero resistivity). Thus, we see that all three resistors are directly connected across the same source of emf ε. Hence, the potential difference across each of the resistors is equal to that across the terminal of the source of emf; i.e., each of the resistors has the same voltage across it.



[b] CORRECT ANSWER: 0.46 amp

This is a very simple, one-loop circuit. Since ε_2 is larger than ε_1 we logically assign a counterclockwise direction to the current. Starting from the + side of ε_2 and going around the loop in a counterclockwise sense, we obtain by Kirchhoff's second rule (we include, of course, the internal resistances)

$$-iR_1 - \varepsilon_1 - ir_1 - iR_2 - ir_2 + \varepsilon_2 = 0$$

or

$$i(R_1 + R_2 + r_1 + r_2) = \varepsilon_2 - \varepsilon_1$$

and finally

$$i = \frac{\varepsilon_2 - \varepsilon_1}{R_1 + R_2 + r_1 + r_2} = \frac{6.0}{13} = 0.46 \text{ amp}$$

[a] CORRECT ANSWER: C

Charge can neither be created nor destroyed. This conservation-of-charge statement is related to Kirchhoff's current rule in the following way: If, say, more current entered a circuit junction than left the junction, some of the current carriers (electrons) would have to accumulate at the junction. The negative charge accumulated there would set up an electric field in a direction which opposes the action of the source of emf, eventually stopping the flow of charge (current). This situation prevails, of course, in the process of charging a capacitor. As we shall see in a later volume the current charging a capacitor diminishes exponentially. Electrons cannot jump from one plate to the other; thus, they accumulate on one plate setting up an electric field which "pushes" back additional electrons that try to come to the plate.

We may show the relation between conservation of charge and Kirchhoff's current rule mathematically as well. Using the definition of current

$$i \equiv dq/dt$$

we may express the rule as follows:

$$\sum_{k=1}^{N} i_k \equiv \sum_{k=1}^{N} \frac{dq_k}{dt} = 0$$

or

$$\frac{d}{dt} \sum_{k=1}^{N} q_k = 0$$

giving

$$\sum_{k=1}^{N} q_k = \text{constant}$$

[b] CORRECT ANSWER: 2 volts

In the preceding problem we calculated the current delivered by the source of emf to be 0.5 amp. Since the resistors are connected in series this current is present in each resistor. From Ohm's law

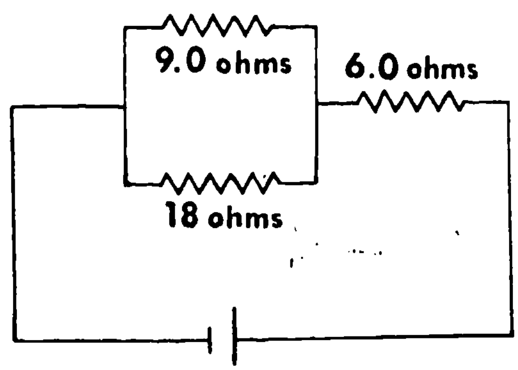
$$V = iR$$

thus for the 4-ohm resistor

$$V = 0.5 \times 4 = 2 \text{ volts}$$

[a] CORRECT ANSWER: A

The 9.0-ohm and 18-ohm resistors are in parallel. Their equivalent resistance R is given by



$$\frac{1}{R} = \frac{1}{9} + \frac{1}{18} = \frac{3}{18} = \frac{1}{6}$$

or

R = 6.0 ohms

Since the same current flows through this equivalent resistance and the additional 6.0-ohm resistor, the voltage drop across each one is the same.

TRUE OR FALSE? The parallel group in this problem could be replaced by a 6.0-ohm resistor without changing V_{AB}.

[b] CORRECT ANSWER: B

The directions of the various currents involved correspond to the direction in which we go around the loop (provided we go in the clockwise direction). Thus, all potential changes across the resistors in this loop are negative (voltage drops). The source of emf is traversed in the positive direction, so ε_1 enters the equation with a plus sign.

TRUE OR FALSE? Any time you traverse any loop in a clockwise sense, the voltage drops across the resistors are all negative.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL — The Ammeter

OBJECTIVE

To study the method of modifying a moving-coil galvanometer to construct an ammeter; to solve problems in ammeter applications.

The design of most laboratory ammeters is based on the basic moving-coil meter movement. An ammeter of this type must be inserted in series with the circuit in which the current is to be measured. If the addition of the instrument is not to change the initial current seriously, the equivalent resistance of the meter must be very small. The resistance of a typical pivoted coil in a modern galvanometer may range from about 10 ohms to 100 ohms, and a current of the order of a fraction of a milliampere to a few milliamperes will ordinarily produce a full-scale deflection of the indicating needle.

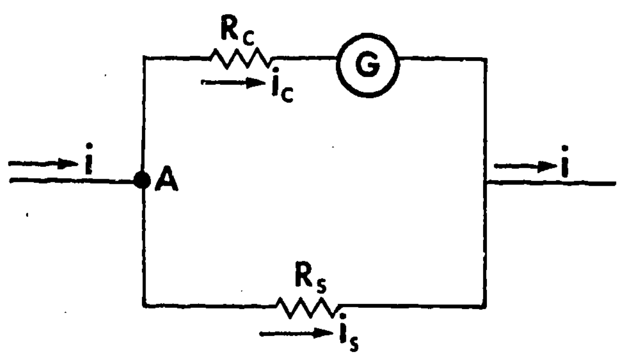


FIGURE 1

To convert the basic galvanometer to an ammeter with a specified full-scale deflection, a low resistance called a *shunt* is connected in parallel with the coil as indicated in Figure 1. In a typical practical situation, one may know the following values:

i ≡ the total *line current* to be measured, or at least its maximum value for given changes in circuit conditions;

R_c ≡ the resistance of the galvanometer coil;

i_c ≡ the coil current that will cause a full-scale deflection.

It would then be necessary to calculate the resistance that the shunt must have to insure that a given maximum line current will produce a full-scale deflection of the instrument.

next page

continued

To obtain the equation needed for calculating the shunt resistance R_s, we apply Kirchhoff's rules as follows: (For junction point A)

(current rule) $i - i_c - i_s = 0$ or $i_s = i - i_c$

(voltage rule) $i_cR_c + i_sR_s = 0$ or $i_sR_s = i_cR_c$

Solving the latter equation for R_s:

$$R_s = i_cR_c/i_s$$

and substituting the equivalent of the shunt current as obtained from the current rule:

$$R_s = \frac{i_c}{i - i_c} R_c$$

For example, suppose we have a galvanometer which deflects full-scale for a coil current of 1.00 milliampere. The resistance of the coil is, say 20.0 ohms, and we wish to convert it to an ammeter that will read full-scale when the line current is 2.00 amperes. The required shunt resistance is then:

$$R_s = \frac{1.00 \times 10^{-3}}{2.00 - 1.00 \times 10^{-3}} \times 20$$

$$R_s = 1.00 \times 10^{-2} \text{ ohms (to 3 significant digits)}$$

The resistance of the shunt is, therefore, very small; this helps us meet the requirement that the equivalent resistance of the ammeter be very small, hence such an instrument inserted in series with the circuit to be measured will not make the current *after* insertion very different from the current *before* the meter was inserted.

The basic Kirchhoff rule equations may be used to solve virtually every problem involving ammeters and ammeter applications.

PROBLEMS

1. The resistance of the coil of a pivoted coil galvanometer is 10.0 ohms and a current of 0.0200 amp causes a full-scale deflection. It is desired to convert this galvanometer into an ammeter reading 10.0 amps full-scale. The only shunt available has a resistance of 0.0300 ohms. What resistance R must be connected in series with the coil so that the ammeter will read properly?

2. An ideal ammeter would have

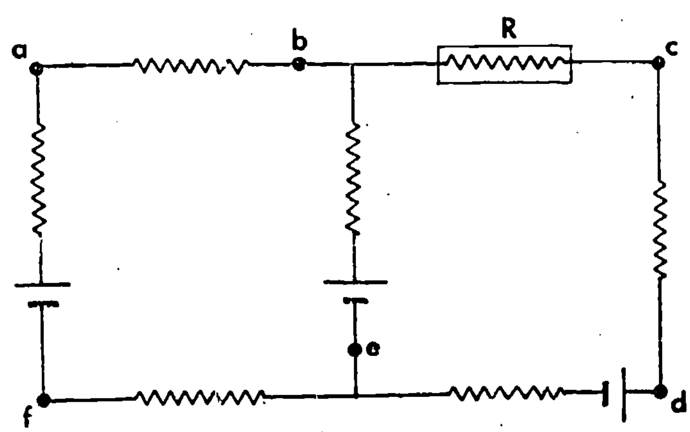
A. infinite resistance

B. zero resistance

C. a lower resistance than the sum of all other resistances in the circuit

D. a high resistance connected in parallel with it

3. Suppose we want to measure the current thru the resistor R of the accompanying circuit, i.e., the resistor enclosed in the box. Of the six points, a, b, c, d, e, and f, shown in the circuit, select the point(s) at which an ammeter must be connected for measuring this current.



A. any of the points will do
B. either b or c
C. either c or d
D. only c

4. A low resistance (shunt) is connected across the coil of an ammeter

A. to protect the meter from high current surges

B. to reduce the resistance of the meter-shunt combination below the resistance of the circuit resistors

C. to enable the ammeter to read higher currents

D. to prevent overheating of the meter

5. A milliammeter with a maximum deflection of 100×10^{-3} amp is to be converted to read in the range 0 to 1 amp by means of a shunt. What must be the resistance of the shunt if the milliammeter has an internal resistance of 45 ohms?

6. A microammeter which has a maximum deflection for $i = 10.0 \times 10^{-6}$ amp is converted to read in the range of 0 to 1.00×10^{-3} amp by means of a 5.00-ohm shunt. What is the internal resistance of the meter?

INFORMATION PANEL — The Voltmeter

OBJECTIVE

To study the method of modifying a moving-coil galvanometer to construct a voltmeter; to solve problems in voltmeter applications.

The moving-coil galvanometer described in the previous Information Panel in this Segment may be converted to a voltmeter by connecting a relatively high resistance called a *multiplier* in series with the coil as shown in Figure 1.

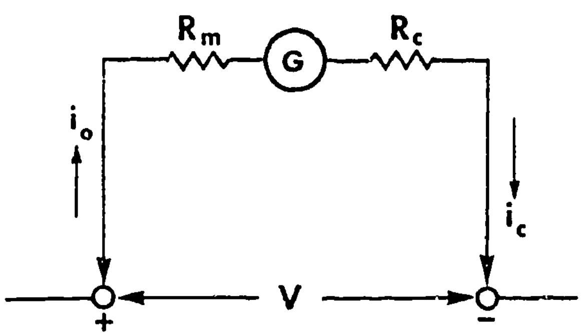


FIGURE 1

Since a voltmeter measures the potential difference between two points, its terminals are joined to these points directly by connecting wires. Thus, to use a voltmeter it is not necessary to break into a circuit to insert it; it is connected in parallel with the component across which the potential difference is to be measured.

When a seat of emf is connected to an *open* circuit, its terminal voltage is equal to its emf. A voltmeter with its associated multiplier resistance, however, is a *closed* circuit in which a current must exist in order for the instrument to operate. As a result of the voltmeter current, a voltage drop will occur across the internal resistance r of the source of emf so that the voltmeter will read *less* than the actual value of the emf, that is,

$$\text{(voltmeter reading) } V = \varepsilon - i_V r$$

where i_V is the voltmeter current. Like the ammeter, the voltmeter can seriously change the voltage it is supposed to be measuring unless it is designed to take an insignificant amount of current. This means that the equivalent resistance of the instrument (coil resistance plus multiplier resistance) must be made as high as possible.

next page

continued

To calculate the required value of the multiplier, the following characteristic values must be known:

$V \equiv$ the total potential difference or voltage to be measured, or at least the maximum value anticipated;

$R_c \equiv$ the resistance of the galvanometer coil;

$i_c \equiv$ the coil current required for full-scale deflection.

The current in the voltmeter coil (and, of course, in the multiplier resistance in series with the coil) is given by:

$$i_c = \frac{V}{R_m + R_c}$$

where R_m = the multiplier resistance. Thus:

$$R_m = \frac{V}{i_c} - R_c$$

To illustrate the use of this relationship, suppose we calculate the multiplier resistance R_m required to convert the galvanometer used in the example in the first Information Panel to a 0 to 10-volt voltmeter. This galvanometer has a full-scale coil current of 1.00 milliampere, and a coil resistance of 20.0 ohms. Substituting:

$$R_m = \frac{10.0}{1.00 \times 10^{-3}} - 20$$

$$R_m = 10{,}000 \text{ ohms (to 3 significant digits)}$$

In this section, you will be asked to

(a) determine the value of a certain resistance, given a specific voltmeter, its internal resistance, and its reading when connected in series with the resistance across a given line voltage;

(b) calculate the reading of a voltmeter, given the resistance across which it is connected, a resistor in series with it, and the current in the circuit;

(c) calculate the ratio of a "true" voltage to the voltage measured by a given voltmeter in a specific circuit.

7. A 150-volt voltmeter has a resistance of 20,000 onms. When connected in series with a large resistance R across a 110-volt line the meter reads 5.0 volts. Find the resistance R.

8. A voltmeter generally consists of a galvanometer with a

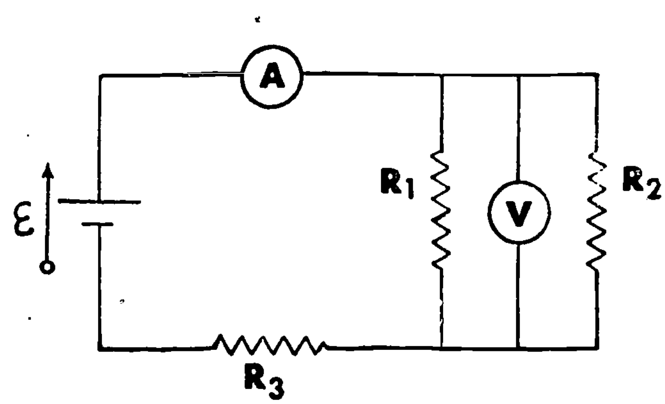
A. high resistance connected in parallel with the coil

B. low resistance connected in series with the coil

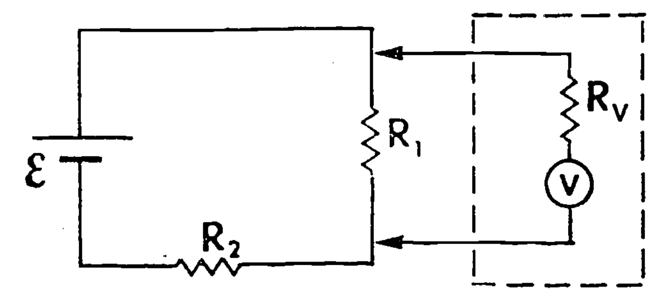
C. low resistance connected in parallel with the coil

D. high resistance connected in series with the coil

9. In the diagram shown below ε = 110 volts and the internal resistance of the source of emf is negligible. Let R_1 = 100 ohms, R_2 = 151 ohms, R_3 = 50 ohms, and the reading of the ammeter be 1.0 amp. If the internal resistance of the voltmeter is 80,000 ohms (or 80 kilohms), what is the reading of the voltmeter?



10. In the accompanying diagram, ε = 200 volts and the internal resistance of the source of emf is negligible. Let R_1 = 100 ohms and R_2 = 100 ohms. If an inexpensive voltmeter of internal resistance R_V = 100 ohms is connected across resistor R_1, calculate the ratio of true voltage across R_1 (without the voltmeter) to the voltage read by the voltmeter.



INFORMATION PANEL — The Potentiometer

OBJECTIVE

To study the principle of the potentiometer; to apply this principle to problems involving the measurement of unknown emf's.

As its name implies, the potentiometer is a device for measuring potential differences. The fundamental principle of the potentiometer is the balancing of one voltage against another in parallel with it. In the circuit of Figure 1, there is a branch circuit containing a galvanometer in which there is a current i_2 because of the potential difference across the tapped section r of the slide wire resistor R. This potential difference results from the current i in R due to the emf of the *working battery* ε_1.

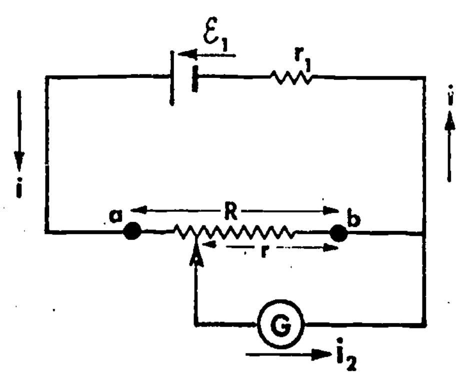


Figure 1

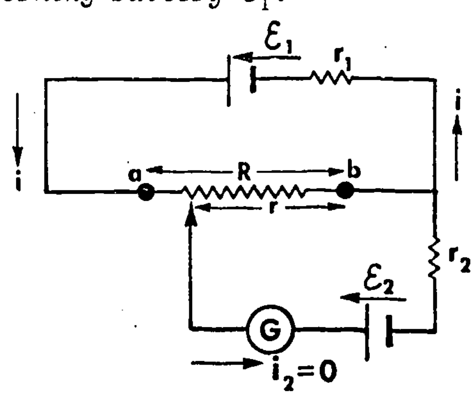


Figure 2

next page

continued

In Figure 2, a second source of emf ε_2 has been introduced into the lower branch. The direction of this emf is such that the current in the branch can be reduced to zero (i.e., $i_2 = 0$) by a balancing process achieved by adjusting the resistance r to the required value, so that the potential difference across r is exactly equal to the emf of the added cell ε_2.

The great advantage of the potentiometer over an ordinary voltmeter is that at the moment of balance there is no current in the seat of emf being measured. Thus, the wires carry no current so that errors due to line drop or contact resistance do not occur. In addition--and of greater importance--there is no voltage drop across the internal resistance of the seat of emf being measured, hence the potentiometer enables the user to ascertain true emf rather than terminal voltage.

In practice, an arrangement such as that illustrated in Figure 3 is used to measure the emf of an unknown source with high precision. In this diagram, we have replaced ε_2 with ε_x to denote the unknown cell; we have added a standard cell ε_s whose emf is known to four or more significant digits; we have omitted the internal resistances of both cells since, at balance, the current in this branch is zero, hence the voltage drop across the internal resistances is zero. The basic process for measurement consists of adjusting the variable resistor with the switch in the ε_x position unitl the galvanometer deflection is zero. This resistance setting is recorded as r_x. The switch is then reset to the ε_s position, thus inserting the standard cell into the branch circuit. The variable resistor is again adjusted until the galvanometer deflection is zero, and the resistance value recorded as r_s. Provided that the current in the working battery circuit (not shown in Figure 3) remains constant, it can be shown that the following relationship now holds:

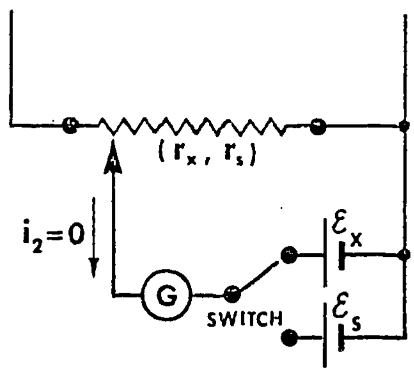


Figure 3

$$\varepsilon_x = \varepsilon_s \frac{r_x}{r_s}$$

Note that this equation can be used only when the emf of the unknown source is compared with that of a standard cell. When the latter is not used, then the current in the working battery loop must be known, or the emf of the working battery must be given.

next page

continued

Referring to Figure 2, the value of ε_2 may be calculated by applying Kirchhoff's rules in the absence of a standard cell. For the working battery loop we have:

$$\varepsilon_1 - i(R + r_1) = 0$$

and for the lower loop:

$$\varepsilon_2 - ir = 0$$

Note that r_2 may be omitted from the calculations because $i_2 = 0$.

Solving for ε_2:

$$\varepsilon_2 = ir = \frac{r}{R + R_1}\varepsilon_1$$

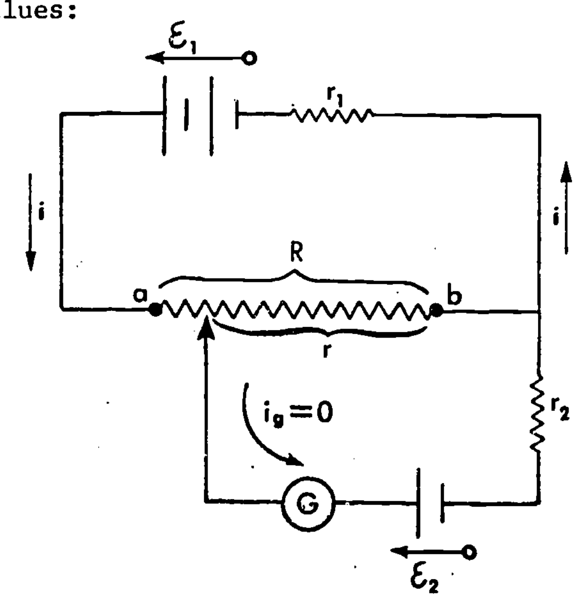
The core problem in this section is based on this last relationship. The remaining problems require that

(a) you recognize the correct circuit of a potentiometer;

(b) you demonstrate understanding of a voltage divider circuit;

(c) you be able to solve a second problem similar to the core problem.

11. In the circuit below, the various elements have the following values:



R = 100 ohms ε_1 = 9 volts

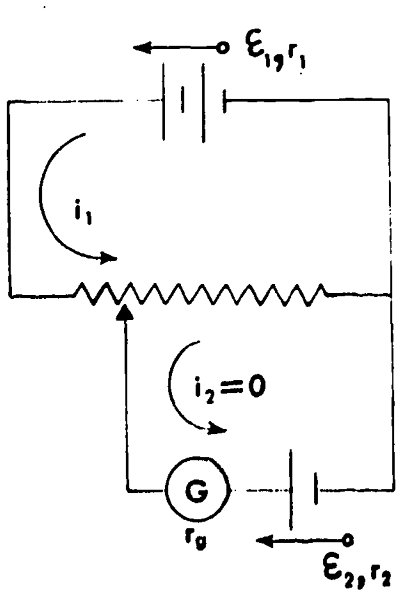
r = 68 ohms ε_2 = ?

$r_1 = r_2 = 2$ ohms

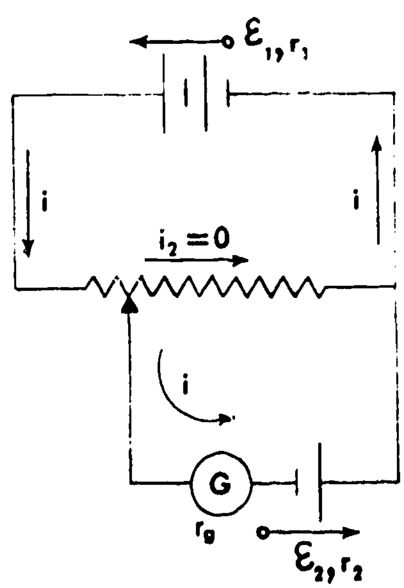
Calculate the value of ε_2.

12. Which of the following circuit diagrams best describes the potentiometer and its use?

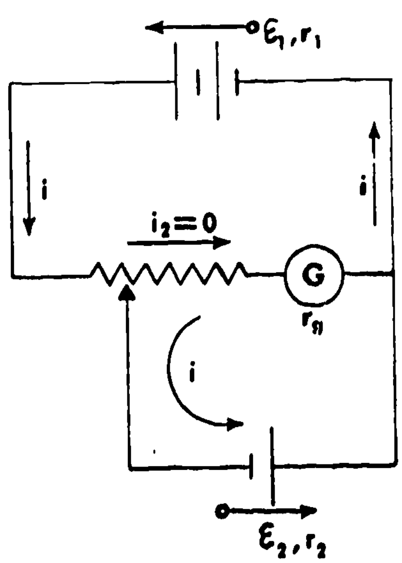
A.



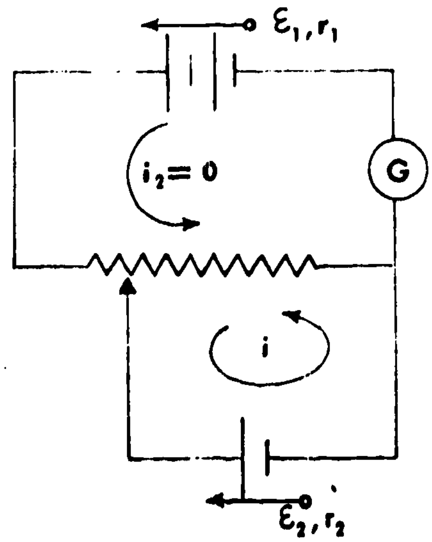
B.



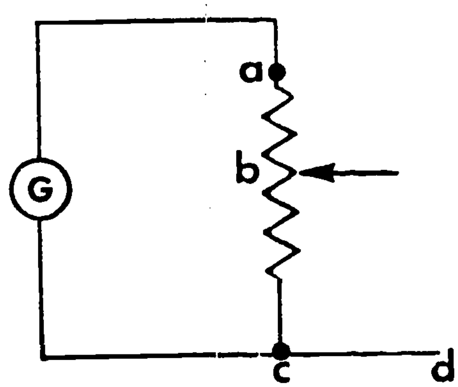
C.



D.



13. The diagra[illegible] sistor with a movable tap. The [illegible]alvanometer has a resistance of 90 ohms. The resistance between a and [illegible] is 75 ohms. What is the resistance between b and c if the current in the galvanometer is to be 1/3 the current in wire dc?



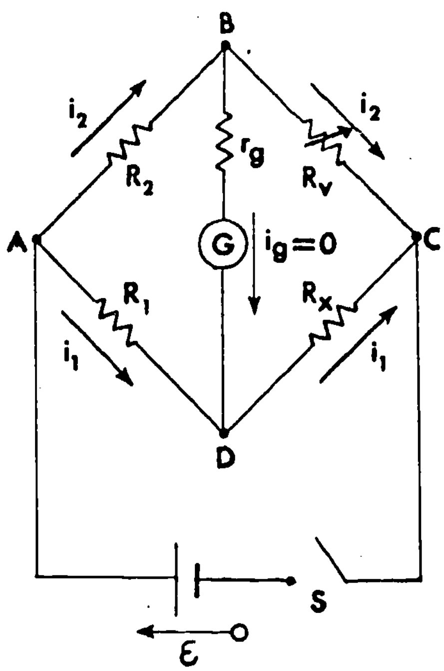
14. The slide wire of a potentiometer has a total resistance of 200 ohms and is 200 cm long. When it is balanced with a standard cell whose emf is 1.0183 volts, the length of wire needed is 121.5 cm. When a dry cell is substituted for the standard cell, the contact point is at 185.0 cm. What is the emf of the cell to 3 significant digits?

INFORMATION PANEL The Wheatstone Bridge

OBJECTIVE

To study the principle of the Wheatstone Bridge; to solve problems in which the Wheatstone Bridge is used to determine resistance.

The resistance of a circuit component may be measured by the voltmeter-ammeter method or by means of an ohmmeter. The precision of these methods, however, often leaves much to be desired. To improve precision of resistance measurements, the Wheatstone Bridge is frequently employed. For your convenience, we are presenting here a brief discussion of the bridge and its principle of operation. For a more extended description, particularly of commercial instrument packages, you are referred to your assigned reading.



As illustrated in the drawing, the bridge consists of three known resistors R_1, R_2, and R_v connected in a continuous loop which includes the unknown resistance R_x. A sensitive galvanometer G with internal resistance r_g joins opposite corners of the diamond configuration, while a source of emf ε and a switch S are connected in series to the remaining corners.

Resistor R_v is variable. In operation, R_v is adjusted until the galvanometer current becomes zero. Actually, the switch S is a spring loaded pushbutton which is tapped sharply to observe the galvanometer action; this avoids what might be damaging currents through the delicate coil of the instrument. As balance is closely approached, the switch is held down while the final fine adjustment is made on R_v to achieve zero current in the galvanometer.

next page

continued

At balance points B and D are at the same potential. Thus, the voltage drops across [illegible] and R_2 are equal (point A is common to R_1 and R_2.) Similarly, the voltage drops across R_v and R_x are equal. In equation form this may be written:

$$i_1 R_1 = i_2 R_2 \quad (1)$$

and

$$i_1 R_x = i_2 R_v \quad (2)$$

or

$$\frac{R_1}{R_x} = \frac{R_2}{R_v} \quad (3)$$

Finally

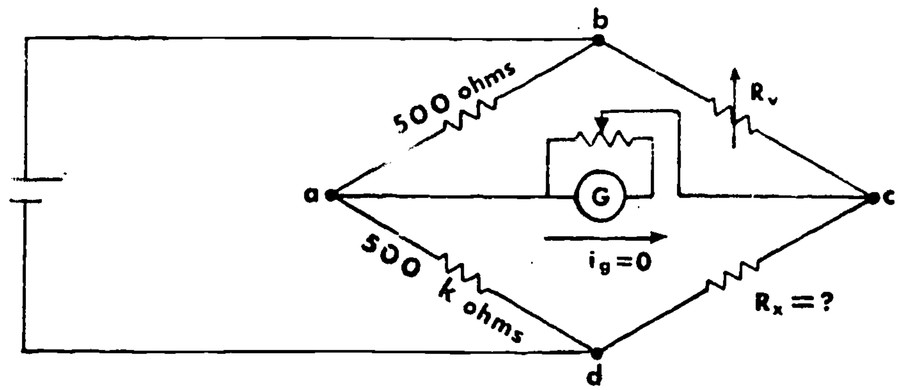
$$R_x = \frac{R_1}{R_2} R_v \quad (4)$$

With the values of R_1, R_2, and R_v known, the value of R_x is then easily calculated. In the laboratory, the Wheatstone Bridge is arranged so that R_1 and R_2 are exactly equal to one another. Equating R_1 and R_2 to each other in equation (4):

$$R_x = R_v$$

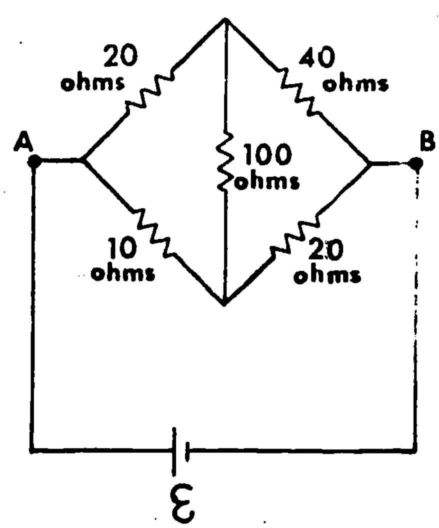
Resistor R_v may be permanently calibrated in ohms so that the resistance R_x may be read directly from the dial of R_v.

15.



In the Wheatstone Bridge illustrated, the variable resistor R_v is adjusted to 1550 ohms in order to make the galvanometer current (i_g) equal to zero. What is the value of R_x in ohms?

16. Calculate the equivalent resistance between points A and B of the circuit shown below.



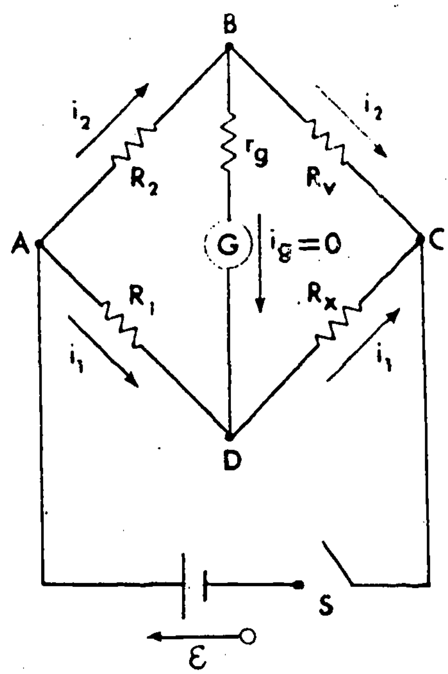
17. In the Wheatstone bridge shown here let R_1 = 10 ohms, ε = 10 volts and i_1 = 0.40 amp. If the internal resistance of the battery is negligible, what is the value of R_x?

A. not enough information given

B. 15 ohms

C. 10 ohms

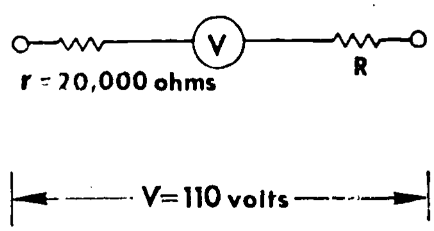
D. 25 ohms



18. For the bridge of the preceding question, if R_2 = 20 ohms, what will be the reading of R_v when the bridge is balanced? (To balance the bridge means to adjust R_v until i_g becomes zero.)

[a] CORRECT ANSWER: 4.2×10^5 ohms

We are asked to find the resistance R that causes the voltmeter to read 5.0 volts, when the voltage V is 110 volts. We can find this resistance if we know the value of the current.



We know that the voltmeter has been calibrated with only the resistance r in the circuit. Therefore, when the voltmeter reads 5.0 volts there is a current

$$i = \frac{5.0}{20,000} = 0.25 \times 10^{-3} \text{ amp}$$

through it. Returning to the above circuit, we have

$$i(r + R) = V$$

or

$$R = \frac{V}{i} - r = \frac{110}{0.25 \times 10^{-3}} - 20 \times 10^3$$

$$= 4.2 \times 10^5 \text{ ohms}$$

TRUE OR FALSE? The current in r is substantially smaller than the current in R.

[b] CORRECT ANSWER: C

An ammeter must be connected in series with the portion of the circuit at which we want to measure the current. This does not mean, however, that the ammeter must be connected right next to the particular element (usually resistor) for measuring the current through that element. The ammeter can be connected at any point such that there is no branch point between it and the element of interest. For the case at hand, the current in R can be measured with an ammeter connected at any point in the circuit that lies to the right of the two branch points of the circuit. Therefore, both points, c or d, are suitable for the connection of an ammeter which will measure the current in R.

[a] CORRECT ANSWER: 1.5

The true voltage V_T across R_1 may be found from

$$V_T = i_T R_1 \quad (1)$$

where

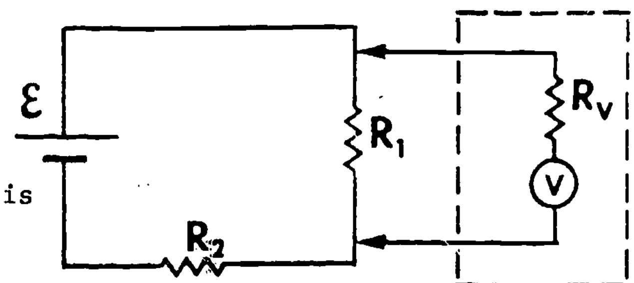
$$i_T = \frac{\varepsilon}{R_1 + R_2}$$

Therefore

$$V_T = \frac{\varepsilon R_1}{R_1 + R_2} \quad (2)$$

When the voltmeter is connected across R_1, the current through R_2 is

$$i = \frac{\varepsilon}{R_2 + \frac{R_1 R_V}{R_1 + R_V}}$$



and the voltage drop across R_2 is

$$V_2 = iR_2$$

$$= \frac{\varepsilon R_2 (R_1 + R_V)}{R_1 R_2 + R_2 R_V + R_1 R_V} \quad (3)$$

Therefore, the voltage across R_1 read by the voltmeter is

$$V = \varepsilon - V_2 \quad (4)$$

The required ratio

$$\frac{V_T}{V} = \frac{V_T}{\varepsilon - V_2} \quad (5)$$

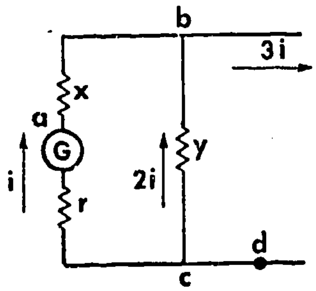
Substituting numerical values in equations (2), (3), (4), and (5) we find

$$\frac{V_T}{V} = 1.5$$

TRUE OR FALSE? The true voltage across R_1 is larger than the voltage read by the instrument.

[a] CORRECT ANSWER: 55 ohms

If we denote the resistance between a and b as x and the resistance between b and c as y, the original circuit can be represented by the following equivalent circuit:



r = 90 ohms
x + y = 75 ohms

Let us assume that the current through dc is $3i$. This means that the current through the galvanometer is i. Using Kirchhoff's loop rule, we have

$$i(r + x) - 2i\,y = 0$$

or

$$r + x = 2y \qquad (1)$$

Combining (1) with the given data

$$x + y = 75$$

we obtain the resistance between b and c (i.e., y):

$$y = 55 \text{ ohms}$$

[b] CORRECT ANSWER: B

An ammeter is inserted in a circuit in series with the other resistors in the circuit. An ideal ammeter, therefore, would have zero internal resistance so that its insertion in a circuit would not change the current in the circuit. Since an ammeter has a coil in its mechanism, a zero-resistance ammeter is nonexistent (there are no zero-resistivity conductors except for certain metals which at extremely low temperatures become "superconductors" with zero resistivity). A good ammeter, however, has a very low internal resistance so its effect on the current in circuits in which it is used is negligible.

[a] CORRECT ANSWER: 1.55×10^6

Since the current in the galvanometer is zero, points *a* and *c* are equipotential. Therefore, $V_{ba} = V_{bc}$ and $V_{ad} = V_{cd}$. Let the current through *bad* and *bcd* be i_1 and i_2 respectively.

Therefore,

$$i_2 R_V = i_1(500) \tag{1}$$

and

$$i_2 R_x = i_1(5.00 \times 10^5) \tag{2}$$

Dividing (2) by (1), we obtain

$$\frac{R_x}{R_V} = \frac{5.00 \times 10^5}{500}$$

or

$$R_x = \frac{5.00 \times 10^5}{500} R_V = \frac{5.00 \times 10^5}{500} (1550) = 1.55 \times 10^6 \text{ ohms}$$

TRUE OR FALSE? At balance, the potential difference between points b and d is zero since $i_g = 0$.

[b] CORRECT ANSWER: B

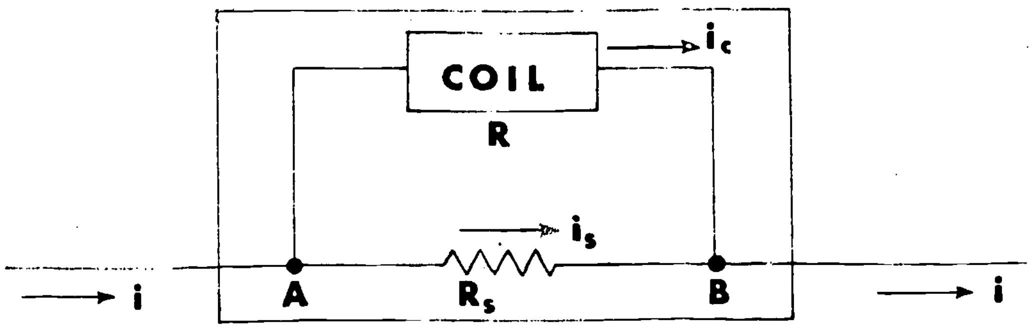
We need not concern ourselves with the upper path of the bridge. We know that the potential difference between points A and C is $\varepsilon = 10$ volts. Since resistors R_1 and R_x are connected in series we obtain

$$i_1(R_1 + R_x) = \varepsilon = 10 \text{ volts}$$

Thus,

$$R_x = \frac{\varepsilon - i_1 R_1}{i_1} = \frac{10 - 4.0}{0.40} = 15 \text{ ohms}$$

[a] CORRECT ANSWER: 5 ohms



Given Data:

R_c = 45 ohms

i_c (max) = 100×10^{-3} amp = 0.1 amp

i (max) = 1 amp

We know that for parallel combinations of resistors

$$V_{AB} = i_c R_c = i_s R_s \qquad (1)$$

and therefore

$$R_s = \frac{i_c}{i_s} R_c \qquad (2)$$

From Kirchhoff's current rule,

$$i = i_c + i_s \qquad (3)$$

so

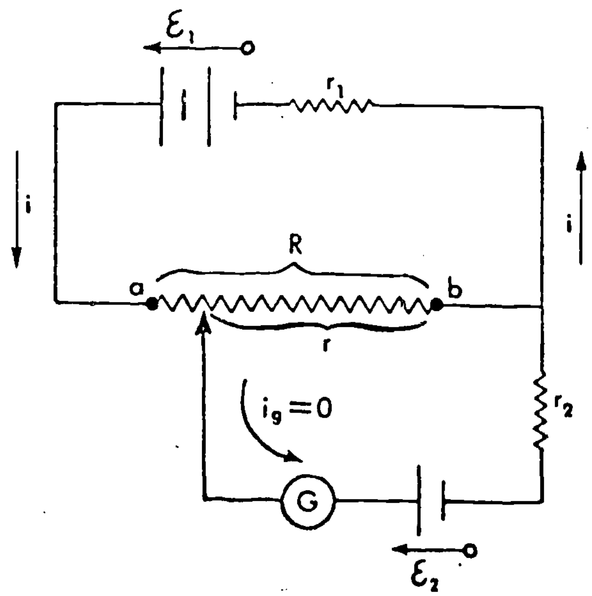
$$i_s = i - i_c = 1 - 0.1 = 0.9 \text{ amp} \qquad (4)$$

Substituting in (2) we obtain

$$R_s = \frac{0.1}{0.9} \times 45 = 5 \text{ ohms}$$

[a] CORRECT ANSWER: 6 volts

The current illustrated is that of a potentiometer.



Kirchhoff's voltage rule for the upper loop is

$$\varepsilon_1 - i(R + r_1) = 0 \tag{1}$$

and for the lower loop is

$$\varepsilon_2 - ir = 0 \tag{2}$$

Note that there is no potential drop across r_2 since the current in it is zero. ($i_g = 0$)

Solving for ε_2,

$$\varepsilon_2 = ir = \frac{\varepsilon_1 r}{R + r_1} = 6 \text{ volts}$$

TRUE OR FALSE? For the conditions specified, the value of r has been adjusted to yield zero galvanometer current.

[a] CORRECT ANSWER: 30 ohms

Since the current in the galvanometer is zero, points B and D are equipotential, hence

$$V_{AB} = V_{AD} \qquad \text{and} \qquad V_{BC} = V_{DC}$$

Therefore

$$i_2 R_2 = i_1 R_1 \tag{1}$$

$$i_2 R_v = i_1 R_x \tag{2}$$

Dividing (2) by (1) we obtain

$$\frac{R_v}{R_2} = \frac{R_x}{R_1} \tag{3}$$

Thus,

$$R_v = \frac{R_x}{R_1} R_2 = 30 \text{ ohms}$$

TRUE OR FALSE? When the bridge is balanced, the current in R_1 is equal to the current in R_2.

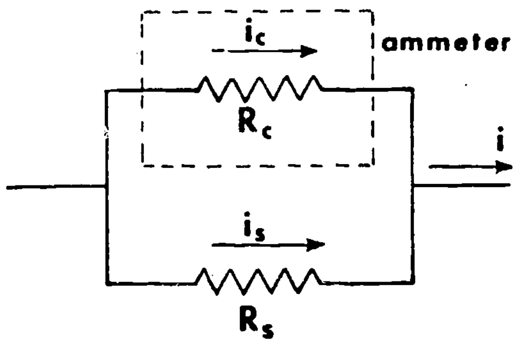
[b] CORRECT ANSWER: 60 volts

The voltage across the source of emf is 110 volts (no internal resistance). Since the total current in the circuit is 1 amp, the voltage drop across R_3 is

$$V_3 = i\,R_3 = 1.0 \times 50 = 50 \text{ volts}$$

Thus the potential difference across R_1, R_2 and the voltmeter is 110 - 50 = 60 volts. This value then is what the voltmeter reads.

[a] CORRECT ANSWER: 495 ohms



We wish to find R_c, the resistance of the ammeter, given

$$i = 1.00 \times 10^{-3} \text{ amp}$$

$$i_c = 10.0 \times 10^{-6} \text{ amp}$$

and

$$R_s = 5.00 \text{ ohms}$$

Since both i_s and Rc are unknown, we need two equations in order to find either quantity. These equations are

$$i_s + i_c = i \qquad \text{(Kirchhoff's first rule)}$$

$$i_c R_c = i_s R_s \qquad \text{(Kirchhoff's second rule)}$$

Solving for R_c, we obtain

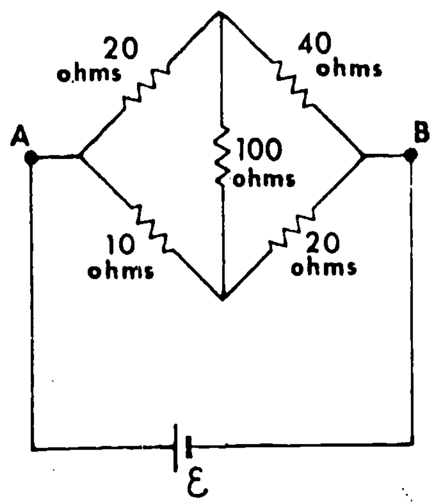
$$R_c = \frac{i_s R_s}{i_c} = \frac{(i - i_c)R_s}{i_c} = \frac{(0.99) \times 10^{-3} \times 5.00}{(0.0100) \times 10^{-3}}$$

$$= 495 \text{ ohms}$$

TRUE OR FALSE? After the shunt is added, the current through the shunt will be smaller than the current through the meter coil.

[a] CORRECT ANSWER: 20 ohms

The given circuit is redrawn here with a voltage ε applied across points A and B. Except for a missing galvanometer this circuit is the same as that of a Wheatstone Bridge.



Furthermore, from the values of the resistors we note that the bridge is balanced. Thus, the current through the 100-ohm resistor is zero which means that the resistor can be removed without changing the characteristics of the circuit. Once the 100-ohm resistor is removed, the remaining circuit is composed of two branches connected in parallel. The upper branch has a resistance of 20 + 40 = 60 ohms. The resistance of the lower branch is 10 + 20 = 30 ohms. Thus, the equivalent resistance of the circuit is given by

$$\frac{1}{R} = \frac{1}{60} + \frac{1}{30} = \frac{3}{60}$$

or

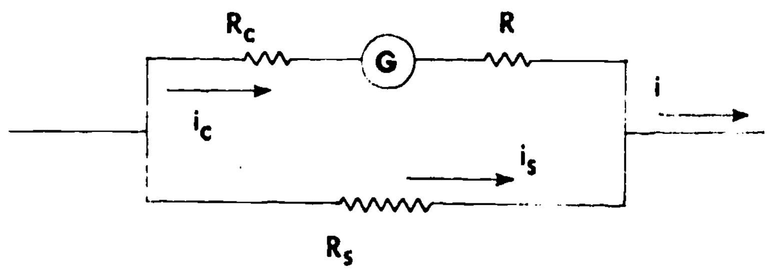
$$R = 20 \text{ ohms}$$

[b] CORRECT ANSWER: C

If an ammeter deflects full-scale for a small current, it can be modified to deflect full-scale for a large current. Most of the large current can be diverted with a shunt resistor, and the ammeter will read full-scale for the total current passed by the ammeter-shunt combination, when, in fact, a very small current is passing through the ammeter itself.

[a] CORRECT ANSWER: 4.97 ohms

The two unknowns for this circuit are i_s and R. We can use Kirchhoff's rules to eliminate i_s and find R.



R_c = 10.0 ohms

R_s = 0.0300 ohms

i = 10.0 amp

i_c = 0.0200 amp

The resulting equations are

$$i = i_s + i_c$$

and

$$i_c(R_c + R) - i_s R_s = 0$$

Solving for R, we find

$$R = \frac{i\ R_s}{i_c} - (R_c + R_s)$$

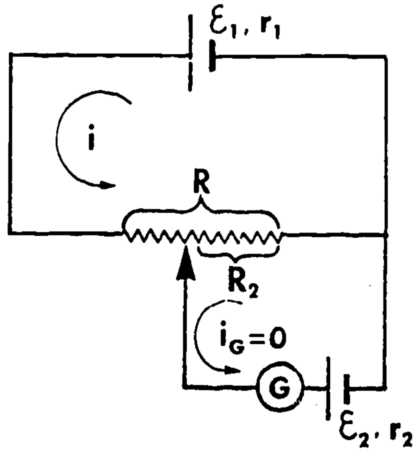
$$= \frac{(10)(0.03)}{0.02} - (10 + 0.03)$$

$$= 4.97 \text{ ohms}$$

TRUE OR FALSE? The voltage drop across R_s is equal to the voltage drop across R_c + R.

[a] CORRECT ANSWER: 1.55 volts

The circuit should be drawn as follows:



From Kirchhoff's voltage rule, we can write the following equations:

$$ir_1 + iR = \varepsilon_1 \qquad (1)$$

and

$$iR_2 = \varepsilon_2 \qquad (2)$$

From these, we obtain

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{iR_2}{i(r_1 + R)}$$

or

$$\varepsilon_2 = \frac{R_2}{r_1 + R}\,\varepsilon_1 \qquad (3)$$

In this case, ε_1, r_1 and R are unknown. When $\varepsilon_2 = 1.0183$ volts, then $R_2 = 121.5$ ohms. However, when $\varepsilon_2 = x =$ the emf of the dry cell, then $R_2 = 185.0$ ohms. (Note 1 cm of the wire has resistance of 1 ohm.)

Thus,

$$\frac{1.0183}{x} = \frac{121.5}{185.0}$$

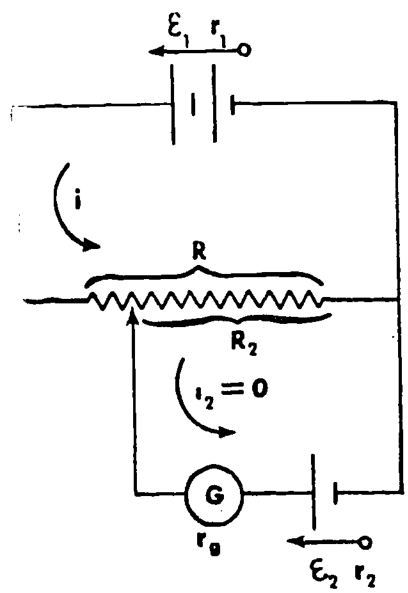
and

$$x = 1.55 \text{ volts}$$

TRUE OR FALSE? The internal resistance of the dry cell (r_2) has been omitted from the Kirchhoff calculations because, at balance, the current in it is zero.

[a] CORRECT ANSWER: A

The galvanometer is a null instrument; i.e., it makes measurement by giving a zero reading. In this case, ε_2 is measured by adjusting the variable resistance until $i_2 = 0$, as measured by the galvanometer. By writing down Kirchhoff's voltage rule for the upper and lower loops, we obtain



$$ir_1 + iR = \varepsilon_1 \qquad (1)$$

and

$$iR_2 = \varepsilon_2 \qquad (2)$$

Dividing equation (2) by equation (1) yields

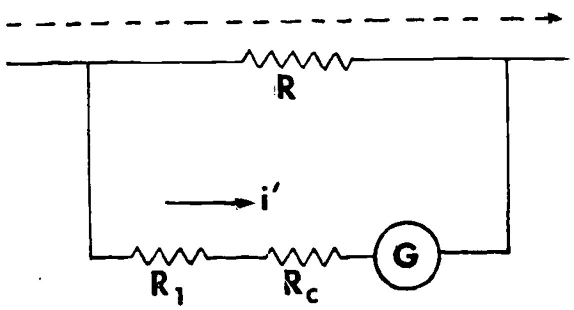
$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{iR_2}{i(r_1 + R)}$$

or

$$\varepsilon_2 = \frac{R_2}{r_1 + R}\,\varepsilon_1$$

..ORRECT ANSWER: D

.he figure shows a high resistance R_1 nnected in series with a galvanometer of (low) resistance R_C. Most of the current will follow the path through R, so the voltmeter (R_1 plus galvanometer) will not modify the voltage V across R appreciably. For the current i' through the galvanometer we have



$$i' = \frac{V}{R_1 + R_C}$$

or

$$V = (R_1 + R_C)i'$$

Thus, a galvanomete- that reads in amperes (an ammeter) can be converted to read in volts by multiplying the ammeter readings by $(R_1 + R_C)$.