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ABSTRACT

Four study segments of the Self-Paced Physics Course materials are presented in this fifth problems and solutions book used as a part of student course work. The subject matter is related to work in electric fields, potential differences, parallel plates, electric potential energies, potential gradients, capacitances, and capacitor circuits. Contained in each segment are information panels, core problems enclosed in a box, core-primed questions, scrambled problem solutions, and true-false questions. Study guides are provided and used to answer the true-false questions and to reveal directions for reaching solutions. When the core problem is answered incorrectly, the study guide requires students to follow the remedial or enabling loop, leading to the solutions of core-primed questions. Also included is a sheet of problem numbers with corresponding page numbers which locate correct answers. (Related documents are SE 016 065 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

UNITED STATES

Self-Paced PHYSICS

U.S.
NAVAL ACADEMY
EX SCIENTIA TRIDENS

NAVAL ACADEMY

SEGMENTS 24-27

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NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY

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13	17b
14	19b
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16	18a
17	17a
18	20a
19	26a
20	17a

Reading:
*HR 29-1/29-5
SW 28-3
SZ 25-3;26-4
AB 37-1;37-2

1	[illegible]
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14	21a
15	24a
16	[illegible]
17	19a
18	* 17b

Reading:
*HR 29-6/25-9
*SW 29-3,28-4
SZ 26-2;*25-6

[illegible]	[illegible]
12	[illegible]
13	13b
14	25a
15	23a
16	17b
17	[illegible]
18	24b
19	20a
20	26b
21	10b
22	20a

Reading:
*HR 30-1;30-2
SW 30-1,30-2
SZ 27-1/27-2
AB 37-4,37-5

[illegible]	26a
[illegible]	21b
[illegible]	17a
[illegible]	19c
[illegible]	25a

Reading:
[illegible] 30-3,30-4
30-3,*30-[illegible]
SW 30-3,*30-4
[illegible] 27-4,*27-5
27-8

P	STEP	NAME
	0.1	Reading: *HR 29-1/29-5 SW 28-3 SZ 26-3; 26-4 AB 37-1; 37-2
	0.2	Information Panel, "Work in an Electric Field"
1		T F (ans)
	1.1	If correct, advance to 5.1; if not, continue sequence.
2		(ans)
3		(ans)
4		(ans)
5		A B C D T F
	5.1	Information Panel, "Potential Difference"
6		T F (ans)
	6.1	If correct, advance to 10.1; if not, continue sequence.

P	STEP	SECTION SEGMENT 24
7		A B C D
8		A B C D
9		A B C D
10		T F (ans)
	10.1	Information Panel, "Potential Due to a Point Charge"
11		A B C D T F
	11.1	If your first choice was correct, advance to 14.1; if not, continue sequence.
12		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 24
13		A B C D	19		A B C D
14		A B C D T F	20		A B C D T F
	14.1	Information Panel, "Potential Due to Combination of Charges"		20.1	Homework: HR 29-8
15		A B C D T F			
	15.1	If your first choice was correct, advance to 20.1; if not, continue sequence.			
16		______________________(ans)			
17		______________________(ans)			
18		A B C D			

P	STEP	NAME
	0.1	Reading: *HR 29-6/29-9 *SW 28-3, 28-4 SZ 26-2; *26-6
	0.2	Information Panel, "Potential Gradient"
1		A B C D T F
	1.1	If your first choice was correct, advance to 5.1; if not, continue sequence.
2		A B C D
3		A B C D
4		______ (ans)
5		______ (ans) T F
	5.1	Information Panel, "Potential Due to Distributed Charges"
6		A B C D T F

P	STEP	SECTION SEGMENT 25
	6.1	If your first choice was correct, advance to P 10; if not, continue sequence.
7		A B C D
8		A B C D
9		A B C D T F
10		A B C D T F
	10.1	If your first choice was correct, advance to 13.1; if not, continue sequence.
11		A B C D
12		A B C D
13		A B C D T F
	13.1	Information Panel, "Applications of the Concept of Electric Potential Energy"

P	STEP	NAME	P	STEP	SECTION SEGMENT 25
14		[ ] T [ ] F [ ] ________ (ans)			
	14.1	If correct, advance to 18.1; if not, continue sequence.			
15		A [ ] B [ ] C [ ] D [ ]			
16		A [ ] B [ ] C [ ] D [ ]			
17		[ ] ________ (ans)			
18		[ ] T [ ] F [ ] ________ (ans)			
	18.1	Homework: HR 29-16			

P	STEP	NAME
	0.1	Reading: HR 30-1, 30-2 SW 30-1, 30-2 SZ 27-1/27-3 AB 37-4, 37-5
	0.2	Information Panel, "The Meaning of Capacitance"
	0.3	Audiovisual, CAPACITORS
1		A B C D T F
	1.1	If your first choice was correct, advance to 5.1; if not, continue sequence.
2		A B C D
3		A B C D
4		______ (ans)
5		T F ______ (ans)
	5.1	Information Panel, "Calculation of Capacitance"
6		A B C D T F

P	STEP	SECTION SEGMENT 26
	6.1	If your first choice was correct, advance to 9.1; if not, continue sequence.
7		A B C D
8		A B C D
9		A B C D T F
	9.1	Information Panel, "Equivalent Capacitance - Series and Parallel"
	9.2	Audiovisual, THE CAPACITOR IN ACTION
10		T F ______ (ans)
	10.1	If correct, advance to 14.1; if not, continue sequence.
11		A B C D
12		______ (ans)

P	STEP	NAME	P	STEP	SECTION SEGMENT 26
13		(ans)	20		A B C D
14		T F (ans)	21		A B C D
	14.1	Information Panel, "Analysis of Capacitor Circuits"	22		T F (ans)
15		T F (ans)		22.1	Homework: HR 30-10
	15.1	If correct, advance to 22.1; if not, continue sequence.			
16		A B C D			
17		A B C D			
18		A B C D			
19		A B C D			

P	STEP	NAME
	0.1	Reading: HR *30-3, 30-4, 30-5; *30-7 SW 30-3, *30-4 SZ 27-4, *27-5; 27-8
	0.2	Information Panel, "Energy Storage in Capacitors"
1		T F (ans)
	1.1	If correct, advance to 5.1; if not, continue sequence.
2		A B C D
3		A B C D
4		A B C D
5		T F (ans)
	5.1	Information Panel, "Transfer of Energy in Capacitors"
6		T F (ans)
	6.1	If correct, advance to 9.1; if not, continue sequence.

P	STEP	SECTION SEGMENT 27
7		(ans)
8		(ans)
9		T F (ans)
	9.1	Information Panel, "Effect of Capacitor Dielectric"
10		A B C D T F
	10.1	If first choice was correct, advance to P 16; if not, continue sequence.
11		A B C D
12		A B C D
13		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 27
14		[ ] ______ (ans)			
15		A [ ] B [ ] C [ ] D [ ] T [ ] F [ ]			
16		[ ] T [ ] F [ ] ______ (ans)			
	16.1	If correct, advance to 18.1; if not, continue sequence.			
17		A [ ] B [ ] C [ ] D [ ]			
18		[ ] T [ ] F [ ] ______ (ans)			
	18.1	Homework: HR 30-28			

INFORMATION PANEL

Work in an Electric Field

OBJECTIVE

To calculate the work done by an external agent in moving a charge over a given distance in an electric field.

One of the principal sources of difficulty for students starting their work in this subject is confusion with respect to the algebraic signs given to the various vectors involved in the equations. We shall try to clarify some of these concepts in this Information Panel.

Starting with a test charge q_o immersed in a uniform electric field of intensity $\vec{E}$ as shown in Figure 1, we can draw on our past studies to state that the force $\vec{F}'$ exerted *by* the field *on* the charge is given by:

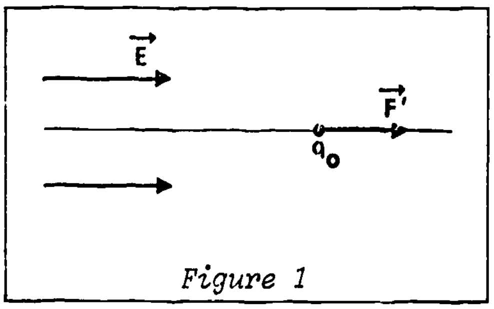


Figure 1

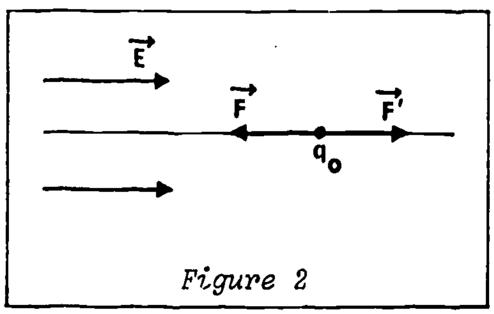


Figure 2

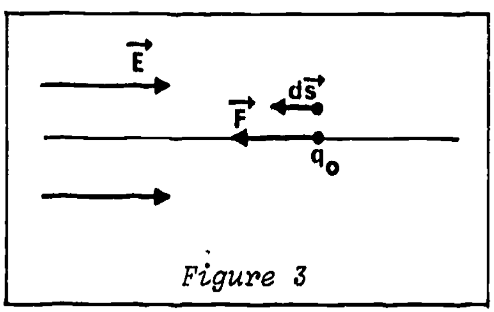


Figure 3

$$\vec{F}' = q_o\vec{E} \qquad (1)$$

We shall be interested in the work required to bring the charge from some distant point toward the source of the field; that is, in the work done *against* the field by some external agent which exerts a force $\vec{F}$ on the charge in a direction opposite that of the field. Referring to Figure 2, the external force is shown as $\vec{F}$, having a magnitude equal to that of $\vec{F}'$ but opposite in direction. This force has been made equal in magnitude to the force $\vec{F}'$ because we are interested in the work done *without accelerating the test charge.* The qualification is that there be no change in the kinetic energy of the particle during its journey inward. Since $\vec{F}$ is equal and opposite $\vec{F}'$, we can then write:

$$\vec{F} = -q_o\vec{E} \qquad (2)$$

next page

continued

To find the work done on the particle, it is necessary to sum up all the elements of work involved in moving through all the elements of displacement $d\vec{s}$ as shown in Figure 3. That is:

$$W = \vec{F}\cdot d\vec{s} \tag{3}$$

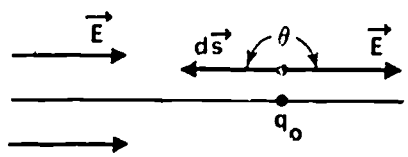


Figure 4

Substituting equation (2) into equation (3) gives us:

$$W = -q\int \vec{E}\cdot d\vec{s} \tag{4}$$

Now, to put this equation into scalar form, it is noted (Figure 4) for this simple case that the angle θ between the electric intensity vector and the displacement vector is 180°, hence:

$$\vec{E}\cdot d\vec{s} = Eds\ \cos 180° \tag{5}$$

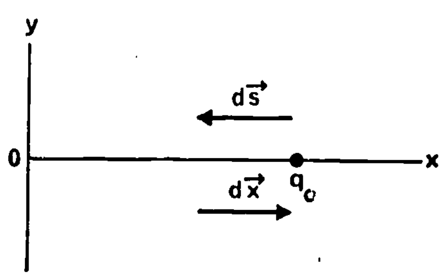


Figure 5

Refer now to Figure 5. When the system is referred to coordinate axes, the element of displacement inward ($d\vec{s}$) is equal in magnitude but opposite in direction to the distance element dx along the x-axis so that we may say:

$$d\vec{s} = -d\vec{x} \tag{6}$$

and substituting equation (6) into equation (5):

$$\vec{E}\cdot d\vec{s} = E(-dx)(-1) \tag{7}$$

hence,

$$\vec{E}\cdot d\vec{s} = Edx \tag{8}$$

so that equation (4) may now be written in scalar form as:

$$W = -q_o\int_{x_1}^{x_2} E\ dx$$

This is the form of the work equation which is most often used in problems in this course. You may now proceed to the core problem which deals with the work required to move a given charge from some position x_1 to another position x_2 against an electric field.

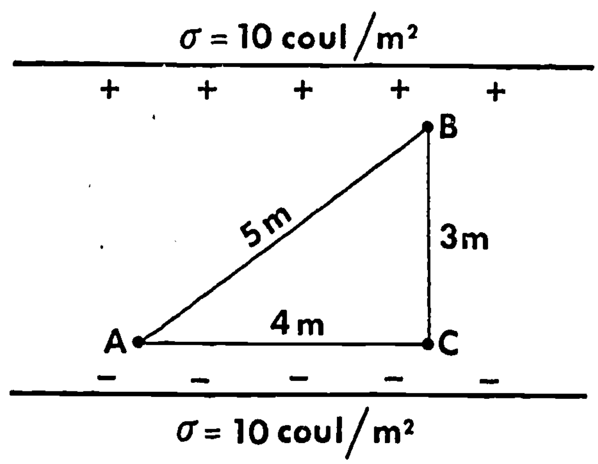
PROBLEMS

1. A particular electric field can be described by the following equation:

$$\vec{E} = \frac{10}{x}\,\hat{i}$$

How much work must be performed to move a charge q = +1 coul from x = 10 m to x = 5 m?

2. A charge q = +30 coul is moved from point A to C and then to B as shown in the diagram. How much work is performed in moving the charge if triangle ABC is 3 meters by 4 meters by 5 meters?



3. A charge q = +30 coul is moved from point A to B as shown in the diagram of the preceding question. How much work is performed in moving the charge if triangle ABC is 3 meters by 4 meters by 5 meters?

4. Refer to the diagram in question 2 above. How much work must be done by an outside agent to move an alpha particle from A to B to C and back to A again (a complete loop)? The triangle ABC is 3 meters by 4 meters by 5 meters.

5. A charge Q of 10.0 coulombs is located at the origin of an x, y, z coordinate system. How much work in joules must be performed to place another charge q = +1.00 coulomb at a point located on the positive x-axis 9.00 m the origin?

A. [illegible]

B. 3.72×10^{2}

C. -10.0×10^{10}

D. $-3.72 \times 10^{?}$

INFORMATION PANEL — Potential Difference

OBJECTIVE

To calculate the potential difference between two points in an electric field.

If a charge is moved between two points in an electric field, work is generally done. Specifically, for the simple electric field shown in Figure 1, a positive test charge moved from point a to point b in some random path, the motion will be against a component of the electric field--against an outward force--and work will have to be done by some outside agency to move the charge. It is one of the most important properties of the static field that this work is completely independent of the path taken by the test charge. If the charge moves back from b to a along the path shown *or any other path,* work will be done on it by the field. If there is no restraining force on the charge, it will accelerate and gain kinetic energy; if its energy is to be kept constant, a restraining force will have to be applied to it and work will then be done on the agency that supplies this force. This work will be identical in magnitude with that needed to move it from a to b, hence the electric field is a *conservative field* so that the law of conservation of energy applies to the movement of charged bodies in such a field.

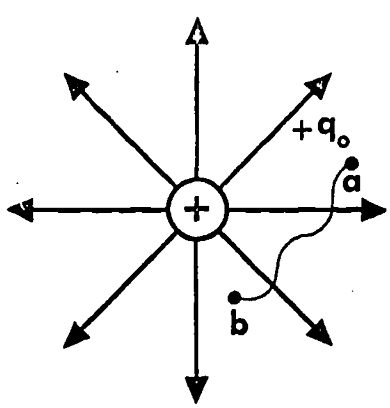


Figure 1

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continued

Since the work required to move a charge between two given points in a field of constant intensity is always the same, and since the work varies in proportion [illegible] the magnitude of the charge moved, we may now define a n[illegible] two points in a field in these terms.

The potential difference V_{ab} between points a and b in a steady electric field is the ratio of the total work done and the magnitude of the charge moved.

$$V_{ab} = W_{ab}/q_o$$

Clearly, this is essentially the same as stating that *potential difference is work per unit charge.*

Conventionally, q_o is always taken to be a positive test charge of small magnitude.

In the MKS system, work is measured in joules (abb. J) and charge is measured in coulombs (coul), so that the unit of potential difference is the joule per coulomb, or the *volt*. One v[illegible], therefore, is the potential difference between points in an electric field such that one joule of work must be done to move a charge of one coulomb between the points considered.

Work done *against* the electric field by the outside agency is considered positive while work done by the electric field *on* the outside agency is taken as negative. If there is no difference of potential between the two considered points in the field, the work required to move the charge between them is zero.

In general, calculating the potential difference between two points requires that the work from *a* to *b* first be determined by properly applying the relation:

$$W_{ab} = -q_o \int \vec{E} \cdot d\vec{s}$$

and then dividing the work thus obtained by the magnitude of the test charge q_o. Thus,

$$V_b - V_a = - \int \vec{E} \cdot d\vec{s}$$

next page

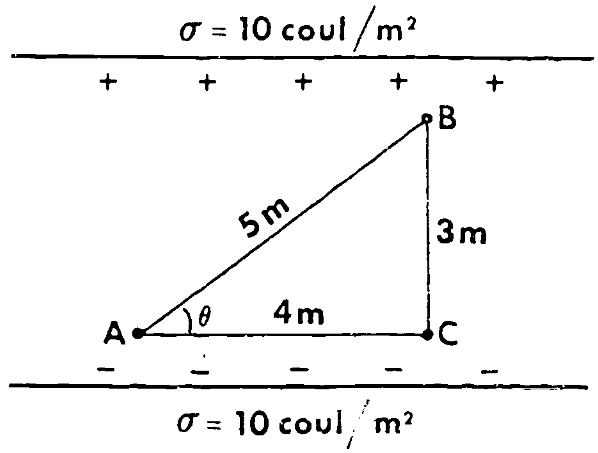
continued

The problems in this section require that you

(a) calculate the potential difference between two points in a uniform electric field when the line connecting these points is not parallel to the field;

(b) be able to define clearly the meaning of potential difference and understand the units used to express it.

6. Two parallel plates each with a surface charge density $\sigma = 10$ coul/m² form a region of uniform electric field as shown in the diagram. Calculate the potential difference $V_{AB} \equiv V_B - V_A$ in volts.



7. Write an expression for the electric potential difference between two points a and b in terms of the work required to move a test charge q_o from a to b, W_{ab}. (Recall that a test charge is defined as a small positive charge.)

A. $V_b - V_a \equiv V_{ab} = W_{ab}/q_o$

B. $V_b - V_a \equiv V_{ab} = W_{ab}$

C. $V_b - V_a \equiv V_{ab} = q_o W_{ab}$

D. $V_a - V_b \equiv V_{ab} = W_{ab}/q_o$

8. The work term that appears in the previous question is

A. positive
B. negative
C. zero
D. any of the above

9. In the MKS system, the unit of electric potential is the *volt* (V). The volt can be expressed as the

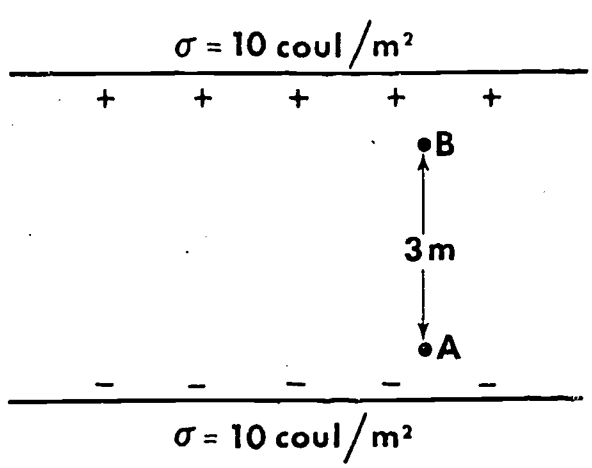
A. J/coul

B. J-coul

C. coul/J

D. J^2/coul

10. Two parallel plates each with a surface charge of $\sigma = 10$ coul/m^2 form a region of uniform electric field as shown in the diagram. Calculate the potential difference V_{AB} between points A and B in volts.



INFORMATION PANEL — Potential Due to a Point Charge

OBJECTIVE

To determine the potential of a point in space immersed in an electric field due to a point charge.

Instead of considering the potential difference between two points, it is often advantageous to think in terms of the potential V_b at a given point. The *potential at a point* is the difference between the electric potential at that point and some arbitrarily chosen reference zero. Thus, the potential at the point is the work done per unit charge during the motion from the arbitrary zero reference to the point in question.

For many situations, the zero reference level for electric potential is taken at infinity. Thus, the definition of potential stems directly from the expression for potential difference, as follows:

Potential difference: $$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s}$$

but by taking point a at infinity, this may be rewritten:

Potential at point b: $$V = -\int_\infty^b \vec{E}\cdot d\vec{s}$$

since V_a is zero by convention.

Now consider a point charge q enveloped by its own electric field. This field is radial and the potential at some position at a distance r from the point charge is given by:

$$V = q/4\pi\varepsilon_0 r$$

In this section, the problems deal with

(a) evaluation of the potential of a point in a field due to a point charge;

(b) determination of the magnitude of a point charge given the potential it produces at a specific distance.

11. Recalling that the potential difference between two points A and B is given by the expression

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s} \qquad (1)$$

we can define the electric potential by taking point A to be at infinity, so that $V_A = 0$

$$V = -\int_\infty^B \vec{E}\cdot d\vec{s} \qquad (2)$$

Using this definition, calculate the potential due to a point charge q at a distance r from it.

A. $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$

B. $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$

C. $V = \frac{1}{4\pi\varepsilon_o} qr$

D. $V = \frac{1}{4\pi\varepsilon_o} r^2$

12. Which of the following is the conventional definition of the electric potential at a point?

A. The potential difference between that point and a point at infinity, the latter taken as the infinite-potential reference point.

B. The potential difference between that point and some arbitrary reference point, the potential at the reference point taken to have an infinity value.

C. The potential difference between that point and a point at infinity, the latter taken as the zero-potential reference point.

D. The potential difference between that point and the origin of a coordinate system, that origin taken as the zero-potential reference point.

13. What is the electric potential at a distance of 5.0×10^{-5} m from a point charge $3q_e$? (Recall that $q_e = -1.6 \times 10^{-19}$ coul.)

A. 8.6×10^{-10} volts

B. 9.6×10^{-15} volts

C. 8.6×10^{-5} volts

D. 1.1×10^{-24} volts

14. What is the value of an isolated positive point charge producing a potential of 1.0×10^{6} volts at a distance of 1.0 meter?

A. 1.1×10^{-4} coul

B. 1.1×10^{-16} coul

C. 9.0×10^{-6} coul

D. $4\pi \times 10^{-3}$ coul

INFORMATION PANEL — Potential Due to Combination of Charges

OBJECTIVE

To calculate the potential at a point located between or outside of two or more point charges.

The potential at a point, or the potential difference between two points, is defined in terms of work per unit charge. Since work and charge are both scalar quantities, it follows that potential is a scalar quantity as well.

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continued

The potential at a point in space due to the proximity of two or more point charges is, therefore, merely the *algebraic* sum of the potentials produced by each individual point charge. The value of each individual potential is calculated without referring to the other charges that may be present at the same time.

A second consequence of the scalar nature of potential is that the rule for algebraic addition is valid for configurations of charges whether or not they and the point in question lie along the same straight line. It is important to remember, however, that attention must be given to the algebraic sign of each potential thus computed.

Since the potential due to a single point charge is:

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

Then the addition process would be written as follows:

$$V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \ldots$$

which reduces to:

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \ldots\right)$$

The charges indicated by q_1, q_2, etc., may be either (+) or (-). Clearly, this makes it possible for the quantity inside the parenthesis to equal zero for the proper combination of charge values and distances. As a result, the net potential at the point would then become zero. To find the point or points where such a potential null exists, it is necessary only to set the parenthetical quantity equal to zero and solve for the distance.

The problems in this section involve the

(a) determination of the position of two points on a line joining two given charges where the potential $V = 0$;

(b) determination of the potential at a point on a line joining two charges, given the magnitudes of the charges and the required separation distances;

next page

continued

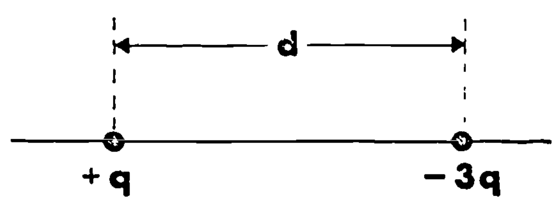
(c) calculation of the potential of a point on the y-axis, given the magnitudes of two charges and their separation distances on the x-axis,

(d) selection of the proper equation for finding the potential at a point that is randomly placed with respect to an electric dipole,

(e) selection of the proper equation for finding the potential at a point that is randomly placed with respect to three individual point charges,

(f) selection of the proper equation for finding the potential at a point lying outside, but on the same line as, an electric quadrupole.

15. Two charges of magnitude q and -3q are separated by a distance of 2 m. Find the two points on the line joining the two charges where the potential V = 0.



A. 1 m left of +q, 0.5 m right of +q

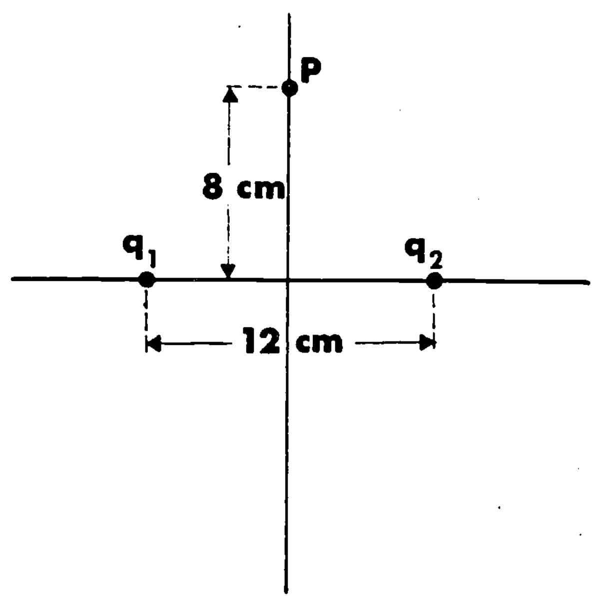
B. 0.5 m left of +q, 1 m right of +q

C. 0.5 m right of +q

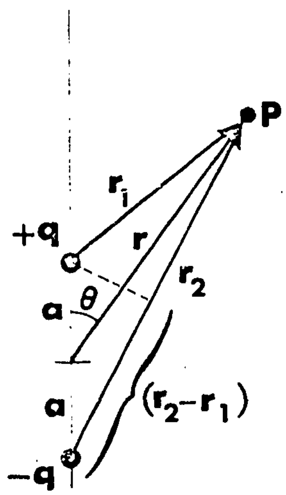
D. 1 m left of +q

16. Two charges $q_1 = 1.0 \times 10^{-8}$ coul and $q_2 = -2.0 \times 10^{-8}$ coul are separated by a distance of 5 cm. What is the electric potential in volts at point P shown in the diagram below.

17. Two charges $q_1 = 1.0 \times 10^{-8}$ coul and $q_2 = -2.0 \times 10^{-8}$ coul are 12 cm apart. Find the electric potential in volts due to the system of charges at point P shown below.



18. Two equal charges q, of opposite sign, are separated by a very short distance 2a. This system of charges is called a dipole. What is the potential at a point P at distances r_1 and r_2 from the positive and the negative charges, respectively?



A. $V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r_1^2} - \frac{q}{r_2^2}\right)$

B. $V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$

C. $V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r_1} + \frac{q}{r_2}\right)$

D. $V = 0$

19. What is the potential at a point P due to three point charges q_1, q_2, and q_3? The distances between the charges and P are r_1, r_2, and r_3, respectively.

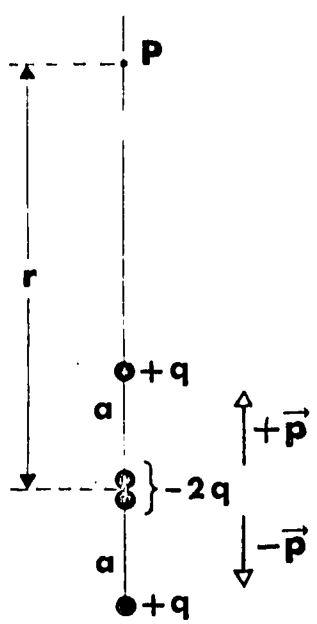
A. $\frac{1}{4\pi\varepsilon_o}(q_1 + q_2 + q_3)$

B. $\frac{1}{4\pi\varepsilon_o}\frac{(q_1 + q_2 + q_3)}{(r_1 + r_2 + r_3)}$

C. $\frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$

D. $\frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + \frac{q_3}{r_3^2}\right)$

20. A system of charges, consisting of two electric dipoles are so arranged that they almost, but not quite, cancel each other in their electric effects at distant points. This system of charges is called an "electric quadrupole."



Calculate the potential at a point P on the axis of the quadrupole.

A. $\frac{q}{4\pi\varepsilon_o} \frac{2a^2}{r(r^2 - a^2)}$

B. $\frac{q}{4\pi\varepsilon_o} \frac{2a^2}{r^2}$

C. $\frac{q}{4\pi\varepsilon_o} \frac{2(2r^2 - a^2)}{4(r2 - a2)}$

D. $\frac{q}{4\pi\varepsilon_o} \left(\frac{1}{(r - a)^2} + \frac{2}{r^2} + \frac{1}{(r + a)^2} \right)$

[a] CORRECT ANSWER: 3.4×10^{12} volts

The potential difference between points A and B is defined to be the work required to move a test charge q_o from A to B divided by q_o. Thus,

$$V_{AB} = V_B - V_A = W_{AB}/q_o \qquad (1)$$

Work done can be found from

$$dW = \vec{F}\cdot d\vec{s}$$

where $\vec{F}$ is the force applied to move the charge q_o from A to B. Therefore,

$$dW = F\ ds\ \cos\theta$$

$$= q_oE\ \cos\theta\ ds$$

Integrating, we obtain

$$W_{AB} = q_oE\ \cos\theta\ S_1$$

where

$$S_1 = 3\ m$$

$$E = \frac{\sigma}{\varepsilon_o}$$

and

$$\cos\theta = 0.8$$

Therefore,

$$V_{AB} \equiv V_B - V_A = W_{AB}/q_o = E\ \cos\theta\ S_1 \qquad (2)$$

Substituting the numerical values, we obtain

$$V_{AB} = 3.4 \times 10^{12}$$

Note that the potential differen. is independent of the test charge q_o. In fact, the potential difference is equal to work done on . unit positive charge.

TRUE OR FALSE? The magnitude of q_o must be unity to obtain this answer.

[a] CORRECT ANSWER: A

We simply write down the potential due to each charge and then add them up to obtain

$$V = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r - a} - \frac{2}{r} + \frac{1}{r + a}\right)$$

$$= \frac{q}{4\pi\varepsilon_o}\,\frac{2a^2}{(r - a)r(r + a)} = \frac{q}{4\pi\varepsilon_o}\,\frac{2a^2}{r(r^2 - a^2)}$$

If P is far away; i.e., r >> a, this potential becomes

$$V = \frac{1}{4\pi\varepsilon_o}\,\frac{2a^2q}{r^3}$$

which may be written as

$$V = \frac{Q}{4\pi\varepsilon_o r^3}$$

where $Q \equiv 2a^2q$. Q is known as the quadrupole moment.

TRUE OR FALSE? The quadrupole moment and the moment of an electric dipole are evaluated by exactly the same expressions.

[b] CORRECT ANSWER: C

The potential due to a point charge is given by the expression

$$V = -\int_\infty^r \vec{E}\cdot d\vec{s} = -\int_\infty^r E\,dr = -\int_\infty^r \frac{3q_e\,dr}{4\pi\varepsilon_o r^2} = \frac{3q_e}{4\pi\varepsilon_o r}\Bigg|_\infty^r$$

Substituting the given numerical values

$$V = \frac{[illegible]\ 1.6 \times 10^{-19}}{5.0 \times 10^{-5}} = 8.6 \times 10^{-5} \text{ volts}$$

[a] CORRE[illegible]: A

The e[illegible]ield intensity due to a point charge q at a distance r from [illegible] is

$$[illegible]\varepsilon_0 \frac{q}{r^2}$$

Noting [illegible]e path $d\vec{s}$ is in opposite direction to both $d\vec{r}$ and $\vec{E}$, we obt[illegible]

$$[illegible] = E\,dr$$

Thus,

$$-\int_{\infty}^{B} \vec{E}\cdot d\vec{s} = -\int_{\infty}^{r} E\,dr = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r} \frac{1}{r^2}\,dr$$

$$-\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$$

TRUE OR F[illegible]E? The expression $V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$ is dimensionally incorrect.

[b] CORREC[illegible]R: -1200

In g[illegible]e can write the potential at any point due to two charges q_1 a[illegible]

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{x_1} + \frac{q_2}{x_2}\right)$$

where x_1 and x_2 are the distances of q_1 and q_2 from the point under consideration.

Substituting numerical values, we get

$$V = 9 \times 10^9 \left(\frac{1 \times 10^{-8}}{.15} - \frac{2 \times 10^{-8}}{.1}\right) = -1200 \text{ volts}$$

[a] ANSWER: Zero

Isolate the problem into three steps by ing each side of the triangle ABC separately. Therefore,

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} \quad (1)$$

In a prior problem we found that

$$W_{AB} = W_{CB}$$

Therefore,

$$W_{AB} = -W_{BC}$$

Also, we found that $W_{AC} = 0$. Using this information in equation (1), we find that

$$W_{ABCA} = 0$$

This is an important finding because it shows us that electrical forces are conservative forces. In other words, work done by a conservative force along a closed path is zero. Another example of a conservative force is the gravitational force.

[b] CORRECT ANSWER: A

The expression for the potential due to a point charge at distance r is

$$V = \frac{q}{4\pi\varepsilon_o r}$$

Thus,

$$q = 4\pi\varepsilon_o r\ V$$

$$= \frac{10^6}{9 \times 10^9} = 1.1 \times 10^{-4}\ \text{coul}$$

TRUE OR FALSE? The fraction q [illegible] ε_o may be measured in volts.

[a] CORRECT ANSWER:

The potential due to this two-charge system is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\,\frac{r_2 - r_1}{r_1 r_2}$$

If we consider points where r >> 2a, so that

$$r_2 - r_1 \approx 2a\cos\theta \qquad \text{and} \qquad r_1 r_2 \approx r^2$$

then the potential reduces to

$$V = \frac{q}{4\pi\varepsilon_0}\,\frac{2a\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\,\frac{p\cos\theta}{r^2}$$

where p = 2aq is called the dipole moment.

[b] CORRECT ANSWER: A

Let us recall the definition of electric potential difference

$$V_B - V_A = \frac{W_{AB}}{q_o}$$

The units of work and charge are the joule (J) and the coulomb (coul), respectively. Thus

1 Volt = 1 J/coul

[c] CORRECT ANSWER: A

The electric potential difference between two points is defined as the work required per unit charge in moving a charge from one point to the other (without changing the kinetic energy of the charge). The reason a test (small positive) charge was brought into the question is that we don't want the introduction of the charge which we use to measure the potential difference to change the environment significantly.

[a] CORRECT ANSWER: 3.4×10^{12}

The potential difference between points B and A is de...ned as work required to move a test charge q_o from A to point B ... ...ded by the test charge q_o. Thus,

$$V_{AB} = V_B - V_A = \frac{W_{AB}}{q_o} \qquad (1)$$

The work done to bring charge q_o from point A t... B is

$$W_{AB} = \vec{F} \cdot \vec{S}$$

where $\vec{F}$, the force exerted by the external agen... is equal to $-q_o\vec{E}$ and $\vec{S}$ is the displacement. However,

$$E = \frac{\sigma}{\varepsilon_o}$$

for the region between parallel plates. Therefore,

$$W_{AB} = -q_o\vec{E} \cdot \vec{S}$$

$$= -q_oE(-S)$$

$$W_{AB} = q_oES$$

$$= q_o \frac{\sigma}{\varepsilon_o} S \qquad (2)$$

Substituting equation (2) in equation (1) we find

$$V_{AB} = V_B - V_A = \frac{q_o\sigma S}{\varepsilon_o q_o}$$

$$= \frac{\sigma S}{\varepsilon_o}$$

$$= 3.4 \times 10^{12} \text{ volts}$$

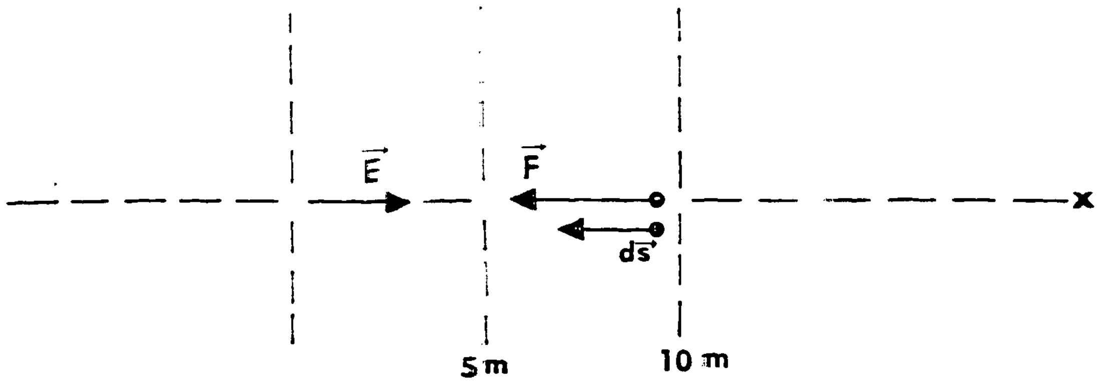
where S = 3 m.

Note that the potential difference is independent of the test charge q_o. In fact, the potential difference is equal to work done on a unit positive charge.

TRUE OR FALSE? Potential difference may be measured in work units.

[a] CORRECT ANSWER: 6.9 J

We can imagine the following situation:



We would like to calculate the work done by the external agent in moving a charge q from x = 10 m to 5 m. The work done may be expressed as

$$W = \int \vec{F} \cdot d\vec{s}$$

where $\vec{F}$ is the force the external agent must apply to keep the charge q from accelerating and is exactly equal to $-q\vec{E}$.

Thus, work done may be expressed as

$$W = \int \vec{F} \cdot d\vec{s} = -q \int \vec{E} \cdot d\vec{s}$$

Since

$$ds = -dx$$

we have

$$\vec{E} \cdot d\vec{s} = E ds \cos 180° = E(-dx)(-1)$$

$$= E dx$$

Thus, the integral becomes

$$W = -q \int_{10}^{5} E\, dx = -\int_{10}^{5} \frac{10q}{x}\, dx$$

$$= 10q \ln\left(\frac{10}{5}\right) = 10q \ln 2$$

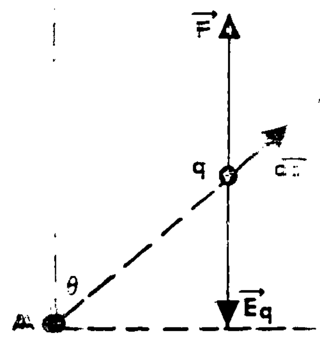
and the work done is

$$W = 6.9 \text{ joule}$$

TRUE OR FALSE? In this solution, the velocity of the charge q is constant.

[a. CORRECT ANSWER: 1.0 × 10^{14} J

A drawing of the charge en route to point B would be a big help in the solution of this problem.



In order to calculate work, one must very carefully outline the force performing the work. In other words, in this problem the field applies a force to the charge q in the downward direction. The agent doing the work applies a force up to move the charge from A to B. Draw a diagram including these two forces and the displacement vector.

Work done can be found from

$$dW = \vec{F} \cdot \vec{ds}$$

where F is the force applied to move the charge from A to B. In any instance, $|F| = Eq$, therefore,

$$dW = |F|\,|ds|\,\cos\theta$$

$$= Eq\,\cos\theta\,ds$$

Integrating, we obtain

$$W = Eq\,\cos\theta\,S$$

where

$$S = 5\text{ m}$$

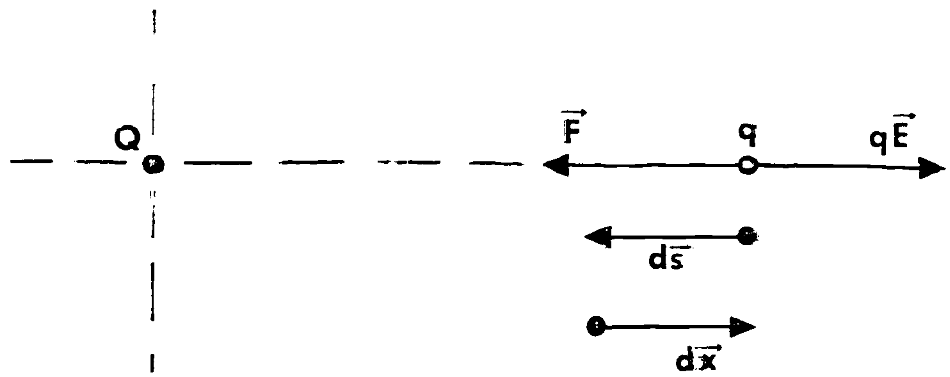
and

$$E = \frac{\sigma}{\varepsilon_0}$$

Substituting the known values results in

$$W = (36\pi \times 10^9)(10)(30)(.6)(5) = 1.0 \times 10^{14}\text{ J}$$

[a] CORRECT ANSWER: A



The orientation of the various vectors involved in this problem are shown above. $\vec{F}$ is the force the external agent must apply to keep the charge q from accelerating and is exactly equal to $-q\vec{E}$. Notice that $\vec{F}$ and $d\vec{s}$ are in the same direction; however, $d\vec{s}$ and $d\vec{x}$ are in opposite directions. Therefore, $d\vec{s} = -d\vec{x}$. We wish to calculate the work required to move the +1 coulomb charge from infinity to the point on the x-axis 9 cm from the origin. Therefore,

$$W = \int_{\infty}^{0.09} \vec{F}\cdot d\vec{s} = -q\int_{\infty}^{0.09} \vec{E}\cdot d\vec{s}$$

and

$$\vec{E}\cdot d\vec{s} = E\,ds\,\cos 180^\circ = E\,dx$$

and

$$E = \frac{Q}{4\pi\epsilon_0 x^2}$$

Substituting, we obtain

$$W = \int_{\infty}^{0.09} \frac{-Qq\,dx}{4\pi\epsilon_0 x^2}$$

Integration of this term leads to

$$W = \left[\frac{Qq}{4\pi\epsilon_0 x}\right]_{x=\infty}^{x=0.09}$$

Substituting the quantities given in this problem yields

$$W = +1.00 \times 10^{12} \text{ joules}$$

The plus sign is important since it indicates that the outside force has to do work *on* the system of charges.

TRUE OR FALSE? In this solution, the angle between **dx** and ds is 180°.

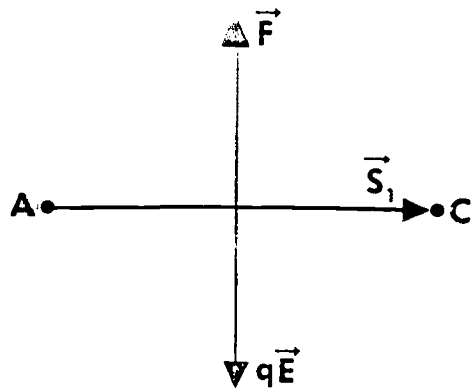
[a] CORRECT ANSWER: 1×10^{14} j

The total work in going from A to B to C can best be calculated in parts. For example,

$$W_{ACB} = W_{AC} + W_{CB}$$

The electric field exerts a force $q\vec{E}$ on the charge as shown. To keep the charge from accelerating an external agent must apply a force $\vec{F}$ chosen to be exactly $-q\vec{E}$ for all positions on the charge. The magnitude of the electric field for parallel plates is

$$E = \frac{\sigma}{\varepsilon_o}$$



Therefore,

$$W_{AC} = -q\vec{E}\cdot\vec{S}_1$$

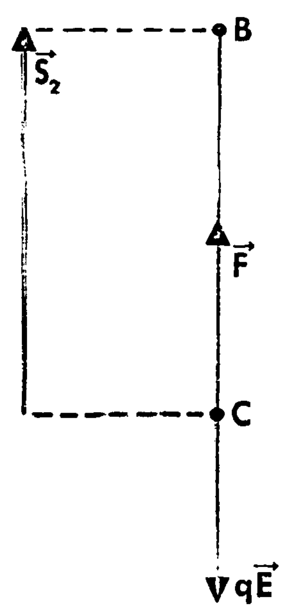
$$= qE \times S_1 \times \cos 90°$$

$$= 0$$

Since

$$\vec{F} = -q\vec{E}$$

The work W_{CB} is



$$W_{CB} = -q\vec{E}\cdot\vec{S}_2$$

$$= qE \times S_2 \times \cos 0°$$

$$= q\frac{\sigma}{\varepsilon_o}S_2 = \frac{4\pi q\sigma S_2}{4\pi\varepsilon_o}$$

$$= 1 \times 10^{14} \text{ j}$$

where

$$S_2 = 3 \text{ m}$$

[a] CORRECT ANSWER: A

First let us assume that the point P where V = 0 is at the left of +q. Then we have

$$\frac{q}{x} - \frac{3q}{x+2} = 0 \qquad (1)$$

giving

$$x = 1 \text{ m}$$

Considering now the possibility that P will lie to the right of +q, we obtain

$$\frac{q}{x} - \frac{3q}{2-x} = 0 \qquad (2)$$

yielding

$$x = 0.5 \text{ m}$$

Thus we find that there are two points where V = 0, one at 1 m to the left of +q, and the other at 0.5 m to the right of +q.

A more formal, but also more complex, way of solving this problem follows. You may skip it if you so desire.

If x is the position of the sought point(s) with respect to charge +q, the position of that (those) point(s) with respect to the -3q charge will be x - d. Thus, the potential there is

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{|x|} + \frac{(-3q)}{|x-d|}\right] = 0$$

or

$$\frac{1}{|x|} = \frac{3}{|x-d|}$$

Squaring and cross-multiplying, we obtain

$$x^2 + d^2 - 2xd = 9x^2$$

or

$$8x^2 + 2xd - d^2 = 0$$

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continued

Thus,

$$x = \frac{-d \pm \sqrt{d^2 + 8d^2}}{8}$$

or

$$x_1 = d/4 = 0.5 \text{ m} \qquad \text{(to the right of } +q)$$

$$x_2 = -d/2 = -1 \text{ m} \qquad \text{(to the left of } +q)$$

TRUE OR FALSE? In equation (1), x is the distance of q from the point where V = 0.

[a] CORRECT ANSWER: -900

The expression for the potential due to the two charges is

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

where

$$r_1 = r_2 = \sqrt{8^2 + 6^2} = 10 \text{ cm} = 0.1 \text{ m}$$

Substituting the numerical values, we obtain

$$V = 9 \times 10^9 \left(\frac{1 \times 10^{-8}}{.1} - \frac{2 \times 10^{-8}}{.1}\right) = -900 \text{ volts}$$

[b] CORRECT ANSWER: D

The work required to move a positive charge from *a* to *b* may be positive, negative or zero. Its sign depends on whether the electric potential at *b* is respectively higher than, lower than, or equal to the potential at *a*.

[a] CORRECT ANSWER: C

The potential at P is sum of the potentials due to each of the charges. Therefore,

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

[b] CORRECT ANSWER: C

We are free to assign any value to the potential at a point and then measure the potential at other points in terms of that assigned value. The choice of the reference value as well as the reference point, however, should be a convenient one. The reference value is chosen to be zero for simplicity. The reference point is taken at infinity since the electric field due to a point charge is also zero there. This, of course, is not general. Quite often the Earth is taken as the zero-potential reference; hence, the term "ground".

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

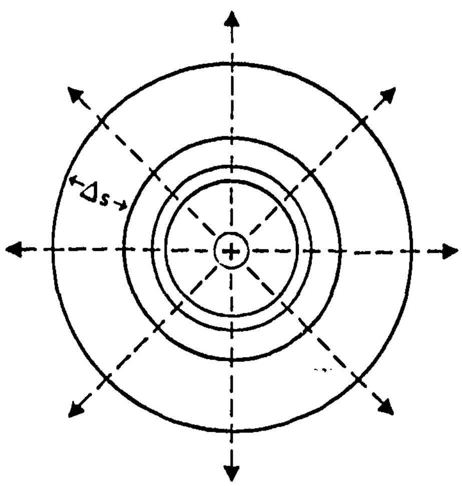
ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Potential Gradient

OBJECTIVE

To solve problems involving the relationship between electric intensity and electric potential in various field configurations.

An electric field can be mapped by means of a network of lines of force or by a series of equipotential surfaces. For example, the electric field around a point charge is radial outward from the point if the latter is positive as in the diagram, and the equipotential surfaces are concentric spheres with the charge at the common center. Imagine that the equipotentials in this example have been drawn with the electrical spacing between them equal to some constant potential difference ΔV. Suppose further that we let Δs represent the perpendicular distance between any two equipotentials.



The potential difference between any two equipotentials, as previously defined, is merely the work per unit charge to move the test charge from one to the other, or:

$$V = w/q = F\Delta s \qquad (1)$$

Also, the electric intensity at any point in the field is:

$$E = F/q \qquad (2)$$

or force per unit charge. Solving both these expressions for q and equating enables us to write:

$$\Delta V = E\Delta s \quad \text{or} \quad \Delta s = \Delta V/E \qquad (3)$$

The last expression indicates that as the electric intensity E is made larger, the smaller becomes the perpendicular distance Δs between the equipotentials. Thus one can visualize a strong electric field as one in which the equipotentials are very closely spaced and a weak field as one in which the equipotentials are more widely separated.

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continued

Equation (3) may be rewritten as:

$$E = \Delta V/\Delta s \qquad (4)$$

This may be interpreted verbally as follows: As we move through an electric field *along a line of force*, the rate of change of potential with distance traveled is equal to the magnitude of the electric field in that direction. In addition, since the direction of E (outward from the point charge in our example) is that of *decreasing* potential, a negative sign precedes the right member:

$$E = -\Delta V/\Delta s \qquad (5)$$

and, in the differential limit, we can say that:

$$E_s = -dV/ds$$

As shown by this expression, an alternative unit for electric intensity is the volt per meter; thus:

$$1\ \frac{\text{volt}}{\text{meter}} = 1\ \frac{\text{newton}}{\text{coulomb}}$$

In summary:

> *Electric lines are perpendicular to equipotential surfaces.*
>
> *The direction of the electric intensity vector is from higher to lower potential.*
>
> *The magnitude of the electric intensity is the space rate of change of potential along an electric line of force.*

Extensive use is made of these concepts in the solutions of the problems in the section that follows.

1. At a point P the electric potential due to a dipole located at the origin of an xy-plane system is given by

$$V = \frac{1}{4\pi\varepsilon_o} \frac{p \cos\theta}{r^2}$$

where $p = 2\ aq$ and $r^2 = x^2 + y^2$ and θ is measured from +y axis.

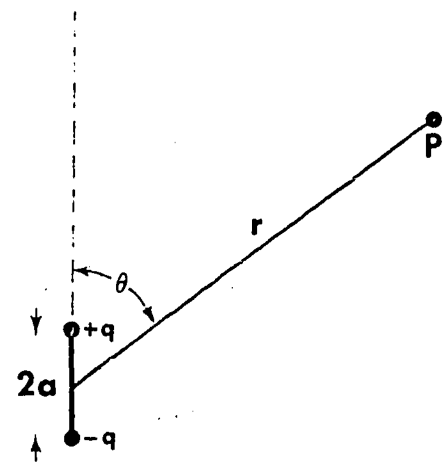
What is the y component of the electric field E_y at P?

A. $E_y = -\frac{p}{4\pi\varepsilon_o}\left[\frac{x^2 - 2y^2}{(x^2 + y^2)^{3/2}}\right]$

B. $E_y = -\frac{p}{4\pi\varepsilon_o}\left[\frac{x^2 - 2y^2}{(x^2 + y^2)^{5/2}}\right]$

C. $E_y = -\frac{p}{4\pi\varepsilon_o}\left[\frac{y^2}{(x^2 + y^2)^{3/2}}\right]$

D. $E_y = -\frac{p}{4\pi\varepsilon_o}\left[\frac{x}{(x^2 + y^2)^{3/2}}\right]$



2. E_s is a component of the electric field intensity at a point on a differential path element $d\vec{s}$ in the direction of the path. E_s may be found from the potential V from the relationship

A. $E_s = dV/ds$

B. $E_s = -dV/ds$

C. $E_s = (1/4\pi\varepsilon_o)\ dV/ds$

D. $E_s = (-1/4\pi\varepsilon_o)\ dV/ds$

3. The potential difference between points A and B can be calculated from the electric field $\vec{E}$ by the line integral

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$$

The value of this integral for an electric field depends

A. upon the choice of reference point

B. only upon the length of the path of integration

C. upon the integration path

D only upon the end points

4. The electric potential V at a point P (x,0) on the x-axis due to a charge q at the origin is

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{x}$$

If x = 2 m and q = 2 × 10^{-6} coul, find the magnitude of electric field E_x at point P.

5. The electric potential V at a point P (x,y) due to a charge q at the origin of the coordinate system is

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{(x^2 + y^2)^{1/2}}$$

If x = 3 m, y = 4 m and q = 5 × 10^{-6} coul, find the magnitude of the y-component of the electric field E_y at the point P (3 m, 4 m).

OBJECTIVE

To solve a group of problems in which potential or potential difference is to be determined for various distributions of charge.

Problems 5 through 13 are varied in nature and requirements, but they have in common the objective of determining potentials due to charges distributed in different ways.

To assist you in these solutions, we have listed a group of equations with which you have already had some contact. Although this list is neither complete, nor is it without some overlap, it should provide a basis for operations in this section. To avoid undesirable cueing, the order of listing is not necessarily that of sequential development.

Potential due to a point charge:	$V = \frac{1}{4\pi\varepsilon_o} \frac{q}{r}$
Potential due to any type of continuous charge:	$V = \frac{1}{4\pi\varepsilon_o} \int \frac{dq}{r}$
Potential due to a dipole:	$V = \frac{q}{4\pi\varepsilon_o} \frac{p \cos\theta}{r^2}$ $(p = 2aq)$
Potential difference between two points in line with an isolated point charge:	$V_B - V_A = \frac{q}{4\pi\varepsilon_o} \left(\frac{1}{r_B} - \frac{1}{r_A}\right)$
Relation of potential difference and electric intensity:	$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{l}$
Electric intensity between oppositely charged parallel conducting plates:	$E = \frac{\sigma}{\varepsilon_o}$

6. Two concentric, conducting spherical shells have radii r and R, respectively (R > r). The respective charges in the shells are +q and -q. What is the potential difference between the two spheres?

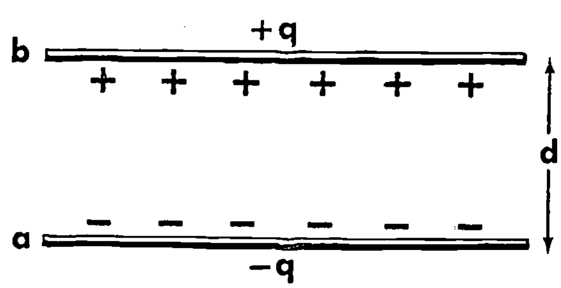
A. $V_r - V_R = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{R} - \frac{1}{r}\right)$

B. $V_r - V_R = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{R}\right)$

C. $V_r - V_R = 0$

D. $V_r - V_R = \frac{1}{4\pi\varepsilon_o}\frac{2q}{r}$

7. Two oppositely charged, parallel plates each of area A are separated by distance d. If charges are +q and -q, find the potential difference V_{ab} between the two plates. (Neglect edge effect.)



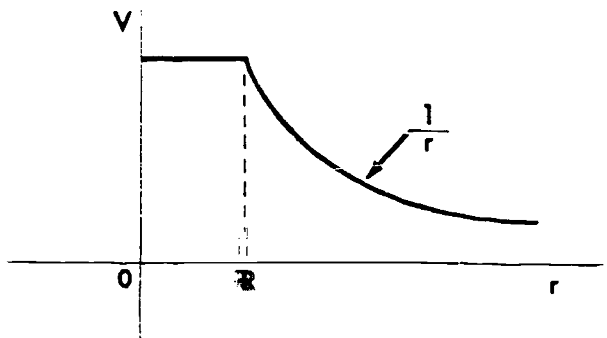
A. $\varepsilon_o qA$

B. $\frac{q}{A\varepsilon_o}$

C. $\frac{qd}{2\varepsilon_o A}$

D. $\frac{qd}{\varepsilon_o A}$

8. The diagram below shows an electric potential plotted as a function of distance. Which of the following objects could produce such a potential?



A. A uniformly charged, non-conducting sphere

B. A uniformly charged conducting spherical shell

C. An infinitely long charged conducting wire

D. A charged conducting cylindrical shell

9. Calculate the potential difference between two coaxial cylinders of radii a and b (b > a) and length L. The cylinders carry charges of +q and -q respectively.

A. $\frac{q(b - a)}{2\pi\varepsilon_o L}$

B. $\frac{1}{4\pi\varepsilon_o} \frac{q}{(b - a)}$

C. $\frac{1}{4\pi\varepsilon_o} q \left(\frac{1}{b} - \frac{1}{a}\right)$

D. $V = \frac{q}{2\pi\varepsilon_o L} \ln \frac{b}{a}$

10. The potential at a point a distance r from the center of a non-conducting sphere of radius R, charged uniformly with a total charge Q, is proportional to

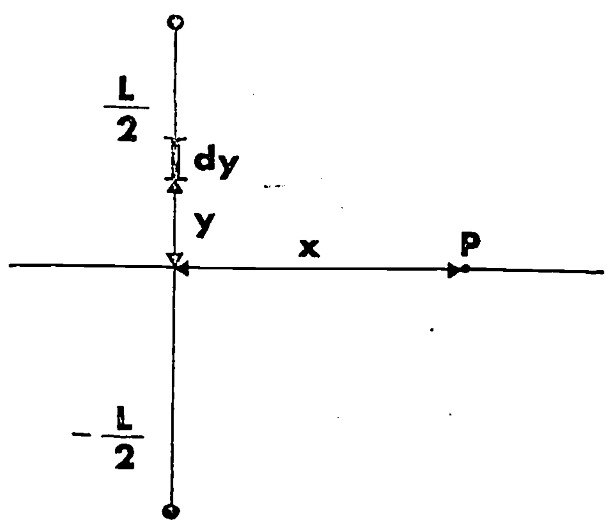
A. r^2 for $r < R$; $1/r$ for $r > R$

B. $1/r^2$ for $r < R$; $1/r$ for $r > R$

C. r for $r < R$; $1/r^2$ for $r > R$

D. constant for $r < R$; $1/r$ for $r > R$

11. Derive an expression for the electric potential at a point P on the perpendicular bisector of a line charge of length L and total charge q. P is a distance x from the line.



A. $V = \frac{q}{4\pi\varepsilon_o L} \int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2 + y^2}}$

B. $V = \frac{q}{4\pi\varepsilon_o L} \int_{-L/2}^{L/2} \frac{dx}{x^2 + y^2}$

C. $V = \frac{q}{4\pi\varepsilon_o L} \int_{-L/2}^{L/2} \frac{dy}{x^2 + y^2}$

D. $V = \frac{q}{4\pi\varepsilon_o L} \int_{-L/2}^{L/2} \frac{dy}{\sqrt{x^2 + y^2}}$

12. A circular metal ring has a radius a and total charge q uniformly distributed over the ring. What is the electric potential at a point on the axis at distance y from the plane of the ring?

A. $V = \frac{1}{4\pi\varepsilon_o} \frac{q}{y^2 + a^2}$

B. $V = \frac{1}{4\pi\varepsilon_o} \frac{q}{(y^2 + a^2)^{1/2}}$

C. $V = \frac{1}{4\pi\varepsilon_o} \frac{q}{(y^2 + a^2)^{3/2}}$

D. $V = \frac{1}{4\pi\varepsilon_o} q\ (y^2 + a^2)^{1/2}$

13. Calculate the electric potential at point P at a distance R on the axis of a uniformly charged circular disk of radius a whose surface charge density is σ.

A. $V = \frac{\sigma}{4\pi\varepsilon_o} \frac{R}{\sqrt{a^2 + R^2}}$

B. $V = \frac{\sigma}{2\varepsilon_o} (\sqrt{a^2 + R^2} - R)$

C. $V = \frac{\sigma}{2\varepsilon_o \sqrt{a^2 + R^2}}$

D. $V = \frac{\sigma}{2\varepsilon_o (a^2 + R^2)}$

INFORMATION PANEL

Applications of the Concept of Electric Potential Energy

OBJECTIVE

To study the potential energy of various charge distributions; to solve problems involving both mechanical and electrical energy.

Since electric forces are conservative, it is possible to base the calculation of the *electric potential energy* of a given charge configuration on the work required to assemble the charges in forming the configuration. In the simplest case, a point charge (positive) brought from infinity into the neighborhood of another similar positive point charge, we can at once start the discussion by giving the electric potential V at point P separated from an isolated charge q_1 by a distance r as in Figure 1. The potential at P is

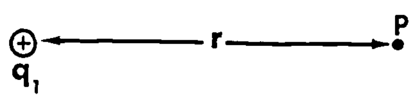


Figure 1

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r} \qquad (1)$$

Now consider a second charge q_2 initially located at infinity and brought to point P where it comes to rest. The agency that moves q_2 must do positive work in the process of moving it to point P since both charges are positive. From the definition of electric potential

$$V = W/q \qquad (2)$$

the work done on q_2 by the external agency is:

$$W = Vq_2 \qquad (3)$$

Combining equations (1) and (3):

$$W = \frac{1}{4\pi\varepsilon_o}\frac{q_1\, q_2}{r} \qquad (4)$$

next page

continued

and since this is a conservative action, the work W is precisely the same as the energy stored in the configuration. Hence, the electric potential energy is:

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r} \quad (5)$$

To extend this concept to configurations consisting of three or more charges, it is necessary to calculate the work done in assembling each charge separately. The total work done is then the *algebraic sum* of the individual works since energy is a scalar quantity. Thus, the total potential energy of the configuration is the sum of the individual energies of the particles.

For example, to assemble the configuration shown in Figure 2, we first picture q_1 in position with all the other point charges at infinity. We then calculate the work needed to bring q_2 into position where the distance separating the charges is r_{12}. Next, we find the work required to bring q_3 into the position where it is separated from q_1 by r_{13} and from q_2 by r_{23}. Finally, we compute the work needed to bring q_4 into position against the forces produced by the three charges already there.

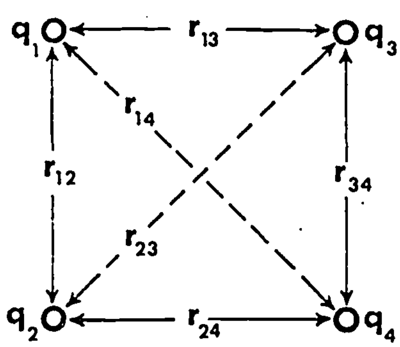


Figure 2

All of these are then added algebraically:

$$U = U_{12} + U_{13} + U_{23} + U_{14} + U_{24} + U_{34}$$

The relationship given in equation (5) is used to evaluate each of these.

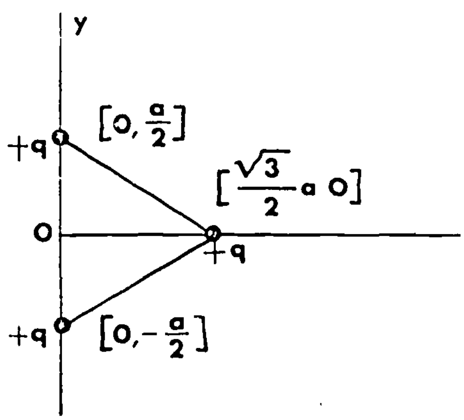
The problems in this section involve

(a) determining the electric potential energy in specific charge configurations;

(b) combining the mechanical energy of moving charges at a specific point with the electrical potential energy at the same point to determine escape velocity or the point of reversal of the motion of one of the charges.

14. A proton (mass $m_p = 1.67 \times 10^{-27}$ kg and charge $q_p = 1.6 \times 10^{-19}$ coul) with an initial velocity $v = 2.00 \times 10^7$ m/sec is directed towards a fixed charge $Q = 1.00 \times 10^{-4}$ coul a distance $r = 1.00$ m from the initial position of the proton. Find the distance of closest approach for the proton to the fixed charge Q.

15. What is the electric potential energy of the following charge configuration?



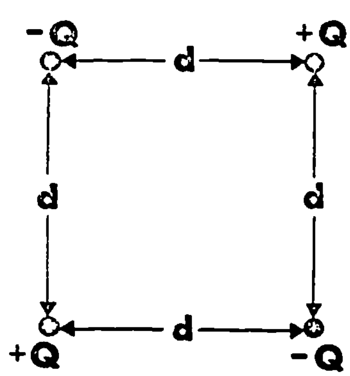
A. $U = q^2/4\pi\varepsilon_o a$

B. $U = 3\ q^2/4\pi\varepsilon_o a$

C. $U = 0$

D. $U = q^2/2\pi\varepsilon_o a$

16. Calculate the work required to assemble the four charges shown in the following diagram, starting with the charges at infinity.



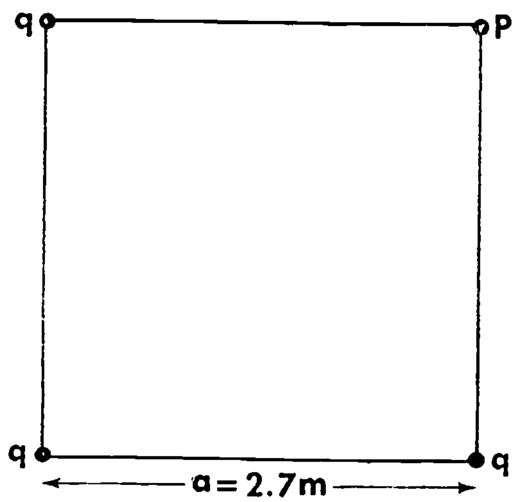
A. $Q^2/\varepsilon_o d$

B. $-.21\ Q^2/\varepsilon_o d$

C. 0

D. $-Q^2/\varepsilon_o d$

17. Three positive charges each of magnitude $q = 3.0 \times 10^{-}$ coul are situated at three corners of a square of side $a = 2.7$ m as shown in the diagram. What is the work required to bring a charge $Q = 4 \times 10^{-3}$ coul to the fourth corner P of the square from infinity?



18. Consider a system of a fixed proton ($m_p = 1.64 \times 10^{-24}$ kg and $q_p = 1.6 \times 10^{-19}$ coul) and an electron ($m_e = 9.10 \times 10^{-31}$ kg and $q_e = -1.6 \times 10^{-19}$ coul) separated by a distance $r = 5.0$ m. Find the minimum speed of the electron at that point where it will just escape from the attraction of the proton. (Neglect the gravitational effects.)

[a] CORRECT ANSWER: b

The relation between the field intensity and potential is given by

$$V = -\int \vec{E} \cdot \vec{ds} \qquad \text{or} \qquad E_s = -\frac{\partial V}{\partial s} \qquad (1)$$

We note from the illustration

$$V = \text{constant for } r < R$$

and

$$V \propto -\int_{\infty}^{r} \frac{dr}{r^2} = \frac{1}{r} \quad \text{for } r > R$$

We can obtain $\vec{E}$ from equations (1)

$$E = 0 \quad r < R$$

and

$$E \propto \frac{1}{r^2} \quad r > R$$

These field relations are true for a charged conducting sphere. In a conductor, the excess charge distributes itself on the surface of the conductor. A charged conducting sphere, therefore, will consist of a neutral body with all the charge distributed on the surface; hence will be the same as a charged spherical shell.

[b] CORRECT ANSWER: $E_x = 4.5 \times 10^3$ nt/coul

The electric field in the x-direction is given by the equation

$$E_x = -\frac{dV}{dx}$$

Differentiation of the given potential with respect to x yields

$$E_x = \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2}$$

Substitution of the numerical data results in

$$E_x = 4.5 \times 10^3 \text{ nt/coul}$$

[a] CORRECT ANSWER: B

The work required to assemble a charge configuration is equal to the potential energy of the configuration. In computing the potential energy we must take every possible pair into account. The potential energy between neighboring charges is (for each pair)

$$U_1 = -\frac{1}{4\pi\varepsilon_o}\frac{Q^2}{d}$$

and there are four such pairs.

The potential energy for each pair across the diagonal is

$$U_2 = \frac{1}{4\pi\varepsilon_o}\frac{Q^2}{\sqrt{2}\,d}$$

and there are two such pairs. Thus,

$$U = 4U_1 + 2U_2 = -\frac{Q^2}{4\pi\varepsilon_o}\left(\frac{4}{d} - \frac{2}{\sqrt{2}\,d}\right)$$

$$= -.21\ Q^2/\varepsilon_o d$$

[b] CORRECT ANSWER: D

Since the electric field is constant between the charged parallel plates, the potential difference is given by the expression

$$V_{ab} = V_b - V_a = -\int \vec{E}\cdot d\vec{s} = Ed$$

Therefore

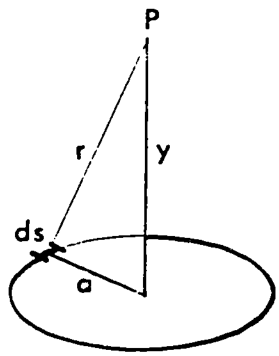
$$V_{ab} = \frac{\sigma}{\varepsilon_o} d \qquad (1)$$

However, the total charge $q = \sigma A$. Substituting the value of σ into (1) yields

$$V_{ab} = \frac{qd}{\varepsilon_o A}$$

[a] CORRECT ANSWER: B

Let us write down the potential at P due to a segment of the ring



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(y^2 + a^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{2\pi a}\frac{ds}{(y^2 + a^2)^{1/2}}$$

The total contribution to the potential due to the ring is obtained by integrating over the entire ring

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{2\pi a}\int_0^{2\pi a}\frac{ds}{(a^2 + y^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2\pi a}\frac{1}{(y^2 + a^2)^{1/2}}\int_0^{2\pi a} ds$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(y^2 + a^2)^{1/2}}$$

[b] CORRECT ANSWER: 1.44×10^3 nt/coul

The electric field in the y-direction is given by

$$E_y = -\frac{\partial V}{\partial y}$$

Therefore

$$E_y = -\frac{\partial}{\partial y}\left(\frac{q}{4\pi\varepsilon_0}\frac{1}{(x^2 + y^2)^{1/2}}\right)$$

$$= \frac{qy}{4\pi\varepsilon_0(x^2 + y^2)^{3/2}}$$

$$\approx 1.44 \times 10^3 \text{ nt/coul}$$

TRUE OR FALSE? The electric potential at point P (3 m, 4 m) varies directly as the charge magnitude at the origin of the coordinate system.

[a] CORRECT ANSWER: B

It is evident from the diagram that

$$r^2 = x^2 + y^2 \quad \text{and} \quad \cos\theta = \frac{y}{r} = \frac{y}{(x^2 + y^2)^{1/2}}$$

Thus, the potential becomes

$$V = \frac{p}{4\pi\varepsilon_o} \frac{y}{(x^2 + y^2)^{3/2}}$$

The y-component of the field is obtained by

$$E_y = -\frac{\partial V}{\partial y} = -\frac{p}{4\pi\varepsilon_o}\left[\frac{+1}{(x^2 + y^2)^{3/2}} - \frac{3}{2}\frac{2y^2}{(x^2 + y^2)^{5/2}}\right]$$

$$= -\frac{p}{4\pi\varepsilon_o}\left[\frac{x^2 + y^2 - 3y^2}{(x^2 + y^2)^{5/2}}\right] = -\frac{p}{4\pi\varepsilon_o}\left[\frac{x^2 - 2y^2}{(x^2 + y^2)^{5/2}}\right]$$

TRUE OR FALSE? For this problem, $p^2 = 2\,aq$.

[b] CORRECT ANSWER: 10 m/sec

The excape speed is the speed which makes the total energy of a particle zero at that point. Therefore,

$$\frac{1}{2} m_e v^2 + \frac{q_p\, q_e}{4\pi\varepsilon_o r} = 0$$

where v = escape speed, or

$$v = \left[-\frac{2q_p\, q_e}{4\pi\varepsilon_o rm}\right]^{1/2} \qquad (1)$$

$$= \left[\frac{2 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19} \times 9 \times 10^{9}}{5 \times 9.1 \times 10^{-31}}\right]^{1/2}$$

$$= 10 \text{ m/sec}$$

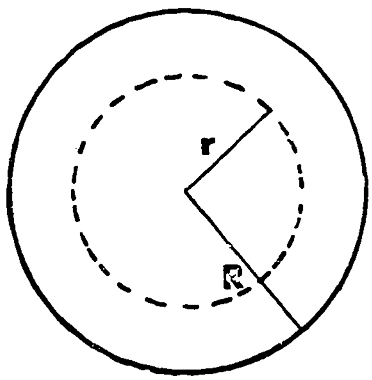
Note that the negative sign in equation (1) when multiplied by the negative sign of the electron charge yields a positive number.

TRUE OR FALSE? At the instant when a charged particle achieves escape speed, enabling it to move out to infinity, its potential energy is zero.

[a] CORRECT ANSWER: A

We may use Gauss's law to determine the dependence of E on the distance r from the center of the sphere. For r > R the situation is the same as if all of the charge were concentrated at the center. Thus,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad \text{for } r > R \qquad (1)$$



For r < R from Gauss's law we obtain

$$\varepsilon_0 \int \vec{E}\cdot d\vec{S} = q$$

or

$$\varepsilon_0 E(4\pi r^2) = q$$

or

$$E = q/4\pi\varepsilon_0 r^2 \qquad (2)$$

where q is the charge inside the Gaussian surface. Since the charge is uniformly distributed we have (ρ is the charge density)

$$q = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi r^3 \frac{Q}{(4/3)\pi R^3} = Q\frac{r^3}{R^3} \qquad (3)$$

Thus, substituting (3) into (2)

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} \quad \text{for } r < R \qquad (4)$$

We can use the relationship

$$V = -\int_{\infty}^{r} \vec{E}\cdot d\vec{s} \qquad (5)$$

to determine V. For r > R, however, we know that

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \quad (r > R) \qquad (6)$$

To find an expression for V inside the sphere (r < R) we use Equation (4) and (5). Thus,

next page

continued

$$V(\text{at } r) - V(\text{at } R) = -\int_R^r \vec{E}\cdot d\vec{s} = -\frac{Q}{4\pi\varepsilon_o R^3}\int_R^r r\,dr$$

$$= -\frac{Q}{4\pi\varepsilon_o R}\left[\frac{1}{2}r^2\right]_R^r = -\frac{Q}{8\pi\varepsilon_o R^3}(r^2 - R^2)$$

Finally, using the fact that V(at R) = $Q/4\pi\varepsilon_o R$ we obtain

$$V = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right)$$

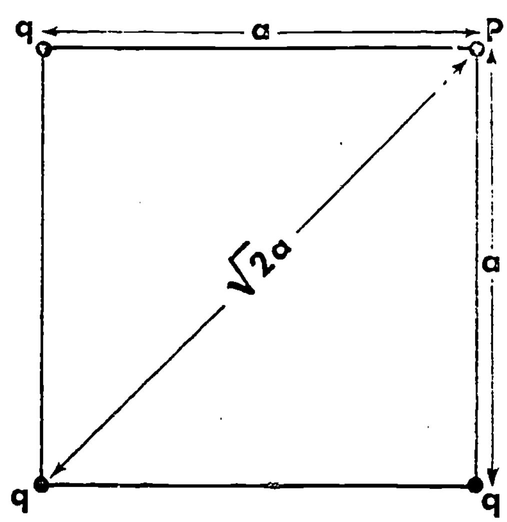
$$= \frac{3Q}{8\pi\varepsilon_o R} - \frac{Qr^2}{8\pi\varepsilon_o R^3}$$

TRUE OR FALSE? For $r < R$, we can consider that all of the charge Q is located at the geometric center of the sphere.

[a] CORRECT ANSWER: 1.1×10^5 j

The work required to bring the charge Q to point P is given by $W = VQ$, where V is the electric potential at the point P. An electric potential at a distance r from a given point q is given by

$$\frac{q}{4\pi\varepsilon_o r}$$



Therefore

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{a} + \frac{q}{a} + \frac{q}{\sqrt{2}a}\right)$$

or

$$V = \frac{q}{4\pi\varepsilon_o a}\left(\frac{2}{1} + \frac{1}{\sqrt{2}}\right)$$

and

$$W = VQ = \frac{qQ}{4\pi\varepsilon_o a}\left(\frac{2}{1} + \frac{1}{\sqrt{2}}\right)$$

$$= 1.1 \times 10^5 \text{ j}$$

[a] CORRECT ANSWER: B

The relation between the potential V and the field E is given by

$$V = -\int \vec{E}\cdot d\vec{s}$$

Thus, their relation in a differential form becomes

$$E_s = -\frac{dV}{ds}$$

The subscript s simply denotes that E_s is the electric field intensity in the direction of the vector $\vec{s}$.

The general relationship between $\vec{E}$ and V, in rectangular coordinates, is

$$\vec{E} = -\vec{\nabla}V \equiv -\left(\frac{\partial V}{x}\hat{i} + \frac{\partial V}{y}\hat{j} + \frac{\partial V}{z}\hat{k}\right)$$

[b] CORRECT ANSWER: D

The electric field in the region between the two cylinders can be obtained by the application of Gauss's law (the Gaussian surface will be a cylindrical surface between the two charged cylinders).

$$\varepsilon_o \oint \vec{E}\cdot d\vec{S} = q$$

For the cylindrical Gaussian surface of radius r, the surface area is $2\pi rL$. From this E is equal to

$$E = \frac{q}{2\pi\varepsilon_o Lr}$$

Now using the definition of potential to calculate the potential difference between the two cylinders, we obtain

$$V = -\int_b^a \vec{E}\cdot d\vec{r} = -\frac{q}{2\pi\varepsilon_o}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_o L}\ln\frac{b}{a}$$

TRUE OR FALSE? In the above solution, the electric potential was obtained by differentiating the expression for electric field intensity.

[a] CORRECT ANSWER: 30.1 cm

As the proton approaches the positive charge Q it is decelerated until its velocity is zero. It will then turn back due to the repulsive force. The distance of closest approach is, therefore, achieved when the kinetic energy of the proton is zero. Therefore, using the conservation of energy principle we find

$$\frac{1}{2} m v^2 + \frac{qQ}{4\pi\varepsilon_o r} = \frac{qQ}{4\pi\varepsilon_o R} \qquad (1)$$

where R is the distance of closest approach. Substituting numerical values in each term in the left hand side of equation (1) yields

$$\frac{1}{2} m_p v^2 = \frac{1}{2}\, 1.67 \times 10^{-27} \times 4 \times 10^{14} = 3.34 \times 10^{-13} \text{ j}$$

and

$$\frac{qQ}{4\pi\varepsilon_o r} = \frac{1.6 \times 10^{-19} \times 1 \times 10^{-4} \times 9 \times 10^{9}}{1} = 1.44 \times 10^{-13} \text{ j}$$

Therefore

$$\frac{qQ}{4\pi\varepsilon_o R} = (3.34 + 1.44) \times 10^{-13} \text{ j}$$

$$= 4.78 \times 10^{-13} \text{ j}$$

Solving for R we obtain

$$R = \frac{qQ}{4\pi\varepsilon_o \times 4.78 \times 10^{-13}}$$

or

$$R = \frac{1.6 \times 10^{-19} \times 1 \times 10^{-4} \times 9 \times 10^{9}}{4.78 \times 10^{-13}} \text{ m}$$

$$R = 30.1 \text{ cm}$$

TRUE OR FALSE? In this solution, the quantity $1/2\ mv^2$ is the kinetic energy of the proton at the point of closest approach to Q.

[a] CORRECT ANSWER: B

The potential on the smaller shell is produced in part by the charge on it and in part by the charge on the outer shell. The contribution of the latter is

$$-\frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

since the potential inside a shell due to its own charge is the same as that on the shell itself. Thus, the total potential on the smaller shell is

$$V_r = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} + \frac{1}{4\pi\varepsilon_o}\frac{(-q)}{R} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{R}\right)$$

On the other hand, the potential of the outer shell is caused in part by its own charge, i.e., $\frac{-q}{4\pi\varepsilon_o R}$ and in part by the charge on the smaller shell, i.e., $\frac{q}{4\pi\varepsilon_o R}$. Therefore the total potential of the outer shell is $V_R = 0$. Thus,

$$V_r - V_R = V_r = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{R}\right)$$

TRUE OF FALSE? As R is increased, q and r remaining unchanged, V_r decreases.

[b] CORRECT ANSWER: D

The relation between work W_{AB} and the potential difference is

$$\frac{W_{AB}}{q_o} = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{\ell}$$

To define the electric potential difference uniquely, W_{AB} and $V_B - V_A$ must be independent of the path and depend on the end-points only.

[a] CORRECT ANSWER: D

The electric potential at a point due to a collection of charges is found by calculating the contribution of each charge to the potential and then summing up the contributions; i.e.,

$$V = \sum_i \frac{1}{4\pi\varepsilon_o} \frac{q_i}{r_i} \qquad (1)$$

where r_i is the distance of charge q_i from the point in question. In the case of a continuous charge distribution we consider the contributions of infinitesimal charge elements dq_i. The summation in (1) becomes an integration

$$dV = \frac{1}{4\pi\varepsilon_o} \frac{dq}{r} \qquad (2)$$

In the present problem the line charge is along the y-axis, so we may write

$$dq = \lambda \, dy = (q/L) \, dy \qquad (3)$$

Also the distance of dq from the field point P is

$$r = \sqrt{x^2 + y^2} \qquad (4)$$

Thus, the integral in (2) becomes

$$V = \int dV = \int_{-L/2}^{L/2} \frac{1}{4\pi\varepsilon_o} \frac{q \, dy}{L\sqrt{x^2 + y^2}} \qquad (5)$$

The constant factor, $q/4\pi\varepsilon_o L$ may be taken outside the integral.

From integral tables we find that

$$\frac{dy}{\sqrt{x^2 + y^2}} = \ell n \, (x + \sqrt{x^2 + y^2}) \qquad (6)$$

Using this in (5) above we find

$$V = \frac{q}{4\pi\varepsilon_o L} \ell n \, (x + \sqrt{x^2 + y^2}) \Bigg|_{-L/2}^{L/2}$$

$$= \frac{q}{4\pi\varepsilon_o L} \ell n \left[\frac{L/2 + \sqrt{(L/2)^2 + x^2}}{-L/2 + \sqrt{(L/2)^2 + x^2}} \right]$$

[a] CORRECT ANSWER: B

There are three possible pairs of positive charges, and the distance between the charges in each pair is a:

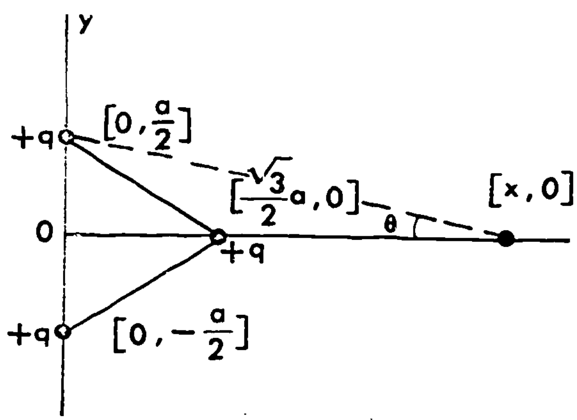
$$d = \sqrt{\left(\frac{\sqrt{3}}{2}a\right)^2 + \left(\frac{a}{2}\right)^2} = a$$

Thus,

$$U = 3 \times \frac{1}{4\pi\varepsilon_o}\frac{q^2}{a} = \frac{3q^2}{4\pi\varepsilon_o a}$$

A detailed calculation of the potential energy in terms of the work required to assemble the charges follows. It is offered as optional enrichment *material.*

This configuration can be assembled in three steps, starting from the premise that all the charges are initially located at infinity.



i) A charge ("+q") is brought from infinity to the point (0, -a/2). Since no forces act on this charge, the required work is equal to zero (why?)

ii) Another charge ("+q") is brought to the point (0, a/2) from $y = +\infty$. Now, however, one must do work against the (repulsive) force exerted by the first charge. The force on a charge +q located at (0, y) due to a charge +q at (0, -a/2) is

$$F_1 = \frac{1}{4\pi\varepsilon_o}\frac{q^2}{(y + (a/2))^2} \qquad (1)$$

next page

continued

Remembering the sign convention

(work done on a system) = - (work done by the system)

and using the work-energy theorem, one gets the work supplied to the system. Note that the path of integration is taken to be parallel to the force so that $\vec{F}_1 \cdot d\vec{s} = F_1 ds$; so

$$W_1 = -\int \vec{F}_1 \cdot d\vec{s} = -\int_{y=\infty}^{y=(a/2)} F_1 dy = -\int_{y=\infty}^{y=(a/2)} \frac{q^2}{4\pi\varepsilon_0} \frac{dy}{(y+(a/2))^2}$$

$$= \frac{q^2}{4\pi\varepsilon_0} \frac{1}{(y+(a/2))} \Bigg|_{\infty}^{(a/2)} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a} \qquad (2)$$

iii) one now brings the third charge to the point $(\frac{\sqrt{3}}{2} a, 0)$ along the x-axis from $x = +\infty$

At a point $(x,0)$ the force on this charge due to the other two charges already assembled:

$$F_2 = \frac{2}{4\pi\varepsilon_0} \frac{q^2}{r^2} \cos\theta = \frac{2}{4\pi\varepsilon_0} \frac{q^2}{r^2} \frac{x}{r} = \frac{2}{4\pi\varepsilon_0} \frac{q^2 x}{(x^2 + \frac{a^2}{4})^{3/2}} \qquad (3)$$

Thus, the work required to move this charge into position

$$W_2 = -\int_{\infty}^{\frac{\sqrt{3}}{2}a} \frac{q^2}{4\pi\varepsilon_0} \frac{2x\,dx}{(x^2 + \frac{a^2}{4})^{3/2}} = -\frac{q^2}{4\pi\varepsilon_0} \frac{(-2)}{(x^2 + \frac{a^2}{4})^{1/2}} \Bigg|_{\infty}^{\frac{\sqrt{3}}{2}a}$$

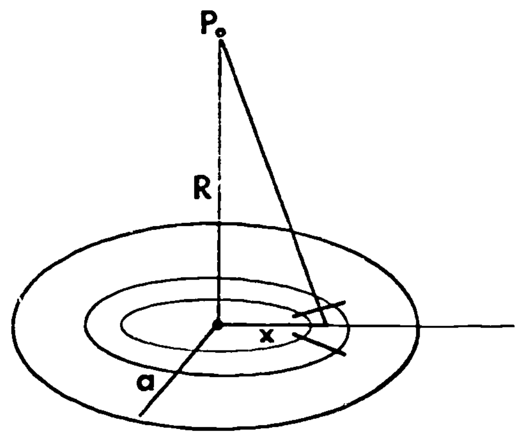
$$= \frac{2}{4\pi\varepsilon_0} \frac{q^2}{a} \qquad (4)$$

The total potential energy of the configuration is given by the sum of equations (2) and (4); namely,

$$U = W_1 + W_2 = \frac{3}{4\pi\varepsilon_0} \frac{q^2}{a} \qquad (5)$$

[a] CORRECT ANSWER: B

The contribution to dV due to a charge element $dq = \sigma dA$ consisting of a flat circular strip of radius x and width dx is



$$dV = \frac{dq}{4\pi\varepsilon_o \sqrt{x^2 + R^2}}$$

and dq may be expressed as

$$dq = \sigma dA = \sigma(2\pi x)\, dx$$

Thus,

$$dV = \frac{\sigma(2\pi x)\, dx}{4\pi\varepsilon_o \sqrt{x^2 + R^2}}$$

The potential V is found by integrating over the area of the disk:

$$V = \int dV = \frac{\sigma}{2\varepsilon_o}\int_o^a \frac{x\,dx}{\sqrt{x^2 + R^2}} = \frac{\sigma}{2\varepsilon_o}\left(\sqrt{a^2 + R^2} - R\right)$$

Note also that this is the only dimensionally correct choice.

TRUE OR FALSE? The area of the charge element is taken as the product of the circumference and width of the element.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL

The Meaning of Capacitance

OBJECTIVE

To define capacitance; to become familiar with the units of capacitance; to solve simple problems involving capacitance.

If several charged conductors are brought near each other, the potential of each one will be determined partially by its own charge and partially by the charge, size, shape, and positions of every other nearby conductor. For example, the potential of an insulated conductor will be raised if a positively charged body is brought close to it; the potential of an insulated conductor will be reduced if a negatively charged body approaches it.

Any device in which a static charge resides or can be made to reside may be called a capacitor and may be said to have capacitance. Thus, any insulated conductor has capacitance. In general, however, for its simplicity and utility, the special case of two near-by conductors given equal amounts of charge of opposite sign is usually discussed in much detail. The simplest form of a capacitor made up in this way comprises two flat, parallel plates separated from one another by a dielectric material. Qualitatively, the capacitance of a capacitor is a measure of its ability to store electrical charge. For a structure of given dimensions and dielectric material, capacitance is a constant and does not depend upon applied potential differences or any other external influences.

A capacitor may be charged by connecting its plates through wires to any source of potential difference such as an electrostatic machine, a rotary generator, or a battery. During the charging process, charges are transferred from one plate to the other, the work being done by the source of potential difference. The ratio of the amount of charge transferred (q) to the potential difference across the plates (V) at any instant during the charging process is constant and, by definition, is the capacitance C of the capacitor

$$C = q/V$$

In the MKS system where q is in coulombs and V is in volts, the unit of capacitance is the *coulomb per volt,* renamed the *farad* (f). Since one farad is a tremendous capacitance from the point of view of practical capacitors, you will find sub-units of the farad commonly used:

1 microfarad (μf) = 10^{-6} f
1 nanofarad (nf) = 10^{-9} f
1 picofarad (pf) = 10^{-12} f

next page

continued

For many years, the work micromicrofarad ($\mu\mu f$) was used instead of pico-farad and is still to be found in much of the literature you will read. Thus,

$$1 \text{ micromicrofarad} = 1 \text{ picofarad} = 10^{-12} \text{ farad}$$

The capacitance of a parallel plate capacitor is given by:

$$C = \varepsilon_0 A/d$$

in which A is the area of *one of the plates* and d is the distance separating them. This expression is valid for a capacitor having a vacuum dielectric but may be used with little error when air is present between the plates. For capacitors in which other dielectrics are employed, the value of the constant in the above equation depends upon the nature of the material. The relationship is generally written:

$$C = K\varepsilon_0 A/d$$

for capacitors other than air or vacuum types, in which K is specific for the dielectric material and is called the dielectric constant. Tables of dielectric constants are readily available in textbooks and in the standard handbooks of physics.

The problems in this section call for the

(a) the definition of capacitance in terms of q and V;

(b) understanding of the units used to measure capacitance;

(c) calculations required to relate C, q, V, d, and A.

1. A parallel plate capacitor consists of two parallel conducting plates of area A separated by a distance d. The plates carry charge +q and -q respectively. *Derive* the expression for capacitance in terms of ε_o, plate area, and distance between plates, then select the correct answer.

(Note: Unless you can derive the required equation without help, you are to work problems 2, 3, 4, and 5 which follow.)

A. $C = \frac{d}{\varepsilon_o A}$

B. $C = \frac{\varepsilon_o A}{d}$

C. $C = \varepsilon_o Ad$

D. $C = \varepsilon_o d$

2. Consider two charged conductors separated by a non-conductor. The charge on one conductor is +q and that on the other is -q. The potential difference between the conductors is V. Which of the following equations defines the capacitance of the system?

A. $C = 1/qV$

B. $C = V/q$

C. $C = qV$

D. $C = q/V$

3. In the MKS system, the basic unit of capacitance is the farad. One farad (1 f) is equivalent to

A. 1 coul/volt

B. 1 coul - volt

C. 1 volt/coul

D. 1 coul/volt^2

4. A 1.2-μf television set capacitor is subject to a 3000-volt potential difference across its terminals. What is the magnitude of the charge in coulombs on each plate of the capacitor?

5. A parallel-plate capacitor consists of two circular plates of 30 cm radius separated by 1.0 mm. What charge in coulombs will appear on the plates if a potential difference of 400 volts is applied?

INFORMATION PANEL

Calculation of Capacitance

OBJECTIVE

To derive equations for the capacitance of capacitors having various geometries.

Recognizing that

$$C = q/V$$

one can usually approach the problem of deriving an equation for the capacitance of a capacitor made of conducting surfaces of any shape by first writing the electric field equation for the particular case, second writing the potential difference between the surfaces in terms of the electric field, and finally substituting the value for V thus obtained in the above equation. In many cases, the second step may be omitted if the potential difference for the particular geometry has already been derived.

For example, the potential difference between two concentric, conducting spherical shells is

$$V = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{R}\right) \text{ where } R > r$$

Thus, to obtain the capacitance of a capacitor made up of two concentric, spherical conductors with a vacuum or air dielectric, this value for potential difference need be substituted in the first equation above to obtain the correct relationship.

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It is worth remarking here that the insertion of conducting material between the plates of a capacitor has the effect of altering the effective spacing between the plates. For example, if a good conductor like copper is formed into a slab of thickness t and inserted midway between the plates of a parallel-plate capacitor, the effective spacing between the plates is then d - t, where d is the actual measured distance.

You will be required to *derive* the expression for the capacitance of

(a) two concentric, conducting, cylinders

(b) two conducting, concentric shells;

(c) an isolated conducting sphere;

(d) a parallel-plate capacitor with a slab of metal between plates.

6. *Derive* the equation for the capacitance of a capacitor formed by two concentric hollow cylinders of length L with radii a and b (b > a); then select the correct answer.

(Note: Unless you can derive this equation without help, you must work problems 7, 8, and 9 which follow.)

A. $C = 4\pi\varepsilon_0 (b - a)$

B. $C = 2\pi\varepsilon_0 L \,\ell n\, (b/a)$

C. $C = \dfrac{2\pi\varepsilon_0 L}{\ell n\, (b/a)}$

D. $C = \dfrac{\ell n\, (b/a)}{2\pi\varepsilon_0 L}$

7. What is the capacitance of a charged, isolated conducting sphere of radius a carrying charge q?

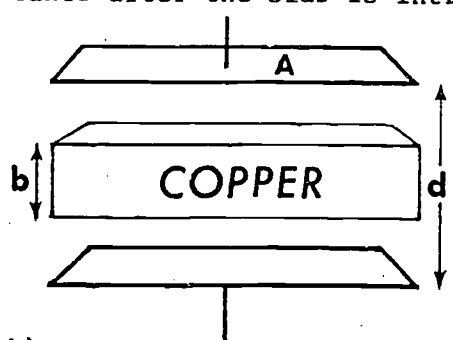
A. $\varepsilon_o a$

B. $4\pi\varepsilon_o a$

C. $\frac{q}{4\pi\varepsilon_o a}$

D. $4\pi\varepsilon_o \frac{q}{a^2}$

8. A uniform slab of copper of thickness b is thrust into a parallel-plate capacitor as shown in the figure. It is exactly halfway between the plates. The capacitance after the slab is introduced is



A. $C = \varepsilon_o A (d - b)$

B. $C = \frac{\varepsilon_o A}{d - b}$

C. $C = 0$

D. $C = \frac{\varepsilon_o A}{d}$

9. What is the capacitance of the capacitor formed by the two concentric conducting, spherical shells of radii r and R (R > r)?

A. $C = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$

B. $C = 4\pi\varepsilon_0 \frac{rR}{R - r}$

C. $C = 4\pi\varepsilon_0 r$

D. $C = 4\pi\varepsilon_0 R$

INFORMATION PANEL — Equivalent Capacitance - Series and Parallel

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OBJECTIVE

To determine the equivalent capacitance of a number of given capacitors connected in series; to determine the equivalent capacitance of a number of given capacitors in parallel.

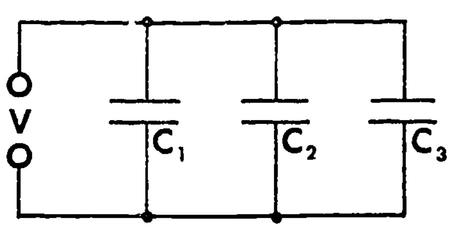

Figure 1

PARALLEL CONNECTION: As indicated in Figure 1, each terminal of each capacitor may be considered to be joined to the source terminal through a resistanceless conductor. Under these conditions, there will be no fall of potential along the connecting wires, hence the potential difference across each capacitor will be that of the source. The equivalent capacitance of a parallel configuration of capacitors may be found from:

$$C = C_1 + C_2 + C_3 + . . . C_n$$

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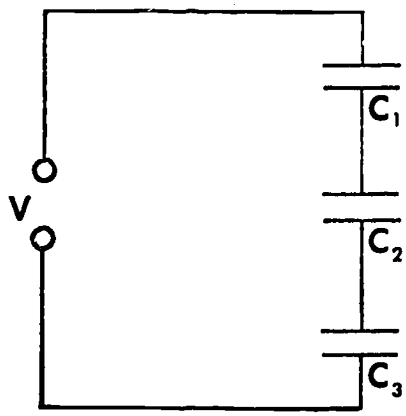


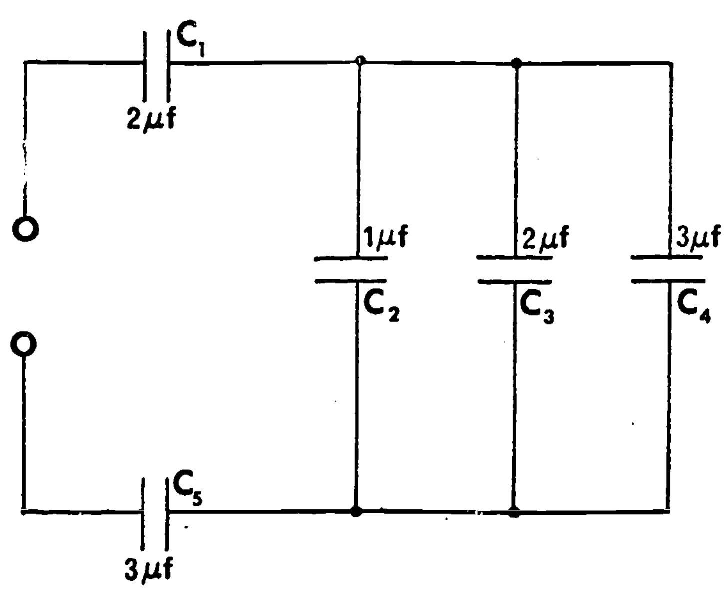
Figure 2

SERIES CONNECTION: (Figure 2)
Again specifying resistanceless conductors between source and capacitors, and between individual capacitors, all of the voltage drops that appear in the circuit will be present across the individual capacitors. The sum of these voltage drops is equal to the source voltage. The equivalent capacitance of a group of capacitors connected in series is given by:

$$1/C = 1/C_1 + 1/C_2 + 1/C_3 + \ . \ . \ . \ 1/C_n$$

When capacitors are connected in series-parallel, the equivalent capacitance may be determined by reducing the circuit to either straight series or straight parallel (depending on the nature of the configuration) by finding the equivalent capacitance of individual groups that make up the complex circuit. Two of the problems in this section deal with such circuits in order to provide basic practice in this simplification process.

10. For the circuit shown below, what is the equivalent capacitance in μf?



11. The equivalent capacitance of a combination of capacitors can be defined as:

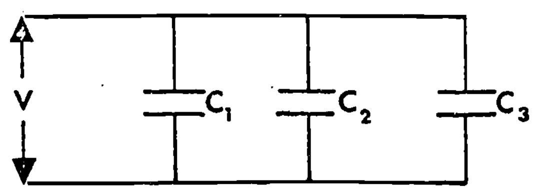
A. the capacitance of several capacitors wired in series with the same capacitance as the original combination.

B. the capacitance of several capacitors wired in parallel with the same capacitance as the original combination.

C. the capacitance of a single capacitor which could replace a group of capacitors in an electrical circuit without changing the performance of the circuit.

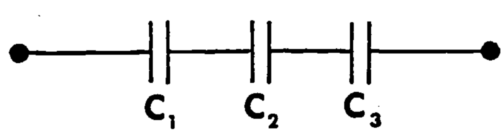
D. the capacitance of the single capacitor which has capacitance equal to the sum of capacitance of all the original capacitors.

12. In the network shown, the capacitors C_1, C_2, and C_3 have values 4 μf, 8 μf, and 16 μf respectively.

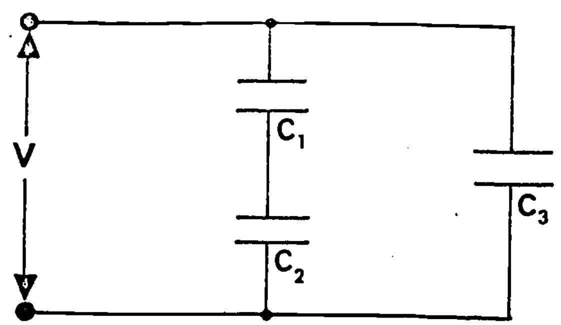


What is the equivalent capacitance of this combination of capacitors?

13. In the network shown, the capacitors C_1, C_2, and C_3 have values 4.0 μf, 8.0 μf, and 16 μf respectively. What is the equivalent capacitance of this combination of capacitors?



14. For the circuit shown below, what is the equivalent capacitance?



$C_1 = 10.0\ \mu f$

$C_2 = 5.00\ \mu f$

$C_3 = 4.00\ \mu f$

INFORMATION PANEL — Analysis of Capacitor Circuits

OBJECTIVE

To study various capacitor circuits in terms of total and individual charges, potential differences, and equivalent capacitances.

Analysis of capacitor circuits can be substantially facilitated by working with a logical itemization of certain individual characteristics of series and parallel circuits. A useful listing appears below:

Capacitors in Parallel

Potential difference (V): The potential difference across each capacitor is the same as that of the source of potential difference.

$$V = V_1 = V_2 = V_3 = \ldots$$

Charge (q): The total charge provided by the battery or other source of electrical energy is *shared* among the capacitors in the parallel circuit and is equal to the sum of the individual charges

$$q = q_1 + q_2 + q_3 + \ldots$$

The charge acquired by each individual capacitor is directly proportional to the capacitance so that

$$q_1 = C_1V, \quad q_2 = C_2V, \quad q_3 = C_3V, \quad \text{etc.}$$

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Equivalent capacitance (C): The equivalent capacitance is the sum of the individual capacitances:

$$C = C_1 + C_2 + C_3 + \dots$$

Capacitors in Series

Potential difference (V): The sum of the individual potential differences is equal to the potential difference of the source:

$$V = V_1 + V_2 + V_3 + \dots$$

Furthermore, the potential difference that appears across the terminals of each of the capacitors in series is inversely proportional to the capacitance of the capacitor:

$$V_1 = q/C_1, \quad V_2 = q/C_2, \quad V_3 = q/C_3, \quad \text{etc.}$$

Charge (q): The charge acquired by each capacitor is the same as that transferred by the source from one of its terminals to the other:

$$q = q_1 = q_2 = q_3 = \dots$$

Thus, each capacitor in a series group acquires the same charge as every other in the group, and this charge is the same as that supplied by the battery to the circuit as a whole.

Equivalent capacitance (C): The equivalent capacitance is given by:

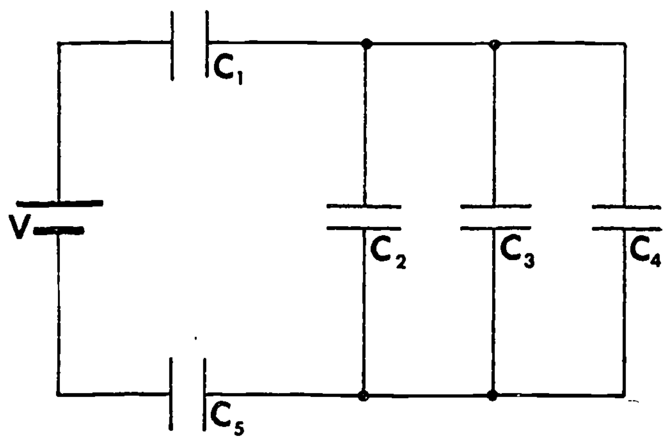
$$1/C = 1/C_1 + 1/C_2 + 1/C_3 + \dots$$

The arithmetic may sometimes be simplified by using the relationship below for *two* (and two only) capacitors in series:

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

The problems in this section of your work involve the relationships given in this Information Panel, plus the techniques used to reduce series-parallel circuits to simple series or simple parallel arrangements, whichever is the more logical.

15. For the circuit shown below, what is the total charge in micro-coulombs supplied by the battery?



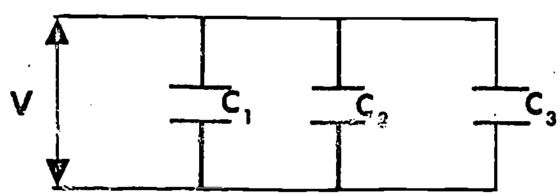
V = 12 volts

$C_1 = C_3 = 2.0\ \mu f$

$C_2 = 1.0\ \mu f$

$C_4 = C_5 = 3.0\ \mu f$

16. Consider the combination of capacitors in parallel shown in the diagram:



where $C_1 = C_2 > C_3$

A voltage V is maintained across the terminals of the combination. The relationship among the voltages across each capacitor is:

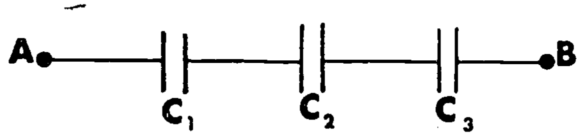
A. voltage across C_1 > voltage across C_2 > voltage across C_3

B. voltage across each of the capacitors is the same

C. voltage across C_3 > voltage across C_2 = voltage across C_1

D. voltage across C_1 = voltage across C_2 > voltage across C_3

17. In a series array of capacitors, as shown below,



which of the following is true?

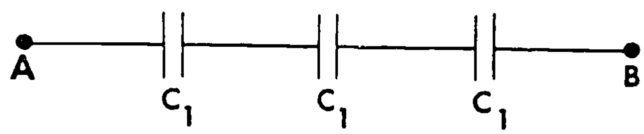
A. The charge stored by the combination is equal to the sum of the charges stored by the individual capacitors.

B. The voltage drop across the combination is equal to the sum of voltage drops across the individual capacitors.

C. More charge can be stored in this combination, for the same *overall* voltage drop, than in any of the single capacitors.

D. The voltage drop across C_2 is zero since the net charge, supplied to it by the battery connected across points A and B, is zero.

18. A series array of capacitors is shown in the diagram. With a given



overall potential difference V across points A and B,

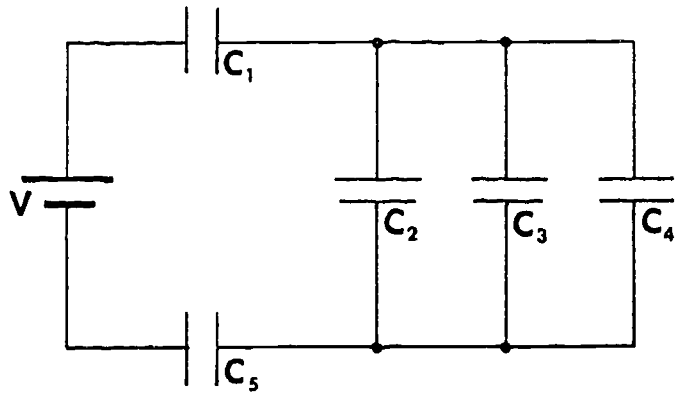
A. less charge is stored by the combination than would be stored by any one of the elements if it had the same potential difference V across its terminals, because the capacity of the combination is less than that of any one of its constituents.

B. more charge is stored by the combination than would be stored by any one of the elements if it had the same potential difference V across its terminals, because the capacity of the combination is more than that of any one of its constituents.

C. since that portion of C_1 which is joined to C_2, and that of C_2 which is joined to C_3, are not connected to the battery, they have *no* net charge, and so do not influence the charge storage capacity of the network.

D. the charge stored by the combination is equal to the average of the charge which would be stored by each one of the components if the same potential difference V were applied across it.

19. In the diagram below, the charges on C_1, C_2, C_3, C_4, and C_5 respectively are (all in μcoulombs)



V = 12 volts

$C_1 = C_3 = 2$ μf

$C_2 = 1$ μf

$C_4 = C_5 = 3$ μf

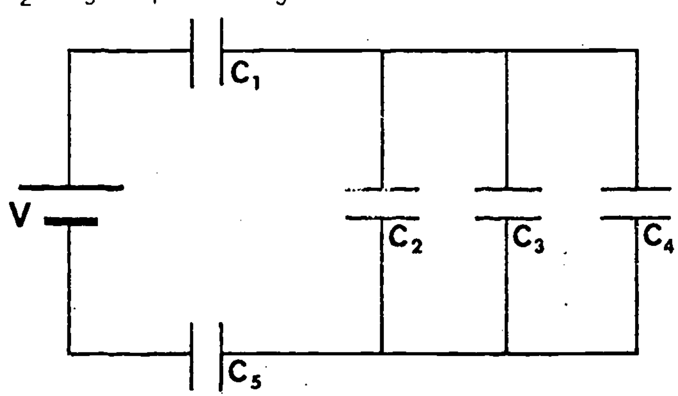
A. 12, 6, 4, 2, 12

B. 12, 2, 4, 6, 12

C. 12, 12, 12, 12, 12

D. 6, 12, 6, 4, 4

20. In the diagram below, the potential differences (in volts) across C_1, C_2, C_3, C_4, and C_5 are respectively



V = 12 volts

$C_1 = C_3 = 2$ μf

$C_2 = 1$ μf

$C_4 = C_5 = 3$ μf

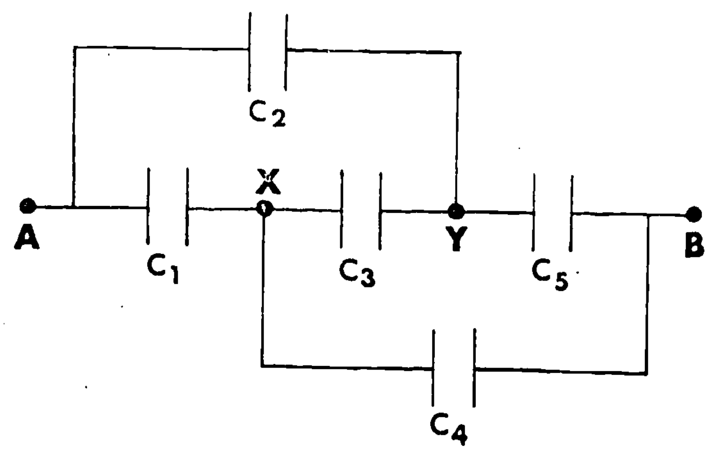
A. 12, 12, 12, 12, 12

B. 24, 4, 8, 12, 24

C. 6, 12, 6, 4, 4

D. 6, 2, 2, 2, 4

21. A potential difference of 10 volts is applied between points



$C_1 = C_2 = 3\ \mu f$

$C_3 = 8\ \mu f$

$C_4 = C_5 = 5\ \mu f$

A and B of the circuit shown above. Calculate the potential difference across capacitor C_3 (between points X and Y).

A. Zero

B. 5 volts

C. 10 volts

D. 16 volts

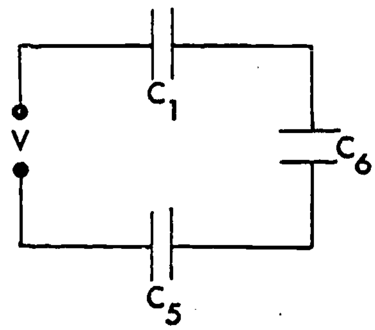
22. A 100-μμf capacitor (1 μμf = 10^{-12} f) is charged to a potential difference of 100 volts; then the charging battery is disconnected and the charged capacitor is connected to a second capacitor. If the potential difference drops to 50 volts, what is the capacitance (in μμf) of this second capacitor?

[a] CORRECT ANSWER: 1

The arrangement shown in the drawing can be broken into three parts, one of them consisting of capacitors C_2, C_3, and C_4. The latter capacitors are connected in parallel so their equivalent capacitance may be found by adding their capacitances arithmetically; i.e.,

$$C_6 = C_2 + C_3 + C_4 = 6\ \mu f \qquad (1)$$

Thus, the original circuit may now be replaced by the one below.



This new arrangement clearly involves three capacitors connected in series; their equivalent capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_6} + \frac{1}{C_5} = \frac{1}{2} + \frac{1}{6} + \frac{1}{3} = 1 \qquad (2)$$

Therefore

$$C = 1\ \mu f$$

TRUE OR FALSE? If C_2, C_3, and C_4 had been 2 μf each, the final answer for C would have come out the same, 1 μf.

[b] CORRECT ANSWER: B

In diagrams we connect circuit elements (such as capacitors) with lines. These lines are to represent conductors with zero "resistance"; i.e., charged particles can move along these conductors without expending any energy. These conductors, therefore, are equipotential lines, giving the same potential difference across each capacitor.

[a] CORRECT ANSWER: B

The capacitance in general is given by the expression

$$C = \frac{q}{V}$$

The potential of a charged, isolated conducting sphere is given by

$$V = \frac{q}{4\pi\varepsilon_0 a}$$

Thus

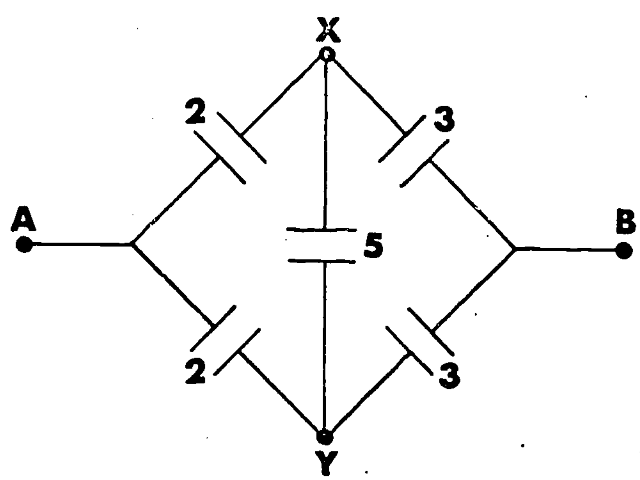
$$C = 4\pi\varepsilon_0 a$$

In other words, the capacitance of a charged sphere is proportional to its radius.

Usually, we calculate the potential difference between two conductors. In this case, however, we found the potential difference between the sphere and a point at infinity.

[b] CORRECT ANSWER: A

We redraw the given circuit in the form shown here. We note the complete symmetry between the upper and lower branch of the diamond ("bridge"). Since the two branches are completely symmetrical we expect the charge stored in each of the capacitors with equal capacitance to be the same. If each of the 3 μf capacitor has the same charge stored in it, the potential difference across each of them will be the same. Thus, points X and Y are at the same potential. The potential difference across the 8 μf capacitor, therefore, is zero.



[a] CORRECT ANSWER: B

The capacitance of a capacitor is given by

$$C = \frac{q}{V} \qquad (1)$$

Since the electric field is constant between the plates of the parallel-plate capacitor, the potential difference is given by the expression

$$V = -\int \vec{E} \cdot d\vec{S} = Ed$$

where the electric field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$$

Thus

$$V = \frac{qd}{\varepsilon_0 A} \qquad (2)$$

From (1) and (2), we obtain

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}$$

TRUE OR FALSE? For a given separation d, the potential difference between the plates varies inversely as the area of one plate.

[b] CORRECT ANSWER: 2.3 μf

Since the capacitors are connected in series, the equivalent capacitance C is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$$

or

$$C = 2.3\ \mu f$$

[a] CORRECT ANSWER: B

You have seen this configuration before. The equivalent capacitance of C_2, C_3, and C_4 in parallel is 6 μf. This equivalent capacitance in series with C_1 (2 μf) and C_5 (3 μf) yields a total circuit equivalent of 1 μf. Hence, from

$$q = CV$$

the total charge stored by the 12-volt battery is

$$q = 1 \text{ μf} \times 12 \text{ volts} = 12 \text{ μcoul}$$

Capacitors C_1 and C_5 will have a charge equal to that supplied by the battery, namely 12 μcoul. The three capacitors in parallel (C_2, C_3, and C_4) must share 12 μcoul. The charges on these capacitors are proportional to their capacitances which are in the ratio 1/6:2/6:3/6 respectively. The corresponding charges become 2, 4, and 6 μcoul.

[b] CORRECT ANSWER: 10^{-6}

The capacitance of a parallel-plate capacitor is given by

$$C = \frac{\varepsilon_o A}{d}$$

where A is the plate area, and d is the distance between the plates. The capacitance is related to the potential difference and charge by

$$C = \frac{q}{V} = \frac{\varepsilon_o A}{d}$$

Thus, the charge on the plate is

$$q = \frac{\varepsilon_o A}{d} V$$

Substituting numerical values with the area $A = \pi r^2 = \pi \times 9 \times 10^{-2}$ we get

$$q = \frac{\varepsilon_o \times \pi \times 9 \times 10^{-2} \times 100}{10^{-3}} = 4\pi\varepsilon_o \times 9 \times 10^3$$

$$= \frac{1}{9 \times 10^9} \times 9 \times 10^3 = 10^{-6} \text{ coul}$$

TRUE OR FALSE? If both the area and separation of the plates of a parallel-plate capacitor are tripled, the capacitance will be tripled.

[a] CORRECT ANSWER: A

The definition of the capacitance of a capacitor is

$$C = \frac{q}{V}$$

i.e., the amount of charge which raises the potential of the capacitor by one unit.

Since the unit of charge is the coulomb (coul) and that of potential is the volt, we have

$$1 \text{ f} = 1 \text{ coul/volt}$$

[b] CORRECT ANSWER: B

The potential difference between two spherical shells is

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

Using the definition of capacitance

$$C = q/V$$

we obtain

$$C = 4\pi\varepsilon_0\left[\frac{1}{r} - \frac{1}{R}\right]^{-1} = 4\pi\varepsilon_0\left[\frac{R - r}{rR}\right]^{-1}$$

$$= 4\pi\varepsilon_0 \frac{rR}{R - r}$$

TRUE OR FALSE? As the final expression indicates, if R = r then the capacitance would become zero.

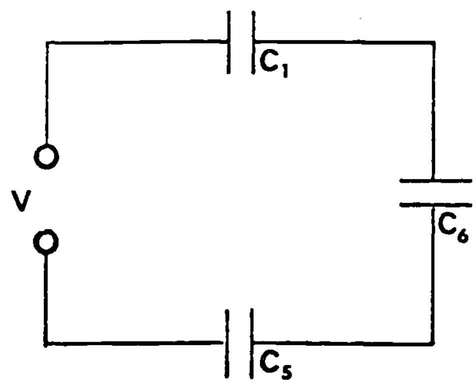
[c] CORRECT ANSWER: 28 μf

Since the capacitors are in parallel, the equivalent capacitance C is

$$C = C_1 + C_2 + C_3 = (4 + 8 + 16)\ \mu\text{f} = 28\ \mu\text{f}$$

[a] CORRECT ANSWER: 12

The original circuit may be replaced by the new one shown below:



where

$$C_6 = C_2 + C_3 + C_4 = 6.0\ \mu f$$

This new arrangement clearly involves three capacitors connected in series and the new capacitance C is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_6} + \frac{1}{C_5} = 1.0$$

$$C = 1.0\ \mu f$$

Since the equivalent capacitance of the circuit is equal to 1.0 μf, i.e., C = 1.0 μf and we know that the battery maintains a potential of 12 volts, we can easily determine the charge from the equation

$$q = CV = (1.0 \times 10^{-6}\ f) \times 12\ \text{volts} = 1.2 \times 10^{-5}\ \text{coul}$$

$$= 12\ \mu\text{coul}$$

TRUE OR FALSE? It is clear from this solution that, in general, μf × volts = μcoul.

[b] CORRECT ANSWER: B

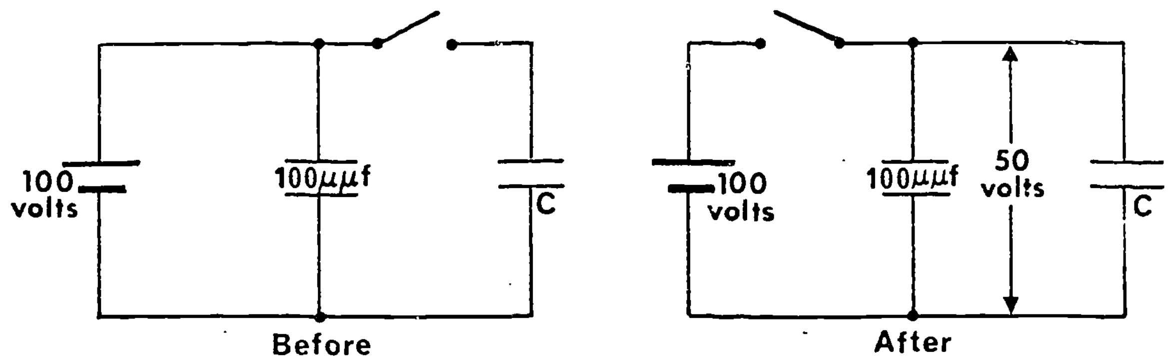
Potential drops are additive in a series combination. If V_1 equals the potential drop across the first capacitor, and V_2 across the second capacitor, and so on, the overall potential drop across the combination is

$$V_{AB} = V_1 + V_2 + V_3$$

[a] CORRECT ANSWER: 100 μμf

The figure below shows the situation described in this problem. We must first find the initial charge on the 100-μμf capacitor by the use of the definition

$$C = \frac{q}{V}$$



Thus,

$$q = (100 \times 10^{-12}\ f) \times (100\ volts) = 10^{-8}\ coul$$

After the charged capacitor is connected to the second capacitor, the potential will be the same on both capacitors; namely, 50 volts and consequently the initial charge will be distributed between the two capacitors. Therefore,

$$10^{-8} = q_1 + q_2 = (C_1 + C_2)\ V$$

$$= (10^{-10} + C_2) \times 50$$

and

$$C_2 = 100\ \mu\mu f$$

TRUE OR FALSE? If we had started with a battery of 50 volts, we would have found a potential difference of 25 volts across C at the conclusion of the action.

[a] CORRECT ANSWER: C

The potential difference between the two cylindrical surfaces can be obtained as follows:

$$V = -\int_a^b \vec{E}\cdot d\vec{S} = \int_a^b E\,dr$$

Using

$$E = \frac{q}{2\pi\varepsilon_o r L}$$

this becomes

$$V = \int_a^b \frac{q}{2\pi\varepsilon_o L}\frac{dr}{r} = \frac{q}{2\pi\varepsilon_o L}\ln\frac{b}{a}$$

Thus, from the definition of capacitance, we get

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_o L}{\ln (b/a)}$$

TRUE OR FALSE? In this solution, $q/2\pi\varepsilon_o L$ is constant for any cylinders that may be selected for use.

[b] CORRECT ANSWER: A

Since the value of the equivalent capacitance of a series chain of capacitors is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

it follows that this value is less than that of any one of the components C_1, C_2, C_3, etc. Thus, the charge storing *capacity* of the combination is less than that of any individual component, provided that same potential difference is applied across them in both cases.

Let us check this with a simple example. Suppose $C_1 = 2$ μf, $C_2 = 3$ μf, and $C_3 = 6$ μf, then

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$$

Thus, C = 1 μf, which is obviously less than any individual component.

[a] CORRECT ANSWER: 3.6×10^{-3}

From the definition of capacitance

$$C = q/V$$

we obtain

$$q = CV$$

Thus,

$$q = 1.2 \times 10^{-6} \times 3 \times 10^{3} = 3.6 \times 10^{-3} \text{ coul}$$

[b] CORRECT ANSWER: B

Since copper is a good conductor, the electric field inside the copper slab is zero. Thus the potential difference between the two capacitor plates is simply

$$V = E\,(d - b) \qquad (1)$$

and

$$E = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A} \qquad (2)$$

From equations (1) and (2), we may write

$$V = \frac{q}{\varepsilon_o A}(d - b) \qquad (3)$$

Substituting (3) into the defining equation for capacitance of a capacitor, we obtain

$$C = \frac{q}{V} = \frac{\varepsilon_o A}{d - b}$$

[a] CORRECT ANSWER: $C = 7.33\ \mu f$

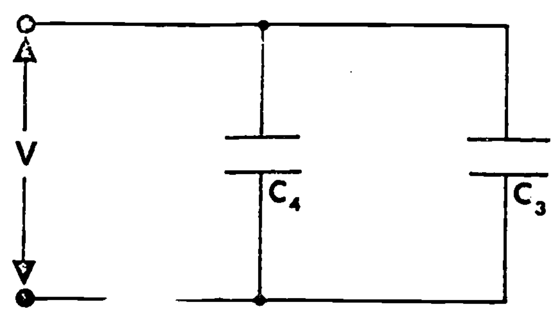
The series combination of C_1 and C_2 can be replaced by a capacitor with an equivalent capacitance C_4 which is given by the equation

$$\frac{1}{C_4} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C_4 = 3.33\ \mu f$$

Thus the original circuit may be replaced by the circuit below:



The equivalent capacitance for this circuit and, therefore, for the original circuit is

$$C = C_3 + C_4 = 4.00 + 3.33 = 7.33\ \mu f$$

TRUE OR FALSE? The equivalent capacitance of C_1 and C_2 *must* turn out to be smaller than either C_1 or C_2 alone.

[b] CORRECT ANSWER: D

The capacitances of the five capacitors are given (in μf) as 2, 1, 2, 3, and 3, respectively. In addition, we found that the respective charges stored in these capacitors are (in μcoul) 12, 2, 4, 6, and 12. Using the relationship

$$V = q/C$$

we obtain the respective potential differences (in volts) the values 6, 2, 2, 2, and 4.

[a] CORRECT ANSWER: D

A capacitor can be thought of as a device which stores electric charge and thus, establishes an electric field around it. Therefore, potential difference, V, exists between the two conductors of the capacitor indicating the storage of electric potential energy. By definition, the capacitance of a capacitor is the amount of charge required to raise the potential of the capacitor by unity, or better, *capacitance is charge per unit potential difference.*

[b] CORRECT ANSWER: C

This correct answer has a subtle point that you should note. The equivalent capacitor is to be capable of replacing a group of capacitors "without changing the performance of the circuit." This means that the equivalent capacitor must have the same capacitance as the group of capacitors which it might replace. Specifically, for the same electric potential difference, the equivalent capacitor must be able to hold the same charge.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL — Energy Storage in Capacitors

OBJECTIVE

To relate the work done in charging a capacitor to the magnitude of the energy stored by it; to solve problems in which this relationship is involved.

During the charging process involving a capacitor and a source of electrical energy such as a battery, charges are transferred from one of the two capacitor conductors (plates) to the other. Since each element of charge transferred causes the potential difference V between plates to increase, and since transfer occurs in a direction opposite that of the electric field being developed between conductors, work must be done to accomplish the transfer. Furthermore, as the charge increases, the work required to transfer each additional element of charge also increases. This is clearly a case of work done against a varying force.

Consider a capacitor that already possesses a charge q' as a result of previous transfers of charge elements. An additional charge element dq' is now to be transferred in the same direction so that the amount of work required to do this is:

$$dW = V\,dq' \qquad (1)$$

where V is the already established potential difference between the conductors (i.e., energy per unit charge already present).

Making use of the relationship for a capacitor, namely,

$$V = q'/C \qquad (2)$$

in which C is the capacitance, we may rewrite equation (1) as follows:

$$dW = (q'/C)dq' \qquad (3)$$

To find the *total work* W required to transfer a total charge q in this manner, it is necessary to integrate equation (3) between the limits of zero and q, thus:

$$W = \int_0^q \frac{q'}{C}\,dq'$$

or

$$W = \frac{1}{2}\frac{q^2}{C} \qquad (4)$$

next page

continued

Equation (4) is a much-used relationship in electrostatics. If the charging process described above has been carried on in a conservative way, that is, if care has been taken to avoid loss of energy due to heating effects, it may be assumed that the energy stored in the capacitor is equal to the work done in charging it so that

$$U = \frac{1}{2}\frac{q^2}{C} \qquad (5)$$

where U is electric potential energy.

Knowing that q = CV enables us to write equation (5) in the alternative form:

$$U = \frac{1}{2}CV^2 \qquad (6)$$

And finally, the fact that C = q/V yields a third form:

$$U = \frac{1}{2}qV \qquad (7)$$

In this section, you will be required to solve problems in which it will be necessary to

(a) find the work done in charging a specific capacitor to produce a specified charge;

(b) find the work done in charging a specific capacitor to a specified potential difference;

(c) make use of the relationship between potential difference, charge, and capacitance.

PROBLEMS

1. Find the work done in charging a parallel plate capacitor to produce a final charge magnitude $Q = 5 \times 10^{-3}$ coul on each plate and a potential difference between the plates of $V = 100$ volts.

2. If a charge q is moved from one plate to the other of an initially uncharged capacitor with capacitance C, the potential difference V across the capacitor plates will be equal to:

A. $V = q/C$

B. $V = C/q$

C. $V = Cq$

D. none of the above

3. If, in the capacitor of the previous question, an additional charge increment of charge "dq" is moved from one plate to the other, in the same direction as q was moved, the work required to make this transfer is given by:

A. $dW = dq/V = (q/C)dq$

B. $dW = Vdq = (q/C)dq$

C. $dW = dq/V = (C/q)dq$

D. $dW = Vdq = (C/q)dq$

4. Consider the process outlined in the previous question until a total charge Q has been transferred from one plate to the other. The total work required to charge the capacitor (i.e., starting with an initial zero charge to end up with a charge Q) is given by

A. $W = \frac{Q^2}{C}$

B. $W = \frac{1}{2}\frac{Q^2}{C}$

C. $W = \frac{1}{2}\frac{Q^2 - q^2}{C}$

D. $W = \frac{1}{2}\frac{q^2}{C}$

5. A parallel plate capacitor of 200 μf capacitance is charged to 500 volts by means of a battery. Find the energy stored in the capacitor.

INFORMATION PANEL — Transfer of Energy in Capacitors

OBJECTIVE

To study the laws that govern the transfer of energy from one charged capacitor system to another, or when capacitor connections are changed from series to parallel and vice versa.

In actual electrical circuits containing capacitors, the charging and discharging processes always carry with them some energy losses in the form of heat generated by the moving charges in the connecting wires (and other components that may form part of the circuit). The problems in the forthcoming section illustrate this effect.

We shall be interested in a number of different energy-transfer situations. These are outlined below; the listing has been arranged to conform with the order in which corresponding problems appear in this section.

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continued

(1) Capacitors may be charged in series, disconnected from the charging source, and then reconnected in parallel. When this is done, charges will move through the connecting wires as they redistribute themselves in the new configuration. An important consideration in this type of energy transfer is that the *total charge of the system remains the same*; charge is neither gained nor lost as a result of the redistribution. In general, to solve a problem of this variety, one must calculate the total charge that resides on the equivalent capacitor formed by the newly connected units. Once this has been determined, the final stored energy is obtained from:

$$E = \frac{1}{2}\frac{q^2}{C}$$

where q = the total charge and C = the equivalent capacitance of the combination.

(2) A capacitor or group of capacitors may be charged from a known voltage source, disconnected from the source, and then connected to an *uncharged* second capacitor or group of capacitors. After the charges have been redistributed, the total charge is again the same as it was before the connections were changed, but in this case it will be shared between the two capacitors instead of residing only in one of them. To find the difference in stored energy for the two conditions, one determines the original stored energy and the final stored energy, and then subtracts one from the other. Here, again, one finds that less energy appears in the final configuration because some energy has been dissipated in the form of heat in the connecting wires.

(3) Two capacitors of different capacitance may be individually charged to different voltages, and then connected to one another with oppositely charged plates joined. In this case, the total charge at the end of the transfer will *not* be equal to the original total charge because some of it will be neutralized. To find the loss of stored energy in an action of this type, the initial energy of the two separate capacitors is first calculated. The residual charge magnitude is then obtained by subtracting the smaller initial charge from the larger initial charge on the individual capacitors (i.e., determining what is left after one charge has partially neutralized the other). The final stored energy is then calculated in the usual manner, and the loss of energy then obtained by subtraction as before.

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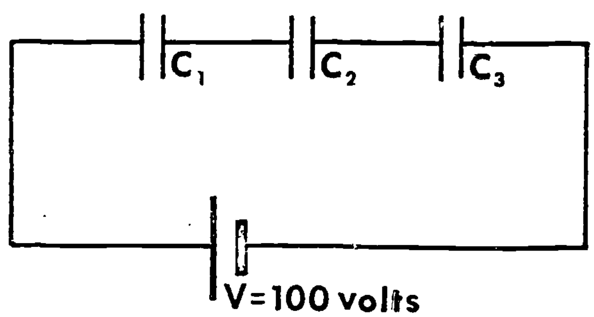
(4) Two capacitors of different capacitance may be charged in parallel from a given source of voltage. They may then be disconnected from the source and each other, and reconnected with plates of opposite sign joined together. The final stored energy may be determined by first calculating the magnitudes of the charge on each capacitor before the reconnection is accomplished, finding the residual charge due to partial neutralization after reconnection, and then determining the final stored energy by properly applying:

$$E = \frac{1}{2}\frac{q^2}{C}$$

6.



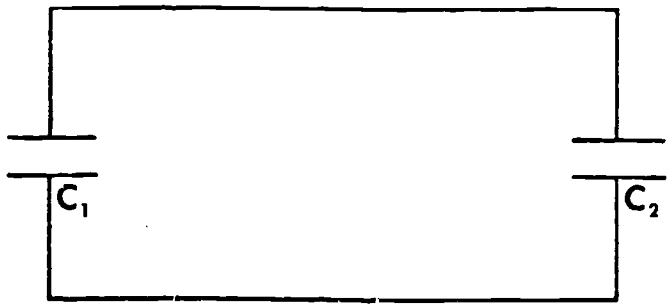
C_1 = 400 μf

C_2 = 400 μf

C_3 = 200 μf

Three large capacitors having capacitances of C_1 = 400 μf, C_2 = 400 μf and C_3 = 200 μf are connected in series across a 100-volt battery. After the capacitors are charged, the battery is disconnected and the capacitors are connected in parallel with the positively charged plates connected together. Find the *difference* in stored energy in the system of three capacitors in the two situations described above.

7. A capacitor having capacitance $C_1 = 10\ \mu f$ is charged to a potential difference $V = 120$ volts. The battery is disconnected and the charged capacitor C_1 is connected to another uncharged capacitor of capacitance $C_2 = 30\ \mu f$ as shown in the diagram. Find the difference in the energy stored.



8. A capacitor having capacitance $C_1 = 1\ \mu f$ is charged to $V_1 = 200$ volts and another capacitor of capacitance $C_2 = 2\ \mu f$ is charged to $V_2 = 400$ volts. If the charged capacitors are connected, positive plate of each to negative plate of the other, find the loss of stored energy.

9. Two capacitors having capacitances $C_1 = 60\ \mu f$ and $C_2 = 30\ \mu f$ are connected in parallel across a 180-volt battery. After the capacitors are charged, the battery is disconnected and the capacitors are reconnected in parallel with plates of opposite sign together. Find the energy stored in the final system.

INFORMATION PANEL Effect of the Capacitor Dielectric

OBJECTIVE

To define dielectric constant; to observe the effect of changing dielectric constant upon capacitance; to use the dielectric constant in capacitor calculations.

The equation

$$C = \varepsilon_o \frac{A}{d}$$

giving the capacitance of a capacitor in terms of plate area A and plate separation distance d has been derived for a capacitor in which the space between plates contains a vacuum only, or air for a close approximation. When a solid or liquid dielectric fills the space between the plates, it is found that the capacitance C increases to an extent determined by the nature of the dielectric substance.

The *dielectric constant* κ may be defined operationally as

$$\kappa = \frac{\text{capacitance with dielectric}}{\text{capacitance in vacuum}}$$

or

$$\kappa = \frac{C_d}{C_o} \qquad (1)$$

Since C_d is always larger than C_o, the dielectric constant is always greater than 1 for material substances.

Several types of experimental observation should be mentioned at this point.

(1) Suppose we place a given charge q on a vacuum capacitor C_o. The voltage across this capacitor would then be

$$V_o = q/C_o \qquad (2)$$

next page

continued

We now slip a slab of some dielectric such as mica or polystyrene between the plates to fill the gap completely. The capacitance will then be larger than before (C_d) and, since the charge does not change, the new voltage will be

$$V_d = q/C_d \qquad (3)$$

Dividing equation (2) by equation (3) then gives us

$$\frac{V_o}{V_d} = \frac{C_d}{C_o} = \kappa \qquad (4)$$

so that it is evident that the dielectric constant is also expressed as the ratio of the voltage in vacuum to the voltage with dielectric (same charge).

(2) Suppose now that we place a given charge on a vacuum capacitor in order to obtain some predetermined voltage. With the voltage measuring device connected across the plates, we then slide in a slab of dielectric and observe that the *voltage decreases*. We then raise the charge magnitude from q_o to q_d in order to reestablish the same voltage that was present before the dielectric was inserted. The relationship is:

(initial) $$q_o = C_oV \qquad (5)$$

(final) $$q_d = C_dV \qquad (6)$$

Dividing equation (6) by equation (5) yields:

$$\frac{q_d}{q_o} = \frac{C_d}{C_o} = \kappa \qquad (7)$$

thus showing that the dielectric constant is also given by the ratio of the charge on the capacitor with dielectric to the charge in vacuum for for the same potential difference between plates.

Since the dielectric constant κ may be defined as in equation (1), we can also write:

$$C = \kappa\varepsilon_o \frac{A}{d}$$

as the capacitance of a capacitor in which the space between plates is filled with a material of dielectric constant κ.

10. A dielectric slab of thickness b and dielectric constant κ is inserted between the plates of a parallel-plate capacitor of plate separation d and area A. What is the capacitance of the capacitor?

A. $C = \frac{\varepsilon_o A}{d - b}$

B. $C = \frac{\kappa \varepsilon_o A}{\kappa d - b(\kappa - 1)}$

C. $C = \frac{\kappa \varepsilon_o A}{d}$

D. $C = \frac{\varepsilon_o A}{\kappa (d - b)}$

11. What is an expression for the capacitance of a parallel-plate capacitor in terms of the following quantities?

dielectric constant = κ
permittivity of free space = ε_o
plate area = A
plate separation = d

A. $C = \frac{\kappa A}{\varepsilon_o d}$

B. $C = \frac{\kappa A d}{\varepsilon_o}$

C. $C = \frac{\kappa}{\varepsilon_o A d}$

D. $C = \frac{\kappa \varepsilon_o A}{d}$

12. An air-capacitor is connected to the terminals of a battery which places a potential difference V_o across its terminals. The battery is then disconnected and a slab of dielectric material having a dielectric constant κ is inserted between the plates. A voltmeter connected across the capacitor now reads a potential difference V_d. Select the true statement:

A. $V_o = V_d$

B. $V_o > V_d$

C. $V_d > V_o$

D. $V_d = 0$

13. If a dielectric is placed in an electric field, induced surface charges appear which tend to

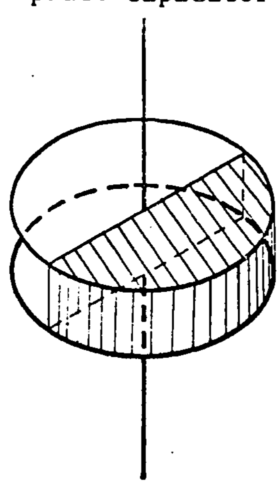
A. increase the electric field by a factor of κ

B. decrease the field by a factor of $1/\kappa$

C. leave the field unaffected

D. increase the potential by a factor of κ

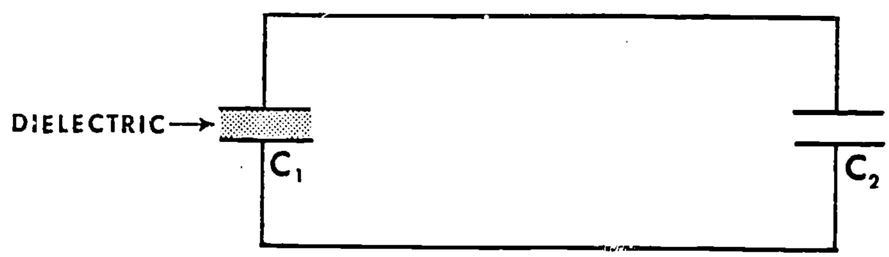
14. A parallel plate capacitor is made up of two circular plates of radius 6 cm separated by a distance of 1 cm. A 1-cm thick semicircular plate of dielectric material with dielectric constant $\kappa = 4.0$ fills half of the space between the plates. Calculate the capacitance of this capacitor. (Give your answer in picofarads (pf) where 1 pf = 1 μμf (micromicrofarad) = 10^{-12} f.)



15. Two identical capacitors each of 3 μf are given identical charges of 900 μcoul each. Each has its plates initially separated by a layer of air. Now a sheet of mica ($\kappa = 8$) is inserted into the air space of the second capacitor, completely filling it. What is the ratio of the potential difference across the capacitor with dielectric to that of the other; namely, V_d/V_a?

A. 8:1
B. 1:8
C. 8:3
D. 3:8

16. An air capacitor having capacitance $C_1 = 1.5$ μf is connected to a 100-volt battery. After the capacitor is fully charged it is disconnected from the battery and filled with a dielectric material of dielectric constant $\kappa = 3.0$. If the capacitor with the dielectric is now connected to another uncharged capacitor $C_2 = 3.0$ μf as shown in the diagram, find the energy stored in the final system.



17. Which is the correct statement concerning the insertion of a dielectric into the air gap of a charged capacitor?

A. The dielectric is pulled into the air gap by attractive forces.

B. Work is done by an external agent, i.e., the dielectric must be pushed into the gap.

C. No work is done either on or by the external agent inserting the dielectric.

D. Initially an external agent must exert a force to get the dielectric halfway into the gap. Thereafter, the dielectric is pulled into place by an attractive force which, on the average, is equal to the force required to push it halfway into the gap.

18. A capacitor has a capacitance $C = 1.5\ \mu f$ when its plates are separated by a layer of air. It is fully charged to $V = 600$ volts by a battery. If the charged capacitor is first disconnected from the battery and then immersed in an oil of dielectric constant $\kappa = 3$, find the energy stored in the capacitor.

[a] CORRECT ANSWER: 0.05 j

The initial energy E_i stored in series situation is

$$E_i = \frac{1}{2} CV^2 \qquad (1)$$

where equivalent capacitance C is given by

$$C = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1} \qquad (2)$$

$$= \frac{4 \times 4 \times 2 \times 10^{-12}}{(16 + 8 + 8) \times 10^{-8}} = 10^{-4} \text{ f}$$

Substituting the numerical values in equation (1) we obtain

$$E_i = \frac{1}{2} \times 10^{-4} \times 10^{4} = 0.5 \text{ j} \qquad (3)$$

In order to calculate the energy E_f in the parallel configuration, we will calculate the total charge Q that will reside on an equivalent capacitor $C' = C_1 + C_2 + C_3$. However, since all the positively charged plates are connected together no charge is neutralized and, hence,

$$Q = Q_1 + Q_2 + Q_3$$

where Q_1, Q_2, and Q_3 are the charges on capacitors C_1, C_2, and C_3 initially. Since C_1, C_2, and C_3 were connected in series initially, the charges

$$Q_1 = Q_2 = Q_3 = Q_s$$

Therefore,

$$Q = 3Q_s$$

The charge Q_s is obtained from

$$V = \frac{Q_s}{C}$$

$$= Q_s \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

or

$$Q_s = VC = 100 \times 10^{-4} = 10^{-2} \text{ coul}$$

next page

continued

The final energy is

$$E_f = \frac{1}{2}\frac{Q^2}{C'} = \frac{1}{2}\frac{(3Q_s)^2}{(C_1 + C_2 + C_3)}$$

$$= \frac{9 \times 10^{-4}}{2 \times 10 \times 10^{-4}}$$

$$= 0.45 \text{ j}$$

The loss in stored energy is given by

$$E_i - E_f = 0.5 - 0.45 = .05 \text{ j}$$

The energy difference appears as heat in the connecting wires as the charges move through them. We shall learn about this heating process in later segments.

TRUE OR FALSE? In this solution we make use of the knowledge that the total charge stored by a group of capacitors in series is equal to the sum of the individual charges.

[a] CORRECT ANSWER: D

The fact that the capacitance of a capacitor is multiplied by κ makes certain materials with high dielectric constant extremely useful in the construction of capacitors. Other factors, of course, must be considered, as mentioned earlier.

In general, the capacitance of a capacitor depends upon:

(1) the geometry of each element of the capacitor. (For parallel plates the area, A, and separation of the plates, d, enter in the expression for the capacitance.)

(2) the material filling the space between the elements of the capacitor. (The dielectric constant κ enters the expression.)

a] CORRECT ANSWER: B

Previously we found that for a capacitor with a charge q, the work required to transfer an additional charge dq across the plates is

$$dW = Vdq$$

Now if we want to find the *total* work required to charge the capacitor from zero charge to a charge Q, we must recognize that q will vary as the capacitor is being charged. Thus,

$$dW = Vdq = \frac{q}{C}\,dq$$

and

$$W = \int_0^Q \frac{q}{C}\,dq$$

Thus,

$$W = \frac{1}{2}\frac{Q^2}{C}$$

b] CORRECT ANSWER: A

This answer stems directly from our definition of capacitance,

$$C = V/q$$

You should remember that capacitance depends upon the following:

(a) geometry of each conductor making up the capacitor,

(b) the geometrical arrangement of the conductors with respect to each other, and

(c) the electrical properties of the material between the conductors making up the capacitor.

[a] CORRECT ANSWER: 0.12 j

The initial charges on the capacitors C_1 and C_2 are

$$Q_1 = C\ [illegible] \times 2 \times 10^2 = 2 \times 10^{-4} \text{ coul} \qquad (1)$$

and

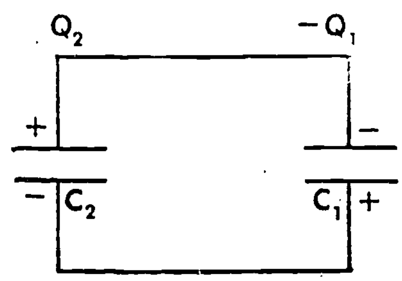
$$Q_2 = C_2V_2 = 2 \times 10^{-6} \times 4 \times 10^2 = 8 \times 10^{-4} \text{ coul}$$

The initial energy E_i of the system of two separate capacitors is

$$E_i = \frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2V_2^2 \qquad (2)$$

$$= 18 \times 10^{-2} \text{ j}$$

When the capacitors are connected with plates of opposite sign together, some of the charge is neutralized and the net charge $Q = Q_2 - Q_1$ will be distributed on the plates of capacitors C_1 and C_2. In terms of equivalent capacitance $C = C_1 + C_2$ the energy stored in the final system E_f is



$$E_f = \frac{1}{2} \frac{Q^2}{C}$$

$$= \frac{1}{2} \frac{(6 \times 10^{-4})^2}{3 \times 10^{-6}} = 6 \times 10^{-2} \text{ j} \qquad (3)$$

Therefore,

$$E_i - E_f = 12 \times 10^{-2} \text{ j} = 0.12 \text{ j}$$

the loss in stored energy..

[a] CORRECT ANSWER: B

The dielectric constant κ can be defined as the ratio of the capacitance of a capacitor with its gap filled with the dielectric to the capacitance of the same capacitor with its gap evacuated (or filled with air, since for air $\kappa \simeq 1$). Since the battery has been removed the charge on the capacitor will remain constant. Thus,

$$V_o = q/C_o$$

and

$$V_d = q/C_d$$

Dividing the two equations, we obtain

$$\frac{V_o}{V_d} = \frac{q/C_o}{q/C_d} = \frac{C_d}{C_o} \equiv \kappa$$

Therefore

$$V_o = \kappa V_d$$

so

$$V_o > V_d$$

since for any material

$$\kappa > 1$$

[b] CORRECT ANSWER: B

The work required to move a charge dq across a potential difference V is

$$dW = V\ dq$$

Now, using the result of the preceding question we find for the potential difference across a capacitor with capacitance C carrying a charge q

$$V = \frac{q}{C}$$

Therefore,

$$dW = \frac{q}{C}\ dq$$

[a] CORRECT ANSWER: A

To verify this let us compare the energy stored in the capacitor before and after the dielectric is inserted. The situation considered is a charged isolated air gap capacitor (i.e., disconnected from the battery). The energy stored in the capacitor is

$$U_a = \frac{1}{2}\frac{q^2}{C_a} \qquad (1)$$

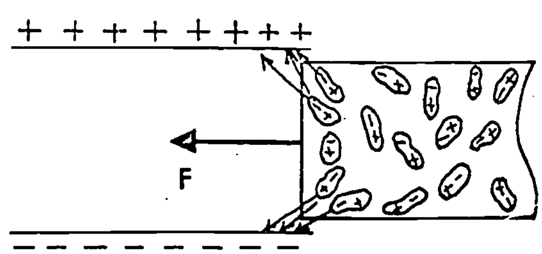
The dielectric is now inserted in the capacitor. Since the charge remains fixed, the new energy stored in the capacitor is

$$U_d = \frac{1}{2}\frac{q^2}{C_d} = \frac{1}{2}\frac{q^2}{\kappa C_a} = \frac{U_A}{\kappa} \qquad (2)$$

or

$$U_a = \kappa U_d$$

Since $\kappa > 1$, it follows that $U_a > U_d$. Thus, some of the electric energy stored in the capacitor originally has been lost. This lost energy was used to do work on the agent placing the dielectric into the gap. The figure below may help you visualize how this comes about. As the dielectric is brought near the capacitor, it gets polarized roughly as shown schematically in the figure. Thus at the top, the "-" charges of the internal dipoles come closer to the "+" charges of the capacitor with similar arrangement at the bottom. This brings about a new inward force since, on the average, unlike charges (attraction) come closer than like charges (repulsion). The dielectric is thus pulled in. The net force will be zero when the dielectric ends are equidistant from the ends of the capacitor plates.



[a] CORRECT ANSWER: B

In order to find the capacitance of the capacitor, we have to find the potential difference between the two capacitor plates. The electric field strength in the gap between the plates and dielectric slab is

$$E_0 = \frac{q}{\varepsilon_0 A}$$

and the electric field strength inside the dielectric slab is

$$E = \frac{E_0}{\kappa} = \frac{q}{\varepsilon_0 A \kappa}$$

Thus, the potential difference between two plates is

$$V = -\int \vec{E} \cdot d\vec{s} = E_0(d - b) + Eb$$

$$= \frac{q}{\varepsilon_0 A}(d - b) + \frac{q}{\varepsilon_0 A \kappa} b$$

$$= \frac{q}{\varepsilon_0 A \kappa}[k(d - b) + b]$$

Thus, from the equation

$$C = \frac{q}{V}$$

we obtain

$$C = \frac{q}{V} = \frac{\varepsilon_0 A \kappa}{\kappa(d - b) + b} = \frac{\varepsilon_0 A \kappa}{\kappa d - b(\kappa - 1)}$$

This problem can also be solved by considering the total capacitance as that of two capacitors in series, one of capitance

$$C_1 = \frac{\varepsilon_0 A}{d - b}$$

the other of capacitance

$$C_2 = \frac{\kappa \varepsilon_0 A}{b}$$

TRUE OR FALSE? The electric field intensity inside the dielectric slab is less than the field intensity in the air gap between plate and slab.

[a] CORRECT ANSWER: 25 j

The energy stored in a capacitor is equal to work done in charging the capacitor. Therefore,

$$\text{Energy stored} = W = \frac{1}{2} CV^2$$

Substituting, we have

$$\text{Energy} = \frac{1}{2} 200 \times 10^{-6} \text{ f} \times (500)^2$$

$$\text{Energy} = 25 \text{ j}$$

Note that the expression

$$\frac{1}{2} CV^2$$

was used for the work done since the data was given in terms of C and V.

TRUE OR FALSE? If the data had been presented in terms of Q and V, then the relationship would have been:

$$\text{Energy} = \frac{1}{2} QV^2$$

[b] CORRECT ANSWER: B

The insertion of a dielectric does not change the charge on the capacitor. Thus, the ratio of the potential differences is

$$\frac{V_d}{V_a} = \frac{Q/C_d}{Q/C_a} = \frac{C_a}{C_d} = \frac{\varepsilon_0 A/d}{\kappa \varepsilon_0 A/d} = \frac{1}{\kappa} = \frac{1}{8}$$

where the subscript *d* and *a* refer to dielectric and air respectively. (We have taken κ for air to be equal to 1.)

TRUE OR FALSE? If the air-dielectric capacitor had been a vacuum-dielectric type, the ratio V_d/V_a would have been much smaller than 1:8.

[a] CORRECT ANSWER: 0.16 j

The initial charges on the capacitor plates C_1 and C_2 are

$$Q_1 = C_1V$$

and

$$Q_2 = C_2V$$

respectively. When the capacitors are reconnected with plates of opposite sign together, some of the charge is neutralized and the net charge $Q = Q_1 - Q_2$ will be redistributed on the plates of capacitors C_1 and C_2. In terms of equivalent capacitance, $C = C_1 + C_2$, the energy stored in the final system E_f is

$$E_f = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{1}{2}\frac{[V(C_1 - C_2)]^2}{C_1 + C_2}$$

$$= \frac{1}{2}V^2\frac{(C_1 - C_2)^2}{C_1 + C_2}$$

$$= \frac{1}{2}\frac{180 \times 180 \times 900 \times 10^{-12}}{90 \times 10^{-6}}\ \text{j}$$

$$= 0.16\ \text{j}$$

TRUE OR FALSE? The net charge present in the system after reconnecting the capacitors as described is greater than it was just prior to reconnection.

[b] CORRECT ANSWER: B

If a dielectric slab is introduced into an electric field, the following relationship may be applied:

$$\frac{E_o}{E} = \frac{V_o}{V} = \kappa$$

where E_o and V_o represent the values of electric field and potential before the insertion of a dielectric.

Thus, $E = E_o/\kappa$. The original electric intensity is reduced by a factor of $1/\kappa$.

[a] CORRECT ANSWER: 0.25 j

The charging process is carried out by transferring small positive charges from the plate at lower potential to the plate at higher potential. During this process, when the total quantity of charge has reached q the potential difference between the plates has reached V, where q and V are related by

$$q = CV \qquad (1)$$

The symbol C in equation (1) is the capacitance of the parallel plates and is a constant depending upon the geometry and the nature of the dielectric. The work dW to transfer an element of charge dq is

$$dW = Vq \qquad (2)$$

Substituting the expression for V from equation (1) yields

$$dW = \frac{1}{C}\,qdq \qquad (3)$$

The total work done W when the charge on the capacitor plate reaches Q is

$$W = \frac{1}{C}\int_0^Q qdq$$

$$= \frac{1}{2}\,\frac{Q^2}{C} \qquad (4)$$

However, the given data is in terms of Q and V and not C. The expression for C in terms of Q and V is

$$C = \frac{Q}{V} \qquad (5)$$

Therefore, equation (4) becomes

$$W = \frac{1}{2}\,\frac{Q^2V}{Q} = \frac{1}{2}\,QV$$

$$= \frac{1}{2}\,(5 \times 10^{-3}\text{ coul}) \times 100\text{ volts}$$

$$= 0.25\text{ j}$$

TRUE OR FALSE? As equation (4) indicates, the work required to transfer a charge Q from one plate to another is inversely proportional to the capacitance between plates.

[a] CORRECT ANSWER: 0.054 j

Initial energy stored in capacitor C_1 is

$$E_i = \frac{1}{2} C_1 V^2 \quad (1)$$

The original charge $Q - C_1V$, will now be shared by the two capacitors so that

$$Q_1 + Q_2 = Q = C_1V \quad (2)$$

where Q_1 and Q_2 are the new charge distributions on the plates of C_1 and C_2, respectively. However, Q_1 and Q_2 may also be computed from

$$Q_1 = C_1V_f \quad \text{and} \quad Q_2 = C_2V_f \quad (3)$$

where V_f is the new potential difference across C_1 and C_2.

Combining equations (2) and (3) and solving for V_f we obtain

$$C_1V_f + C_2V_f = C_1V$$

or

$$V_f = \frac{C_1}{C_1 + C_2} V \quad (4)$$

The final stored energy E_f is

$$E_f = \frac{1}{2} C_1V_f^2 + \frac{1}{2} C_2V_f^2$$

Substituting equation (4) for V_f,

$$E_f = \frac{1}{2} (C_1 + C_2) \frac{C_1^2V^2}{(C_1 + C_2)^2}$$

or

$$E_f = \frac{1}{2} C_1 V^2 \frac{C_1}{(C_1 + C_2)} \quad (5)$$

Therefore, the difference in stored energy is

$$E_i - E_f = \frac{1}{2} C_1V^2 - \frac{1}{2} C_1V^2 \frac{C_1}{C_1 + C_2}$$

$$= \frac{1}{2} C_1V^2\left(1 - \frac{C_1}{C_1 + C_2}\right)$$

$$= 0.054 \text{ j}$$

next page

continued

Alternate Solution:

The final energy E_f may be written as

$$E_f = \frac{1}{2}\frac{Q^2}{C}$$

where C is the equivalent capacitance, that is $C = C_1 + C_2$ and Q is the charge on the plates of equivalent capacitor which in this case is the original charge $Q = C_1V$. Therefore,

$$E_f = \frac{1}{2}\frac{{C_1}^2V^2}{C_1 + C_2}$$

which is the same as equation (5) above.

[a] CORRECT ANSWER: .09 j

The energy E_f stored by the capacitor after immersion in oil may be calculated from

$$E_f = \frac{1}{2}\frac{Q^2}{C_f} \qquad (1)$$

where C_f is the capacitance of the capacitor when it is immersed in oil, that is

$$C_f = \kappa C \qquad (2)$$

and Q, the original charge on the capacitor, is given by

$$Q = VC \qquad (3)$$

Substituting expressions (2) and (3) in equation (1), we obtain

$$E_f = \frac{1}{2}\frac{V^2C^2}{\kappa C}$$

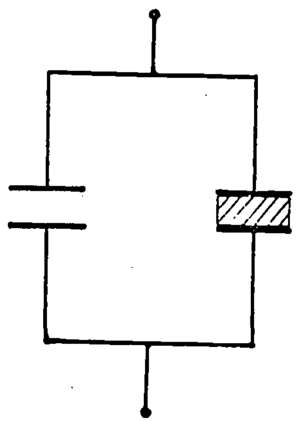
$$= \frac{1}{2}\frac{CV^2}{\kappa}$$

$$= 0.09 \text{ j}$$

TRUE OR FALSE? The amount of stored electrical energy decreases when a charged, isolated air capacitor is immersed in an oil dielectric.

[a] CORRECT ANSWER: 25 pf

We can consider the given capacitor as made up of two capacitors, one with the dielectric filling its gap. Since the plates of the given capacitor "short out" the respective sides of the two "halves," the arrangement is similar to the one shown here; i.e., two capacitors connected in parallel. The area of each circular plate is



$$A_o = \pi r^2 \quad (1)$$

and the area of each semicircular plate is

$$A = A_o/2 = \pi r^2/2 \quad (2)$$

Now for a parallel plate capacitor

$$C = \frac{\kappa\varepsilon_o A}{d} \quad (3)$$

where A is the area as given in (2), d is the separation of the plates and is the same for both halves, ε_o is the permittivity constant given by $\varepsilon_o = 8.85 \times 10^{-12}$ or

$$\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \quad (4)$$

and κ is the dielectric constant; namely $\kappa_1 \simeq 1$ and for the dielectric $\kappa_2 = 4$.

Using equations (2), (3) and (4), we obtain

$$C_1 = \frac{\kappa_1 \varepsilon_o \pi r^2/2}{d} = 4\pi\varepsilon_o \frac{\kappa_1 r^2}{8d}$$

$$= \frac{1}{9 \times 10^9} \times \frac{1 \times 36 \times 10^{-4}}{8 \times 10^{-2}} = 0.5 \times 10^{-11} \text{ f} = 5 \text{ pf} \quad (5)$$

For computing C_2 we must use $\kappa_2 = 4$, so the result will be

$$C_2 = 20 \text{ pf} \quad (6)$$

Recalling that for capacitors in parallel the capacitances are added arithmetically we obtain

$$C = C_1 + C_2 = 25 \text{ pf}$$

[a] CORRECT ANSWER: 1.5×10^{-3} j

The total charge in the capacitor system remains unaltered since C_2 is originally uncharged. Thus, the net charge on each plate of an equivalent capacitor C is

$$Q = C_1 V \tag{1}$$

The equivalent capacitance of the final system is

$$C = \kappa C_1 + C_2 \tag{2}$$

where κC_1 is the capacitance of the capacitor C_1 with the dielectric of dielectric constant κ added. Thus the final stored energy of the system may be obtained from

$$E_f = \frac{1}{2}\frac{Q^2}{C} \tag{3}$$

Substituting the values of Q and C from equations (1) and (2) into equation (3) yields

$$E_f = \frac{C_1{}^2 V^2}{2(\kappa C_1 + C_2)} = 1.5 \times 10^{-3} \text{ j}$$

TRUE OR FALSE? The final stored energy in the system is the same as the initial energy stored in C_1.