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ABSTRACT

Three review segments of the Self-Paced Physics Course materials are provided in this volume which is arranged to match study segments 1 through 14. Each of the three segments is composed of a set of problems and solutions, and accompanied by its own individual study guide. The problem set is designed as a back-referencing system, and the scrambling method is used in solution presentation. Directions for reaching solutions are revealed through the use of latent image study guides. The purpose of review problem sets is to help students in isolating and organizing essential physics concepts which are common to problem situations. Also included is a sheet of problem numbers with corresponding page numbers which locate correct answers. (Related documents are SE 016 065 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

DEVELOPED AND PRODUCED UNDER THE UNITED STATES NAVY CONTRACT #N00600-68C-0749 AND UNITED STATES OFFICE OF EDUCATION, BUREAU OF RESEARCH, PROJECT #8-0446, FOR THE UNITED STATES NAVAL ACADEMY

NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY, LONG ISLAND, NEW YORK 11568

Course Materials - Fall, 1970

UNITED STATES NAVAL ACADEMY

Self-Paced PHYSICS

SEGMENTS 15-17

DEVELOPED AND PRODUCED UNDER THE
U.S. OFFICE OF EDUCATION, BUREAU OF RESEARCH,
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[illegible] S[illegible]11 Self-Paced Physics [illegible]

[illegible] AND THEIR CORRECT ANSWER PAGES

Segments 15, 16 and 17

SEGMENT 15

Problem No.	Solution Page
1	18b
2	19b
3	7a
4	14a
5	9a
6	6a
7	16a
8	11a
9	13a
10	13a
11	8b
12	15a
13	10a
14	12b
15	8a
16	17a
17	12a

SEGMENT 16

Problem No.	Solution Page
1	13b
2	10a
3	15a
4	19b
5	9a
6	11a
7	20a
8	18b
9	14a
10	16b
11	16a
12	14b
13	17a
14	8a
15	20b
16	12a
17	18a
18	8b
19	15b
20	19a
21	17b
22	15a

SEGMENT 17

Problem No.	Solution Page
1	9b
2	12[illegible]
3	8a
4	[illegible]
5	13a
6	11a
7	14a
8	[illegible]
9	12b
10	15b
11	6a
12	15a
13	7a
14	8b
15	9a

P	STEP	NAME	P	STEP	SECTION SEGMENT 15
1		1) mi/hr (ans)	8		(ans)
		2) ft/sec (ans)	9		(ans)
		3) m/sec (ans)	10		(ans)
2		A B C D	11		(ans)
3		1) (ans)	12		A B C D
		2) (ans)	13		(ans)
4		(ans)	14		(ans)
5		(ans)	15		(ans)
6		(ans)	16		(ans)
7		(ans)	17		(ans)

STUDY GUIDE — SELF-PACED PHYSICS

P	STEP	NAME
1		a) ______ (ans)
		b) ______ (ans)
2		a) ______ (ans)
		b) ______ (ans)
3		______ (ans)
4		______ (ans)
5		______ (ans)

P	STEP	SECTION — SEGMENT 16
6		a) ______ (ans)
		b) ______ (ans)
7		______ (ans)
8		______ (ans)
9		a) ______ (ans)
		b) ______ (ans)
10		______ (ans)
11		______ (ans)

STUDY GUIDE

SELF-PACED PHYSICS

P	STEP	NAME	P	STEP	SECTION	SEGMENT 16

12

____________________(ans)

13

____________________(ans)

14

____________________(ans)

15

A B C D

16

____________________(ans)

17

____________________(ans)

____________________(ans)

18

____________________(ans)

19

____________________(ans)

20

a)____________________(ans)

b)____________________(ans)

21

____________________(ans)

22

____________________(ans)

STUDY GUIDE

SELF-PACED PHYSICS

P	STEP	NAME
1		______ (ans)
2		______ (ans)
3		______ (ans)
4		______ (ans)
		______ (ans)
5		______ (ans)
6		______ (ans)
		______ (ans)

SEGMENT 17

P	STEP	SECTION
7		______ (ans)
8		______ (ans)
9		A B C D
10		A B C D
11		A B C D
12		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 17
13		a) ______ (ans)			
		b) ______ (ans)			
		c) ______ (ans)			
14		A B C D			
15		______ (ans)			

PREFACE TO REVIEW SEGMENTS

This volume of your Self-Paced Physics course contains three Review Segments which have been carefully arranged to match corresponding sections of your PROBLEMS AND SOLUTIONS. You may find the grouping below helpful in organizing your review time.

Review Segment	Covers Topics in Study Segments
15	1 through 5
16	6 through 10
17	11 through 14

The review problems are numbered in sequence in each Segment. Previous study problem(s) to which each review problem relates is shown in parentheses following the review problem number.

For example, review problem 9 in Segment 15 identifies the related material as

9 (3-12; 1-10)

which means that the topical substance of review problem 9 contains concepts and operations which are also involved in both Segment 3, Problem 12 and Segment 1, Problem 10. In this particular case, problem 9 is concerned with projectile motion in common with problem 3-12, and with rectangular components of vectors in common with problem 1-10.

We recommend that you establish a pattern of review which will make full use of this back-referencing system. Despite the fact that there is almost an infinite number of ways to state a problem in a given field, the number of relevant concepts and operations are limited. By referring to the original problems before or after solving the review problem, you will profitably view essential operations and procedural sequences from several different vantage points. It will enable you to isolate and organize the essential concepts that are common to so many problem situations.

next page

continued

Each Segment is accompanied by its own individual STUDY GUIDE to be retained in your files as usual. And, as in the learning Segments, the problems are in numerical order but the solutions are scrambled so that the latent image STUDY GUIDE must be used to locate the solution in which you are interested. You will at once observe, however, that the STUDY GUIDES for the Review Segments differ from those you have used previously in that there are no references to Information Panels, Audiovisuals, assigned reading, or homework. All review problems are to be solved by everyone: there are no alternative paths. Finally, no provision is made for true-false follow-up questions or answers in the solutions or STUDY GUIDE.

Completed STUDY GUIDES should be submitted for evaluation to your instructor in accordance with previously established procedures. These will be returned to you after they have been examined.

Good luck.

1 (1-1). A car moving on a straight road covers a distance of 0.7500 mi in exactly 90.0 sec. Compute the average speed of the car (write your answer with the correct number of significant figures)

1) in mi/hr
2) in ft/sec
3) in m/sec

2 (1-13). The speed of a particle in meters per second is given by $v = a + bt$. The units of constants a and b are

A. a = m/sec b = m/sec^2

B. a = m b = sec

C. a = sec/m b = sec^2/m

D. a = m/sec b = m/sec

3 (2-6, 2-1). The magnitudes of vectors $\vec{P}$ and $\vec{Q}$ in a given system of units are 20 and 40 respectively, and the angles which the vectors make with the positive direction of the x-axis are 15° and 75° respectively.

1) What is the scalar product of these two vectors?
2) What is the magnitude of their cross (vector) product?

4 (2-10, 1-6). A train moving at a constant speed of 60 mi/hr moves eastward for 45 min, then in a direction 37° north of east for 15 min, and finally westward for 30 min. What is the average velocity of the train during this run?

5 (2-14). A particle moves along a straight line with a time-dependent velocity given by

$$v = \alpha t + \beta t^2$$

If in 1 sec the particle has traveled 2 m and at the end of the 1st second its speed is 5 m/sec, how far does the particle travel in 5 sec?

6 (2-17). A man wishes to cross a river 500 m wide. His rowing speed relative to the water is 3.0 km/hr. The river flows at a speed of 2 km/hr. The man's walking speed on shore is 5.0 km/hr. If the man wishes to reach a point directly opposite his starting point, at what angle to the line connecting start and finish must he head the boat in order to reach his destination in minimum time?

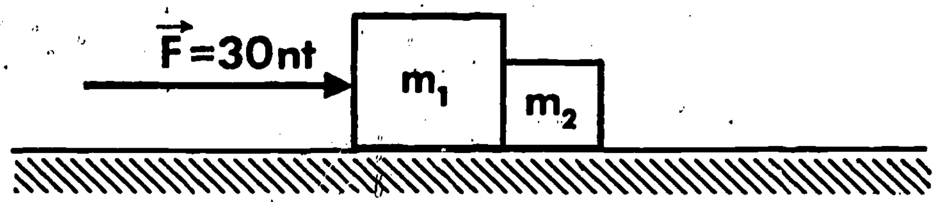
7 (3-6). A stone is dropped into the water from a bridge 144 ft high. Another stone is thrown vertically down from the same bridge 1 sec after the first is dropped. Both stones strike the water at the same time. Calculate the initial speed of the second stone.

8 (3-9, 3-1). After jumping, a parachutist falls 50 m with his chute unopened. (Assume zero air resistance.) When the parachute opens, he decelerates at 2.0 m/sec^2. He reaches the ground with a speed of 3.0 m/sec. Find the time the parachutist is in the air.

9 (3-12, 1-10). A projectile is fired from a point on a flat plane and just clears a 25-ft fence a distance x_1 ft away. If $x_1 = 125$ ft and $\theta_0 = 45°$, find the initial speed.

10 (3-18). A particle is projected from the origin with an initial velocity of 40 m/sec at 37° above the horizontal. At the same time a second particle is projected from the point (100 m, 0) with an initial velocity $\vec{v}_o$. After 4.55 seconds the two particles collide in midair. What is $\vec{v}_o$?

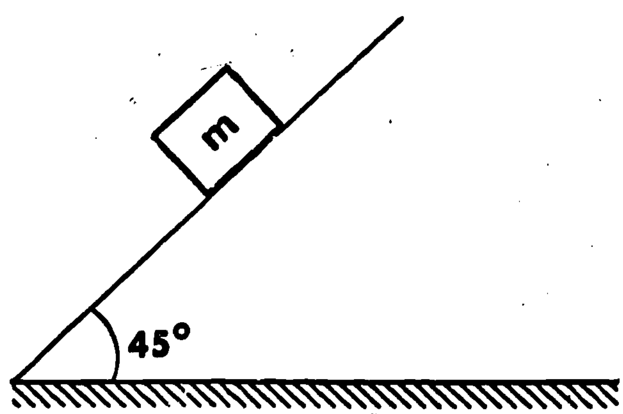
11 (5-1, 4-16). Two blocks of masses m_1 = 10 kg and m_2 = 5 kg are in contact on a frictionless table as shown in the diagram.



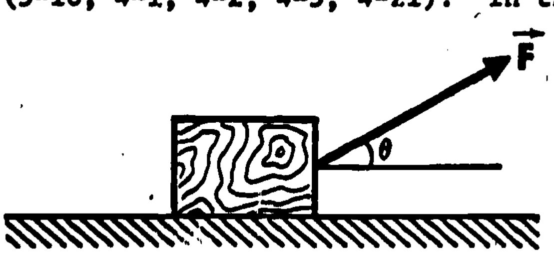
A horizontal force of 30 nt is applied to the block of mass m_1. Find the force acting on mass m_2.

12 (5-5, 4-32). A block of mass m is sliding down a plane inclined at 45° to the horizontal. If the acceleration of the block is 2.0 m/sec^2, find the coefficient of kinetic friction.

A. 0.81
B. 1.1
C. 0.71
D. 0.51

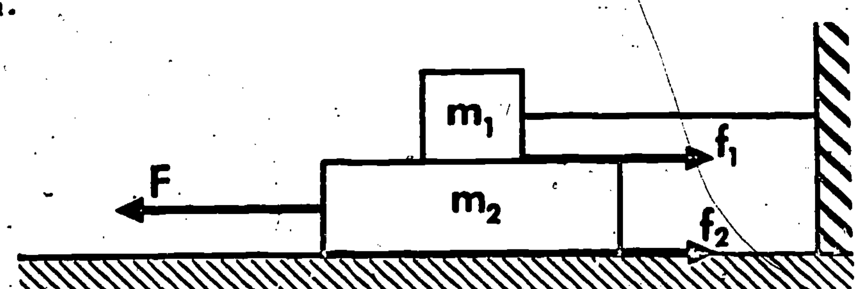


13 (5-10, 4-1, 4-2, 4-5, 4-21). In the figure, a force $\vec{F}$ is used to pull the box at constant speed along a horizontal surface that has a coefficient of kinetic friction μ_k = 0.60. What is the angle θ at which the magnitude F of the required force is minimum?



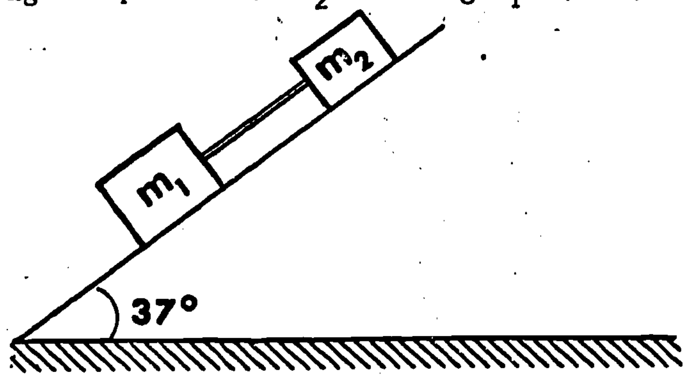
14 (5-11). What is the minimum value of acceleration of an airplane on a runway that will cause a 30-kg block placed against the rear vertical wall of the plane to stick there? The coefficients of static and kinetic friction are 0.40 and 0.30, respectively, for all surfaces inside the plane.

15 (5-12). A block of mass $m_1 = 2.0$ kg is attached to the wall by a string and lies on top of a block of mass $m_2 = 6.0$ kg as shown in the diagram.



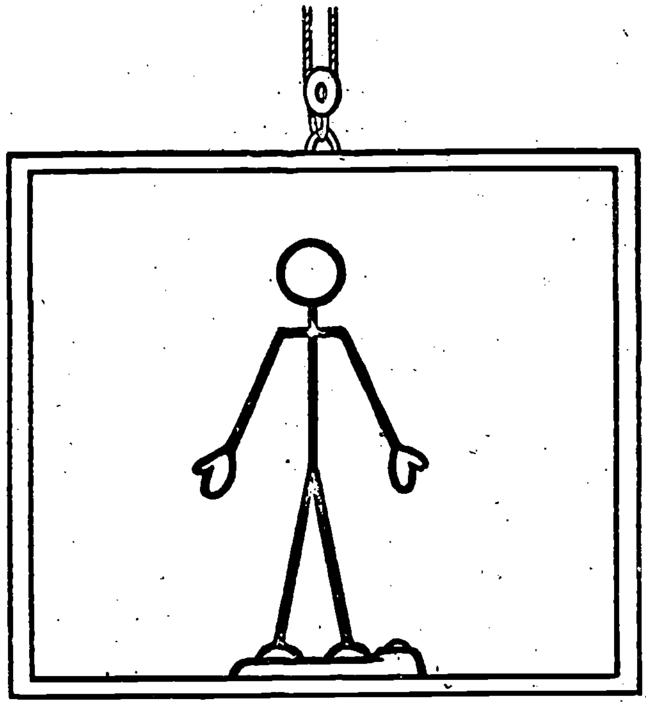
The coefficient of static friction between the blocks is 0.60 and that between the block of mass m_2 and the floor is 0.40. What is the minimum magnitude of the horizontal force F that will cause m_2 to start moving?

16 (5-13, 5-2, 4-6). Two masses $m_1 = 4.0$ kg and $m_2 = 2.0$ kg, attached by a rod of negligible mass parallel to the incline on which both slide, travel down along the plane with m_2 trailing m_1. (See diagram.)



The angle of the incline is $\theta = 37°$. The coefficient of kinetic friction between m_1 and the incline is $\mu_1 = 0.10$, between m_2 and the incline is $\mu_2 = 0.20$. Find the common acceleration of the two masses.

17 (5-18) A man stands on a scale in an elevator. The elevator initially [illegible]nward at a constant speed; then it starts decelerating at 4.0 ft/sec^2. What is the reading of the scale during the deceleration?



[a] CORREC… …WER: 25.4°

… us …st solve the problem symbolically. The symbols used are:

$v_{RE} = v_o$ = speed of the river relative to the Earth

$v_{BW} = v_b$ = speed of the boat relative to the water

$v_{ME} = u$ = speed of the man relative to the Earth

d = width of the river

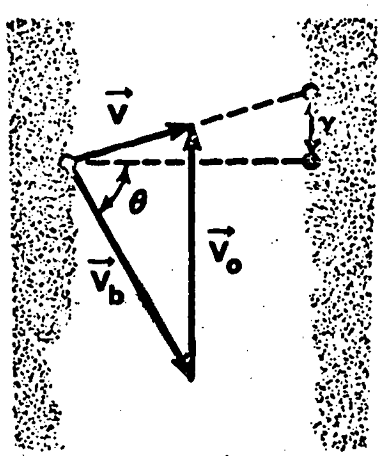
Let θ be the angle at which the boat heads. The components of the resultant velocity v_x and v_y are

$$v_x = v_b \cos\theta \qquad (1)$$

$$v_y = v_o - \ \sin\theta \qquad (2)$$

Therefore, the time t_1 the boat takes t… cross the river is

$$t_1 = \frac{d}{v_x} = \frac{d}{\ \cos\theta} \qquad (3)$$



The distance y (this is the distance downstream between the man's landing point and his destination) is given by

$$y = v_y t_1$$

$$= (v_o - v_b \sin\theta)\ t_1 \qquad (4)$$

The time, t_2, required to walk back to his destination is

$$t_2 = \frac{y}{u}$$

Substitution of values for y and t_1 from equations (4) and (3) yields

$$t_2 = \frac{(v_o - v_b \sin\theta)\ d}{u v_b \cos\theta} \qquad (5)$$

To obtain total time T, we add t_1 and t_2. Therefore,

$$T = t_1 + t_2 = \frac{d}{v_b}\left(1 + \frac{v_o}{u}\right)\frac{1}{\cos\theta} - \frac{d}{u}\tan\theta \qquad (6)$$

Note: In equation (5) the quantity $v_o - v_b \sin\theta$ must be positive.

next page

continued

To minimize the function, we differentiate T with respect to θ and equate to zero.

$$\frac{dT}{d\theta} = \frac{d}{v_b}\left(1 + \frac{v_o}{u}\right)\sec\theta \tan\theta - \frac{d}{u}\sec^2\theta = 0$$

or

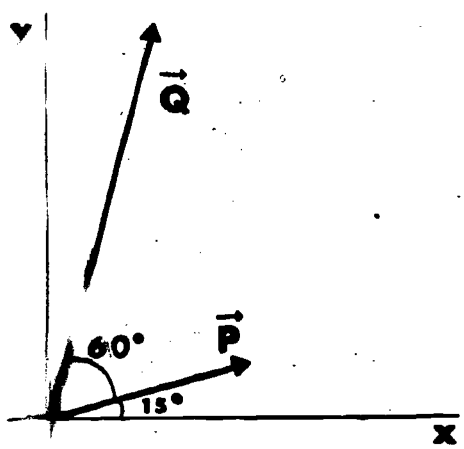
$$\sin\theta = \frac{v_b}{u + v_o} \qquad (7)$$

$$\sin\theta = \frac{3.0}{7.0}$$

Therefore $\theta = 25.4°$

[a] CORRECT ANSWER: 1) 400 2) 693

1) The given vectors are oriented as shown in the diagram. Note that the angle between $\vec{P}$ and $\vec{Q}$ is 60° so the scalar product is given by



$$\vec{P}\cdot\vec{Q} = PQ\cos\theta$$

$$= (20)(40)(\cos 60°)$$

$$= 400$$

2) The cross product is a vector of magnitude

$$|\vec{P}\times\vec{Q}| = PQ\sin\theta = (20)(40)(0.866)$$

$$= 693$$

[a] CORRECT ANSWER: 43 nt

If m_2 is to start moving, the applied force F must be greater than the sum of frictional forces, i.e.,

$$F > f_1 + f_2$$

where

$$f_1 = \mu_{s1}N_1 = \mu_{s1}m_1g$$

$$f_2 = \mu_{s2}N_2 = \mu_{s2}(m_1 + m_2)\ g$$

Thus the minimum necessary force is

$$F = f_1 + f_2 = \mu_{s1}m_1g + \mu_{s2}(m_1 + m_2)\ g$$

$$= 43.1 \text{ nt} = 43 \text{ nt}$$

[b] CORRECT ANSWER: 10 nt

First, we treat the blocks as a group to find the acceleration; then we isolate the block of mass m to calculate the force of contact. Since the force of friction is ignored, the sum of the forces in the x-direction is

$$\sum F_x = F = (m_1 + m_2)\ a$$

Therefore

$$a = 2 \text{ m/sec}^2$$

Now for the block of mass m_2 to accelerate at 2 m/sec^2, an unbalanced force must act on it. Therefore,

$$\sum F_x = F = m_2a$$

$$= 10 \text{ nt}$$

the force acting on mass m_2.

[a] CORRECT ANSWER: 150 m

From the velocity equation,

$$v = \alpha t + \beta t^2 \qquad (1)$$

the displacement is

$$s = \int v\,dt = (\alpha t + \beta t^2)\,dt = \frac{1}{2}\alpha t^2 + \frac{1}{3}\beta t^3 \qquad (2)$$

Setting t = 1 sec in equations (1) and (2) and using the given data we obtain

$$2 = s\ (\text{at } t = 1) = \frac{1}{2}\alpha + \frac{1}{3}\beta$$

$$5 = v\ (\text{at } t = 1) = \alpha + \beta$$

Solving the system of equations we obtain

$$\alpha = 2 \qquad \text{and} \qquad \beta = 3$$

Therefore, at t = 5 sec, the displacement is

$$s\ (\text{at } t = 5) = \frac{1}{2}(2)(25) + \frac{1}{3}(3)(125) = 150 \text{ m}$$

[b] CORRECT ANSWER: A

As stated in the problem, the speed is measured in meters per second. Every term in the expression for speed must have the same units.

Therefore,

$$a = \text{m/sec}$$

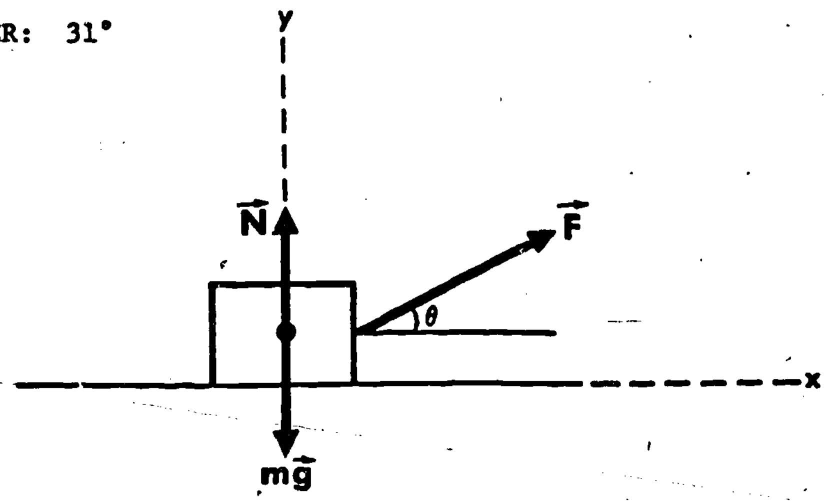
and

$$bt = \text{m/sec}$$

or

$$b = \text{m/sec}^2$$

[a] CORRECT ANSWER: 31°



When the box is moving at a constant speed, the horizontal and vertical components of the net force on the box must be zero:

$$F_x = F\cos\theta - f = 0 \qquad (1)$$

$$F_y = F\sin\theta + N - mg = 0 \qquad (2)$$

and

$$f = \mu_k N \qquad (3)$$

Solving for F, we obtain

$$F = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta} \qquad (4)$$

To minimize F, we put $dF/d\theta = 0$, i.e.,

$$\frac{dF}{d\theta} = -\frac{\mu_k mg}{(\cos\theta + \mu_k \sin\theta)^2}(-\sin\theta + \mu_k \cos\theta) = 0$$

or

$$\mu_k = \tan\theta$$

Thus $\theta = \tan^{-1}0.6 = 31°$

[a] CORRECT ANSWER: 17 sec

The speed when the parachute opens and the time t_1 during free fall are

$$v = \sqrt{2gs} \quad (1)$$

where $s = 50$ m, and

$$t_1 = \frac{v}{g} \quad (2)$$

respectively.

The time t_2 required for the remaining part of the trip may be obtained from

$$v_f = v - at_2 \quad (3)$$

where $v_f = 3.0$ m/sec and $a = 2.0$ m/sec^2.

Therefore

$$t_2 = \frac{v - v_f}{a}$$

$$= \frac{\sqrt{2gs} - v_f}{a}$$

$$= 14.2 \text{ sec}$$

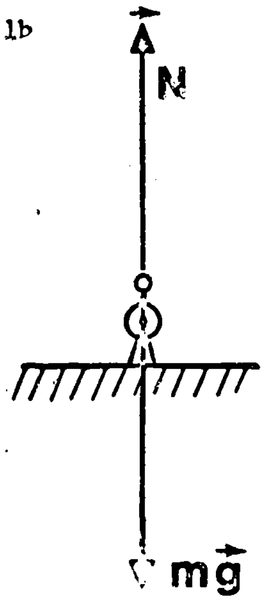
and

$$t_1 = \frac{\sqrt{2gs}}{g}$$

$$= 3.2 \text{ sec}$$

Therefore, total time $= t_1 + t_2 = 17.4$ sec ≈ 17 sec

[a] CORRECT ANSWER: 180 lb



A free-body diagram shows all the forces on the man. The force of gravity, $\vec{w}$, is directed downward. The reaction force, $\vec{N}$, of the scale-platform on the man (which is equal in magnitude to the reading of the scale) is directed upward.

Now, deceleration in the downward direction is equivalent to positive (upward) acceleration. Thus,

$$\vec{N} + \vec{w} = m\vec{a}$$

or

$$N - mg = ma,$$

and

$$\text{Reading of the scale} = N = m(g + a) = \frac{160}{32}(32 + 4.0) = 180 \text{ lb}$$

[b] CORRECT ANSWER: 25 m/sec^2

In order for the block to remain stationary, the frictional force must be at least equal to its own weight:

$$f = \mu N = mg$$

or

$$\mu ma = mg$$

Thus,

$$a = \frac{g}{\mu} = \frac{9.8}{0.40} = 24.5 \text{ m/sec}^2 = 25 \text{ m/sec}^2$$

[a] CORRECT ANSWER: 26 m/sec and $\theta = 67°$

At the time of collision, both particles have the same coordinates, i.e.,

$$\left.\begin{array}{l} x_1 = x_2 \\ y_1 = y_2 \end{array}\right. \quad \text{at } t = 4.55 \text{ sec} \qquad (1)$$

The equations for coordinates are

$$x_1 = v_1 \cos\theta_1 \; t \qquad (2)$$

$$y_1 = v_1 \sin\theta_1 \; t - \frac{1}{2} g t^2 \qquad (3)$$

and

$$x_2 = x_o + v_2 \cos\theta_2 \; t \qquad (4)$$

$$y_2 = v_2 \sin\theta_2 \; t - \frac{1}{2} g t^2 \qquad (5)$$

From (1), (3) and (5), we get

$$v_2 \sin\theta_2 = v_1 \sin\theta_1 \qquad (6)$$

and from (1), (2) and (4)

$$v_2 \cos\theta_2 = v_1 \cos\theta_1 - \frac{x_o}{t} \qquad (7)$$

From (6) and (7)

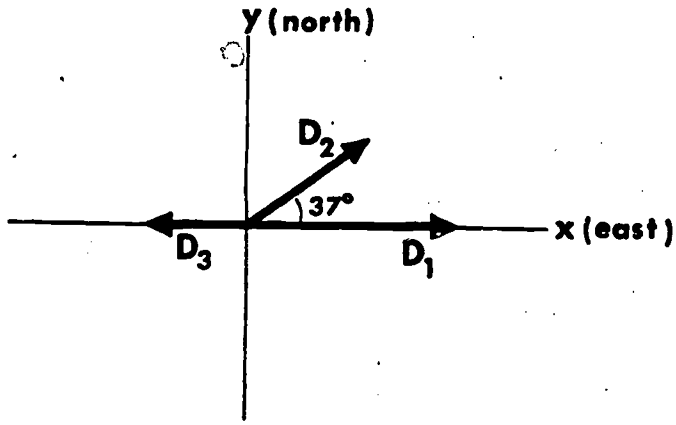
$$v_2 = \sqrt{v_1{}^2 - 2v_1 \cos\theta_1 \frac{x_o}{t} + \frac{x_o{}^2}{t^2}} = 26 \text{ m/sec}$$

and

$$\tan\theta_2 = \frac{v_1 \sin\theta_1}{v_1 \cos\theta_1 - \frac{x_o}{t}} = 2.4$$

$$\theta_2 = 67.4° \approx 67°$$

[a] CORRECT ANSWER: 19 mi/hr, 19° north of east



The average velocity is

$$\vec{\bar{v}} = \frac{\sum \vec{D}_i}{\sum t_i} = \frac{\vec{D}}{t} \qquad (1)$$

where

$$D = \sqrt{D_x^{\,2} + D_y^{\,2}} \qquad (2)$$

and

$$D_x = D_{1x} + D_{2x} + D_{3x}$$
$$= v_o t_1 + v_o t_2 \cos 37° - v_o t_3$$
$$= 27 \text{ mi}$$

$$D_y = D_{1y} + D_{2y} + D_{3y}$$
$$= 0 + v_o t_2 \sin 37° + 0$$
$$= 9 \text{ mi}$$

Therefore

$$D = 28.4 \text{ mi}$$

and

$$t = t_1 + t_2 + t_3$$
$$= 1.5 \text{ hr}$$

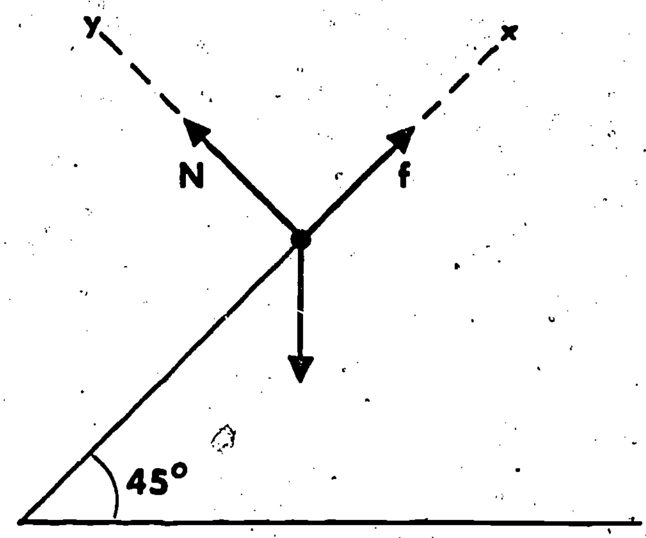
$$\bar{v} = 18.9 \text{ mi/hr} = 19 \text{ mi/hr}$$

and makes an angle of

$$\tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{1}{3}\right)$$

with the easterly direction; in other words, makes an angle of 19° with the easterly direction.

[a] CORRECT ANSWER: C



The sums of the forces in the x and y directions are

$$\sum F_x = \mu_k N - mg\ \sin 45° = -ma$$

and

$$\sum F_y = N - mg\ \cos 45° = 0$$

where $\mu_k N = f$, the force of kinetic friction.

Solving the simultaneous equations for μ_k, one obtains

$$uk = \frac{g\ \sin 45° - a}{g\ \cos 45°}$$

$$= \frac{(9.8\ m/sec^2)(0.707) - 2\ m/sec^2}{(9.8\ m/sec^2)(0.707)}$$

= 0.71, the coefficient of kinetic friction

Note that the coefficient can be calculated without reference to the mass of the block.

[a] CORRECT ANSWER: 40 ft/sec

Let t be the time required for the first stone to reach the water. Then

$$144 = \frac{1}{2} gt^2 \quad (1)$$

For the second stone, we have

$$144 = v_o(t - 1) + \frac{1}{2} g(t - 1)^2 \quad (2)$$

From (1) and (2), we obtain

$$\frac{1}{2} gt^2 = v_o(t - 1) + \frac{1}{2} g(t - 1)^2$$

or

$$v_o = \frac{gt - \frac{1}{2} g}{t - 1} \quad (3)$$

Solving for t in (1), we obtain

$$t = 3 \text{ sec}$$

Thus,

$$v_o = 40 \text{ ft/sec}$$

[b] CORRECT ANSWER: 1) 30.0 mi/hr 2) 44.0 ft/sec 3) 13.4 m/sec

1) From $\bar{v} = \Delta s/\Delta t$ we find

$$\bar{v} = \frac{0.75}{90} \frac{\text{mi}}{\text{sec}} \times 3600 \frac{\text{sec}}{\text{hr}} = 30 \text{ mi/hr}$$

The distance was given in 4 significant figures and the time in 3. Therefore, the answer must be written in 3 significant figures; i.e.,

$$\bar{v} = 30.0 \text{ mi/hr}$$

2) Using the relationship 60 mi/hr = 88 ft/sec we find

$$\bar{v} = 44.0 \text{ ft/sec}$$

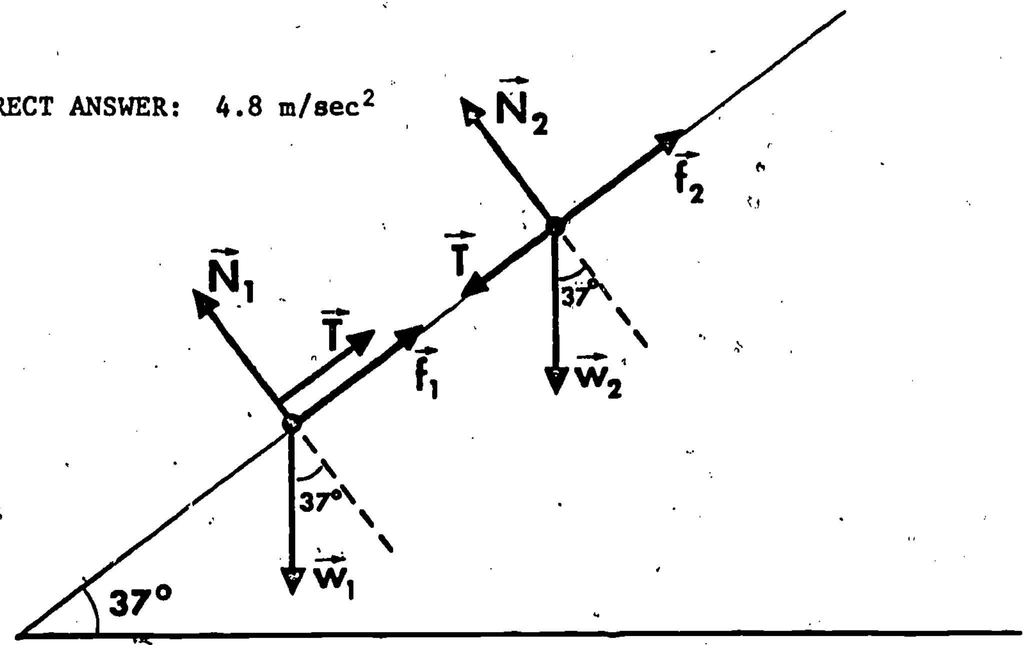
3) In the MKS system

$$\bar{v} = 44.0 \frac{\text{ft}}{\text{sec}} \times 12.0 \frac{\text{in}}{\text{ft}} \times 2.54 \frac{\text{cm}}{\text{in}} \times \frac{1}{100} \frac{\text{m}}{\text{cm}}$$

or

$$\bar{v} = 13.4 \text{ m/sec}$$

[a]. CORRECT ANSWER: $4.8\ m/sec^2$



Mass m_1:

The equations of motion in the directions parallel to and normal to the incline are

$$w_1 \sin\theta - T - f_1 = m_1 a \qquad (1)$$

and

$$N_1 = w_1 \cos\theta$$

where

$$f_1 = \mu_1 N_1 = \mu_1 w_1 \cos\theta \qquad (2)$$

Substituting the equivalent of f_1 and solving equation (1) for T, we obtain

$$T = w_1 \sin\theta - \mu_1 w_1 \cos\theta - m_1 a \qquad (3)$$

Mass m_2:

$$\sin\theta + T - f_2 = m_2 a \qquad (4)$$

and

$$N_2 = w_2 \cos\theta$$

where

$$f_2 = \mu_2 N_2 = \mu_2 w_2 \cos\theta \qquad (5)$$

Solving equation (4) for T after substituting the equivalent of f_2 from equation (5), we find that

$$T = -w_2 \sin\theta + \mu_2 w_2 \cos\theta + m_2 a \qquad (6)$$

next page

continued

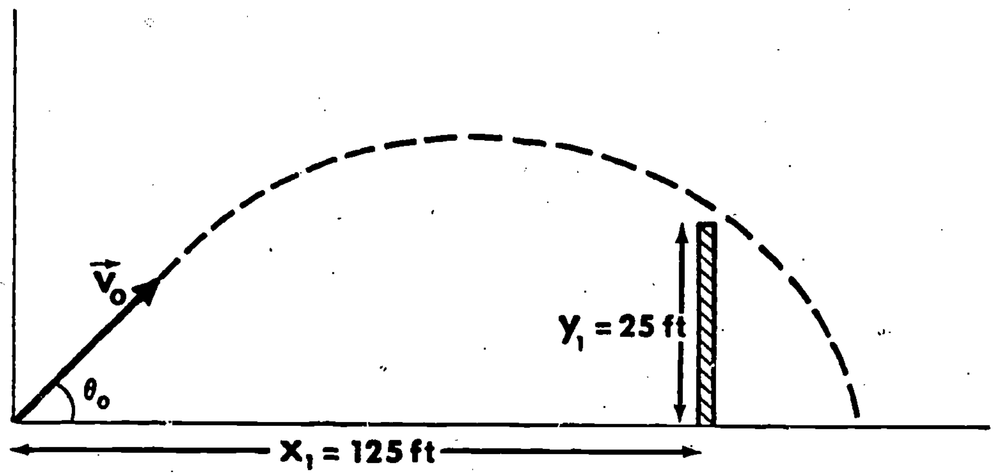
Equating equations (3) and (6) and solving for a, it is seen that

$$a = \frac{1}{m_1 + m_2}\left[(w_1 + w_2)\ \sin\theta - (\mu_1 w_1 + \mu_2 w_2)\ \cos\theta\right]$$

$$= \frac{g}{m_1 + m_2}\left[(m_1 + m_2)\ \sin\theta - (\mu_1 m_1 + \mu_2 m_2)\ \cos\theta\right]$$

$$= 4.8\ \text{m/sec}^2$$

[a] CORRECT ANSWER: 71 ft/sec



$$x_1 = v_{ox} t_1$$

$$= v_o(\cos\theta_o)t_1 \qquad (1)$$

where t_1 is the time the projectile takes to reach point (x_1, y_1), and

$$y_1 = v_{oy}t_1 - \frac{1}{2}\,g t_1^{\,2}$$

$$= v_o(\sin\theta_o)t_1 - \frac{1}{2}\,g t_1^{\,2} \qquad (2)$$

Solving equation (1) for t_1 and substituting the value obtained in equation (2) yields

$$y_1 = x_1\ \tan\theta_o - \frac{1}{2}\,g\,\frac{x_1^{\,2}}{v_o^{\,2}\ \cos^2\theta_o} \qquad (3)$$

Therefore,

$$v_o = \left[\frac{(1/2)g x_1^{\,2}}{(x_1\ \tan\theta_o - y_1)\ \cos^2\theta_o}\right]^{1/2}$$

$$= 70.7\ \text{ft/sec} = 71\ \text{ft/sec}$$

D STATES
-Paced
Physics
ACADEMY

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

1 (6-8, 6-2). A 2.0-kg sphere is tie[illegible] one end of a string 1-meter long. The sphere is whirled around in a horizontal circle with an angular speed of 2.0 rad/sec.

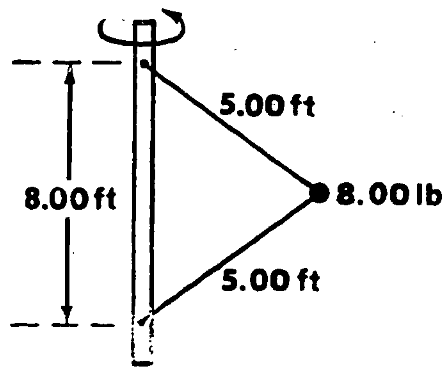
a. Calculate the magnitude of the sphere's acceleration.

b. Calculate the tension in the string.

2 (6-9, 6-1, 4-29, 4-26). The 8.00 lb particle in the figure below is attached to a vertical rod by means of two strings of equal length--each 5.00 ft. When the system rotates about the axis of the rod the strings become extended as shown.

a. How many revolutions per minute must the system make in order for the tension in the upper cord to be 15.0 lb?

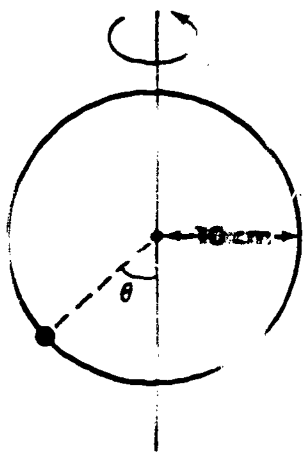
b. What is then the tension in the lower cord?



3 (6-14). A highway exit is a circular curve of radius 300 ft. If the road is level, what minimum coefficient of friction between the automobile tires and the road will prevent skidding at 30 mph?

[illegible] (6-15). The pilot of an airplane diving vertically at a velocity of [illegible] mi/hr pulls out of the dive by changing his course to a circle in a vertical plane. What is the minimum radius of the circle in feet which will insure that the acceleration at the lowest point will not exceed 7 g?

5 (6-16) bead can slide without friction on a circular hoop of radius 10 cm in a vertical plane. The hoop rotates at a constant rate of 2.0 rev/sec about the vertical diameter shown in the diagram. Find the angle θ at which the bead is in equilibrium.



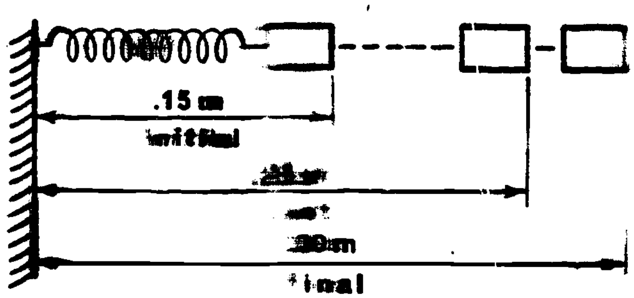
6 (7-2, 7-1). A man pushes a 60-lb block along a level floor with a force directed 45° below the horizontal. If the coefficient of kinetic friction is 0.20

(a) how much work must the man do in order to move the block through 30 ft at constant speed?

(b) How much work is done by the normal force

7 (7-5). The magnitude of the force in pounds required to stretch a certain spring a distance of x feet beyond its unstretched length is given by $F = 10x$. How much work is required to stretch the spring by 1.0 ft?

8 (7-9). A 1.0-kg block is attached to a spring on a frictionless horizontal surface. The spring when unstretched has a length of 0.25 m (including the block) and its force constant is 10 nt/m. What is the work done in stretching the spring when the block is moved from $x_1 = 0.15$ m to $x_2 = 0.30$ m?



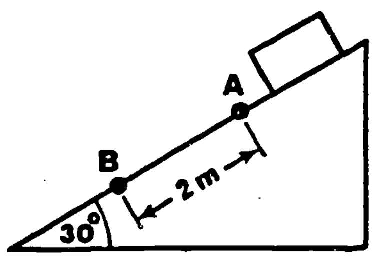
9 (7-10). A constant horizontal force o[illegible]nt is used to push a 50-kg block along a horizontal floor.

(a) How much work does the force do [illegible] moving the block through 10 m?

(b) If the block moves at a constant [illegible]eed of 2 m/sec, what is the power delivered?

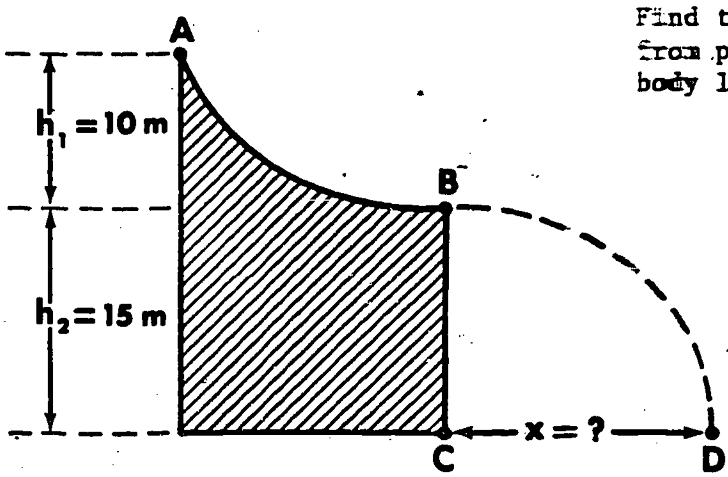
10 (7-15). A particle of mass m is pr[illegible]ected from ground level at an angle θ to the horizontal with an ini[illegible] speed of v_0. The kinetic energy of the particle at the highest point [illegible] that at the ground level. Find the angle of projection. (Neglec[illegible] resistance.)

11 (7-24, 8-1). A 4-kg block slides [illegible] 30° inclined plane. The speed of the block at point A is 1 m/sec, and at B it is 3 m/sec. If the distance from A to B is 2 m, what is the work done by friction between A and B?

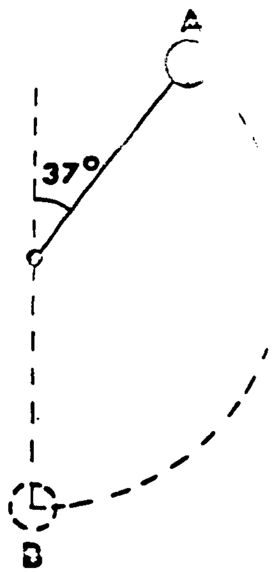


12 (7-27). A 5.00-kg block starts to slide up an ~~inclined plane with~~ a speed of 40.0 m/sec at the lowest point. The block stops momentarily near the top of the incline and then slides down again. The angle of the incline is 37°. If, on return, the speed of the block at the lowest point on the incline is 30.0 m/sec, how much energy is dissipated due to friction?

13 (8-5, 7-18). A 3-kg body at rest at point A slides down a smooth track as shown in the diagram. Find the horizontal distance x from points C to D where the body lands.



14 (8-18) A 40-kg ball is attached to one end of a rigid, massless rod. The other end of the rod is connected to a frictionless pivot. The ball is released from rest from position A as shown in the diagram. What is the tension in the rod in newtons when the ball swings through the lowest point (B)?



15 (8-9). For a force

$$F(x) = -\frac{K}{x^7}$$

where K is a constant and $U(\infty) = 0$, what is the potential energy $U(x)$ of a particle located at an arbitrary point x?

A. $-\frac{K}{x^7}$

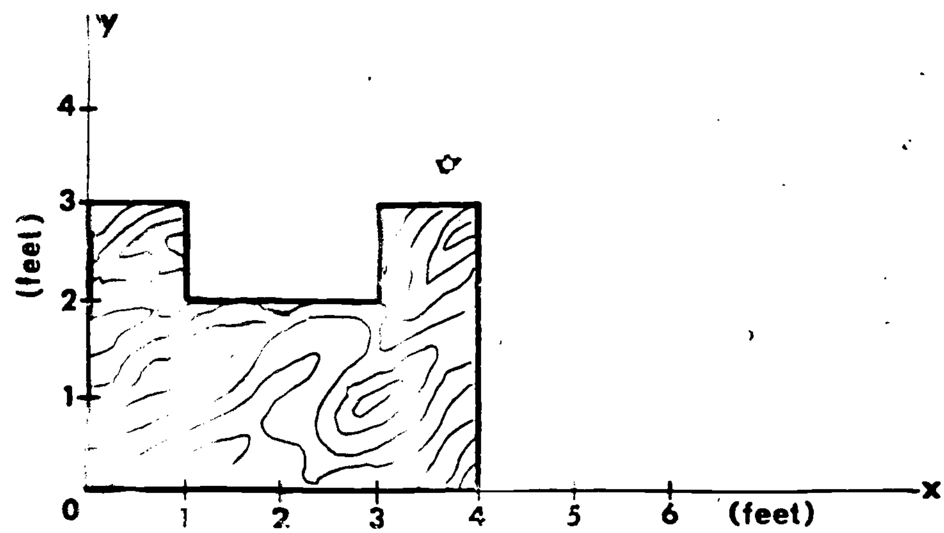
B. $-\frac{K}{6}\frac{1}{x^6}$

C. $\frac{K}{6}\frac{1}{x^6}$

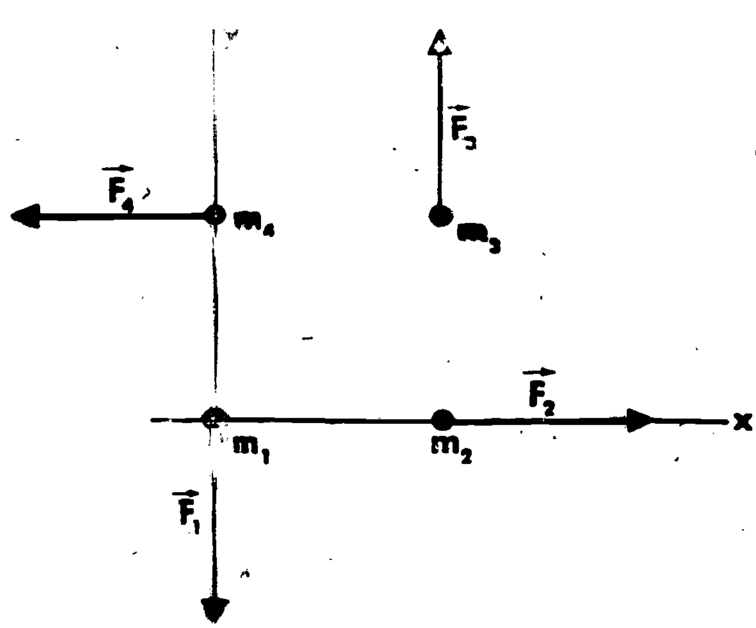
D. $-\frac{K}{7}\frac{1}{x^6}$

16 (8-13). A 0.. .. block is presse against, but not attached to, a light spring having a spring const... ≈ 1.2×10^3 nt/m. When the spring has been compressed 0.05 m ... e block is released. The block slides along a frictionless incli... surface. What maximum height does the block reach before sliding ba... down the plane?

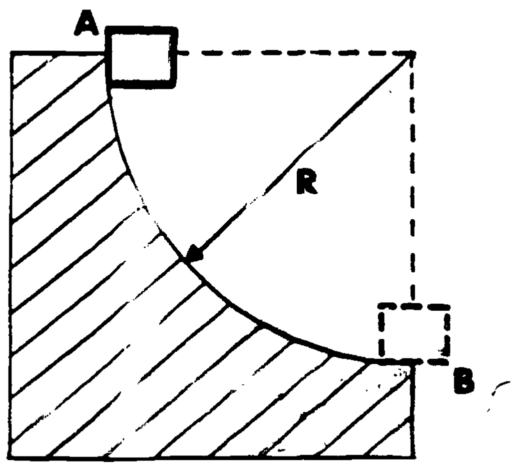
.. (9-4). A piece of 3/4-inch plyw... has been cut into the shape shown. If uniform mass density and thickness are assumed for this piece of wood, find the coordinates of the center ... mass.



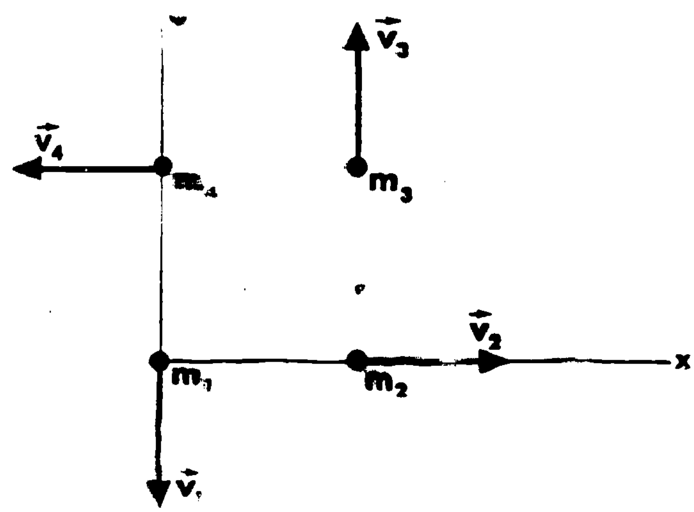
18 (9-6) ...our particles with respective masses $m_1 = m_2 = 3$ kg and $m_3 = m_4 = 1$ kg occupy the four corners of a 4 m × 4 m square as shown in the diagram. Each particle is subject to a 15-nt force ... the direction shown ... the diagram. Calculate the acceleration of the center of mass of this system.



19 (10-1). A body with a mass of 3 kg slides down a curved track which is one quadrant of a circle of radius m. If the track is frictionless and the block starts from rest, what is the momentum of the block at the bottom of the track.



20 (10-5; 9-1). Four particles, each of mass 3 kg, occupy the four corners of a 4 m 4 m square as shown in the diagram. Each particle is moving with a speed of 10 m/sec in the direction shown in the diagram.



(a) Locate the coordinates of the center of mass of the four-particle system in the coordinate system shown in the diagram.

(b) Calculate the velocity of the center of mass of this system.

21 (10-10). A force of constant direction given by

$$F = \frac{1}{2} kt^2 \text{ nt} \qquad (k = 8 \text{ nt/sec}^2)$$

is exerted on a 2-kg particle which is initially moving at a speed of 10 m/sec. Find the momentum of the particle at the end of 3 seconds.

22 (10-13). A nucleus, originally at rest, decays radioactively by emitting an electron of momentum 9.22×10^{-16} gm-cm/sec, and at right angles to the direction of the electron a neutrino with momentum 5.33×10^{-16} gm-cm/sec. What is the magnitude of the momentum of the residual nucleus?

[a] CORRECT ANSWER: 1800 nt

In addition to counteracting the weight of the ball, mg, the rod must provide the required centripetal force, mv^2/ℓ, where ℓ is the distance from the pivot to the center of the ball. Thus, the tension in the rod will be

$$T = mg + \frac{mv^2}{\ell} \qquad (1)$$

We can use conservation of energy to eliminate the speed v from equation (1). The initial position of the ball is

$$h = \ell + \ell \cos 37° = 1.8\ell \qquad (2)$$

above the lowest point B. If B is taken as the zero potential energy reference, then its potential energy at A is $mgh = 1.8\ mg\ell$. This must be equal to the kinetic energy of the ball at B. Thus,

$$1.8\ mg\ell = (1/2)mv^2$$

or

$$mv^2 = 3.6\ mg\ell$$

Substituting this in (1) we obtain

$$T = mg + 3.6\ mg = 4.6\ mg$$

$$= 4.6 \times 40 \times 9.8 = 1800\ \text{nt}$$

[b] CORRECT ANSWER: Zero

The sum of the four forces is

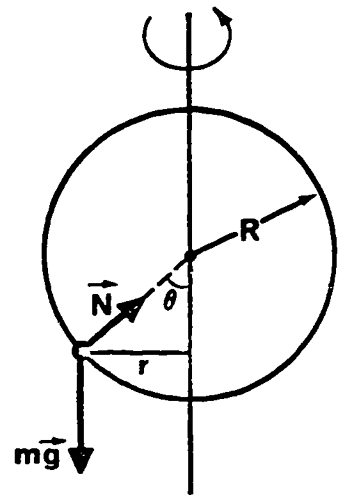
$$\vec{F} = \sum_i \vec{F}_i = -15\ \hat{j} + 15\ \hat{i} + 15\ \hat{j} - 15\ \hat{i} = 0$$

Therefore, the acceleration of the center of mass is

$$\vec{a}_{cm} = 0$$

Each particle is accelerating in the direction of the force exerted on it, but the directions and magnitudes of the four accelerations are such that the center of mass remains stationary.

[a] CORRECT ANSWER: 52°



The equations of motion in horizontal and vertical directions are, respectively

$$N \sin\theta = m \frac{v^2}{r} \qquad (1)$$

where v is the linear velocity of the bead, and

$$N \cos\theta - mg = 0 \qquad (2)$$

Substituting the expression for N given in equation (2) into equation (1) yields

$$\frac{mg \sin\theta}{\cos\theta} = m \frac{v^2}{r}$$

However, $v = r\omega = 2\pi fr$, where f = number of revolutions per second. Therefore,

$$4\pi^2 f^2 r = g \frac{\sin\theta}{\cos\theta} \qquad (3)$$

However, $r = R \sin\theta$

Therefore

$$\cos\theta = \frac{g}{4\pi^2 f^2 R}$$

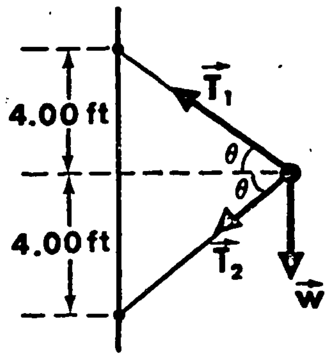
$$= \frac{980}{4 \times (3.14)^2 \times 4 \times 10}$$

$$= 0.62$$

Therefore

$$\theta = 52°$$

[a] CORRECT ANSWER: 38.2 rev/min; 5.00 lb



a) The components of the forces in the horizontal and vertical directions, respectively, are

$$\frac{mv^2}{r} = T_1 \cos\theta + T_2 \cos\theta \qquad (1)$$

$$T_1 \sin\theta = T_2 \sin\theta + w \qquad (2)$$

From equation (2),

$$T_2 = T_1 - \frac{w}{\sin\theta} \qquad (3)$$

Thus

$$\frac{mv^2}{r} = 2T_1 \cos\theta - w \frac{\cos\theta}{\sin\theta} \qquad (4)$$

But

$$v = r\omega = 2\pi fr \qquad (5)$$

where f is the number of revolutions per second.

Therefore, we have

$$m(2\pi f)^2 r = 2T_1 \cos\theta - w \cot\theta$$

$$f^2 = \frac{2T_1 \cos\theta - w \cot\theta}{4\pi^2 mr} \qquad (6)$$

Substituting numerical data we obtain

$$f = \frac{2}{\pi} \text{ rev/sec}$$

$$= 38.2 \text{ rev/min}$$

b) From equation (3)

$$T_2 = T_1 - \frac{w}{\sin\theta} = 5.00 \text{ lb}$$

[a] CORRECT ANSWER: 450 ft-lb; 0

a) Since the block moves at constant speed, the net force on it is zero. Therefore, the frictional force must be equal to the horizontal component of the applied force; that is

$$f = \mu N = F\cos\theta \qquad (1)$$

The magnitude of the normal force N is the sum of the block's weight and the vertical component of the applied force, so

$$N = mg + F\sin\theta \qquad (2)$$

Substituting (2) into (1) we obtain

$$\mu mg + \mu F\sin\theta = F\cos\theta \qquad (3)$$

We solve (3) for F, using the fact that for the present problem $\theta = 45°$ so $\sin\theta = \cos\theta$, to obtain

$$F = \frac{\mu mg}{(1 - \mu)\cos\theta}$$

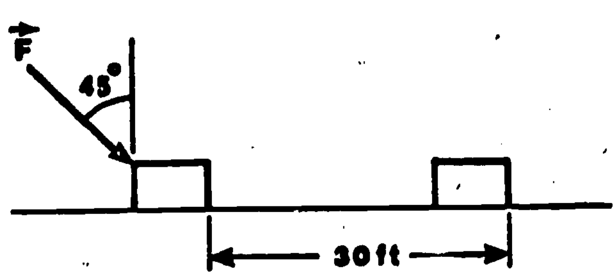
The work done on the block is

$$W = \vec{F}\cdot\vec{s} = Fs\cos\theta = \frac{\mu mgs}{1 - \mu}$$

$$= \frac{0.2 \times 60 \times 30}{1 - 0.2} = 450 \text{ ft-lb}$$

b) The work done by the normal force is

$$W_N = \vec{N}\cdot\vec{s} = Ns\cos 90° = 0$$



[a] CORRECT ANSWER: 38 cm

The compressed spring-plus-block represents a system at one energy state. In this state the total mechanical energy is the potential energy of the spring plus the potential energy of the block.

State One: Kinetic Energy + Potential Energy

$$K_1 + U_1 = 0 + \frac{1}{2} kx^2 + mgh_1$$

where h_1 is the initial height of the block.

After the spring is released, the block slides (without friction) up the plane until it stops momentarily. At that point we may write,

State Two:

$$K_2 + U_2 = 0 + mgh_2$$

Since the total mechanical energy of the system is conserved, we may equate the energy of the first state with the energy of the second state. Therefore,

$$K_1 + U_1 = K_2 + U_2$$

or

$$\frac{1}{2} kx^2 + mgh_1 = mgh_2$$

The maximum height would be

$$(h_2 - h_1) = \frac{1/2\ kx^2}{mg}$$

or

$$h_{max} = 38\text{ cm}$$

[a] CORRECT ANSWER: 0.202

A centripetal force equal to mv^2/r is required to provide the centripetal acceleration if the car is to remain on the (circular) road. Since the road is level, the force must be provided by friction. The frictional force is

$$F = \mu N = \mu mg$$

and must be equal to the required centripetal force; i.e.,

$$\mu mg = \frac{mv^2}{r}$$

or

$$\mu = \frac{v^2}{gr} = \frac{(44)^2}{32 \times 300} = \frac{1936}{9600} = 0.202$$

(Note: Use was made of the relationship that 30 mi/hr = 44 ft/sec)

[b] CORRECT ANSWER: 9 m/sec^2; 18 nt

a) Since the angular speed of the sphere is constant, the only acceleration experienced by the sphere is the centripetal acceleration directed toward the center of rotation. The magnitude of this acceleration is

$$a_c = v^2/r = \omega^2 r = (9 \text{ rad}^2/\text{sec}^2) \times (1 \text{ m}) = 9 \text{ m/sec}^2$$

b) The tension in the string is equal to the magnitude of the required centripetal force. Thus, using Newton's second law of motion,

$$T = ma_c = (2 \text{ kg}) \times (9 \text{ m/sec}^2) = 18 \text{ nt}$$

[a] CORRECT ANSWER: 1000 j; 200 w

a) By definition, the work done by a constant force is

$$W = \vec{F}\cdot\vec{s}$$

In this case the force is directed along the displacement so

$$W = Fs = 100 \times 10 = 1000 \text{ j}$$

b) From the definition of power

$$P = \frac{dW}{dt}$$

the expression

$$P = \vec{F}\cdot\vec{v}$$

can be derived.

Since the force and velocity are along the same direction, we obtain

$$P = 100 \times 2 = 200 \text{ w}$$

[b] CORRECT ANSWER: 1750 j

Energy lost due to friction

$$= (KE + U)_i - (KE + U)_f$$

But

$$U_i = U_f$$

Thus, energy lost due to friction

$$= (KE)_i - (KE)_f$$

$$= \frac{1}{2} mv_i^2 - \frac{1}{2} mv_f^2$$

$$= 1750 \text{ j}$$

[a] CORRECT ANSWER: 1.06×10^{-15} gm-cm/sec

When there is no external force acting in the system, we know that the momentum of the system is conserved. Since the momentum of the nucleus is zero before the decay, the total momentum of the electron, the neutrino, and the nucleus must be zero after decay. Therefore, we may write:

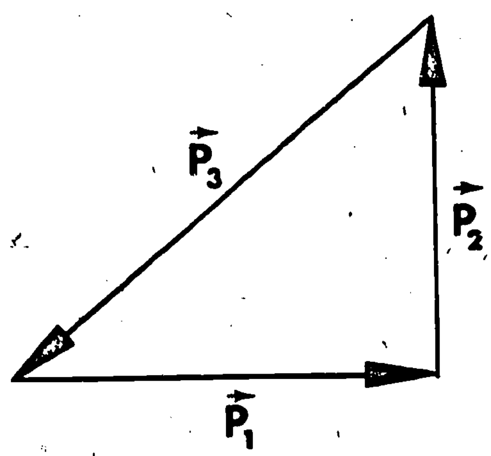
$$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0$$

where $\vec{P}_1$ is the momentum of the electron, $\vec{P}_2$ that of the neutrino, $\vec{P}_3$ that of the nucleus. The sum of $\vec{P}_1 + \vec{P}_2$ can be calculated easily because of geometry. Therefore,

$$|\vec{P}_1|^2 + |\vec{P}_2|^2 = |\vec{P}_3|^2$$

or

$$|P_3| = 1.06 \times 10^{-15} \text{ gm-cm/sec}$$



[b] CORRECT ANSWER: 13.3 kg-m/sec

Taking the bottom of the track as the zero potential energy reference, and using conservation of energy we find

Potential Energy at A = Kinetic Energy at B

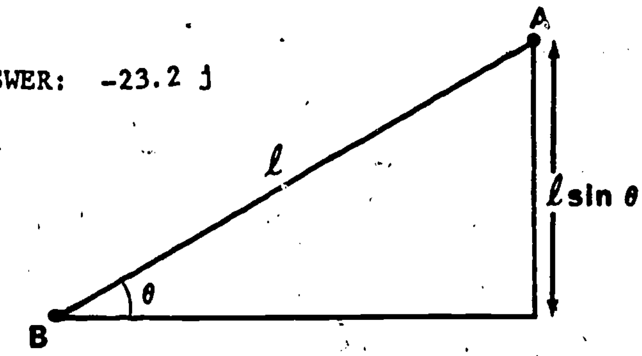
or

$$mgR = \frac{1}{2}\frac{p^2}{m} \quad \text{where } p = mv$$

Thus,

$$p = m\sqrt{2gR} \quad 3\sqrt{2(9.8)(1)} = 13.3 \text{ kg-m/sec}$$

[a] CORRECT ANSWER: -23.2 j



$$W = \Delta E = (KE + U)_f - (KE + U)_i$$

$$= \left(\frac{1}{2} mv_f^2 + 0\right) - \left(\frac{1}{2} mv_i^2 + mg\ell \sin\theta\right)$$

$$= \frac{1}{2} m\left(v_f^2 - v_i^2 - 2g\ell \sin\theta\right)$$

$$= \frac{1}{2} 4\left[3^2 - 1^2 - 2(9.8)(2)\left(\frac{1}{2}\right)\right]$$

$$= 2(-11.6) = -23.2 \text{ j}$$

[b] CORRECT ANSWER: 45°

$$(KE)_{ground} = \frac{1}{2} mv_o^2$$

$$(KE)_{highest\ point} = \frac{1}{2} mv_x^2$$

sincy v_y at the highest point is zero

Given: $(KE)_{ground} = 2(KE)_{highest\ point}$

Substitution yields

$$\frac{1}{2} mv_o^2 = 2\left(\frac{1}{2} mv_x^2\right)$$

Therefore

$$v_x = \frac{1}{\sqrt{2}} v_o$$ But, in addition

$$v_x = v_o \cos\theta_o$$

Therefore

$$\theta_o = 45°$$

[a] CORRECT ANSWER: 24.5 m

Using conservation of energy, the speed of the particle at point B is

$$v_o = v_{ox} = \sqrt{2gh_1} \quad (1)$$

The time of flight may be obtained from

$$h_2 = v_{oy}t + \frac{1}{2}gt^2 \qquad \text{or} \qquad h_2 = \frac{1}{2}gt^2$$

where

$$v_{oy} = 0$$

Therefore

$$t = \sqrt{\frac{2h_2}{g}} \quad (2)$$

The required distance is

$$x = v_{ox}t$$

$$= v_o t$$

$$= \sqrt{\frac{2gh_1 2h_2}{g}} = 2\sqrt{h_1h_2}$$

$$x = 24.5 \text{ m}$$

[b] CORRECT ANSWER: 56 kg-m/sec

$$P_f - P_i = \int_0^3 F dt$$

$$P_f = P_i + \int_0^3 \left(\frac{1}{2}kt^2\right) dt$$

$$= mv_i + \frac{1}{2}\left(\frac{1}{3}kt^3\right)\Bigg|_0^3$$

$$= (20 + 36) \text{ kg-m/sec} = 56 \text{ kg-m/sec}$$

[a] CORRECT ANSWER: $x_{cm} = 2$ ft; $y_{cm} = 1.3$ ft

Divide the piece of wood into a rectangle and two squares of areas 8 ft^2, 1 ft^2 and 1 ft^2 respectively. The masses and the coordinates of the centers of mass of these pieces are 8ρ, 1ρ and 1ρ and (2.0, 1.0), (0.5, 2.5) and (3.5, 2.5), respectively, where

$$\rho = \text{mass units/ft}$$

Therefore

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{16\rho + 0.5\rho + 3.5\rho}{10\rho}$$

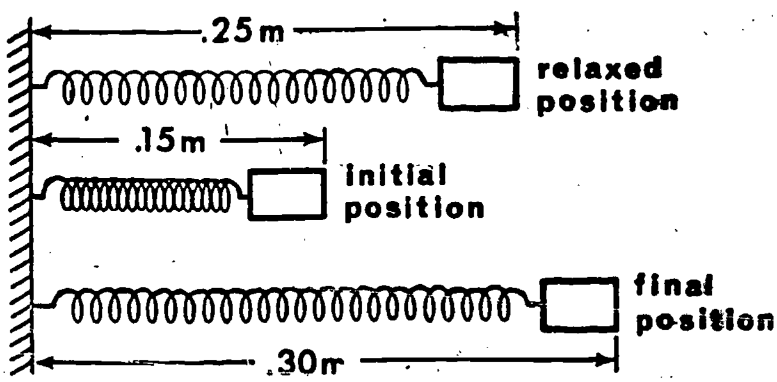
$$= 2 \text{ ft}$$

and

$$y_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{8\rho + 2.5\rho + 2.5\rho}{10\rho}$$

$$= 1.3 \text{ ft}$$

[b] CORRECT ANSWER: −.375 j



Work done on spring $= \int_0^x kx\,dx = \frac{1}{2}kx^2$ generally.

Work done <u>on</u> spring in "stretching" from 0.15 m to 0.25 m (a displacement of 0.1 m)

$$= -\frac{1}{2}kx^2 = -\frac{1}{2}(100)(0.1)^2$$

$$= -0.5 \text{ j}$$

Work done on spring from 0.25 to 0.30 (displacement of 0.05 m)

$$= +\frac{1}{2}kx^2 = +\frac{1}{2}(100)(0.05)^2$$

$$= +.125 \text{ j}$$

Total work done on spring = −.375 j

[a] CORRECT ANSWER: (a) $x_{cm} = 2m$; $y_{cm} = 2m$ (b) Zero

(a) The position vectors of the four particles are

$$\vec{r}_1 = 0 \qquad \vec{r}_2 = 4\hat{i} \qquad \vec{r}_3 = 4\hat{i} + 4\hat{j} \qquad \vec{r}_4 = 4\hat{j}$$

Therefore

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{0 + 12\hat{i} + (12\hat{i} + 12\hat{j}) + 12\hat{j}}{12}$$

or

$$x_{cm} = 2m \quad \text{and} \quad y_{cm} = 2m$$

(b) The momentum vectors of the four particles are

$$\vec{p}_1 = -30\hat{j} \qquad \vec{p}_2 = 30\hat{i} \qquad \vec{p}_3 = 30\hat{j} \qquad \vec{p}_4 = -30\hat{i}$$

Therefore

$$v_{cm} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{-30\hat{j} + 30\hat{i} + 30\hat{j} - 30\hat{i}}{12}$$

$$= 0$$

[b] CORRECT ANSWER: 1530 ft

$$\frac{mv^2}{R} = ma$$

$$= m(7g)$$

Thus

$$R = \frac{v^2}{(7g)} = \frac{\left(400 \times \frac{5280}{3600}\right)^2}{7 \times 32} = 1530 \text{ ft}$$

[a] CORRECT ANSWER: 45 ft-lb

By definition

$$W \equiv \int_{s_1}^{s_2} F_s \, ds = \int_{x_1}^{x_2} F_x \, dx$$

Using the given data, with $x_1 = 0$ and $x_2 = 3.0$ ft, we obtain

$$W = \int_0^{3.0} 10x \, dx = 5x^2 \Big|_0^{3.0} = 45 \text{ ft-lb}$$

[b] CORRECT ANSWER: B

Since

$$dU = -F \, dx$$

$$\int_{U(\infty)}^{U(x)} = -\int_{\infty}^{x} F \, dx$$

we obtain

$$U(x) = \int_{\infty}^{x} \frac{K}{x^7} \, dx = -\frac{K}{6} \frac{1}{x^6}$$

UNITED STATES NAVAL ACADEMY

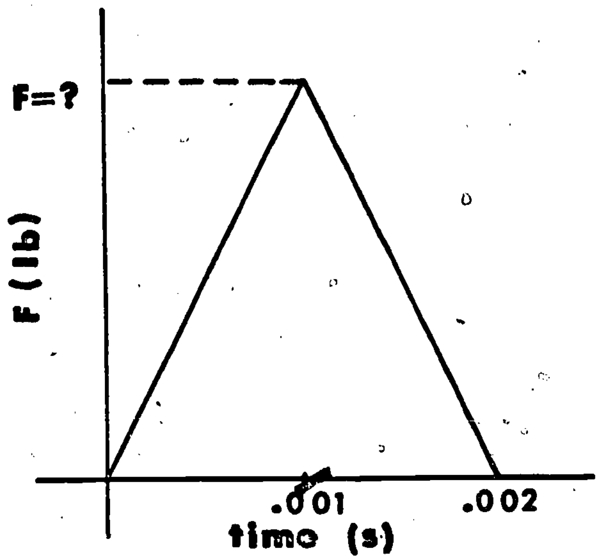
Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

1 (11-1). The diagram shows the dependence of the force applied by a mallet to a croquet ball during the time of contact. If the magnitude of the total impulse imparted to the croquet ball by the mallet is 0.4 lb-sec, what is the maximum value of the magnitude of this impulsive force?



2 (11-5). A falling tennis ball of mass 4 oz strikes the floor vertically at a speed of 16 ft/sec and rebounds upward at 16 ft/sec. The ball was in contact with the floor for 0.01 sec. What was the magnitude of average force during contact?

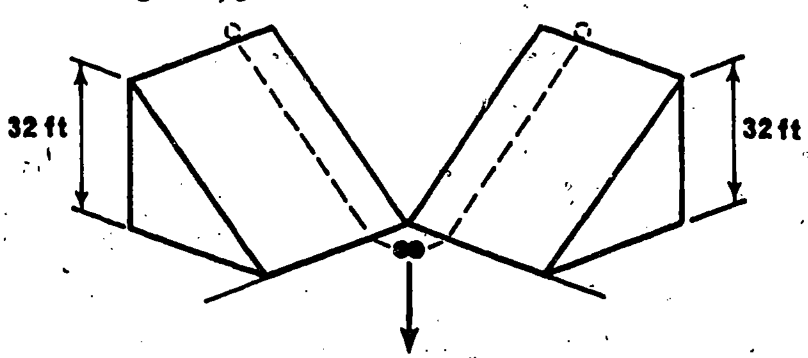
3 (11-11). A 500-kg flat car can roll without friction along a straight horizontal track. A 100-kg man is standing still on the car when it moves to the right at 5 m/sec. The man starts running to the left and he picks up a speed of 6 m/sec relative to the car before jumping off the car at the left end. What is the speed of the car at the moment the man jumps?

4 (12-1, 11-18). In a one-dimensional elastic collision between two particles, an 18-kg mass is initially moving to the right with a speed of 5 m/sec. The second particle with a mass of 2 kg is initially behind the first and is also moving to the right with a speed of 20 m/sec. What is the speed of each particle immediately after the collision?

5 (12-6, 11-15). A 15-gm wood block rests on a frictionless horizontal surface. A 5-gm bullet is fired horizontally and becomes embedded in the block. What fraction of the bullet's kinetic energy is dissipated (converted to other forms of energy)?

6 (12-10). A particle of mass m moving with an initial speed of 7 m/sec collides elastically with an identical particle which is initially at rest. After collision, the path of the first particle is observed to make an angle of 45° with the initial direction. Find the speeds of the two particles after collision

7 (12-14). A pair of frictionless inclined planes are so oriented that the paths of two particles, one sliding down each plane, intersect at right angles on the level surface at the instant of collision. If each particle has a mass of 2.0 slugs and they both start 32 ft above the level surface, find the speed of the combined mass after a perfectly inelastic collision.



8 (13-1). Two particles with respective masses of 1.0 kg and 9.0 kg are held fixed at points 1.0 m apart. How far from the 1.0-kg particle must a 2.0-kg particle be placed so that it will remain at rest while the gravitational force due to the fixed masses is acting upon it?

9 (13-4). Derive an expression for the speed of a satellite of mass m in circular orbit around the Earth in terms of the mass of the earth, M_e, and the radius of the satellite's orbit, r.

A. $v = G\sqrt{M_e r}$ B. $v = \sqrt{\frac{GM_e}{r}}$

C. $v = \sqrt{Gr}$ D. $v = \sqrt{\frac{Gm}{r}}$

10 (13-10, 4-11). A mass m is placed on the surface of the Earth. The gravitational attraction of the Earth for m can be written as

$$F = G \frac{mM_e}{R_e^2} \qquad (1)$$

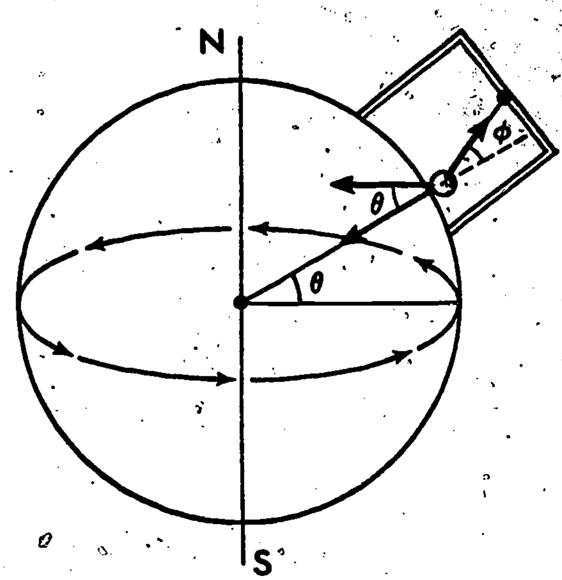
On the other hand, the weight of a body with mass m' can be written as

$$w = m'g \qquad (2)$$

where g is the acceleration with which the body falls toward the Earth. Do the symbols m and m' in (1) and (2) stand for inertial or gravitational mass?

A. both gravitational
B. both inertial
C. m gravitational, m' inertial
D. m inertial, m' gravitational

11 (13-11). Because of the rotation of the Earth, a plumb bob does not hang exactly along the direction of the gravitational pull but deviates from this direction by a small angle ϕ.



In the diagram, the bob swings southward in the plane of the paper. If $a = \omega^2 R \cos\theta$ is the radial acceleration of the body at latitude θ and g is the acceleration due to gravity at the surface of the Earth, the relationship between ϕ and latitude (θ) is:

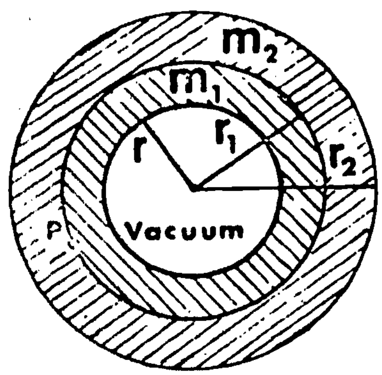
A. $\cot\phi = \cot\theta$

B. $\cot\phi = \dfrac{g\cos\theta - a}{g\sin\theta}$

C. $\tan\phi = \dfrac{\omega^2 R\cos\theta\sin\theta}{g + \omega^2 R\cos^2\theta}$

D. $\tan\phi = \dfrac{g\cos\theta - a}{g\sin\theta}$

12 (13-19). A spherical body is made up of two concentric shells of different mass densities. The shells have masses m_1 and m_2, respectively. The inner and outer radius of m_1 are r and r_1, those of m_2 are r_1 and r_2 as shown in the diagram. The density of each shell is uniform. Derive an expression for the magnitude of the gravitational field strength at a point P located between the two shells.



A. $\frac{Gm_2}{r_2^2}$

B. $\frac{Gm_1}{r_1^2}$

C. $\frac{Gm_1}{r_1^2} + \frac{Gm_2}{r_2^2}$

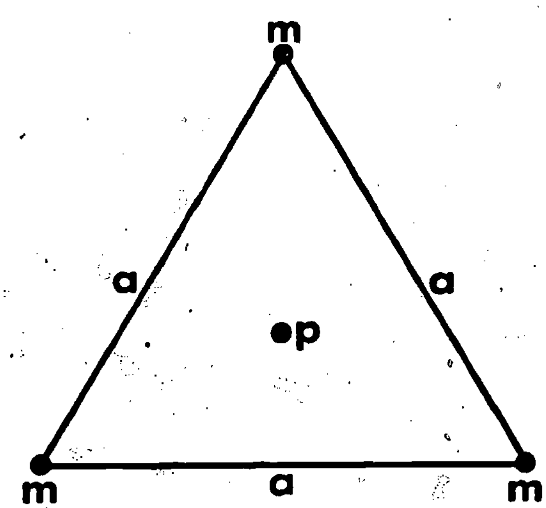
D. zero

13 (14-6, 14-1, 13-15). Three particles of equal mass m = 2.0 kg are placed at the vertices of an equilateral triangle of side a = 2.0 m as shown. Calculate

a) the field at the center of mass of the system (point p)

b) the gravitational potential at the same point

c) the work that must be done by an outside agent in order to move a 3.0-kg particle from a point at infinity to point p



14 (14-10). Two equal point masses each of mass M are placed a distance 4r apart. Midway between the two masses at point p, the gravitational field is zero and a point particle of mass m is trapped there. However, if m is disturbed and allowed to escape towards one of the larger masses M, the kinetic energy of mass m when it is a distance r from M is

A. $\frac{GMm}{r}$

B. $\frac{GMm}{2r}$

C. 0

D. $\frac{GMm}{3r}$

15 (14-14). What is the minimum speed necessary for a Lunar Module to escape the moon's gravitational field (escape velocity)?

GIVEN: M = moon's mass = 7.78×10^{22} kg

R = moon's radius = 1080 miles

m = mass of Lunar Module = 1000 kg

G = Universal Gravitational Constant = 6.67×10^{-11} nt-m^2/kg^2

[a] CORRECT ANSWER: C

Everybody on the surface of the Earth rotates in a circle with center on the Earth's axis. It therefore has a radial acceleration $a = \omega^2 r$ towards the axis ($r = R\cos\theta$). The equation of motion for the bob is

$$\vec{T} + m\vec{g} = m\vec{a}$$

where T = tension in the plumb line.

We choose a set of axes oriented in the direction of the gravitational force.

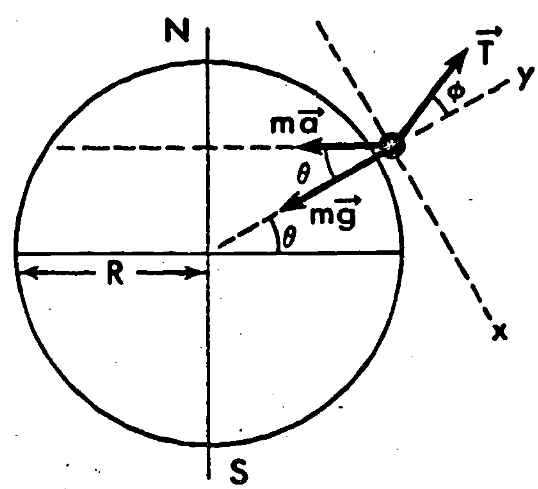
The scalar equations for the x and y directions are

$$T_x = m\omega^2 R \cos\theta \sin\theta$$

$$T_y = g + m\omega^2 R \cos^2\theta$$

Combining the last two equations we obtain

$$\tan\phi = \frac{Tx}{Ty} = \frac{\omega^2 R \cos\theta \sin\theta}{g + \omega^2 R \cos^2\theta}$$



[b] CORRECT ANSWER: D

Using conservation of energy principle for particle m we obtain

KE + PE at center = KE + PE at new location

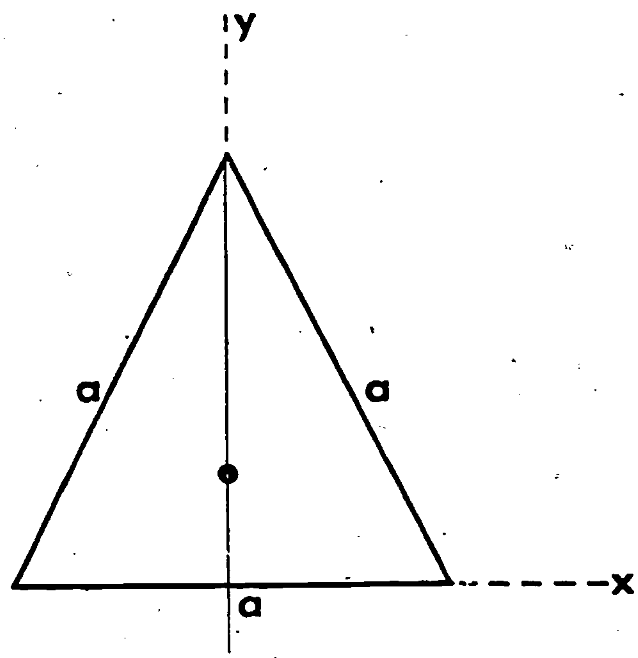
$$0 - \frac{GMm}{2r} - \frac{GMm}{2r} = KE - \frac{GMm}{r} - \frac{GMm}{3r}$$

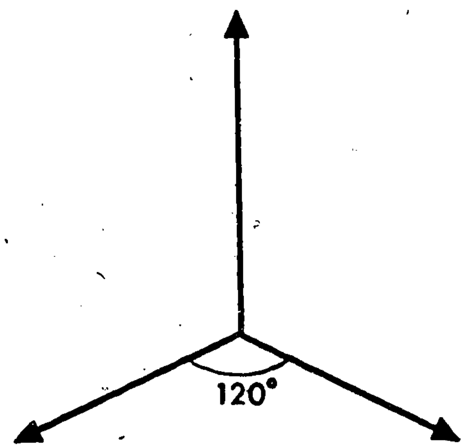
Therefore

$$KE = \frac{GMm}{3r}$$

[a] CORRECT ANSWER: a) Zero b) -3.5×10^{-10} j/kg c) -11×10^{-10} j

a)





With the chosen coordinate system as shown in the figure, the coordinate of the center of mass of the system is

$$\left(0,\ \frac{a}{2\sqrt{3}}\right)$$

The field at that point may be represented as the sum of three vectors with the same magnitude. The angle between any two of them is 120°. Thus the field at that point is zero.

b) The potential at the center of mass can be calculated as the algebraic sum of three equal terms

$$V = -\frac{Gm}{\left(\frac{a}{\sqrt{3}}\right)} - \frac{Gm}{\left(\frac{a}{\sqrt{3}}\right)} - \frac{Gm}{\left(\frac{a}{\sqrt{3}}\right)} = -3\sqrt{3}\,\frac{Gm}{a} = -3.5 \times 10^{-10} \text{ j/kg}$$

c) The work that must be done against gravity is equal to the potential energy of the 3.0-kg particle in the field of the other three particles. This is equal to the mass times the potential at p, i.e.,

$$U = m'V = -10.5 \times 10^{-10} \text{ j}$$

$$= -11 \times 10^{-10} \text{ j}$$

[a] CORRECT ANSWER: 6 m/sec

Since there are no external forces momentum must be conserved. Taking the ground as our frame of reference, we obtain for the momentum of the system before the man starts running

$$p_i = (m + M)v_o \qquad (1)$$

where m and M are the masses of the man and car respectively, and v_o is the initial speed of the car. Let u be the speed of the man relative to the car, and v be the speed of the car relative to ground. The velocity of the man relative to ground is $\vec{v} + \vec{u}$ or, in scalar form. v - u (recall that $\vec{v}$ is to the right and $\vec{u}$ is to the left). Thus the total final momentum becomes

$$p_f = Mv + m(v - u) = (m + M)v - mu \qquad (2)$$

Conservation of momentum ($p_i = p_f$) gives

$$(m + M)v_o = (M + m)v - mu$$

or

$$v = v_o + \frac{m}{m + M} u = 5 + \frac{100}{600} \times 6 = 6 \text{ m/sec}$$

[b] CORRECT ANSWER: 400 lb

From the definition of impulse

$$\vec{J} = \int \vec{F}\, dt$$

we know that the magnitude of the impulse is area under the shown curve. Thus,

$$J = \frac{1}{2} \text{(height)} \times \text{(base)}$$

and

$$F = \text{(height)} = 2J/\text{(base)}$$

$$= \frac{2 \times 0.4}{0.002} = 400 \text{ lb}$$

[a] CORRECT ANSWER: 1.53 miles/sec

The gravitational potential energy of the Lunar Module on the moon's surface is

$$U_1 = -\frac{GMm}{R}.$$

We know that the gravitational potential energy of the Lunar Module will be zero at an infinite distance from the moon. Basically then, we want to give the Lunar Module enough initial velocity to carry it all the way to "infinity" and, when it gets there, its velocity should just be zero. In equation form, this becomes

on the moon *at infinity*

$$K_m + U_m = K_\infty + U_\infty$$

but

$$U_\infty = 0 \quad \text{(reference point of potential energy)}$$

and

$$K_\infty = 0 \quad \text{(see discussion above)}$$

Therefore,

$$\frac{1}{2} mv^2_{escape} - \frac{GMm}{R} = 0$$

and thus,

$$v^2 = \frac{2GM}{R}$$

or

$$v = 1.53 \text{ miles/second}$$

[a] CORRECT ANSWER: 8 m/sec; -7 m/sec

Let m_1 and m_2 be the masses of the small and large particles, respectively. Let u_1 and u_2 be the initial speeds of m_1 and m_2, respectively.

From conservation of momentum we obtain

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad (1)$$

and from conservation of kinetic energy we obtain

$$\frac{1}{2} m_1 u_1{}^2 + \frac{1}{2} m_2 u_2{}^2 = \frac{1}{2} m_1 v_1{}^2 + \frac{1}{2} m_2 v_2{}^2 \quad (2)$$

Rewriting (1) and (2) as

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \quad (3)$$

$$m_1 (u_1{}^2 - v_1{}^2) = m_2 (v_2{}^2 - u_2{}^2) \quad (4)$$

and dividing (4) by (3) we obtain

$$u_1 + v_1 = v_2 + u_2 \quad (5)$$

or

$$u_1 - u_2 = v_2 - v_1 \quad (6)$$

From (6)

$$v_2 = u_1 - u_2 + v_1 \quad (7)$$

Substituting (7) into (3) we find

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2$$

Substituting the given numerical values, we obtain

$$v_1 = \frac{2 - 18}{2 + 18}(20) + \frac{2(18)}{20}(5)$$

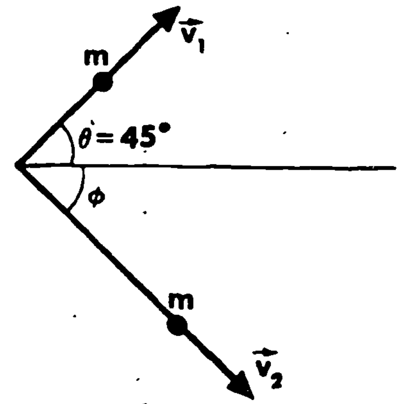
$$= -7 \text{ m/sec}$$

$$v_2 = 20 - 5 - 7 = 8 \text{ m/sec}$$

[a] CORRECT ANSWER:: 5 m/sec; 5 m/sec

Before Collision

After Collision



Applying the conservation of momentum for the x and y directions, we obtain

$$u_1 = v_1 \cos\theta + v_2 \cos\phi \qquad (1)$$

and

$$0 = v_1 \sin\theta - v_2 \sin\phi \qquad (2)$$

Squaring equations (1) and (2) and adding yields

$$u_1{}^2 = v_1{}^2 + v_2{}^2 - 2v_1v_2 \sin\theta \sin\phi + 2\, v_1v_2 \cos\theta \cos\phi \qquad (3)$$

(noting that $\sin^2 x + \cos^2 x = 1$)

The conservation of kinetic energy for identical particles yields,

$$u_1{}^2 = v_1{}^2 + v_2{}^2 \qquad (4)$$

Comparing equations (3) and (4), we obtain

$$\sin\theta \sin\phi = \cos\theta \cos\phi \qquad (5)$$

Therefore

$$\theta + \phi = 90°$$

Hence

$$\phi = 45°$$

Substituting numerical values in equations (1) and (2) we obtain

$v_1 = 5$ m/sec and $v_2 = 5$ m/sec

[a] CORRECT ANSWER: 25 lb

$$\vec{J} = \Delta\vec{p} = \int \vec{F}\, dt$$

$$\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) = \overline{\vec{F}}\, \Delta t$$

where $\vec{v}_f$ = rebound velocity and $\vec{v}_i$ = impact velocity

Thus

$$\Delta p = m\, v_f - (-v_i) = \overline{F}\, \Delta t$$

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{m(v_f + v_i)}{\Delta t} = 25 \text{ lb}$$

[b] CORRECT ANSWER: B

A satellite in circular orbit experiences a centripetal acceleration $a_c = v^2/r$. The gravitational force on the satellite of magnitude GmM_e/r^2 is the only force acting on it and the force must be equal to the required centripetal acceleration, mv^2/r. Thus

$$G\frac{mM_e}{r^2} = \frac{mv^2}{r}$$

or

$$v = \sqrt{G\, M_e/r}$$

[a] CORRECT ANSWER: 0.75

In the horizontal direction, there are no external forces acting on the wooden block or the bullet. Consequently, the initial horizontal momentum of the bullet and block before collision must be equal to the final horizontal momentum of the bullet and block after the bullet becomes embedded in the block. Therefore, we can write that

$$mv = (M + m)V$$

where m = mass of bullet

M = mass of wooden block

v = initial velocity of bullet

V = final velocity of bullet plus block

Consequently,

$$v = \frac{(M + m)}{m} V \qquad (1)$$

The fraction f of the bullet's initial kinetic energy that was dissipated during the collision can be determined by

$$f = \frac{\text{initial KE} - \text{final KE}}{\text{initial KE}}$$

Therefore,

$$f = \frac{\frac{1}{2} mv^2 - \frac{1}{2}(M + m)V^2}{\frac{1}{2} mv^2}$$

Rearranging, we obtain,

$$f = 1 - \left(\frac{M + m}{m}\right)\left(\frac{V}{v}\right)^2 \qquad (2)$$

Now we can insert the expression for v from equation 1 into equation 2 to yield,

$$f = 1 - \left(\frac{M + m}{m}\right)\left(\frac{Vm}{(M + m)V}\right)^2$$

next page

continued

or

$$f = \frac{M}{M + m}$$

Notice that this fraction does not depend upon any velocities. Substituting the known values we obtain

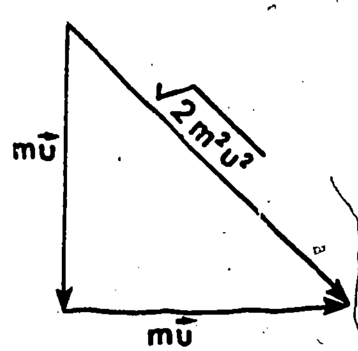
$$f = 0.75$$

[a] CORRECT ANSWER: 32 ft/sec

Using the principle of conservation of energy, the speed of each particle just before collision is

$$u = \sqrt{2gh} \quad \text{for both}$$

Applying conservation of momentum, we obtain



momentum before collision = momentum after collision

$$2m^2u^2 = 2mV$$

where V = speed of combined mass after collision

Hence

$$V = \frac{\sqrt{u}}{\sqrt{2}} = \frac{\sqrt{2gh}}{\sqrt{2}}$$

$$V = \sqrt{gh} = 32 \text{ ft/sec}$$

[a] CORRECT ANSWER: B

The gravitational field strength inside a spherical shell of uniform mass distribution is zero. The outer shell, therefore, will not contribute to the field at P.

The contribution of the inner shell will be the same as if all of its mass, m_1, were concentrated at its center. Thus, the magnitude of the field at P is

$$\gamma = Gm_1/r_1^2$$

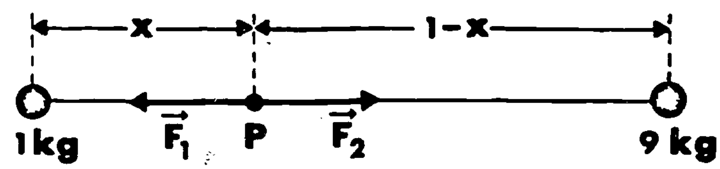
[b] CORRECT ANSWER: C

In the expression

$$F = \frac{GmM_e}{R_e^2}$$

m clearly stands for the gravitational mass. On the other hand m' represents an inertial mass as g is given as an acceleration and (2) is merely a statement of Netwon's second law.

[a] CORRECT ANSWER: 0.25 m



Since gravitational forces are attractive the point in question must lie between the two given particles so that the respective forces on the third particle are equal in magnitude but *opposite* in direction. Let this point, P, lie a distance x from the 1.0-kg particle. Then

$$|\vec{F}_1| = \frac{Gm_1m_3}{x^2}$$

and

$$|\vec{F}_2| = \frac{Gm_2m_3}{(1 - x)^2}$$

We must have

$$\vec{F}_1 = -\vec{F}_2 \quad \text{or} \quad |\vec{F}_1| = |\vec{F}_2|$$

Thus,

$$\frac{Gm_1m_3}{x^2} = \frac{Gm_2m_3}{(1 - x)^2}$$

or

$$\sqrt{\frac{m_1}{m_2}}\frac{1}{x} = \frac{1}{(1 - x)}$$

or

$$\sqrt{\frac{m_2}{m_1}}\, x = 1 - x$$

Finally,

$$x = \frac{1}{1 + \sqrt{m_2/m_1}} = \frac{1}{1 + 3} = 0.25 \text{ m}$$

that is, the point in question lies a distance

x = 0.25 m from the 1.0-kg particle