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ABSTRACT

These models are based on the concept that school district decisionmakers seek an optimum balance between education program levels and tax burdens imposed on the community, subject to a budget constraint. A model is derived in which spending depends positively on community income and lump-sum grants and negatively on the relative price of educational inputs, the pupil/household ratio, and the local share of matching grants. Extensions take account of nonschool taxes, the composition of the property tax base (residential vs. business property), equalization features of State aid formulas, categorical grants, and enrollment growth. Since the models include lump-sum aid, matching aid, and interactions between them, they are potentially useful for selecting optimal aid formulas for accomplishing a grantor's objectives. Explicit functional forms of the expenditure relationships are developed and variants of the model that can be tested with different data bases are described. (Author)

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# Theoretical Models of School District Expenditure Determination and the Impact of Grants-In-Aid

S. M. Barro

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# PREFACE

This report is part of a Rand study of federal-state-local fiscal relations, sponsored by the Ford Foundation. The theoretical models presented here are intended to provide a foundation for empirical studies of the fiscal behavior of local public school districts and, ultimately, for efforts to predict impacts of alternative state and federal grant-in-aid programs. Several empirical studies are now under way and will be published as separate Rand reports.

## SUMMARY

This report presents a theoretical analysis of the determinants of expenditures by public school districts, with special emphasis on the influence of state and federal grants-in-aid. The analysis is carried out within the framework of the economic theory of constrained maximizing behavior: Decisionmakers in each school district are assumed to arrive at an optimal balance between the level of educational programs and the tax burdens they must impose on the community. The model is developed in two stages: first, a basic model focusing on a few key variables; then a series of "extensions," in which other variables and relationships are incorporated into the basic formulation.

### THE BASIC MODEL

It is assumed that a school district seeks to maximize a utility function having real educational expenditure per pupil and real tax burden per household as its principal arguments. Tax burden is defined as a positive function of the amount of school tax per household and a negative function of household income. The budget constraint specifies that total current outlay by the district must equal the sum of local school tax receipts plus grants-in-aid from the state and federal governments. Outside aid is assumed, in general, to consist of a combination of lump-sum and matching grants.

Utility maximization implies that the marginal rate of trade-off between spending per pupil and taxes per household (analogous to the marginal rate of substitution of consumer demand theory) must equal the price ratio between those two quantities. In this case, that ratio depends not only on the price of educational inputs relative to prices in general, but also on the ratio of pupils to households and on the local share parameter of state or federal matching grants.

If appropriate properties of the marginal rate of trade-off function are assumed (or derived from assumptions about district preferences), it can be shown that the model implies the following about school district fiscal behavior: (1) positive relationships between per pupil spending and both income per household and lump-sum aid per pupil; (2)

unequal local responses to equal increments in income and aid, with the aid effect being considerably greater than the income effect under normal circumstances; (3) negative relationships between per pupil spending and the relative price of educational inputs, the ratio of pupils to households, and the local share parameter of matching grants; (4) equal elasticities of expenditures with respect to all three of the last-mentioned variables.

According to the model, lump-sum grants-in-aid should always be partly additive to per pupil spending and partly substitutive for local school taxes. The same may be true of matching aid, but it is also possible that matching grants will stimulate higher local taxes than would have been levied in their absence. Lump-sum and matching grants interact, the impact of one depending on the level of the other. Since the models include both kinds of aid and allow for interaction, they are potentially useful for selecting optimal combinations of lump-sum and matching aid to accomplish an aid grantor's objectives.

The assumption that the pupil/household ratio has a direct effect on decisionmakers' preferences leads to two variations on the basic model. In each, the relative price variable and the pupil/household ratio enter differently into the expenditure and tax equations. The expenditure elasticity of the former is less negative than that of the latter and the sign of the latter's effect on spending is uncertain.

Provision is made in the analysis for unspecified community characteristics ("taste variables") to appear in the expenditure and tax equations. Any such variable that is positively related to the willingness of a district to raise incremental taxes to support increments in education will also be positively related to real per pupil spending.

Finally, the model is recast in terms of a number of specific functional forms: the linear expenditure system and several nonlinear forms that correspond to linear and exponential forms of the marginal rate of trade-off function.

## EXTENSIONS

The basic model is extended to include the effects of (1) variations in nonschool taxes, (2) variations in the composition of the local property tax base, (3) equalization features of state aid formulas, (4) categorical versus noncategorical grants-in-aid, and (5) variations in the rate of growth of pupil enrollment.

Nonschool taxes include state and federal income taxes, sales taxes, and noneducational property taxes. Income taxes can be treated as deductions from household personal income. If school taxes are deductible from the income tax base, however, it is necessary to take account of the positive impact of deductibility on school spending and taxing. The effect of differential sales taxes can be translated into differences in the price of "other goods" and, therefore, in the relative price of educational inputs. Nonschool property taxes can be treated in the same manner as income taxes [illegible] negative grants-in-aid [illegible], or as a separate variable in the expenditure equation with a negative effect on spending. The preferred specification must be determined empirically.

The composition of the local property tax base as between business and residential property has an effect on spending if decisionmakers care about the relative burdens imposed on the two types of taxpayers. If only taxes on residents count, then the presence of business property in a community affects expenditure behavior in the same way as would a matching grant with a local share parameter equal to the ratio of residential to total property; i.e., the smaller the proportion of residential property, the higher the level of spending. If both types of taxes count, but with different weights in the eyes of the decisionmaker, the same result holds as long as the decisionmakers derive more disutility from incremental taxes on homeowners than on businesses.

Equalization provisions of state aid formulas typically distribute funds in inverse relation to assessed property value per pupil. This may or may not have the desired equalizing effect, depending on the sources of tax base variation. If interdistrict variations are attributable to differences in business property per pupil, the effect will be in the correct direction. If variations are attributable to differences in household incomes and, hence, in residential property values, the desired effect will probably, though not necessarily, be obtained. If, however, the differences are due to locational or other environmental factors that cause property values to vary, the effect of existing equalization formulas can be perverse: More aid may be given to districts that would have spent above-average amounts even in the absence of aid.

Categorical grants will have a different effect on spending than non-categorical grants when the categorical constraint is binding, i.e., when the aid recipient is forced to spend more on a function than he would have spent voluntarily, given the same level of total funding. In that event, separate categorical and noncategorical aid [illegible] should appear in the [illegible] equation and, in general, the two coefficients should be unequal.

The effect of enrollment growth is to impose capital outlay requirements on a district, which, in general, will be partly competitive with outlays for current operations. Whether capital is financed out of bond receipts or out of current funds, the model predicts a negative enrollment growth effect proportional to the annual growth rate. If borrowing is the mode of finance, the magnitude of the growth effect will also depend positively on both the rate of bond interest and the existing stock of debt.

## MODELS APPLICABLE TO DIFFERENT DATA BASES

Variants of the models apply to studies that have the individual school district as the unit of observation and to those that focus on larger units such as states or counties. The individual district model can be aggregated to the state level by weighting each state observation by a factor that reflects the distribution of pupils among districts within the state. Certain other differences between state and local models (and between cross-sectional and time series models) occur because certain magnitudes that vary among states (or over time) may show no variation within states (or across observational units).

CONTENTS

# I. INTRODUCTION

This report presents the results of a theoretical inquiry into the factors that determine the fiscal behavior of local school districts. It has been prepared as part of a larger study aimed at providing analytical tools for planning intergovernmental grant-in-aid programs. The most immediate application of the findings is in developing empirical models of the determinants of public school expenditures and taxes for use in evaluating alternative programs of state and federal aid to education. It is hoped, also, that similar theoretical approaches will prove applicable to fields other than education and that this work will help to build a foundation for policy-relevant analyses of a variety of intergovernmental aid programs.

## BACKGROUND

Public elementary and secondary education in the United States is provided by approximately 18,000 local school districts. Most of these units have the status of independent local governments with corresponding authority in fiscal affairs. Others, though subordinate to municipal or county governments, are generally controlled by independent local boards and also have considerable financial autonomy. While public education is ultimately a state responsibility, the system is structured so that state authority in the financial sphere is exercised indirectly, primarily through provision of state funds and imposition of constraints. The wide variation that exists in levels of per pupil spending and local tax rates within states attests to the extent to which fiscal decisionmaking is decentralized to local school boards and voters.

School district expenditures are financed partly out of locally raised funds (predominantly property tax receipts) and partly out of grants-in-aid from the states, and, to a lesser extent, from the federal government. Grants-in-aid are the primary means by which higher levels of government seek to influence the level, distribution, and utilization of educational resources. State governments use various aid allocation formulas to supplement local revenues and, in most states, to reduce the inequality among districts that would exist if districts had to rely solely on their own resources. Both state and federal authorities also use grants to direct resources to activities within districts that they especially value. Conditions are often attached to such grants in an effort to assure that aid funds will be used as intended by the grantor.

Policymakers at the state level frequently attempt to use changes in state aid formulas as the means for effecting changes in the level or distribution of education expenditures. Proposals for new formulas may be motivated, for example, by a desire to reduce inequality among districts, to stimulate greater local spending, or to provide property tax relief. In choosing among alternative proposals, however, or in designing a formula to achieve a specific objective, officials are seriously handicapped by uncertainty about the nature of the link between grants-in-aid and fiscal outcomes. Local fiscal autonomy and the availability of local revenue sources enable school districts to behave *adaptively* in response to changes in state or federal support. Unless such adaptation is correctly anticipated, outcomes can be quite different from what the grantor intended. Consider, for example, the effect of an attempt by a state government to increase spending $100 per pupil by granting that much additional aid to each district. The net increase in spending could easily be much less than intended if districts were to react by substituting the state money for funds they would otherwise have raised locally. Moreover, differential responses by the aid recipients could have the unintended side effect of worsening existing expenditure disparities. With the information currently available, there is no reliable or systematic way of predicting the fiscal impacts of grant actions. This makes it virtually impossible to do intelligent evaluations of grant alternatives and is a serious impediment to effective policymaking for state and federal support of education.

## THE PROBLEM

The problem, then, is to develop methods for predicting the consequences of grant-in-aid decisions so that alternatives can be compared and choices made in a more informed manner. The central analytical task is to construct models capable of simulating the response of local fiscal behavior to changes in outside aid. Since, in general, the impact of aid on local spending and taxing will vary not only according to the type of aid formula, but also according to characteristics of the particular district, a model that encompasses all the major influences on local spending is required. Inclusion of major determinants of spending other than aid is also necessary to permit empirical estimation. Thus, a more complete statement of the objective of the analysis is to *develop models of the determinants of local school district expenditures that allow the effects of changes in the level or form of outside aid to be estimated.* A first step, development of a theoretical foundation for these models, is the subject of this report. The second step is empirical estimation and validation of the models. Efforts in that direction will be discussed in a series of separate reports. The last step, of course, is application of the empirical models to actual policy alternatives. Several cycles of theoretical and empirical refinement are likely to be needed before the models are sufficiently reliable to be used for that purpose.

## THE ANALYTICAL APPROACH

The models presented here are based on the economic theory of constrained maximizing behavior. Each school district is viewed as a decisionmaking unit that faces certain trade-off possibilities between the level of support for its educational programs and the burden of educational taxes it must impose on the community.* The district seeks to reach an optimal balance between the benefits of higher per pupil expenditures, on one hand, and the disadvantages (political and other) of a higher tax rate. If it is assumed that districts behave "rationally," as the term is used in the theory of consumer demand, the effort to select the "best" expenditure-tax combination can be treated as an attempt to maximize a utility function with expenditure and tax burden

*The local school board and administration, collectively, may be thought of as "the decisionmaker." In some cases, decisions on educational spending and taxing must be approved by local voters, in which case the district electorate must also be considered part of the decisionmaking unit.

as arguments. Provision is also made for other local characteristics ("taste variables") that cause preferences to vary from one district to another. The maximization hypothesis leads to a number of empirically testable propositions about the response of district spending to changes in various economic and demographic variables, including changes in the level and form of grants-in-aid.

The models presented here are similar but not identical to those developed in connection with studies of consumer demand. In deriving the latter, it is generally assumed that the consumer has a fixed budget, or income, to use in purchasing different commodities. A school district, like any other independent unit of government, is free within limits to determine both its expenditures and income (revenue). Community income enters into the model as a major determinant of school district decisions, but not as a direct constraint on spending. Rather, as will be shown, it plays the indirect role of influencing decisonmakers' valuations of the burden (disutility) of a given tax.

A direct analog to the consumer demand model would be to assume that the school district strikes a balance between education and all other goods purchased by the community.* However, that would preclude in advance the possibility that the school district behaves differently from the way consumers would behave if they purchased education directly. We will show that the education-versus-other goods model is actually a special case of the more general model to be presented here.

## ORGANIZATION OF THE REPORT

To make the discussion as clear as possible, we begin by examining a basic model containing only the most important variables. Then a number of extensions are developed in which additional variables, relationships, and institutional factors are added to the basic structure.

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*Models of school district expenditure behavior that assume a tradeoff between education and other goods have been proposed, *inter alia*, in James A. Wilde, "The Expenditure Effects of Grant-in-Aid Programs," *National Tax Journal*, Vol. XXI, No. 3, September, 1968; and Gail S. R. Wilensky, *State Aid to Education*, Ph.D. dissertation, University of Michigan, 1968. Both studies rely on indifference curve diagrams identical to those used in two-good consumer choice models.

The basic model permits inferences to be made about effects on school expenditures and taxes of changes in community income, costs of education, the general price level, the number of pupils per household in a district, and the level and form of state and federal aid. Effects of the composition of the local tax base, noneducational taxes, categorical versus noncategorical grants, equalization features of aid formulas, and enrollment growth are dealt with in the extensions. At several points it is impossible to choose, *a priori*, among alternative theoretical specifications. In those cases, the implications of each alternative are examined, and a comparison, stressing differences that may be empirically observable, is provided. Also, as both an expository device and a step toward an econometric formulation, some specific functional forms for the preference and expenditure relationships are introduced and examined along with the more general model. Finally, it is shown that different combinations of the theoretical specifications apply to the different types of data bases that may be encountered in empirical studies. Variants of the model suitable for time series and cross-section data, local data versus state aggregates, and intrastate versus interstate studies are described.

## THE BASIC MODEL

The analysis ca [illegible] either with a set of assumptions about school district preferences (a utility function) or a more direct formulation of district behavior in "trading off" marginal school expenditures and tax burdens. The utility approach will be presented first because it is more familiar and in some ways theoretically more satisfactory. We will then take up the trade-off approach, which is more convenient to work with and easier to relate to empirically testable forms of the model. The two are equivalent if appropriate assumptions are made, and in fact, the former leads naturally to the latter.

### VARIABLES

The model represents the behavior of a school district that has $A$ pupils attending school, $N$ households, and $Y$ current dollars of personal income per year. The district spends $E$ current dollars per year for current operations of its schools and collects $T$ current dollars per year of local school property taxes. It receives $S$ current dollars of grants-in-aid per year from the state and $F$ dollars from the federal government. In developing the basic model, we assume that there are no other taxes, all school taxes are paid by households, and there are no capital outlays for the schools.

Two price variables are needed: $p_e$, an index of prices of inputs into public schooling (e.g., teachers' salaries), and $p_x$, an index of prices of all goods other than public education. Also, the symbol, $z$, will be used to represent unspecified characteristics of local communities (e.g., income distribution, ethnic composition, occupational mix) that are associated with interdistrict variations in preferences for education.

From the foregoing variables, we define the following real magnitudes:

$e = E/(p_e A)$ = real educational expenditure per pupil. This will be the measure of the level of public school services.

$t = T/(p_x N)$ = real school property taxes per household. The significance of using $p_x$ as the deflator is that taxes are measured erms of the loss of purchasing power for goods other ion.

$y = Y/(p_x N)$ = real personal income per household. Again, the unit of measure is purchasing power for other goods.

$s = S/(p_e A)$ = real state aid per pupil, measured in units of *educational* purchasing power.

$f = F/(p_e A)$ = real federal aid per pupil, measured in the same units as $s$.

DISTRICT PREFERENCES

Each local school district is assumed to have a preference function of the form

$$U = U[e, b(t, y), z]. \tag{1}$$

Real educational expenditure ("education"), $e$, is a positive good to the district.* The burden of school property taxes on each household--a function that we denote by $b$--is a negative good. It is assumed that

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*The reader may question the use of educational expenditures per pupil, rather than a measure of educational output, as an argument in the utility function. Apart from the practical consideration that there are no suitable output measures, this specification may be justified on either of the following grounds:

1. Educational output, $q$, can be assumed to be related to $e$ according to $q = F(e)$, where $F$ is an education production function. The underlying preference function is $U' = U'[F(e), b(t, y), z]$. Assume that prices are measured "correctly" so that $e$ is a true measure of physical inputs. Assume also that $F'(e) > 0$. We are only able, however, to observe the overall dependence of $U$ on $e$, i.e., $(\partial U'/\partial F)(\partial F/\partial e)$, not to infer the shape of the production function, $\partial F/\partial e$, or the marginal utility of educational output, $\partial U'/\partial F$. Therefore, we work with Eq. (1), which combines the two effects.

2. Alternatively, it can be assumed that the absence of output measures affects educational decisionmakers as well as economists, and that, for want of anything better, district officials actually use expenditures per pupil as a proxy for educational output or quality. Under that assumption, Eq. (1) is clearly the appropriate form; in fact, the relationship between educational spending and output ceases to have any logical connection to the question of how educational spending is determined.

the burden of taxes depends on both the amount of taxes collected per household, $t$, and on household income, $y$. Given $y$, the burden function increases with $t$. Given $t$, it decreases with $y$. That is, the disutility of a given level of school property taxes is assumed to decline with increasing income. These assumptions are expressed by the following specifications of the signs of first derivatives of $U$ and $b$:

$$\frac{\partial U}{\partial e} > 0, \quad \frac{\partial U}{\partial b} < 0; \quad \frac{\partial b}{\partial t} > 0, \quad \frac{\partial b}{\partial y} < 0.$$

We also assume diminishing marginal utility of education, increasing marginal disutility of property tax burdens, an increasing marginal burden of increased taxes, and a decreasing marginal burden of taxes as income rises; or, in symbols,

$$\frac{\partial^2 U}{\partial e^2} < 0, \quad \frac{\partial^2 U}{\partial b^2} < 0; \quad \frac{\partial^2 b}{\partial t^2} > 0, \quad \frac{\partial^2 b}{\partial y \partial t} < 0.$$

A final important assumption is that the preference function is additively separable in education and tax burden. That is, $U$ can be written

$$U = U_1(e, z_1) + U_2(t, y, z_2), \tag{1a}$$

where $z_1$ and $z_2$ are subsets of the set of taste variables, $z$.* Separability, of course, implies that $\partial^2 U/\partial e \partial t = \partial^2 U/\partial e \partial y = 0$. The marginal utility of an increment in educational spending is not affected by the level of taxes or income, nor is the marginal disutility of an increment in taxes affected by the level of school spending. It will be shown that the separability assumption and the assumptions about signs of the second derivatives suffice to fix the signs of effects of the exogenous variables on the level of expenditure.

*Some taste variables may be included in both $z_1$ and $z_2$.

## THE BUDGET CONSTRAINT

Assuming that no borrowing is allowed and that districts do not accumulate cash balances, the budget constraint is that school expenditure equals tax collections plus state and federal aid. In terms of the previously defined variables,

$$E = T + S + F,$$

or

$$p_e Ae = p_x Nt + p_e As + p_e Af.$$

The *form* of aid as well as the amount per pupil is important. In general, $s$ and $f$ should be interpreted as aid *functions* or formulas, rather than magnitudes. Aid may be allocated according to such factors as the district tax base or local "tax effort."* For simplicity, we will assume that federal education aid comes to districts in the form of a lump-sum grant of $f$ real dollars per pupil. However, we let state aid consist of two components: a lump-sum grant of $g$ real dollars per pupil and a matching grant that pays the district a fraction $(1 - \alpha)$, the "state share," of each dollar per pupil in excess of the state and federal lump-sum grants. Total real state aid per pupil is then given by

$$s = g + (1 - \alpha)(e - f - g). \qquad (2)$$

The parameter $\alpha$ may be interpreted as the "local share" of expenditures under the matching grant.

With the state aid formula incorporated, the budget constraint becomes

$$p_e Ae = p_x Nt + p_e A[f + g + (1 - \alpha)(e - f - g)],$$

or

$$(p_e/p_x)(A/N)\ \alpha(e - f - g) = t.$$

*Formulas that distribute aid in an inverse relationship to the local property tax base ("equalization") are discussed on pp. 52-58.

To write this more compactly, we define the following new variables: $p = p_e/p_x$, the relative price of education, $a = A/N$, the number of pupils per household. The budget constraint may then be expressed as

$$t = \alpha pa(e - f - g). \tag{3}$$

This is the form to be used in developing the basic model.

MAXIMIZATION

Maximization of $U$ subject to the budget constraint is equivalent to maximization of the Lagrangian,

$$U[e,\ b(t,\ y),\ z] - \lambda[\alpha pa(e - f - g) - t].$$

The two first-order conditions for a maximum are, for given $y$, $z$, $f$, $g$, and $\alpha$,*

$$\frac{\partial U}{\partial e} - \lambda \alpha pa = 0,$$

$$\frac{\partial U}{\partial b}\frac{\partial b}{\partial t} + \lambda = 0.$$

Combining them, we obtain

$$\frac{-\dfrac{\partial U}{\partial e}}{\dfrac{\partial U}{\partial b}\dfrac{\partial b}{\partial t}} = \alpha pa. \tag{4}$$

Equations (3) and (4) make up the basic behavioral model. If we had knowledge of the exact form of $U$, it would be possible to calculate

*This formulation depends on the assumption that $y$ and $z$ are exogenous. In the long run, it might be hypothesized that the level of school spending will play a role in attracting families of given characteristics ($y$ and $z$) to a community. If that were the case, we would have $y = y(e)$ and $z = z(e)$, both relationships, presumably, operating with a considerable lag. Such relationships are not included in the models developed here. Therefore, the results should be interpreted as applying to adjustment in the short run, during which the makeup of the community remains constant.

the derivatives in Eq. (4) and to solve the pair of equations explicitly for expenditures, $e$, and taxes, $t$. Before introducing explicit functional forms, however, it is of interest to investigate more general implications of the model concerning local expenditure and tax behavior. That task becomes easier when the model is recast in a somewhat different form.

## FORMULATION IN TERMS OF THE MARGINAL RATE OF TRADE-OFF

The expression on the left of Eq. (4) plays a role directly analogous to that of the marginal rate of substitution in the theory of consumer demand. We will term it the *marginal rate of trade-off between school spending and taxes* and denote it by $m$.* Equation (4) may be written in terms of $m$ as

$$m[e,\ b(t,\ y),\ z] = \alpha pa. \tag{5}$$

The marginal rate of trade-off may be interpreted as the amount of additional tax per household that the district would be willing to impose in order to obtain one additional dollar of educational spending per pupil. Properties of $m$ may be derived from the assumptions made previously about first and second derivatives of $b$ and $U$ and the stipulation that $U$ is additively separable. Referring to Eq. (1a), derivatives of $m$ with respect to $e$, $b$, $t$, and $y$, respectively, are

$$\begin{aligned}
\frac{\partial m}{\partial e} &= \frac{-\left(\partial^2 U_1/\partial e^2\right)}{(\partial U_2/\partial b)(\partial b/\partial t)} < 0, \\
\frac{\partial m}{\partial b} &= \frac{(\partial U_1/\partial e)\left[(\partial b/\partial t)(\partial^2 U/\partial b^2)\right]}{\left[(\partial U_2/\partial b)(\partial b/\partial t)\right]^2} < 0, \\
\frac{\partial m}{\partial t} &= \frac{\partial m}{\partial b}\frac{\partial b}{\partial t} < 0, \\
\frac{\partial m}{\partial y} &= \frac{\partial m}{\partial b}\frac{\partial b}{\partial y} < 0.
\end{aligned} \tag{6}$$

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*In consumer demand theory, the marginal rate of substitution (*MRS*) measures the number of units of one good that a consumer could give up

These inequalities allow unambiguous inferences to be made about the signs of the effects of most exogenous variables on levels of spending and taxes. Among other things, they assure that education will not be an inferior good.*

The nature of the marginal rate of trade-off function is shown graphically in Fig. 1. Each contour in the figure is an indifference

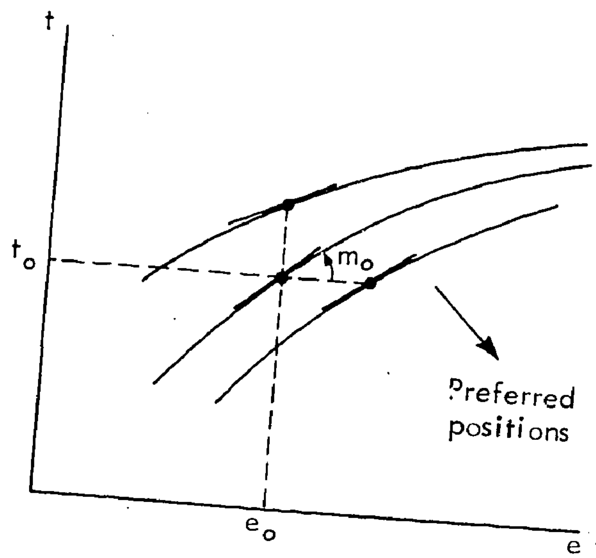


*Fig. 1--The marginal rate of trade-off between school spending and taxes (holding real income constant)*

curve, representing constant-utility combinations of $e$ and $t$ for a given level of $y$. At any point, such as $(e_0, t_0)$, the *slope* of the preference

for one unit of a second good without reducing his utility. If $x_1$ and $x_2$ are quantities of the two goods and $U = U(x_1, x_2)$, the condition of constant utility is

$$dU = \frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0, \quad \text{or} \quad \frac{dx_2}{dx_1} = -\frac{\partial U}{\partial x_1} \Big/ \frac{\partial U}{\partial x_2} = MRS.$$

In this model, the condition of constant utility while trading off education against taxes is

$$dU = \frac{\partial U}{\partial e} de + \frac{\partial U}{\partial b} \frac{\partial b}{\partial t} dt = 0, \quad \text{or} \quad \frac{dt}{de} = -\frac{\partial U}{\partial e} \Big/ \left(\frac{\partial U}{\partial b} \frac{\partial b}{\partial t}\right)$$

$$= \text{marginal rate of trade off.}$$

*The assumption of separability is sufficient, but not necessary, to establish the signs of $\partial m/\partial e$, $\partial m/\partial t$, and $\partial m/\partial y$ and to assure that $e$ is not an inferior good.

contour represents the marginal rate of trade-off, $m$, between real spending per pupil, $e$, and real taxes per household, $t$. As shown, the slope decreases with increases in either $e$ or $t$. Movement "southeastward" in the diagram represents progress towards preferred positions, i.e., combinations of higher spending and lower taxes. In terms of the diagram, the effect of an increase in $y$ would be to increase the slope of the indifference curves (value of $m$) at every point.

Heuristically, the rationale for the properties of $m$ is as follows: As educational outlay per pupil, $e$, increases, each marginal increment becomes less urgent than the preceding one and the district becomes willing to impose a progressively smaller tax to obtain it. Similarly, as the level of school taxes per household, $t$, increases, each marginal increment becomes more burdensome than the preceding one and the district becomes progressively more reluctant to raise taxes still higher. On the other hand, as income, $y$, increases, the burden of a given level of taxes becomes less, and the district becomes more willing to impose an incremental tax to finance an increment in public school support.

As was mentioned earlier, it would have been possible to begin the analysis by defining the marginal rate of trade-off function and postulating its properties. We would then have observed that in equilibrium the marginal rate of trade-off between spending and taxing must be equal to the price ratio between them. Thus, we would have arrived by a different route at Eq. (5). As we turn to examining expenditure and tax behavior, it becomes much more convenient to carry out the analysis in terms of $m$ than $U$. Therefore, Eq. (5) will be taken as the starting point for the remainder of the discussion.

## EFFECTS OF EXOGENOUS VARIABLES ON PER PUPIL EXPENDITURES

The most direct way to determine the effects on spending of changes in the exogenous variables is to substitute the budget constraint relationship into Eq. (5), thereby eliminating $t$, then to differentiate totally and solve for the change in spending, $de$. Substitution of Eq. (3) into Eq. (5) results in

$$m\{e, b[\alpha_t a(e - f - g), y], z\} = \alpha_e a.$$

Total differentiation yields

$$\frac{\partial m}{\partial e}\,de + \frac{\partial m}{\partial b}\frac{\partial b}{\partial t}\Big(\alpha pa[de - d(f + g)] + (e - f - g)d(\alpha pa)\Big) + \frac{\partial m}{\partial b}\frac{\partial b}{\partial y}\,dy + \frac{\partial m}{\partial z}\,dz = d(\alpha pa).$$

The solution for $de$ is

$$de = \frac{-\frac{\partial m}{\partial b}\frac{\partial b}{\partial y}\,dy + \alpha pa\,\frac{\partial m}{\partial b}\frac{\partial b}{\partial t}\,d(f + g) - \frac{\partial m}{\partial z}\,dz + \left[1 - (e - f - g)\frac{\partial m}{\partial b}\frac{\partial b}{\partial t}\right]d(\alpha pa)}{\frac{\partial m}{\partial e} + \alpha pa\,\frac{\partial m}{\partial b}\frac{\partial b}{\partial t}}. \quad (7)$$

Since $\frac{\partial m}{\partial e}$ and $\frac{\partial m}{\partial b}$ are both negative and $\frac{\partial m}{\partial y}$ is positive, signs of the income, aid, and relative price effects are

$$\frac{de}{dy} > 0, \qquad \frac{de}{d(f + g)} > 0, \qquad \frac{de}{d(\alpha pa)} < 0.$$

The general form of the expenditure equation is

$$e = e[\underset{+}{y},\ \underset{+}{\alpha pa(f + g)},\ \underset{-}{\alpha pa},\ \underset{+}{z}]. \quad (8)$$

where the plus and minus signs show the expected direction of the effect of each independent variable on per pupil spending. The effect of $z$ on $e$ will be positive for all $z$'s for which $\frac{\partial m}{\partial z} > 0$ and negative for all $z$'s for which $\frac{\partial m}{\partial z} < 0$.

Three implications of the model are of special interest:

1. *Partial Substitution of Lump-Sum Aid for Local Funds.* The effect of a change in state or federal lump-sum aid is always positive, but less than one, as can be seen from Eq. (7). The effect of a change in lump-sum aid, holding other exogenous variables constant, is

$$\frac{de}{d(f + g)} = \frac{\alpha pa\,\frac{\partial m}{\partial b}\frac{\partial b}{\partial t}}{\frac{\partial m}{\partial e} + \alpha pa\,\frac{\partial m}{\partial b}\frac{\partial b}{\partial t}},$$

in which all three terms in the expression on the right have the same sign. This result means that whenever a school district is given a lump-sum grant, the grantor can expect some fraction of the grant funds to go for increased per pupil spending and the remaining fraction to be used for local tax reduction. In other words, the model implies that lump-sum grants will always be partly additive to spending, partly substitutive for local taxes.

2. *Unequal Income and Aid Effects*. The model implies that the effect of an increase in lump-sum aid to a school district will be *different*, in general, from the effect of an equal increase in the personal income of district residents. Since income and aid are deflated by different price indexes, it is easiest to show this by considering the response of total district spending in current dollars to changes in current dollar amounts of income and lump-sum aid. Total district spending is, by definition, $P_eAe$; total income and total lump-sum aid are $P_xNy$ and $P_eA(f + g)$, respectively. From Eq. (7), the respective effects are

$$\frac{d(P_eAe)}{d(P_xNy)} = \frac{P_eA}{P_xN}\frac{de}{dy} = -\,pa\,\frac{1}{D}\,\frac{\partial m}{\partial b}\,\frac{\partial b}{\partial y},$$

and

$$\frac{d(P_eAe)}{d[P_eA(f + g)]} = \frac{de}{d(f + g)} = \alpha pa\,\frac{1}{D}\,\frac{\partial m}{\partial b}\,\frac{\partial b}{\partial t},$$

where $D$ represents the denominator of the right-hand side of Eq. (7). The factor $\alpha$ appears in the expression for $de/d(f + g)$ because the aid formula (Eq. 2) was written so that lump-sum and matching aid interact. That, however, is not the important difference. Even with $\alpha = 1$ (no matching aid), the two effects differ by the ratio, $-\,[(\partial b/\partial y)/(\partial b/\partial t)]$. In general, that ratio will not equal one; therefore, the income and aid effects on spending will not be identical.

For a range of reasonable assumptions about the nature of $b(t, y)$, it can be shown that the effect of a dollar increase in aid will be *greater* than the effect of a dollar increase in income. For example, if the burden function depends only on the tax *rate*, i.e., $b(t, y) = b(t/y)$, then, since $t < y$,

$$-\frac{\partial b/\partial y}{\partial b/\partial t} = \frac{t/y^2}{1/y} = \frac{t}{y} < 1.$$

If the disutility of a given *tax rate* is assumed to decline with income--which is the assumption implicit in progressive taxation--then the income effect becomes stronger. However, an absurdly rapid decrease with income would be needed to make the income effect as great as the effect of aid.*

As was pointed out in the Introduction, the implication of unequal income and aid effects is contrary to what would be obtained from a direct analog of the usual model of consumer demand. That model, in its simplest form, would have the district maximize a utility function $U(e, x)$, where $x$ = consumption of all goods other than education, subject to an income constraint, $p_e Ae + p_x Nx = p_x Ny + p_e A(s + f)$,† or $pa(e - s - f) + x = y$. Maximization of $U$ subject to the constraint would lead to the condition $m(e, x) = pa$, where $m = -[(\partial U/\partial e)/(\partial U/\partial x)]$ = marginal rate of substitution between $e$ and $x$. Using the same method as before, we would obtain as the solution for $de$,

$$de = \frac{\frac{\partial m}{\partial x}\, dy + pa\, \frac{\partial m}{\partial x}\, d(s + f) + \left[1 - (e - f)\frac{\partial m}{\partial x}\right] d(pa)}{\frac{\partial m}{\partial e} - pa\, \frac{\partial m}{\partial x}}.$$

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*To illustrate, suppose $b(t, y) = k(t/y)^{\gamma} y^{-\beta}$, where both $\gamma$ and $\beta$ are positive. Then $\partial b/\partial t = (\gamma b/t)$ and $\partial b/\partial y = -(\gamma + \beta)(b/y)$. In order to have $-[(\partial b/\partial y)/(\partial b/\partial t)] > 1$, it would be necessary to have $[(\gamma + \beta)/\gamma](t/y) > 1$, or $\beta > \gamma(y/t - 1)$. Actual values of $t$ are on the order of five percent of $y$ or less, which means $y/t \approx 20$. Therefore, assuming that $\gamma = 1$ (which is very conservative, since it implies constant rather than increasing marginal disutility of taxes), $\beta$ would have to be 19. To see how absurd this value is, consider how much of a rise in the tax rate would be needed to offset the decrease in tax burden attributable to a ten percent increase in income if $b = k(t/y)^1 y^{-19}$. The required increase would be $(1.1)^{19} = 6.1$; i.e., more than a 600 percent increase in $t$ to offset a ten percent increase in $y$.

†For simplicity, matching aid is omitted.

Effects on total spending of a dol[illegible] increase in either total income or total aid would be identical: [illegible], with the denominator represented by $D$,

$$\frac{d(P_e Ae)}{d(P_x Ny)} = pa\,\frac{de}{dy} = pa\,\frac{1}{D}\,\frac{\partial n}{\partial x} = \frac{de}{d(s + f)} = \frac{d(P_e Ae)}{d[P_e A(s + f)]}\,.$$

Just such a model has been proposed by several writers on the effects of aid on state-local spending.* Therefore, an empirical test of the inequality of income and aid effects will be critical in choosing between the model proposed here and a number of available rivals.

A final note on this issue: It is easy to see that the education versus other goods model is, in fact, nothing more than a special case of the education versus tax burden model developed here. The two budget constraints, $pa(e - s - f) + x = y$ and $pa(e - s - f) = t$, for the "other goods" and tax burden models, respectively (assuming no matching aid), are obviously identical because $x = y - t$ by definition. The respective preference functions are $U = U(e, x) = U(e, y - t)$ and $U = U[e, b(t, y)]$. The special case of the latter in which $\partial b/\partial t = - \partial b/\partial y$ is exactly equivalent to the former. That is, the two models become equivalent under the restrictive assumption that the welfare of school district decisionmakers is decreased just as much by a decrease in overall community income as by an equal increase in school taxes.

3. *Equivalence of Matching Grant and Price Effects*. The third important implication of the model is that a change in either the relative price of education, $p$, or in the pupil/houshold ratio, $a$, should be indistinguishable from an equal-proportionate change in the *local share*, $\alpha$, of expenditures under a matching grant. Since $p$, $a$, and $\alpha$ enter into the model identically, it is evident that the elasticity of expenditures with respect to all three must be the same. The important practical consequence is that quantification of the composite price effect ($\alpha pa$) will make it possible to estimate impacts of matching grants,

*See footnote, page 4.

even where there is no past experience with matching grants to observe. Since most state and federal aid to school districts in the past has taken the form of lump-sum grants ($\alpha = 1$), it is only this characteristic of the model that permits predictions of the effects of an important class of aid alternatives.

## EFFECTS OF EXOGENOUS VARIABLES ON PER PUPIL TAXES

The response of local school taxes to changes in income, aid, and prices can be calculated directly from Eq. (7) and the budget constraint, Eq. (3). From the latter,

$$t = \alpha pa(e - f - g),$$

$$dt = \alpha pa[de - d(f + g)] + (e - f - g)d(\alpha pa).$$

Substitution of $de$ from Eq. (7) into the above leads, after rearrangement, to

$$dt = \frac{-\alpha pa\,\frac{\partial m}{\partial b}\frac{\partial b}{\partial y}\,dy - \alpha pa\,\frac{\partial m}{\partial e}\,d(f + g) + \left[\alpha pa + (e - f - g)\frac{\partial m}{\partial e}\right]d(\alpha pa) - \alpha pa\,\frac{\partial m}{\partial z}\,dz}{\frac{\partial m}{\partial e} + \alpha pa\,\frac{\partial m}{\partial b}\frac{\partial b}{\partial t}}.$$

The income effect, $de/dy$, is positive; the aid effect, $de/d(f + g)$, is negative; the sign of the price effect, $de/d(\alpha pa)$, is indeterminate. Uncertainty about $de/d(\alpha pa)$ is due to what, in consumer demand theory, would be called opposing "pure price" and "income" effects. The pure price effect makes other goods more attractive relative to education when $p$ increases, thereby tending to reduce both educational spending and taxes. On the other hand, the price increase reduces the value of revenue (the analog of the "income" effect in consumer demand theory), which tends to stimulate a tax increase. There is no way to determine, *a priori*, which effect predominates.

The general form of the tax equation is

$$t = t[\underset{+}{y}, \underset{-}{\alpha pa(f + g)}, \underset{?}{\alpha pa}, \underset{+}{z}], \tag{9}$$

where plus and minus signs represent the expected polarity of each effect. The positive sign associated with $z$ applies if $z$ is defined so that $\partial m/\partial z > 0$.

EFFECTS OF LUMP-SUM AND MATCHING AID

Responses of per pupil spending to changes in the amount of lump-sum aid per pupil and the local share parameter of a matching grant were discussed above. We now ask the following somewhat different question about the effects of aid: What is the change in real per pupil spending per dollar of real per pupil aid when incremental dollars of aid are provided (1) by increasing the amount of lump-sum aid, or (2) by decreasing the local share under a matching grant? This question is different from the ones answered previously for two reasons. First, with respect to matching grants, we have thus far shown only the response of spending to a change in the local share parameter of the aid formula, but not to a change in the actual *amount* of aid. The translation from the former to the latter is not trivial because the amount of aid depends on both the formula and the local response. Second, we have not yet investigated the interaction between lump-sum aid and matching grants. As was noted earlier, when both types of aid exist, an increase in lump-sum aid will be partially offset by a decline in matching funds. It follows that the increase in spending per dollar of total aid will be greater than the increase per dollar of lump-sum aid; however, that relationship needs to be quantified. The interaction also works in the opposite direction: The change in spending per incremental dollar of matching funds will depend, among other things, on the amount of lump-sum aid provided to the district.

From the general expenditure equation, Eq. (8), we have

$$de = \frac{\partial e}{\partial y}\, dy + \frac{\partial e}{\partial (f + g)}\, d(f + g) + \frac{\partial e}{\partial (\alpha p a)}\, d(\alpha p a),$$

which, with personal income, prices, and the pupil/household ratio held constant, becomes

$$de = \frac{\partial e}{\partial(f + g)}\, d(f + g) + pa\, \frac{\partial e}{\partial(\alpha pa)}\, d\alpha\,. \tag{10}$$

Total state and federal aid to the district is, from Eq. (2),

$$aid = f + g + (1 - \alpha)(e - f - g),$$

from which

$$\begin{aligned} d(aid) &= d(f + g) + \Big\{(1 - \alpha)\big[de - d(f + g)\big]\Big\} - (e - f - g)d\alpha \\ &= \alpha d(f + g) + (1 - \alpha)de - (e - f - g)d\alpha. \end{aligned} \tag{11}$$

Looking first at the effect of a change in lump-sum aid, $f + g$, let $d\alpha = 0$. Then, substituting Eq. (11) into Eq. (10),

$$de = \frac{\partial e}{\partial(f + g)}\left[\frac{d(aid) - (1 - \alpha)de}{\alpha}\right],$$

or

$$\frac{de}{d(aid)} = \frac{\dfrac{1}{\alpha}\dfrac{\partial e}{\partial(f + g)}}{\dfrac{1}{\alpha}\dfrac{\partial e}{\partial(f + g)} + 1 - \dfrac{\partial e}{\partial(f + g)}}\,. \tag{12}$$

Earlier we showed that $\partial e/\partial(f + g)$ is always positive and less than one. The same is true, therefore, of the quantity $1 - \partial e/\partial(f + g)$ in the denominator above. It follows that $de/d(aid)$, the change in real per pupil expenditure per dollar of real per pupil aid, is also positive but less than one. Notice that the larger the value of $\alpha$, the smaller the value of $de/d(aid)$. In particular, when $\alpha = 1$ (no matching aid), $de/d(aid) = \partial e/\partial(f + g)$, which is the pure lump-sum aid effect.

Looking next at the effect of matching grants, let $d(f + g) = 0$. Substitution of Eq. (11) into Eq. (10) yields

$$de = pa \frac{\partial e}{\partial(\alpha pa)} \left[\frac{(1 - \alpha)de - d(aid)}{e - f - g}\right],$$

or

$$\frac{de}{d(aid)} = \frac{-[pa/(e - f - g)][\partial e/\partial(\alpha pa)]}{1 - [(1 - \alpha)pa/(e - f - g)][\partial e/\partial(\alpha pa)]}. \quad (13)$$

In this relationship, $de/d(aid)$ represents the increase in spending per dollar of aid when the means of increasing aid is a reduction in the local share parameter, $\alpha$. The sign of the aid effect is always positive since $\partial e/\partial(\alpha pa) < 0$. However, the magnitude of $de/d(aid)$ may be less than or greater than one, which means that a matching grant may be partly additive and partly substitutive as is lump-sum aid, or it may be *stimulative* of greater local revenue effort than would have existed in the absence of matching. A matching grant will be stimulative if $de/d(aid) > 1$, which, from Eq. (13) is equivalent to

$$-\frac{pa}{e - f - g}\frac{\partial e}{\partial(\alpha pa)} > 1 - \frac{(1 - \alpha)pa}{e - f - g}\frac{\partial e}{\partial(\alpha pa)},$$

or

$$-\frac{\partial e}{\partial(\alpha pa)} > \frac{e - f - g}{\alpha pa}.$$

This becomes more meaningful when written in terms of the elasticity of $e$ with respect to $\alpha pa$ as follows:

$$elasticity = \frac{\alpha pa}{e}\frac{\partial e}{\partial(\alpha pa)} < -\frac{e - f - g}{e}.$$

A matching grant will be stimulative if the absolute value of the elasticity is greater than the ratio of spending less lump-sum aid to total spending per pupil. If lump-sum aid is zero, the condition is simply that the elasticity be less than minus one. The effect of increasing lump-sum aid is to mak the effect of a matching grant more stimulative; i.e., a less ela ic response of $e$ to $\alpha pa$ is needed for stimulation when $f + g$ becomes greater

AID PROGRAM OPTIMIZA

The same approach may be used to determine what combinations of matching grant and lump-sum aid are optimal for achieving given objectives of the aid grantor. Possible objectives include (1) maximization of per pupil spending for a given amount of aid, (2) maximization of the local contribution, given a fixed target level of per pupil spending, (3) maximization of per pupil spending, given a fixed ratio between local contributions and aid, or (4) optimization of the distribution of spending and/or taxes among a number of districts according to some specified criterion, given the characteristics of each district and a fixed total amount of aid. To illustrate the optimality calculations, consider the first objective--selection of the lump-sum aid amount, $f + g$, and the local share parameter, $\alpha$, that maximize per pupil spending for a fixed total amount of aid.

The constraint on total aid may be represented by setting $d(aid) = 0$ in Eq. (11). The resulting equation, solved for $d(f + g)$, is

$$d(f + g) = \frac{1}{\alpha}\left[(e - f - g)d\alpha - (1 - \alpha)de\right].$$

Substituting the expression for $d(f + g)$ into Eq. (10) yields

$$de = \frac{\partial e}{\partial(f + g)}\,\frac{1}{\alpha}\left[(e - f - g)d\alpha - (1 - \alpha)de\right] + pa\,\frac{\partial e}{\partial(\alpha pa)}\,d\alpha.$$

To find the value of $\alpha$ that maximizes $e$, set $de/d\alpha = 0$, or

$$\frac{de}{d\alpha} = \frac{[(e - f - g)/\alpha][\partial e/\partial(f + g)] + pa[\partial e/\partial(\alpha pa)]}{1 + [(1 - \alpha)/\alpha][\partial e/\partial(f + g)]} = 0.$$

The optimal value of $\alpha$ would be calculated from

$$\alpha = -\frac{e - f - g}{pa} \frac{\partial e/\partial(f + g)}{\partial e/\partial(\alpha pa)},$$

provided that the functional form of $e(y, f + g, \alpha pa)$ were known. The corresponding optimal value of $f + g$ would then be obtained from the aid equation, given on p. 20.

Of the objectives listed above, item (4), which concerns the distribution of aid among a number of districts, is by far the most interesting It is also the most difficult to accomplish since the quantity to be maximized is a function of per pupil spending in *each* district and the aid constraint applies to the *sum* of lump-sum and matching grants to all districts. A general theoretical solution to the multi-district problem would be too complicated to interpret. Such a discussion is better couched in the context of a specific functional form of the district expenditure equation.

## VARIATIONS IN ASSUMPTIONS

One of the distinctive features of the model is that two of the key exogenous variables, the relative price of education, $p$, and the ratio of pupils to households, $a$, enter into the expenditure and tax equations only in the combined form $pa$. This restriction on the form of the equations derives from the basic assumption that school districts trade off public school outlays *per pupil* against tax burdens *per household*. That assumption is plausible, but not the only one worth considering. Therefore, we will briefly examine two other assumptions--also plausible *a priori*--that imply somewhat different forms of the expenditure and tax relationships.

### The Pupil/Household Ratio as an Argument in the Preference Function

A reasonable case can be made for inclusion of the pupil/household ratio, $a$, among the arguments of the school district preference function. A larger value of $a$ means either that more taxpaying households have children in school, or each taxpaying unit has more children in school, or both. That is, either more of the electorate will have a direct interest in providing an adequate level of school support or the interest of those directly involved will be more intense. Comparing across communities, therefore, we would expect--holding other things equal--that those communities with the highest values of $a$ would be most inclined to support school taxes and to vote for school board members inclined to increase educational spending.

To represent this factor, we can postulate a direct, positive effect of $a$ on the marginal rate of trade-off between expenditures and taxes. The marginal rate of trade-off would then be written,

$$m = m[e,\ b(t,\ y),\ a,\ z],$$

with $\partial m/\partial a > 0$. Referring back to the definition of $m$ as $-(\partial U/\partial e)/(\partial U/\partial t)$, it can be seen that this specification is compatible with the assumptions

$$\frac{\partial^2 U}{\partial a \partial e} > 0, \qquad \frac{\partial^2 U}{\partial a \partial t} > 0;$$

that is, the marginal utility of an increment in $e$ increases and the marginal disutility of an increment in $t$ decreases when $a$ becomes larger.

The effect of this assumption is, of course, to make $p$ and $a$ enter differently into the expenditure and tax functions. The general form of the expenditure function becomes

$$e = e(y,\ f + g,\ a,\ \alpha pa,\ z). \qquad (14)$$

The sign of $\partial e/\partial a$ (holding $\alpha pa$ constant) is positive. If the equation is written in the alternative form,

$$e = e(y,\ f + g,\ a,\ \alpha p,\ z),$$

then the sign of $\partial e/\partial a$ is uncertain because the positive effect of $a$ exerted through the preference function may or may not outweigh its effect on the ratio of exchange between taxes per household and spending per pupil. Even if the net effect is negative, we would expect the elasticity of $e$ with respect to $a$ to be less negative than the elasticity of $e$ with respect to $\alpha p$.

## Trade-off Between Education per Household and Taxes per Household

Another alternative is to assume that decisionmakers are concerned with the balance between school expenditures per household (not per pupil) and school taxes per household, but with the pupil/household ratio entering into the preference function, as above. This is somewhat more akin to a conventional consumer model than is the original formulation in that it gives prominence to the proportion of income to be spent on education. It recognizes, however, that the pupil/household ratio may function as a taste variable in affecting that decision.

Define $e' = ae$, the real amount spend on education per household, measured in units of educational purchasing power. The assumption about $U$ is that

$$U = U[e', b(t, y), a, z] = U[ae, b(t, y), a, z].$$

Maximization with respect to the budget constraint (which does not change) yields the marginal rate of trade-off equation

$$-\frac{\partial U/\partial e'}{\partial U/\partial t} = m[ae, b(t, y), a, z] = \alpha p. \qquad (15)$$

Note that only $\alpha p$ appears on the right-hand side of the equation. The factor $a$ is not present because both $e$ and $t$ are measured in units per household.

Expenditure equations derived from this model have the general form

$$e = e(y, f + g, \alpha p, a, z).$$

This is the same form as was shown on p. 24 for the first variant assumption; however, there are certain differences that emerge when the models are cast in terms of a specific functional form, as will be shown below.

## EXPLICIT FUNCTIONAL FORMS

To obtain empirically testable expenditure and tax equations, it is necessary to assume an explicit functional form for either $m$ or $U$. If the utility function is the starting point, and if a differentiable function is specified, then the marginal rate of trade-off function, $m$, can always be derived. The reverse is not always true. But whichever function is taken as the starting point, derivatives of $m$ must satisfy Eq. (6). Other than that, the main criteria for selection of a functional form are (1) empirical convenience, which means that the resulting expenditure and tax equations should be linear in the parameters, if possible; and (2) avoidance of artificial *a priori* restrictions on values of the parameters. The three criteria are not easy to satisfy. Only one functional form has been found, thus far, that conforms to Eq. (6) and is linear in all parameters. Several other forms satisfy Eq. (6) and are "nearly" linear--i.e., they are nonlinear in one variable (the effective price variable, $\alpha p a$) and in one parameter. These forms are presented below. The expenditure equations associated with several of the forms will be used in the following section to make the discussion of "extended" models more concrete.

### The Linear Expenditure Model

The preference function that satisfies Eq. (6) and yields linear expenditure and tax equations is a modified form of the additive-logarithmic utility function used to derive the linear expenditure system of consumer economics.* The form is

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*The linear expenditure system is discussed in detail in R. A. Pollak and T. J. Wales, "Estimation of the Linear Expenditure System," *Econometrica*, Vol. 37, No. 4, October 1969, pp. 611-628; and in A. S. Goldberger, "Functional Form and Utility: A Review of Consumer Demand Theory," unpublished draft, University of Wisconsin, 1968.

$$U = b_1 \log (e - c_1) + b_2 \log (c_2 - t),$$

where the $b$'s and $c$'s are parameters and $b_1 + b_2 = 1$. The parameter $c_1$ can be interpreted as a "minimum required" level of educational outlay per pupil; $c_2$ can be interpreted as the "maximum tolerable" tax per household. $U$ is undefined for $e < c_1$ or $t > c_2$. The parameters $b_1$ and $b_2$ are the incremental shares of additional lump-sum aid that the district will allocate to educational spending and tax reduction, respectively.* The marginal rate of trade-off function is

$$m(e, t) = -\frac{\partial U/\partial e}{\partial U/\partial t} = \frac{b_1}{b_2}\left(\frac{c_2 - t}{e - c_1}\right).$$

Derivatives of $m$ with respect to $e$ and $t$ are

$$\frac{\partial m}{\partial e} = -\frac{b_1}{b_2}\frac{c_2 - t}{(e - c_1)^2} < 0 \qquad \frac{\partial m}{\partial t} = -\frac{b_1}{b_2}\left(\frac{1}{e - c_1}\right) < 0.$$

Both have the required negative signs. The simplest way to include the effect of income is to let the parameter $c_2$ depend positively on $y$, say, by letting $c_2 = d_0 + d_1 y$, with $d_1 > 0$. Then the derivative of $m$ with respect to $y$ is

$$\frac{\partial m}{\partial y} = \frac{b_1}{b_2}\frac{d_1}{e - c_1} > 0,$$

which has the required positive sign.

Expenditure and tax equations are obtained by substituting the expression for $m$ into Eq. (5), then solving Eqs. (3) and (5) for $e$ and $t$. Making use of the specification, $b_1 + b_2 = 1$, the pair of equations is

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*These properties of the linear expenditure model are analogous to properties of the consumer demand model described by Goldberger, ibid, p. 47.

$$\frac{b_1}{1 - b_1}\left(\frac{d_0 + d_1 y - t}{e - c_1}\right) = \alpha pa,$$

$$t = \alpha pa(e - f - g).$$

Solutions for $e$ and $t$, omitting intermediate steps, are

$$e = c_1 + b_1\left[f + g + d_0(1/\alpha pa) + d_1(y/\alpha pa) - c_1\right],$$

$$t = d_0 + d_1 y - b_2\left[\alpha pa(f + g) - \alpha pac_1 + d_0 + d_1 y\right].$$

The linear equations that would be used for empirical estimation are

$$\begin{aligned} e &= \beta_0 + \beta_1(f + g) + \beta_2(1/\alpha pa) + \beta_3(y/\alpha pa), \\ t &= \gamma_0 + \gamma_1 \alpha pa(f + g) + \gamma_2 \alpha pa + \gamma_3 y, \end{aligned} \tag{16}$$

where
$$\begin{aligned} \beta_0 &= c_1(1 - b_1), \\ \beta_1 &= b_1, \\ \beta_2 &= b_1 d_0, \\ \beta_3 &= b_1 d_1, \\ \gamma_0 &= d_0(1 - b_2), \\ \gamma_1 &= - b_2, \\ \gamma_2 &= b_1 c_1, \\ \gamma_3 &= d_1(1 - b_2). \end{aligned}$$

An Exponential Form of the Marginal Rate of Trade-off Function

A form of $m$ that satisfies Eq. (6) is the exponential function

$$m = \beta_0 \varepsilon^{-\beta_1 e - \beta_2 t} \phi(y), \qquad \text{all } \beta\text{'s} > 0,$$

where $\epsilon$ is the base of the natural logarithms and $\phi(y)$ is any function of $y$ with properties $\phi(y) > 0$ and $\phi'(y) > 0$.* Derivation of the expenditure and tax equations corresponding to this form is very simple. Taking logarithms of both sides,

$$\log m = \log \beta_0 - \beta_1 e - \beta_2 t + \log \phi(y).$$

From Eq. (3) and Eq. (5), respectively, $t = \alpha pa(e - f - g)$ and $m = \alpha pa$. Therefore,

$$\log (\alpha pa) = \log \beta_0 - \beta_1 e - \beta_2 \alpha pa(e - f - g) + \log \phi(y).$$

Solving for $e$, we obtain

$$e = \frac{\log \beta_0 + \log \phi(y) + \beta_2 \alpha pa(f + g) - \log (\alpha pa)}{\beta_1 + \beta_2 \alpha pa}.$$

A convenient specification of the income term is to set $\phi(y) = \epsilon^{\beta_3 y}$. Then $\log \phi(y) = \beta_3 y$. Redefine parameters as follows: $\gamma_0 = (1/\beta_1) \log \beta_0$; $\gamma_1 = - 1/\beta_1$; $\gamma_2 = \beta_2/\beta_1$; and $\gamma_3 = \beta_3/\beta_1$. The resulting expenditure equation is

$$e = \frac{\gamma_0 + \gamma_3 y + \gamma_2 \alpha pa(f + g) + \gamma_1 \log (\alpha pa)}{1 + \gamma_2 \alpha pa}. \tag{17}$$

From the budget constraint, the tax equation is,

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*Another exponential form of $m$,

$$m = \beta_0 e^{-\beta_1} t^{-\beta_2} \phi(y), \qquad \text{all } \beta\text{'s} > 0$$

also satisfies Eq. (6). However, it leads to a transcendental equation that can not be solved explicity for $e$ and $t$.

$$t = \frac{\alpha pa\left[\gamma_0 + \gamma_3 y - (f + g) + \gamma_1 \log(\alpha pa)\right]}{1 + \gamma_2 \alpha pa} . \qquad (17a)$$

Both equations are nonlinear in the price variable, $\alpha pa$, and in the parameter $\gamma_2$. Although the equations conform to Eq. (8) with respect to the signs of the income, aid, and price effects, one new factor is added: Both income and aid effects vary in magnitude according to the value of $\alpha pa$. The nonlinear form creates some difficulties, but it is certainly feasible to estimate the equation by iteration over the single nonlinear parameter, $\gamma_2$. We judge the form to be sufficiently manageable to pose a viable alternative to the linear expenditure model discussed above.

## A Linear Marginal Rate of Trade-off Function

Perhaps the simplest functional form to analyze is the linear marginal rate of trade-off function,

$$m = \beta_0 - \beta_1 e - \beta_2 t + \beta_3 y, \qquad \text{all } \beta\text{'s} > 0.$$

Strictly speaking, this form is inadmissable because $m$ becomes negative for some values of $e$ and $t$. However, viewing it as a linear approximation of $m$, valid only within a certain range, we can derive expenditure and tax equations that are very similar to those of the exponential model.

Making use of Eqs. (3) and (5), we have immediately,

$$\beta_0 - \beta_1 e - \beta_2 \alpha pa(e - f - g) + \beta_3 y = \alpha pa.$$

This yields,

$$e = \frac{\beta_0 + \beta_3 y + \beta_2 \alpha pa(f + g) - \alpha pa}{\beta_1 + \beta_2 \alpha pa} .$$

Redefining variables as $\gamma_0 = \beta_0/\beta_1$,

$$\gamma_1 = -1/\beta_1,$$
$$\gamma_2 = \beta_2/\beta_1,$$
$$\gamma_3 = \beta_3/\beta_1,$$

we obtain the expenditure equation

$$e = \frac{\gamma_0 + \gamma_3 y + \gamma_2 \alpha p a(f + g) + \gamma_1 \alpha p a}{1 + \gamma_2 \alpha p a}. \tag{18}$$

This is identical to Eq. (17) except that the price term in the numerator is linear instead of logarithmic. We omit the tax equation since it is identical to Eq. (17a) except for the same log-to-linear transformation.

Note that with either the exponential or linear form of $m$, it would be possible to obtain linear expenditure and tax relationships by assuming $\gamma_2 = 1$ (i.e., $\beta_1 = \beta_2$). However, that would constrain the lump-sum aid effect, $de/d(f + g)$, to be unity, which contradicts the results of the general analysis and is clearly unduly restrictive.

## Effects of Variations in Assumptions

Finally, we examine the effects of the variations in assumptions discussed on pages 23-26 when specific functional forms are applied to the model. As an illustration, we use the last functional form discussed, which corresponded to the linear form of the marginal rate of trade-off function.

The first alternative assumption was inclusion of the pupil/household ratio, $a$, among the arguments of the preference function. The effect on the linear marginal rate of trade-off function would be to add a positive term in $a$, i.e.,

$$m = \beta_0 - \beta_1 e - \beta_2 t + \beta_3 y + \beta_4 a, \qquad \text{all } \beta\text{'s} > 0.$$

The solution for $e$ is the same as Eq. (18) except that a term in $a$ appears in the numerator. The equation to be estimated is

$$e = \frac{\gamma_0 + \gamma_4 a + \gamma_3 y + \gamma_2 \alpha p a(f + g) + \gamma_1 \alpha p a}{1 + \gamma_2 \alpha p a}. \qquad (19)$$

In fact, it is clear that any taste variable or community characteristic appearing in the preference funtion would, like $y$ and $a$, reappear in an additional linear term in the expenditure equation.

The second alternative was to include educational expenditure per household rather than per pupil in the preference function. That would make the linear marginal rate of trade-off function

$$m = \beta_0 - \beta_1 ae - \beta_2 t + \beta_3 y + \beta_4 a, \qquad \text{all } \beta\text{'s} > 0.$$

The equation to be solved for $e$ is

$$\beta_0 - \beta_1 ae - \beta_2 \alpha pa(e - f - g) + \beta_3 y + \beta_4 a = \alpha p,$$

and the expenditure equation is

$$e = \frac{\gamma_0 + \gamma_4(1/a) + \gamma_3(y/a) + \gamma_2 \alpha p(f + g) + \gamma_1(\alpha p/a)}{1 + \gamma_2 \alpha p}. \qquad (20)$$

Equations (18), (19), and (20) constitute three alternative models to be estimated, all conforming to the assumption that the marginal rate of trade-off function is linear. It is possible to formulate similar alternatives of the linear expenditure model, the model derived by assuming an exponential trade-off function, and any other form that satisfies the requirements of the basic theoretical model. There is, therefore, a considerable array of alternatives to test in any empirical study. That array will increase several fold as we consider possible extensions to the model in the following section.

## III. EXTENSIONS OF THE MODEL

A number of factors not considered in developing the basic model clearly have a bearing on school district financial decisions. The following additional variables and relationships will now be examined:

1. *Variations in nonschool taxes* from one district to another will lead to differences in per pupil spending. In general, other things being equal, the higher the level of each such tax, the lower will be local ability and willingness to spend for education. However, each major type of tax has a different effect and each must be examined separately.
2. *Variations in the composition of local property tax bases* will also lead to differences in fiscal behavior. The most important distinction to draw is between property owned by district residents and voters (primarily residential property) and property owned by nonresidents (primarily business property). Other things being equal, the higher the proportion of the latter, the more willing the district is likely to be to tax for education. However, there are a number of alternative hypotheses about the nature of the tax base composition effect, and these require analysis and comparison.
3. *Equalization features of state aid formulas* cause amounts of per pupil aid to vary from one district to another within a state, usually in inverse relationship to the local tax base. This tends to reduce the interdistrict expenditure variations that would be observed if each district were left to rely solely on its local tax base and a uniform per pupil grant. The analysis of equalization provisions must take account of their interaction with tax base composition effects and other relationships in the basic model.
4. *Categorical and noncategorical grants-in-aid* may have different impacts on spending. Thus far, the discussion has focused, by implication, on noncategorical grants, since nothing has been said about restrictions on the use of aid funds. In this section, we will also investigate the effects of grant conditions that impose binding constraints on the aid recipients.

5. *Growth in district enrollment* is likely to have a negative effect on per pupil spending for current operations because it diverts funds to capital outlay (building construction). No attempt will be made to develop a complete model of school district capital outlay and debt financing, but a simple analysis will be presented of the effect of growth-induced capital spending on current expenditures.

Each of the five factors implies certain modifications of the basic model. In discussing these "extensions" of the model, we will treat each factor separately except where interactions among factors are critical. Later, we will discuss the combinations of factors that must be incorporated into the model when it is applied to different bodies of empirical data.

## EFFECTS OF NONSCHOOL TAXES

There are likely to be variations among school districts in the amounts of federal and state income taxes, state and local sales taxes, and local, nonschool property taxes paid by their residents. Each category of "other" taxes has a different effect on district fiscal behavior. Also, certain interactions among taxes and between school and nonschool taxes must be taken into account.

### Federal and State Income Taxes

The effect of income taxes is to reduce the amount of community income, which, in turn, increases the burden of any given level of school taxes. The simplest way to take account of this effect is to redefine the income variable in the district preference function as $y_D$, disposable income, where $y_D = y - t_S - t_F$, and $t_S$ and $t_F$ are real state and federal income taxes per household, respectively. Use $t_e$ to denote school property taxes per household, formerly represented by $t$. The district preference function then becomes

$$U = U\left[e,\ b(t_e,\ y_D),\ z\right] = U\left[e,\ b(t_e,\ y - t_s - t_F),\ z\right] .$$

The effects of increases in $t_S$ and $t_F$ are exactly equal, but opposite, to the effects of identical increases in $y$.

*Deductibility of School Taxes.* This simple formulation does not suffice if we wish to take account of the deductibility of school property taxes from the taxable income on which federal and some state income taxes are paid. Instead, we must work with income tax *schedules.* Let $t_F(y)$ and $t_S(y)$ be the average federal and state income tax rates payable by the residents of a community with average taxable income $y$.* Then state and federal income taxes per household with deductible property taxes are $t_S = t_S(y - t_e)$ and $t_F = t_F(y - t_e)$. This makes the amount of disposable income per household an endogenous rather than exogenous variable of the model and a variable that is itself dependent on $t_e$; i.e.,

$$y_D = y - t_S - t_F = y - t_S(y - t_e) - t_F(y - t_e).$$

The form of the preference function becomes

$$\begin{aligned} U &= U\{e,\ b[t_e,\ y_D(y,\ t_e)],\ z\} \\ &= U\{e,\ b[t_e,\ y - t_S(y - t_e) - t_F(y - t_e)],\ z\}. \end{aligned}$$

The effect of deductibility of school property taxes is to reduce the marginal disutility of a given school tax increa . This can be seen from

$$\frac{\partial U}{\partial te} = \frac{\partial U}{\partial b}\left\{\frac{\partial b}{\partial t_e} + \left[\frac{\partial t_S}{\partial (y - t_e)} + \frac{\partial t_F}{\partial (y - t_e)}\right]\frac{\partial b}{\partial y_D}\right\}.$$

*In general, the average income tax payable by residents of a community depends on the distribution of income among residents as well as the average income. In this analysis, we neglect the distributional factor and assume, in effect, that the income of each household equals the average income per household of the community.

The direct negative effect of the tax increase, represented by $\partial b/\partial t_e$, is partly offset by the increase in $y_D$ that occurs by virtue of the deductibility of higher school taxes from the income tax base. Spending per pupil and school taxes per household will both be higher because of deductibility than they would have been otherwise. Moreover, the effect of progressive federal taxation $[t_F''(y) > 0]$ is to provide a relatively greater stimulus to school spending in higher income communities.

*An Alternative Treatment of Deductibility.* Inman* has proposed an alternative treatment of deductibility. His approach implies that it is not the total amount of school taxes per household, $t_e$, that should enter into the preference function, but the *net* amount after deductibility of income taxes is taken into account. The net school property tax, which we denote by $t_e'$, is

$$t_e' = t_e - \Delta t_S - \Delta t_F,$$

where $\Delta t_S = t_S(y) - t_S(y - e)$, and $\Delta t_F = t_F(y) - t_F(y - e)$. The corresponding form of the preference function would be

$$U = U[e, b(t_e', y_D), z].$$

This is a more manageable form than that given on p. 35 because the dependent variable only enters in once.

The general implications of this model are similar to those of the initial formulation, especially with respect to the stimulatory effect of property tax deductibility on school spending. However, it is obvious that different forms of the expenditure and tax equations are implied. The same results are not obtained by viewing deductibility as something that reduces the property tax bill as by treating it as something that augments disposable income. The point at issue is, Which more accurately measures the disutility of taxes to district decisionmakers:

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*Robert P. Inman, *Four Essays on Fiscal Federalisim*, Ph.D. dissertation, Harvard University, 1971. Inman applied his model to cities, not school districts. We have translated his approach into the terminology used in developing our models.

the total amount of property taxes levied or the net amount when deductibility is taken into account? If disutility is assumed to hinge on adverse voter reaction to taxes, the question becomes, Which is more closely associated with voter displeasure: the gross or net amount of the property tax? If voters appreciate that the "real" impact of school taxes depends on the income tax offset, then the Inman version would be correct; but if they respond to the total school tax bill and then react separately to income taxes, the initial version of the model would be more appropriate. Clearly, the issue can only be settled by empirical comparison of the two versions.

## Sales Taxes

The effect of a sales tax is to raise the price of "other goods." Therefore, if sales taxes are higher in one school district that in another, the relative price of education is lower in the first than in the second. The effect on school spending and taxes of imposing a sales tax should be indistinguishable from the effect of an increase in $p_x$, the price of other goods, by the effective sales tax rate.

Let $t_{SLS}$ be the amount of sales tax paid per household per year. Then $t_{SLS}/(y_D - t_e)$ is the effective sales tax rate on all goods other than public schooling.* Denote that rate by $r_{SLS}$. The price of other goods, counting the sales tax in the price, is $p_x(1 + r_{SLS})$. The relative price of education is $p_e/[p_x(1 + r_{SLS})]$, or $p/(1 + r_{SLS})$. Therefore, imposition of a sales tax with an effective rate $r_{SLS}$ is equivalent to a reduction in the relative price of public school inputs by a fraction $1/(1 + r_{SLS})$.

Unlike the income tax, which enters into the model only through the preference function, the sales tax enters both through the preference function and the budget constraint. Instead of $y_D$ in the preference function, we now have $y_D/(1 + r_{SLS})$, which is the amount of other goods that disposable income will buy, given the existence of the sales tax. The modified budget constraint is

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*The effective sales tax rate is different from the nominal sales tax because not all other goods are taxed. In particular, there are no sales taxes on public services (other than education) that enter into the "other goods" category.

$$t = \left[\alpha pa/(1 + r_{SLS})\right](e - f - g),$$

where $\alpha pa/(1 + r_{SLS})$ is the new ratio at which it is possible to trade off units of other goods for units of public school inputs. The effect on expenditure and tax equations--both the general ones and those corresponding to specific functional forms of $U$--is to replace $p$ with $p/(1 + r_{SLS})$ and $y_D$ with $y_D/(1 + r_{SLS})$ whenever those variables occur.

The sign of the net effect on real per pupil spending of an increase in the sales tax rate is uncertain because the effects on price and income terms of the expenditure equation work in opposite directions. The net effect depends on relative magnitudes of the respective coefficients.

Nonschool Property Taxes

Property tax levies by other local governments (cities, counties, special districts) may affect school district behavior in either or both of the following ways: (1) by reducing community disposable income, thereby increasing the burden of any given amount of school property taxes; or (2) by increasing the total of school plus nonschool property taxes, thereby increasing the disutility of any school tax increment.

The first effect is identical to the effect of income taxes and would be represented within the model in the same way. That is, we would write the preference function as

$$U = U[e, b(t_e, y_D), z],$$

where $y_D$ would be defined by $y_D = y - t_S - t_F - t_0$, with the new variable, $t_0$, being other local property taxes per household.

The second effect would exist under the assumption that the welfare of school district decisionmakers depends on the *total* local property tax bill, not on the school tax bill alone. It would then be appropriate to replace $t_e$ in the preference function with total property taxes, $t_e + t_0$. The preference function would then become

$$U = U\left[e,\ b(t_e + t_0,\ y_D),\ z\right].$$

There is an important difference in implications. Under the first assumption, an increase in nonschool property taxes would have exactly the same effect on school spending as an equivalent decrease in community income. Under the second assumption, the effect would be the same as that of an equivalent decrease in the dollar amount of lump-sum aid. An important feature of the basic model is that the magnitudes of the two effects are different.

Neither assumption seems fully satisfactory. On one hand, it does not seem adequate to treat nonschool property taxes like any other subtraction from income because a competitive relationship probably exists between the two property tax levies.* On the other hand, it is implausible to expect that school officials would react only to the total property tax bill since the voters, whose reaction presumably counts, are informed of the magnitude of the school tax component and are able to make the distinction.

A third, but vaguer, alternative is to treat the level of nonschool property taxes as a separate argument in the utility function; i.e., to write

$$U = U\left[e,\ b(t_e,\ y_D),\ t_0,\ z\right],$$

where $y_D = y - t_S - t_F$ and $\partial U/\partial t_0 < 0$. This information allows the effect of an increase in nonschool property taxes to fall in between the effects of equivalent changes in income or lump-sum aid. In sum, there

*If fiscal decisions by school district officials depend on tax rates set by officials of other local juridictions, then it is reasonable to assume that the relationship is reciprocal; i.e., the level of non-school property taxes should be affected by decisions on school tax levies. That would make nonschool taxes an endogenous variable of the model. In this analysis, however, we will neglect that feedback relationship (and similar relationships that could be hypothesized in connection with other "exogenous" variables) and continue to develop the closed model of a school district responding to external influences that it cannot control.

are three alternative hypotheses about the effects of nonschool property taxes to be tested, each of which can be translated into forms corresponding to different functional forms of $U$.

Tax Financing of State Aid

Inclusion of state taxes in the model serves as a reminder that state aid is not "free," but must be financed out of state taxes, which ultimately must be payed by taxpayers in the local districts. It follows that if a given increment in state aid to a district is accompanied by an increment in state taxes, the positive effect of the aid on local spending will be offset to a degree by the reduction in local disposable income.

Assuming no federal aid, no state matching aid, and state taxes consisting of income taxes only, we may write the general form of the expenditure equation as

$$e = e(y - t_S, pag, pa, z).$$

Suppose that an increment in aid is fully financed, on a statewide basis, by an increment in state income taxes. Then, in an "average" district, we would have $dt_S = d(pag)$

$$de = \left(\frac{\partial e}{\partial (pag)} - \frac{\partial e}{\partial (y - t_S)}\right) d(pag).$$

As long as the aid effect exceeds the income effect* there would be a net positive effect on real per pupil spending even though the same amount of money is taxed away from local citizens as is returned to the school district in the form of aid. This result--which sharply contradicts what would be predicted by a direct analog of the consumer demand model--derives from our initial decision to formulate the model in terms of preferences of district decisionmakers, rather than those of the community.

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*See p. 15 ff.

More generally, of course, we would not expect the increment in income taxes paid by the community to be exactly equal to the increment in state aid to the district. With a progressive state tax system, districts with above average household income would be likely to pay more in taxes than they would receive in aid. Moreover, if aid funds were distributed on an "equalized" basis (see the discussion of equalization below), districts with high tax bases would receive lower-than-average per pupil grants. A district with both a high tax base and a high level of household income might have to pay several times as much in additional taxes as it received in additional aid. In such a case, the combined effect of the aid-income tax increase could be to reduce per pupil spending. In the aggregate, however, the model predicts that the effect of an aid increase will be positive even when the increase is fully financed out of taxes. The effect will not be as positive, obviously, as in the case where more aid is provided without a corresponding tax increase.

## THE LOCAL PROPERTY TAX BASE AND ITS COMPOSITION

Although we have acknowledged that local school taxes consist of levies on property, that fact has not yet influenced the form of the model. The tax variable, $t_e$, could equally well have represented the amount of any other form of tax, and we would have written the preference function and budget constraint equation exactly the same way. However, the form of tax *does* make a difference. It is just that the main effect appears not to be due to the use of property taxation *per se*, but to the fact that property tax bases are not homogeneous from one district to another. Therefore, our analysis will focus on the tax base composition effect.

### Tax Base and Tax Rate

One can readily account for interdistrict variations in tax rate by introducing tax rate and property tax base variables into the basic model. Recall that the two equations of the basic model are

$$m\left[e,\ b(t,\ y_D),\ z\right] = \alpha pa, \tag{5}$$

$$t = \alpha pa(e - f - g). \tag{3}$$

Under the original assumption that all school property taxes are paid by district residents, we can write a tax identity,

$$t = rv,$$

where $v$ is the real assessed value of taxable property per household* and $r$ is the property tax rate. Once Eqs. (3) and (5) are solved for $t$, the district tax rate can be calculated as $r = t/v$. However, it is evident that the substitution of $rv$ for $t$ has no effect whatever on the determination of school district spending. The model simply says that the value of $v$ will determine the tax rate needed to raise the predetermined tax per household, $t$. The amount of property per household, $v$, exerts no independent influence on levels of school spending or taxes.

## The Tax Rate as an Argument in the Preference Function

It may be of interest to examine an alternative model in which the local property tax base does have an independent effect, contrary to what was said above. That model is one in which the *tax rate*, rather than the amount of tax per household, enters into the utility function. A possible rationale for such a formulation is that citizens may judge the fiscal performance of their school officials by comparing the school tax rate in their district with the prevailing rate in other districts.

A form of $U$ that embodies this rationale is

$$U = U\left[e,\ b(r,\ r_0,\ y),\ z\right],$$

where $r$ is the tax rate in the district in question and $r_0$ is the prevailing tax rate in the surrounding area. The burden of a given tax

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*It is assumed that assessed value is a constant fraction of market value across districts. Real assessed value is defined by deflating the current dollar value by the general price index, $p_x$.

rate, $r$, is assumed to increase with $r$ and to decrease with $r_0$ and $y$. The budget constraint, obtained by substituting $rv$ for $t$ in Eq. (3), is

$$r = \frac{\alpha pa}{v}(e - f - g).$$

Maximization yields a marginal rate of trade-off equation,

$$m\left[e,\ b(r,\ r_0,\ y),\ z\right] = \frac{\alpha pa}{v}.$$

The expenditure and tax rate equations obtained by solving the above pair of equations will have the forms

$$e = e\left(r_0,\ y,\ z,\ f + g,\ \frac{\alpha pa}{v}\right),$$

$$r = r\left(r_0,\ y,\ z,\ f + g,\ \frac{\alpha pa}{v}\right).$$

Differences in $v$ will affect expenditure levels through the price-like term, $\alpha pa/v$. The model implies that, other things being equal, per pupil spending will tend to increase with $v$ because higher $v$ makes it possible to support a given level of spending with a lower tax rate.

The shortcoming of this model is that it depends on a "tax rate illusion." Decisionmakers in low tax rate districts are assumed to be better off even though they may be taxing away as much of their citizens' incomes as officials of districts with lower tax bases and higher rates.* The intuitively pleasing feature of the analysis is that it

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*The implausibility of this model emerges when we compare districts that differ in the ratio of assessed property value to income. Suppose that two districts are identical with respect to disposable income per household and all other characteristics except assessed value. Suppose that assessed value per household is one and one-half times as much in the second district as in the first (say, for example, because land values are higher in the second district). A model that had tax rate as the argument in the preference function would predict equal tax rates in the two districts; but equal tax rates would mean that taxpayers in implies a positive relationship between assessed property value and school spending, which, as is well known, exists in the real world. We will show, however, that the same result is implied by a model that takes tax base composition into account without the assumption that tax rate is the proper measure of the burden of school taxes on the citizens.

## Composition of the Tax Base

The property tax base of each district consists of both property owned by district residents and property owned by nonresidents. Most residential property is likely to be owned by local residents and voters. The same is not nearly as likely to be true of business property, such as commercial, industrial, and agricultural holdings. It is reasonable to assume that it makes a difference to district officials whether school taxes fall on local residents and voters or on outsiders. Under that assumption, differences in the composition of the tax base (proportion of property owned by nonresidents to total property) wi‑. lead to differences in levels of school taxes and expenditures.

As a practical matter, it is impossible to obtain data on resident versus nonresident ownership of property. Therefore, we will assume that residential property is owned by local residents and business property by outsiders and conduct the analysis in terms of the residential versus business distinction. Accordingly, we define two new variables, $v_R$, the real assessed value of residential property per household, and $v_B$, the real assessed value of business property per household in the district. The two are related to total assessed value per household, $v$, by $v = v_R + v_B$. It is also convenient to decompose the property tax into two components, the school property tax on residential property

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the second district would be sacrificing one and one-half times as much of their incomes for education as the taxpayers in the first, even though incomes and all prices were the same. If anything, the effect should be in the opposite direction because home owners in the high tax base district have to spend more for housing, leaving less for all other purposes including education. Even if that effect were ignored, however, it is difficult to see a rationale for dissimilar behavior in the two districts of the kind predicted by the tax rate versus spending model when the real trade-off possibilities are so nearly identical.

per household, $t_R$, and the school property tax on business property (also per household, for symmetry), $t_B$. These are defined by $t_R = rv_R$ and $t_B = rv_B$. It is assumed that the same tax rate, $r$, must be applied to both residential and business property. The total school property tax per household, $t_e$, is related to the two components by $t_e = t_R + t_B$.

The Effect of Tax Base Composition on School Spending

We examine local fiscal behavior under each of two assumptions about district preferences with respect to property taxes:

- *Case 1:* Only taxes on district residents count. There is no disutility associated with taxes on business property.
- *Case 2:* Both components of the property tax count, but they have different weights in the eyes of district decisionmakers.

Case 1 leads to a simple model in which the tax base composition variables play a role similar to that of the local share parameter of a matching grant. Case 2 requires a more elaborate analysis in which two interrelated tax burden arguments enter into the district preference function.

*The Simple Model (Case 1).* The assumption that only taxes on residents count can be expressed by substituting $t_R$ for $t_e$ in the preference function. That makes

$$U = U[e,\ b(t_R,\ y_D),\ z].$$

The budget constraint remains

$$t_e = \alpha p a(e - f - g);$$

however, by making use of the identities that interrelate components of school taxes and the property tax base, we have

$$t_e = rv = \frac{t_R}{v_R}\, v,$$

which permits us to write the constraint in terms of $t_R$ as

$$t_R = \frac{v_R}{v}\alpha pa(e - f - g).$$

The quantity $v_R/v$ is, of course, the fraction of the property tax base that is residential.

By comparison with Eqs. (5) and (3), it is apparent that these equations have exactly the same form as the equations of the basic model except that $t_R$ replaces $t$ and the composite price term of the basic model, $\alpha pa$, has become $\alpha pav_R/v$. The resulting expenditure equation has the same form as Eq. (8) except for the transformed price term; i.e.,

$$e = e[y_D, f + g, z, (v_R/v)\alpha pa].$$

The equation for $t_R$ is the same as the equation for $t_e$ in the basic model except for the same transformation:

$$t_R = t_R[y, f + g, z, (v_R/v)\alpha pa].$$

Total educational taxes per household is given by

$$t_e = (v/v_R)t_R = (v/v_R)t_R[y, f + g, z, (v_R/v)\alpha pa].$$

There is a simple interpretation of the results. With $v_B$ dollars of business property for every $v_R$ dollars of residential tax base, each one dollar of tax levied on residential property brings in a total of $1 + v_B/v_R$ dollars in property tax revenue. The share of property taxes paid by residents is, therefore, $1/(1 + v_B/v_R) = v_R/(v_R + v_B) = v_R/v$. Each dollar of local revenue raised by residents is matched by revenues collected from business property owners in such a way that the residents' share of property taxes is $v_R/v$. That ratio enters into the model in exactly the same way as the local share parameter, $\alpha$, of a state matching grant. Therefore, the effect of variations in $v_R/v$ among districts should be indistinguishable from the effect--in the absence of tax base variations--of a program of matching aid in which each district was assigned a local share, $v_R/v$.

According to this model, the effect of an increase in $v$ with $v_R$ held constant (i.e., an increase in $v_B$) will be to increase per pupil spending. The effect of an increase in $v_R$ with $v_B$ held constant is to increase the ratio $v_R/v$, which implies a decrease in $e$. Therefore, the answer to the question, "Does per pupil spending increase with assessed property value?" appears indeterminate. The model says "yes" if the increase is in business property; "no" if it is in residential property. However, this answer neglects the fact that residential property and disposable income are likely to be closely related--a phenomenon that modifies the result, as we will show below.

*A Two-Tax Model (Case 2).* A model in which residential and business property taxes both count can be developed from the preference function

$$U = U\left[e,\ b_R(t_R,\ y),\ b_B(t_B,\ w)\right],$$

where $b_R$ and $b_B$ are property tax burdens on residential and business property (both measured per household), and $w$ is some measure of business income (e.g., sales, profits), which is assumed to affect the burden of a given level of business property tax. From the budget constraint and the identities defining $t_R$, $t_B$, $v_R$, and $v_B$, we can make the substitutions, $t_R = (v_R/v)t$, $t_B = (v_B/v)t$, and $t = \alpha pa(e - f - g)$. Assuming that $U$ is separable in $e$, $b_R$, and $b_B$, that gives us a preference function in which $e$ is the only dependent variable:

$$U = U_1(e) + U_2\left(\frac{v_R}{v}\, pa(e - f - g),\ y\right) + U_3\left(\frac{v_B}{v}\, pa(e - f - g),\ w\right). \quad (21)$$

The first-order condition for maximization of $U$ is

$$\frac{\partial U_1}{\partial e} + \frac{\partial U_2}{\partial t_R}\frac{v_R}{v}pa + \frac{\partial U_3}{\partial t_B}\frac{v_B}{v}pa = 0.$$

Since we are primarily interested in the response of per pupil spending to change in the tax base composition variables, $v_R/v$ and $v_B/v$, we hold $y$, $w$, $f$, $g$, and $pa$ constant while differentiating the above equation. Then, noting that $v_B/v = 1 - v_R/v$ and $d(v_R/v) = -d(v_B/v)$ by definition, we obtain as the expression r $de/d(v_R/v)$,

$$\frac{de}{d\left(\frac{v_R}{v}\right)} = \frac{-pa(e-f-g)\left[\frac{v_R}{v}\frac{\partial^2 U_2}{\partial t_R^2} - \left(1-\frac{v_R}{v}\right)\frac{\partial^2 U_3}{\partial t_B^2}\right] - \frac{\partial U_2}{\partial t_R} + \frac{\partial U_3}{\partial t_B}}{\frac{\partial^2 U_1}{\partial e^2} + (pa)^2\left[\left(\frac{v_R}{v}\right)^2\frac{\partial^2 U}{\partial t_R^2} + \left(1-\frac{v_R}{v}\right)^2\frac{\partial^2 U_3}{\partial t_B^2}\right]}.$$

Since $v_R/v$ and $v_B/v$ entered symmetrically into the preference function and since the effect on $e$ is the resultant of opposing changes in the two variables, the sign of $de/d(v_R/v)$ is, naturally, indeterminate. However, note the following: As the fraction of business property, $v_B/v$ or $1 - v_R/v$, becomes small, the terms in $U_2$ tend to dominate, making $de/d(v_R/v)$ negative. That is to say, when the fraction of business property is small, increases in that fraction tend to raise per pupil spending. Conversely, when the fraction of residential property is small, $de/d(v_R/v)$ tends to become positive, which means that reductions in the fraction of business property will have a positive effect on spending. The only general conclusion that can be stated is that per pupil spending depends on the proportion of $v_R$ in the tax base; i.e.,

$$e = e(y,\ w,\ g + f,\ pa,\ v_R/v), \tag{22}$$

but the sign of the effect of $v_R/v$ is not specified *a priori*.

In practice, the amount of residential property per household, $v_R$, is likely to be closely related to household income, $y$, and the amount of business property, $v_B$, is likely to be linked to business income, $w$. Therefore, it should be possible to translate the tax base composition variable, $v_R/v$, into some function of $y$ and $w$. A plausible hypothesis for empirical testing is that $e$ will vary according to the *ratio*, $y/w$, of personal to business income. In accord with the preceding analysis, we would expect to find $e$ increasing with $y/w$ up to a point, but decreasing thereafter. The greater the weight that decisionmakers assign to taxes on district residents relative to taxes on business, the greater will be the range over which per pupil spending is a decreasing function of $v_R/v$ or $y/w$.

## The Relationship Between Residential Property Value and Personal Income

Unless the relationship between disposable personal income and residential property value is explicitly included in the model, there is likely to be a confounding of income and tax base influences on spending. In particular, the operation of the tax base composition effect is likely to be obscured. Therefore, we briefly examine the consequences of incorporating an explicit relationship between $v_R$ and $y$ into the expenditure equation, Eq. (22). We consider the case where decisionmakers weigh taxes on residents more heavily than taxes on business and would choose, if they were able, to impose a higher tax rate on business than on residential property.

The amount of housing services consumed in a community (rent payed for rental housing plus imputed rent of owner-occupied homes) is, like consumption in general, likely to be closely correlated with income. The ratio of housing services to the value of residential property depends on locational and environmental factors and other characteristics of each community. We can express the relationship between $v_R$ and $y$ as

$$v_R = v_R(y, z_1),$$

where $\partial v_R/\partial y > 0$ and $z_1$ represents the aforementioned community characteristics. The tax base composition variable, $v_R/v$, then becomes $v_R(y, z_1)/[v_R(y, z_1) + v_B]$, the value of which increases with $y$ and decreases with $v_B$. Taking account of that relationship, the effect of $v_R/v$ on spending can be decomposed into three terms:

$$de = \frac{\partial e}{\partial(v_R/v)}\, d(v_R/v) = \frac{\partial e}{\partial(v_R/v)}\left[\frac{\partial(v_R/v)}{\partial y}\, dy + \frac{\partial(v_R/v)}{\partial z_1}\, dz_1 + \frac{\partial(v_R/v)}{\partial v_B}\, dv_B\right],$$

where $\partial e/\partial(v_R/v) < 0$, $\partial(v_R/v)/\partial y > 0$, $\partial(v_R/v)/\partial v_B < 0$, and $\partial(v_R/v)/\partial z_1 > 0$ if $z_1$ is defined in such a way that $\partial v_R/\partial z < 0$.

The effect of income on expenditure in the above expression is, of course, distinct from the ordinary income effect represented in Eq. (22). Therefore, the net effect of an increase in income is the sum of a positive direct effect and a negative effect operating indirectly through the tax base composition variable. The effect of an increase in $v_B$ is unambiguously positive and the effect of an increase in $z_1$, if $z_1$ is defined as specified above, is unambiguously negative. Therefore, our conclusions regarding the effect of tax base differences on spending must encompass the following three cases:

1. Districts A and B are identical except that district A has more business property per household. The model predicts that spending will be higher in district A than in district B.
2. Districts A and B are identical in all respects except that district A has higher residential property value per household than district B because of differences in some of the locational or environmental characteristics included in $z_1$. The model predicts that district A will spend less per pupil than district B.
3. District A has more income per household than district B and, because of that, also has higher residential property value per household. The sign of the expenditure difference is ambiguous. If the direct income effect outweighs the tax base composition effect, district A will spend more. If the composition effect predominates, district B will spend more.

The question, What is the relationship between per pupil spending and tax base? does not have a simple answer. It depends on the factors causing tax base differences and their interactions.

Illustration Using the Linear Expenditure Model

The impact of the tax base composition effect and the income-property value relationship on empirically testable forms of the model can be illustrated with the previously discussed linear expenditure model.*
From Eq. (21), it is clear that the tax base composition effect is associated with the tax term rather than the expenditure term of the preference function. Therefore, a suitable form of the additive-logarithmic preference function is

$$U = b_1 \log (e - c_1) + b_2 \log \left\{c_2\left[y,\ v_R(y,\ z_1)/v\right] - t\right\},$$

or, if $c_2$ is assumed to depend linearly on the indicated variables and if $v_R$ is linear in $y$ and $z_1$,

$$U = b_1 \log (e - c_1) + b_2 \log \left(d_0 + d_1 y + d_2 \frac{1}{v} + d_3 \frac{y}{v} + d_4 \frac{z_1}{v} - t\right),$$

where $d_1 > 0$ and $d_2$, $d_3$, $d_4$ are all negative. This leads to an expenditure equation,

$$e = c_1 + b_1\left(f + g - c_1 \frac{1}{pa} + d_1 \frac{y}{pa} + d_2 \frac{1}{pav} + d_3 \frac{y}{pav} + d_4 \frac{z_1}{pav}\right).$$

The two terms containing $y$ have opposite signs, as expected. The effect of an increase in $v$, holding $y$ and $z_1$ constant (which is to say, an increase in $v_B$) is positive. Property value interacts with income and the price variable, $pa$, as well as with $z_1$.

*See pp. 26-28.

A potentially important feature of the equation is that only values of $y$ and $v$ are needed to estimate the tax base composition effect, at least under the linearity assumptions concerning $v_R$. Since data on $v_B$ and $v_R$ are likely to be difficult to obtain, this characteristic of the model may be the only thing that will permit empirical tests.

## EQUALIZATION PROVISIONS OF AID FORMULAS

In most states, one of the important purposes of state aid to compensate, or "equalize" for interdistrict differences in ability to support public schools. To that end, states allocate aid in inverse relationship to what they term the "financial ability" of each district, which---almost universally--is defined as the total local property tax base per pupil ($v/a$ in our notation). Given the results of our analysis of tax base and tax base composition effects, it is evident that such a concept of "ability" raises some serious questions. If the analysis shows anything, it is that the relationship between total tax base and educational spending is neither simple nor, necessarily, unidirectional. Therefore, the issue arises: How does an aid formula based on property value actually affect interdistrict variations in spending? Apart from that, we must deal with the technical question of how equalization provisions affect the form of empirically testable expenditure equations.

### Forms of Equalization

Throughout this report, we have assumed that state aid to public schools has the form

$$s = g + (1 - \alpha)(e - g),$$

i.e., lump-sum aid plus matching of all local spending in excess of the lump-sum aid amount. Equalization means that either the lump-sum amount, $g$, or the matching parameter, $\alpha$, or both, vary from one district to another according to the property tax base per pupil. In general, therefore, we must allow for the relationships

$$g = g(v/a), \quad \text{where} \quad g'(v/a) < 0,$$

and

$$\alpha = \alpha(v/a), \quad \text{where} \quad \alpha'(v/a) > 0.$$

Recall that $\alpha$ is the local share of matching aid, and that *lower* $\alpha$ means a larger grant from the state.

The Foundation Aid Formula

The aid formula in most common use in the United States is based on a concept called the *foundation program*. Briefly, the idea is that each district should be able to spend a specified minimum amount per pupil--no matter how poor it is or how lacking in taxable property--provided only that it is willing to make a specified minimal "fiscal effort." Just as "fiscal ability" is measured by the amount of taxable property per pupil, "fiscal effort" is measured by the tax *rate*. Therefore, the operational meaning of the foundation formula is that the state will pay to each district the difference (if greater than zero) between the specified minimum, or "foundation," amount per pupil and the amount raised locally by imposing the stipulated minimum property tax rate. That difference is greater the lower the local tax base; it vanishes for districts that have more than a certain amount of property value per pupil. Any district that wishes to tax at more than the specified minimum rate may do so, but the proceeds from the added levy will be strictly proportional to local assessed value per pupil.

The foundation aid formula may be written

$$s = g\left(\frac{v}{a}\right)$$

$$= e_0 - \frac{r_0 v}{pa}.$$

where $e_0$ is the real value of the foundation amount and $r_0$ is the minimum required property tax rate. The $pa$ term is needed because $v$ was defined as property value per household, deflated by $p_x$, while $s$ and $e_0$ are measured in dollars per pupil, deflated by $p_e$.

Actual foundation aid formulas in the several states contain many other features, few of which are of theoretical interest. The only addition that we will acknowledge is the m[illegible] [illegible] one, inclusion of a "floor" under the amount of aid per pupil to be received by each district. If the real value of that floor is represented by $b$, the aid formula becomes

$$g = \max \left(b,\ e_0 - \frac{r_0 v}{pa}\right).$$

According to this formula, aid always has the form of a lump-sum grant, but the amount of aid may be either the flat grant amount, $b$, or the variable amount, $e_0 - r_0 v/pa$.

If the model is applied to a set of data from districts that receive aid under a foundation formula, it will be of interest to estimate the effects of changes in the formula parameters, $e_0$, $r_0$, and $b$. Two subsets of observations must be considered: those corresponding to districts that receive the flat grant amount of aid, $b$, per pupil, and those from districts that receive equalization aid according to the formula given above. If the linear expenditure model is used, the flat grant case can be accommodated by Eq. (16) simply by making the substitution $g = b$, and the equalization case by making the substitution $g = e_0 - r_0 v/pa$. An expenditure equation that permits the full set of data to be used can be written if we define a dummy variable, $\delta$, such that $\delta = 1$ if $e_0 - r_0 v/pa > b$; $\delta = 0$, otherwise. Then, the expression to be substituted for $g$ in the expenditure equation, Eq. (16), is

$$(1 - \delta)b + \delta\left(e_0 - \frac{r_0 v}{pa}\right).$$

Variable Matching Formulas

A formula used in only a few states at present, but important in school finance thinking, is a variable matching ("percentage equalization") formula that makes the local share, $\alpha$, an increasing function of the local tax base. That formula can be written,

$$s = \left[1 - \alpha_0 \left(\frac{v/a}{(v/a)_0}\right)\right](e - f),$$

where $(v/a)_0$ is the average value of assessed property per pupil in the state and $\alpha_0$ is the local share parameter applicable to a district whose tax base per pupil equals the statewide average. The parameter $\alpha_0$ is a policy instrument that sets the average statewide ratio between state and local support for the schools. The rationale for the formula is that it provides the same level of per pupil support to all districts that levy the same tax rate, no matter how small or how large their per pupil tax base.* The aid formula may also stipulate a minimum amount of aid per pupil to be provided to each district and either an aid "ceiling" or a maximum amount of spending to be matched. Assuming the latter, we can write the aid formula, including upper and lower bounds, as

$$s = \max \left\{ b, \left[1 - \alpha_0 \left(\frac{v/a}{(v/a)_0}\right)\right] \min\ (e - f,\ e_m) \right\},$$

where $b$ is the aid floor and $e_m$ is the maximum amount of per pupil spending that the state is willing to share.

If districts receive aid under a formula of this type, three subcases must be considered in estimating the model: districts that receive the flat grant amount $b$; those that receive a variable lump-sum grant, $\{1 - \alpha_0[v/a(v/a)_0]\}e_m$; and those that receive matching aid with

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*This is true for all but the wealthiest districts, which would have to receive negative amounts of aid to be placed on an equal footing with the others.

a local share parameter equal to $\{1 - \alpha_0[v/a(v/a)_0]\}$. Only in the third subcase will aid appear to have the form of a matching grant. To estimate the model where observations include all three subcases, we would substitute for $g$ and $\alpha$ in Eq. (16),

$$g = \delta_1 b + \delta_2 \left[1 - \alpha_0 \left(\frac{v/a}{(v/a)_0}\right)\right] e_m,$$

$$\alpha = (1 - \delta_1 - \delta_2) \left[1 - \alpha_0 \left(\frac{v/a}{(v/a)_0}\right)\right],$$

where $\delta_1$ = 1 if the district receives the flat grant amount $b$, zero otherwise,

$\delta_2$ = 1 if the district spends over the expenditure ceiling, $e_m$ (excluding federal aid), zero otherwise.

## Assessment of Formulas that Equalize According to Property Value per Pupil

Since the purpose of equalization is to reduce expenditure disparities among districts, the efficacy of a formula based on property value depends on the relationship between the property tax base and spending. If spending per pupil tends to be positively related to property value per pupil in the absence of equalization aid, then the equalization formulas will tend to work in the right direction.

From the analysis in the last section, we can conclude that the equalizing effect will operate in the right direction in most situations; in some instances, however, the theory predicts that the effect will be perverse. Moreover, even where the direction of the effect is correct, relative magnitudes of aid may be inappropriate because of the disparate expenditure tendencies of districts with different tax base compositions.

Consider the different sources of tax base inequality: First, to the extent that tax base variations reflect differences in the amount of business property per pupil, districts with higher tax bases will tend to spend more. Therefore, distribution of aid in an inverse relationship to tax base will have the desired equalizing effect. Second, where districts vary in household income and, for that reason, in the value of residential property per pupil, there may still be a positive relationship between property and spending, as the formula assumes. If the income effect predominates over the tax base composition effect,* equalization will work in the right direction. Third, where districts are equal in business property per pupil and in household income, but unequal in residential property per pupil because of locational or other factors, the theoretical analysis says that equalization will operate in the wrong direction. Districts with higher residential property values would tend to spend *less* on schools, other things being equal, both because of the adverse tax base composition effect and because larger shares of residents' incomes would probably go for housing. Yet such districts will actually receive *less* aid from the state. Equalization according to property value per pupil will have an effect opposite to that intended.

Approaching the problem somewhat differently, we can distinguish three cases in which the equalization formulas will either fail to adjust adequately for expenditure disparities or will actually make the disparities worse:

1. When residential property value per pupil varies among districts because of locational or other factors, the aid formulas may actually cause greater disparities than would have existed with flat grants.
2. When districts have equal amounts of assessed property value per pupil, but different mixes of residential and business property, the aid formulas will do nothing to relieve expenditure disparities stemming from differences in tax base composition.
3. Finally--a point not yet covered--when districts are equal in property value per pupil, but not in property value per household, the aid formula will ~~do~~ nothing to compensate for expenditure variations that may arise because of differences in

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*See p. 50.

numbers of pupils and income per household, both of which are important expenditure determinants.

In sum, there are enough intervening variables in the tax base-expenditure relationship to make equalization according to property value a highly imperfect instrument for achieving fiscal equality.

## CATEGORICAL VERSUS NONCATEGORICAL AID

The way in which aid was treated in the basic model is valid only if (1) the aid is noncategorical (not "earmarked" for any specific district program or function) or (2) categorical, but with a nonbinding constraint (i.e., the aid recipient would have spent at least the stipulated amount for the aided function even with no earmarking). Categorical aid with a *binding* constraint--one that forces the recipient to spend more for a function than he would otherwise have spent willingly--must be treated differently.

The standard way to analyze the effect of a categorical grant is to formulate a two-good model, with the aided function as one good and all other district activities as the second good. Such a model can be used to show how different amounts and forms of aid and different types of restrictions influence expenditures for both aided and unaided functions.* We will make use of a simple, special-purpose two-good model in which the level of district expenditure for the categorically-aided function is assumed to be set administratively by the grantor. The purpose of the analysis is to find out how the level of total district spending is affected by that form of restriction.

We use the following notation:

$e_c$ = real per pupil spending for the categorically-aided program or function,

$e_u$ = real per pupil spending for all other district activities,

$g_c$ = real categorical aid per pupil (assumed to be a lump-sum grant),

$g_u$ = real noncategorical aid per pupil (also a lump-sum grant).

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*See, for example, Alan Williams, *Public Finance and Budgetary Policy*, George Allen & Unwin Ltd., London, 1963, pp. 176-180.

It is assumed that the same cost-of-education index applies to both $e_c$ and $e_u$ and that there is no matching aid. Total expenditure per pupil, $e$, is given by $e = e_c + e_u$.

District preferences with respect to different combinations of $e_c$, $e_u$, and taxes are represented by the preference function

$$U = U[e_u,\ e_c,\ b(t,\ y)].$$

The key assumption that the value of $e_c$ is administratively constrained will be expressed by requiring $e_c = kg_c$. The case where $k > 1$ corresponds to a closed-ended matching grant, under which the district is required to spend $k - 1$ dollars on $e_c$ for every dollar provided by the aid grantor. The case of $k < 1$ may be thought of as a situation in which the district is nominally expected to spend the full amount of categorical aid for the stated purpose, but in which imperfect enforcement permits a fraction $1 - k$ of aid funds to be diverted to other uses. Alternative interpretations are also possible.

Taking account of the binding constraint on $e_c$, the preference function is

$$U = U[e_u,\ kg_c,\ b(t,\ y)].$$

The budget constraint equation is

$$t = pa(e_u + e_c - g_u - g_c) = pa[e_u - (1 - k)g_c - g_u].$$

The marginal rate of trade-off equation obtained by maximizing $U$ subject to the constraint (after substituting the above expression for $t$ into $U$) is

$$m\left[e_u,\ kg_c,\ b\left\{pa\left[e_u - (1 - k)g_c - g_u\right],\ y\right\}\right] = pa.$$

By differentiating the latter, first with respect to $g_c$ and then with respect to $g_u$, and rearranging terms appropriately, we obtain the following effects of categorical and noncategorical aid.

$$\frac{de}{dg_c} = \frac{k\left(\partial m/\partial e_u - \partial m/\partial e_c\right) + pa(\partial m/\partial b)(\partial b/\partial t)}{\partial m/\partial e_c + pa(\partial m/\partial b)(\partial b/\partial t)} ,$$

$$\frac{de}{dg_u} = \frac{pa(\partial m/\partial b)(\partial b/\partial t)}{\partial m/\partial e_u + pa(\partial m/\partial b)(\partial b/\partial t)} .$$

Since $k < 0$, the categorical aid effect, $de/dg_c$, will be greater than the noncategorical aid effect, $de/dg_u$ provided that $\partial m/\partial e_u - \partial m/\partial e_c$ is negative. We know that $\partial m/\partial e_u$ is negative, since it is analogous to $\partial m/\partial e$ in the basic model.* Therefore, the question is: What is the sign and magnitude of $\partial m/\partial e_c$? That depends on the relationship between $e_u$ and $e_c$. If $e_u$ and $e_c$ are *complements*, then an increase in $e_c$ would tend to make a district more willing to tax itself. That would make $\partial m/\partial e_c$ positive and $de/dg_c > de/dg_u$. If $e_u$ and $e_c$ are *substitutes*, then $\partial m/\partial e_c$ will be negative and the relative magnitudes of $de/dg_c$ and $de/dg_u$ will depend on the strength of the substitution relationship. In general, the sign is indeterminate, but the effects of categorical and noncategorical aid will be unequal. This result must be allowed for in empirical studies.

To illustrate the effect on an empirically-testable expenditure equation, suppose that we were using the nonlinear form represented by Eq. (18). Noting that the forms of $de/dg_c$ and $de/dg_u$ are similar except for the extra term in the numerator of $de/dg_c$, we can write the expenditure equation with one term in *total* aid, $g_u + g_c$, and another term in $g_c$ only. The modified form of Eq. (18) would be

$$e = \frac{\gamma_0 + \gamma_1 y + \gamma_2 \alpha pa\left(g_u + g_c\right) + k\gamma_3 g_c + \gamma_4 \alpha pa}{1 + \gamma_2 \alpha pa} .$$

The test for a differential effect of categorical aid is that the coefficient of the $g_c$ term, which is the product $k\gamma_3$, not be equal to zero.

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*See Eq. (6).

A final point is that the analysis of categorical versus noncategorical aid may provide a rationale for hypothesizing that state and federal aid coefficients are different in the expenditure equations. Most state aid takes the form of general-purpose grants for current operations. Most federal aid, to date, has been for specific programs or resources. Therefore, even if no breakdown on amounts of categorical and noncategorical grants is available, it is reasonable to assume that federal aid is "more e" a categorical grant than state aid and, therefore, that the coefficients may be different.

## THE EFFECT OF ENROLLMENT GROWTH

Individual school districts, as well as entire states, vary widely in the rate at which their pupil enrollments have grown. Some districts have had virtually constant pupil populations; others have experienced rates of increase of up to ten percent per year. It can be hypothesized that these variations in growth rates will have affected per pupil expenditures in at least two ways: First, district requirements for school construction funds are positively related to enrollment growth. These funds must be financed out of present or future taxes, which, like the nonschool taxes discussed earlier, will probably substitute to a degree for taxes for current operations. Therefore, rapid growth will tend to be associated with lower per pupil outlay. Second, when enrollment is growing, it is possible that the district will not be able to adjust taxes rapidly enough to keep up with the increase in the number of pupils. Thus, per pupil outlays may lag behind their equilibrium value. In this section, we will discuss only the first hypothesis. The second has to be treated within the framework of a dynamic model, which lies outside the scope of this paper.

A comprehensive analysis of determinants of school construction outlays is not contemplated. However, when enrollment is growing rapidly, it is not possible to ignore the drain of capital requirements on current or future property taxes and the consequent depressing effect on local willingness to spend for current operations. To get at that effect without studying capital outlay in detail, we examine the implications for current spending of a very simple, fixed-coefficient model of capital requirements.

Suppose that the number of pupils in a district increases by $\Delta A$ from one year to the next. Assume that each pupil "requires" $h$ physical units of capital (school buildings). The quantity $h$ is measured by deflating the current dollar value of new capital per pupil by a cost of construction price index, $p_h$. The total capital outlay requirement for the district during a year (assuming that all construction to accommodate the annual increase in pupils is carried out in a single year) is $p_h h\Delta A$. The requirement per household is $p_h h\Delta A/N$, which we can write as $p_h h(\Delta A/A)(A/N)$ or $p_h ha(\Delta A/A)$.

Suppose, first, that capital outlays are financed out of current revenues. The overall district budget constraint would then be

$$t = \alpha pa(e - f - g) + p_h^* ha(\Delta A/A),$$

where $p_h^* = p_h/p_x$, the relative price of school construction. Comparing the terms in this equation, it can be seen that the capital outlay requirement affects current spending like a negative lump-sum grant proportional to the growth rate, $\Delta A/A$. The general form of the resulting expenditure equation would be

$$e = e\left[y,\ \alpha pa(f + g),\ p_h^* ha(\Delta A/A),\ \alpha pa,\ z\right],$$

where $de/d(p_h^* ha\Delta A/A) < 0$. Using the functional form of Eq. (18) as an illustration, a possible expenditure equation is

$$e = \frac{\gamma_0 + \gamma_1 y + \gamma_2 \alpha pa(f + g) + \gamma_3 p_h^* a(\Delta A/A) + \gamma_4 \alpha pa + \gamma_5 z}{1 + \gamma_2 \alpha pa}.$$

In reality, expenditures for school construction are rarely financed out of current taxes, although minor capital outlays may be. Rather, construction is financed by bond issues, which is to say, out of future property taxes. Taking this into account, it is inappropriate to include school construction costs in the current budget constraint. Instead, we must focus on the impact of bond debt on decisionmakers' willingness to trade off current expenditures and taxes.

If we make the reasonable assumption that decisionmakers' are averse to both current and future taxes, then a separable preference function can be written as

$$U = U_1(e) + U_2(t, y) + U_3(t_N, y_N),$$

where the subscript $N$ denotes values of variables in future years.* The future tax variable, $t_N$, represents taxes to be levied in later years as a result of current and past bond issues. It does not include projected taxes for school operating expenses.

Assume that school bonds are perpetuities. Let $B_0$ represent the real stock of bonds per household outstanding before the current year and $\rho_0$ the average rate of interest on them. Let $\Delta B$ be the real value of new school construction bonds per household to be issued during the current year and $\rho$ the applicable rate of interest. The annual burden of bond finance in future years will be

$$t_N = \rho_0 B_0 + \rho\Delta B = t_{N0} + \rho\Delta B,$$

where $t_{N0}$ is the level of future tax determined by past actions and, as has already been shown,

$$\Delta B = p_h^* ha(\Delta A/A).$$

Therefore,

$$t_N = t_{N0} + \rho p_h^* ha(\Delta A/A).$$

Maximization of the separable utility function subject to the usual current budget constraint leads to a marginal rate of trade-off equation,

$$m(e, t, t_N, y, y_N) = \alpha pa,$$

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*$t_N$ and $y_N$ are treated as tax and income streams that have the same value in each future year. $y_N$ can be thought of as the constant income stream that would have the same present value as the actual (projected) future income stream.

with $\partial m/\partial e$, $\partial m/\partial t$, and $\partial m/\partial t_N$ all negative. The negative value of $\partial m/\partial t_N$ means that a school district becomes less willing to pay current taxes for an increment in $e$ as the level of future taxes, $t_N$, increases. The corresponding expenditure equation has the general form

$$e = e(y,\ y_N,\ f + g,\ t_N,\ \alpha pa)$$

$$= e(y,\ y_N,\ f + g,\ t_{N0} + \rho p_h^{\prime} ha\Delta A/A,\ \alpha pa),$$

with $de/dt_N < 0$.

Just as in the previous case, we expect $e$ to be lower where $\Delta A/A$ is higher. But when bond financing is taken into account, we also find that the impact of enrollment growth depends on both the interest rate and the initial level of debt. Higher values of either will tend to reduce current outlays per pupil. Possible refinements of the analysis would include allowance for debt redemption, as well as debt issuance, and relaxation of the rigid, fixed-coefficient capital requirement. However, this version suffices to demonstrate the negative impact of growing enrollment on the level of the educational program.

## IV. MODELS APPLICABLE TO DIFFERENT DATA BASES

Empirically-testable expenditure models take on somewhat different forms depending on the type of data base to which they are to be applied. The main distinction is between studies that have the individual school district as the unit of observation and studies that focus on larger units such as states or counties. Also, there are certain differences between time series and cross-section models and between interstate studies and those that are limited to a single state.

### INDIVIDUAL DISTRICT MODELS

Since the whole analysis was couched in terms of the behavior of a single district, all the theoretical results apply except where the data base precludes variations in the relevant exogenous variables. In the case of a time-series model, where financial behavior of one or more districts is observed over several years, no variables need be excluded. Data permitting, the model to be tested should allow for all of the effects discussed in the "Extensions"--nonschool taxes, tax base composition, equalization, categorical aid, and enrollment growth.

Assuming (1) that the *net* school property tax, allowing for deductibility, is what counts (p. 36), (2) that the *simple* model of the tax base composition effect applies (p. 45), and (3) that the district receives only flat grant aid; and applying the functional form of Eq. (18), a possible model to use in an individual district, time series study is

$$e_i = \frac{\beta_0 + \beta_i y_{D,i} + \beta_2 Q_i(f_i + g_i) + \beta_3 Q_i + \beta_4 \rho_i p'_{h,i} a_i \left(\frac{\Delta A}{A}\right)_i}{1 + \beta_2 Q_i} + u_i,$$

where the subscript $i$ denotes the year of the observation, $u_i$ is a stochastic error term, and $y_D = y - t_S - t_F - t_0$. The variable $Q_i$ is defined by

$$Q_i = [1 - r_i(y_{D,i})]\left(\frac{v_R}{v}\right)_i p_i a_i / (1 + r_{SLS,i}),$$

where $r(y_D)$ is the combined state plus federal marginal income tax rate. The composite price term, $Q_i$, takes in the price-reducing effect of deductibility, the effect of the tax rate composition variable, $v_R/v$, and the effect of a sales tax.

If a cross-sectional analysis were conducted under similar assumptions, using observations of a number of districts within a single state during a single year, the model would differ from that shown above in the following particulars: (1) The sales tax rate, $r_{SLS}$, would be the same for all districts (assuming no local sales tax); (2) the bond interest rate, $\rho$, would be a constant; (3) the amount of flat grant aid from the state, $g$, would be a constant, although variations in federal aid might still permit variations in $f + g$ to be observed; and (4) the price variables, $p$ and $p_h^*$ would probably best be treated as constants (even under the unlikely circumstance that local prices could be observed) under the assumption that a state is a single market and that interdistrict price differences reflect quality variations. Therefore, $Q_i$ would become

$$Q_i = [1 - r_i(y_{D,i})]\left(\frac{v_R}{v}\right)_i a_i,$$

where the subscript $i$ is now an identifier of individual districts in the cross section.

The assumption of a constant price in all districts focuses special attention on the alternative form of the model represented by Eq. (20), in which $p$, but not $\alpha$, appears in the denominator. If $p$ is assumed constant across districts, that equation translates into the straighforward linear model,

$$e_i = \beta_0 + \beta_1 \frac{y_{D,i}}{a_i} + \beta_2(f_i + g_i) + \beta_3 \frac{1}{a_i} + u_i.$$

Additions of tax rate composition and deductibility effects to the previously used forms would destroy the linearity, but alternative assumptions would allow it to be maintained. The effect of enrollment growth could be added with no difficulty.

An individual district cross-sectional analysis that cuts across states (e.g., a cross-sectional analysis of large city school districts in the U.S.) differs from the within-state analysis in that differences in state flat grant aid, sales tax rates, and (possibly) prices can be observed. Therefore, the model would have essentially the same form as that used for an individual district time series study. A possible difference is that the tax base composition variable, $v_R/v$, may prove impossible to measure consistently across states because of difference in definitions of taxable property, categories of property, and assessment ratios. Therefore, that variable might have to be omitted.

## STATES OR OTHER AGGREGATES AS OBSERVATIONAL UNITS

### Aggregation

If counties, states, metropolitan areas, or other multi-district groupings are the units of observation, it becomes necessary to aggregate financial and other variables over all districts within each group. From a purely theoretical point of view, all that is necessary is to use average values of expenditure, taxes, and the exogenous variables for the multi-district units. However, because of the presence of the stochastic error term, direct use of averages would introduce heteroskedasticity into the estimation. This can be avoided by applying appropriate weighting factors to the state, county, or metropolitan area observations. To show this, we consider the case of a cross-sectional analysis across states, with statewide averages as the data, and with the assumption that the simple linear expenditure equation, Eq. (16), is the "correct" specification of the individual district model.

We first write a stochastic version of the individual district model, introducing subscripts $i = 1 \ldots N_j$ to identify each of the $N$ local school districts within a state and $j = 1 \ldots M$ to identify each of $M$ states. For the $i^{th}$ district in the $j^{th}$ state, the expenditure equation is

$$e_{ij} = \beta_0 + \beta_1(f_{ij} + g_{ij}) + \beta_2 \frac{1}{(\alpha pa)_{ij}} + \beta_3 \frac{y_{ij}}{(\alpha pa)_{ij}} + \beta_4 z_{ij} + u_{ij} .$$

The customary assumptions will be made about the stochastic error term; that is, $E(u_{ij}) = 0$, $E(u_{ij})^2 = \sigma^2$. Let $A_{ij}$ be the number of pupils in the $i^{th}$ district of state $j$ and let $A_j$ be the total number of pupils in all districts of state $j$. Real expenditure per pupil in state $j$ is then given by

$$\frac{\sum_i A_{ij}e_{ij}}{A_j} = \beta_0 \frac{\sum_i A_{ij}}{A_j} + \beta_1 \frac{\sum_i A_{ij}(f_{ij} + g_{ij})}{A_j} + \beta_2 \frac{\sum_i A_{ij}[1/(\alpha pa)_{ij}]}{A_j}$$

$$+ \beta_3 \frac{\sum_i [A_{ij}y_{ij}/(\alpha pa)_{ij}]}{A_j} + \beta_4 \frac{\sum_i A_{ij}z_{ij}}{A_j} + \frac{\sum_i A_{ij}u_{ij}}{A_j} .$$

Denote the average level of real per pupil expenditure in state $j$ by $e_j$. Then, by definition, $e_j = \sum_i (A_{ij}e_{ij})/A_j$. Similarly, we can define average values of lump-sum aid, the inverse price variable $1/\alpha pa$, income divided by $\alpha pa$, and the taste variable, $z$. Labeling these $f_j + g_j$, $1/(\alpha pa)_j$, $y_j/(\alpha pa)_j$, and $z_j$, respectively (and noting that $\sum_i A_{ij}/A_j = 1$), we can rewrite the equation for statewide per pupil spending as

$$e_j = \beta_0 + \beta_1(f_j + g_j) + \beta_2 \frac{1}{(\alpha pa)_j} + \beta_3 \frac{y_j}{(\alpha pa)_j} + \beta_4 z_j + \sum_i (A_{ij}/A_j)u_{ij} .$$

It is evident that this equation violates the linear regression assumptions. Using $u_j$ to represent the stochastic error term in the state-level equation, where $u_j = \sum_i [(A_{ij}/A_j)u_{ij}]$, we have

$$E(u_j)^2 = \sigma^2 \sum (A_{ij}/A_j)^2$$
$$\neq \text{constant.}$$

This is a case of heteroskedasticity in which the expected error term for each state depends on the *distribution* of pupils among districts within the state, as reflected in the quantity $\sum_i (A_{ij}/A_j)^2$. Note that if there is only one district, $\sum_i (A_{ij}/A_j)^2 = 1$, and the expenditure equation becomes identical to the individual district equation.

The heteroskedasticity can be removed by weighting each observation by $1/\sqrt{\sum_i (A_{ij}/A_j)^2}$. If that factor is designated $W_j$, the actual equation to be estimated is

$$W_j e_j = \beta_0 W_j + \beta_1 W_j (f_j + g_j) + \beta_2 \frac{W_j}{(\alpha pa)_j} + \beta_3 \frac{W_j y_j}{(\alpha pa)_j} + \beta_4 W_j z_j + W_j u_j,$$

where $E(W_j u_j)^2 = \sigma^2$.

In general, the weighting factor, $W_j$, depends on the whole distribution of pupils among districts within the state. As an aid in interpreting $W_j$, consider the special case in which the state has a certain number of districts, all with the same pupil enrollment. Letting $N_j$ be the number of districts, we have $\sum_i A_{ij}/A_j = 1/N_j$ for all $i$. The weighting factor becomes

$$W_j = 1/\sqrt{\sum_i (A_{ij}/A_j)^2}$$

$$= 1/\sqrt{\sum_i (1/N_j)^2}$$

$$= 1/\sqrt{N_j (1/N_j)^2}$$

$$= \sqrt{N_j}.$$

That is, if all school districts within a state had the same number of pupils, heteroskedasticity would be eliminated by weighting the observed data for each state by the square root of the number of school districts in the state. The effect of inequality in the distribution of pupils among districts is to increase $\sum_i (A_{ij}/A_j)^2$ and, therefore, to reduce $W_j$. Therefore, the value of $W_j$ depends positively on the number of school districts in the state and negatively on the skewedness of the distribution of pupils. Interestingly, $W_j$ is unaffected by variations in the pupil population itself; only the distribution of pupils among districts matters.

## Other Aspects of State-level Studies

Aggregation to the state level will reduce the ability of the models to represent certain features of state grants-in-aid. It will not be possible, for example, to include equalization provisions in the model since only statewide average levels of aid will be observed. It remains possible, in principle, to distinguish lump-sum aid from matching grants and to include a matching parameter (the marginal local share) in the expenditure and tax equations. In practice, however, it is unlikely that, in an interstate study, sufficient individual district data would be available to calculate the effective marginal local share for each state. Fortunately, nearly all states have, in the past, operated aid programs that provide lump-sum aid to districts, regardless of their nominal form. Therefore, the assumption that all state aid is of the lump-sum variety--an assumption that the analyst may be forced to make in an interstate study--is likely to be very nearly correct.

Differences between cross-section and time series models with states as the observational units are parallel to the differences encountered in individual district models. In particular, it is unlikely that the tax base composition variable can be measured across states using presently available data, which means that variable may have to be omitted from interstate studies. The variable can be included, however, in time series studies limited to a single state.

An especially troublesome problem in cross-sectional studies is measurement of interstate variations in the relative price of education. There is no basis, at present, for making the input quality comparisons that would allow construction of an interstate index of prices of educational inputs. Moreover, satisfactory general price indexes that can be used to approximate the price of "other goods" are not available for individual states. The best present strategy may be to test a number of alternative, simple assumptions about interstate price variations (e.g., that all observed teacher salary differences represent price variations *or* that they represent quality variations). It then becomes possible, at least, to examine the sensitivity of results to the alternative sets of price data.

Because of gaps in available data, it will probably not be possible to test all the hypotheses embedded in the theoretical model in any single empirical study. Certain hypotheses (e.g., effects of equalization provisions and tax base composition) can best be tested by examining variations among individual districts. Other propositions (e.g., effects of variations in income and nonschool taxes) mav be easier to test in interstate studies. Still other effects (e.g., effects of relative price variations) may be most easily recognized in studies that focus on individual districts or states over time. In sum, verification of the theoretical models will depend on a series of empirical studies using different data bases, different units of observation, and different forms of the expenditure and tax equations.

# V. CONCLUSIONS

## DETERMINANTS OF SCHOOL DISTRICT EXPENDITURES

Taking the basic model and theoretical extensions together, the analysis accounts for effects on school district taxing and spending of the following major economic and demographic variables: (1) disposable personal income, (2) state and federal grants-in-aid, (3) the number of pupils per household in a district, (4) the price of educational inputs in relation to the general price level, (5) levels of non-school taxes, including state and federal income taxes, sales taxes, and non-educational property taxes, (6) the composition of the local property tax base, as between residential and business property, and (7) the rate of growth in school enrollment. Provision is also made for incorporating effects of unspecified community characteristics that affect local willingness to support education ("taste variables") into the models. Except for state and federal grants-in-aid, which are discussed separately, conclusions regarding the effects of these variables on per pupil expenditures by school districts are summarized immediately below.

### Disposable Personal Income

Disposable income of households enters into the model as a variable that determines the burden of a given level of local property taxes. From the assumption that the burden of a given tax per household declines as income increases, it follows that there will be a positive relationship between disposable income per household and school district expenditure per pupil. In this context, disposable income is defined as personal income per household less state and federal income taxes and--at least in one version of the model--less noneducational local property taxes. There is no *a priori* restriction on the size of the income effect except relative to the grant-in-aid effect (see below).

### The Number of Pupils Per Household

This variable appears in the expenditure equations because the analysis was framed in terms of a trade-off between *spending per pupil* and *taxes per household.* Other things being equal, a higher value of the pupil/household ratio represents a greater sacrifice by each household to provide a given level of per pupil support. Therefore, it follows that higher values of the ratio will tend to be associated with lower values of per pupil spending. An alternative specification of the model introduces a possible offsetting effect: namely, that communities with higher pupil/household ratios may have stronger preferences for education. If this hypothesis is included, the sign of the overall effect becomes indeterminate.

## The Relative Price of Educational Inputs

The relative price of inputs to education enters into the model just like any commodity price variable in conventional consumer demand theory. Other things being equal, the model predicts that real per pupil spending will be negatively related to the relative price of inputs to schooling. The price elasticity may be greater or less than minus one. If the assumption about a taste effect of the pupil/household ratio is excluded, that ratio and the relative price ratio enter the expenditure equation as a product, which means that the expenditure elasticity is the same with respect to both variables.

## Nonschool Taxes

State and federal income taxes appear in the expenditure equations as deductions from personal income; therefore, an increase in those taxes reduces per pupil spending by the same amount as an identical decrease in household income. The effect must be modified if school taxes are deductible from the income tax base.

The effect of a sales tax is to lower the relative price of education. The magnitude of the reduction is the same as would be generated by an increase in the price of "other goods" by the effective sales tax rate. However, since the income variable is defined in terms of power to purchase "other goods," a higher sales tax also translates into lower real disposable income per household. Therefore, the overall effect of a sales tax increase is the resultant of a positive relative price effect and a negative income effect. The sign of the net effect is indeterminate.

The model predicts a negative relationship between levels of nonschool property taxes and real expenditure per pupil. However, those taxes can enter into the equations in any one of three ways: (1) as a component of a total property tax (school plus nonschool), which implies that the effect on spending is equal but opposite to the effect of a lump-sum grant; (2) as a deduction from household income, in which case the effect is the same as [illegible] of an income tax, or (3) as a separate argument in the preference function, which leads to a negative nonschool property tax term in the expenditure equation.

## Composition of the Local Property Tax Base

The analysis examines implications of two possible assumptions about district preferences with respect to residential and business property taxes. The simpler assumption is that only taxes on district residents (residential property) have disutility to the local decisionmaker. That implies a negative response of per pupil spending to increases in the ratio of residential to total property. The elasticity of expenditures with respect to the tax base composition variable (residential/total property) should be the same as the elasticity with respect to the relative price of education.

The more general assumption is that taxes on both residential and business property count, but that they are assigned different weights by the decisionmaker. In that case, the tax base composition variable enters independently into the expenditure equation; however, the sign remains negative so long as the decisionmaker would prefer to tax businesses at a higher rate than residences, if he were not constrained against doing so.

## Growth in Enrollment

If it is assumed that either (1) buildings to house increased numbers of pupils must be financed out of current revenues, or (2) buildings are financed out of future property tax revenues through issuance of bonds, then there will be a negative relationship between the rate of enrollment growth and per pupil expenditure. Under the former assumption, the effect is similar to that of a negative lump-sum grant proportional to the enrollment growth rate. Under the latter, more realistic assumption, the magnitude of the effect also depends on the bond interest rate and the existing stock of debt. The effect becomes stronger (more negative) as either of those variables increases.

Taste Variables

Effects of specific demographic, social, political, and other characteristics of communities (except for the pupil/household ratio) were not dealt with explicitly in the analysis. However, the model does provide for inclusion of such "taste variables" in the expenditure and tax equations. The general hypothesis pertaining to these variables is that any factor believed to be positively (negatively) associated with district willingness to trade-off incremental taxes for increments in school spending should appear in the equations with a positive (negative) coefficient.

THE IMPACT OF GRANTS-IN-AID

The analysis deals with the impact on school spending of lump-sum aid, matching grants, and combinations of the two. The main theoretical conclusions are as follows:

With respect to lump-sum aid, the models predict that the impact of increments in aid will always be partly additive, partly substitutive: Some fraction of each incremental dollar of aid will go to increase per pupil spending; the remainder will be used to reduce local taxes below what they would have been in the absence of the aid increment. In general, the effect on spending of an increment in lump-sum aid will be different from the effect of an equal increment in community income; under reasonable assumptions about preferences, the aid effect will be considerably greater.

The effect of matching grants is to reduce the price of educational inputs to the school district. The model predicts that the response of per pupil spending to a change in the local share parameter of a matching grant formula will be identical to the response to an equal-proportionate change in the relative price of educational inputs. Depending on the price elasticity of per pupil spending, matching grants may be partly additive, and partly substitutive, as are lump-sum grants, or they may be stimulative of higher local taxes than would have been raised in the absence of aid.

It is also shown that lump-sum and matching grants interact. As the level of lump-sum aid increases, matching aid becomes more stimulative; as matching aid becomes greater (i.e., as the local share parameter becomes lower) the responsiveness of expenditures to lump-sum increments becomes greater. The model makes it possible to calculate optimal mixes of lump-sum aid and matching grants for achieving various goals of grant-in-aid programs.

Analysis of state aid equalization formulas, in which aid is distributed among districts in an inverse relationship to property value per pupil, shows that the equalization objective may or may not be furthered, depending on the causes of interdistrict tax base variations. Where differences are due to unequal allotments of business property, equalization formulas will tend to work in the proper direction. Where differences are due to variations in personal income and residential property, the result will probably, but not necessarily, also be in the desired direction. But where differences are due to locational and other factors that affect property values, the effects of equalization aid may actually be perverse: The pattern of aid may actually amplify existing disparities. In general, formulas based on property value will not "equalize" consistently where there are significant interdistrict differences in tax base composition.

Finally, one of the conclusions from the analysis is that the effect of categorical grants (for which the grantor-imposed restrictions are binding) will be different, in general, from the effect of noncategorical, or unrestricted, grants. An implication is that the federal and state aid coefficients in the expenditure equations may differ because federal aid consists primarily of categorical grants while most state aid is provided in unrestricted form.