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ABSTRACT

The harmonic mean and Kramer (1956) unequal n forms of the Tukey multiple comparison statistic were investigated for Monte Carlo Type I and Type II errors under conditions of assumption violations. The two major questions concerning the sensitivity of multiple comparison statistics for different types of pairwise contrasts and the affect of increasing the number of treatment levels are discussed. Neither procedure consistently out-performs the other; the choice of test depends upon the population condition(s) and the pattern of unequal cell frequencies. (Author)

THE HARMONIC MEAN AND KRAMER UNEQUAL n FORMS OF THE TUKEY STATISTIC[1]

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ABSTRACT

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THE HARMONIC MEAN AND KRAMER UNEQUAL n FORMS OF THE TUKEY STATISTIC

The Tukey (T) multiple comparison statistic is considered appropriate for probing pairwise differences among means (Kirk, 1968; Miller, 1966; Scheffé, 1959). A crucial assumption of the T statistic is that the variances of the contrasts be equal (Scheffé, 1959). To satisfy this assumption there must be an equal number of observations per cell.

Smith's (1971) Monte Carlo investigation compared three unequal n procedures that can be used with the T statistic: the harmonic mean, the Kramer (1956) method, and the procedure suggested by Miller (1966). The harmonic mean, the most commonly cited procedure (Kirk, 1968; Winer, 1972), utilizes all the sample sizes from the K treatment levels ($[K/ 1/n_1 + 1/n_2 +,\ldots,+ 1/n_k]$). Miller suggests using an average or median value of the group sizes, whereas Kramer's procedure utilizes the sample sizes of the largest and smallest (range) groups. Smith (1971) found that the harmonic mean procedure and the procedure suggested by Kramer generated Type I estimates that were closer to theoretical alpha than the procedure cited by Miller. When choosing any statistical test both types of errors should be considered. Using the Kramer method, as Smith suggests solely on the basis of Type I estimates places too great an emphasis on the Type I error.

Recent studies (Games, 1971; Keselman and Toothaker, 1973) indicate two important factors that need to be investigated for the T statistic. Keselman and Toothaker (1973) point out that the power of multiple comparison tests is a function of the magnitude of the comparison deviating from the null hypothesis that $\Psi = \Sigma c_k \mu_k = 0$. For example, in their investigation the four treatment levels were made to differ by .67 σ-units, consequently, $\mu_1 = 0.00$, $\mu_2 = 0.67$, $\mu_3 = 1.34$, and $\mu_4 = 2.01$. The contrast that compares means one and four is the maximum contrast and its power would be greater than any of the remaining five pairwise contrasts. Their results corroborate the correspondence between the sensitivity of the analysis of variance (ANOVA) F test and the contrast that compares the maximum difference in a set of K means suggested by Scheffé (1959). Games (1971) shows that theoretical Type I error rates will vary, depending upon the number of treatment levels. A major question that remains unanswered is, "whether differences between extremes in a large set of means will be more sensitive to form and heterogeneity of variance than is the two means case" (Games, 1971, p.100).

To examine this question the harmonic mean (H) and the Kramer (K) unequal n forms of the T statistic were investigated for the empirical probability of a Type I and Type II error. The Monte Carlo estimates were generated under conditions of assumption violations for varying numbers of treatment levels when the true magnitude of deviation from the multiple comparison null hypothesis of zero was considered.

Procedure

Pseudo-random numbers were selected using a pseudo-random number generator. Depending upon the assumption violation, the numbers were selected from either a normal or exponential distribution. The normal deviates with $\mu = 0$ and $\sigma^2 = 1$, were generated by a technique developed by Box and Muller (1958). To sample from the exponential distribution (Lehman and Bailey, 1968, p.227),

$$f(t) = pe^{-pt} \tag{1}$$

with $p = 1$, $E(t) = 1/p = 1$, and $var(t) = 1/p^2 = 1$, pseudo-random exponential variables were generated by multiplying the negative of the mean, $-E(t) = -1$, times the natural logarithm of uniform random variates distributed on the unit interval (IBM, 1970). The exponential variates were then scaled so that the mean would be zero and the variance σ_k^2, k = 1,...,K (for the unequal variance conditions). The resulting skewed population has mean zero, variance σ_k^2, skewness measure $\gamma_1 = 2$ and kurtosis measure $\gamma_2 = 6$.

Differences in means were obtained by adding multiples of the constant δ ($\delta = 0.67$) to the successive observations within the K treatment levels starting with the second level. Depending upon whether the sampling procedure was restricted to four, six, or eight treatment populations, the generated samples have population means of: $\mu_1 = 0.00$, $\mu_2 = 0.67$, $\mu_3 = 1.34$, $\mu_4 = 2.01$, $\mu_5 = 2.68$, $\mu_6 = 3.35$, $\mu_7 = 4.02$, and $\mu_8 = 4.69$.

From the Pearson and Hartley (1951) tables for noncentrality value of $\phi = (\Sigma\beta_k^2/k)^{\frac{1}{2}}/\sigma/(n)^{\frac{1}{2}} = 1.98$, seven observations per cell were required to obtain 86% power for rejecting the ANOVA null hypothesis.[3] Only after rejecting the ANOVA F hypothesis was the T statistic simulated. The probability of a Type II error was simulated with the observations that had the mean differences built-in($\delta = 0.67$), whereas the probability of a Type I error was evaluated with the original randomly generated observations ($\delta = 0.00$).

Given four, six, and eight levels of the treatment variable there were six, fifteen, and twenty-eight pairwise contrasts, respectively. For the six, fifteen, and twenty-eight contrasts there were three, five, and seven respective magnitudes of deviation from a true psi of zero for 0.67 σ-unit differences between adjoining means. The contrasts were coded as to the extent to which they differed from the multiple comparison null hypothesis. Consequently, the probability of a Type I and Type II error was counted only for contrasts that had the same magnitude of deviation from zero.

For comparisons involving unequal variances the variances were in the ratio of 1:2:3:4 (K = 4), 1:2:3:4:5:6 (K = 6), and 1:2:3:4:5:6:7:8 (K = 8). The unequal variances were achieved by multiplying the generated number by a value for each level such that the average variance was one for the given experiment.

Unequal variances and unequal sample sizes were combined when sampling from the normal distribution to explore the probability of a Type I and Type II error under conditions of assumption violations. The five combinations examined were (1) equal observations per treatment level - equal variances, (2) equal observations per treatment level - unequal variances, (3) unequal observations per treatment level - equal variances, (4) unequal observations per treatment level - unequal variances (positively related) and (5) unequal observations per treatment level - unequal variances (negatively related). These five conditions were also investigated for the non-normal exponential population. The unequal variances and sample sizes for the nine combinations of treatment levels and sample sizes investigated are contained in Table 1.

Table 1 about here

The procedure of generating K random samples with n_k observations per cell and calculating the T statistics constituted one experiment; the procedure was repeated for 1,000 experiments.

Monte Carlo Type I Errors: Because the levels contain different numbers of contrasts there is a difference between the Type I probabilities (Tables 2-4). For example, for four treatment levels there are six pairwise contrasts which are separated into three magnitudes of deviation from zero, that is, for .67 σ-unit differences between adjoining means there is one population contrast (Ψ_I) equal to 2.01 two contrasts equal to 1.34 (Ψ_{II}), and three contrasts equal to 0.67 (Ψ_{III}). The experimentwise error rate fluctuates depending upon the number of contrasts that define the experiment. For the Tukey tests (Table 2, condition 1) over 1000 experiments there are six experiments containing one false rejection when each experiment contains one contrast (Ψ_I), nineteen experiments with at least one false rejection when each experiment contains two contrasts (Ψ_{II}), and forty-two experiments with at least one erroneous statement of rejection when the experiment is defined over three contrasts (Ψ_{III}). When the probability of a Type I experimentwise error is defined over all contrasts comprising the experiment, there are fifty-two experiments with at least one false rejection (Ψ_A). When considering all contrasts comprising the experiment, there are six contrasts for four treatment levels, fifteen for six treatment levels, and twenty-eight when there are eight levels of the treatment variable.

Tables 2-4 about here

Conditions (1) and (2) are not comparison conditions of crucial interest. The probabilities reported for (1) only reflect the procedure of dichotomizing the contrasts into levels of deviation from a true psi of zero whereas, the probabilities tabled under condition (2) not only reflect the above but also, the effect of heterogeneity of variance. In both conditions the H and K probabilities must be equal since, $n_k = n$. Considering conditions (3), (4), and (5) a distinct pattern of comparison emerges. For the combinations of unequal observations per treatment level - equal variances (3), and unequal observations per treatment level - unequal variances (negatively related) (5), the K probabilities are consistently less than the H probabilities. When positively pairing unequal sample sizes and unequal variances (4) the probabilities for the H procedure are smaller. The above pattern holds across the four, six, and eight treatment levels.

Differences in extremes in a large set of means are not adversly affected by form and heterogeneity of variance with increases in the number of treatment levels. If Games (1971) meant by extremes the contrast comparing the two most disparate means, that is Ψ_I, then the probability of a Type I error is inversly related to increases in the number of treatment levels for the three sample sizes investigated. This general inverse relationship holds for each level of deviation (Ψ_I, Ψ_{II},..., Ψ_{VII}) but not when considering all the contrasts (Ψ_A).

Sampling from the exponential distribution does not cause the Type I probabilities to substantially deviate from the probabilities when sampling from the normal distribution, though, the exponential probabilities are, more often than not, less than the corresponding normal distribution probabilities.

Monte Carlo Type II Errors: The H and K Type II probabilities (Tables 5-7) compliment the reported Type I error comparison pattern, though, the pattern here is very weak. The H Type II probabilities are, more often than not, less than and/or generally equal to the K probabilities for conditions (3) and (5). For condition (4), the H probabilities are more often than not, slightly larger and/or generally equal to the tabled K estimates.

Tables 5-7 about here

For comparisons other than the maximum contrast the T statistics are not powerful for the smallest sample condition (n = 7). Increases in sample size and/or an increase in nominal alpha (from .05 to .10) can substantially increase the power for some of the non-maximum contrasts. The greater the true deviation from zero, the more pronounced is the affect.

The exponential and normal distribution probabilities do differ from one another and there are cases in which the discrepancies are large but, no distinctive pattern of difference is specifiable.

Discussion

The harmonic mean and Kramer (1956) unequal n forms of the Tukey multiple comparison statistic were simulated under conditions of assumption violations. Monte Carlo Type I and Type II probabilities for various combinations of population form and heterogeneity of variance were tabled.

The pairwise contrasts were differentiated in terms of their magnitude of deviation from the multiple comparison null hypothesis that $\psi = \sum^{k} c_k \mu_k = 0$. Consequently, Type II errors were evaluated with contrasts that have the same magnitude of deviation. As a consequence of separating the pairwise contrasts, the similarity of the sensitivity of the ANOVA F test and the contrast that compares the maximum difference in a set of K means, as suggested by Scheffé (1959), is corroborated.

The Tukey statistic controls the probability of a Type I error experimentwise and is considered therefore most appropriate when several contrasts are to be computed. The effect of non-normality and heterogeneity of variance is similar to the consequences reported for the ANOVA F test (Scheffé, 1959). Increasing the number of treatment levels under conditions of assumption violations does not generally affect the Type I error probabilities. The probabilities whether extremely conservative, conservative, or liberal, are typically within sampling variability of one another across the four, six, and eight treatment levels.

While selection of an appropriate statistical test is always specific to an experimental question, the Monte Carlo investigation delineates general frames of reference leading to the following recommendations. If there is evidence that suggests that the population variances are not equal, or if the number of observations per cell are not equal and negatively related to unequal variances, the Kramer form of the Tukey statistic would appear to be most appropriate since it better controls for the probability of a Type I error and is not substantially less powerful than the harmonic mean procedure. If suspecting heterogeneity of population variances and sample sizes are positively paired, the data favors the use of the harmonic mean procedure. Although the Type I error probabilities are larger than the K Type I probabilities they are nonetheless still quite conservative, yet the H procedure generally commits fewer Type II errors and is therefore the more powerful statistic.

Significant Tukey contrasts can occur in the absence of a significant F test, though infrequently, since it is based on the distribution of the Studentized range and is not therefore mathematically related to an ANOVA F test. The Tukey tests were only simulated when the ANOVA null hypothesis was rejected in order to mirror the popularized accepted procedure (Glass and Stanley, 1970; Hopkins and Chadburn, 1967; Hays, 1963). Assuming that the tabled Type II probabilities may be slightly inflated estimates, the Tukey tests still lack substantial power for detecting non-maximum contrasts when alpha equals .05. Therefore, regardless of which form of the Tukey test is used to increase power for non-maximum contrasts it is suggested that the level of significance be at least .10.

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Footnotes

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3. $\sum^{k} \beta_k^2$ = squared treatment effects

   σ = square root of the mean square error from the ANOVA

   n = number of observations per cell

   k = number of means

TABLE 1

UNEQUAL VARIANCES AND SAMPLE SIZES FOR EIGHT COMBINATIONS OF TREATMENT LEVELS AND SAMPLE SIZES

	TREATMENT LEVELS (K)		
	4	6	8
Variances:	.4, .8, 1.2, 1.6	.28571, .57142, .85713, 1.14284, 1.42855, 1.71426	.22222, .44444, .66666, .88888, 1.11110, 1.33332, 1.55554, 1.77776
Sample Sizes:			
$n_k = 7$	4, 5, 7, 12	6, 6, 7, 7, 7, 9	6, 6, 7, 7, 7, 7, 8, 8
$n_k = 16$	12, 14, 17, 21	15, 15, 16, 16, 17, 17	15, 15, 16, 16, 16, 16, 17, 17
$n_k = 25$	21, 23, 26, 30	24, 24, 25, 25, 26, 26	24, 24, 25, 25, 25, 25, 26, 26

TABLE 2

MONTE CARLO TYPE I EXPERIMENTWISE ERRORS FOR THE HARMONIC MEAN (H) AND KRAMER (K) UNEQUAL FORMS OF THE TUKEY STATISTIC FOR FOUR TREATMENT LEVELS WHEN DEVIATION FROM ZERO IS CONSIDERED

			CONDITIONS*																			
			NORMAL DISTRIBUTION										EXPONENTIAL DISTRIBUTION									
			1		2		3		4		5		1		2		3		4		5	
			H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K
$\alpha = .05$	$n_k=7$	ψ_I	0.006	0.006	0.009	0.009	0.005	0.005	0.001	0.001	0.036	0.037	0.009	0.009	0.016	0.016	0.012	0.012	0.001	0.001	0.030	0.030
		ψ_{II}	0.019	0.019	0.026	0.026	0.024	0.021	0.006	0.006	0.058	0.050	0.019	0.019	0.017	0.017	0.030	0.027	0.008	0.010	0.063	0.056
		ψ_{III}	0.042	0.042	0.048	0.048	0.045	0.033	0.016	0.023	0.107	0.074	0.026	0.026	0.028	0.028	0.040	0.033	0.011	0.016	0.100	0.069
		ψ_A	0.052	0.052	0.064	0.064	0.058	0.049	0.018	0.027	0.141	0.106	0.041	0.041	0.049	0.049	0.056	0.050	0.015	0.024	0.136	0.099
	$n_k=16$	ψ_I	0.012	0.012	0.011	0.011	0.013	0.013	0.002	0.002	0.023	0.023	0.010	0.010	0.014	0.014	0.010	0.010	0.007	0.007	0.020	0.020
		ψ_{II}	0.024	0.024	0.031	0.031	0.021	0.020	0.016	0.016	0.042	0.037	0.016	0.016	0.038	0.038	0.017	0.018	0.014	0.013	0.030	0.033
		ψ_{III}	0.031	0.031	0.041	0.041	0.024	0.022	0.015	0.024	0.064	0.044	0.024	0.024	0.034	0.034	0.033	0.029	0.021	0.023	0.064	0.044
		ψ_A	0.057	0.057	0.063	0.063	0.052	0.049	0.028	0.035	0.093	0.080	0.038	0.038	0.066	0.066	0.047	0.043	0.035	0.038	0.092	0.073
	$n_k=25$	ψ_I	0.007	0.007	0.023	0.023	0.011	0.011	0.017	0.017	0.029	0.029	0.012	0.012	0.012	0.012	0.014	0.014	0.011	0.011	0.014	0.015
		ψ_{II}	0.023	0.023	0.035	0.035	0.025	0.025	0.014	0.019	0.059	0.051	0.020	0.020	0.018	0.018	0.014	0.012	0.018	0.018	0.038	0.030
		ψ_{III}	0.033	0.033	0.033	0.033	0.025	0.024	0.025	0.037	0.054	0.045	0.030	0.030	0.025	0.025	0.018	0.017	0.027	0.032	0.051	0.044
		ψ_A	0.053	0.053	0.069	0.069	0.050	0.049	0.043	0.058	0.096	0.085	0.052	0.052	0.047	0.047	0.036	0.035	0.046	0.051	0.082	0.071
$\alpha = .10$	$n_k=7$	ψ_I	0.021	0.021	0.023	0.023	0.012	0.013	0.004	0.004	0.059	0.059	0.016	0.016	0.022	0.022	0.021	0.021	0.008	0.009	0.061	0.062
		ψ_{II}	0.050	0.050	0.054	0.054	0.044	0.041	0.025	0.023	0.107	0.102	0.041	0.041	0.054	0.054	0.054	0.051	0.021	0.025	0.100	0.098
		ψ_{III}	0.057	0.057	0.069	0.069	0.072	0.069	0.032	0.034	0.156	0.131	0.057	0.057	0.074	0.074	0.070	0.063	0.028	0.036	0.151	0.110
		ψ_A	0.092	0.092	0.103	0.103	0.095	0.092	0.043	0.047	0.195	0.177	0.082	0.082	0.099	0.099	0.097	0.088	0.042	0.054	0.214	0.181
	$n_k=16$	ψ_I	0.022	0.022	0.040	0.040	0.023	0.023	0.016	0.016	0.055	0.055	0.023	0.023	0.028	0.028	0.027	0.027	0.009	0.009	0.055	0.055
		ψ_{II}	0.042	0.042	0.054	0.054	0.040	0.040	0.031	0.036	0.094	0.087	0.040	0.040	0.052	0.052	0.039	0.039	0.033	0.035	0.087	0.083
		ψ_{III}	0.061	0.061	0.068	0.068	0.069	0.070	0.049	0.057	0.122	0.101	0.069	0.069	0.091	0.091	0.055	0.051	0.042	0.053	0.115	0.100
		ψ_A	0.093	0.093	0.112	0.112	0.103	0.108	0.073	0.086	0.177	0.162	0.097	0.097	0.129	0.129	0.092	0.090	0.067	0.082	0.174	0.154
	$n_k=25$	ψ_I	0.022	0.022	0.042	0.042	0.013	0.013	0.035	0.036	0.053	0.054	0.017	0.017	0.022	0.022	0.021	0.021	0.014	0.014	0.029	0.030
		ψ_{II}	0.055	0.055	0.067	0.067	0.032	0.032	0.032	0.037	0.092	0.082	0.048	0.048	0.058	0.058	0.057	0.052	0.030	0.033	0.061	0.055
		ψ_{III}	0.064	0.064	0.079	0.079	0.044	0.049	0.065	0.082	0.107	0.102	0.058	0.058	0.074	0.074	0.060	0.057	0.042	0.055	0.102	0.088
		ψ_A	0.103	0.103	0.137	0.137	0.074	0.079	0.100	0.114	0.173	0.164	0.093	0.093	0.112	0.112	0	0.096	0.069	0.083	0.143	0.127

$\alpha = .05$, $\sigma_p = .007$; $\alpha = .10$, $\sigma_p = .009$

*Conditions: (1) equal n's - equal σ^2's (2) equal n's unequal σ^2's (3) unequal n's - equal σ^2's (4) unequal n's - unequal σ^2's (positively related) (5) unequal n's - unequal σ^2's (negatively related).

**ψ_I Maximum contrast; ψ_{II} 2nd largest contrast(s); ψ_{III} 3rd largest contrast(s); ψ_A all contrasts.

TABLE 3

MONTE CARLO TYPE I EXPERIMENTWISE ERRORS FOR THE HARMONIC (H) AND KRAMER (K) UNEQUAL FORS OF THE TUKEY STATISTIC FOR SIX TREATMENT LEVELS WHEN DEVIATION FROM ZERO IS CONSIDERED

			CONDITIONS*																			
			NORMAL DISTRIBUTION										EXPONENTIAL DISTRIBUTION									
			1		2		3		4		5		1		2		3		4		5	
		ψ**	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	L	H	K	H	K
$\alpha = .05$	$n_k=7$	ψ_I	0.005	0.005	0.008	0.008	0.004	0.006	0.003	0.003	0.012	0.013	0.004	0.004	0.005	0.005	0.006	0.006	0.002	0.002	0.005	0.005
		ψ_{II}	0.018	0.018	0.014	0.014	0.006	0.005	0.008	0.008	0.018	0.020	0.009	0.009	0.014	0.014	0.014	0.015	0.011	0.009	0.017	0.015
		ψ_{III}	0.016	0.016	0.020	0.020	0.017	0.019	0.009	0.009	0.035	0.026	0.006	0.006	0.026	0.026	0.019	0.014	0.012	0.014	0.032	0.026
		ψ_{IV}	0.016	0.016	0.030	0.030	0.014	0.012	0.015	0.019	0.038	0.034	0.019	0.019	0.024	0.024	0.024	0.026	0.020	0.022	0.045	0.036
		ψ_V	0.026	0.026	0.033	0.033	0.013	0.015	0.025	0.030	0.058	0.052	0.021	0.021	0.041	0.041	0.029	0.026	0.024	0.028	0.057	0.042
		ψ_A	0.058	0.058	0.062	0.062	0.039	0.041	0.040	0.049	0.096	0.085	0.041	0.041	0.067	0.067	0.050	0.048	0.046	0.052	0.092	0.070
	$n_k=16$	ψ_I	0.004	0.004	0.004	0.004	0.003	0.003	0.001	0.001	0.002	0.002	0.005	0.005	0.004	0.004	0.007	0.007	0.005	0.005	0.006	0.006
		ψ_{II}	0.007	0.007	0.009	0.009	0.009	0.009	0.006	0.006	0.008	0.008	0.008	0.008	0.014	0.014	0.005	0.005	0.012	0.012	0.009	0.009
		ψ_{III}	0.017	0.017	0.022	0.022	0.011	0.011	0.015	0.016	0.022	0.020	0.016	0.016	0.021	0.021	0.018	0.017	0.016	0.018	0.023	0.021
		ψ_{IV}	0.013	0.013	0.032	0.032	0.012	0.013	0.012	0.016	0.029	0.027	0.015	0.015	0.030	0.030	0.023	0.022	0.026	0.024	0.030	0.028
		ψ_V	0.014	0.014	0.032	0.032	0.015	0.013	0.033	0.038	0.057	0.052	0.023	0.023	0.037	0.037	0.025	0.028	0.036	0.041	0.055	0.046
		ψ_A	0.047	0.047	0.070	0.070	0.040	0.040	0.055	0.061	0.083	0.077	0.045	0.045	0.079	0.079	0.048	0.049	0.071	0.075	0.086	0.076
	$n_k=25$	ψ_I	0.007	0.007	0.004	0.004	0.006	0.006	0.005	0.005	0.010	0.010	0.010	0.010	0.002	0.002	0.005	0.005	0.006	0.006	0.007	0.007
		ψ_{II}	0.018	0.018	0.004	0.004	0.012	0.012	0.012	0.012	0.016	0.016	0.015	0.015	0.011	0.011	0.013	0.012	0.008	0.008	0.008	0.008
		ψ_{IIII}	0.021	0.021	0.022	0.022	0.015	0.014	0.016	0.017	0.020	0.019	0.025	0.025	0.021	0.021	0.009	0.009	0.012	0.012	0.024	0.023
		ψ_{IV}	0.019	0.019	0.027	0.027	0.018	0.019	0.027	0.030	0.038	0.038	0.022	0.022	0.034	0.034	0.018	0.017	0.029	0.029	0.038	0.034
		ψ_V	0.018	0.018	0.036	0.036	0.023	0.022	0.031	0.034	0.053	0.049	0.035	0.035	0.043	0.043	0.023	0.023	0.033	0.036	0.046	0.039
		ψ_A	0.062	0.062	0.067	0.067	0.055	0.054	0.065	0.070	0.095	0.091	0.072	0.072	0.086	0.086	0.049	0.048	0.070	0.073	0.085	0.075
$\alpha = .10$	$n_k=7$	ψ_I	0.004	0.004	0.012	0.012	0.015	0.015	0.009	0.009	0.022	0.024	0.021	0.021	0.012	0.012	0.008	0.010	0.004	0.006	0.014	0.022
		ψ_{II}	0.015	0.015	0.023	0.023	0.022	0.019	0.016	0.015	0.034	0.033	0.028	0.028	0.028	0.028	0.017	0.017	0.013	0.016	0.040	0.037
		ψ_{III}	0.030	0.030	0.036	0.036	0.035	0.033	0.030	0.032	0.057	0.052	0.031	0.031	0.037	0.037	0.037	0.039	0.014	0.024	0.052	0.044
		ψ_{IV}	0.038	0.038	0.051	0.051	0.041	0.038	0.034	0.038	0.078	0.071	0.040	0.040	0.041	0.041	0.040	0.035	0.020	0.034	0.072	0.058
		ψ_V	0.043	0.043	0.066	0.066	0.056	0.058	0.047	0.055	0.104	0.085	0.053	0.053	0.052	0.052	0.047	0.045	0.036	0.048	0.078	0.068
		ψ_A	0.091	0.091	0.117	0.117	0.115	0.111	0.081	0.089	0.175	0.154	0.103	0.103	0.108	0.108	0.091	0.091	0.063	0.095	0.147	0.129
	$n_k=16$	ψ_I	0.010	0.010	0.018	0.018	0.009	0.009	0.009	0.009	0.010	0.003	0.009	0.009	0.015	0.015	0.012	0.012	0.006	0.006	0.020	0.019
		ψ_{II}	0.013	0.013	0.028	0.028	0.018	0.018	0.019	0.019	0.020	0.003	0.031	0.031	0.022	0.022	0.020	0.020	0.018	0.018	0.026	0.025
		ψ_{III}	0.023	0.023	0.048	0.048	0.023	0.024	0.030	0.029	0.048	0.010	0.041	0.041	0.038	0.038	0.030	0.031	0.035	0.037	0.052	0.050
		ψ_{IV}	0.035	0.035	0.052	0.052	0.032	0.029	0.043	0.048	0.059	0.013	0.043	0.043	0.054	0.054	0.038	0.035	0.048	0.050	0.064	0.062
		ψ_V	0.035	0.035	0.068	0.068	0.052	0.052	0.063	0.070	0.080	0.021	0.043	0.043	0.080	0.080	0.041	0.044	0.060	0.066	0.097	0.087
		ψ_A	0.085	0.085	0.140	0.140	0.093	0.092	0.111	0.121	0.142	0.042	0.098	0.098	0.139	0.139	0.094	0.093	0.105	0.109	0.157	0.147
	$n_k=25$	ψ_I	0.016	0.016	0.006	0.006	0.015	0.015	0.007	0.007	0.012	0.012	0.008	0.008	0.015	0.015	0.012	0.012	0.009	0.008	0.017	0.017
		ψ_{II}	0.018	0.018	0.019	0.019	0.015	0.015	0.022	0.022	0.029	0.029	0.018	0.018	0.029	0.029	0.018	0.018	0.020	0.020	0.031	0.031
		ψ_{III}	0.029	0.029	0.033	0.033	0.027	0.029	0.031	0.032	0.038	0.036	0.032	0.032	0.042	0.042	0.029	0.029	0.035	0.036	0.046	0.044
		ψ_{IV}	0.040	0.040	0.052	0.052	0.030	0.028	0.043	0.044	0.054	0.055	0.041	0.041	0.045	0.045	0.041	0.042	0.047	0.048	0.062	0.062
		ψ_V	0.049	0.049	0.077	0.077	0.039	0.037	0.066	0.072	0.077	0.078	0.052	0.052	0.066	0.066	0.050	0.052	0.053	0.057	0.072	0.069
		ψ_A	0.099	0.099	0.130	0.130	0.096	0.095	0.108	0.112	0.135	0.133	0.102	0.102	0.122	0.122	0.108	0.110	0.110	0.114	0.143	0.139

$\alpha = .05$, $\sigma_p = .007$; $\alpha = .10$, $\sigma_p = .009$

*Conditions: (1) equal n's - equal σ^2's (2) equal n's unequal σ^2's (3) unequal n's - equal σ^2's (4) unequal n's - unequal σ^2's (positively) related) (5) unequal n's - unequal σ^2's (negatively related).

**ψ_I Maximum contrast; ψ_{II} 2nd largest contrast(s); ψ_{III} 3rd largest contrast(s); ψ_{IV} 4th largest contrast(s); ψ_V 5th largest contrast(s); ψ_A all contrasts.

TABLE 4

MONTE CARLO TYPE I EXPERIMENTWISE ERRORS FOR THE HARMONIC MEAN (H) AND KRAMER (K) UNEQUAL FORMS OF THE TUKEY STATISTIC FOR EIGHT TREATMENT LEVELS WHEN DEVIATION FROM ZERO IS CONSIDERED

			CONDITIONS*																			
			NORMAL DISTRIBUTION										EXPONENTIAL DISTRIBUTION									
			1		2		3		4		5		1		2		3		4		5	
		ψ**	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K
$\alpha = .05$	$n_k = 7$	ψ_I	0.002	0.002	0.002	0.002	0.001	0.001	0.002	0.002	0.006	0.005	0.005	0.005	0.003	0.003	0.001	0.001	0.005	0.003	0.006	0.006
		ψ_{II}	0.003	0.003	0.008	0.008	0.003	0.003	0.003	0.003	0.009	0.009	0.007	0.007	0.008	0.008	0.007	0.006	0.009	0.009	0.011	0.011
		ψ_{III}	0.004	0.004	0.014	0.014	0.006	0.007	0.008	0.007	0.031	0.029	0.009	0.009	0.012	0.012	0.004	0.003	0.011	0.012	0.018	0.017
		ψ_{IV}	0.016	0.016	0.016	0.016	0.006	0.007	0.016	0.015	0.032	0.030	0.012	0.012	0.013	0.013	0.010	0.012	0.014	0.014	0.030	0.027
		ψ_V	0.012	0.012	0.021	0.021	0.010	0.009	0.018	0.023	0.043	0.039	0.012	0.012	0.028	0.028	0.012	0.015	0.018	0.020	0.033	0.030
		ψ_{VI}	0.010	0.010	0.024	0.024	0.005	0.006	0.022	0.028	0.042	0.037	0.014	0.014	0.029	0.029	0.008	0.009	0.028	0.033	0.047	0.038
		ψ_{VII}	0.012	0.012	0.040	0.040	0.019	0.022	0.031	0.038	0.054	0.042	0.020	0.020	0.030	0.030	0.014	0.011	0.029	0.035	0.049	0.049
		ψ_A	0.045	0.045	0.077	0.077	0.038	0.044	0.061	0.073	0.117	0.100	0.036	0.036	0.073	0.073	0.031	0.036	0.057	0.064	0.109	0.099
	$n_k = 16$	ψ_I	0.000	0.000	0.002	0.002	0.001	0.001	0.003	0.003	0.005	0.005	0.004	0.004	0.002	0.002	0.000	0.000	0.002	0.002	0.003	0.003
		ψ_{II}	0.005	0.005	0.008	0.008	0.005	0.004	0.002	0.002	0.011	0.011	0.004	0.004	0.006	0.006	0.005	0.005	0.011	0.011	0.007	0.007
		ψ_{III}	0.011	0.011	0.012	0.012	0.007	0.008	0.006	0.007	0.009	0.009	0.007	0.007	0.009	0.009	0.007	0.007	0.009	0.009	0.008	0.008
		ψ_{IV}	0.011	0.011	0.021	0.021	0.018	0.017	0.014	0.016	0.022	0.020	0.009	0.009	0.020	0.020	0.007	0.007	0.012	0.014	0.011	0.011
		ψ_V	0.010	0.010	0.031	0.031	0.014	0.014	0.020	0.021	0.034	0.031	0.010	0.010	0.020	0.020	0.013	0.013	0.013	0.014	0.030	0.029
		ψ_{VI}	0.020	0.020	0.038	0.038	0.021	0.019	0.018	0.023	0.033	0.030	0.012	0.012	0.032	0.032	0.018	0.017	0.030	0.030	0.029	0.029
		ψ_{VII}	0.023	0.023	0.042	0.042	0.017	0.018	0.031	0.039	0.037	0.033	0.014	0.014	0.038	0.038	0.013	0.013	0.024	0.026	0.040	0.034
		ψ_A	0.053	0.053	0.093	0.093	0.062	0.059	0.067	0.079	0.082	0.092	0.035	0.035	0.080	0.080	0.043	0.042	0.058	0.060	0.077	0.073
	$n_k = 25$	ψ_I	0.004	0.004	0.001	0.001	0.003	0.003	0.002	0.002	0.001	0.001	0.003	0.003	0.005	0.005	0.001	0.001	0.000	0.000	0.003	0.003
		ψ_{II}	0.003	0.003	0.005	0.005	0.004	0.004	0.002	0.002	0.007	0.007	0.002	0.002	0.008	0.008	0.004	0.004	0.004	0.004	0.008	0.008
		ψ_{III}	0.006	0.006	0.010	0.010	0.009	0.008	0.014	0.015	0.010	0.009	0.011	0.011	0.012	0.012	0.013	0.012	0.009	0.009	0.011	0.011
		ψ_{IV}	0.011	0.011	0.014	0.014	0.013	0.011	0.013	0.014	0.014	0.014	0.006	0.006	0.013	0.013	0.017	0.017	0.005	0.005	0.017	0.017
		ψ_V	0.013	0.013	0.017	0.017	0.011	0.013	0.024	0.025	0.021	0.019	0.014	0.014	0.018	0.018	0.011	0.009	0.018	0.019	0.031	0.029
		ψ_{VI}	0.012	0.012	0.032	0.032	0.011	0.011	0.025	0.027	0.042	0.040	0.020	0.020	0.017	0.017	0.015	0.015	0.024	0.027	0.035	0.035
		ψ_{VII}	0.012	0.012	0.046	0.046	0.018	0.018	0.040	0.041	0.040	0.035	0.024	0.024	0.038	0.038	0.021	0.022	0.028	0.031	0.053	0.051
		ψ_A	0.046	0.046	0.083	0.083	0.048	0.047	0.084	0.086	0.096	0.088	0.054	0.054	0.070	0.070	0.058	0.058	0.068	0.073	0.099	0.098
$\alpha = .10$	$n_k = 7$	ψ_I	0.009	0.009	0.008	0.008	0.003	0.003	0.007	0.006	0.011	0.010	0.012	0.012	0.002	0.002	0.003	0.003	0.006	0.006	0.009	0.009
		ψ_{II}	0.010	0.010	0.012	0.012	0.009	0.008	0.013	0.012	0.024	0.023	0.013	0.013	0.013	0.013	0.010	0.009	0.010	0.010	0.019	0.018
		ψ_{III}	0.014	0.014	0.019	0.019	0.020	0.022	0.018	0.021	0.031	0.029	0.022	0.022	0.022	0.022	0.012	0.014	0.011	0.013	0.028	0.027
		ψ_{IV}	0.020	0.020	0.026	0.026	0.024	0.028	0.027	0.031	0.048	0.036	0.028	0.028	0.033	0.033	0.027	0.024	0.020	0.025	0.033	0.027
		ψ_V	0.028	0.028	0.051	0.051	0.028	0.027	0.032	0.033	0.070	0.062	0.041	0.041	0.038	0.038	0.023	0.022	0.021	0.026	0.058	0.052
		ψ_{VI}	0.029	0.029	0.050	0.050	0.026	0.029	0.037	0.048	0.079	0.072	0.043	0.043	0.045	0.045	0.031	0.035	0.040	0.044	0.066	0.061
		ψ_{VII}	0.032	0.032	0.068	0.068	0.041	0.041	0.052	0.066	0.106	0.088	0.050	0.050	0.054	0.054	0.026	0.027	0.043	0.054	0.092	0.078
		ψ_A	0.083	0.083	0.131	0.131	0.099	0.104	0.108	0.126	0.197	0.167	0.117	0.117	0.117	0.117	0.073	0.069	0.087	0.107	0.159	0.139
	$n_k = 16$	ψ_I	0.005	0.005	0.001	0.001	0.005	0.005	0.005	0.005	0.011	0.011	0.006	0.006	0.001	0.001	0.004	0.004	0.003	0.002	0.008	0.008
		ψ_{II}	0.011	0.011	0.007	0.007	0.014	0.014	0.015	0.015	0.018	0.017	0.011	0.011	0.015	0.015	0.011	0.011	0.012	0.012	0.010	0.010
		ψ_{III}	0.015	0.015	0.018	0.018	0.018	0.017	0.022	0.023	0.018	0.019	0.020	0.020	0.022	0.022	0.012	0.013	0.016	0.018	0.020	0.019
		ψ_{IV}	0.022	0.022	0.036	0.036	0.017	0.019	0.026	0.027	0.034	0.032	0.017	0.017	0.019	0.019	0.020	0.020	0.034	0.036	0.032	0.029
		ψ_V	0.023	0.023	0.034	0.034	0.026	0.024	0.037	0.037	0.048	0.042	0.031	0.031	0.044	0.044	0.021	0.021	0.038	0.039	0.048	0.047
		ψ_{VI}	0.026	0.026	0.042	0.042	0.026	0.027	0.045	0.049	0.069	0.066	0.030	0.030	0.049	0.049	0.027	0.028	0.042	0.045	0.058	0.056
		ψ_{VII}	0.030	0.030	0.058	0.058	0.044	0.041	0.062	0.068	0.071	0.060	0.028	0.028	0.050	0.050	0.031	0.030	0.058	0.066	0.075	0.069
		ψ_A	0.089	0.089	0.117	0.117	0.101	0.102	0.117	0.130	0.149	0.137	0.091	0.091	0.117	0.117	0.081	0.081	0.116	0.129	0.140	0.129
	$n_k = 25$	ψ_I	0.010	0.010	0.004	0.004	0.003	0.003	0.009	0.009	0.006	0.006	0.008	0.008	0.005	0.005	0.005	0.005	0.010	0.010	0.014	0.014
		ψ_{II}	0.007	0.007	0.011	0.011	0.011	0.011	0.010	0.010	0.021	0.021	0.012	0.012	0.014	0.014	0.010	0.010	0.015	0.015	0.022	0.022
		ψ_{III}	0.017	0.017	0.013	0.013	0.020	0.019	0.016	0.016	0.029	0.029	0.019	0.019	0.014	0.014	0.018	0.017	0.018	0.017	0.031	0.028
		ψ_{IV}	0.016	0.016	0.035	0.035	0.022	0.019	0.026	0.028	0.039	0.039	0.031	0.031	0.025	0.025	0.025	0.029	0.032	0.035	0.051	0.043
		ψ_V	0.017	0.017	0.046	0.046	0.021	0.022	0.040	0.039	0.052	0.050	0.028	0.028	0.038	0.038	0.035	0.035	0.045	0.047	0.073	0.073
		ψ_{VI}	0.028	0.028	0.044	0.044	0.027	0.028	0.045	0.046	0.064	0.060	0.035	0.035	0.056	0.056	0.025	0.025	0.051	0.052	0.072	0.070
		ψ_{VII}	0.035	0.035	0.056	0.056	0.035	0.036	0.059	0.059	0.079	0.074	0.042	0.042	0.050	0.050	0.033	0.035	0.069	0.075	0.089	0.086
		ψ_A	0.091	0.091	0.127	0.127	0.101	0.099	0.131	0.134	0.182	0.176	0.102	0.102	0.121	0.121	0.096	0.098	0.136	0.145	0.178	0.169

$\alpha = .05, \sigma_p = .007; \quad \alpha = .10, \sigma_p = .009$

*Conditions: (1) equal n's - equal σ^2's (2) equal n's unequal σ^2's (3) unequal n's - equal σ^2's (4) unequal n's - unequal σ^2's (positively related) (5) unequal n's - unequal σ^2's (negatively related).

**ψ_I Maximum contrast; ψ_{II} 2nd largest contrast(s); ψ_{III} 3rd largest contrast(s); ψ_{IV} 4th largest contrast(s); ψ_V 5th largest contrast(s); ψ_{VI} 6th largest contrast(s); ψ_{VII} 7th largest contrast(s); ψ_A all contrasts.

TABLE 5

MONTE CARLO TYPE II EXPERIMENTWISE ERRORS FOR THE HARMONIC MEAN (H) AND KRAMER (K) UNEQUAL FORMS OF THE TUKEY STATISTIC HAVING .67 σ-UNIT MEAN DIFFERENCES FOR FOUR TREATMENT LEVELS WHEN DEVIATION FROM ZERO IS CONSIDERED

			CONDITIONS*																			
			NORMAL DISTRIBUTION										EXPONENTIAL DISTRIBUTION									
			1		2		3		4		5		1		2		3		4		5	
		ψ**	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K
α = .05	$n_k = 7$	ψ_I	0.037	0.037	0.047	0.047	0.086	0.084	0.112	0.106	0.082	0.081	0.065	0.065	0.058	0.058	0.098	0.094	0.108	0.104	0.110	0.107
		ψ_{II}	0.763	0.763	0.746	0.746	0.820	0.807	0.890	0.895	0.723	0.709	0.675	0.675	0.743	0.743	0.738	0.741	0.889	0.892	0.595	0.575
		ψ_{III}	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.998	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.993	0.994
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.995
	$n_k = 16$	ψ_I	0.001	0.001	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.004	0.004	0.003	0.003	0.010	0.010	0.001	0.001	0.016	0.016
		ψ_{II}	0.255	0.255	0.251	0.251	0.260	0.264	0.301	0.300	0.264	0.268	0.235	0.235	0.231	0.231	0.262	0.264	0.276	0.288	0.277	0.281
		ψ_{III}	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	0.998	0.999	0.992	0.992	0.999	0.999	0.996	0.995	1.000	0.999	0.973	0.972
		ψ_A	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	0.998	0.999	0.992	0.992	0.999	0.999	0.996	0.995	1.000	0.999	0.973	0.972
	$n_k = 25$	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
		ψ_{II}	0.055	0.055	0.012	0.012	0.058	0.062	0.027	0.020	0.138	0.143	0.050	0.050	0.029	0.029	0.058	0.065	0.041	0.041	0.081	0.090
		ψ_{III}	0.955	0.955	0.936	0.936	0.953	0.953	0.957	0.961	0.960	0.957	0.949	0.949	0.983	0.983	0.958	0.961	0.992	0.993	0.915	0.910
		ψ_A	0.955	0.955	0.936	0.936	0.953	0.953	0.957	0.961	0.960	0.957	0.949	0.949	0.983	0.983	0.958	0.961	0.992	0.993	0.915	0.910
α = .10	$n_k = 7$	ψ_I	0.033	0.033	0.018	0.018	0.073	0.072	0.045	0.045	0.085	0.084	0.055	0.055	0.040	0.040	0.092	0.089	0.088	0.081	0.071	0.069
		ψ_{II}	0.569	0.569	0.532	0.532	0.631	0.627	0.657	0.680	0.542	0.548	0.564	0.564	0.628	0.628	0.652	0.659	0.784	0.778	0.513	0.516
		ψ_{III}	0.999	0.999	0.997	0.997	1.000	0.998	1.000	1.000	0.992	0.990	0.999	0.999	1.000	1.000	0.997	0.997	0.999	0.999	0.982	0.978
		ψ_A	0.999	0.999	0.997	0.997	1.000	0.999	1.000	1.000	0.992	0.991	0.999	0.999	1.000	1.000	0.999	0.999	1.000	1.000	0.983	0.979
	$n_k = 16$	ψ_I	0.001	0.001	0.000	0.000	0.003	0.003	0.000	0.000	0.018	0.018	0.003	0.003	0.001	0.001	0.004	0.004	0.001	0.001	0.010	0.010
		ψ_{II}	0.170	0.170	0.051	0.051	0.169	0.175	0.083	0.079	0.254	0.255	0.170	0.170	0.132	0.132	0.194	0.195	0.167	0.174	0.182	0.188
		ψ_{III}	0.975	0.975	0.970	0.970	0.981	0.981	0.981	0.987	0.978	0.983	0.980	0.980	0.995	0.995	0.985	0.985	0.998	0.998	0.942	0.943
		ψ_A	0.975	0.975	0.970	0.970	0.981	0.981	0.981	0.987	0.978	0.983	0.980	0.980	0.995	0.995	0.985	0.985	0.998	0.998	0.942	0.943
	$n_k = 25$	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.039	0.039	0.005	0.005	0.022	0.022	0.006	0.007	0.094	0.095	0.028	0.028	0.021	0.021	0.038	0.036	0.010	0.011	0.053	0.053
		ψ_{III}	0.887	0.887	0.838	0.838	0.892	0.904	0.878	0.872	0.925	0.923	0.899	0.899	0.930	0.930	0.903	0.907	0.944	0.944	0.871	0.862
		ψ_A	0.887	0.887	0.838	0.838	0.892	0.904	0.878	0.872	0.925	0.923	0.899	0.899	0.930	0.930	0.903	0.907	0.944	0.944	0.871	0.862

$\alpha = .05$, $\sigma_p = .007$; $\alpha = .10$, $\sigma_p = .009$

*Conditions: (1) equal n's - equal σ^2's (2) equal n's unequal σ^2's (3) unequal n's - equal σ^2's (4) unequal n's - unequal σ^2's (positively related) (5) unequal n's - unequal σ^2's (negatively related).

** ψ_I Maximum contrast; ψ_{II} 2nd largest contrast(s); ψ_{III} 3rd largest contrast(s); ψ_A all contrasts.

TABLE 6

MONTE CARLO TYPE II EXPERIMENTWISE ERRORS FOR THE HARMONIC MEAN (H) AND KRAMER (K) UNEQUAL FORS OF THE TUKEY STATISTIC HAVING .67 σ-UNIT MEAN DIFFERENCES FOR SIX TREATMENT LEVELS WHEN DEVIATION FROM ZERO IS CONSIDERED

			CONDITION*																			
			NORMAL DISTRIBUTION										EXPONENTIAL DISTRIBUTION									
			1		2		3		4		5		1		2		3		4		5	
		ψ**	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K
α = .05	n_k = 7	ψ_I	0.003	0.003	0.002	0.002	0.001	0.000	0.001	0.000	0.000	0.000	0.009	0.009	0.002	0.002	0.010	0.007	0.003	0.002	0.020	0.019
		ψ_{II}	0.072	0.072	0.052	0.052	0.057	0.063	0.083	0.081	0.051	0.052	0.086	0.086	0.049	0.049	0.113	0.112	0.064	0.070	0.117	0.122
		ψ_{III}	0.535	0.535	0.529	0.529	0.548	0.544	0.599	0.602	0.495	0.502	0.464	0.464	0.457	0.457	0.488	0.503	0.487	0.493	0.449	0.456
		ψ_{IV}	0.991	0.991	0.988	0.980	0.991	0.990	0.991	0.992	0.984	0.983	0.968	0.968	0.971	0.971	0.954	0.956	0.983	0.986	0.918	0.916
		ψ_V	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	n_k = 16	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.006
		ψ_{III}	0.011	0.011	0.011	0.011	0.007	0.008	0.013	0.012	0.014	0.015	0.029	0.029	0.018	0.018	0.030	0.033	0.007	0.007	0.046	0.048
		ψ_{IV}	0.594	0.594	0.611	0.611	0.625	0.621	0.609	0.602	0.571	0.574	0.568	0.568	0.557	0.557	0.538	0.539	0.577	0.577	0.526	0.521
		ψ_V	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.998
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.998
	n_k = 25	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{III}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002
		ψ_{IV}	0.129	0.129	0.128	0.128	0.118	0.119	0.131	0.129	0.129	0.135	0.157	0.157	0.106	0.106	0.153	0.156	0.124	0.121	0.168	0.168
		ψ_V	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999	1.000	1.000	0.999	1.000	1.000	1.000	0.994	0.995
		ψ_A	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999	1.000	1.000	0.999	1.000	1.000	1.000	0.994	0.995
α = .10	n_k = 7	ψ_I	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.001	0.005	0.005	0.000	0.000	0.002	0.002	0.000	0.000	0.015	0.011
		ψ_{II}	0.024	0.024	0.026	0.026	0.020	0.022	0.035	0.035	0.030	0.035	0.062	0.062	0.027	0.027	0.058	0.061	0.036	0.035	0.087	0.091
		ψ_{III}	0.382	0.382	0.346	0.346	0.370	0.333	0.414	0.422	0.349	0.355	0.372	0.372	0.318	0.318	0.342	0.349	0.356	0.362	0.350	0.353
		ψ_{IV}	0.977	0.977	0.969	0.969	0.970	0.971	0.972	0.974	0.966	0.964	0.913	0.913	0.946	0.946	0.924	0.922	0.970	0.972	0.850	0.844
		ψ_V	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	n_k = 16	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
		ψ_{III}	0.009	0.009	0.004	0.004	0.003	0.003	0.007	0.006	0.010	0.010	0.018	0.018	0.006	0.006	0.016	0.017	0.008	0.007	0.034	0.035
		ψ_{IV}	0.426	0.426	0.418	0.418	0.448	0.439	0.451	0.448	0.404	0.415	0.435	0.435	0.409	0.409	0.413	0.414	0.411	0.409	0.421	0.419
		ψ_V	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	n_k = 25	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{III}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
		ψ_{IV}	0.049	0.049	0.077	0.077	0.065	0.064	0.082	0.079	0.076	0.084	0.079	0.079	0.072	0.072	0.080	0.079	0.068	0.066	0.111	0.112
		ψ_V	0.999	0.999	1.000	1.000	0.999	0.999	1.000	1.000	0.999	0.999	0.999	0.999	0.998	0.998	0.995	0.996	1.000	1.000	0.985	0.986
		ψ_A	0.999	0.999	1.000	1.000	0.999	0.999	1.000	1.000	0.999	0.999	0.999	0.999	0.998	0.998	0.995	0.996	1.000	1.000	0.985	0.986

$\alpha = .05$, $\sigma_p = .007$; $\alpha = .10$, $\sigma_p = .009$

*Conditions: (1) equal n's – equal σ^2's (2) equal n's unequal σ^2's (3) unequal n's – equal σ^2's (4) unequal n's – unequal σ^2's (positively related) (5) unequal n's – unequal σ^2's (negatively related).

** ψ_I Maximum contrast; ψ_{II} 2nd largest contrast(s); ψ_{III} 3rd largest contrast(s); ψ_{IV} 4th largest contrast(s); ψ_V 5th largest contrast(s); ψ_A all contrasts.

TABLE 7

MONTE CARLO TYPE II EXPERIMENTWISE ERRORS FOR THE HARMONIC MEAN (H) AND KRAMER (K) UNEQUAL FORMS OF THE TUKEY STATISTIC HAVING .67 σ-UNIT MEAN DIFFERENCES FOR EIGHT TREATMENT LEVELS WHEN DEVIATION FROM ZERO IS CONSIDERED

			CONDITION*																			
			NORMAL DISTRIBUTION										EXPONENTIAL DISTRIBUTION									
			1		2		3		4		5		1		2		3		4		5	
		ψ**	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K	H	K
α = .05	n_k = 7	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.004
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.00	0.000	0.002	0.002	0.000	0.000	0.008	0.009
		ψ_{III}	0.005	0.005	0.007	0.007	0.003	0.004	0.010	0.010	0.007	0.010	0.015	0.015	0.008	0.008	0.024	0.023	0.010	0.010	0.054	0.059
		ψ_{IV}	0.144	0.144	0.130	0.130	0.158	0.156	0.166	0.165	0.143	0.154	0.170	0.170	0.129	0.129	0.194	0.203	0.143	0.148	0.212	0.221
		ψ_V	0.813	0.813	0.777	0.777	0.805	0.802	0.828	0.827	0.744	0.741	0.700	0.700	0.680	0.680	0.700	0.709	0.702	0.699	0.623	0.628
		ψ_{VI}	0.998	0.998	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.996	0.996	0.996	0.993	0.991	1.000	1.000	0.975	0.973
		ψ_{VII}	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	n_k = 16	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{III}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{IV}	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.004	0.004
		ψ_V	0.026	0.026	0.032	0.032	0.018	0.018	0.035	0.033	0.034	0.035	0.053	0.053	0.046	0.046	0.053	0.052	0.032	0.033	0.088	0.090
		ψ_{VI}	0.864	0.864	0.848	0.848	0.825	0.820	0.837	0.838	0.805	0.808	0.763	0.763	0.763	0.763	0.759	0.765	0.794	0.790	0.722	0.726
		ψ_{VII}	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	n_k = 25	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{III}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{IV}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_V	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.007	0.007
		ψ_{VI}	0.249	0.249	0.244	0.244	0.234	0.238	0.270	0.267	0.232	0.238	0.261	0.261	0.211	0.211	0.259	0.257	0.247	0.245	0.286	0.286
		ψ_{VII}	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
α = .10	n_k = 7	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.000	0.000	0.003	0.003	0.000	0.000	0.006	0.006
		ψ_{III}	0.002	0.002	0.004	0.004	0.000	0.000	0.005	0.004	0.005	0.005	0.014	0.014	0.006	0.006	0.016	0.015	0.004	0.004	0.032	0.030
		ψ_{IV}	0.067	0.067	0.079	0.079	0.072	0.076	0.084	0.092	0.084	0.096	0.131	0.131	0.089	0.089	0.118	0.123	0.102	0.096	0.154	0.156
		ψ_V	0.674	0.674	0.645	0.645	0.661	0.660	0.697	0.691	0.626	0.627	0.585	0.585	0.587	0.587	0.544	0.545	0.602	0.600	0.553	0.562'
		ψ_{VI}	0.999	0.999	0.997	0.997	0.999	0.999	0.999	0.999	0.994	0.994	0.981	0.981	0.992	0.992	0.983	0.983	0.995	0.995	0.967	0.964
		ψ_{VII}	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	n_k = 16	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{III}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{IV}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_V	0.012	0.012	0.019	0.019	0.013	0.013	0.024	0.022	0.010	0.010	0.025	0.025	0.026	0.026	0.027	0.028	0.023	0.023	0.044	0.045
		ψ_{VI}	0.699	0.699	0.668	0.668	0.686	0.688	0.698	0.697	0.683	0.684	0.648	0.648	0.617	0.617	0.628	0.632	0.672	0.668	0.587	0.586
		ψ_{VII}	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	n_k = 25	ψ_I	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{II}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{III}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_{IV}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		ψ_V	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001	0.007	0.007
		ψ_{VI}	0.145	0.145	0.159	0.159	0.156	0.158	0.157	0.152	0.183	0.186	0.181	0.181	0.165	0.165	0.181	0.180	0.173	0.168	0.211	0.215
		ψ_{VII}	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
		ψ_A	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999

$\alpha = .05,\ \sigma_p = .007;\ \alpha = .10,\ \sigma_p = .009$

*Conditions: (1) equal n's - equal σ^2's (2) equal n's unequal σ^2's (3) unequal n's - equal σ^2's (4) unequal n's - unequal σ^2's (positively related) (5) unequal n's - unequal σ^2's (negatively related).

**ψ_I Maximum contrast; ψ_{II} 2nd largest contrast(s); ψ_{III} 3rd largest contrast(s); ψ_{IV} 4th largest contrast(s); ψ_V 5th largest contrast(s); ψ_{VI} 6th largest contrast(s); ψ_{VII} 7th largest contrast(s); ψ_A all contrasts.