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ABSTRACT

This report presents the problem and a solution to the treatment of students by age interactions. It considers the plight of an educational researcher who wishes to demonstrate that a particular treatment effect changes from age to age. In analysis of variance language, he is seeking a treatment by age interaction. The report presents a rationale for considering treatment by age interactions in a relative rather than in an absolute sense. The problem considered is, "How can the interaction question be answered when treatment differences at one age level are based on a different variability from those at another age level?" An appropriate data transformation is derived that enables a researcher to interpret such interactions statistically. (Author/WS)

TREATMENT BY AGE INTERACTIONS: THE PROBLEM AND A SOLUTION

WISCONSIN RESEARCH AND DEVELOPMENT

CENTER FOR
COGNITIVE LEARNING

Theoretical Paper No. 41

TREATMENT BY AGE INTERACTIONS: THE PROBLEM AND A SOLUTION

by
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Report from the
Program 1 Component:
Individual Differences Among Learners

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Statement of Focus

Individually Guided Education (IGE) is a new comprehensive system of elementary education. The following components of the IGE system are in varying stages of development and implementation: a new organization for instruction and related administrative arrangements; a model of instructional programing for the individual student; and curriculum components in prereading, reading, mathematics, motivation, and environmental education. The development of other curriculum components, of a system for managing instruction by computer, and of instructional strategies is needed to complete the system. Continuing programmatic research is required to provide a sound knowledge base for the components under development and for improved second generation components. Finally, systematic implementation is essential so that the products will function properly in the IGE schools.

The Center plans and carries out the research, development, and implementation components of its IGE program in this sequence: (1) identify the needs and delimit the component problem area; (2) assess the possible constraints—financial resources and availability of staff; (3) formulate general plans and specific procedures for solving the problems; (4) secure and allocate human and material resources to carry out the plans; (5) provide for effective communication among personnel and efficient management of activities and resources; and (6) evaluate the effectiveness of each activity and its contribution to the total program and correct any difficulties through feedback mechanisms and appropriate management techniques.

A self-renewing system of elementary education is projected in each participating elementary school, i.e., one which is less dependent on external sources for direction and is more responsive to the needs of the children attending each particular school. In the IGE schools, Center-developed and other curriculum products compatible with the Center's instructional programing model will lead to higher student achievement and self-direction in learning and in conduct and also to higher morale and job satisfaction among educational personnel. Each developmental product makes its unique contribution to IGE as it is implemented in the schools. The various research components add to the knowledge of Center practitioners, developers, and theorists.

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Abstract

A rationale is presented for considering treatment by age interactions in a relative rather than in an absolute sense. An appropriate data transformation is derived that enables a researcher to interpret such interactions statistically.

Treatment by Age Interactions: The Problem and a Solution[1]

Consider the plight of an educational researcher who wishes to demonstrate that a particular treatment effect (say, treatment A vs. treatment B) changes from age to age (say, age 1 to age 2). In analysis of variance language, he is seeking a treatment by age interaction. If a wide enough age range is covered—in particular, if very young children (say, nursery schoolers) are compared with adolescents—two generally nonindependent outcomes may be anticipated:[1] (a) mean improvements in both treatment groups will be observed from age 1 to age 2 and (b) this will be accompanied by differences in within-treatment variability at the two age levels.

The Problem

Given that the second outcome does occur with some regularity in educational research, the problem can now be stated: How can the interaction question be answered when treatment differences at one age level are based on a different variability from those at another age level? Before pursuing the issue further, however, let us indicate courses of action typically taken by researchers.

Many investigators simply regard the differing within-age variabilities as noise in the system, merely biasing the F test for interaction in the two (treatments) by two (ages) design either negatively or positively, depending on whether the larger or smaller variance is perceived as "real." Indeed, this is generally not viewed as a problem of consequence by those of us who were weaned on the dictum that the F test is robust with regard to inhomogeneity of variance, especially when sample sizes are equal. When variance differences are perceived as a threat to the statistical assumptions, a variety of well-known procedures ranging from data transformations to nonparametric analyses to "F-like" tests (Li, 1964) are commonly employed.

But even when concern is expressed (and behavior modified) to meet the assumptions of the analysis of variance model, are we really attending to the issue at hand? Let us consider it briefly now.

As is true of most measured variables, differences are interpretable only with regard to some index of their variability. Thus, while differences of 10 units are considered to be equal in the analysis of variance model (since it is assumed that the variances underlying each are also equal), it should not be taken for granted that this is true in the general case. A difference of 10 units based on an underlying standard deviation of 5 units is relatively larger than the same 10-unit difference based on an underlying 20-unit standard deviation. When we equate (standardize) each difference by putting it on a scale with a common standard deviation (z units), it is readily apparent that in the former case the two means are farther apart ($10/5 = 2$ units) than they are in the latter ($10/20 = 1/2$ unit). That is to say, the former difference is relatively larger than the latter, even though the two are equal in an absolute sense. It would certainly seem desirable for an analysis of differential effects to recognize this fact.

[1]In this discussion we will not consider differential outcomes that are attributable to differences in Ss' ability to follow instructions, familiarity with the task, attentiveness, and other age by task-related variables.

A Solution

Choose constants c_1 and c_2 in age groups 1 and 2 respectively such that when applied to every score in the two groups, they will transform absolute treatment differences into relative treatment differences. Assume that within each age group the variances associated with treatments are equal; that is,

$$\sigma^2_{1A} = \sigma^2_{1B} = \sigma^2_1 \text{ and } \sigma^2_{2A} = \sigma^2_{2B} = \sigma^2_2 ,$$

where the subscripted number indicates the particular age group and the letter indicates the treatment. Now, let $c_1 = \frac{1}{\sigma_1}$ and $c_2 = \frac{1}{\sigma_2}$. Thus if every score (in both treatment groups) is multiplied by the reciprocal of the within-treatment standard deviation for children of that age, then each individual's transformed score in the four age/treatment combinations may be represented as follows: $\frac{1}{\sigma_1} \cdot X_{1A}$ and $\frac{1}{\sigma_1} \cdot X_{1B}$ for treatments A and B at age 1 and $\frac{1}{\sigma_2} \cdot X_{2A}$ and $\frac{1}{\sigma_2} \cdot X_{2B}$ for treatments A and B at age 2, where X refers to a given raw score. Mean treatment differences based on this transformation would yield

$$\bar{D}_1 = \frac{1}{\sigma_1} (\bar{X}_{1A} - \bar{X}_{1B}) = \frac{\bar{X}_{1A} - \bar{X}_{1B}}{\sigma_1} \text{ and}$$

$$\bar{D}_2 = \frac{1}{\sigma_2} (\bar{X}_{2A} - \bar{X}_{2B}) = \frac{\bar{X}_{2A} - \bar{X}_{2B}}{\sigma_2}$$

in the two age groups, and an F test of the difference in the transformed means $(\bar{D}_1 - \bar{D}_2)$ would produce the desired test of equal relative treatment differences at the two ages.

The only obstacle remaining, therefore, is to obtain good estimates of σ_1 and σ_2. The optimal strategy is straightforward: collect sufficient data based on independent samples or subsamples to permit stable estimates of these two parameters. To the extent that good estimates are obtained (and large enough samples are used in the experiment proper), c_1 and c_2 will approximate their respective parameters, σ_1 and σ_2. As the correspondence becomes closer, it is easy to demonstrate that the standard error of the treatment difference in each age group will approach

$$\sqrt{\frac{1}{N_A} + \frac{1}{N_B}} ,$$

the denominator reflecting the sample sizes for treatments A and B in each age group. (More typically, this quantity reduces to $\sqrt{\frac{2}{n}}$ when $N_A = N_B = n$.)

What if independent estimates of σ_1 and σ_2 cannot be conveniently obtained? Intuitively, one would turn to the experiment proper and employ the sample estimates it provides. This cannot be recommended at present, however, due to the unknown consequences of within-sample correlations, sample size, and their joint influence on the appropriate reference distribution for hypothesis testing. Clearly the robustness of the statistical procedures following various manipulations of these conditions needs to be determined empirically.[2]

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[2] Note that the preceding problem and solution are applicable to any treatment by subject classification (e.g., sex, IQ, social class) design for which the K subject classification levels would be expected to exhibit differing variability on the dependent variable, and one's interaction question is best conceptualized in terms of relative treatment differences. It is important to remember, though, that in the development of the recommended procedures, an assumption of equal treatment variances within subject classification levels has been imposed.

References

Li, C. C. Introduction to Experimental Statistics. New York: McGraw-Hill, 1964.

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