DOCUMENT RESUME

ED 072 977 SE 015 695

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TITLE	Development of a Self-Instructional Course in Engineering Statics. Final Report.
INSTITUTION	Washington Univ., Seattle.
SPONS AGENCY	National Center for Educational Research and Development (DHEW/OE), Washington, D.C.
BUREAU NO	BR-1-J-042
PUB DATE	Aug 72
CONTRACT	OEC-X-71-0042(057)
NOTE	165p.
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	*Autoinstructional Programs *College Science; *Course Descriptions; Curriculum Development; Engineering; *Engineering Education; *Instruction; Instructional Materials; Program Descriptions

ABSTRACT

Reported is the development of a self-instructional course in engineering statics designed for engineering students that has been implemented in several institutions. There are 15 unit modules in the course divided into three different levels. Each unit begins with a description of general objectives. The unit is then divided into several subunits each of which has terminal objectives. A self-assessment test is given after finishing each subunit. After the completion of all subunits in a unit, an achievement test is given to assess the student's grasp of the material in the complete unit. Only pass/fail grades were given on the achievement test. Final letter grades were given on the basis of the number of units covered by each student. The course is aimed at enabling students to learn at their own pace. The 15 units included in the course were Fundamentals, Equilibrium Diagrams, Equilibrium Analysis, Components Superposition Cantilever Beams, Vector Algebra, Friction, Engineering Frames, Non-Coplaner Systems, Equivalent Systems, Trusses, Properties of Surfaces, Energy Methods, Hydrostatics, Beam Diagrams, and Slide Rule. Evaluation of the course was favorable. A description of most units is included in the report. (PS)

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Final Report

Project No. 1-J-042
Contract No. OEC-X-71-0042(057)

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DEVELOPMENT OF A SELF-INSTRUCTIONAL COURSE IN ENGINEERING STATICS

August 1972

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE

Office of Education

SE 015 695

ABSTRACT

For Contract DEC - X - 71 - 0042 (057)
by Associate Professor D.E. Alexander

The objective of this project was to design a course where each student learns engineering statics using self-paced modular units. This course has been constructed and used here at the University of Washington and at some nearby community colleges. The course has been organized into a learning hierarchy of 14 modules. Each module begins with a behaviorable objective for the complete module and is then broken down into units with assessment tasks and terminal objectives for each unit. Each student decides when he wants to take an achievement test on each module. When he passes the test (there is no grade, only pass or fail for each module, also no time limit in any test) he proceeds to the next higher module. Successful achievement tests on 10 modules gives a "C" grade, 12 a "B" grade and all 14 an "A".

All classroom lectures have been replaced by classroom consulting by the instructors. The results of student surveys are included in the body of this report. Since this technique has been developed, many other instructors are presenting classes in this self-study manner. This quarter I am offering a course in digital computation and numerical methods using the same format.

Final Report

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DEVELOPMENT OF A SELF-INSTRUCTIONAL COURSE IN ENGINEERING STATICS

Daniel E. Alexander

University of Washington

Seattle, Washington

August 12, 1972

The research reported herein was performed pursuant to a contract with the Office of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

August 12, 1972

Innovative Project in Engineering Statics

Daniel E. Alexander
Department of Mechanical Engineering

1. INTRODUCTION

The objective of this project was to construct a course in which the student learns engineering statics using self-instructional units. All of this material is to be presented using a three-dimensional approach. This course has been constructed and used for the last four quarters here at the University of Washington and one quarter at Everett Community College.

2. COURSE FORMAT

A detailed format of the course is enclosed with this report. Some units are enclosed with this report.

3. INSTRUCTORS AND STUDENTS INVOLVED IN THE COURSE

	Instructors	Sections
Fall Quarter	M. Ekse, H. Strausser, D. Alexander	4
Winter Quarter	H. Chenoweth	1
Spring Quarter	W. Chalk, J. Morrison H. Chenoweth, D. Alexander W. Zimmerman at Everett Community College	5
Summer	D. Alexander	1

4. STUDENT PROGRESS

Records were kept of the time needed for each student to complete each unit and also the time needed to complete the course. The time needed to finish a unit and be prepared for an achievement test on the unit averaged out to be between 4 and 6 hours. The fastest student finished the course in 3 weeks. Almost all of the students finished the course before finals week. The fastest student had an overall grade point of 3.93 for two years. The next fastest student took 3 1/5 weeks, his overall grade point after 7 quarters is 1.86. The next fastest student needed 6 weeks to complete the course. During 3 of the 4 quarters, a girl finished first. The slowest students have needed 2 quarters to achieve C grades.

5. GRADES EARNED

For all 11 sections

A	116
B	39
C	21
D	3
E	5
PW	18

6. STUDENT EVALUATION

All students were asked to evaluate the course in the Fall quarter. No formal evaluations were taken during the other quarters, but spot evaluations were given to some students and the evaluations were the same as during the Fall quarter.

7. COMMENTS

Careful notes were kept of feed-back from the students and instructors using the units. This feed-back has been used to rebuild most of the units and it was found necessary to add a basic unit on the slide rule. Also a teachers manual with a comprehensive set of achievement tests for each unit has been constructed.

The course has been presented without using formal timed tests. This made it impossible to measure objectively the student achievement in the course vs the achievement in the regular courses. However the individual tests given have all been much more difficult than the timed conventional tests. The amount of material covered in this course has been about 40% greater than in the conventional course.

It is obvious that all students do not and can not learn at the same rate. Many students have to repeat units at the first of the course and some even toward the end of the course. The instructors and students all like the idea of keeping a student in a unit until he can actually demonstrate that he knows the material.

Other instructors in the college of engineering are now presenting courses in this self-instructional manner. EE has at least three courses, ME has one, Chem. E one, and the college courses have at least three. All of these instructors have either attended one of the Engr. 180 sessions or attended a seminar where I presented the course. Everett Community College has adopted our Engr. 180 course for next year. Also as a direct result of a teaching institute that I sponsored in 1971, almost all the community colleges, universities, and colleges that teach engineering in the northwest have one or more self-instructional courses.

In my opinion many courses will be presented as self-instructional courses in the future. The students like them and the instructors who have been involved like them. Some faculty and administrators have told me they cannot justify spending their time on this type of educational research as against the traditional engineering and scientific research. I believe both types of research are needed at the university level.

FORMA -

ENGINEERING 180

1. Professors Alexander, Chalk, Chenoweth, and Morrison

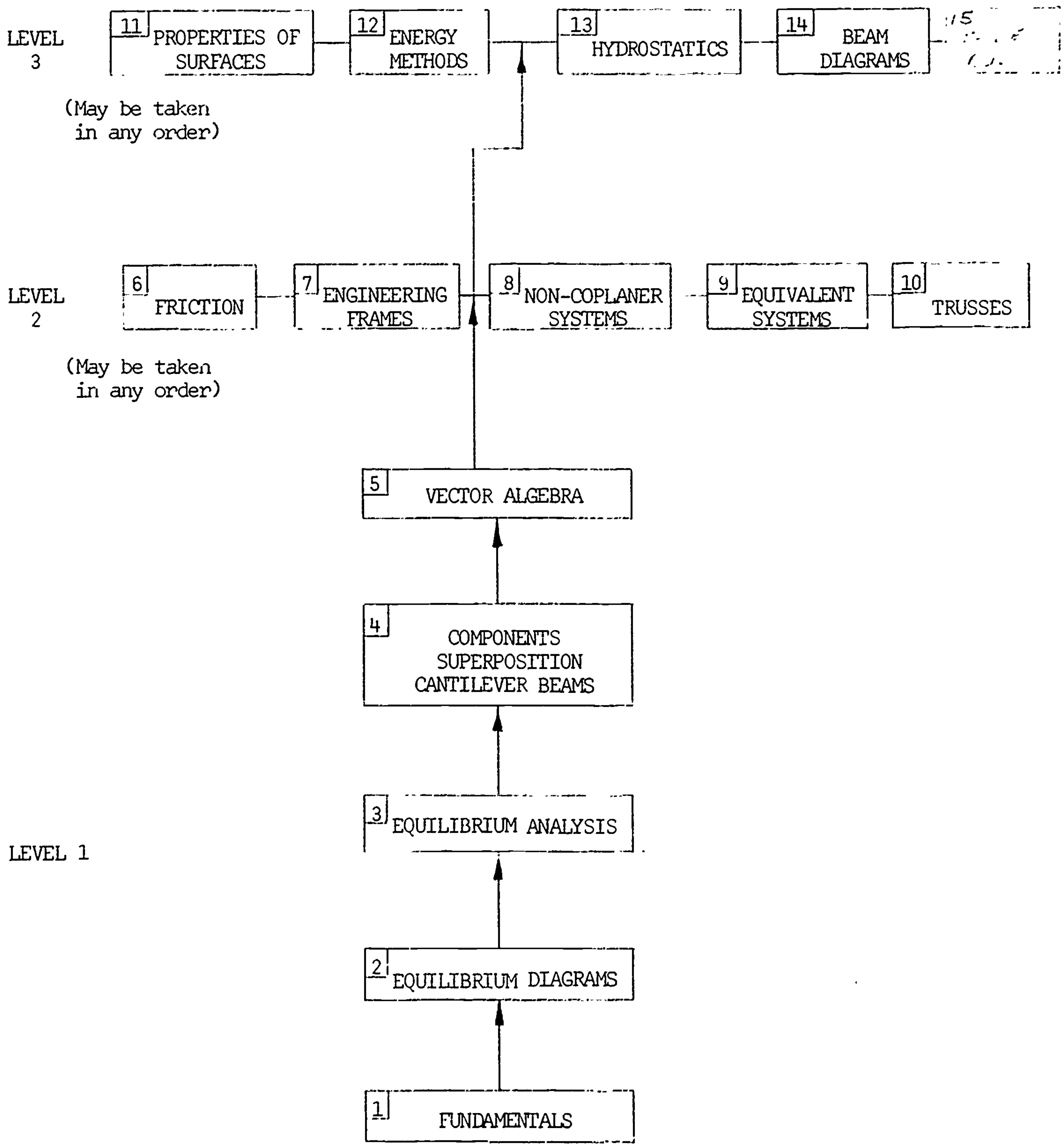
2. <u>Engineering Statics</u> Traditionally an engineering student studies a series of courses that leads to an engineering design course. A typical series is shown below.

Engineering Problems → Engineering Statics → Dynamics → Mechanics of Materials → Material Science → Engineering Lab. → Design

The first course is an introduction to engineering analysis and problem solving. Engineering statics is concerned with constructing equilibrium diagrams and determining the external loads that act on stationary structures. Dynamics includes the studies of pure motion and the external loads that act upon members that are not at rest. Mechanics of materials begins the study of the internal conditions of a body acted upon by external loads. Material science is taught in the Department of Metallurgy and is concerned with the internal structures of different materials and how materials react to loads, temperature changes, etc. In the engineering laboratory course the student tests actual members for deflections, failures, etc. Then in the design course the student can draw on all of these courses plus math, chemistry, physics, etc. to design actual engineering structures or members. Each of the courses depends upon the ability of the student to visualize and construct good equilibrium diagrams and to find the external loads acting upon any member, this is the material learned in engineering statics.

3. <u>The Course</u> A flow chart of the course is drawn on page 2. The course has been organized into 14 units at three levels. Each unit has been designed to be self-instructional and each student works through each unit at his own pace. Each unit begins with an objective for the complete unit. This tells the student exactly what is to be learned in the unit. The unit is then broken down into learning sub-units with a terminal objective written for each sub-unit. When a student decides that he fully understands the terminal objective, he then works the self-assessment task written for the learning sub-unit. When he completes the task, he checks it with one of the professors. When he has finished a complete unit and decides that he is ready to demonstrate his grasp of the material in the unit, he takes an achievement test on the complete unit. When he has passed the test (there is no grade, only pass), he can then proceed to the next unit.

ENGINEERING STATICS



4. Course Conduct The professors will be in Room 326 during the following hours:

	9:30 - 10:30	10:30 - 11:30	11:30 - 12:30
Monday	Chalk	Chalk	
Tuesday	Morrison	Morrison	Morrison
Wednesday	Chalk	Chenoweth	
Thursday	Chenoweth	Chenoweth	
Friday	Alexander	Alexander	

The instructors will work with the students on an individual basis to check units and give tests. No lectures will be given. Mostly the professors will be consultants helping the students in any way. During the first five weeks of the course, it is important that the students come to Room 326 and work with the instructors at least four times per week including one day for testing. Achievement tests will be given and graded in Room 327 only during the course hours.

5. Achievement Tests Each student will take an individual test on any unit when he decides he is ready. All achievement tests will be given in Room 327. All the tests will be prepared by Professor Alexander and administered by Professors Alexander, Chalk, Chenoweth, and Morrison. There will be no time limit on any test, all tests will be closed book, and each test will be graded by one of the instructors with the student before the student leaves the testing room. Before being allowed to take an achievement test on a unit, the student must have one of the professors sign the unit. No penalty will be given for a non-passed test.

6. Grades Grades will be given for the number of successful units completed as shown below.

Total Units	5	6	8	10	12
Level 3				2	3
Level 2		1	3	3	4
Level 1	5	5	5	5	5
Grade	E	D	C	B	A

Level 1 units must be taken in order. Level 2 and level 3 units can be taken in any order except three level 2 units must be completed before taking any level 3 units. Also the units must be completed as shown on the following schedule:

Unit	1	Friday	March 31
	2	Thursday	April 6
	3	Wednesday	April 12
	4	Tuesday	April 18
	5	Monday	April 24
	6 - 12	At least one per week	

If a unit is not completed on time, it will not count toward a grade and the student will have to complete an extra unit for his grade. Also a pass must be earned for all level 1 units. It is hoped that all students will get ahead of the minimum schedule.

7. Office Hours The four professors will be available in their offices during their regular office hours. These times will be announced later.

8. Study Rooms Room 326 is available for study M W F 1:30-2:30 and T 1:30-3:30. Room 327 is available all day every day.

9. Student Supplies Each student must have a slide rule, some engineering paper, two triangles, a protractor, an engineers scale, a mechanical pencil, and a 2H drafting pencil.

10. Text The text is "A Self-Learning Course in Engineering Statics" by D.E. Alexander. It costs $9.00 and will be given out in units as the student progresses through the course.

Student Evaluation

D. E. ALEXANDER — ENGINEERING 180 — Fall 1971

Student evaluation of self study course designed and constructed by D. E. Alexander
Seventy-nine (79) students from four classes.
Two classes by D. E. Alexander 38 students
Two classes by M. Ekse and H. Strausser 41 students

You are just now finishing a course in engineering statics where you learned the material using self-paced self-study units with written objectives, no lectures, and individual non-timed tests.

	Yes	No
1) Did you like the units approach as compared to the conventional course:	79	0
2) Do you feel that you knew the material when you finished each unit?	79	0
3) Do you like the testing program?	78	1
4) Do you like the grading system?	79	0
5) Do you like the non-lecture consulting type teacher relationship?	79	0
6) Would you like to take dynamics using this system?	79	0
7) Would you recommend that Engr. 180 be taught this way next quarter?	79	0

	A	B	C	D	E
8) What grade would you assign the course compared to other freshman and sophomore courses?	76	3			

Comments - All good. Typical listed below.

Best course I've ever taken.

Liked it all the way.

Want more.

Learn more.

Good feedback.

Excellent student-teacher relationship.

Learned much more.

Almost too good.

Want this to continue.

This is the first time I have been in a class like this, and it is very refreshing to experience a class with such a free attitude. The learning experience is left up to the student.

Engineering 180 (Cont.)

Good change from conventional class structure.

Less pressureyet you still learn more than normal.

Superior to conventional course - can work at own speed.

After the problem set and similar test, you may even over learn it which is better for retention.

Testing program was flexible and often I felt that I learned something instead of merely reciting back information. Also, I like to have lots of time.

Grading system great, unambiguous, you can get what you are willing to work for.

Liked non-lecture consulting type teacher relationship. Get to talk to professors and get questions answered. Normally in a lecture class, you don't know the instructor, his goals for the class are unknown and the prof. gives out pearls of wisdom which are usually redundant, whereas this course is much more applied and less superfluousknowledge, which you don't use anyway.

It is very complete, step by step approach to the problem solving of statics. I feel very grateful to be able to take this course in such a manner. I hope to see dynamics taught this way.

Much less pressure and I've learned more in this course than any conventional course.

Don't like Prof. Strausser. He talks down like he's too good to teach this low level class.

You learn more this way,only a couple of spots in units hard to understand. Good work, Prof. Alexander.

This course allows for much feedback to the student, letting him know how he stands in relation to the course at all times.

Good--almost too good--takes desire away from other courses.

Best part of the course was the few minutes the instructor would spend each day with the individual student.

Course excellent. Course makes studying a learning process rather than a competitive one. I would recommend all science and math courses being taught in the same way.

Student-teacher relationship gives more meaning to the class thus stimulating more interest in the course.

This method of teaching a difficult subject matter is very easy to follow, informative and interesting.

I was very impressed with the class in general. At all times I knew exactly where I stood. The units were clear and the work was difficult, but interesting.

Undoubtedly, the best overall course I've taken at the University.

UNIT 1

FUNDAMENTALS

AT THE END OF UNIT 1, YOU WILL BE ABLE TO VISUALIZE FORCE FIELDS ACTING UPON ENGINEERING MEMBERS, CONSTRUCT POINT FORCE RESULTANTS OF THESE FORCE FIELDS, AND DEMONSTRATE ADDITION AND RESOLUTION OF POINT FORCES WITH THE PARALLELOGRAM LAW AND WITH MOMENT EQUATIONS.

Introduction

Engineering statics is the study of forces acting upon stationary structures. The space surrounding the structures is assumed to have a constant air pressure and temperature. Lengths within the space are measured in feet, and forces are measured in pounds.

The action between two members that affects the size, shape or motion of the members is called a force. Forces are classified as contact forces (when two members actually contact each other) or as distant forces (when two members are attracted to each other by magnetism or gravity). In engineering statics only actions between stationary members will be considered.

The study of forces acting upon engineering members involves four principles which will be developed: (1) the principle of a force field, (2) the principle of a point force, (3) the parallelogram law for the addition of point forces, and (4) the principle of moments.

FD

The Principle of a Force Field

Figure FD 1(a) shows a student's hand which has gradually pushed directly against a round block attached to a compression scale until the scale reads 12 pounds. A method of identifying the action against the student's hand and against the block will be developed when the scale is held at 12 pounds.

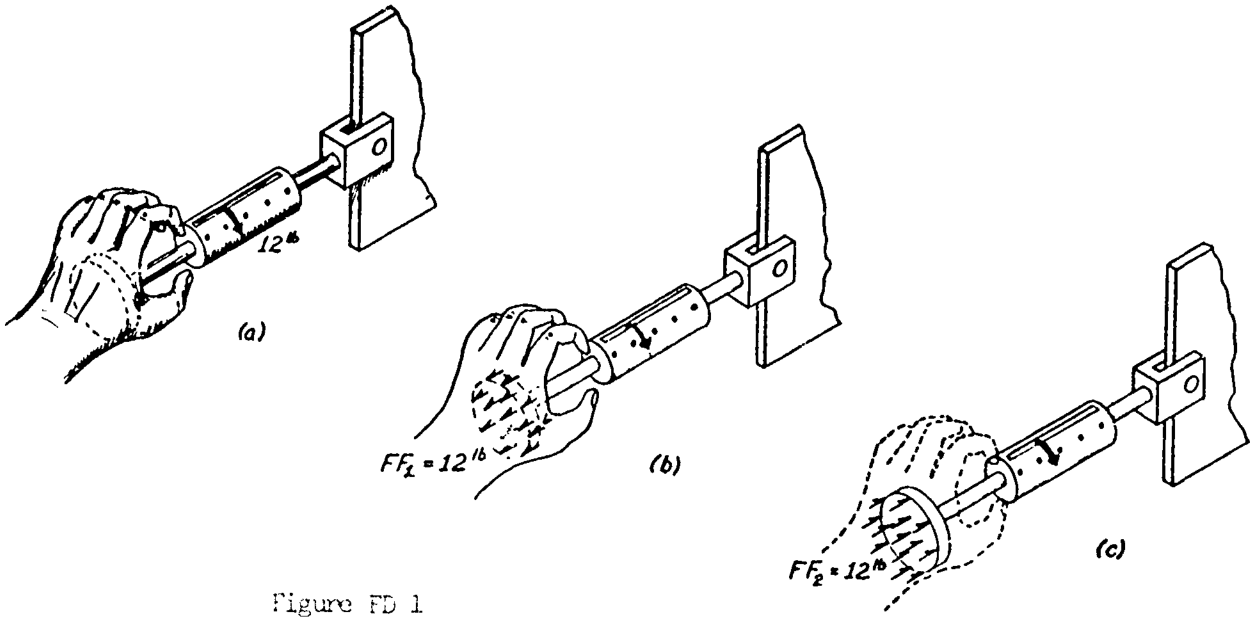


Figure FD 1

Wherever the student's hand contacts the block, contact forces are built up. It will be assumed here that the contact forces are uniformly distributed over the entire contact area and that the intensity of the contact forces is uniform over the entire contact area. These contact forces will be represented by arrows acting against the student's palm as shown in (b). With the assumption that the contact forces are uniform in intensity and uniformly distributed over the entire contact area, there would be an infinite number of arrows, all of the same length. For convenience, only a limited number are drawn. The distributed force system acting against the student's palm is called a force field and is labeled FF_1. Force field arrows are always drawn with half arrowheads, as shown in (b). When the spring scale reads 12 pounds, the magnitude of the total force field FF_1 is 12 pounds. No attempt is made to draw the lengths of the arrows to scale, but with the assumption that the contact is uniform, the arrows will all be of the same lengths in the same direction.

The action of the student's palm against the block is equal and opposite to the action of the block against the student's palm. Figure FD 1(c) shows force field FF_2 acting against the block. FF_2 is equal and opposite to FF_1, with a magnitude of 12 pounds.

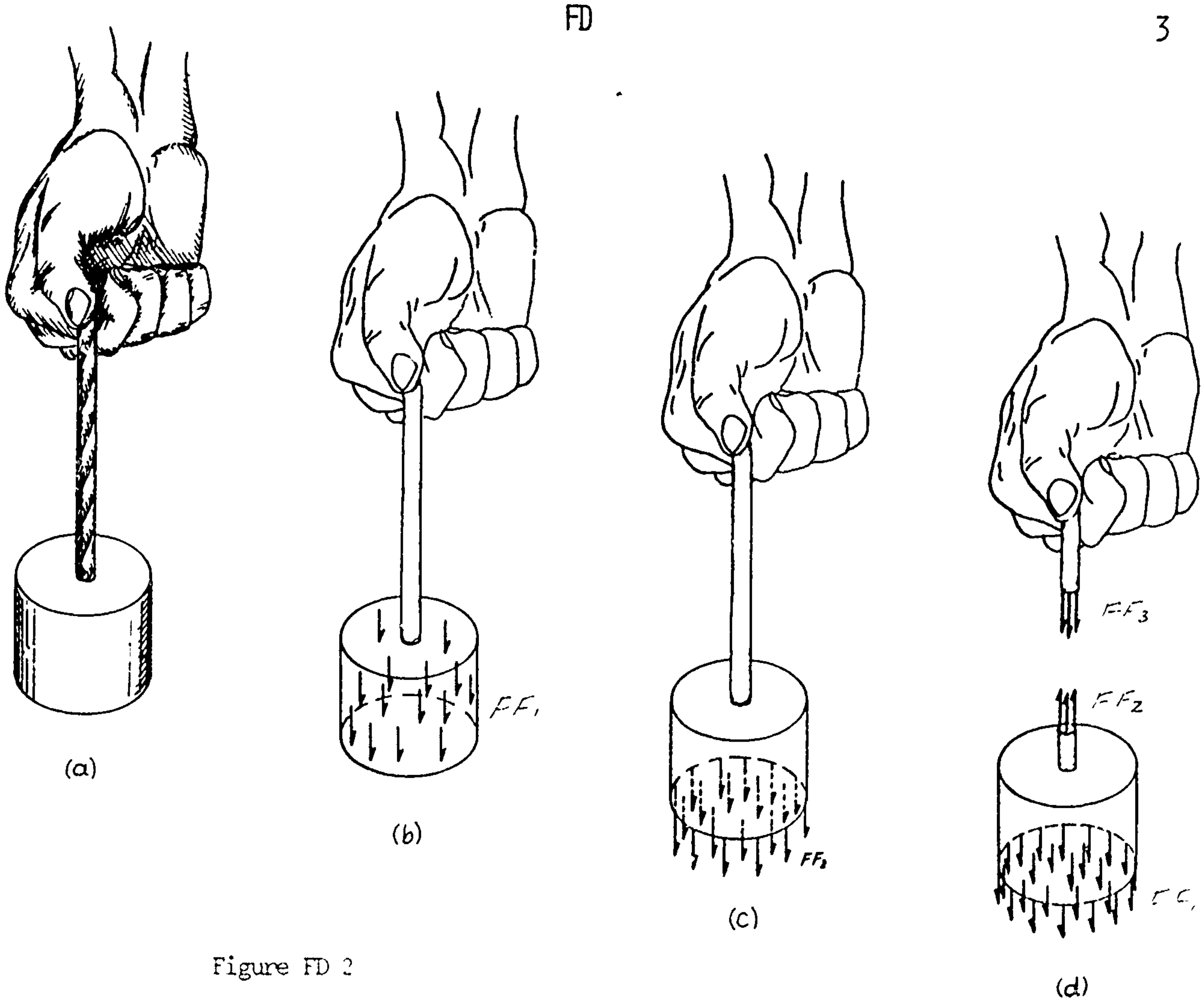


Figure FD 2

Figure FD 2(a) shows a student holding a homogeneous cylinder with a weightless cord. This time the distant attraction of the earth for the cylinder is acting on the cylinder. This attraction acts uniformly throughout the cylinder. It is called gravity and is represented by force arrows acting downward as shown in (b). Again, the force would be most accurately represented with an infinite number of arrows, but only a limited number can be drawn. If the cylinder weighs 4 pounds, FF_1 = 4 pounds. In (c) FF_1 is drawn as a uniform force field acting on the lower surface of the cylinder. For convenience, gravity force fields are usually drawn either on the top or bottom of a member. Although it is not shown, a force field equal and opposite to FF_1 will be acting upon the earth.

In (d) some transverse sections of the supporting cord are shown exposed with their acting force fields. It is assumed that each force field is evenly distributed over the exposed surface. Notice that although the magnitude of all the force fields in (d) are equal, the force fields are not drawn to scale.

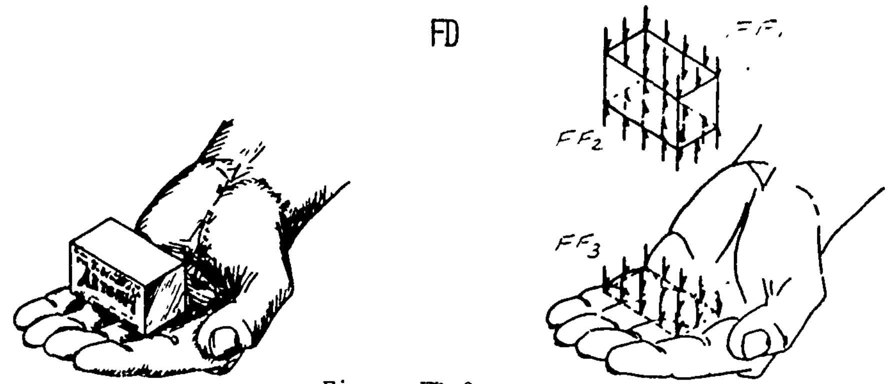


Figure FD 3

(a) (b)

In figure FD 3(a) a student is holding an eraser in his hand. The force fields acting upon the eraser are to be shown. Usually the object being analyzed is drawn as if it is isolated from its surroundings as shown in (b), then the force fields that are acting against it are shown. FF_1 is the force field due to gravity. FF_2 acts as shown in (b) to support the eraser. Force field FF_3 acting against the hand is equal and opposite to FF_2.

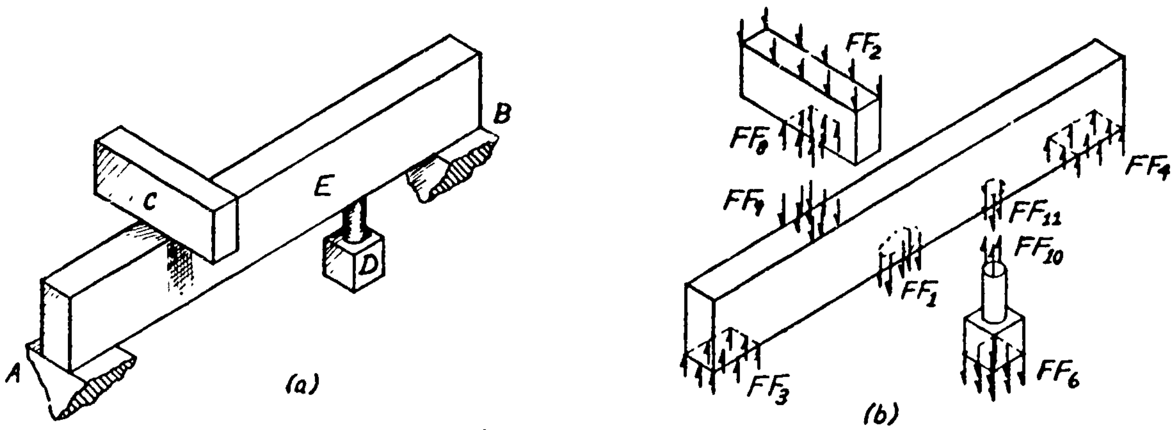


Figure FD 4

In figure FD 4(a) a uniform horizontal beam E supports symmetrical loads C and D and rests on smooth horizontal supports at A and B. The force fields acting on all the members are to be constructed. In (b) the members are isolated from each other with their force fields shown. Although FF_1, which is due to the weight of beam E, acts over the entire beam, for convenience only a small cut-out section is shown. It is assumed that all the members are rigid; for rigid members the shapes of the contact force fields can be assumed to be symmetrical. The actual shape of the force field depends upon the loading and material in contact, but only the simplest symmetrical force fields will be used in this presentation.

AT THIS POINT YOU SHOULD BE ABLE TO VISUALIZE FORCE FIELDS ACTING ON SIMPLE ENGINEERING MEMBERS AND BE ABLE TO CONSTRUCT THE FORCE FIELDS ON THREE-DIMENSIONAL DRAWINGS.

The Principle of a Point Force

Any force acting upon an object is distributed over a contact surface or a volume. In this presentation these distributed force systems are called force fields. After identifying a force field acting upon an object, the force field is sometimes replaced by a single point force.

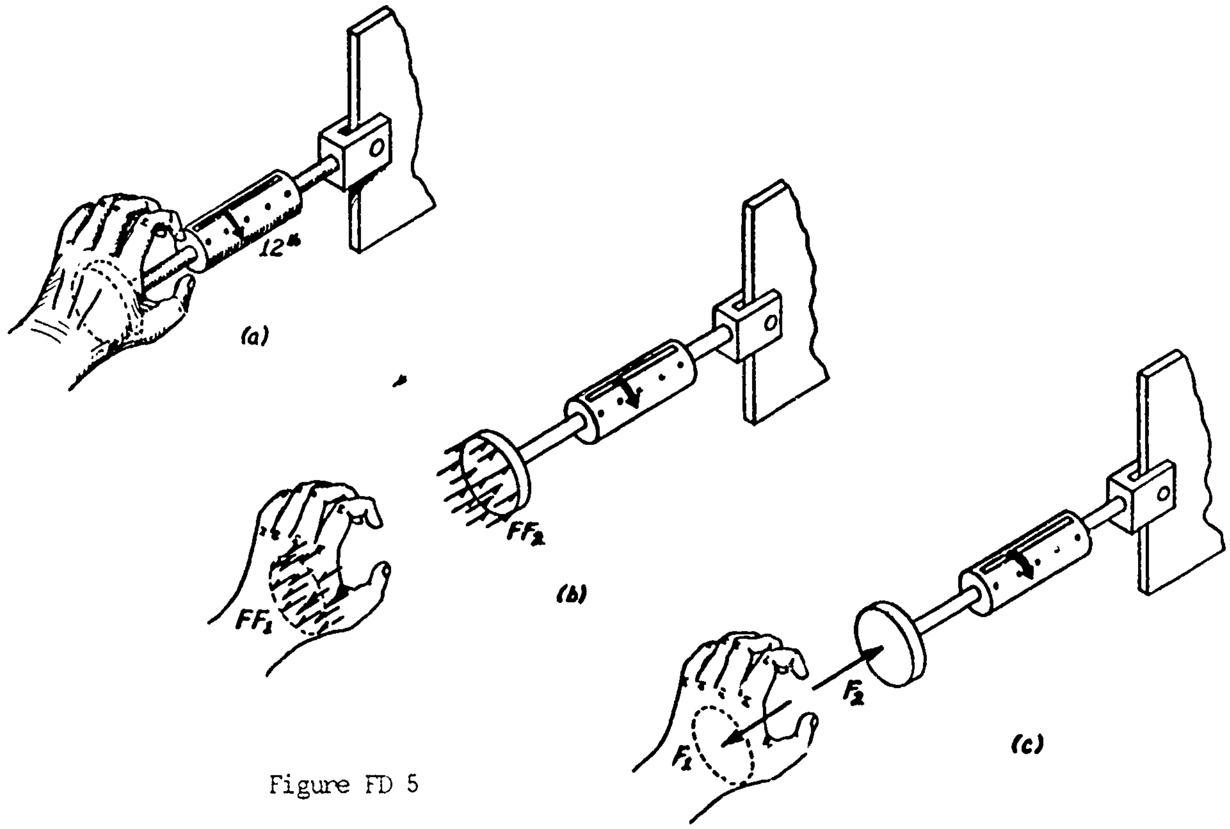


Figure FD 5

An example of a point force replacing a force field is shown in figure FD 5. In (a) a student is pushing with his palm directly against a smooth surface until the compression spring registers 12 pounds. In (b) the student's hand and the scale are redrawn separated from each other with their acting force fields shown. Now in (c) each force field is replaced by an arrow passing through the center of its field. Each arrow represents a point force which has the same magnitude and direction as the force field it replaced. Each point force is called the resultant of its force field, thus the resultant of FF_1 is F_1 with a magnitude of 12 pounds. Point force arrows are drawn with full arrowheads to distinguish them from the force field arrows with the half-arrowheads.

Since FF_1 and FF_2 are equal and opposite to each other, it would follow that their point force resultants F_1 and F_2 are also equal and opposite to each other. The placement on an object of a point force resultant cannot be an exact placement, since the point force is purely imagninary and abstract. It must only pass through the center of the force field it replaces.

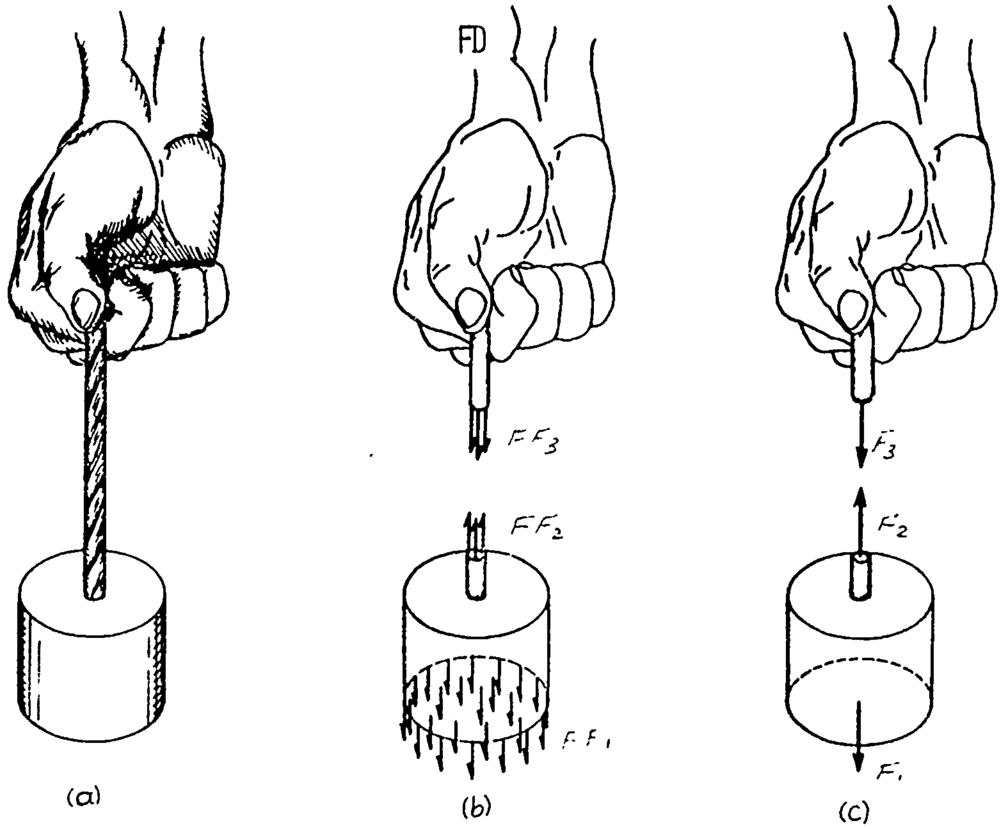


Figure FD 6

Figure FD 6(a) shows a weight hanging on a cord. The force fields shown in (b) have been replaced by their point force resultants in (c). Again, these point force resultants pass through the centers of their force fields. F_2 passes through a point called the center of gravity of the cylinder.

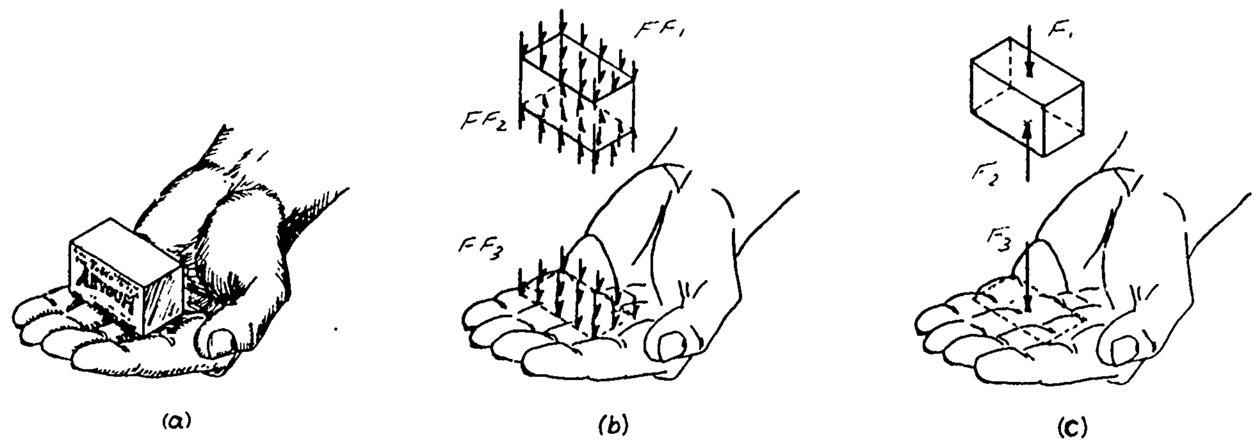


Figure FD 7

The eraser being held by a student's hand in figure FD 7(a) is shown with its force fields in (b) and its point force resultants in (c). Try picking up an eraser and feeling the point force resultant of the eraser's weight. This should help to show that any point force is purely imaginary.

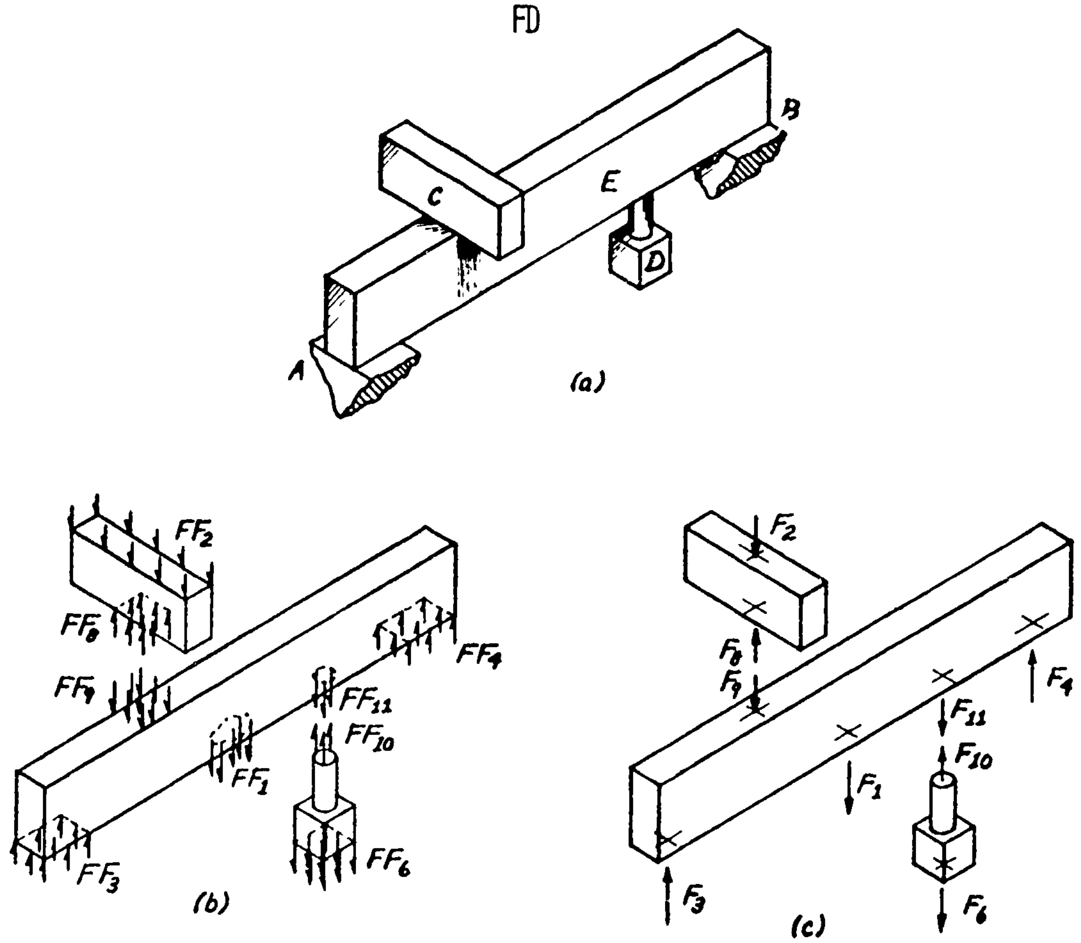


Figure FD 8

The symmetrically loaded engineering structure shown in figure FD 8(a) is redrawn in (b) with the force fields shown and in (c) with the point force resultants of these force fields.

NOW IF YOU ARE GIVEN A SIMPLE ENGINEERING STRUCTURE, YOU SHOULD BE ABLE TO CONSTRUCT THE FORCE FIELDS ACTING UPON THE INDIVIDUAL MEMBERS AND BE ABLE TO REPLACE THOSE FORCE FIELDS BY THEIR POINT FORCE RESULTANTS.

FD - 2

Figure FD 9 is an isometric drawing of a horizontal stationary ring with four vertical holes a, b, c, and d drilled through it. The ring is assumed to be weightless and the holes frictionless. Spring scales S1 and S2 are attached through holes a and b and gradually pulled while maintaining the ring in its original horizontal position until S1 reads 20 lbs. S2 also reads 20 lbs and it is found by trial and error that the centerlines of S1 and S2 must be horizontal and colinear (on the same straight line) or the ring will not stay in its original position. Force fields FF_1 and FF_2 are shown on the drawing with their point forces F_1 and F_2. F_1 and F_2 are equal, opposite, colinear and horizontal.

$\overline{F}_3(17.4 \text{ lb}) + \overline{F}_4(12.2 \text{ lb}) = \overline{F}_2(20 \text{ lb})$

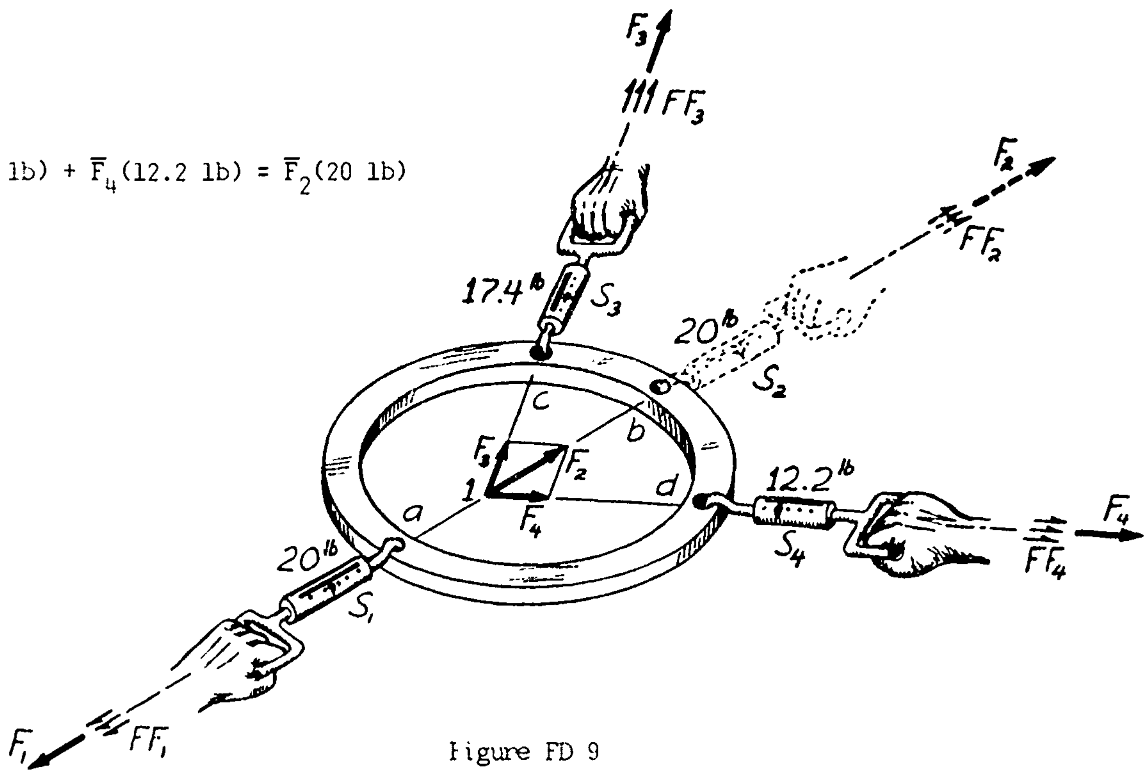


Figure FD 9

Now an experiment is to be performed. S1 and S2 are to be released, then two other spring scales are to be attached through holes c and d. These two scales and S1 are to be gradually pulled while keeping S1 and the ring in their original positions, until S1 again reads 20 lb. S2 has now been effectively replaced by the new scales.

It is found by experimentation that the two new scales must always be pulled in the same horizontal plane as S1. However, they can be pulled in a variety of directions, and for each set of directions will read different magnitudes. One combination is shown in the drawing where S3 and S4 replace S2. When S3 and S4 are pulled in the directions shown and S1 registers 20 lbs, S3 reads 17.4 lbs and S4 reads 12.2 lbs.

FF_3 and FF_4 are shown in figure FD 9 with their point forces F_3 = 17.4 lbs and F_4 = 12.2 lbs drawn to scale. When the action lines of point forces F_3 and F_4 are extended, they are found to meet at point 1 on the line of action of F_2. Now, if a parallelogram is drawn to scale at point 1 with F_3 and F_4 as its sides, it is found purely from graphical measuring that the diagonal of the parallelogram is equal to F_2. In order for the point forces to intersect each other in this manner, each one must be transmitted along its line of action. This is sometimes called the principle of transmissibility of a point force.

Since the point forces all act in the same plane, a plan view of part of the ring can be drawn to a larger scale in figure FD 10. Notice that the parallelogram for F_3 and F_4 can be drawn two ways. The solution will be the same when the arrowheads come together at one end of the parallelogram (dotted example) as it is when the tails meet (shown with solid lines).

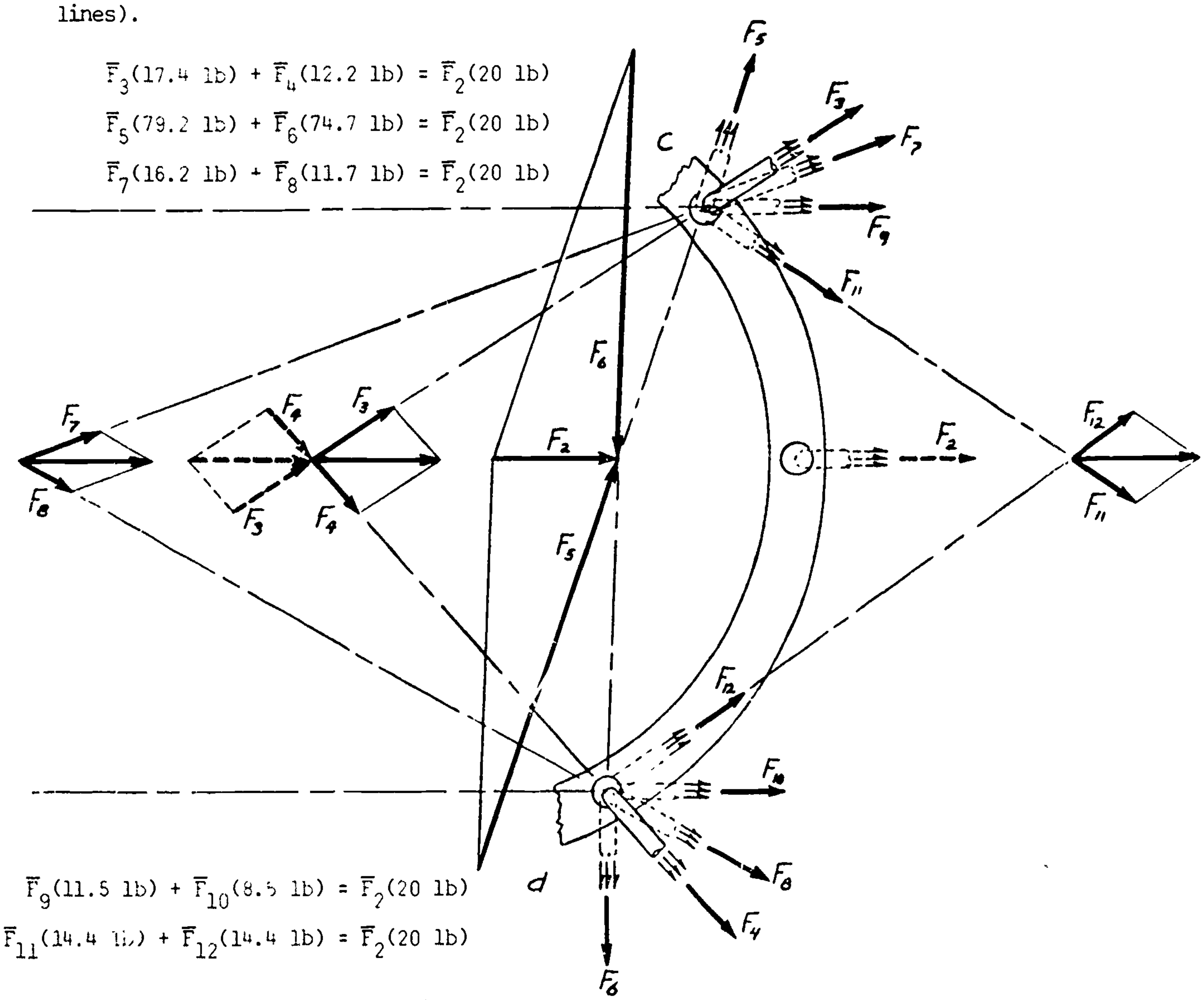


Figure FD 10

The centerlines of other spring combinations that can be found by trial and error are also shown in figure FD 10. When pulled as shown, S5 and S6 replace S2. Their point forces F_5 and F_6 are found to be coplanar (in the same plane) and concurrent (the lines of action meet at the same point) with F_2 and form the sides of a parallelogram which has a diagonal equal to F_2. Other sets that can replace S2 are S7 and S8, S9 and S10, and S11 and S12. All of these sets are found to have point forces that are coplanar with F_2 and all are concurrent and form parallelograms with diagonals equal to F_2, except the parallel set of S9 and S10. The case of parallel forces will be treated later in a special section.

Of course, each of the experiments could have been performed in the reverse order. For instance, the three springs S1, S3 and S4 could have been applied first, then the right-handed pair, S3 and S4, could have been replaced by S2. When S2 is replaced by S3 and S4, point force F_2 is replaced by F_3 and F_4. In reversing the order, F_3 and F_4 would be replaced by F_2. In either case, F_2 is called the sum or resultant of F_3 and F_4, while F_3 and F_4 are called components of F_2 along their direction lines. F_9 and F_{10} are not usually referred to as components of F_2.

The results of experiments are usually written in the form of a law. The law for the addition of forces is called the parallelogram law. It states that when a single load is to be replaced by two loads, their three point forces (1) must have lines of action that are coplanar and concurrent, and (2) must form a parallelogram at the point of concurrency, with the diagonal equal to the single point force (called the resultant) and its sides equal to the other two point forces (called the components). Conversely, when two loads are to be replaced by a single load, their two point forces (called components) must (1) be coplanar and concurrent, and (2) form the sides of a parallelogram at their concurrent point, which has a diagonal (called the resultant) equal to the point force of the total load.

Quantities that obey the parallelogram law when added are called vector quantities or vectors. A vector has these characteristics: (1) a magnitude, (2) a line of action, a direction, and a sense, that is, it is a directed quantity, and (3) a vector obeys the parallelogram law when added to another of its kind or when replaced by others of its kind. A point force is therefore a vector, as represented by a full arrow. The length of the arrow represents the magnitude of the point force, the position of the body of the arrow represents the line of action and direction of the point force, the position of the arrowhead represents the sense of the point force, and these point forces are combined only with parallelogram addition. Vectors will be written in this presentation with bars over them (example - $\bar{F}_3$), and the magnitudes of the vectors will be written with capital letters (examples - F_3, F_4).

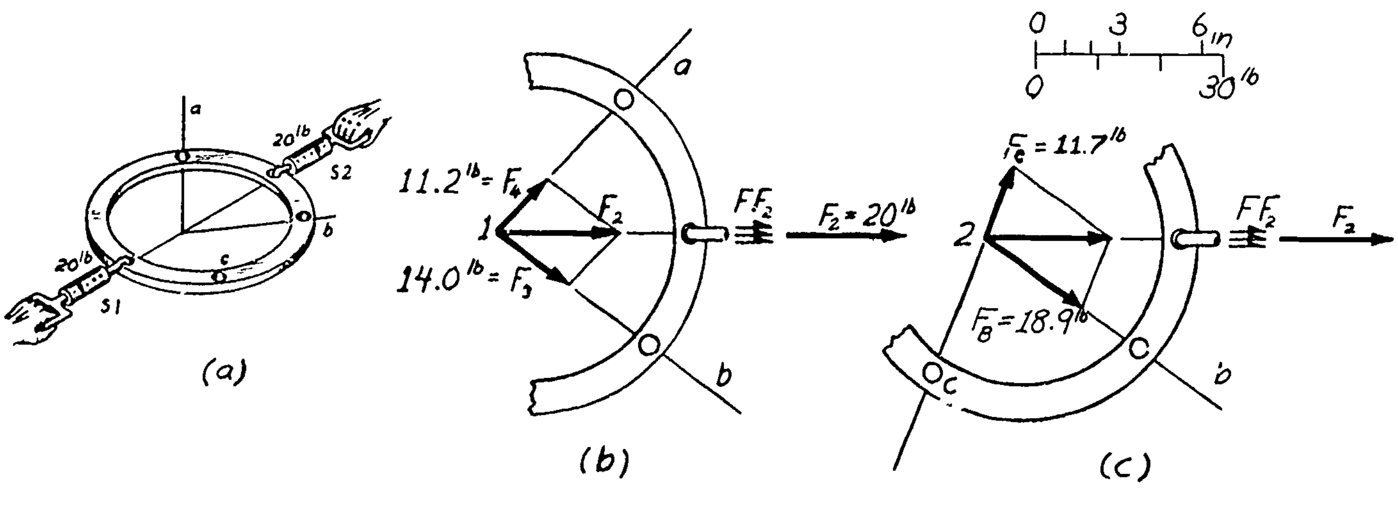


Figure FD 11

Figure FD 11(a) shows a horizontal ring held by S1 and S2. The parallelogram law is to be used: (1) to replace S2 by point force components along the action lines a and b, and (2) to replace S2 by a set of components, one along b and the other through point c.

For case (1) a plan view is drawn to scale in (b) of the right half of the ring with FF_2 and $\overline{F}_2$ shown along with lines a and b. Next, a parallelogram is drawn to scale at point 1 with $\overline{F}_2$ as its diagonal and F_3 and F_4 as its sides. F_3 and F_4 are the point forces of two loads acting at a and b.

For case (2) a plan view is drawn to scale in (c). FF_2 and $\overline{F}_2$ are shown, and the action line of $\overline{F}_b$ can be drawn. $\overline{F}$ and $\overline{F}_b$ intersect at point 2, therefore $\overline{F}_c$ must pass through point c and point 2. $\overline{F}_c$ can now be placed on the diagram. Now $\overline{F}_2$ can be replaced by components $\overline{F}_b$ and $\overline{F}_c$ as shown by the parallelogram. Replacing a point force by components is sometimes called resolution.

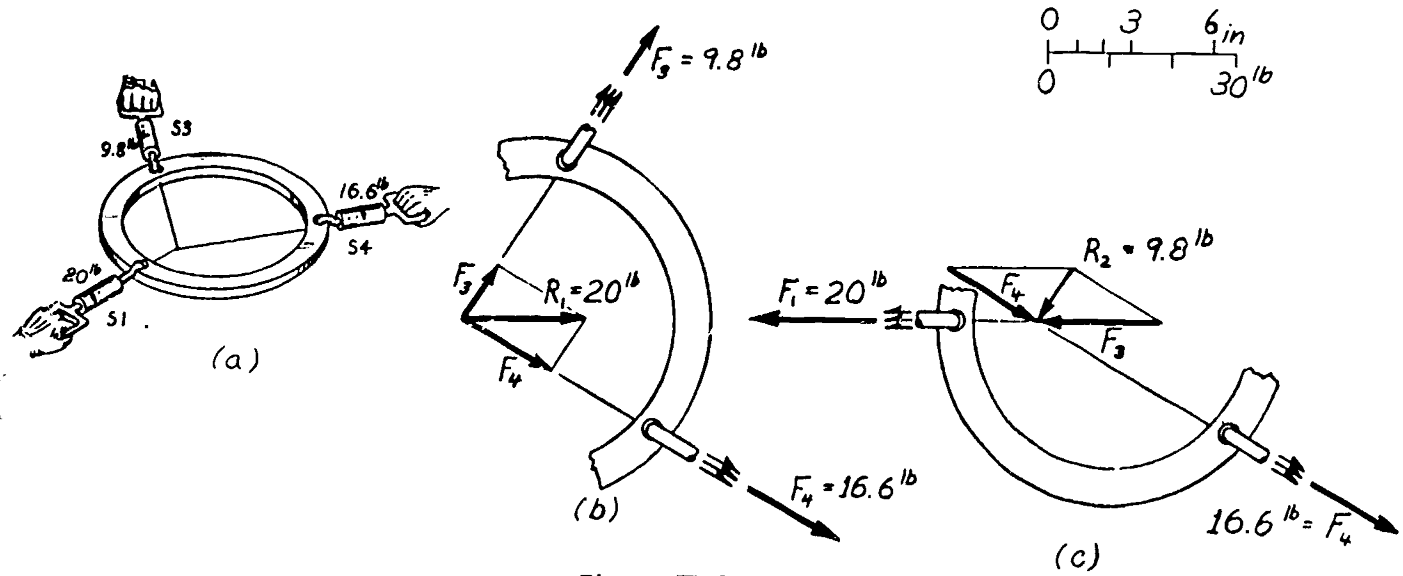


Figure FD 12

The three loads S3, S4, and S1 hold the ring in a stationary position in figure FD 12(a). In (b), loads S3 and S4 are to be replaced by their point force resultant using the parallelogram law, then in (c) the point force resultant of S1 and S4 is to be found using the parallelogram law.

In (b) $\overline{F}_4$ and $\overline{F}_3$ are combined at their intersection point. In (c) $\overline{F}_1$ and $\overline{F}_4$ intersect as shown and combine with a parallelogram to give their resultant $\overline{R}_2$.

AT THIS POINT, IF YOU ARE GIVEN AN OBJECT ACTED UPON BY TWO OR MORE LOADS, YOU SHOULD BE ABLE TO REPLACE ANY ONE OF THESE LOADS BY POINT FORCE COMPONENTS USING THE PARALLELOGRAM LAW. ALSO, IF YOU ARE GIVEN AN OBJECT ACTED UPON BY THREE OR MORE COPLANAR NON-PARALLEL LOADS, YOU SHOULD BE ABLE TO FIND THE POINT FORCE RESULTANT OF ANY TWO OF THE LOADS USING THE PARALLELOGRAM LAW.

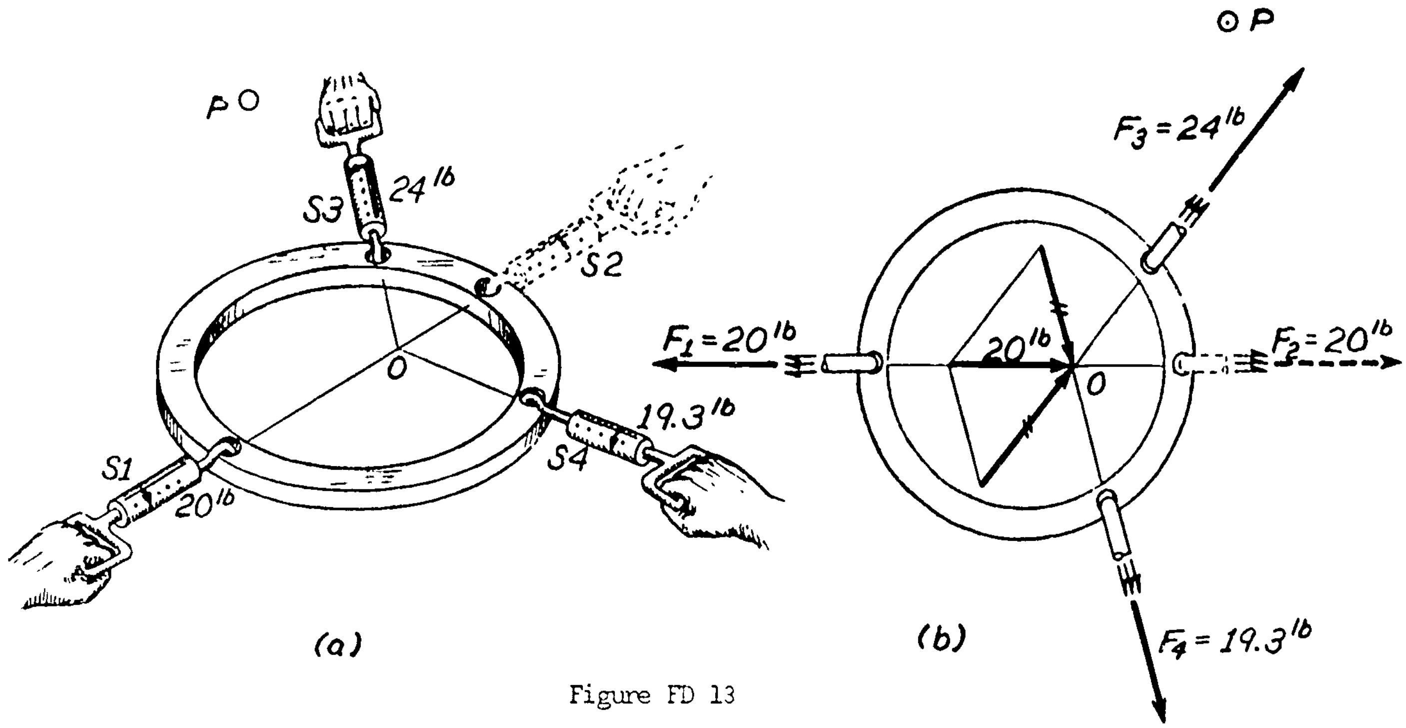


Figure FD 13

Figure FD 13(a) shows a ring which was originally held in a stationary horizontal position by loads S1 and S2. S2 was then replaced by S3 and S4, and, while maintaining the ring in its original horizontal position, the three scales were pulled until S1 registered its original magnitude of 20 lbs. It was found that S1 and S2 were colinear, and that S2, S3, and S4 were coplanar and concurrent. In (b) a plan view of the ring is constructed with $\overline{F}_1$, $\overline{F}_2$, $\overline{F}_3$, and $\overline{F}_4$ drawn to scale along their action lines on the outside of the ring. $\overline{F}_2$, $\overline{F}_3$, and $\overline{F}_4$ are related to each other as shown by the force parallelogram drawn to scale at their point of concurrency 0. Now a point P is chosen in the plane of $\overline{F}_2$, $\overline{F}_3$, and $\overline{F}_4$. Some new relationships are to be developed between $\overline{F}_2$ and its components $\overline{F}_3$ and $\overline{F}_4$ with respect to point P.

A plan view of the right half of the ring is drawn in figure FD 14. Point P is shown, and the point forces $\overline{F}_2$, $\overline{F}_3$, and $\overline{F}_4$ are drawn to scale on their lines of action. The point of concurrency O is also shown.

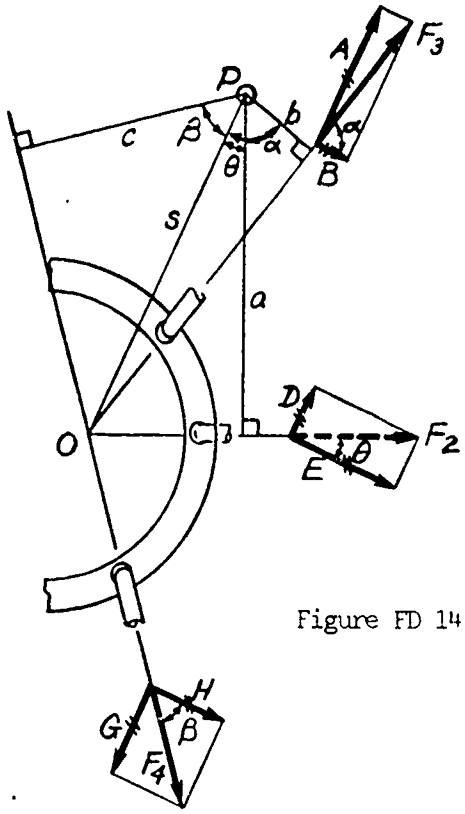


Figure FD 14

First, a relationship between $\overline{F}_2$ and point P is to be developed. A straight line s is drawn from P to 0. Using the parallelogram law, $\overline{F}_2$ is replaced with components $\overline{D}$ and $\overline{E}$, which are parallel and perpendicular, respectively, to line s. Line a is next drawn from P perpendicular to the line of action of $\overline{F}_2$. Notice that the angles labeled θ are equal, so from similar triangles,

$$\frac{s}{a} = \frac{F_2}{E}$$

Next, $\overline{F}_3$ is to be related to point P. Components $\overline{A}$ and $\overline{B}$, which are parallel and perpendicular to s, replace $\overline{F}_3$. Line b is drawn from P perpendicular to the action line of $\overline{F}_3$. Angles labeled α are equal, so from similar triangles,

$$\frac{s}{b} = \frac{F_3}{B}$$

$\overline{F}_4$ is now to be related to point P. $\overline{H}$ and $\overline{G}$, perpendicular and parallel to s, replace $\overline{F}_4$, line c is drawn from P perpendicular to $\overline{F}_4$, and the angles β are equal, so

$$\frac{s}{c} = \frac{F_4}{H}$$

$\overline{F}_2$ is the resultant of $\overline{F}_3$ and $\overline{F}_4$, so the component of $\overline{F}_2$ perpendicular to line s must equal the sum of the components of $\overline{F}_3$ and $\overline{F}_4$ which are perpendicular to s. This means that

$$\overline{E} = \overline{B} + \overline{H}$$

These are colinear, so $E = B + H$

Substituting the first three equations into this equation gives

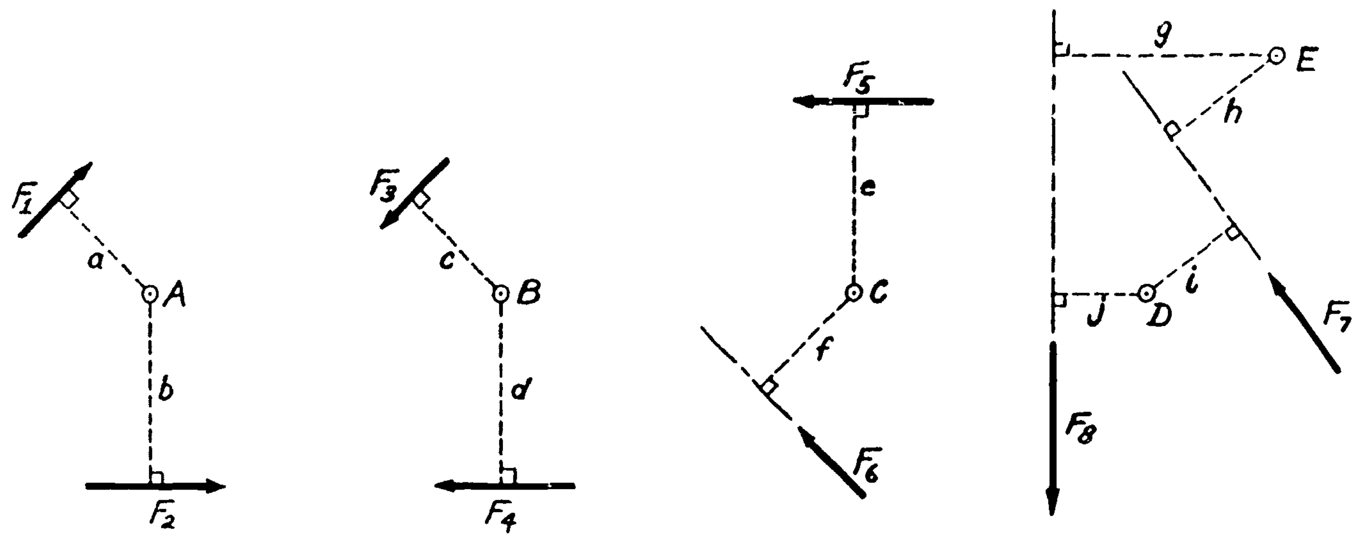
$$\frac{aF_2}{s} = \frac{bF_3}{s} + \frac{cF_4}{s}$$

s is the denominator for each term and can be cancelled, leaving

$$aF_2 = bF_3 + cF_4$$

This equation relates the resultant $\overline{F}_2$ and its components $\overline{F}_3$ and $\overline{F}_4$ with point P. The product aF_2 is called the moment of $\overline{F}_2$ with respect to point P and can be written $M_{F_2/P}$. The product bF_3 is called the moment of $\overline{F}_3$ with respect to P, or $M_{F_3/P}$. $M_{F_4/P}$ is the product cF_4. In the use of this equation, P is called the moment center, a is called the moment arm (or lever arm) of $\overline{F}_2$ with respect to P, and b and c are the moment arms of $\overline{F}_3$ and $\overline{F}_4$ with respect to P. If the lengths are in inches and the forces in pounds, the units for moments are inches times pounds or inch-pounds.

The relationships between the point force F and its components F_3 and F_4 with respect to point P resulted in an equation. The terms in any equation must have signs. The usual sign convention for moments is that a moment will be called positive if the force arrow points counterclockwise on the lever arm and negative if the force arrow points clockwise on the moment arm. Some examples of positive and negative moments of point forces are shown in figure FD 15.



$$M_{F_1/A} = -aF_1$$
$$M_{F_2/A} = +bF_2$$

$$M_{F_3/B} = +cF_3$$
$$M_{F_4/B} = -dF_4$$

$$M_{F_5/C} = +eF_5$$
$$M_{F_6/C} = -fF_6$$

$$M_{F_7/D} = +iF_7$$
$$M_{F_7/E} = -hF_7$$
$$M_{F_8/D} = +jF_8$$
$$M_{F_8/E} = +gF_8$$

Figure FD 15

AT THIS POINT, IF YOU ARE GIVEN COPLANAR POINT FORCES AND A POINT IN THEIR PLANE, YOU SHOULD BE ABLE TO IDENTIFY THE MOMENTS OF THE POINT FORCES WITH RESPECT TO THE POINT USING CORRECT SIGNS.

Measured Moments

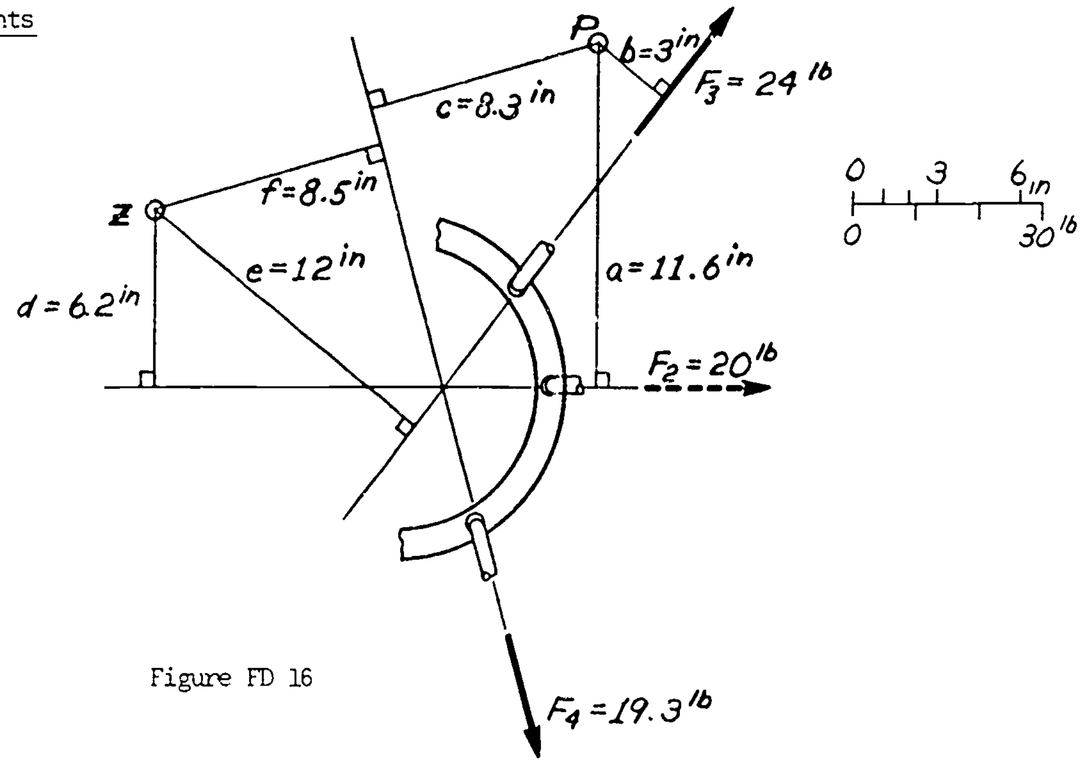


Figure FD 16

The right half of the ring from figure FD 14 is redrawn in figure FD 16. The mathematical moments are to be found using measured lever arms for the moments of F_2, F_3, and F_4 with respect to point P.

$$M_{F_2/P} = +(11.6)(20) = +232 \text{ in-lb}$$

$$M_{F_3/P} = +(3)(24) = +72 \text{ in-lb}$$

$$M_{F_4/P} = +(8.3)(19.3) = +160 \text{ in-lb}$$

$$M_{F_2/P} = M_{F_3/P} + M_{F_4/P} = +72 + 160 = +232 \text{ in-lb}$$

In the derivation of the moment equation, the distance s was cancelled. This means point P is a random point and the moment of the resultant $\overline{F}_2$ will equal the moment of its components $\overline{F}_3$ and $\overline{F}_4$ with respect to any point in the plane of $\overline{F}_2$, $\overline{F}_3$, and $\overline{F}_4$.

To illustrate this, a point Z is also shown in the plane of $\overline{F}_2$, $\overline{F}_3$, and $\overline{F}_4$ in figure FD 16. Perpendicular lines d, e, and f are drawn and measured.

$$M_{F_2/Z} = M_{F_3/Z} + M_{F_4/Z}$$

Therefore,

$$+ (6.2)(20) = + (12.0)(24) - (8.5)(19.3)$$

$$+ 124 = + 288 - 164$$

NOW IF YOU ARE GIVEN A DIAGRAM SHOWING COPLANAR POINT FORCES ACTING ON AN OBJECT, YOU SHOULD BE ABLE TO FIND THE MOMENTS OF THE POINT FORCES WITH RESPECT TO ANY POINT IN THEIR PLANE.

Using Moments to find Components

1 inch = 10 inches

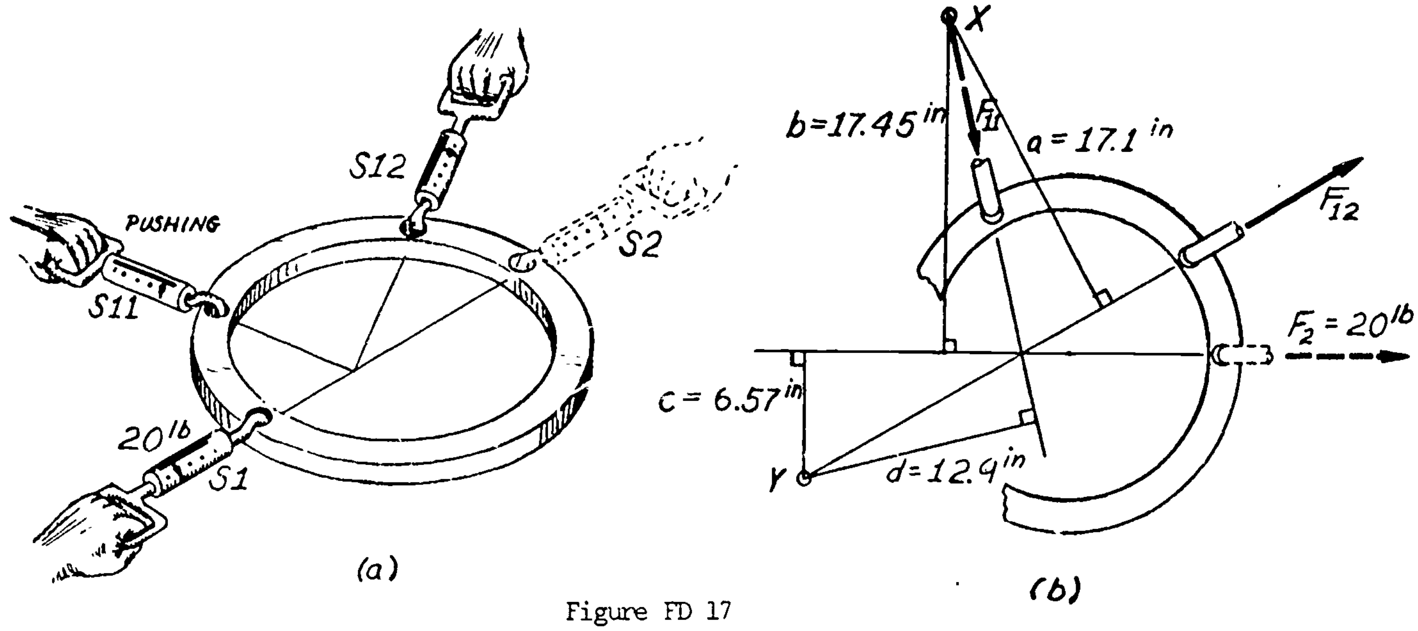


Figure FD 17

In figure FD 17 a ring is acted upon by loads S1 and S2. S11 and S12 are to replace S2 while keeping the ring in its original position. The magnitudes of S11 and S12 are to be found using moment equations. A portion of the plan view of the ring is drawn in (b), showing $\overline{F}_2$, $\overline{F}_{11}$, and $\overline{F}_{12}$. Remember that the moment of the resultant $\overline{F}_2$ equals the moment of its components $\overline{F}_{11}$ and $\overline{F}_{12}$ with respect to any point.

First, $\overline{F}_{12}$ will be found. A point X is chosen anywhere along the line of action of $\overline{F}_{11}$; the equation of the moments of the point forces with respect to X can now be written. The lever arms for the three point forces are drawn and scaled from the drawing.

$$M_{F_2/X} = M_{F_{11}/X} + M_{F_{12}/X}$$

$$+ (17.45)(20) = (0)(F_{11}) + (17.1)(F_{12})$$

$$F_{12} = 20.4 \text{ lb}$$

To find the magnitude of $\overline{F}_{11}$, a point y is chosen anyplace on the action line of $\overline{F}_{12}$. Perpendicular moment arms are drawn and scaled.

$$M_{F_2/Y} = M_{F_{12}/Y} + M_{F_{11}/Y}$$

$$-(6.57)(20) = (0)(F_{12}) - 12.9(F_{11})$$

$$F_{11} = 10.4 \text{ lb}$$

NOW YOU SHOULD BE ABLE TO REPLACE A POINT FORCE BY POINT FORCE COMPONENTS ALONG DESIGNATED DIRECTIONS BY USING MOMENT EQUATIONS.

Using Moments to find Resultants

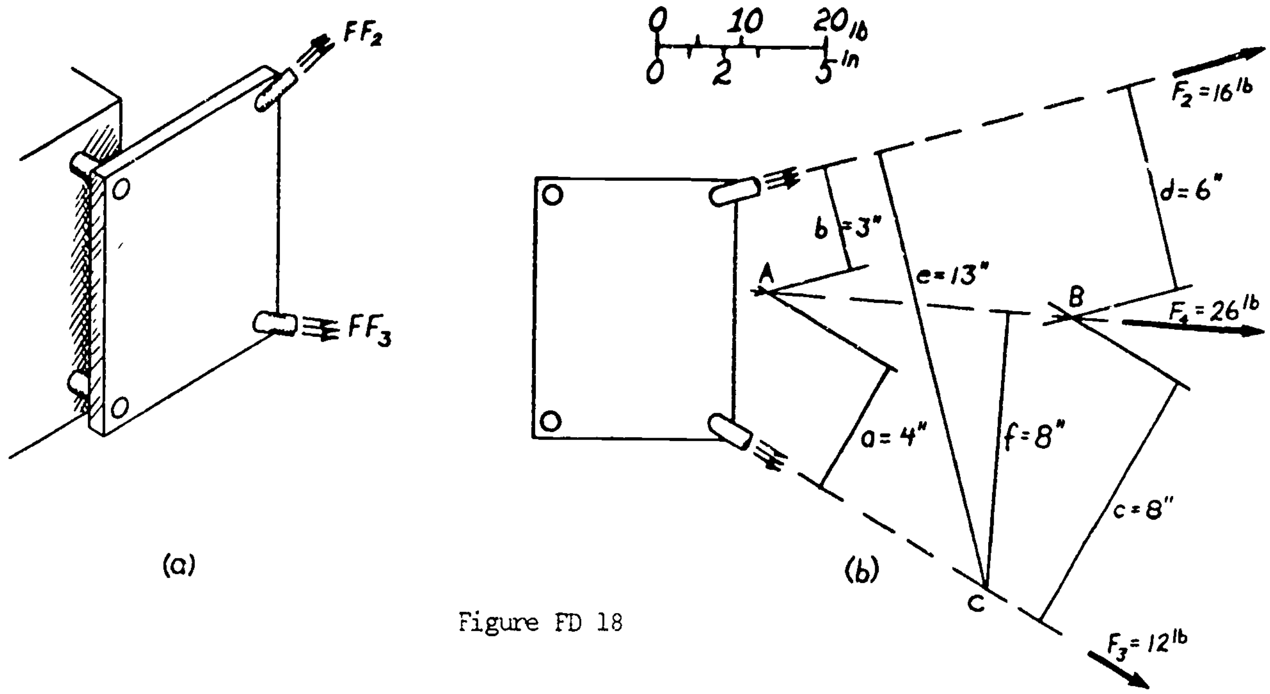


Figure FD 18

Figure FD 18(a) shows a vertical slab acted upon by two loads. The single point force resultant $\overline{F}_4$ of loads S2 and S3 is to be found, using moment equations.

A plan view of the slab is drawn to scale in (b), with $\overline{F}_2$ and $\overline{F}_3$ shown. The line of action of their resultant $\overline{F}_4$ will be found first.

A point A is located 3 inches from the line of action of $\overline{F}_2$ and 4 inches from the line of action of $\overline{F}_3$. Point A is on the line of action of the resultant of $\overline{F}_2$ and $\overline{F}_3$, since

$$M_{F_4/A} = M_{F_2/A} + M_{F_3/A}$$

$$0 = + (3)(16) - (4)(12)$$

Another point B is located 6 inches and 8 inches from the action lines of $\overline{F}_2$ and $\overline{F}_3$. The action line of $\overline{F}_4$ has now been established, since it must pass through points A and B.

The magnitude of $\overline{F}_4$ now can be found by choosing an arbitrary point C on the line of action of $\overline{F}_3$. Perpendiculars e and f are drawn and measured.

$$M_{F_4/C} = M_{F_2/C} + M_{F_3/C}$$

$$(f)(F_4) = (e)(F_2) + 0$$

$$(8)(F_4) = (13)(16)$$

$$F_4 = 26 \text{ lb}$$

NOW YOU SHOULD BE ABLE TO FIND THE SINGLE POINT FORCE RESULTANT OF ANY TWO COPLANAR LOADS BY USING MOMENT EQUATIONS.

The acting loads on the apparatus shown in figure FD 19(a) are the weights of the two blocks and the spring pull. The system is symmetrical about the centerline of the spring scale. The single point force resultant of these acting loads is to be found (1) using the parallelogram law directly, and (2) using a moment equation.

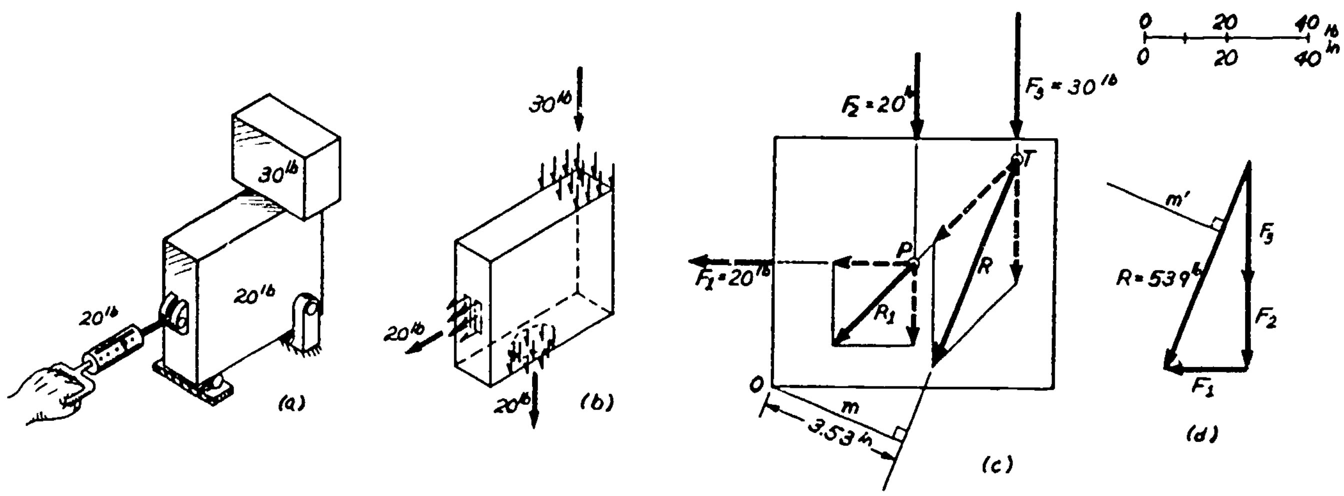


Figure FD 19

A 3-D diagram is drawn in (b) showing the three force fields. Their point force resultants are superimposed on the diagram. Since the point forces are coplanar, a 2-D diagram showing the point forces can be drawn to scale in (c). The action lines of $\overline{F}_1$ and $\overline{F}_2$ meet at point P, so $\overline{F}_1$ and $\overline{F}_2$ are added at P to find their resultant $\overline{R}_1$. Next, the action lines of $\overline{R}_1$ and $\overline{F}_3$ are extended and found to meet at point T. $\overline{R}$ is the sum of $\overline{R}_1$ and $\overline{F}_3$ at T. The magnitude, line of action, direction, and sense of $\overline{R}$ are now known.

Method (2) begins with diagram (c) drawn to scale with $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ shown. Now in (d) $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ are added with a triangle (this technique can easily be derived from the parallelogram law) to find R = 53.9 lb. Next, the line of action of $\overline{R}$ must be found in (c). Line m' is first drawn perpendicular to $\overline{R}$ in (d). Next, line m is drawn from reference point 0 in (c) parallel to m'. $\overline{R}$ acts perpendicular to m in (c) at an exact distance from 0. This distance will be called e.

The moment of $\overline{R}$ equals the sum of the moments of $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ with respect to 0.

$$M_{R/0} = M_{F_1/0} + M_{F_2/0} + M_{F_3/0}$$

$$-(e)(53.9) = (3)(20) - (3.5)(20) - (6)(30)$$

$$e = 3.53 \text{ inches.}$$

The line of action of $\overline{R}$ can now be placed perpendicular to line m 3.53 inches from 0. This checks with the first method.

WHEN YOU ARE GIVEN AN OBJECT ACTED UPON BY LOADS THAT REDUCE TO COPLANAR NON-PARALLEL POINT FORCES, YOU SHOULD BE ABLE TO FIND THE SINGLE POINT FORCE RESULTANT OF THE LOADS USING EITHER DIRECT PARALLELOGRAM ADDITIONS OR MOMENT EQUATIONS.

Many times in engineering statics two parallel loads are replaced by one load. When the two loads are replaced by their force fields and these in turn are replaced by their point forces, these point forces do not intersect. Therefore, the parallelogram law cannot be used directly to add parallel point forces. Graphical and moment equation techniques will be developed now for finding the resultant of two parallel point forces.

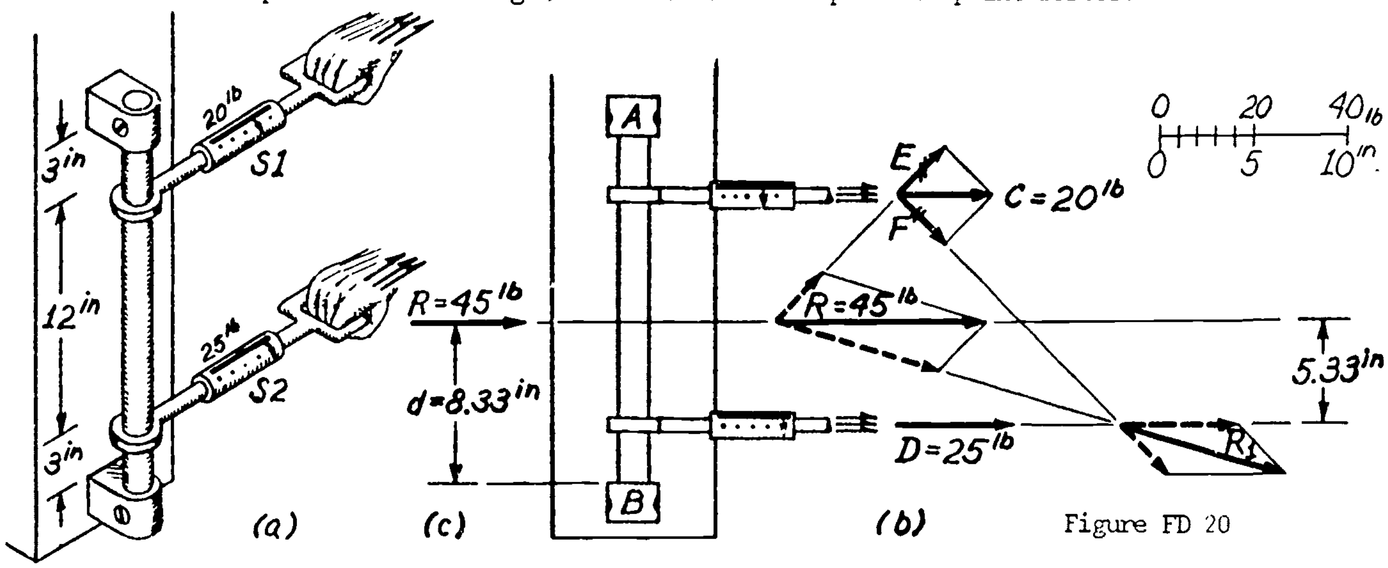


Figure FD 20

Figure FD 20(a) shows a vertical member acted upon by two horizontal loads S1 and S2 that are to be replaced by a single load.

In (b) the force fields that represent the two loads are shown together with the point forces that replace them. $\bar{C}$ and $\bar{D}$ are coplanar but not concurrent. Point force $\bar{C}$ can be replaced by two components $\bar{E}$ and $\bar{F}$ which are coplanar with $\bar{D}$. $\bar{F}$ can be added with $\bar{D}$ to give $\bar{R}_1$. $\bar{R}_1$ can now be added to $\bar{E}$ to give $\bar{R}$ which is the single point force resultant of $\bar{C}$ and $\bar{D}$. $\bar{R}$ is scaled and found to equal 45 lbs, with a line of action 5.33 inches above $\bar{D}$. This point force resultant $\bar{R}$ represents a single load that could replace S1 and S2.

The point force resultant of S1 and S2 can also be found using moments, as shown in (c). The three vectors are coplanar, $\bar{R}$ acts parallel to $\bar{C}$ and $\bar{D}$ towards the right, and $\bar{R} = \bar{C} + \bar{D}$, so the magnitude of $\bar{R} = 20 + 25 = 45$ lbs. Also, the moment of $\bar{R}$ equals the moment of $\bar{C}$ plus the moment of $\bar{D}$ with respect to any point. Taking the top of bracket B as a reference, distance d can be found:

$$M_{R/B} = M_{D/B} + M_{C/B}$$

$$-(d)(45) = -(3)(25) - (15)(20)$$

$$d = 8.33 \text{ inches.}$$

$\bar{R}$ can now be placed on diagram (c).

The two methods check each other. S1 and S2 can be replaced by a 45 lb single spring that is coplanar and parallel with them and positioned 5.33 inches above S2.

NOW YOU SHOULD BE ABLE TO FIND THE SINGLE POINT FORCE RESULTANT OF TWO PARALLEL LOADS, BOTH GRAPHICALLY AND WITH MOMENTS.

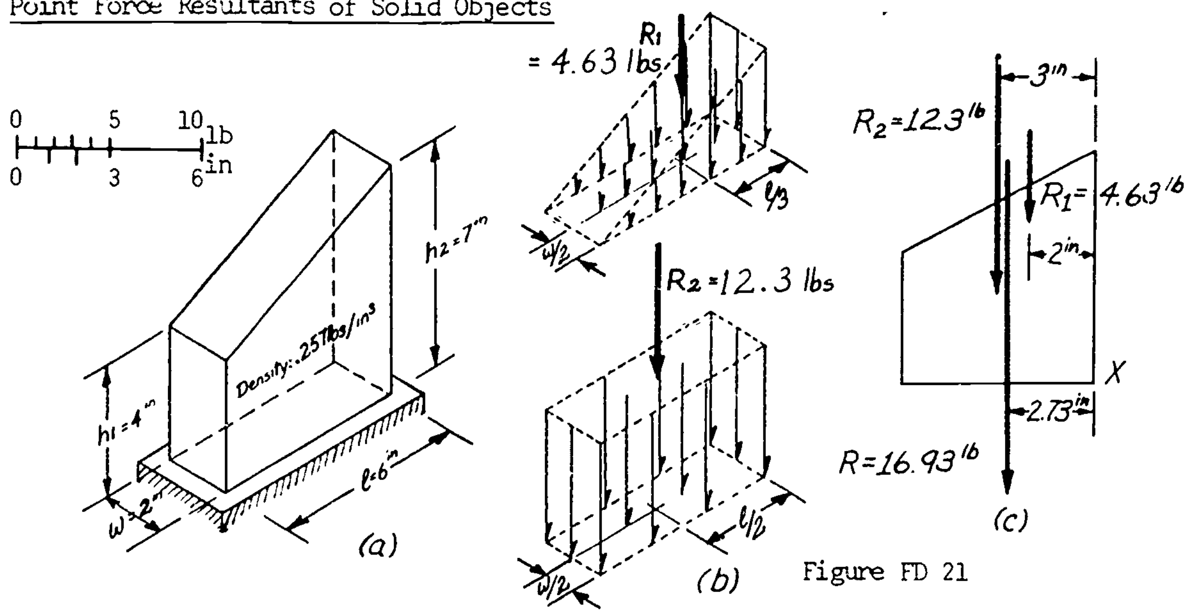


Figure FD 21

The single point force resultant of the solid object in figure FD 21 is to be found.

The object can be mentally pictured as if it were two simple objects, as shown in (b). This type of object is called a composite object. (The composite objects presented in this unit will be made up only of rectangular, triangular, and circular blocks. The centers of gravity of these common objects can be found in an engineering handbook.) Each simple object has a gravitational force field acting downward that has the same shape as the object.

For the lower rectangle the point force resultant acts through the center of gravity of the rectangle (at w/2 and 1/2 from any corner) and has a magnitude of

$$R_2 = (6)(2)(4)(.257) = 12.3 \text{ lb}$$

The triangular shape has a point force resultant that acts through its center of gravity (1/3 and w/2 from the right front corner) and has a magnitude of

$$R_1 = \frac{(6)(2)(3)(.257)}{2} = 4.63 \text{ lb}$$

The two resultants are parallel, so a 2-D diagram (c) can be used to find the single resultant $\overline{R}$ of $\overline{R}_1$ and $\overline{R}_2$.

$$\overline{R} = \overline{R}_1 + \overline{R}_2 = 4.63 + 12.3 = 16.93 \text{ lb}$$

$$M_{R/X} = M_{R_1/X} + M_{R_2/X}$$

$$(d)(16.93) = (2)(4.63) + (3)(12.3)$$

$$d = 2.73 \text{ inches.}$$

$\overline{R}$ can now be placed with its correct magnitude, direction, sense, and line of application in (c).

The composite member shown in figure FD 22(a) also has a single resultant. The resultant is to be found using a moment equation. First, in (b), the member is mentally replaced by three known shapes. The block and wedge are considered to be solid with their resultants

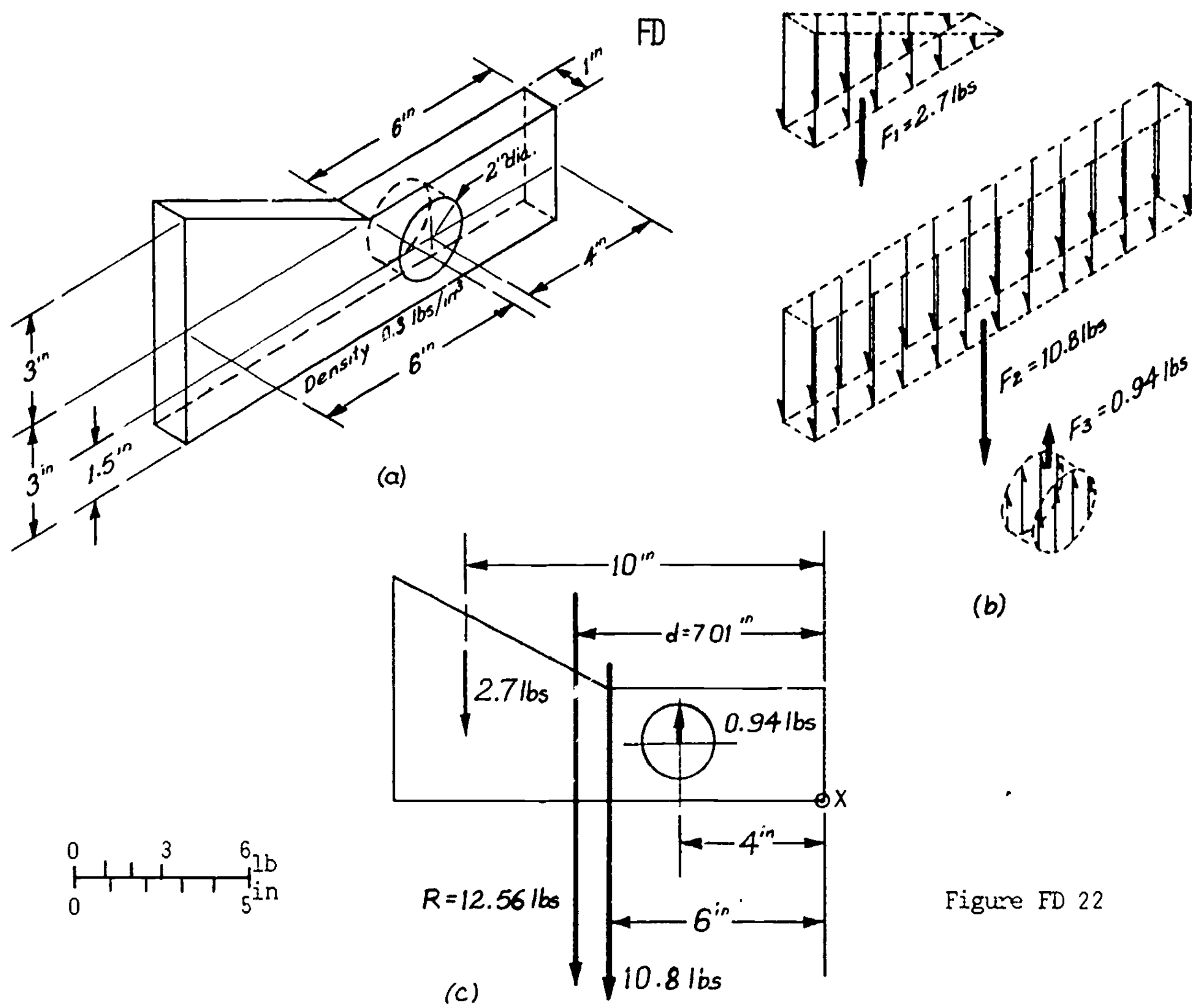


Figure FD 22

acting downward. The resultant of the cylindrical hole is considered to act upward, and all three resultants are coplanar. In (c) the three point force resultants are drawn in 2-D with their correct magnitudes, directions, senses, and lines of application. The magnitude of the single vector resultant must be equal to the sum of the magnitudes of the vertical forces.

$R = 10.8 + 2.7 - 0.94 = 12.56$ lbs acting downward in the plane of the three point forces.

In addition, the moment of the resultant about some point X must equal the sum of the moments of the individual parts about the same point.

$$M_{R/X} = M_{F_2/X} + M_{F_1/X} - M_{F_3/X}$$

$$(d)(12.56) = (6)(10.8) + (10)(2.7) - (4)(0.94)$$

$d = 7.01$ inches from the right edge of the member.

WHEN A COMPOSITE OBJECT HAS POINT FORCE RESULTANTS THAT ARE COPLANAR, YOU SHOULD BE ABLE TO FIND THE SINGLE POINT FORCE RESULTANT FOR THE COMPOSITE OBJECT.

UNIT 2

EQUILIBRIUM DIAGRAMS

WHEN YOU HAVE FINISHED UNIT 2, IF YOU ARE GIVEN AN ENGINEERING STRUCTURE, (1) YOU WILL BE ABLE TO VISUALIZE THE 3-D FORCE FIELDS ACTING ON THE COMPLETE STRUCTURE AND ON ANY INDIVIDUAL MEMBER OF THE STRUCTURE, (2) YOU WILL BE ABLE TO CONSTRUCT FREE-BODY (F-B) DIAGRAMS IN 3-D OF THE STRUCTURE OR ANY MEMBER OF THE STRUCTURE USING POINT FORCES IN PLACE OF THE FORCE FIELDS, AND (3) YOU WILL BE ABLE TO CONSTRUCT 2-D F-B DIAGRAMS OF THE STRUCTURE OR ANY MEMBER OF THE STRUCTURE USING POINT FORCES.

Introduction

In Unit 1 you learned how to visualize force fields acting upon members and then how to replace these force fields by their point forces. In this unit more involved engineering structures and members will be analyzed. These structures will be shown in 3-D diagrams with force fields and point forces. 2-D diagrams will then be drawn of each member with the point forces shown. Only the relative positions of the force fields and point forces will be considered in this unit. All the structures analyzed in this unit will be in static equilibrium, that is, when they are acted upon by forces, they will not deflect or move from their stationary positions.

Forces Between Bodies

Force fields acting upon a body are caused either by direct contact with another body or by a magnetic or gravitational attraction between the body being studied and another body. The force fields acting between bodies are always equal and opposite. This means that the action (a force field) of body A on another body B will be equal and opposite to the action (a force field) of the body B on body A. The point forces that replace the force fields will be colinear, equal, and opposite to each other.

Figure EQD 1(a) shows an isometric drawing of a horizontal stationary beam H that is loaded with a wooden block A, another beam B, and a concrete block C. Beam H is resting upon two horizontal smooth supports D and E which are assumed to be rigidly attached to the ground. All the force fields acting upon each member are to be shown in 3-D. Each member is then to be shown in 3-D and 2-D with point forces replacing the force fields they represent. Members A, B, D, and H are assumed to be rigid, homogeneous and symmetrical about their vertical centerlines (℄'s). These vertical ℄'s are in the same vertical plane.

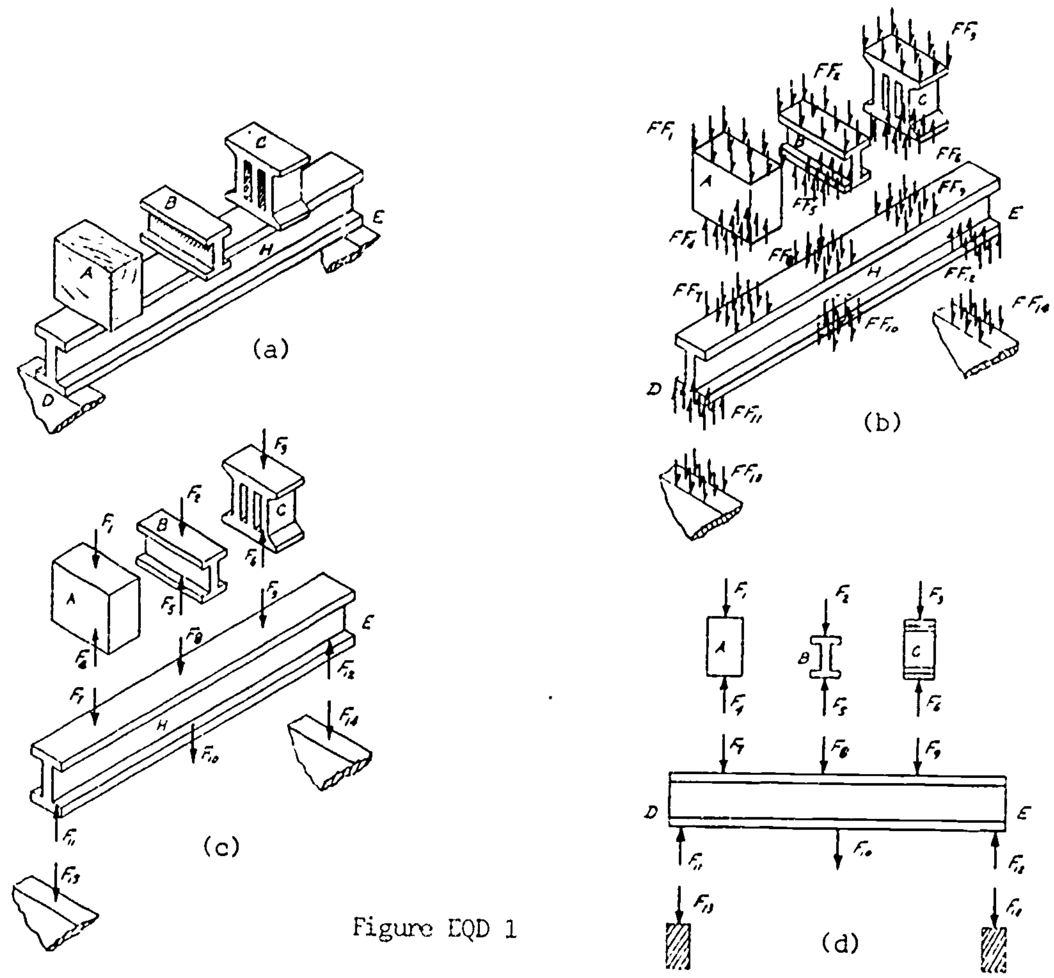


Figure EQD 1

The force fields acting upon each member are shown in (b). FF_{10} represents the weight of H but only a small section of this force field is shown. Each contact force field in (b) is an evenly distributed force field. In (c) all the force fields have been replaced by their point forces. Each point force acts through the center of its force field and all the point forces are in the same vertical plane, that is, they are coplanar. Although, of course, the four members are themselves 3-D and could not be in a single plane, the point forces that represent the force fields between them are coplanar. This type of system is called a coplanar system. The members are shown in (d) in 2-D with point forces.

Engineering Pins

Many times engineering members are connected to other members with round pins. Figure EQD 2 will be used to analyze the force fields acting upon a pin. The force fields acting upon weight W, pin A, fitting B, and cord C will be found. It is assumed that A, B, and C are weightless and rigid and that pin A is frictionless. The system is also symmetrical about the vertical ℄ of cord C.

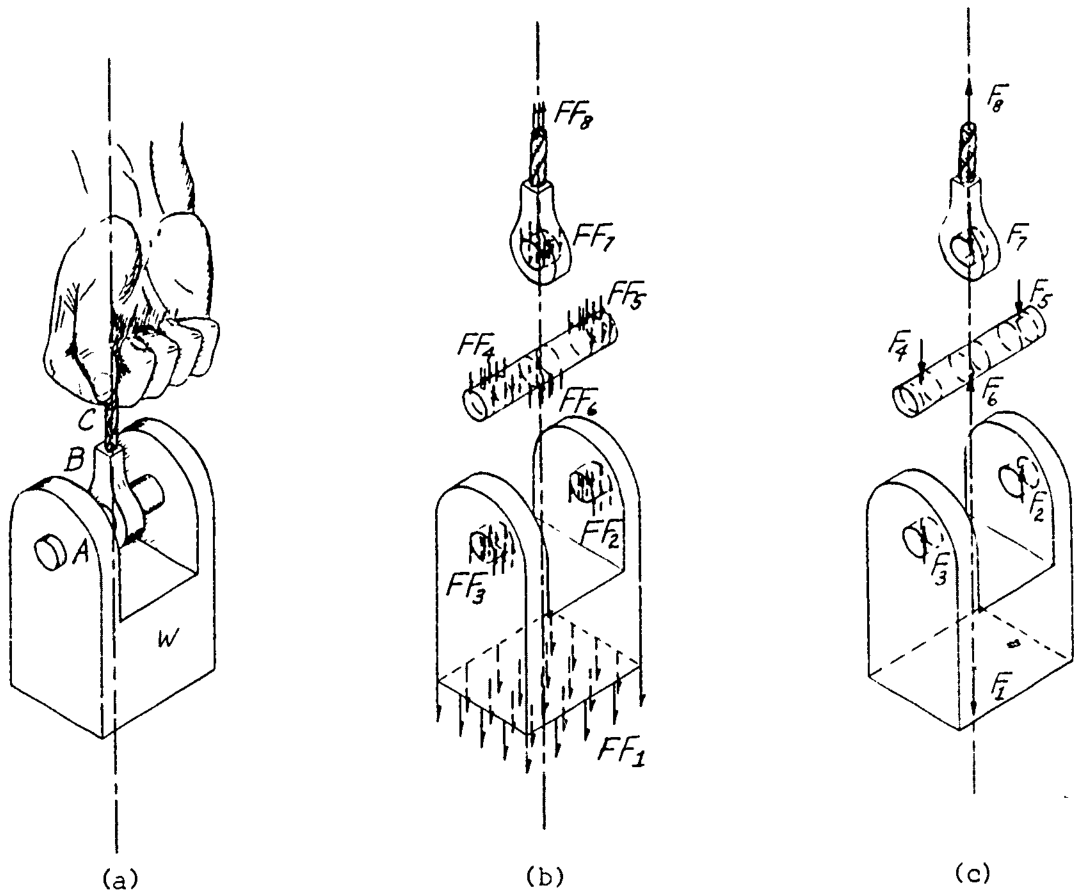


Figure EQD 2

In (b) the members are separated. It is further assumed that pin A has snug fits with W and B and that uniform force fields act at each contact surface. FF_6 then acts upward against the pin and is uniformly distributed over its contact area as shown. FF_7 acting against the fitting is equal and opposite to FF_6. FF_8 acts uniformly over its area and must balance FF_7. Since the pin is symmetrical about a vertical ℄, FF_4 and FF_5 are equal to each other as shown. FF_2 and FF_3 are equal and opposite to FF_5 and FF_4. Remember again that all the force fields are assumed to be evenly distributed over their contact areas.

In (c) the force fields have been replaced by point forces with each point force acting through the center of its force field.

EQD

An isometric space diagram of a horizontal beam H, this time supporting a load P with a pinned yoke B, is shown in figure EQD 3(a). Beam H is supported at its right end by a roller C and at its left end by a pin E held in a bracket D. The system is coplanar, the pins are friction free with snug fits, all members are weightless except H and P, and all members are rigid. All the members are to be drawn in 3-D with force fields, in 3-D with point forces, and in 2-D with point forces.

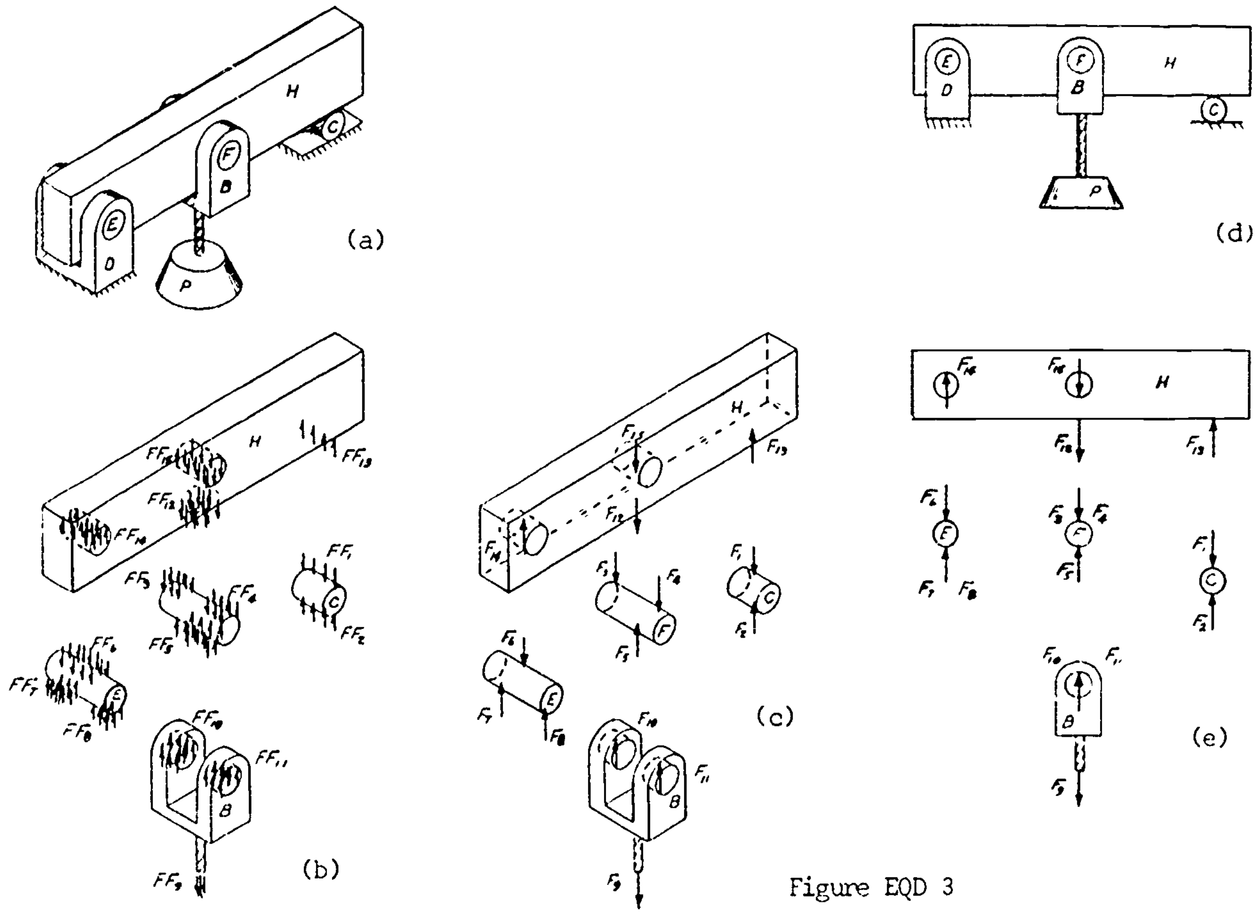


Figure EQD 3

In (b) all the members are drawn in exploded 3-D and all the force fields that act against all the members are shown. All the force fields are evenly distributed. Notice that roller C has line contact with its support and member H so its force fields FF_1 and FF_2 are line fields.

In (c) the isometric drawings show the members with the point force resultants of the force fields. All of these point forces act through the centers of the fields they represent.

The 2-D space diagram is drawn in (d) and the 2-D diagrams with point forces in (e). Notice that some of the arrows in (e) actually represent two force fields.

AT THIS TIME IF YOU ARE GIVEN AN OBJECT THAT IS LOADED WITH VERTICAL LOADS AND CONSTRAINED BY PINS AND ROLLERS, YOU SHOULD BE ABLE TO PLACE ON 3-D DRAWINGS THE FORCE FIELDS ACTING UPON ALL THE MEMBERS, THEN SHOW THE EQUIVALENT POINT FORCES ON 3-D DIAGRAMS, AND FINALLY SHOW THE POINT FORCES ON 2-D DIAGRAMS.

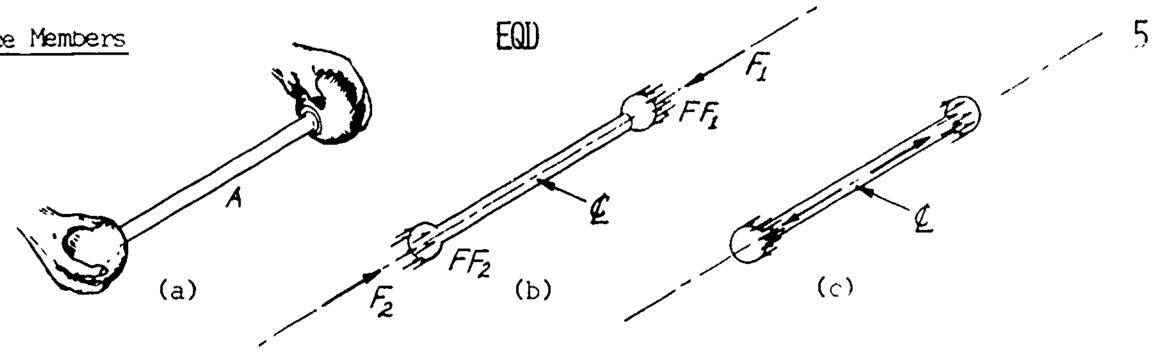


Figure EQD 4

An experiment is to be performed with rod A held by a student, as shown in figure EQD 4(a). The student will load member A by pushing or pulling through the ball and socket joints without moving member A. He finds from experimenting that he can (1) push his hands toward each other along the ℄ of A or (2) pull his hands apart again along A's ℄ . If he pushes or pulls in any other direction, member A will not remain stationary.

Diagram (b) shows the force fields and the corresponding point forces that are the results of pushing on the rod. The force fields FF_1 and FF_2 are equal and opposite. The point forces F_1 and F_2 are also equal and opposite and colinear along the ℄ of A. Diagram (c) shows the force fields and point forces caused by pulling on A. Again the point forces are equal, opposite and colinear along the ℄ of A.

The type of member shown in figure EQD 4 is called a two-force (2-F) member. A 2-F member can be described as a member loaded only at two places by evenly distributed force fields, and must therefore be weightless with frictionless supports at two places and no loads between the two supports. When the two force fields are replaced by their point forces, these two point forces must be equal and opposite and colinear with the ℄ of the member.

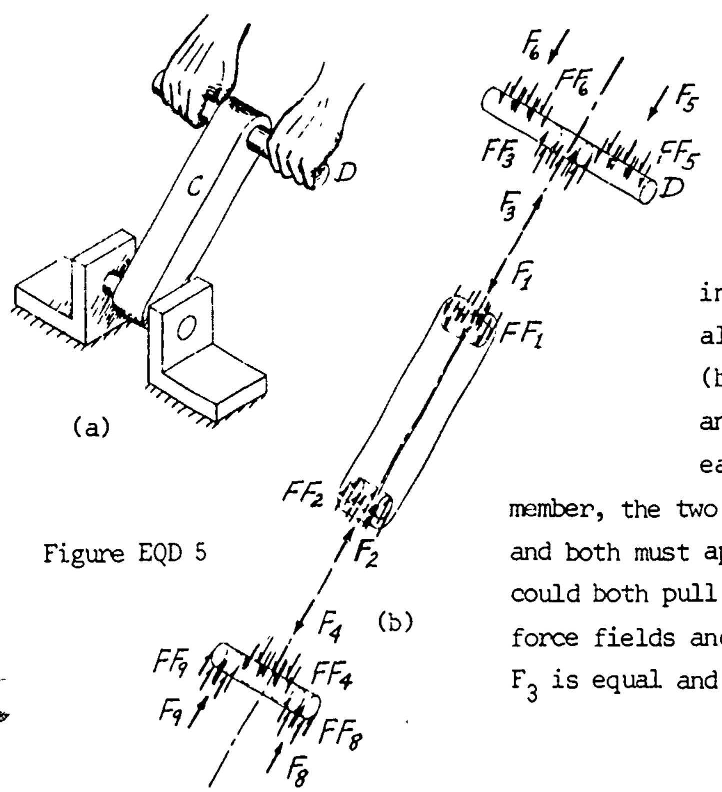


Figure EQD 5

Another example of a 2-F member is shown in EQD 5. F_1 and F_2 must be equal and opposite along the ℄ of the 2-F member C, as shown in (b). This means that FF_1 and FF_2 must be equal and opposite. FF_5 and FF_6 on pin D are equal to each other. This means that for C to be a 2-F member, the two hands must be equal distances from the ℄ of C and both must apply the same pressure. Of course, the hands could both pull and the result would be the same with all the force fields and point forces in (b) reversed. In addition, F_3 is equal and opposite to F_4 and F_9 is equal to F_8.

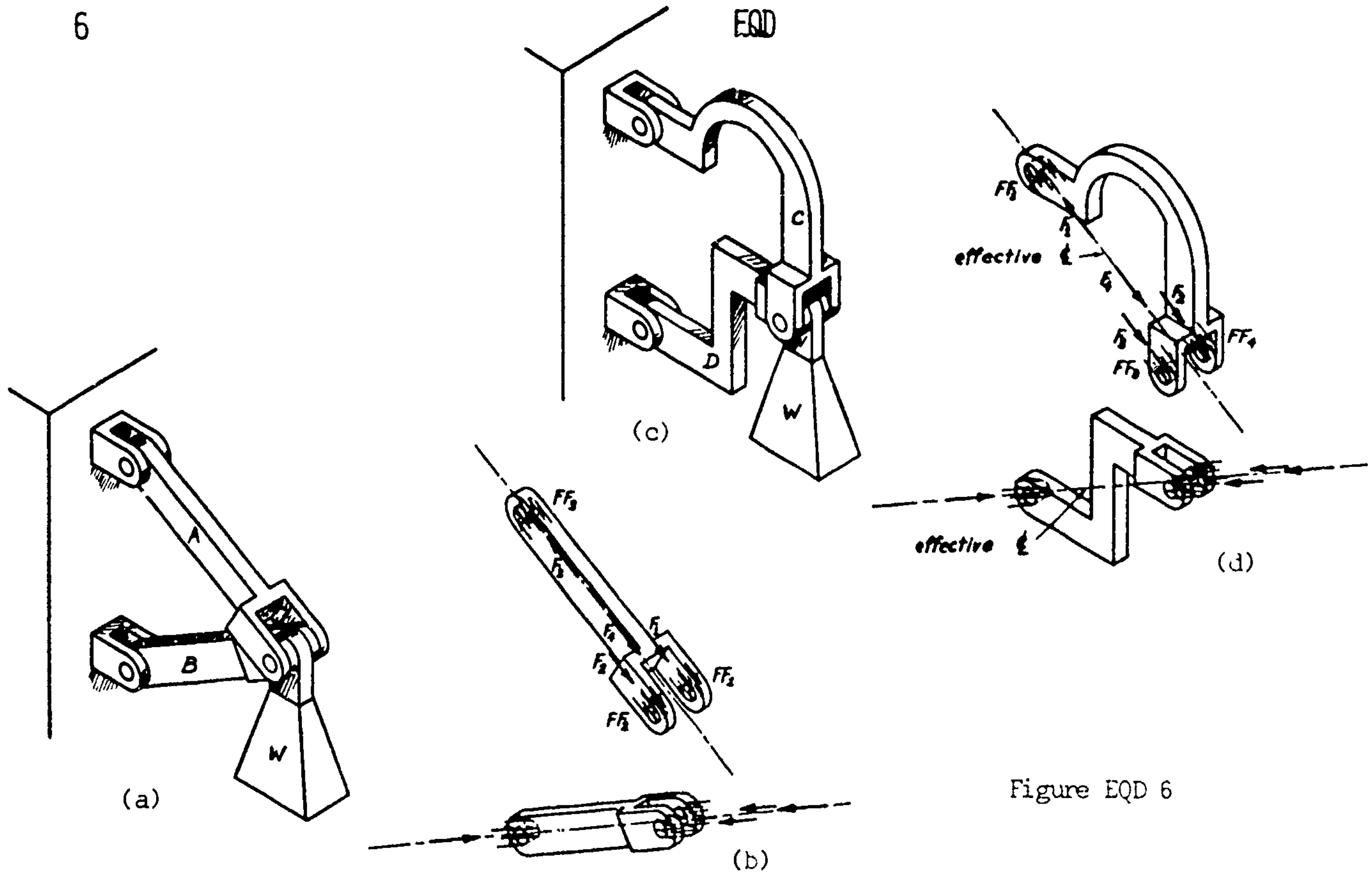


Figure EQD 6

The structure shown in figure EQD 6(a) shows a weight W being supported by two members A and B. The system is coplanar, that is, the ℄'s of A, B, and W are coplanar. A and B are considered to be weightless and the connecting pins friction free.

Now, in (b), diagrams of A and B are drawn with their force fields shown. FF_3 is replaced by F_3, which is on the ℄ of A. FF_1 and FF_2 are equal, so the corresponding point forces F_1 and F_2 can be replaced by F_4. F_4 is equal and opposite to F_3 and is also on the ℄ of A. Member A is also called a 2-F member, even though it is loaded at 3 places. Wherever the loads on a member can be replaced by two point forces that are colinear with the ℄ of the member, the member is called a 2-F member. Member B can be analyzed in a similar manner. It is called a 2-F compression member.

The structure shown in figure (c) is also coplanar and supports load W. In (d) FF_1 is a uniform force field. Its corresponding point force is F_1. FF_2 and FF_3 are equal, so the point forces F_2 and F_3 are also equal and can be replaced by F_4. F_4 must be colinear, equal, and opposite to F_1. A ℄ of the member can be drawn along the action lines of F_1 and F_4; this is called the effective ℄ of the member. Because this ℄ can be drawn, member C is a 2-F member. Member D is also a 2-F member with an effective ℄ as shown. The actual shape of a 2-F member is not important as long as an effective ℄ can be found.

AT THIS POINT YOU SHOULD BE ABLE TO IDENTIFY 2-F MEMBERS
AND THEIR EFFECTIVE ℄'S.

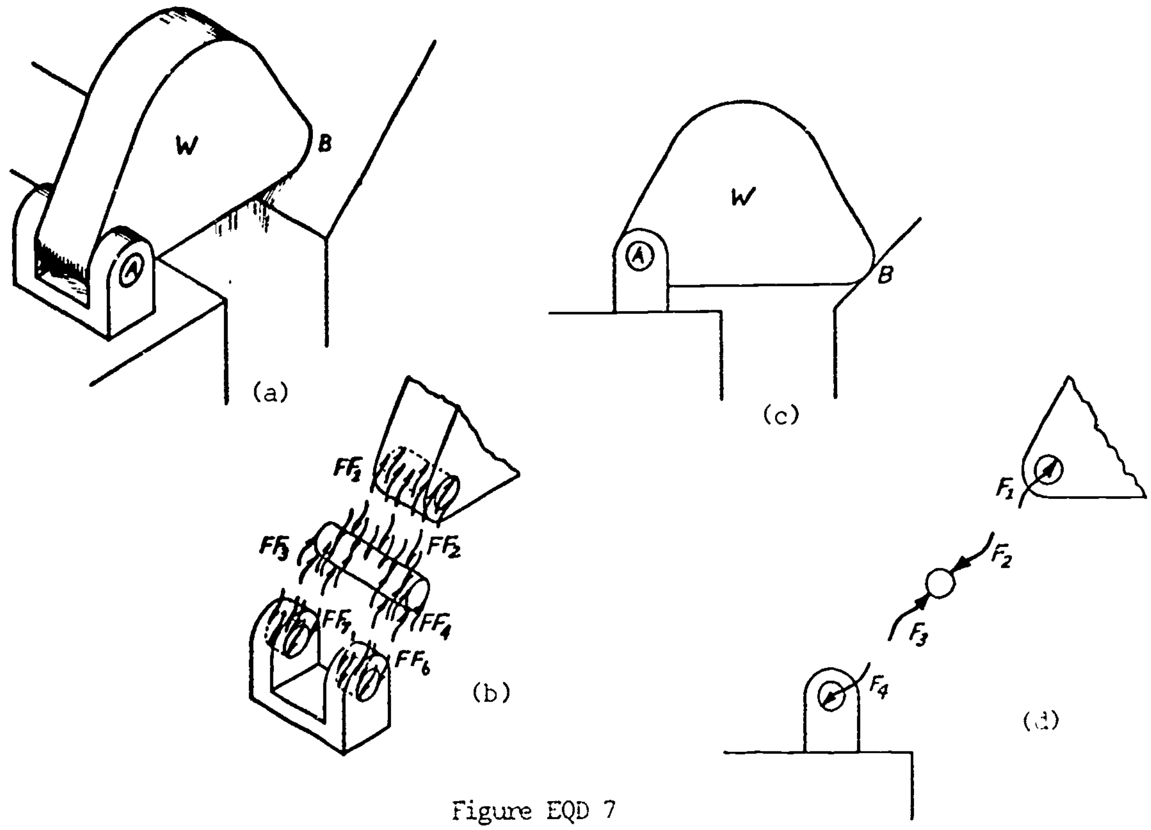


Figure EQD 7

Up until now members have been loaded so that the directions of the loads could be determined by inspection. In figure EQD 7(a) and (b) weight W is constrained by a smooth sloping surface at B and a frictionless snug pin A. The system is coplanar.

Because the surface at B is smooth, the reaction there is a line field perpendicular to the sloping surface (this reaction is the same as it would be if W rested on a roller at B). However, the direction of the force field acting on W at A is unknown. FF_1 is therefore drawn with wavy arrows in the cut-away view shown in (c), to illustrate that it has no known direction. For the same reason, FF_2, FF_3, FF_4, FF_5, and FF_6 on pin A and its bracket are also drawn with wavy arrows. In (d), the 2-D diagram, the point forces F_1, F_2, F_3, and F_4 are also drawn with wavy arrows, since their directions are unknown.

In engineering statics, members are connected by smooth surfaces, rough surfaces, frictionless pins, rollers, ball-and-socket joints, friction surfaces and other means. On the next two pages, figure EQD 8 illustrates some types of contact and distant force relationships. Whenever the directions are unknown, wavy arrows are shown for both the force fields and the point forces. The examples marked with an asterisk (*) will be studied in later cases; all others should be analyzed and understood at this time.

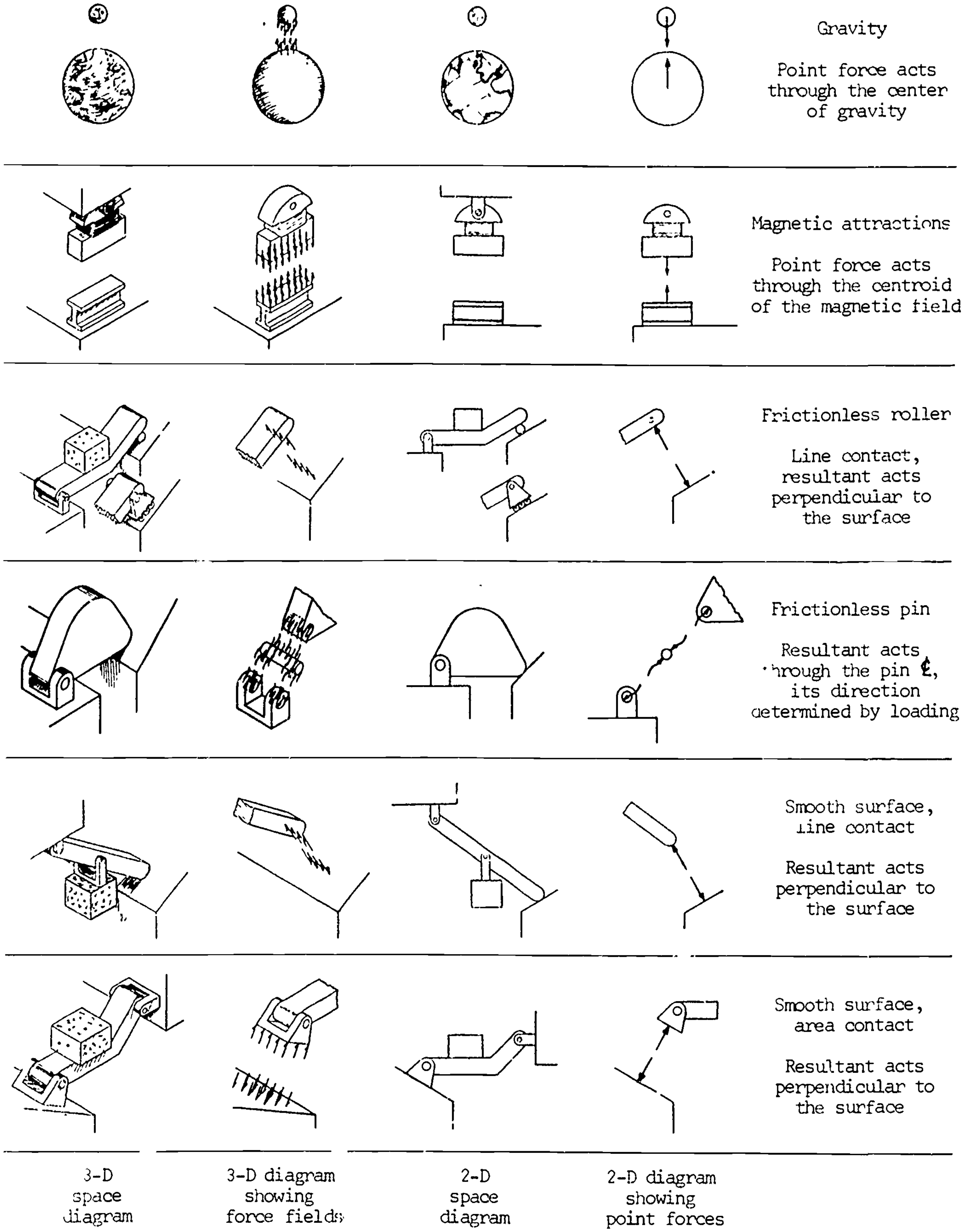

Gravity
Point force acts through the center of gravity
Magnetic attractions
Point force acts through the centroid of the magnetic field
Frictionless roller
Line contact, resultant acts perpendicular to the surface
Frictionless pin
Resultant acts through the pin ℄, its direction determined by loading
Smooth surface, line contact
Resultant acts perpendicular to the surface
Smooth surface, area contact
Resultant acts perpendicular to the surface
3-D space diagram
3-D diagram showing force fields
2-D space diagram
2-D diagram showing point forces

Figure EQD 8

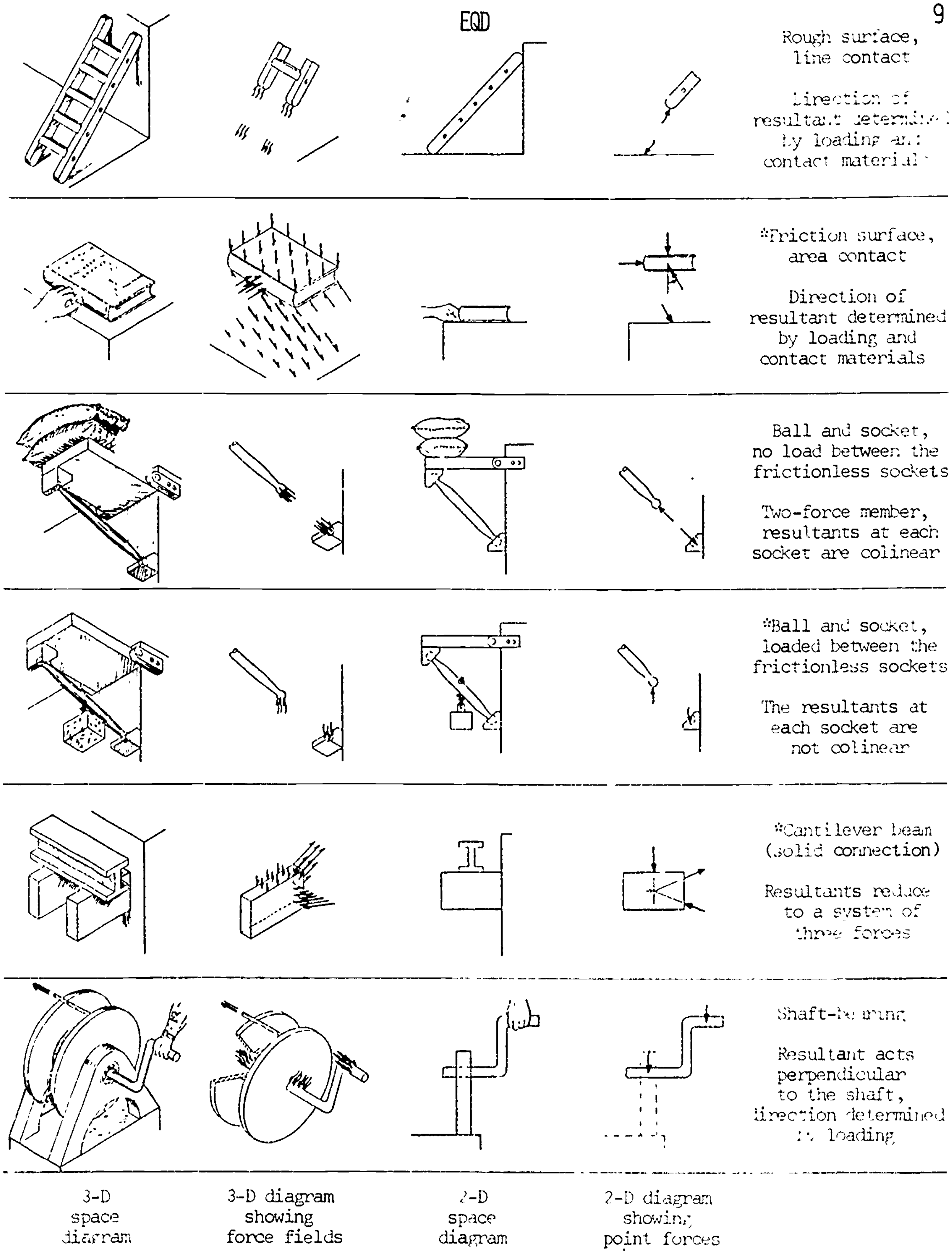

Rough surface, line contact
Direction of resultant determined by loading and contact materials
*Friction surface, area contact
Direction of resultant determined by loading and contact materials
Ball and socket, no load between the frictionless sockets
Two-force member, resultants at each socket are colinear
*Ball and socket, loaded between the frictionless sockets
The resultants at each socket are not colinear
*Cantilever beam (solid connection)
Resultants reduce to a system of three forces
Shaft-bearing
Resultant acts perpendicular to the shaft, direction determined by loading
3-D space diagram
3-D diagram showing force fields
2-D space diagram
2-D diagram showing point forces

Figure EQD 8

Free-Body (F-B) Diagrams

Figure EQD 9(a) shows a frame supporting two loads. These loads cause other loads to be built up on each member of the frame. The loads upon each individual member can all be represented by force fields. Assumptions are made that all the members of the frame are weightless, all the pins are friction free with snug fits, and all the members of the frame have ℄'s that are coplanar, thus all the force fields are evenly distributed over their areas of application.

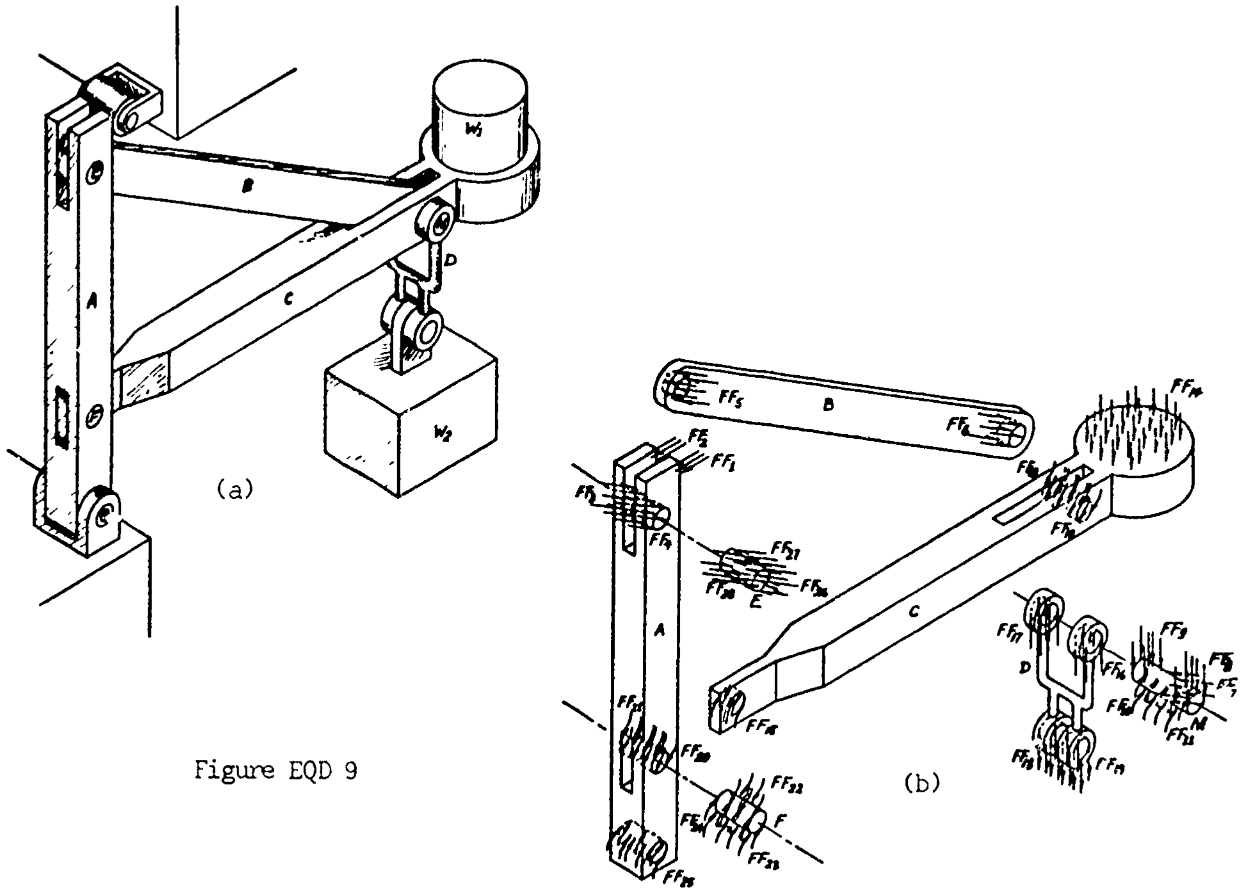


Figure EQD 9

In (b) each member of the frame is drawn as if it is isolated with only the force fields that act against it. These diagrams are called free-body (F-B) diagrams. Each of these F-B diagrams will now be analyzed.

Member B is a 2-F tension member so FF_5 and FF_6 must be equal and opposite with known directions. The force fields on pin E are FF_{28}, which is equal and opposite to FF_5, and FF_{26} and FF_{27}, which are equal to each other.

D is also a 2-F tension member. FF_{17} and FF_{16} are equal to each other; in addition, FF_{18} equals FF_{19}.

Pin M is acted upon by three other members: B, C, and D. FF_7 is equal and opposite to FF_6 on B. FF_9 and FF_8 are equal and opposite to FF_{17} and FF_{16} on D. Although FF_{11} and FF_{10} are equal and opposite to FF_{13} and FF_{12} on C, their directions are unknown so all are drawn with wavy arrows.

Member C is acted upon by FF_{14}, which is caused by weight W_1; FF_{13} and FF_{12} which are equal and opposite to FF_{11} and FF_{10} from pin M; and FF_{15} (unknown direction) caused by pin F.

Pin F is acted upon by FF_{22}, which is equal and opposite to FF_{15}, with FF_{23} and FF_{24}, all of which are represented by wavy arrows.

Member A has seven acting force fields. These are FF_1 and FF_2, caused by the roller; FF_3 and FF_4, from pin E; FF_{21} and FF_{20} (unknown direction) from pin F; and FF_{25} (unknown direction) caused by pin G.

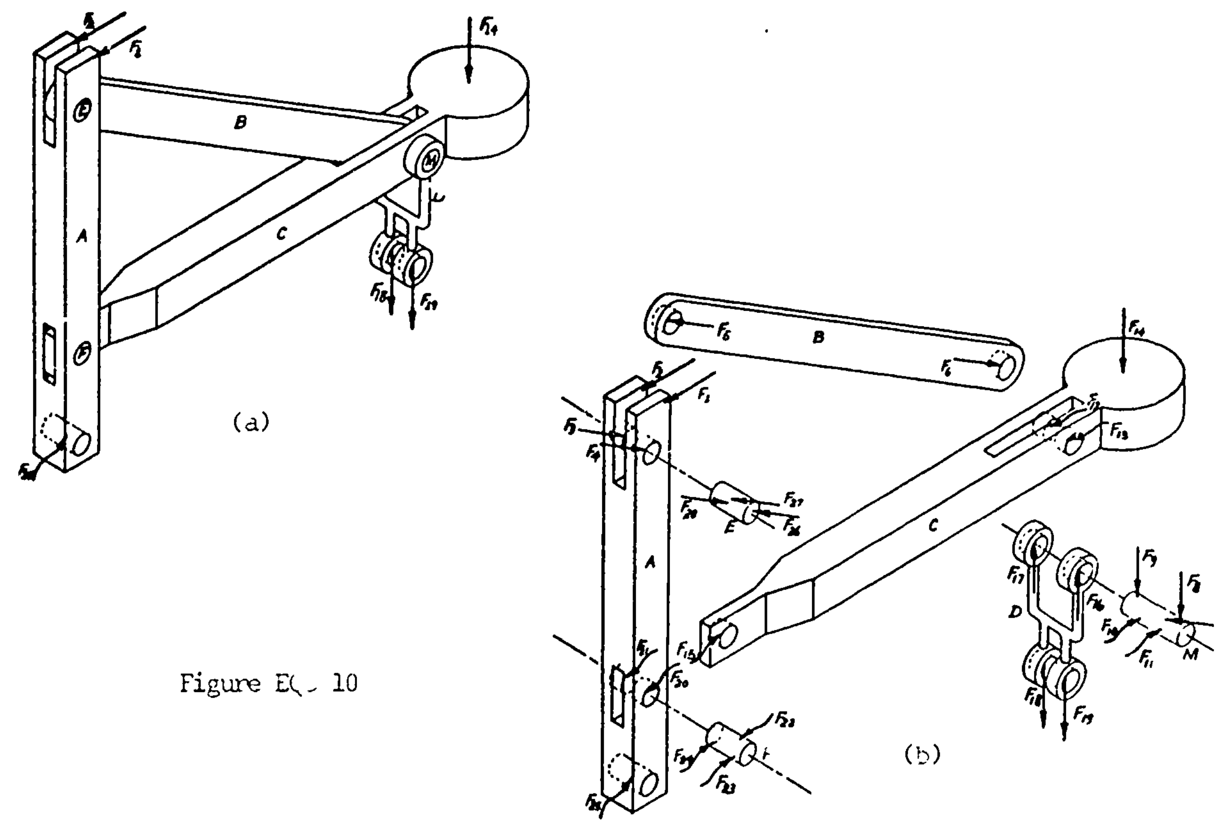


Figure EQD 10

The complete frame is drawn in figure EQD 10(a) as a 3-D F-B diagram with point forces replacing the force fields. 3-D F-B diagrams of each of the members are drawn with point forces in (b).

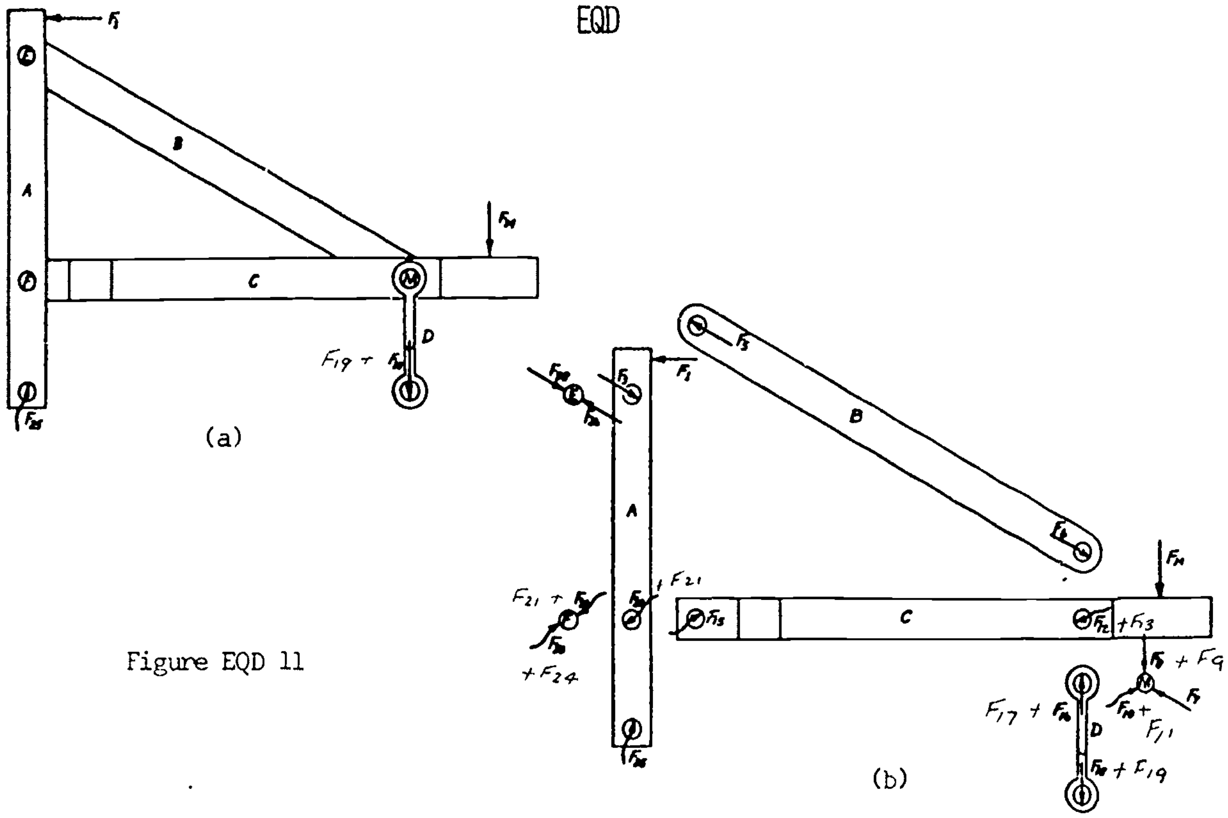


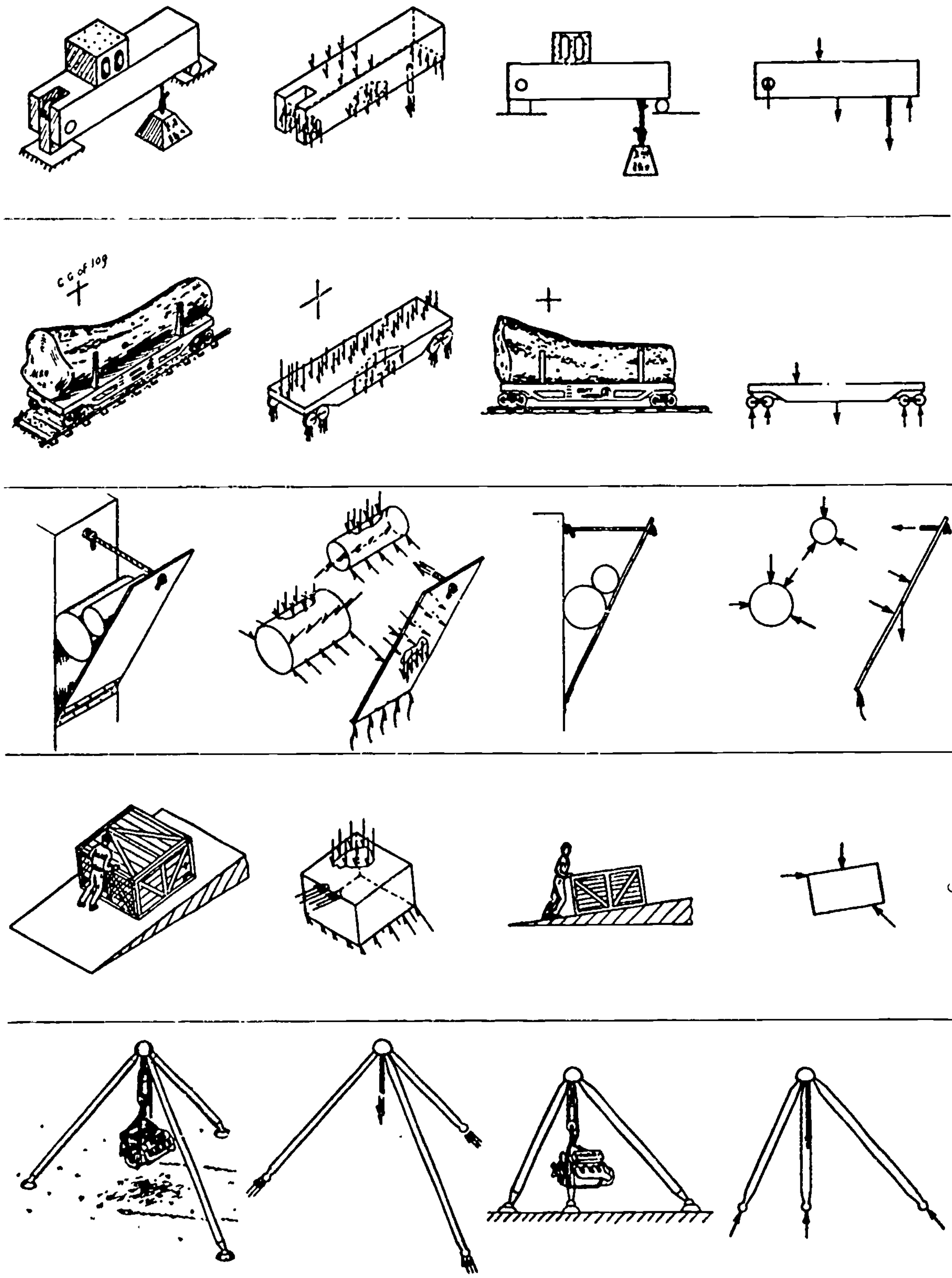
Figure EQD 11

The complete frame is drawn as a 2-D F-B diagram in figure EQD 11(a) with point forces; in (b) 2-D F-B diagrams are drawn with point forces on all the members. This type of 2-D F-B diagram is widely used in engineering statics. To properly construct and use the 2-D F-B diagrams, it is essential that you are able to visualize: (1) the 3-D F-B diagrams with force fields, and (2) the 3-D F-B diagrams with point forces. Notice that single arrows in the 2-D F-B diagrams can actually represent two separated force fields.

Some basic F-B diagrams are shown on the next two pages in figure EQD 12. They should be studied at this time.

NOW, IF YOU ARE GIVEN A 3-D SPACE DIAGRAM OF A STATIONARY STRUCTURE, YOU SHOULD BE ABLE TO: (1) VISUALIZE 3-D F-B DIAGRAMS OF EACH MEMBER OF THE STRUCTURE USING FORCE FIELDS, (2) CONSTRUCT 3-D F-B DIAGRAMS OF EACH MEMBER OF THE STRUCTURE USING POINT FORCES, AND (3) CONSTRUCT 2-D F-B DIAGRAMS OF EACH MEMBER OF THE STRUCTURE USING POINT FORCES.

In free-body diagrams such as these, all pins are assumed to be frictionless and all members weightless, unless shown otherwise



Parallel point force system

Non-uniform loading reduces to a single resultant through the log's center of gravity

Forces acting at contact surfaces are equal and opposite

Lower force field is trapezoidal in shape, with its direction dependent upon loading and contact surface conditions

Since all the constraints are ball and socket, all the members are two-force

3-D space diagram	3-D free-body diagram showing force fields	2-D space diagram	2-D free-body diagram showing point forces

Figure EQD 17

In free-body diagrams such as these, all pins are assumed to be frictionless and all members weightless, unless shown otherwise

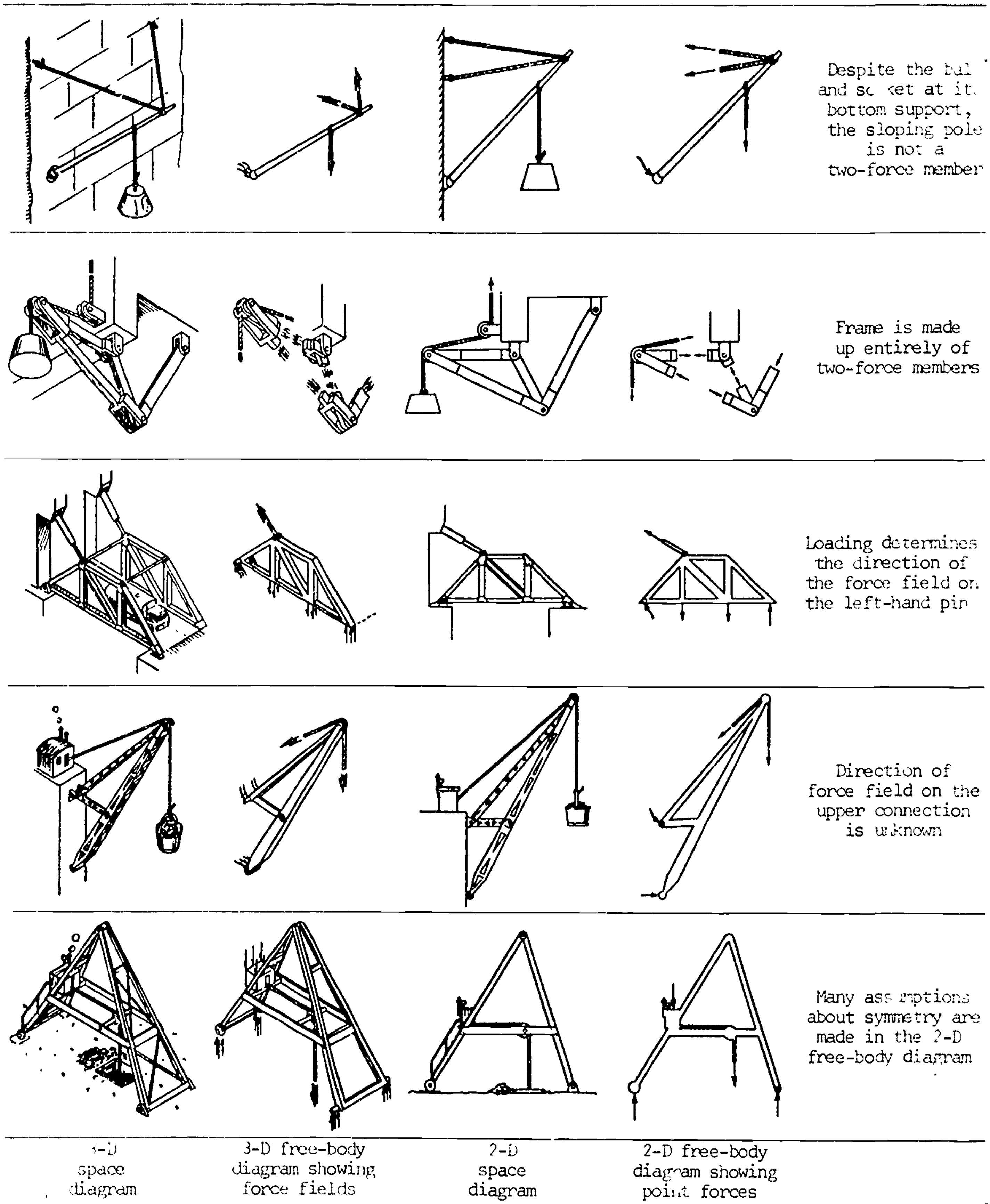
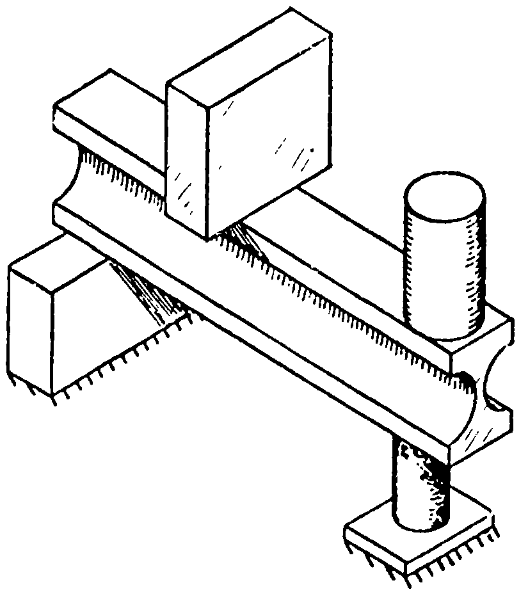
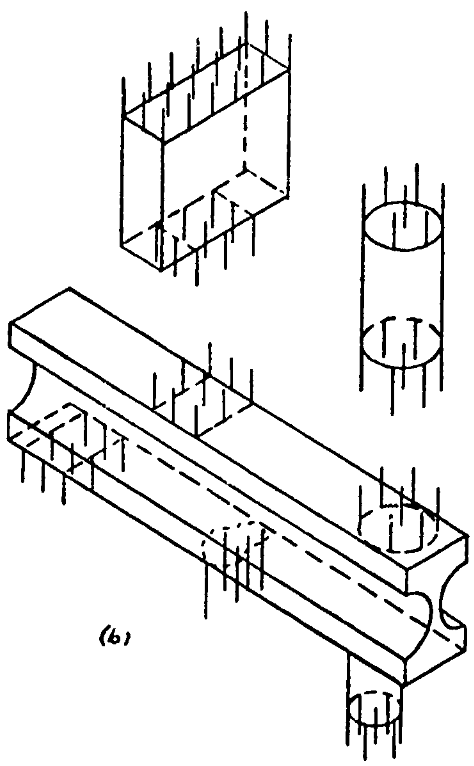


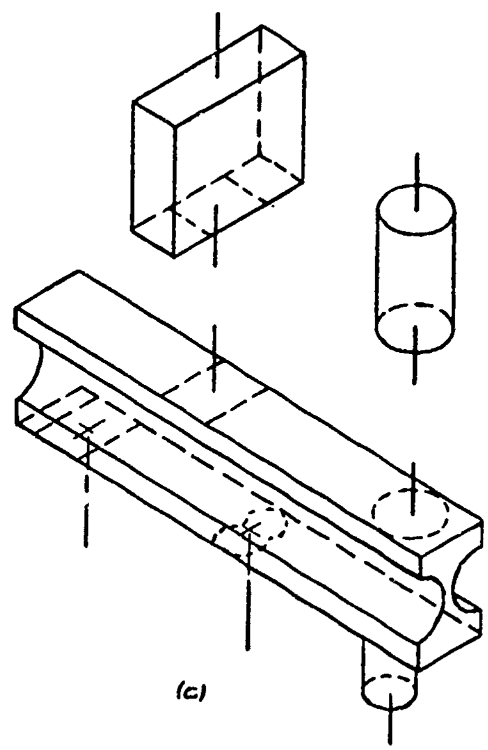
Figure EQD 12



(a)



(b)



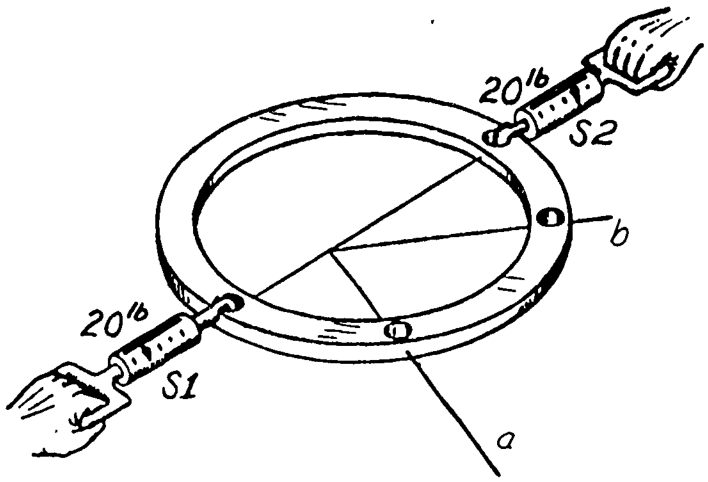
(c)

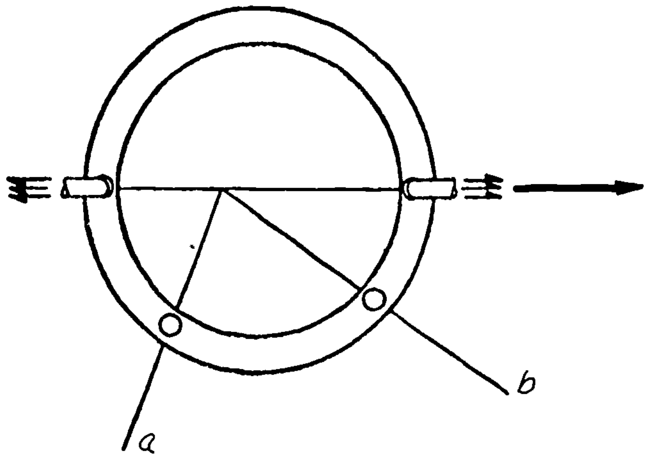
FD - 1

Complete the force fields shown in (b) above.

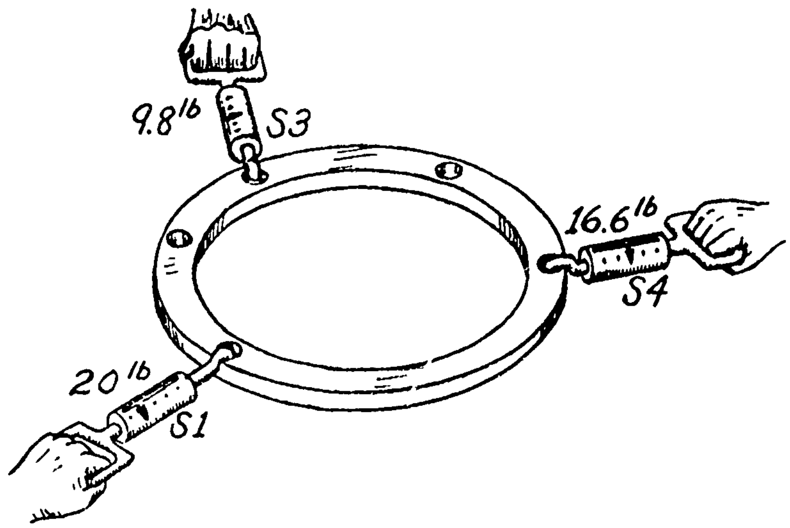
FD - 2

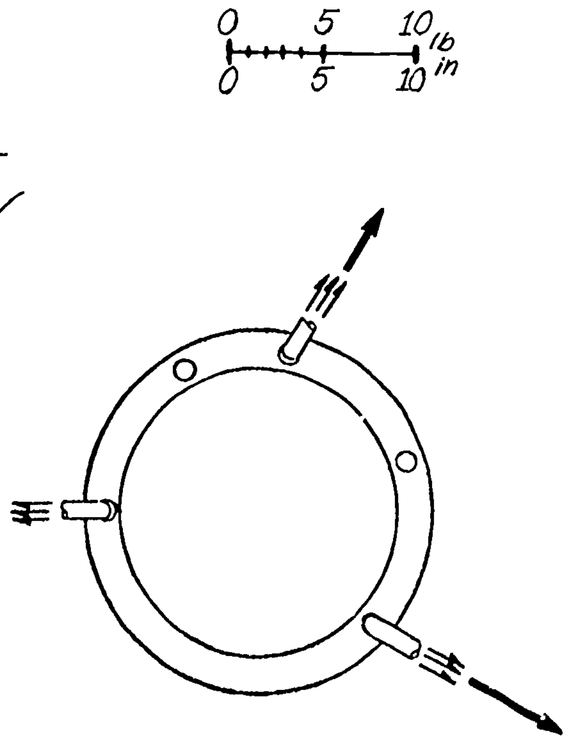
Complete the point force resultants shown in (c).



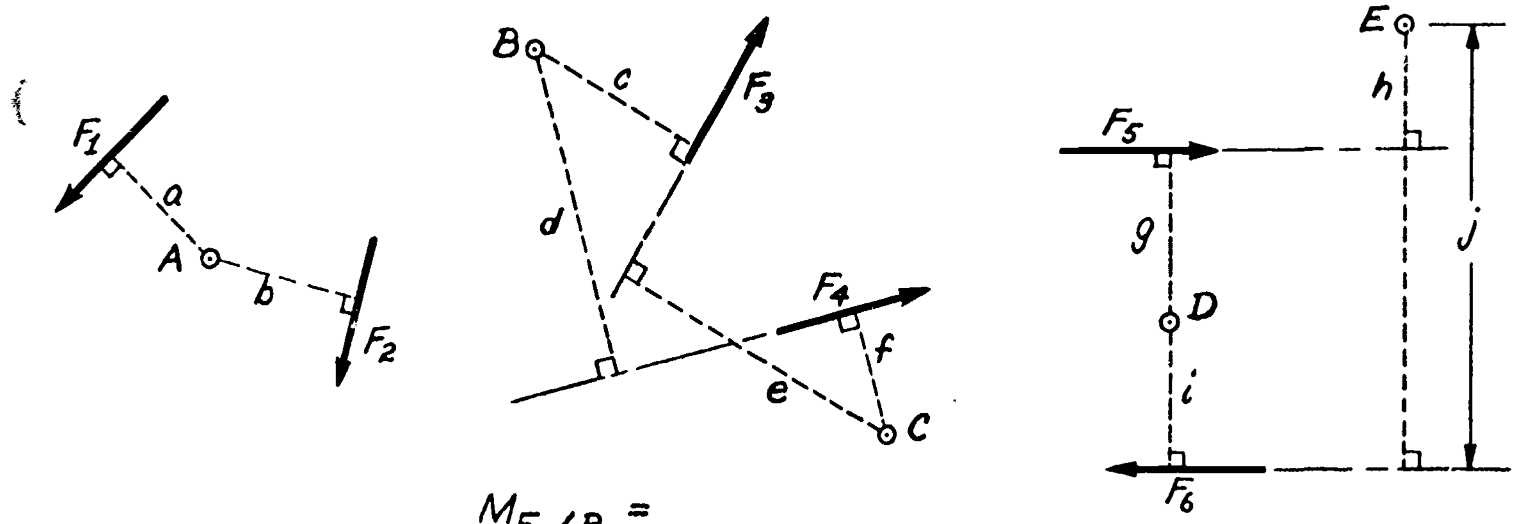


FD - 3(A) Replace S2 by point forces acting along lines a and b.





FD - 3(B) Find the point force resultant of S3 and S4.



$M_{F_1/A} =$

$M_{F_2/A} =$

$M_{F_3/B} =$

$M_{F_3/C} =$

$M_{F_4/B} =$

$M_{F_4/C} =$

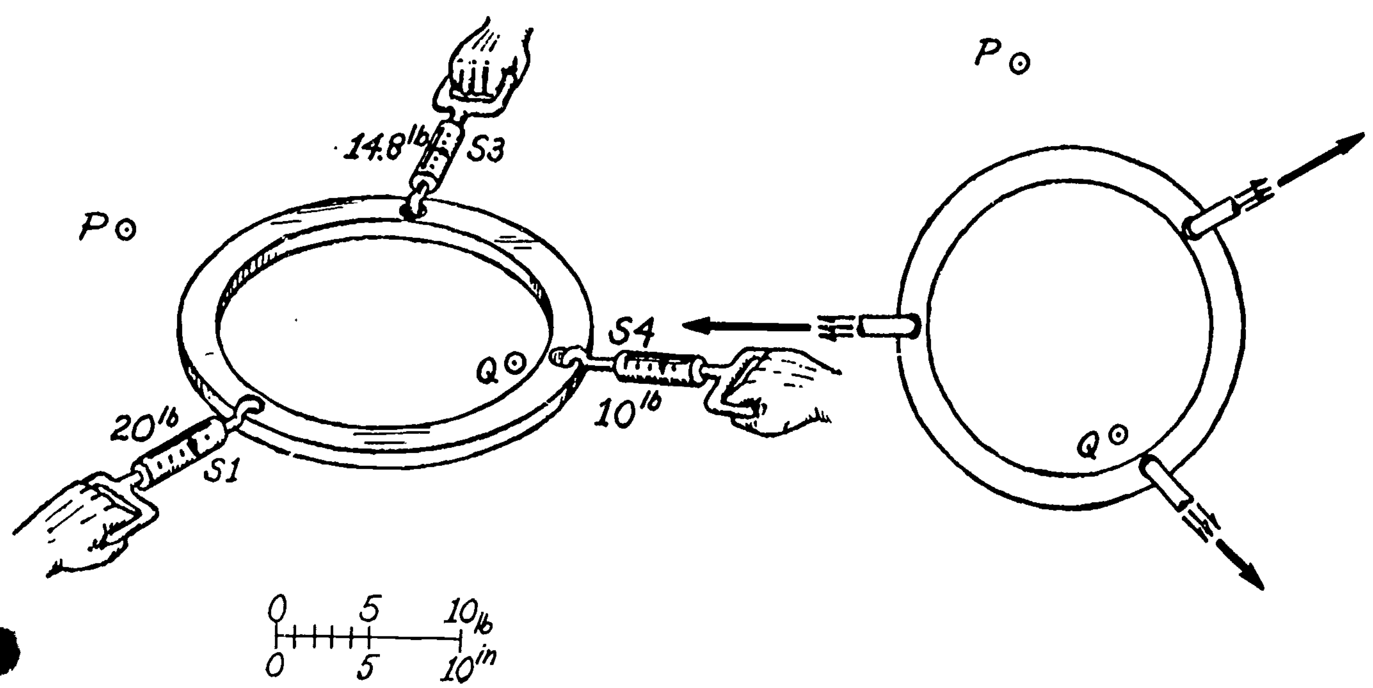
$M_{F_5/D} =$

$M_{F_5/E} =$

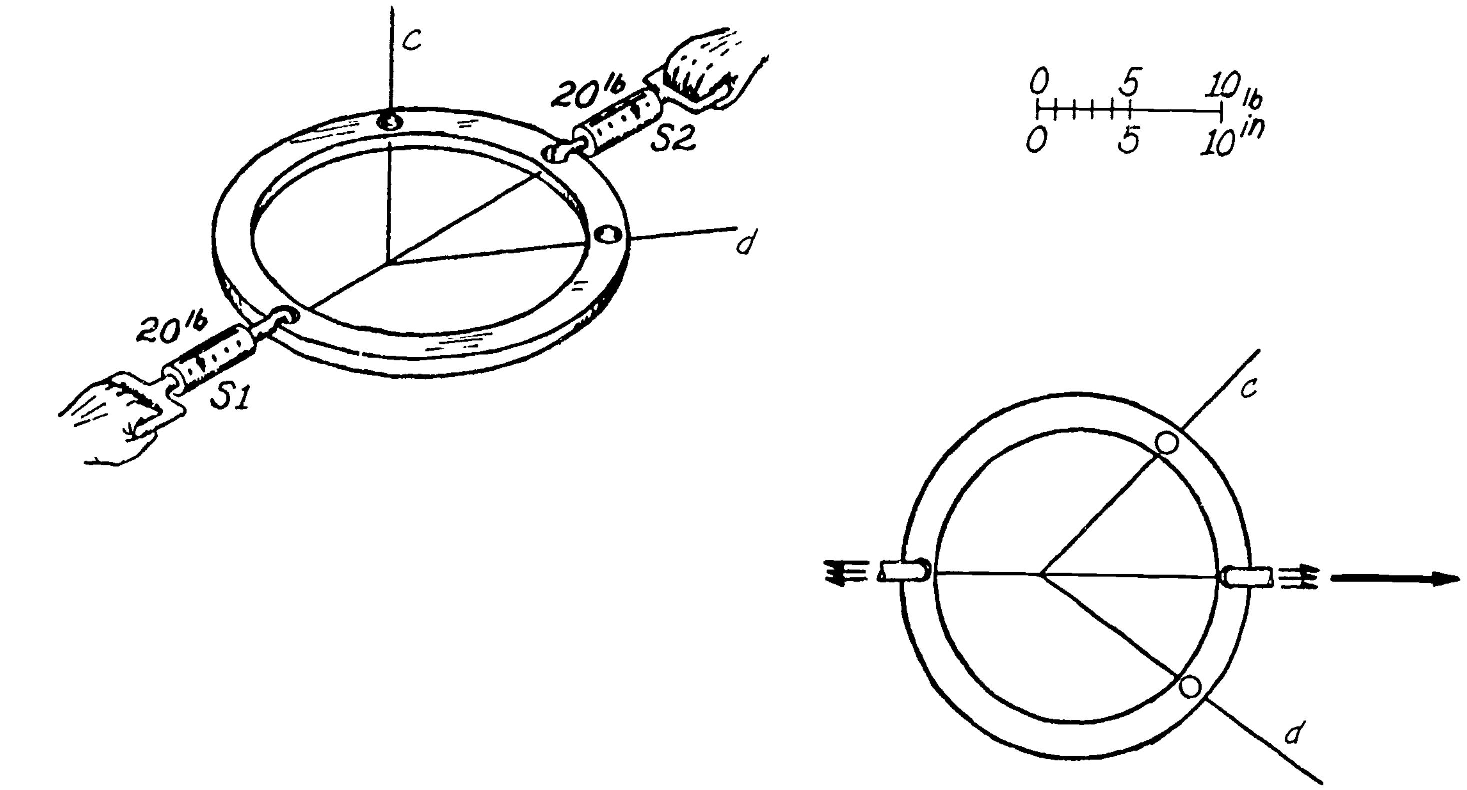
$M_{F_6/D} =$

$M_{F_6/E} =$

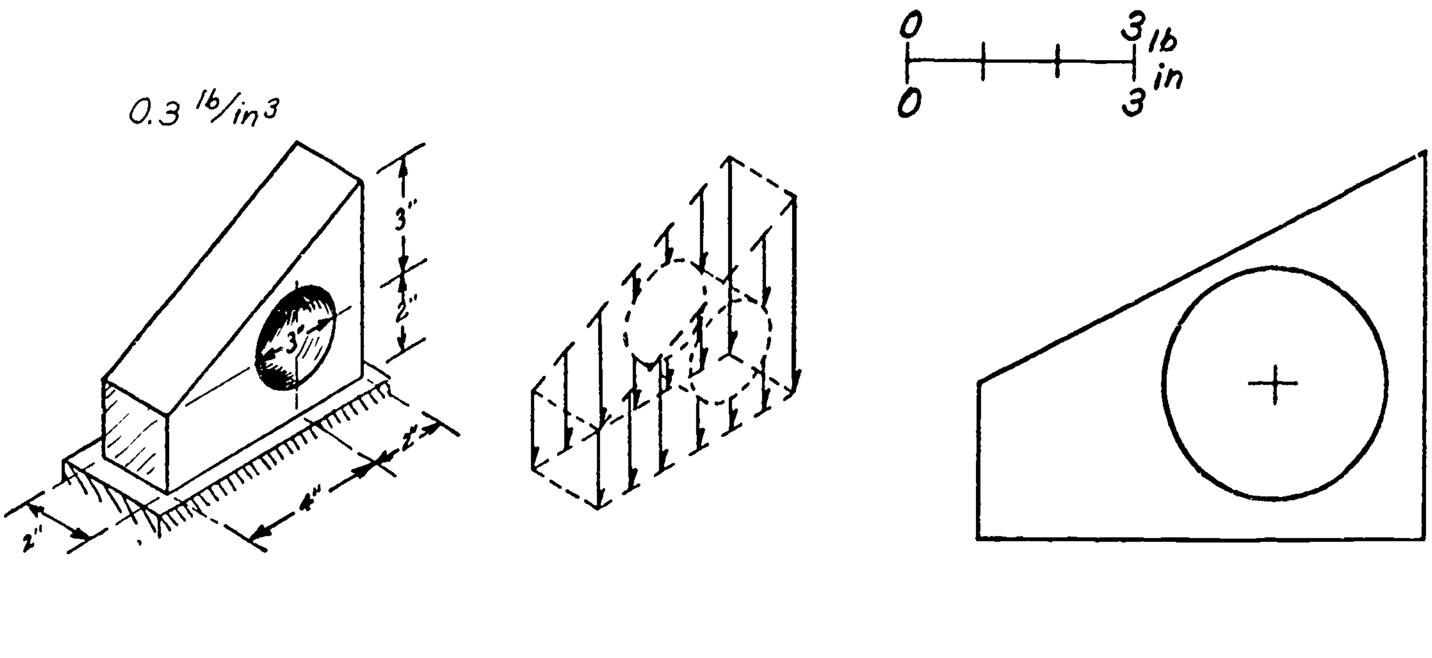
FD - 4 Finish the moment equations using the correct signs.



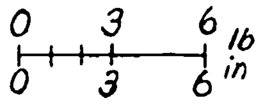
FD - 5 Find the moments of S1, S2, and S3 with respect to points P and Q.

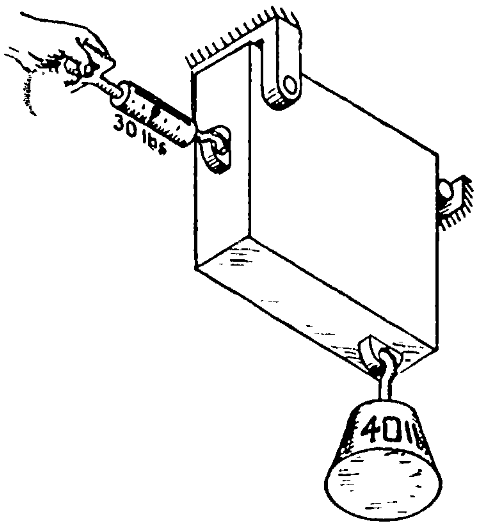


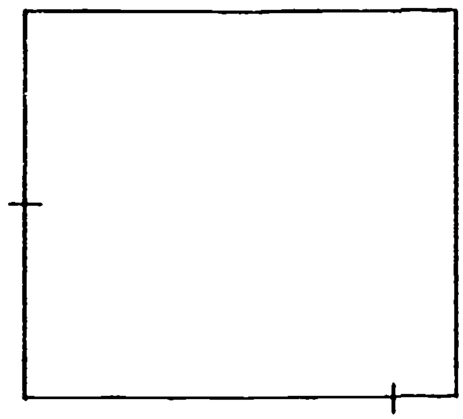
FD - 6 Replace S2 by point force components along lines c and d using moment equations.



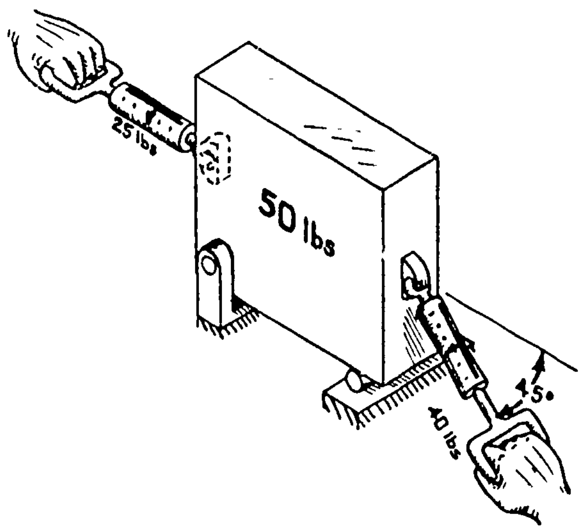
FD - 10 Find the point force resultant of the composite object.

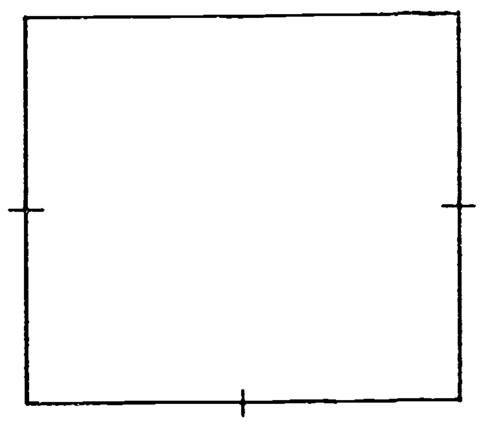




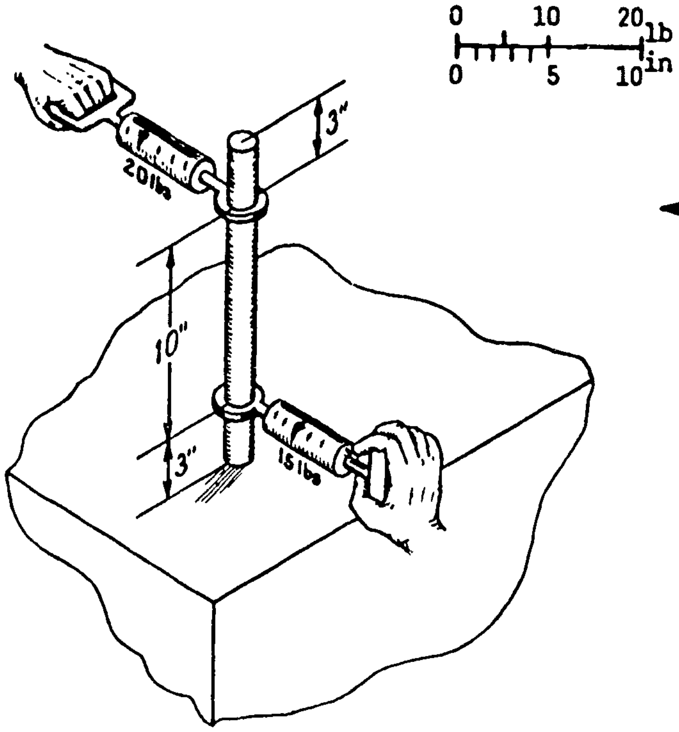


FD - 7 Find the single point force resultant of the 30 lb and 40 lb loads. Use moment equations only.

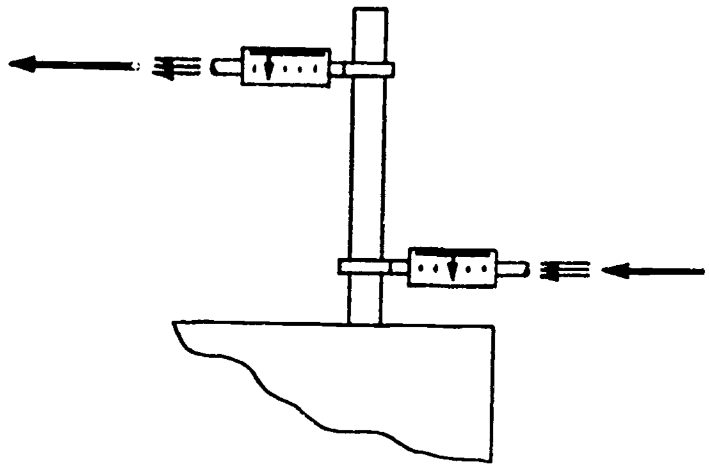




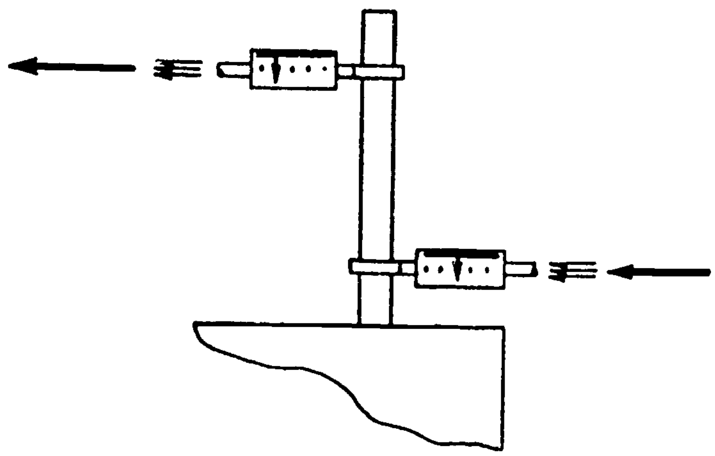
FD - 8 Find the point force resultant of the 25, 50, and 40 lb loads using a force triangle and a moment equation. Check with direct parallelogram addition.



(a)



(b)



(c)

FD - 9(A) Use the parallelogram law to replace the acting loads with a single point force in (b).

FD - 9(B) Check your answer by using a moment equation in (c).

UNIT 3

EQUILIBRIUM ANALYSIS

AT THE END OF UNIT 3 IF YOU ARE GIVEN A 3-D SPACE DIAGRAM OF A COPLANAR STRUCTURE THAT IS LOADED WITH COPLANAR LOADS AND MADE UP OF A COMBINATION OF TWO-FORCE (2-F) AND THREE-FORCE (3-F) MEMBERS AND IF THE STRUCTURE IS CONSTRAINED AT TWO PLACES SO THAT THE DIRECTIONS OF ONE CONSTRAINT IS KNOWN, YOU WILL BE ABLE TO FIND THE REACTIONS ON THE STRUCTURE AT ITS CONSTRAINTS USING F-B DIAGRAMS WITH FORCE POLYGONS AND F-B DIAGRAMS WITH MOMENT EQUATIONS.

Introduction

The equilibrium diagrams developed in Unit 2 were used to visualize in 3-D the force fields acting upon complete structures and their individual members. F-B diagrams were then constructed in 3-D and 2-D with the force fields replaced by their point force resultants. In this unit F-B diagrams will be used with the parallelogram law and moment equations to determine the lines of action, directions, senses, and magnitudes of the point force resultants of the force fields that are acting on a coplanar structure when the structure is in static equilibrium.

Two-Force Members

In unit 2 you learned what a two-force (2-F) member is. When only two loads act upon an object which is in equilibrium, the two point force resultants of the loads must balance each other. To do this, they must be colinear with the effective ℄ of the object, opposite in sense, and equal in magnitude. This is called the two force principle.

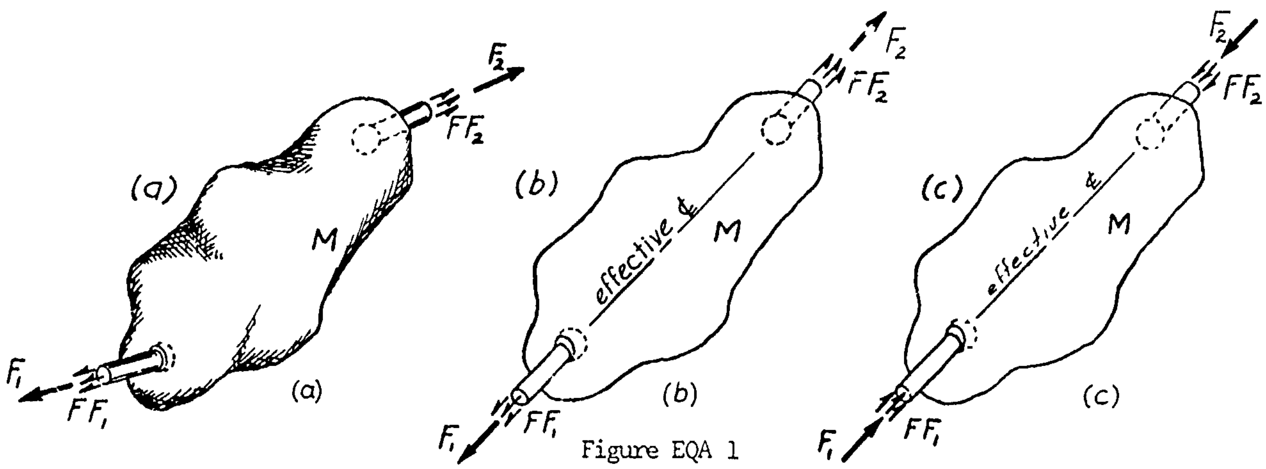


Figure EQA 1

The 2-F principle is illustrated in figure EQA 1. In (a) object M is assumed to be weightless and loaded through two frictionless ball-and socket joints. FF_1 and FF_2 represent the loads. Equilibrium of the object can be established by either pulling as shown in (b) or pushing as shown in (c). $\bar{F}_1$ and $\bar{F}_2$ must be colinear, opposite in sense, and equal in magnitude. $\bar{F}_1$ and $\bar{F}_2$ are equal and opposite vectors, so vector equations can be written for them.

$$\bar{F}_1 = -\bar{F}_2 \quad \text{or} \quad \bar{F}_1 + \bar{F}_2 = 0 \quad \text{or} \quad \Sigma \bar{F} = 0$$

Three-Force Members

Object M in figure EQA 2(a) is assumed to be weightless. It is loaded at three frictionless ball-and-socket joints A, B, and C. Loads which are represented by FF_1, FF_2, and FF_3 [illegible] gradually applied while keeping the member M in stationary equilibrium. [illegible] point force resultants of their force fields.

It can be deduced using the parallelogram law [illegible] the addition of point [illegible] when M is in equilibrium, $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ 1) must be coplanar and either [illegible] parallel, 2) add with the parallelogram law to a zero resultant, and 3) the moments of $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ with respect to any point in their plane must sum to zero.

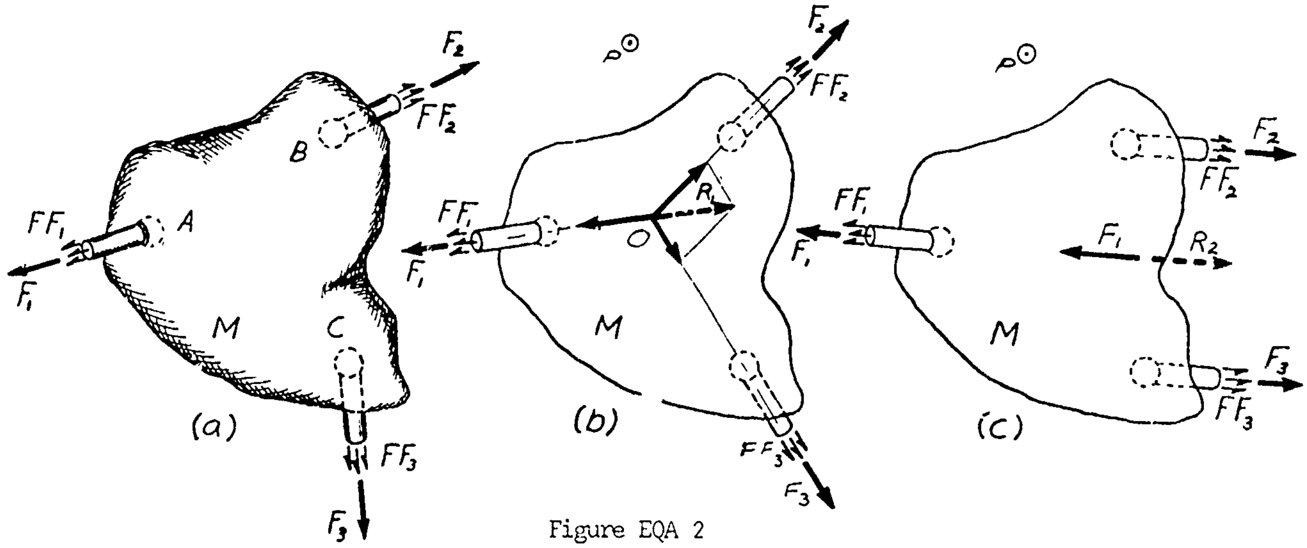


Figure EQA 2

The system is in equilibrium, so any one of the loads must balance the [illegible] two loads, that is, when the point forces replace their loads, $\overline{F}_1$ must b[illegible] $\overline{F}_2$ and $\overline{F}_3$. $\overline{F}_2$ must add with $\overline{F}_3$ using the parallelogram law to give a resultant $\overline{F}_1$ which must be by the 2-F principle, colinear with and equal, but opposite to $\overline{F}_1$. It has [illegible] been shown that $\overline{F}_2$, $\overline{F}_3$, and their resultant $\overline{R}_1$ must be coplanar and either concurrent or parallel. Since $\overline{F}_1$ and $\overline{R}_1$ must be colinear then $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ must be coplanar and either concurrent or parallel.

The results of these deductions are shown in EQA 2(b) and (c). $\bar{F}_2$ and $\bar{F}_3$ are shown as being concurrent with $\bar{F}_1$ at O in (b) or parallel to it in (c). You already know that since $\bar{F}_2$ and $\bar{F}_3$ are concurrent at O, a parallelogram can be drawn at O showing $\bar{F}_2 + \bar{F}_3 = \bar{R}_1$, as in (b). $\bar{F}_1$ must be colinear with, and equal, but opposite to $\bar{R}_1$. For the parallel case in (c), $\bar{F}_2 + \bar{F}_3$ equals $\bar{R}_2$ which is also colinear with, and equal, but opposite to $\bar{F}_1$.

For (b) $\overline{F}_2 + \overline{F}_3 = \overline{R}_1$, but $\overline{R}_1 = -\overline{F}_1$, so $\overline{F}_2 + \overline{F}_3 = -\overline{F}_1$. This becomes $\overline{F}_1 + \overline{F}_2 + \overline{F}_3 = 0$ that can be written $\Sigma \overline{F} = 0$. In (c) $\overline{F}_2 + \overline{F}_1 = \overline{R}_2$, $\overline{R}_2 = -\overline{F}_1$, so $\overline{F}_2 + \overline{F}_3 + \overline{F}_1 = 0$ or $\Sigma \overline{F} = 0$.

A random point P can be chosen in the plane of $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ as in (b) and (c). You know in (b) that $M_{F_2/P} + M_{F_3/P} = M_{R_1/P}$ and since $\overline{R}_1 = -\overline{F}_1$ then $M_{R_1/P} = -M_{F_1/P}$.

Combining these gives $M_{F_2/P} + M_{F_3/P} = -M_{F_1/P}$ or using correct signs $M_{F_1/P} + M_{F_2/P} + M_{F_3/P}$ = 0 which can be written $\Sigma M_P = 0$. Again in (c) $M_{F_1/P} + M_{F_2/P} + M_{F_3/P} = 0$ or $\Sigma M_P = 0$. So for either (b) or (c), $\Sigma \bar{F} = 0$ and $\Sigma M_P = 0$ where P can be any point in the plane of $\bar{F}_1$, $\bar{F}_2$, and $\bar{F}_3$.

A principle that applies to a member which is loaded at only three places can now be stated. When an object is in equilibrium under the action of three loads, the three point force resultants of the loads must (1) be coplanar, (2) be concurrent or parallel, (3) add with the parallelogram law to a zero resultant ($\Sigma \bar{F} = 0$), and (4) the summation of the moments of the three point forces with respect to any point in their plane must equal zero ($\Sigma M_{any\ point} = 0$). This is called the three-force (3-F) principle. This principle will now be applied to find the reactions on some simple structures.

Three-Force Member Reactions

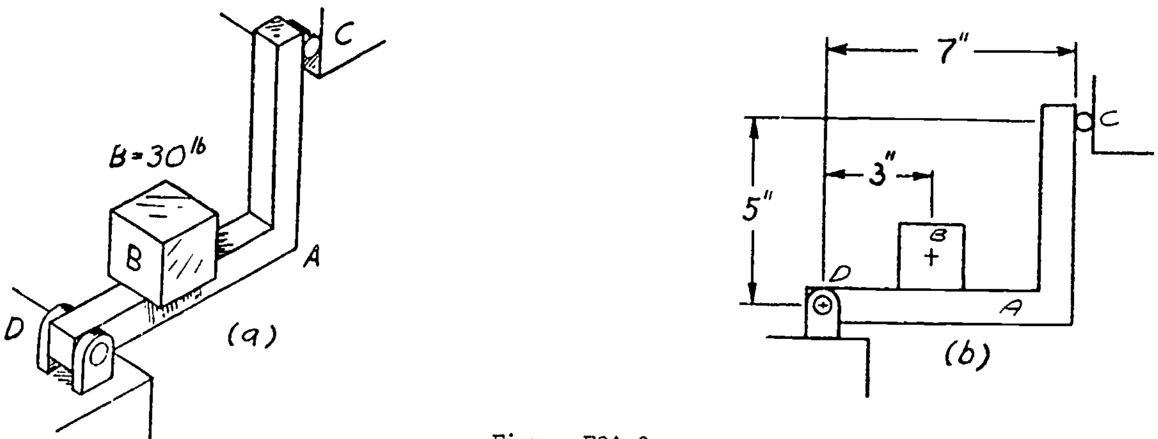
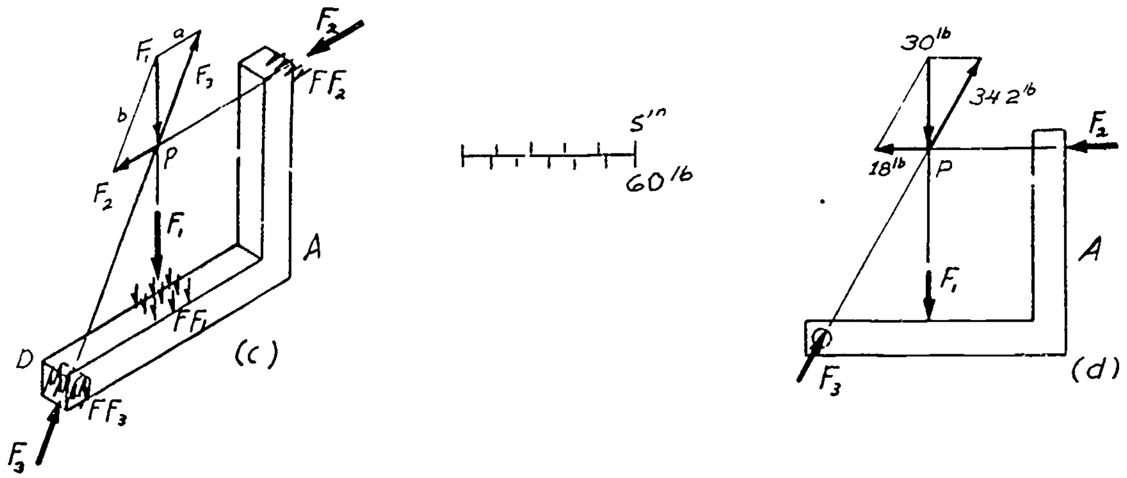


Figure EQA 3

Member A shown in figure EQA 3(a) and (b) is symmetrically loaded by weight B, constrained by a frictionless roller C and a frictionless pin D, and is assumed to be weightless. Since member A is loaded at three places, it is called a three-force (3-F) member. The acting load is weight B. The loads acting against A at C and D are called reacting loads or reactions, and the point force resultants of these loads are also usually referred to as the reactions on member A. The point forces representing the reactions on member A at C and D will be found using the 3-F principle.

Remember that if member A is symmetrically loaded by weight B that the vertical ℄'s of A and B must be coplanar.



First a 3-D diagram is drawn to scale of A in (c). This 3-D diagram of member A is drawn with pin D, roller C, and weight B removed. FF_1 and FF_2 can be placed in the diagram. Next $\overline{F}_1$ and $\overline{F}_2$ are superimposed upon the diagram. Each point force replaces its force field and of course acts through the center of its force field. Now $\overline{F}_1$ and $\overline{F}_2$ intersect at point P. $\overline{F}_3$ must therefore act through the ¢ of hole D and point P, so $\overline{F}_3$ can be placed in the diagram. FF_3 has the same direction as $\overline{F}_3$ and can also be placed on the diagram. Diagram (c) is now a 3-D F-B diagram of member A.

The directions, senses, and lines of action of $\overline{F}_2$ and $\overline{F}_3$ are now known, their magnitudes will now be found using parallelogram addition. $\overline{F}_1$ is laid out to scale at point P. The action lines of $\overline{F}_2$ and $\overline{F}_3$ are extended through point P. Line a is drawn from the tail of $\overline{F}_1$ parallel to $\overline{F}_2$, line b is drawn from the tail of $\overline{F}_1$ parallel to $\overline{F}_3$. Next arrowheads are placed on $\overline{F}_2$ and $\overline{F}_3$ at point P. A parallelogram has now been constructed at point P in (c) where $\overline{F}_1$ is equal and opposite to the single point force resultant of $\overline{F}_2$ plus $\overline{F}_3$. That is, the parallelogram at P shows $\overline{F}_2 + \overline{F}_3 = \overline{R}_1$ (not shown) $= -\overline{F}_1$ or $\overline{F}_1 + \overline{F}_3 + \overline{F}_2 = 0$: $\Sigma\overline{F}$.

Although the parallelogram in (c) is drawn to scale, it is difficult to measure the magnitudes of the point forces in an isometric view. For this reason 2-D diagram (d) is drawn. $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ are coplanar in (d). Now the same procedure in (d) as in (c) will give the directions, senses, lines of action, and magnitudes of $\overline{F}_2$ and $\overline{F}_3$. 2-D diagram (d) is drawn to scale. $\overline{F}_1$ and $\overline{F}_2$ are placed on their correct lines of applications, and extended to meet at P, $\overline{F}_3$ is drawn through the ¢ of hole D and P. This completes the 2-D F-B diagram. Now a parallelogram is constructed to scale at P with $\overline{F}_1$ as its reversed diagonal and $\overline{F}_2$ and $\overline{F}_3$ as its sides. $\overline{F}_2$ and $\overline{F}_3$ are measured giving $F_2 = 18$ lb and $F_3 = 34.2$ lb. $\overline{F}_2$ and $\overline{F}_3$ are now known, they are the point force resultants of the force fields acting against member A at C and D.

The magnitudes of $\overline{F}_2$ and $\overline{F}_3$ can also be found using 2-D F-B diagrams drawn to scale and moment equations. The 2-D F-B diagram in figure EQA 4(a) is drawn to scale and will be used to find the magnitude of $\overline{F}_2$.

In (a) $\overline{F}_1$ and $\overline{F}_2$ are placed in the diagram and as before intersect at P. $\overline{F}_3$ can then be drawn on the diagram as it acts through the ¢ of D and point P.

$M_{F_1} + M_{F_2} + M_{F_3}$ must equal zero with respect to any point. Point D (the ¢ of the pin) will be used as a moment center to find F_2.

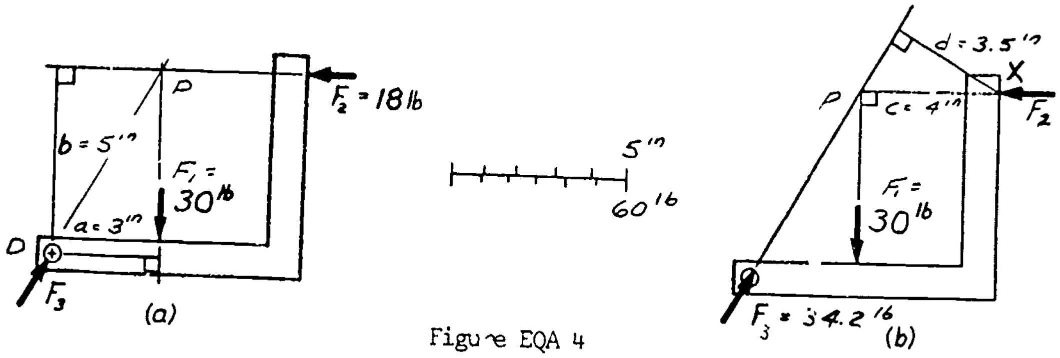


Figure EQA 4

Line a is drawn from the ℄ of D perpendicular to the line of action of $\overline{F}_1$, line b is drawn from the ℄ of D perpendicular to the line of action of $\overline{F}_2$. The lengths of a and b are 3 inches and 5 inches as shown in figure EQA 3(b).

Now $\quad M_{F_3/D} + M_{F_1/D} + M_{F_2/D} = 0 \qquad$ or $\Sigma M_D = 0$

$$(0)\ (F_3)\ -\ (a)\ (F_1)\ +\ (b)\ (F_2) = 0$$

$$(0)\ (F_3)\ -\ (3)\ (30)\ +\ (5)\ (F_2) = 0 \qquad F_2 = \frac{(3)(30)}{(5)} = 18\ \text{lb}$$

F-B diagram (b) will be used to find the magnitude of $\overline{F}_3$. As before $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ intersect at P. Next a point X is chosen as the line of action of $\overline{F}_2$. Perpendicular line c can be drawn from X to the line of action of $\overline{F}_1$ and its length scaled. Next line d is drawn from X perpendicular to the line of action of $\overline{F}_3$. Line d is scaled as 3.5 inches.

$\Sigma M_X = 0 \quad$ or $M_{F_2/X} + M_{F_1/X} + M_{F_3/X} = 0$

$$(0)\ (F_2) + (4)\ (30)\ -\ (3.5)\ (F_3) = 0 \qquad F_3 = \frac{(4)(30)}{(3.5)} = 34.2\ \text{lb}$$

These answers check with those found by the direct parallelogram law addition.

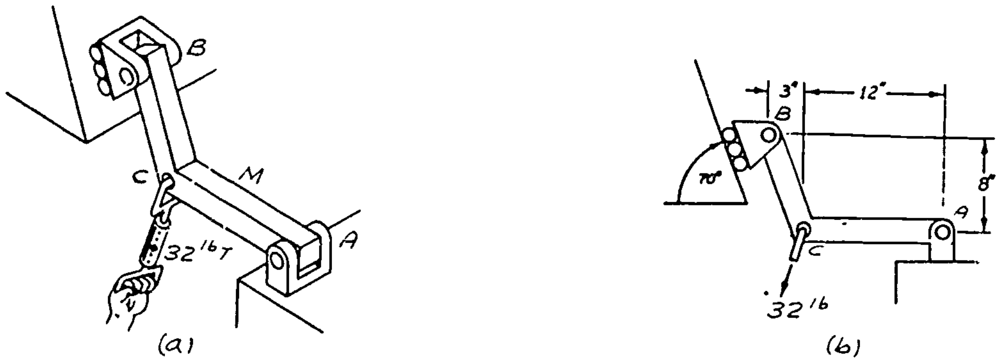


Figure EQA 5

The 3-F principle will now be used to find the reactions at A and B for 3-F member shown in figure EQA 5(a) and (b). The system is coplanar, that is the ℄'s of M, B, A, and T are coplanar. Member M is weightless, and the connections of B, C, and A are frictionless.

It is not necessary to drawn a 3-D F-B diagram as the system is coplanar, however it is always necessary to visualize the 3-D F-B diagram.

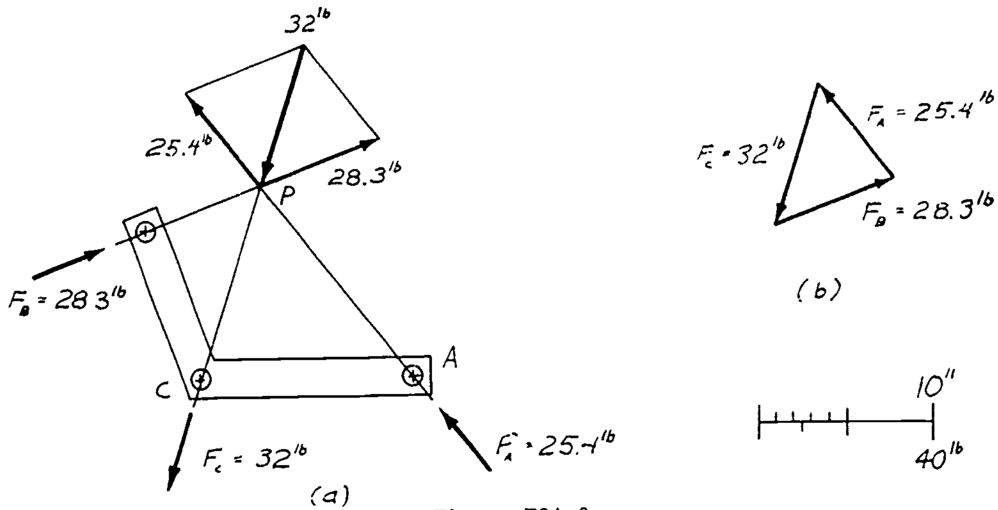


Figure EQA 6

A 2-D F-B is drawn to scale in figure EQA 6(a) with $\bar{F}_A$ and $\bar{F}_B$ placed on their correct lines of action. The action line of $\bar{F}_A$ must pass through points A (the pin ℄) and P (the intersection point of $\bar{F}_B$ and $\bar{F}_C$). A force parallelogram can be drawn at P with $\bar{F}_A + \bar{F}_B$ balancing the known $\bar{F}_C$. Scaling the force parallelogram gives F_A = 25.4 lb and F_B = 28.3 lb.

Instead of constructing a parallelogram at the point of concurrency, it is also possible to find the magnitudes of $\bar{F}_A$, $\bar{F}_B$, and $\bar{F}_C$ with the device shown in (b), called a closed force polygon. To construct such a polygon for these forces, a line representing the known point force $\bar{F}_C$ is first laid out to scale anywhere on the figure but parallel to $\bar{F}_C$'s line of action. From the head of $\bar{F}_C$ a line is constructed parallel to $\bar{F}_B$ and from the tail of $\bar{F}_C$ another line is drawn parallel to $\bar{F}_A$. The intersection point of these two lines determines the head of $\bar{F}_B$ and the tail of $\bar{F}_A$. $\bar{F}_B$ and $\bar{F}_A$ can be directly measured on the polygon. Comparison of figures (a) and (b) indicates that the closed force polygon method is actually one half of the force parallelogram, and that the results found with the two methods will be identical. However the closed force polygon method is more convenient since the polygon requires fewer lines and need not be constructed at the point of concurrency of the forces.

To find F_A and F_B using moment equations, another F-B diagram is drawn to scale in figure EQA 7. Again the action lines of $\bar{F}_B$, $\bar{F}_C$, and $\bar{F}_A$ are concurrent at point P. To find F_B, construct and measure lever arms a and b and take moments about the ℄ of pin A.

a = 11.4 inches and b = 12.9 inches

$$\Sigma M_A = 0$$

$$+(11.4)(32) - (12.9)(F_B) = 0$$

$$F_B = 28.3 \text{ lb}$$

To find F_A, construct and measure lever arms c and d and take moments about the ℄ of pin B.

c = 5.4 inches and d = 6.8 inches

$$\Sigma M_B = 0$$

$$-(5.4)(32) + (6.8)(F_A) = 0$$

$$F_A = 25.4 \text{ lb}$$

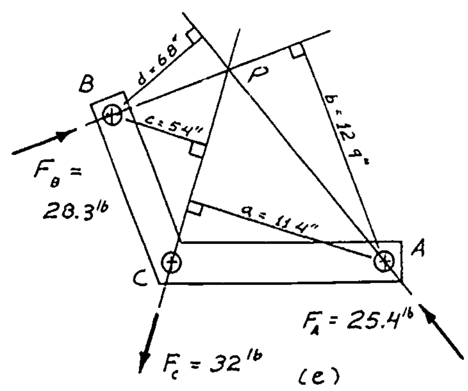


Figure EQA 7

GIVEN A 3-F MEMBER LOADED AT ONE PLACE AND CONSTRAINED AT TWO PLACES ONE OF WHICH HAS A REACTION OF KNOWN DIRECTION, YOU SHOULD BE ABLE TO DRAW THE NECESSARY F-B DIAGRAMS AND FIND THE REACTIONS AT THE CONSTRAINTS USING A FORCE POLYGON OR MOMENT EQUATIONS. EQA - 1

Reactions on Structures Containing 2-F and 3-F Members

The coplanar structure shown in figure EQA 8(a) and (b) consists of two 2-F members loaded at their concurrent point. The loads on members AB and BC are to be found.

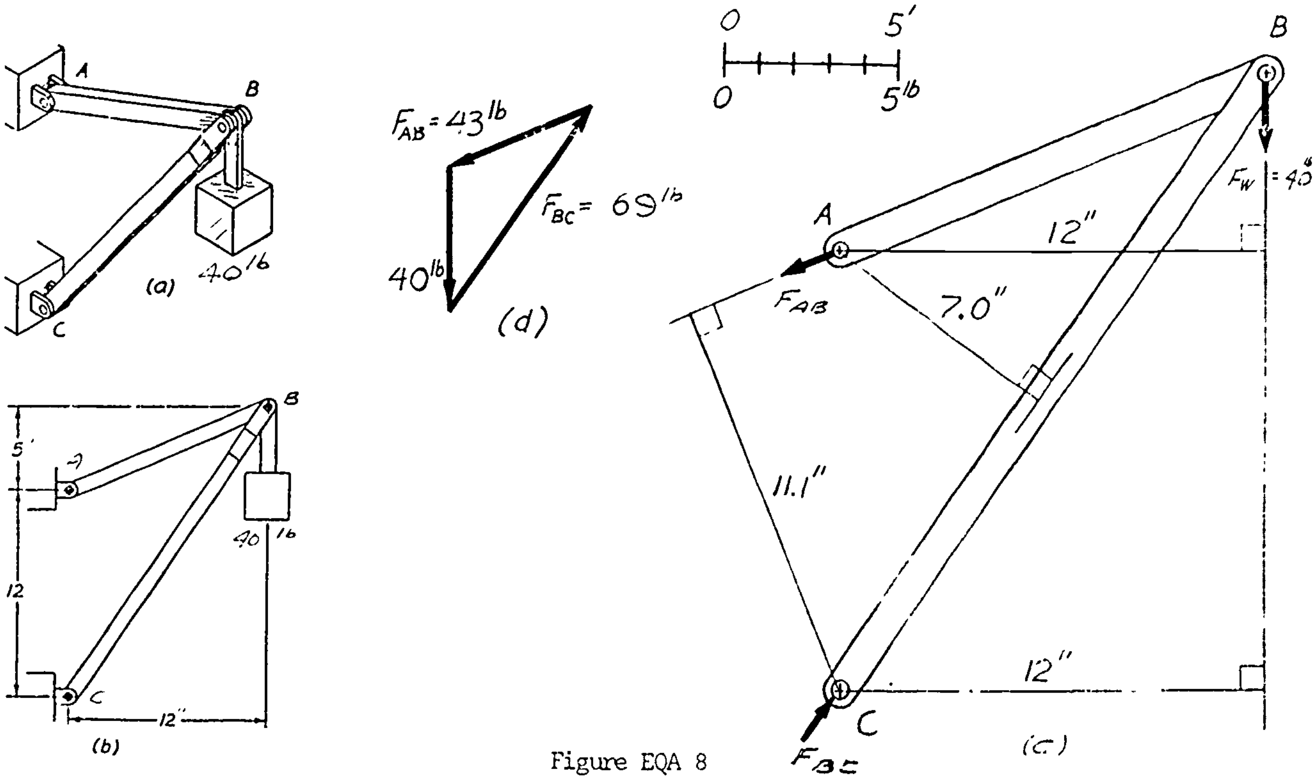


Figure EQA 8

A 2-D F-B diagram of the structure is drawn to a larger scale in (c). $\overline{F}_{AB}$ and $\overline{F}_{BC}$ act along the effective ¢'s of their members as shown. At B the pin is left in the structure, the load W and its vertical supporting shank are removed, and $\overline{F}_W$ is shown acting downward through the pin. The closed force polygon in (d) can be constructed to scale, and the magnitudes of forces $\overline{F}_{AB}$ and $\overline{F}_{BC}$ can be measured. F_{AB} = 43 lb and F_{BC} = 69 lb.

Also in (c) moment equations can be used to check the values found for $\overline{F}_{AB}$ and $\overline{F}_{BC}$. The action lines of all the point forces are known, so perpendicular lever arms can be drawn and measured, and the equations can be completed.

To find F_{AB} $\quad \Sigma M_C = 0$

$$+(11.1)(F_{AB}) - (12)(40) = 0$$
$$F_{AB} = 43.2 \text{ lb}$$

To find F_{BC} $\quad \Sigma M_A = 0$

$$-(12)(40) + (7.0)(F_{BC}) = 0$$
$$F_{BC} = 68.6 \text{ lb}$$

The answers check with those found by parallelogram addition. $\overline{F}_{AB}$ and F_{BC} are of course the point force resultants of force fields that act on any transverse section of AB and BC.

The structure shown in figure EQA 9(a) and (b) is used to support a 50 lb weight. The reactions at B and D are wanted. The members are assumed to be weightless, coplanar, and joined with frictionless pins.

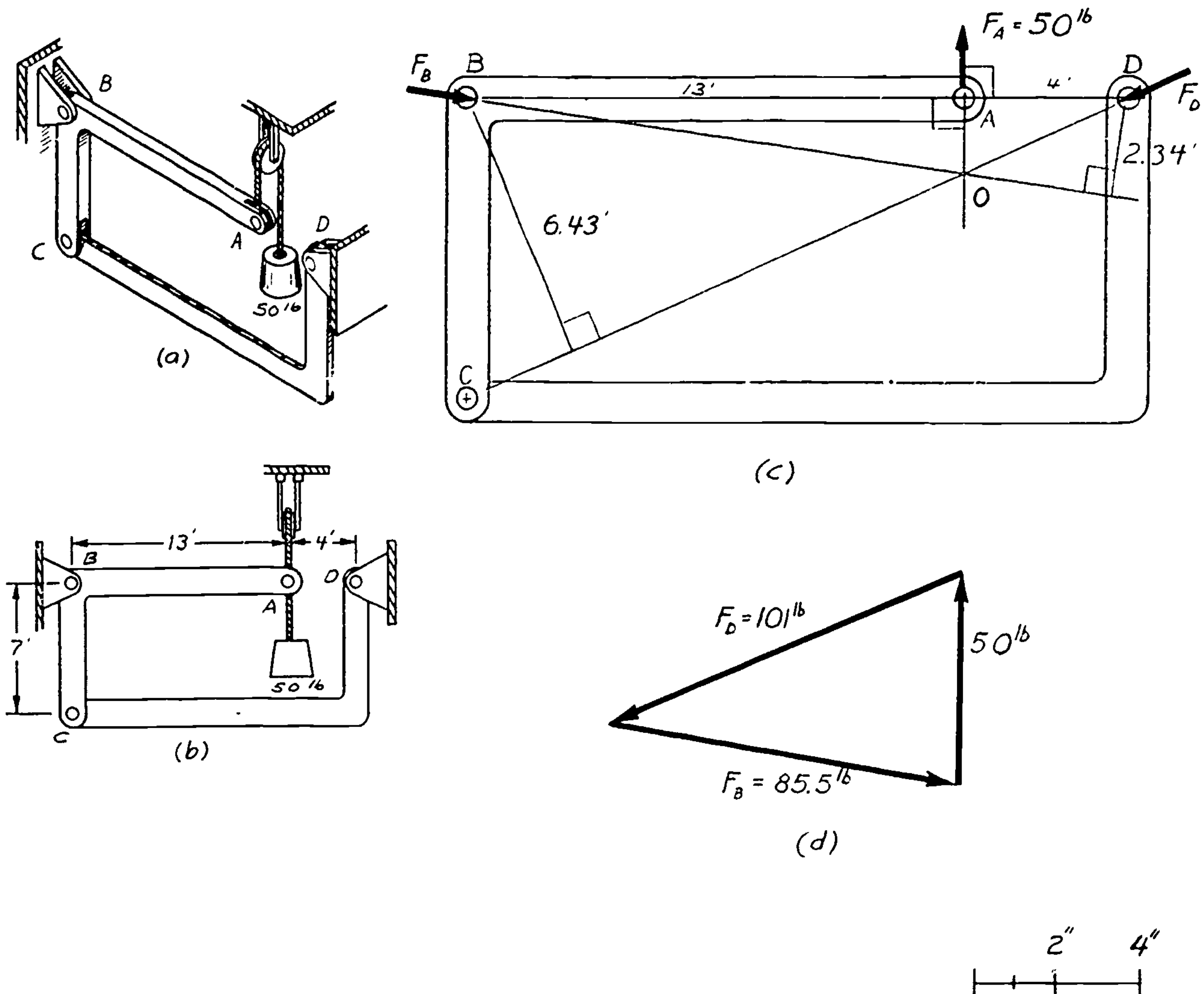


Figure EQA 9

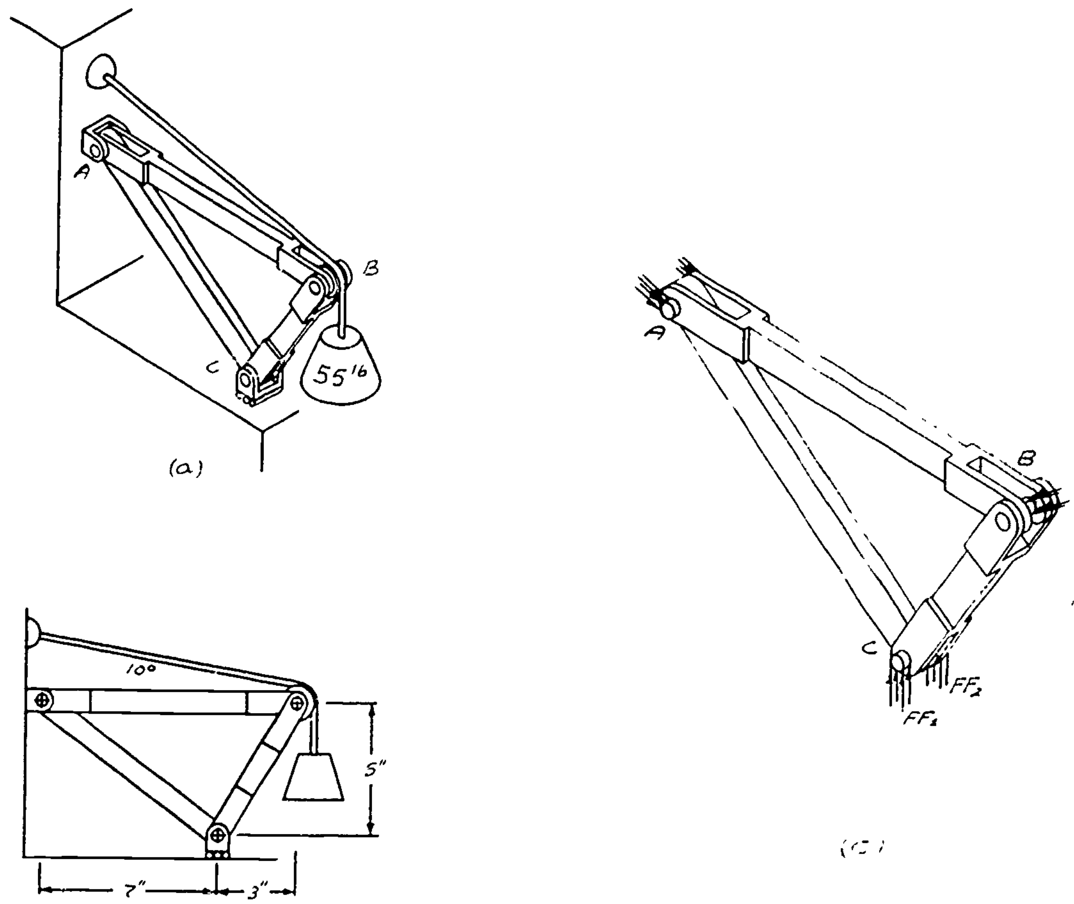
The F-B diagram shown in (c) is drawn to a larger scale. $\bar{F}_A$ is known, so it can be placed on this F-B diagram. Since member CD is a 2-F member, the line of action of $\bar{F}_D$ can be placed on the F-B diagram, and because $\bar{F}_B$ must be concurrent with $\bar{F}_A$ and $\bar{F}_D$ at point 0, its line of action is also known. The polygon drawn in (d) shows the magnitudes and senses of $\bar{F}_B$ and $\bar{F}_D$.

$$F_B = 85.5 \text{ lb} \quad \text{and} \quad F_D = 101 \text{ lb}$$

The same F-B diagram in (c) can be used to find lever arms to use in moment equations:

$\Sigma M_B = 0$	$\Sigma M_D = 0$
$-(6.43)(F_D) + (13)(50) = 0$	$(2.34)(F_B) - (4)(50) = 0$
$F_D = 101$ lb	$F_B = 85.5$ lb

Frame ABC supports an 55 lb load through a pulley at B. The system is considered weightless with frictionless pins. The point force resultants of the reactions at A and C and the load on member AC are to be found.



(b)

Figure EQA 10

A 3-D diagram of member ABC is drawn to scale in (c). The pins at A, B, and C are removed in this diagram but are left in the frame. The brackets are removed at A and C and the pulley is removed at B.

Notice in (c) that if the pin at C is removed, the reaction at C would consist of one force field pushing against member AC in one direction ↖ and two force fields pushing against BC in another direction ↗. Now if the pin is left in the structure at C, two parallel force fields FF_1 and FF_2 act vertically upward against the pin, since the bracket at C is supported by horizontal rollers. These two force fields FF_1 and FF_2 are called the reactions of the bracket against the structure at constraint C. The pins then are left in the structure when finding reactions whenever more than one member is attached to a pin at a constraint. The directions of the force fields acting against the pins at A and B cannot be determined by inspection, the directions of their point force resultants will be found using 2-D F-B diagrams and then 3-D F-B diagram (c) will be completed.

$\Sigma M_A = 0$

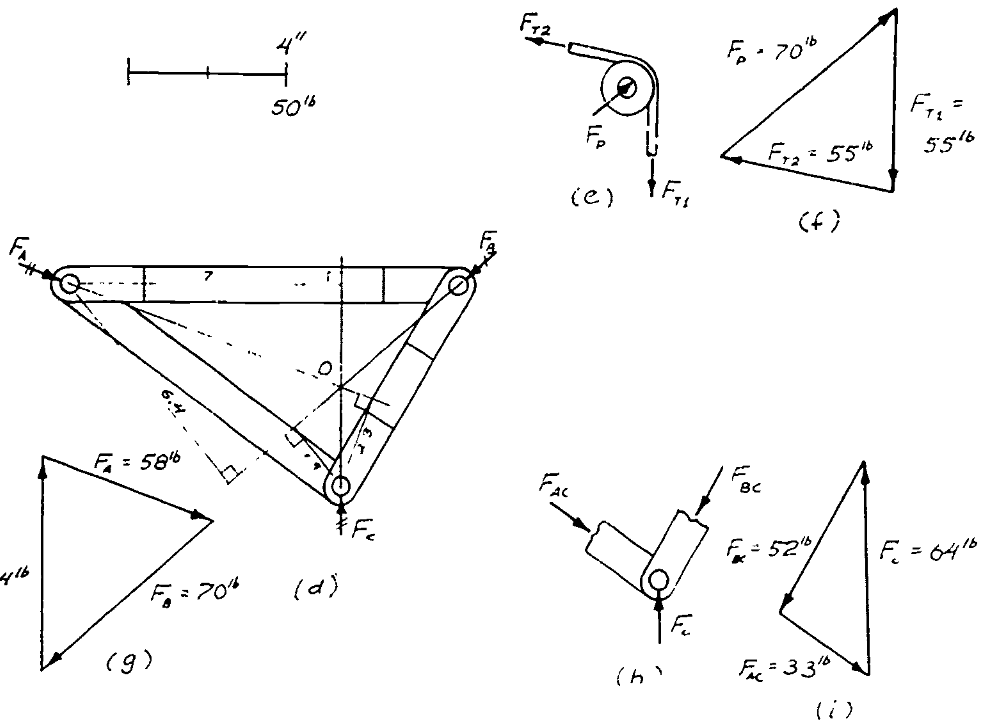
$7(F_c) = 64(70)$

$F_c = 64^{lb}$

$\Sigma M_c = 0$

$2.3(F_A) = 1.9(70)$

$F_A = 58^{lb}$



2-D F-B diagram (d) is now drawn to scale. $\overline{F}_C$, which replaces [illegible] (c), is shown in (d) acting vertically upward against the pin. $\overline{F}_C$ is double slashed to show that it actually represents two separate force fields.

Next F-B (e) of the pulley is drawn to scale. $\overline{F}_{T1}$ and $\overline{F}_{T2}$ are equal and are placed in (e) on their correct lines of action. Force polygon (f) can now be drawn of $\overline{F}_{T2}$, $\overline{F}_{T1}$, and $\overline{F}_P$. $\overline{F}_B$ in (d) is equal and opposite to $\overline{F}_P$ in (e).

F-B (d) can now be completed. $\overline{F}_B$ and $\overline{F}_C$ intersect at O, $\overline{F}_A$ can be placed in its correct line of action, force polygon (g) can be drawn, so $\overline{F}_C$ and $\overline{F}_A$ can be measured in (g). $\overline{F}_A$ is double slashed to show that it represents two force fields.

Using the directions of the point forces in (d) for the directions of their force fields in (c), diagram (c) can now be completed. 3-D diagram (c) is usually not drawn but it must be visualized from (a) to really understand what the point forces in the 2-D F-B diagram (d) actually represent.

2-D F-B diagram (h) is drawn to scale to find the point force resultant of the load acting on member AC. Members AC and BC are 2-F members so their point forces act along their effective ℄'s. $\overline{F}_{AC}$ and $\overline{F}_{BC}$ are placed on their correct lines of action in (h). Force polygon (i) are now be drawn and the magnitude of $\overline{F}_{AC}$ can be measured giving F_{AC} = 33 lb.

As soon as the directions of the point forces are established in (d) and (h), moment equations could be used in place of the force polygon. The magnitudes of $\overline{F}_A$ and $\overline{F}_B$ are found in the figure using moment equations with measured lever arms.

NOW IF YOU ARE GIVEN A COPLANAR STRUCTURE THAT IS SUPPORTING A COPLANAR LOAD AND IS COMPOSED OF 2-F AND 3-F MEMBERS AND CONSTRAINED AT TWO PLACES ONE OF WHICH HAS A KNOWN DIRECTION, YOU SHOULD BE ABLE TO FIND THE STRUCTURE'S REACTIONS OR THE LOADS ON ANY 2-F MEMBER OF THE STRUCTURE USING FORCE POLYGONS OR MOMENT EQUATIONS WITH THE NECESSARY FREE-BODY DIAGRAMS.

Combinations of 3-F Members -- Combined Diagrams

Figure EQA 11(a) is an isometric drawing to scale of a large pipe A being supported by two smaller pipes B and C. Each pipe is assumed to be of uniform weight per foot. It is also assumed that each pipe is rigid so that the line contacts between A, B, and C and the supporting structure are uniform. The point force resultants of the reactions of B and C on A and of the supporting walls on B and C are to be found, first by using moment equations and then with force polygons.

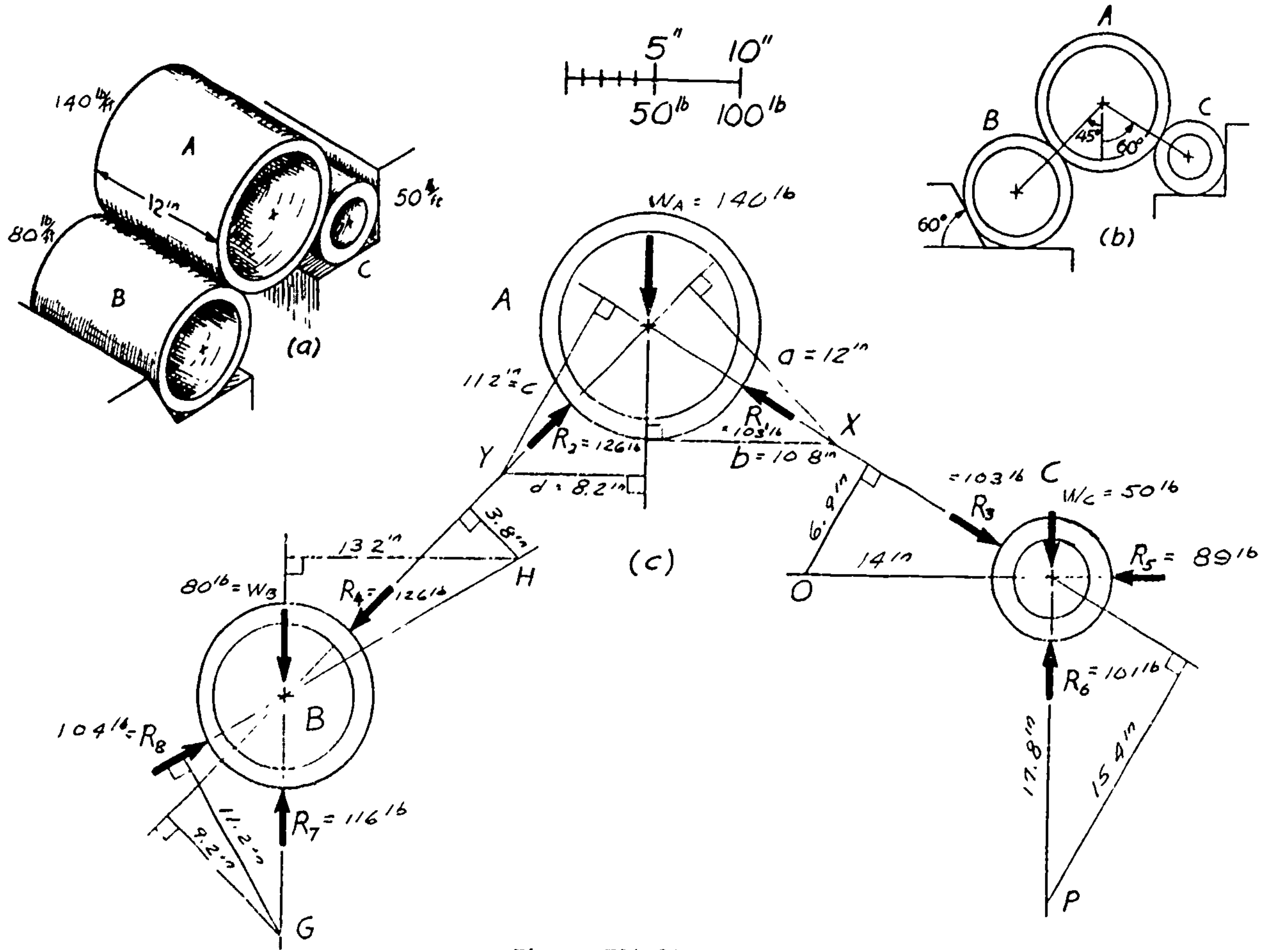


Figure EQA 11

End views of the pipes are drawn to scale in 2-D in (b). Since the pipes are of uniform weight per foot and all have the same length, the point force resultants of all the loads acting upon A, B, and C are coplanar. In (c) F-B diagrams of each pipe are drawn to scale in 2-D.

Pipe A has three point forces on its 2-D F-B diagram, $\bar{W}_A$ is the weight of A, $\bar{R}_2$ is caused by B on A and acts perpendicular to the contact surface of B and A, and $\bar{R}_1$ which is caused by C pushing against A is perpendicular to the contact surface of A and C.

Pipe B has four point forces on its 2-D F-B diagram. $\bar{W}_B$ is the weight of B, $\bar{R}_4$ is equal and opposite to $\bar{R}_2$, and $\bar{R}_7$ and $\bar{R}_8$ are caused by the left support acting against B.

The forces acting on the F-B diagram of pipe C are its weight $\bar{W}_C$, $\bar{R}_3$ equal and opposite to $\bar{R}_1$, and $\bar{R}_5$ and $\bar{R}_6$ caused by the right support pushing against it.

Now moment equations will be used to find the magnitudes of $\bar{R}_1$ and $\bar{R}_2$ on A, $\bar{R}_7$ and $\bar{R}_8$ on B, and $\bar{R}_5$ and $\bar{R}_6$ on C.

To find R_2 on A, a point X is chosen on the line of action of $\bar{R}_1$ as shown in the F-B diagram of pipe A. Perpendicular lever arms a and b are drawn and measured.

a = 12 inches and b = 10.8 inches

$\Sigma M_X = 0 \qquad (0)(R_1) - (12)(R_2) + (10.8)(140) = 0 \qquad R_2 = 126 \text{ lb}$

To find R_1 a point Y is chosen on the line of action of R_2. Lever arms c and d are drawn and measured.

c = 11.2 inches and d = 8.2 inches

$\Sigma M_Y = 0 \qquad (0)(R_2) + (11.2)(R_1) - (8.2)(140) = 0 \qquad R_1 = 103 \text{ lb}$

The F-B diagram of B can now be solved. R_4 = 126 lb and W_b = 80 lb, so the two unknowns R_7 and R_8 can be found.

$\Sigma M_G = 0 \qquad (0)(R_7) + (0)(80) - (11.2)(R_8) + (9.2)(126) = 0 \qquad R_8 = 104 \text{ lb}$

$\Sigma M_H = 0 \qquad -(13.2)(R_7) + (13.2)(80) + (3.8)(126) = 0 \qquad R_7 = 116 \text{ lb}$

Now the F-B diagram of C can be used to solve for R_5 and R_6.

$\Sigma M_O = 0 \qquad (14.0)(R_6) - (14.0)(50) - (6.9)(103) = 0 \qquad R_6 = 101 \text{ lb}$

$\Sigma M_P = 0 \qquad (17.8)(R_5) - (15.4)(103) = 0 \qquad R_5 = 89 \text{ lb}$

To find the reactions on each pipe with force polygons, the F-B diagram of A is redrawn in figure EQA 12(a) and its force polygon is drawn in (b). $\bar{R}_1$ and $\bar{R}_2$ are then scaled. The F-B diagram of pipe B is redrawn in (c) and will be used to find $\bar{R}_7$ and $\bar{R}_8$. Since $\bar{W}_B$ and $\bar{R}_4$ are known ($\bar{R}_4$ is equal and opposite to $\bar{R}_2$ found in (b)), these two point forces can be added to give $\bar{R}_9$. A four-sided polygon can be used to find $\bar{R}_7$ and $\bar{R}_8$ directly. To do this, $\bar{W}_B$ is first laid out to scale, then $\bar{R}_4$ is laid out from its arrowhead. The polygon can then be closed with $\bar{R}_7$ and $\bar{R}_8$, just as you would close a three-sided polygon.

Now a F-B diagram of C can be used to find $\bar{R}_5$ and $\bar{R}_6$ using a four-sided force polygon as shown in (f). $\bar{R}_3$ is first laid out to scale, then $\bar{W}_C$ is drawn to scale from its tip. $\bar{R}_5$ and $\bar{R}_6$ close the polygon and can be measured.

When drawing four-sided polygons, the forces are usually added in clockwise order around the concurrent point, always starting with the known forces.

Notice that $\bar{R}_2$ and $\bar{R}_4$ in *(b)* and *(d)* are equal and opposite. The two force polygons for A and B can be superimposed upon each other as in the top half of (g). That is, force polygon $\bar{W}_A$, $\bar{R}_1$, and $\bar{R}_2$ from A can be drawn as before. Then $\bar{W}_B$ and $\bar{R}_4$ can be laid out with $\bar{R}_4$ superimposed on $\bar{R}_2$ and pointing in the opposite direction. $\bar{R}_7$ and $\bar{R}_8$ then complete the force polygon for B. In the same manner force polygon $\bar{R}_3$, $\bar{W}_C$, $\bar{R}_5$, and $\bar{R}_6$ can be superimposed on the figure. This is called a combined force diagram.

1 in = 100 lb
1 in = 10 in

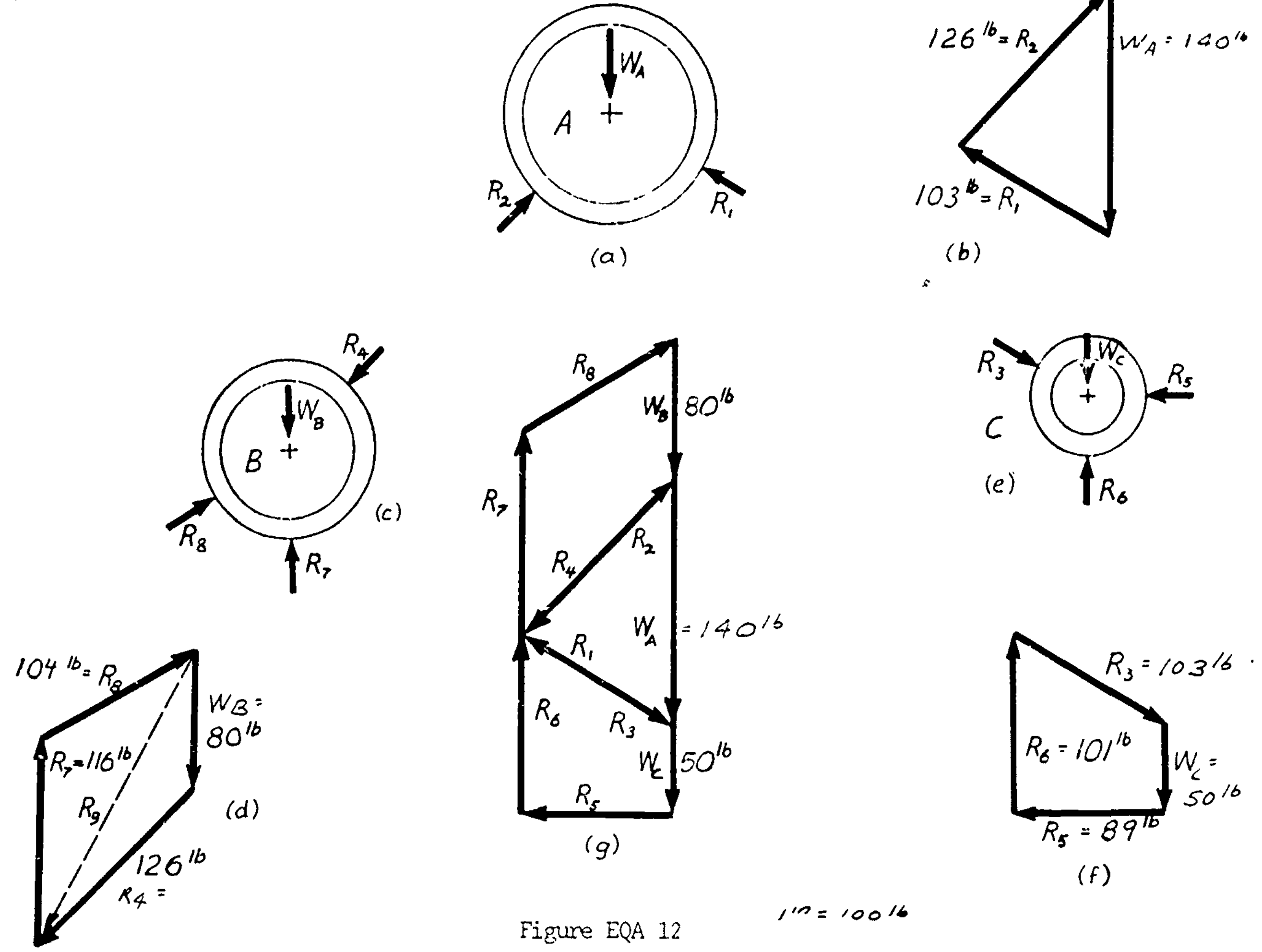
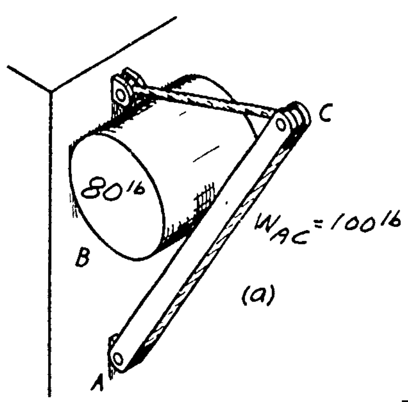


Figure EQA 12

Also notice that A is a 3-F member since it has one acting load and two reaction loads. Although B has two acting loads and two reaction loads, the two acting loads $\bar{W}_3$ and $\bar{R}_4$ can be combined into one acting load. The same applies to pipe C. Pipes B and C are then also called 3-F members. A 3-F member may be acted upon by any number of coplanar loads, as long as it has only two reactions.

As another example, the reactions on the cylinder B and the member AC in figure EQA 13(a) and (b) will be found using a single combined diagram and then checked using moment equations.



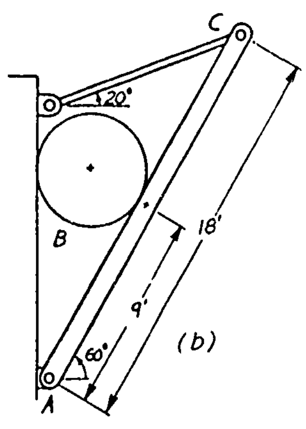


Figure EQA 13

2-D F-B diagrams of cylinder B and member AC are drawn to scale in figure EQA 14(a) and (b). The outside reactions on B, $\bar{R}_1$ and $\bar{R}_2$, are found using the triangular force polygon shown in (c). $\bar{W}_{AC}$ and $\bar{R}_3$ are then added to give $\bar{R}_5$ as shown in (b). The line of action of $\bar{R}_A$ is now known since $\bar{R}_5$, $\bar{T}$, and $\bar{R}_A$ are concurrent at P. Now the force polygon in (c) can be completed with $\bar{W}_{AC}$, $\bar{R}_A$, and $\bar{T}$ and the magnitudes measured.

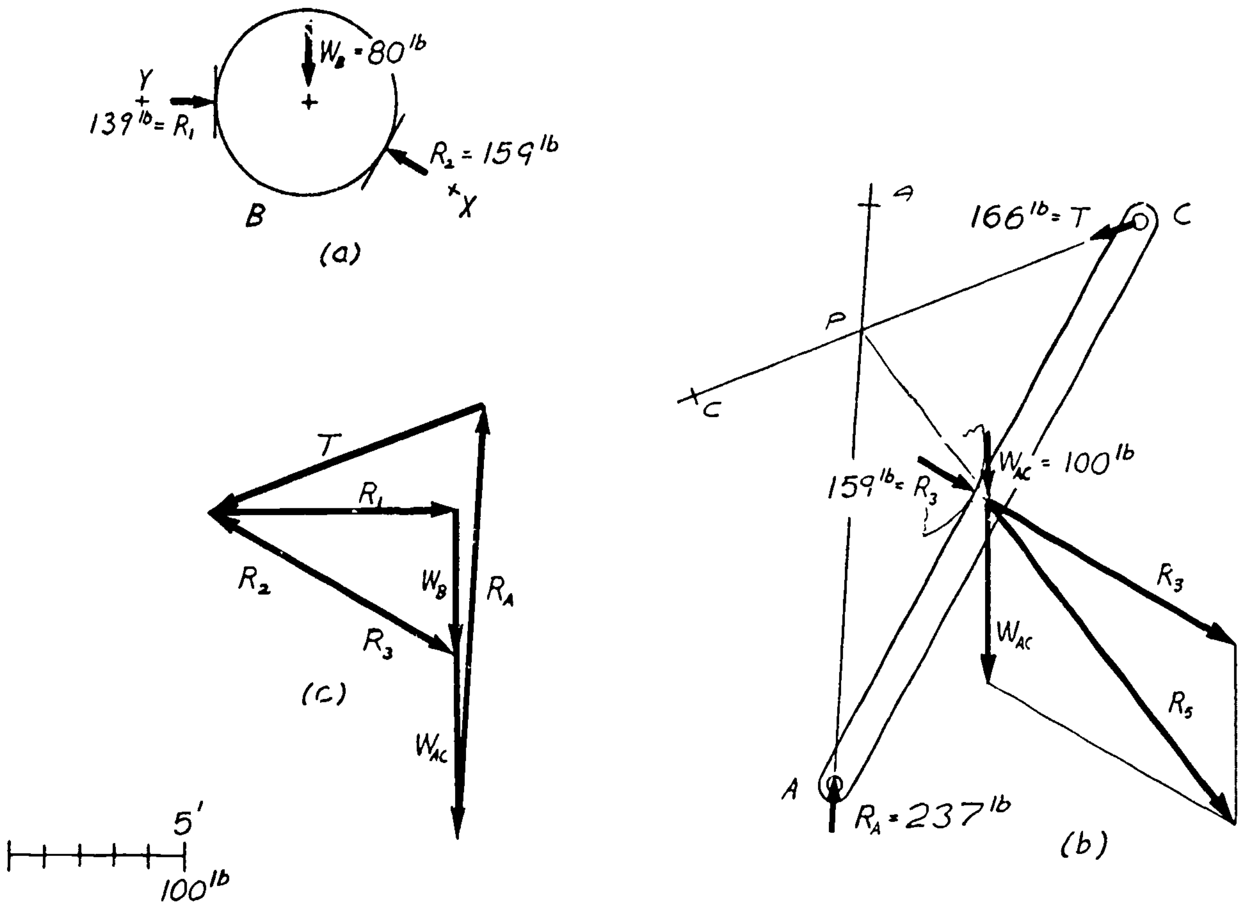


Figure EQA 14

Note that the unknowns in F-B (a) can be found by summing moments about points X and Y and the unknowns in FB (b) can be found by taking moments about points A and C.

GIVEN A COMBINATION OF 3-F MEMBERS, YOU SHOULD NOW BE ABLE TO FIND THEIR REACTIONS USING 2-D F-B DIAGRAMS WITH COMBINED FORCE POLYGONS OR MOMENT EQUATIONS.

CSC

UNIT 4

COMPONENTS - SUPERPOSITION - CANTILEVER BEAMS

WHEN YOU HAVE COMPLETED THIS UNIT, YOU WILL BE ABLE TO: (1) FIND THE REACTIONS ON A STRUCTURE MADE UP OF a) A 2-F AND A 3-F MEMBER OR b) TWO 2-F MEMBERS USING MUTUALLY PERPENDICULAR POINT FORCE COMPONENTS, (2) FIND THE REACTIONS ON A STRUCTURE MADE UP OF TWO 3-F MEMBERS USING THE METHOD OF SUPERPOSITION, AND (3) FIND THE REACTION ON ANY VERTICAL SECTION OF A CANTILEVER BEAM WHICH IS SUPPORTING A TRANSVERSE LOAD.

Introduction

In Units 2 and 3 you learned how to construct F-B diagrams and find the unknown reaction on coplanar structures that are in equilibrium using graphical and moment equation forms of the parallelogram law. In this unit the technique of drawing F-B diagrams and using the parallelogram law to find reactions on coplanar structures will be expanded to include the use of mutually perpendicular (orthogonal) components of the forces in arithmetic force and moment equations. Another basic tool called superposition will be developed and used, and the direct parallelogram force equations and moment equations will be used to find the reactions on the rigid connection of a cantilever beam.

Use of Mutually perpendicular Components for Finding Reactions

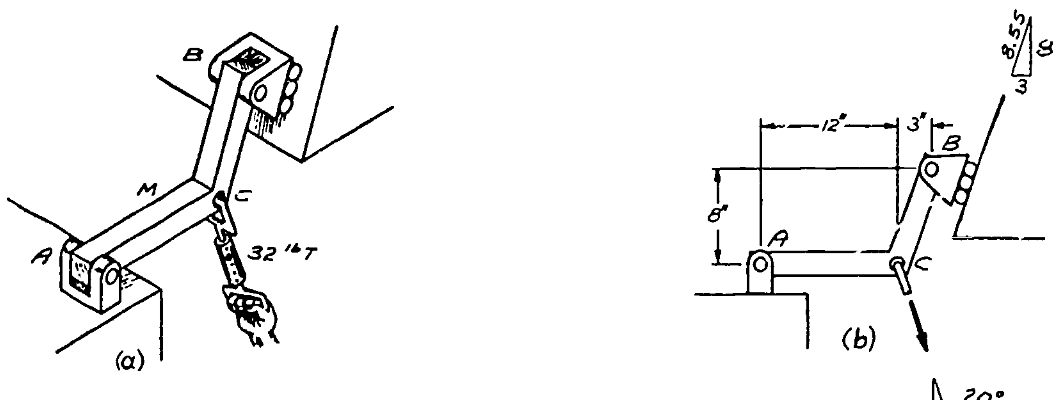


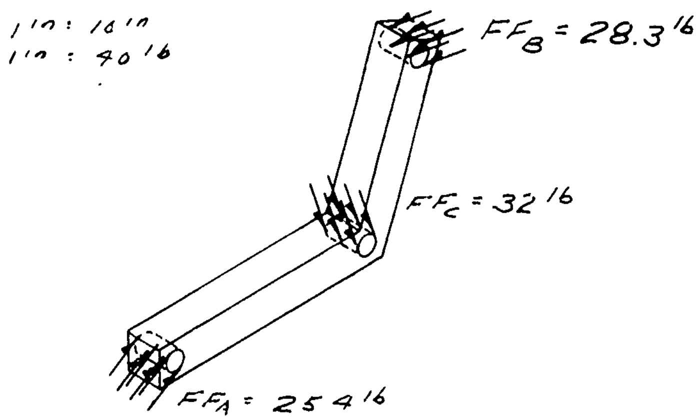
Figure CSC 1

A weightless member M is loaded with a coplanar 32 pound pull and is supported by a roller at B and a frictionless pin at A. The reactions at A and B are to be found. These reactions can be found only with the three force principle as developed in this presentation. The three force principle will be used to find the reactions using five different methods:

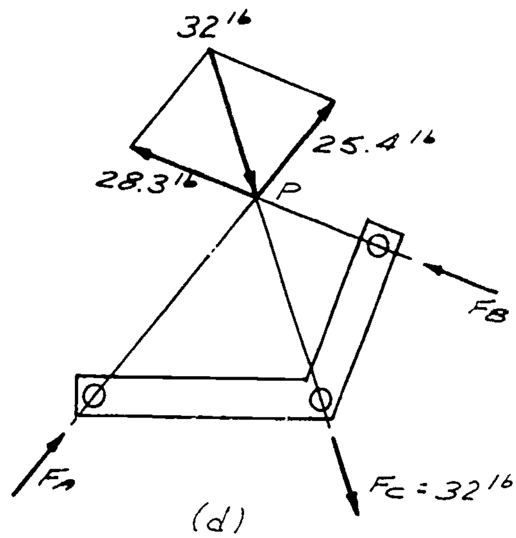
1. By direct summation of the point forces.
2. By direct summation of the moments of the point forces.
3. By summation of mutually perpendicular components of the point forces.
4. By summation of the moments of mutually perpendicular components of the point forces.
5. By a combination of 3 and 4 summation of mutually perpendicular components and summation of the moments of mutually perpendicular components.

CSC

Direct Summation of the Point Forces



(c)



(d)

The first method has already been developed. The free body diagram (d) is drawn to scale with $\overline{F}_C$ and $\overline{F}_B$ as shown. $\overline{F}_C$ and $\overline{F}_B$ intersect at point P so $\overline{F}_A$ can be placed on the free-body diagram. $\overline{F}_C$ is known, the force parallelogram can be constructed to scale and the magnitudes of $\overline{F}_A$ and $\overline{F}_B$ can be measured. The three dimensional fre... (c) can now be drawn.

Direct Summation of the Moments of the Point Forces

Measuring $a = 11.4''$ $b = 12.9''$

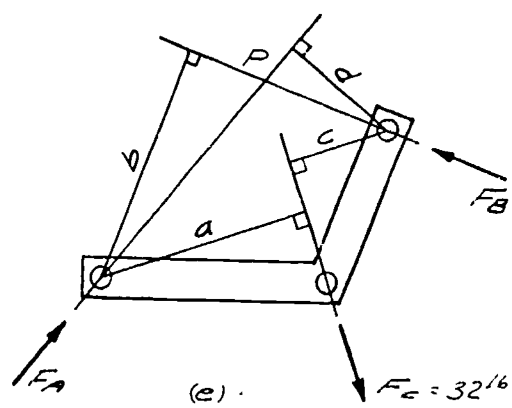
$\Sigma M_A = 0 \qquad -aF_C + bF_B + 0F_A = 0$

$-(11.4)(32) + (12.9)(F_B) = 0 \qquad F_B = 28.3$ lb

Measuring $c = 5.4''$ $d = 6.8''$

$\Sigma M_B = 0 \qquad cF_C - dF_A + 0F_B = 0$

$(5.4)(32) - (6.8)(F_A) = 0 \qquad F_A = 25.4$ lb

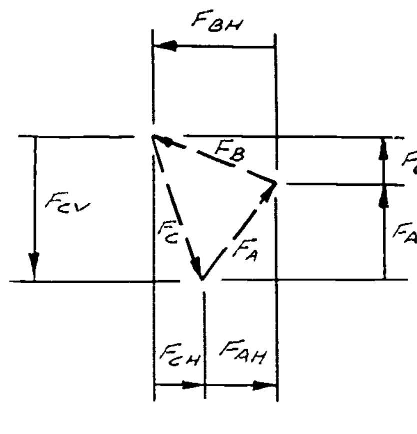


(e)

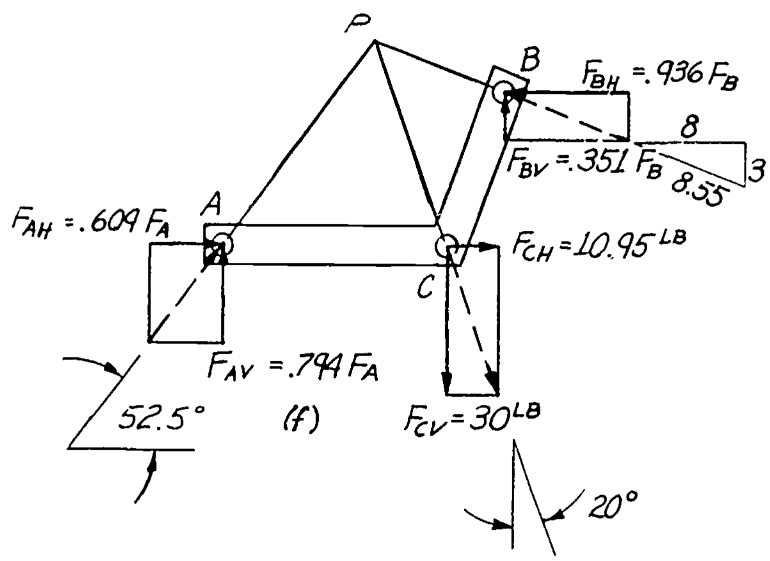
The second method has been developed also and is shown in (e). Again a free-body diagram (e) is constructed to scale with $\overline{F}_C$ and $\overline{F}_B$ shown. To find F_B (the magnitude of $\overline{F}_B$) moments are taken of $\overline{F}_B$ and $\overline{F}_C$ with respect to point A as shown next to (e). $\overline{F}_A$ is now drawn in its correct location with its line of action passing through points A and P. Moments with respect to B as shown next to (e) give the reaction at A. These magnitudes of $\overline{F}_A$ and $\overline{F}_B$ can be seen to check with those found by the direct summation of forces. The three dimensional free-body diagram for this solution is (c), exactly as in the other solution.

Summation of Mutually Perpendicular Components of the Point Forces

The third method uses the principle that the three loads acting upon M can be replaced by their three point force resultants and these point force resultants can be replaced by point force components without altering the equilibrium conditions of the system. Mutually perpendicular components or orthogonal components are used because mutually perpendicular components depend only on their resultant and are independent of each other. It is customary to make one of the components horizontal and the other vertical, as measurements are usually taken from horizontal or vertical lines. Horizontal $\overline{F}_H$ and vertical $\overline{F}_V$ (mutually perpendicular perpendicular components) of any point force $\overline{F}$ are independent of each other because $\overline{F}_H$ cannot be replaced using the parallelogram law with any set of orthogonal components where either one of the set is vertical.



(g)



Member M is first drawn to scale (f). $\overline{F}_C$, $\overline{F}_B$, and $\overline{F}_A$ will be placed in order in (f) and then their horizontal and vertical components will be shown with them.

$\overline{F}_C$ has a magnitude of 32 lb and a direction of 20° from the vertical so its horizontal and vertical components can be found with arithmetic equations. $\overline{F}_C$ is drawn dashed at C. Then a parallelogram is drawn showing $\overline{F}_C$ being replaced by horizontal ($\overline{F}_{CH}$) and vertical ($\overline{F}_{CV}$) components. The slope (θ = 20°) of $\overline{F}_C$ is known so

$$\sin \theta = \frac{F_{CH}}{F_C} \qquad \cos \theta = \frac{F_{CV}}{F_C}$$

$$F_{CH} = (32)(\sin 20^\circ) = 10.95 \text{ lb} \qquad F_{CV} = (32)(\cos 20^\circ) = 30 \text{ lb}$$

These components can be placed at point C It is not necessary to construct them to an exact scale, but it is best to show their relationships to $\overline{F}_C$ to scale as shown in (f).

$\overline{F}_B$ acts perpendicular to the wall so its direction is known. Using the slope box of 3/8/8.55 the following relationships hold

$$\frac{F_B}{8.55} = \frac{F_{BH}}{8} = \frac{F_{BV}}{3} \qquad \text{or} \qquad F_{BH} = .936\, F_B \qquad F_{BV} = .351\, F_B$$

These components written in terms of F_B can be placed on the free-body diagram at B.

$\overline{F}_A$ acts through points A and P. The slope of $\overline{F}_A$ can be measured and is found to be 52.5°. These relationships also hold

$$\cos 52.5^\circ = \frac{F_{AH}}{F_A} \qquad F_{AH} = .609\, F_A \qquad \sin 52.5^\circ = \frac{F_{AV}}{F_A} \qquad F_{AV} = .794\, F_A$$

$\overline{F}_{AH}$ and $\overline{F}_{AV}$ can be placed at A as shown in (f).

$\overline{F}_A$, $\overline{F}_B$, and $\overline{F}_C$ are concurrent at point P and can be imagined to act at P. At P for equilibrium to exist, the three point forces add to a zero resultant $\overline{R}$, that is $\overline{F}_B + \overline{F}_C + \overline{F}_A = \overline{R} = 0$. This zero resultant will have a zero horizontal componer $\overline{R}_H$ and a zero vertical component $\overline{R}_V$ where $\overline{R}_H = \overline{F}_{BH} + \overline{F}_{CH} + \overline{F}_{AH}$ and $\overline{R}_V = \overline{F}_{BV} + \overline{F}_{CV} + \overline{F}_{AV}$.

The horizontal and vertical component sets are colinear, so using signs of ↑+ and +→ , the magnitudes of the components relate to each other as

$$-F_{BH} + F_{CH} + F_{AH} = R_H = 0 \quad (\Sigma F_H = 0) \qquad + F_{BV} - F_{CV} + F_{AV} = R_V = 0 \quad (\Sigma F_V = 0)$$

Another way to visualize that $F_{AH} + F_{CH} - F_{BH} = 0$ and $F_{AV} - F_{CV} + F_{BV} = 0$ is shown in (g). First, since $\overline{F}_C$, $\overline{F}_B$, and $\overline{F}_A$ add to zero and intersect at P, then $\sum M_{\text{any point}}$ must equal zero for the three point forces. Next the force polygon for the three point forces is drawn in (g). Notice that when $\overline{F}_C$, $\overline{F}_B$, and $\overline{F}_A$ are replaced by their vertical and horizontal components in (g) that $F_{AV} + F_{BV} - F_{CV} = 0$ and $F_{CH} + F_{AH} - F_{BH} = 0$

Now using signs of $\overset{+}{\rightarrow}$ and $+\uparrow$

$$+ F_{BV} - F_{CV} + F_{AV} = 0 \qquad\qquad - F_{BH} + F_{CH} + F_{AH} = 0$$

Substituting

$F_{BV} = .351\ F_B$ $\qquad$ $F_{BH} = .936\ F_B$

$F_{CV} = 30\ lb$ $\qquad$ $F_{CH} = 10.95\ lb$

$F_{AV} = .794\ F_A$ $\qquad$ $F_{AH} = .609\ F_A$

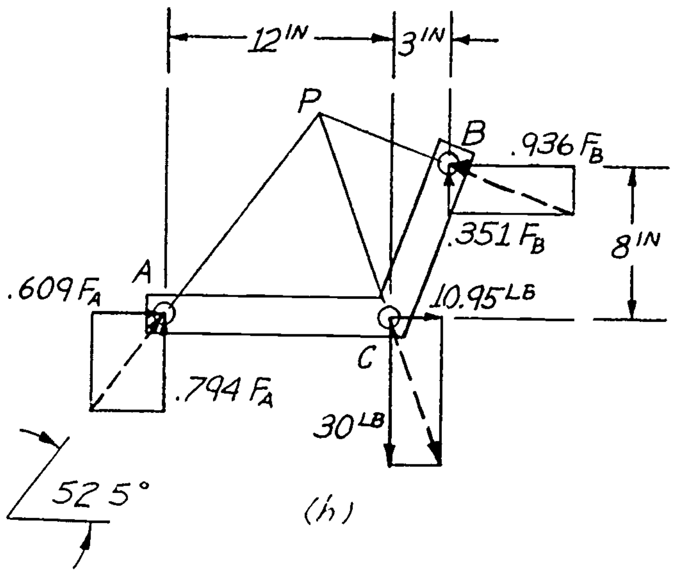
$$.351\ F_B - 30 + .794 F_A = 0 \qquad\qquad - .936 F_B + 10.95 + .609 F_A = 0$$

Solving the two independent equations for the two unknowns gives the magnitudes of $\overline{F}_A$ and $\overline{F}_B$. $\qquad F_A = 25.4$ lb $\qquad F_B = 28.3$ lb

These answers check with the other methods. Again the actual 3-D F-B diagram for this solution is still (c).

Summation of the Moments of Mutually Perpendicular Components of the Point Forces

It has been developed in the fundamentals section that the moment of a point force is equal to the sums of the moments of its components. If the direct moments of the point forces equal zero with respect to any point, then the moments of the components of the point forces must also equal zero with respect to the same point. This principle will be used in method four. First a free-body diagram (h) is drawn with the three point forces shown replaced by their components as in method 3. To find F_B moments are taken of all the components with respect to point A. Distances used are shown on the free-body diagram.



$\sum M_A = 0$

$$(0)(F_{AH}) + (0)(F_{AV}) - (12)(F_{CV}) + (0)(F_{CH}) + (8)(F_{BH}) + (15)(F_{BV}) = 0$$

$$- (12)(30) + (8)(.936F_B) + (15)(.351\ F_B) = 0$$

$F_B = 28.3$ lb

To find F_A moments are taken with respect to point B.

$\sum M_B = 0$

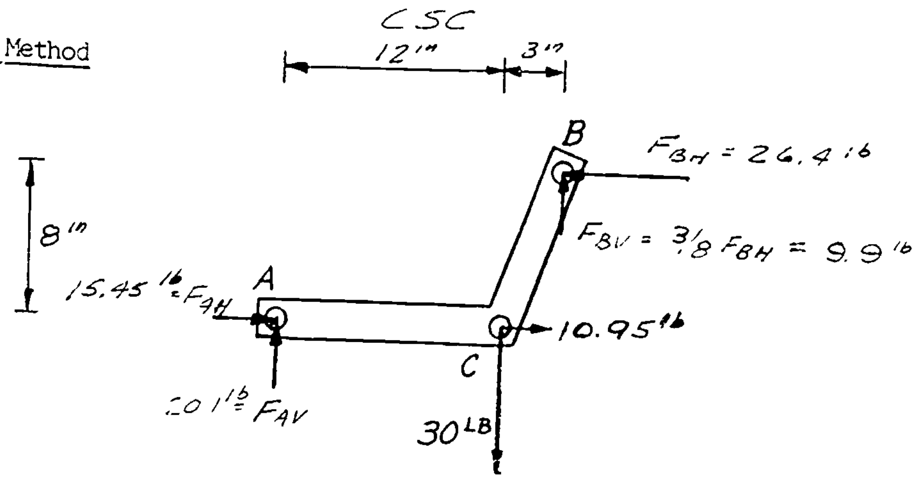
$$(0)(F_{BH}) + (0)(F_{BV}) + (8)(F_{CH}) + (3)(F_{CV}) + (8)(F_{AH}) - (15)(F_{AV}) = 0$$

$$+ (8)(10.95) + (3)(30) + (8)(.609F_A) - (15)(.794F_A) = 0$$

$F_A = 25.4$ lb

As always the results check with the other methods and the actual free-body diagram is (c)

Combination Method



(a)

This method is a combination of 3 and 4 without using point P. F-B diagram (a) is drawn and $\overline{F}_C$, $\overline{F}_B$, and $\overline{F}_A$ are replaced by horizontal and vertical components. $\overline{F}_{CH}$ and $\overline{F}_{CV}$ are shown on the F-B but their resultant $\overline{F}_C$ is not shown. $\overline{F}_{CH}$ and $\overline{F}_{CV}$ are drawn so that $\overline{F}_{CV}$ is approximately three times as long as $\overline{F}_{CH}$. As before F_{CH} = 10.95 lb and F_{CV} = 30 lb.

Now $\overline{F}_B$ is replaced by $\overline{F}_{BH}$ and $\overline{F}_{BV}$ in a slightly different manner than before. Instead of relating both of them to their resultant, $\overline{F}_{BH}$ and $\overline{F}_{BV}$ are related to each other. $\overline{F}_{BH}$ is labeled and F_{BV} = 3/8 F_{BH} (remember the slope of $\overline{F}_B$ is 3/8/8.55).

Next the direction of $\overline{F}_A$ is guessed and $\overline{F}_A$ is replaced by $\overline{F}_{AH}$ and $\overline{F}_{AV}$. No relationships have been established yet between $\overline{F}_A$, $\overline{F}_{AH}$, and $\overline{F}_{AV}$, so $\overline{F}_{AH}$ and $\overline{F}_{AV}$ are merely shown acting through the ℄ of pin A.

$\Sigma M_A = 0$ will give F_{BH}

$$(12)(30) = (15)(3/8\,F_{BH}) + (8)(F_{BH})$$

$$F_{BH} = 26.4 \text{ lb} \qquad F_{BV} = (3/8)(26.4) = 9.9 \text{ lb}$$

$$F_B = \sqrt{F_{BH}^2 + F_{BV}^2} = \sqrt{26.4^2 + 9.9^2} = 28.3 \text{ lb}$$

The magnitudes of $\overline{F}_{BH}$ and $\overline{F}_{BV}$ are now placed on the F-B diagram.

$$\Sigma F_H = 0 \quad 10.95 + F_{AH} = 26.4 \qquad F_{AH} = 15.45 \text{ lb}$$

$$\Sigma F_V = 0 \quad F_{AV} + 9.9 = 30 \qquad F_{AV} = 20.1 \text{ lb}$$

$$F_A = \sqrt{20.1^2 + 15.45^2} = 25.4 \text{ lb}$$

The slope angle α that F_A makes with a horizontal line at A can be found

$$\cos\alpha = \frac{F_{AH}}{F_A} = \frac{15.45}{25.4} = .608 \qquad \alpha = 52.5^\circ$$

Notice in this last method that only components are drawn on the F-B diagram. It is best to put the magnitudes of the components on the F-B for a quick visual check. Also the 2-D F-B need not be drawn to an exact scale as no measuring is done on the F-B diagram, all measurements can be taken from the original space diagram (b). As before however the actual F-B diagram is still (c).

Combination methods will now be used to find the point force resultants of the reactions on three coplanar systems.

Example 1

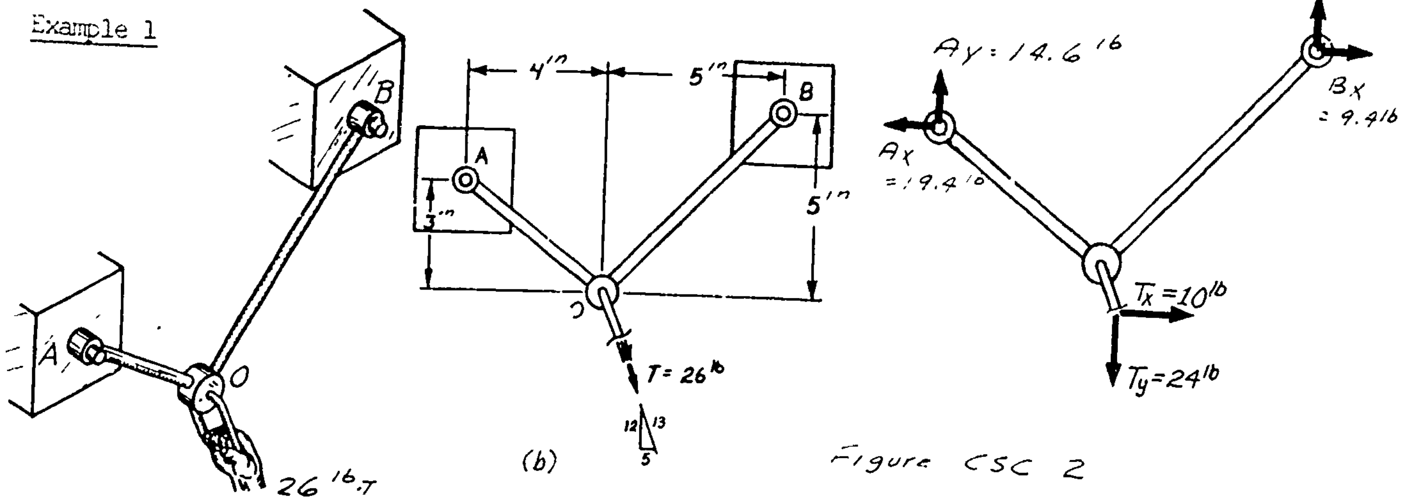


Figure CSC 2

Weightless cables OA and OB support a 26 lb load. The reactions at A and B are to be found. FB (c) is drawn, $\overline{T}$ is replaced by $\overline{T}_x$ (10 lb) and $\overline{T}_y$ (24 lb), $\overline{B}$ (actually $\overline{F}_B$) by $\overline{B}_x$ and $\overline{B}_y$ ($B_y = B_x$ as the slope of OB is 1/1), and $\overline{A}$ by $\overline{A}_y$ and $\overline{A}_x$ ($A_x = 4/3\ A_y$)

To find B

$\Sigma M_A = 0$ assume $\overline{T}$ acts at point O

$(3)(10) - (4)(24) + (9)(B_x) - (2)(B_x) = 0$ ($B_x = B_y$)

$B_x = 9.4$ lb $B_y = 9.4$ lb $B = 13.3$ lb

Check- $\Sigma F_H = 0$ $19.4 = 9.4 + 10$

$\Sigma F_V = 0$ $14.6 + 9.4 = 24$

To find A

$\Sigma M_B = 0$

$(5)(10) + (5)(24) - (9)(A_y) - (2)(4/3\ A_y) = 0$ (A_x)

$A_y = 14.6$ lb

$A_x = 19.4$ lb

$A = 24.4$ lb

Example 2

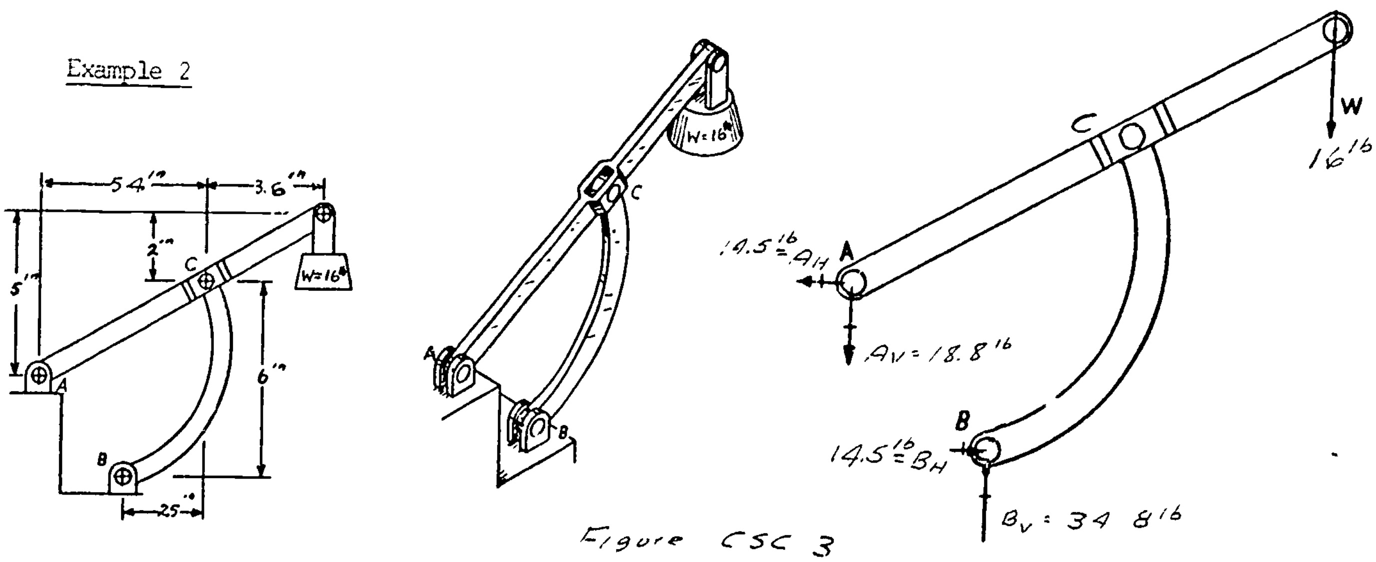


Figure CSC 3

3-F member ACD and 2-F member BC are weightless and coplanar. The pins are frictionless. The reactions at A and B are to be found.

F-B (c) is drawn. BC has a slope of 6/2.5 so F_B can be replaced by $\overline{B}_H$ and $\overline{B}_V$ (6/2.5 B) (2.4). $\overline{A}_V$ and $\overline{A}_H$ complete the F-B diagram.

$\Sigma M_A = 0$ (2.4 B_H)

$(2.9)(B_V) + (3)(B_H) = (9)(16)$

$B_H = 14.5$ lb

$B_V = (2.4)(14.5) = 34.8$ lb

$\Sigma F_H = 0$

$A_H = 14.5$ lb

$\Sigma F_V = 0$

$A_V = 34.8 - 16 = 18.8$ lb

Check- $\Sigma M_W = 0$ $(9)(18.8) + (8)(14.5) = (5)(14.5) + (6.1)(34.8)$

169 + 116 = 72.5 + 212.5

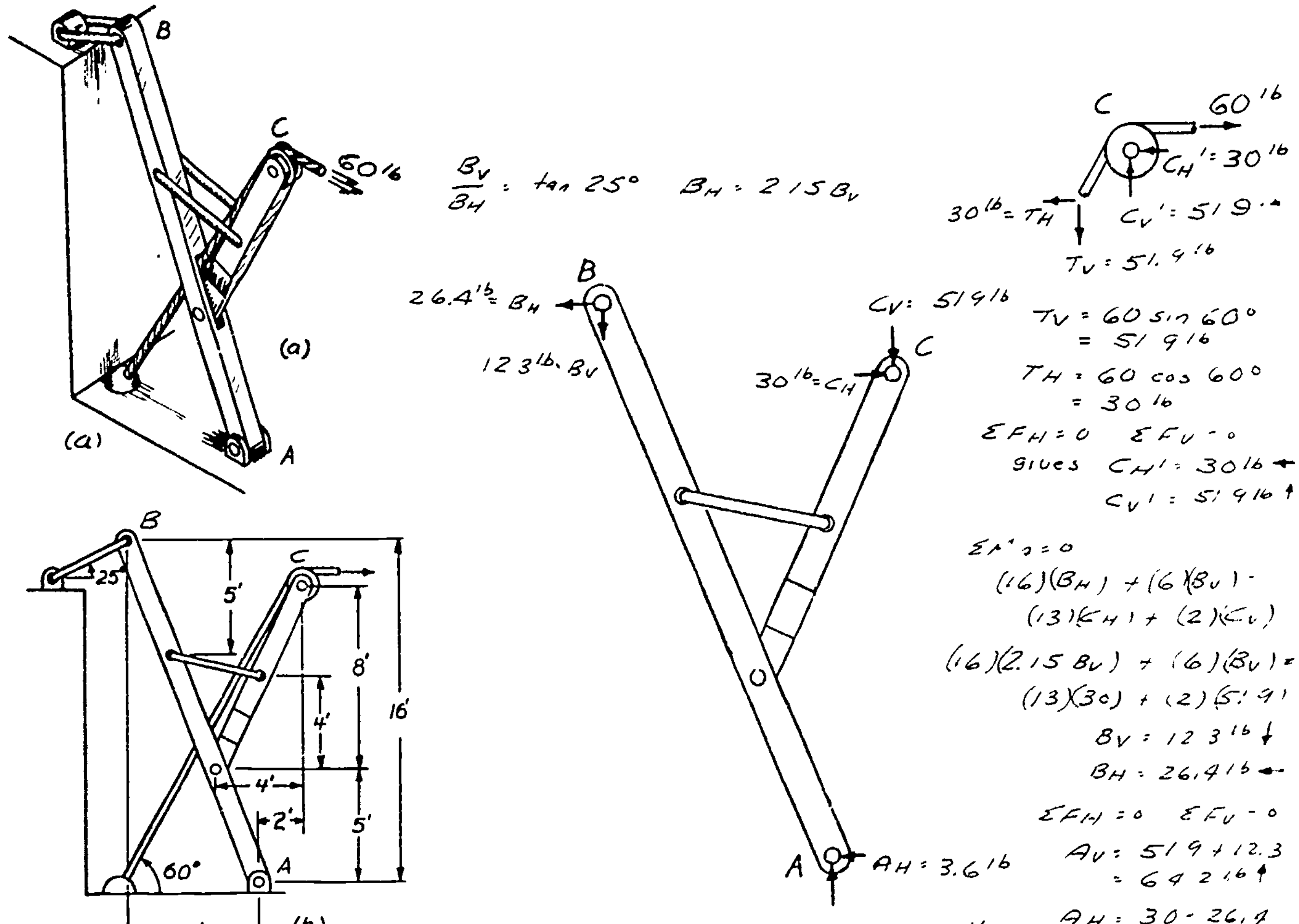


Figure CSC 4

The coplanar frame supports a 60 lb load. The reactions at A and B are wanted if all members are weightless and frictionless.

F-B diagrams (c) and (d) are drawn, the pin at C is left in (c). All forces are replaced by horizontal and vertical components. It is not necessary that the components are drawn "pushing" against the frame, for instance at B components B_H and B_V can be drawn "pulling" on the frame.

F-B (c) can now be completed as shown in the figure. Then the components at C (C_V and C_H) at C on the frame (d) are equal and opposite to the components (C_V and C_H) at C on the pulley (c). So with C_V and C_H known, F-B (c) can be completed.

Check. $\Sigma M_C = 0$

$(13)(3.6) + (2)(64.2) = (8)(12.3) + (3)(26.4)$

$176 \approx 177$

$A = \sqrt{A_H + A_V} = 64.3$ lb

$B = \sqrt{B_H + B_V} = 29.1$ lb

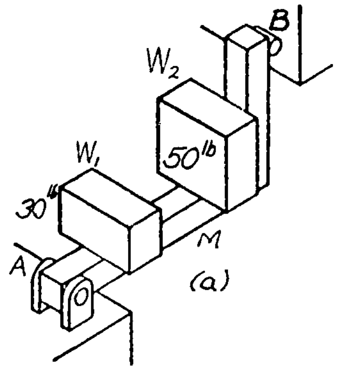
α for $A = \tan^{-1} \frac{A_V}{A_H} = 3.2°$

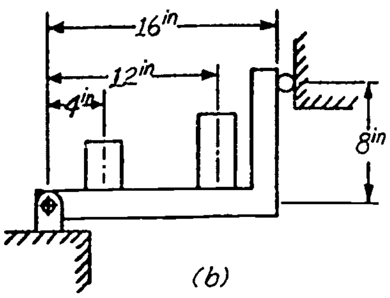
Some cardinal rules in engineering statics. Remember when solving for reactions using $\Sigma F_H = 0$, $\Sigma F_V = 0$, and $\Sigma M = 0$. 1) Always draw F-B diagrams; put all component magnitudes as they are found on the F-B diagrams, this will give you a quick visual check of your work. Do not put spatial dimensions on the F-B diagrams, use the dimensions from a space diagram. 2) Always check your work with an independent method. 3) Always do neat work that you can be proud of.

AT THIS POINT, GIVEN A COPLANAR STRUCTURE MADE UP OF 2-F AND 3-F MEMBERS, YOU SHOULD BE ABLE TO CONSTRUCT A 2-D F-B DIAGRAM OF THE STRUCTURE, REPLACE THE LOADS BY HORIZONTAL AND VERTICAL POINT FORCE COMPONENTS, AND WRITE THE FORCE AND MOMENT EQUATIONS NEEDED TO FIND THE REACTIONS.

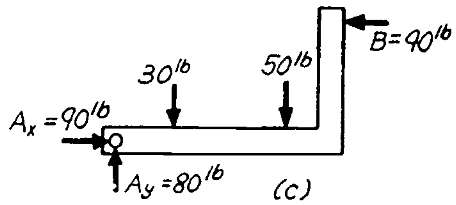
Superposition

Superposition is a technique used in engineering statics for finding the reactions on members or structures that have more than one external load. Partial reactions are found for each load acting alone and these partial reactions are then added by the parallelogram law to find the total reaction at each constraint.





The system is coplanar.



$\Sigma M_A = 0$

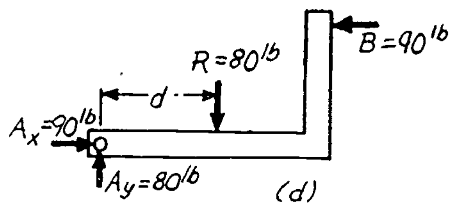
$(4)(30) + (12)(50) = (8)(B)$

$B = 90$ lb

$\Sigma F_x = 0$ & $\Sigma F_y = 0$

$A_x = 90$ lb $A_y = 80$ lb

$A = 120$ lb



R = 30 + 50 = 80 lb at d

$(4)(30) + (12)(50) = (d)(80)$

$d = 9$ in

$\Sigma M_A = 0$

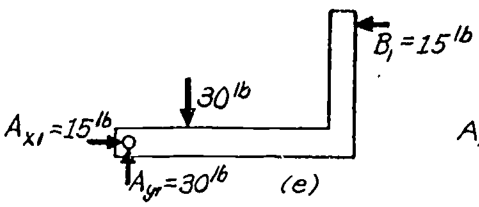
$(9)(80) = (8)(B)$

$B = 90$ lb

$\Sigma F_x = 0$ & $\Sigma F_y = 0$

$A_x = 90$ lb $A_y = 80$ lb

$A = 120$ lb



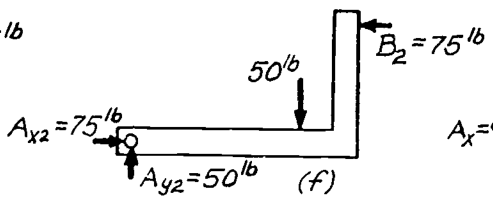
-1-

$\Sigma M_A = 0$ $(4)(30) = (8)(B_1)$

$B_1 = 15$ lb

$A_{y1} = 30$ lb

$A_{x1} = 15$ lb



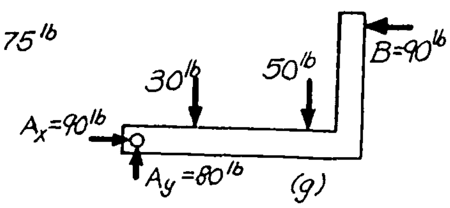
-2-

$\Sigma M_A = 0$ $(12)(50) = (8)(B_2)$

$B_2 = 75$ lb

$A_{x2} = 75$ lb

$A_{y2} = 50$ lb



Superposition of -1- and -2-

$B = 15 + 75 = 90$ lb

$A_x = 15 + 75 = 90$ lb

$A_y = 30 + 50 = 80$ lb

$A = 120$ lb

Figure CSC 8

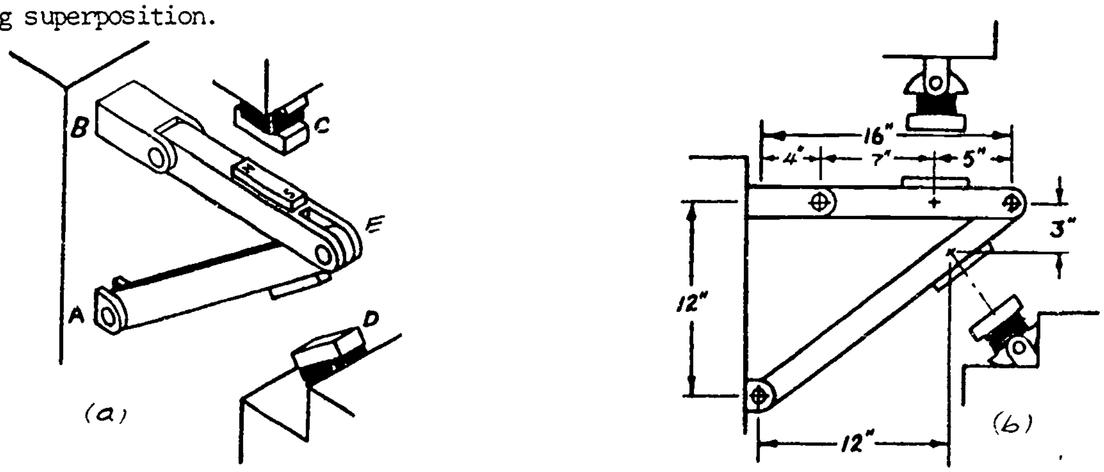
When an object such as the one shown in CSC 8(a) and (b) is loaded at two places and the reactions are wanted at A and B, three techniques can be used.

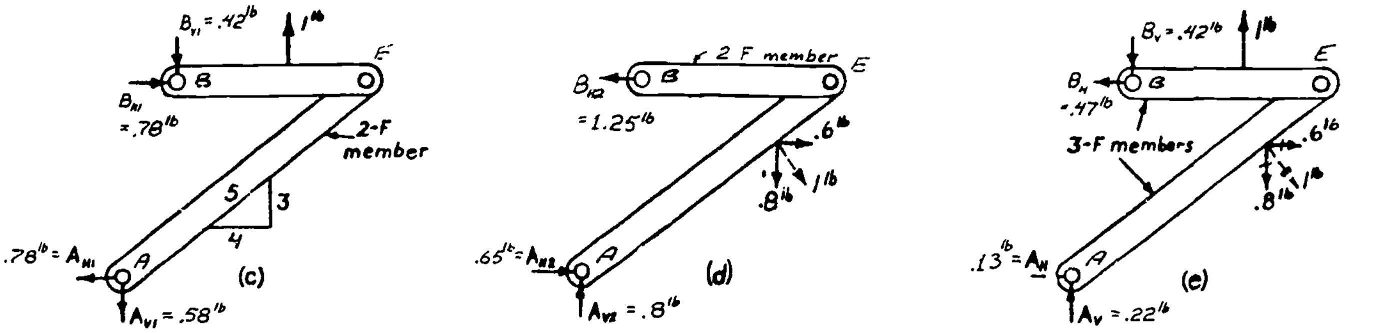
(1) A F-B diagram can be drawn to scale with both of the forces and solved as shown in (c).

(2) The two acting loads can first be replaced by their resultant as in (d) and the problem solved as shown.

(3) The reactions can be found by first assuming that W_1 acts alone as shown in (e) and finding the partial reactions caused by it. The partial reactions caused by W_2 acting alone can then be found as shown in (f). By the principle of superposition the two sets of partial reactions can be added to give the **total** reactions as shown in (g).

The frame shown in figure CSC 9(a) and (b) is loaded by twc electromagnets C and D. It is assumed that the structure is weightless with frictionless pins, and each magnet is assumed to set up an evenly distributed force field of 1 lb. The reactions at A and B are to be found using superposition.





$\Sigma M_B = 0$

$(7)(1) + (4)(A_{V1}) = (12)(A_{H1})$

$A_{V1} = 3/4\ A_{H1}$

$(7)(1) + (4)(3/4\ A_{H1}) = (12)(A_{H1})$

$A_{H1} = .78$ lb

$A_{V1} = .58$ lb

$B_{H1} = .78$ lb $B_{V1} = .42$ lb

$\Sigma M_A = 0$

$(12)(.8) + (9)(6) = (12)(B_{H2})$

$B_{H2} = 1.25$ lb

$B_{V2} = 0$

$A_{H2} = .65$ lb $A_{V2} = .8$ lb

Superimposed

$B_V = .42 + 0 = .42$ lb

$B_H = 1.25 - .78 = .47$ lb

$A_H = .78 - .65 = .13$ lb

$A_V = .8 - .58 = .22$ lb

Check-

$\Sigma F_H = 0$ $.47 + .13 = .6$

$\Sigma F_V = 0$ $.22 + 1 = .42 + .8$

$\Sigma M_E = 0$ $(5)(1) + (12)(.13) + (16)(.22) = (3)(.6) + (4)(.8) + (12)(.42)$

$10.08 \approx 10.04$

Figure CSC 9

F-B diagram (c) is drawn and completed with magnet C on and magnet D off. Notice that with D off, AO is a 2-F member, and the reaction at A therefore lies along the ℄ of A .

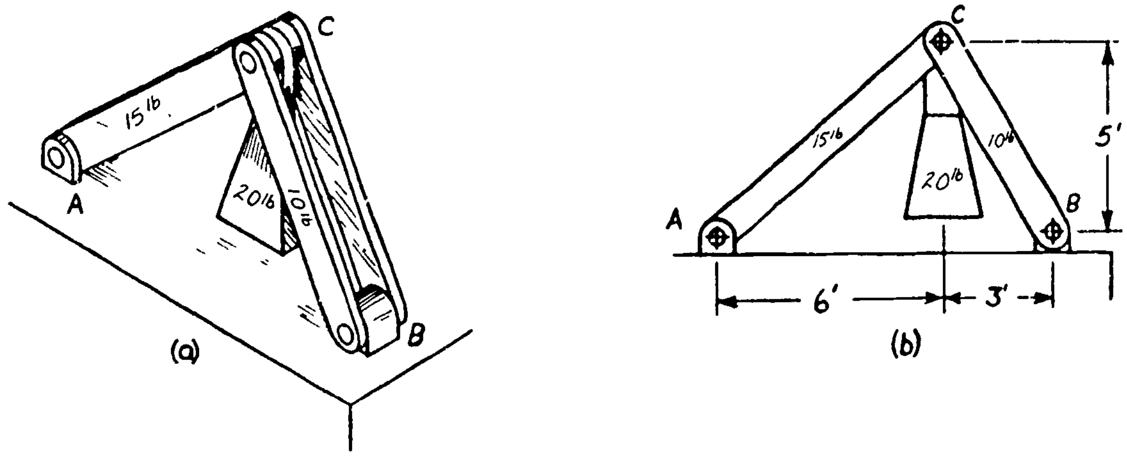
F-B diagram (d) is drawn and completed with magnet D on and magnet C off. BE becomes a 2-F member, and the reaction at B therefore lies along the ℄ of BE.

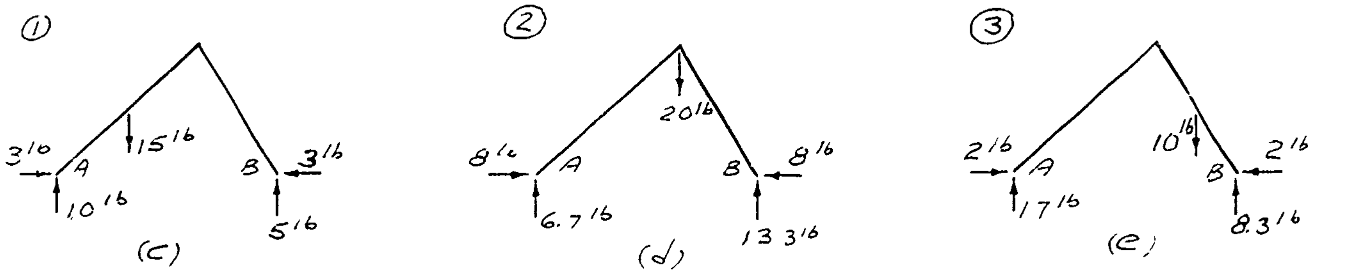
The partial reactions are superimposed in (e) and can be visually checked by summation of forces. It is suggested that you prove to yourself that this example cannot be solved with C and D both on together.

It should be apparent to you now that with multi-membered frames, the loads cannot be added to a single resultant acting upon one member of the frame.

CSC

In figure CSC 10(a) and (b) a 20 lb weight is being supported by two coplanar members. AC is a uniform member weighing 15 lb and BC weighs 10 lb. The reactions at A and B are to be found by superposition.

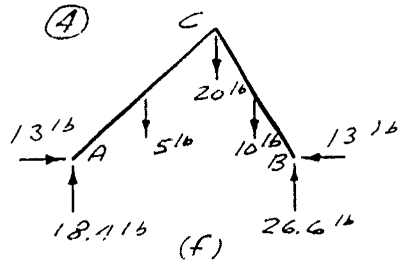




$\Sigma M_A = 0$
$(3)(15) = (9)(B_{V1})$
$B_{V1} = 5$ lb
$B_{H1} = 3$ lb $= (3/5)(5)$
$A_{V1} = 10$ lb
$A_{H1} = 3$ lb

$\Sigma M_B = 0$
$(3)(20) = (9)(A_{V2})$
$A_{V2} = 6.7$ lb
$A_{H2} = (6/5)(6.7) = 8$ lb
$B_{H2} = 8$ lb
$B_{V2} = 13.3$ lb

$\Sigma M_B = 0$
$(1.5)(10) = (9)(A_{V3})$
$A_{V3} = 1.7$ lb
$A_{H3} = (6/5)(1.7) = 2$ lb
$B_3 = 8.3$ lb
$B_3 = 2$ lb



Check -
$\Sigma F_H = 0$
$13 = 13$

$\Sigma F_V = 0$
$18.4 + 26.6 = 15 + 20 + 10$
$45 = 45$

$\Sigma M_B = 0$
$(3)(15) + (5)(13) + (3)(26.6) = (1.5)(10) + (5)(13) + (6)(18.4)$
$124.9 \approx 125.1$

Superimposed
$A = 22.4$ lb @ $54.6°$
$B = 29.6$ lb @ $63.9°$

Figure CSC 10

Simplified F-B diagrams showing only the ℄'s of the members are drawn (c), (d), and (e). ① is solved with the 15 lb load acting alone, ② with the 20 lb load only, and F-B ③ with the 10 lb load. The partial reaction components are sumperimposed in (f). A force and moment check is made of (f).

NOW IF YOU ARE GIVEN A STRUCTURE COMPOSED OF TWO 3-F MEMBERS, YOU SHOULD BE ABLE TO FIND THE REACTIONS USING SUPERPOSITION.

Cantilever Beam Reactions

Frequently in engineering statics it is necessary to analyze the external forces that would act on an imaginary cut section of a member. When a symmetrical two-force member has a cross section exposed in a F-B diagram, the external force field acting on this imaginary section is uniform, symmetrical and acts parallel to the center line of the member. The sense of the field is determined by the external loads. If the loads push on the member, the field on the exposed surface also pushes and the member is said to be in compression. If the external forces pull on the member, the field on the exposed section also pulls from the exposed surface and the member is said to be in tension.

Exposed transverse sections are necessary to analyze the reactions at the constraint of a cantilever beam . A cantilever is a member that is rigidly constrained at one end and loaded somewhere along its length (e.g. a diving board with a diver about to dive). Figure CSC 11 shows a uniform horizontal cantilever beam which is assumed to be weightless. A method of finding the forces acting on any vertical section when a vertical load is placed on the beam is to be developed.

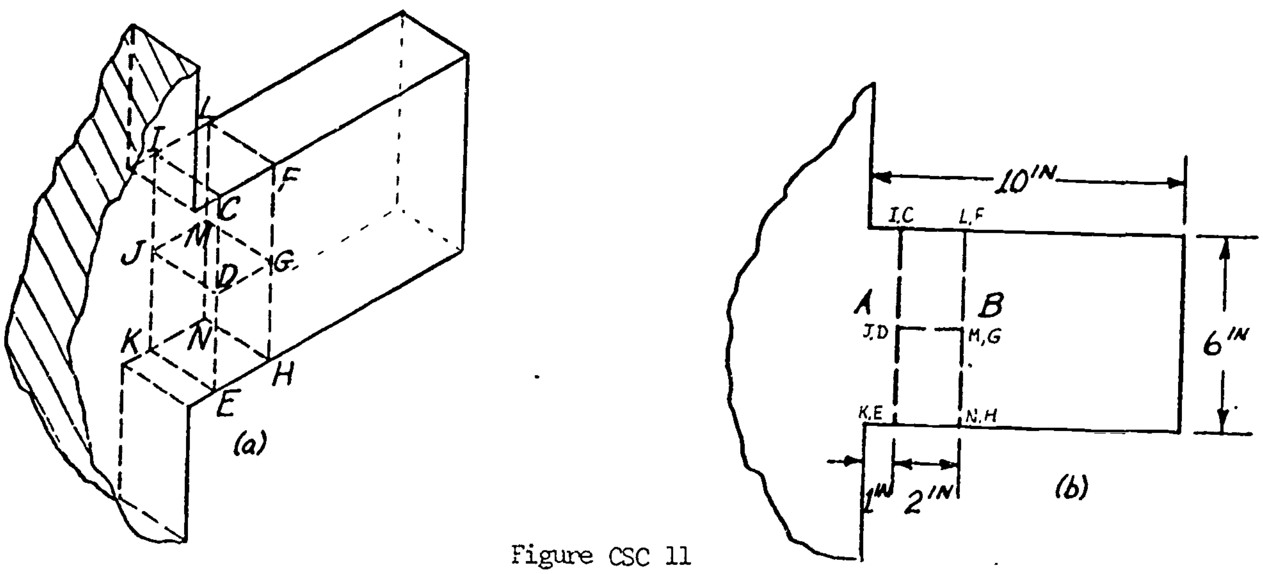


Figure CSC 11

The part of the beam to be analyzed extends from an imaginary vertical plane A, which is perpendicular to the axis of the beam, to the right end of the beam. Lines CE and IK indicate where plane A cuts the sides of the beam. Another vertical plane B, which cuts the sides of the beam along lines FH and LN, is shown two inches to the right of plane A. D, G, J, and M are the mid-points of their respective lines and indicate the points of intersection of a horizontal plane which passes through the center line of the beam.

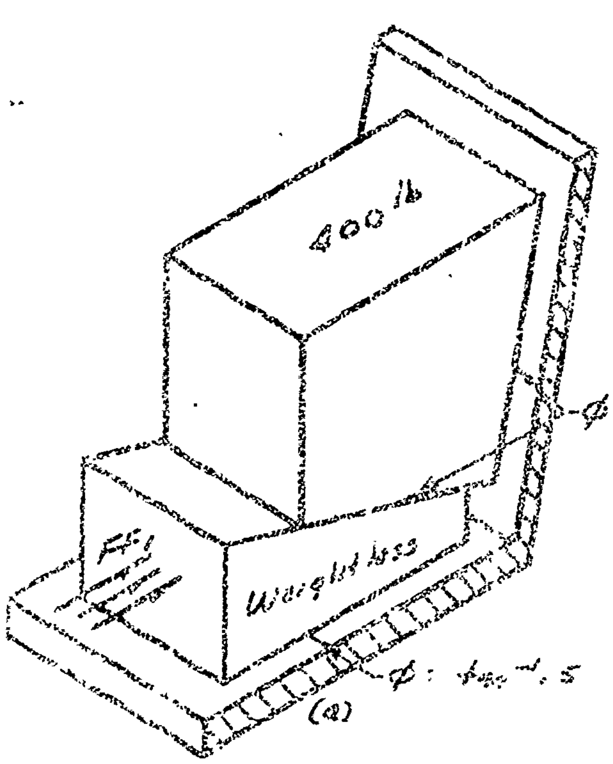
Figure CSC 12

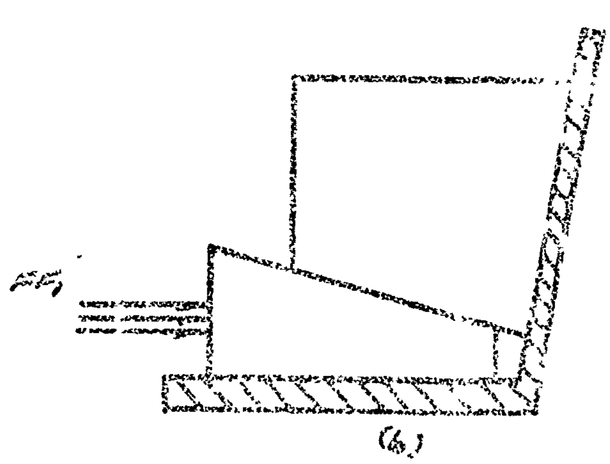
If a 50 lb load is applied to the end of the beam as shown in figure CSC 12, the beam will tend to bend. Careful measurement of a real beam loaded in this way would show that the distance between C and F (two inches) has increased a small increment $\triangle$ x whereas the distance between E and H (two inches) has shortened by exactly the same incremental distance $\triangle$ x. The distance between D and G, however, will not change, and the two cross sections A and B will both remain as rectangular planes perpendicular to the axis of the beam.

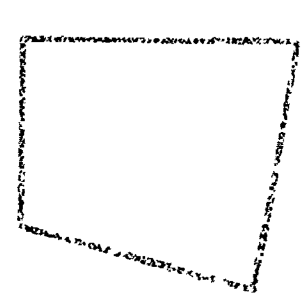
Another experiment can be performed showing that when a tension load is applied to a sample of material, the sample elongates some distance $\triangle$ D. When the load is increased or decreased, the change in $\triangle$ D is directly proportional to the change in the load (doubling the load doubles the deflection, halving the load halves the deflection, etc.). This relationship is also found for compression tests. Furthermore, it is found experimentally that a given load gives the same deflection to the sample whether it is applied as a tension load or a compression load. From this experimental evidence it can be said that a member deflects in direct proportion to the load applied.

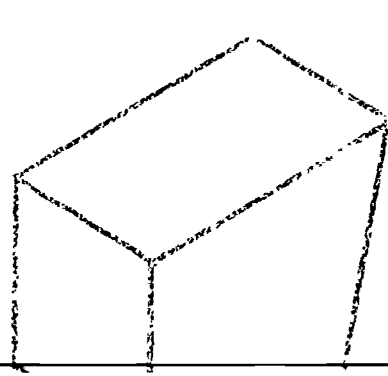
F-B diagrams of the beam with cross section A exposed are drawn in figure CSC 13. These diagrams show no deflections as the deflections are very small and do not significantly alter the shape of the F-B diagram. FF_W and $\bar{F}_W$ are known and can be placed on the two F-B diagrams.

The first experiment showed that maximum deflection occurs at both the top and bottom of the beam while no deflection occurs at the middle. In addition the elongation of the upper half varies uniformly from DG to CF and the shortening of the lower half varies uniformly from DG to EH. This is true because cross sections A and B remain as rectangular planes perpendicular to the axis of the beam. Therefore a force field varying from zero at JD to a maximum at IC must be acting upon the top half of the beam at section A to put it in tension









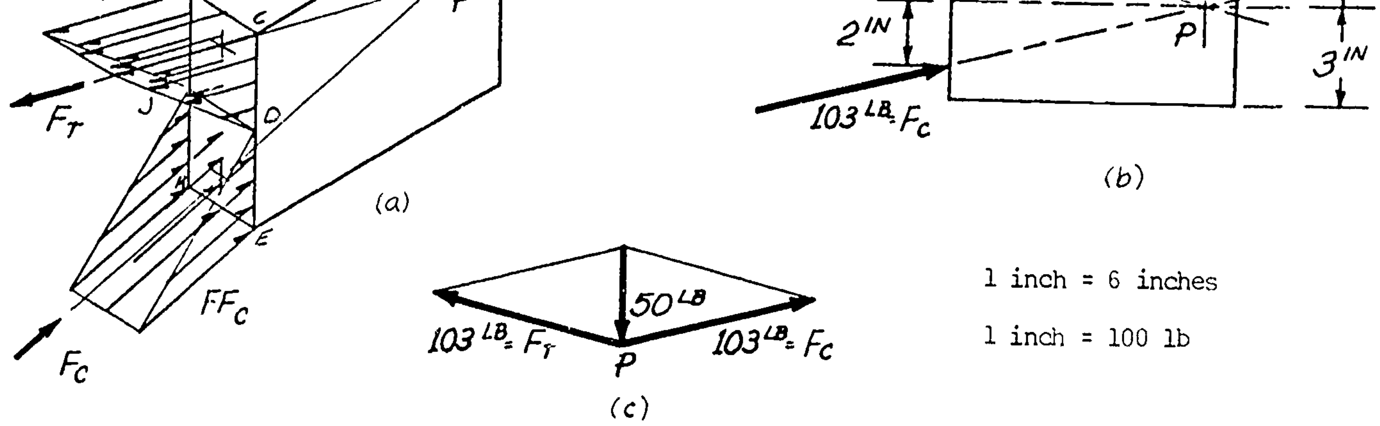
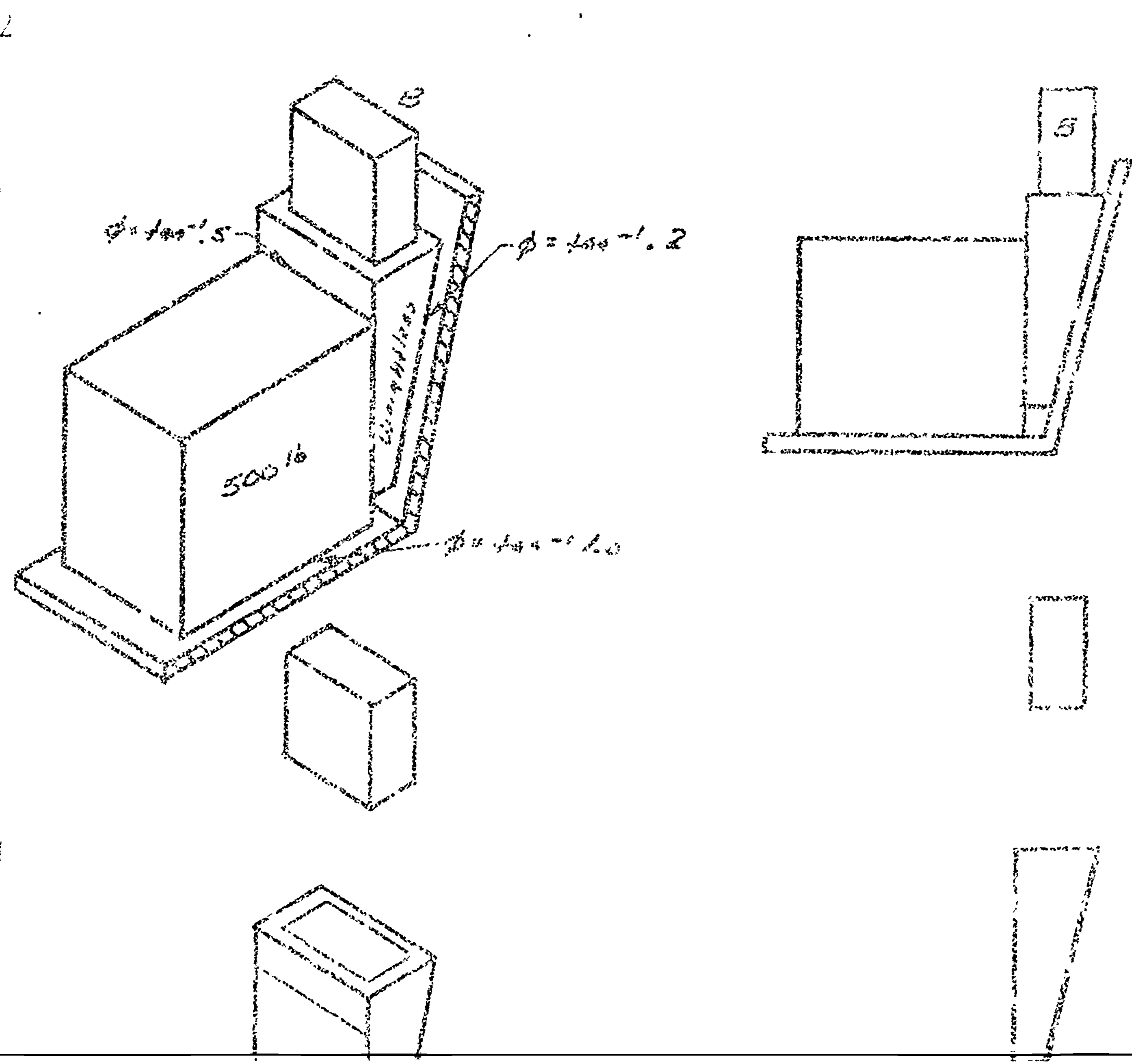


Figure CSC 13

Each of the force fields can be replaced by a point force resultant acting through its effective center, which in this case is two inches from the centerline of the beam. These two point force resultants, together with the point force resultant of the load, establish a system of three point forces which must, by the three force principle, be concurrent at some point on the line of action of $\bar{F}_W$. $\bar{F}_T$, the resultant of the upper force field, and $\bar{F}_C$, the resultant of the lower force field, are equal in magnitude and act through points two inches above and below the center line of the beam respectively. Their lines of action must therefore meet with the action line of $\bar{F}_W$ at point P, which is on the centerline of the beam, as shown in (a) and (b). The magnitudes of $\bar{F}_T$ and $\bar{F}_C$ can then be found by constructing a parallelogram of forces as shown in (c). Force fields FF_T and FF_C have the same slopes as the point force resultant $\bar{F}_T$ and $\bar{F}_C$.

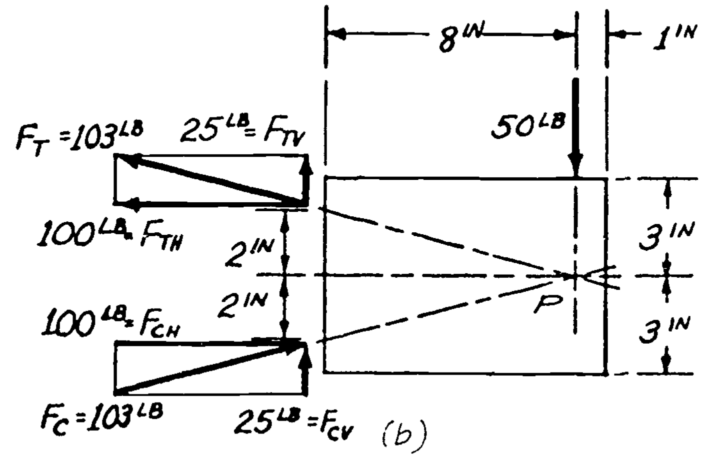
AT THIS POINT, GIVEN A CANTILEVER BEAM LOADED ON ONE END, YOU SHOULD BE ABLE TO VISUALIZE THE FORCE FIELDS ACTING ON A CROSS-SECTION OF THE BEAM AND DETERMINE THEIR POINT FORCE RESULTANTS USING THE PARALLELOGRAM LAW.



Vertical Shears and Bending Moment Couples on Cantilever Beams

1 inch = 6 inches

1 inch = 100 lb



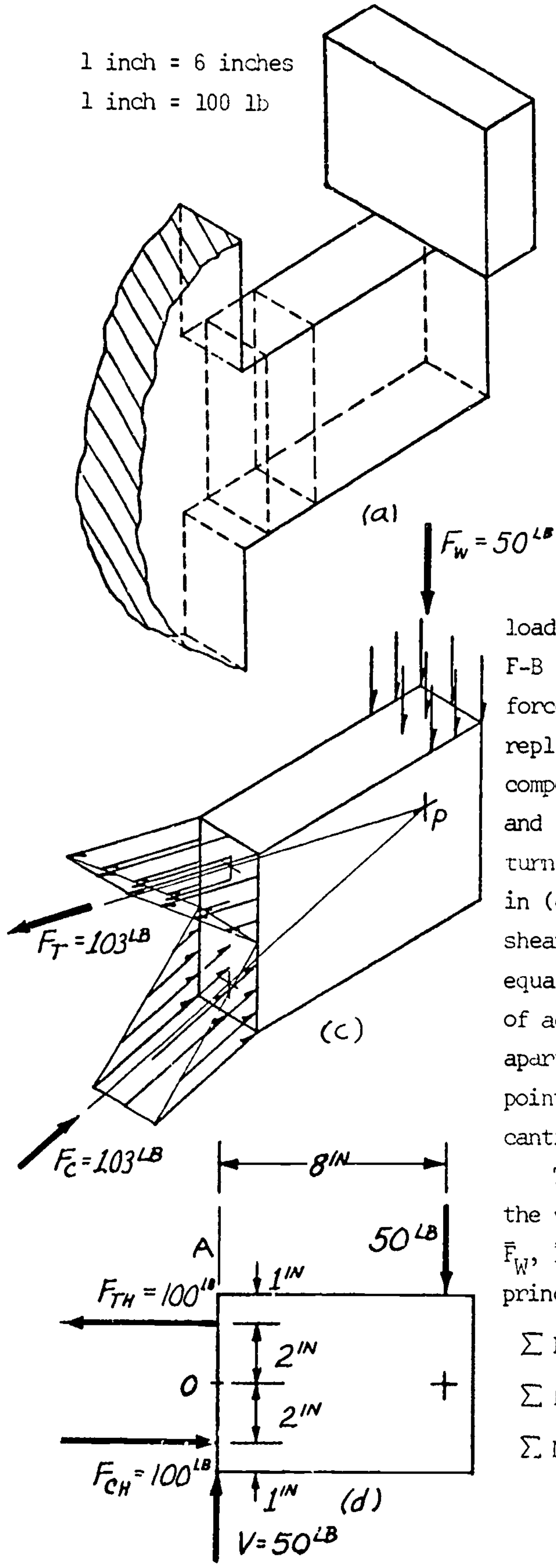


Figure CSC 14

The 3-D F-B diagram of the cantilever beam with its loads is redrawn in figure CSC 13(a) and the two-dimensional F-B diagram is redrawn in (b) with the three acting point forces $\bar{F}_W$, $\bar{F}_T$, and $\bar{F}_C$. Sometimes it is convenient to replace the point force reactions by mutually perpendicular components as in (b) where $\bar{F}_T$ is replaced by $\bar{F}_{TH}$ and $\bar{F}_{TV}$ and $\bar{F}_C$ is replaced by $\bar{F}_{CH}$ and $\bar{F}_{CV}$. $\bar{F}_{CV}$ and $\bar{F}_{TV}$ can in turn be replaced by a single vertical component $\bar{V}$, as shown in (d). This vertical component is called the vertical shear. The remaining components $\bar{F}_{TH}$ and $\bar{F}_{CH}$ are coplanar, equal in magnitude, and opposite in sense, with their lines of action displaced a distance (in this case four inches) apart. The two horizontal point force components form a point force system that is called a couple which for a cantilever beam is called a bending moment couple.

The point force components representing the couple and the vertical shear can be found using arithmetic equations. $\bar{F}_W$, $\bar{F}_T$, and $\bar{F}_C$ and their components obey the three-force principle. Therefore in (d) for $\bar{F}_{TH}$, $\bar{F}_{CH}$, $\bar{V}$, and $\bar{F}_W$,

$\sum F_V = 0 \qquad V = 50 \text{ lb}$

$\sum F_H = 0 \qquad F_{TH} = F_{CH}$

$\sum M_O = 0 \qquad (2)(F_{TH}) + (2)(F_{CH}) = (8)(50)$

$4 F_{CH} = 400$

$F_{CH} = 100 \text{ lb} \qquad F_{TH} = 100 \text{ lb}$

Another cross section seven inches from the right end of the beam is to be investigated for the magnitudes of its vertical shear and couple. A three-dimensional F-B diagram is drawn in figure CSC 15(a) showing FF_W and $\overline{F}_W$, and a two-dimensional F-B diagram is drawn in (b) showing $\overline{F}_W$. At the exposed section, $\overline{V}$ is shown acting vertically, and $\overline{F}_{TH}$ and $\overline{F}_{CH}$ are drawn horizontally 2 inches from the beam ℄ .

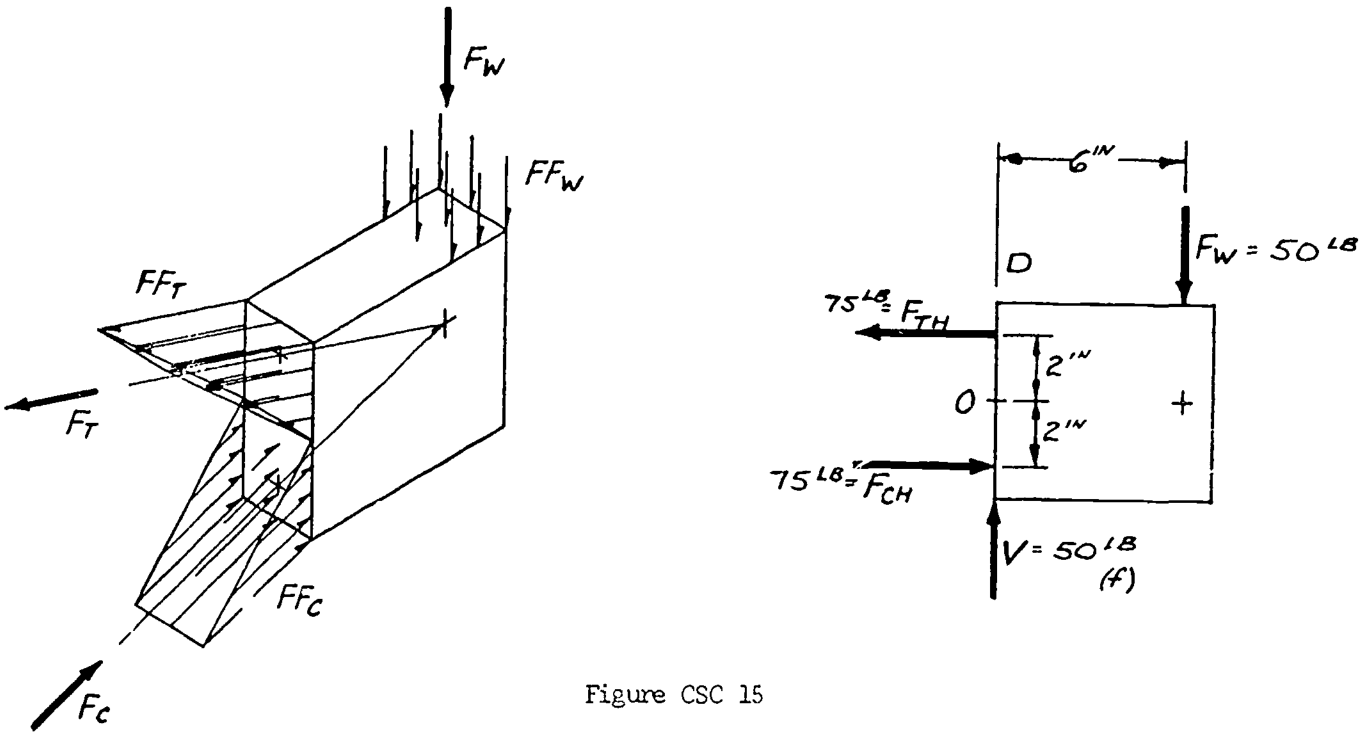


Figure CSC 15

Now

$$\sum F_V = 0 \qquad V = 50 \text{ lb}$$

$$\sum F_H = 0 \qquad F_{TH} = F_{CH}$$

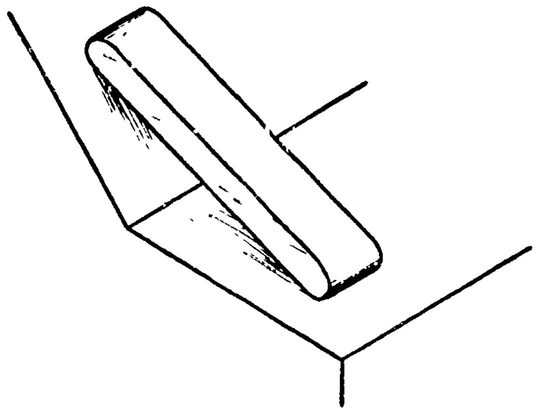
$$\sum M_O = 0 \qquad (2)(F_{TH}) + (2)(F_{CH}) = (6)(50)$$

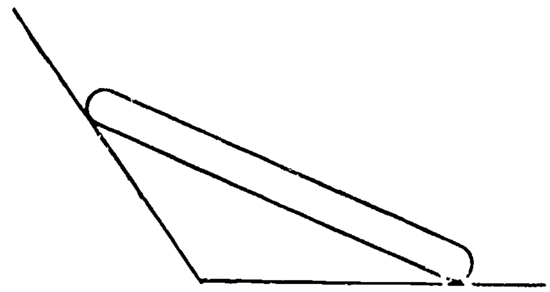
$$4\, F_{TH} = 360$$

$$F_{TH} = F_{CH} = 75 \text{ lb}$$

The vertical shear is found to be 50 lb acting vertically upward, and the bending moment couple consists of two 75 lb parallel point forces acting horizontally 4 inches apart as shown in (b).

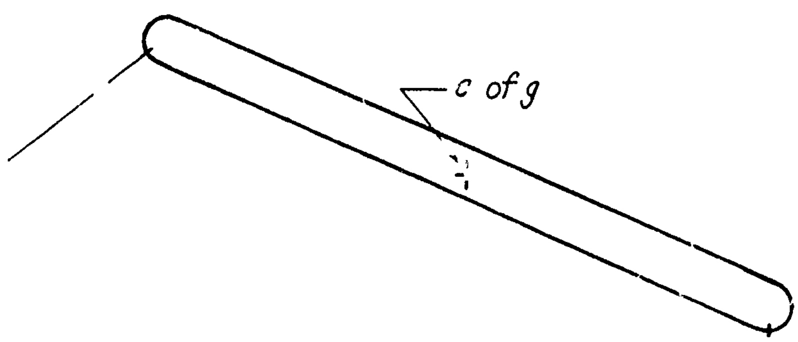
AT THIS POINT, GIVEN A CANTILEVER BEAM LOADED ON ONE END, YOU SHOULD BE ABLE TO FIND THE VERTICAL SHEAR AND THE BENDING MOMENT COUPLE ACTING ON ANY VERTICAL CROSS-SECTION USING FORCE AND MOMENT EQUATIONS.





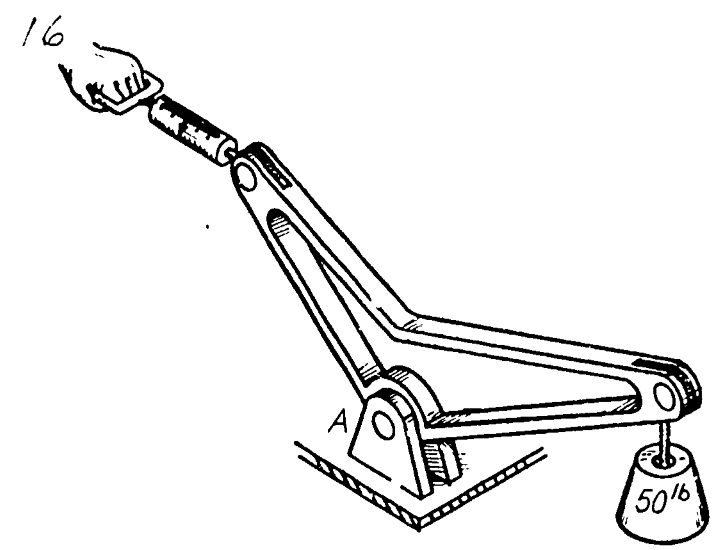
$1^{in} = 4^{in}$

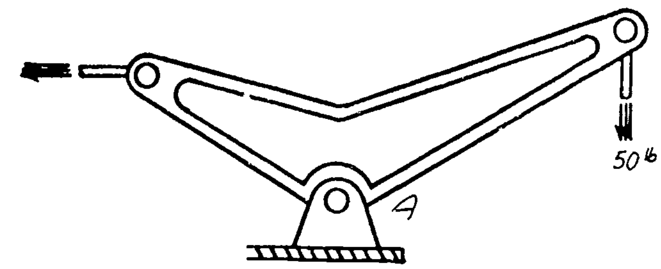
$1^{in} = 10^{lb}$



EQA - 1(A) The object is in equilibrium as shown with a rough horizontal surface and a smooth sloping surface. If the object weighs 10 lb, find the point force reactions on the two supporting surfaces (a) using a force polygon, and (b) using moment equations.

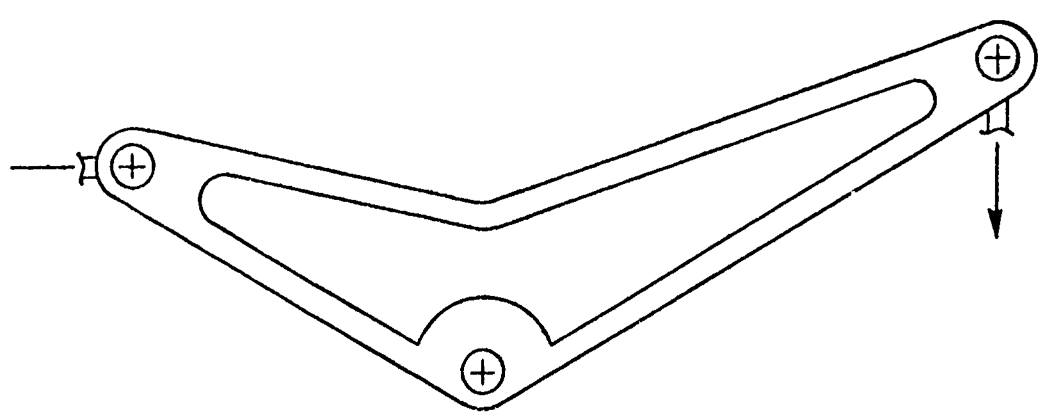
EGi



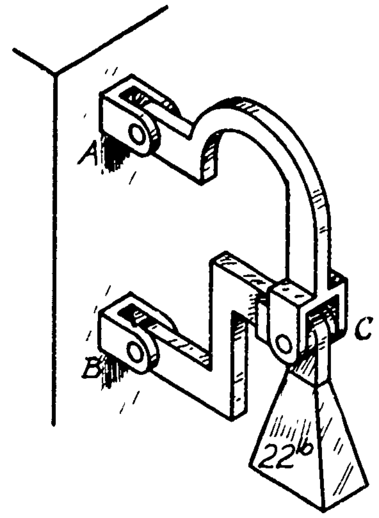


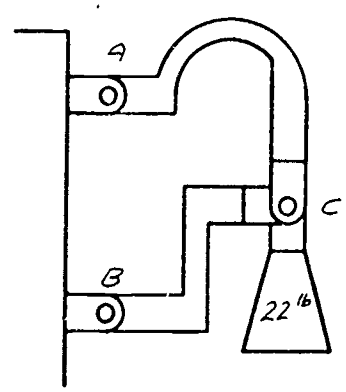
$1^{in} = 30^{lb}$

$1^{in} = 5^{in}$

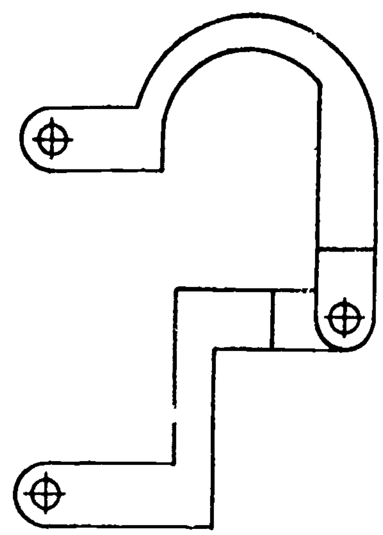


EQA - 1(B) Find the reaction on pin A using a force polygon. Check with a moment equation. The member is weightless and the pins are frictionless.



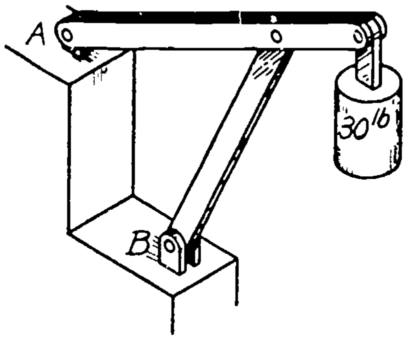


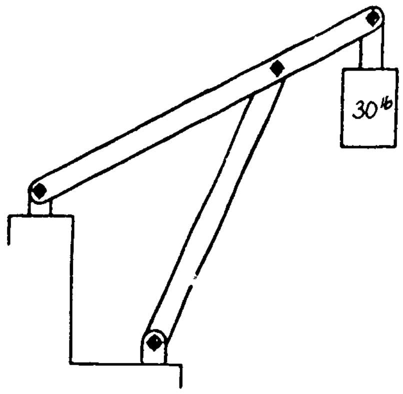
$1^{in} = 6^{in}$



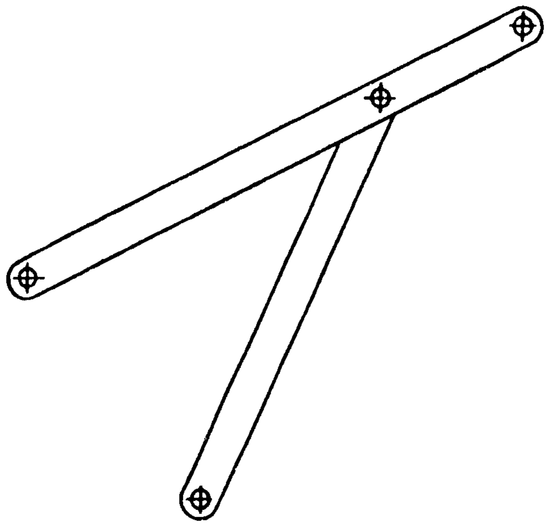
EQA - 2(A) Find the reactions at A and B using moment equations. Members AC and BC are weightless and pins A, B, and C are frictionless.

EQA

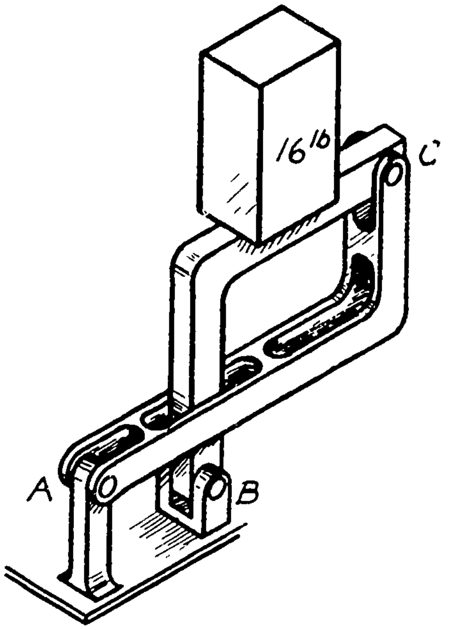


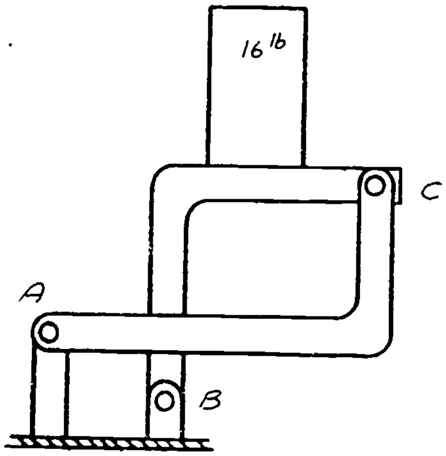


$1^{in} = 10^{lb}$

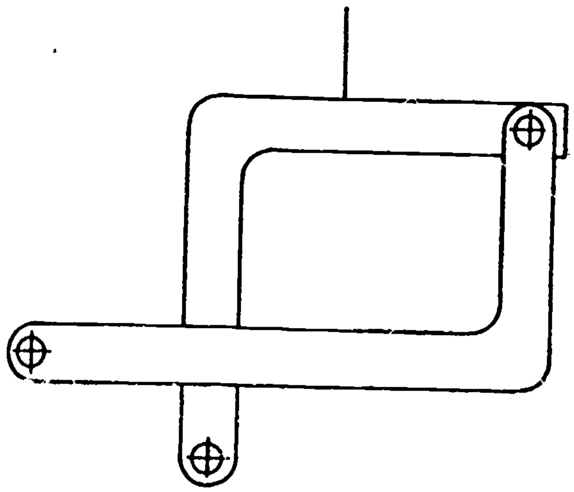


EQA - 2(B) If the members are weightless and the pins are frictionless, what are the reactions at A and B?



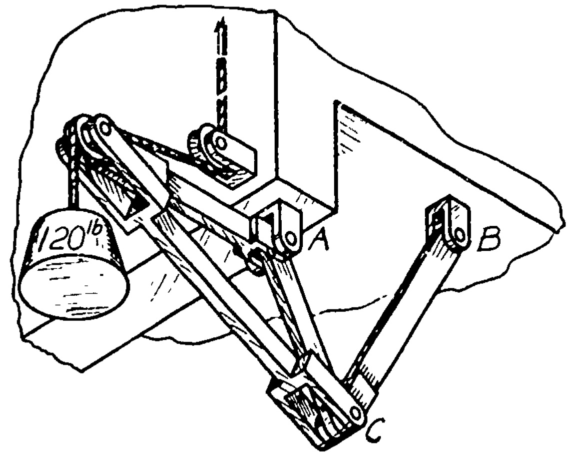


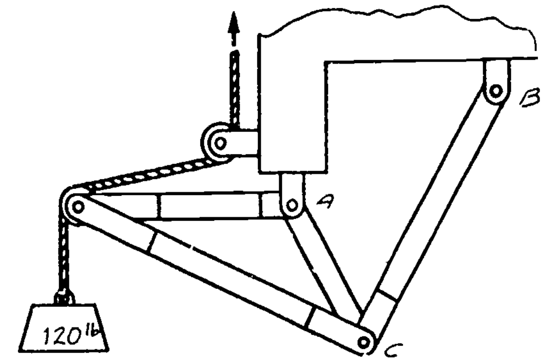
1 in = 5 in



EQA - 2(C) Find the reactions at A and B for the frame shown. State the assumptions needed in your solution.

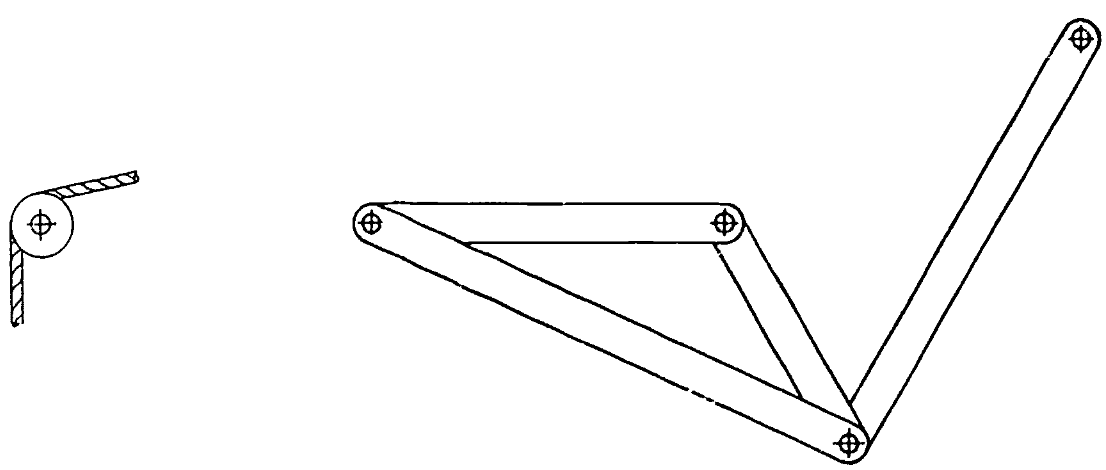
EQA



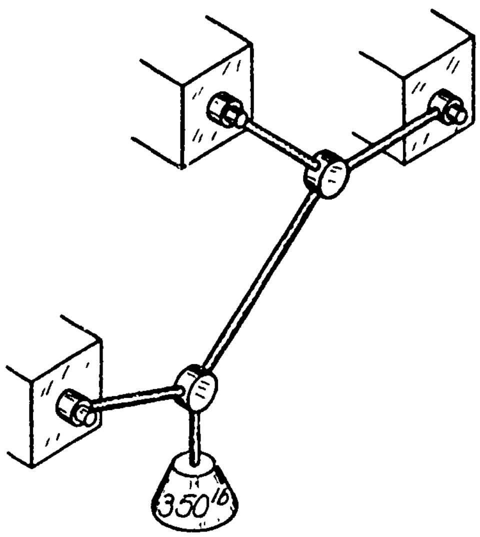


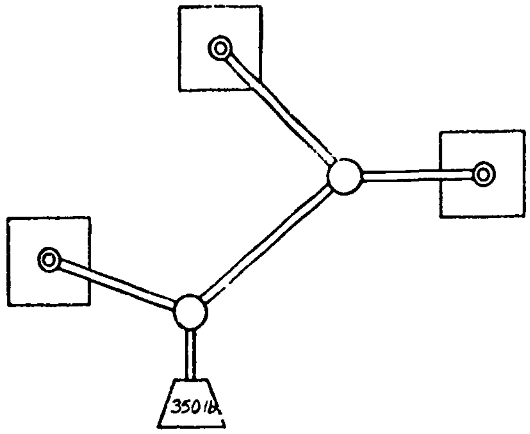
Assumptions: Weightless members and frictionless pins.

$1^{in} = 5^{in}$



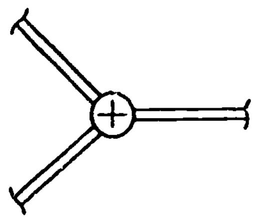
EQA - 2(D) Find the reaction at B and the direction of the reaction on the pin at A. What force is acting on member AC?

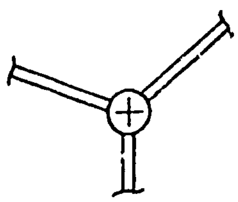




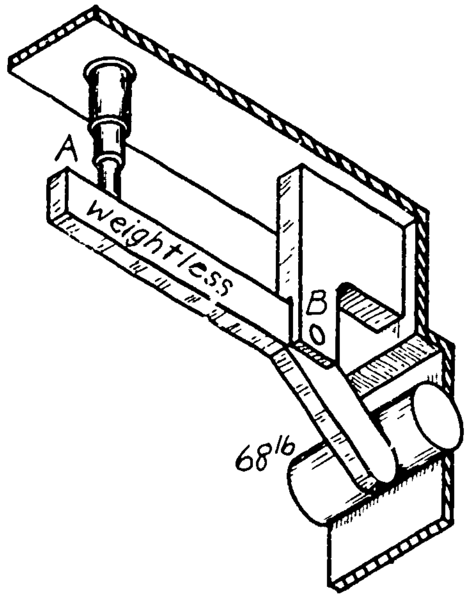
$1^{in} = 5^{in}$

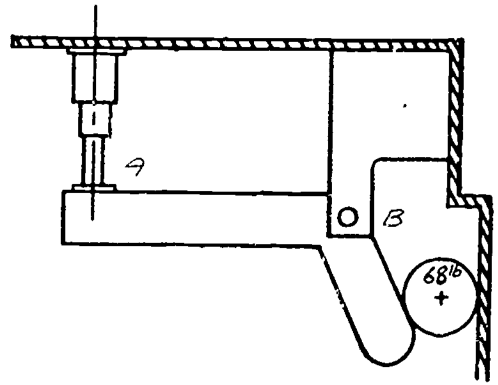
$1^{in} = 200^{lb}$





EQA - 3(A) Find the point forces acting on each weightless cord using one combined force diagram.

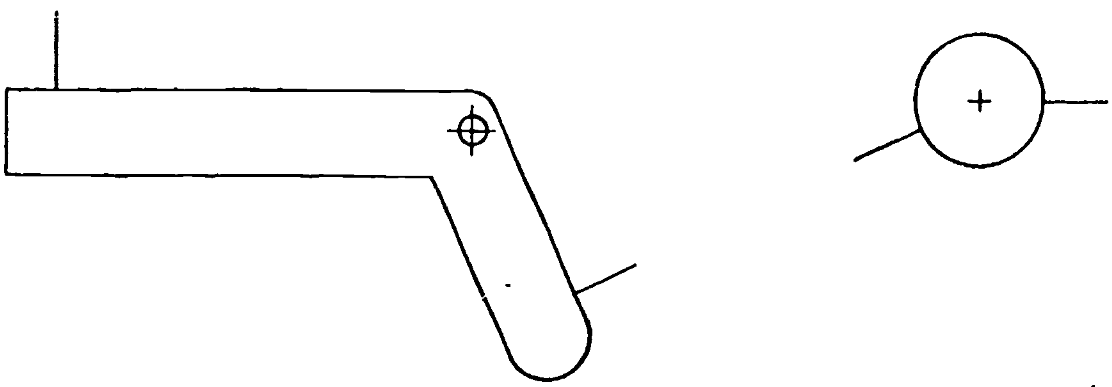




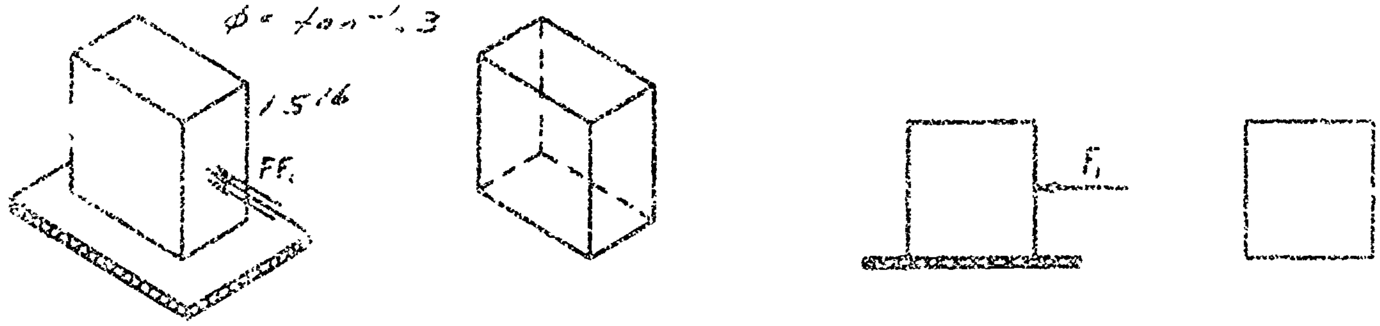
Pins and surfaces are frictionless.

$1^{in} = 60^{lb}$

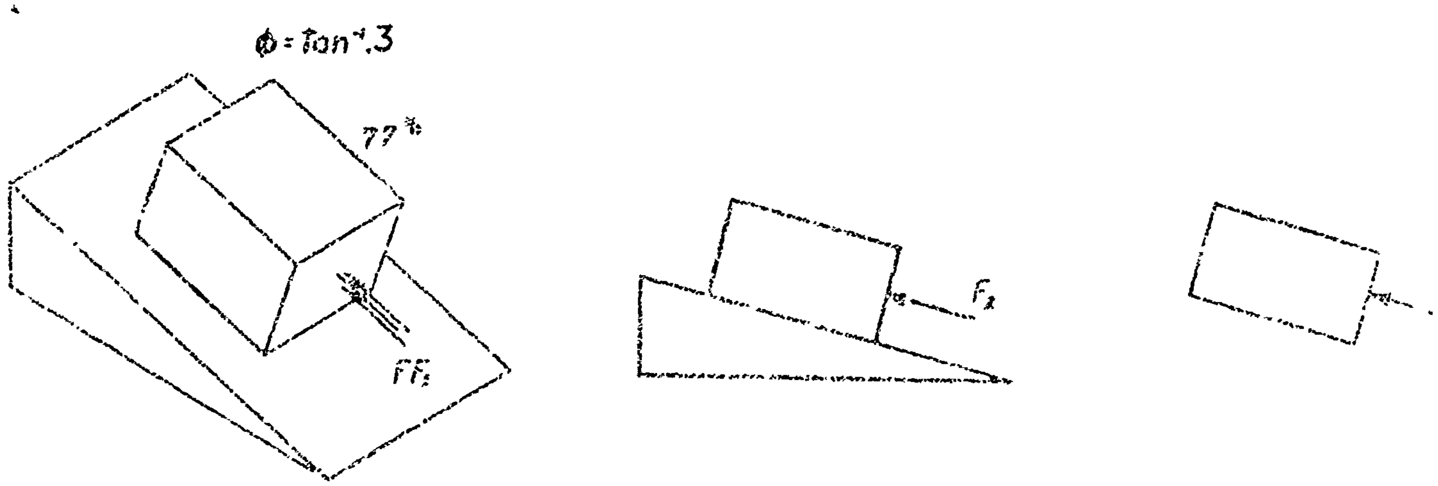
$1^{in} = 5^{in}$



EQA - 3(B) Find the reactions at A and B using one combined diagram.

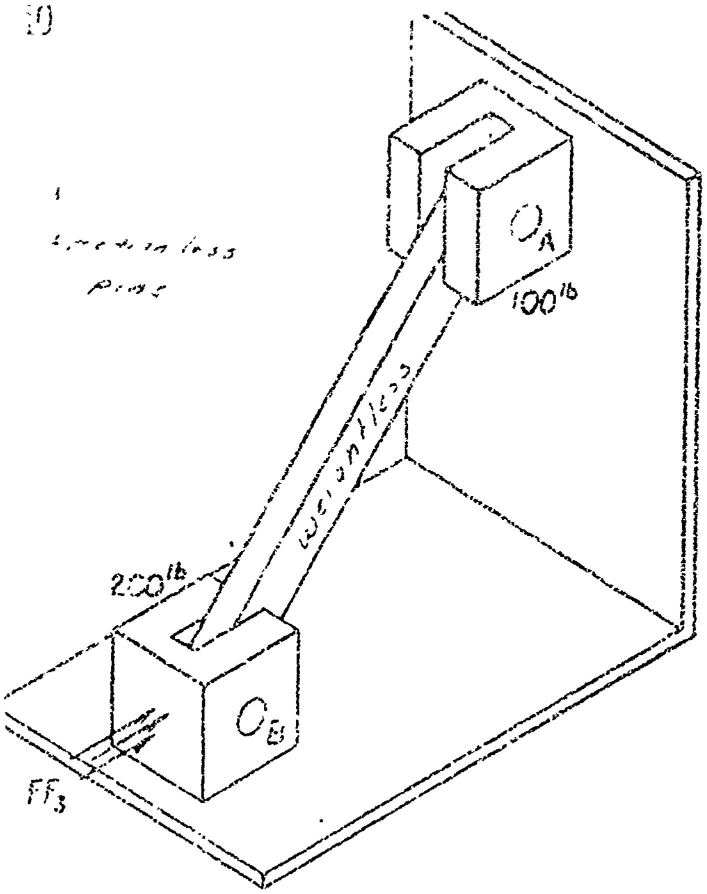


FR - 1(A) Find the magnitude of FF_1 needed to cause impending motion of the block. Sketch the acting force fields in (b).

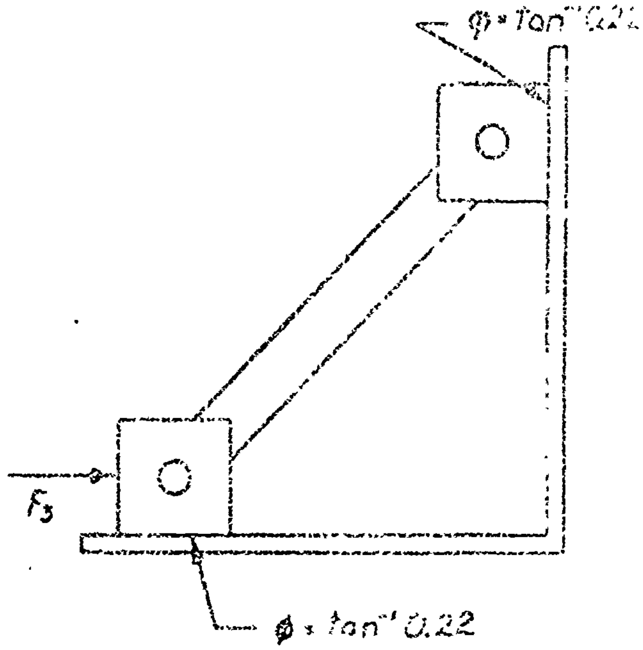


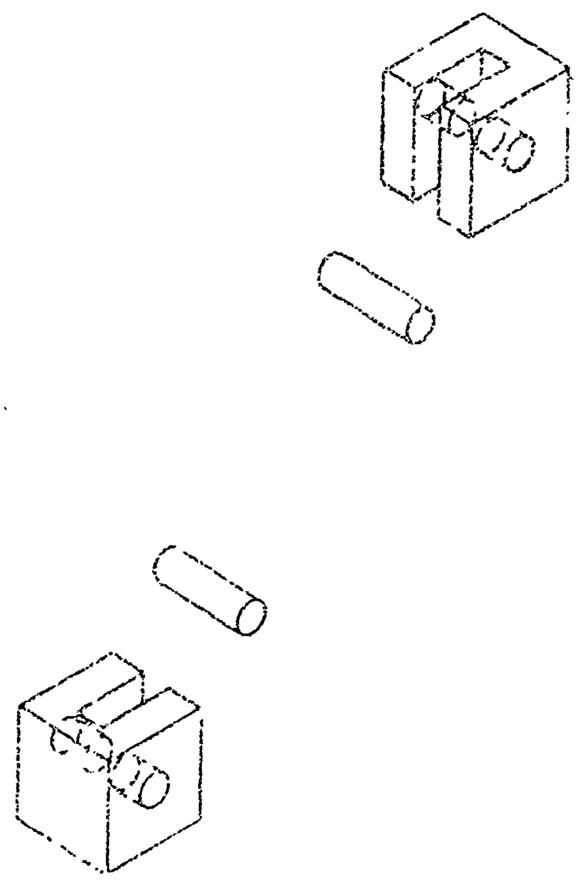
FR 1(B) Find the magnitude of F_2 for impending motion up the incline

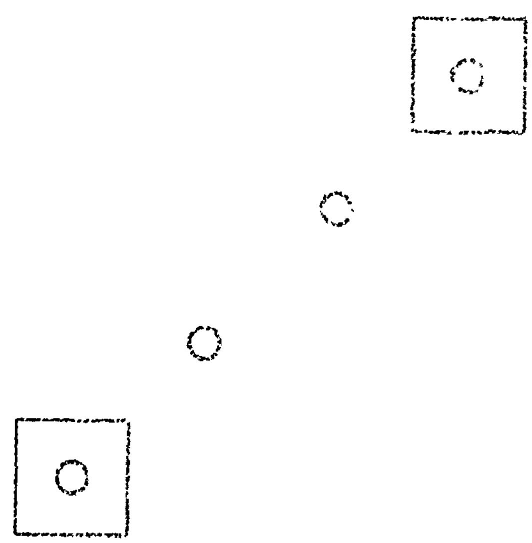
I



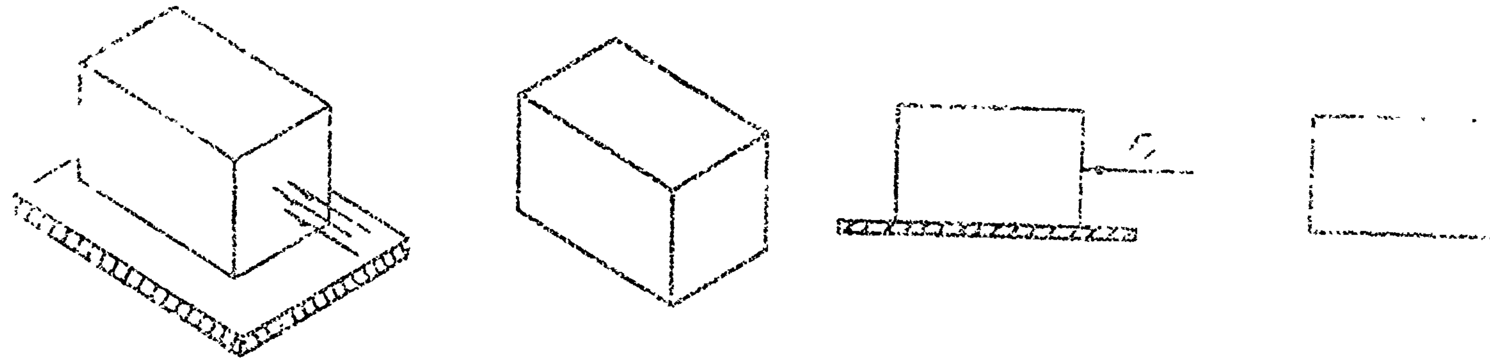
II



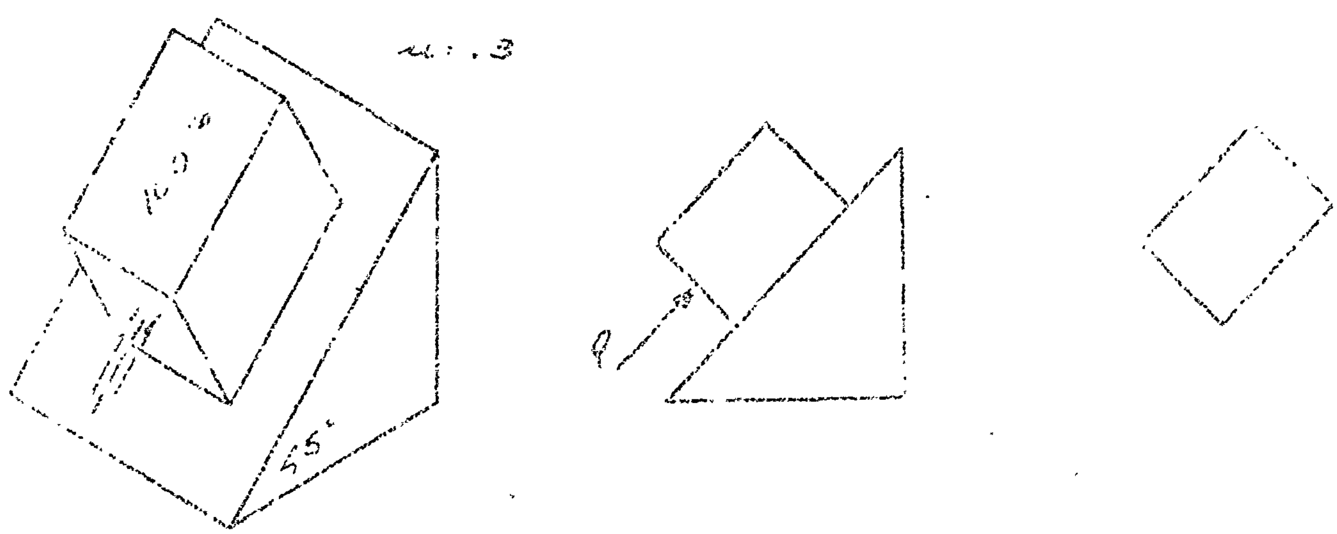




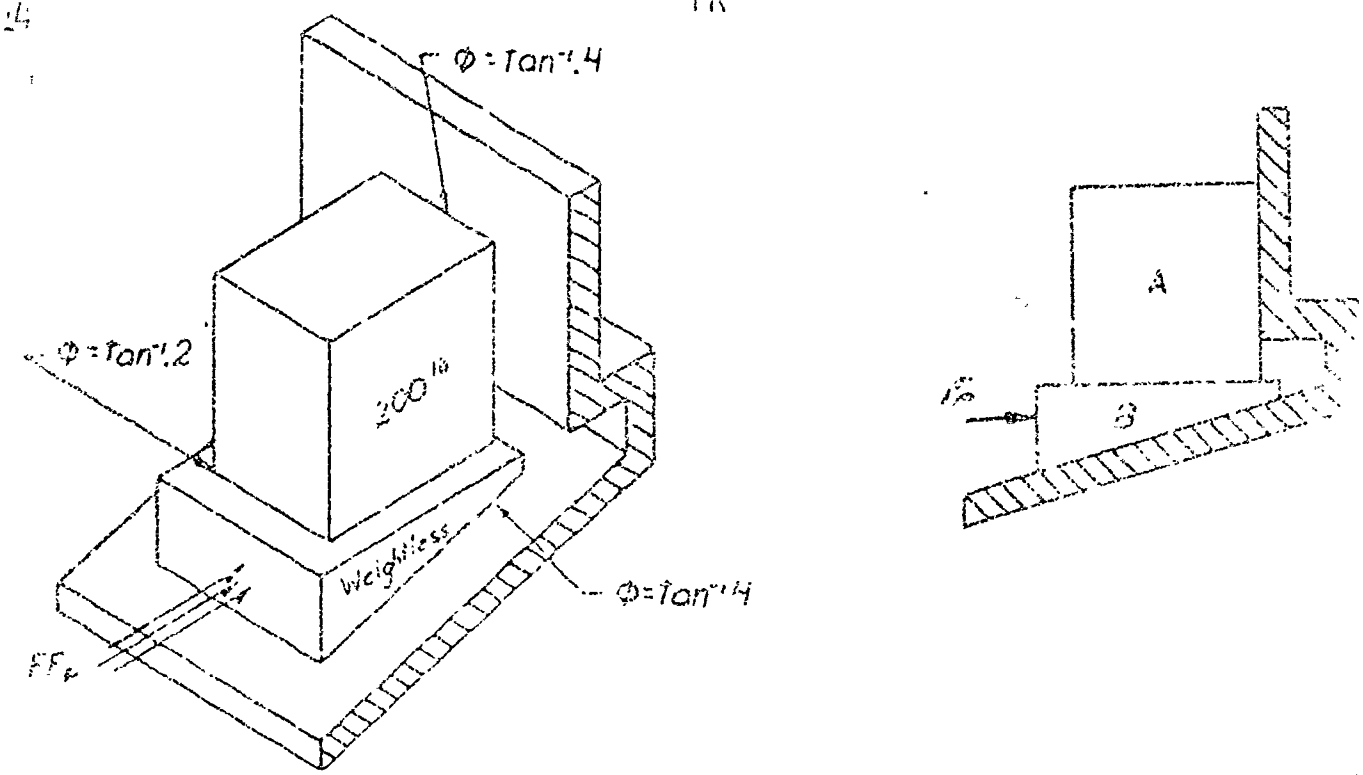
FR - 1(C) Find the magnitude of FF needed to cause impending motion in the assembly. Place all point forces on the [illegible]



FR - 3(A) Use components to find F_1 to give impending motion if $\mu = 0.45$ and the block weighs 64 lb. Place the components in the 2-D F-B diagram and the point force resultant in the 3-D F-B diagram.

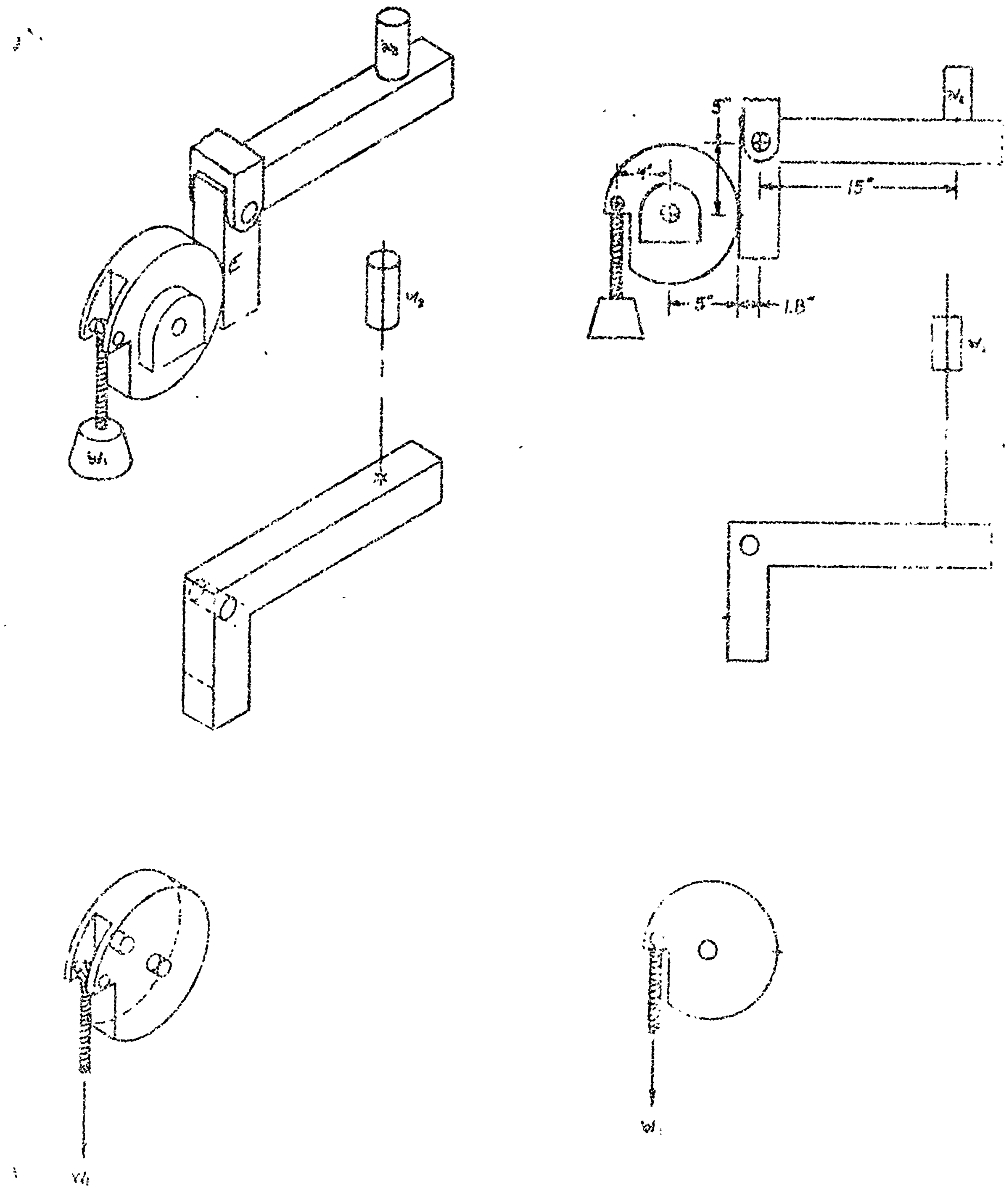


FR - 3(B) Find the minimum value [illegible] equilibrium. Use components [illegible] your solution and check [illegible] parallelogram



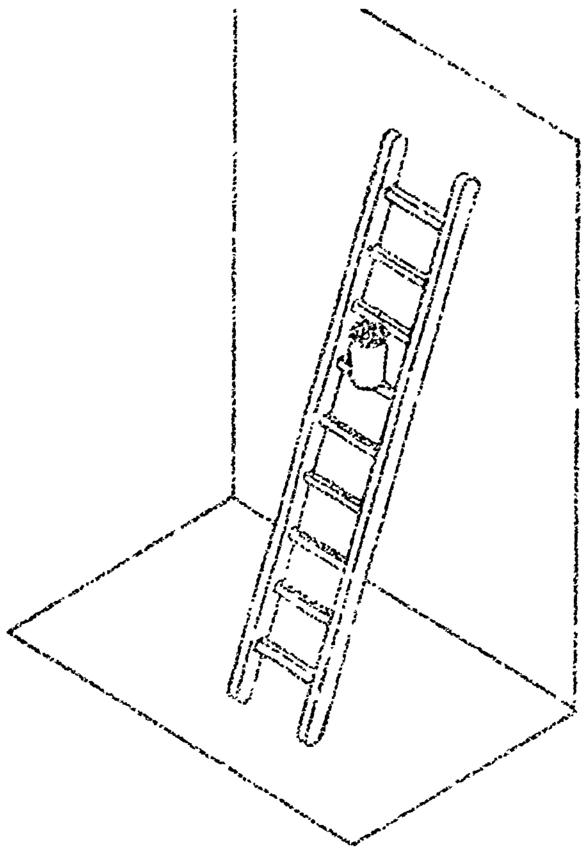
FR - 3(C)

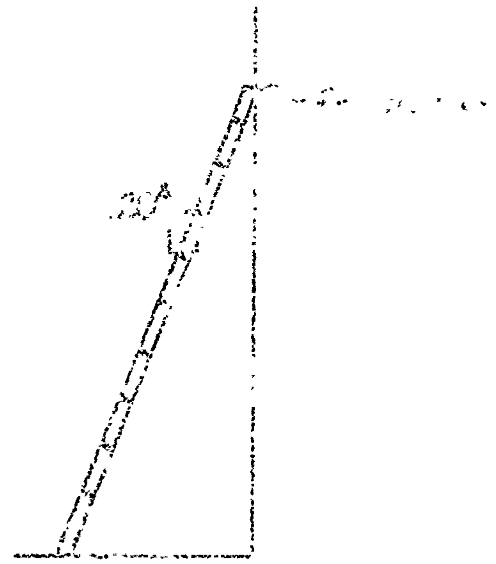
The 2-D drawing is to scale. Use components to find the magnitude of FF_p needed to cause impending motion of B to the right. Draw 2-D F.B. diagrams of B and A complete with all the components shown.

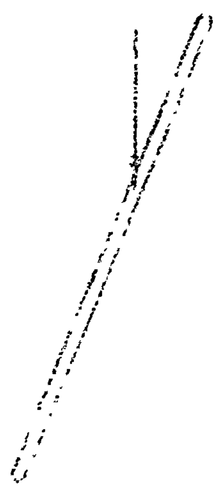


FR - 4(A) W_1 weighs 100 lb and μ at E = 0.3. Find the minimum value of [illegible] maintain equilibrium. Complete all the [illegible] diagrams [illegible]

[illegible]

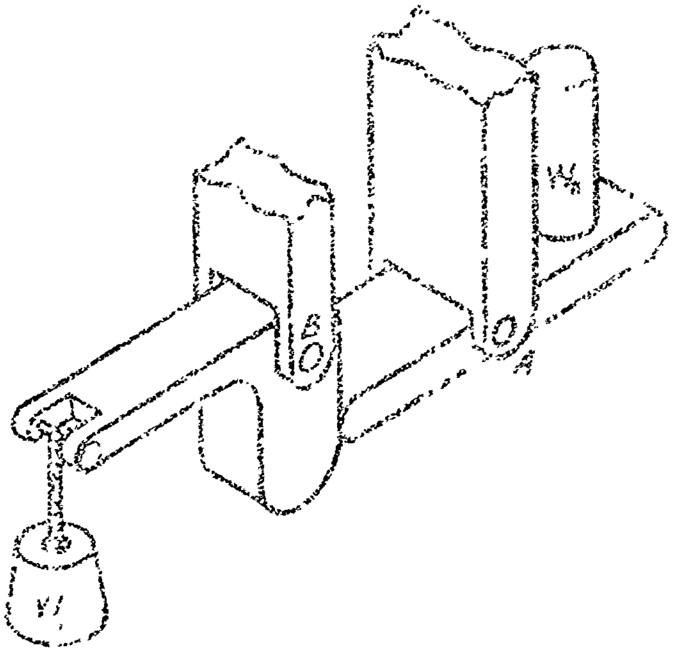


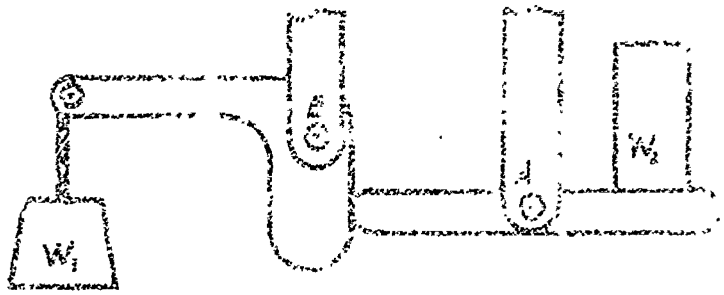


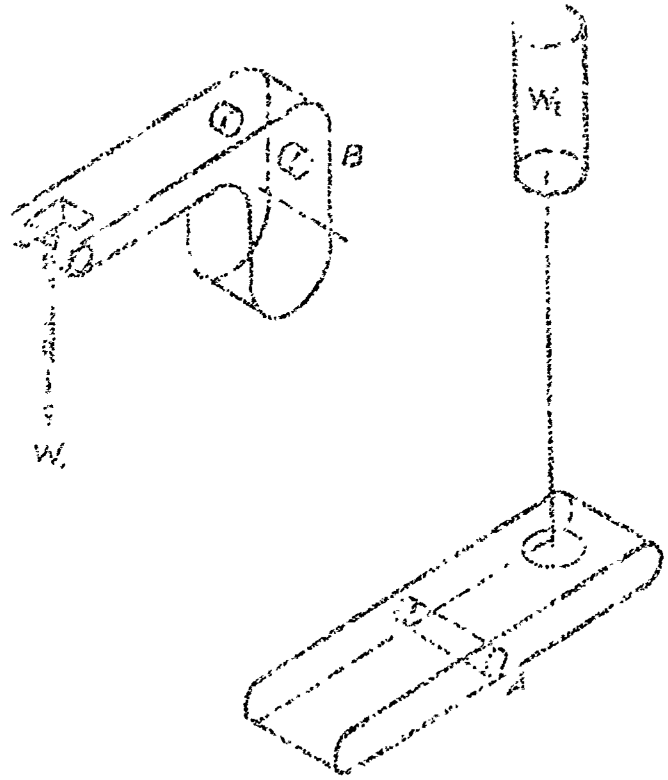


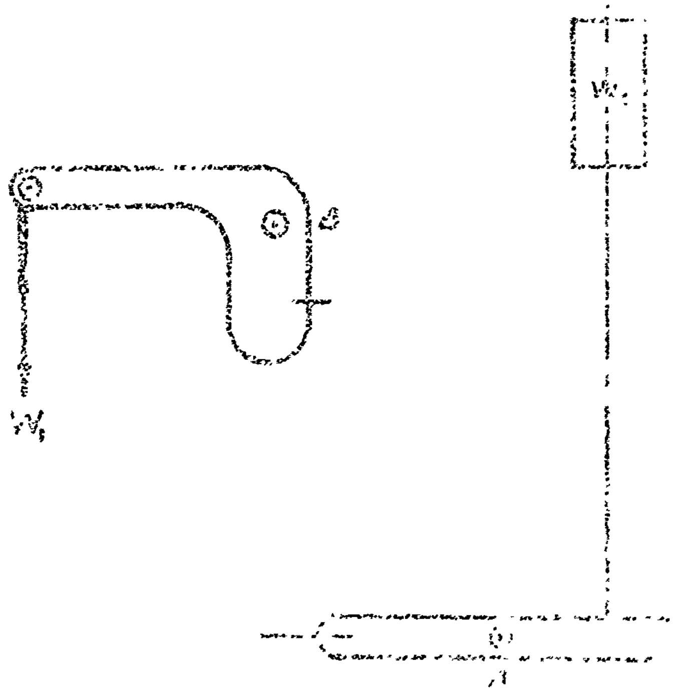
FR - 4(b)

If μ = [illegible] the weightless [illegible] equilibrium [illegible]
If not, what minimum value of μ [illegible] drawing [illegible]









III - 4(C) [illegible] weighs [illegible] minimum value [illegible] for equilibrium. Use free [illegible] diagrams [illegible]

UNIT 7

ENGINEERING FRAMES

AT THE END OF THIS UNIT, GIVEN A 3-D DIAGRAM OF AN ENGINEERING FRAME SUPPORTING A LOAD, YOU WILL BE ABLE TO FIND THE HORIZONTAL AND VERTICAL POINT FORCE COMPONENTS OF THE FORCE FIELDS ACTING ON THE CONTACT SURFACES OF ALL THE MEMBERS OF THE FRAME.

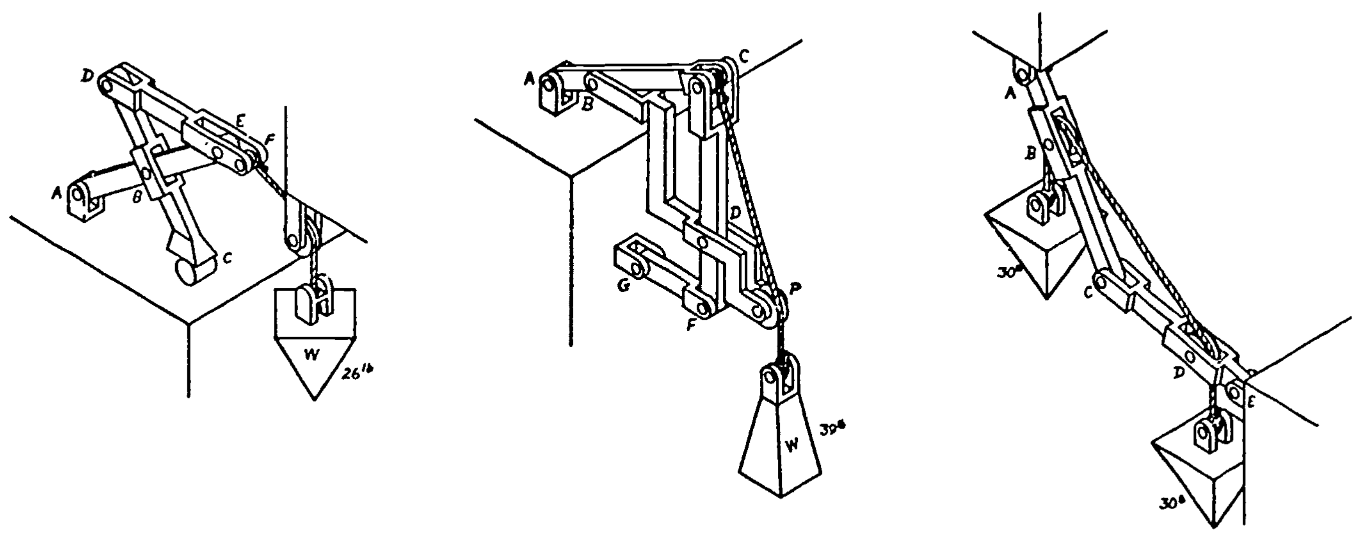


Figure EF 1

The three stationary loaded structures in figure EF 1 are engineering frames. Frames are used to support loads and are assumed to be made up of weightless and rigid 2-F and 3-F members joined with frictionless pins. The frames in this unit are symmetrical about a vertical plane which contains the centerline of each member of the frame including the load, the three frames then are called coplanar systems.

The horizontal and vertical point force components of the force fields acting upon each member including the pins of the three frames in figure EF 1 will now be found.

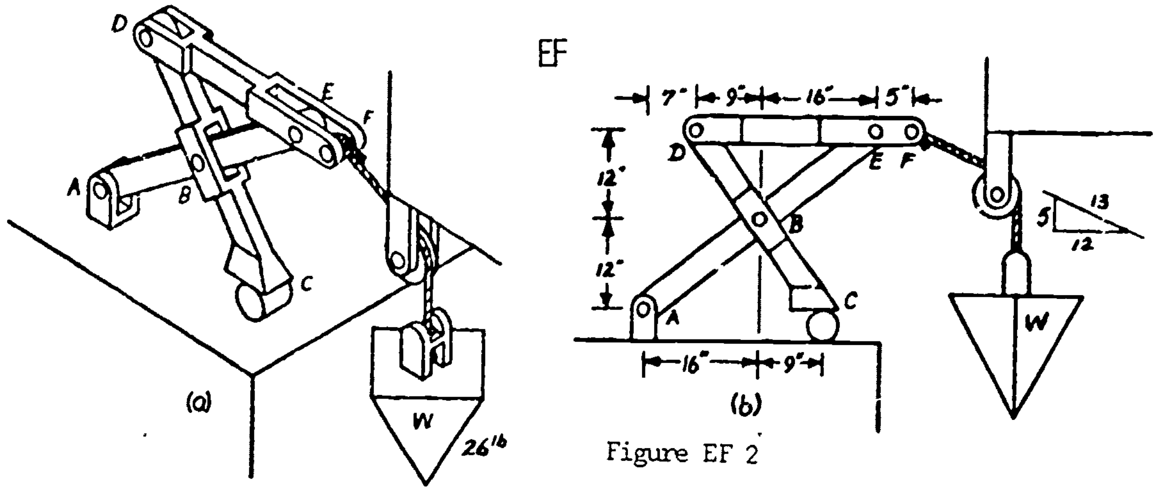


Figure EF 2

The first frame to be analyzed is drawn to scale in figure EF 2 in 3-D (a) and in 2-D (b) with dimensions. Next 2-D F-B diagrams of the whole frame and each member are drawn in figure EF 3. The first step in drawing a F-B diagram is to draw each member to scale without showing any components. At each contact surface a force field acts. Each force field is mentally replaced by a single point force and this point force is in turn replaced by horizontal and vertical components using the parallelogram law . Only these components are shown in the F-B diagrams. Although these components are abstract, it is easier to imagine that the components are actually active on the members.

Assume now that the F-B diagrams in EF 3(a) and (b) are drawn without any components shown. Each set of components at each contact surface will now be added to each F-B diagram. Arrowheads and magnitudes will be omitted except for the load until later, since it cannot be determined by inspection which direction the components are "pushing" against the members. The directions and magnitudes will be then found using arithmetic equations. It is only necessary at this time to show the lines of action of the components.

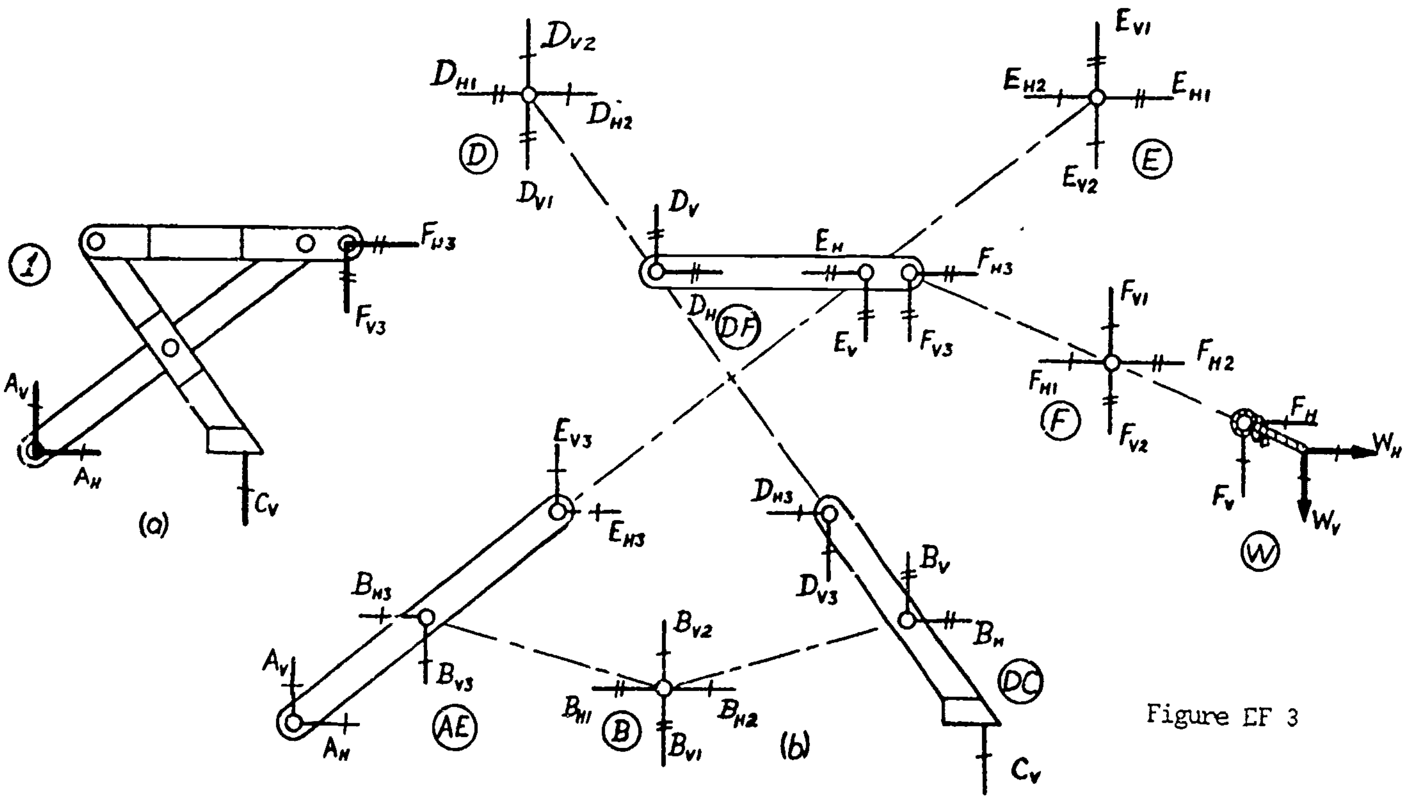


Figure EF 3

F-B (W) will be analyzed first. $\bar{W}_V$ and $\bar{W}_H$ act on the exposed surface of the weightless rope. Both $\bar{W}_V$ and $\bar{W}_H$ are single components and are given a single slash as shown. At the left end of (W) a single load acts. It is replaced by $\bar{F}_H$ and $\bar{F}_V$ but without arrowheads. $\bar{F}_V$ and $\bar{F}_H$ actually represent a single force field which acts against the rope, however, they are drawn as if they act through the ℄ of the removed pin F. $\bar{F}_H$ and $\bar{F}_V$ are single slashed. The lengths of the components need not be drawn to scale.

The F-B diagram for pin F is next. It has a load caused by the rope and two symmetrical loads caused by member DF. The load caused by the rope has two components which must be equal and opposite to $\bar{F}_H$ and $\bar{F}_V$ on (W) . These are drawn as $\bar{F}_{H1}$ and $\bar{F}_{V1}$ and act through the ℄ of the pin. $\bar{F}_{V2}$ and $\bar{F}_{H2}$ are double slashed to show that they actually represent two force fields. Figure EF 3(c) is an exploded isometric drawing of the joint at pin F and clearly shows how pin F is loaded by one force field caused by the rope and by two force fields from member DF.

F-B (DF) has a set of double components $\bar{F}_{H3}$ and $\bar{F}_{V3}$ that are equal and opposite to $\bar{F}_{H2}$ and $\bar{F}_{V2}$ on pin F. $\bar{E}_V$ and $\bar{E}_H$ are caused by AE acting through the removed pin E. At pin D double slashed point force components $\bar{D}_V$ and $\bar{D}_H$ are caused by member DC.

It is not possible to determine whether the components come from single or double force fields with 2-D drawings, you must refer to the 3-D diagram figure EF 2(a).

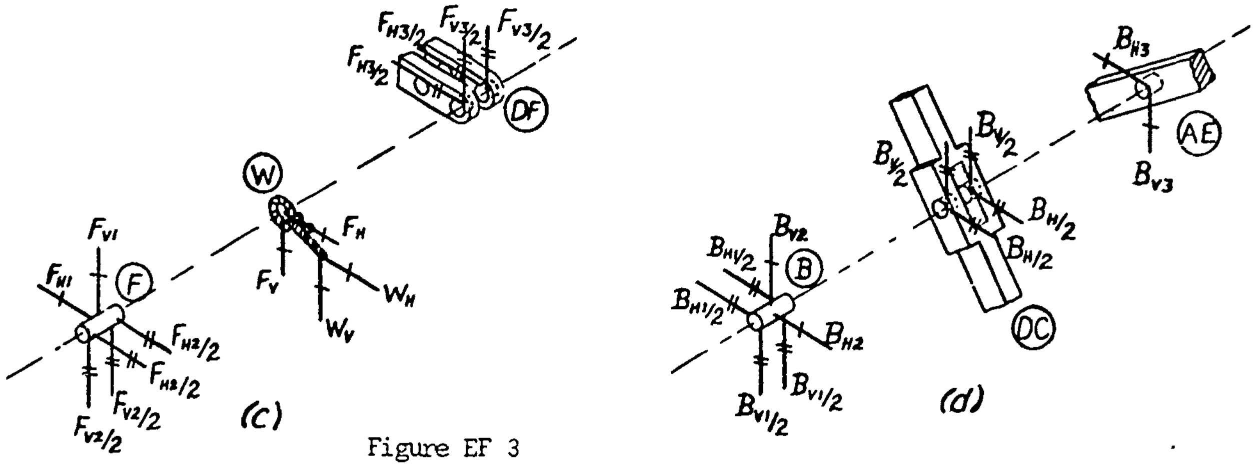


Figure EF 3

The point force components $\bar{C}_V$, $\bar{A}_V$, and $\bar{A}_H$ can now be placed in F-B diagram (1) in figure EF 3(a). $\bar{C}_V$, $\bar{A}_V$, and $\bar{A}_H$ are all single slashed.

Following the above procedure, you should now be able to analyze the rest of the F-B diagrams yourself. The joint at pin B is shown in isometric in figure EF 3(d) to help you. Four things should be apparent to you in your study of the F-B diagrams: (1) all the loads are replaced by vertical and horizontal components, (2) the H and V components are left without arrowheads except the external load and are not drawn with their lengths to scale, (3) wherever a component represents a single force field, it is single slashed, and wherever a component actually represents two force fields, it is double slashed, (4) the loads between connecting members (usually members only connect with pins) are equal and opposite force fields, so their sets of components are equal and opposite. For instance, $\bar{B}_V$ and $\bar{B}_H$ on DC are equal and opposite to $\bar{B}_{V1}$ and $\bar{B}_{H1}$ on pin B, and $\bar{B}_{H3}$ and $\bar{B}_{V3}$ on AE are equal and opposite to $\bar{B}_{H2}$ and $\bar{B}_{V2}$ on pin $\bar{B}$.

4 EF

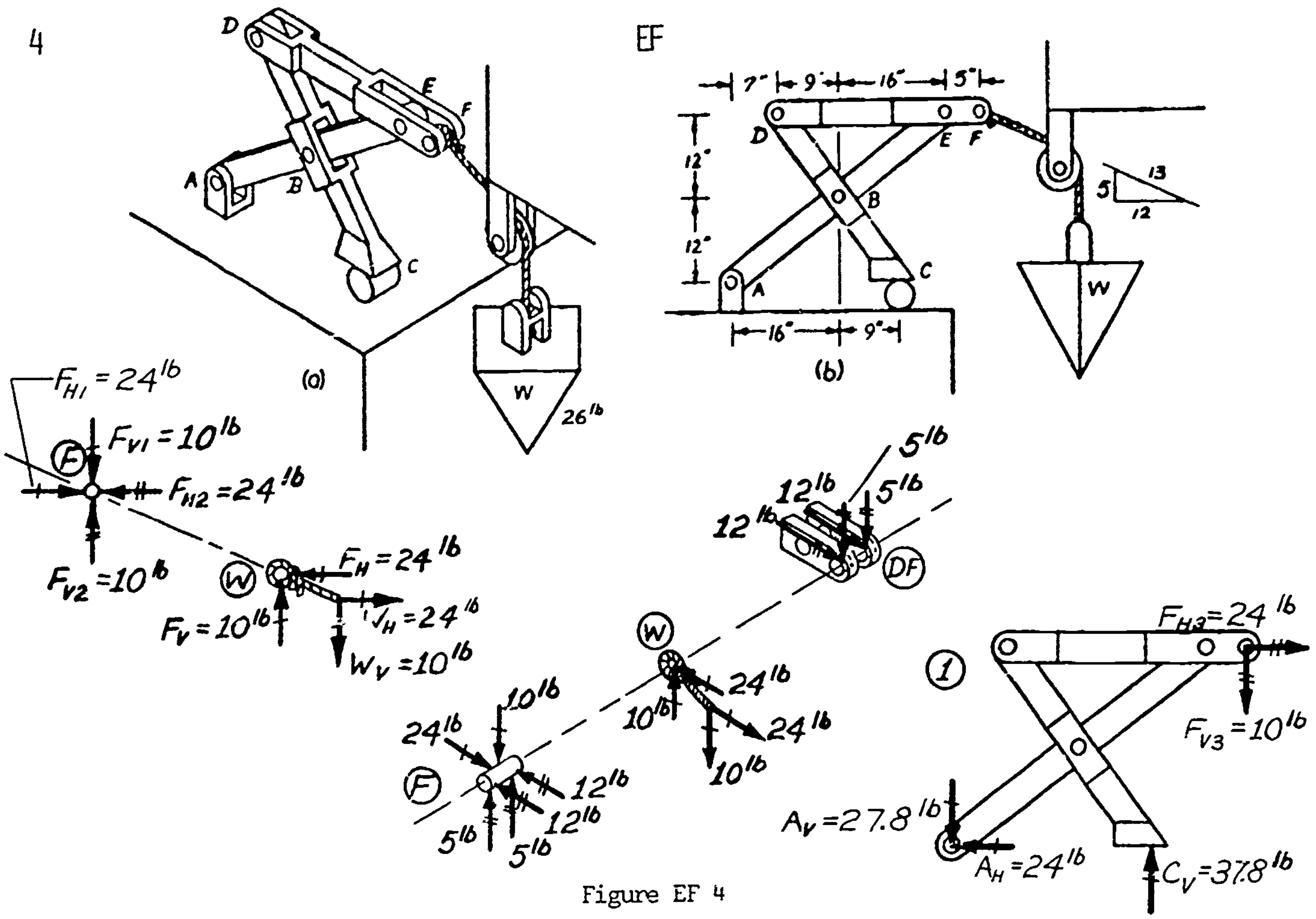


Figure EF 4

The frame and all the F-B diagrams are redrawn in figures EF 4 and EF 5. The magnitudes and senses of all the components will now be found using the parallelogram law in force and moment equation form. As the components on each F-B diagram are found, their magnitudes and senses will be placed on the drawings.

F-! (W) $\frac{W}{13} = \frac{W_H}{12} = \frac{W_V}{5}$ $W_V = 10$ lb $W_H = 24$ lb

$\Sigma F_X = 0$ $F_H = 24$ lb $\Sigma F_Y = 0$ $F_V = 10$ lb

The arrowheads and magnitudes are now placed on the F-B diagrams.

F-B (F) $\bar{F}_{V1}$ and $\bar{F}_{H1}$ are equal and opposite to $\bar{F}_V$ and $\bar{F}_H$ on W, so

$F_{V1} = 10$ lb $F_{H1} = 24$ lb

ΣF_X and $\Sigma F_Y = 0$ $F_{H2} = 24$ lb $F_{V2} = 10$ lb

F-B diagram (F) can now be completed.

F-B (1) $\bar{F}_{H3}$ and $\bar{F}_{V3}$ are equal and opposite to $\bar{F}_{H2}$ and $\bar{F}_{V2}$ on F. ($\bar{F}_{H3}$ and $\bar{F}_{V3}$ "push" against (1) but can be shown "pulling")

$\Sigma M_A = 0$ (A is the center of pin A.)

$(37)(F_{V3}) + (24)(F_{H3}) = (25)(C_V)$

$(37)(10) + (24)(24) = (25)(C_V)$ $C_V = 37.8$ lb

ΣF_X and $\Sigma F_Y = 0$ $A_H = 24$ lb $A_V = 27.8$ lb

F-B (DF) F_{H3} = 24 lb F_{V3} = 10 lb $\Sigma M_D = 0$ (D is the center of pin D.)

$(30)(F_{V3}) = (25)(E_V)$ $(30)(10) = (25)(E_V)$ E_V = 12 lb

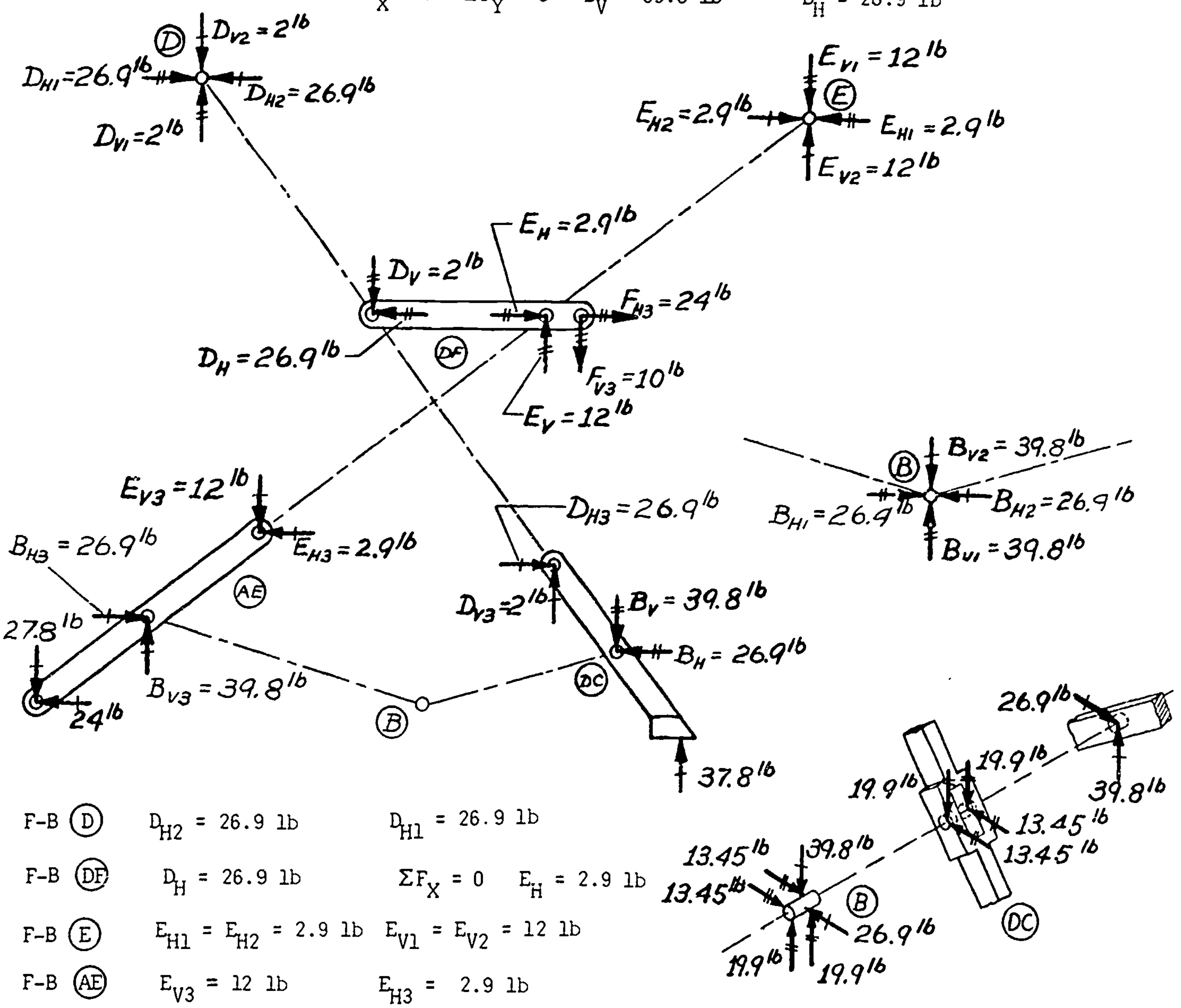
$\Sigma F_Y = 0$ D_V = 2 lb D_H and E_H cannot be found yet.

F-B (D) D_{V1} = 2 lb D_{V2} = 2 lb D_{H1} and D_{H2} are unknown now.

F-B (DC) D_{V3} = 2 lb $\Sigma M_B = 0$ $(9)(D_{V3}) + (12)(D_{H3}) = (9)(C_V)$

$(9)(2) + (12)(D_{H3}) = (9)(37.8)$ D_{H3} = 26.9 lb

ΣF_X and $\Sigma F_Y = 0$ B_V = 39.8 lb B_H = 26.9 lb



F-B (D) D_{H2} = 26.9 lb D_{H1} = 26.9 lb

F-B (DF) D_H = 26.9 lb $\Sigma F_X = 0$ E_H = 2.9 lb

F-B (E) $E_{H1} = E_{H2}$ = 2.9 lb $E_{V1} = E_{V2}$ = 12 lb

F-B (AE) E_{V3} = 12 lb E_{H3} = 2.9 lb

ΣF_X and $\Sigma F_Y = 0$ B_{H3} = 26.9 lb B_{V3} = 39.8 lb

Figure EF 5

F-B (B) B_{H1} = 26.9 lb B_{V1} = 39.8 lb both from (DC)

B_{H2} = 26.9 lb B_{V2} = 39.8 lb both from (AE)

$\Sigma F_X = 0$ $\Sigma F_Y = 0$ Checks.

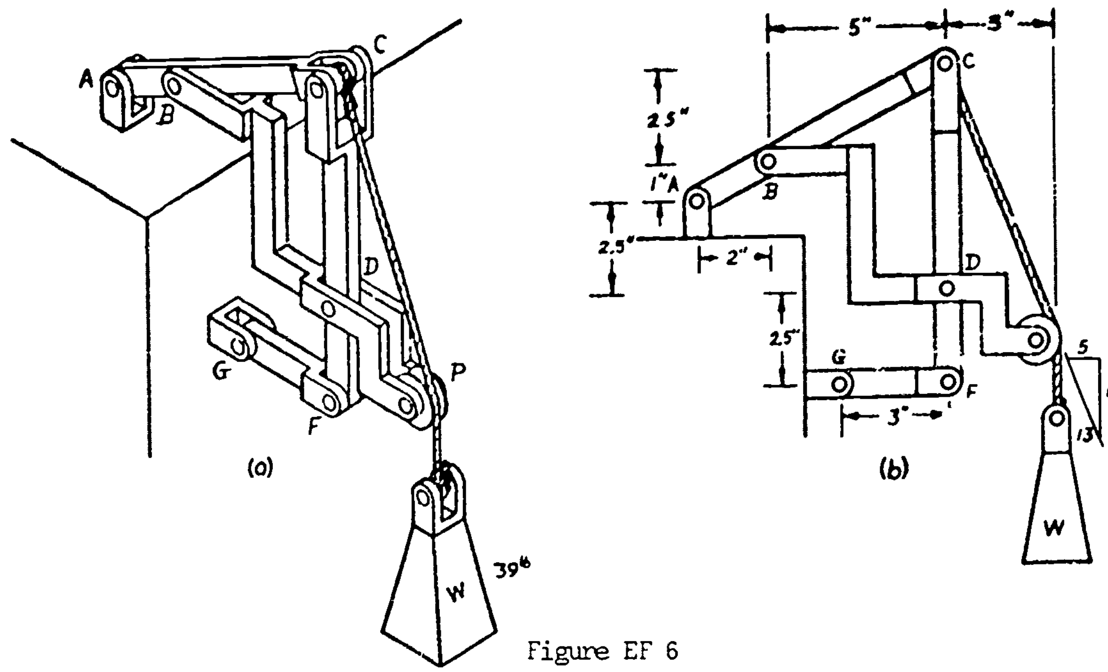


Figure EF 6

The second frame from EF 1 is redrawn in EF 6(a) and (b). Again the frame is considered to be rigid and weightless, the pins frictionless, and the frame and all its members symmetrical about a vertical plane through the ℄ of the rope. The components of the loads acting on all the members are to be found. F-B diagrams are drawn in EF 7 and EF 8. Pin C is shown in isometric with its connecting members in EF 8(a).

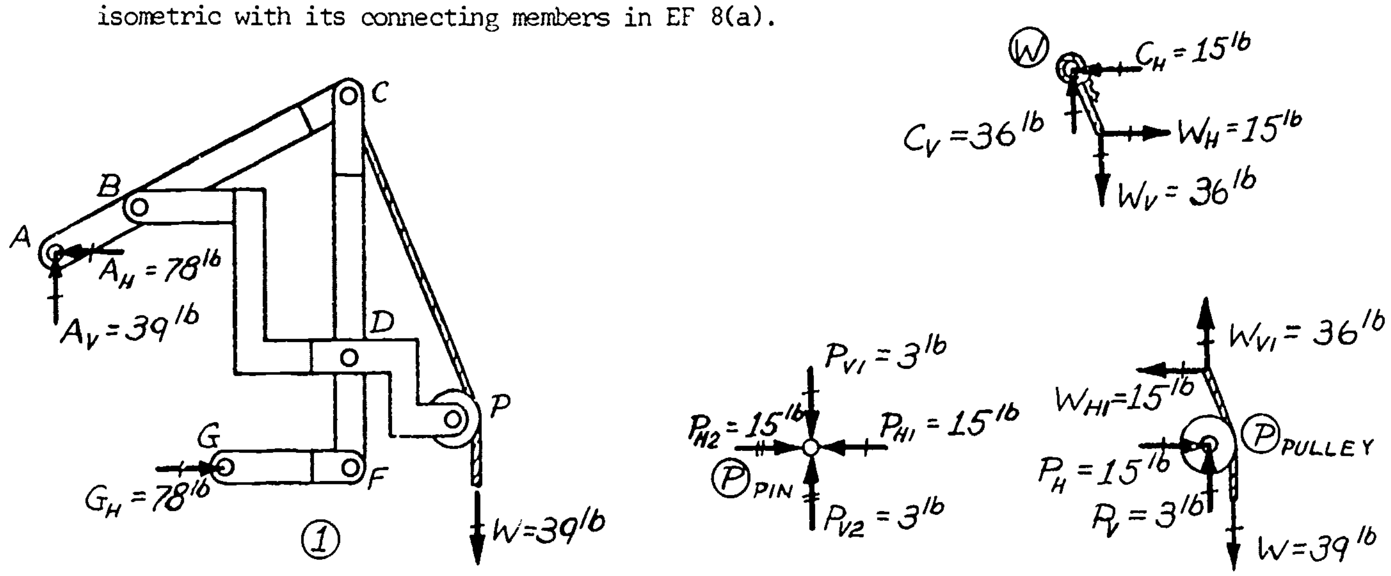


Figure EF 7

(1) GF is a 2-f member.

$\Sigma M_A = 0 \qquad (10)(39) = (5)(G_H) \qquad G_H = 78$ lb

$A_V = 39$ lb $\qquad A_H = 78$ lb

(W) It is known by inspection that $\frac{W}{13} = \frac{W_V}{12} = \frac{W_H}{5}$

(P) pulley $\quad W_{V1} = 36$ lb $\qquad W_{H1} = 15$ lb $\qquad P_V$ and P_H are found by inspection

(P) pin $\quad$ By inspection

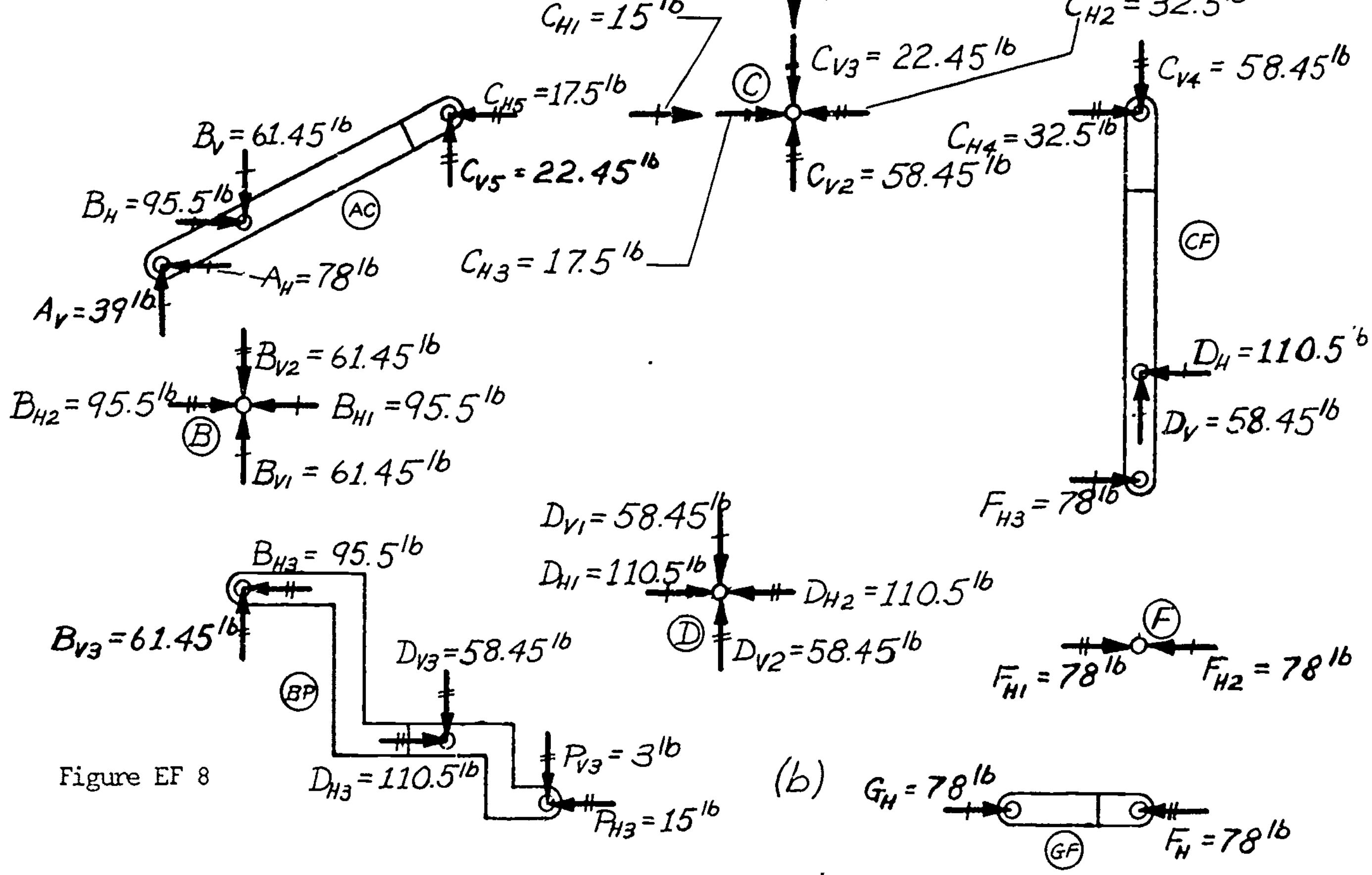
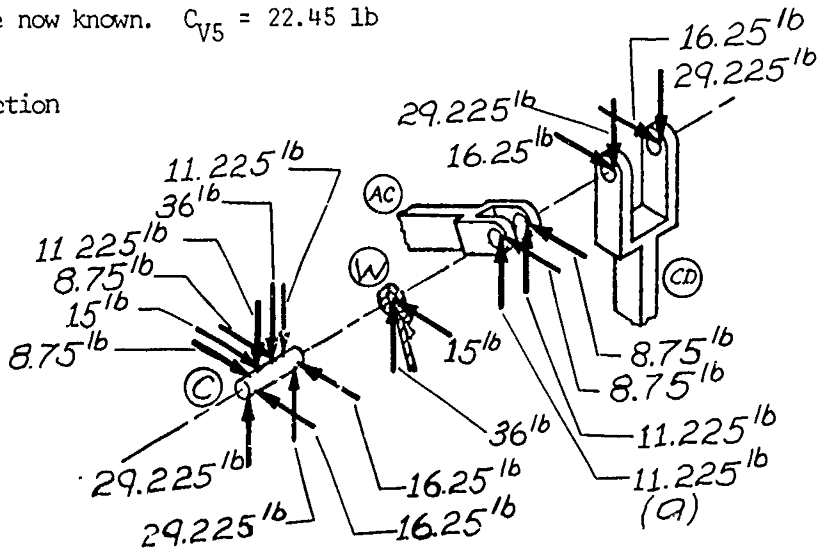


Figure EF 8

(GF) & (F)	By inspection
(CF)	$\Sigma M_C = 0 \qquad (8.5)(78) = (6)(D_H) \qquad D_H = 110.5$ lb
	$C_{H4} = 32.5$ lb $\qquad D_V$ and C_{V4} cannot be found yet.
(C)	C_{V1} and C_{H1} are equal and opposite to C_V and C_H in (W)
	$C_{H1} = 15$ lb $\qquad C_{V1} = 36$ lb
	C_{H2} is equal and opposite to C_{H4} from (CF) $\quad C_{H2} = 32.5$ lb $\quad C_{H3} = 17.5$ lb
(AC)	$C_{H5} = 17.5$ lb $\quad \Sigma M_B = 0 \quad (2.5)(17.5) + (5)(C_{V5}) = (1)(78) + (2)(39)$
	B_V and B_H are now known. $C_{V5} = 22.45$ lb
(B), (C), (CF), (D), (BV)	All by inspection
(BP)	Checks



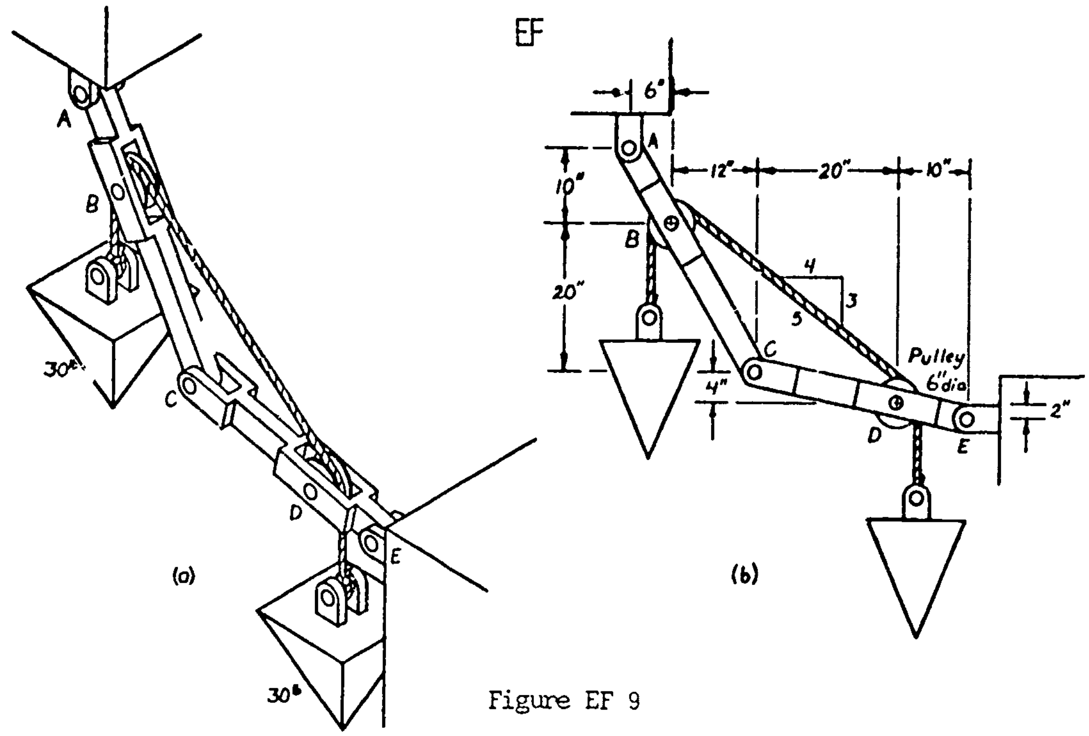


Figure EF 9

The third frame is redrawn in figure EF 9, and F-B diagrams of all the members are shown in figure EF 10. The horizontal and vertical components on pulleys D and B and pins D and B can be found as before. However, when F-B diagrams (1), (AC), and (CE) are drawn, no moment equation can be written that will have only one unknown. On F-B (1) a moment equation $\Sigma M_A = 0$ will relate E_H and E_V. On F-B (CE) $\Sigma M_C = 0$ will also relate E_H and E_V. These two equations with two unknowns can then be solved. The procedure is shown below.

(D)	pulley	$W_V = 18$ lb	$W_H = 24$	Rest by inspection.
(L)	pin	All known now.		
(B)	pulley	$B_V = 48$	$B_H = 24$	By inspection
(B)	pin	Known now.		
(AC)		B_{H3} and B_{V3} known from pin B.		
(CE)		D_{V3} and D_{H3} known from pin D.		

(1) $\Sigma M_A = 0$

$$(3)(30) + (41)(30/ = (36)(E_H) + (48)(E_V)$$

(CE) $\Sigma M_C = 0$

$$0)(12) + (4)(24) = (6)(E_H) + (30)(E_V)$$

Slight rearrangement of the above equations yields

$6\,E_H + 8\,E_V = (3)(5) + (41)(5) = 220$ from (1), and

$6\,E_H + 30\,E_V = (20)(12) + (4)(24) = 336$

Simultaneous solution of these equations gives

$E_V = 5.3$ lb and $E_H = 29.6$ lb

All the F-B diagrams can now be completed and the actual loads at each contact surface can be found. For instance, $(E_V^2 + E_H^2)^{1/2} = E$, where E is the magnitude of the force field acting at constraint E.

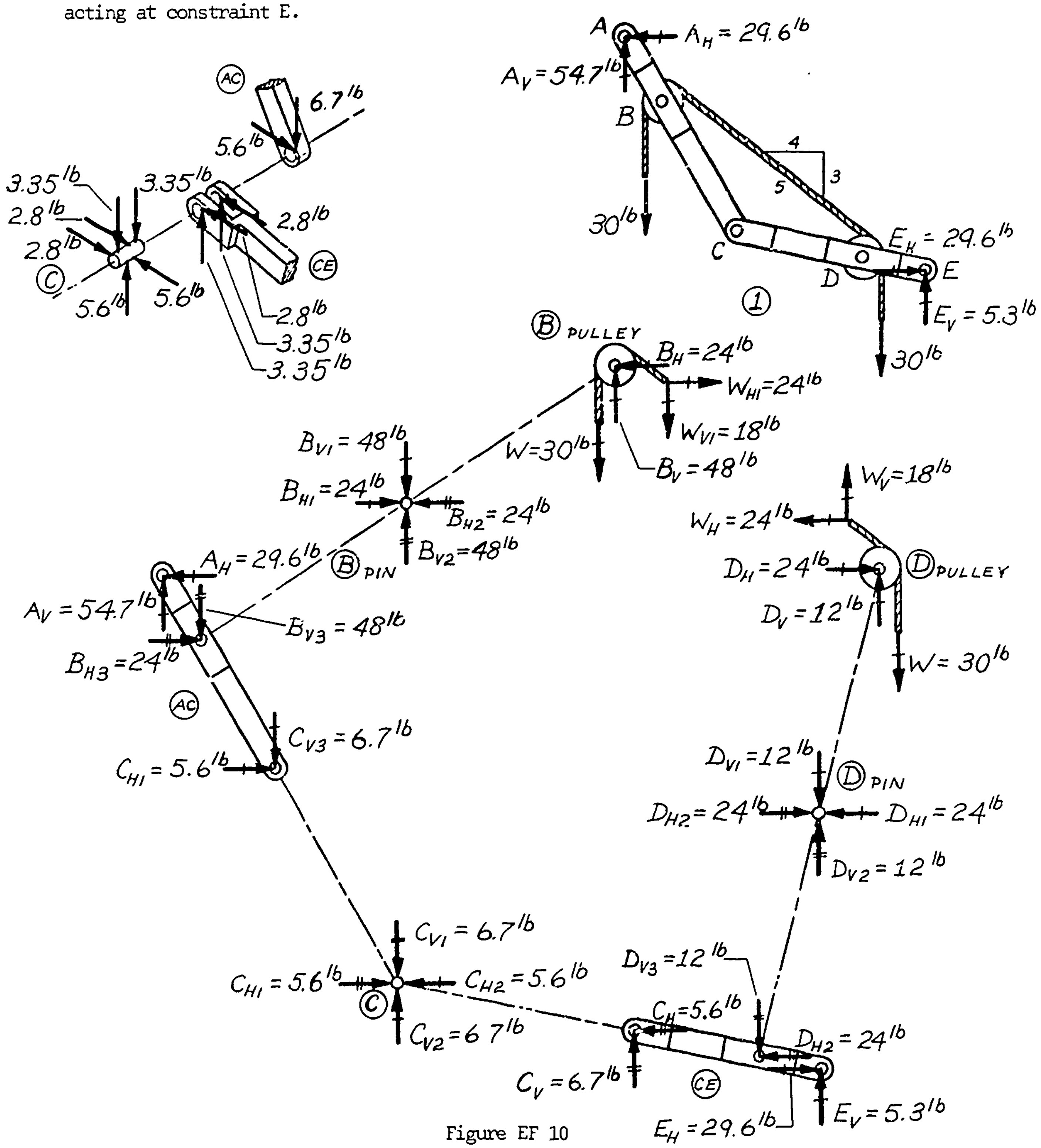


Figure EF 10

NOW IF YOU ARE GIVEN A LOADED ENGINEERING FRAME, YOU SHOULD BE ABLE TO FIND THE POINT FORCE COMPONENTS OF THE FORCE FIELDS ACTING UPON EVERY MEMBER OF THE FRAME.

UNIT 10

TRUSSES

AT THE END OF THIS UNIT IF YOU ARE GIVEN A SPACE DIAGRAM OF A TRUSS, YOU SHOULD BE ABLE TO SOLVE FOR THE FORCES IN THE TRUSS MEMBERS USING (1) THE METHOD OF JOINTS, (2) THE METHOD OF SECTIONS, AND (3) A COMBINED DIAGRAM.

Introduction

A framework composed of I-beams, channels, .s, bars, and other special shaped engineering members that are joined only at their ends to form a rigid structure is called a truss. Bridges, roof supports, and cranes are usually made up of combinations of trusses. Each truss is essentially a series of connected triangular shapes. In this unit only coplanar trusses will be considered. An example of a typical coplanar truss is shown in figure TR 1(a). The bridge actually consists of two coplanar trusses joined by the cross beams. The bridge deck has been omitted in (b) to show how the members supporting it transfer the loads from the deck to the bottom joints of the two trusses.

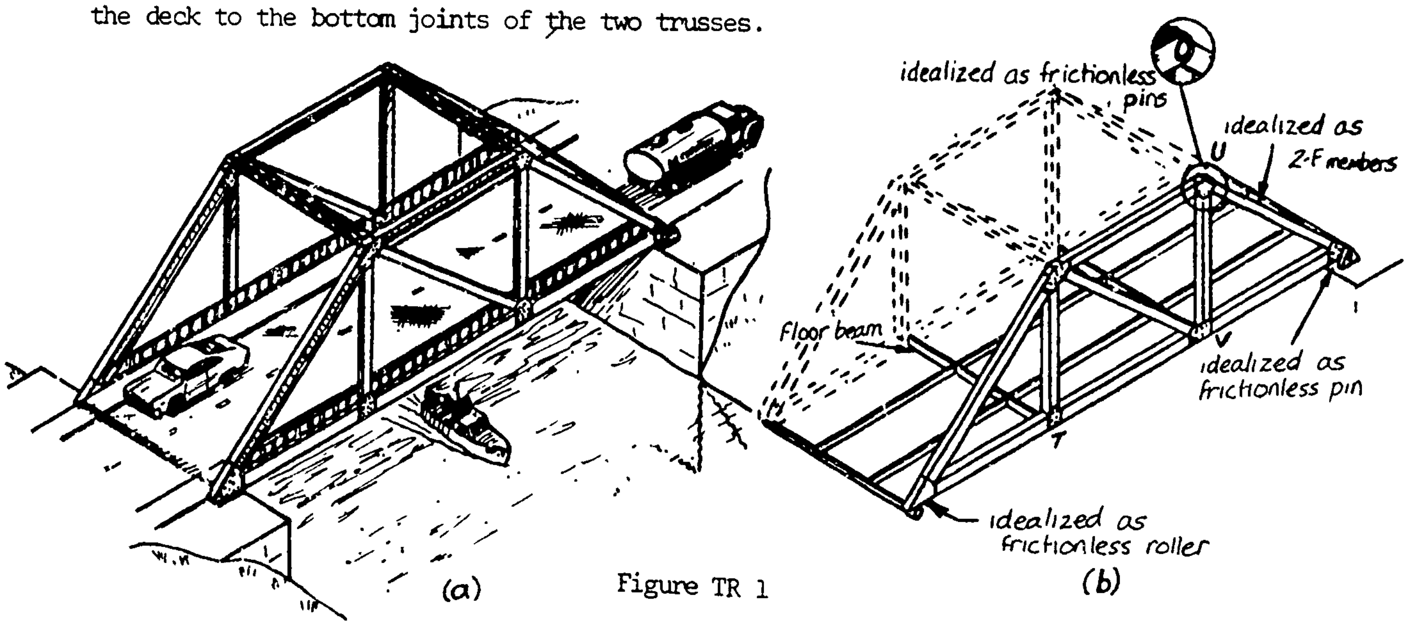


Figure TR 1

For purposes of analysis, the truss is idealized by making some assumptions. (1) The individual members are considered to be weightless, rigid, and coplanar. (2) The members are connected at each joint by a single frictionless pin (as shown in the isometric, the actual connection could be a welded or riveted gusset plate), and no member is continuous through a joint. (3) All the loads are assumed to be applied only on the pins and at each joint the ℄'s of the members and the external loads are coplanar and concurrent with the ℄ of the connecting pin. With these assumptions and idealizations, each truss is a coplanar system, each joint is a concurrent system, and each member is a 2-F member capable of resisting either pure tension or compression.

Some external loads have been assumed at joints T, U, and V. These loads are caused by vehicles, wind, and the weight of the road bed. The point force resultants of the loads acting on the members will now be found using three different methods.

Method of Joints with Components

The F-B of the idealized truss is shown in figure TR 2. Since every member is considered to be 2-F and the forces at each joint are concurrent, only the ¢'s of the members are drawn. The loads are replaced by their point force resultants and are in kip (1,000 lb) units. Before the forces in the members can be determined, the unknown support reactions $\bar{F}_R$ and $\bar{F}_W$ must be found.

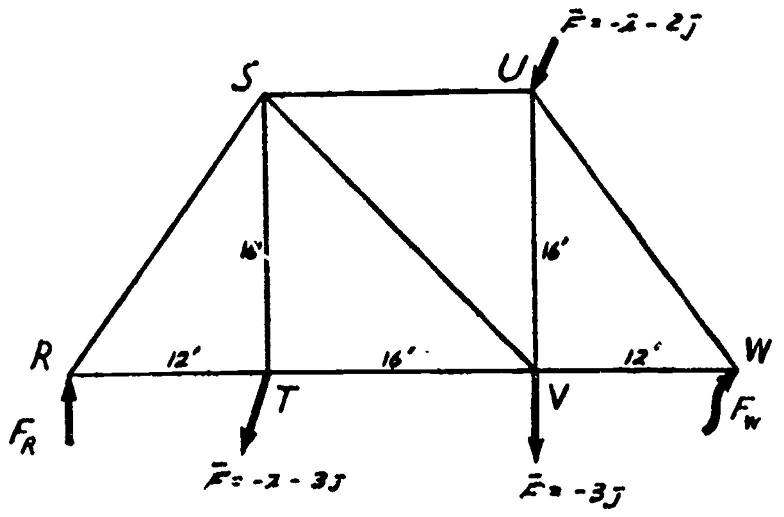


Figure TR 2

The connection at R is a smooth roller so the direction of $\bar{F}_R$ is known to be vertically upward, but the direction of the reaction at W is unknown by inspection so $\bar{F}_W$ is drawn as a wavy arrow. F_R can be found by taking moments about W.

$$\Sigma M_W = 0 \qquad -12\bar{i} \times -3\bar{j} + (-12\bar{i} + 16\bar{j}) \times (-\bar{i} - 2\bar{j}) + -28\bar{i} \times (-\bar{i} - 3\bar{j}) + -40\bar{i} \times F_R\bar{j} = 0$$

which gives $F_R = 4^k$ and $\bar{F}_R = 4\bar{j}$

$\bar{F}_W$ can be found by summing forces. $\Sigma F = 0 \quad \bar{F}_W + (\bar{i} - 2\bar{j}) + (-3\bar{j}) + (-\bar{i} - 3\bar{j}) + (4\bar{j}) = 0$

$\bar{F}_W = 2\bar{i} + 4\bar{j}$ and $F_W = 4.47^k$

The F-B diagram is redrawn in figure TR 3(a). A convenient system for referring to all the point forces acting on the members of the truss is called Bow's notation. This consists of placing a small letter in any space that is between two outside forces or two members as shown. For joint R the forces reading in a clockwise order are called $\overline{af}$, $\overline{fe}$, and $\overline{ea}$ as shown in (b). For S the forces are $\overline{ah}$, $\overline{hg}$, $\overline{gf}$, and $\overline{fa}$ as shown in (c). Joint point forces are always read in a clockwise order.

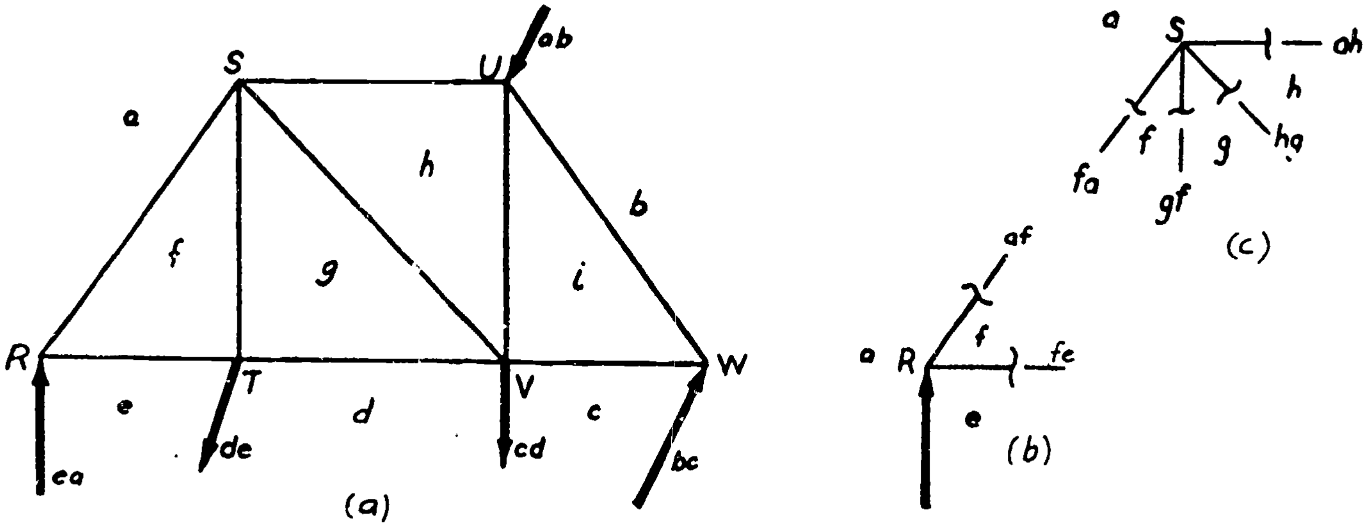


Figure TR 3

Joint R will be analyzed first since it contains only two unknown forces. The point forces acting on its two members due to the 4^k load will be found.

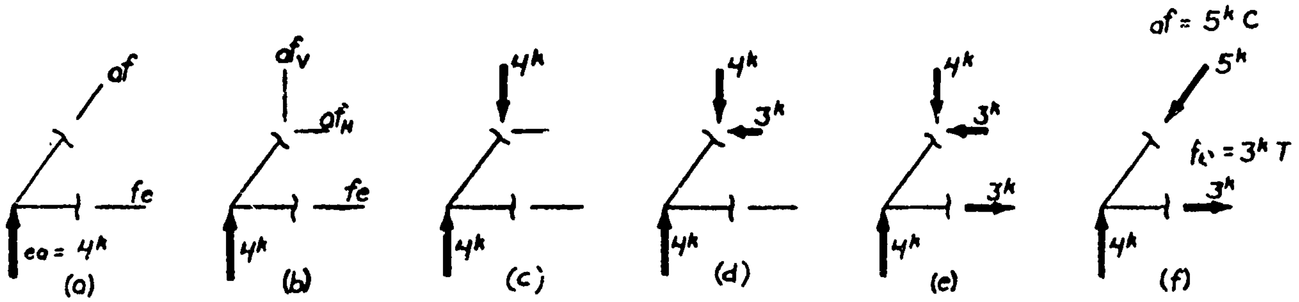


Figure TR 4

In figure TR 4(a) a F-B diagram is drawn of joint R with the 4^k load $\overline{ea}$ and the ℄'s of the two unknown forces $\overline{af}$ and $\overline{fe}$. Next $\overline{af}$ is replaced by horizontal and vertical components as in (b). $\Sigma F_V = 0$ gives $af_V = 4^k$ as in (c). The slope of $\overline{af}$ is 4/3 so $af_H = 3^k$ as shown in (d). $\Sigma F_H = 0$ in (e) gives $fe = 3^k$. F-B (f) shows the actual point force resultants $\overline{af}$ and $\overline{fe}$ that act on members RS and RT. Force $\overline{af}$ acts toward joint R, so it puts member RS in compression (C). Force $\overline{fe}$ acts away from joint R, so it puts RT in tension (T). All of this work could be done on one F-B diagram.

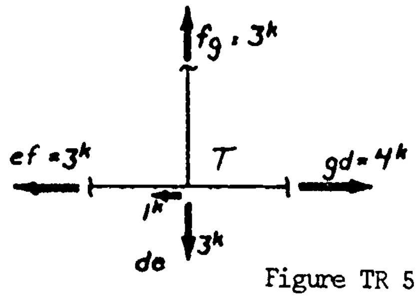


Figure TR 5

Now joint T can be solved as it has only two unknowns. First external load $\overline{de}$ is replaced by horizontal and vertical components. Member RT is in tension so and its force arrow on joint T still points away from the joint, so point force $\overline{ef}$ on joint T is equal and opposite to $\overline{fe}$ on joint R. Now $\overline{fg}$ and $\overline{gd}$ can be found by summing forces horizontally and vertically.

Joints S, U, V, and W can be solved in the same manner. A joint can be solved when it has two or less unknown point forces. All the joint F-B diagrams are shown related to each other in figure TR 6.

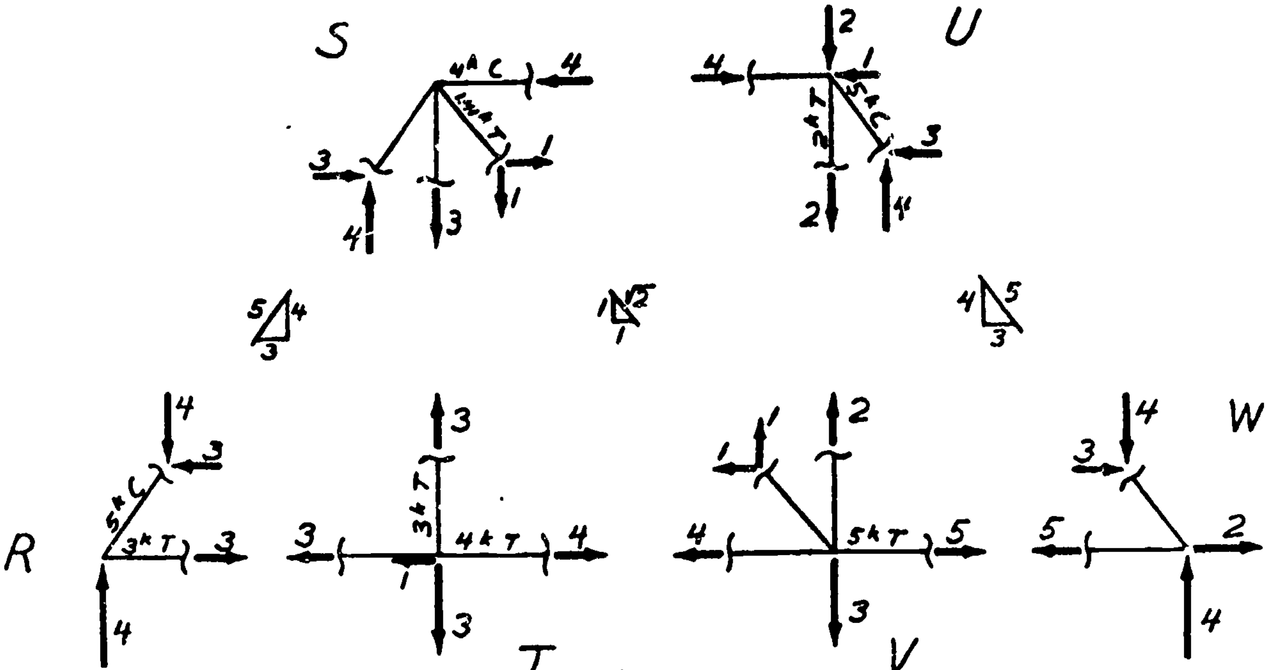


Figure TR 6

The horizontal and vertical components are usually left on the F-B diagrams and the actual point forces with their T or C labels are placed on each member.

AT THIS TIME, GIVEN A COPLANAR TRUSS, YOU SHOULD BE ABLE TO FIND THE POINT FORCE RESULTANT OF THE LOAD ON ANY MEMBER USING THE METHOD OF JOINTS. TR - 1

Method of Sections

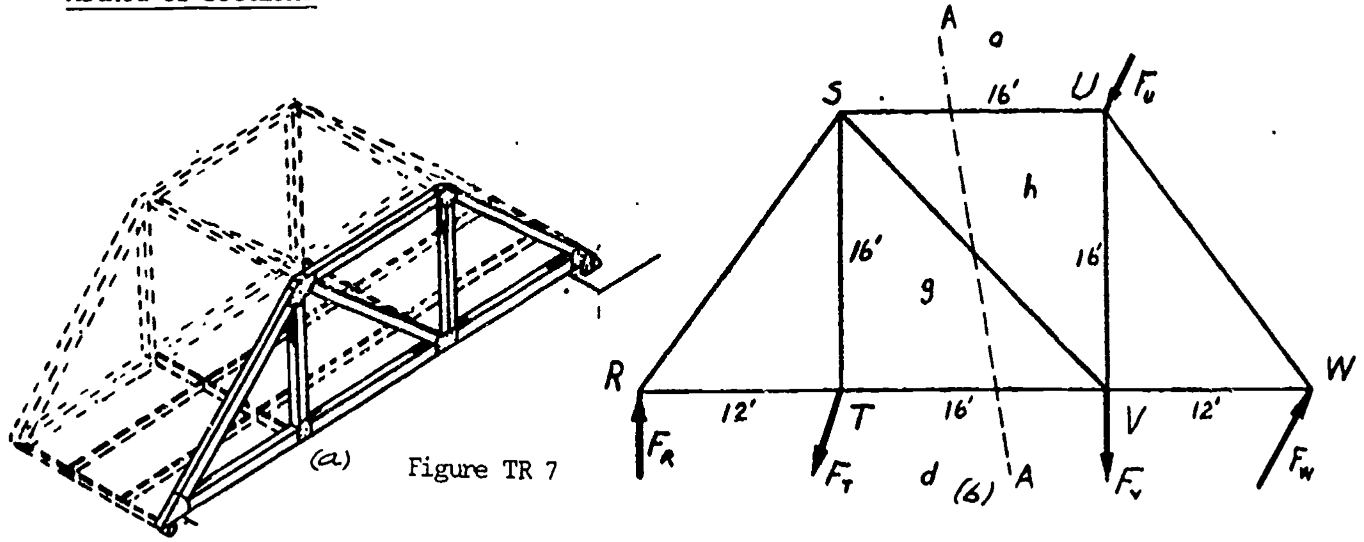
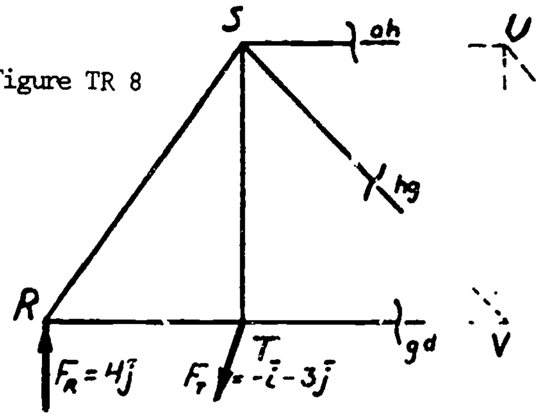


Figure TR 7

The 3-D space diagram of the truss from figure TR 1 is redrawn in figure TR 7(a) and the 2-d F-B diagram is redrawn in (b) with all the outside loads shown. This time the point force resultants of the loads on members SU, SV and TV ($\overline{ah}$, $\overline{hg}$ and $\overline{gd}$) are to be found using the method of sections.

A cutting plane A-A is drawn through the F-B diagram as shown in (b). Another F-B diagram is then drawn of the portion to the left of the cutting plane as shown in figure TR 8. This F-B diagram is in equilibrium so $\overline{ah}$, $\overline{hg}$, and $\overline{gb}$ become the point force resultants of external loads and can be found using force and moment equations.

Figure TR 8



Taking moments about point S will involve only one unknown $\overline{gd}$. Assume TV to be in tension.

$$\Sigma M_S = 0$$

$$-16\bar{j} \times (-\bar{i}-3\bar{j}) + (-16\bar{j} - 12\bar{i}) \times (4\bar{j}) + (-16\bar{j}) \times (gd\bar{i}) = 0 \quad \text{or} \quad -16\bar{k} - 48\bar{k} + 16gd\bar{k} = 0$$

$$gd = 4^k \text{ and } \underline{\overline{gd} = 4\bar{i}}$$

Member TV is therefore a tension member with a force magnitude of 4^k.

Assume SU is in tension. $\Sigma M_V = 0$

$$(-16\bar{j}) \times (-\bar{i} -3\bar{j}) + (-16\bar{i} + 16\bar{j}) \times (ah\bar{i}) + (-28\bar{i}) \times (4\bar{j}) = 0$$

$$48\bar{k} - 16ah\bar{k} - 112\,\bar{k} = 0 \qquad ah = 4^k \quad \underline{\overline{ah} = -4\bar{i}}$$

(- sign means wrong direction assumed for $\overline{ah}$)

Member SU is a compression member, its point force resultant is 4^k.

$\Sigma F = 0$ will give $\overline{hg}$ and hg $\quad (4\bar{j}) + (-\bar{i} - 3\bar{j}) + (4\bar{i}) + (-4\bar{i}) + (\overline{hg}) = 0$

$\underline{\overline{hg} = +\bar{i} - \bar{j}} \qquad hg = 1^2 + 1^2 = 1.41^k$

Member SV is therefore a tension member with a force magnitude of 1.41^k. The completed F-B diagram is shown in figure TR 9(a). This F-B should check by summing forces $\Sigma F = 0$

$$\overline{ah} + \overline{hg} + \overline{gd} + \bar{F}_T + \bar{F}_R = 0$$

$$-4\bar{i} + \bar{i} - \bar{j} + 4\bar{i} - \bar{i} - 3\bar{j} + 4\bar{j} = 0$$

i components $\quad -4 + 1 + 4 - 1 = 0$

j components $\quad -1 - 3 + 4 = 0$

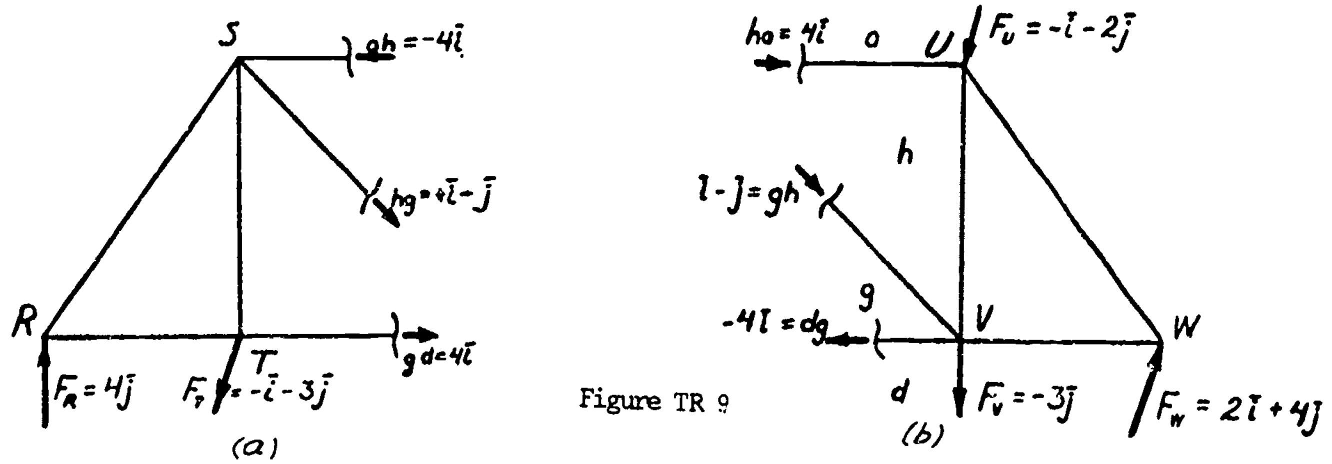


Figure TR 9

Of course the other half of the truss is also in equilibrium as shown in (b). Summation of forces and moments will show that F-B (b) is correct.

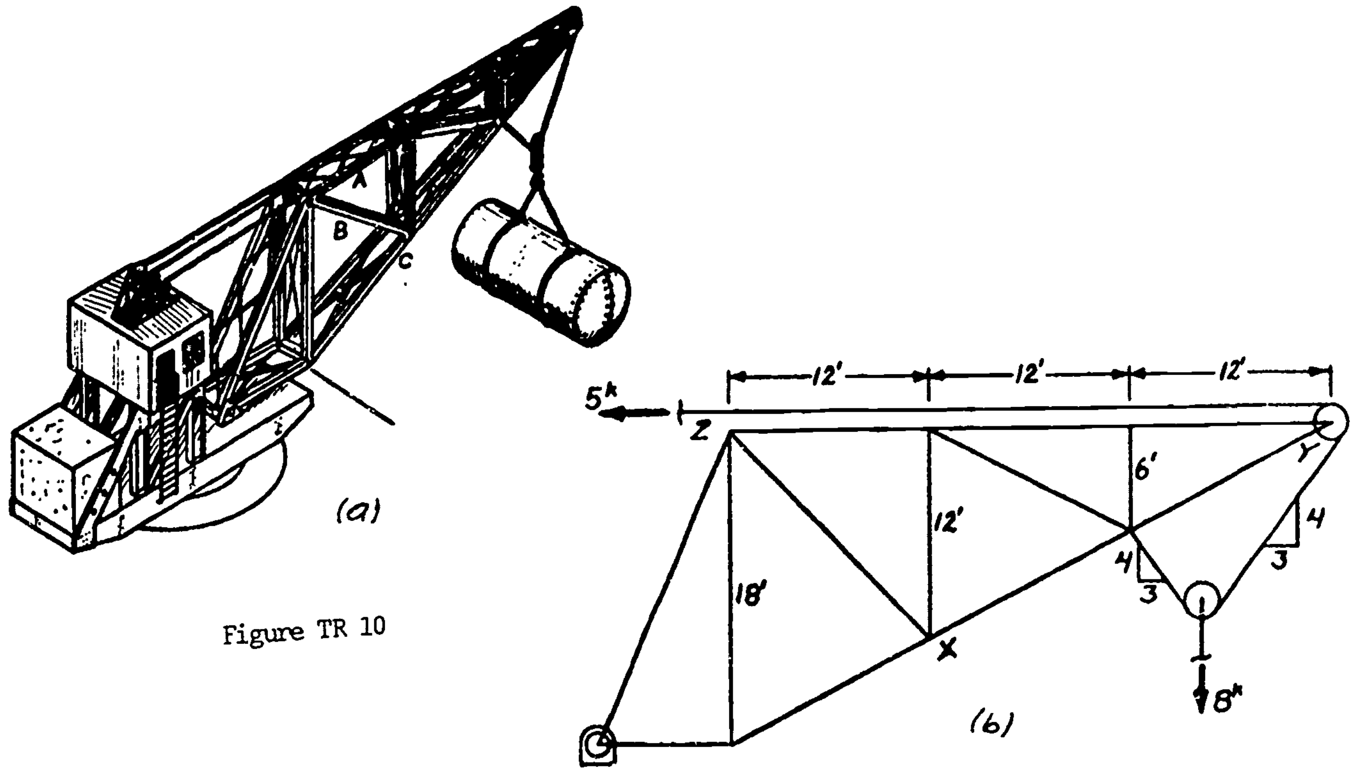


Figure TR 10

The method of sections will now be used to find the point force resultants of the loads acting on members A, B, and C of the derrick in figure TR 10(a). The load of 16^k is being raised with no acceleration. An idealized 2-D space diagram of the coplanar truss that contains members A, B, and C is shown with dimensions in (b).

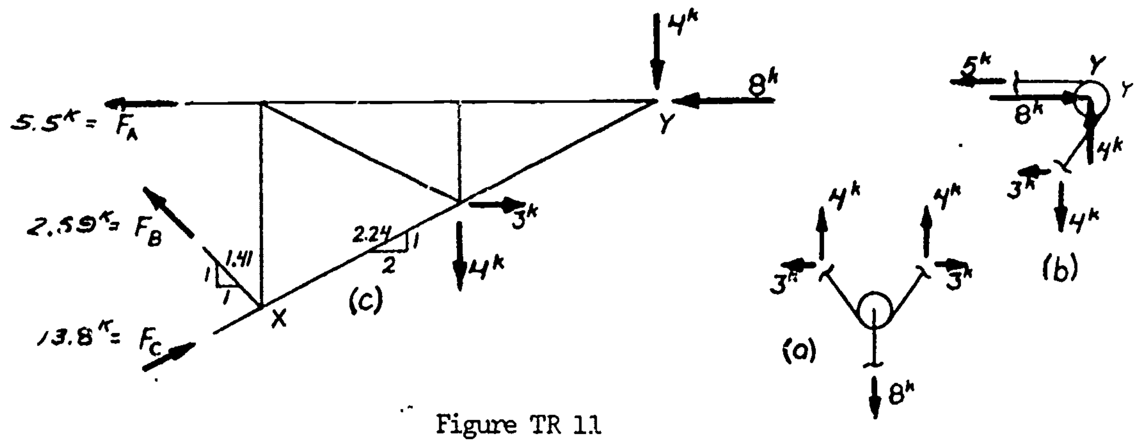


Figure TR 11

The F-B in figure TR 11(a) is used to find the components of the loads on the cable, and the F-B in (b) is used to find the components of the load on the pin at Y. Now the F-B diagram shown in (c) can be drawn and the method of sections used to find the point force resultants of the loads on A, B, and C.

To find F_A assume A is a tension member and take moments about X.

$$\Sigma \overline{M}_X = 0 \quad (24\bar{i} + 12\bar{j}) \times (-8\bar{i} - 4\bar{j}) + (12\bar{i} + 6\bar{j}) \times (3\bar{i} - 4\bar{j}) + (12\bar{j}) \times (-F_A\bar{i}) = 0$$

$$-96\bar{k} + 96\bar{k} - 48\bar{k} - 18\bar{k} + 12F_A\bar{k} = 0$$

$$F_A = \frac{+66}{12} \qquad \underline{F_A = 5.5^{k}T}$$

To find F_B assume it is a tension force and take moments about Y.

$$\overline{F}_B = \frac{-1}{1.41} F_B\bar{i} + \frac{1}{1.41} F_B\bar{j} = -.707F_B\bar{i} + .707 F_B\bar{j}$$

$$\Sigma \overline{M}_Y = 0 \quad (-12\bar{i} - 6\bar{j}) \times (3\bar{i} - 4\bar{j}) + (-24\bar{i} - 12\bar{j}) \times (-.707F_B\bar{i} + .707F_B\bar{j}) = 0$$

$$48\bar{k} + 18\bar{k} - 17F_B\bar{k} - 8.5F_B\bar{k} = 0$$

$$\underline{F_B = 2.59^{k}T}$$

To find F_C assume it is a compression force and take moments about Z.

$$\overline{F}_C = \frac{2}{2.24} F_C\bar{i} + \frac{1}{2.24} F_C\bar{j} = .894F_C\bar{i} + .447F_C\bar{j}$$

$$\Sigma \overline{M}_Z = 0 \quad (24\bar{i} - 6\bar{j}) \times (3\bar{i} - 4\bar{j}) + (36\bar{i}) \times (-8\bar{i} - 4\bar{j}) + (12\bar{i} - 12\bar{j}) \times (.894F_C\bar{i} + .447 F_C\bar{j}) = 0$$

$$-96\bar{k} + 18\bar{k} - 144\bar{k} + 5.37F_C\bar{k} + 10.74F_C\bar{k} = 0$$

$$\underline{F_C = 13.8^{k}C}$$

Check by summing point forces.

$$\overline{F}_A + \overline{F}_B + \overline{F}_C + (3\bar{i} - 4\bar{j}) + (-8\bar{i} - 4\bar{j}) = 0$$

$$\overline{F}_A = -5.5\bar{i}$$

$$\overline{F}_B = -.707\,(2.59)\,\bar{i} + .707(2.59)\bar{j} = -1.82\bar{i} + 1.82\bar{j}$$

$$\overline{F}_C = .894\,(13.8)\,\bar{i} + .447(13.8)\bar{j} = 12.4\bar{i} + 6.2\bar{j}$$

$$-5.5\bar{i} - 1.82\bar{i} + 1.82\bar{j} + 12.3\bar{i} + 6.2\bar{j} + 3\bar{i} - 4\bar{j} - 8\bar{i} - 4\bar{j} = 0$$

i components $\quad -5.5 - 1.82 + 12.4 + 3 - 8 = 0$

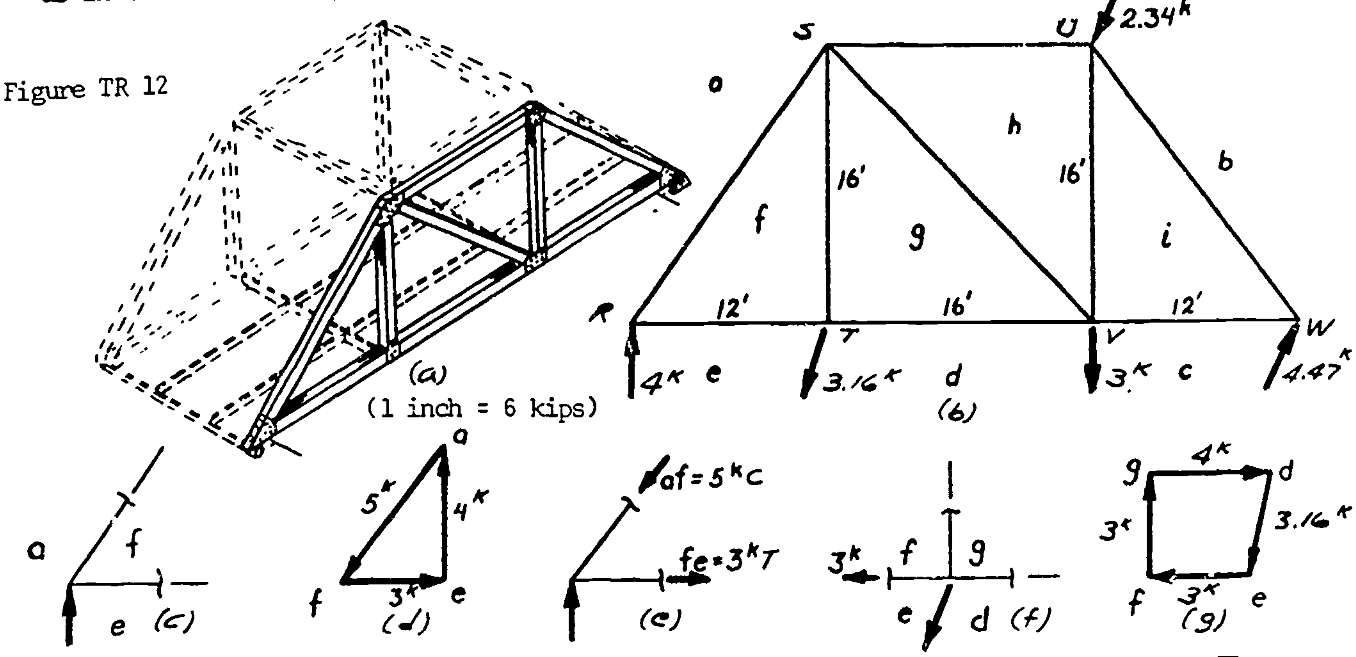
j components $\quad +1.82 + 6.2 - 4 - 4 = 0$

The values of $\overline{F}_A$, $\overline{F}_B$, and $\overline{F}_C$ are usually placed on the F-B diagram after they are checked as shown in (c).

NOW YOU SHOULD BE ABLE TO USE THE METHOD OF SECTIONS TO FIND THE POINT FORCE RESULTANTS OF THE LOADS ACTING UPON MEMBERS OF A GIVEN COPLANAR TRUSS.

The point force resultants of the loads acting on all the members of the truss in figure TR 12(a) will now be found by using a graphical technique called a Maxwell diagram. The 2-D F-B diagram is drawn to scale in (b) and the reactions at the constraints found previously are placed on it as single point forces with their correct directions. First a 2-D F-B diagram of joint R is drawn as shown in (c). Next a force polygon of the three point forces is drawn to scale starting with the known force $\overline{ea}$ and continuing in a clockwise order with $\overline{af}$ and $\overline{fe}$ as shown in (d). Forces $\overline{af}$ and $\overline{fe}$ are then scaled and put on the F-B diagram as in (e). Force $\overline{af}$ puts member RS in compression and force $\overline{fe}$ puts member RT in tension.

Figure TR 12



The F-B diagram for joint T can be solved as it has two unknown forces. Force $\overline{de}$ is known and since $\overline{ef} = -\overline{fe}$, force polygon (g) can be drawn beginning with de followed by $\overline{ef}$, $\overline{fg}$, and $\overline{gd}$. Forces $\overline{fg}$ and $\overline{gd}$ can be scaled. Members ST and TV are tension members with loads of 3^kT and 4^kT.

This procedure can be continued joint by joint as shown in figure TR 13(a). Each force polygon is drawn around its joint. The scales have been enlarged in figure 13 for clarity. All of the force polygons will now be combined into force polygon.

The procedure starts by drawing to scale the point force polygon of the outside loads as shown in (b). This consists of point forces $\overline{ab}$, $\overline{bc}$, $\overline{cd}$, $\overline{de}$, and $\overline{ea}$ plotted in a clockwise with their arrowheads drawn.

Now beginning with $\overline{ea}$, polygon eafe ① can be drawn superimposed upon (b) as shown in (c). No arrows are put on $\overline{af}$ and $\overline{fe}$, but their directions must be visualized as indicated by the half arrows in (c). The magnitudes can then be measured and the forces listed as af = 5^kC and fe = 3^kT.

Polygon ② defgd can be drawn upon (c) as shown in (d). Note that $\overline{ef}$ is opposite to $\overline{fe}$ in ①. The polygon gives fg = 3^kT and gd = 4^kT.

Polygon ③ can be superimposed upon (d) as shown in (e). This process can be continued with polygons ④, ⑤, and ⑥ until (f) is complete. This combined set of force polygons is called a Maxwell diagram. With only a single F-B diagram of the truss, this combined diagram gives graphically the point forces acting upon each member of the truss.

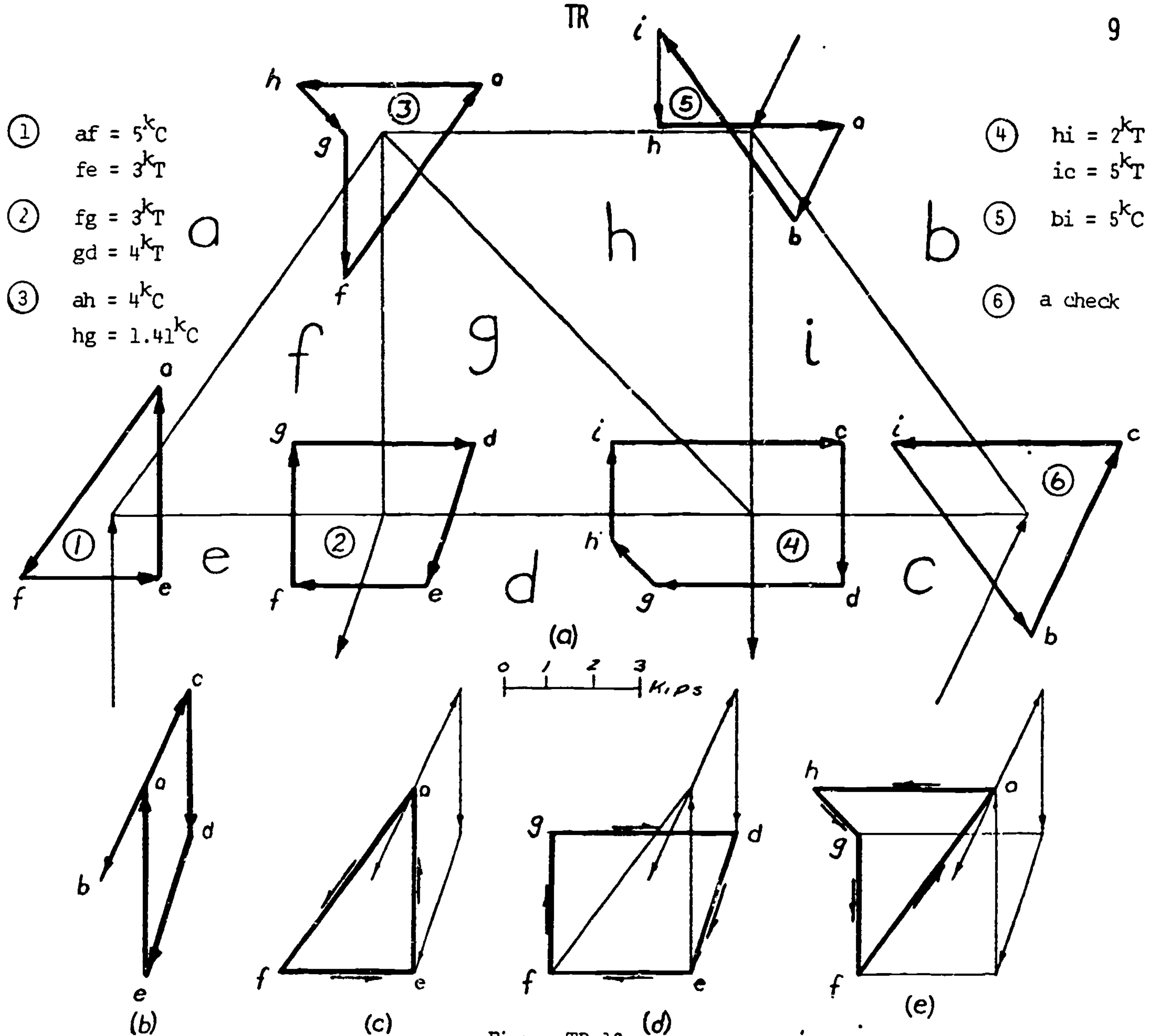
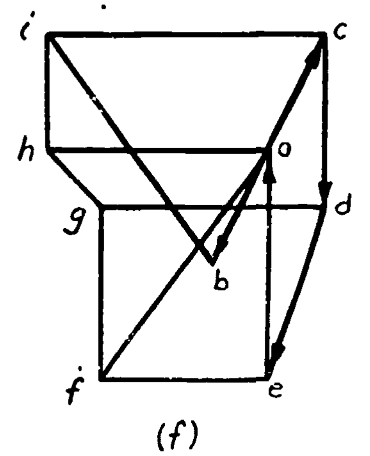


Figure TR 13



The main difficulty students have when constructing a Maxwell diagram is in determining whether the members are in compression (C) or tension (T). You must visualize the individual joint F-B diagrams as in TR 13 and mentally place the arrows representing the forces on these F-B diagrams. If the arrow points toward a joint, the member is in compression. If the arrow points away from the joint, the member is in tension.

Always start with the outside force polygon as in TR 13(b). This polygon has arrowheads, and it must close or you have made a mistake in calculating the reactions. If you are neat and careful, the last joint polygon ⑥ when superimposed on (f) will also close.

Another brief example of the construction of a Maxwell diagram for a truss is shown in figures TR 14 and TR 15. This example should be carefully studied before you attempt to solve any problems this way. The isometric space diagram and its 2-D F-B diagram are drawn to scale in figure TR 14(a) and (b) with the loading shown on (b). In (c) and (d) the reactions are found graphically.

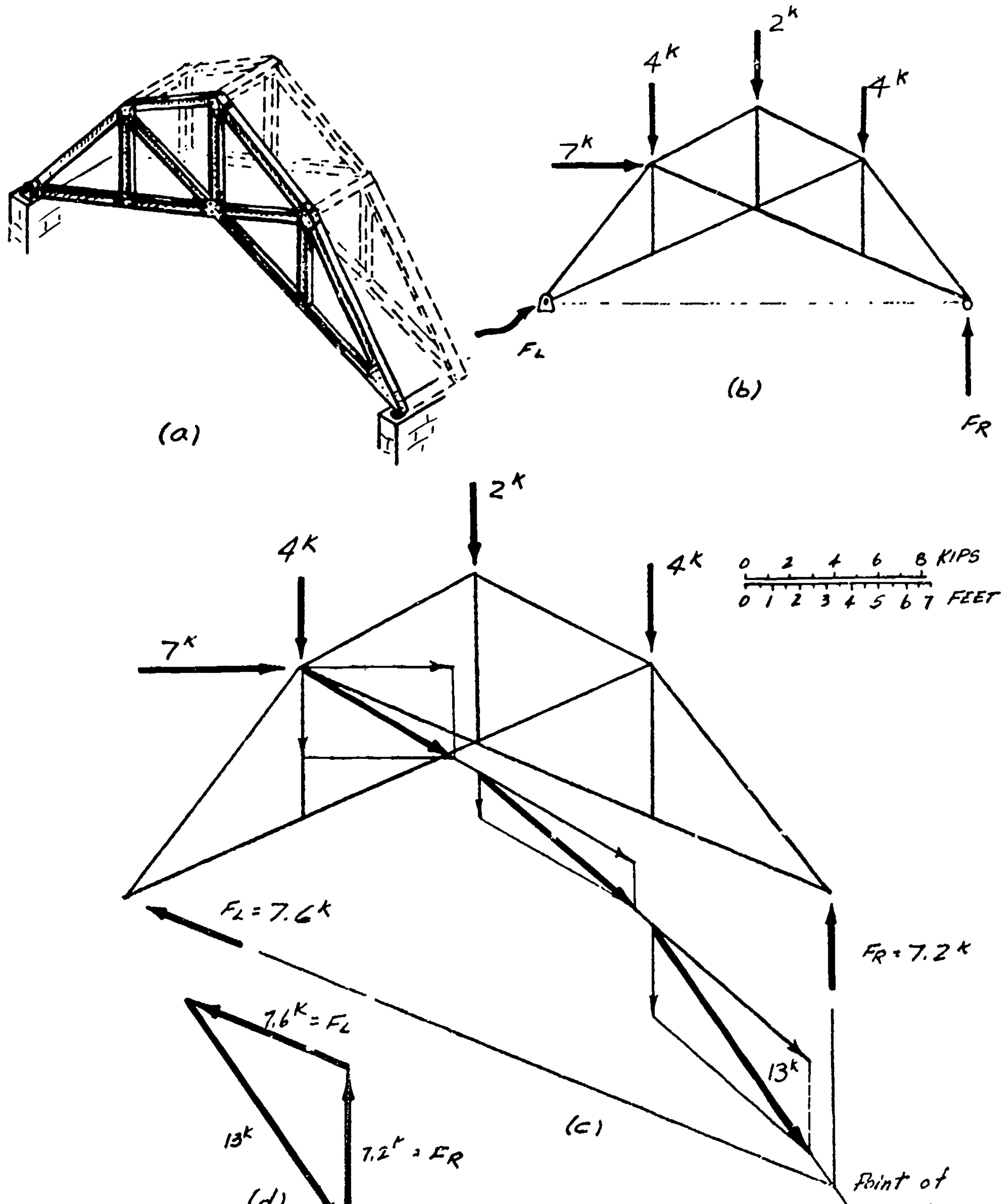


Figure TR 14

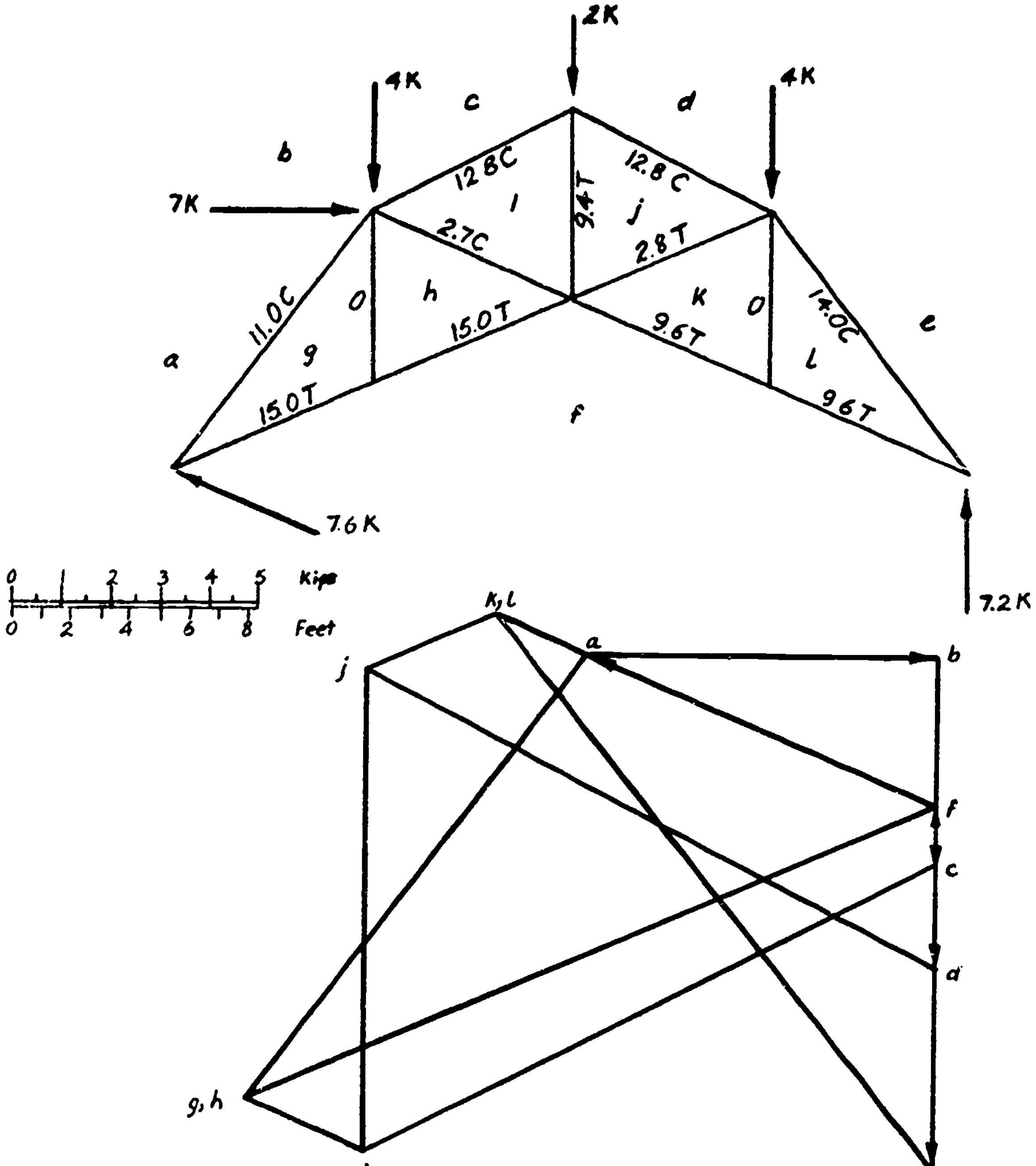
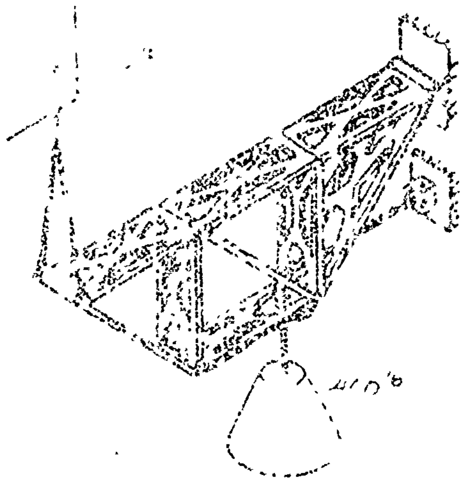


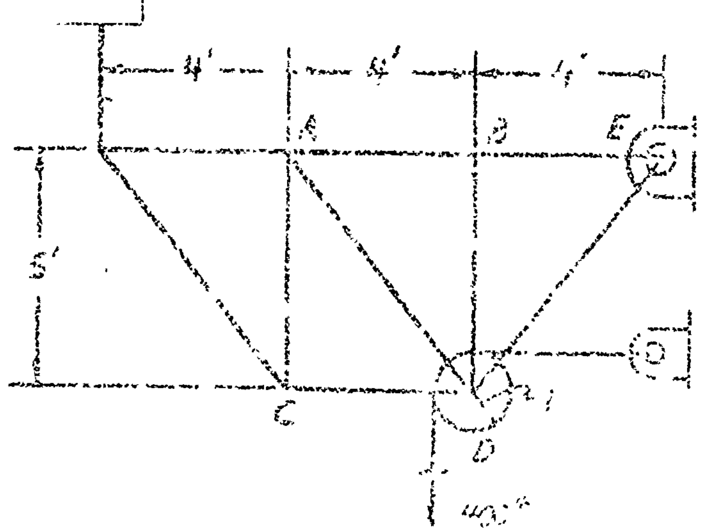
Figure TR 15

In figure TR 15(a) the 2-D F-B diagram is drawn to a larger scale and Bow's notation is applied. The combined diagram is constructed (f). First the outside force polygon abcdefa is constructed to scale with its arrowheads shown. Either joint fagf or efle can be drawn next, and the remaining joint force polygons drawn one by one. The forces acting on each member are shown in (a) with their magnitudes and proper C or T notations.

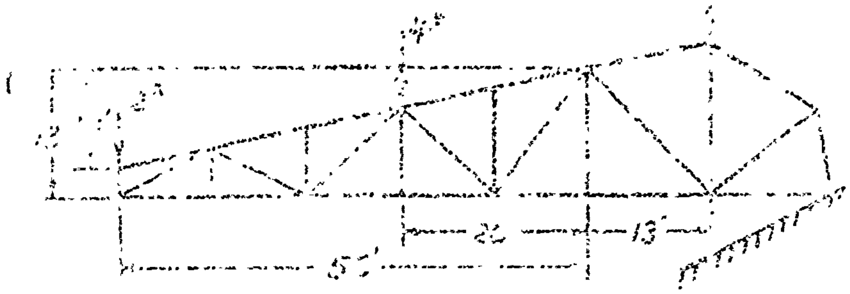
AT THIS TIME WHEN YOU ARE GIVEN A LOADED COPLANAR TRUSS, YOU SHOULD BE ABLE TO FIND ITS REACTIONS AND THEN FIND THE FORCES ACTING ON EACH MEMBER OF THE TRUSS USING A MAXWELL DIAGRAM.

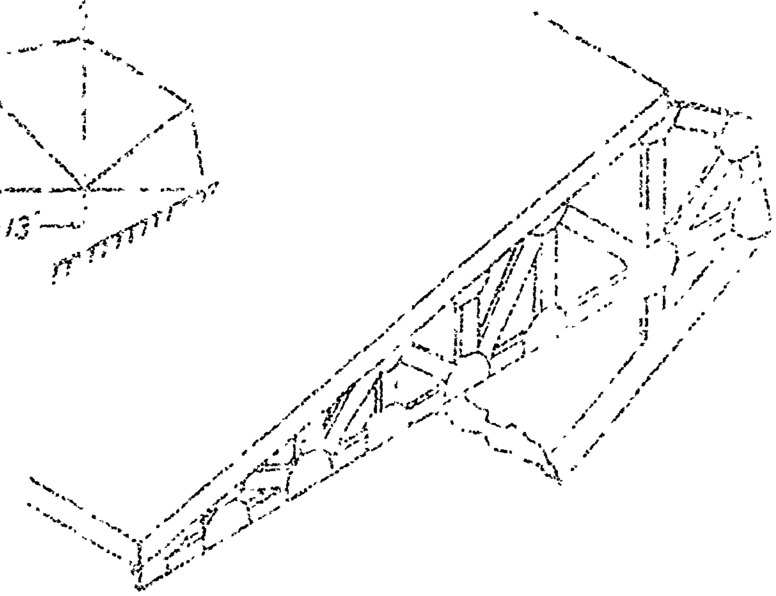


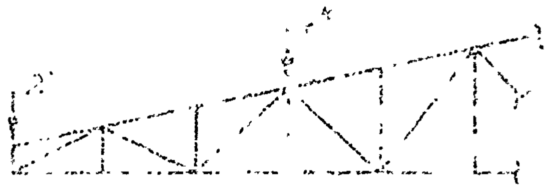
TR



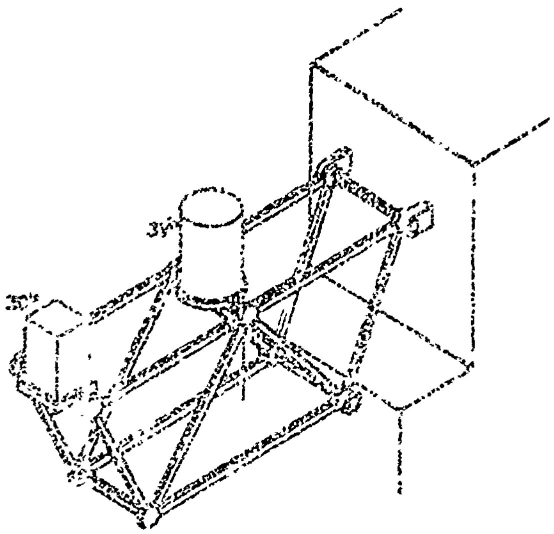
TR - 1 Find the point force resultants of the loads acting on all the members of the truss using the method of joints

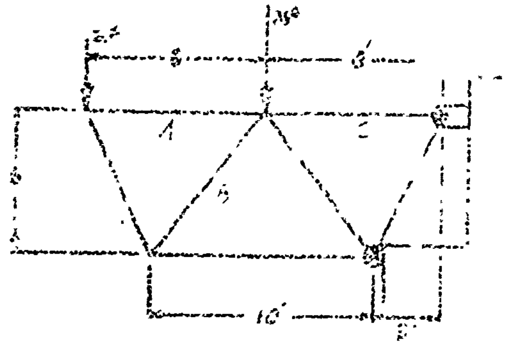




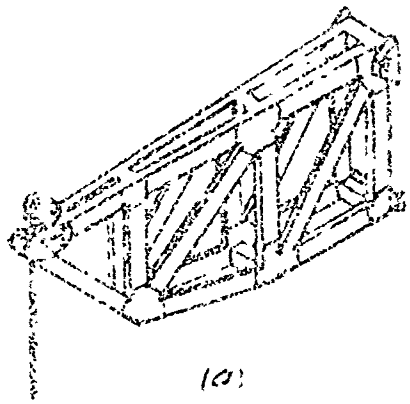


TR

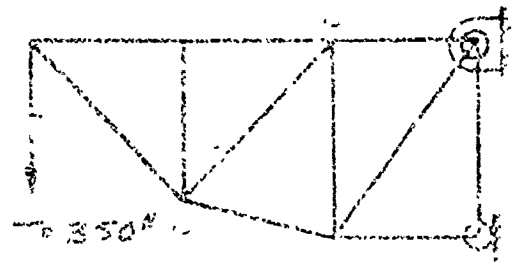




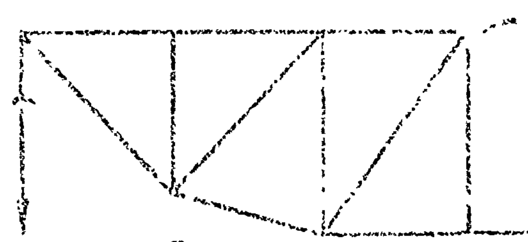
TP - 2(B) Find the loads acting on members A, B, and C of the truss using the [illegible]



(a)



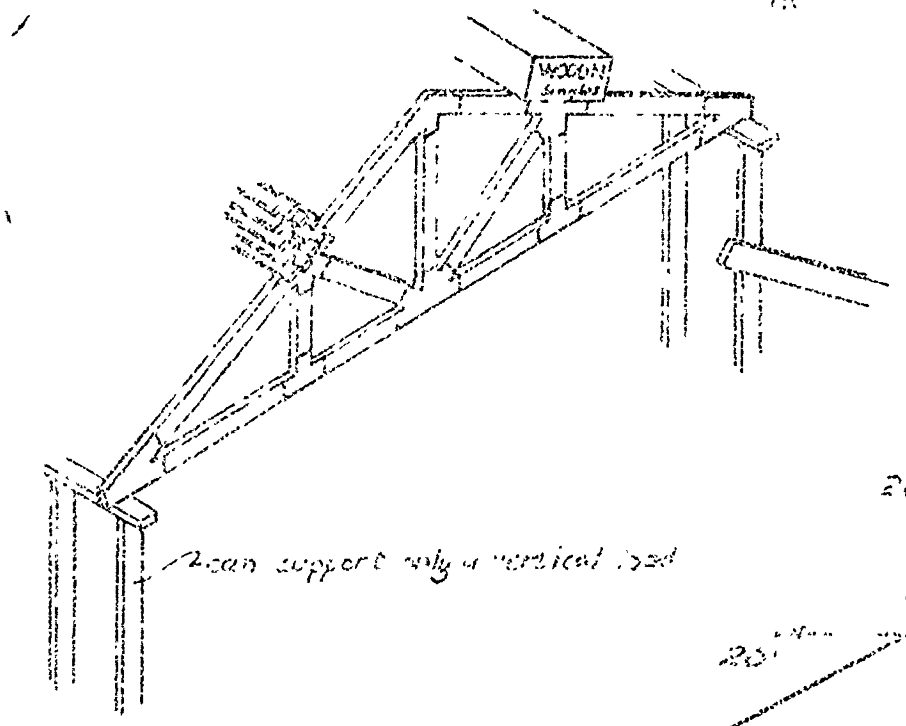
(b)



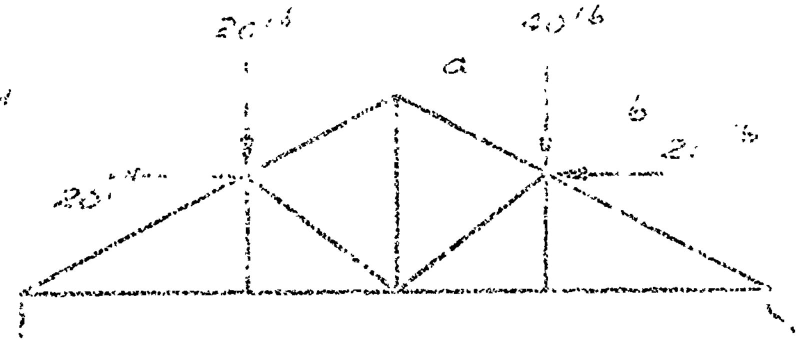
1" = 200 lb

start here

TR - 36. [illegible] the loads acting on all the members of the truss using a Maxwell diagram.



1" = 20 lb



TR - 3(B) Use a combined diagram to find the joint force resultants of the loads on all the members of the truss.

UNIT 12

HYDROSTATICS

AT THE END OF THIS UNIT IF YOU ARE GIVEN A FLAT COMPOSITE SURFACE SUBMERGED IN A STATIONARY FLUID, YOU WILL BE ABLE TO FIND THE POINT FORCE RESULTANT OF THE FLUID PRESSURE ACTING AGAINST THE SURFACE.

Introduction

In the work so far in engineering statics, loads have been applied to solid objects by other solid objects or by gravity. These loads were always distributed loads and in most cases for analysis were replaced by their point force resultants. In this unit flat surfaces will be submerged under a fluid and the distributed loads caused by the fluid will be replaced by their point force resultants. The fluids used will be stationary and non-compressible. A non-compressible stationary fluid is found from experimentation to be unable to exert any friction forces, that is it can exert only normal forces on any surface.

Developement

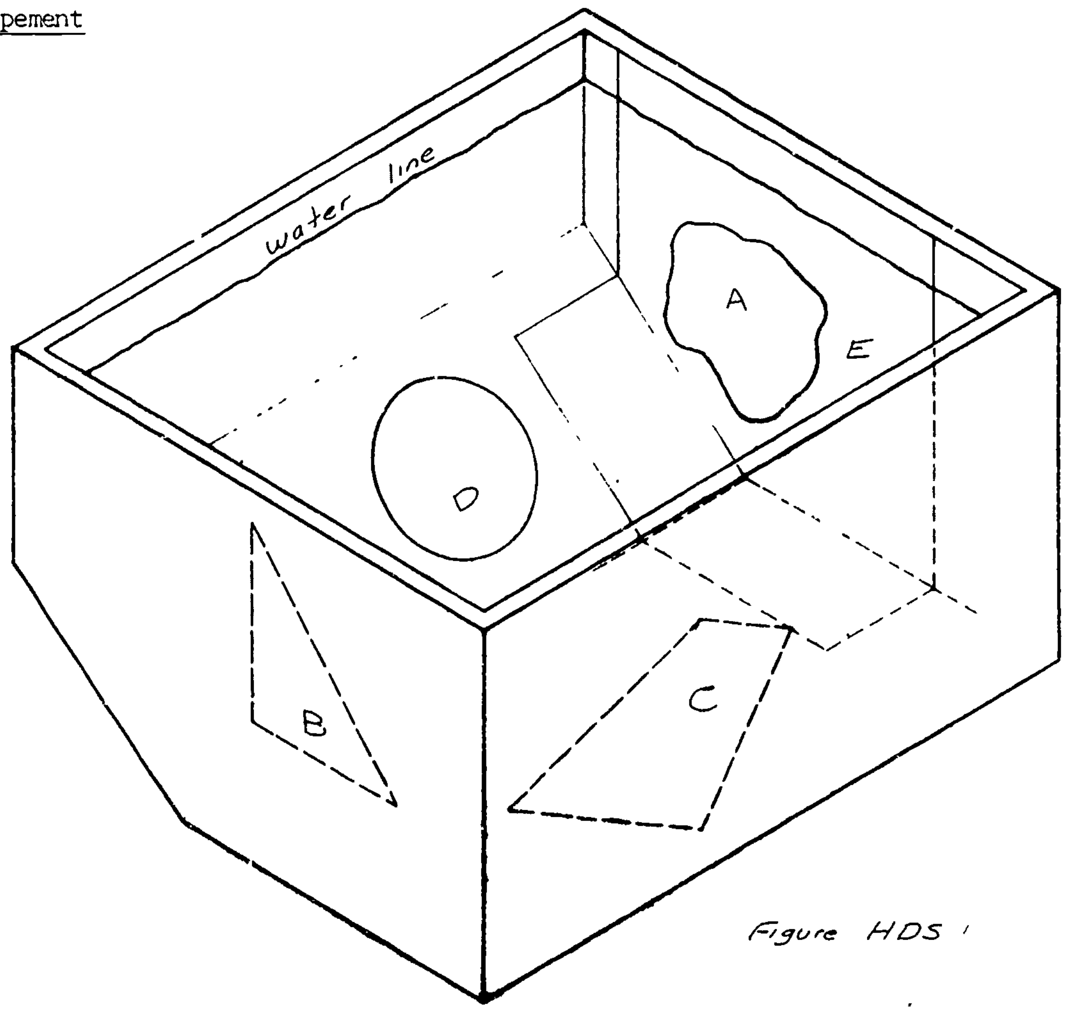


Figure HDS 1

The tank in figure HDS 1 is filled with water. The point force resultants of the distributed force fields that act upon flat surfaces A, B, C, and D are to be found. The dimensions and locations of each surface will be given as it is analyzed.

Water is a non-compressible fluid. Any stationary surface submerged in stationary water has a distributed load (a force field) acting upon it due to the water. This distributed load is generally called a hydrostatic load. The intensity of this force field is called pressure. This pressure (force field intensity) depends only upon the depth and the density of the water above it. Also it is found from experimentation that the pressure at any depth acts perpendicular to any surface at the same depth with the same intensity. That is the pressure at any depth, for a static fluid is constant in any direction.

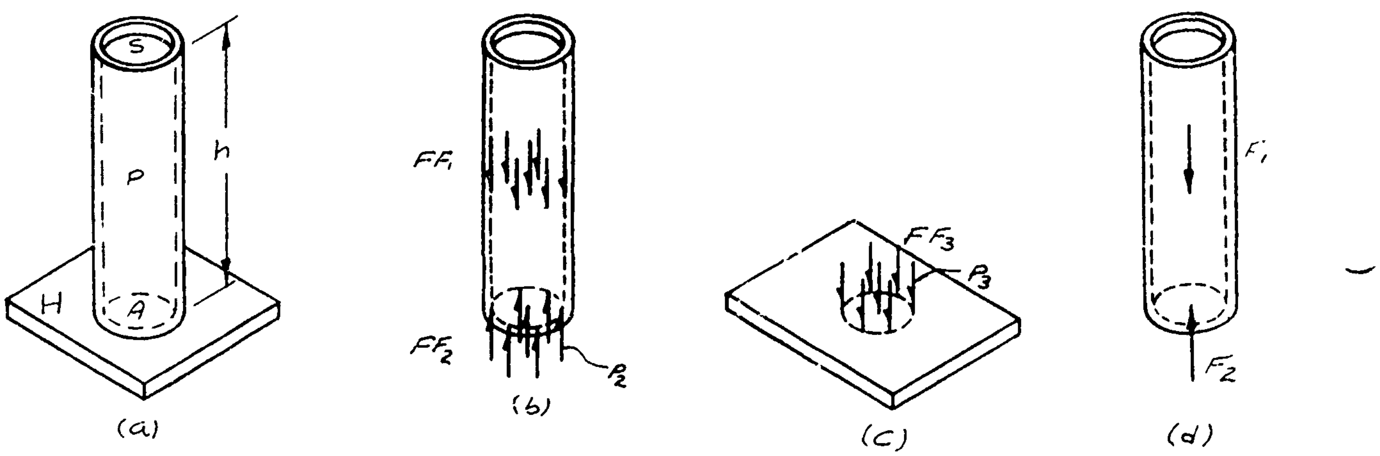


Figure HDS 2

The weightless pipe P has been inserted into the tank of water to a depth h. Then the lower end has been sealed by plate H. The pipe full of water is then removed from the tank. Plate H is then mentally removed in (b) and a F-B diagram of the pipe filled with water in drawn.

It is assumed that the air pressure is negligible so there is no force field acting on the upper free surface S. One force field caused by gravity acts vertically downward, it is distributed throughout the complete volume of the water but is represented by FF_1 acting as shown. When plate H is mentally removed, force field FF_2 acts upward to keep the water in the pipe. FF_2 is uniformily distributed over area A. In (c) force field FF_3 is shown acting against plate H. FF_3 is equal and opposite to FF_2. Now looking at FF_2, each individual half arrow can be thought of as the intensity (pressure) of the force field. Pressure p_2 on FF_2 is equal to pressure p_3 on FF_3. F-B diagram (d) can now be drawn of the pipe filled with water with F_1 and F_2 replacing FF_1 and FF_2.

Now letting V equal the volume of the water, A the lower surface of the water, γ the density of the water (.0361 lb/cu-in), p_2 the pressure of FF_2 ($p_2A = FF_2$), and p_3 the pressure of FF_3 ($p_3A = FF_3$), the relationships will developed between p_2, p_3, A, h, and γ.

In (6) $FF_1 = V\gamma = Ah\gamma$

But $FF_1 = FF_2$ so $FF_2 = Ah\gamma$

Also $FF_2 = p_2A$ so $p_2A = Ah\gamma$ and $p_2 = h\gamma$

Since $FF_3 = FF_2$ $p_3 = h\gamma$

That is the pressure at any distance h under water is equal to h times the density of the water. This is a linear relationship because doubling h doubles the pressure, halving h halves p, etc.

$p_1 = h\gamma$
$p_2 = h\gamma$
$p_3 = h\gamma$
$p_4 = h\gamma$
$p_5 = h_1\gamma$
$p_6 = h_2\gamma$
$p_7 = h_3\gamma$
$p_8 = h_4\gamma$
$p_9 = h\gamma$

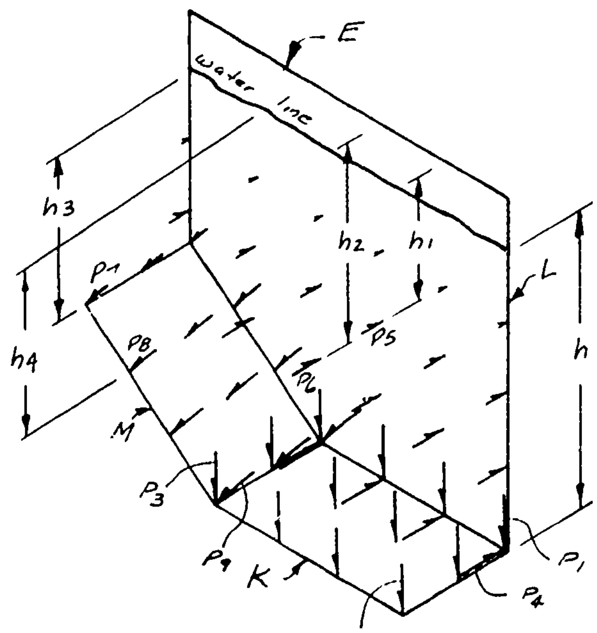


Figure HDS 3

Now portion E of the tank is redrawn in figure HDS 3. The force fields and their intensities (pressures) acting against surfaces K, L, and M will be analyzed.

Surface K is at a constant depth h, so its force field FF_1 will be a uniform force field acting perpendicular to K. Its pressure $p = h\gamma$ and of course is uniform over area K. That is $p_1 = p_2 = p_3 = p = h\gamma$.

Vertical surface L has a distributed force field FF_2 acting perpendicularly against it. This force field intensity (pressure) varies linearly from zero at the free water surface to pressure $p_4 = h\gamma$ at height h. Also pressure $P_5 = h_1\gamma$ and pressure $p_6 = h_2\gamma$.

Sloping surface M has a force field FF_3 that acts perpendicular to its surface as shown. Since the intensity (pressure) at any point is directly proportional to the height to the free surface and γ of the fluid, pressure $p_7 = h_3\gamma$, pressure $p_8 = h_4\gamma$, and pressure $p_9 = h\gamma$. Some of the pressures on the three surfaces are summarized in the figure.

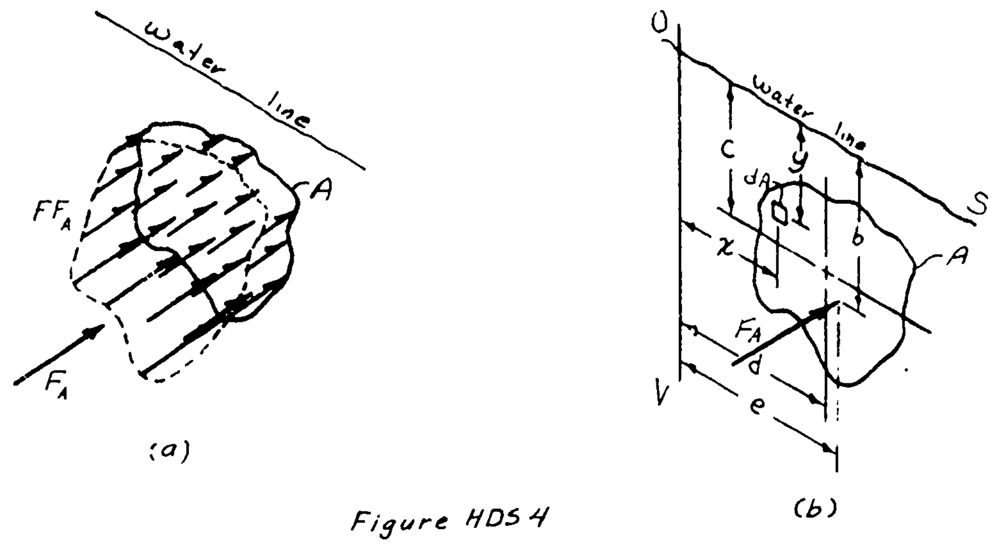


Figure HDS 4

Now looking at surface A in figure HDS 1, the point force resultant of the force field acting against surface A due to the water pressure will be found.

3-D diagram HDS 4(a) is drawn of surface A. Force field FF_A acts against the surface with its intensity (pressure) being a minimum on the top of the surface and varying linearly to a maximum at the bottom of the surface. The point force resultant F_A of FF_A is shown, its magnitude and line of application are to be found.

In (b) the surface is redrawn in 3-D with the force field deleted for convenience and dA is drawn on the surface dimensioned with y and x as shown. Line OS is the intersection of surface A and the free water surface. Vertical line OV is any arbitrary vertical line that intersects OS.

$dF \text{ on } dA = p\,dA = \gamma y\,dA$

$F \text{ on } A = \int_A dF = \gamma \int_A y\,dA$ which is γ times the first moment of A with respect to OS

So the magnitude of F_A is γ (the density of the fluid) times c (the distance from the free fluid line to the centroid of the surface A) times the submerged area A.

Now the distances b and e to the line of action of F_A will be found. Vertical distance b will be found first. Remember for force field FF_A that the moment of the resultant point force F_A equals the sum of the moments of the distributed forces in the field with respect to any point or line.

So with respect to OS

$$M_{F_A/OS} = M_{FF_A/OS}$$

$$(b)(F_A) = \int_A (y)\,dF = \int_A (y)\,p\,dA = \int_A y\,\gamma y\,dA = \gamma \int_A y^2\,dA$$

$$b = \frac{\gamma \int_A y^2\,dA}{F_A} = \frac{\gamma \int_A y^2\,dA}{\gamma c A} = \frac{I_x}{cA}$$

I_x is called the moment of inertia of surface A with respect to OS (the free surface). The term cA is the first moment of surface A with respect to OS.

Next the arbitrary vertical line OV is drawn. Distance e will now be found.

$$M_{F_A/OT} = M_{FF_A/OT}$$

$$e F_A = \int_A (x)(dF) = \int_A x\, p\, dA = \int_A x \gamma y\, dA = \gamma \int_A xy\, dA$$

$$e = \frac{\gamma \int_A xy\, dA}{F_1} = \frac{\gamma \int_A xy\, dA}{\gamma \int_A y\, dA} = \frac{\int_A xy\, dA}{\int_A y\, dA} = \frac{I_{xy}}{cA}$$

I_{xy} is called the product of inertia of the area with respect to axes OS and OV at point O (the intersection of OS and OV), cA is the first moment of the area with respect to line OS.

The two terms $b = \frac{I_x}{cA}$ and $e = \frac{I_{xy}}{cA}$ actually locate the centroid of the volume of force field FF_A.

Now a statement can be made. The magnitude, direction, and line of action of the point force resultant of the distributed load acting upon a vertical flat surface A submerged in a static fluid can be found. Its magnitude is $F = \gamma cA$ where γ is the density of the fluid, c is the vertical distance from the free surface of the fluid to the centroid of surface A, and A is the total area of surface A.

The point force resultant F has the same direction as its force field FF. Its line of action passes through the centroid of its force field. The centroid can be found by

$b = \frac{I_x}{cA}$ I_x is the moment of inertia of the submerged surface with respect to the intersection of the submerged surface and the free fluid surface. This line will be called OS.

cA is the first moment of the submerged surface with respect to line OS.

b is the vertical distance from line OS to the point force resultant F.

$e = \frac{I_{xy}}{cA}$ I_{xy} is the product of interia of the submerged surface with respect to the intersection of OS and any arbitrary vertical line OV that intersects OS.

cA is the first moment of the surface with respect to line OS.

e is the horizontal distance from line OV to point force resultant F.

Vertical Surfaces

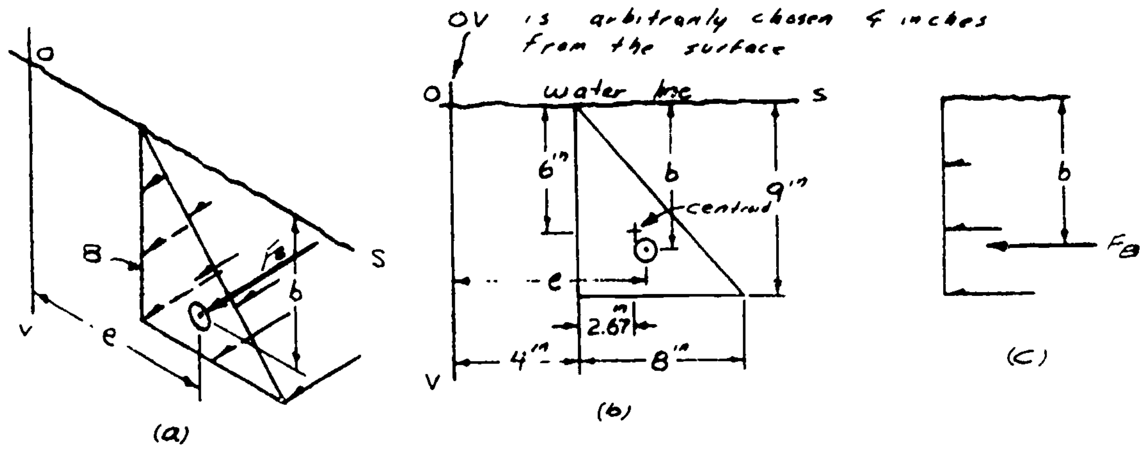


Figure HDS 5

The single point force resultant of the hydrostatic load acting on the triangular surface B will now be found. Surface B is redrawn in 3-D and 2-D in figure HDS 5 and its dimensions are shown. The top of B lies on the free water surface. Its force field FF_B is shown with its resultant F_B. Now the magnitude and line of action of F_B are to be found.

$F_B = \gamma c A \qquad \gamma = .0361 \text{ lb/cu-in} \qquad F_B = (.0361)(6)(36)$

$c = 6 \text{ in} \qquad = 7.8 \text{ lb}$

$A = \frac{(8)(9)}{(2)} = 36 \text{ in}^2$

$b = \frac{I_x}{cA} \qquad I_x = I_{os} = I_{cc} + Ad^2$ (the transfer formula) $(d = 6 \text{ in})$

$I_{cc} = \frac{bh^3}{12} = \frac{(8)(9)^3}{(12)} = 162 \text{ in} \qquad Ad^2 = (36)(6)^2 = 1300 \text{ in}^4$

$I_x = I_{os} = 162 + 1300 = 1462 \text{ in}^4$

$c = -6 \text{ in}$ (measured down from OS)

$A = 36 \text{ in}^2$

$b = \frac{1462}{(-6)(36)} = -6.78$ inches Sign is negative ∴ b is measured down from OS

$e = \frac{I_{xy}}{cA} \qquad I_{xy} = I_{os\text{-}ov} = I_{xy\,cc} + xyA$ (the transfer formula)

$I_{xy\,cc} = -\frac{b^2h^2}{72} = -72 \text{ in}^4 \qquad x = 6.67 \text{ in} \qquad y = -6 \text{ in}$

$I_{xy} = I_{os\text{-}ov} = -72 + (6.67)(-6)(36) = -1552 \text{ in}^4$

$c = -6 \text{ in} \qquad A = 36 \text{ in}$

$e = \frac{(-1552)}{(-6)(36)} = +7.2 \text{ in}$ Sign is positive ∴ e is measured to the right from OV

The single point force resultant of the hydrostatic load acting upon composite surface C will now be found. The surface is redrawn in figure HDS 6. It is then replaced by the rectangular and triangular surfaces.

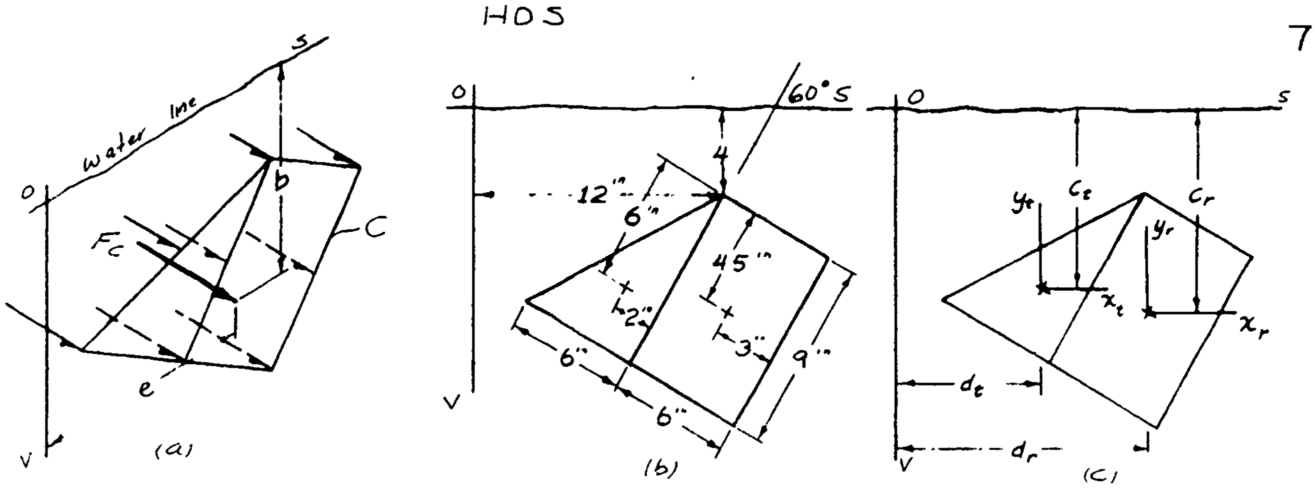


Figure HOS 6

First the magnitude of the force is found.

$$F = \gamma c A = \gamma(c_t A_t + c_r A_r)$$

using $y' = y\cos\theta - x\sin\theta$, its found that $c_t = 8.19''$ and $c_r = 9.39''$

$$F = (.0361)[(8.19)27 + (9.39)54] = 26.3 \text{ lb}$$

Now the position of the force must be found.

$$b = \frac{I_x}{cA} = \frac{I_x}{c_t A_t + c_r A_r} \text{ where } I_x = I_{os} = I_{x'_t} + c_t^2 A_t + I_{x'_r} + c_r^2 A_r$$

$I_{x'_t}$ and $I_{x'_r}$ are found using $I_{x'} = \frac{I_x + I_Y}{2} + \frac{I_x - I_Y}{2}\cos 2\theta - I_{XY}\sin 2\theta$

$$I_{x'_t} = \frac{9^3(6)/36 + 6^3(9)/36}{2} + \frac{9^3(6)/36 - 6^3(9)/36}{2}(.5) - \frac{9^2 6^2}{72}(.867) = 69\ 8 \text{ in}^4$$

$$I_{x'_r} = \frac{9^3(6)/12 + 6^3(9)/12}{2} + \frac{9^3(6)/12 - 6^3(9)/12}{2}(.5) - (0)(.867) = 314 \text{ in}^4$$

$$I_x = 69.8 + (^-8.19)^2\, 27 + (^-9.39)^2\, 54 = 7{,}966 \text{ in}^4$$

$$b = \frac{7{,}966}{(8.19)27 + (^-9.39)57} = {}^-10.9 \text{ in}, \quad F \text{ acts } 10.9 \text{ in below the water surface}$$

$$e = \frac{I_{xy}}{cA} \quad \text{where} \quad I_{xy} = I_{x'_t y'_t} + c_t d_t A_t + I_{x'_r y'_r} + c_r d_r A_r$$

d_t and d_r are determined from $x' = x\cos\theta - y\sin\theta$, $d_t = 7.27''$ $d_r = 10.34''$

The formula $I_{xy'} = \frac{I_x - I_Y}{2}\sin 2\theta + I_{XY}\cos 2\theta$ is used to find $I_{x'_t y'_t}$ and $I_{x'_r y'_r}$

$$I_{x'_t y'_t} = \frac{9^3(6)/36 - 6^3(9)/36}{2}(.867) + \frac{9^2 6^2}{72}(.5) = 49\ 5 \text{ in}^4$$

$$I_{x'_r y'_r} = \frac{9^3(6)/12 - 6^3(9)/36}{2}(867) + (0)(.5) = 87.5 \text{ in}^4$$

$$I_{xy} = 49.5 + (^-8.19)(7.27)27 + 875 + (^-9\ 39)(10.34)54 = {}^-6715 \text{ in}^4$$

$$e = \frac{^-6715}{(^-8.19)27 + (^-9.39)(54)} = 9.21 \text{ in}, \quad F \text{ acts } 9.21 \text{ in to the right of OV}$$

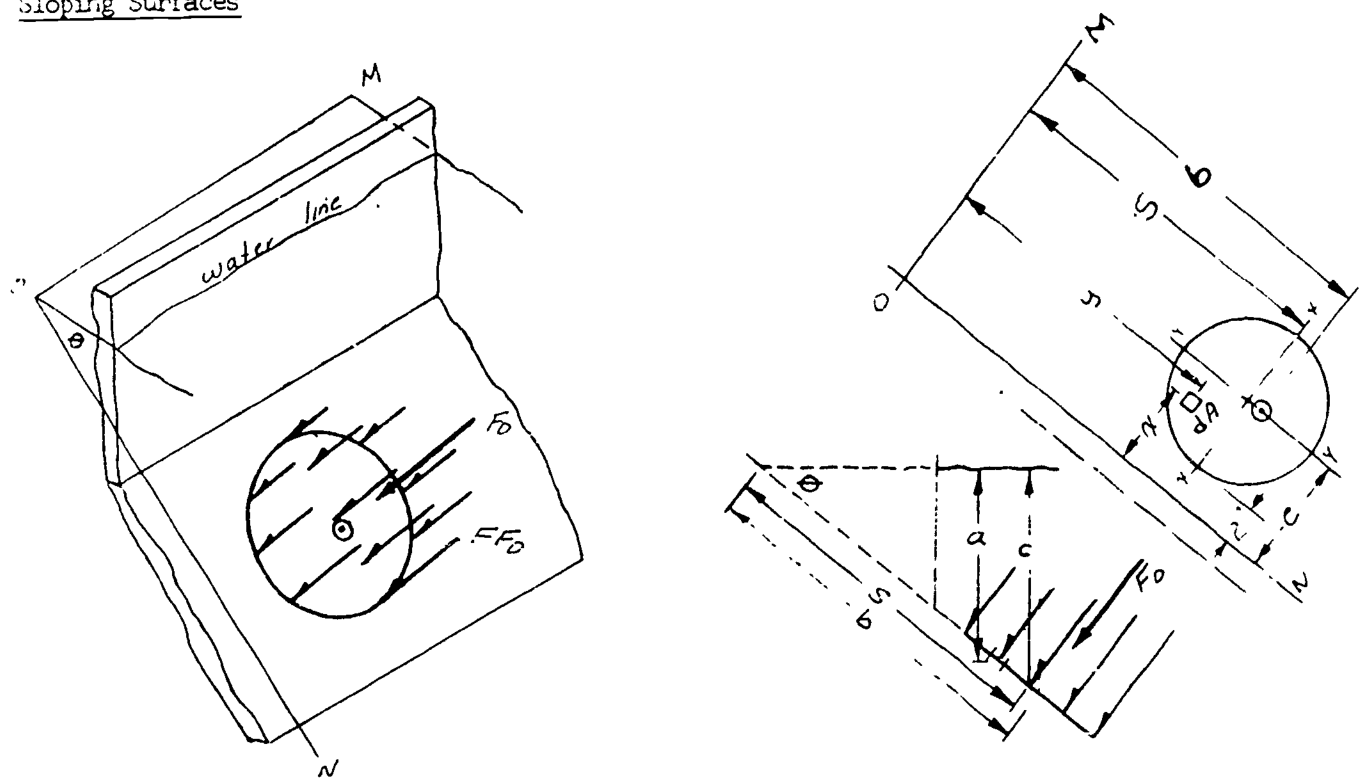


Figure HDS 7

The point force resultant of the distributed load acting on sloping surface D will now be found. First 3-D and 2-D diagrams are drawn in figure HDS 7. Next the point force resultant and its location will be derived analytically. The intersection of surface D with an extension of the free water surface is labeled OM. Line ON is a line parallel to surface D. Element dA is chosen a distance a below the water surface

$$dF \text{ on } dA = p\,dA = \gamma a\,dA = \gamma y \sin\theta\,dA$$

$$F_D \text{ on } D = \gamma\int_A y \sin\theta\,dA = \gamma c A$$

As before for a vertical surface the magnitude of F on a sloping surface is γ times c (the vertical distance to the centroid of the surface) times the submerged area.

The line of action of F_D is needed.

To find b

$$M_{F_D/OM} = M_{FF_D/OM} \qquad b F_D = \int_A y\,\gamma y \sin\theta\,dA = \gamma \sin\theta \int_A y^2 dA$$

$$\text{where } F_D = \gamma\int_A y \sin\theta\,dA$$

$$b = \frac{\gamma \sin\theta \int_A y^2 dA}{\gamma \sin\theta \int_A y\,dA} = \frac{\int_A y^2\,dA}{\int_A y\,dA} = \frac{I_{OM}}{SA}$$

To find e

$$M_{F_D/ON} = M_{FF_D/ON} \qquad e F_D = \int_A x\,\gamma y \sin\theta\,dA = \gamma \sin\theta \int_A xy\,dA$$

$$e = \frac{\gamma \sin\theta \int_A xy\,dA}{\gamma \sin\theta \int y\,dA} = \frac{\int_A xy\,dA}{\int_A y\,dA} = \frac{I_{OM\text{-}ON}}{SA}$$

Now the point force resultant of the hydrostatic load acting on surface D will be found when c = 11 inches, θ = 30°, and the diameter of D is 7 inches. Distance s to OM is then 11 / sin 30° = 22 inches, ON is drawn 2 inches from the edge of surface D.

$$F = \gamma c A = (.0361)(11)(38.4) = 15.3 \text{ lb} \qquad A = (\pi)(3.5)^2 = 38.4 \text{ in}^2$$

$$b = \frac{I_{OM}}{sA} \qquad I_{OM} = I_{cc} + As^2 \qquad I_{cc} = \frac{\pi r^4}{4} = 118 \text{ in}^4$$

$$= (118) + (38.4)(22)^2$$

$$= 18716 \text{ in}^4$$

$$b = \frac{(18716)}{(-22)(38.4)} = -22.2 \text{ inches} \qquad b \text{ is downward from OM}$$

$$e = \frac{I_{OM\text{-}ON}}{sA} \qquad I_{OM\text{-}ON} = I_{xy} + xy\,dA$$

$$= 0 + (2 + 3.5)(-22)(38.4)$$

$$= -4660 \text{ in}^4$$

$$= \frac{(-4660)}{(-22)(38.4)} = 5.5 \text{ inches} \qquad e \text{ is to the right from ON}$$

Distances b and e locate the centroid of force field FF_D. Whenever the surface is symmetrical (I_{xy} = 0), its force field will be symmetrical and distance e can be found by inspection. That is in this example F_D will lie on yy.

Engineering Applications

Example 1

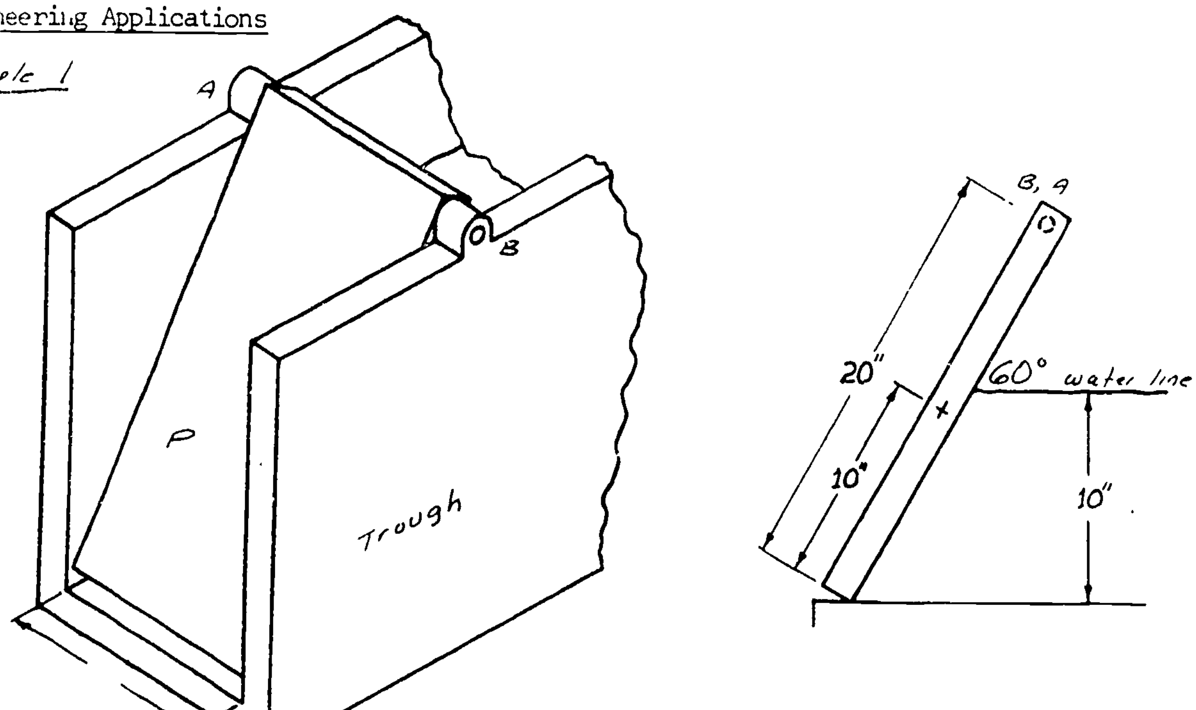


Figure HDS 8

Plate P in figure HDS 8 weighs 85 lb and is held in a wooden trough by frictionless pins at A and B. The plate makes a snug fit with the sides and bottom of the trough. The pin reactions are to be found with the water level as shown if all contact surfaces between P and the trough are considered to be frictionfree.

.0 HDS

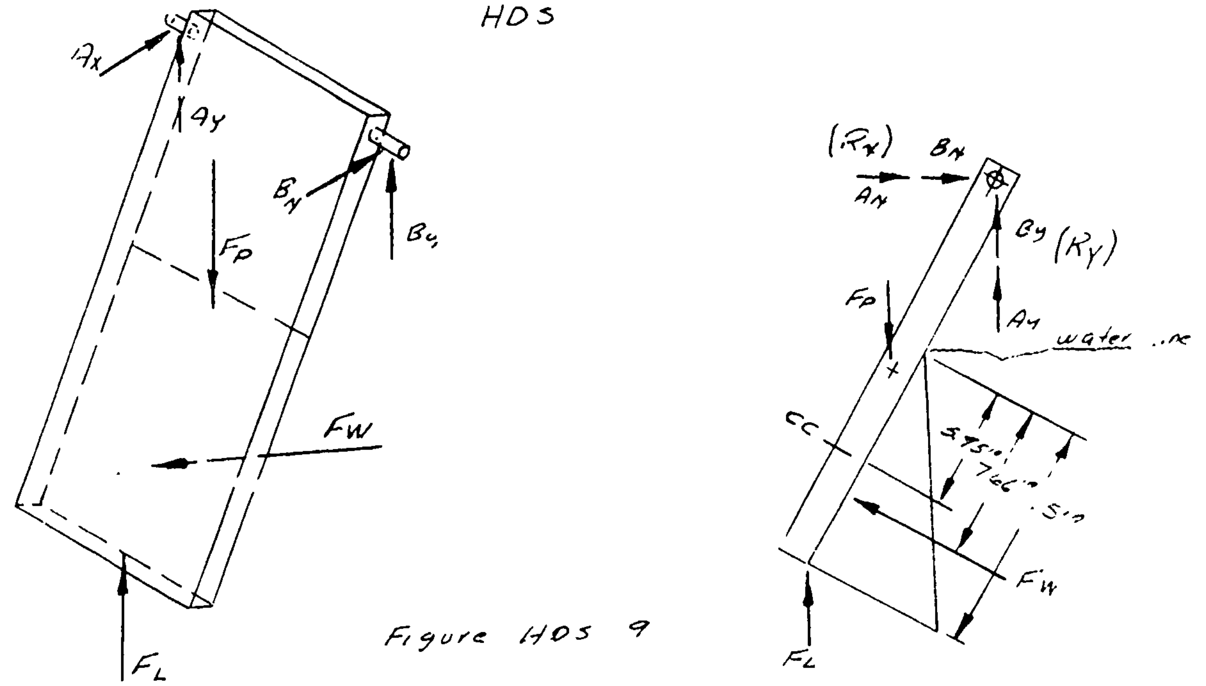


Figure HDS 9

3-D and 2-D F-B diagrams are drawn. F_P is known. F_W must be found and placed in the F-B diagrams.

$F_W = \gamma c A \qquad \gamma = .0361 \text{ lb/cu.in} \qquad c = 5\text{ in} \qquad A = (9)\left(\frac{10}{\cos 60°}\right) = (9)(11.5)$

$= (.0361)(5)(9)(11.5) = 18.7 \text{ lb}$

$b = \frac{I_x}{sA} \qquad I_x = I_{cc} + Ad^2 = \frac{bh^3}{12} + Ad^2 \qquad b = 9\text{ in} \quad h = 11.5\text{ in}$

$= \frac{(9)(11.5)^3}{12} + (9)(11.5)(5.75)^2 \qquad A = (9)(11.5)\text{ in}^2$

$d = 5.75\text{ in} \qquad s = 5.75\text{ in}$

$= 4580 \text{ in}^4$

$b = \frac{(4580)}{(5.75)(9)(11.5)} = 7.66$ inches which is $\left(\frac{2}{3}\right)(11.5)$ inches by inspection

c = 4.5 inches from either trough side by inspection

The system can reduce to a coplanar system in (b) by letting $\bar{R} = \bar{A} + \bar{B}$ where $R_x = A_x + B_x$, $R_y = A_y + B_y$, and $A_x = B_x$ and $A_y = B_y$

$\Sigma M_{A,B} = 0 \qquad (5)(85) = (10)(F_L) + (10 + 7.66)(18.7)$

$F_L = 9.5 \text{ lb}$

$\Sigma F_V = 0 \qquad R_y + 9.5 + (18.7)(.5) = 85 \qquad R_y = 66.2 \text{ lb}$

$\Sigma F_H = 0 \qquad R_x = (18.7)(.866) = 16.2 \text{ lb}$

$A_x = B_x = 16.2/2 = 8.1 \text{ lb} \qquad A_y = B_y = 66.2/2 = 33.1 \text{ lb}$

$A = B = \sqrt{8.1^2 + 33.1^2} = 33.2 \text{ lb}$

So the point force resultant of the force field acting at A or B is 33.2 lb.

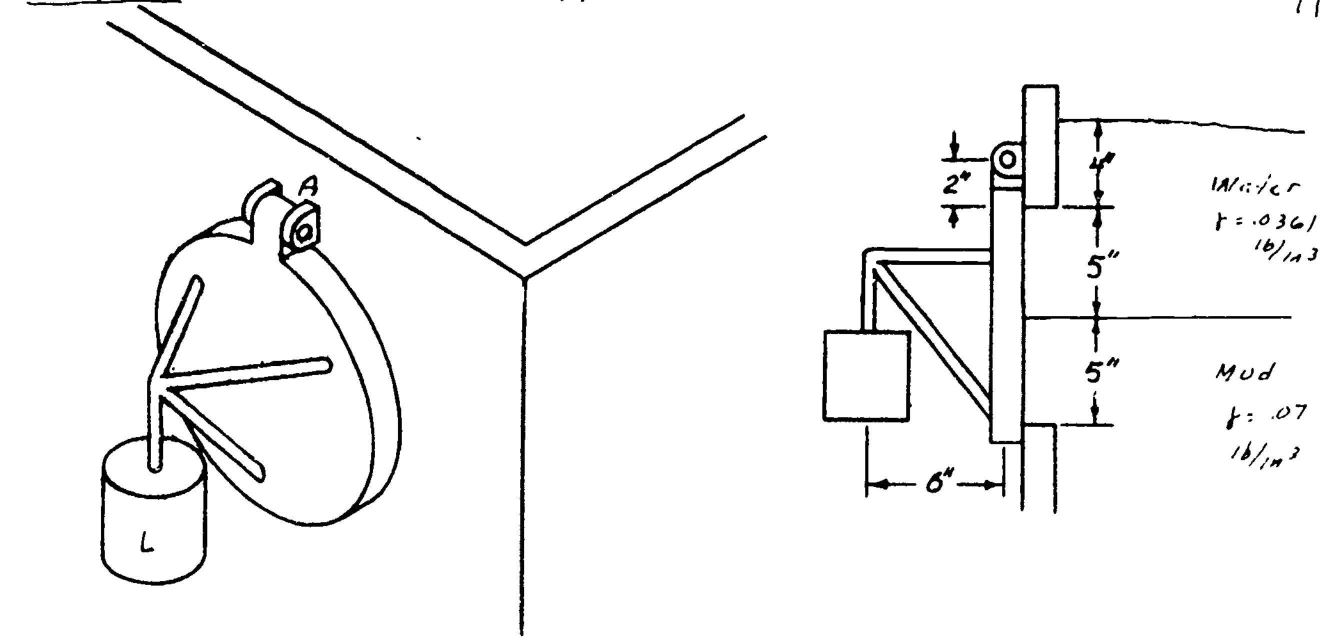


Figure HOS 10

The symmetrical gate is centered over a 10 inch diameter opening. The upper half of the gate is acted upon by water, the lower by fluid mud. The weight L needed to keep the gate c closed is to be found, also the components of the load on A. The gate is assumed to be weig weightless and the pin at A frictionless.

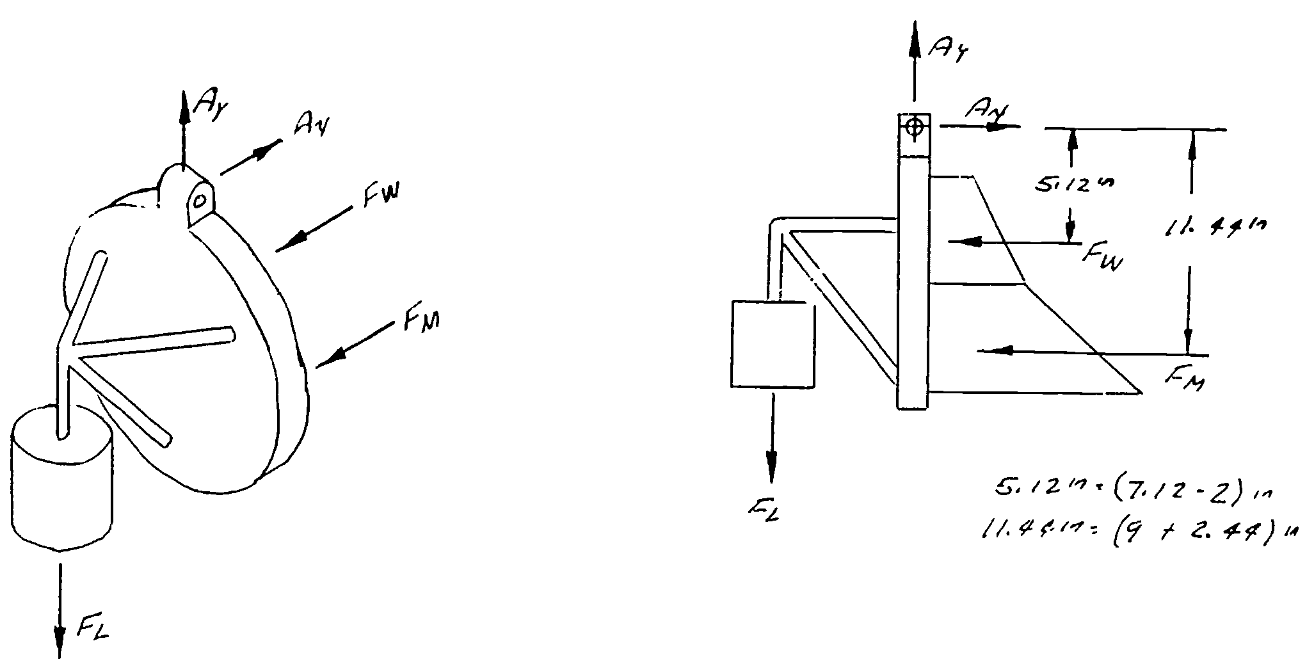


Figure HOS 11

3-D and 2-D F-B diagrams are constructed of the gate with the pin at A removed. The loads from the water and the mud will be handled separately.

FW will be found first.

$F_W = \gamma \bar{c} A \qquad \gamma = .0361 \text{ lb/in}^3 \qquad \bar{c} = 9 - \frac{(4)(5)}{(3)(\pi)} = 6.88 \text{ in}$

$= (.0361)(6.88)(39.2) \qquad A = \frac{(\pi)(5)^2}{(2)} = 39.2 \text{ in}^2$

$= 9.75 \text{ lb}$

$b = \frac{I_x}{\bar{c}A} \qquad I_x = .11 r^4 + A d^2$

$= \frac{(.11)(5)^4 + (39.2)(6.88)^2}{(6.88)(39.2)} = 1929 \text{ in}^4$

$= \frac{(1929)}{(6.88)(39.2)}$

$= -7.15$ in down from the water line

$e = 5$ inches from either side of the opening by inspection

The force field for the fluid mud pressure has a pressure at the intersection of the water and the mud of (9)(.0361) = .324 lb/sq-in. This is equivalent to a height of .324/.07 = 4.65 inches of mud. F_M will be found as if 4.65 inches of mud is acting. Line OM will be 4.65 inches above the plane separating the mud and the water.

$F_M = \gamma \bar{c} A = (.07)\left(4.65 + \frac{(4)(5)}{(3)(\pi)}\right)(39.2) = (.07)(6.77)(39.2) = 18.6 \text{ lb}$

$b \text{ (from OM')} = \frac{I_{OM'}}{\bar{c}A} = \frac{(.11)(5)^4 + (39.2)(6.77)^2}{(-6.77)(39.2)} = 7.05$ in down from OM'

F_W & F_M can now be located on the F-B diagram

Using the dimensions on the 2-D F-B diagram

To find F_L

$\Sigma M_A = 0$

$(5.12)(9.75) + (11.4)(18.6) = (6)(F_L)$

$F_L = 43.8 \text{ lb}$

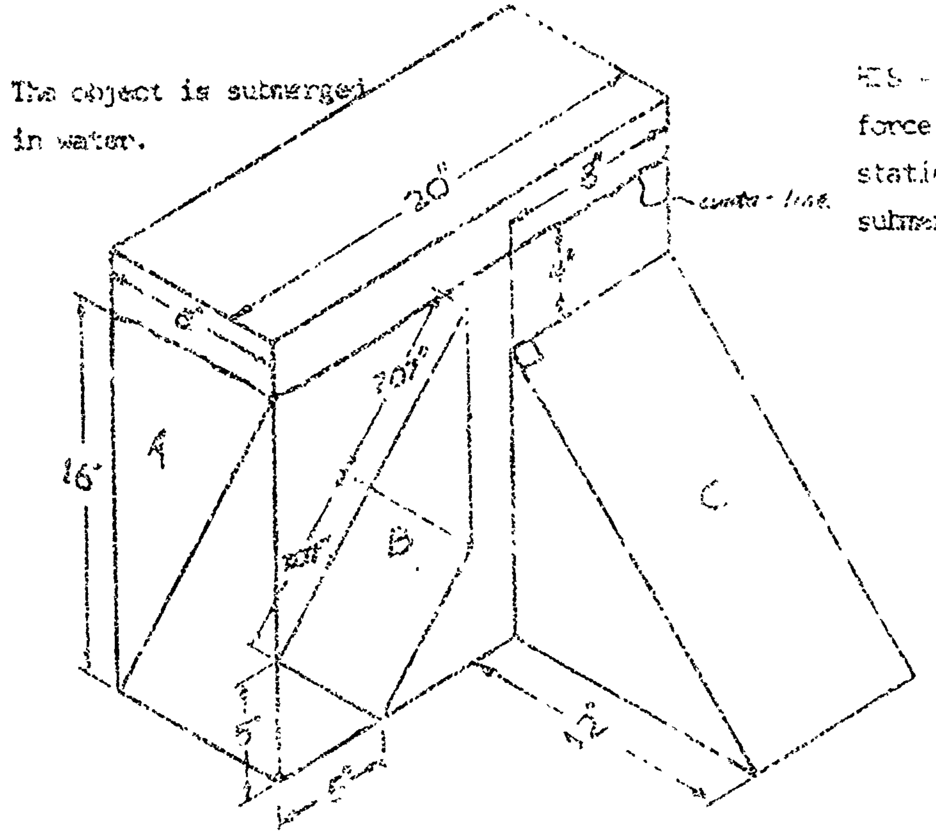
To find A_V & A_H

$\Sigma F_V = 0 \qquad A_V = 43.8 \text{ lb} \uparrow \qquad A = \sqrt{43.8^2 + 28.35^2} = 52.2 \text{ lb}$

$\Sigma F_H = 0 \qquad A_H = 9.75 + 18.6 = 28.35 \text{ lb} \rightarrow$

NOW IF YOU ARE GIVEN A FLAT SURFACE SUBMERGED IN A STATIC FLUID, YOU SHOULD BE ABLE TO FIND THE MAGNITUDE AND LINE OF APPLICATION OF THE POINT FORCE RESULTANT OF THE HYDROSTATIC FORCE FIELD THAT ACTS UPON THE SURFACE.

The object is submerged in water.



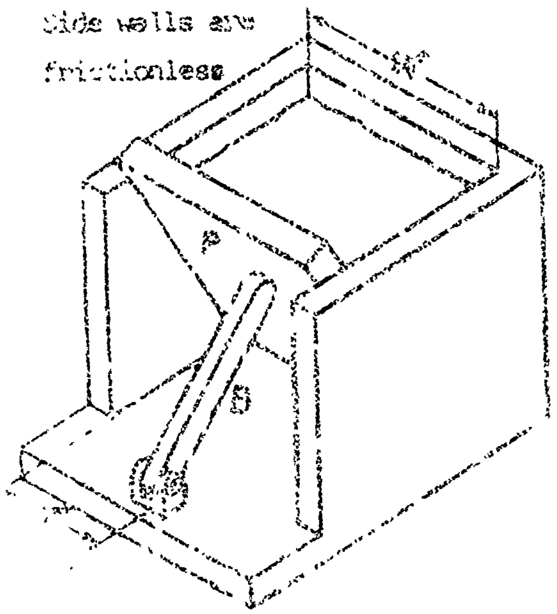
HDS - 1(a) Find the single point force resultants of the hydrostatic pressures acting on submerged surfaces A, B and C.

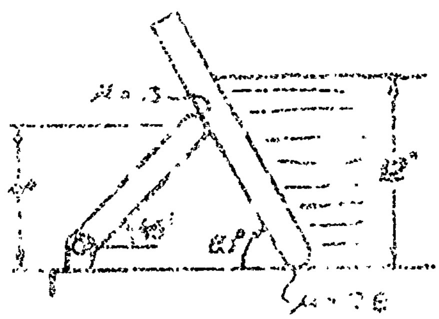
F_A = [illegible]
8 in below water line
2 in from left edge

F_B = 25.3 lb
10.4 in below water [illegible]
5.71 in from left edge

F_C = [illegible] lb
11.[illegible] in from top edge
on vertical centroidal axis

Side walls are frictionless

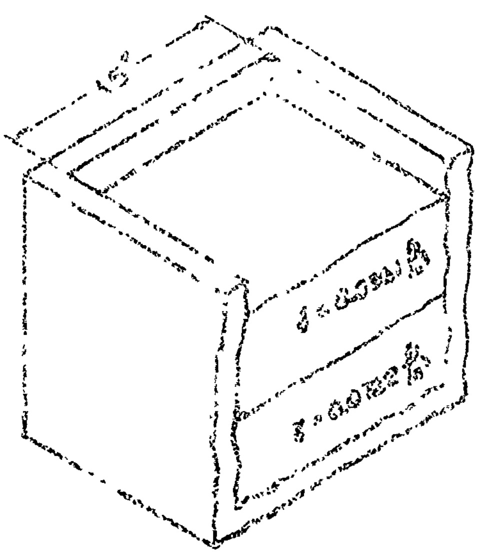


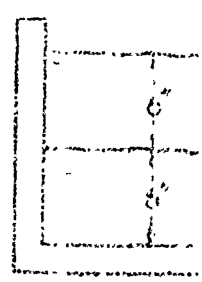


HDS - 1(b) [illegible] supports [illegible] weightless plate [illegible] which is acted on by a hydrostatic force. Find the force on B.

The force is 15.7 [illegible]

HDS - 1(c) Find the magnitude and position of the single point force resultant exerted by the composite fluid on the end wall.





The point force equals 48.8 lb acting 8.42 in from the top fluid surface.

UNIT 13
VIRTUAL WORK

AT THE END OF THIS UNIT IF YOU ARE GIVEN A SINGLE MEMBER OR A COMBINATION OF TWO OR THREE MEMBERS ACTED UPON BY COPLANAR LOADS OR COUPLES, YOU WILL BE ABLE TO FIND THE STATIC EQUILIBRIUM POSITION OF THE SYSTEM USING THE PRINCIPLE OF VIRTUAL WORK.

Introduction

In your work in equilibrium in the previous units, you have been concerned with finding the reactions on stationary engineering members using the parallelogram law in its force and moment equation ms. In this unit you will learn how to find the equilibrium position for a system that is acted upon by external loads. To find this equilibrium position, active-force diagrams, displacements, and work relationships will be used in place of free-body diagrams with force and moment equations.

Work

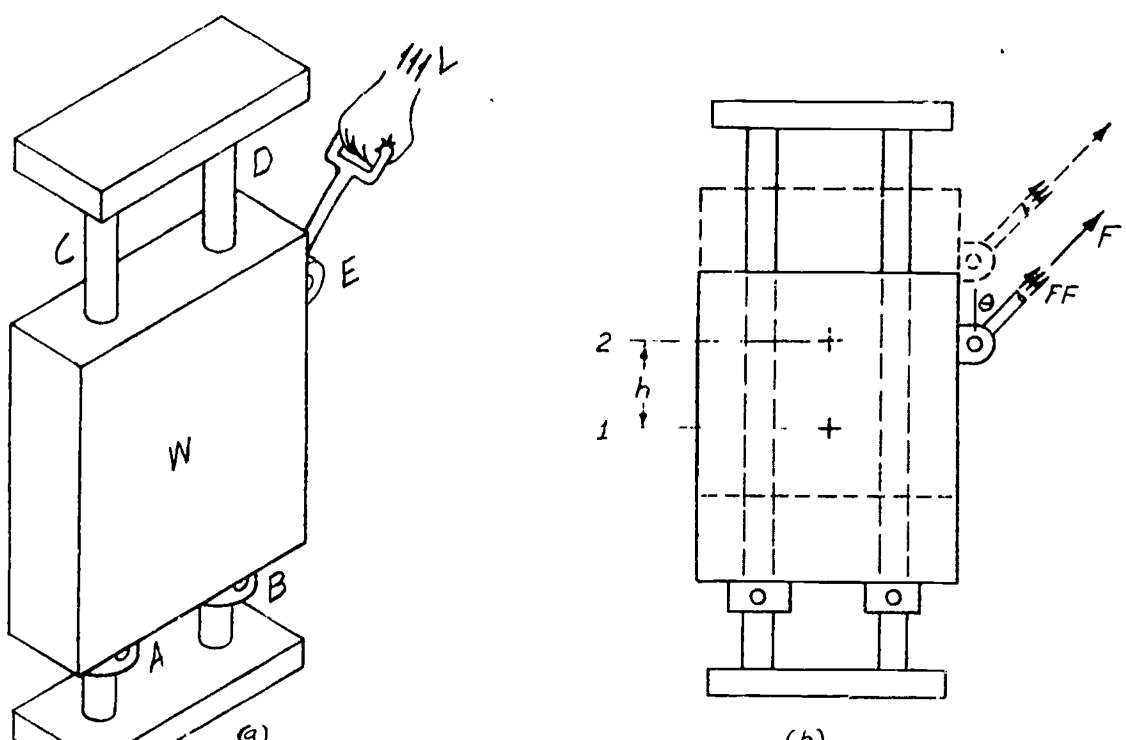


Figure VW 1

Weight W in figure VW 1 is resting in position 1 on collars A and B. C and D are frictionless vertical guides. Next a load L is gradually applied to block W through the frictionless connection E in the direction shown. L is increased until weight W is being held in position 1, at this time load L becomes constant. Then L is pulled until block W moves with no acceleration to position 2 as shown in (b). Weight W has now been moved through the vertical distance h.

When weight W is moved a vertical distance h, it is said that work has been done on W. The magnitude of this work is (W)(h). W is in lb and h in inches, so the units for work are in - lb.

The work to raise W was done by load L. In other words pulling in the direction of L with a force field FF will give the work equal to (W)(h). This work was done by FF when moving from 1 to 2. It is found from experimentation that if L is measured, replaced by FF and then F, then (F cos θ)(h) is equal to (W)(h). That is the work done by L moving vertically a distance h is (F cos θ)(h). It is also found from experimentation that work is not a vector quantity but is a scalar quantity, so when work equations are used, all the manipulations of real numbers can be used. This means that work quantities have signs and can be added or multiplied using real number manipulations.

It is easy to confuse effort with work. For instance as L is gradually increased until weight W is not supported by the collars, effort must be applied to E, but no work is done. Effort is needed to simply hold the weight. Now as L is pulled until W is in position 2, the effort of F sin θ does no work. Also as θ is varied, the effort needed to raise weight W varies. With a large θ much more effort is needed than with a small θ.

<u>Virtual Displacement and Virtual Work</u>

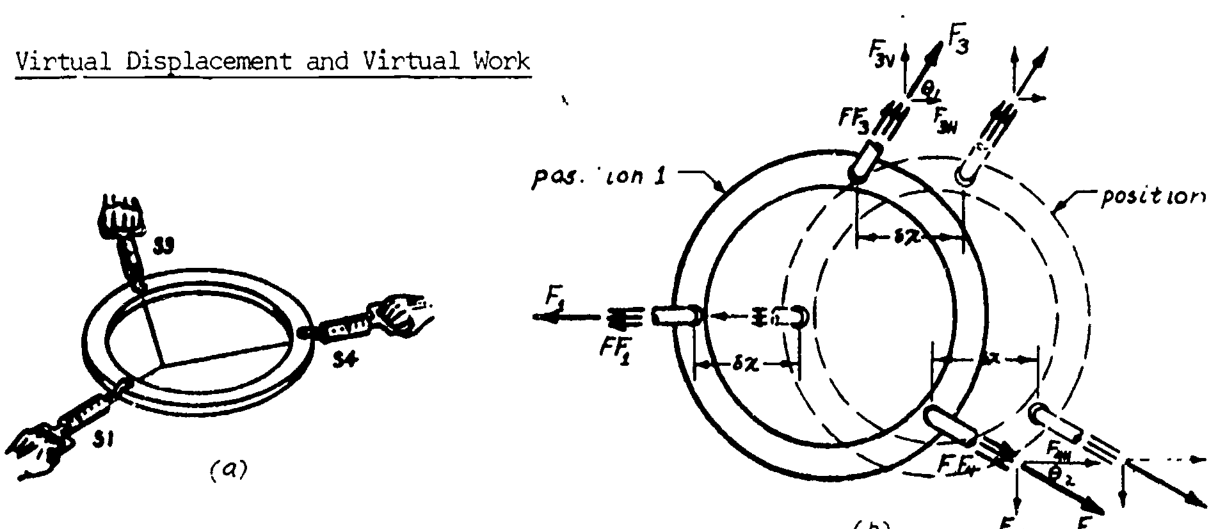


Figure VW 2

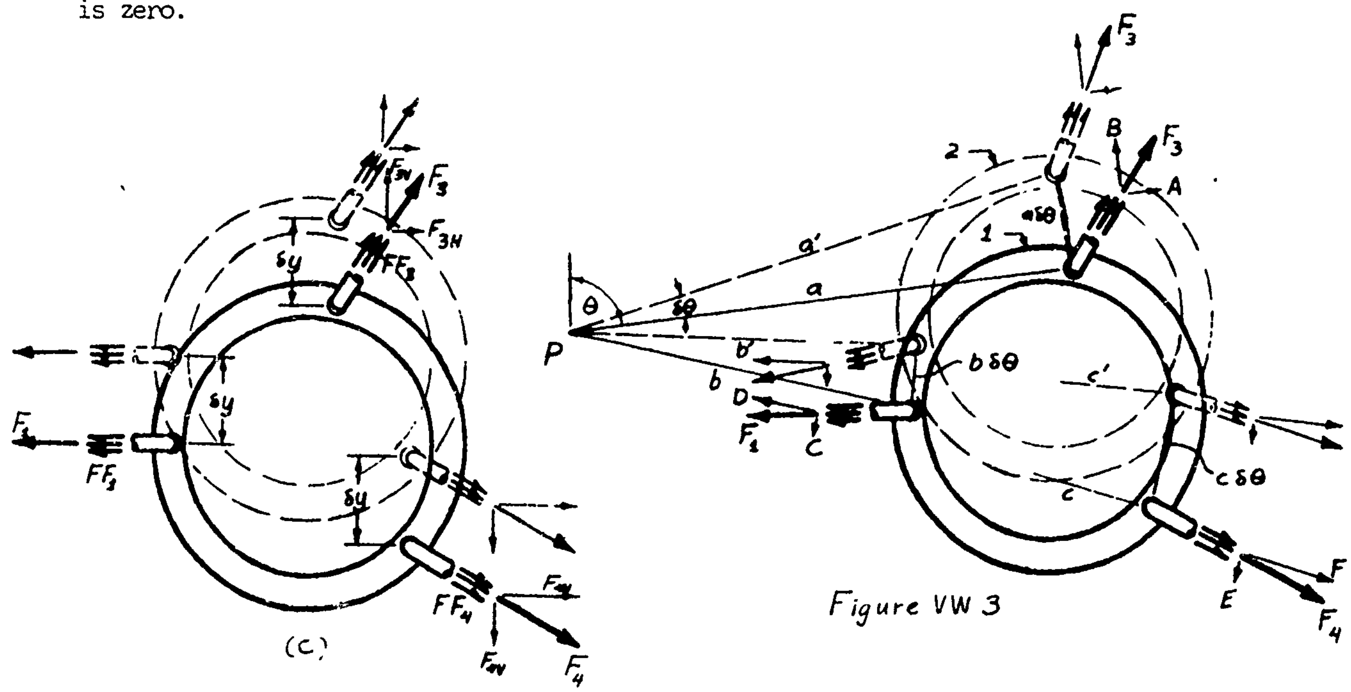
A 3-D drawing figure VW 2(a) shows a ring held in equilibrium in a horizontal position by loads S1, S3, and S4. A plan view of the ring is drawn next (b). Now the whole system is displaced in (b) a distance δx to the right from the original position 1 to position 2 while keeping the ring in equilibrium with S1, S3, and S4. Distance δx is on the order of dx and is called a virtual displacement.

The work done by FF_3 during displacement δx is (F_3 cos θ_1)(δx) or (F_{3H})(δx), this will be called positive work. Notice that F_{3V} does no work during the δx displacement. The work done by FF_4 when displaced is (F_4 cos θ_2)(δx) or (F_{4H})(δx) and is also positive. Now FF_1 acts toward the left and is displaced to the right. The work done by the horizontal force field FF_1 during displacement δx is ($-F_1$ cos θ_3)(δx) or - (F_1)(δx).

You know the system is in equilibrium whether at rest at 1, moving with no acceleration between 1 and 2, or at rest at 2. So $\Sigma F = 0$ for the system and the sum of the horizontal components equals 0 for the system. $\Sigma F_X = 0 \quad F_{3H} + F_{4H} - F_{1H} = 0$

Multiplying each term by δx gives $(F_{3H})(\delta x) + (F_{4H})(\delta x) - (F_{1H})(\delta x) = 0$

This states that the work done by the three loads during the virtual displacement δx is zero.



Now in (c) the ring is kept in equilibrium and displaced a vertical displacement δy upward. FF_3 does work of $(F_{3V})(\delta y)$, FF_4 of $-(F_{4V})(\delta y)$, and FF_1 does work of $F_{1V}(\delta y)$ = zero work.

Using $\Sigma F_y = 0 \quad F_{3V} - F_{4V} + F_{1V} = 0$

Multiplying by $\delta y \quad (F_{3V})(\delta y) - (F_{4V})(\delta y) + (F_{1V})(\delta y) = 0$

This states that the work done by the three loads during a virtual displacement $\delta y = 0$.

The ring could be displaced while in equilibrium through any virtual displacement in any direction and the work done would be zero.

Now in figure VW 3, the ring is displaced through a small angle $\delta\theta$ which is on the order of $d\theta$. The work done by each point force when the ring is rotated through $\delta\theta$ will be found.

First the work done by F_3 when it is rotated through $\delta\theta$ will be found. Line a (solid) is drawn from P to F_3 at position 1. Then the ring is rotated through $\delta\theta$ to position 2. Line a′ (dotted) is then drawn. Next F_3 is replaced by A and B where A is parallel to line a and B is perpendicular to line a. For a small $\delta\theta$ chord $a\delta\theta$ is equal to arc $a\delta\theta$. When F_3 is rotated a small $\delta\theta$ from position 1 to position 2, A does no work. The only work done is $(B)(a\,\delta\theta)$. But aB is the moment of F_3 with respect to point P, so the work done is $(aB)(\delta\theta) = (M_{F_3/P})(\delta\theta)$.

Similarily F_1 does work of $(-C)(b\,\delta\theta) = (M_{F_1/P})(\delta\theta)$.

And F_2 does work of $(-E)(c\,\delta\theta) = (M_{F_2/P})(\delta\theta)$.

These are all virtual work quantities as $\delta\theta$ is a virtual rotation.

Since $\Sigma M_p = 0$ for the system while in equilibrium, correct signs can be used to give

$$M_{F_3/P} - M_{F_1/P} - M_{F_4/P} = 0$$

Multiplying by $\delta\theta$, $(M_{F_3/P})(\delta\theta) - (M_{F_1/P})(\delta\theta) - (M_{F_4/P})(\delta\theta) = 0$

This states that when a body in equilibrium under the action of external loads is displaced through a virtual angle $\delta\theta$, the work done by the external loads is zero.

Looking back at the derivation, you can see why $\delta\theta$ must be on the order of θ. $\delta\theta$ must be small enough so that the physical geometry remains constant when $\delta\theta$ takes place. The work done during a virtual displacement, either translation or rotation, is called virtual work and written δU.

Now a principle can be stated. When a body is acted upon by external loads and is in equilibrium, the extra external work needed to give the body a virtual displacement is zero. Or stated another way, if a body is in equilibrium under the action of external loads and is displaced a virtual displacement, the external loads do no work. This is called the principle of virtual work and is written $\delta U = 0$.

Equilibrium Position of a Single Member Acted Upon by Coplanar Loads

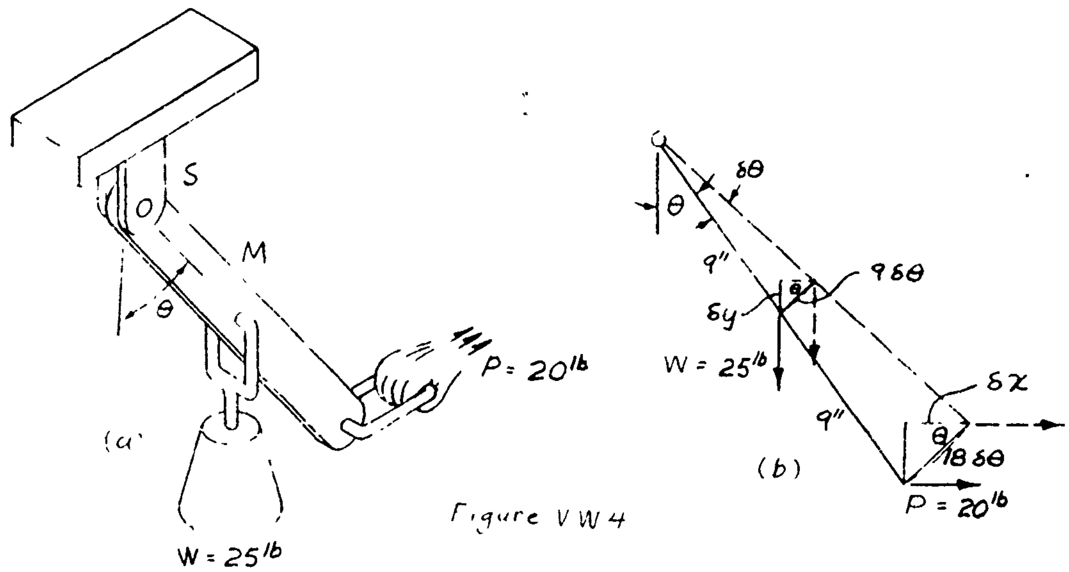


Figure VW 4

Member M in figure VW 4(a) is weightless, supports loads W and P through frictionless connections, and is supported at S with a frictionless pin. The equilibrium position is to be determined (angle θ) using the principle of virtual work.

In (b) a diagram is drawn of the ℄ of M. The equilibrium position is assumed (angle θ) as shown with the full line diagram. Now M is displaced through $\delta\theta$ to the dotted position. Since M is in equilibrium, $\delta U = 0$ for the virtual displacement $\delta\theta$. The only forces that do work during the virtual displacement $\delta\theta$ are W and P, so only W and P are shown on the diagram. Diagram (b) is called an active-force diagram.

Now the work done by P during $\delta\theta$ is $P\delta x$, also during $\delta\theta$ work is done by W of $-W\delta y$.

$\delta U = 0 \qquad P\,\delta x - W\,\delta y = 0$

Notice that $\delta x = 18\,\delta\theta\cos\theta$ and $\delta y = 9\,\delta\theta\sin\theta$

so $20(18\,\delta\theta\cos\theta) - 25(9\,\delta\theta\sin\theta) = 0$

Dividing by $\delta\theta \qquad 20(18\cos\theta) - 25(9\sin\theta) = 0$

Solving for $\theta \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{(20)(18)}{(25)(9)} = 1.6 \qquad \theta = 58^\circ$

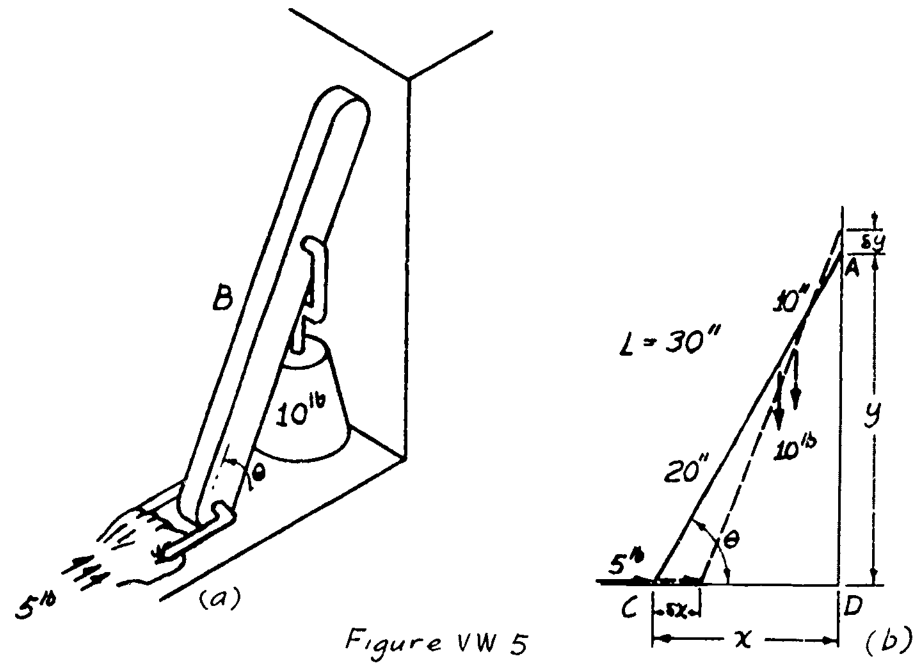


Figure VW 5

The weightless bar B on VW 5(a) supports a 10 lb weight and is held in equilibrium by a horizontal push of 5 lb. The vertical and horizontal walls are frictionless. Angle is to be found for static equilibrium using $\delta U = 0$.

The active force diagram (b) is drawn (solid line) with θ assumed. From C to D is labeled x, from D to A is y. Now B is given a horizontal virtual displacement δx, this gives A a vertical virtual displacement δy.

$\delta U = 0 \qquad 5\,\delta x - 10(\tfrac{2}{3}\,\delta y) = 0$

Now δx and δy must be related to each other.

$x^2 + y^2 = 30^2$ differentiating yields $2x\,dx + 2y\,dy = 0$

Now let $\delta x = dx$ and $\delta y = dy$ so $2x\,\delta x + 2y\,\delta y = 0$ or $\delta y = -\frac{x}{y}\,\delta x$

The minus sign shows that as y gets longer x gets shorter.

Substituting in $\delta U = 0$, $\quad 5\,\delta x - 10(2/3)(x/y\;\delta x) = 0$

$5 - 10(2/3)(x/y) = 0 \qquad$ or $\quad x/y = .75$

$\cot\theta = x/y = .75 \qquad \theta = \cot^{-1}(.75) = 53^\circ$

NOW IF YOU ARE GIVEN A MEMBER THAT IS IN EQUILIBRIUM UNDER THE ACTION OF COPLANAR LOADS, YOU SHOULD BE ABLE TO DETERMINE ITS EQUILIBRIUM POSITION USING THE PRINCIPLE OF VIRTUAL WORK WITH AN ACTIVE-FORCE DIAGRAM.

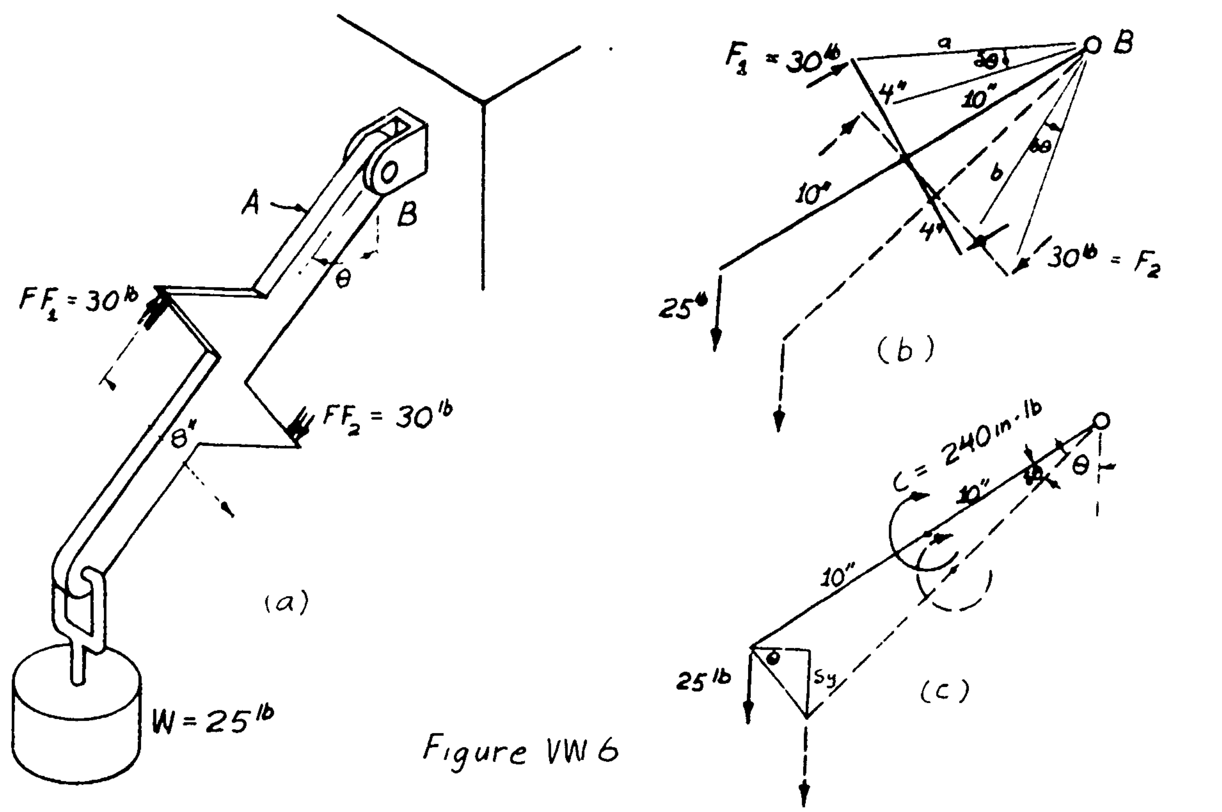


Figure VW 6

Figure VW 6(a) shows a weightless member A loaded by weight W (25 lb) and force fields FF_1 (30 lb) and FF_2 (30 lb). Angle θ is to be found when the system is in static equilibrium. Notice that FF_1 and FF_2 form a couple.

Active force diagram (b) is drawn with the three loads replaced by their point forces. The system is then displaced through the virtual angle $\delta\theta$. Point Force F_1 does work of $(M_{F_1/B})(\delta\theta)$ when it is rotated through $\delta\theta$. F_2 does work of $(M_{F_2/B})(\delta\theta)$ during this rotation. The total work done by F_1 and F_2 during $\delta\theta$ are $(M_{F_1/B} + M_{F_2/B})(\delta\theta)$. $M_{F_1/B} + M_{F_2/B}$ is equal to the moment of the couple formed by F_1 and F_2 with respect to B. This is also the moment of the couple with respect to any point. It can be said then that when the couple is rotated through a $\delta\theta$ virtual rotation angle, the virtual work done is $(M_C)(\delta\theta)$ or $(C)(\delta\theta)$. It can also be deduced that when a couple is translated through a straight line translation δx, it does no work as the work done by one of the forces would be equal and opposite to the work done by the other force.

Now active-force diagram (c) is drawn with load W and couple C. The system is in equilibrium and is again rotated through $\delta\theta$. The couple does negative work against the rotation.

$$\delta U = 0 \qquad -C\,\delta\theta + W\,\delta y = 0$$

$$-(240)\,\delta\theta + (25)(20\,\delta\theta \sin\theta) = 0$$

$$\sin\theta = \frac{240}{500} = .48 \qquad \theta = 28.6^\circ$$

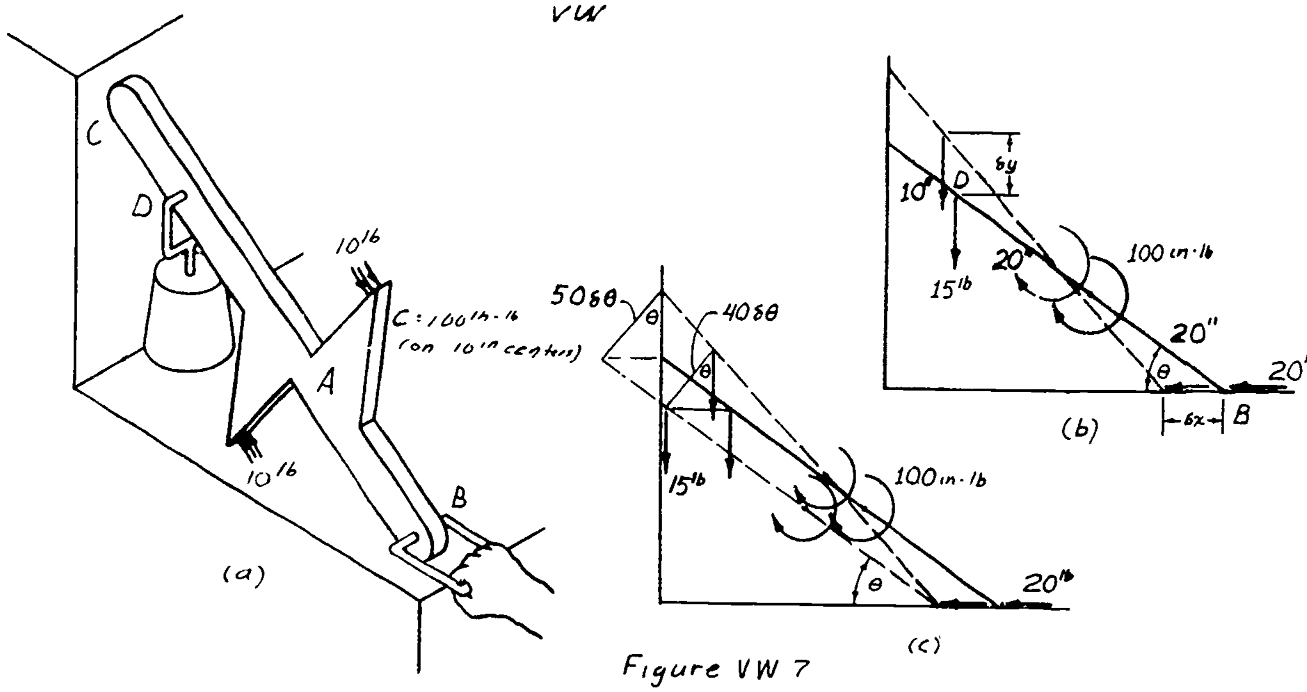


Figure VW 7

Sloping member A in figure VW 7(a) is loaded as shown. The floor and wall are frictionless and angle θ is to be found when the system is in static equilibrium.

Active-force diagram (b) is drawn with an assumed θ. Now if the 20 lb force is given a δx to the left at B, the bar at D will rise a δy. The virtual distances the 20 and 15 lb point forces move can be found, but the virtual angle through which the couple rotates is not known. A new technique will be developed to find this $\delta\theta$.

The new technique is shown on the active-force diagram (c). This time the bar is first given a horizontal displacement δx as shown. Then the bar is rotated about B until it matches the position of the dashed bar in (b). During the δx displacement the 20 lb force does work of $20\delta x$. During the rotation the 15 lb force does work of $15\delta y$ and the couple does work of $100\delta\theta$.

$$\delta U = 0 \qquad 20\,\delta x + 15\,\delta y + 100\,\delta\theta = 0$$

From the geometry, $\delta x = 50\,\delta\theta\,\sin\theta$ and $\delta y = 40\,\delta\theta\cos\theta$

$$20(50\,\delta\theta\sin\theta) - 15(40\,\delta\theta\cos\theta) + 100\,\delta\theta = 0$$

$$10\sin\theta - 6\cos\theta + 1 = 0$$

$$10\sin\theta - 6\sqrt{1-\sin^2\theta} + 1 = 0$$

$$(10\sin\theta + 1)^2 = (6\sqrt{1-\sin^2\theta})^2$$

$$100\sin^2\theta + 20\sin\theta + 1 = 36(1-\sin^2\theta)$$

$$\sin\theta = \frac{-20 \pm \sqrt{20^2 + 4(136)(35)}}{2(136)} = .44 \qquad \theta = 26^\circ$$

NOW IF YOU ARE GIVEN A SINGLE MEMBER IN EQUILIBRIUM UNDER THE ACTION OF LOADS AND COUPLES, YOU SHOULD BE ABLE TO FIND ITS EQUILIBRIUM POSITION USING THE PRINCIPLE OF VIRTUAL WORK.

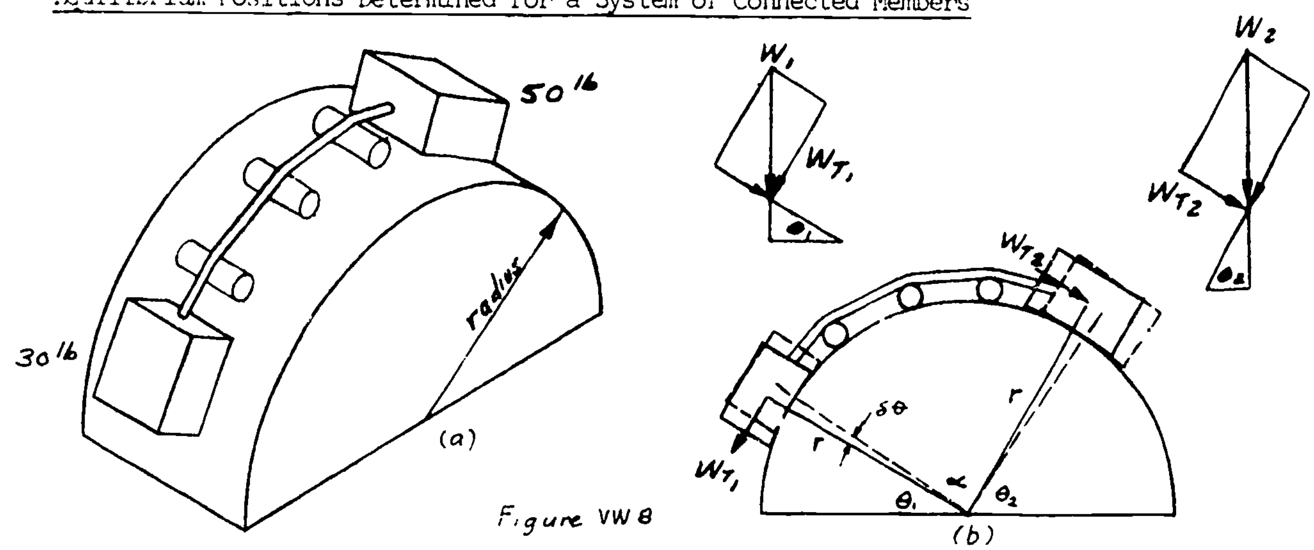


Figure VW 8

The two blocks in figure VW 8(a) are in static equilibrium. The equilibrium position of the system is to be determined using $\delta U = 0$. In (b) an active-force diagram is drawn with θ_1 assumed. The length of the weightless rope is such that angle α is 80° in (b). Angle θ_1 will be found.

First for each weight the tangential component is found in terms of W.

$$W_{T_1} = W_1 \cos\theta_1 = 30\cos\theta_1 \qquad W_{T_2} = 50\cos\theta_2$$

The system is now displaced through $\delta\theta$.

$$\delta U = 0 \quad -W_{T_1} r\delta\theta + W_{T_2} r\delta\theta = 0 \qquad -30\cos\theta_1 r\delta\theta + 50\cos\theta_2 r\delta\theta = 0$$

$$-30\cos\theta_1 + 50\cos\theta_2 = 0$$

Also $\theta_1 + 80^\circ + \theta_2 = 180^\circ$ or $\theta_2 = 100^\circ - \theta_1$

Substituting, $-30\cos\theta_1 + 50\cos(100 - \theta_1) = 0$

$$-30\cos\theta_1 + 50[\cos 100^\circ \cos\theta_1 + \sin 100^\circ \sin\theta_1] = 0$$

$$-30 - 8.68 - 49.2\tan\theta_1 = 0 \qquad \tan\theta_1 = .433 \qquad \theta_1 = 23.8^\circ$$

Three frictionless links in figure VW 9(a) weigh as shown. The three angles θ_1, θ_2 and θ_3 are to be found when the 10 lb horizontal load is applied.

First active-force diagram (b) is drawn for link A and angle θ_1 is found. It is assumed that the 5 lb point force acts through the center of the link.

Now active-force diagram (c) is drawn with links B and A joined with frictionless pin D. Link B is given a virtual displacement $\delta\theta$. Remember that link A has a known equilibrium angle θ_1 with a vertical line. This angle will not change during the virtual displacement $\delta\theta$.

Angle θ_3 can be found with active-force diagram (d). This time when the system is displaced through $\delta\theta$, angles θ_1 and θ_2 remain constant with a vertical line.

$\delta U = 0$

$$10\delta x - 5\delta y/2 = 0 \qquad 10\, l\delta\theta_1 \sin\theta_1 - 5\, l\delta\theta_1 \cos\theta_1/2 = 0$$

(b) $\tan\theta_1 = \frac{2.5}{10} = .25 \qquad \theta_1 = 14^\circ$

$$10\delta x - 5\delta y - 6\delta y/2 = 0 \qquad 10\, l\delta\theta_2 \sin\theta_2 - 5\, l\delta\theta_2 \cos\theta_2 - 6\, l\delta\theta_2 \cos\theta_2/2 = 0$$

(c) $\tan\theta_2 = \frac{8}{10} = .8 \qquad \theta_2 = 38.7^\circ$

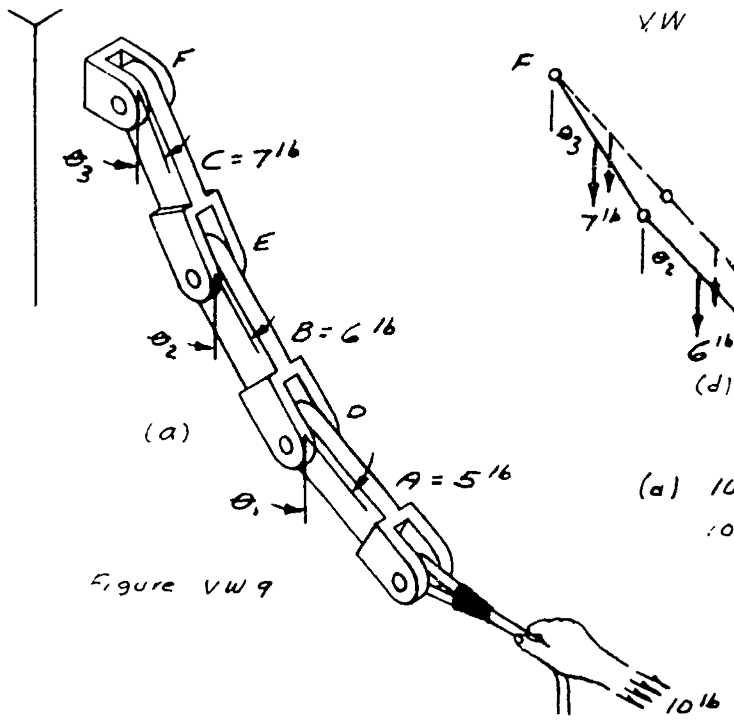
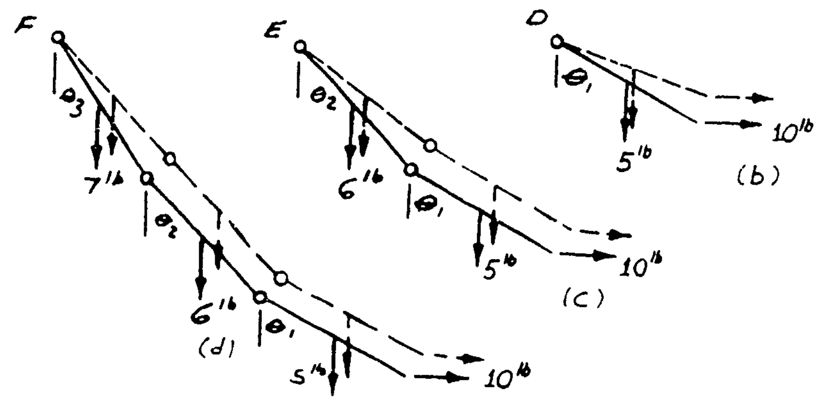


Figure VW 9



(a) $10\delta x - 5\delta y - 6\delta y - 7\,\delta y/2 = 0$

$10 l \delta\theta_3 \sin\theta_3 - 5 l \delta\theta_3 \cos\theta_3 - 6 l \delta\theta_3 \cos\theta_3 - 7 l \delta\theta_3 \cos\theta_3/2 = 0$

$\tan\theta_3 = \frac{14.5}{10} = 1.45$

$\theta_3 = 55.4°$

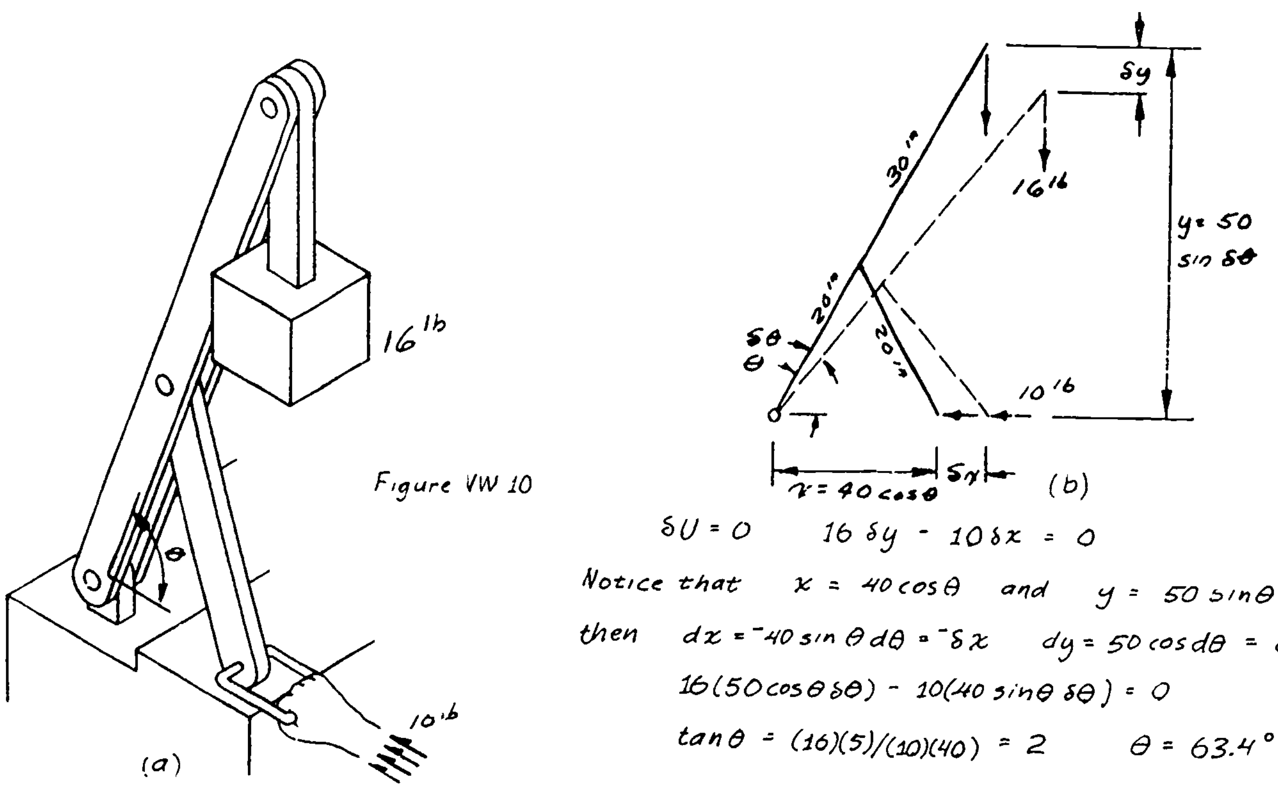


Figure VW 10

$\delta U = 0 \qquad 16\,\delta y - 10\,\delta x = 0$

Notice that $x = 40\cos\theta$ and $y = 50\sin\theta$

then $dx = -40\sin\theta\, d\theta = -\delta x \qquad dy = 50\cos d\theta = \delta y$

$16(50\cos\theta\,\delta\theta) - 10(40\sin\theta\,\delta\theta) = 0$

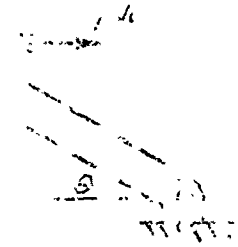
$\tan\theta = (16)(5)/(10)(40) = 2 \qquad \theta = 63.4°$

The 16 lb weight in figure VW 10(a) is being held in equilibrium by a horizontal 10 lb push. The equilibrium angle θ is to be found when the dimensions are as shown on the active-force diagram (b).

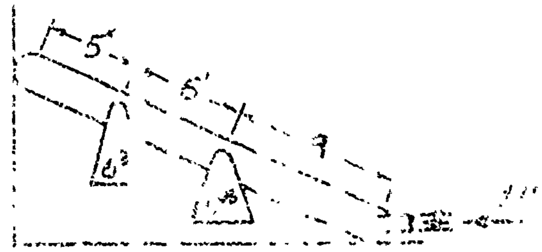
As always angle θ is assumed to be the equilibrium angle and the system is given a $\delta\theta$.

NOW IF YOU ARE GIVEN A SYSTEM OF TWO OR THREE MEMBERS IN EQUILIBRIUM, YOU SHOULD BE ABLE TO FIND THE EQUILIBRIUM POSITON OF THE SYSTEM USING VIRTUAL WORK.

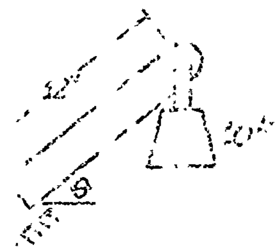
... are frictionless.



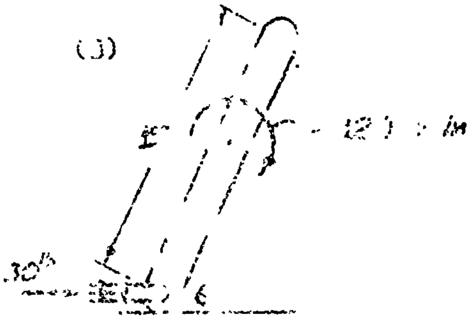
(B)



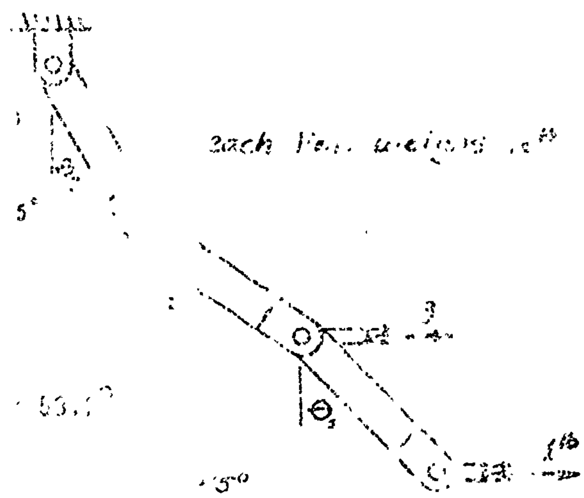
(C)



(D)



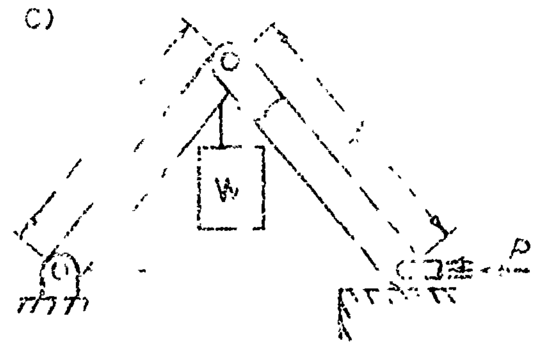
Each bar weighs ...



(H)

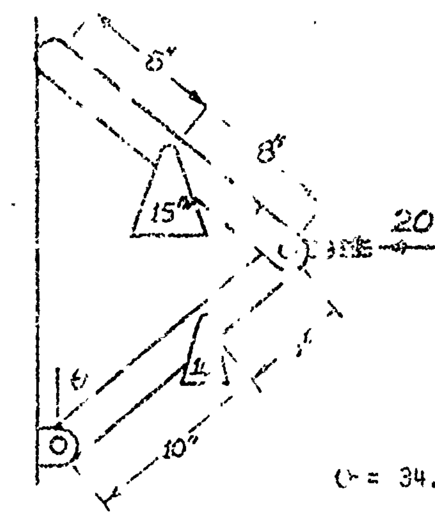
θ = 52.8°

(G)



$\theta = \tan^{-1}\frac{W}{2P}$

(I)



θ = 34.8°

UNIT 14

BEAM DIAGRAMS

GIVEN A BEAM SUPPORTING TRANSVERSE LOADS, AT THE END OF THIS UNIT YOU WILL BE ABLE TO CONSTRUCT LOAD, SHEAR, AND BENDING MOMENT DIAGRAMS FOR THE BEAM.

Introduction

The structural members labeled B in figure BD 1 are called beams. Beams are generally long, slender members used for supporting loads that are transverse to their ℄'s. Each of the beams shown below will be analyzed in this unit for the external reactions at its constraints and for the internal reactions on any vertical section of the beam.

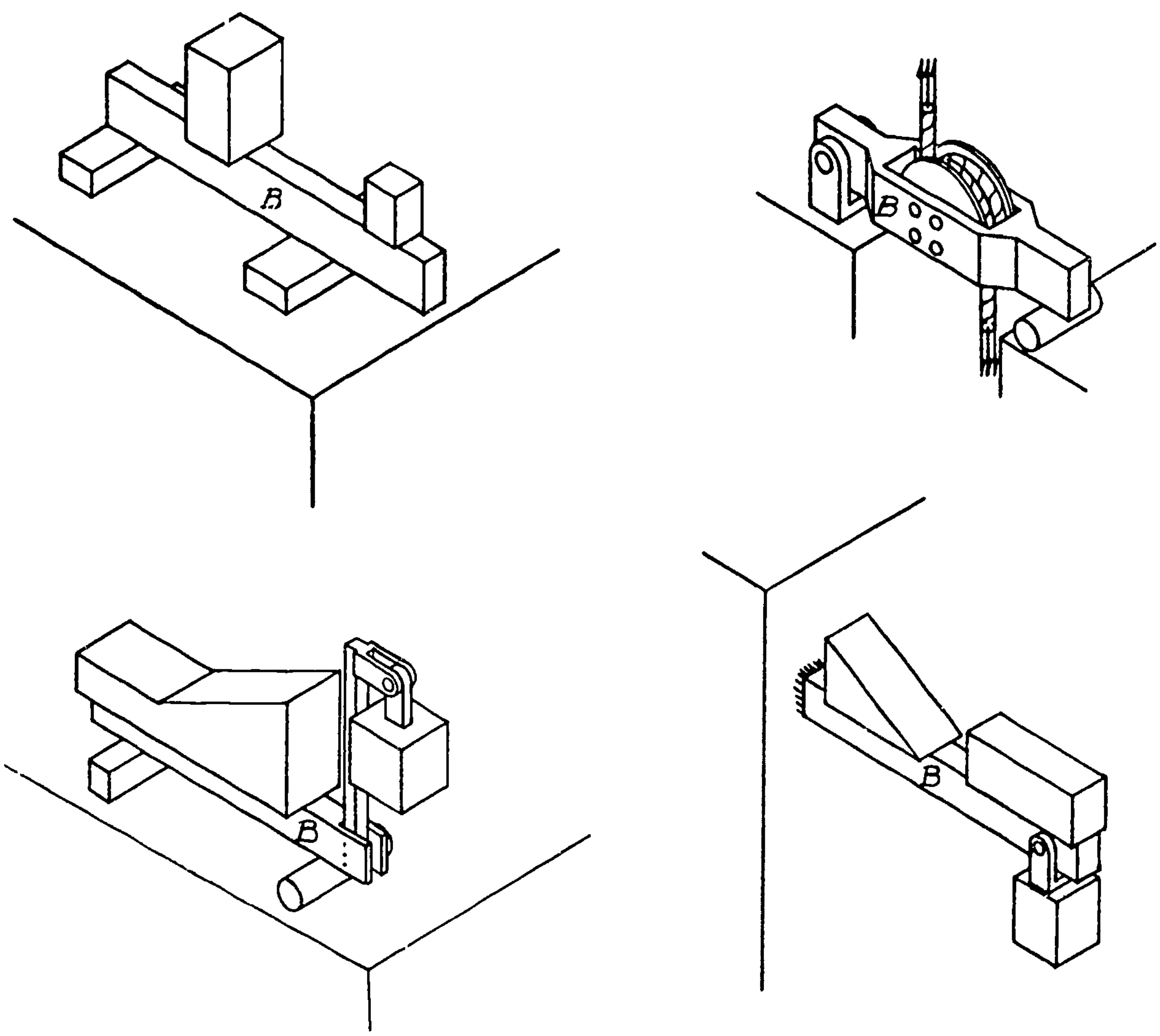


Figure BD 1

Uniformly Loaded Beams

The first beam to be analyzed in this unit is shown in figure BD 2(a). The reactions at its supports will be found, then internal reactions will be found for any vertical section of the beam. The beam is assumed to be weightless.

To find the reactions, F-B diagram (b) is drawn. The larger block weighs (15)(8)(10)(.1) = 120 lb, the smaller one (6)(8)(5)(.1) = 24 lb. Each block is replaced in (b) by its force field which is assumed to be evenly distributed. It is further assumed that the 6 inch and 8 inch supports have evenly distributed force fields. The magnitudes of FF_3 and FF_4 will be found by replacing each force field with its point force resultant and finding the resultants using moment equations.

$\Sigma M_{F_4} = 0$ gives F_3 $\quad (19)(120) + (49)(24) = (36)(F_3) \quad F_3 = 96$ lb

$\Sigma M_{F_3} = 0 \quad (13)(24) + (36)(F_4) = (17)(120) \quad F_4 = 48$ lb

Now an imaginary vertical section is to be exposed 19 inches from the left end. A F-B diagram is drawn in figure BD 2(c) of the portion of the beam from the exposed section to the left end of the beam. You have learned in the development of a cantilever beam that the force fields that act on the exposed section must balance the acting force fields on the rest of the beam, that is FF_5 and FF_6 must balance FF_4 and FF_7. FF_7 = 1/8 of FF_1 = (1/8)(120) = 15 lb.

The resultant of FF_4 and FF_7 is $\bar{R}$ where R = 48 - 15 = 33 lb↑.

In addition the moment of $\bar{R}$ equals the sum of the moments of FF_4 and FF_7 with respect to the exposed section.

$$M_{R/S} = M_{F_7/S} + M_{F_4/S} \qquad -(d)(33) = +(.5)(15) - (16)(48)$$

d = 23.05 inches to the left of the exposed surface.

For equilibrium to be maintained, this 33 lb force must be balanced by the resultants of FF_5 and FF_6 acting at the exposed surface. Remember from the cantilever development that the force fields on an exposed section vary from zero at the neutral axis to a maximum at the top and bottom, the two force fields must be equal, they are triangular in shape, their resultants pass through their effective centers, and their resultants balance the acting loads' resultant.

FF_5, $\bar{F}_5$, FF_6, and $\bar{F}_6$ can now be drawn with their correct directions in (c).

Now in (d) the point force resultants are drawn on the exposed surface. In (e) they are replaced by vertical and horizontal components, that is, $\bar{A} + \bar{B} = \bar{F}_5$ and $\bar{D} + \bar{C} = \bar{F}_6$. Remember from the cantilever section that $\bar{B} + \bar{C} = \bar{V}$ which is called the vertical shear, and $\bar{A}$ and $\bar{D}$ form a couple called the bending moment couple. $\bar{V}$ can be represented by one vertical point force. The couple formed by $\bar{A}$ and $\bar{D}$ has no resultant force but only a resultant moment, so a curved arrow is used to represent this moment. In (f) $\bar{V} = \bar{B} + \bar{C}$ and the curved arrow $\bar{M}$ represents the bending moment couple. The magnitudes of $\bar{V}$ and $\bar{M}$ can be found using F-B (c). Imagine that the F-B diagram (c) is constructed only with FF_4, FF_7, $\bar{V}$, and $\bar{M}$ acting on it.

$\Sigma F_V = 0$

V = 48 - 15 = 33 lb↓

$\Sigma M_{\text{neutral axis}} = 0$

M = -(.5)(15) + (16)(48)

= 760.5 in-lb ↺

All dimensions in inches

For both blocks, Density = 0.1 lb/in^3

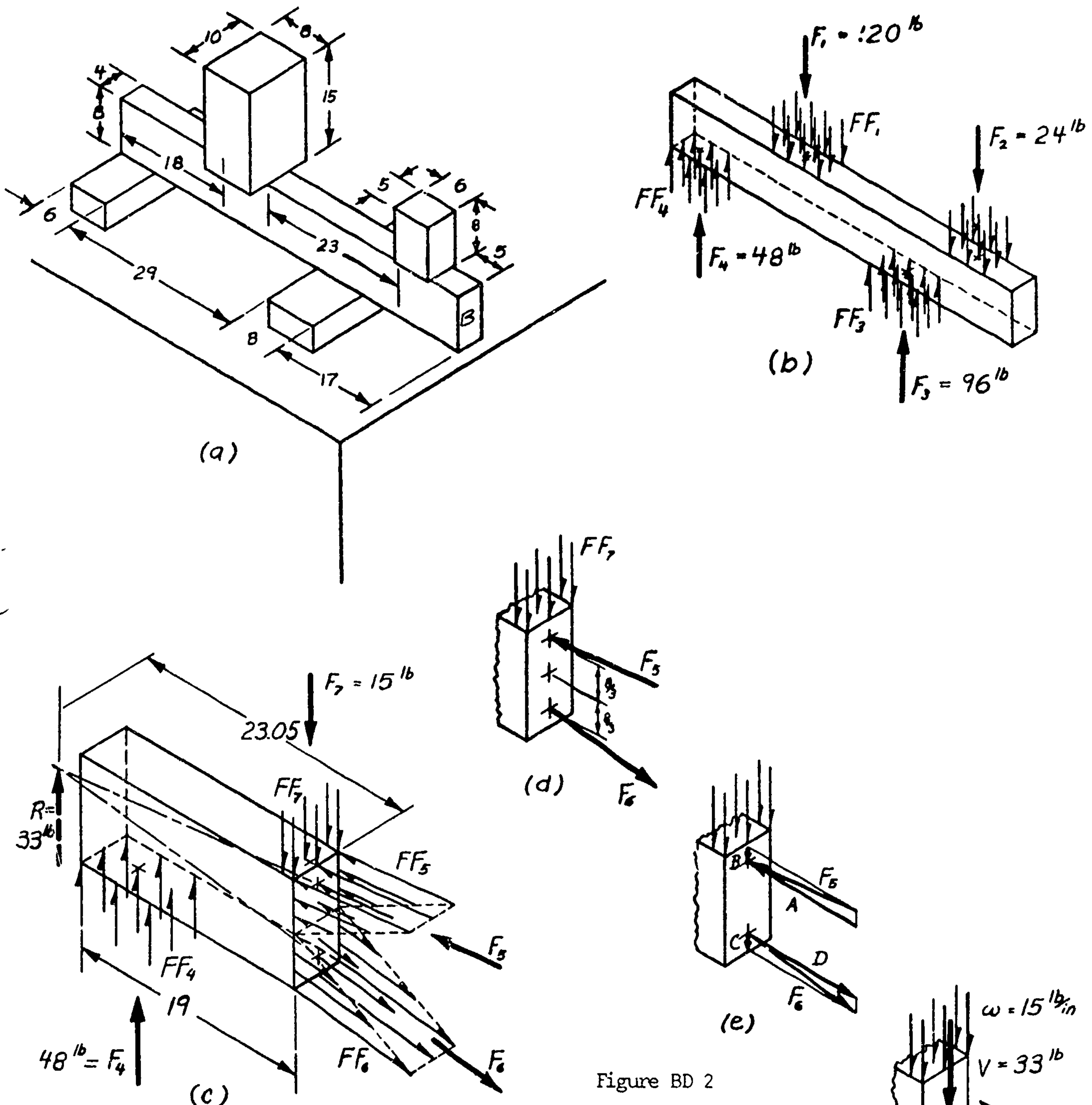
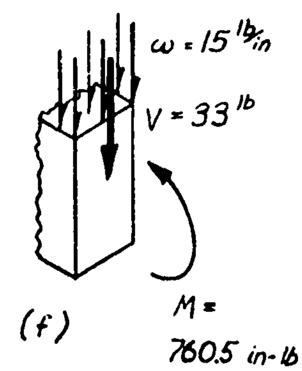


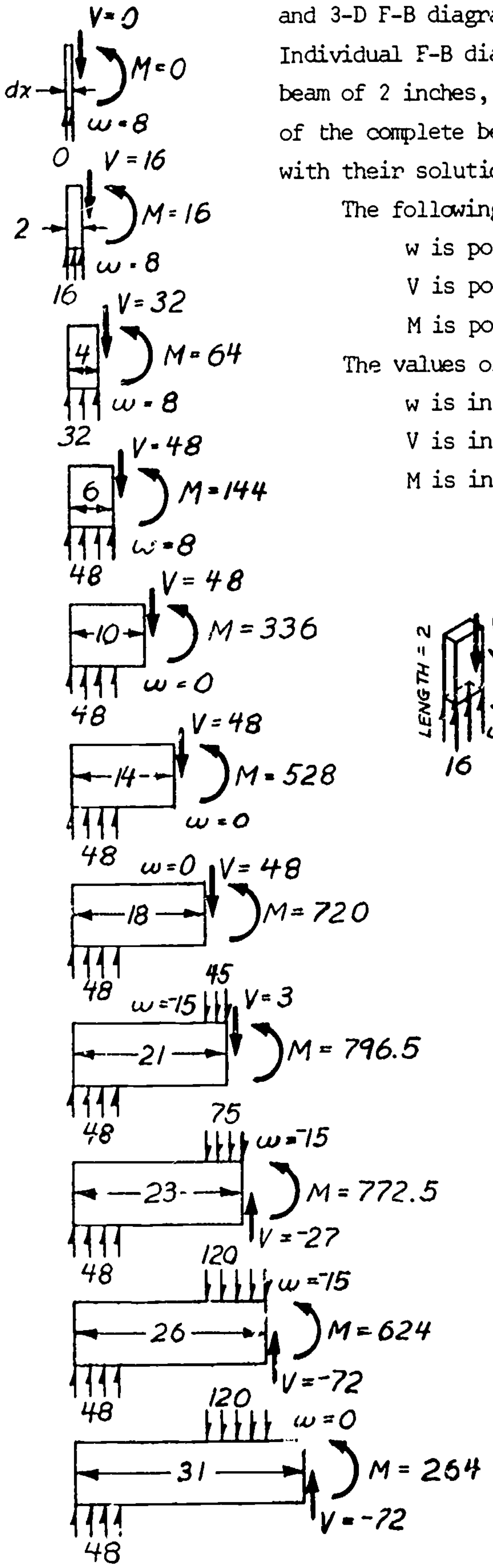
Figure BD 2



Also acting on the exposed section is a uniformly distributed line force field caused by the direct application of the 120 lb load. This line force field is called w and is equal to the weight of the block divided by the length of the block, w = 120 lb/8 inches = 15 lb/in.

These three types of reactions will now be found for a number of vertical sections of the beam using F-B diagrams.

Beginning with a section a distance dx from the left end of the beam, 2-D and 3-D F-B diagrams are constructed and solved for w, V, and M in figure BD 3. Individual F-B diagrams are drawn in 2-D for lengths from the left end of the beam of 2 inches, 4 inches, 6 inches, 10 inches, etc., until the last F-B is of the complete beam. F-B diagrams of some of the sections are drawn in 3-D with their solutions worked out.



The following sign conventions will be used for the values of w, V, and M:

w is positive when ↑, negative when ↓

V is positive when ↓, negative when ↑

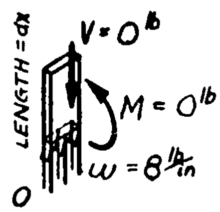
M is positive when), negative when (

The values of w, V, and M are in the following units:

w is in lb/in

V is in lb

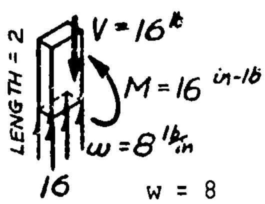
M is in in-lb



$w = \frac{48}{6} = 8$ lb/in

$V = wdx \approx 0$ as $dx \to 0$

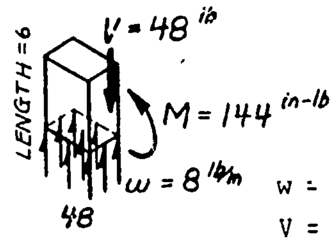
$M = wdx\frac{dx}{2} \approx 0$ as $dx^2 \approx 0$



w = 8

V = 2(8) = 16

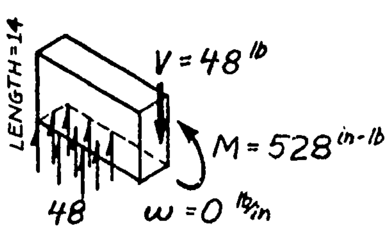
M = 1(16) = 16



w = 8

V = 6(8) = 48

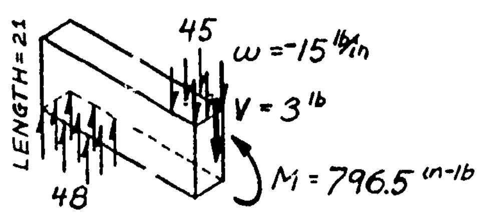
M = 3(48) = 144



w = 0

V = 48

M = 11(48) = 528

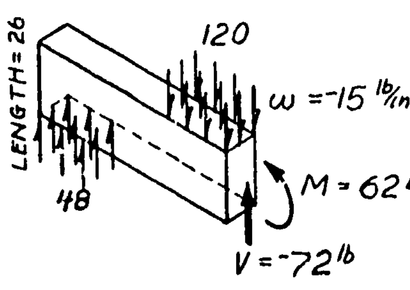


w = -15

V = 48 - 3(15) = 3

M = 18(48) - 1.5(45)

= 864 - 67.5 = 796.5



w = -15

V = 48 - 8(15) = -72

M = 23(48) - 4(120)

= 1104 - 480 = 624

Figure BD 3

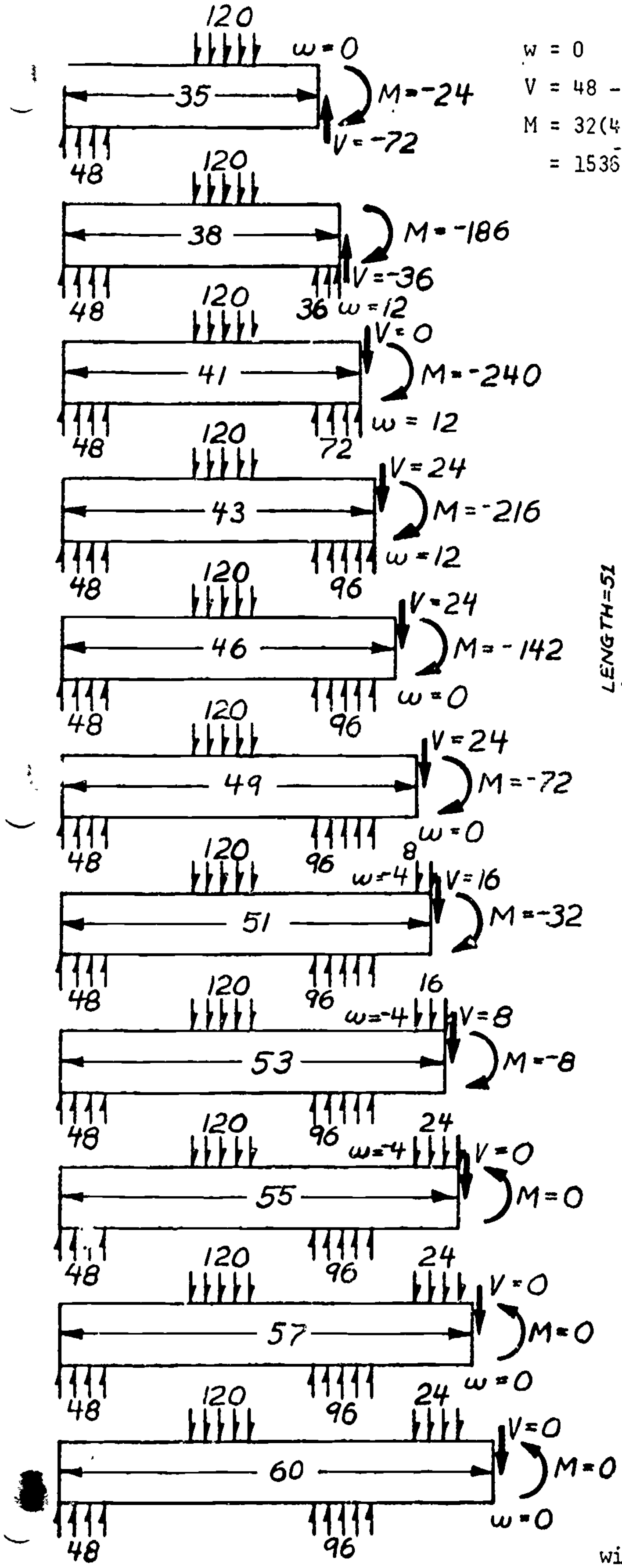
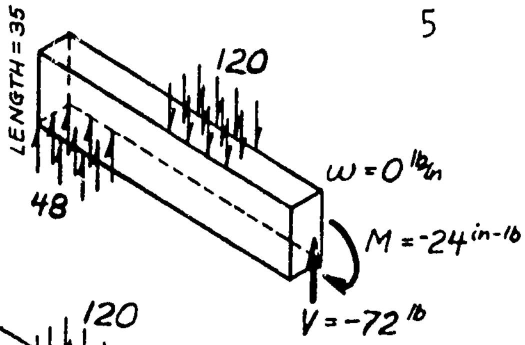


Figure BD 3

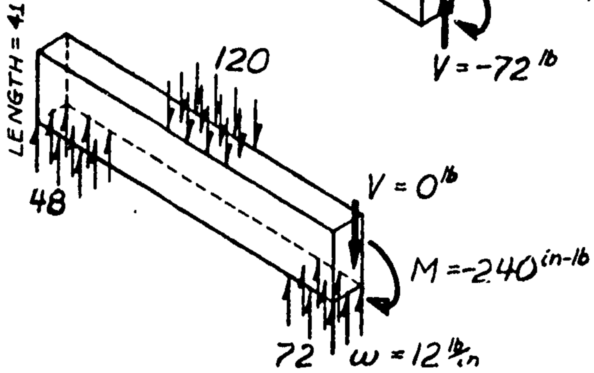


w = 0

V = 48 - 120 = -72

M = 32(48) - 13(120)

= 1536 - 1560 = -24

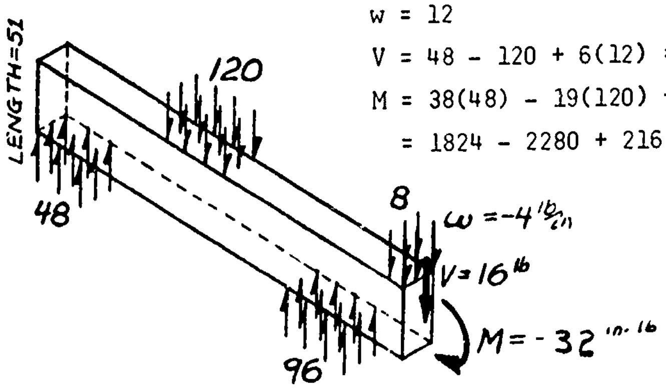


w = 12

V = 48 - 120 + 6(12) = 0

M = 38(48) - 19(120) + 3(72)

= 1824 - 2280 + 216 = -240

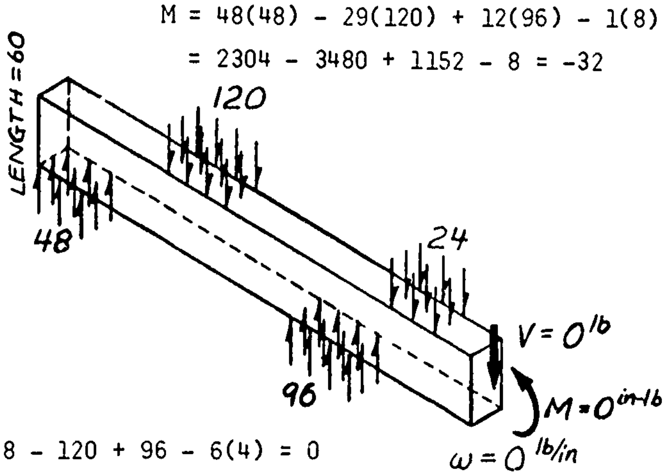


w = -4

V = 48 - 120 + 8(12) - 2(4) = 16

M = 48(48) - 29(120) + 12(96) - 1(8)

= 2304 - 3480 + 1152 - 8 = -32



w = 0

V = 48 - 120 + 96 - 6(4) = 0

M = 57(48) - 38(120) + 21(96) - 8(24)

= 2736 - 4560 + 2016 = 192 = 0

w, V, and M for each exposed vertical beam section will now be plotted as ordinates vs the beam lengths as abscissas using the established sign conventions of + w ↑ , + V ↓ , and + M ↶ .

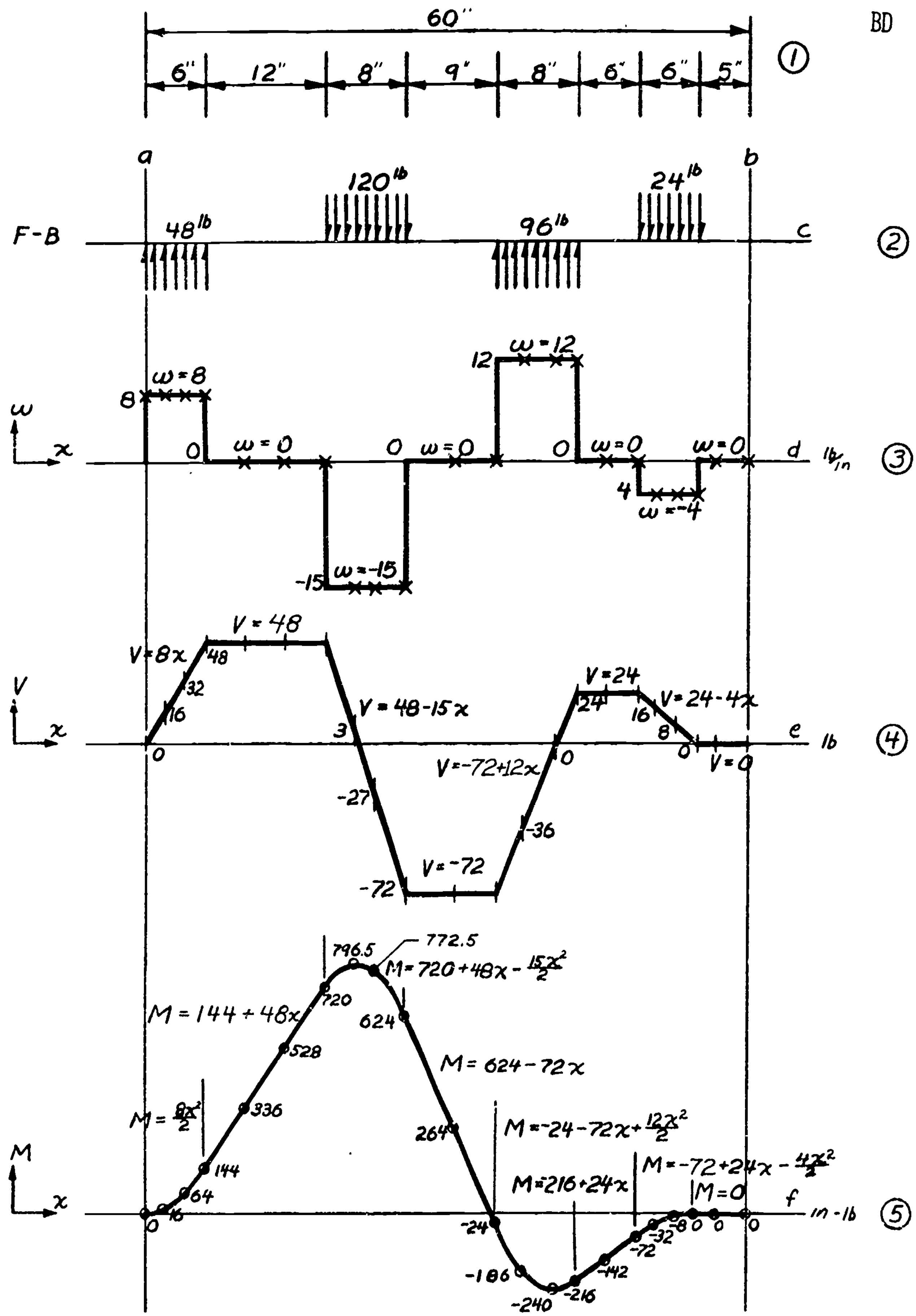

60"
6"
12"
8"
9"
8"
6"
6"
5"
①
a
b
F-B
48 lb
120 lb
96 lb
24 lb
c
②
w
x
ω=8
ω=0
ω=12
ω=-15
ω=-4
d
lb/in
③
V
x
V=8x
V=48
V=48-15x
V=-72
V=-72+12x
V=24
V=24-4x
V=0
e
lb
④
M
x
M = 8x²/2
M = 144 + 48x
M = 720 + 48x - 15x²/2
M = 624 - 72x
M = -24 - 72x + 12x²/2
M = 216 + 24x
M = -72 + 24x - 4x²/2
M = 0
796.5
772.5
f
in-lb
⑤

Figure BD 4

BD Figure BD 4 is now drawn. The beam length is laid out to scale horizontally in (1) then vertical lines a and b are drawn. Next a base line c is drawn and the force fields from the F-B diagram of the entire beam are drawn in (2) . This will be called the F-B diagram. 7

Next a base line d is drawn and the values of w from the F-B diagrams in BD 3 are plotted to scale vs the length x from the left end to give curve (3) .

Base line e is then drawn and the V values from the F-B diagrams in BD 3 are plotted to scale to form curve (4) .

Next on the f base line the values of M are plotted for each of the F-B diagrams to give curve (5) .

The curves in BD 4 will now be analyzed. First the three curves will be compared in the region where x varies from 0 to 6 inches from the left end of the beam. Looking at curve (3) you can see that w is a straight line since w = 8 lb/in for any value of x. The V curve between 0 and 6 inches also forms a straight line with the equation $V = 0 + mx = 0 + \frac{48}{6}x = 8x$ lb. For M between 0 and 6 inches, M = 0 when x = 0, 16 when x = 2, 64 when x = 4, and 144 when x = 6.

The equation for w (from x = 0 to x = 6) is w = 8, and V (x = 0 to x = 6) is V = 8x. Comparing them V = 8x is the integral of w from x = 0 to x = 6 written

$$V = \int w\,dx = \int 8\,dx = 8\int dx = 8x$$

Now try integrating the V curve and see if it matches the answers from the F-B diagrams for M.

$$M = \int V\,dx = \int 8x\,dx = \frac{8x^2}{2} = 4x^2$$

When x = 0 M = 0

x = 2 $M = 4x^2 = 16$

x = 4 $M = 4x^2 = 64$

x = 6 $M = 4x^2 = 144$

All calculated values from the F-B diagrams check with those found by the integration method.

Between x = 0 and x = 6 the w, V, and M curves are related to each other as

$$V = \int w\,dx \quad \text{and} \quad M = \int V\,dx \quad \text{or} \quad w = \frac{dV}{dx} \quad \text{and} \quad V = \frac{dM}{dx}$$

In other words the V curve is the integral (area under) the w curve, the M curve is the integral (area under) the V curve, the w curve is the derivative (slope) of the V curve, and the V curve is the derivative (slope) of the M curve.

Letting x = 0 at 6 inches, the three curves from 6 inches to 18 inches (now x goes from 0 to 12) relate as w = 0 V = 48 M = 144 + 48x .

All the equations are now placed on the diagram in figure BD 4. You should be able to verify all the rest of the equations.

This set of curves, that is w vs x, V vs x, and M vs x, is called a set of beam curves or beam diagrams. It is a single diagram that shows the values of w, V, and M for all sections of the beam.

The relationships between the curves are sometimes referred to as laws. If the w curve is called a lower curve, V is the next higher curve relative to w, and M is the next higher curve relative to V.

Law #1: The length of the ordinate at any point on any curve is equal to the slope of the next higher curve at the same corresponding point.

Law #2: Between any two ordinates on any lower curve, the area underneath the curve is equal to the change in length of the ordinates on the next higher curve, between the same corresponding ordinates.

A section of the beam 21 inches from the left end and dx long has been drawn in a 3-D F-B diagram in figure BD 5. Relationships will be derived between w, V, and M on this F-B.

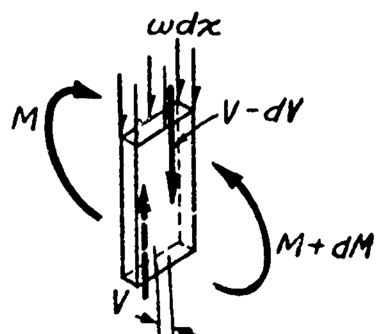


Figure BD 5

On the left face of the section V = 3 lb and acts upward. M = 796.5 in-lb and acts clockwise and w = 15 lb/in and acts downward. On the right face w still is 15 lb/in acting downward. V acts downward and is 3 lb minus some dV as x increases. V can therefore be drawn on the right face of the section as V - dV. M is counterclockwise and equals 796.5 in-lb plus some dM, so it can be placed on the diagram as M + dM.

$$\Sigma M_{\text{right face}} = 0 \quad -M - Vdx + (wdx)\frac{dx}{2} + M + dM = 0$$

$$-Vdx + dM = 0, \text{ since } w\,\frac{(dx)^2}{2} \text{ is negligible}$$

$$V = \frac{dM}{dx} \text{ or } Vdx = dM \text{ or } M = Vdx$$

$$\Sigma F_V = 0 \quad V - (wdx) - (V - dV) = 0 \quad wdx = dV$$

$$w = \frac{dV}{dx} \text{ or } wdx = dV \text{ or } V = wdx$$

These are the same equations that were developed using the F-B diagram approach.

Knowing how the w, V, and M curves are related, it is not necessary to draw F-B diagrams to find the reactions on any vertical surface, only a F-B diagram of the complete beam is needed. The w, V, and M curves can then be plotted from this one F-B diagram. Figure BD 6 shows a loaded beam. The F-B diagram, w, V, and M curves are to be plotted below the beam.

The F-B diagram is drawn first. Reactions FF_1 and FF_2 are found by imagining each force field being replaced by its point force and taking moments about each point force. The lengths of the force fields are not plotted to scale on the F-B diagram.

Next the w curve is plotted using the F-B diagram. In interval 1, w = 120/12 = - 10, in 2 w = 0, in 3 w = 108/3 = + 36, in 4 w = 0 , in 5 w = 108/9 = + 12, in 6 w = 0, and in 7 w = 96/6 = -16. The w curve need not be drawn to an exact vertical scale. Equations can be written for w in each interval.

Next the V curve can be plotted using the w curve and the two derived curve laws. V has a zero ordinate when x = 0. In interval 1 w has a constant negative ordinate so V has a constant negative slope. Also in 1 the area under the w curve (- 10 x 12 =-120) is the change in the V curve so at x = 12, V = - 120. In interval 2 w = 0 so V remains constant at - 120. In 3 w is positive so V has a positive slope and the positive change in V is (3)(36) = + 108, so at x = 3 in interval 3, V = - 120 + 108 = - 12. In 4 V is constant at - 12. In 5 V has a positive slope and V = - 12 + 108 = + 96 at x = 9. In 6 V remains constant. In 7 w = - 16 so V has a negative slope. The change in V in 7 is (6)(-16) = - 96 which brings V to 0 at the free end of the beam. The V curve has units of lbs but need not be drawn to an exact scale vertically. The equations for V for each interval are shown below.

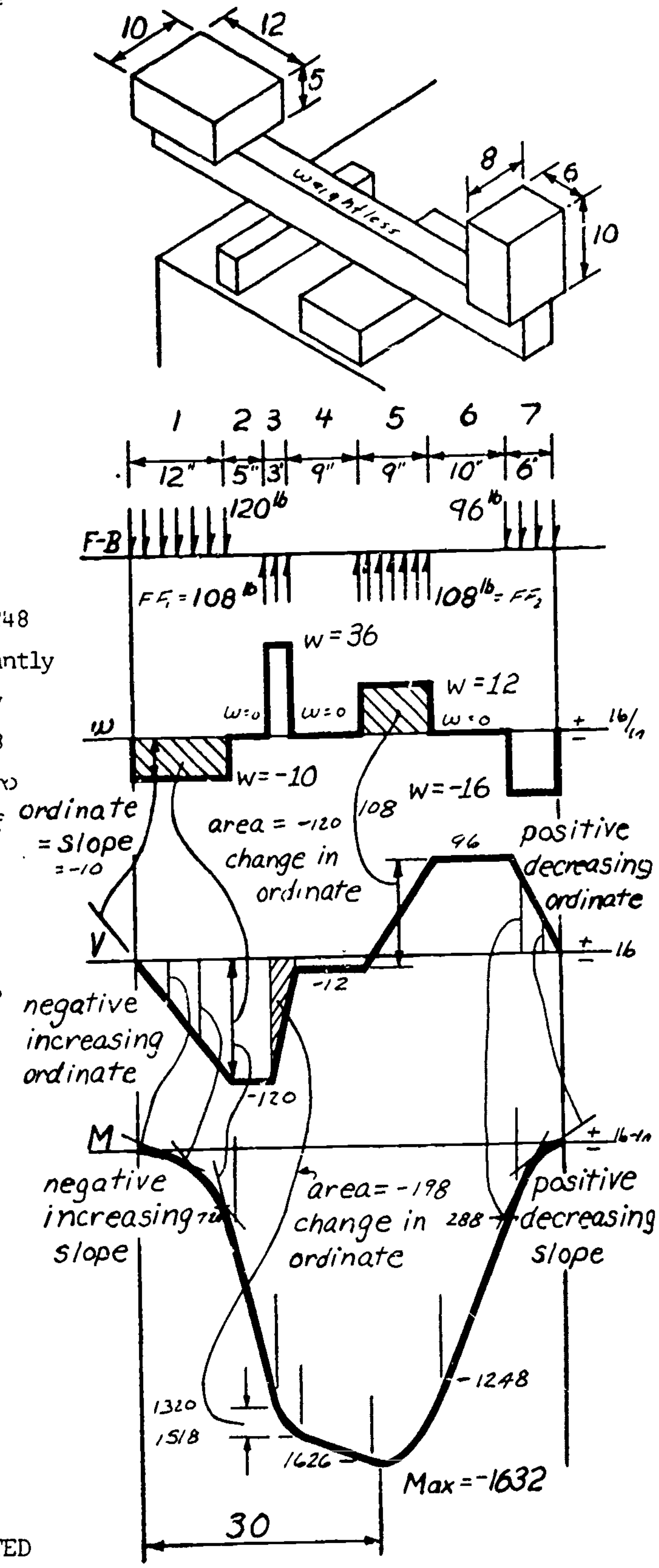
The M curve can now be plotted using the V curve and the two derived curve laws. M begins with a zero ordinate at x = 0. In interval 1 V has a negative increasing ordinate, so M must have a negative increasing slope. The area under the M curve in 1 is (-12)(120) 12 = -720 so M = -720 at x = 12. In 2 V has a constant negative ordinate so M has a constant negative slope, also the area under the V curve is (5)(-120) = - 600 so at x = 5 in 2 M = - 720 - 600 = - 1320. In 3 V has a negative decreasing ordinate so M has a negative decreasing slope. At the beginning of 3 M has a negative slope of - 120 , at the end M has a slope of - 12. The ordinate of M at the end of 3 is - 1320 - 198 = - 1518. In 4 V has a constant negative ordinate so M has a constant negative slope. At x = 9 in 4 M = - 1518 - 108 = -1626.

V has a negative ordinate at the beginning of 5, the ordinate becomes 0 and equals 96 at the end of 5. The M curve in 5 begins with a negative slope, decreases to a zero slope, and then increases to a positive slope of 96. The best way to find the ordinate for M at x = 9 in interval 5 is to write the equation for V ($V = -12 + 12x$), then integrate it to get M ($M = -1626 - 12x + 6x^2$) and set x = 9. This gives M = -1248 at x = 9.

In 6 V is positive so M is positive and = -1248 + (96)(10) = - 288 at x = 10. In 7 V has a constantly decreasing positive ordinate so M has a constantly decreasing positive slope. At x = 6 in 7 M = -288 + (96)(6) /2 = 0. This checks as M must equal zero at the free end of the beam. Again M has units of in-lb and need not be plotted to scale. All the equations for M can be found by integrating the appropriate V curves.

To find the maximum value of M in interval 5, set $V = -12 + 12x = 0$ and get x = 1. Then $M_{max} = -1626 - 12(1) + 6(1)^2 = -1632$.

DENSITY = 0.2 $^{LB}/_{IN^3}$



Shear and bending moment equations.

1	$V = -10x$	$M = -5x^2$
2	$V = -120$	$M = -720 - 120x$
3	$V = -120 + 36x$	$M = -1320 + 18x^2 - 120x$
4	$V = -12$	$M = -1518 - 12x$
5	$V - 12 + 12x$	$M = -1626 + 6x^2 - 12x$
6	$V = 96$	$M = -1248 + 96x$
7	$V = +96 - 16x$	$M = -288 - 8x^2 + 96x$

AT THIS POINT, IF YOU ARE GIVEN A 3-D DIAGRAM OF A BEAM LOADED WITH UNIFORM LOADS AND SUPPORTED AT TWO SUPPORTS WHICH ALSO HAVE UNIFORM LOADING, YOU SHOULD BE ABLE TO CONSTRUCT THE 2-D F-B DIAGRAM AND THE w, V, AND M CURVES FOR THE BEAM.

Figure BD 6

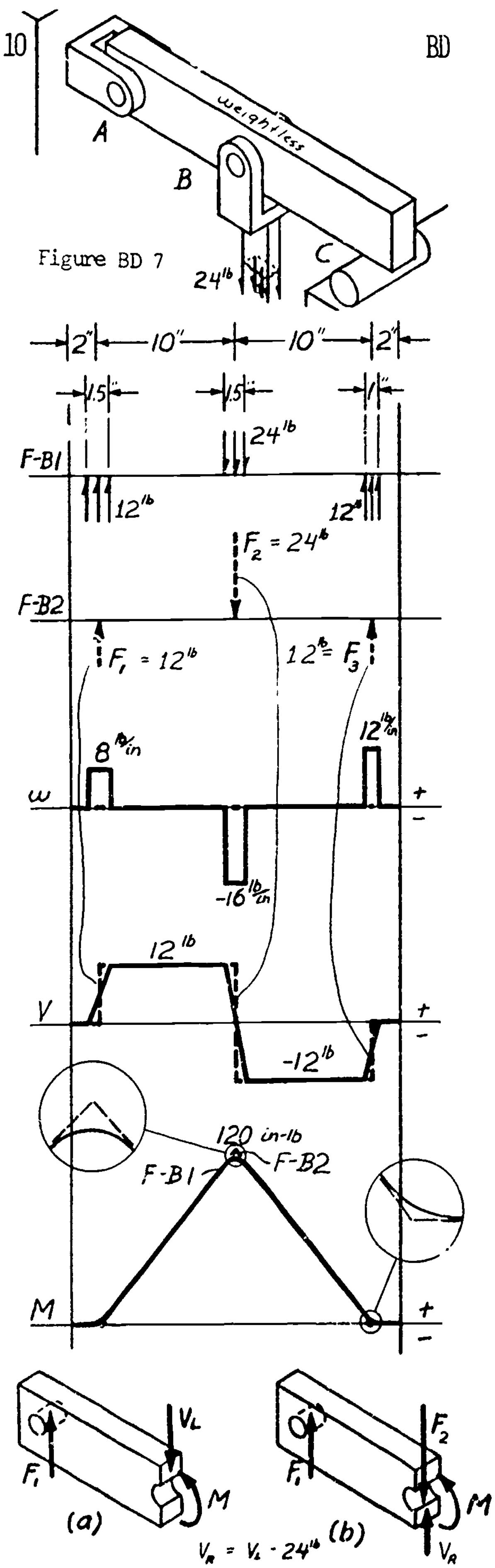


Figure BD 7

Beams with Idealized Point Forces and Point Couples

In figure BD 7, F-B 1 is the F-B diagram of the given beam showing the distributed loads that act on the beam. The w, V, and M curves that show the reactions occurring on any cut section of the beam are constructed as before. It is sometimes convenient to replace the distributed loads with point forces as shown in F-B 2. To illustrate this in figure BD 7 the w, V, and M curves derived from F-B 2 are superimposed on the curves from F-B 1. The curves derived from F-B 1 (distributed loads) are constructed with solid lines, and those from F-B 2 (point forces) are constructed with dotted lines.

The construction of the dotted w, V, and M curves will now be explained. Since there are no distributed loads in F-B 2, w = 0 over the entire length of the beam. (This makes sense, since a point force on the F-B diagram would have to be plotted as an infinitely tall and vanishingly narrow area on the w curve.)

Since w = 0 for F-B 2, it is necessary to develop a new technique to draw the V curve. Starting from the left end of the beam, V = 0 for 2 - $\triangle$x inches, since there are no loads on the beam in this interval. Then at x = 2 + $\triangle$x inches, V = 12 lb acting downward, which is positive on the V curve. V continues to be equal to a positive 12 lb until x = 12 - $\triangle$x inches. At x = 12 + $\triangle$x inches, the application of the point force F_2 from F-B 2 causes V to become equal to 12 lb acting upward, so V = -12 lb. V remains constant at this magnitude until X = 22 - $\triangle$x inches. In the interval between x = 22 + $\triangle$x inches and the end of the beam at x = 24 inches, V = 0. To visualize this method of drawing the V curve for F-B 2, imagine that F-B diagrams are drawn of beam sections just to the left and just to the right of a point force load. The two isometric drawings below show how V changes between x = 12 - $\triangle$x inches and x = 12 + $\triangle$x inches.

Actually, the V curve is discontinuous at each point force, and is plotted Δx to the left and Δx to the right of a point force. This results in a vertical change in the V curve equal to the magnitude of the point force in the F-B diagram, as indicated by the connecting lines between F-B 2 and the dashed V curve in figure BD 7.

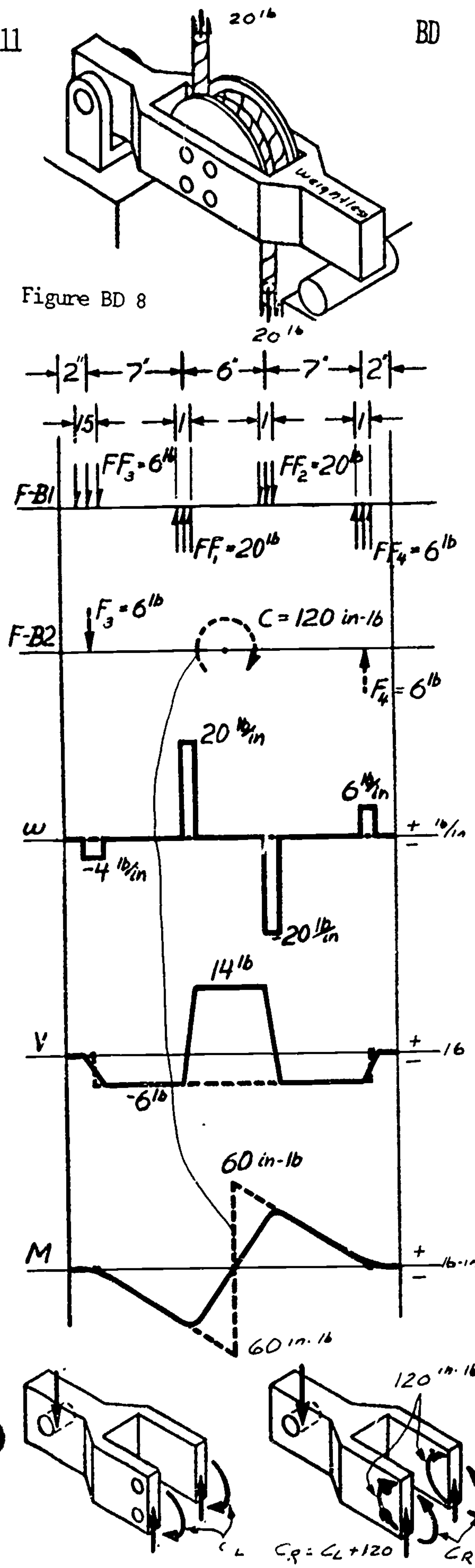


Figure BD 8

Once the V curve for F-B 2 has been found, it can be integrated to find the corresponding M curve, just as the w curve for F-B 1 is integrated to find its corresponding V curve. Comparison of the moment curves for F-B 1 and F-B 2 indicates that a point force approximation of the loads causes only slight changes in the M curve. The critical intervals on the M curves are magnified below to show the differences between the two curves.

Figure BD 8 shows another beam for which the w, V, and M curves derived from a F-B diagram showing the point force resultants of the loads will be superimposed on the curves from a F-B diagram showing the distributed loads on the beam.

F-B 1 is the F-B diagram showing the distributed loads on the beam in figure BD 8. The w, V, and M curves derived from it are constructed with solid lines using the methods developed in previous examples. F-B 2 shows the point force resultants of these loads, and the curves from it are constructed with dotted lines. Notice that the point force resultants of FF_1 and FF_2 are replaced by a point couple on F-B 2. The w and V curves for F-B 2 are drawn with the same techniques used in figure BD 7. The point couple has no effect on either the w curve or the V curve.

The M curve for F-B 2 is plotted with a technique similar to that used for plotting a V curve in the interval around a point force , between x = 0 inches and $x = 12 - \Delta x$ inches, the M curve is the integral of the V curve, just as it has been in previous examples. However between $x = 12 - \Delta x$ inches and $x = 12 + \Delta x$ inches, the M curve undergoes a sudden increase equal to the magnitude of the point couple in F-B 2. Between $x = 12 + \Delta x$ inches and the end of the beam at x = 24 inches, the M curve is again equal to the integral of the V curve. The discontinuity in the M curve at x = 12 inches can be explained by imagining that F-B drawings are drawn of sections of the beam just to the left and just to the right of the point of application of the couple in F-B 2 as shown in the isometric drawings below. The ordinate change on the M curve is not C = 120 lb-in acting ↷, but is a couple of 120 lb-in acting ↶ that is suddenly needed to keep the second isometric F-B in equilibrium, this is a positive change.

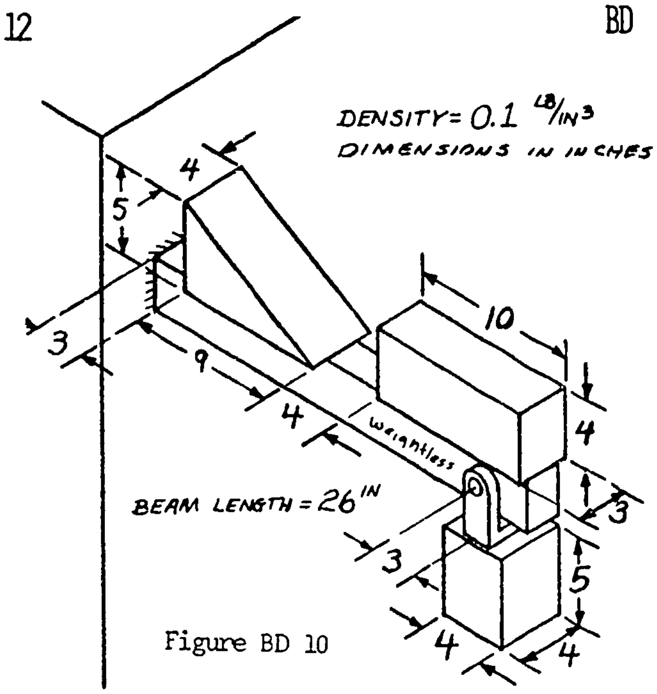
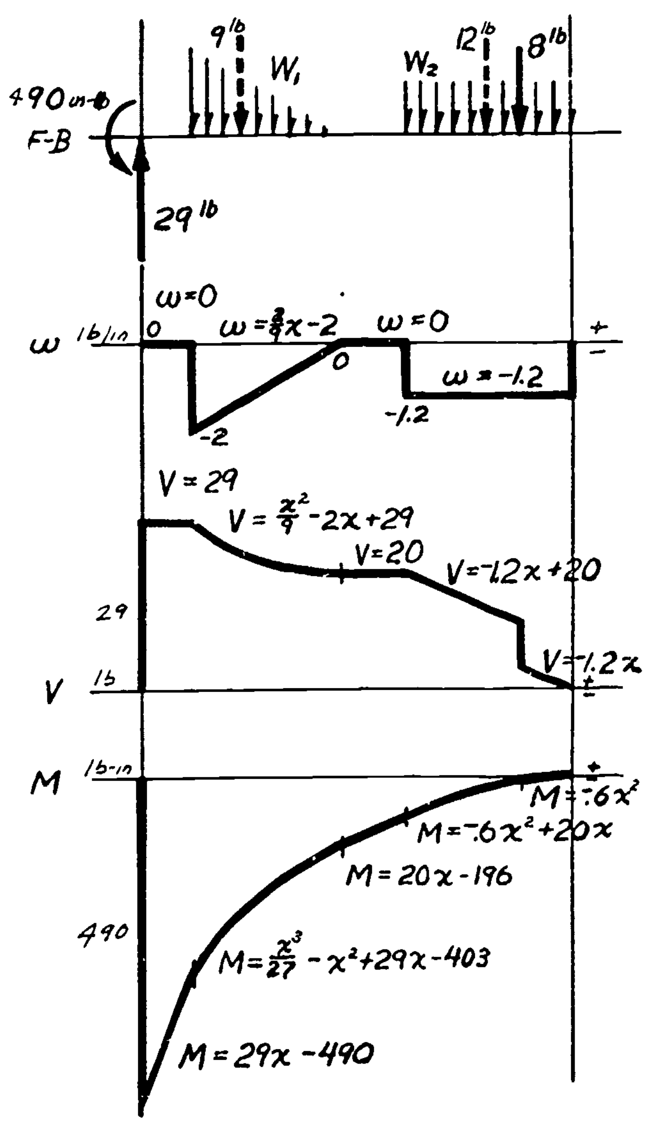


Figure BD 10



It is frequently necessary to analyze beams that are loaded both by distributed loads and loads that can be approximated with point forces and point couples. The following two examples demonstrate how w, V, and M curves for such beams are to be drawn.

Figure BD 10 shows the w, V, and M curves for a cantilever beam. The hanging weight is represented by a point force on the F-B diagram, and the two blocks are represented by distributed loads. The beam itself is considered to be weightless.

To find the reactions at the left end of the beam, it is necessary to mentally represent W_1 and W_2, the weights of the blocks, with their point force resultants, as shown on the F-B diagram in Figure BD 10. These dotted point forces on the F-B are used only to find reactions.

$$\Sigma M_{\text{left end}} = 0 \quad M_1 = +6(9) + 21(12) + 23(8)$$
$$M_1 = +490 \text{ in-lb}$$

$$\Sigma F_V = 0 \quad V_1 = 9 + 12 + 8$$
$$V_1 = 29 \text{ lb}$$

To construct the w curve, it is necessary to remember that the shape of a distributed load is the same as the shape of the block that causes it. Since the block on the left is triangular, this means that the w curve must be triangular in the interval between x = 3 inches and x = 12 inches from the left end of the beam. At x = 3 w = (9)(4)(5)(.1) /9 = 2 lb/in, at x = 12 w = 0, so between x = 3 and x = 12 $w = -2 + \frac{2}{9}x$.

The initial ordinate on the V curve is not the 29 lb point force acting ↑, but is a 29 lb point force that balances it on a F-B diagram so it must be 29 lb ↓. The initial ordinate on the M curve is not 490 in-lb acting ↺, but 490 in-lb acting ↻, so it is negative.

BD 13

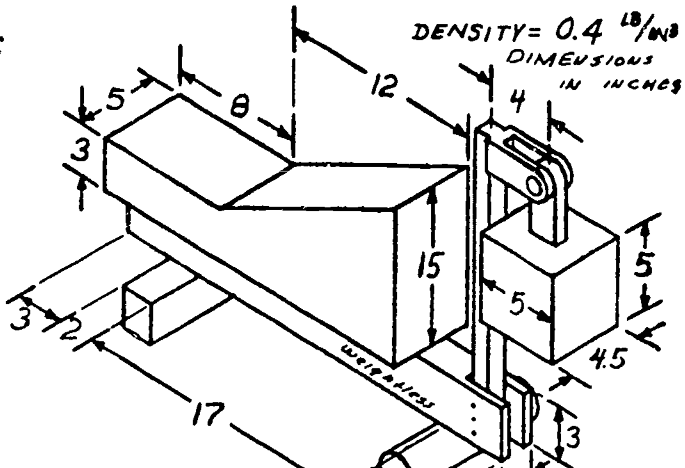
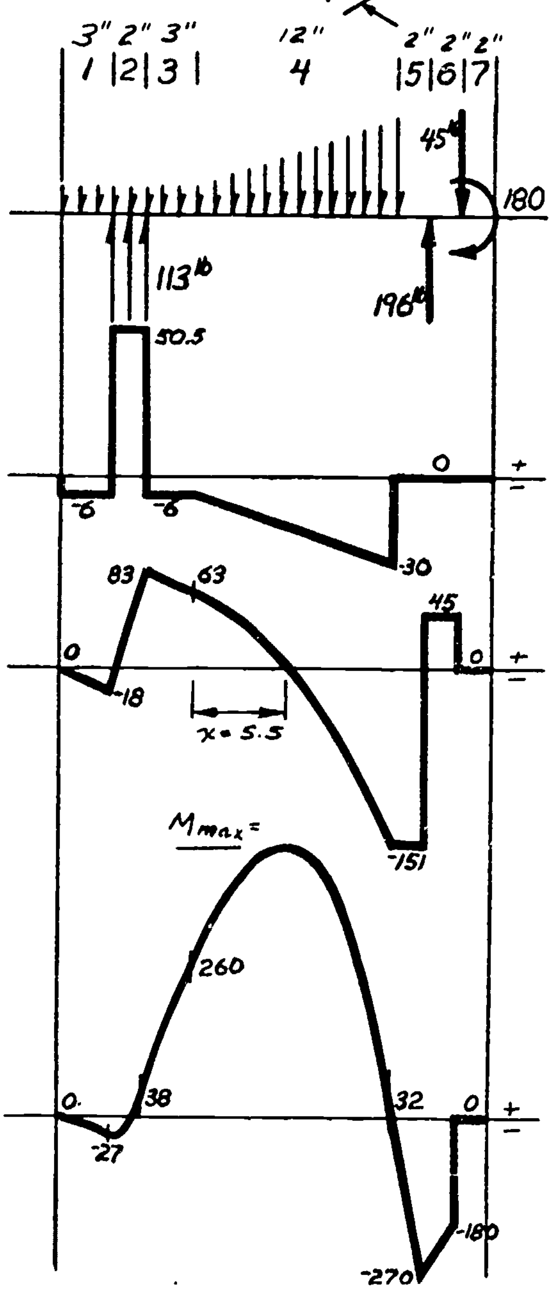


Figure BD 11

In the F-B diagram of the beam shown in figure BD 11, the left-hand reaction is represented by an evenly distributed force field and the right-hand reaction is represented by a point force resultant since the roller causes a line force field to act on the beam. The weight of the block is distributed over its contact area as shown in the F-B diagram. The hanging weight is assumed to cause a 45 lb point force and a 4(45) = 180 in-lb point couple, both acting 2 inches from the right end of the beam.

Once the F-B diagram has been established, the w, V, and M curves can be drawn using the techniques developed in previous examples.



Most of the time the curves are used to find the maximum shear and maximum bending moment.

The maximum value of the vertical shear can be found by inspection. It is − 151 lb in interval 5. It is not − 151 + 48.

The maximum value of M is either in interval 4 or − 270 in-lb at the end of interval 5. In interval 4

$$w = -6 - 2x$$

$$V = 63 - 6x - x^2$$

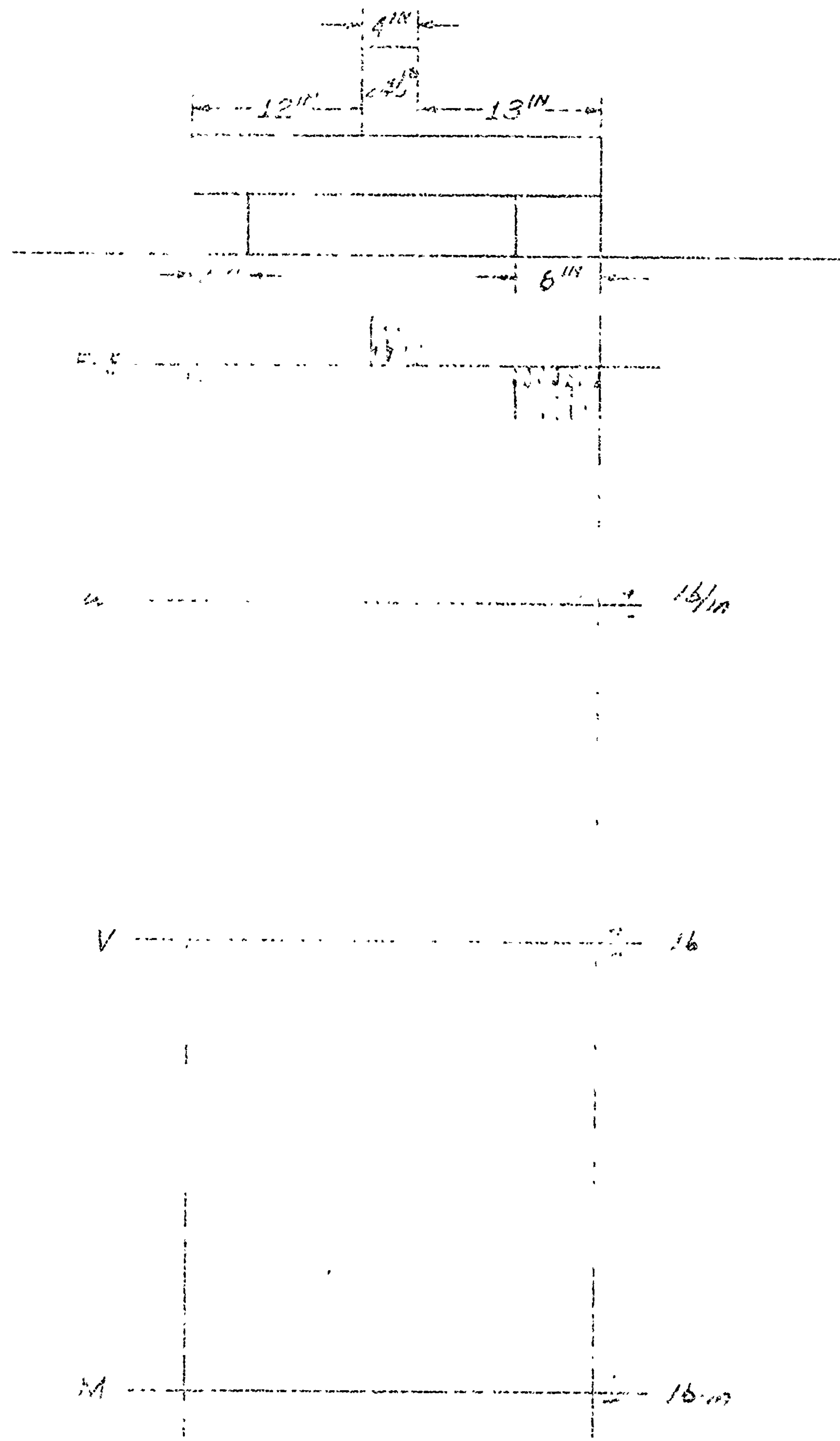
$$M = 260 + 63x - 3x^2 - x^3/3$$

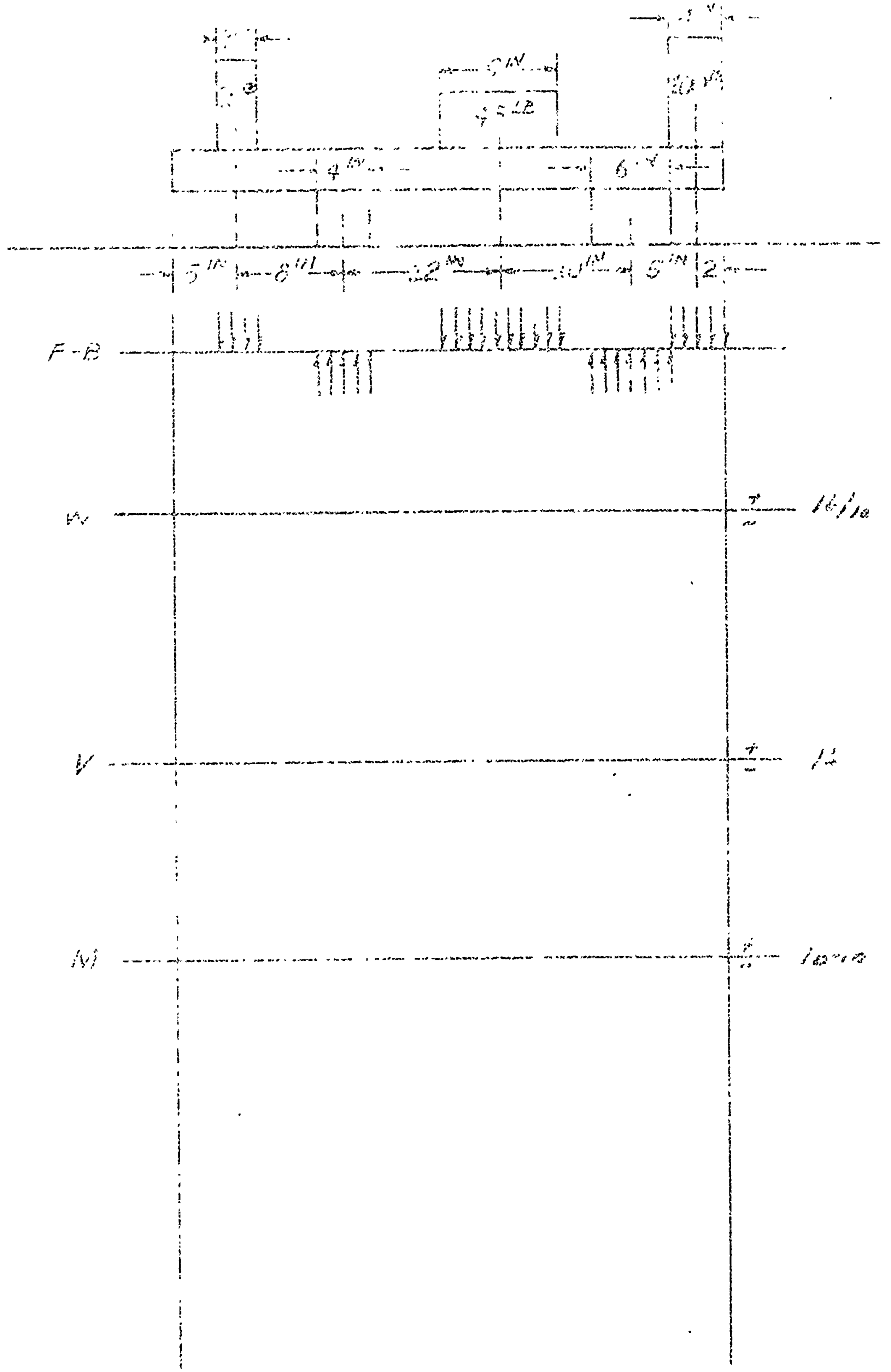
Setting V = 0 gives x or $V = 63 - 6x - x^2 = 0$ gives x = 5.6 inches.

$M_{max} = 260 + 63x - 3x^2 - x^3/3$ when x = 5.5 inches

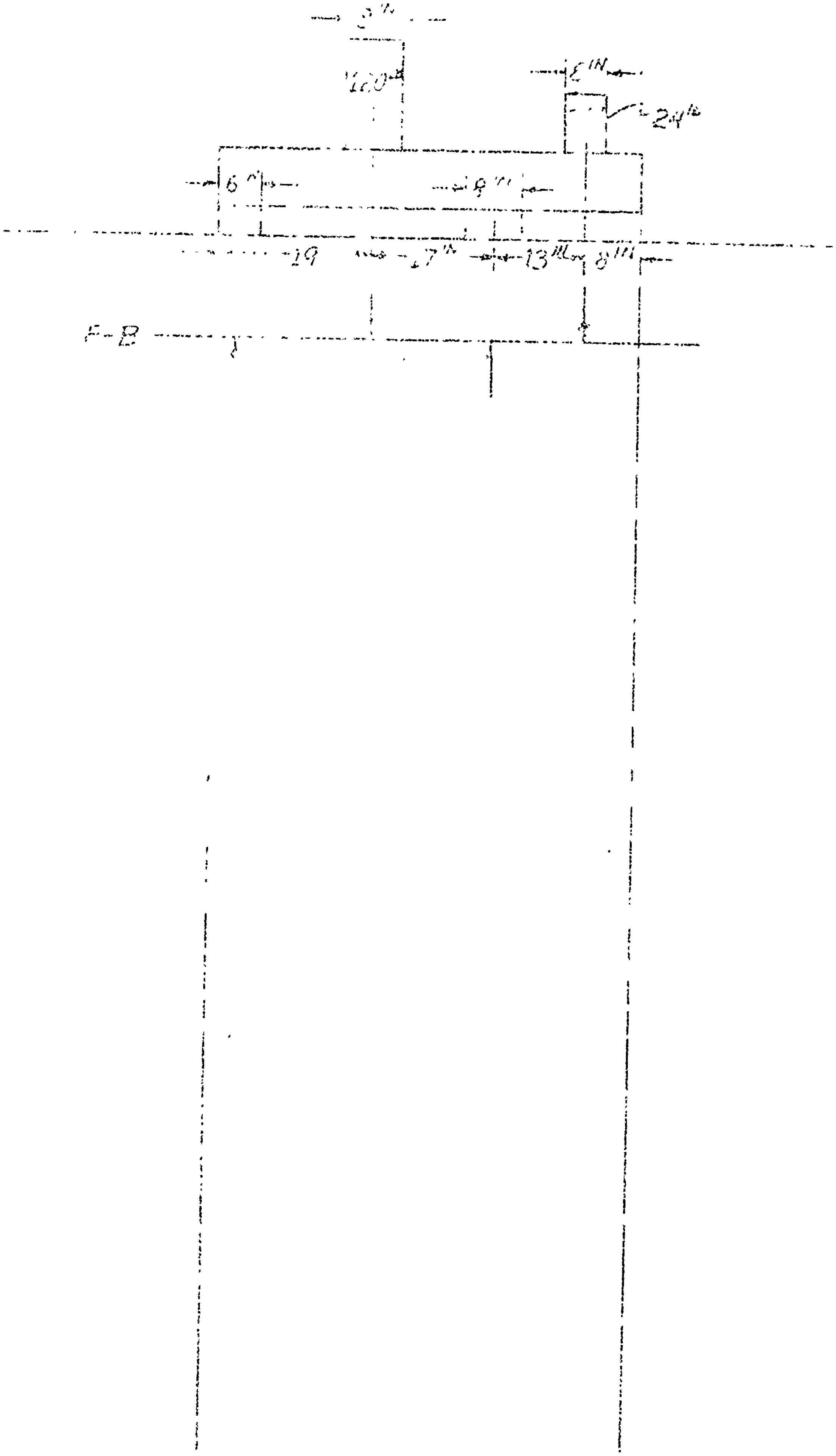
$= 459.4$ in-lb.

NOW IF YOU ARE GIVEN AN ISOMETRIC DRAWING OF A LOADED CANTILEVER OR A SIMPLY SUPPORTED BEAM, YOU SHOULD BE ABLE TO CONSTRUCT w, V, AND M CURVES FOR THE BEAM. YOU SHOULD BE ABLE TO DRAW THESE CURVES FOR DISTRIBUTED LOADS, IDEALIZED POINT FORCES, OR POINT COUPLES.

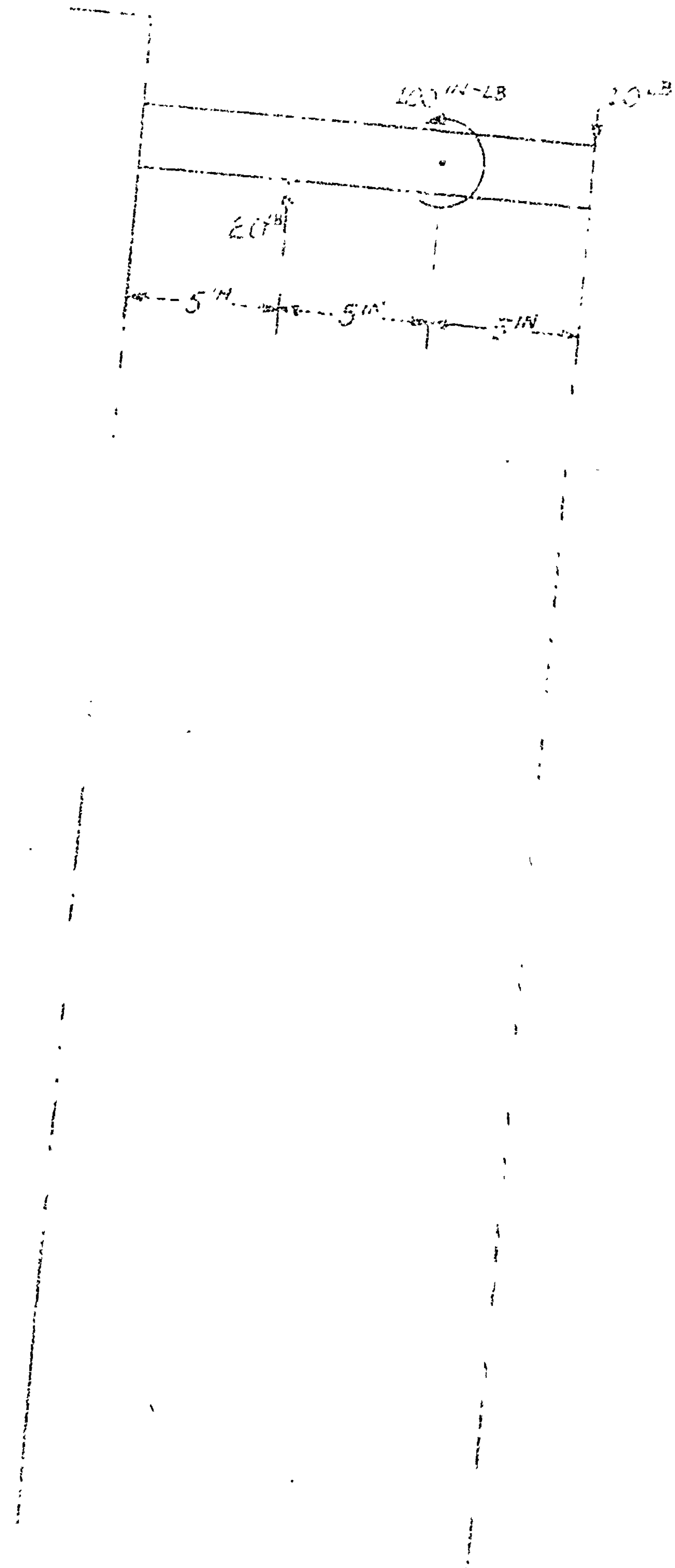

4 IN
12 IN
13 IN
6 IN
15/in
V
16
M
16 in

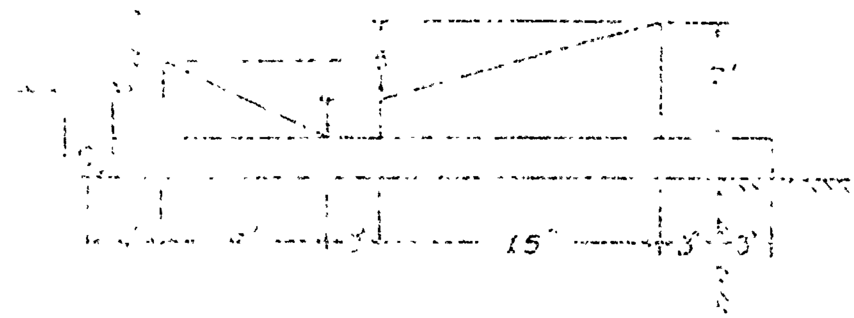

F-B
W
V
M

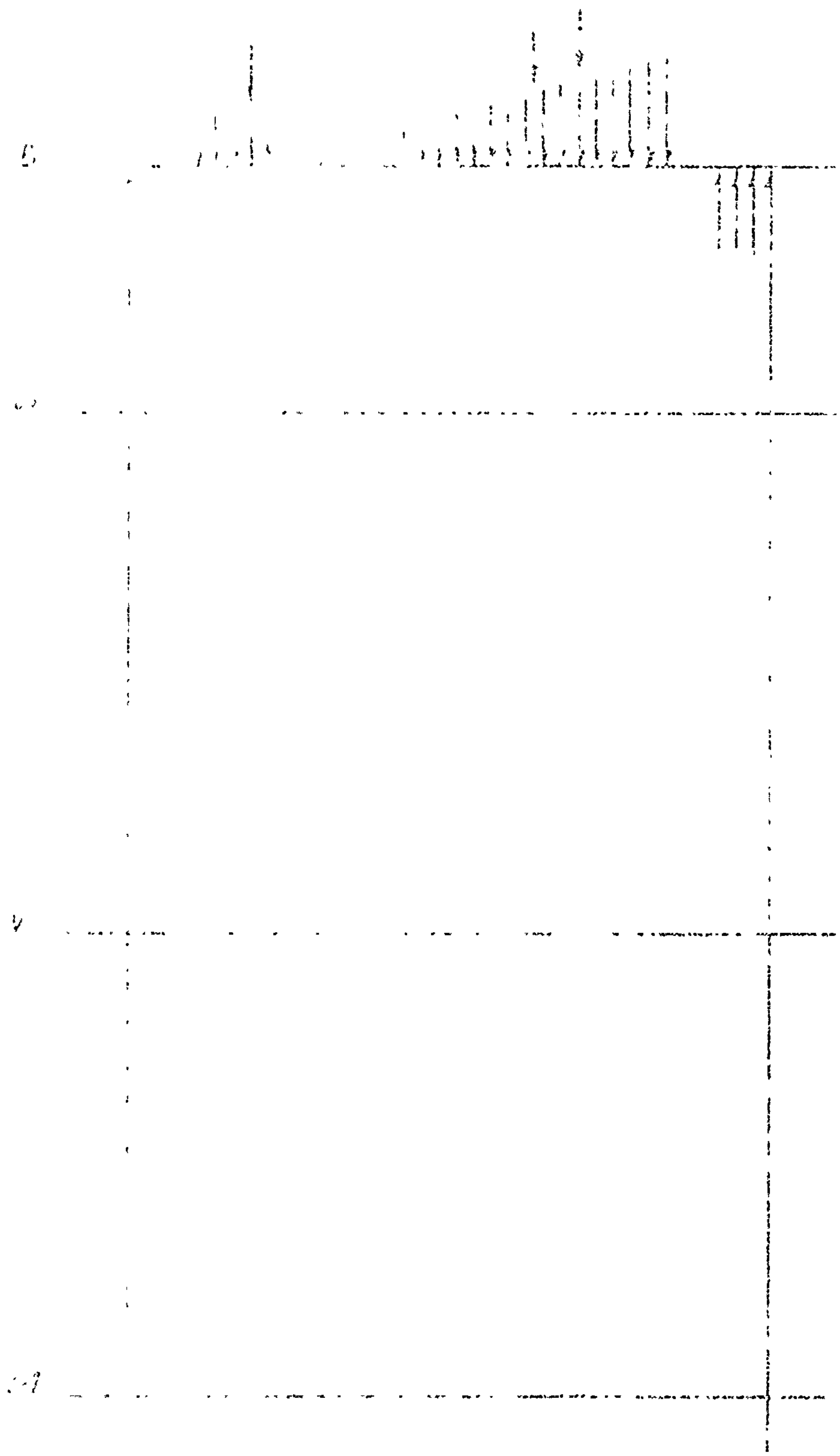
[illegible]



[illegible] curves [illegible] point




7'
15'




50 LB
10 LB
20
F-L