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ABSTRACT

Nine experiments and 17 activities are presented in this handbook. The experiments are related to the earth's size and orbit, Piton height, telescope operations, Mars and Mercury orbits, stepwise approximation, and models of comet orbits. Further naked-eye observations in astronomy are designed in connection with the sun, moon, and planet positions. The activities are concerned with sunspots, Foucault pendulum, three-dimensional orbits, satellite and comet orbits, Galileo's work, forces on a pendulum, angular measurements, analemma, epicycles, retrograde motions, armillary sphere, sidereal days, scale model of the solar system, and summary of physics learning in the Japanese haiku form. Self-directed instruction, demonstrations, and construction projects are stressed in these activities. The four chapters in the handbook are arranged to correspond to the text materials, with complete instructions in each experiment. Some experiments and activities are suggested for assignment, and the remaining are used at student discretion. Besides illustrations and film loop notes for explanation use, a table of planet longitudes, a guide for planet and eclipse observations, and a set of review problems are included. Additional suggestions for activities are given in the appendix. The work of Harvard Project Physics has been financially supported by: the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University. (CC)

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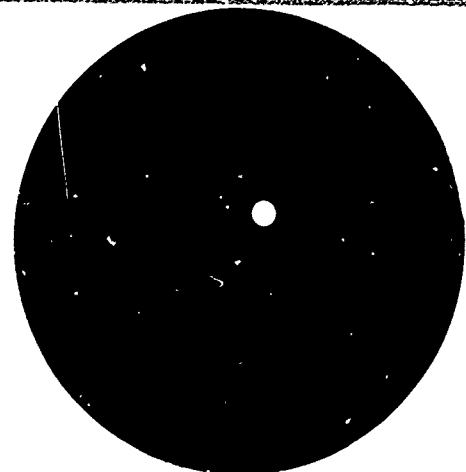
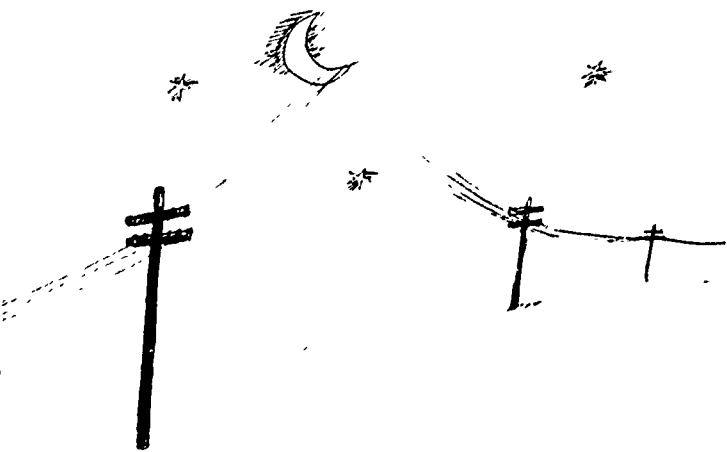
Project Physics Handbook

An Introduction to Physics

Motion in the Heavens



SE015 526



This handbook is the authorized interim version of one of the many instructional materials being developed by Harvard Project Physics, including text units, laboratory experiments, and teacher guides. Its development has profited from the help of many of the colleagues listed at the front of the text units.

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Project Physics **Handbook**

An Introduction to Physics **2** Motion in the Heavens



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Chapter 5 Where is the Earth?—The Greeks' Answers

Experiments

- | | |
|---|---|
| 1. Naked Eye Astronomy (continued from Unit 1 Student Handbook) | 1 |
| 13. The Size of the Earth | 7 |
| 14. The Height of Piton, a Mountain on the Moon | 9 |

Activities

- | | |
|---------------------------------|----|
| Review Problems | 14 |
| Making Angular Measurements | 15 |
| Plotting an Analemma | 16 |
| Epicyles and Retrograde Motion | 17 |
| Celestial Sphere Model | 19 |
| Armillary Sphere | 21 |
| How Long is a Sidereal Day? | 21 |
| Scale Model of the Solar System | 22 |

Chapter 6 Does the Earth Move?—The Works of Copernicus and Tycho

Experiments

- | | |
|---|----|
| 15. The Shape of the Earth's Orbit | 23 |
| 16. Using Lenses, Making a Telescope, Using the Telescope | 26 |

Activities

- | | |
|---|----|
| Observing Sunspots | 30 |
| Planetary Positions Relative to the Sun | 30 |
| Foucault Pendulum: The Earth Rotates | 30 |

Chapter 7 A New Universe Appears—The Work of Kepler and Galileo

Experiments

- | | |
|------------------------------------|----|
| 17. The Orbit of Mars | 31 |
| 18. The Inclination of Mars' Orbit | 36 |
| 19. The Orbit of Mercury | 39 |

Activities

- | | |
|---------------------------------------|----|
| Three-Dimensional Model of Two Orbits | 41 |
| Demonstrating Satellite Orbits | 41 |
| Galileo | 42 |

Chapter 8 The Unity of Earth and Sky—The Work of Newton

Experiments

- | | |
|--|----|
| 20. Stepwise Approximation to an Orbit | 43 |
| 21. Model of a Comet Orbit | 48 |

Activities

- | | |
|----------------------|----|
| Forces on a Pendulum | 54 |
| Haiku | 54 |
| Other Comet Orbits | 55 |

Film Loop Notes

- | | |
|---|----|
| Film Strip—Retrograde Motion of Mars | 56 |
| 10a. Retrograde Motion of Mercury and Mars, Shown
by Animation | 56 |
| 10. Retrograde Motion—Geocentric Model | 57 |
| 11. Retrograde Motion—Heliocentric Model | 59 |
| 12. Jupiter Satellite Orbit | 61 |
| 13. Program Orbit I | 64 |
| 14. Program Orbit II | 65 |
| 15. Central Forces (Computer Program) | 66 |
| 16. Kepler Laws for Two Planets (Computer Program) | 67 |
| 17. Perturbation | 68 |

Appendix

- | | |
|---------------------------------------|----|
| Additional Suggestions for Activities | 71 |
|---------------------------------------|----|

This Student handbook is different from laboratory manuals you may have worked with before. There are far more things described in this handbook than any one student can possibly do. Only a few of the experiments and activities will be assigned. You are encouraged to pick and choose from the rest any of the activities that appear interesting and valuable for you. A number of activities may occur to you that are not described in the handbook and that you would prefer to do instead. You should feel free to pursue these in consultation with your teacher.

There is a section corresponding to each chapter of the text. Each section is composed of two major subsections—Experiments and Activities.

The Experiments section contains complete instructions for the experiments your class will be doing in the school laboratory. The Activities section contains suggestions for demonstrations, construction projects and other activities you can do by yourself. (The division between Experiments and Activities is not hard and fast; what is done in the school laboratory and what is done by the student on his own may vary from school to school.)

The Film Loop Notes give instructions for the use of the film loops which have been prepared for this course.

EXPERIMENT 1 Naked-Eye Astronomy (continued from Unit 1 Student Handbook)

At the very beginning of this course, it was suggested that you might want to begin making some basic astronomical observations in order to become familiar with the various objects in the heavens and with the ways in which these objects move. Now the time has come to analyze your data more carefully and to continue your observations. From observations much like your own, scientists in the past have developed a remarkable sequence of theories. The more aware you are of the motions in the sky, the more easily you can follow the development of these theories.

If you have been careful and thorough in your data-taking, (and if the weather has been mostly favorable), you have your own data for analysis. If, however, you do not have your own data, similar results are provided in the following sections.

a) One Day of Sun Observations

One student made the following observations of the sun's position during September 23.

<u>Eastern Daylight Time</u>	<u>Sun's Altitude</u>	<u>Sun's Azimuth</u>
7:00 A.M.	---	---
8:00	08°	097°
9:00	19	107
10:00	29	119
11:00	38	133
12:00	45	150
1:00 P.M.	49	172
2:00	48	197
3:00	42	217
4:00	35	232
5:00	25	246
6:00	14	257
7:00	03	267

If you plot altitude vs. azimuth and mark the hours for each point, you will be able to answer these questions.

1. What was the sun's greatest altitude during the day?
2. What was the latitude of the observer?
3. At what time (EDT) was the sun highest?
4. When during the day was the sun's direction (azimuth) changing fastest?
5. When during the day was the sun's altitude changing fastest?
6. Remember that daylight time is an hour ahead of standard time. On September 23 the apparent sun, the one you see, gets to the meridian 8 minutes before the mean sun. Can you determine the longitude of the observer? Near what city was he?

b) A Year of Sun Observations

One student made the following monthly observations of the sun through a full year. (He had remarkably good weather!)

<u>Dates</u>	<u>Sun's Noon Altitude</u>	<u>Sunset Azimuth</u>	<u>Interval to Sunset After Noon</u>
Jan 1	20°	238°	4 ^h 25 ^m *
Feb 1	26	245	4 50
Mar 1	35	259	5 27
Apr 1	47	276	6 15
May 1	58	291	6 55
Jun 1	65	300	7 30
Jul 1	66	303	7 40
Aug 1	61	295	7 13
Sep 1	52	282	6 35
Oct 1	40	267	5 50
Nov 1	31	250	5 00
Dec 1	21	239	4 30

*h = hour, m = minutes.

In terms of the dates make three plots (different colors or marks on the same sheet of graph paper) of the sun's noon altitude, direction at sunset and time of sunset after noon.

1. What was the sun's noon altitude at the equinoxes (March 21 and September 23)?
2. What was the observer's latitude?
3. If the observer's longitude was 79°W, near what city was he?

Experiments

4. Through what range (in degrees) did his sunset point change during the year?
5. By how much did the observer's time of sunset change during the year?
6. If the interval between sunrise and noon equalled the interval between noon and sunset, how long was the sun above the horizon on the shortest day? On the longest day?

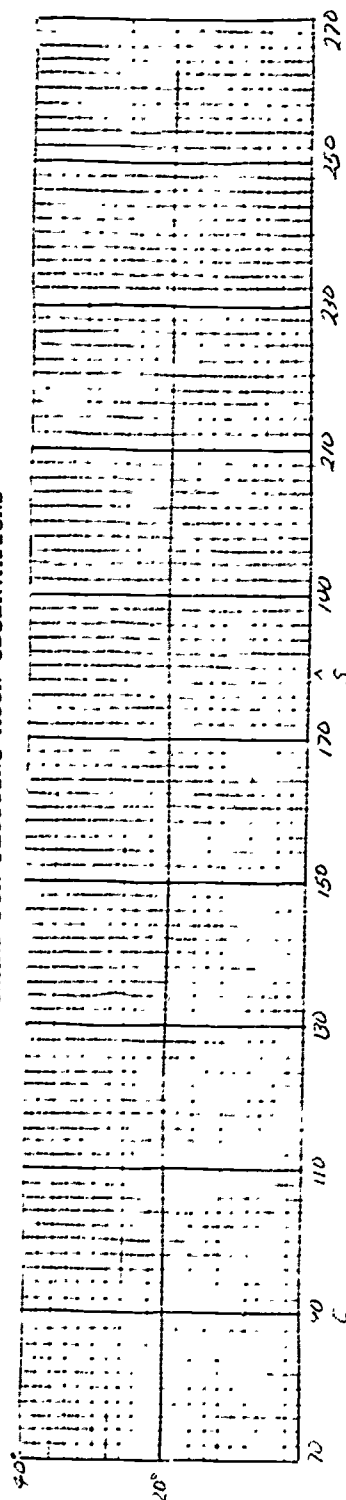
c) Moon Observations

During October 1966 a student in Las Vegas, Nevada made the following observations of the moon at sunset when the sun had an azimuth of about 255°.

Date	Angle from Sun to Moon	Moon Altitude	Moon Azimuth
Oct. 16	032°	17°	230°
18	057°	25	205
20	081°	28	180
22	104°	30	157
24	126°	25	130
26	147°	16	106
28	169°	05	083

1. Plot these positions of the moon on the chart.
2. From the data and your plot, estimate the dates of new moon, first quarter moon, full moon.
3. For each of the points you plotted, sketch the shape of the lighted area of the moon.

CHART FOR PLOTTING MOON OBSERVATIONS



d) Locating the Planets

Table I tells you where to look in the sky to see each of the planets whose wanderings puzzled the ancients. One set of positions is given, accurate to the nearest degree, for every ten-day interval; by interpolation you can get the planets' positions on any given day.

The column headed "J.D." shows the corresponding Julian Day calendar date for each entry. This calendar is simply a consecutive numbering of days that have passed since an arbitrary "Julian Day 1" in 4713 B.C.: September 26, 1967, for example, is the same as J.D. 2,439,760.

Look up the sun's present longitude in the table. Locate the sun on your SC-1 Constellation Chart: its path, the ecliptic, is the curved line marked off in 360 degrees; these are the degrees of longitude.

A planet that is just to the west of the sun's position (to the right on the chart) is "ahead of the sun," that is, it rises just before the sun does. One that is 180° from the sun rises near sundown and is in the sky all night.

When you have decided which planets may be visible, locate them along the ecliptic. Unlike the sun, they are not exactly on the ecliptic, but they are never more than eight degrees from it. The Constellation Chart shows where to look among the fixed stars.

e) Graphing the Position of the Planets

Here is a useful graphical way to display the information in the planetary longitude table.

On ordinary graph paper, plot the sun's longitude versus time. Use Julian Day numbers along the horizontal axis, beginning as close as possible to the present date. The plotted points should fall on a nearly straight line, sloping up toward the right until they reach 360° and then starting again at zero. (See Fig. 1.)

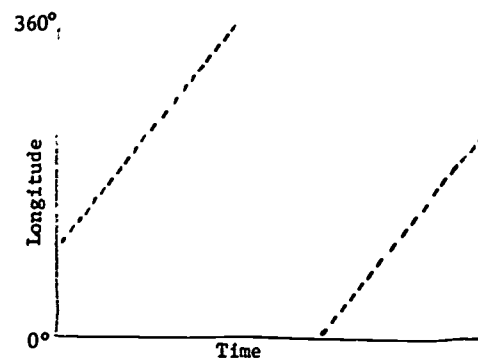


Fig. 1 The sun's longitude

How long will it be before the sun again has the same longitude as it has today? Would the answer to that question be the same if it were asked three months from now? What is the sun's average angular speed over a whole year? By how much does its speed vary? When is it fastest?

Plot Mercury's longitudes on the same graph (use a different color or shape for the points). According to your plot, how far (in longitude) does Mercury get from the sun? (This is Mercury's maximum elongation.) At what interval does Mercury pass between the earth and the sun (forget for the time being about latitudes)? (This interval is Mercury's synodic period: the period of phases.)

Plot the positions of the other planets (use a different color for each one). The resulting chart is much like the data that puzzled the ancients. In fact, the table of longitudes is just an updated version of the tables that Ptolemy, Copernicus and Tycho made.

The graph contains a good deal of useful information. For example, when will Mercury and Venus next be close enough to each other so that you can use bright Venus to help you find Mercury? Can you see from your graphs how the ancient astronomers decided on the relative sizes of the planetary spheres? (Hint: look at the relative angular speeds of the planets.) Where are the planets, relative to the sun, when they go through their retrograde motions?



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☉Sun

♀Venus

♃Jupiter

☿Mercury

♂Mars

♄Saturn

Table 1 PLANET LONGITUDES

Table with columns: Yr. Date, J.D., ☉, ♀, ♁, ♂, ♃, ♄. Contains planetary longitude data for years 1985 through 1990.

Table with columns: Yr. Date, J.D., ☉, ♀, ♁, ♂, ♃, ♄. Contains planetary longitude data for years 1988 through 1991.

Table with columns: Yr. Date, J.D., ☉, ♀, ♁, ♂, ♃, ♄. Contains planetary longitude data for years 1971 through 1973.

Arranged by Owen Gingerich

Table 2

A GUIDE FOR PLANET AND ECLIPSE OBSERVATIONS

Check your local newspaper for local eclipses times and extent of eclipse in your locality.

Mercury Visible for about one week around stated time.	Venus Visible for several months around stated time.	Mars Very bright for one month on each side of given time. Observable for 16 months surrounding given time.	Jupiter Especially bright for sev- en months be- yond stated time	Satur- n Especially bright for two months on each side of given time. Visible 13 months.	Lunar Eclipses	Solar Eclipses
Mercury and Venus are best viewed the hour before dawn when indicated as a.m. and the hour after sunset when indicated as p.m.						
1 9 early Oct.: p.m. 6 mid Nov.: a.m. 7	early Oct.: morning star			late Oct.: overhead around midnight	Oct. 18	
early Feb.: p.m. 1 mid March: a.m. 9 late May: p.m. 6 mid July: a.m. 8 late Sept.: p.m. late Oct.: a.m.			late Mar.: overhead around midnight	mid Nov.: overhead at midnight	Apr. 13 Oct. 6	
mid Jan.: p.m. 1 late Feb.: a.m. 9 early May: p.m. 6 mid June: a.m. 9 early Sept.: p.m. mid Oct.: a.m. late Dec.: p.m.	late Feb.: p.m. mid May: a.m.	mid July: overhead at midnight	late Apr.: overhead at midnight	late Nov.: overhead at midnight		Sept. 11: partial in western U.S.
mid Feb.: a.m. 1 late Apr.: p.m. 9 early June: a.m. 7 mid Aug.: p.m. 0 late Sept.: a.m. early Dec.: p.m.	early Nov.: p.m. mid Dec.: a.m.		late May: overhead at midnight	early Dec.: overhead at midnight	Feb. 21 Aug. 17	Mar. 7: total in Fla., partial in eastern and southern U.S.
mid Jan.: a.m. 1 late Mar.: p.m. 9 mid May: a.m. 7 late July: p.m. 1 mid Sept.: a.m. late Nov.: p.m.		early Sept.: overhead at midnight	late June: overhead at midnight	late Dec.: overhead at midnight	Feb. 10	
early Jan.: a.m. 1 mid Mar.: p.m. 9 early May: a.m. 7 mid July: p.m. 2 late Aug.: a.m. early Nov.: p.m. mid Dec.: a.m.	mid May: p.m. early Aug.: a.m.		late July: overhead at midnight		Jan. 30 July 26	July 10: partial in northern U.S.
late Feb.: p.m. 1 late Apr.: a.m. 9 late June: p.m. 7 early Aug.: a.m. 3 mid Oct.: p.m. early Dec.: a.m.	late Dec.: p.m.	late Nov.: overhead at midnight	early Sept.: overhead at midnight	early Jan.: overhead at midnight	Dec. 10	
mid Feb.: p.m. 1 late Mar.: a.m. 9 early June: p.m. 7 mid July: a.m. 4 late Sept.: p.m. early Nov.: a.m.	early Mar.: a.m.		mid Oct.: overhead at midnight	late Jan.: overhead at midnight	June 4 Nov. 29	
late Jan.: p.m. 1 early Mar.: a.m. 9 mid May: p.m. 7 early July: a.m. 5 mid Sept.: p.m. late Oct.: a.m.	mid-late July: p.m. early Oct.: a.m.		early Nov.: overhead at midnight	early Feb.: overhead at midnight	May 25 Nov. 18	
mid Jan.: p.m. 1 late Feb.: a.m. 9 early May: p.m. 7 mid June: a.m. 6 late Aug.: p.m. early Oct.: a.m. mid Dec.: p.m.		late Jan.: overhead at midnight	early Dec.: overhead at midnight	late Feb.: overhead at midnight		
early Feb.: a.m. 9 early Apr.: p.m. 7 late May: a.m. 7 mid Aug.: p.m. 7 late Sept.: a.m.	early Mar.: p.m. mid Apr.: a.m.				Apr. 4	

EXPERIMENT 13 The Size of the Earth

The first recorded approximate measurement of the size of the earth was made by Eratosthenes in the third century B.C. His method was to compare the lengths of shadows cast by the sun at two different points rather far apart but nearly on a north-south line on the earth's surface. The experiment described here uses an equivalent method. Instead of measuring the length of a shadow you will measure the angle between the vertical and the sight line to a star or the sun.

You will need a colleague at least 200 miles away due north or south of your position to take simultaneous measurements. You will need to agree in advance on the star, the date and the time for your observations.

Theory

The experiment is based on the assumptions that

- 1) the earth is a perfect sphere.
- 2) a plumb line points towards the center of the earth.
- 3) the distance from stars and sun to the earth is great compared with the earth's diameter.

The two observers must be located at points nearly north and south of each other (i.e., they are nearly on the same meridian). They are separated by a distance s along that meridian. You (the observer at A) and the observer at B both sight on the same star at the pre-arranged time, preferably when the star is on or near the meridian.

Each of you measures the angle between the vertical of his plumb line and the sight line to the star.

Light rays from the star reaching locations A and B are parallel (this is implied by assumption 3).

You can therefore relate the angle at A, θ_A , to the angle at B, θ_B , and the angle between the two radii, ϕ , as shown in Fig. 1.

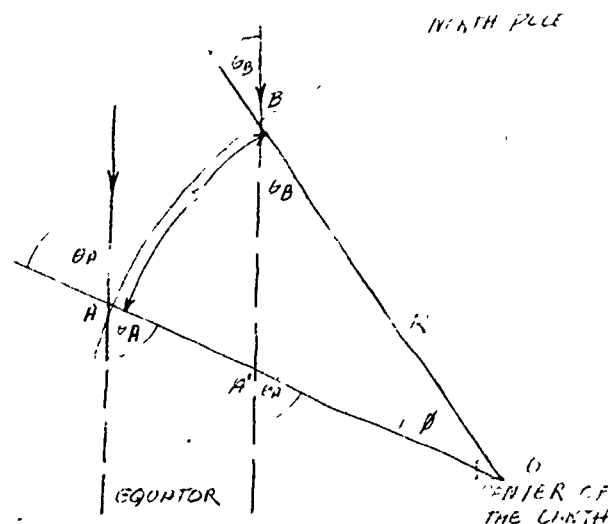


Fig. 1

In the triangle A'BO

$$\phi = (\theta_A - \theta_B) \quad (1)$$

If C is the circumference of the earth, and s is an arc of the meridian, then

$$\frac{s}{C} = \frac{\phi}{360} \quad (2)$$

Combining Eqs. (1) and (2),

$$C = \left(\frac{360}{\theta_A - \theta_B} s \right)$$

where θ_A and θ_B are measured in degrees.

Procedure

For best results, the two locations A and B should be directly north and south of each other. The observations are made just as the star crosses the local meridian, that is, when it reaches its highest point in the sky.

You will need some kind of instrument to measure the angle θ . One such instrument is an astrolabe. One can be made fairly easily from the design in Fig. 2.

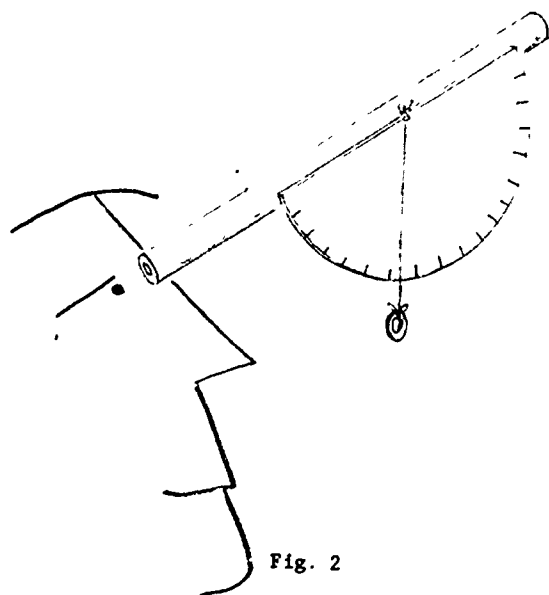


Fig. 2

Align your astrolabe along the meridian (north-south line) and measure the angle from the vertical to the star as it crosses the meridian.

If the astrolabe is not aligned along the meridian, the altitude of the star will be observed before or after it is highest in the sky. An error of a few minutes from the time of transit will make little difference.

An alternative method would be to measure the angle to the sun at (local) noon. (Remember that this means the time when the sun is highest in the sky, and not necessarily 12 o'clock.) You could use the shadow theodolite described in Experiment 1. Remember that the sun, seen from the earth, is itself $\frac{1}{2}^\circ$ wide. Do not try sighting directly at the sun. You may damage your eyes.

An estimate of the uncertainty in your measurement of θ is important.

Take several measurements on the same star (at the same time) and take the average value of θ . Use the spread in values of θ to estimate the uncertainty of your observations and of your result.

If it can be arranged, you should exchange instruments with your colleague at the other observing position and repeat the measurements on the same star on another night. (Remember that it will cross the meridian 4 minutes earlier every night, so your time of observation will be different.) By taking the average of the two values of θ given by the different instruments you can eliminate errors due to differences in construction between the two.

The accuracy of your value for the earth's circumference also depends on knowing the over-the-earth distance between the two points of observation. See: "The Shape of the Earth," *Scientific American*, October, 1967, page 67.

Q1 How does the uncertainty of the over-the-earth distance compare with the uncertainty in your value for θ ?

Q2 What is your value for the circumference of the earth and what is the uncertainty of your value?

Q3 Astronomers have found that the average circumference of the earth is about 24,900 miles. What is the percentage error of your result?

Q4 Is this acceptable, in terms of the uncertainty of your measurement?

B.C.

by Johnny Hart



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EXPERIMENT 14 The Height of Piton, A Mountain on the Moon

You have probably seen photographs of the moon's surface radioed back to earth from an orbiting space ship or from a vehicle that has made a "soft landing" on the moon. The picture on page 95 of the Unit 1 text shows an area about 180 miles across and was taken by Lunar Orbiter II from a height of 28.4 miles.

But even before the space age, astronomers knew quite a lot about the moon's surface. Galileo's own description of what he saw when he first turned his telescope to the moon is reprinted in Sec. 7.8 of Unit 2.

From Galileo's time on, astronomers have been able, as their instruments and techniques improved, to learn more and more about the moon, without ever leaving the earth.

In this experiment you will use a photograph taken with a 36-inch telescope in California to estimate the height of a mountain on the moon. Although you will use a method similar in principle to Galileo's, you should be able to get a much more accurate value than he could working with a small telescope (and without photographs!).

Materials

You are supplied with a photograph of the moon taken at the Lick Observatory near the time of the third quarter. The North Pole is at the bottom of the photograph. This is because an astronomical telescope, whether you look through it or use it to take a photograph, gives an inverted image.

Why Choose Piton?

Piton, a mountain in the northern hemisphere, is fairly easy to measure because it is a slab-like pinnacle in an otherwise fairly flat area. It is

quite close to the terminator (the line separating the light portion from the dark portion of the moon) at third quarter phase.

You will find Piton on the moon photograph above (to the south) and left of the large crater Plato at the moon latitude of about 40° N. Plato and Piton are both labeled on the moon map.

The important features of the photograph are shown in Fig. 1. The moon is a sphere of radius R . Piton (P) is a distance s from the terminator and casts a shadow of apparent length l .

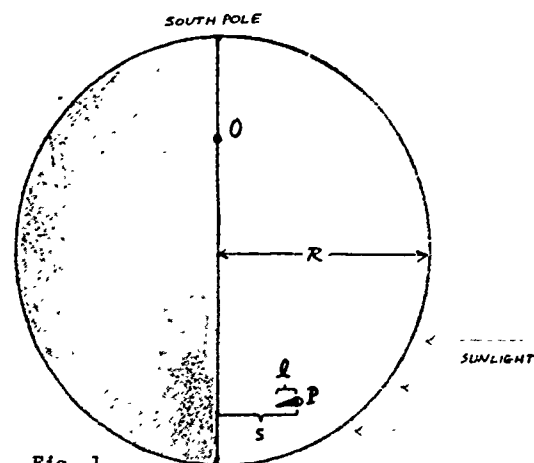


Fig. 1

Figure 2 shows how the moon would appear if viewed from above the point O which is on the terminator and 90° from Piton. From this point of view Piton is seen on the edge of the moon's disc. Its size is exaggerated in the sketch.

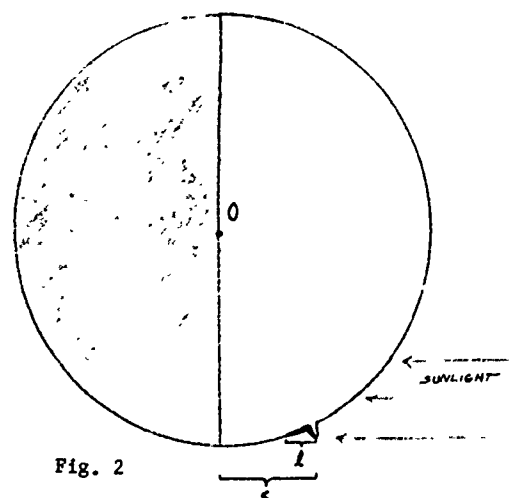


Fig. 2

Experiments

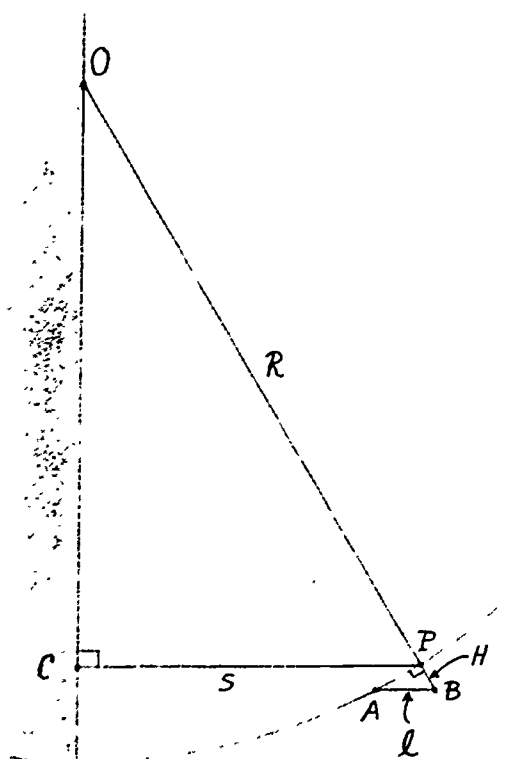


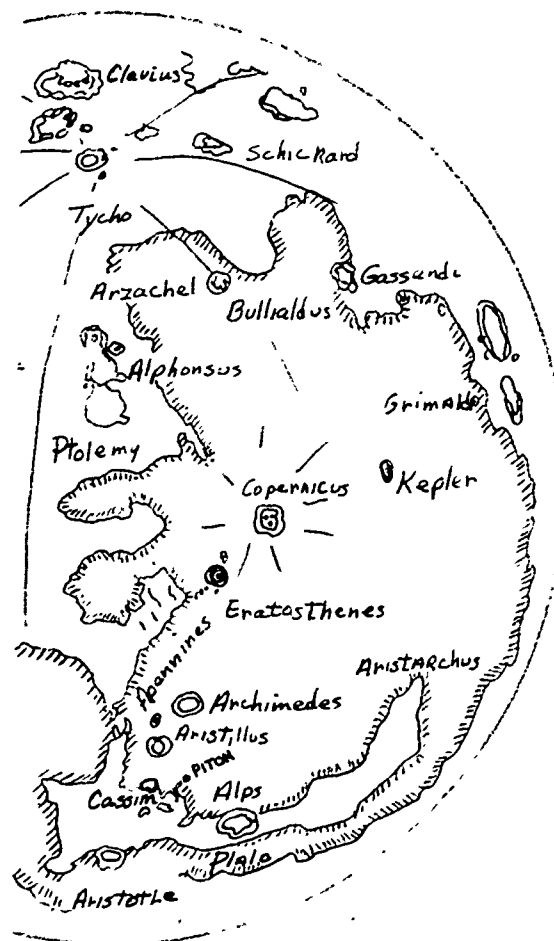
Fig. 3

Some simplifying assumptions

You can readily derive the height of Piton from measurements made on the photograph if you assume that:

- 1) the shadow l is short compared to the lunar radius R . This allows you to neglect the curvature of the moon under the shadow—you can approximate arc AP by a straight line.
- 2) in Fig. 1 you are looking straight down on the top of the peak.
- 3) the moon was exactly at third quarter phase when the photograph was made.

Q1 How big an error do you think these assumptions might involve?



The geomet. c model

Look at the triangles OCP and APB in Fig. 3. Both have a right angle (at C and P respectively) and AB is parallel to CP . From this it can be shown that the angles \hat{COP} and \hat{PAB} are equal. The two triangles are therefore similar. Corresponding sides of similar triangles are proportional, so we can write $\frac{s}{R} = \frac{H}{l}$.

To determine H , you must measure l , s and R from the photograph in arbitrary units, and then establish the scale of the photograph by comparing the measured diameter of the moon photo (in mm) with its given diameter (3476 km).

Experiments



A photograph of the moon taken at the third quarter phase. (Lick Observatory, University of California)

Measurements and Calculations

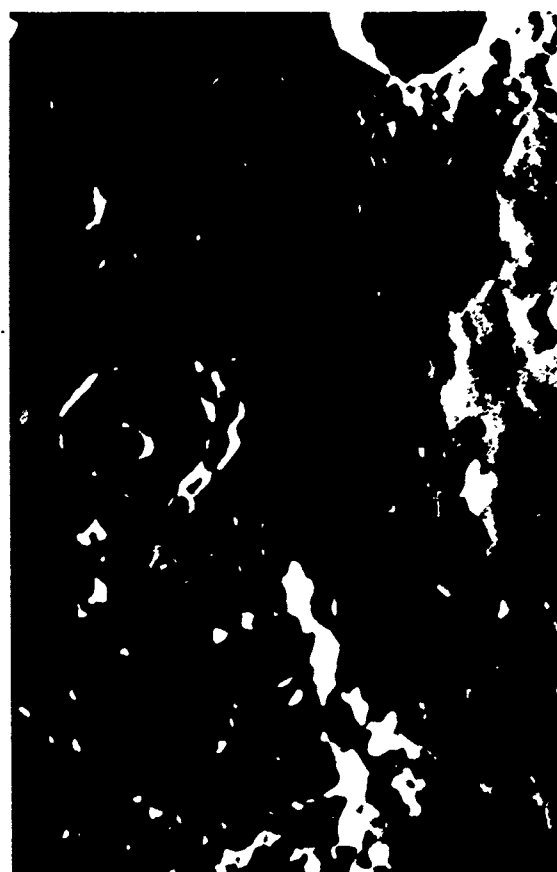
The first problem is to locate the terminator. Because the moon has no atmosphere, there is no twilight zone. The change from sunlit to dark is abrupt. Those parts of the moon which are higher than others remain sunlit longer. Thus the shadow line, or terminator, is a ragged line across the moon's surface. To locate the terminator use a white thread or string stretched tight and seek an area close to Piton where the moon's surface is flat. Move the thread until it is over the boundary between the totally dark and partly illuminated areas.

Use a 10X magnifier to measure s and l on the pictures provided. Or you can use a scale to measure the length of the shadow and the distance from Piton to the terminator on the 10X enlargement; the values of l and s will be one tenth of these measured lengths. To measure R in millimeters find the moon's diameter along the terminator and divide by two. The diameter of the moon is 3476 km.

Q2 How much does one millimeter on the photograph represent on the moon's surface?

Use this scale factor and the equation given above to find the height of Piton, in km.

Below is a picture of the area enclosed by the white line, enlarged exactly ten times. On these telescope pictures north is at the bottom, east is to the right, as in the map opposite.



Experiments

Discussion

1. What is your value for the height of Piton?
2. Which is the least certain of your measurements? What is your estimate of the uncertainty of your final result?
3. Astronomers, using methods considerably more sophisticated than those you

use here, calculate the height of Piton to be about 2.3 km (and about 22 km across at its base). How does your value compare with this?

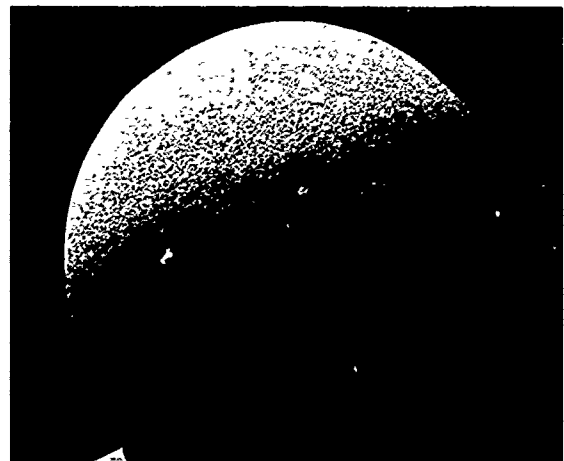
4. Does your value differ from this by more than your experimental uncertainty? If so, can you suggest why?



Tenth lunar surface picture taken by Surveyor I spacecraft on June 2, 1966, shows a moon rock six inches high and twelve inches long. Surveyor I, America's first lunar soft-landing spacecraft, touched down in the Ocean of Storms at 11:17 p.m., Pacific Daylight Time, June 1, 1966. Bright spots at left are reflections of the sun. The picture was received at the National Aeronautics and Space Administration's Jet Propulsion Laboratory.

Can you estimate the size of some of the smaller objects?

Can you estimate the angle that the sun's rays make with the moon's surface (angle θ in Fig. 2)? (Photo credit: NASA).

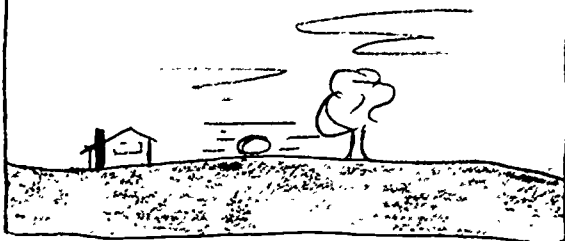


These photographs are of a monumental stone sphere, dating from the precolumbian period, found in Costa Rica. The photographs were taken from different angles with respect to the sun, giving an effect similar to different phases of the moon. As for the moon, the greatest surface detail is visible along the shadow line. The photograph at the left shows a practical check being made on the method of measuring the height of a moon mountain.

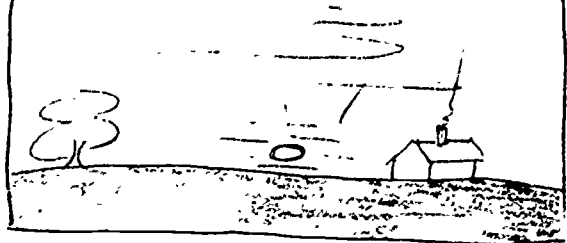
Activities

Review problems

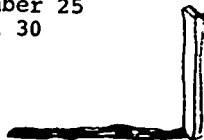
- 1 The sketch below illustrates sunrise on March 21. Sketch, approximately, the positions of sunrise on each of the following dates:
- a) June 21
 - b) September 23
 - c) December 22



- 2 The sketch below illustrates sunset on March 21. Sketch, approximately, the positions of sunset on each of the following dates:
- a) June 21
 - b) September 23
 - c) December 27



- 3 At noon on October 5 a vertical stake casts a shadow as shown below. Sketch, approximately, where the tip of the shadow will be on the following dates:
- a) June 17
 - b) December 25
 - c) March 30



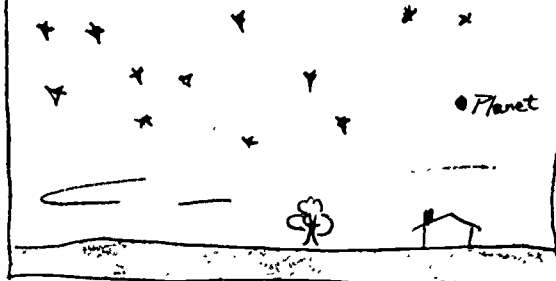
- 4 The shadow of the stake is shown for noon of October 5. In what direction is the sun? How high in the sky is the sun at noon on
- a) December 25?
 - b) March 30?
 - c) June 17?



- 5 In the twilight an hour after sunset, you see a pattern of stars in the western sky. Where will you find these stars at the same time two weeks later?



- 6 A planet nearing opposition is at the position indicated. Indicate how its position will change among the stars as it passes through opposition. In what direction will the planet be moving on the day of opposition?



Making Angular Measurements

For astronomical observations, and often for other purposes, you need to measure the angle between two objects as viewed from your location. Sometimes you need to be able to make quick estimates of the location of an object, such as where a meteor was sighted.

Approximate measurements

Hold your hand out in front of you at arm's length. You have several instant measuring devices handy once you calibrate them. They are the angular sizes of:

- 1) your thumb
- 2) your fist not including thumb knuckle
- 3) two of your knuckles
- 4) your hand from thumb-tip to tip of little finger when your hand is opened wide.

For a first approximation, your fist is about 8° , and thumb-tip to little finger is between 15° and 20° .

However, since the length of people's arms and the size of their hands vary, you can calibrate yourself using the following method.

Calibration

Stand at least 5 meters away from a chalkboard or wall (Fig. 1). The accuracy of your calibration increases with increasing distance. Have someone make a mark on the chalkboard. Hold your hand out at arm's length and line up the

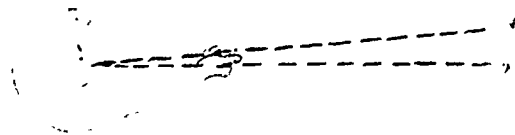


Fig. 1

left edge of your thumb with the mark on the board. Have the other person make another mark on the chalkboard in line with the right edge of your thumb. Measure the distance between the two marks. Find the circumference of a circle whose radius is equal to the distance from you to the chalkboard. The ratio of the distance between the two marks to the circumference of the circle is the ratio of the number of degrees subtended by your thumb to 360 degrees.

Example: for an average physics student at an eye-chalkboard distance of 5.0 meters, the distance between marks for a thumb width was 0.19 m.

$$\frac{0.19 \text{ m}}{2 \times \pi \times 5 \text{ m}} = \frac{x \text{ degrees}}{360 \text{ degrees}}$$

Therefore, $x = 2.2$ degrees. A similar procedure gave 8.3° for a fist, and 16° for the angle subtended by the thumb to little finger tip.

Mechanical aids

You can make a simple instrument for measuring other angles from a 3×5 file card and a meter stick or yard stick. When an object is placed 57.4 diameters away, it subtends an angle of 1° . That is, a one-inch diameter coin at 57.4 inches cuts off an angle of 1° . Since 57.4 inches would make a rather cumbersome instrument, we can scale down the diameter and distance to a half. Then $1/2$ inch at 27.7 inches (call it 28 inches) subtends an angle of 1° . At the same distance from your eye, 2° would be subtended by 1 inch and 5° subtended by $2 \frac{1}{2}$ inches. Now you can make a simple device with which you can estimate angles of a few degrees (Fig. 2).

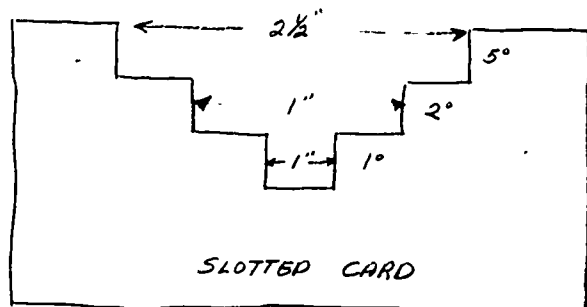
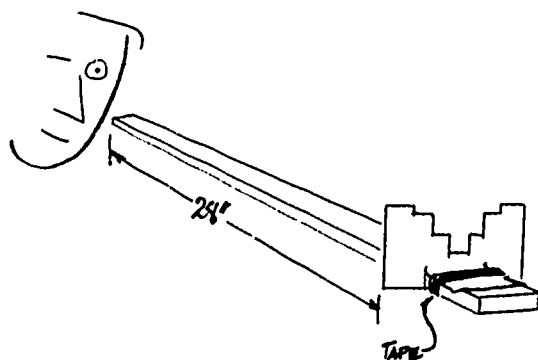


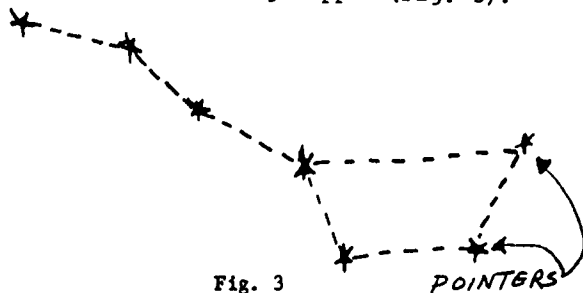
Fig. 2

Cut a series of step-wise slots in a file card, as indicated in Fig. 2. Mount the card vertically at the 28-inch mark on a yardstick. Cut flaps in the bottom of the card, fold them and tape the card to the yardstick. Then hold the zero end of the yardstick against your upper lip—and observe. (Keep a stiff upper lip!)



Things to observe

1. What is the angle between the pointers of the Big Dipper (Fig. 3)?



2. What is the length of Orion's belt?

3. What is the angular diameter of the sun? (Measure the sun at sunrise or sunset, when it is not bright enough to hurt your eyes. Do NOT stare at it for long. Even then, use dark sunglasses!

4. How many degrees away from the sun is the moon? Observe on several nights.

5. What is the angular diameter of the moon? Does it change between the time the moon rises and the time when it is highest in the sky on a given day? To most people the moon seems larger when near the horizon. Is it? See: "The Moon Illusion," *Scientific American*, July, 1962, p. 120, which describes some methods used to study this illusion.

Plotting an Analemma

Have you seen an analemma? Examine a globe of the earth, and you will usually find a graduated scale in the shape of a figure 8, and having dates on it. This figure is called an analemma, and is used to summarize the changing positions of the sun during the year.

You can plot your own analemma. Place a small square mirror on a horizontal surface so that the reflection of the sun at noon falls on a south-facing wall. Make observations each day at exactly the same time, such as noon, and mark the position of the reflection on a sheet of paper fastened to the wall. If you remove the paper each day, you must be sure to replace it in exactly the same position. Record the date beside the point. The north-south motion is most evident during September-October and March-April. You can find more about the east-west migration of the marks in astronomy texts and encyclopedias under the heading Equation of Time.

Epicyles and Retrograde Motion

The manually operated epicycle machine allows you to explore the patterns of motion produced by various ratios of the radii of the two motions. You can vary both the length of the epicycle arm compared to the radius of the deferent and the ratio of the turning rates to find the forms of the different curves which may be traced out.

The epicycle machine supplied (Fig. 1) has three possible gear ratios: 2 to 1 (producing two loops per revolution), 1 to 1 (one loop per revolution) and 1 to 2 (one loop per two revolutions). To change the ratio simply slip the drive band (twisted in a figure 8 so the main arm and the secondary arm rotate in the same sense) to another set of pulleys.

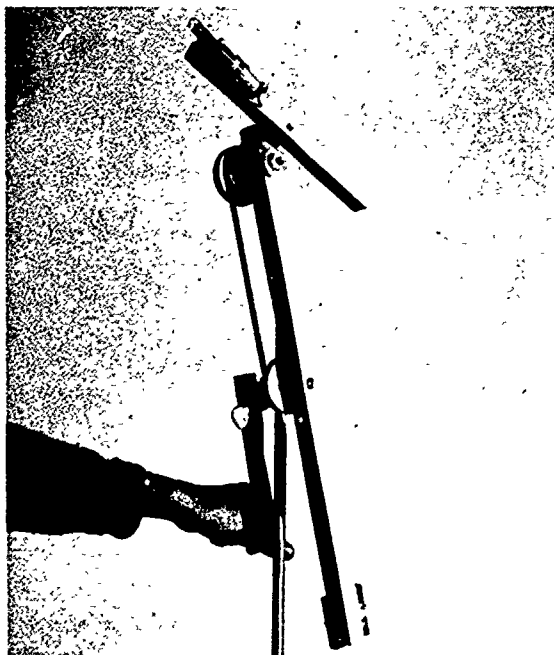


Fig. 1

Tape a light source (pen-light cell, holder and bulb) securely to one end of the secondary arm and counter-weight the other with, say a second, unilluminated light source. If you use a fairly high rate of rotation in a darkened room, you and other observers should be able to see the light source move in an epicycle.

The form of the curve traced out depends not only on the gear ratio but also on the relative lengths of the radii. As the light is moved closer to the center of the secondary arm, the loop decreases in size until it becomes a cusp. When the light is very close to the center of the secondary arm, the curve will be a slightly distorted circle (Fig. 2), like the motion of the moon around the earth as seen from the sun.

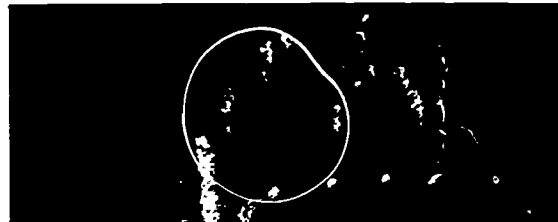


Fig. 2

With the correct choices of gear ratio, relative radii and direction of motion, the light source will move in either an eccentric circle or an ellipse. These special cases have been known from antiquity and require that the epicycle turn at exactly the same rate as the deferent. If the motions are both in the same direction, the result is equivalent to an eccentric circle. On one side of the primary circle the epicycle adds to the radius of the deferent and on the other side it subtracts from it (Fig. 3).

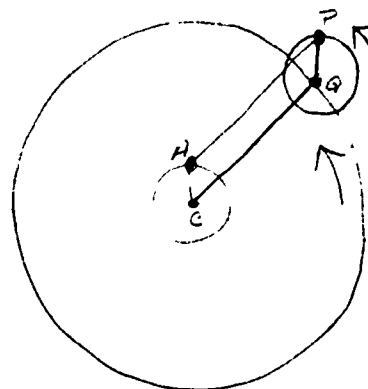


Fig. 3

Activities

If the epicycle moves opposite to the deferent, the result is an ellipse (Fig. 4).

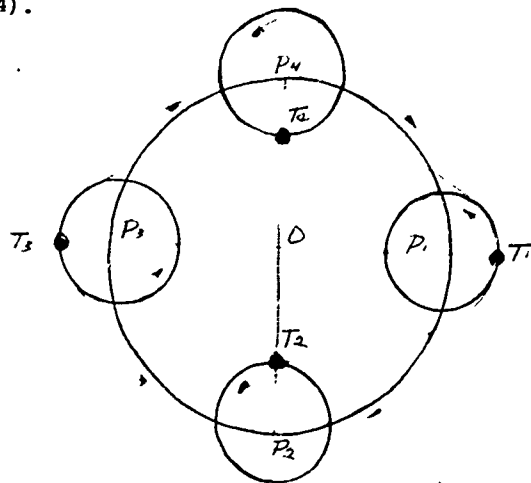


Fig. 4

To relate this machine to the Ptolemaic model, where planets move in epicycles around the earth as a center, one should really stand at the main axis (earth) and view the lamp against the fixed background. The size of the machine, however, does not allow this so one must instead view the motion from the side. The lamp then goes into retrograde each time an observer in front of the machine sees a loop. The retrograde motion is most pronounced with the light source far from the center of the secondary axis (Fig. 5).

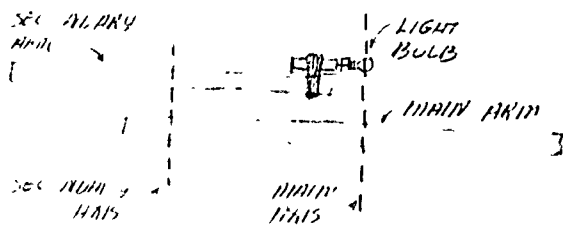


Fig. 5

Photographing epicycles

The motion of the light source can be photographed by mounting the epicycle machine on a turntable and holding the

center pulley stationary with a clamp (Fig. 6). Alternately, the machine can be held in a buret clamp on a ringstand and turned by hand.

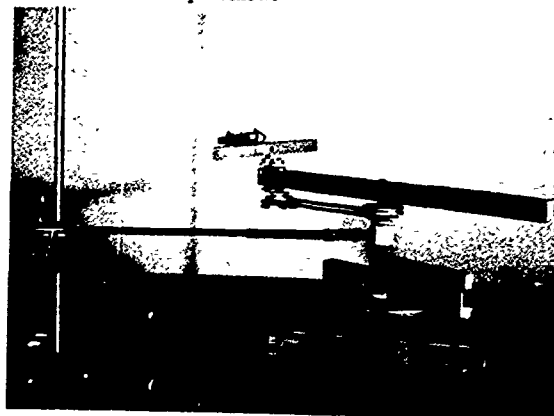


Fig. 6

Running the turntable for more than one revolution will show that the traces do not exactly overlap (Fig. 7). This

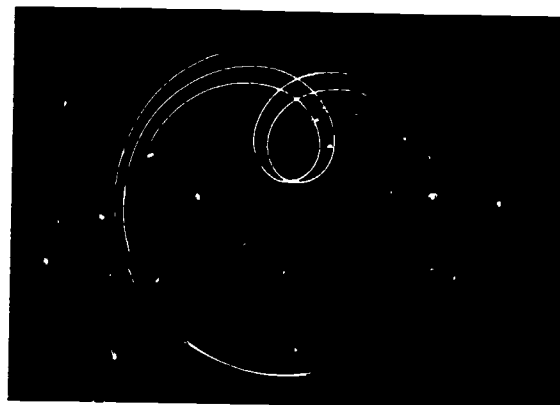


Fig. 7

is probably caused by using a drive band which is not of uniform thickness. As the joint runs over either pulley, the ratio of speeds changes momentarily and a slight displacement of the axes takes place. By letting the turntable rotate for some time, the pattern will eventually begin to overlap.

A time photograph of this motion has a good bit of esthetic appeal, and you might enjoy taking such pictures as an after-class activity. Figures 8, 9 and 10 are examples of the different patterns which can be produced.

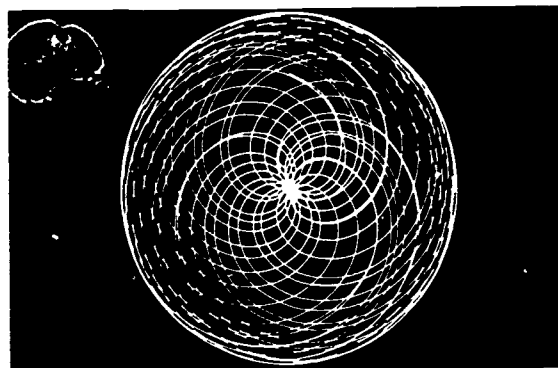


Fig. 8

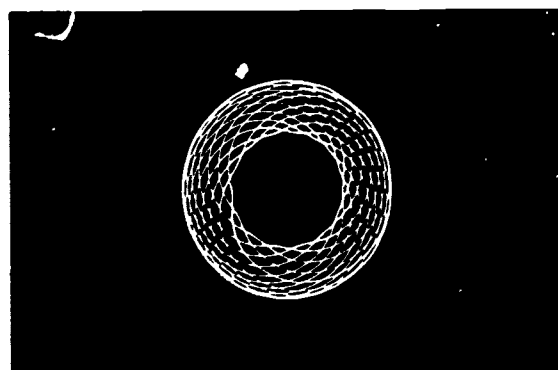


Fig. 9

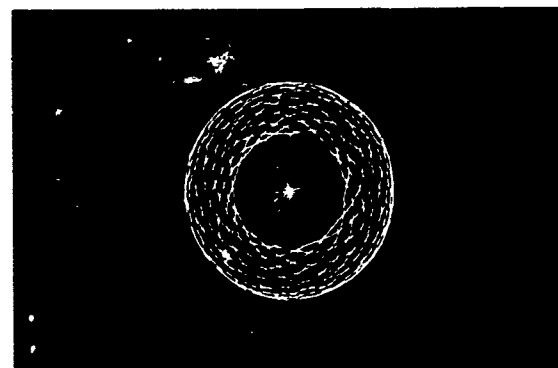


Fig. 10

Celestial Sphere Model*

You can make a model, using a round-bottom flask, of the celestial sphere. From it you can see how the appearance of the sky changes if you go northward or southward and how the stars appear to rise and set.

*From YOU AND SCIENCE, by Paul F. Brandwein, et al., copyright 1960 by Harcourt, Brace and World, Inc.

To make this model all you will need in addition to the round-bottom flask is a one-hole rubber stopper to fit its neck, a piece of glass tubing, paint, a fine brush or grease pencil, a star map or a table of star position, and considerable patience.

On the bottom of the flask locate the point opposite the center of the neck. Mark this point North Pole. With a string or tape, determine the circumference of the flask—the greatest distance around it. This will be 360° in your model. Then, starting at the North Pole, mark points that are $1/4$ of the circumference, or 90° , from the North Pole point. These points lie on a line around the flask, which is the equator. You can mark in the equator with a grease pencil (china-marking pencil), with paint, or India ink.

To locate the stars accurately on your "globe of the sky," you will need a coordinate system. If you do not wish to have the coordinate system marked permanently on your model, put on the lines with a grease pencil. Through the North Pole (N.P.) draw a line to the equator. The intersection is the vernal equinox. Perpendicular to this line, as Fig. 1 shows, mark a point $23\ 1/2^\circ$ from the North Pole (about $1/4$ of 90°). This will be the pole of the ecliptic (Fig. 1).

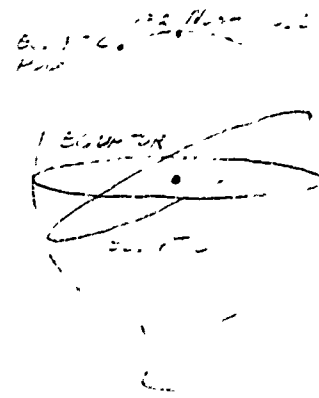


Fig. 1

Activities

The ecliptic, path of the sun, will be a great circle 90° from the ecliptic pole. Where the ecliptic crosses the equator from south to north is the vernal equinox (Fig. 2), the position of the sun on March 21. All positions in the sky, are located eastward from this point, and north or south from the equator.

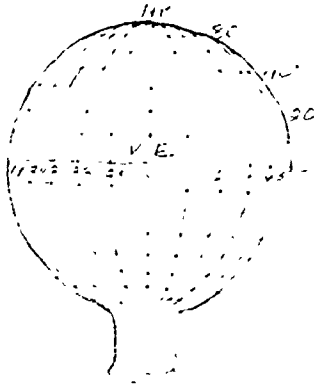


Fig. 2

To set up the north-south scale, which is like latitude on the earth but called declination in the sky, measure off points $1/9$ of the way from the North Pole to the equator and draw through these points little circles that run east and west. These lines will be 10° apart, which is close enough for spotting in your stars.

To set up the east-west scale, mark from the vernal equinox intervals of $1/24$ of the total circumference. These marks will be 15° apart. We use this interval rather than 10° marks because the sky turns through 15° each hour and star positions are recorded in hours eastward from the vernal equinox, called their right ascension.

Now your globe looks like a big piece of curved graph paper. From a table of star positions (listed under Tin Can Planetarium in the Student Handbook for Unit 1) or a star map you can spot in a star's position north or south of the

equator. All east-west positions are expressed eastward from the vernal equinox—as you face your globe these will be to the right of the vernal equinox.

To finish the model, put the glass rod into the stopper so that it almost reaches across the flask and points to your North Pole point. Then put in the flask enough inky water so that, when you hold the neck straight down, the water just comes up to the line of the equator. For safety, wrap wire around the neck of the flask and over the stopper so it will not fall out (Fig. 3).

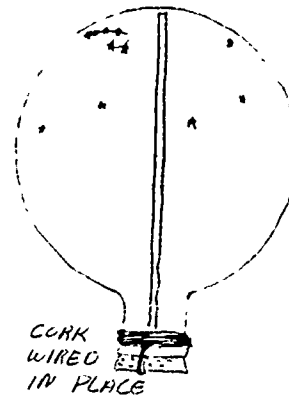


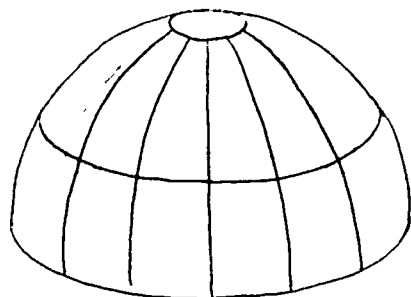
Fig. 3

Now, no matter how you tip the flask you have a model of the sky as you would see it from outside. If the North Pole is straight up, you are at the north pole of the earth and you see stars in only the northern half of the sky. As you rotate the globe from right to left, the stars will move as you would see them from the earth. If you tip the pole down halfway, you are at latitude 45° N. As you turn the globe stars will rise above the sea in the east and set in the west. If you hold the globe with the earth's axis horizontal, you are at the equator. Then all the stars can be seen as they rise and set.

Armillary Sphere

An armillary sphere is a mechanical device which shows the various coordinate systems used in the sky. Metal arcs are used to represent the horizon, the celestial equator, the ecliptic and the coordinate systems. You will find such a device very helpful as you try to visualize these imaginary lines in the sky.

Armillary spheres, and plastic spheres which can serve the same function, are available from several suppliers, for example Welch Scientific Co., or Damon Educational Inc. However, you can make a reasonably satisfactory substitute for \$2.00 or less from a "hanging basket," a hemispherical wire basket purchased from a garden supply store.



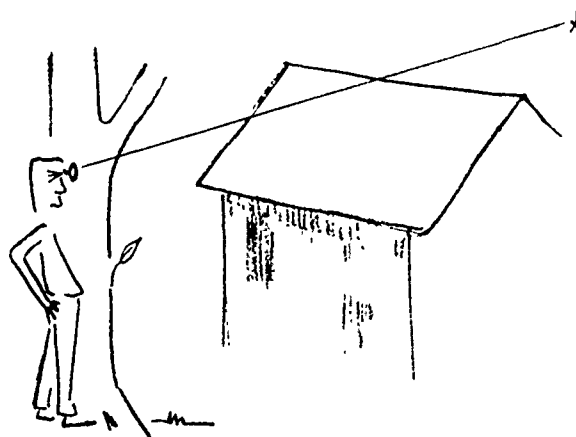
These baskets, 10 or 12 inches in diameter, are used for hanging plants. They will have an equator with twelve ribs (meridians) going toward the bottom (pole). One or two small circles of wire paralleling the great circle help support the ribs (meridians). You can add wire circles for other coordinates. For example, if the great circle represents the equator, add another great circle tipped at $23\frac{1}{2}^\circ$ to show the ecliptic. Use bits of paper to locate some of the brighter stars.

Two such baskets can be used to represent the whole sky.

How Long is a Sidereal Day?

A sidereal day is the time interval between two successive instants when a star passes the same meridian in the sky. To measure a sidereal day you need an electric (synchronous motor) clock and a screw-eye.

Choose a neighboring roof, fence, etc. towards the west. Then fix a screw-eye as an eyepiece in some rigid support such as a tree so that a bright star, when viewed through the screw-eye, will be a little above the roof.



For several successive nights, record the time when the star just disappears behind the roof. By averaging the time intervals you will have determined the length of a sidereal day.

Activities

Scale Model of the Solar System

Most drawings of the solar system are badly out of scale, because it is so hard to show both the sizes of the sun and planets and their relative distance on an ordinary-sized piece of paper. Constructing a simple scale model will help you develop a better picture of the true dimensions of the solar system.

Let a three-inch tennis ball represent the sun. The distance of the earth from the sun is 107 times the sun's diameter, or for this model, about 27 feet. You can confirm this easily. In the sky the sun has a diameter of half a degree—about half the width of your thumb when held upright at arm's length in front of your nose. Check this, if you wish, by comparing your thumb to the angular diameter of the moon (which is nearly equal to that of the sun); both have diameters of $\frac{1}{2}^\circ$. Now hold your thumb in the same

upright position and walk away from the tennis ball until its diameter is about half the width of your thumb. You will be between 25 and 30 feet from the ball!

The distance between the earth and sun is called the "astronomical unit" (AU). This unit is used for describing distances within the solar system. Since the diameter of the sun is about 1,400,000 kilometers (870,000 miles), in the model one inch represents about 464,000 kilometers. From this scale the proper scaled distances and sizes of all the other planets can be derived.

The moon has an average distance of 384,000 kilometers from the earth and has a diameter of 3,476 kilometers. Where is it on the scale model? How large is it? Completion of the column for the scale model distances will yield some surprising conclusions.

A Scale Model of the Solar System

Object	Solar Distance		Diameter		Sample Object
	AU	Model (ft)	Km (approx.)	Model (inches)	
Sun	-	-	1,400,000		tennis ball
Mercury	0.39		4,000		
Venus	0.72		12,000		
Earth	1.00	27	13,000		pin head
Mars	1.52		6,600		
Jupiter	5.20		140,000		
Saturn	9.45		120,000		
Uranus	19.2		48,000		
Neptune	30.0		45,000		
Pluto	39.5		6,000		
Nearest Star	2.7×10^5				

EXPERIMENT 15 The Shape of the Earth's Orbit

Two questions to think about before starting to plot the orbit

Q1 How long does it take the sun to make one complete cycle (360°) through the sky against the background of stars? (See Sec. 5.1 if you have difficulty answering this.) How fast does the sun move along the ecliptic in degrees per day (to the nearest degree)?

Q2 The further away you are from an object the smaller it appears to be. Suppose you photograph a friend at a distance of ten meters, and then again at a distance of twenty meters. Can you find a formula that relates his apparent size to his distance?

Ptolemy, and most of the Greeks, thought that the earth was at the center of the universe, and that the sun revolved around the earth. From the time of Copernicus the idea gradually became accepted that the earth and other planets revolve around the sun. You probably believe this, just as you believe that the earth is round. But from the evidence of our senses—how we see the sun move through the sky during the year—there is no reason to prefer one model over the other.

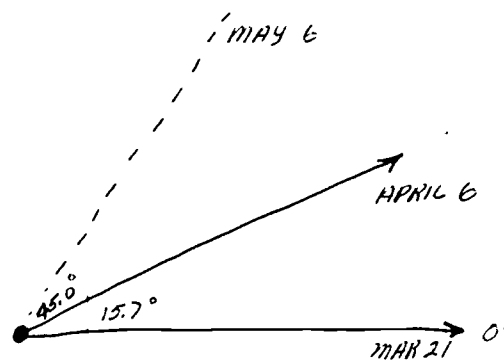
Plotting the orbit

The raw material for this experiment is a series of sun photographs taken at approximately one-month intervals and printed on a film strip.

Q3 The photograph in Frame 5 shows halves of the images of the January sun and July sun placed adjacent. How can you account for the obvious difference in size?

Assume that the earth is at the center of the universe. (This, after all, is the "common sense" interpretation; it is what our senses tell us.)

Take a large piece of graph paper (20" x 20" or four 8 1/2 x 11" pieces pasted together) and put a mark at the center to represent the earth. Take the 0° direction, the direction of the sun as seen from the earth on March 21st, to be along the grid lines toward the left.



The dates of all the photographs, and the direction to the sun measured from this zero direction are given in the table below. Use a protractor to draw a spider web of lines radiating out from the earth in these different directions. The angles are measured counterclockwise from the zero line.

Table

Date	Direction from earth to sun
March 21	0°
April 6	15.7
May 6	45.0
June 5	73.9
July 5	102.5
Aug. 5	132.1
Sept. 4	162.0
Oct. 4	191.3
Nov. 3	220.1
Dec. 4	251.4
Jan. 4	283.2
Feb. 4	314.7
March 7	346.0

Experiments

Measure carefully the projected diameters of each of the frames in the film strip. What is the relationship between these measurements and the relative distance of sun from earth? Adopt a scale factor (a constant) to convert the diameters to distances from the earth in arbitrary units. Your plot of the earth-sun distance should have a radius of about 10 cm. If your measurement for the sun's diameter is about 50 cm, you should use a scale factor of 500 cm.

$$\text{Distance to sun} = \frac{\text{constant}}{\text{sun's diameter}} = \frac{500}{\text{diameter}}$$

(For instance, if the measured diameter of the sun is 49.5 cm the relative distance will be $\frac{500}{49.5} = 10.1$ cm; and for a measured diameter of 51.0 cm the relative distance would be $\frac{500}{51.0} = 9.8$ cm.) Make a table of the relative distances for each of the twelve dates.

Along each of the direction lines you have drawn measure off a length corresponding to the relative distance to the sun on that date. Draw a smooth curve through these points using a compass or a set of French curves. This is the orbit of the sun relative to the earth. The distances are relative; we cannot find the actual distance in miles from the earth to the sun from this plot.

Q4 Is the orbit a circle? If so, where is the center of the circle? If the orbit is not a circle, what shape is it?

Q5 Locate the major axis of the orbit, through the points where the sun passes closest to and farthest from the earth! What are the approximate dates of closest approach (perihelion) and greatest distance (aphelion)? What is the ratio of aphelion distance to perihelion distance?

A heliocentric system

Copernicus and his followers adopted the sun-centered model only because the solar system could be described more simply that way. They had no new data that could not be accounted for by the old model. If you are not convinced that the two models, geocentric and heliocentric, are equally valid descriptions of what we see, try one of the activities at the end of the notes on this experiment.

You can use the same data to plot the earth's orbit around the sun, if you make the Copernican assumption that the earth revolves above the sun. You already have a table of the relative distances of the sun from the earth. Clearly there's going to be some similarity between the two plots. The dates of aphelion and perihelion won't change, and the table of relative distances is still valid because you didn't assume either model when deriving it. Only the angles used in your plotting change.

When the earth was at the center of the plot the sun was in the direction 0° (to the right) on March 21st.

Q6 What is the direction of the earth as seen from the sun on that date? What is the earth's longitude as seen from the sun? (The answer to this question is given at the end of the notes on this experiment. Be sure that you understand it before going on.)

At this stage the end is in sight. Perhaps you can see it already, without doing any more plotting. But if not, here is what you must do:

Make a new column in your table giving the angle from sun to earth. (These are the angles given on the last frame of the film strip.) Place a mark for the sun at the center of another sheet of paper on each of the twelve dates. Use the

Experiments

new angles to plot the orbit of the earth. When you have marked the earth's position on a few dates, compare the new plot with the first one you made around the sun. You may now see the relationship between the two plots. Before you finish the second plot you will probably see what the relationship is, and how you can use the same plot to represent either orbit. It all depends on whether you initially assume a geocentric or a heliocentric model.

Answer to Q6

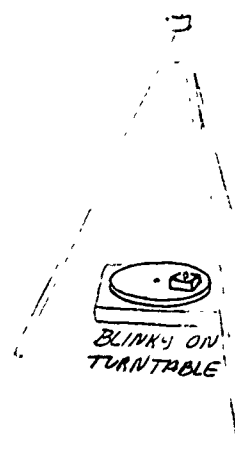
The direction of the earth on March 21st, as seen from the sun, is to the left on your plot. The angle is 180° . If you do not understand the answer read the following:

I am standing due north of you. You must face north to see me. When I look at you I am looking south. If the direction north is defined as 0° the direction south is 180° . Any two "opposite" directions (north and south, south-west and north-east) differ by 180° .

Two activities on frames of reference

(1) You and a classmate take hold of opposite ends of a meter stick or a piece of string a meter or two long. You stand still while he walks around you in a circle at a steady pace. You see him moving around you. But how do you appear to him? Ask him to describe what he sees when he looks at you against the background of walls, furniture, etc. You may not believe what he says; reverse your roles to convince yourself. In which direction did you see him move—toward your left or your right? In which direction did he see you move—toward his left or his right?

(2) The second demonstration uses a camera, tripod, blinky and turntable. Mount the camera on the tripod and put the blinky on a turntable. Aim the camera straight down.



Take a time exposure with the camera at rest and the blinky moving one revolution in a circle. If you do not use the turntable, move the blinky by hand around a circle drawn faintly on the background. Then take a second print, with the blinky at rest and the camera moved steadily by hand about the axis of the tripod. Try to move the camera at the same rotational speed as the blinky moved in the first photo.

Can you tell, just by looking at the photos, whether the camera or the blinky was moving?

Experiments

EXPERIMENT 16 Using Lenses, Making a Telescope, Using the Telescope

In this experiment you will first examine some of the properties of single lenses, then combine these lenses to form a telescope which you can use to observe the moon, the planets and other heavenly (as well as earth-bound) objects.

The simple magnifier

You certainly know something about lenses already—for instance, that the best way to use a magnifier is to hold it immediately in front of the eye and then move the object you want to examine until its image appears in sharp focus.

Examine some objects through several different lenses. Try lenses of a variety of shapes and diameters. Separate any lenses that magnify from those that don't. Describe the difference between lenses that magnify and those which do not.

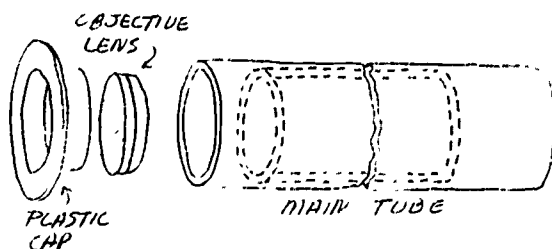
Q1 Which lens has the highest magnifying power? Arrange the lenses in order of their magnifying powers.

Q2 What physical feature of a lens seems to determine its power—is it diameter, thickness, shape, the curvature of its surface?

Sketch side views of a high-power lens, of a low-power lens and of the lowest power lens you can imagine.

Real images

With one of the lenses you have used, project an image of a ceiling light or a (distant) window on a sheet of paper.



Describe all the properties of the image that you can observe. Such an image is called a real image.

Q3 Do all lenses give real images?

Q4 How does the image depend on the lens?

Q5 If you want to look at a real image without using the paper, where do you have to put your eye?

Q6 Why isn't the image visible from other positions?

Q7 The image (or an interesting part of it) may be quite small. How can you use a second lens to inspect it more closely? Try it.

Q8 Try using other combinations of lenses. Which combination gives the maximum magnification?

Making a telescope

Parts list

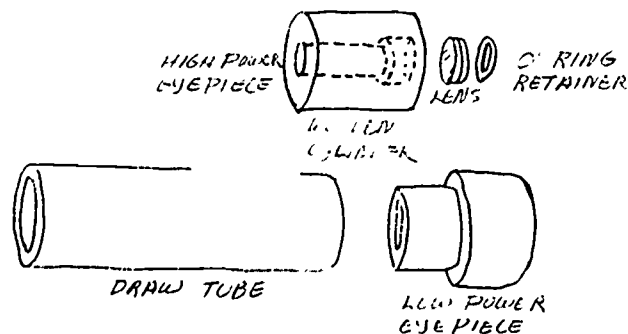
- 1 large lens
- 1 magnifier
- 1 small lens, mounted in a wooden cylinder
- 2 cardboard tubes
- 1 plastic cap

Note the construction of the largest lens. This lens is called the objective lens (or simply the objective).

Q9 How does its magnifying power compare with the other lenses in the kit?

Assembling the telescope

The sketches show how the various parts go together to make your telescope.



Notes on assembly

1. If you lay the objective down on a flat clean surface, you will see that one surface is more curved than the other: the more curved surface should face outward.
2. Clean dust, etc., off lens (using lens tissue or clean handkerchief) before assembling telescope.
3. Focus by sliding the draw tube, not by moving the eyepiece in the tube.
4. To use high power satisfactorily, a firm support—tripod—is essential.
5. Be sure that the lens lies flat in the high-power eyepiece. Low power gives about 12X magnification. High power gives about 30X magnification.

Try out your telescope on objects inside and outside the lab. The next section suggests some astronomical observations you can make.

Telescope or Binocular Observations

Introduction

You should look at terrestrial objects with and without the telescope to develop handling skill and familiarity with the appearance of objects.

Mounting

Telescopic observation is difficult when the mounting is unsteady. If a swivel-head camera tripod is available, the telescope can be held in the wooden saddle by rubber bands, and the saddle attached to the tripod head by the head's standard mounting screw. Because camera tripods are usually too short for comfortable viewing from a standing position, it is strongly recommended that the observer be seated in a reasonably comfortable chair. The telescope should be grasped as far forward and as far back

as possible, and both hands rested firmly against a car roof, telephone pole, or other rigid support.

Aiming

Even with practice, you may have trouble finding objects, especially with the high-power eyepiece. Sighting over the top of the tube is not difficult, but making the small searching movements following the rough sighting takes some experience. One technique is to sight over the tube, aiming slightly below the object, and then tilt the tube up slowly while looking through it. Or, if the mounting is firm and the eyepiece tubes are not too tight, an object can be found and centered in the field with low power, and then the high-power eyepiece carefully substituted. Marks or tape ridges placed on the eyepieces allow them to be interchanged without changing the focus.

Focussing

Pulling or pushing the sliding tube tends to move the whole telescope. Use the fingers as illustrated in Fig. 1 to push the tubes apart or pull them together. Turn the sliding tube while moving it (as if it were a screw) for fine adjustment.



Fig. 1

Experiments

An observer's eyeglasses keep his eye much farther from the eyepiece than the optimum distance. Far-sighted or near-sighted observers are generally able to view more satisfactorily by removing their eyeglasses and refocussing. Observers with astigmatism have to decide whether or not the astigmatic image (without glasses) is more annoying than the reduced field of view (with glasses).

Many observers find that they can keep their eye in line with the telescope while aiming and focussing if the brow and cheek rest lightly against the forefinger and thumb. When using a tripod mounting, the hands should be removed from the telescope while actually viewing to minimize shaking the instrument. (See Fig. 2.)

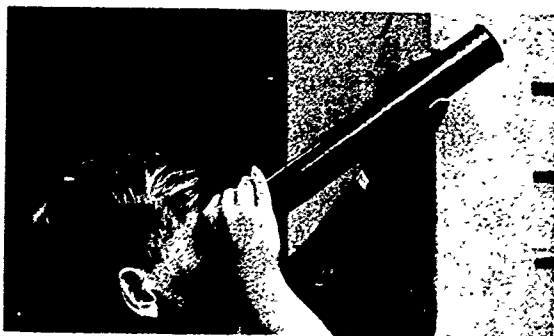


Fig. 2

Limitations

By comparing the angular sizes of the planets with the resolving power of the telescope, you can get some idea of how much fine detail to expect when observing the planets.

For a 30X telescope to distinguish between two details, they must be at least 0.001° apart. The low-power Project Physics telescope gives a 12X magnification, and the high power gives 30X. (Note: Galileo's first telescope gave 3X magnification, and his "best" gave about 30X. You should find it challenging to see whether you can observe all the phenomena mentioned in Sec. 7.8 of your text.)

The angular sizes of the planets as viewed from the earth are:

Mars:	0.002° (minimum)
	0.005° (maximum)
Saturn:	0.005° (average)
Uranus:	0.001° (average)
Venus:	0.003° (minimum)
	0.016° (maximum)
Jupiter:	0.012° (average)

Observations

The following group of suggested objects have been chosen because they are (1) fairly easy to find, (2) representative of what is to be seen in the sky, and (3) quite interesting. You should observe all objects with the low power first and then the high power. For additional information on current objects to observe, see the Handbook of the Heavens, the last few pages of each issue of "Sky and Telescope," "Natural History," or "Science News."

Venus: It will appear as a featureless disc, but you can observe its phases, as shown on page 68 of your text. When it is very bright you may need to reduce the amount of light coming through the telescope in order to tell the true shape of the image. A paper lens cap with a hole in the center will reduce the amount of light.

Saturn: It is large enough that you can resolve the projection of the rings beyond the disc, but you probably can't see the gap between the rings and the disc with your 30X telescope. Compare your observations to the sketches on page 69 of the text.

Jupiter: Observe the four satellites that Galileo discovered. If you use the low-power eyepiece with the ruled scale, you can record the relative radii of the orbits for each of the moons. By keeping detailed data over about 6 months' time, you can determine the period for each of the moons, the radii of their orbits, and

then the mass of Jupiter. See the notes for film loop 12, Jupiter Satellite Orbit, in Chapter 8 of the Student Handbook for directions on how to analyze your data.

Jupiter is large enough that some of the surface detail—like a broad, dark, equatorial stripe—can be detected (especially if you know it should be there!)

Moon: Best observations are made between four days after new moon and four days after first quarter. Make sketches of your observations, and compare them to Galileo's sketch on page 66 of your text. Look carefully for walls, central mountains, peaks beyond the terminator, craters in other craters, etc.

The Pleiades: A beautiful little group of stars which is located on the right shoulder of the bull in the constellation Taurus. They are almost overhead in the evening sky during December. The Pleiades were among the objects Galileo studied with his first telescope. He counted 36 stars, which the poet Tennyson described as "a swarm of fireflies tangled in a silver braid."

The Hyades: This group is also in Taurus, near the star Aldebaran, which forms the Bull's eye. The high power may show several double stars.

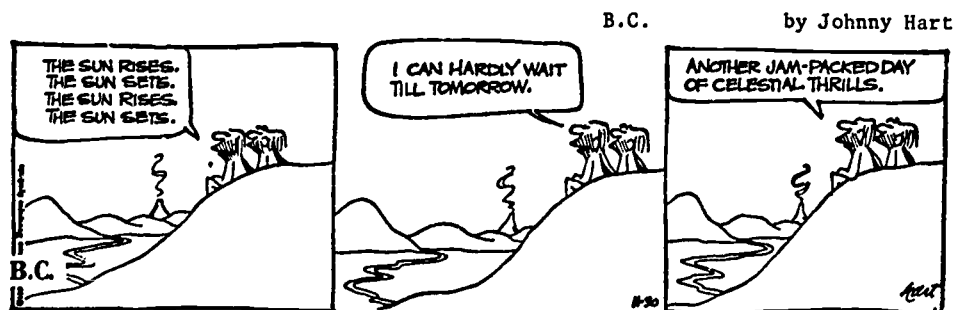
The Great Nebula in Orion: Look about half way down the row of stars which hangs like a sword in the belt of Orion. It is in the southeastern sky during December and January.

Algol, a famous variable star, is in the constellation Perseus, south of Cassiopeia. Algol is high in the eastern sky in December, and near the zenith during January. Generally it is a second-magnitude star, like the Pole Star. After remaining bright for almost 2 1/2 days, it fades for 5 hours and becomes a fourth-magnitude star, like the faint stars of the Little Dipper. Then it brightens during 5 hours to its normal brightness. From one minimum to the next the period is 2 days, 20 hours, 49 minutes.

The Double Cluster in Perseus: Look for the two star clusters at the top of the constellation Perseus near Cassiopeia. High power should show two magnificent groups of stars.

Great Nebula in Andromeda: Look high in the western sky in December, for by January it is on the way toward the horizon. It will appear like a fuzzy patch of light, and is best viewed with low power. The light from this nebula has been on its way for 1.5 million years.

The Milky Way: It is particularly rich in Cassiopeia and Cygnus (if air pollution in your area allows it to be seen).



By permission of John Hart and Field Enterprises Inc.

Activities

Observing Sunspots

Figure 1 shows an arrangement of a tripod, the low-power telescope and a sheet of paper for viewing sunspots. Cut a hole in a piece of cardboard so it fits tightly over the end of the telescope.



Fig. 1

This acts as a shield so there is a shadow area where you can view the sunspots. Project the image of the sun on a piece of white paper. Focus the image by moving the tube holding the eyepiece in and out. DO NOT LOOK THROUGH THE TELESCOPE. IT WILL INJURE YOUR EYES. When the image is in focus, you will see some small dark spots on the paper. To separate marks on the paper from sunspots, jiggle the paper back and forth. By focussing the image farther from the telescope you can make the image larger and not so intense.

Sunspot activity follows an 11-year cycle. Maximum activity will occur in 1969 or 1970. If you make a series of drawings of the sunspots over several weeks, you can conclude for yourself that the sun is rotating. How can you be sure that the motion of the spots isn't an illusion due to the earth moving about the sun?

Planetary Positions Relative to the Sun

It is easy to turn the graph you made in the last section of Experiment 1 (continued) into one like the illustration on page 47 of the text. Simply subtract the longitude of the sun from the longitude of each planet and plot the difference against time. The subtractions can be performed directly on your original graph, with dividers or a ruler. If the difference is greater than 180 degrees, subtract 360 degrees from it; if it is less than minus 180 degrees, add 360 degrees.

Once you have made this new graph, answer the questions in Study Guide 6.1 for each of the planets.

Foucault Pendulum

You can make a small-scale Foucault pendulum by suspending a large mass from the classroom ceiling if a sturdy support is available; however, a great deal of care is needed to reduce the effects of air currents and friction at the support. Articles in the Amateur Scientist section of Scientific American, June 1958 and February 1964, describe details of construction.

You can demonstrate the principle of the Foucault pendulum by constructing a small model as shown in Fig. 1.

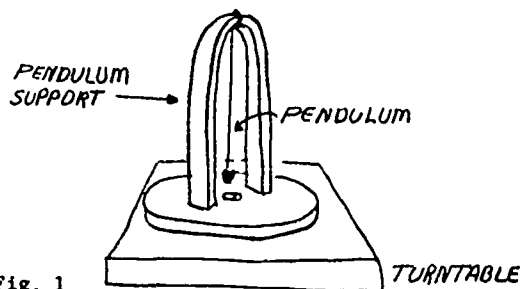


Fig. 1

Start the pendulum swinging, then turn on the turntable. The pendulum continues swinging in the same plane in space, although friction at the support will tend to rotate the plane of swing slightly.

EXPERIMENT 17 The Orbit of Mars

(See Student Activity, Three-Dimensional Model of Two Orbits, in this section, before starting this experiment.)

In this laboratory activity you will derive an orbit for Mars around the sun by the same method that Kepler used. Because the observations are made from the earth, you will need the orbit of the earth which you developed in Experiment 15.

Make sure that the plot you use for this experiment represents the orbit of the earth around the sun, not the sun around the earth.

If you did not do the earth's orbit experiment, you may use, for an approximate orbit, a circle of 10 cm radius drawn in the center of a large sheet of graph paper. Because the eccentricity of the earth's orbit is very small (0.017) you can place the sun at the center of the orbit without introducing a significant error in this experiment.

From the sun (at the center) draw a line to the right, parallel to the grid of the graph paper. Label the line 0° . This line is directed toward a point on the celestial sphere called the Vernal Equinox and is the reference direction from which angles in the plane of the earth's orbit (the ecliptic plane) are measured.

The earth crosses this line on September 23. On March 21 the sun is between the earth and the Vernal Equinox.

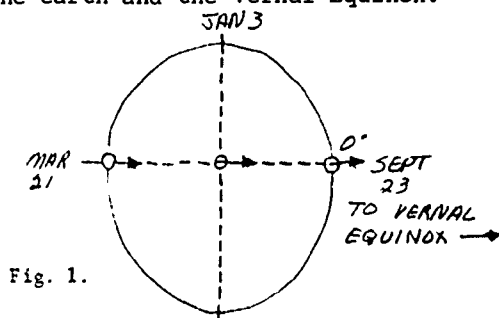


Fig. 1.

The photographs

You will use a booklet containing sixteen enlarged photographs of the sky showing Mars among the stars at various dates between 1931 and 1950. All were made with the same small camera used for the Harvard Observatory Sky Patrol. On some of the photographs Mars was near the center of the field. On many other photographs Mars was near the edge of the field where the star images are distorted by the camera lens. Despite these distortions the photographs can be used to provide positions of Mars which are satisfactory for this study.

Changes in the positions of the stars relative to each other are extremely slow. Only a few stars near the sun have motions large enough to be detected from observations with the largest telescopes after many years. Thus we can consider the pattern of stars as fixed.

Theory

Mars is continually moving among the stars but is always near the ecliptic. From several hundred thousand photographs at the Harvard Observatory sixteen were selected, with the aid of a computer, to provide pairs of photographs separated by 687 days—the period of Mars around the sun as determined by Copernicus. During that interval the earth has made nearly two full cycles of its orbit, but the interval is short of two full years by 43 days. Therefore the position of the earth, from which we can observe Mars, will not be the same for the two observations of each pair. But Mars will have completed exactly one cycle and will be back in the same position. If we can determine the direction from the earth toward Mars for each observation, the sight lines will cross at a point on the orbit of Mars. This is a form of triangulation in which we use the different

Experiments

positions of the earth to provide a base-line (Fig. 2). Each pair of photographs in the booklet (A and B, C and D, etc.) is separated by a time interval of 687 days.

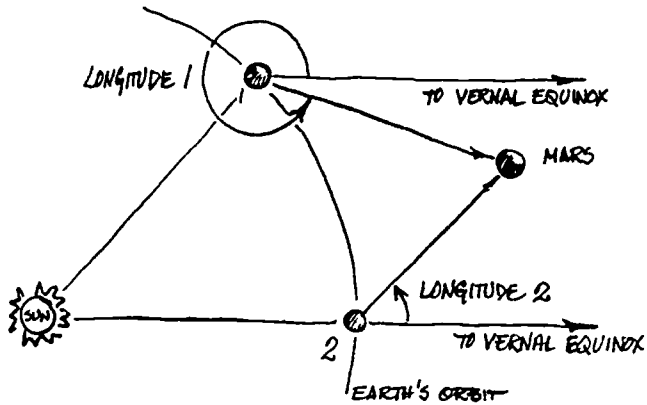


Fig. 2 Points 1 and 2 on the earth orbit are 687 days apart. This is the period of Mars. Mars is therefore back in the same position. Observations of the direction of Mars on these two dates enable us to find its position.

Coordinate system

When we look into the sky we see no coordinate system. We create coordinate systems for various purposes. The one we wish to use here centers on the ecliptic. Remember that the ecliptic is the imaginary line along which the sun moves on the celestial sphere during the year.

Along the ecliptic we measure longitudes always eastward from the 0° point; the direction towards the Vernal Equinox. Perpendicular to the ecliptic we measure latitudes north or south to 90° . The small movement of Mars above and below the ecliptic is considered in the next experiment, The Inclination of Mars' Orbit.

To find the coordinates of a star or of Mars we must project the coordinate system upon the sky. To do this you are provided with transparent overlays which show the coordinate system of the ecliptic for each frame, A to P. The positions of various stars are circled. Adjust the overlay until it fits the star posi-

tions. Then you can read off the longitude and latitude of the position of Mars. Figure 3 shows how you can interpolate between marked coordinate lines. Because we are interested in only a small section of the sky on each photograph, we can draw each small section of the ecliptic as a straight line. Since the values you obtain are to be used for plotting, an accuracy of 0.5° is quite sufficient. Record your results in the table provided.

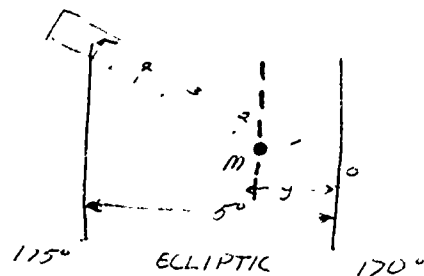


Fig. 3 Interpolation between coordinate lines. In the sketch Mars (M) is at a distance y° from the 170° line. The distance between the 170° line and the 175° line is 5° .

Take a piece of paper or card at least 10 cm long. Make a scale divided into 10 equal parts and label alternate marks 0, 1, 2, 3, 4, 5. This gives a scale in $\frac{1}{2}^\circ$ steps. Notice that the numbering goes from right to left on this scale.

Place the scale so that the edge passes through the position of Mars. Now tilt the scale so that 0 and 5 marks each fall on a grid line. Read off the value of y from the scale.

In the sketch $y = 1\frac{1}{2}^\circ$ and so the longitude of M is $170^\circ + 1\frac{1}{2}^\circ = 171\frac{1}{2}^\circ$.

For a simple plot of Mars' orbit around the sun you will only use the first column—the longitude of Mars. You will use the columns for latitude, Mars' distance from the sun, and the sun-centered coordinates if you derive the inclination, or tilt, or Mars' orbit in Experiment 18. Record the latitude of Mars; you might use it later on.

Observed Positions of Mars

Frame	Date	Mars		Mars'	Dist	Heliocentric	
		Long.	Lat.	Earth	Sun	Long.	Lat.
A	Mar. 21, 1931						
B	Feb. 5, 1933						
C	Apr. 20, 1933						
D	Mar. 8, 1935						
E	May 26, 1935						
F	Apr. 12, 1937						
G	Sept. 16, 1939						
H	Aug. 4, 1941						
I	Nov. 22, 1941						
J	Oct. 11, 1943						
K	Jan. 21, 1944						
L	Dec. 9, 1945						
M	Mar. 19, 1946						
N	Feb. 3, 1948						
O	Apr. 4, 1948						
P	Feb. 21, 1950						

Now you are ready to locate points on the orbit of Mars.

1. On the plot of the earth's orbit, locate the position of the earth for each date given in the 16 photographs. You may do this by interpolating between the dates given for the earth's orbit experiment. Since the earth moves through 360° in 365 days, you may use 1° for each day ahead or behind the date given in the previous experiment. (For example, frame A is dated March 21. The earth is at 166° on March 7 and 195.7° on April 6. You now add 14° (14 days) to 166° or subtract 17° (17 days) from 196° .) Always work from the earth-position-date nearest the date of the Mars photograph.

2. Through each earth position point draw a "0° line" parallel to the line you drew from the sun towards the Vernal Equinox (the grid on the graph paper is helpful). Use a protractor and a sharp pencil to establish the angle between the 0° line and the direction to Mars as seen from the earth (longitude of Mars). Lines drawn from the earth's posi-

tions for each pair of dates will intersect at a point. This is a point on Mars' orbit. Figure 4 shows one point on Mars' orbit obtained from the data of the first pair of photographs. By drawing the intersecting lines from the eight pairs of positions, you establish eight points on Mars' orbit.

3. Draw a smooth curve through the eight points you have established. Perhaps you can borrow a French curve or long spline (e.g., from the mechanical drawing department). You will notice that there are no points in one section of the orbit. But since the orbit is symmetrical about its major axis you can fill in the missing part.

Now that you have plotted the orbit you have achieved what you set out to do: you have used Kepler's method to determine the path of Mars around the sun.

If you have time to go on, it is worthwhile to see how well your plot agrees with Kepler's generalization about planetary orbits.

Experiments

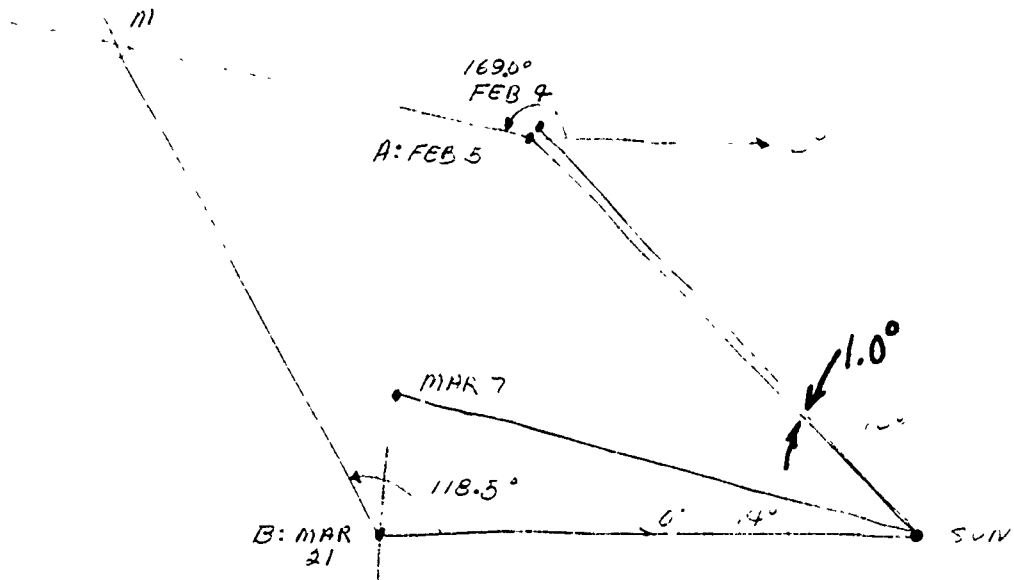


Fig. 4

Kepler's laws from your plot

- Q1 Does your plot agree with Kepler's conclusion that the orbit is an ellipse?
- Q2 What is the mean sun-to-Mars distance in AU?
- Q3 As seen from the sun, what is the direction (longitude) of perihelion and of aphelion for Mars?
- Q4 During what month is the earth closest to the orbit of Mars? What would be the minimum separation between the earth and Mars?
- Q5 What is the eccentricity of the orbit of Mars?
- Q6 Does your plot of Mars' orbit agree with Kepler's law of areas, which states that a line drawn from the sun to the planet, sweeps out areas proportional to the time intervals? From your orbit you see that Mars was at point B' on February 5, 1933, and at point C' on April 20, 1933.

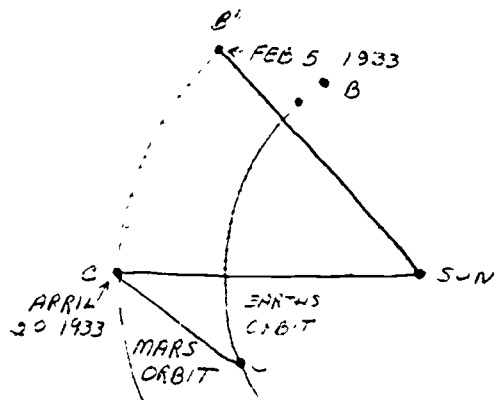


Fig. 5 In this example the time interval is 74 days.

There are seven such pairs of dates in your data. The time intervals are different for each pair.

Connect these pairs of positions with a line to the sun. Find the areas of these sectors by counting blocks of squares on the graph paper (count a square when more than half of it lies

within the area). Divide the area (in squares) by the number of days in the interval to find an "area per day" value. Are these values nearly the same?

Q7 How much (what percentage) do they vary?

Q8 What is the uncertainty in your area measurements?

Q9 Is the uncertainty the same for large areas as for small?

Q10 Do your results bear out Kepler's law of areas?

This is by no means all that you can do with the photographs you used to make the plot of Mars' orbit. If you want to do more, look at Experiment 18.

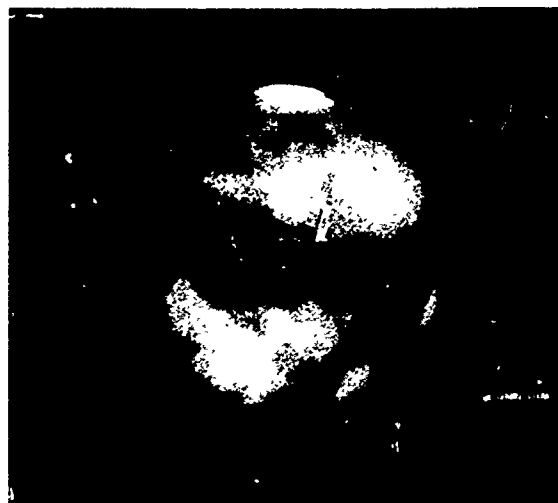
MARS 1956

Taken with 60-inch telescope



August 10

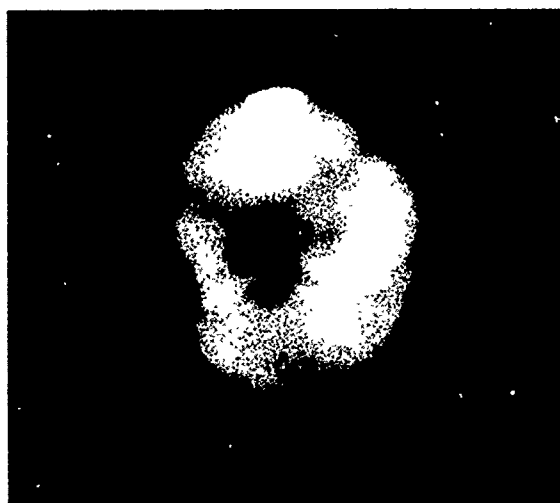
ORANGE



August 22

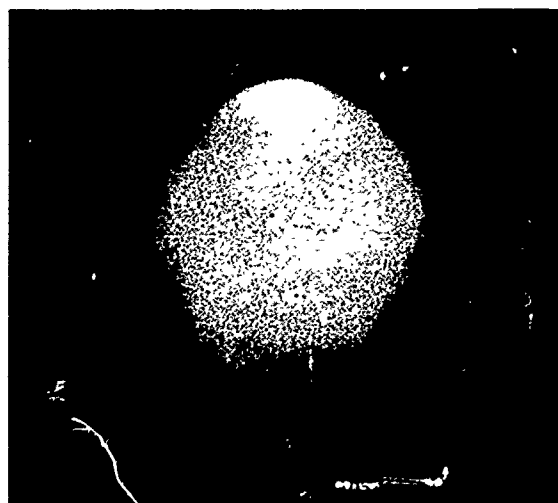
RED

Showing opposite hemispheres



September 11

ORANGE



September 11

BLUE

Haze in the Martian atmosphere obscured surface detail in September

Experiments

EXPERIMENT 18 The Inclination of Mars' Orbit

When you plotted the orbit of Mars in Experiment 17 you ignored the slight movement of the planet above and below the ecliptic. This movement of Mars north and south of the ecliptic shows that the plane of its orbit is slightly inclined to the plane of the earth's orbit. In this experiment you will measure the angular elevation of Mars from the ecliptic and so determine the inclination of its orbit.

Theory

From each of the photographs in the set of 16 you can find the latitude (angle from the ecliptic) of Mars as seen from the earth at a particular point in its orbit. Each of these angles must be converted into an angle as seen from the sun (heliocentric latitude).

Figure 1 shows that we can represent Mars by the head of a pin whose point is stuck into the ecliptic plane. We see Mars from the earth to be north or south of the ecliptic, but we want the N-S angle of Mars as seen from the sun. An example shows how the angles at the sun can be derived.

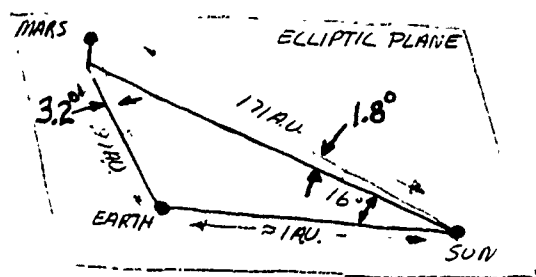


Fig. 1

In plate A (March 21, 1933) in the booklet of photographs Mars is about 3.2° north of the ecliptic as seen from the earth. But the earth was considerably closer to Mars on March 21, 1933 than the sun was. The angular elevation of Mars above the ecliptic plane as seen from the sun will therefore be considerably less than 3.2° .

Measurement on the plot of Mars' orbit (Experiment 17) gives the distance earth-Mars as 9.7 cm (0.97 AU) and the distance sun-Mars as 17.1 cm (1.71 AU) on the date of the photograph. The heliocentric latitude of Mars is therefore

$$\frac{9.7}{17.1} \times 3.2^\circ = 1.8^\circ$$

You can get another value for the heliocentric latitude of this point in Mars' orbit from photograph B (February 5, 1933). The earth was in a different place on this date so the geocentric latitude and the earth-Mars distance will both be different, but the heliocentric latitude should be the same to within your experimental uncertainty.

Making the measurements

With the interpolation scale used in Experiment 17 measure the latitude of each image of Mars. If necessary, place the edge of a card across the 5° latitude marks. Remember that the scale factor is not the same on all the plates.

On your Mars orbit plot from Experiment 17 measure the corresponding earth-Mars and sun-Mars distances. From these values calculate the heliocentric latitudes as explained above. The values of heliocentric latitude calculated from the two plates in each pair (A and B, C and D, etc.) should agree within the limits of your experimental technique.

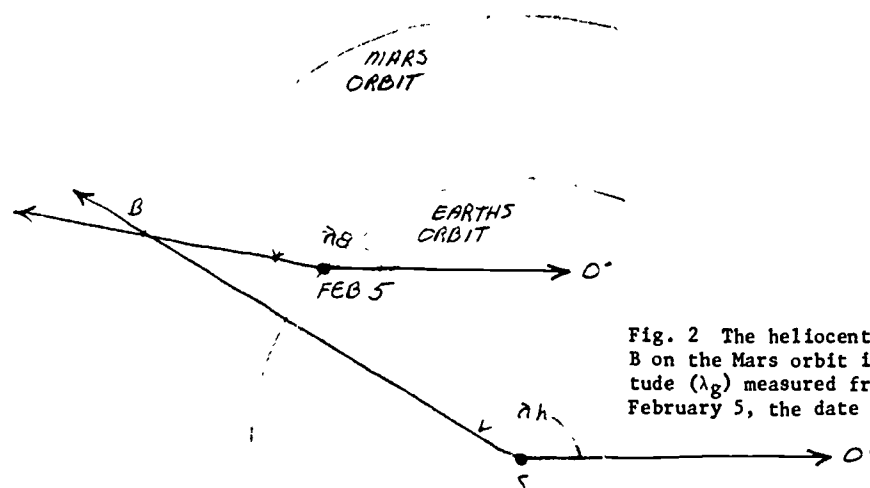


Fig. 2 The heliocentric longitude (λ_h) of Point B on the Mars orbit is 150° ; the geocentric longitude (λ_g) measured from the earth's position on February 5, the date of the photograph, was 169° .

On the plot of Mars' orbit measure the heliocentric longitude λ_h for each of the eight Mars positions. Heliocentric longitude is measured from the sun, counter-clockwise from the 0° direction (direction towards Vernal Equinox), as shown in Fig. 2.

Complete the table given in Experiment 17 by entering the earth-to-Mars and sun-to-Mars distances, the geocentric and heliocentric latitudes, and the geocentric and heliocentric longitudes for all sixteen plates.

Make a graph, like Fig. 3, that shows how the heliocentric latitude of Mars changes with its heliocentric longitude.

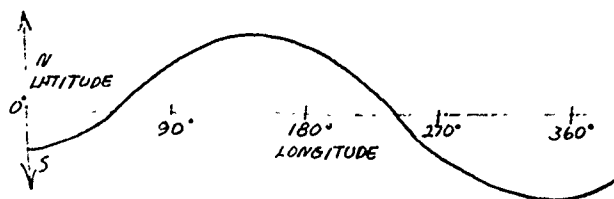


Fig. 3

From this graph you can determine two of the elements that locate the orbit of Mars with respect to the ecliptic. The point at which Mars crosses the ecliptic from south to north is called the ascending node, Ω . (The descending node is the point at which Mars crosses the ecliptic from north to south.) The planet reaches its maximum latitude above the ecliptic 90° beyond the ascending node. This maximum latitude equals the inclination of the orbit i , which is the angle between the plane of the earth's orbit and the plane of Mars' orbit.

Two angles, the longitude of the ascending node, Ω , and the inclination, i , locate the plane of Mars' orbit with respect to the plane of the ecliptic. One more angle is needed to locate the orbit of Mars in its orbital plane. This is the "argument of perihelion" ω and is the angle in the orbit plane between the ascending node and perihelion point. On your plot of Mars' orbit measure the angle from the ascending node Ω to the direction of perihelion to obtain the argument of perihelion, ω .

Experiments

Parameters of an orbit

If you have worked along this far, you have done well. You have determined five of the six elements or parameters that define any orbit:

- a - semi-major axis, or average distance (determines the period)
- e - eccentricity (shape of orbit)
- i - inclination (tilt of orbital plane)
- Ω - longitude of ascending node (where orbital plane crosses ecliptic)
- ω - argument of perihelion (orients the orbit in its plane).

These elements fix the orbital plane of any planet or comet in space, tell

the size and shape of the orbit, and also report its orientation within the orbital plane. To compute a complete timetable, or ephemeris, for the body we need only to know T, a zero date when the body was at a particular place in the orbit. Generally this is given as the date of a perihelion passage. Photograph G was made on September 16, 1939. From this you can estimate a date of perihelion passage for Mars.

The instructions for Experiment 21 tell how to make a three-dimensional model of an orbit. You could use the same procedures to make a model of the orbit of Mars. Cardboard, plastic sheets, or wire could be used.

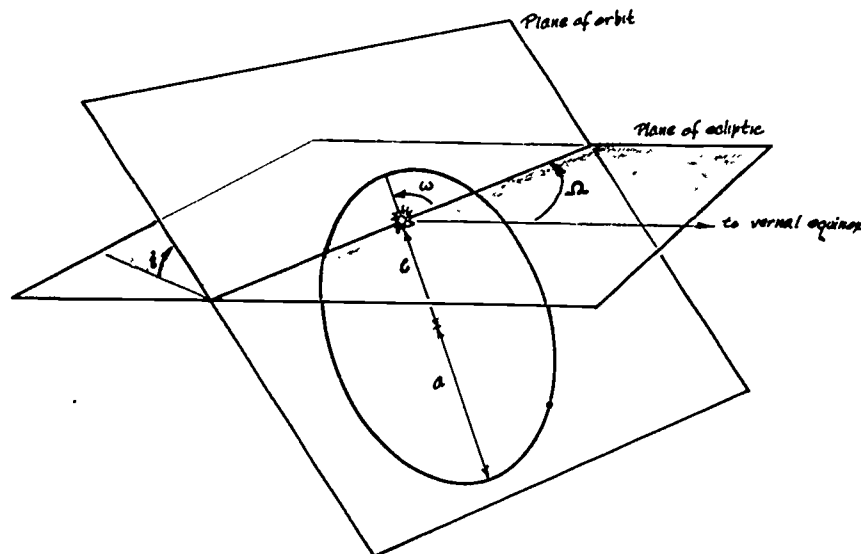


Fig. 4

EXPERIMENT 19 The Orbit of Mercury

Mercury, the innermost planet, is never very far from the sun in the sky. It can only be seen at twilight close to the horizon, just before sunrise or just after sunset, and viewing is made difficult by the glare of the sun. Except for Pluto, which differs in several respects from the other planets, Mercury's orbit is the most eccentric planetary orbit in our solar system ($e = 0.206$). The large eccentricity of Mercury's orbit has been of particular importance, since it has led to one of the tests for the general theory of relativity.

Procedure

Let us assume a heliocentric model for the solar system. Mercury's orbit can be found from Mercury's maximum angles of elongation east and west from the sun as seen from the earth on various known dates.

The angle θ (Fig. 1), measured at the earth between the earth-sun line and the earth-Mercury line, is called the "elongation angle." Note that when θ reaches its maximum value, the elongation sight-lines from the earth are tangent to Mercury's orbit.

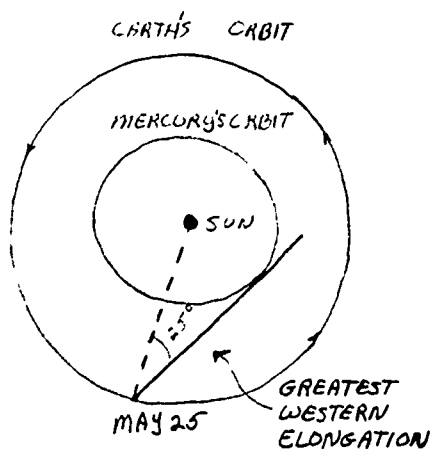
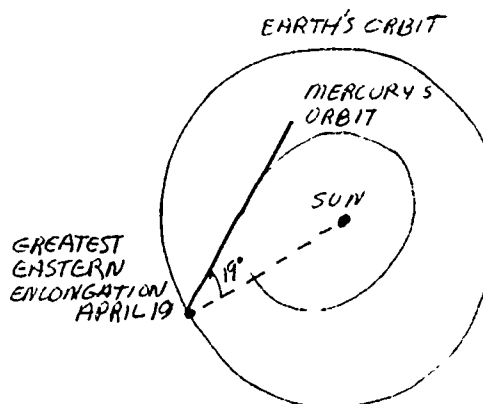


Fig. 1



Since the orbits of Mercury and the earth are both elliptical, the greatest value of θ varies from revolution to revolution. In Fig. 5.5(a) in the text, the 28° elongation angle given for Mercury refers to the maximum possible value of θ for that planet.

Plotting the orbit

Table 1

Some Dates and Angles of Greatest Elongation for Mercury (From the American Ephemeris and Nautical Almanac)

Date	θ
Jan. 4, 1963	19° E
Feb. 14	26° W
Apr. 26	20° E
June 13	23° W
Aug. 24	27° E
Oct. 6	18° W
Dec. 18	20° E
Jan. 27, 1964	25° W
Apr. 8	19° E
May 25	25° W

You can work from the plot of the earth's orbit that you established in Experiment 15. Make sure that the plot you use for this experiment represents the orbit of the earth around the sun, not of the sun around the earth.

If you did not do the earth's orbit experiment, you may use, for an approximate orbit, a circle of 10 cm radius drawn in the center of a sheet of graph paper. Because the eccentricity of the earth's orbit is very small (0.017) you can place the sun at the center of the orbit without introducing a significant error in the experiment.

Experiments

Draw a reference line horizontally from the center of the circle to the right. Label the line 0° . This line points towards the Vernal Equinox and is the reference from which the earth's position in its orbit on different dates can be established. The point where the 0° line from the sun crosses the earth's orbit is the earth's position in its orbit on September 23.

The earth takes 365 days to move once around its orbit (360°). Use the rate of 1° per day, or 30° per month to establish the position of the earth on each of the dates given in Table 1. Remember that the earth moves around this orbit in a counter-clockwise direction, as viewed from the north celestial pole. Draw radial lines from the sun to each of the earth positions you have located.

Now draw sight-lines from the earth's orbit for the elongation angles. Be sure to note from Fig. 1 that for an eastern elongation, Mercury is to the left of the sun as seen from the earth. For a western elongation it is to the right of the sun.

You know that on a date of greatest elongation Mercury is somewhere along the sight line, but you don't know exactly where on the line to place the planet. You also know that the sight line is tangent to the orbit. A reasonable assumption is to put Mercury at the point along the sight line closest to the sun.

You can now find the orbit of Mercury by drawing a smooth curve through, or close to, these points. Remember that the orbit must touch each sight line without crossing any of them.

Calculating the semi-major axis a

To find the size of the semi-major axis of Mercury's orbit, relative to the earth's semi-major axis, you must first find the aphelion and perihelion points

of the orbit. You can use your drawing compass to find the points on the orbit farthest from and closest to the sun.

Measure the size of the orbit along the line perihelion-sun-aphelion. Since 10.0 cm corresponds to one AU (the semi-major axis of the earth's orbit), you can now obtain the semi-major axis of Mercury's orbit in AU's.

Kepler's second law

You can test the equal-area law on your Mercury orbit in the same way that is described in Experiment 17, The Orbit of Mars. By counting squares you can find the area swept out by the radial line from the sun to Mercury between successive dates of observation (e.g., January 4 to February 14, June 13 to August 24). Divide the area by the number of days in the interval to get the "area per day." This should be constant, if Kepler's law holds for your plot. Is it?

Calculating orbital eccentricity

Eccentricity is defined as $e = c/a$ (Fig. 2). Since c , the distance from the center of Mercury's ellipse to the sun, is small on our plot, we lose accuracy if we try to determine e directly.

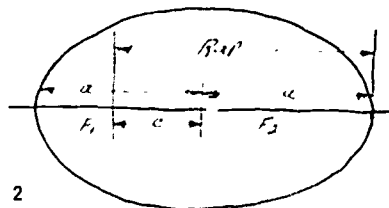


Fig. 2

From Fig. 2, R_{ap} , the aphelion distance, is the sum of a and c .

$$R_{ap} = (a + c);$$

but since $c = ae$,

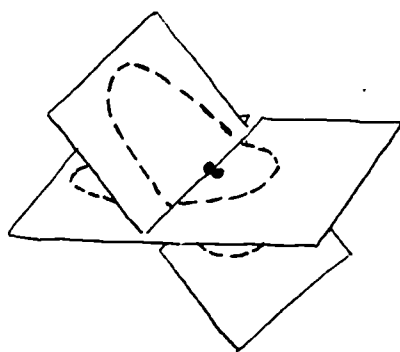
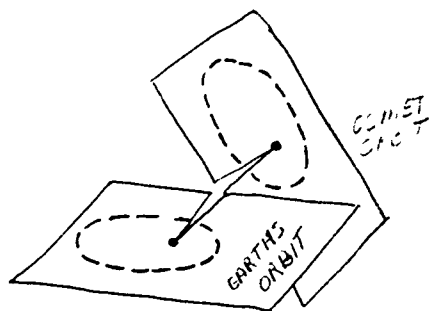
$$R_{ap} = (a + ae) = a(1 + e).$$

Now, solving for e ,

$$e = \frac{R_{ap}}{a} - 1.$$

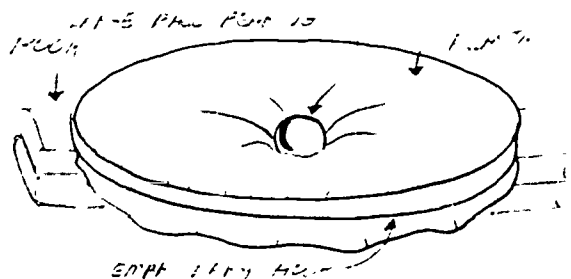
Three Dimensional Model of Two Orbits

You can make a three-dimensional model of two orbits quickly with two small pieces of cardboard (or 3 x 5 cards). On each card draw a circle or ellipse, but have one larger than the other. Mark clearly the position of the focus (sun) on each card. Make a straight cut to the sun, on one card from the left, on the other from the right. Slip the cards together until the sun points coincide. Tilt the two cards (orbit planes) at various angles.



Demonstrating Satellite Orbits

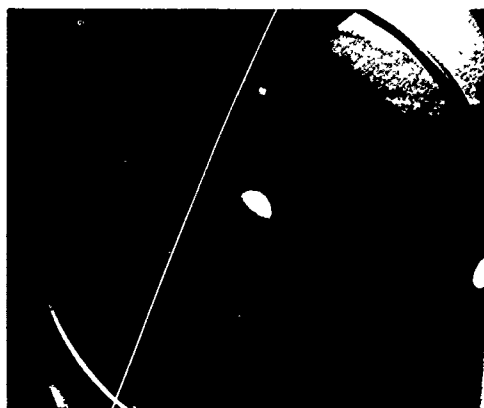
A piece of clear plastic (the 29¢ variety which is sold for storm windows) or a rubber sheet can be stretched tight and clamped in an embroidery hoop about 22" in diameter. Place the hoop on some books and put a heavy ball, for example, a 2"-diameter steel ball bearing, in the middle of the plastic. The plastic will



sag so that there is a greater central force on an object when it is closer to the center than when it is far away. Use small ball bearings, marbles, or beads as "satellites" to roll around in the "gravitational field."

You can use a smaller hoop (about 14") on the stage of an overhead projector. You will have a shadow projection of the large central mass, with the small satellites racing around it. Be careful not to drop the ball through the glass.

If you take electronic strobe photos of the motion, you can check whether Kepler's three laws are satisfied; you can see where satellites travel fastest in their orbit, and how the orbit itself turns in space. To take the picture, set up the hoop on the floor with black paper under it. Place a camera directly over the hoop and the strobe at the side, slightly above the plane of the hoop, but so that the floor under the hoop is not well lighted when the strobe is flashing.



Activities

A ball bearing or marble will make the best pictures. Remember that this is only a crude model.

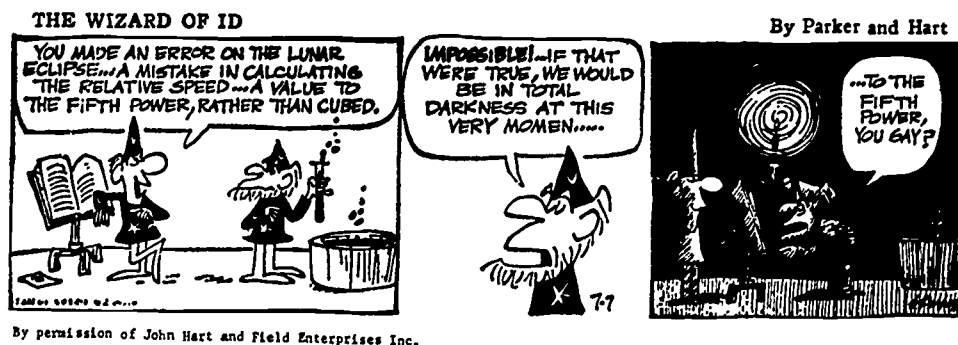
Here are some starter questions to think about:

1. Does our model give a true representation of the gravitational force around the earth? In what ways does it fail (other than suffering from finger-nail holes in the plastic)?
2. Is it much harder to put a satellite into a perfectly circular orbit rather than an elliptical one? What conditions must be satisfied for a circular orbit?
3. Are Kepler's three laws really verified? Should they be?
4. Can you represent the gravitational fields of the planets in the solar system by taping appropriate weights in the positions of each of the planets at some instant of time? You can then roll a ball bearing and see how its path is changed by the gravitational force of each planet.

For additional detail and ideas see "Satellite Orbit Simulator," Scientific American, October, 1958.

Galileo

Read Bertoldt Brecht's play, Galileo, and present a part of it for the class. It was Brecht's last play, and there is some controversy about whether it truly reflects what historians feel were Galileo's feelings. For comparison, you should read The Crime of Galileo, by Giorgio de Santillana; Galileo and the Scientific Revolution, by Laura Fermi; the Galileo Quadricentennial Supplement in Sky and Telescope, February, 1964; or articles in the April, 1966 issue of The Physics Teacher, "Galileo: Antagonist," and "Galileo Galilei: An Outline of His Life."



**EXPERIMENT 20 Stepwise Approximation
To An Orbit**

Your textbook describes how Newton analyzed the motions of the planets, using the concept of a centrally directed force. If you have read the discussion in Sec. 8.4, you are now ready to apply Newton's method to develop an approximate orbit of a satellite or a comet around the sun. You can also, from your orbit, check Kepler's law of areas and other relationships discussed in the text.

Imagine a ball rolling over a smooth, level surface such as a piece of plate glass.

Q1 What would you predict for the path of the ball, based on your knowledge from Unit 1 of Newton's laws of motion?

Q2 Suppose you were to strike the ball from the side. Would the path direction change?

Q3 Would the speed change? Suppose you gave the ball a series of "sideways" blows as it moves along, what do you predict its path might be?

Reread Sec. 8.4 if you have difficulties answering these questions.

A planet or satellite in orbit has a continuous force acting on it. But as the body moves, the magnitude and direction of the force change. To predict exactly the orbit under the application of this constantly changing force requires advanced mathematics. However, you can get a reasonable approximation of the orbit by plotting a series of separate points. In this experiment, therefore, you will assume a series of sharp "blows" acting at 60-day intervals on a moving comet and explore what orbit the body would follow.

The application of repeated steps is known as "iteration." It is a powerful

technique for solving problems. Modern high-speed digital computers use repeated steps to solve complex problems, such as the best path (or paths) for a Mariner probe to follow between earth and Mars.

Make these additional assumptions:

1) The force on the comet is a radial attraction toward the sun.

2) The force of the blow varies inversely with the square of the distance from the sun.

3) The blows occur regularly at the ends of equal time intervals, in this case 60 days. The magnitude of each brief blow has been chosen to equal the total effect of the continuous attraction of the sun throughout a 60-day interval.

The effect of the central force on the comet's motion

From Newton's second law you know that the gravitational force will cause the comet to accelerate towards the sun. If a force \vec{F} acts for a time interval Δt on a body of mass m , we know that

$$\vec{F} = m\vec{a} = m \frac{\Delta\vec{v}}{\Delta t}$$

$$\therefore \vec{F} \Delta t = m \Delta\vec{v}.$$

This equation relates the change in the body's velocity to its mass, the force, and the time for which it acts. The mass m is constant. So is Δt (assumption 3 above). The change in velocity is therefore proportional to the force $\Delta\vec{v} \propto \vec{F}$. But remember that the force is not constant: it varies inversely with the square of the distance from comet to sun.

Q4 Is the force of a blow given to the comet when it is near to the sun greater or smaller than one given when the comet is far from the sun?

Q5 Which blow causes the biggest velocity change?

Experiments

In Fig. 1 the vector \vec{v}_0 represents the comet's velocity at the point A. You want to plot the position of the comet. You must use its initial velocity (\vec{v}_0) to derive its displacement (Δd_0) during the first sixty days: $\Delta d_0 = \vec{v}_0 \times 60$ days. Because the time intervals between blows is always the same (60 days) the displacement along the path is

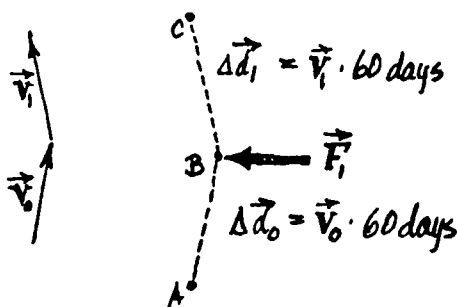


Fig. 1

proportional to the velocity— $\Delta \vec{d} \propto \vec{v}$. We can therefore use a length proportional to the comet's velocity in a given 60-day period to represent its displacement during that time interval.

During the first sixty days, then, the comet moves from A to B (Fig. 1). At B a blow provides a force \vec{F}_1 which causes a velocity change $\Delta \vec{v}_1$. The new velocity after the blow is $\vec{v}_1 = \vec{v}_0 + \Delta \vec{v}_1$, and is found by completing the vector triangle (Fig. 2).

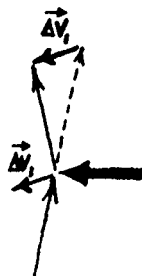


Fig. 2

The comet therefore leaves point B with velocity \vec{v}_1 and continues to move with this velocity for another 60-day interval. The displacement $\Delta \vec{d}_1 = \vec{v}_1 \times 60$ days establishes the next point, C, on the orbit.

The scale of the plot

The shape of the orbit depends on the initial position and velocity, and on the force acting. Assume that the comet is first spotted at a distance of 4 AU's from the sun. Its velocity at this point is $\vec{v} = 2$ AU per year (about 20,000 miles per hour) at right angles to the sun-comet radius.

The following scale factors will reduce the orbit to a scale that fits conveniently on a 20" x 20" piece of graph paper.

1. Let 1 AU be scaled to 2.5 inches, so 4 AU becomes 10 inches (SA, in Fig. 3).
2. Since the comet is hit every 60 days, it is convenient to express the velocity in AU's per 60 days. We will adopt a scale factor in which a velocity of 1 AU/60 days is represented by a vector 2.5 inches long.

The comet's initial velocity of 2 AU per year can be given as $\frac{2}{365}$ AU per day, or $\frac{2}{365} \times 60 = 0.33$ AU per 60 days. This scales to a vector 0.83 inches long.

The displacement of the comet in the first 60 days ($\Delta d_0 = \vec{v}_0 \times 60$) is $(0.33 \text{ AU}/60 \text{ days}) \times 60 \text{ days} = 0.33 \text{ AU}$. This displacement scales to 0.83 inches.

Notice the AB is perpendicular to SA, the line from the sun to A (Fig. 3).

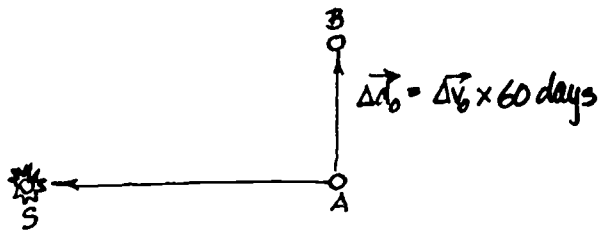


Fig. 3
Computing Δv

On the scale and with the 60-day iteration interval that we have chosen the force field of the sun is such that the Δv given by a blow when the comet is 1 AU from the sun is 1 AU/60 days.

Q7 What will the Δv be at a distance of 2 AU from the sun?

Values of Δv for other distances from the sun which have been calculated according to the inverse-square law are given in Table 1.

Table 1

R, from sun		Δv	
AU	Inches	AU/60 days	Inches
0.75	1.87	1.76	4.44
0.8	2.00	1.57	3.92
0.9	2.25	1.23	3.07
1.0	2.50	1.00	2.50
1.2	3.0	0.69	1.74
1.5	3.75	0.44	1.11
2.0	5.0	0.25	0.62
2.5	6.25	0.16	0.40
3.0	7.50	0.11	0.28
3.5	8.75	0.08	0.20
4.0	10.00	0.06	0.16

To avoid arithmetic computation, plot the above data on 8" x 10" graph paper. Plot Δv (in inches) against sun distance R (in inches), with the sun at the origin (see Fig. 4). Carefully connect the points with a smooth curve. You can make this curve into a simple graphical computer. Cut off the bottom margin of the graph paper, or fold it under along the R axis. Lay this computer on the orbit plot and adjust its position until the sun points on both sheets coincide. Rotate this computer around the sun point so that the horizontal (R) axis passes through the point where the blow is applied (e.g., point B). Read off the value of R at B. Pick off the value of Δv corresponding to this R from the computer with dividers. Lay off this distance (Δv) inwards along the radius line towards the sun (see Fig. 5 on next page).

Making the plot

1. Mark the position of the sun S half-way up the large graph paper and 12 inches from the right edge.
2. Locate a point 10 inches (4 AU) to the right from the sun S. This is point A where we find the comet.
3. Draw vector \vec{AB} 0.83 inches (0.33 AU) long through point A, perpendicular to SA. This vector represents the comet's

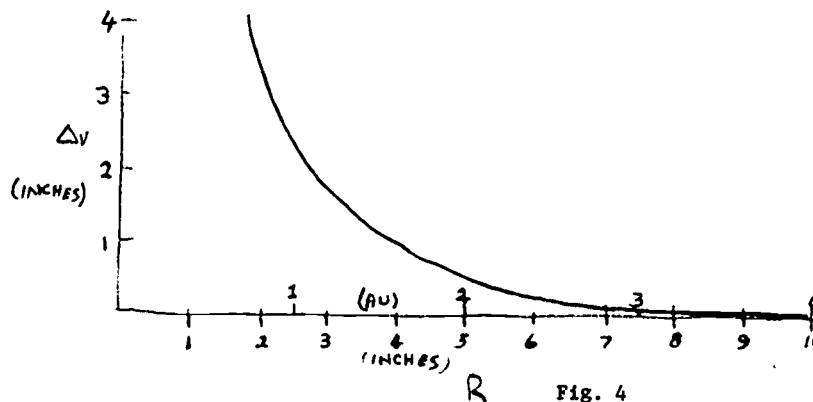


Fig. 4

Experiments

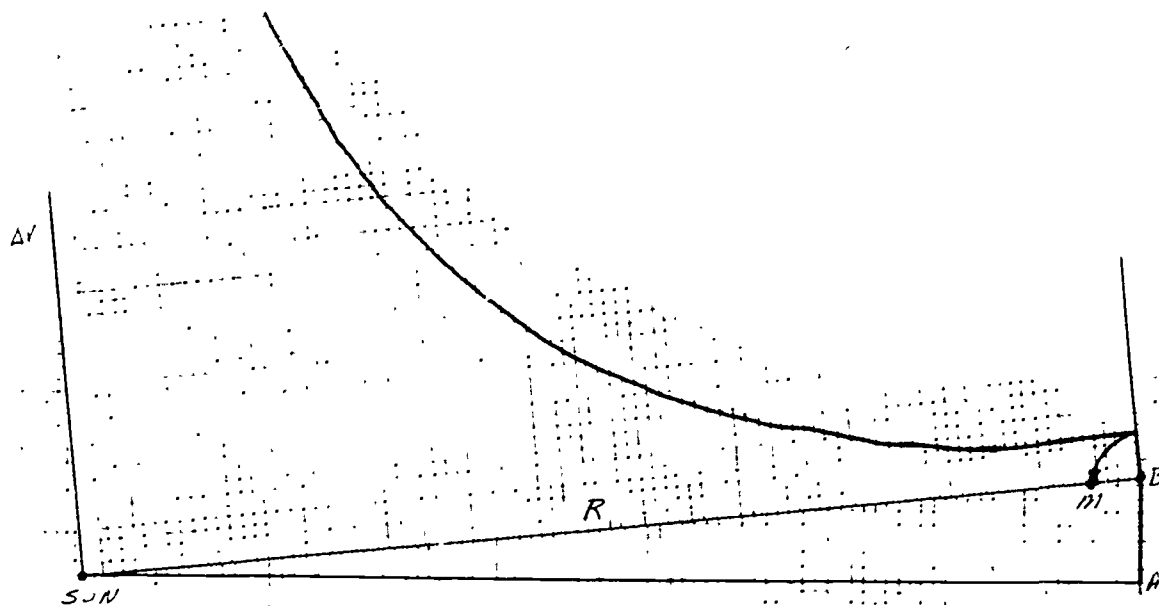


Fig. 5

velocity (0.33 AU/60 days), and B is its position at the end of the first 60-day interval. At B a blow is struck which causes a change in velocity $\Delta\vec{v}_1$.

4. Use your Δv computer to establish the distance of B from the sun at S, and to find $\Delta\vec{v}_1$ for this distance (Fig. 5).

5. The force, and therefore the change in velocity, is always directed towards the sun. From B lay off $\Delta\vec{v}_1$ towards S. Call the end of this short line M.

6. Draw the line BC' which is a continuation of AB and has the same length as AB.

7. To find the new velocity \vec{v}_1 use a straightedge and triangle to draw the line C'C parallel to BM and of equal length. The line BC represents the new velocity vector \vec{v}_1 , the velocity with which the comet leaves point B (Fig. 6).

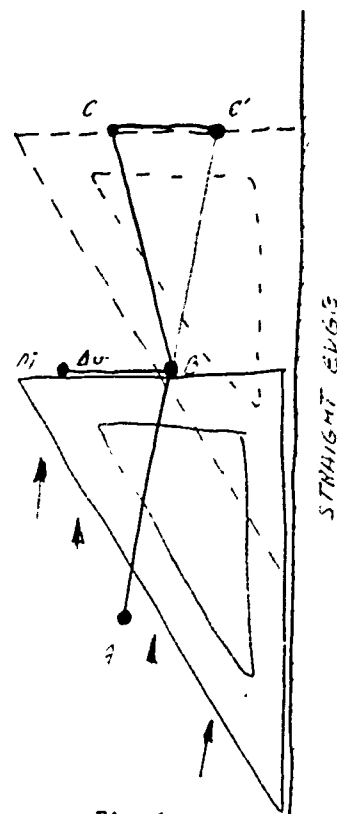


Fig. 6

AM is actually the new velocity from B to C.

8. Again the comet moves with uniform velocity for 60 days. Its displacement in that time is $\Delta d_1 = \vec{v}_1 \times 60 \text{ days}$ and because of the scale factor we have chosen, the displacement is represented by the line BC. C is therefore a point on the comet's orbit.

9. Repeat steps 1 through 8 to establish point D and so forth, for 14 or 15 steps (25 steps gives the complete orbit).

10. Connect points A, B, C... with a smooth curve. Your plot is finished.

Prepare for discussion

Since you derived the orbit of this comet, you may name the comet.

From your plot, find the perihelion distance.

Q8 What is the length of the semi-major axis of the ellipse?

Q9 Find the center of the orbit and calculate the eccentricity.

Q10 What is the period of revolution of your comet? (Refer to text, Sec. 7.3.)

Q11 How does the comet's speed change with its distance from the sun?

If you have worked this far, you have learned a great deal about the motion of this comet. It is interesting to go on to see how well the orbit obtained by iteration obeys Kepler's laws.

Q12 Is Kepler's first law confirmed? (Can you think of a way to test your curve to see how nearly it is an ellipse?)

The time interval between blows is 60 days, so the comet is at positions B, C, D..., etc., after equal time intervals. Draw a line from the sun to each of these points (include A), and you have a set of triangles.

Find the area of each triangle. The area of a triangle is given by $A = \frac{1}{2}ab$ where a and b are altitude and base, respectively. Or you can count squares to find the areas.

Q13 Is Kepler's second law (the Law of Equal Areas) confirmed?

More things to do

1. The graphical technique you have practiced can be used for many problems. You can use it to find out what happens if different initial speeds and/or directions are used. You may wish to use the $1/R^2/\text{force}$ computer, or you may construct a new computer, using a different law (e.g., force proportional to $1/R^3$, or to $2/R$ or to R) to produce different paths; actual gravitational forces are not represented by such force laws of course.

2. If you use the same force computer (graph) but reverse the direction of the force (repulsion), you can examine how bodies move under such a force. Do you know of the existence of any such repulsive forces?

Based on similar experiment developed by Leo Lavatelli
Am. J. Phys. 33, 605, 1967.

Experiments

EXPERIMENT 21 Model of a Comet Orbit

The complete orbit of a comet can be derived from only three observations of its position, but this is quite an intricate process. Here we reverse the problem and make a three dimensional model of the orbit from the six elements that describe the comet's orbit.

From this model, you will be able to construct at least a rough timetable (ephemeris) for the apparent positions of the comet and can check these against reported observations. Halley's comet has been considered several times in the text and its orbit has several interesting features. When you have constructed the model you can compare it to the plot of observations across the sky during its last return, Fig. 6.10, p. 43 of your text.

The elements of a comet's orbit

Six elements are needed to describe a comet's orbit. Three of these—semi-major axis, eccentricity and perihelion date—are already familiar from planetary orbits.

The orbits of the earth and the other planets are all in or very close to the same plane—the plane of the ecliptic. (If you did Experiment 18, you will remember that Mars' orbit is inclined at about 1.8 degrees to the ecliptic.) But this is not so for comets. The orbit of a comet can be inclined to the ecliptic plane at any angle. Three more elements—inclination, longitude of ascending node and angle from node to perihelion are needed to describe the inclination. These elements are illustrated in Fig. 1.

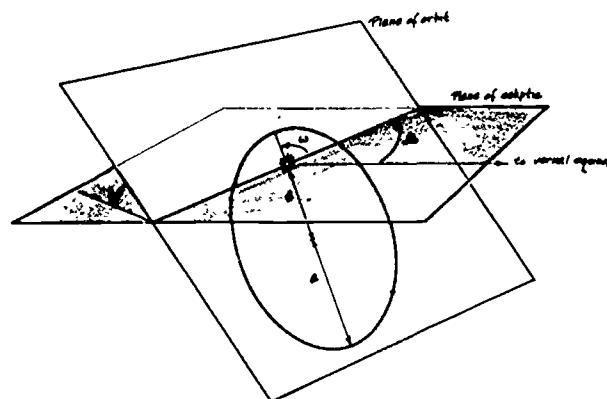


Fig. 1

The elements of Halley's comet are approximately:

a (semi-major axis)	17.9 AU
e (eccentricity)	0.967
i (inclination)	162°
Ω (longitude of ascending node)	57°
ω (angle from ascending node to perihelion)	112°
T (perihelion date)	April 20, 1910

From these data we also know that period $P = 76$ years, and perihelion distance $q = a(1-e) = 0.59$ AU.

Plotting the orbit

In the center of a large sheet of stiff cardboard draw a circle 10 cm in radius for the orbit of the earth. Also draw approximate (circular) orbits for Mercury (radius 0.4 AU) and Venus (radius 0.7 AU). For this plot, you can consider all of the planets to lie roughly in the one plane. Draw a line from the center, the sun, and mark this line as 0° longitude. Now, as in Experiments 15 and 17, you can establish the position of the earth on any date. You will need these positions of the earth later in the experiment.

Use another large sheet of stiff cardboard to represent the orbital plane of the comet. Down the middle draw a line for the major axis of the orbit. Choose a point for the sun on this line about 15 cm from one edge of the sheet.

For this experiment, we can consider the orbit of Halley's comet as being essentially a parabola. In fact the small near-sun section of the large ellipse does have almost exactly that shape. Now you want to construct a parabola.

You have an orbital plane with the major axis drawn and the position of the sun marked. Use the same scale as for the earth's orbit (1 AU equals 10 cm), and mark a point on the major axis at a distance q from the sun. This is the perihelion point, one point on the orbit. The orbit will be symmetrical around the major axis and will flare out and away from the perihelion point (see Fig. 2).

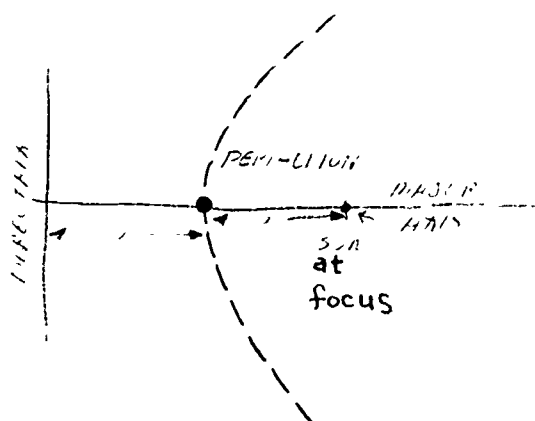


Fig. 2

Mark another point on the major axis at a distance q beyond the perihelion point, or at a distance $2q$ from the sun. Draw a line perpendicular to the axis at this point: this construction line is known in analytical geometry as the "directrix." A parabola has the property that each point on it is equidistant

from a straight line (the directrix) and from a fixed point (the focus).

Here is one way to draw a parabola: if you know another, try it. Draw a line parallel to the directrix and at a distance R from it. Use a drawing compass centered at the focus to swing two arcs of radius R , one above and one below the major axis.

The intersections of the arcs and the line are two points on the parabola. Repeat the process with arcs of different sizes to locate more points on the parabola.

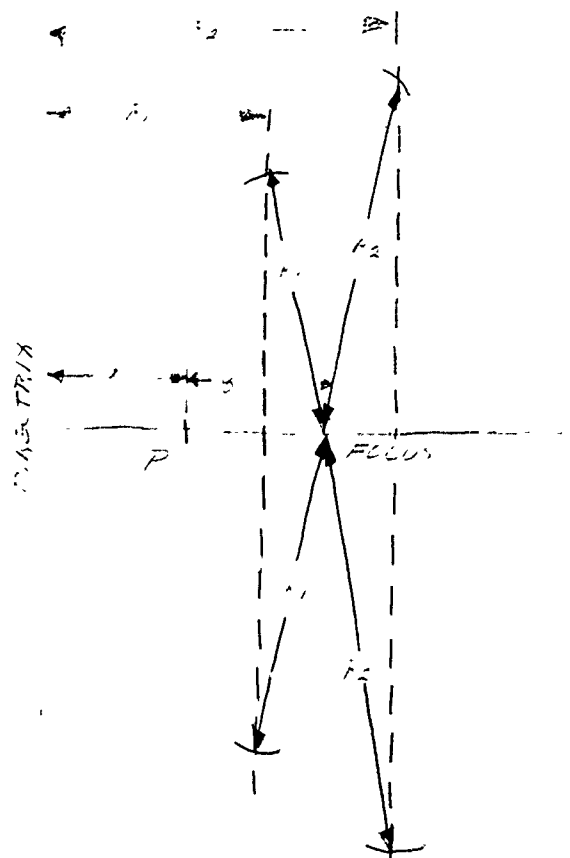


Fig. 3

Determine a sufficient number of points so that a smooth curve can be drawn through them. This curve is the comet's (parabolic) curve.

Experiments

Now we have the two orbits, the comet's and the earth's, in their planes, each of which contains the sun. You need only to fit the two together.

The line along which the orbital plane cuts the ecliptic plane is called the "line of nodes." Since you have the major axis drawn, you can locate the ascending node, in the orbital plane, by measuring ω from perihelion in a direction opposite to the comet's motion (see Fig. 4).

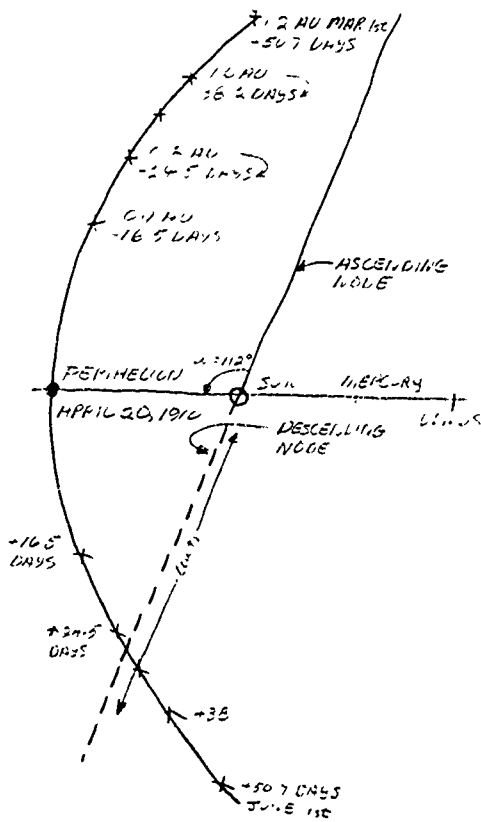


Fig. 4

To fit the two orbits together, cut a narrow slit in the ecliptic plane (earth's orbit) along the line of the ascending node in as far as the sun. Then slit the comet's orbital plane on the side of the descending node in as far as the sun (see Figs. 4 and 5). Slip these two together until the sun-points on the two planes come together.

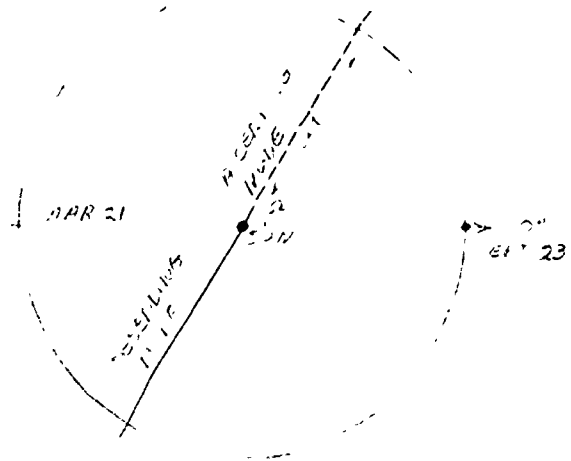


Fig. 5

To establish the model in three dimensions you must now fit the two planes together at the correct angle. Remember that the inclination i is measured upward (northward) from the ecliptic from the longitude $\Omega + 90^\circ$ (see Fig. 1).

You can construct a small tab to support the orbital plane in the correct position. In the ecliptic plane draw a line in the direction $\Omega + 90^\circ$. From this line measure off the angle of inclination i towards the descending node.

If the inclination is less than 90° , draw a line from the sun at the angle of inclination. Then with a razor blade cut a section of the tab as shown in the sketch (Fig. 6). The model will be

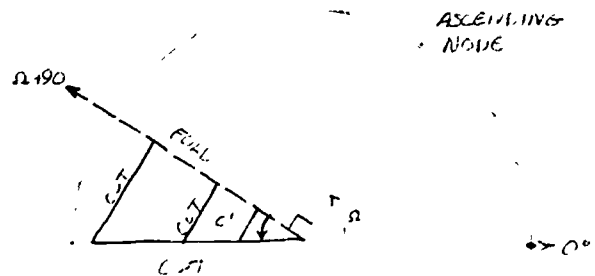


Fig. 6

stronger if you do not cut the triangular wedge all the way into the sun's position. Fold the tab up vertically along the line ($\Omega + 90^\circ$) and tape it into place.

If the inclination exceeds 90° , as it does for Halley's Comet, cut the tab and bend it up along the line $\Omega + 270^\circ$ (see Fig. 7).

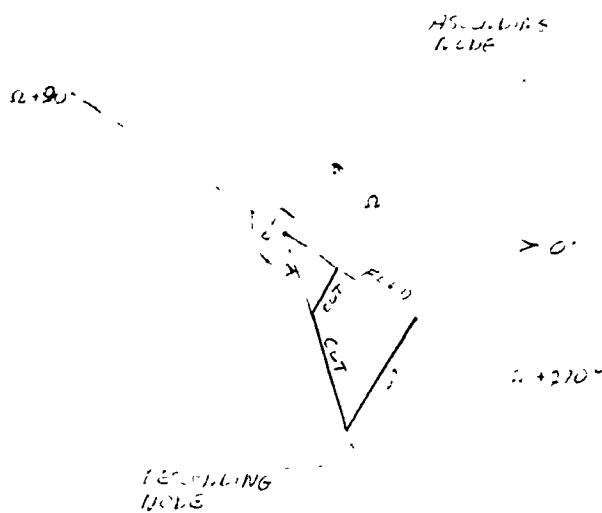


Fig. 7

When you fit the two planes together you will find that the comet's orbit is on the underside of the cardboard. The simplest way to transfer it to the top is to prick through with a pin at enough points to draw a smooth curve.

Finally, you can develop the timetable, or ephemeris, of the comet in its parabolic orbit. Because all parabolas have the same shape and eccentricity ($e = 1$) this calculation is simple. The time t (days) required for a body to move from a solar distance r (AU) to perihelion is given by:

$$t = 27.4(r + 2q)(r - q)^{\frac{1}{2}}$$

where q is the perihelion distance.

Times for certain values of r and q are given in Table 1. If plotted, these data produce an interesting set of curves.

Table 1

Time (days) to perihelion passage from different solar distances (r) for parabolic orbits of different perihelion distances (q)

(From Between the Planets,
F.G.Watson, pp. 215-216)

Solar dist.	Perihelion distance, q (AU)						
r (AU)	0.0	0.2	0.4	0.6*	0.8	1.0	1.2
2.0	77.5	88.1	97.1	103.8	108.0	109.6	107.8
1.8	66.1	76.2	84.3	90.0	93.2	93.0	88.6
1.6	56.1	64.8	72.0	76.7	78.2	76.0	69.4
1.4	45.4	54.0	60.3	63.6	63.4	59.0	46.6
1.2	36.0	43.9	48.9	50.7	48.5	38.0	0.0
1.0	27.4	34.3	38.0	38.2	31.9	0.0	
0.8	19.6	25.4	27.8	24.5	0.0		
0.7	16.1	21.3	22.5	16.5			
0.6	12.8	17.3	17.2	0.0			
0.5	9.7	13.5	11.3				
0.4	6.9	9.8	0.0				
0.3	4.5	6.1					
0.2	2.5	0.0					
0.1	0.9						

*This column was used in determining dates on Fig. 4.

Since the date of perihelion passage of Halley's Comet T (April 20, 1910) is given, you can use this table to find the dates at which the comet was at various solar distances before and after perihelion passage. Mark these dates along the orbit.

Finally locate the earth's position in its orbit for each of these dates.

For each date make a sightline from the earth to the comet by stretching a thread between the two points.

You can make a timetable for the apparent positions of the comet in the sky by measuring the longitude of the comet around the ecliptic plane as seen from the earth on each date. With a protractor estimate the latitude of the comet (its angular height from the ecliptic).

Plot these points on a star map having ecliptic coordinates or plot them roughly relative to the ecliptic on a map having equatorial coordinates, such as the constellation chart SC-1.

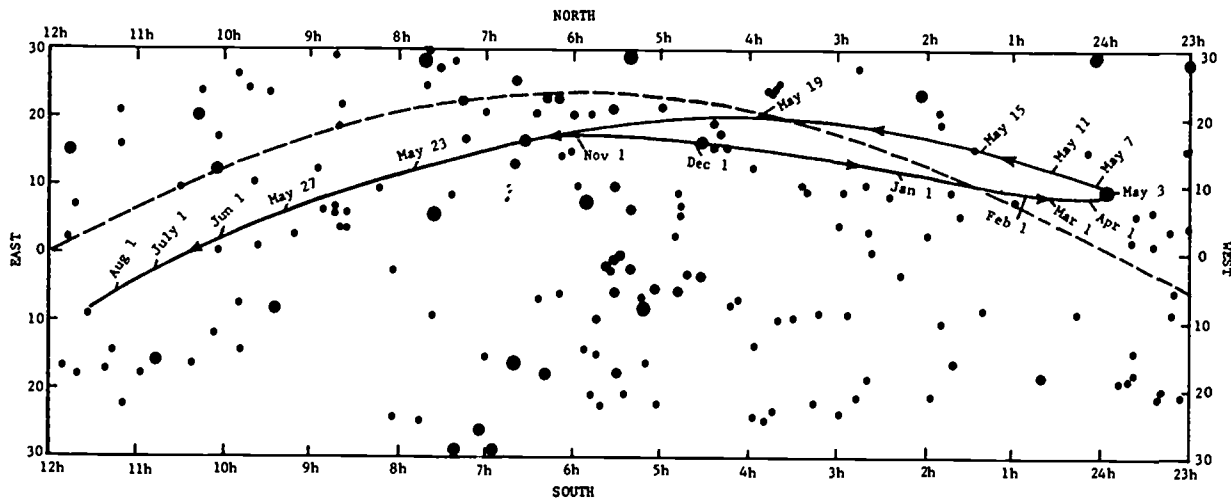


Fig. 6.10 Motion of Halley's Comet in 1909-10.

If you have persevered this far, and your model is a fairly accurate one, it should be easy to explain the comet's motion through the sky shown in Fig. 6.10. The dotted line in the figure is the ecliptic.

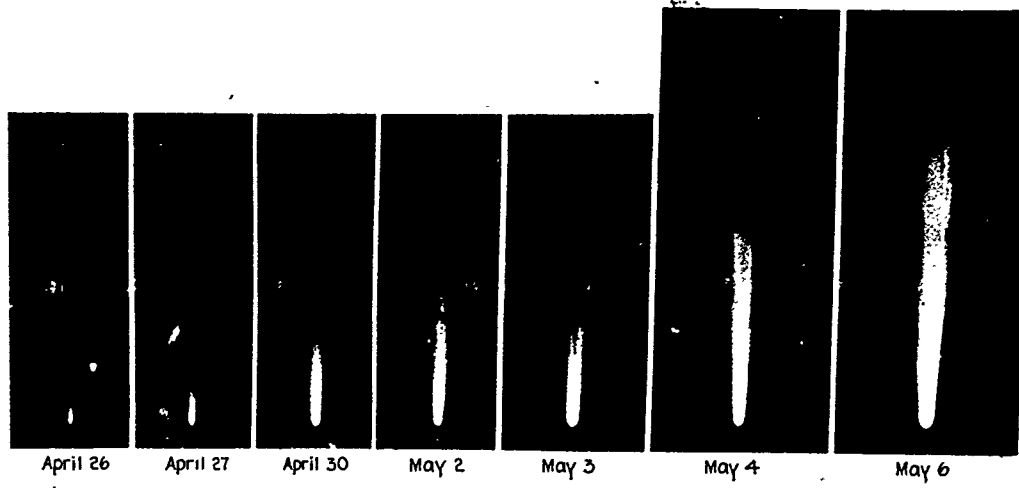
With your model of the comet orbit you can now answer some very puzzling questions about the behavior of Halley's Comet in 1910, as shown in Fig. 6.10.

1. Why did the comet appear to move westward for many months?
2. How could the comet hold nearly a stationary place in the sky during the month of April 1910?
3. After remaining nearly stationary for a month, how did the comet move nearly halfway across the sky during the month of May 1910?
4. What was the position of the comet in space relative to the earth on May 19th?
5. If the comet's tail was many millions of miles long on May 19th, is it likely that the earth passed through part of the tail?
6. Were people worried about the effect a comet's tail might have on life on the earth? (See newspapers and magazines of 1910!)

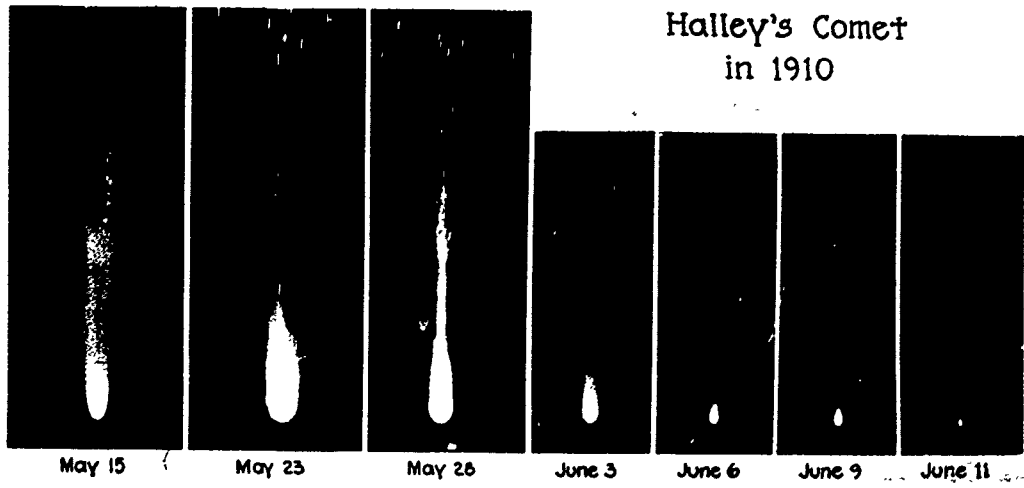
7. Did anything unusual happen? How dense is the material in a comet's tail? Would you expect anything to have happened?



M. BABINET PREVENU PAR SA PORTIERE
DE LA VISITE DE LA COMETE
Lithograph. Honore Daumier, French, 1808-1879
MUSEUM OF FINE ARTS, BOSTON



Halley's Comet
in 1910



Activities

Forces on a Pendulum

If a pendulum is drawn aside and released with a small sideways push, it will move in an almost elliptical path. This looks vaguely like the motion of a planet about the sun, but there are some differences.

To investigate the shape of the orbit and see whether the motion follows the law of areas, you can make a strobe photo as shown by the setup in Fig. 1. Use

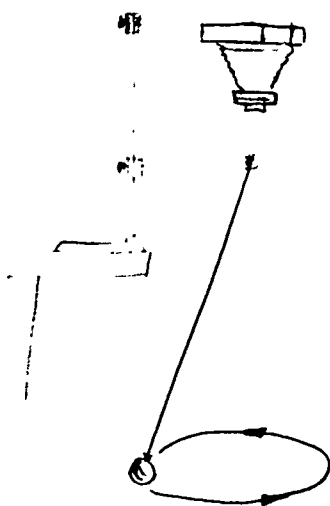


Fig. 1

either an electronic strobe flashing from the side, or use a small light and AA cell on the pendulum and a motor strobe disc in front of the lens. If you put tape over one slot so it is half as wide as the rest, it will make every 12th dot fainter (if 12 slits are open), giving a handy time marker, as shown in Fig. 2. You can also set the camera on its back on the floor with the motor strobe above it, and suspend the pendulum overhead.

Are the motions and the forces really the same for the pendulum and the planets? The center of force for planets is located at one focus of the ellipse. Recall your experiences in Unit 1 and decide where the center of force is for the pendulum. Measure your photos to determine whether



Fig. 2

the pendulum bob follows the law of areas relative to the center of force.

In the case of the planets the force varies inversely with the square of the distance between the sun and the planet. From the sketch in Fig. 1 conclude how the restoring force on the pendulum changes with distance R from the rest point. If you have a copy of Newton's Principia available, read Proposition X.

Haiku

If you are of a literary turn of mind, try your hand at using Japanese haiku, a form of poetry, to summarize what you have learned so far in physics. The rules are quite simple: a haiku must have three lines, the first and third having five syllables and the second having seven syllables. No rhyming is necessary. Several student haiku (the plural is pronounced and spelled the same as the singular) are given below:

Maze of eccentrics,
Scientists, telescope armed,
Ignored, machines die.

An epicycle
Those most complicated things
Were rid by Kepler.

The orbit of Mars
Was a great task to achieve
From that of the earth.

Kepler took star dates,
The shining stars in the sky,
Physics came this way.

Other Comet Orbits

If you enjoyed Experiment 21 on the orbit of Halley's Comet, you may want to make models of some other comet orbits. Data are given below for several others of interest.

Encke's comet is interesting because it has the shortest period known for a comet, only 3.3 years. In many ways it is representative of all short-period comet orbits. All have orbits of low inclination and pass near the orbit of Jupiter, where they are often strongly deviated. The full ellipse can be drawn at the scale of 10 cm for 1 AU. The orbital elements for Encke's comet are:

$$\begin{aligned} a &= 2.22 \text{ AU} \\ e &= 0.85 \\ i &= 15^\circ \\ \Omega &= 335^\circ \\ \omega &= 185^\circ \end{aligned}$$

From these data we know that $P = 3.3$ years and $q = 0.33$ AU (q' , the aphelion distance, is 4.11 AU).

The comet of 1680 is discussed extensively in Newton's *Principia*, where approximate orbital elements are given. The best parabolic orbital elements known are:

$$\begin{aligned} T &= \text{Dec. 18, 1680} \\ \omega &= 350.7^\circ \\ \Omega &= 272.2^\circ \\ i &= 60.16^\circ \\ q &= 0.00626 \text{ AU} \end{aligned}$$

Note that this comet passed very close to the sun. At perihelion it must have been exposed to intense destructive forces like the comet of 1965.

Comet Candy (1960N) had the following parabolic orbital elements:

$$\begin{aligned} T &= \text{Feb. 8, 1961} \\ \omega &= 136.3^\circ \\ \Omega &= 176.6^\circ \\ i &= 150.9^\circ \\ q &= 1.06 \text{ AU} \end{aligned}$$

During its appearance in 1960-61 it moved from Leo, up through Ursa Major (the Big Dipper), then around west of Cassiopeia southward through Pegasus into Sculptor, Grus, Indus and Telescopium in the southern sky.

Film Loops

FILM STRIP—Retrograde Motion of Mars

You should view this film strip before viewing Film Loop 10a on retrograde motions. The film loop is done by animation, but the film strip shows actual photographs of the night sky.

Photographs of the positions of Mars, from the files of the Harvard College Observatory, are shown for three oppositions of Mars, in 1941, 1943, and 1946.

The first series of twelve frames shows the positions of Mars before and after the opposition of October 10, 1941. The series begins with a photograph on August 3, 1941 and ends with one on December 6, 1941.

The second series shows positions of Mars before and after the opposition of December 5, 1943, beginning October 28 and ending February 19, 1944.

The third set of eleven pictures, showing Mars during 1945-46 around the opposition of January 14, 1946, begins with October 16, 1945 and ends with February 23, 1946.

Uses:

- a) The star fields for each series of frames have been carefully positioned so that the background star positions are nearly identical in each frame. If you flick the frames of each series through the projector in rapid succession, the stars will be seen as stationary on the screen, while the motion of Mars among the stars is quite apparent.
- b) You can project the frames on a paper screen and mark the positions of various stars and Mars. If you adjust the star positions for each frame to match the positions of the previous frame, the positions of Mars can be marked for the various dates. By drawing a continuous line through

these points, you will get a plot like that shown in Fig. 5.7 of the Unit 2 text. By estimating the dates for the beginning and end of the retrograde motion, you can determine the duration of the retrograde motion. With the scale of 10° shown in one frame, you can also find the angular size of the retrograde loop.

Jupiter also appears in the frames for 1943-44 and 1945-46 and shows part of its retrograde motion. You can plot the positions for Jupiter, and estimate the duration and size of its retrograde loop. Average values are listed in Table 5.1 of the text for comparison.

The opposition dates for Jupiter were:

1941 - Dec. 8	1944 - Feb. 11
1942 - None	1945 - Mar. 13
1943 - Jan. 11	1946 - Apr. 13

FILM LOOP 10a Retrograde Motion of Mercury and Mars, shown by Animation

This film shows the retrograde motions of Mercury and Mars which are pictured in Fig. 5.7 of the text. The animation shows a background of fixed stars. Although no star except the sun is close enough for us to see as a visible disk even with the largest telescopes, to show the differences in magnitude small disks are used whose sizes are related to the brightness of the stars. (The same is true in Fig. 5.7.)

1. Motion of Mercury and Sun starting April 16, 1963, with time markers at 5-day intervals. The field of view includes portions of the constellations of Aries and Taurus; the familiar group of stars called the Pleiades cluster is at the upper left of the picture. The sun's motion is steadily eastward (to the left) due to the earth's orbital motion. During

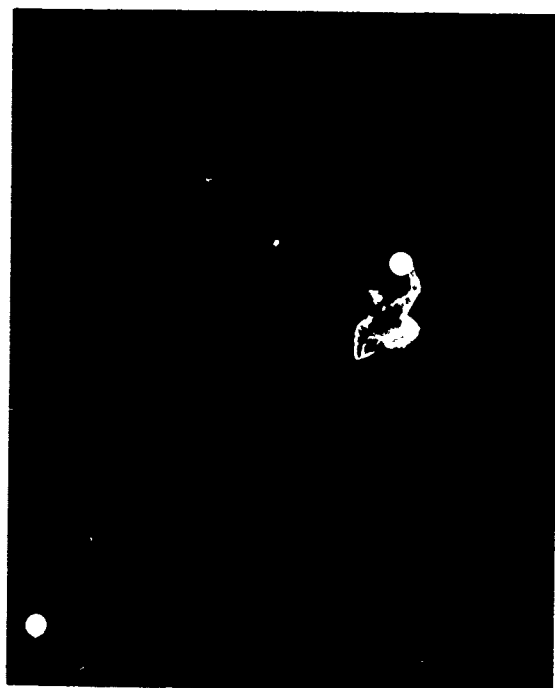
its retrograde motion, Mercury passes between the earth and the sun (inferior conjunction).

2. Motion of Mars starting October 17, 1962, with time markers at 10-day intervals. The retrograde motion occurs around the time when Mars passes through opposition. The field of view includes parts of the constellations Leo and Cancer; the cluster at the upper right is Praesepe (the Beehive), faintly visible to the naked eye on a moonless night.

You can use the time markers to determine the approximate date of the opposition of Mars (center of the retrograde portion).

FILM LOOP 10 Retrograde Motion—Geocentric Model

Using a specially-constructed large "epicycle machine" as a model of the Ptolemaic system, the film shows the motion around the earth of a planet such as Mars.



First we see the motion from above, with the characteristic retrograde motion during the "loop" when the planet is closest to the earth. It was to explain this loop that Ptolemy devised the epicycle system. When the studio lights go up, we see how the motion is created by the combination of two circular motions.

The earth is then replaced by a camera which points in a fixed direction in space from the center of the machine. (This means that we are ignoring the rotation of the earth on its axis, and are concentrating on the motion of the planet relative to the fixed stars.) For an observer viewing the stars and planets from a stationary earth, this would be equivalent to looking always toward one constellation of the zodiac (ecliptic); for instance, toward Sagittarius or toward Taurus. With the camera located at the center of motion the planet, represented by a white globe, is seen along the plane of motion. A planet's retrograde motion does not always occur at the same place in the sky, so not all retrograde motions are visible in any chosen direction.

Several examples of retrograde motion are shown. In interpreting these scenes, imagine that you are facing south, looking upward toward the selected constellation. East is on your left, and west is on your right. The direct motion of the planet, relative to the fixed stars, is eastward, i.e., toward the left. First we see a retrograde motion which occurs at the selected direction (this is the direction in which the camera points). Then we see a series of three retrograde motions; smaller bulbs and slower speeds are used to simulate the effect of viewing from greater distances.

Film Loops

Note the changes in apparent brightness and angular size of the globe as it sweeps close to the camera. While the actual planets show no disk to the unaided eye and appear as points of light, certainly a marked change in brightness would be expected. This was, however, not considered in the Ptolemaic system, which focussed only upon the timetable of the angular motions and positions in the sky.

B.C.

by Johnny Hart



By permission of John Hart and Field Enterprises Inc.

**FILM LOOP 11 Retrograde Motion -
Heliocentric Model**

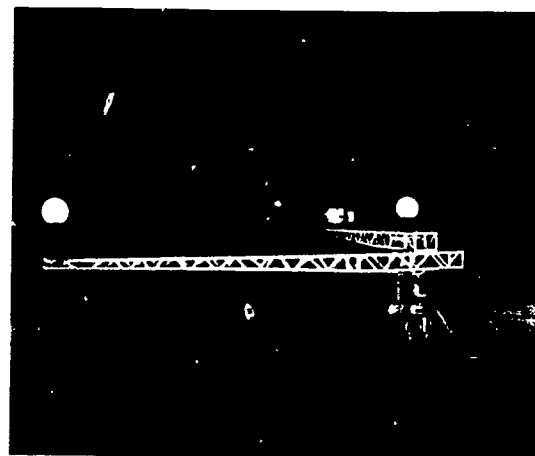
The machine used in Loop 10 was reassembled to give a heliocentric model with the earth and the planet moving in concentric circles around the sun. The earth (represented by a light blue globe) is seen to pass inside a slower moving outer planet such as Mars (represented by a white globe). The sun is represented by a yellow globe.

Then the earth is replaced by a camera, having a field about 25° wide, which points in a fixed direction in space. The arrow attached to the camera shows this fixed direction. (As in Loop 10, we are ignoring the daily rotation of the earth on its axis and are concentrating on the motion of the planet relative to the sun and the fixed stars.)

Several scenes are shown. Each scene is viewed first from above, then viewed along the plane of motion. Retrograde motion occurs whenever Mars is in opposition; this means that Mars is opposite the sun as viewed from the earth. But not all these oppositions take place when Mars is in the sector toward which the camera points.

1. Mars is in opposition; retrograde motion takes place.
2. The time between oppositions averages about 2.1 years. The film shows that the earth moves about 2.1 times around its orbit (2.1 years) between one opposition and the next one. You can, if you wish, calculate this value, using the length of the year (sidereal period) which is 365 days for the earth and 687 days for Mars. This is the "chase problem" discussed on page 31 of Unit 2.

In one day the earth moves $1/365$ of 360° , Mars moves $1/687$ of 360° , and the motion of the earth relative to Mars is



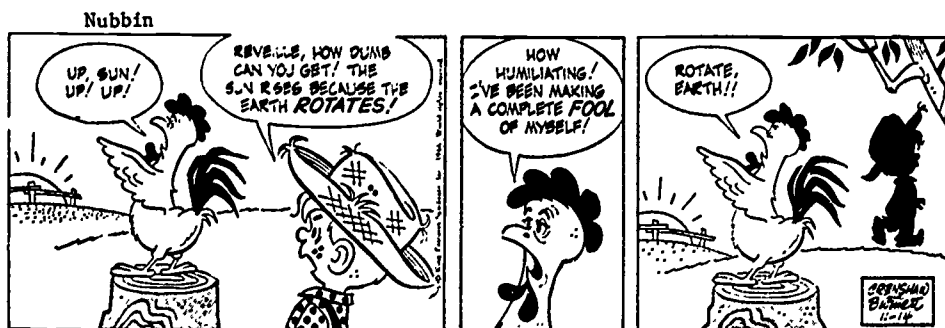
$(1/365 - 1/687)$ of 360° . But $1/365 - 1/687 = 0.00274 - 0.00146 = 0.00128 = 1/780$. Thus in one day the earth gets ahead of Mars by $1/780$ of 360° ; it will take 780 days for the earth to catch up to Mars again. The "phase period" of Mars is, therefore, 780 days, or 2.14 years. This is an average value.

3. The view from the moving earth is shown for a period of time greater than 1 year. First the sun is seen in direct motion, then Mars comes to opposition and undergoes a retrograde motion loop, and finally we see the sun again in direct motion.

Note the changes in apparent size and brightness of the globe representing the planet when it is nearest the earth (in opposition). Viewed with the naked eye, Mars does in fact show a large variation in brightness (ratio of 50:1). The angular size also varies as predicted by the model, although the disk of Mars, like that of all the planets, can be seen only with telescopic aid. The heliocentric model illustrated in this film is simpler than the geocentric model of Ptolemy, and it does give the main features observed for Mars and the other planets: retrograde motion and variation

Film Loops

in brightness. However, detailed numerical agreement between theory and observation cannot be obtained using circular orbits. With the proposal by Kepler of elliptical orbits, better agreement with observation was finally obtained, using a modified heliocentric system.



FILM LOOP 12 Jupiter Satellite Orbit

The innermost of the four largest satellites of Jupiter, discovered by Galileo in 1610, is Io, which moves in a circular orbit with a period of $42^{\text{h}} 28^{\text{m}}$. The film shows most of the orbit of Io in time-lapse photography done at the Lowell Observatory in Flagstaff, Arizona, using a 24-inch refractor (Fig. 1). Exposures were made at 1-minute intervals

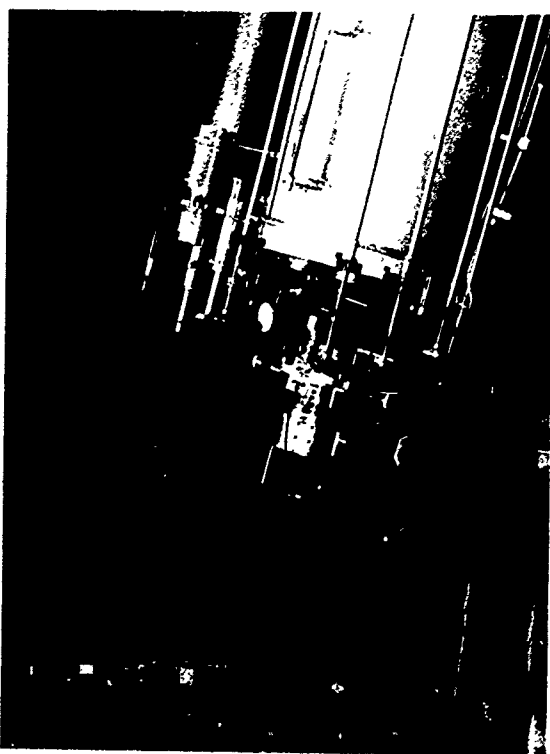


Fig. 1

during seven different nights early in 1967. The orbit had to be photographed in segments for an obvious reason: the rotation of the earth caused Jupiter to rise and set, and also, of course, caused interruptions due to daylight periods.

First, the film shows one segment of the orbit just as it was photographed at the focal plane of the telescope; a clock shows the passage of time. Due to small errors in guiding the telescope and also atmospheric turbulence, the highly mag-

nified images of Jupiter and its satellites dance about. To remove this unsteadiness, each of the images of Jupiter—over 2100 of them—was optically reprinted at the center of the frame, and the clock was masked out. The films with stabilized images were then joined together to give a continuous record of the motion of satellite I (Io). Some variation in the brightness of the satellites was caused by occasional light haze or cloudiness.

Table 1

Satellites of Jupiter

Name	Period	Radius of Orbit (mi)	Eccentricity of Orbit	Diameter (mi)
I Io	$1^{\text{d}} 18^{\text{h}} 28^{\text{m}}$	262,000	0.0000	2,000
II Europa	$3^{\text{d}} 13^{\text{h}} 14^{\text{m}}$	417,000	0.0003	1,800
III Ganymede	$7^{\text{d}} 3^{\text{h}} 43^{\text{m}}$	666,000	0.0015	3,100
IV Callisto	$16^{\text{d}} 16^{\text{h}} 32^{\text{m}}$	1,171,000	0.0075	2,800

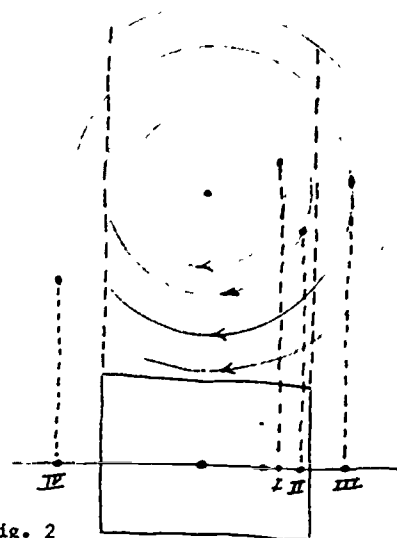


Fig. 2

The four Galilean satellites are listed in Table 1. On Feb. 3, 1967, they had the configuration shown in Fig. 2. The satellites move nearly in a plane which we view almost edge-on; thus they seem to move back and forth along a line. The field of view is large enough to include the entire orbits of I and II, but III and IV are outside the camera field when they are farthest from Jupiter.

Film Loops

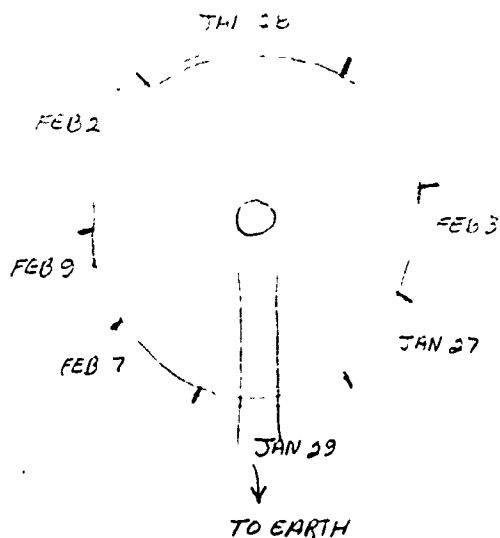


Fig. 3

The seven segments of film were spliced together as indicated in Fig. 3 to give a synthesis of the motion of satellite I (Io). For example, the position of I in the last frame of the Jan. 29 segment matches the position of I in the first frame of the Feb. 7 segment. However, since these were photographed 9 days apart, the other three satellites had moved varying distances around their orbits. Therefore, when viewing the film you will see satellites II, III and IV "pop in" and "pop out" while the image of I remains in a continuous path. Marking lines have been added to help you identify Io when each new section of film begins. Fix your attention on the steady motion of I and ignore the comings and goings of the other satellites.

Here are some interesting features that you can see when viewing the film:

- a) At the start of the film, I is almost stationary at the right side of the camera field (it is almost at its greatest eastern elongation—see Fig. 2); satellite II is moving toward the left and overtakes I.
- b) As I moves toward the left it passes in front of Jupiter and becomes

invisible for a while. This is called a transit. Satellite III (Ganymede, the largest of the satellites) also has a transit at about the same time. Also, II moves toward the right and disappears behind Jupiter (this is called an occultation). It is a very active scene! Figure 4 shows these three satellites at the start of this segment; satellite IV is out of the picture, far to the right of Jupiter.

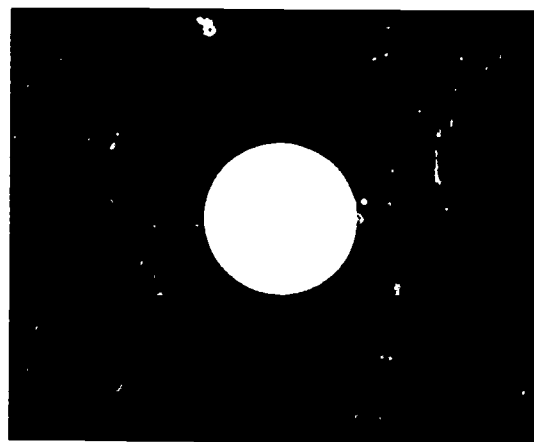


Fig. 4

If you look closely during the transit period, you can see the shadow of Ganymede and perhaps that of Io, on the left part of the surface of Jupiter.

- c) Near the end of the film loop, I (moving toward the right) disappears at D in Fig. 5 as an occultation begins. Look for its reappearance—it emerges from an eclipse and suddenly appears in space at a point E to the right of Jupiter.
- d) The image of Jupiter is not a perfect circle. Just as for the earth, the rotation of the planet on its axis causes it to flatten at the poles and bulge at the equator. The effect is quite noticeable for Jupiter, which is large and has a rapid rotation period of about $9^{\text{h}} 55^{\text{m}}$. The equatorial

diameter is 89,200 miles and the polar diameter is 83,400 miles. Occasionally you may notice the broad bands of clouds on Jupiter, but generally the pictures are too overexposed to show the bands.

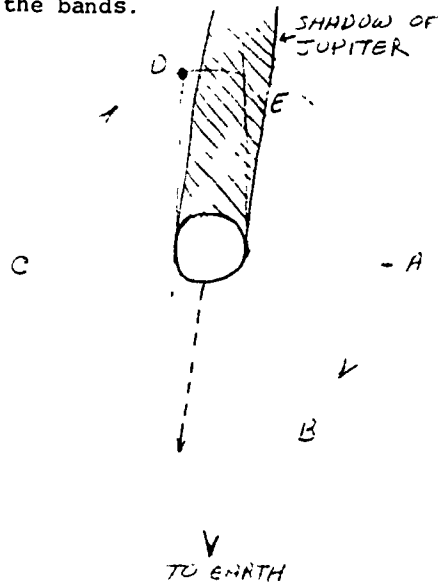


Fig. 5

Measurements

You can make two measurements from this film and from them find your own value for the mass of Jupiter.

1. Period of orbit. Use a clock or watch with a sweep second hand to time the motion of the satellite between points B and D (Fig. 5). This is half a revolution, so in this way you can get the period, in apparent seconds. To convert to real time, use the speed-up

factor. Since film was exposed at 1 frame/minute and is projected at 18 frames/sec, the speed-up factor is 18×60 , or 1080. (For a more precise value, calibrate your projector. A punch mark at the start of the loop gives a flash of light as it passes the lens. Measure the time interval between two flashes. Divide the total number of frames in the loop by the time interval to get the number of frames/sec for your projector.) In this way obtain the period T for one complete revolution of the satellite. How does your result compare with the value listed in Table 1?

2. Radius of orbit. Project the film on a piece of paper and mark the two extreme positions of the satellite, when it is farthest to the right (at A) and farthest to the left (at C). This gives the diameter of the orbit. To convert to miles, use the fact that Jupiter's equatorial diameter is 89,200 miles (about 11 times that of the earth). Now you can find the radius R of the satellite's orbit, in miles. How does your measurement compare with the value listed in Table 1?

3. Mass of Jupiter. You can use your previous calculations to find the mass of Jupiter relative to that of the sun (a similar calculation based on the satellite Callisto is given in Sec. 8.15 of the text). How does your experimental result compare with the accepted value, which is $m_J/m_S = 1/1048$?

Film Loops

FILM LOOP 13 Program Orbit I

This film is the first in a series in which a computer is used to help us understand the applications of Newton's laws to planetary motions. We use a computer for two reasons. First, the burden of calculation is removed, so we can immediately see the effect of a change in the data or in the assumed force law. Second, computers are a part of contemporary culture, so it is important to learn what a computer can do and cannot do.

A student is plotting the orbit of a planet, as in Experiment 20, Stepwise Approximation to an Orbit (Fig. 1). While the student is working, his teacher is preparing the computer program for the same problem by punching a set of cards. Then the computer is fed the program and instructed to solve the problem.

The computer's output can be presented in many ways. A table of X and Y values can be typed or printed. For a more easily interpreted display, the computer output is fed to an X-Y plotter, which prints a graph from the table of values. This X-Y plot is similar to the hand-constructed graph made by the student. The computer output can also be shown visually on a cathode-ray tube (CRT). The CRT display will be used in Loop 14.

Let us compare the work of the student and that of the computer. The student chooses an initial position and velocity. Then he calculates the force on the planet from the inverse-square law of gravitation; then he imagines a "blow" to be applied toward the sun, and uses Newton's

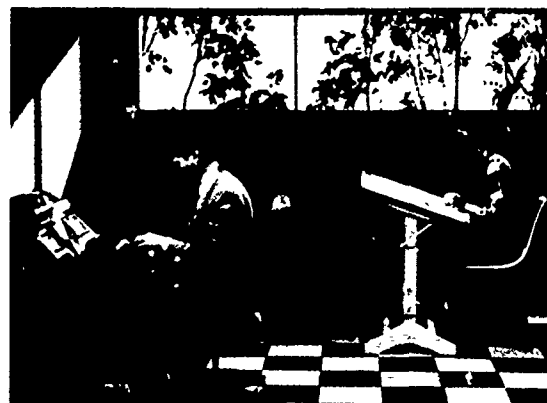


Fig. 1

laws of motion to calculate how far and in what direction the planet moves.

The computer does exactly the same steps. The initial values of X and Y are selected and also the initial components of velocity XVEL and YVEL. (We are beginning to use computer terminology here; XVEL is the name of a single variable, rather than a product of four! The computer "language" we will use is FORTRAN.) The FORTRAN program (represented by the stack of punched cards) consists of the "rules of the game"—which are the laws of motion and the law of gravitation. The "dialogue" between operator and computer takes place after the program has been stored in the computer's memory. During the dialogue, the operator types instructions in response to computer requests; these requests have been "built in" the program by the programmer so that there will be no pieces of data missing or overlooked. Finally, the dialogue indicates a choice of mode of display (X-Y plotter in this case). The calculations are made and displayed.

The dialogue for trial 1 is as follows:
(the series of dots at the end of a machine statement represent a request for data):

(machine)

PROGRAM HAS NOW BEEN TRANSLATED INTO
MACHINE INSTRUCTIONS

PROGRAM ORBIT
SUBROUTINE GRAPH
READY....

(operator)

YES

GIVE ME INITIAL POSITION IN AU....

X = 4.
Y = 0.

GIVE ME INITIAL VELOCITY IN AU/YR....

XVEL = 0.
YVEL = 2.

GIVE ME CALCULATION STEP IN DAYS....

60.

GIVE ME NUMBER OF STEPS FOR EACH POINT
PLOTTED....

1.

GIVE ME DISPLAY MODE....

X-Y PLOTTER.

We see that the orbit displayed on the X-Y plotter is in every way like the student's graph. Both orbits fail to close exactly because the stepwise approximation is too coarse; the blows are too infrequent near perihelion to be a good approximation to a continuously acting force. In the next loop we will see if this explanation is in fact correct.

Program Orbit, in the section of Additional Suggestions for Activities, is an actual FORTRAN II program which was used on an IBM 1620 computer. If you have access to a computer, you can try it with a different force law.

FILM LOOP 14 Program Orbit II

This is a continuation of Loop 13. We were left with the feeling that the orbit of trial 1 failed to close because the blows were spaced too far apart. A direct way to test this would be to calculate the orbit using many more blows—but to do this by hand would require much more pencil-pushing and a lot of time. Now we see one way in which a computer quickly solves a complex problem. The operator merely needs to change the dialogue slightly, giving a smaller interval between the calculated points. The laws of motion are the same as before, so the same program is used; only the dialogue is different. A portion of the new dialogue for trial 2 is as follows:

READY....

YES

GIVE ME INITIAL POSITION IN AU....

X = 4.
Y = 0.

GIVE ME INITIAL VELOCITY IN AU/YR....

XVEL = 0.
YVEL = 2.

GIVE ME CALCULATION STEP IN DAYS....

3.

GIVE ME NUMBER OF STEPS FOR EACH POINT
PLOTTED....

7.

GIVE ME DISPLAY MODE....

X-Y PLOTTER.

Note that only minor changes in the dialogue have been made. Points are now calculated every 3 days (20 times as many calculations as for trial 1), and only 1 out of 7 of the calculated points are plotted (to avoid a graph that is crowded with too many points).

Film Loops

The final instruction can also be modified to obtain a display on the face of the cathode-ray tube which exactly duplicates the X-Y plotter display:

GIVE ME DISPLAY MODE....

CRT

The CRT display has the advantage of speed and flexibility; plotted points can be erased if desired (as in Loop 17, on perturbations). On the other hand, the permanent record afforded by the X-Y plotter is more convenient and has better precision than a photographic record of a CRT display.

We will use the CRT display in the other films in this series, Loops 15, 16 and 17.

FILM LOOP 15 Central Forces (Computer Program)

Section 8.4 of the text shows that a body acted on by a central force will move in such a way that Kepler's law of areas applies. It doesn't matter whether the force is constant or variable, or whether the force is attractive or repulsive. The law of areas is a necessary consequence of the fact that the force is central, directed toward or away from a point. The proof in Sec. 8.4 follows that of Newton in the Principia.

The initial scene in the film shows a dry-ice puck bouncing on a circular bumper. This is one way in which a central force can be visualized as demonstrating the motion of a body under the action of repeated blows of equal duration, all directed toward a center. The rest of the film is made by photographing the face of a CRT which displays the output of a computer.

It is important to realize the role of the computer program: it controls the change in direction and change in

speed of the "mass" as a result of a "blow." This is how the computer program uses Newton's laws of motion to predict the result of applying a brief impulsive force, or blow. The program remains the same for all parts of the loop, just as Newton's laws remain the same during all experiments in a laboratory. However, at one place in the program the operator must specify how he wants the force to vary with the distance from the central point. The basic program (laws of nature) remains the same throughout.

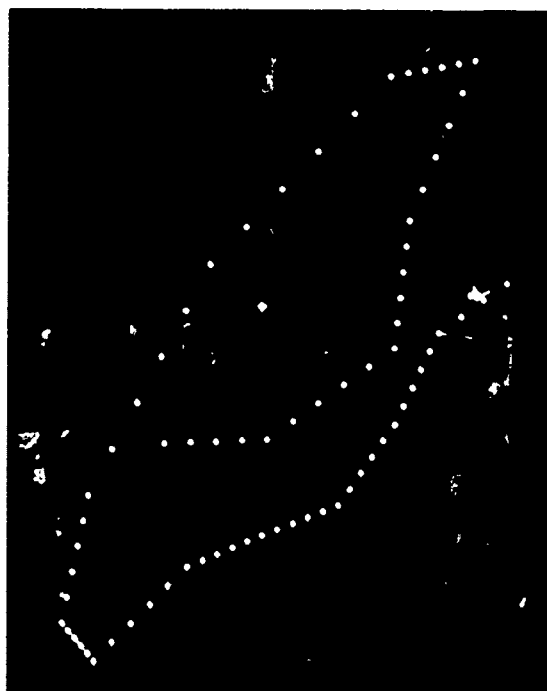


Fig. 1

1. Random blows. The photograph (Fig. 1) shows part of the motion of the mass as blows are repeatedly applied at equal time intervals. No one decided in advance which blows to use; the program merely told the computer to select a number at random to represent the magnitude of the blow. The directions toward or away from the center were also selected at random, although a slight preference for attractive blows was built in so the pattern would, on the whole,

stay on the face of the CRT. Study the photograph. How many blows were attractive? How many were repulsive? Were any blows so small as to be negligible?

You can see if the law of areas applies to this random motion. Project the film on a piece of paper, mark the center and mark the points where the blows were applied. Now measure the areas of the triangles. Does the moving mass sweep over equal areas in equal time intervals?

2. Force proportional to distance.

Perhaps your teacher has demonstrated such a force by showing the motion of a weight on a long string. If the weight is pulled back and released with a sideways shove, it moves in an elliptical orbit with the force center (lowest point) at the center of the ellipse. Notice in the film how the force is largest where the distance from the center is greatest. The computer shows how a relatively smooth orbit is built up by having the blows come at shorter time intervals. In 2a, only 4 blows are used to describe an entire orbit; in 2b there are 9 blows, and in 2c, 20 blows give a good approximation to the ellipse that would be obtained if the force acted continuously.

3. Inverse-square force. Finally, the same program is used for two planets simultaneously, but this time with a force which varies inversely as the square of the distance from the center of force. Notice how the force on each planet depends on the distance from the sun. For these ellipses, the sun is at one focus (Kepler's first law), not at the center of the ellipse.

In this loop, the computer has done for us what we could do for ourselves (using Newton's laws) if we had great patience and plenty of time. The computer reacts so quickly that we can change

the conditions rather easily, and thus investigate many different cases and display the results as a diagram. And it makes fewer errors than a person!

FILM LOOP 16 Kepler's Laws for Two Planets (Computer Program)

The computer program described in the notes for Loop 15 was used to display the motion of two planets. According to this program, each planet was acted on at equal time intervals by an impulsive force of "blows" of equal duration, directed toward a center (the sun). The force exerted by the two planets on each other is ignored in this program. In using the program, the operator selected a force law in which the force varied inversely as the square of the distance from the sun (Newton's law of universal gravitation). Figure 1 (taken from Loop 15 on central forces) shows the two planets and the forces acting on them. For



Fig. 1

clarity, the forces are not shown in this loop. The initial positions and initial velocities for the planets were selected, and the positions of the planets were shown on the face of the cathode-ray tube at regular intervals. (Only representative points are shown, although many more points were calculated in between those that were displayed.) This procedure is illustrated in more detail in the notes for Loops 13 and 14. The film is spliced into an endless loop,

Film Loops

each planet's motion being repeated indefinitely.

You can check all three of Kepler's laws by projecting this film on a sheet of paper and marking the position of each planet at each of the displayed orbit points. The law of areas is checked immediately, by drawing suitable triangles and measuring their areas. For example, you can check the constancy of the areas swept over at three places: near perihelion, near aphelion and at a point approximately midway between perihelion and aphelion.

To check Kepler's law of periods (third law), use a ruler to measure the distances of perihelion and aphelion for each orbit. To measure the periods of revolution, use a clock or watch with a sweep second hand; an alternative method is to count the number of plotted points in each orbit.

To check the first law, you must see if the orbit is an ellipse with the sun at one focus. Perhaps as good a way as any would be to use a string and two thumb tacks to draw an ellipse. On a large copy of the projected orbit of either planet, locate the empty focus, point S' , which is symmetrical with respect to the sun's position S . Tie a piece of string in a loop which will just extend from P to S' and back again, and place the loop around the thumb tacks (Fig. 2). Then put your pencil

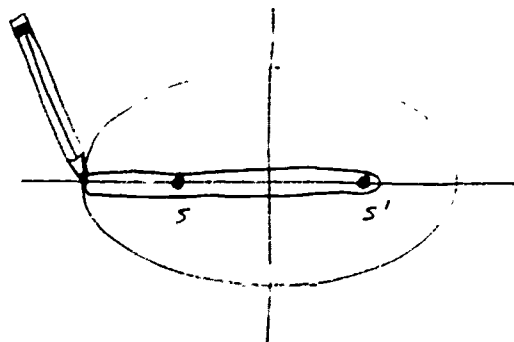


Fig. 2

point in the loop and draw the ellipse, always keeping the string taut. How well does this ellipse (drawn assuming Kepler's first law) match the observed orbit of the planet?

You may think that these are not measurements in the true sense of the word; after all, didn't the computer "know" about Kepler's laws and display the orbits accordingly? Not so—the computer "knew" (through the program we gave it) only Newton's laws of motion and the inverse-square law of gravitation. What we are measuring here is whether these laws of mechanics have as their consequence the Kepler laws which describe, but do not explain, the planetary orbits. This is exactly what Newton did, but without the aid of a computer to do the routine work. Our procedure in its essentials is the same as Newton's, and our results are as strong as his.

FILM LOOP 17 Perturbation

The word "perturbation" refers to a small force which slightly disturbs the motion of a celestial body. For example, the planet Neptune was discovered because of its gravitational pull on Uranus. The main force on Uranus is the gravitational pull of the sun, and the force exerted on it by Neptune is a perturbation which changes the orbit of Uranus very slightly. By working backward, astronomers were able to predict the position and mass of the unknown planet from its small effect on the orbit of Uranus. This spectacular "astronomy of the invisible" was rightly regarded as a triumph for the Newtonian law of universal gravitation.

A typical result of perturbations is that a planet's orbit rotates slowly, always remaining in the same plane.

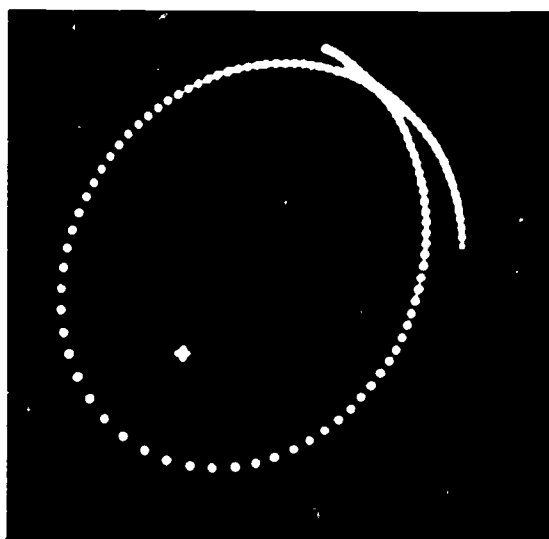


Fig. 1(a)

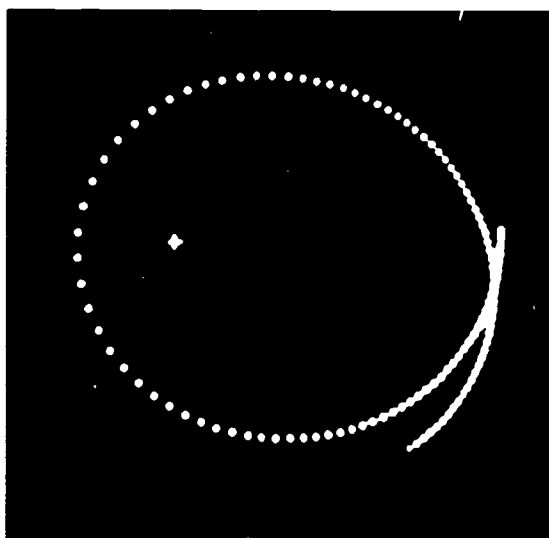


Fig. 1(b)

This effect is called the advance of perihelion, illustrated in Fig. 1. The earth is closest to the sun about Jan. 3 each year; but the perihelion point is slowly advancing. This slow rotation of the earth's orbit (not to be confused with the precession of the direction of the earth's axis) is due to the combined effect of many perturbations: small gravitational forces of the other planets (chiefly Jupiter), and the retarding force of friction due to dust in the space through which the earth moves.

Mercury's perihelion advances at the rate of more than 500 seconds of arc per century. Most of this was fully explained by perturbations due to Newtonian mechanics, the inverse-square law of gravitation and the attractions of the other planets. However, about 43 seconds per century remained unaccounted for. These 43 seconds are crucial; we cannot sweep them under the rug any more than Kepler could ignore the 8 minutes' discrepancy of the position of Mars as calculated on the circular-orbit theory (see Sec. 7.1). When Einstein re-examined the nature of space and time in developing the theory of relativity, he found a slight modification of the Newtonian gravitational theory. As discussed at the end of Sec. 8.18, relativity theory is important for bodies moving at high speeds and/or near large masses. Mercury's orbit is closest to the sun and therefore is most affected by Einstein's extension of the law of gravitation. The relativity theory was successful in explaining the extra 43 seconds per century of advance of Mercury's perihelion, but recently this "success" has again been questioned.

In this film we use a modification of the program which is described in the notes to Loop 15 (central forces). The force on the mass is still a central one, but no longer an exact inverse-square force.

The first sequence shows the advance of perihelion caused by a small force proportional to the distance R . This perturbation is added to the usual inverse-square force. The dialogue between operator and computer starts as follows (the dots at the end of machine statements represent requests for data):

Film Loops

(machine)
PROGRAM HAS NOW BEEN TRANSLATED INTO
MACHINE INSTRUCTIONS....

PROGRAM PRECES
SUBROUTINE GRAPH

READY....

(operator)

YES

PRECESSION PROGRAM WILL USE
ACCEL = $G/(R^*R) + P^*R$
GIVE ME PERTURBATION P

P = .66666

GIVE ME INITIAL POSITION IN AU....

X = 2.
Y = 0.

GIVE ME INITIAL VELOCITY IN AU/YR....

XVEL = 0.
YVEL = 3.

GIVE ME CALCULATION STEP IN DAYS....
(etc.)

In this dialogue the symbol * means multiplication; thus $G/(R^*R)$ is the inverse-square force, and P^*R is the perturbing force, proportional to R.

In the second sequence, the inverse-square force law is replaced by an inverse-cube law. The dialogue includes the following:

READY....

YES

GIVE ME POWER OF FORCE LAW....

-3.

THANK YOU. PROGRAM WILL USE
ACCEL = $G/(R^*R^*R)$
GIVE ME INITIAL POSITION IN AU....

X = 1.
Y = 0.

GIVE ME INITIAL VELOCITY IN AU/YR....

XVEL = 0.
YVEL = 6.2832

GIVE ME CALCULATION STEP IN DAYS....
(etc.)

The orbit resulting from the inverse-cube attractive force is not a closed one. The planet spirals into the sun in a "catastrophic" orbit. As the planet approaches the sun it speeds up (law of areas); for this reason the last few plotted points are separated by a large fraction of a revolution.

Our use of the computer is, indeed, experimental science; we are able to see what would happen if the force law changes, but we retain the "rules of the game" expressed by Newton's laws of motion.

Additional Suggestions for Activities

Stonehenge

Stonehenge (see page 2 of your Unit 2 text) has been a mystery for centuries. Some have thought it was a pagan temple, others that it was a monument to slaughtered British chieftains. Legends invoked the power of Merlin to explain how the stones were brought to their present location. Recent studies indicate that Stonehenge may have been an astronomical observatory and eclipse computer.

Read "Stonehenge Physics," in the April, 1966 issue of Physics Today; Stonehenge Decoded, by G. S. Hawkins and J. B. White; or see Scientific American, June, 1953. Make a report and/or a model of Stonehenge for your class.

Photograph the Night Sky

Make a time exposure of the night sky, like Fig. 5.4 on page 8 of your text. The camera must be able to make a time exposure, and it must remain stationary, so you will need a tripod or rigid support. Aim the camera at the North Star, focus for infinity and open the shutter. A small aperture opening (large f-number) will reduce fogging due to reflected sky light. Aim the camera toward the south to get a photo like Fig. 5.3. Open the shutter for 30 seconds, and then close it for several minutes before you make the long (2 or 3 hours) time exposure. A dot at the end of each star trail will then show the start of the picture.

Wire Sculpture

If you don't like plotting orbits on paper or cardboard, make a wire sculpture of the solar system. Show the various orbits of the planets and comets to the correct scale and having the proper inclination to each other.

Literature

The ideas about the universe which you read about in Chapters 5 and 6 of your text influenced the Elizabethan view of the world and the universe a great deal. In spite of the ideas of Galileo and Copernicus, writers, philosophers and theologians continued to use Aristotelian and Ptolemaic ideas in their works. In fact, there are many references to the crystal-sphere model of the universe in the writings of Shakespeare, Donne and Milton. The references often are subtle because ideas of order were commonly accepted by the people for whom the works were written. For a quick overview of this idea, with reference to many authors of the period, read the paperback The Elizabethan World Picture, by E. M. W. Tillyard, Vintage Press, or Basil Willey, Seventeenth Century Background, Doubleday. There are two articles by H. Butterfield and B. Willey in Reader 1. The appendix contains a Resource Letter on Science and Literature with helpful annotations. An interesting specific example is found in Christopher Marlowe's Doctor Faustus where Faustus sells his soul in return for the secrets of the universe. Speaking to the devil, Faustus says:

"...Come, Mephistophilis, let us
dispute again
And argue of divine astrology.
Tell me, are there many heavens
above the moon?
Are all celestial bodies but one
globe,
As is the substance of this cen-
trical earth?..."

Moon Crater Names

Prepare a report about how some of the moon craters were named. See Isaac Asimov's Biographical Encyclopedia of Science and Technology for material about some of the scientists whose names were used for craters.

Appendix

Program Orbit

The computer program is written in FORTRAN II and includes "ACCEPT" statements used on an IBM 1620 input typewriter. With slight modification it worked on a CDC 3100 and CDC 3200, as shown in the film loops. With additional slight modifications (in statement 16 and the three succeeding statements) it can be used for other force laws. The

method of computation is the scheme used in the Feynman selection in Reader 1. A similar program is presented and explained in FORTRAN for Physics (Alfred M. Bork, Addison-Wesley, 1967).

Note that it is necessary to have a subroutine MARK. In our case we used it to plot the point X,Y on a plotter, but MARK could be replaced by a PRINT statement to print the X and Y coordinates.

```
PROGRAM ORBIT
C
C   HARVARD PROJECT PHYSICS ORBIT PROGRAM.
C   EMPIRICAL VERIFICATION OF KEPLERS LAWS
C   FROM NEWTONS LAW OF UNIVERSAL GRAVITATION.
C
      G=40.
      4 CALL MARKF(0.,0.)
      6 PRINT 7
      7 FORMAT(9HGIVE ME Y )
      X=0.
      ACCEPT 5,Y
      PRINT 8
      8 FORMAT(12HGIVE ME XVEL)
      5 FORMAT(F10.6)
      ACCEPT 5,XVEL
      YVEL=0.
      PRINT 9
      9 FORMAT(49HGIVE ME DELTA IN DAYS, AND NUMBER BETWEEN PRINTS)
      ACCEPT 5,DELTA
      DELTA=DELTA/365.25
      ACCEPT 5,PRINT
      IPRINT = PRINT
      INDEX = 0
      NFALLS = 0
      13 CALL MARKF(X,Y)
      PRINT 10,X,Y
      15 IF(SENSE SWITCH 3) 20,16
      20 PRINT 21
      10 FORMAT(2F7.3)
      NFALLS = NFALLS + IPRINT
      21 FORMAT(23HTURN OFF SENSE SWITCH 3 )
      22 CONTINUE
      IF(SENSE SWITCH 3) 22,4
      16 RADIUS = SQRTF(X*X + Y*Y)
      ACCEL = -G/(RADIUS*RADIUS)
      XACCEL = (X/RADIUS)*ACCEL
      YACCEL = (Y/RADIUS)*ACCEL
C FIRST TIME THROUGH WE WANT TO GO ONLY 1/2 DELTA
      IF(INDEX) 17,17,18
      17 XVEL = XVEL + 0.5 * XACCEL * DELTA
      YVEL = YVEL + 0.5 * YACCEL * DELTA
      GO TO 19
C DELTA V = ACCELLRATION TIMES DELTA T
      18 XVEL = XVEL + XACCEL * DLLTA
      YVEL = YVEL + YACCEL * DELTA
C DELTA X = XVELOCITY TIMES DELTA T
      19 X = X + XVEL * DELTA
      Y = Y + YVEL * DELTA
      INDEX = INDEX + 1
      IF(INDEX - NFALLS) 15,15,13
      END
```

Trial of Copernicus

Hold a mock trial for Copernicus. Two groups of students represent the prosecution and the defense. If possible, have English, social studies and language teachers serve as the jury for your trial.

Time Zones

Make a map showing the time zones of the United States. Report why and when time zones were established. There have been spirited debates in many states over the years about the adoption of Daylight Saving Time, especially in those states which lie near the western boundary of each time zone. Find out arguments for and against daylight saving time. It is interesting that many farmers are against it, while city people generally favor the change.

Discovery of Neptune and Pluto

The Project Physics supplementary unit, Discoveries in Physics, describes how Newton's law of universal gravitation was used to predict the existence of Neptune and Pluto before they were observed telescopically. Read it, and decide whether you think it possible that we may yet discover another planet beyond Pluto.

Measuring Irregular Areas

Are you tired of counting squares to measure the area of irregular figures? A device called a planimeter can save you much drudgery. There are several styles, ranging from a simple pocket knife to a complex arrangement of worm gears and pivoted arms. See the Amateur Scientist section of Scientific American, August, 1958 and February, 1959.

Calendars

Report on some of the ancient calendars, and the various methods of designing them. Perhaps more interesting, report on the various calendars still in use today, such as the Gregorian, Mohammedan, Jewish and Chinese calendars. As late as 1937 there were 14 different calendars in use in India. In 1955 the World Calendar Association presented a plan for calendar reform to the United Nations. The International Fixed Calendar League also proposed a 13-month calendar (Fig. 1). Two good sources are Time Measurement, by E. J. Brill, Leiden, Netherlands; and The Romance of the Calendar, P. W. Wilson, Norton.

This Calendar may be inaugurated on the first day of any Gregorian Calendar year of 1938, 1949, 1955, 1966, 1977, 1983 or 1994, with a minimum of inconvenience and may be continued forever thereafter without change, provided the present average solar year is not materially increased or decreased.

ALPHA, 1st day, designated as a year day

WEEK DAY	JAN	FEB	MAR	APR	MAY	JUN	JULY	AUG	SEP	OCT	NOV	DEC	XIII MONTHS
Sunday	1	1	1	1	1	1	1	1	1	1	1	1	1
Monday	2	2	2	2	2	2	2	2	2	2	2	2	2
Tuesday	3	3	3	3	3	3	3	3	3	3	3	3	3
Wednesday	4	4	4	4	4	4	4	4	4	4	4	4	4
Thursday	5	5	5	5	5	5	5	5	5	5	5	5	5
Friday	6	6	6	6	6	6	6	6	6	6	6	6	6
Saturday	7	7	7	7	7	7	7	7	7	7	7	7	7
Sunday	8	8	8	8	8	8	8	8	8	8	8	8	8
Monday	9	9	9	9	9	9	9	9	9	9	9	9	9
Tuesday	10	10	10	10	10	10	10	10	10	10	10	10	10
Wednesday	11	11	11	11	11	11	11	11	11	11	11	11	11
Thursday	12	12	12	12	12	12	12	12	12	12	12	12	12
Friday	13	13	13	13	13	13	13	13	13	13	13	13	13
Saturday	14	14	14	14	14	14	14	14	14	14	14	14	14
Sunday	15	15	15	15	15	15	15	15	15	15	15	15	15
Monday	16	16	16	16	16	16	16	16	16	16	16	16	16
Tuesday	17	17	17	17	17	17	17	17	17	17	17	17	17
Wednesday	18	18	18	18	18	18	18	18	18	18	18	18	18
Thursday	19	19	19	19	19	19	19	19	19	19	19	19	19
Friday	20	20	20	20	20	20	20	20	20	20	20	20	20
Saturday	21	21	21	21	21	21	21	21	21	21	21	21	21
Sunday	22	22	22	22	22	22	22	22	22	22	22	22	22
Monday	23	23	23	23	23	23	23	23	23	23	23	23	23
Tuesday	24	24	24	24	24	24	24	24	24	24	24	24	24
Wednesday	25	25	25	25	25	25	25	25	25	25	25	25	25
Thursday	26	26	26	26	26	26	26	26	26	26	26	26	26
Friday	27	27	27	27	27	27	27	27	27	27	27	27	27
Saturday	28	28	28	28	28	28	28	28	28	28	28	28	28

OMEGA, 366th day of year, is added as a supplemental day at the end of every fourth year, and is not included in any month. Leap Year day is intercalated every fourth year in all years exactly divisible by four except century years, which must be divisible by 400, the year 2000 being exactly divisible by 400 will be a century Leap Year that includes the Omega day, consequently the present series of leap year days will run unchanged from 1904 to 2100 A. D.





The engraving of the French Academy by Sebastian LeClerc (1698) reflects the activity of these learned societies. The picture, of course, does not depict an actual scene, but in allegory shows the excitement of communication that grew out of the informal atmosphere. The dress is symbolic of the Grecian heritage. Although all the sciences are represented, notice that anatomy, botany and zoology with their skeletons and dried leaves, along with alchemy and theology, have been edged aside by mathematics and the physical sciences, including astronomy, which occupy the center stage. How many activities relate to the study of motion? How many relate to astronomy? Several activities relate to ideas about momentum and energy which you will study later. Return to this picture then and see which ones they are.

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P. 74 Print Collection of the Federal Institute of Technology, Zurich.

Acknowledgment

P. 5 The table is reprinted from Solar and Planetary Longitudes from Years -2500 to +2000, prepared by William D. Stahlman and Owen Gingerich (University of Wisconsin Press, 1963).