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ABSTRACT

An expanded abstract and a critical analysis for each of 17 research articles are given. Four of the articles are concerned with evaluating methods of instruction, seven deal with evaluation of achievement and attitudes, two present research in teacher education, and the remaining four articles investigate basic patterns of learning. (Editor/DT)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Expanded Abstracts and Critical Analyses of Recent Research

Center for Science and Mathematics Education
The Ohio State University
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SE 015 513

INVESTIGATIONS IN MATHEMATICS EDUCATION

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Autumn, 1972

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NOTES

...from the Editor

This issue of Investigations in Mathematics Education presents abstracts of a varied array of research articles. Four articles are concerned with evaluating methods of instruction. The Bat-Haee and Ingersoll articles are both concerned with the teaching of fractions at the fifth and sixth grade levels. The Gray article concerns the effectiveness of homework at the sixth grade level, while the Rockhill article explores the use of computer-based resource units at the college level.

Articles by Alspaugh, Coppedge, Lefkowitz, Levine, Olander, Renzulli, and Travers deal with the evaluation of achievement and attitudes. They range from the construction and validation of test instruments (Coppedge, Renzulli, and Travers) to the relationship between test scores and later performance in computer programming (Alspaugh) or college mathematics (Lefkowitz).

Research in teacher education is represented by articles by Cooney and Gall. These studies relate to the observation and classification of teaching acts (Cooney) and to methods of providing feedback in microteaching situations (Gall).

Basic patterns of learning are investigated by the remaining four articles. The Allen article is an example of basic research which may have special implications for mathematics education. The role of conservation in achievement is explored by Cathcart, while Rogers and Scandura are concerned with specific learning situations. We apoligize that the Scandura document is not available from EDRS, due to the marginal legibility of some pages. We have extra copies at our ERIC Center, and will be glad to loan you the document. You may make copies of it if you wish to add it to your library.

Jon L. Higgins
Editor

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EJ 035 323 310 AA 509 099
CONCEPT SELECTION STRATEGIES OF NEW GUINEA STUDENTS Allen, A.L.; Shannon, A.G., Journal of Experimental Education, v39 n3, pp1-4, Spr 71
Descriptors--*Longitudinal Studies, *Concept Formation, *Educational Research, *Thought Processes, *Mathematical Concepts

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Larry Sowder, Northern Illinois University.

1. Purpose

(a) To explore the reception strategies used by indigenous preliminary year Papuan and New Guinea students in conjunctive and disjunctive concept attainment.

(b) To compare qualitatively the results with those for U.S. students.

2. Rationale

Bruner, Goodnow, and Austin (A Study of Thinking, 1956) reported that Harvard and Wellesley undergraduates were more successful in attaining conjunctive concepts (for which positive instances require the presence of all of some set of attributes) than disjunctive concepts (for which positive instances require the presence of at least one attribute from some set of attributes) and conjectured that such results might in part be a result of Western culture. However, for attainment of conjunctive concepts with a time limit, their subjects showed a consistent use of one strategy over another: focusing over scanning, to use the terms of the present study. Further, focusers were more successful than scanners.

3. Research Design and Procedure

Three classes of indigenous students at the University of Papua and New Guinea were randomly chosen from the preliminary year classes, which consist of pre-undergraduate students who have completed a school certificate. These students were deemed to be the least Westernized. Classes had been matched by the administration. Sixty-five were involved. Testing was done "in a group situation."

Concepts to be attained were based on cards like these reported in Bruner, et al. Instances were either positive or negative and for either a conjunctive or a disjunctive concept. Each of 8 instances for a concept was projected on a screen for 20 seconds, with subjects responding after each instance on specially prepared record sheets. (If the study followed Bruner, et al., the instance was identified as positive or negative and the subject's response was his hypothesis as to what the concept was.) Four conjunctive and 4 disjunctive concepts were presented in successive weeks.

On the basis of their responses, the subjects' strategies were categorized as focusing (considering all the attributes of the first positive instance), scanning (considering only one attribute of the first positive instance at a time), or mixed. To see whether the strategies were equally common, chi-squared tests were used.

4. Findings

For conjunctive concepts:

(a) 60% consistently used a scanning strategy, 35% the focusing strategy, and 5% a mixed strategy, yielding rejection of a hypothesis of equal frequencies ($p < .001$).

(b) However, 73% of the focusers were successful in identifying all 4 of the concepts, whereas only 21% of the scanners and 0% of the mixed strategists were.

(c) Breakdowns by type of secondary school, district of primary school education, and proposed undergraduate major yielded roughly the same ratios for choice of strategy as indicated in (a).

For the disjunctive concepts:

(d) 38% were scanners, 16% focusers, and 46% mixed strategists, yielding rejection of a hypothesis of equal frequencies ($p < .01$).

(e) None of the scanners identified all 4 concepts, half the focusers did, as did one-sixth of the mixed strategists. Identifying none of the 4 concepts were 40%, 50%, and 67% of the scanners, focusers, and mixed strategists, respectively.

(f) Conjunctive focusers and scanners did not interchange strategies on the disjunctive tasks but some of each did adopt mixed strategies.

5. Interpretations

(a) The preference for the scanning strategy over the focusing strategy with conjunctive concepts is the opposite of the preference of the US students.

(b) Whether this preference, or the high percentage of students sticking to one strategy for the conjunctive tasks, or the relative success of the focusers can be explained in cultural terms remains to be seen.

(c) Like the US students, these subjects had greater difficulty with disjunctive concepts than with conjunctive concepts.

(d) All subjects from East New Britain were scanners; grade 7 students there had been noted to be weak in set theory and logic.

Abstractor's Notes

As with many journal articles, the necessary lack of detail in this interesting report raises some questions:

(a) How had the classes been "matched"? Three of how many classes were chosen?

(b) Were there three group testings each week, or one? How was the nature of the tasks explained? How many attributes were included in each task? Were warm-ups given?

(c) Were there any measures of general ability available? Although such might not be relevant or appropriate, they might give desirable information when attempting to compare Western Ss' performance.

(d) Is the description of a complementary positive instance over-simplified?

(e) Why wasn't a complete report of Ss' success included?

(f) How theoretically appropriate is the scanning--or the focusing--strategy for disjunctive tasks in a reception setting?

(g) Were, as implied, Ss categorized on the basis of their reactions to the <u>first</u> positive instance only? If so, why (for the conjunctive tasks)? And when did the negative instances occur (for the disjunctive tasks especially)?

Larry Sowder
Northern Illinois University

EJ 053 935 010 SE 505 356
IDENTIFICATION OF SOME COMPONENTS OF COMPUTER PROGRAMMING APTITUDE. Alspaugh, Carol Ann, Journal for Research in Mathematics Education, v3 n2, pp89-98, Mar 72.

Descriptors---*Aptitude, *College Mathematics, *Computer Science Education, *Programming, *Research, Correlation, Mathematics Education, Personality Tests, Prediction

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jack E. Forbes, Purdue University, Calumet Campus

1. Purpose

The existence of "talent" or "aptitude" for mastery of computer programming skills is assumed and an effort is made to identify measurable components of this aptitude.

2. Rationale

It is common practice for schools and colleges to place courses in computer programming languages in mathematics departments. In some instances the departments require that their majors take these courses. The author attributes such placement and requirements to a hidden assumption that mathematical aptitude and aptitude for acquisition of programming skills are the same. She observes that the validity of this assumption can be determined only if major components of programming aptitude can be identified.

3. Research Design and Procedure

During the fall semester, 1969-70, this study was conducted with a sample of fifty students in a beginning course in computer science at the University of Missouri-Columbia. The course content included instruction in both Basic Assembly Language (BAL) for the IBM 360 and FORTRAN IV.

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Scores were available or were obtained for all students on the following:

1. Thurstone Temperament Schedule (seven scores as measures of personality)
2. IBM Programmer Aptitude Test (four scores--number series, figure analogies, arithmetic reasoning, and total)
3. Watson Glaser Critical Thinking Appraisal
4. SCAT -- Verbal
5. SCAT -- Quantitative
6. Mathematics background measure (a number 2 through 6, with 2 denoting three or four units of high school mathematics or college algebra, through 6 denoting at least one course beyond second term calculus.)

Proficiency measures were student scores on a teacher-constructed test in BAL (200 pts.), on teacher-constructed tests in FORTRAN IV (200 pts.), and a "comprehensive measure of proficiency" consisting of the sum of the two scores above.

Correlation coefficients were computed for each of the fifteen independent measures above paired with each of the three proficiency measures, and inter-correlations for all pairings of the eighteen measures were determined. Then, multiple regression analysis was applied to express each of the three proficiency measures as linear combinations of the seven scores in (1), the total in (2), the measure in (6), and for FORTRAN IV only, the measure in (3). Low correlations led to elimination of the other measures from the regression analysis. Only the total in (2) was used because of recommendations concerning the use of the test.

4. Findings

The computation of correlation coefficients for the fifteen independent measures with each of the three proficiency measures produced the following profile of a successful student of programming languages:

> He is characterized as non-impulsive by the Thurstone Temperament Scale ($p < .01$ for all three proficiency measures). He has a "strong" mathematics background ($p < .01$ f[illegible]N IV and Comprehensive measures, $p < .05$ for [illegible] BAL measure). He scores well in arithmetic reasoning, and low on the sociability measure of the TTS ($p < .05$ for all three proficiency measures). He exhibits a reflective ($p < .05$ for BAL proficiency), non-dominant ($p < .05$ for FORTRAN IV proficiency) nature and scores well on number reasoning ($p < .05$ for FORTRAN IV proficiency).

No independent variable correlates significantly higher with either BAL or FORTRAN IV proficiency than with the other. Intercorrelations among the three proficiency measures are:

	2.	3.
1. BAL	.606	.910
2. FORTRAN IV		.881
3. Comprehensive		

For all three, $p < .01$

Regression analyses produced significant multiple correlations for all three proficiency measures ($p < .01$ for Comprehensive, $p < .05$ for the other two). In all three cases mathematics background was the major contributor to the variance accounted for by regression, with other major contributors being low scores on the TTS impulsive and sociability scales, with a low score on the TTS vigorous scale ranking high in the analyses of FORTRAN IV and the Comprehensive, low in the BAL analysis.

5. Interpretations

"The findings of this study indicate that the placement of symbolic and algebraic computer programming classes within the mathematics curriculum would be justified. However, due to the particular personality factors that appear to influence achievement in computer programming it would not be realistic to assume that all students who are talented in mathematics will also be talented in computer programming."

The investigator recommends additional research to determine what specific components of mathematical background account for its contribution to success in programming. She also recommends a study comparable to this one using other programming languages. Further, she observes: "Although the multiple R's in the regression analysis were significant from zero, most of the variance was not explained. There are obviously other components ...yet to be identified."

Abstractor's Notes

As the investigator observes, "most of the variance was not explained." (The largest multiple R was .632. The largest correlation coefficient for mathematics background was .411 when it was paired with the Comprehensive Measure of proficiency.) Thus, one must certainly agree that there are "...other components...yet to be identified."

Several questions are raised by this study. The data include neither student grades in mathematics courses completed nor the semester classification of the students in the study. Perhaps "mathematical background" is really a measure of time spent in college and, therefore, of "course taking" ability. Perhaps it is a measure of survival ability (failure rates in early mathematics courses often exceed those in other fields!) That is, perhaps the mathematical background measure is <u>really</u> a measure of ability to succeed within the system and an indication of stratification of the sample rather than a direct measure of a component of aptitude for programming.

The investigator's apparent inference that the personality traits found to make significant contributions to aptitude for programming are, in fact, different from those which contribute to mathematics aptitude is not supported within this report and is I strongly suspect, insupportable.

Finally, the title of the report seems a bit misleading. It purports to be an analysis of aptitude while, in fact, it is an analysis of the factors which contribute to achievement in an academic setting. While there is no doubt a statistically significant correlation between these, any attempt to infer either from a study of the other will, no doubt, leave much of the variance unexplained.

Jack E. Forbes
Purdue University
Calumet Campus

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EJ 050 203 140 RE 503 507
EFFECTIVENESS OF TWO METHODS OF FINDING THE LEAST COMMON DENOMINATOR OF UNLIKE FRACTIONS AND CHARACTERISTICS OF GRADE 5 Ss. Bat-Haee, Mohammad Ali, Perceptual and Motor Skills, v33 n3, pp732-34, Dec 71

Descriptors--*Mathematics Education, *Grade 5, *Academic Achievement, *Teaching Methods, *Fractions, Learning Theories, Learning Processes, Cognitive Processes, Individual Differences, Performance Factors

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Thomas A. Romberg, University of Wisconsin

1. Purpose

The purpose of this study was "to compare the effectiveness of two methods of teaching fifth-grade children to find the least common denominator" (p. 732).

2. Rationale

The author states three reasons for doing the study. First, "finding the least common denominator is known to be the most difficult step in the addition and subtraction of unlike fractions" (p. 732). Second, "two methods of finding the least common denominator (LCD) are recommended: by trial and error or inspection and by factoring" (p. 732) and third, "there is no empirical evidence for the value of one method over the other" (p. 732). The implication being that this study would give empirical information as to which method was more effective and hence reduce the difficulty of adding and subtracting unlike fractions.

3. Research Design and Procedure

The basic experimental design used was a 2 x 2 completely crossed fixed effects factorial design, the two factors being sex and method of instruction. The 112 subjects used in the study were selected from six classes in four schools (53 boys and 59 girls).

The two treatments each lasted 12 periods of 40 to 45 minutes each. The first five periods were the same for both treatments, and covered primes, factors, the factoring process and the concept of least common multiple (LCM). The last seven periods both dealt with adding and subtracting unlike fractions. One treatment developed by the author emphasized factoring to find LCD and the other treatment adapted from the text used in the schools emphasized inspection to find LCD. The regular fifth grade teachers taught the lessons.

No pretreatment data was collected. Posttest data was gathered on a 20-item test of addition and subtraction of unlike fractions (10 items on each). The items were weighted to give a total of 100 credits on the test. Other data that was collected included arithmetic achievement scores and IQ scores.

To determine the relative effectiveness of the two treatments a two-stage analysis is reported.. First, "the null hypothesis (unstated) was examined by generation of a restricted model against its pre-formulated unrestricted model of multiple linear regression" (p. 733). And second, analysis of variance was used to determine if the treatments had different effects.

4. Findings

The multiple linear regression was used to determine if there were differences in posttest scores between boys and girls. Raw score differences on the posttest, achievement and IQ favored boys, although posttest differences were not significant when achievement and IQ were taken into account. The Analysis of Variance of posttest means gave an F of 5.86 (df 1/110, $p. < .02$), which was significant. The difference in means favored the factoring treatment.

5. Interpretations

The author is willing to make two conclusions. First, "there were no significant differences in mathematics between boys and girls provided both groups have been exposed to the same experience" (p. 734). And second, "that the factoring method is the more effective means of finding the lowest common denominator than inspection" (p. 734). He then interprets this result as "factoring--provides a better understanding of this (LCD) structure" (p. 734).

Abstractor's Notes

Even if I might concede that a study comparing two such methods is worth doing, I must raise several questions about how it was carried out.

First, how was the sample selected? Random selection of Ss from all classes in a population (such as a large school district) would be ideal. But, are there just 6 classes of fifth graders in 4 schools? This seems unlikely since that would be 18 2/3 students per class, and if one were randomly selecting from a population, one would expect a balanced design with equal numbers of Ss in each cell. The following table gives the actual cell sizes of the 2 x 2 design.

Table 1
Cell Sizes in the Study

	Boys	Girls
T_1 (factor)	25	31
T_2 (inspect)	28	28

If not all schools in the district were included in the sample what criteria were used to select schools? Similarly, if not all classes in the selected schools were included in the sample, what criteria were used to select classes? Taken together one can only assume that the selection was based on availability or cooperativeness of teachers and not on any systematic procedure.

Second, how were Ss and teachers assigned to treatments? There is no evidence that random assignment of Ss stratified by sex was employed. Nor is there evidence that teachers were randomly assigned.

Third, how different were the treatments? Both exposed students to primes, factors, factoring, LCM during the first 5 days. What specifically was covered in the next seven, particularly by the inspection treatment? Is seven days sufficient to teach these operations? I doubt it. And, in light of the author's final statement about structure how did the treatments differ structutally? For neither treatment is the method of adding and subtracting fractions or of finding a common denominator adequately characterized.

Fourth, is the data gathered adequate or valid to satisfy the purpose of the study? No pretreatment data was gathered. Thus, we do not know about Ss performance on any prerequisite behaviors such as adding or subtracting fractions with common denominators, reducing fractions to lowest terms, etc., nor even whether some could already find LCD's. To determine the effectiveness of some instructional unit one should only include Ss who have mastered the prerequisites, but have not learned the content of the unit. No data gathered during the study was reported. For example, it would be nice to know how many students could factor or find LCM after the 5 days of common instruction. However, my real concern is with the post-test. What is the sample space from which items were selected? Did one treatment teach to the test? How were the items scored? For example, if $1/3 + 1/6 = \square$ was one of the problems, which is the correct answer 1/2 or 3/6 or even 9/18 or were all scored as correct? How were the 20 items weighted to produce a total score of 100? Five points per item? Or was partial credit given? Under any circumstances using weight scores in ANOVA is incorrect. I can only conclude that the evidence the author presents is neither adequate nor valid to accomplish the purpose of the study.

Fifth, the analysis is weak and misleading at best. The regression analysis is used like covariance to demonstrate that cell groups were equivalent. If random assignment of Ss was used, it confirms expectations and IQ and Math achievement should have been used as covariates. If not, it is as inappropriate as covariance. For this ANOVA as mentioned above the scores are wrong and the df is misleading. Since teachers undoubtedly taught groups not individuals df should be g-2 where g= number of groups. Examining the reported cell means (Table 2) and estimating actual raw means (by dividing by 5) would indicate that T_1 Ss got 2 1/2 more items correct than T_2 Ss which would not be significant in this study.

Table 2

Reported Weighted Cell Means and Estimated Raw Cell Means

Reported Means

	Boy	Girl
T_1 (factor)	86.56	83.16
T_2 (inspect)	75.50	68.30

Estimated Means

	Boy	Girl	Total
T_1	17.31	16.63	16.9
T_2	15.10	13.66	14.4

And, if partial credit was given, differences would be even less than my estimates.

Thus, given the inadequate evidence and the inappropriate analysis the conclusions drawn are not justified. Finally, his interpretation of the results in terms of "structure" is impossible to derive from this study. Only if the characterization of the two treatments was done in terms of structure and appropriate evidence was gathered could this be done. In fact, only if the study had been done from such a perspective would it have been worthy of being published.

Thomas A. Romberg
University of Wisconsin

EJ 046 672 510 PS 501 358
THE RELATIONSHIP BETWEEN PRIMARY STUDENTS' RATIONALIZATION OF CONSERVATION AND THEIR MATHEMATICAL ACHIEVEMENT. Cathcart, W. George, Child Development, v42 n3, pp755-765, Sep 71

Descriptors---*Intellectualization, *Mathematics, *Achievement, *Conservation (Concept), Student Characteristics, Primary Grades

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert E. Reys and Douglas A. Grouws, University of Missouri**

1. Purpose

The expressed purposes were:

a. To investigate the relationship between the various modes of rationalization for conservation and achievement.

b. To determine the frequence with which the different kinds of rationalizations were used.

c. To investigate differences in social and personal characteristics of subjects who preferred different kinds of rationalizations of conservation.

2. Rationale

An understanding of identity, reversibility and compensation seems to play an important role in the acquisition of conservation which in turn seems to affect mathematics achievement. The study is related to Piaget's postulate that conservation is "a necessary condition for any mathematical understanding." Some research has found a positive relationship between conservation and achievement in mathematics, but no previous studies have investigated the direct relationship between the modes of rationalization and mathematics achievement.

**The abstractors wish to thank Donald Balka, Raymond Cornett, Margaret Doerr, Marcy Frick, Dorothy King, Katherine Parli, Paul Rahmoeller, and Nadine Wasserman for their assistance in the preparation of this report.

Since other studies have shown that factors such as age, intelligence, social status, and language comprehension affect the correlation between conservation and mathematics achievement, then these variables should affect the correlation between modes of rationalizations and mathematics achievement.

3. Research Design and Procedure

A random sample of five second and third grade children was chosen from each of 12 schools. The 120 subjects selected tended to be middle class.

An eight-item conservation test, containing items similar to those used by Piaget, was designed to test each subject's ability to conserve properties.

After each item, a subject was asked, "How can you tell that they are still the same?" These responses were placed into one of seven categories.

a. Operational identity: no addition or subtraction of the property had taken place.
b. Substantive identity: property was invariant.
c. Reversibility: transform applied could be reversed.
d. Compensation: amount of property in one dimension was compensated for by an equal amount in another dimension.
e. Other rational responses.
f. Nonclassifiable responses.
g. No response.

An interjudge reliability of .83 was obtained between a panel of judges' categorization of responses and that made by the investigator.

A 39-item mathematics achievement test was devised by the investigator. Tests were also administered to obtain measures of each subject's vocabulary (WISC vocabulary section), intelligence (Raven's Colored Progressive Matrices), and listening ability (Cooperative Primary Tests--Listening, Form 12A). Also the Blishen scale was used to determine the socioeconomic status of each subject.

A Komogorov-Smirnov one-sample test was used to test the difference between the observed frequency with which each mode of rationalization was chosen and a rectangular distribution expected by chance ($\alpha = 0.01$).

A chi-squared analysis was used to examine the relationship between:

a. The mode of rationalization expressed for conservation and the type of conserver (total or partial).
b. The mode of rationalization and intelligence, socioeconomic status, vocabulary, and listening ability.
The modes of rationalization and achievement in various areas of mathematics.

4. <u>Findings</u>

a. Subjects demonstrated a preferred mode of rationalization, namely, identity.
b. There was no significant relationship between mode of rationalization and intelligence, socioeconomic status, vocabulary, and listening ability, age or sex.
c. The major finding of the study is that the kind of verbalization (as classified in this study) given to justify a conservation response is not an indicator of success in mathematics. However, mathematics achievement was higher for subjects who used several rationalizations for conservation than for those who could only give one.

5. Interpretations

The subjects were quite clear in their preference for the identity mode or rationalization. Reasons for this could be that identity is the easiest mode to voice quickly in a testing situation and/or there is less abstraction involved in giving this type of response. It appears that the mode of rationalization given cannot be used as an indicator of success in mathematics.

Since subjects who gave multi-mode responses had higher achievemnet in mathematics, teachers should teach a more general problem-solving approach rather than teaching specific rules for specific situations. This allows the student to draw from a wider background to solve a problem, rather than remembering a specific rule.

Abstractor's Notes

This article is addressed toward a significant issue and represents a contribution to the field of Piagetian research. In reviewing the article, several related research questions surfaced that were not answered in the manuscript. The abstractor's notes reflect these questions.

Although 120 subjects composed the original sample, the usable sample was 95. What accounts for this attrition and what determined a "usable" sample?

An instrument reflecting various conservation tasks was developed by the researcher. However there was no mention of test reliability, which becomes acute when one considers the limited number of tasks (eight).

The author states "the first rationalization verbalized was given the heaviest weight" in categorizing the subject's response. The scheme used to assign weight or even the actual weights is not clear. It appears from later discussion that most of the analyses are based on the subject's initial response.

A panel was used to check the author' categorization of responses. How many and what kinds of people served on the panel? How were the panel members prepared to categorize children's responses? Was the panel used only to judge agreement between panel and reseacher? How were conflicts between the researcher and panel resolved?

Several portions of the analyses involved partial and total conservers. Although objective criteria were stated for placing subjects into these groups, it was not explained why these groups were formed.

One section of the analyses (Table 1) reported the frequency of the different modes of rationalization used by the subjects. Did the subjects consistently use the same mode of rationalization? i.e. Were there total conservers who used reversibility on several questions and operational identity on several others? How were these modes of rationalization determined?

What levels of significance were used? Significance levels are not given for all tests, especially for those relationships found not to be significant.

This research suggests several questions that need further investigation including: Can the implication that subjects using multimodal responses perform better mathematically be generalized to infer that general problem solving techniques rather than specific rules for particular problems be taught? Also are there meaningful relationships between the modes of rationalization and various methods of problem solving? What actual importance does conservation or even the modes of rationization have for a mathematics program?

Robert E. Reys
Douglas A. Grouws
University of Missouri

EJ 050 585 270 SE 504 655

WAYS MATHEMATHICS TEACHERS HELP STUDENTS ORGANIZE KNOWLEDGE.

Cooney, Thomas J.; Henderson, Kenneth B., Journal for Research in Mathematics Education, v3 n1, pp21-31, Jan 72

Descriptors---*Instruction, *Logic, *Mathematics Education, *Research, *Teacher Behavior, Classroom Research, Classroom Techniques, Mathematical Concepts, Secondary School Students, Teaching Styles

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Donald J. Dessart, The University of Tennessee, Knoxville.

1. Purpose

To develop a taxonomy of ways (relations) that mathematics teachers assist students in organizing knowledge.

2. Rationale

The past ten years have witnessed numerous studies designed to classify and quantify through various observational and recording techniques the styles of teachers during sessions of classroom instruction. Some of these investigations have provided descriptions of instructional moves or teaching strategies of teachers, while others have sought to characterize the social skills which teachers may use to encourage learning activities. As the authors of this study observed, such studies have produced a vast amount of information concerning the teaching process, but they have not concentrated specifically upon an identification of relations that teachers find useful in aiding students to organize their knowledge in structures.

3. Research Design and Procedure

Ten teachers who were classified as "good" teachers by a review of information obtained from personal and professional contacts of the investigators were selected for the study. The criteria used to classify these teachers were not reported, but presumably subjective judgments of the contacts were among the major factors. Nine of the teachers taught in three middle-class, suburban schools, and the tenth taught in a small, rural school. The lengths of experience of the teachers ranged from 4 to 34 years with a mean of 14 years. Seven of the group possessed master's degrees or their equivalents, and all ten were fully certified.

Audiotapes were made of a total of 44 teaching sessions in which each teacher had one class taped in one subject during a period of four to five days. The subjects included 7th grade mathematics, 8th grade mathematics, 9th grade Algebra I, 9th grade Geometry, 10th grade Geometry, 11th grade Algebra II, 12th grade Trigonometry, and 12th grade Senior Mathematics. Each of these subjects was the topic of recording during one four- to five-day period with the exception of 9th grade Geometry and 11th grade Algebra II which were taped during two four- to five-day periods. No teacher was recorded in more than one subject area.

Transcripts of each of the tapes were made with insertions of any materials which had been written on the chalkboard during the classroom sessions. Relying upon theories previously developed by Henderson, Smith, Ryle, and others and by modifying these theoretical considerations with evidence gained in analyses of the transcripts, the authors identified nine organizing relations. Each relation is described as an ordered pair of concepts or items of knowledge in which one element of the pair is the concept or item under discussion and the other element is a concept or item to be incorporated by the student into a knowledge structure.

4. Findings

The nine organizing relations which were identified are: 1) the set membership relation (R_{SM}), 2), the set inclusion relation (R_{SI}), 3) the analysis relation (R_A), 4) the specifying relation (R_S), 5) the characterizing relation (R_C), 6) the explaining relation (R_G), and 9) the abstracting relation (R_{Ab}). Descriptions and examples of each of the relations are included in the report.

In addition to the development of the taxonomy of organizing relations, the frequency of occurrence of the relations for each class was reported. It was found that the smallest number of occurrences, seven, was identified in the Algebra I class; whereas, the largest number, 52, was found in the 10th grade Geometry class. It was also noted that the geometry classes represented only 30 percent of the total number of sessions recorded, but yet 54 percent of the relations occurred during those classes. In addition, the total number of relations ranged from 21 to 29 in the geometry classes, whereas, the number in the remaining classes was considerably less.

5. Interpretations

Because of the small sample size and the limited number of sessions recorded, the investigators were reluctant to generalize the results of this study to any other populations. They felt that additional studies might lead to an identification of other organizing relations, but it was not clear to them precisely how such relations might differ from the set obtained in this study

Abstractor's Notes

Due to the advent of reliable and inexpensive audio- and videotaping equipment, one might expect more studies of this kind in the future. Since the taxonomy presented in this report has benefited from a series of previous research studies directed by Henderson, one might feel confident in concluding that the taxonomy has sufficient maturity to have more than a cursory appeal to other researchers. Although the taxonomy will undoubtedly undergo further refinements, it could well become a model for future investigations. There is merit in recommending it as a model, if only to avoid the semantic difficulties which arise when investigators choose to start anew rather than to build upon the work of earlier researchers.

In developing refinements of the taxonomy, the question of its completeness needs further study. The authors were obviously concerned with this question, but perhaps other investigators will see additional relations or more encompassing relations than those defined in this study. For example, the relation of implicating is closely interwoven with the relation of explaining, and possibly a chain of implicating relations should be treated differently from a single implicating relation.

It would seem that studies of the patterns of relations used by teachers could lead to more fruitful results than merely recording the frequency of occurrence of particular relations. Do teachers generally pursue similar patterns of relations when teaching similar concepts or do these patterns differ significantly? If they do differ, which pattern leads to more productive learning? Answering questions of this kind would seem to have important implications for teacher training.

Donald J. Dessart
University of Tennessee
Knoxville

ERIC

EJ 046 714 520 SE 504 303
COMPARISON OF TEACHER-WRITTEN AND EMPIRICALLY DERIVED DISTRACTORS TO MULTIPLE-CHOICE TEST QUESTIONS Coppedge, Floyd L.; Hanna, Gerald S., Journal for Research in Mathematics Education, v2 n4, pp299-303, Nov 71

Descriptors--*Geometry, *Mathematics Education, *Multiple Choice Tests, *Test Construction, Objective Tests, Research, Teacher Developed Materials, Testing

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jeremy Kilpatrick, Teachers College, Columbia University

1. Purpose

To determine whether experienced teachers and student teachers, given a test question in a completion format, can supply the "best" distractors; that is, those erroneous responses that discriminate best between high and low scorers on the test as a whole.

2. Rationale

Constructors of multiple-choice tests who are concerned with students' ability to produce rather than recognize correct answers might well defined the best multiple-choice item distractors as those responses that discriminate best between good and poor students when the items are given in a completion format. Distractors for multiple-choice items are usually written to discriminate performance on a test given in a multiple-choice format, although "scores on multiple-choice tests built tc discriminate in the same way that completion tests discriminate probably correlate more highly with actual completion test scores than do scores on multiple-choice tests built with multiple-choice format discrimination in mind [p. 222]." Authorities who have recommended the use of data from completion items in writing distractors for multiple-choice items have stressed the frequency of an erroneous response but not its discrimination power.

3. Research Design and Procedure

A 33-item geometry test was administered in a completion format, with numerical and algebraic answers, to 357 (presumably tenth-grade) students in 15 classes of five teachers in three midwestern high schools. For each response to an item, a discrimination index was calculated as the difference between the fraction of the high scoring group (upper 27% of scores) who gave the response and the corresponding fraction of the low scoring group (lower 27% of scores).

Eleven experienced secondary mathematics teachers (five or more years of teaching) and 18 student teachers (university seniors) were given the same test, without access to the item-analysis data, and asked to supply the three best distractors for each item; that is, the distractors they thought would best discriminate if used in a multiple-choice format.

4. Findings

Data for seven selected items showed great variation from item to item in the teachers' collective efficiency in providing the most discriminating distractors. "Both experienced teachers and student teachers appeared unable to differentiate popular distractors from best-discriminating distractors [p. 302]."

For all 33 items, the loss of discrimination potential when the teachers' collective judgment was used instead of the best-discriminating incorrect responses averaged 72% for the experienced teachers and 68% for the student teachers. The average percentage of teachers who included the best-discriminating distractor among their three choices was 24 for the experienced teachers and 27 for the student teachers.

5. Interpretations

The slightly lower accuracy of the experienced teachers, compared with the student teachers, in anticipating best-discriminating errors may be due to (1) chance factors, (2) the students teachers' greater conscientiousness regarding research, (3) the student teachers' greater identification with the examinees' mental processes, or (4) the student teachers' more accurate perception of the distinction between distractors' discrimination power and their popularity.

The following procedure is suggested for multiple-choice test construction: (1) administer the items in completion format, (2) item analyze the results, and (3) select as distractors the most discriminating errors. If teachers are unable to follow this time-consuming sequence every time, they might improve their item-writing skills, their ability to anticipate students' errors, and their insight into students' mental processes if they were to use the procedure occasionally in conjunction with their standard procedures.

Empirical comparison is needed of the correlation of scores on completion items with scores on parallel multiple-choice items generated by student responses and by teacher judgments.

Abstracter's Notes

Misprints in the article include "Lore" for "Loree" in line 5 on page 300, "13" for "33" in line 25 on page 300, and "+0.8" for "+.08" (I guess) in Table 1 as the total discrimination of the teachers' responses for item 14.

The authors deserve credit for reminding test constructors that the most popular erroneous responses to an item in a completion format need not be the most discriminating distractors. Since the data were at hand, it is unfortunate that the authors do not report on the relationship between an erroneous responses' popularity and its discrimination power.

A long chain of reasoning (with most of the links in need of empirical verification) is required to argue that because the distractors teachers supply for a multiple-choice item are not the best-discriminating errors made on the associated completion item, the procedure recommended by the authors will yield more valid multiple-choice tests. The reader should understand that the study provided no information on how the various erroneous responses might actually have functioned as multiple-choice distractors.

Descriptive data are lacking on the sample of students and, more important, on the test items. Any conclusion that teachers cannot anticipate best-discriminating distractors should be tempered by the qualification that the study dealt with a restricted sample of items. And of course one should keep in mind that the discrimination power of distractors is not the only, or even the major, consideration in item analysis.

Jeremy Kilpatrick
Teachers College
Columbia University

ED 049 034 SE 010 929

IMPROVING TEACHERS' MATHEMATICS TUTORING SKILLS THROUGH MICROTEACHING: A COMPARISON OF VIDEOTAPE AND AUDIOTAPE FEEDBACK. Gall, Meredith, and others, Far West Lab. for Educational Research and Development, Berkeley, Calif.; Westinghouse Learning Corp., New York, N.Y. Pub. Date, Feb. '71, Note--22p.; Paper presented at the Annual Meeting of the American Educational Research Association (February 4-7, 1971, New York City, N.Y.) EDRS Price MF-$0.65 HC-$3.29

Descriptors--*Mathematics Teachers, *Microteaching, Research, *Teacher Education, *Tutoring, Video Tape Recordings

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Kenneth B. Henderson, University of Illinois, Urbana-Champaign.

1. Purpose

The reported purpose was to compare the relative effectiveness of two types of feedback, viz., videotape and audiotape, which can be provided immediately after a teacher has taught a lesson via microteaching.

2. Rationale

There has been research relating such independent variables in microteaching as lesson length, types and number of students, number and placement of sessions of microteaching, and the use of videotape by the supervisor during the critique with the dependent variable, viz., effectiveness of microteaching in training teachers. Ward (1970) found that feedback via audiotape and videotape were not significantly different in their effectiveness in improving teacher's use of higher cognitive questions. In light of the relative disadvantages of videotape vs. audiotape, e.g., initial expense, expense of maintenance, training of users of the equipment, and the difficulty of moving the equipment, it is worthwhile finding out under what conditions audiotape is just as effective as videotape.

3. Research Design and Procedure

Thirty-five elementary school teachers who were presently teaching were recruited as subjects for two experimental groups. They were assigned randomly either to the audiotape or the videotape feedback version of the treatment.

Prior instruction, called a Minicourse, for the subjects was provided by self-instructional packages of 4 to 6 lessons based on the microteaching procedures developed at the Stanford School of Education (Allen and Ryan, 1969). Each subject (1) viewed instructional and model films demonstrating several behaviorally-defined teaching skills, (2) practiced the skills in a micro-teaching lesson, (3) evaluated the feedback--videotape or audiotape, and repeated steps 2 and 3 to obtain further practice.

The teachers taught seven lessons by microteaching. Each consisted of tutoring one student on number operations, e.g., addition, subtraction, and another student on solving verbal problems. The Minicourse lasted four weeks and involved about thirteen hours of instruction. The only difference between the two treatments was the difference in the kind of feedback provided; one group received it via videotape and the other via audiotape.

A control group consisting of fifteen elementary school teachers similar to the experimental groups was used. Although not explicitly so reported, it is assumed that this control group experienced none of the treatment that the experimental groups experienced.

To get a measure of initial status, i.e., before the Minicourse, teachers in both the two experimental groups and in the control group conducted two videotaped ten-minute tutoring sessions. In the first of these sessions, each teacher tutored a student from his class who was having difficulty with number operations. In the second, he helped another student in solving a verbal reasoning problem.

To get a measure of final status, i.e., after completing the Minicourse, the teachers in the experimental and control groups repeated each of the two sessions. Each viable videotape was scored by two trained raters for occurrences of the tutoring techniques covered in the Minicourse, viz., use of five types of diagnostic questions, use of six demonstration techniques, giving an example to judge whether or not the student being tutored understood, providing practice, and the use of verbal praise for reinforcement. It was reported that the interrater reliability was generally high.

4. Findings

Teachers in both experimental groups made greater use of diagnostic questions that teachers in the control group. About 80% of the teachers in the experimental groups improved their use of this technique of teaching, and the average gain from the premeasure to the final measure was about 50%. Covariance analysis and t-tests revealed significant differences between both experimental groups and the control group, but not between the two experimental groups.

Teachers in both experimental groups improved both in the time spent using the six demonstration techniques presented in the Minicourse and in the variety of these techniques used. No comparison was made between the two experimental groups.

In employing one or more expamples to evaluate the student's learning, the group receiving audio-feedback improved significantly; the group receiving video-feedback improved but not significantly.

Both experimental groups improved significantly in providing practice for the students. Not a single teacher in the control group provided practice.

To judge the use of praise to reward students, the subsets of the three treatment groups were studied using the Wilcoxson signed-ranks test. Both experimental groups made a gain, but neither was statistically significant. The control group showed a loss.

From a questionnaire which both experimental groups answered, it was found that both groups had favorable reactions to the Minicourse. Only one teacher in the group receiving video-feedback would have preferred audio-feedback; eight of the fourteen teachers receiving audio-feedback would have preferred video-feedback.

5. Interpretations

It appears that for the pedagogical skills taught in the particular Minicourse, audio-feedback is as effective as video-feedback. Yet as the researchers state, "One might hypothesize that videotape feedback would be superior for skills involving a substantial 'visual' aspect, but there would be no difference between videotape and audiotape feedback for verbal skills." This conclusion seems confirmed by the finding that for training teachers in demonstration techniques, many of which contained a visual component, the teachers who received the video-feedback did better than those who received only audio-feedback.

Abstractor's Notes

This is useful research. Moreover, the findings are significant inasmuch as there is so much mystique

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about videotaping. The researchers rightly point out that the study needs to be replicated. There are possibilities for spin-off from this research as attempts are made to ascertain the conditions under which the two kinds of feedback are differentially effective.

Kenneth B. Henderson
University of Illinois
Urbana-Champaign

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EJ 038 319 340 SE 503 335
AN EXPERIMENTAL STUDY OF THE RELATIONSHIP OF HOMEWORK TO PUPIL SUCCESS IN COMPUTATION WITH FRACTIONS. Gray, Roland F.; Allison, Donald E., School Science and Mathematics, v71 n4, pp339-346, Apr 71

Descriptors--*Elementary School Mathematics, *Homework, Evaluation, Fractions

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Marilyn Zweng, University of Iowa

1. Purpose

The primary purpose of this study was to determine if drill type homework would improve pupils' skills in performing the four fundamental operations with fractional numbers. Two secondary questions were also studied: (1) Does the effect of homework on review type material differ from the effect of homework on new material? and (2) Is there a difference in the understanding of fractional number concepts between homework and non-homework pupils?

2. Rationale

The authors state that only nine experimental studies of the effect of homework on elementary school pupils' achievement have been reported in the literature in the period 1928-1969. Of these only two dealt specifically with homework in arithmetic. In a 1965 study Koch concluded that homework of the reinforcing type does increase the arithmetic achievement of sixth graders. The results of Maertens' 1969 study do not agree with Koch's. Maertens found that for third graders, homework in arithmetic did not improve either computational skills or problem solving.

Recent surveys indicate that teachers, parents, college professors and students favor homework and that there is a growing trend toward more homework in the elementary school. Hence the question of the contribution of homework to achievement is an important one.

3. Research Design and Procedure

Fifty-five sixth grade students completed all phases of the experimental procedure which took place over a period of 8 weeks. The eight weeks were divided into two experimental periods of 4 weeks each. Experimental Period 1 was devoted to review materials. The content of the lessons was addition and subtraction of fractional numbers. During Experimental Period 2, multiplication and division of fractional numbers were taught. This material was new to the subjects. Two teachers, designated as Teacher A and Teacher B participated in the study. Within each teacher's class the children were randomly assigned to either Treatment I which consisted of three 20 minute homework assignments per week or Treatment II for which no homework was assigned. At the end of the first four week period, children in Treatment I froups were placed in Treatment II groups and vice versa.

Four sets of pre-test scores were used as co-variates. Three were standardized arithmetic achievement tests and the fourth was an IQ test.

The criterion measures were an achievement test on addition and subtraction of fractional numbers administered after completion of Experimental Period I, an achievement test on multiplication and division of fractional numbers administered after the completion of Experimental Period 2 and individual interview tests of arithmetic understanding. Interviews were administered at the end of each of the two Experimental Periods to 5 children randomly selected from each of the four groups, a total of 20 per period. Both the achievement tests and the interview tests were author made.

4. Findings

Analysis of covariance was used to examine the results of the tests of computational skills. The variables considered were method, teacher and sex for each of the two experimental periods. None of the F ratios was significant at the 5% level. In particular, no differences in achievement were attributable to homework either for review material or for new content.

On the interview test of understanding, the responses of all 40 students examined were identical, hence no differences in understanding of fractional number concepts were due to homework. The uniformity of responses was due to the fact that none of the students appeared to have much understanding of underlying principles.

5. Interpretations

The authors conclude that while there was no evidence to suggest that homework was in any way harmful the findings may indicate that drill type homework is, in fact, unrelated to pupil growth in computational skill.

It is also noted that the negative findings for all groups on the concepts test indicate that "... the goal of understanding so prominent in modern mathematics programs is not being met..." and that this question needs further study apart from the homework question.

Abstractor's Notes

The experimenters, themselves, raise two important questions about the design of their study: (1) Should the three day a week homework assignments have been raised to five days a week? and (2) Should homework assignments have been in terms of a constant number of practice examples rather than a constant period of time? The question raised in (1) may be crucial to the results. In the Koch study cited earlier which was also conducted with sixth graders, the two homework treatment groups received 75 minutes and 150 minutes of homework per week, respectively, compared to the 60 minutes of homework per week in this study. Koch reported maximum achievement in the 150 minute per week treatment. Although the abstractor is reluctant to admit this, the amount of time spent on homework may be the determining factor in whether or not homework contributes to achievement.

However, neither the Koch study nor this one considered several other important variables. One of these is attitude. In this study, the statement is made that "there was no evidence that homework was in any way harmful." Only achievement was examined, though, not attitude. This writer strongly suspects for many children attitude towards mathematics might be affected negatively by homework. If, in fact, as the authors contend, homework does not contribute to achievement, a strong case against homework might be built on the attitude question.

Another variable not examined in this study was ability or prior achievement. Frequently slower students have more homework assignments than more able students because they don't complete their work in class. Do they benefit from the additional practice or is their anxiety about mathematics increased?

Finally, how are socio-economic factors related to the homework question? Does the effect of homework on a child's achievement depend on the home environment and whether or not he can get help from parents or siblings?

With the trend toward giving increasing amounts of homework (which is usually heavily weighted with homework in mathematics), the paucity of research in the area, and the conflicting results between the Koch study and this one, this unresolved question certainly needs further study with considerably more attention g `n to important variables not considered in either stu、y.

Marilyn Zweng
University of Iowa

EJ 033 863 340 PS 500 878
AN EXPERIMENTAL STUDY OF TWO METHODS OF PRESENTING THE INVERSION ALGORITHM IN DIVISION OF FRACTIONS Ingersoll, Gary M., California Journal of Educational Research, v22 n1, pp17-25, Jan 71

Descriptors--*Experimental Programs, *Mathematics Instruction, *Teaching Methods, *Fractions, *Grade 6, Algorithms, Testing.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Alan R. Osborne, The Ohio State University.

1. Purpose

To compare the effectiveness of two strategies of teaching the inversion algorithm for division of fractions to sixth grade students. Strategy CF, the complex fractions approach, is based upon rewriting

$$\frac{a}{b} \div \frac{c}{d} \text{ as } \frac{\frac{a}{b}}{\frac{c}{d}}.$$

The numerator and denominator are then multiplied by the reciprocal of the denominator. Strategy A, the associative strategy, assumes division defined as

$$\frac{a}{b} \div \frac{c}{d} = N \iff N \cdot \frac{c}{d} = \frac{a}{b}$$

Children must then do the equivalent of finding an x such that $N = \frac{a}{b} \cdot x$ by examining

$$\left(\frac{a}{b} \cdot x\right) \cdot \frac{c}{d} = \frac{a}{b}$$

and using the associative property and $a \cdot 1 = a$.

2. Rationale

The author develops a rationale based upon 1) the difficulty of children in understanding and using the algorithm and 2) the complexities of meaningful explanations for teachers and curriculum developers.

3. Research Design and Procedure

Three treatments were used each of which was based upon programmed materials:

(a) Treatment CF
(b) Treatment A
(c) "Control". The control treatment was a random selection of items from treatments CF and A randomly sequenced. The programs of treatments CF and A were "deductive" programs.

Two experiments were conducted. In experiment I two schools were used to provide 60 subjects. History indicated differences in school programs corresponded to differences in criterion measures. Experiment II utilized children in three classes within a single school but the classes were homogeneously grouped.

The author reports "the investigation was conducted over a period of four days...." This includes one day for the pretest, one day for a program common to all three treatments, one day for the different treatments, and one day for the posttest.

Analysis of covariance was used to interpret results.

4. Findings

(a) Experiment I. Differences corresponding to the instructional programs of the two schools were more apparent than differences derivative from treatment. Children in the control or random treatment did not perform significantly different than those in the other treatments for some subscale measures of the posttest.

(b) Experiment II. Treatment CF produced significantly higher scores than the random treatment unlike Treatment A.

Abstractor's Notes

The two treatments, CF and A, are based upon contexts which are complex both from a mathematical and a psychological points-of-view. This research is, essentially, based upon one day treatments. Given the complexities of each strategy, how could the researcher expect impact upon these learning tasks in one short day? The relative power of the random treatment in both experiments I and II is a red flag that one should not consider the results interpretable. Teachers have found merits to both instructional strategies, CF and A. No reason for electing one of these strategies over the other can be found in this research.

Alan R. Osborne
The Ohio State University

EJ 033 210 150 SE 502 714

THE FIRST NINE YEARS--A STUDY OF THE ADVANCED PLACEMENT PROGRAM IN MATHEMATICS Lefkowitz, Ruth S., Journal for Research in Mathematics Education, v2 n1, pp23-35, Jan '71

Descriptors--*Advanced Placement, *Followup Studies, *Student Placement, *Secondary School Mathematics, Mathematics

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Stephen S. Willoughby, New York University

1. Purpose

To evaluate the Advanced Placement mathematics program in one New York City high school through a questionnaire sent to graduates of the program.

2. Rationale

The College Entrance Examination Board Advanced Placement mathematics examination has been in existence for more than 15 years. In that time its popularity has increased rapidly, but "little attention has been given to the evaluation of the Advanced Placement program." The article is based on a doctoral dissertation, by the author, conducted at Columbia University.

3. Research Design and Procedure

Between 1957 and 1965 inclusive, a substantial (but unspecified in this report) number of students took the advanced placement course at the New York City high school used for the study. The investigator was able to obtain addresses for 271 of these students. Two page questionnaires were sent to these 271 individuals, and 182 responded. The post office returned 33 of the questionnaires because the individuals had moved and left no forwarding address, and for the remaining 56 there was no reply. The average score on the Advanced Placement (AP) test for the 112 students who took the test and responded was essentially the same (3.1) as the average score for all of those (152) who took the test. School and CEEB records were used to obtain IQs, high school averages, Scholastic Aptitude scores, Mathematics Achievement scores and Advanced Placement scores. All other information was obtained from the questionnaires. On the questionnaire, individuals were asked to provide information regarding college and graduate school attended, major, mathematics courses taken, placement and credit related to the advanced placement program, and their opinion of the program.

4. Findings

Of the 182 students who responded to the questionnaire, 95 were offered advanced placement and 76 accepted; while 58 were offered college credit and 52 accepted. Seventeen of the 70 who did not take the test were offered advanced placement and three were offered college credit. All 16 who scored 5 on the AP test were offered advanced placement and 15 were offered college credit. Only one of the nine students who scored 1 on the test was offered advanced placement and he was not offered college credit. For scores of 2, 3, and 4, offers of placement and credit could not be easily predicted by the score, but the higher the score, the more common were offers of advanced placement and/or credit.

Eighty (44%) of the respondees took Calculus 1 in the first semester of college, 28 took calculus 2, 28 took calculus 3, 20 took no mathematics in the freshman year. Of the remaining 26, 18 took some form of linear algebra, and the other 8 took a more advanced course in the general area of analysis.

Of the 182 students, 50 majored in the natural sciences, 38 majored in mathematics, 28 majored in engineering or accounting, 27 majored in the social sciences, 15 majored in English, literature or foreign languages, and the remainder majored in other subjects or were undecided when the questionnaire was returned.

Fifty-nine of the 73 students who completed the program by or before 1960 went to graduate school, of whom 6 majored in mathematics, although 23 studied some mathematics in graduate school.

Approximately 90% of the respondees stated that they would recommend the Advanced Placement Program in mathematics to present high school students.

In an "additional comments" section at the end of the questionnaire, several of the students indicated their high regard for the teacher of the Advanced Placement course. They also indicated that the Advanced Placement course was not as abstract and theoretical as corresponding college courses. Apparently the students perceived this as a weakness in the advanced placement course.

There was little uniformity in the treatment of the AP students by the colleges, particularly during the early years of the program, though many colleges subsequently formulated and used definite advanced placement policies.

5. Interpretations

The investigator recognizes the fact that the limitations in her study make it difficult to generalize to other students, and that college achievement as well as student opinion is an important factor in evaluating the Advanced Placement program.

Abstractor's Notes

In spite of the obvious limitations of this study as a piece of educational research, its information, combined with other available information, makes clear the fact that the Advanced Placement Program is, and probably ought to be, a strong program that is increasing in size and influence.

Stephen S. Willoughby
New York University

ERIC

EJ 049 905 040 SE 504 658

ATTITUDES OF ELEMENTARY SCHOOL PUPILS AND THEIR PARENTS TOWARD MATHEMATICS AND OTHER SUBJECTS OF INSTRUCTION. Levine, George, Journal for Research in Mathematics Education, v3 n1, pp51-58, Jan 72

Descriptors---*Elementary School Students, *Parent Attitudes, *Questionnaires, *Student Attitudes, English Education, Mathematics Education, Science Education, Social Sciences

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Arthur F. Coxford, University of Michigan

1. Purpose

To investigate and further explore the relationship between the attitudes of elementary school pupils and their parents toward mathematics and other areas of instruction.

2. Rationale

Previous studies of attitudes toward mathematics have concentrated on the relationship between teacher, prospective teacher, and student attitudes. The tacit assumptions which undergird these studies are that attitudes of the teacher and student are related and that attitudes are significantly correlated with achievement.

Other investigators have demonstrated the existence of some relationships between student attitudes and attitudes held by parents. The present study seeks to investigate attitudinal similarities for elementary school students and their parents. Student and parent attitudes toward mathematics were not measured, but rather a relative ranking of mathematics in relation to three other school subjects was sought.

3. Research Design and Procedures

The sample included elementary school youngsters in three sixth grades, two third grades, and one fourth grade (N=144) from one elementary school. The parents of these youngsters comprised the sample of parents.

To measure the student and parent attitudes, two author constructed forced-choice questionnaires were prepared and administered. Both parents and students were asked to rank the subjects of English, mathematics, science, and social studies in response to nine statements. Student questionnaires were administered by the investigator to intact class groups. Parent questionnaires were sent home and returned at a 73% rate. The items included on each questionnaire are given below.

Student Questionnaire

1. I enjoy studying this subject the most.
2. I do my best work in this subject.
3. I think this subject is the most important subject I study in school.
4. My parents are able to help me most in this subject.
5. My parents feel that this should be my best subject.
6. I wish this was my best subject.
7. I feel I need the most help in this subject.
8. I feel my teacher does her (his) best job in teaching this subject.
9. This is my teacher's favorite subject.

Parent Questionnaire

1. When I was a student I enjoyed studying this subject the most.
2. When I was a student I did best in this subject.
3. I now feel that this subject was the most worthwhile subject I studied in school.
4. I feel most competent in helping my child in this subject.

5. I feel that my child has the ability to do best in this subject.
6. I feel it most important that my child do best in this subject.
7. I feel my child could use the most help in this subject.
8. I feel my child is receiving in school the best instruction in this subject.
9. If I had the chance to go to school all over again, I would try hardest to do well in this subject.

To determine the statistical significance of the rankings obtained, a binomial distribution was assumed with success defined as ranking mathematics 1 or 2, and failure a ranking of 3 or 4. The normal approximation to the binomial distribution was employed for the purpose of computation.

4. Findings

The data were grouped on the basis of pupil characteristics into (a) Boys, (b) Girls, (c) Grades 3-4, (d) Grade 6, and (e) Total for both the students (N=144) and the parents (N=105).

1. For all student groups mathematics was ranked highest for all statements by statement seven.
2. The parent groups differed markedly from the comparable student groups. Mathematics was consistently ranked highest for only statements eight and nine. For statements one, two, and four parent groups rated English highest.
3. For all student groups the number of "success" responses for statements 1-6, 8 and 9 were statistically significant ($p < .01$ for all but grade 6, statement 4, which had $p < .05$). The number of success responses was not statistically significant for any student group on statement seven.

4. For parent groups the number of "success" responses were consistently significant ($p < .05$ or $p < .01$) for statements 3, 6, 8 and 9. For no parent group was the "success" response significant for statements 1 and 2. There were mixed responses on the remaining statements.

5. <u>Interpretations</u>

1. Pupils ranked mathematics highest with respect to importance, enjoyment, best subject, and subject for which they believed the teacher did her best teaching. They also believed that parents thought mathematics should be their best subject and that parents were best able to help them in mathematics.

2. Parents ranked both mathematics and English highly. They indicated that English was as important as mathematics and just as important for their children to do well in. Parents recalled doing best in English but also opined that they would try hardest in mathematics if given a chance to return to school.

3. Parents tended to expect more in mathematics from their sons than from their daughters.

4. Both students and parents viewed mathematics in a favorable light in comparison with the subjects of English, science and social studies.

Abstractor's Notes

The author has noted some limitations to this study. One such limitation was that the author administered the questionnaire to students. This could have influenced student responses because the students knew he had special interest in mathematics. There was also no attempt to control possible student influence on parental response, nor was an attempt made to gain responses from the 39 non-responding parents. This is always a matter of concern when one uses survey techniques.

A question which was not answered was which parent, mother or father or both, completed the questionnaire. If Hill's results are valid then mothers and fathers may respond differently to statements concerning the importance of mathematics for their children. Further investigation in this area could provide more accurate information.

This study tacitly assumed that there was a relationship between student and parent opinions. This conjecture could be investigated, and perhaps should be prior to further investigation of the relation between opinions.

Arthur F. Coxford
The University of Michigan

EJ 035 466 340 PS 500 999
WHAT PUPILS KNOW ABOUT VOCABULARY IN MATHEMATICS--1930 AND 1968 Olander, Herbert T.; Ehmer, Charles L., Elementary School Journal, v71 n7, pp361-367, Apr 71

Descriptors--*Mathematical Vocabulary, *Mathematical Concepts, *Comparative Analysis, *Test Results [Buswell John Vocabulary of Arithmetic Test, Contemporary Mathematical Vocabulary Test]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Lewis R. Aiken, Jr., Guilford College

1. Purpose

(a) To compare the understanding of mathematical terms by today's children with that of children several decades ago.

(b) To determine whether today's children understand the terms in modern mathematics textbooks better than the terms in older textbooks.

(c) To determine whether today's children know the mathematical terms taught years ago as well as yesteryear's children.

(d) To compare the achievement in mathematical vocabulary of boys and girls.

2. Rationale

This is the report of an empirical study which compared knowledge of arithmetic vocabulary in a sample of elementary school children in 1968 with that in a sample of elementary school children in 1930. The earlier data were obtained from a study reported by Buswell and John (1931).

3. Research Design and Procedure

The Buswell-John Vocabulary of Arithmetic Test, a 100-item, four-choice test that had been administered in 1930 to 500 pupils in each of grades 4, 5, and 6, was re-administered in 1968 to 400 pupils (200 boys and 200 girls) in each of grades 4, 5, and 6. The latter group, consisting of "randomly" selected classes in six school districts "randomly" selected from 94 districts in western Pennsylvania, was also administered a specially constructed Contemporary Mathematical Vocabulary Test. This test, consisting of 100 four-choice items, was prepared from an initial list of 367 terms obtained in a survey of 15 recently published arithmetic textbook series. The reliabilities of the Buswell-John Vocabulary of Arithmetic Test are reported as .89-.95, and the reliabilities of the Contemporary Mathematical Vocabulary Test as .73-.93. The

1968 group was also administered the arithmetic and reading subtests of the Stanford Achievement Test, Intermediate (1964) and the Kuhlmann-Anderson Intelligence Tests, Seventh Edition (1963).

4. Findings

The results are presented as percentages, arithmetic means, and t ratios. On the Buswell-John Test, a larger percentage of the 1968 group, as compared to the 1930 group, was correct on 74 of the items at the fourth-grade level, 59 items at the fifth-grade level, and 48 items at the sixth-grade level. For purposes of comparison, the mean test scores of the two groups are given in the following table:

Means on Buswell-John and Contemporary Tests of 1930 and 1968 Groups

Grade	Buswell-John Test 1930	Buswell-John Test 1968	Contemporary Test 1968
4	58.07	65.06	49.34
5	69.23	72.59	57.87
6	77.09	78.55	64.02

All differences among the three grade means within each year group are significant at the .01 level by t tests. Results of tests of significance of differences between means on the Buswell-John Vocabulary Arithmetic Test for the 1930 and 1968 groups or between the Buswell-John Test and the Contemporary Mathematics Vocabulary Test for the 1968 group are not reported, but the writers noted that the 1968 group scored higher on the Buswell-John Test than on the Contemporary Test. Also, the mean scores for the 1968 girls were higher than those for the 1968 boys at all grade levels on both tests, although only five of the six mean differences were statistically significant. Finally, on the arithmetic subtest of the Stanford Achievement Test the mean score of the girls was higher than that of the boys at all grade levels on arithmetic computation but only at the fourth-grade level on arithmetic concepts.

5. Interpretations

The writers conclude that one important contribution of the research is the hundred-item Contemporary Mathematical Vocabulary Test, the terms of which, as well as those on the earlier Buswell-John Vocabulary of Arithmetic Test, are presented in a separate table. With respect to comparisons between the 1930 and 1968 groups on the Buswell-John Test, it is concluded that today's children are learning mathematical vocabulary better than children did 40 years ago. This difference seems to disappear, however, as the children grow older, and in any case certain terms were learned better by yesterday's children and others by today's children.

Concerning the relationships of grade level to knowledge of arithmetic vocabulary, knowledge of both traditional and modern mathematical vocabulary increases as a function of intermediate grade level. In addition, today's children seem to have a better understanding of traditional mathematics vocabulary than of contemporary mathematics vocabulary. There are also significant sex differences in knowledge of mathematical vocabulary and arithmetic computational ability, girls being superior to boys at all intermediate grade levels. Finally, the finding, for both sexes, of significantly lower mean scores on arithmetic computation than on other areas of mathematics leads to the question of whether new mathematics programs are emphasizing understanding of number structure at the expense of computational proficiency.

Abstractor's Notes

Not enough information concerning the samples and method used in obtaining the reliability coefficients reported for the Contemporary and Buswell-John Tests is given. Are these split-half or test-retest coefficients?

Precisely how were the 1930 and 1968 samples selected? Some information is given on the procedure for selecting the 1968 sample, but it is not clear what the writers mean by random.

The writers draw conclusions about the meaning of a percentage difference without using an appropriate statistical test. Also, the conclusion, based on percentages, that the 1930 sixth-grade group was slightly better than the 1968 sixth-grade group on the Buswell-John Test is not consistent with the difference between the means of these groups (see table above). In any case, neither difference is statistically significant.

The writers conclude that the 1968 group scored lower on the items of the contemporary test than on the earlier test, but no supporting statistical test is mentioned. They also conclude that the differences between grades are greater for the 1930 pupils than for the 1968 pupils, but again no statistical test is referred to. Finally, there are at least two inconsistencies in the means of the tables in the paper: The figure 64.02 in Table 4 should be the average of the figures 61.38 and 66.34 in Table 7, but it isn't. The figure 72.59 in Table 5 should be the average of the figures 70.87 and 74.24 in Table 7, but it isn't quite.

Lewis R. Aiken, Jr.
Guilford College

EJ 050 329 180 TM 500 454

THE RELIABILITY AND VALIDITY OF THE CONTEMPORARY MATHEMATICS TEST Renzulli, Joseph S.; Shaw, Robert A., Educational and Psychological Measurement, v31 n4, pp973-76, W 71

Descriptors--*Achievement Tests, *Mathematics, *Test Reliability, *Test Validity, Item Analysis, Junior High Schools, Standardized Tests, (*Contemporary Mathematics Test)

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James W. Wilson, University of Georgia

1. Purpose

To obtain additional empirical information relating to the reliability and validity of the Contemporary Mathematics Test--Junior High Level.

2. Rationale

The authors argue that no independent research studies relating to the reliability and validity of the Contemporary Mathematics Test series have been reported, even though critics (Romberg, 1968; Smith. 1967) have expressed concern about the lack of empirical support for the instrument.

3. Research Design and Procedure

A sample of 232 students in grades seven, eight, and nine who were enrolled in a modern mathematics program for a minimum of three years were tested using two forms of the CMT at the beginning and end of a school year. Reliability estimates were computed using Kuder-Richardson formula 20 for each administration and a Pearson product-moment correlation between the pre- and posttests.

Validity data consisted of correlations between the two CMT forms and scores on 1) a comprehensive final examination, 2) final grades in mathematics, 3) Arithmetic Reasoning and Arithmetic Fundamentals subtests of the California Achievement Test, 4) the mathematics portion of the Sequential Test of Educational Progress, and 5) the Lorge-Thorndike Intelligence Test.

The reliability data are reported for each grade level (N=103, seventh; N=96, eighth; N=33, ninth) and for total. The validity data are reported for small subsamples (N=33 to N=70) on which data were available for the particular variables.

4. Findings

The internal consistency estimates for the CMT on the sample, using the Kuder-Richardson formula 20 ranged from .78 to .88, very similar to the range reported for the CMT on the norming samples in 1965. The congruent validity data produced high correlations of the CMT with the STEP and IQ measures only.

5. Interpretations

The investigators concluded the CMT reliability was favorable both for stability and internal consistency. The low correlations of the CMT and either course grades or final examinations were interpreted to suggest "that the test is largely independent of the content to which the subjects in the study were exposed."

Abstractor's Notes

The data in this study did not warrant, and did not receive, much interpretation by the authors. Surely Romberg (1968), Smith (1967), Begle (1972), and O'Brien (1972) have called for further empirical investigations of this test series. But their concern has been more for the adequacy of 1965 norms in 1972, with predictive validity, and relative performance of groups pursuing different programs. This study touches only on the predictive validity and is very limited in that regard.

Technically, the study has some glaring inadequacies --e.g., no information is given on the selection of the sample, the calculation of congruent validity correlations does not describe any grade level separations, no rationale is given for the selection of the various measures used in the validity study, no identification or description was geven for the "modern mathematics program" to which the students had been exposed, etc.

The Contemporary Mathematics Test has been criticized severely for its content validity. Smith and Romberg have each criticized the entire CMT series on this point; Begle and O'Brien have each made the same criticism of the elementary and junior high levels of the CMT. Renzulli and Shaw accept that the CMT "is designed to measure the extent to which students have mastered course content in modern mathematics." That is, they dod not address themselves to the issues of content validity. In my view, unless the content validity of the CMT can be argued, studies such as this one can contribute very little.

James W. Wilson
University of Georgia

References

Begle, E.G. Review of Contemporary Mathematics Test: Elementary and Junior High. In O.K. Boros, The Seventh Mental Measurements Yearbook. Highland Park, N.J.: The Gryphon Press, 1972.

O'Brien, T.C. Review of Contemporary Mathematics Test: Elementary and Junior High. In O.K. Boros, The Seventh Mental Measurements Yearbook. Highland Park, N.J.: The Gryphon Press, 1972.

Romberg, T.A. Test Review: Contemporary Mathematics Test Series. Journal of Educational Measurement, 1968, 5, 349-351.

Smith, F.M. Test Review: Contemporary Mathematics Test. Journal of Educational Measurement, 1967, 4, 123-124.

ED 053 966 24 SE 012 222

THE DEVELOPMENT OF AN INDIVIDUALIZED INSTRUCTIONAL PROGRAM IN BEGINNING COLLEGE MATHEMATICS UTILIZING COMPUTER BASED RESOURCE UNITS. FINAL REPORT. Rockhill, Theron D., State Univ. of New York, Brockport Coll. at Brockport. Spons Agency--Office of Education (DHEW), Washington, D.C. Bureau of Research. Pub Date Jun 71, Note--79p, EDRS Price MF-$0.65 HC-$3.29

Descriptors--Calculus, *College Mathematics, *Computer Assisted Instruction, Individual Instruction, *Individualized Instruction, Individualized Programs, *Mathematics Instruction, *Programed Materials, Set Theory, Trigonometry.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James K. Bidwell, Central Michigan University.

1. Purpose

"To develop and evaluate an individualized instructional program in pre-calculus mathematics. Computer based resource units were developed which produce individualized instructional units based upon the student's background and understanding of each pre-calculus topic."

2. Rationale

Students enter calculus classes with variable mathematics maturity. The problem was to assure that all students have the necessary background by requiring students to study only those topics in which he is deficient. A person learns as an individual. The instructor should diagnose, prescribe, and evaluate the progress of the individual student. "The need for extensive memory and adaptability to repetitive processes suggest that the computer could be used in this individualized approach." "Mathematics is a discipline which should lend itself to computerized individualization of instruction." The development of computer based resource units was based on the research model of J. Fred Weaver (Arithmetic Teacher, May 1969, p. 379 - 382).

3. Research Design and Procedure

During the summer of 1970 four resource units in Algebra; Analytic Geometry; Sets, Relations, and Functions; and Elementary Functions were developed by three experienced calculus instructors and the project director. A resource unit was defined as a collection of suggested learning activities and materials, organized around specific objectives. Objectives were determined for each unit. Five option multiple-choice questions were written for each objective, three or four questions per objective. The fifth option for every question was "I do not know." Resource matierials were selected for independent treatment of objectives, readability, and organization.

Computer programs were written for each unit. The programs were designed to take student pretest answers as input, to determine objectives not satisfied, and to provide printed output of instructional material for each objective not satisfied. Later, a subroutine was added to provide a print-out of unit bibliography with each output. Unit programs were made compatible with a test analysis program, SUPERGRADER. A general program, INFOS, was written to incorporate the above features and permit input of all parameters, headings, and messages.

All objectives, pretest items, computer programs, and print-outs are included as an appendix of the report.

Two pre-calculus classes were taught by investigator; one used project resource materials and the other followed usual class patterns. Two resource centers were provided for the resource unit class. The resource unit class met once a week for fourteen weeks. Unit pretests were given and students worked on assignments for unsatisfied objectives.

The resource units were evaluated in two ways. First, achievement of students using resource units was compared with students in control class using one-way ANOVA with four achievement variables: unit test I, unit test II, Final Exam, and final letter grade. Subjective evaluation of student reaction was also made. A subset of the four pretests was used as a pretest for three hundred beginning calculus students. This pretest was compared with the final letter grade in calculus as a predictor of success. This pretest was also given to the control precalculus class.

4. <u>Findings</u>

The correlation matrix for control group data showed an average correlation of .35 between the pretest and the achievement variables. Considerable variability occurred in the experimental group correlation between unit pretests and achievement variables (from .19 to .70). Means and s.d. for all achievement variables are given. High s.d. were observed (18.5 for the 49 item algebra pretest, for example). The ANOVA for the achievement variables produced an F-ratio of 5.80 for unit test II (significant at 5% level). Others were not significantly different for the two classes.

The mean on the calculus pretest for calculus students was 53.1%; for the pre-calculus control group it was 23.7%. A correlation of .45 was found between final grades and pretest socres by the calculus students.

5. Interpretation

The researcher regrets not being able to use multivariate ANOVA, with a suitable covariate, which was not provided for in the design. This experiment may not have been the best test for the resource unit concept, since no provision was made for accelerating students through the program. The best application may be to provide review and remedial work for students enrolled in a course for which not all prequisites have been met. The general program INFOS could be used in any discipline besides mathematics. For each objective the user must provide a title, the number of questions, the question numbers, the number right required, and the message to the student if the objective is not satisfied.

Abstractor's Notes

It is clear that the sample size (control and experimental classes had a total of 44 students) together with a minimum research design resulted in the study having little statistical significance. No acceptable comparison on learning achievement was possible. What is more important was the development of an approach to the management of individualized instructional programs. The pretests and computer programs were designed to handle large groups of students not just twenty-one. It is regrettable that a larger sample could not have been processed and true individual progress achieved. It is the opinion of the abstractor that only through the use of the computer to manage the instructional process can significant progress be made in individualizing the mathematics curriculum for large numbers of students. This study provides a useable framework for such computer use. Hopefully, INFOS and its refinements will become well known and well used.

It should be noted that given the pretest questions for the objective, the criteria for staisfying the objective, and the list of corresponding readings, an individual student could easily produce his own assignments given only the pretest results. The computer program of this study was merely a convenient way for the instructor to inform the student of his task.

James K. Bidwell
Central Michigan University

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THE EFFECT OF DELAY OF KNOWLEDGE OF RESULTS ON THE ACQUISITION AND RETENTION OF NOVEL MULTIPLICATION FACTS. Rogers, W. Todd, Hopkins, Kenneth D. Colorado Univ., Boulder. Lab. of Educational Research. Pub Date Feb 71.
Note--26p.; Paper presented at the Annual Meeting of the American Educational Research Association (Feb. 4-7, 1971, New York City, N.Y.) EDRS Price MF-$0.65 HC Not Available from EDRS.

Descriptors--Arithmetic, *Elementary School Mathematics, Grade 3, *Learning, *Mathematical Concepts, *Multiplication, Reinforcement, *Retention

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Gerald R. Rising, State University of New York at Buffalo.

1. Purpose

"To determine the effect of delay of knowledge of results on the learning of novel multiplication facts through drill in a classroom situation." (p. 4)

2. Rationale

Although studies with rats suggest that delay of reinforcement impairs acquisition of new learning with no predictable effect on retention (Renner, Psychological Bulletin, 1964, 341-361), the results of similar studies with humans have been mixed and inconclusive. Most of the studies with humans have also been with individual subjects under strict laboratory control restricting the value of application to the classroom setting.

3. Research Design and Procedure

Nine classes of heterogeneously grouped third-grade pupils from five elementary schools in a metropolitan area were assigned three treatments using a stratified ordering procedure. A pretest indicated 14 novel (difficulty index $\leq .05$) multiplication facts. Then on each of five treatment days a brief study period with the list of these facts was followed by a taped (oral) quiz, students recording answers on an answer sheet. The tape treatments were:

Immediate Knowledge of Results: "Number 1: 6 times 7 equals (3 sec. pause). 6 times 7 equals 42. Number 2: 6 times 8. . ."

Delayed Knowledge of Results: "Number 1: 6 times 7 equals (3 sec. pause). Number 2: 6 times 8. . ."

No Knowledge of Results: same as Delayed.

For the first two groups the list of facts were read with answers at the end of the test as well. A post test on the final treatment day and a retention test six calendar days later were administered.

Acquisition, pre-test first post-test difference, and retention, pre-test retention test difference, served as dependent variables. Age and I.Q. scores were found not to affect acquisition on either variable. A four factor mixed model analysis of variance was employed in which 15 pre-selected subjects were nested within each of the three classes, which were in turn nested within each of the three treatments. For the fourth (repeated measures) factor, acquisition and retention scores, a procedure suggested by Greenhouse and Geisser (Psychometrika, 1959, 95-112) for determining significance of obtained F was used. A further pooling strategy was rejected because of interclass differences.

A second four factor mixed model analysis of variance was performed with days as repeated-measures factor, again using score differences.

4. Findings

For neither analysis did treatment means differ significantly. Significant differences were found for classes within treatments only. (The data was also used to explore differences with non-novel facts with similar results.)

5. Interpretations

"The findings obtained failed to support the generalization that where knowledge of results is given to one group, either immediately or delayed, and withheld from a second, comparable group, the former will reach a higher level of proficiency." (p. 16) The task was a difficult one for these children which may account for lack of differences. This suggests replication with more instruction. The delay interval might also be modified.

Abstractor's Notes

This is quite evidently a very careful study of a problem that has practical overtones for the classroom teacher. That no answers were provided does not take away from the value of the search for those answers. One of the difficulties of the study, the interclass differences that prevented the authors from applying additional statistical tools, would be controlled, as they note, by administering the three treatments to randomly selected sub groups within each class. It occurs to me that the tape format provides exactly the vehicle needed for this procedure in a replication.

Gerald R. Rising
State University of New York
at Buffalo

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RELATIONSHIPS OF LEARNING VERBAL AND SYMBOLIC STATEMENTS OF MATHEMATICAL RULES, FINAL REPORT. Scandura, Joseph M., Pennsylvania Univ., Philadelphia. School of Education. Spons Agency--Office of Education (DHEW), Washington, D.C. Bureau of Research. Bureau No.--BR-8-B-046. Pub Date May 70. Contract--OEC-0-8-080046-3679(010). Note--27p. Available from--Available on loan from the ERIC Center for Science and Mathematics Education, 1460 West Lane Ave., Columbus, Ohio 43221. Document Not Available from EDRS.
Descriptors--College Students, *Educational Research, *Learning, Mathematical Logic, Mathematical Vocabulary, *Mathematics Education, *Research, *Symbolic Learning, Symbols (Mathematics).

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert B. Kane, Purdue University.

1. Purpose

(a) To determine whether rules are memorized more easily when stated in mathematical symbolism or when stated verbally in English.
(b) To determine whether the ability to use constituent symbols correctly, assuming mastery of the underlying grammar, is a necessary and/or sufficient condition for applying a memorized rule statement.

2. Rationale

While some studies suggest that aptitude profiles (spatial vs. verbal or symbolic abilities) could be used to predict achievement in mathematics, more indicate that such profiles are not nearly as strong predictors of learning as are immediately requisite abilities. Based on these results, the general validity of the task-analysis procedure associated with learning hierarchies has become widely accepted. However the question remains as to whether aptitude measures may still be the best predictors of which presentation form should be used. Some studies suggest that specific interpretive abilities such as the ability to interpret rules or statements may be more fundamental than any general aptitude measure. Further, familiarity with terms used may play a key role in the

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ability to interpret statements. More particularly, the interpretation of a statement may depend upon certain requisite abilities which can be logically determined directly from the statement itself by asking the question, "What does S need to be able to do in order to interpret...?"

The present study was designed to help clarify the role symbolism plays in learning mathematical rules. Both the symbols used to construct rule statements and the ability of an S to interpret symbols were varied.

3. Research Design and Procedure

A 2 x 2 factorial design with repeated measures was used; each S served as his own control. One factor was the form in which the rules were stated, either succinctly worded English or mathematical symbolism. The second factor was the presence or absence of pretraining on the meaning of the symbols used. Four rules (eight statements) were counter-balanced over the four treatment combinations so that each of 24 ways of assigning 4 rules to 4 treatments was used once. All other factors were randomized. There were 24 Ss (22 female) all enrolled in a mathematics methods course for undergraduate elementary education majors.

The Ss reported one at a time and were given some common pretraining to assure that S understood the way certain terms were to be used. None of this information was judged sufficient for understanding any of the symbolic rule statements.

Next S was required to learn specific information about the symbols used in two of the four rule statements introduced later. The symbols used were defined, an example was given and four tasks similar to the example were given to S for practice.

Just before presenting the rules S was told he would see a card including the name of each rule, an introductory statement typed in black and the rule to learn in red. S's task was to memorize the part in red so that he could write it correctly whenever he saw its name. Practice was provided, then the experiment proceeded. Presentation order was randomized on each trial; S was told whether he was right or wrong after each attempt. Testing for each trial took place after all four rules were presented. On trial 1, 5 seconds study time was given; on Trial 2, 10 seconds; on Trial 3, 15 seconds; etc. As a final review S reproduced all four rules until he made no mistakes. After the criterion was met S was required to apply each rule in two problems. The four rules were in full view of S during this time.

To illustrate the material one of the rule statements, the Integer Rule, is shown.

Symbolic form: $\{[x] \div [y]\}$. Verbal form: (1) Take the greatest integer in x. (2) Take the greatest integer in y. (3) Divide the result in Step 1 by the result of Step 2. (4) Take the greatest integer in the quotient in Step 3.

4. Findings

The rules stated in symbolic form were applied successfully <u>if and only if</u> <u>S</u> had been taught how to apply the constituent symbols. Symbolic rule statements were learned more rapidly than verbal statements and this was true whether or not <u>S</u> learned specific information about the symbols used. Although they took longer to learn, verbal statements were applied equally as well as (in fact, slightly better than) those symbolic statements in which use of the constituent symbols had been learned previously. Moreover these verbal statements were learned at approximately the same rate and the rules were applied equally well whether or not use of the corresponding symbols was learned.

5. Interpretations

Mastery of the constituent mathematical symbols accounted for about 80% of the experimental outcomes on the application test. Much of the remaining 20% can probably be attributed to random errors. Thus general mathematics aptitude and achievement measures can be expected to account for the ability to interpret rules stated symbolically only as they covary with the sort of interpretive prerequisites identified here.

It is likely that symbolic statements were learned more easily because they were shorter. Learning the longer verbal statements probably required a substantially greater degree of recoding. Since providing <u>S</u> with the meanings of the mathematical symbols before he learned the corresponding verbal statements did not increase learning rate, it is likely that <u>Ss</u> tended to use their own preferred bases for recoding the verbal statements rather than the relatively unfamiliar symbols.

Apparently the redundancy of English assisted Ss to apply verbal rule statements even though they were relatively long. This suggests that ordinary English can be effectively used to teach precise mathematical ideas. By extrapolating the results of this study an explicit basis for making one type of branching decision in instructional sequences emerges. Given a rule to learn and an expository mode one might proceed as follows: (a) test whether S can use the constituent symbols, (b) if so, present the rule in symbolic form, (c) if not, present the rule in English. In summary, these results suggest that specific sorts of feedback are needed to make specific kinds of decisions. If this is true, then general feedback measures such as error rates, average latency, etc. will play a diminishing role in instructional sequencing as found, for example, in CAI.

Abstractor's Notes

The strengths of this experiment are evident from the abstract. The clarification of the interactions between modes of instruction and learning would be extremely valuable to designers of instructional sequences using adaptive-potential systems such as CAI. Scandura's efforts to begin charting this map should be well appreciated.

It is unfortunate that the study was limited to 24 Ss. With a larger n it might have been possible to investigate the relationships between the variables quantified in this study and general verbal and symbolic aptitudes. Considering the pool of Ss, it may be the case that most were high verbal--low symbolic individuals. If this were so would it not help explain performance on the application tasks for verbal statements? A point of interest along this line of speculation is that the training on symbol use which preceeded the experimental trials used the verbal mode.

About 20% of the application problems for symbolic statements in which the constituent symbols were mastered were missed. Scandura suggested that much of this could be attributed to momentary lapses and the like. It seems that some measure of the efficacy of this conjecture could have been made. If S were to have turned in his computation together with his answer to each application problems some assessment of the cause of missing a given application problem could have been made. Since fewer errors were made on verbal statement application problems than on symbolic statement problems, this refinement would have been doubly welcome.

Robert B. Kane
Purdue University

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PREFERENCES FOR MODES OF EXPRESSION IN MATHEMATICS.
Travers, Kenneth S. and Others. Stanford Univ., Calif. School of Education. Pub Date May 67, Note-25p.
EDRS Price MF-$0.65 HC-$3.29
Descriptors--Graphs, Instruction, *Learning, *Mathematics Education, *Research, Secondary School Mathematics, *Symbols (Mathematics)

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Merlyn J. Behr, Northern Illinois University.

1. Purpose

The purpose of the work reported in this paper is not clearly delineated. However, it was reported that the work was part of a project in which an attempt was being made to extend psychological research on stylistic differences in cognition to the study of teaching. The emphasis of prior research it was reported, has been toward the learner side of the teacher-learner interactive process. The apparent purpose of this work was to develop and validate an instrument to reliably measure cognitive preferences of both teachers and students.

2. Rationale

Any existing relationship between the theory of the psychological research on stylistic differences in cognition and work reported in this paper is not developed. The brief rationale for the work is related to the "hypothesis (untested in this work) that people--teachers and students--differ in their preference for modes of mathematical expression. If such cognitive preferences exist and an instrument can be developed to measure them, then, according to the authors, the relationship of these preferences to learning and teaching may be studied. In this connection several questions are cited by the authors; among those cited are: 1) Are preferences for modes of expressions of mathematics teachers related to teaching success? 2) Does correspondence between teacher and student preferences facilatate the educational process? 3) What are the relationships between preference for modes of mathematical expression and other aptitude variables?

3. Research Design and Procedure

Three modes of expression were specified: verbal, symbolic, and graphic. A 30-item paper pencil instrument was developed in which each item presented a different mathematical concept in each of the three modes. The respondents were required to indicate their preference from among the three modes. Respondents' scores consisted of triples which indicated the frequency of selection of each mode of presentation.

In March 1967, a trial form fo the instrument was administered to a sample of 115 seventh-grade students. In addition, these students' scores on the Iowa Test of Basic Skills, Form 1, were obtained from school files.

Numerous "reliability" indices were computed and reported. Correlations between respondents' scores for each of the three scales (corresponding to the three modes of expression) and nine Iowa Test of Basic Skills Form 1, scores were computed and reported.

4. Findings

The Piloting of the instrument indicated that: a) The 30-item test can be administered in a forty-minute classroom period, b) the items function as intended and appear to discriminate, and c) a usable balance of scores is produced by the instrument.

The symbolic option was most often preferred. Scores on the verbal and graphic scales formed positively skewed distributions, while scores on the symbolic scale formed a relatively symmetric though bimodal distribution.

5. Interpretations

No conclusions or inferences were suggested by the authors.

Abstractor's Notes

An inherent difficulty in the reporting of this work is rightfully brought out by the authors. That is, the ipsative nature of the preference scales make virtually every statistic reported in the paper uninterpretable. Indeed, this reviewer wonders whether the reported "correlations" between the scale scores and scores on the Iowa Test of Basic Skills are correlations in fact. It is not reported which of several correlational techniques was used. This reviewer was unable to find, in the more standard references, any correlational technique to indicate the relationship between scores on an ipsative scale and scores on a basic skills test (a continuous variable).

It is not clear to this reviewer what the authors had in mind when they reported that "the items on the instrument functioned as intended," nor is it clear what a "usable balance among preference scores" means.

In spite of the inherent statistical problems of the work some interesting questions are raised. Most ATI research has been concerned with the learner side of the educational process. Interactions between modes of presenting mathematical material and teacher aptitudes and preferences may be equally important. The question of the effect due to the match (or mismatch) between teacher and student aptitudes (or preferences) and treatments warrants investigation.

Merlyn J. Behr
Northern Illinois University

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