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ABSTRACT

This self-study program for high-school level contains lessons on: Algebra, Powers and Roots, Geometry, and Number Series. Each of the lessons concludes with a Mastery Test to be completed by the student. (DB)

PM 431 - 57

ADVANCED
GENERAL EDUCATION PROGRAM

A HIGH SCHOOL SELF-STUDY PROGRAM

ALGEBRA
LEVEL: II
UNIT: 8
LESSON: 1

U.S. DEPARTMENT OF LABOR
MANPOWER ADMINISTRATION, JOB CORPS
NOVEMBER 1969

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A man hires you to build a four foot wide flagstone walk around his office building. How much flagstone will the job take?

You are trying to get a loan. A bank offers one rate, a finance company at another, higher rate. But the bank's interest is to be paid in advance. Which loan is better?

As a surveyor you are trying to figure out the width of a river, but there is no way to get across it. What must you do?

You are the route managerof a trucking company. You have to ship a cargo of refrigerators to a city 300 miles away. How many trucks should you use? How long will they be tied up? How much will it cost you, in wages, gas, oil, and other expenses?

Questions like these confront everybody, no matter what their jobs. To solve them, you must have a knowledge of algebra and geometry. Indeed, without these branches of mathematics modern society would fall apart. The trains would not run, the bridges would not be built; the companies would go bankrupt.

Algebra was invented in Arabia a thousand years ago. The idea is simple. Suppose you are trying to solve that problem with the trucks. You must write down the distance of the city from you and the time it takes to drive there, but instead of writing down "distance" and "time" you abbreviate "d" and "t." The capacity load the trucks is important, too, and you abbreviate that "c" for capacity. Now you would find it much easier to compute the answers you need. That's all algebra is - a system of symbols and abbreviations to make figuring easier.

Geometry is much the same, except that it deals with areas, like circles and squares, instead of quantities, like distance and speed. With a little geometry, you would have no trouble figuring out the area of that walkway around the office building.

Now begin the first lesson.

Time completed ______________.

1. PREVIEW FRAME The relations between quantities, such as a car's speed and the distance it travels, can be expressed in handy, compact symbols. These symbolic expressions allow you to make many calculations which would not otherwise be possible. The next lesson introduces you to this new, powerful "language." NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
2. The distance a car travels equals the speed at which it moves times the length of time it travels. The above statement expresses a relation between quantities involved in an automobile's motion. The relation is:	
☐ distance	
☐ distance = speed x time	distance = speed x time
☐ speed	
☐ time	
The quantities involved are:	
☐ distance	distance
☐ distance = speed x time	
☐ speed	speed
☐ time	time

3. When you can express a relation between quantities, the relation is called a formula, and the quantities are called variables. The area of a rectangle equals the length of the rectangle times the width of the rectangle. Which of the following is/are formula(s)? ☐ area ☐ area = length x width ☐ length ☐ width Which of the following is/are variable(s)? ☐ area ☐ length ☐ width	area = length x width area length width
4. For simplicity, variables can be represented as letters, and formulas can be written using the letters. For instance, in an electric wire, the current of electricity equals the voltage divided by the resistance of the wire. Suppose you let I stand for the current, E stand for the voltage, and R stand for the resistance. The formula for current, written with letters, would be: ☐ $E = \frac{I}{R}$ ☐ $I = \frac{E}{R}$ ☐ $I = (E)(R)$ ☐ $(E)(R)$ ☐ I	$I = \frac{E}{R}$

5.	
Amount of money (A) equals the principal (P) plus the product of principal (P), rate (R), and time (T).	
WRITE an F beside the formula(s) below; WRITE a V beside the variable(s):	
_____ A	V
_____ $A = P + PRT$	F
_____ P	V
_____ R	V
_____ T	V
6.	
In a formula multiplication can be represented in three ways. A times B can be represented as: A x B, (A) (B), or AB.	
Which of the following represent the multiplication S times T:	
☐ S, T	
☐ S + T	
☐ (S) (T)	(S) (T)
☐ ST	ST
☐ $\frac{S}{T}$	
☐ S x T	S x T
The letter S is a:	
☐ formula	
☐ variable	variable

7.	
Numbers and letters can be combined in a formula. Suppose that the weight of a rod (W) equals 3 times its length (L).	
This formula can be written in several ways:	
$W = 3 \times L$ $W = 3\ (L)$ $W = (3)\ (L)$ $W = 3L$	
The length of an object in inches (I) equals 12 times its length in feet (F). Which of the following formulas correctly express(es) this relation?	
☐ $I = F$	
☐ $I = 12\ (F)$	$I = 12\ (F)$
☐ $F = 12\ (I)$	
☐ $I = 12 \times F$	$I = 12 \times F$
☐ $I = (12)\ (F)$	$I = (12)\ (F)$
☐ $I = 12F$	$I = 12F$
☐ $I = \frac{F}{12}$	
8.	
Given: the area (A) of a triangle equals one half the base (b) times the height (h).	
Which of the following correctly express(es) this relation?	
☐ $A = b \times h$	
☐ $A = 1/2\ bh$	$A = 1/2\ bh$
☐ $A = (1/2)\ (b)\ (h)$	$A = (1/2)\ (b)\ (h)$
☐ $A = 1/2 \times b \times h$	$A = 1/2 \times b \times h$

9. When a formula contains an equal sign = the formula can be called an equation. CHECK the equation(s) below: ☐ L ☐ $(L)(W)$ ☐ LW ☐ $A = LW$ ☐ $A = (L)(W)$ ☐ $A = (L)(W) + 5$ ☐ $LW + 5$	$A = LW$ $A = (L)(W)$ $A = (L)(W) + 5$
10. Sometimes a formula or equation contains only one variable. When this is so, you can always solve the equation to find the numerical value of the variable. Which of the following equations can be solved to find the numerical value of A? ☐ $A = LW + 5$ ☐ $A = 3(2+1)$ ☐ $A = 1/2 (b \times h)$ ☐ $14 = 3A + 7$	$A = 3(2+1)$ $14 = 3A + 7$
11. An equation can be solved when it contains: ☐ no variables ☐ only one variable ☐ two or more variables	only one variable

12. PREVIEW FRAME You have learned some of the terms used to describe formulas or equations and their contents. In the next section you will learn some rules which will help you to solve equations containing only one variable. NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
13. Consider the equation 18t = 4A + 13. Everything in the equation that lies to the <u>left</u> of the equal sign (=) is called the <u>left-hand side</u> of the equation. Everything in the equation that lies to the <u>right</u> of the equal sign is called the <u>right-hand side</u> of the equation. The left-hand side of the equation 4 x + 3y = 12t + 7 is: ☐ 4x ☐ 4 x + 3y ☐ 12t ☐ 12t + 7	4x + 3y

14. The equation $3x = 9 + 6$ may be solved for the value of the variable x. To solve it, you may divide each side of the equation by 3: $\underbrace{\frac{3x}{3}}_{\text{Left-hand side}} = \underbrace{\frac{9 + 6}{3}}_{\text{Right-hand side}}$ You know that $\frac{3}{3} = 1$. The left-hand side of the above equation therefore equals: ☐ x ☐ $2x$ ☐ $3x$ ☐ $\frac{x}{3}$	x
The expression $\frac{9 + 6}{3}$, when it is reduced: ☐ gives one third the value of x ☐ gives the value of x ☐ gives three times the value of x	gives the value of x
In solving the equation $3x = 9 + 6$, we divided: ☐ only the left-hand side of the equation by 3 ☐ only the right-hand side of the equation by 3 ☐ both sides of the equation by 3	both sides of the equation by 3

15. The basic rule for solving equations is: If you do anything to one side of an equation, you must do the same thing to the other side of the equation. If you divide the right-hand side of an equation by 5, you must: ☐ multiply the left-hand side by 5 ☐ multiply the left-hand side by 2 ☐ divide the left-hand side by 5 ☐ divide the left-hand side by 2	divide the left-hand side by 5
16. $\frac{x}{2} = 15 + 1$ The left-hand side of the above equation may be changed from $\frac{x}{2}$ into x by multiplying it by 2. If you do this, you must also: ☐ divide $\frac{x}{2}$ by 2 ☐ divide 15 + 1 by 2 ☐ multiply 15 + 1 by 2	multiply 15 + 1 by 2

17.

PROBLEM:

Solve the equation $13x = 50 + 15$ for x.

The left-hand side, 13x, will become simply x if you divide it by 13.

If you do the same to the right-hand side you obtain:

☐ $\frac{13x}{13}$

☐ $50 + 15$

☐ $\frac{50 \div 15}{13}$

☐ $(13)\ (50 + 15)$

The value of x is:

☐ 5
☑ 13
☐ 50
☐ 65

$\frac{50 + 15}{13}$

5

18. $x + 7 = 48$	
The left-hand side becomes x when you: ☐ add 7 to it ☐ subtract 7 from it ☐ multiply it by 7 ☐ divide it by 48	subtract 7 from it
At the same time, you must: ☐ add 7 to the right-hand side ☐ subtract 7 from the right-hand side ☐ multiply the right-hand side by 7 ☐ divide the right-hand side by 48	subtract 7 from the right-hand side
The value of x is: ☐ 7 ☐ 41 ☐ 48 ☐ 55	41

19.

PROBLEM:

SOLVE: $3x = 15 + 6$ for x

SOLUTION:

STEP 1: $\frac{3x}{3} = \frac{15 + 6}{3}$

STEP 2: $x = \frac{15 + 6}{3}$

STEP 3: $x = 7$

To solve the above equation, we first changed :

☐ the left-hand side to 3
☐ the left-hand side to x
☐ the right-hand side to 3
☐ the right-hand side to x

We then found:

☐ the value of the left-hand side
☐ the value of the right-hand side

the left-hand side to x

the value of the right-hand side

20.

Using the methods you have learned, SOLVE the equation $x - 6 = 4 + 12$ for x:

x equals:

☐ 6
☐ 10
☐ 16
☐ 22

22

21. PREVIEW FRAME Often equations with one variable are more complex than the ones you studied in the previous section. More complex equations, and the rules for simplifying them, will be taught in the following section. NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
22. Suppose you wish to express 3 times the sum of 5 and 7. The value of this quantity is 3 (12), or 36. If you write (3) (5) + (7), you might evaluate this to be 15 + 7, or 22. This is: ☐ the correct answer ☐ a wrong answer	a wrong answer
The right way to express 3 times the sum of 5 + 7 is: 3 (5 + 7), or (3) (5 + 7). The correct way to express 15 times the difference between x and y is: ☐ 15x − y ☐ 15 + x − y ☐ 15 (x + y) ☐ 15 (x − y)	15 (x − y)

23. The symbols ( ) are used to avoid mistakes like the one shown in the previous frame. The symbol ( ) is called a <u>parentheses</u>. You wish to indicate the product: 4 times the sum of A and B. To avoid mistakes you should place in parentheses: ☐ 4 ☐ only A ☐ only B ☐ 4A + B ☐ A + B	A + B
The expression, correctly written, is: ☐ 4B + A ☐ A + B(4) ☐ 4 (A + B) ☐ 4A + (B)	4 (A + B)
24. You wish to indicate: 3 times the expression $\frac{x + y - A}{53}$. In other words, you want the <u>entire</u> expression $\frac{x + y - A}{53}$ to be multiplied by 3, and not merely some part of it, such as x. To do this, you put the entire expression in parentheses, and then write a 3 in front of it. The correct answer would be: ☐ $\frac{3x + y - A}{53}$ ☐ $\frac{3(x + y - A)}{53}$ ☐ $3\left(\frac{x + y - A}{53}\right)$	$3\left(\frac{x + y - A}{53}\right)$

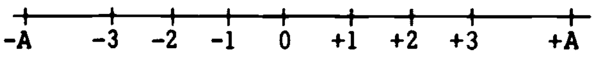
25. Which expression below correctly indicates the quantity 41 times the sum of A and 14 and B? ☐ 41A + 14 + B ☐ 41 (14) (A + B) ☐ 41 (A + 14 + B) ☐ 41 (A) + 14 + B	41 (A + 14 +B)
26. A letter or number written just before an expression in parentheses is called the coefficient of the expression. In 4 (A + B + 10), the coefficient is: ☐ 4 ☐ A ☐ A + B + 10 ☐ 4 (A + B + 10) A coefficient may contain more than one letter or number, or a mixture of letters and numbers. In $2AB\left(\frac{3x + 7y}{5x}\right)$, the coefficient of the expression in parentheses is: ☐ 2 ☐ AB ☐ 2AB ☐ B ☐ $2AB\left(\frac{3x + 7y}{5x}\right)$	4 2AB

27.	
$3 + xy - 7AB + y$	
In the above expression, each part that is separated from the rest of the expression by + or - signs is called a term of the expression.	
In the above, 3 is a term, xy is a term, $7AB$ is a term, and y is a term.	
In the expression $4p + r + 7s + t$, the terms are:	
☐ $4p$	$4p$
☐ $4p + r$	
☐ r	r
☐ $r + 7s$	
☐ $7s$	$7s$
☐ $4p + r + 7s$	
☐ t	t
28.	
$4x (A + 9B - 73C + 6)$	
Referring to the above expression, WRITE a T before terms in the list below, and a C before the coefficient:	
_____ $4x$	C
_____ A	T
_____ $9B$	T
_____ $73C$	T
_____ 6	T
_____ $A + 9B - 73C + 6$	
_____ $4x (A + 9B - 73C +6)$	

29. When a coefficient is written before an expression in parentheses, it may be multiplied times each term in the expression, and the value remains unchanged. For example, 2 (A + B +C) is equal to 2A + 2B + 2C. Likewise, 7x (r - s) equals 7xr - 7xs. Therefore, 15 (x - y) is equal to: ☐ 15x + 15y ☐ 15x - 15y ☐ 15x - y	15x - 15y
30. Example: 8s (t - 3v) equals 8st - 24sv The quantity 4 (A + 12B) is equal to: ☐ 4A + 4B ☐ 4A + 48B ☐ 4 (A + B) Recall that (a) x (a) can be written: a^2. The quantity 3a (t + 4a) is equal to $3at + 12a^2$. The quantity 7s (t - 3s + 4) equals: ☐ 7st - 21s + 28 ☐ 7st - 21s + 28s ☐ $7st - 21s^2 + 28s$	4A + 48B $7st - 21s^2 + 28s$
31. The quantity 4a (a + b - 2c) equals: ☐ $4a^2 + 4ab - 2ac$ ☐ $4a^2 + 4ab - 8ac$ ☐ 4a + 4ab - 8ac ☐ 4a + a + b - 2c	$4a^2 + 4ab - 8ac$

32. PREVIEW FRAME You have learned how to multiply each term in a parenthesis by the coefficient to obtain a quantity that is equal to the original form of the expression. It often happens that every term in an expression contains the same number or letter. . In this case, the common number or letter can be removed from the terms and written as a coefficient to the expression. This is the reverse of the procedure you have just learned. It is explained in the next section. NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
33. Consider the expression: $2A + AB + 3AC$. Each term in the expression contains: ☐ 2 ☐ A ☐ 2A ☐ B The letters or numbers common to all terms can be removed from the terms. The expression may be enclosed in parentheses and the quantity removed may be written before the parentheses as a coefficient. The above expression becomes: $A(2 + B + 3C)$ An expression equivalent to $3x + xy - 7xt$ is: ☐ $3(x + xy - \frac{7}{3} x t)$ ☐ $x(3 + y - 7t)$ ☐ $x(3 + y - 7xt)$	A $x(3 + y - 7t)$

34. The process of removing quantities common to all terms and making them a coefficient is called <u>factoring</u>. The coefficient can also be called the <u>factor</u>. What is the factor in the expression $3ab + 7a^2b - 9ab$? ☐ 3 ☐ a ☐ ab ☐ 3ab	ab
When the expression is factored, it becomes: ☐ $3(ab + 7a^2b - 3ab)$ ☐ $b(3a + 7a^2 - 9a)$ ☐ $ab(3 + 7a - 9)$ ☐ $a(3ab + 7b - 9b)$	$ab(3 + 7a -9)$
35. When you factor the expression $7x^2y - 21\ xy^2 + 14\ xy$ you obtain: ☐ $7(x^2y - 3\ xy^2 + 14\ xy)$ ☐ $7x(xy - 3y^2 + 14y)$ ☐ $7xy(x - 3y + 2)$ ☐ $7xy(x - 3 + 14)$	$7xy(x - 3y + 2)$

36. PREVIEW FRAME You know already that the symbols + and - signify addition and subtraction. You also learned in a previous section that these symbols can have a different meaning. They sometimes indicate whether a number is positive or negative. You will review these, their meaning, and learn how to use them in equations in the following section. NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
37.  The expression -2 signifies two less than zero. The expression +3 means: ☐ three greater than zero ☐ three less than zero	three greater than zero
The expression +A means A units greater than zero. The expression -2A signifies: ☐ A units less than zero ☐ 2 less than zero ☐ 2A units less than zero	2A units less than zero

38. The symbol + or - before a number, letter, term or expression is called the sign of the number, letter, term or expression. $-4+8\left(\frac{-x+12}{-3A}\right)$ In the above expression, the sign of 3A is: ☐ + ☐ -	-
The sign of $8\left(\frac{-x+12}{-3A}\right)$ is: ☐ + ☐ -	+
The sign of 4 is: ☐ + ☐ -	-
39. When a term or coefficient has no sign, the sign is assumed to be +. In the expression 4a(x - 3t + 7c), the sign of the coefficient 4a is: ☐ + ☐ -	+
The sign of the term x is: ☐ + ☐ -	+
The sign of the term 3t is: ☐ + ☐ -	-
The sign of the term 7c is: ☐ + ☐ -	+

PANEL 1

Rule of Signs for Multiplication:

$$(+A)(+B) = +AB$$

$$(-A)(-B) = +AB$$

$$(+A)(-B) = -AB$$

$$(-A)(+B) = -AB$$

Rule of Signs for Division:

$$\frac{(+A)}{(+B)} = +\frac{A}{B}$$

$$\frac{(-A)}{(-B)} = +\frac{A}{B}$$

$$\frac{(+A)}{(-B)} = -\frac{A}{B}$$

$$\frac{(-A)}{(+B)} = -\frac{A}{B}$$

40.	
REFER TO PANEL 1	
The panel shows how signs behave when you multiply and divide.	
When you multiply two quantities, the product will be + when:	
☐ the two quantities are both +	the two quantities are both +
☐ the two quantities are both -	the two quantities are both -
☐ one quantity is +, the other is -	
When the signs of two quantities are different, the sign of their product (after multiplication) will be:	
☐ +	
☐ -	-
41.	
REFER TO PANEL 1	
LOOK at the Rule of Signs for Division.	
When two quantities having the same sign are divided, the result will be:	
☐ +	+
☐ -	
When two quantities having different signs are divided, the result will be:	
☐ +	
☐ -	-
In this respect the Rule of Signs for Division:	
☐ differs from the Rule of Signs for Multiplication	
☐ is the same as the Rule of Signs for Multiplication	is the same as the Rule of . . .

42. REFER TO PANEL 1	
(- 5) (+ 7) equals: ☐ + 35 ☐ - 35 ☐ neither of the above	- 35
x (- y) equals: ☐ + xy ☐ - xy ☐ neither of the above	- xy
$\frac{10}{5}$ equals: ☐ +2 ☐ -2 ☐ neither of the above	+2
$\frac{-4}{-10}$ equals: ☐ $+\frac{2}{5}$ ☐ $-\frac{2}{5}$ ☐ neither of the above	$+\frac{2}{5}$

43.	
DO NOT REFER TO THE PANEL	
- 3 (- x) equals: ☐ + 3x ☐ - 3x ☐ neither of the above	+ 3x
$\frac{14}{-\,yt}$ equals: ☐ $+\frac{14}{yt}$ ☐ $-\frac{14}{yt}$ ☐ neither of the above	$-\frac{14}{yt}$
44.	
When an expression within parentheses has no coefficient, the coefficient can be assumed to be 1.	
EXAMPLES: 5x (- 7t + 6s) (41A - 3B + C)	
In the above expressions, the coefficient of - 7t + 6s is: ☐ 1 ☐ 5 ☐ 5x	5x
The coefficient of 41A - 3B + C is: ☐ 1 ☐ 5 ☐ 5x ☐ 41	1

45. The expression $-(A - B + C)$ is equivalent to: $+(-A + B - C)$. The expression $-5(4 + x)$ is equivalent to: $+5(-4 - x)$. When the coefficient of an expression in parentheses has the sign −, that sign may be changed to +, but you must also: ☐ change the sign of the first term in parentheses to − ☐ change the signs of every term in parentheses ☐ make all terms in the parentheses have a − sign	change the signs of every . . .
46. The expression $-4(6x + 3y - 9)$ is equivalent to: ☐ $+(24x - 12y - 36)$ ☐ $+(-24x - 12y + 36)$ ☐ $+(24x + 12y + 36)$ ☐ $+(-24x - 12y - 36)$	$+(-24x - 12y + 36)$

47. The expression x - (y - 5) is equivalent to x + (- y + 5). The expression - 4 - (- s + 3t) is equivalent to: ☐ - 4 + (s + 3t) ☐ - 4 + (s - 3t) ☐ - 4 + (-s -3t) ☐ - 4 + (- s + 3t)	-4 + (s - 3t)
48. The expression 6x + (- 3x) can be written more simply as a subtraction: 6x - 3x. It equals: ☐ 6 ☐ +6x ☐ +3x ☐ -3x	+3x
The expression AB + (- 3AB) is equivalent to: ☐ AB + 3AB ☐ AB - 3AB ☐ neither of the above	AB - 3AB
49. The expression 4t + (-51t) is equivalent to: ☐ - 4t + 51t ☐ 4t - 51t ☐ - 4t - 51t ☐ 4t + 51t	4t - 51t
The expression -3 -6 (A + x) is equivalent to: ☐ -3 -6A + 6x ☐ -3 -6A -6x ☐ 3 - 6A - 6x	-3 -6A -6x

50. PREVIEW FRAME You have learned how to factor an expression. You have also learned how to deal with signs in expressions. You are now prepared to make an expression simpler by using the techniques you have learned, together with another technique in which several terms containing the same letters may be brought together into one term. NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
51. AB - 3CD + 2AB - BD In the above expression, two of the terms contain the same letters. The two letters found together in both terms are: ☐ AB ☐ AD ☐ BC ☐ CD ☐ BD	AB
When two or more terms contain the same letters, they can be <u>combined</u> into a single term. If the terms with the same letters are to be combined and have the <u>same sign</u>, they can be added together. The two terms in the above example: ☐ have the same sign ☐ do not have the same sign	have the same sign
When you add these two terms together you obtain: ☐ + AB ☐ + 2AB ☐ + 3AB	+ 3AB

52. AB - 3CD + 2AB - BD Two terms in the above expression contain the same letters. They can be added together to form one term. When this is done, the expression becomes: ☐ AB - 3CD + 2AB - BD ☐ AB - 3CD - BD ☐ 3AB - 3CD - BD ☐ 3AB - 3CD Terms can be collected when they contain: ☐ the same variables ☐ different variables ☐ the same numbers ☐ different numbers	3AB - 3CD - BD the same variables
53. Adding together terms with the same variables is called <u>collecting terms</u>. If you collect terms in the expression, 3x + 5y + x + 2y - 7 you obtain: ☐ 4x + 5y + 2y - 7 ☐ 4x + 7y ☐ 4x + 7y - 7	4x + 7y - 7

54. Terms to be collected may be both + and -. You simply add the + terms and subtract the minus terms. + 3AB - 5AB becomes - 2AB - AB - 7AB + C becomes - 8AB + C	
16x - 3y + y - 2 becomes: ☐ 16x - 4y - 2 ☐ 16x - 2y - 2 ☐ 16x + 4y - 2	16x - 2y - 2
-31xyz + xyz - 6 becomes: ☐ - 32xyz - 6 ☐ - 30xyz - 6 ☐ + 30xyz - 6	- 30xyz - 6
- 4t - 6t - A becomes: ☐ 10t - A ☐ - 10t - A ☐ - 2t - A ☐ 2t - A	- 10t - A
55. If you collect terms in the expression 6xy - 2z + 4x - xy + 3 you obtain: ☐ 6xy - 2z + 4x + 3 ☐ 10xy - 2z + 3 ☐ 6xy - 2z - xy + 3 ☐ 5xy - 2z + 3 ☐ 5xy - 2z + 4x + 3	5xy - 2z + 4x + 3

56. If you collect terms in the expression $(4a) + (-5a) - (10a)$ you obtain: ☐ $-a$ ☐ $-19a$ ☐ $-11a$ ☐ $+11a$ ☐ $+19a$	$-11a$
57. You have learned that if two terms contain the same variables, you can collect them. This is only true if each variable has the same power in both terms. The expression $AB + 3AB^2$ does not simplify to $4AB$. The reason is that: ☐ the power of A is different in the two terms ☐ the power of B is different in the two terms In the expression $2x + 3x^2 + 5x$, which terms can be collected? ☐ $2x$ ☐ $3x^2$ ☐ $5x$	the power of B is different . . . $2x$ $5x$
58. If you collect terms in the expression $3xy - 5x^2y + 2xy^2 - xy + x^2y$ you obtain: ☐ $2xy - 5x^2y + 2xy^2$ ☐ $2xy - 4x^2y + 2xy^2$ ☐ $3xy - 4x^2y + 2xy^2$	$2xy - 4x^2y + 2xy^2$

59. PREVIEW FRAME Sometimes an expression is so complicated it needs parentheses within parentheses. When such a case arises, special symbols are used. In the next section you will learn about these special symbols. NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
60. You might wish to indicate the subtraction of $t + 3$ from 4; you might then wish to indicate that this whole expression is to be multiplied by 6. Which of the following would correctly express the above? ☐ $6(4 - t + 3)$ ☐ $6(4 - (t + 3))$ ☐ $24 - (t + 3)$	$6(4 - (t + 3))$
61. $6(4 - (t + 3))$ When you have parentheses within parentheses, as above, it can be confusing. To avoid confusion, the <u>outermost</u> pair of symbols -- ( ) -- is replaced by symbols that are more square in appearance: [ ] When this is done, the above expression becomes: ☐ $6(4 - [t + 3])$ ☐ $[6(4 - (t + 3))]$ ☐ $6[4 - (t + 3)]$	$6\left[4 - (t + 3)\right]$

62.

The expression:

$$5 - (x - (y - 3))$$

should be written as:

☐ $5 - [x - [y - 3]]$

☐ $5 - [x - (y - 3)]$

☐ $5 - (x - [y - 3])$

$5 - [x - (y - 3)]$

63.

The squared off symbols -- $[\ ]$ -- are called <u>brackets</u>.

It is usual for:

☐ brackets to be written within parentheses

☐ parentheses to be written within brackets

☐ neither of the above

parentheses to be written . . .

64.

Collecting terms in order to reduce an expression to the least possible number of terms is called <u>simplifying</u> the expression.

A: $2x$
B: $x - (3x - 4x)$
C: $x - (-x)$

Expressions A, B, and C above are all equal.

Which of the following is/are true?

☐ A simplifies to B

☐ B simplifies to C

☐ A simplifies to C

☐ C simplifies to B

☐ C simplifies to A

B simplifies to C

C simplifies to A

65.

When you simplify an expression, you work from the <u>inside outward</u>.

For instance, in the expression $x - 5\left[y - (x - 2x)\right]$ you would collect terms for $(x - 2x)$, then simplify $y - (x - 2x)$, and lastly simplify the whole expression.

When you simplify a complex expression, you simplify the contents of brackets:

☐ before you simplify the contents of parentheses

☐ after you simplify the contents of parentheses

after you simplify the . . .

When you collect terms for $(x - 2x)$, you obtain:

☐ x
☐ $-x$
☐ $-2x$

$-x$

The expression inside the brackets above becomes:

☐ $y - (+x)$
☐ $y - x$
☐ $y + x$

$y + x$

66.

$$x - 5\left[\, y - (x - 2x)\right]$$

can be simplified to:

☐ $x - 5(y + x)$
☐ $x - 5y - x$
☐ $x - 5(y - x)$

$x - 5(y + x)$

This, in turn, can be simplified to:

☐ $-5y$
☐ $2x - 5y$
☐ $-5y - 4x$
☐ $-5y + 6x$

$-5y - 4x$

67. What is the simplest form of the expression $x - \left[5 - (x - 2) \right]$? ☐ $- 5$ ☐ $x - (7 - x)$ ☐ $x - (7 + x)$ ☐ $2x - 7$ ☐ $2x + 7$	$2x - 7$
68. PREVIEW FRAME Formulas or equations consist of two expressions related by an = sign. You have learned to simplify expressions. You are therefore better able to solve equations. In the next section you will apply what you have learned to the solution of equations. NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
69. Consider the equation: $2x = 5t - 7 (4 - t)$ You wish to solve this equation for x. First, simplify the right-hand side. The right-hand side simplifies to: ☐ $6t - 28$ ☐ $- 2t - 28$ ☐ $12t - 28$	$12t - 28$
You must do the same thing to each side of the equation. Let us divide both sides by 2. We find that x =: ☐ $6t - 28$ ☐ $-2t - 28$ ☐ $12t - 28$ ☐ $24t - 56$ ☐ $6t - 14$	$6t - 14$

70. SOLVE the equation $3x = 6 - \left[12 - (3a - 3) \right]$ for x. x equals: ☐ $- 9 + 3a$ ☐ $- 3 + a$ ☐ $- 6 + 3a$ ☐ $- 2 + a$	$- 3 + a$
71. Consider the equation $ab - x = c$. You wish to solve it for x. Remember that you may do anything to one side of the equation so long as you do it to the other side. First, subtract ab from both sides. What do you obtain ? ________________	$ab - x - ab = c - ab$
72. You obtain $ab - x - ab = c - ab$. This simplifies to: ☐ $2ab - x = c - ab$ ☐ $- x = c - ab$ ☐ $x = c - ab$	$- x = c - ab$
73. Now, change the signs of both sides in the equation $- x = c - ab$. What do you obtain ? ________________	$- (- x) = - (c - ab)$

74. Simplify $-(-x) = -(c - ab)$. You obtain: ☐ $-x = -c + ab$ ☐ $x = c - ab$ ☐ $x = ab - c$	$x = ab - c$
75. SOLVE for x the equation $ax - y = b$. x equals: ☐ $y + b$ ☐ $a(y + b)$ ☐ $y + \frac{b}{a}$ ☐ $\frac{y + b}{a}$ ☐ none of the above	$\frac{y + b}{a}$
76. SOLVE for x in the equation: $2(x - 3) = -5\left[1 - (4 - x)\right]$ x equals: ☐ 3 ☐ 7 ☐ 21 ☐ -3 ☐ -7	3 Time completed ____________

YOU HAVE NOW FINISHED THE FIRST PART OF THIS LESSON. WRITE DOWN THE TIME. THEN, AFTER YOU HAVE REVIEWED THE MAIN IDEAS IN THE FOLLOWING SUMMARY, TAKE THE MASTERY TEST AT THE END OF THE BOOKLET.

MASTERY TEST

Time started ______________

1. Solve for x in the equation:

$$3t - ax = b + 4$$

x equals:

a. ☐ $b + 4 - 3t$

b. ☐ $\dfrac{b + 4 - 3t}{a}$

c. ☐ $\dfrac{3t - b - 4}{a}$

d. ☐ $a(3t - b - 4)$

e. ☐ none of the above

2. The expression $2r - [6 - (5 - 3r)]$ simplifies to:

a. ☐ $2r - (1 - 3r)$

b. ☐ $5r + 6$

c. ☐ $1 - r$

d. ☐ $-r - 1$

3. Solve for x in the equation:

$$6\,(5 - x) = -3\,[x - 2\,(-6 - x)]$$

x equals:

a. ☐ 30

b. ☐ 15

c. ☐ 3

d. ☐ -3

e. ☐ -22

Time completed ______________

WHEN YOU HAVE FINISHED THIS TEST, WRITE DOWN THE TIME. THEN TAKE THE LESSON TO YOUR INSTRUCTOR OR HIS ASSISTANT FOR CHECKING. WAIT UNTIL THE LESSON IS APPROVED BEFORE GOING ON TO THE NEXT LESSON.

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ADVANCED
GENERAL EDUCATION PROGRAM

A HIGH SCHOOL SELF-STUDY PROGRAM

POWERS AND ROOTS
LEVEL: II
UNIT: 8
LESSON: 2

U.S. DEPARTMENT OF LABOR
MANPOWER ADMINISTRATION, JOB CORPS
NOVEMBER 1969

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1.	
PREVIEW FRAME	
Often in mathematics you are given a number to be multiplied with itself two or more times. Also, you are frequently asked to find some number which, when multiplied with itself, would produce the given number.	
You will learn how to deal with these quantities in the following lesson.	
NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
2.	
The multiplication 7 x 7 x 7 x 7 may be written in a briefer way, as $(7)^4$.	
This indicates that how many sevens are multiplied together? ______	4
The multiplication 8 x 8 x 8 may be written more briefly as:	
(8) ↑ WRITE a number here	$(8)^3$
The number you wrote is called the exponent or power of 8.	
CIRCLE the exponent: $(4)^7$	$(4)^{\textcircled{7}}$

3.	
$(7.42)^3 = 7.42 \times 7.42 \times 7.42$	
The number 3 in the above equation is called the:	
☐ multiplying factor ☐ multiplier ☐ power or exponent	power or exponent
$(2)^5 = 2 \times 2 \times 2 \times 2 \times 2$	
The exponent in this example is _____ (number).	5
4.	
The multiplication 6 x 6 x 6 may be written in briefer form as $(6)^3$.	
WRITE the following multiplications in the briefer form:	
6.2 x 6.2 x 6.2 x 6.2 = ______________	$(6.2)^4$
2 x 2 x 2 x 2 x 2 = ______________	$(2)^5$
WRITE out the following expressions as you would write a multiplication in the usual way:	
$(.04)^3$ = .04 x ____ x ____	.04 x .04 x .04
$(4)^4$ = ____ x ____ x ____ x ____	4 x 4 x 4 x 4
$(2.7)^2$ = ______________	2.7 x 2.7

5.	
WRITE out five to the sixth power in the long way:	
____________	5 x 5 x 5 x 5 x 5 x 5
Five to the third power is written in brief form as ______.	$(5)^3$
$(3)^4$ is read, "three to the fourth power."	
$(3)^5$ is read, "three to the ____________ power."	fifth
$(7)^9$ is read, "________ to the ________ power."	seven, ninth
$(4)^{10}$ is read, " ________________."	four to the tenth power
6.	
$(4)^3$ may be written as 4 x 4 x 4. **Its** <u>value</u> is 4 x (4 x 4) or 4 x 16, which is equal to 64.	
$(6)^3$ may be written in the long way as ____________.	6 x 6 x 6
CALCULATE the <u>value</u> of $(6)^3$:	
________	216
What is the exponent in $(6)^3$? _____	3
7.	
5.9 x 5.9 x 5.9 can be written more briefly as _______.	$(5.9)^{\textcircled{3}}$
$(2.6)^4$ can also be written as _______________.	2.6 x 2.6 x 2.6 x 2.6
3.1 x 3.1 x 3.1 can also be written as _______.	$(3.1)^{\textcircled{3}}$
CIRCLE the exponents in your answers.	

8.	
When we want to show that the number 4.3 is multiplied by a coefficient, such as 2, we place 4.3 in parenthesis and write 2(4.3). This is the same as 2 x 4.3.	
Similarly, $2(4.3)^3 = 2(4.3 \times 4.3 \times 4.3)$. Note that the coefficient, 2, is not raised to the 3rd power.	
WRITE out the expression $5(7.1)^2$ in a longer way:	
_____ x ___ x _____	5 x 7.1 x 7.1
CALCULATE the value of the above:	
______________	252.05
WRITE the multiplication 7 x 2 x 2 x 2 x 2 in a briefer form:	
______________	$7(2)^4$
9.	
EXAMPLE:	
$5(2)^3 = 5 \times 2 \times 2 \times 2 = 40$	
CALCULATE the following, using the example as a guide:	
$2(3)^3$ = ______________	2 x 3 x 3 x 3 = 54
$4(2.4)^2$ = ______________	4 x 2.4 x 2.4 = 23.04
$5(.04)^2$ = ______________	5 x .04 x .04 = .008

10.	
CALCULATE the value of:	
$4(3)^4$ = ______________	(4 x 3 x 3 x 3 x 3) = 324
$2.9(5)^2$ = ______________	(2.9 x 5 x 5) = 72.5
$6(.04)^3$ = ______________	(6 x .04 x .04 x .04) = .000384
11.	
A minus sign inside the parentheses may cause the result to be minus.	
EXAMPLE 1: $(-2)^2 = (-2) \times (-2) = +4$	
EXAMPLE 2: $(-2)^3 = (-2) \times (-2) \times (-2) = -8$	
In Example 1, the exponent is:	
☐ an odd number ☐ an even number	an even number
The result of the calculation is:	
☐ positive ☐ negative	positive
In Example 2, the exponent is:	
☐ an odd number ☐ an even number	an odd number
The result of the calculation is:	
☐ positive ☐ negative	negative
When there is a minus sign within the parentheses, the result will be <u>negative</u> if the exponent is:	
☐ an even number ☐ an odd number	an odd number

12.	
Some powers are especially easy to calculate.	
$0 \times 0 = 0$	
Also, $0 \times 0 \times 0 \times 0 = 0$	
From the above you can see that	
$(0)^3$, $(0)^{15}$, $(0)^4$, and $(0)^{10}$	
are all equal to ________.	0 (or) zero
$1 \times 1 = 1$	
$1 \times 1 \times 1 \times 1 = 1$	
From this you can see that	
$(1)^4$, $(1)^{16}$, $(1)^2$, and $(1)^{51}$	
are all equal to ________.	1
One last fact: when the exponent is zero, the value is <u>always</u> equal to 1 (unless the quantity inside the parentheses also is zero).	
Therefore, $(12)^0$ = ________.	1
$(7.003)^0$ = ________.	1
$3(29)^0$ = ________.	$3 \times 1 = 3$
13.	
WRITE a 0 or a 1 in each blank:	
$(1)^6$ = _____	1
$(0)^4$ = _____	0
$(1)^{412}$ = _____	1
$(31)^0$ = _____	1
$(0)^1$ = _____	0
$(1)^0$ = _____	1

14.	
CALCULATE the value of the following:	
$2(6.3)^2$ = ____________	2 x 6.3 x 6.3 = 79.38
$6(4)^3$ = ____________	6 x 4 x 4 x 4 x = 384
$(-7.1)^2$ = ____________	(-7.1) x (-7.1) = 50.41
$(3)^4$ = ____________	3 x 3 x 3 x 3 = 81
$2.4(6)^2$ = ____________	2.4 x 6 x 6 = 86.4
$3(-4)^3$ = ____________	3 x (-4) x (-4) x (-4) = -192

15.

CALCULATE each of the following:

$(6.5)^2$ = ________	42.25
$5(-6)^2$ = ________	180
$(0)^{12}$ = ________	0
$(1)^9$ = ________	1
$(2)^4$ = ________	16
$(-1)^6$ = ________	1
$(-4)^3$ = ________	-64
$(-1)^9$ = ________	-1
$(6.1)^0$ = ________	1
$4(2)^0$ = ________	4

16.

In the expression $14(35.07)^6$, a parenthesis is used for the sake of clarity.

If the quantity raised to a power is not a number but a variable, or an unknown, represented by a letter such as x, the parentheses usually aren't necessary.

For instance:

$$14x^6$$

CHECK the expression(s) below in which the parentheses are <u>not</u> needed:

☐ $(4)^2$	
☐ $4(x)^2$	$4(x)^2$
☐ $2(7.02)^{16}$	
☐ $2.03(x)^{41}$	$2.03(x)^{41}$
WRITE the expression(s) you checked <u>without</u> the parentheses: ________________	$4x^2$, $2.03x^{41}$

17. If the exponent of a quantity is 1, it need not be written. $(2.3)^1$ is the same as 2.3 x^1 is the same as x. CHECK the expression(s) below in which it is <u>unnecessary</u> to write the exponent: ☐ $7(2.3)^5$ ☐ $5x^2$ ☐ $2(27.02)^1$ ☐ x^{49} ☐ $3x^1$ WRITE the expression(s) you checked without using the exponent:	$2(27.02)^1$ $3x^1$ 2(27.02) 3x
18. In the expression 7.3x, the exponent of x is not written. What is this unwritten exponent? __________(number)	1

19.	
What is the exponent of the x-term in each of the following?	
$8x^7$ _____	7
$12x^{35}$ _____	35
$57.2x$ _____	1
20.	
$(3)^5$ is read as "three to the __________ power."	fifth
$(3)^2$ is read as "three to the __________ power."	second
For $(3)^2$, instead of using the above expression, people usually say: "three squared."	
WRITE the expression for "five squared." __________	$(5)^2$
This may also be written as ____ x ____.	5 x 5
The value of "five squared" is __________.	25
The value of three squared is __________.	9
The value of four squared is __________.	16
The value of two squared is __________.	4
WRITE three squared, using the exponent: __________	$(3)^2$

21. Four squared is 16. That is, $16 = (4)^2$, or 4 x 4. We call 4 the square root of 16. We call 16 the square of 4. Consider the equation $(3)^2 = 9$. In this equation, 9 is the ______________ of 3; and 3 is the ______________ of 9. The square of 5 is _____. The square root of 36 is _____.	square square root 25 6
22. What is the square root of 16? ☐ 1 ☐ 2 ☐ 4 ☐ 8 ☐ 6	4

23.	
REVIEW FRAME	
9.5 x 9.5 x 9.5 may be written using an exponent as follows:	
__________	$(9.5)^3$
The exponent is: _____	3
$(7)^4$ is read as: "seven to the ______________"	fourth power
CALCULATE the value of each of the following:	
$5(.04)^2 =$ __________	.008
$(-2)^5 =$ __________	-32
$(1)^9 =$ __________	1
$12(6.1)^0 =$ __________	12
24.	
PREVIEW FRAME	
In the next section you will learn how to find the square root of any number.	
If the square root is a decimal number, the method allows you to calculate it to as many decimal places as you desire.	
NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME

25.	
Before finding the square roots of large numbers, you must understand the term "perfect square."	
A perfect square is a number whose square root is a whole number.	
The square root of 5 is 2.236.	
Is 5 a perfect square? ☐ yes ☐ no	no
The square root of 81 is 9.	
Is 81 a perfect square? ☐ yes ☐ no	yes
CHECK the perfect squares below:	
☐ 4	4 = 2 x 2
☐ 6	
☐ 100	100 = 10 x 10
☐ 116	

26.	
The symbol for the square root of 4 is	
$\sqrt{4}$	
WRITE the square root of 28 in this new way.	
__________	$\sqrt{28}$
The square of 4 is 16. The square root of 4 is 2.	
WRITE the correct value beside the expressions below:	
$\sqrt{4}$ = __________	2
$(4)^2$ = __________	16
WRITE the symbol for the square root of 11:	
__________	$\sqrt{11}$

PANEL 1

FIND the square root of 279841. I.e., find $\sqrt{279841.}$

STEP 1. Beginning with the decimal point and moving left, mark off the number into groups of <u>two digits</u>.

$$\sqrt{27'98'41.}$$

STEP 2. Consider the first group on the far left as if it were a separate number. Find the <u>largest</u> perfect square <u>less</u> than this number and WRITE its root just above the group.

(5) = 25, but (6) = 36, which is too large.

$$\begin{array}{l} \searrow 5 \\ \sqrt{27'98'41.} \end{array}$$

STEP 3. WRITE the largest perfect square itself just <u>below</u> the group.

$$\begin{array}{l} \quad 5 \\ \sqrt{27'98'41.} \\ \nearrow 25 \end{array}$$

STEP 4. SUBTRACT the largest perfect square from the group and write the remainder below.

$$\begin{array}{l} \quad 5 \\ \sqrt{27'98'41.} \\ \underline{-25} \\ \nearrow 2 \end{array}$$

STEP 5. BRING DOWN the next group of two digits from the left; place it beside the remainder obtained in Step 4.

$$\begin{array}{l} \quad 5 \\ \sqrt{27'98'41.} \\ \underline{-25} \downarrow \\ \quad 2\ 98 \end{array}$$

STEP 6. Take the number above the square root sign, <u>double it</u> and <u>add a zero</u> (multiply by 10).

5 x 2 x 10 = 100

CIRCLE the result: (100)

STEP 7. How many times will the <u>circled number</u> go into the new remainder?

$$\begin{array}{r} 2 \\ \textcircled{100}\ \overline{)\,298} \\ \underline{200} \\ 98 \end{array}$$

(It goes two times, with a remainder of 98, but it won't go three times.)

Hence, the number is 2.

(a) ADD the number to the amount in the circle and (b) PLACE the number above the group of digits you brought down.

$$\text{(a)}\quad \begin{array}{r} 100 \swarrow \\ \underline{+\,2} \\ \textcircled{102} \end{array}$$

$$\text{(b)}\quad \begin{array}{rl} & \;\;5\;\;\;2 \swarrow \\ & \sqrt{27'98'41.} \\ & \underline{-\,25} \\ \textcircled{102} & \;\;2\;98 \end{array}$$

NOTE: The number found in this step must be the largest number which, when added to the circled amount, and then multiplied by that sum, is still smaller than the new remainder.

STEP 8. MULTIPLY the new value in the circle by the number of times its old value went into the remainder.

$$\begin{array}{r} \textcircled{102} \\ \underline{\times\,2} \\ 204 \end{array}$$

Place this product beneath the remainder:

$$\begin{array}{l} \;\;5\;\;\;2 \\ \sqrt{27'98'41.} \\ \underline{-\,25} \\ \;\;\;298 \\ \searrow 204 \end{array}$$

STEP 9. SUBTRACT this product from the remainder:

$$\begin{array}{l} \;\;5\;\;\;2 \\ \sqrt{27'98'41.} \\ \underline{-\,25} \\ \;\;\;298 \\ \underline{-\,204} \swarrow \\ \;\;\;\;94 \end{array}$$

STEP 10. REPEAT STEP 5.
That is, BRING DOWN the next group of two digits from the left; place it beside the remainder obtained in Step 9:

$$\begin{array}{rrr} 5 & 2 & \\ \hline \sqrt{\,}27' & 98' & 41. \\ -\underline{25} & & \Big| \\ 2 & 98 & \Big| \\ -\underline{2} & \underline{04} & \downarrow \\ & 94 & 41 \end{array}$$

STEP 11. REPEAT STEP 6.
That is, take the number in the answer space above the square root sign, double it and add a zero (multiply by 10).

$52 \times 2 \times 10 = 1040$

CIRCLE the result: $\enclose{circle}{1040}$

STEP 12. REPEAT STEP 7 EXACTLY.
That is, determine how many times the circled number will go into the new remainder obtained in Step 10:

$$\enclose{circle}{1040}\ \begin{array}{r} 9 \\ )\overline{9441} \\ \underline{9360} \\ 81 \end{array}$$

It will go 9 times.

(a) ADD the number to the amount in the circle and (b) PLACE the number above the group of digits you brought down.

$$\text{(a)}\ \begin{array}{r} 1040 \swarrow \\ \underline{+\ 9}\swarrow \\ \enclose{circle}{1049} \end{array}$$

$$\text{(b)}\ \begin{array}{rrr} 5 & 2 & 9 \swarrow \\ \hline \sqrt{\,}27' & 98' & 41. \\ -\underline{25} & & \\ 2 & 98 & \\ -\underline{2} & \underline{04} & \\ & 94 & 41 \end{array}$$

STEP 13. REPEAT STEP 8.
Multiply the new value in the circle by the new digit in the answer space.

$$\begin{array}{r} \enclose{circle}{1049} \\ \underline{\times\ 9} \\ 9441 \end{array}$$

PLACE their product beneath the remainder:

$$\begin{array}{crrr} & 5 & 2 & 9 \\ \hline & \sqrt{\,}27' & 98' & 41. \\ & -\underline{25} & & \\ \enclose{circle}{102} & 2 & 98 & \\ & -\underline{2} & \underline{04} & \\ & \searrow & 94 & 41 \\ \enclose{circle}{1049} & & \underline{94} & \underline{41} \end{array}$$

STEP 14. REPEAT STEP 9.
That is, SUBTRACT this product from the remainder:

$$\begin{array}{r} 5\ \ 2\ \ 9. \\ \sqrt{27'98'41.} \\ -25 \\ \hline 2\ 98 \\ -2\ 04 \\ \hline 94\ 41 \\ -94\ 41 \\ \hline \searrow 0 \end{array}$$

The process may be continued as long as necessary, until the remainder is zero or the answer has the desired number of decimal places (digits after the decimal point).

THE ANSWER, the square root, appears above the square root sign.

$\sqrt{279841} = 529$

27. REFER TO PANEL 1 READ carefully each step in Panel 1. When you have finished, return to Step 1. Suppose you wanted to take the square root of 4553. MARK OFF this number into groups, as required in Step 1: __________	45'53.
Beginning with the decimal point, you should mark off the groups as you move: ☐ to the left ☐ to the right	to the left
You wish to find the square root of 23159. MARK this number off into groups of two digits, working left from the decimal point. __________	2'31'59.
The first group on the left, in this case, contains _____ (how many) digits.	1
NOTE NOTE NOTE NOTE Skip two(2) pages to find page 21.	

28.

REFER TO PANEL 1, Step 1. (Page 16)

You may wish to take the square root of a <u>decimal</u> number such as 359.720193.

In this case, you mark the number off in groups of two digits, moving to the left from the decimal point, and then <u>to the right</u> from the decimal point, as follows:

←——— 3' 59.72'01'93 ———→

↑

decimal
point

MARK OFF the following decimal number:	
7948261.001908	7'94'82'61.00'19'08
MARK OFF the following:	
23.08	23.08 (no marks needed)

29. REFER TO PANEL I, Step 1. You may wish to take the square root of numbers such as 801, or 78, or 3 which are not perfect squares. For instance, $\sqrt{3}$ to three decimal places is 1.732 To calculate a square root to three decimal places, there must be three groups of two digits to the right of the decimal in the square itself. $\sqrt{3} = \sqrt{3.00'00'00} = 1.732$ For each decimal place in the root, <u>two zeros</u> must be placed after the decimal point at the beginning of the calculation. WRITE DOWN and MARK OFF the $\sqrt{3}$ so as to obtain a root with four decimal places: ______________	$\sqrt{3.00'00'00'00}$
30. REFER TO PANEL I Using the panel as a guide, find the square root of 249.64. You will do this step-by-step in the following frames. BEGIN with Step 1. MARK OFF the above number: ______________	2'49.64'

31. REFER TO PANEL 1, Step 2 What is the leftmost group in your number? ______ The largest perfect square less than this number is: __________ WRITE the original number beneath a square root sign, MARK IT OFF, and place the root of the largest perfect square in the position indicated in Step 2: ______________	2 1 (1 x 1 = 1) $\overset{1}{\sqrt{2'\,49.\,64}}$
32. REFER TO PANEL 1, Steps 3 and 4 Perform each of the indicated operations on the number 249.64:	$\begin{array}{l} \quad 1 \\ \sqrt{2'49.64} \\ -1 \quad \downarrow \\ \hline \quad 1\,49 \end{array}$

33.	
REFER TO PANEL 1, Step 5	
OBTAIN and CIRCLE the desired number:	⑳
REFER TO Steps 6 and 7	
The "new remainder" in your problem is _______(number).	149
DIVIDE the circled number into the new remainder:	7 ⑳) 149 140 9
The circled number goes into the new remainder _____ times, with _____ left over.	7 9
34.	
REFER TO PANEL 1, Step 7 (Page 17)	
WARNING! LOOK CAREFULLY at the Note.	
The number you found in Step 6 is 7. ADD it to the circled amount:	7 + ⑳ = 27
If the sum just obtained is multiplied by 7, the product is:	27 x 7 = 189
However, the new remainder is only 149. The product of the number obtained in Step 7 and the "new circled amount" must be <u>less than</u> 149.	
Therefore, instead of 7, try 6. ADD 6 to the circled amount: _______________ MULTIPLY the sum by 6: _______________ .	6 + ⑳ = 26 26 x 6 = 156
Is this product smaller than the "new remainder"? ☐ yes ☐ no	no

35.	
REFER TO PANEL 1, Steps 7 and 8	
LOOK at the Note.	
Both 7 and 6 produce too large a product by comparison with the new remainder.	
Try 5: ADD it to the circled amount: ______________	5 + ⑳ = 25
MULTIPLY this sum by 5: The product is __________.	125
Is this product less than the new remainder? ☐ yes ☐ no	yes
REFER TO Step 9	
PLACE this number in the indicated position above the square root sign, and ADD it to the amount in the circle:	
$\begin{array}{l} \quad 1 \\ \sqrt{2'\,49.64} \\ -1 \\ \hline \quad 149 \end{array}$	$\begin{array}{l} \quad 1\ \ 5 \\ \sqrt{2'49.64} \end{array}$ ㉕

36. REFER TO PANEL 1, Steps 8 and 9 The number found in Step 7 was 5. MULTIPLY the <u>new</u> value in the circle by this number:	25 x 5 = 125
PLACE this product in the location shown in Step 8, and SUBTRACT, as indicated in Step 9: $\sqrt{249.64}$ ㉕	$\begin{array}{r} 1\ \ 5\,. \\ \sqrt{2'49.64} \\ -1 \\ \hline 149 \\ -125 \\ \hline 24 \end{array}$
37. REFER TO PANEL 1, Step 10 (Page 18) PERFORM the indicated operation: $\begin{array}{r} 1\ \ 5\,. \\ \sqrt{2'49.64} \\ -1 \\ \hline 149 \\ -125 \\ \hline 24 \end{array}$ ㉕	$\begin{array}{r} 1\ \ 5\,. \\ \sqrt{2'49.64} \\ -1 \\ \hline 149 \\ -125 \\ \hline 24\ \ 64 \end{array}$
Now REFER to Step 11 PLACE the required number in the circle: ○	2(15) = 30◌ ↓ (300)
Now CONSIDER Step 12 The "new remainder" is __________ The circled number is __________ The circled number goes into the "new remainder" ______ (how many) times.	2464 300 8 (with a remainder of 64)

38.

REFER TO PANEL 1, Step 12

In Step 12, two operations are performed with the number found in the preceding frame.

PERFORM them:

```
         1  5.
       √2'49.64
        - 1
         149
(300)   - 125
          2464
```

```
   1  5 . 8
 √2'49.64
 - 1
  149
 - 125
   2464

  (308)
```

39.

REFER TO PANEL 1, Steps 13 and 14. REPEAT Steps 1 through 12, then PERFORM the operations shown in Steps 13 and 14:

```
             1  5 . 8
           √2'49.64
           - 1
 ㉕          149
           - 125
              2464
(308)       - 2464
                0
                ↓
             ______
```

The final remainder is _____.

0

The exact square root of 249.64 is _________.

15.8

40.

Using Panel 1 as little as possible, FILL IN the incomplete parts of the calculation below.

Make sure you understand each step.

FIND the square root of 51:

__. 1 __

√51.00'00

\- 49

(14◌) _00

___

5900

(142◌) - 5696

___

---

7. 1 4

√51.00'00

\- 49

2 00

(141) - 1 41

5900

(1424) - 5696

204

41.

DO NOT REFER TO THE PANEL

EXAMPLE: Find the square root of 27:

```
                 5. 1  9  6
               √27.00'00'00
               - 25
  101             200
                - 101
                  9900
  1029          - 9261
                  63900
  10386           62316
                   1584
```

You may use this example as a guide for the following problem:

FIND $\sqrt{17}$ to 3 decimal places:

$\sqrt{17}$ = __________

```
              4. 1  2  3
            √17.00 00 00
            - 16
               1 00
   (81)          81
                 19 00
   (822)         16 44
                  2 56 00
   (8243)         2 47 29
                     8 71
```

4.123

42.

DO NOT REFER TO THE PANEL

FIND $\sqrt{18}$ and CHECK it below:

$\sqrt{18.00'00}$

___

___

___

☐ 4

☐ 4.20

☐ 4.24

☐ 4.34

☐ 5

$$\begin{array}{rr} & 4.\ 2\ \ 4 \\ & \sqrt{18.00'00} \\ & -\ 16 \\ & 200 \\ (82) & -\ 164 \\ & 3600 \\ (844) & -\ 3376 \\ & 224 \end{array}$$

4.24

43.

FIND $\sqrt{529}$:

$\sqrt{529}$ = ______

$$\begin{array}{rl} & \quad 2\ \ 3. \\ & \sqrt{5'29.} \\ & \underline{-\ 4\qquad} \\ & \quad 1\ 29 \\ \text{㊸} & \quad \underline{1\ 29} \\ & \quad\ \ 0 \end{array}$$

23

Time completed ______________

YOU HAVE NOW FINISHED THE FIRST PART OF THIS LESSON. WRITE DOWN THE TIME. THEN, AFTER YOU HAVE REVIEWED THE MAIN IDEAS IN THE FOLLOWING SUMMARY, TAKE THE MASTERY TEST AT THE END OF THE BOOKLET.

MASTERY TEST

Time started ________________

1. CALCULATE each of the following powers:

   a. $(5.2)^2$ = __________
   b. $(-.04)^2$ = __________
   c. $(-1)^7$ = __________
   d. $6(3)^2$ = __________
   e. $(0)^3$ = __________

2. What is the exponent of the x term in each of the following?

   a. $7x^3$ __________
   b. $5x$ __________

3. What is the square root of 16? (CHECK one)

   a. ☐ 1
   b. ☐ 2
   c. ☐ 4
   d. ☐ 8
   e. ☐ 16

NOTE: Continue with question 4 on the next page.

4. What is the square root of 19 ?

   a. ☐ 4.00

   b. ☐ 4.30

   c. ☐ 4.36

   d. ☐ 4.46

   e. ☐ 5.00

Time completed ______________

WHEN YOU HAVE FINISHED THIS TEST, WRITE DOWN THE TIME. THEN TAKE THE LESSON TO YOUR INSTRUCTOR OR HIS ASSISTANT FOR CHECKING. WAIT UNTIL THE LESSON IS APPROVED BEFORE GOING ON TO THE NEXT LESSON.

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PM 431 - 59

ADVANCED
GENERAL EDUCATION PROGRAM

A HIGH SCHOOL SELF-STUDY PROGRAM

GEOMETRY
LEVEL: II
UNIT: 8
LESSON: 3

U.S. DEPARTMENT OF LABOR
MANPOWER ADMINISTRATION, JOB CORPS
NOVEMBER 1969

78

U.S. DEPARTMENT OF LABOR
MANPOWER ADMINISTRATION, JOB CORPS
NOVEMBER 1969

PANEL 1

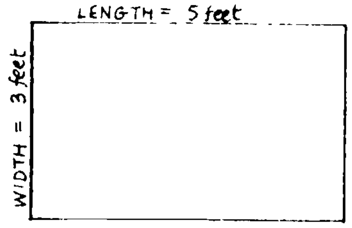


Figure A: Rectangle

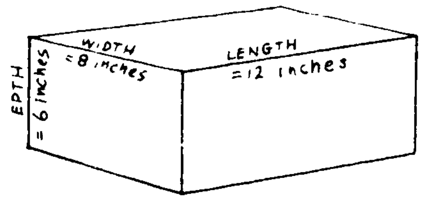


Figure B: Box

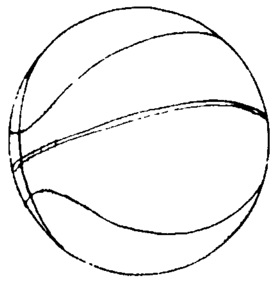


Figure C: Sphere - such as a basketball

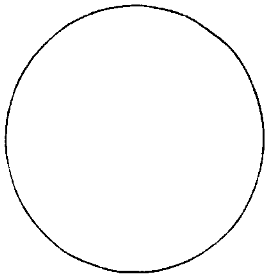


Figure D: Circle

1.

PREVIEW FRAME

In your studies, before you began the high school equivalency curriculum, you learned certain facts about triangles, squares, circles, and rectangles. In this lesson you will learn to find how much area is contained within these figures. You will also learn some properties of three-dimensional objects, such as boxes and spheres. First, however, you will learn to distinguish between such three-dimensional objects which have dimensions of length and width, but also height, and two-dimensional figures which can be drawn on a flat surface and have dimensions of only length and width.

NO RESPONSE REQUIRED

GO ON TO THE NEXT FRAME

2.

REFER TO PANEL I

In Figure A, the rectangle is described by the two quantities 3 feet and 5 feet. In Figure B, the box is described by the three quantities 6 inches, 8 inches, and 12 inches.

These quantities are called the "dimensions" of the figures shown.

The box:

☐ can be fully measured by the dimensions of length and width

☐ cannot be fully measured by the dimensions of length and width

Which of the following is true? (CHECK one)

☐ It takes more dimensions to measure a box than a rectangle.

☐ It takes fewer dimensions to measure a box than a rectangle.

☐ A box and a rectangle may be measured using the same number of dimensions.

cannot be fully measured . . .

It takes more dimensions . . .

3. REFER TO PANEL 1	
How many dimensions are involved in Figure A? ☐ 1 ☐ 2 ☐ 3	2
What are they? ☐ height ☐ length ☐ width	length width
How many dimensions are involved in Figure B? ☐ 0 ☐ 1 ☐ 2 ☐ 3	3
What are they? ☐ height ☐ length ☐ width	height length width
4. REFER TO PANEL 1 A flat surface, such as this page, has as many dimensions as the rectangle. An object, such as an orange crate, has as many dimensions as the box.	
A flat surface has: ☐ 0 dimensions ☐ 1 dimension ☐ 2 dimensions ☐ 3 dimensions	2 dimensions
A box-like object has: ☐ 0 dimensions ☐ 1 dimension ☐ 2 dimensions ☐ 3 dimensions	3 dimensions

5.	
REFER TO PANEL 1	
Figure C shows something which has: ☐ 0 dimensions ☐ 1 dimension ☐ 2 dimensions ☐ 3 dimensions	3 dimensions
Figure D is a circle which can be drawn on a piece of paper. A circle has: ☐ 0 dimensions ☐ 1 dimension ☐ 2 dimensions ☐ 3 dimensions	2 dimensions
The building you are in has: ☐ 0 dimensions ☐ 1 dimension ☐ 2 dimensions ☐ 3 dimensions	3 dimensions

6. REFER TO PANEL 1 A flat surface is called a plane. An object is called a solid.	
Figure A is a: ☐ plane figure ☐ solid figure	plane figure
Figure B is a: ☐ plane figure ☐ solid figure	solid figure
Figure C is a: ☐ plane figure ☐ solid figure	solid figure
Figure D is a: ☐ plane figure ☐ solid figure	plane figure
Solid figures have: ☐ 0 dimensions ☐ 1 dimension ☐ 2 dimensions ☐ 3 dimensions	3 dimensions
Plane figures have: ☐ 0 dimensions ☐ 1 dimension ☐ 2 dimensions ☐ 3 dimensions	2 dimensions

7.

DO NOT REFER TO THE PANEL

MATCH the terms to the numbered items below by writing one letter in each blank.

P plane figure

S solid figure

1. _____ can be drawn on a flat surface
2. _____ a circle
3. _____ a globe
4. _____ has three dimensions
5. _____ has two dimensions
6. _____ an ice cube
7. _____ a shoebox
8. _____ a square
9. _____ a triangle

1. P
2. P
3. S
4. S
5. P
6. S
7. S
8. P
9. P

8.

PREVIEW FRAME

You have learned the difference between plane figures and solid figures. Before studying these figures in more detail, there are some facts you must know about lines and angles. In the next section you will learn some mathematical terms used to describe two straight lines drawn on a flat surface.

NO RESPONSE REQUIRED

GO ON TO THE NEXT PAGE

PANEL 2

Figure A

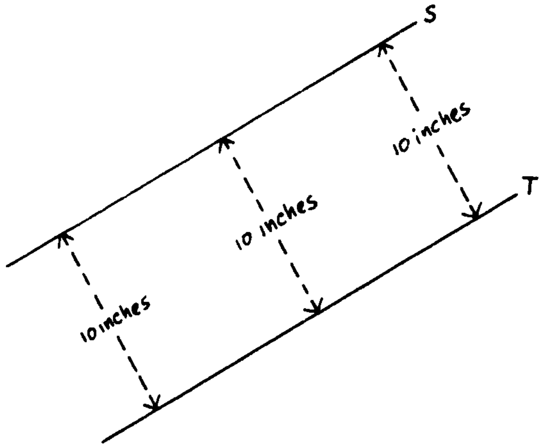


Figure B

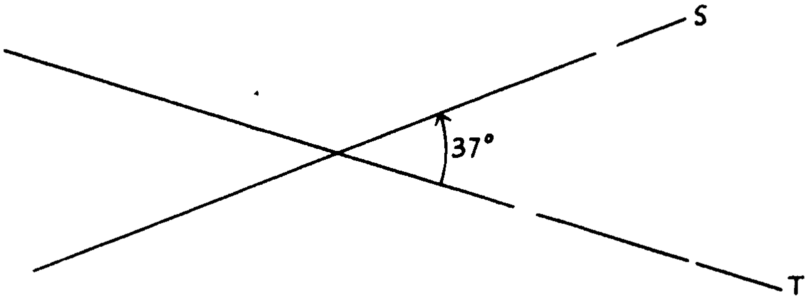
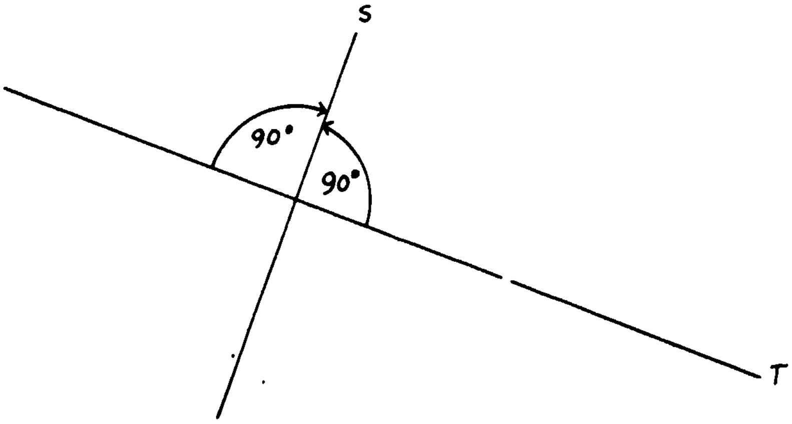


Figure C



9.	
REFER TO PANEL 2	
If the lines in Figure A were extended, they would:	
☐ become further apart	
☐ remain the same distance apart	remain the same distance apart
If the lines in Figure B were extended, they would:	
☐ become further apart	become further apart
☐ remain the same distance apart	
If the lines in Figure C were extended, they would:	
☐ become further apart	become further apart
☐ remain the same distance apart	
In which figures do lines S and T cross?	
☐ Figure A	
☐ Figure B	Figure B
☐ Figure C	Figure C
10.	
REFER TO PANEL 2	
Lines which never cross and stay the same distance apart are called <u>parallel</u> lines.	
Lines which cross at an angle of exactly ninety degrees are called <u>perpendicular</u> lines.	
Parallel lines are illustrated in Figure:	
☐ .	A
☐ B	
☐ C	
Perpendicular lines are illustrated in Figure:	
☐ A	
☐ B	
☐ C	C

11.

REFER TO PANEL 2

Lines which cross at an angle of 90° are called:

☐ parallel lines
☐ perpendicular lines

Lines which do not cross but stay the same distance apart are called:

☐ parallel lines
☐ perpendicular lines

perpendicular lines

parallel lines

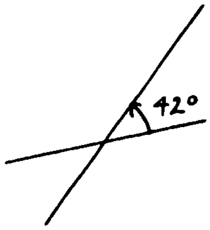
12.

DO NOT REFER TO THE PANEL

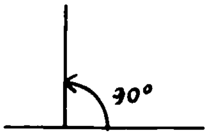
MATCH the terms to the examples below by writing a term in a blank where appropriate:

parallel lines
perpendicular lines

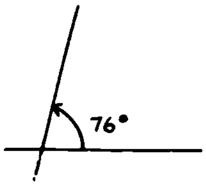
1. ____________________



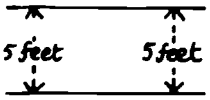
2. ____________________



3. ____________________



4. ____________________



2. perpendicular lines

4. parallel lines

13.

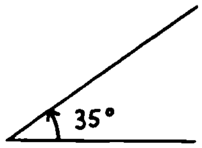
The angle between two perpendicular lines is called a <u>right angle</u>.

A right angle has:

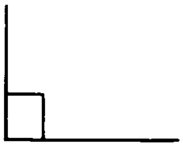
- ☐ 0 degrees
- ☐ 30 degrees
- ☐ 76 degrees
- ☐ 90 degrees
- ☐ 180 degrees

90 degrees

14.



A



B

Figure B above illustrates two perpendicular lines. The angle between the two lines in Figure B contains:

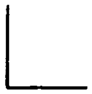
- ☐ 0 degrees
- ☐ 30 degrees
- ☐ 90 degrees
- ☐ 270 degrees

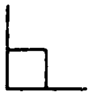
The angle in Figure B is called a:

- ☐ parallel angle
- ☐ perpendicular angle
- ☐ right angle

A special symbol is used to denote a right angle. From the two figures above, you can see that the symbol for a right angle is:

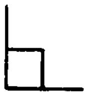
☐ 

☐ 

☐ 

90 degrees

right angle



15.

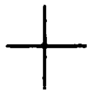
A right angle is found between two:

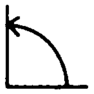
☐ parallel lines
☐ perpendicular lines

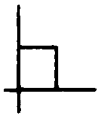
A right angle contains:

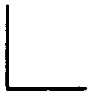
☐ 0 degrees
☐ 90 degrees
☐ 180 degrees
☐ 270 degrees

The symbol for a right angle is:

☐ 

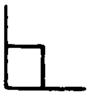
☐ 

☐ 

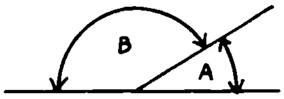
☐ 

perpendicular lines

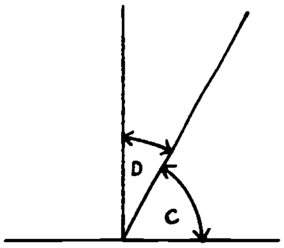
90 degrees



16.



1



2

Angle A = 30°
Angle B = 150°
Angle C = 60°
Angle D = 30°

In which figure do the two angles add up to a right angle:

☐ Figure 1
☐ Figure 2

Figure 2

In which figure do the angles add up to 180 degrees?

☐ Figure 1
☐ Figure 2

Figure 1

17.

Two angles which add up to a right angle (90°) are called <u>complementary angles</u>.

Two angles which add up to 180° are called <u>supplementary angles</u>.

Angle a = 50°
Angle b = 110°
Angle c = 30°
Angle d = 40°
Angle e = 70°

Which two angles are complementary? (CHECK two)

- ☐ Angle a
- ☐ Angle b
- ☐ Angle c
- ☐ Angle d
- ☐ Angle e

Angle a

Angle d

Which two angles are supplementary? (CHECK two)

- ☐ Angle a
- ☐ Angle b
- ☐ Angle c
- ☐ Angle d
- ☐ Angle e

Angle b

Angle e

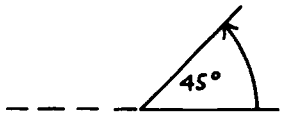
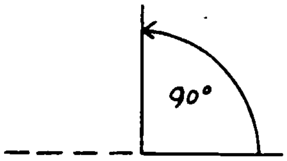
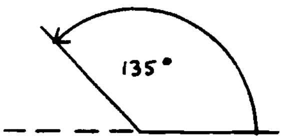
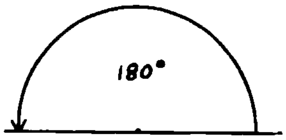
18.

WRITE a C in the blank below when angles a and b are complementary. WRITE an S in the blank when angles a and b are supplementary:

1. ______ angle a = 60°, angle b = 40°
2. ______ angle a = 30°, angle b = 150°
3. ______ angle a = 17°, angle b = 253°
4. ______ angle a = 17°, angle b = 73°

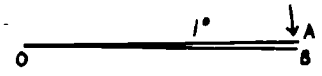
2. S

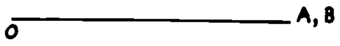
3. C

19. Complementary angles: ☐ add up to a right angle ☐ add up to 180° ☐ add up to 360°	add up to a right angle
Supplementary angles: ☐ add up to a right angle ☐ add up to 180° ☐ add up to 360°	add up to 180°
20.     From the above figures you can see that a straight line can be considered to be an angle of: ☐ 45° ☐ 90° ☐ 135° ☐ 180°	180°
Angles which add up to a straight line are: ☐ complimentary angles ☐ equal angles ☐ supplementary angles	supplementary angles

21.

FOOTNOTE FRAME



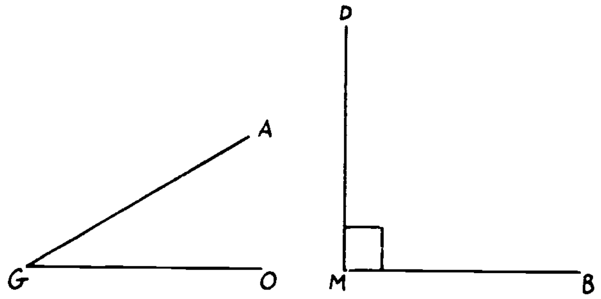


An angle of 180° is not the only angle which forms a straight line. Figure 1 represents an angle of 1°. Line OA is being brought down toward OB, and the angle grows smaller. In Figure 2 the lines coincide, that is, they form a single straight line, and the angle equals zero degrees.

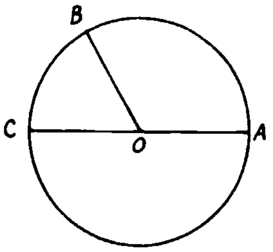
NO RESPONSE REQUIRED

GO ON TO THE NEXT FRAME

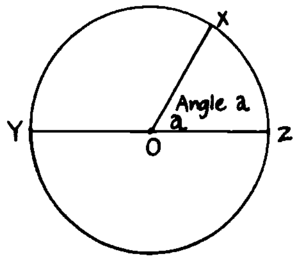
22.



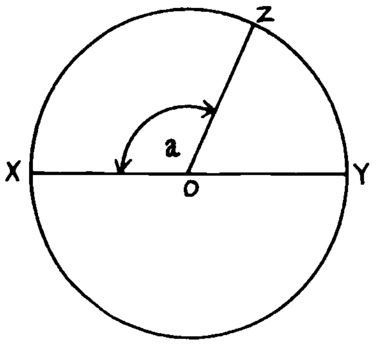
The above angles can be referred to as angle AGO and angle DMB, or angle OGA and angle BMD.



The smallest angle above is referred to as: ______________	COB or BOC
The largest angle is referred to as: ______________	COA or AOC

23.	
	
The above represents a circle, center at 0, with diameter YZ and radius OX.	
Angle a can be referred to as angle:	
☐ YOZ	
☐ XZO	
☐ ZOX	ZOX
☐ XOZ	XOZ
The angle which is supplementary to angle a can be referred to as angle:	
☐ XYZ	
☐ XOY	XOY
☐ YOZ	
☐ YXO	
☐ YOX	YOX

24.



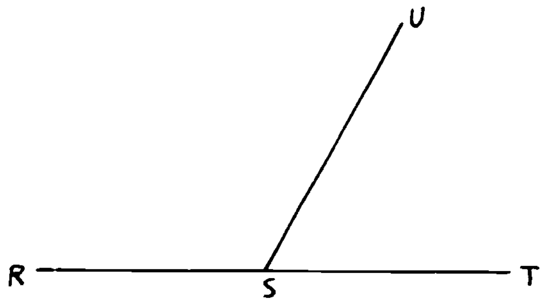
Given: a circle, center at 0, diameter XY, radius OZ, and angle YOZ = 65°.

Angle a has:

☐ 25°
☐ 45°
☐ 115°
☐ 165°
☐ 265°
☐ none of the above

115°

25.



Straight line US meets the straight line RT at S. How are the two angles UST and USR related?

☐ They are complementary.
☐ They are equal.
☐ They are supplementary.

They are supplementary.

26.

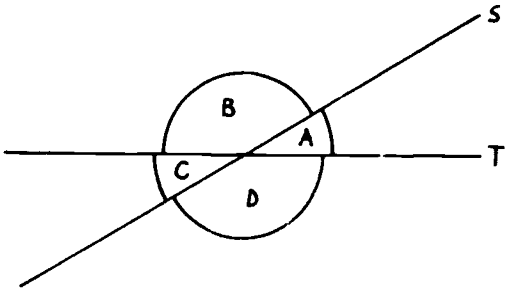
PREVIEW FRAME

You now know the difference between complementary angles and supplementary angles. In the next section you will learn some important facts about the angles formed when two straight lines cross each other.

NO RESPONSE REQUIRED

GO ON TO THE NEXT FRAME

27.



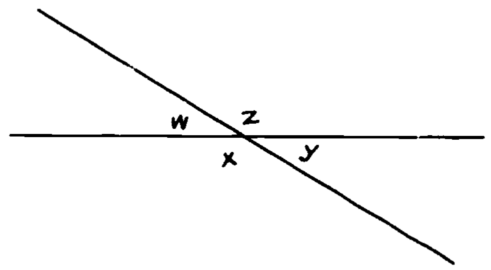
When two straight lines cross, the opposite angles are equal. In the above figure, angles a and c are equal.

Which other pair(s) of angles is/are equal ?

☐ a and b
☐ b and c
☐ b and d
☐ c and d

b and d

28.



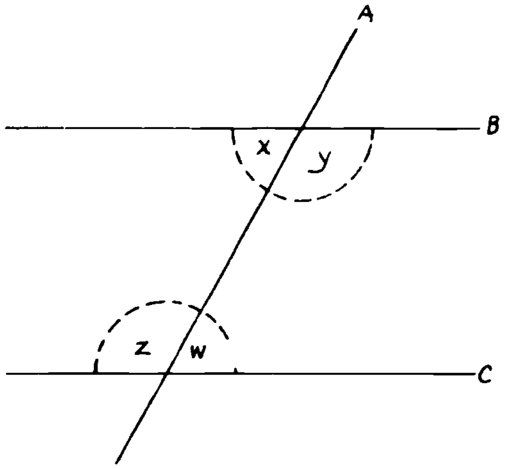
In the above figure, which pairs of angles are equal?

☐ x and y
☐ x and z
☐ x and w
☐ y and z
☐ y and w
☐ z and w

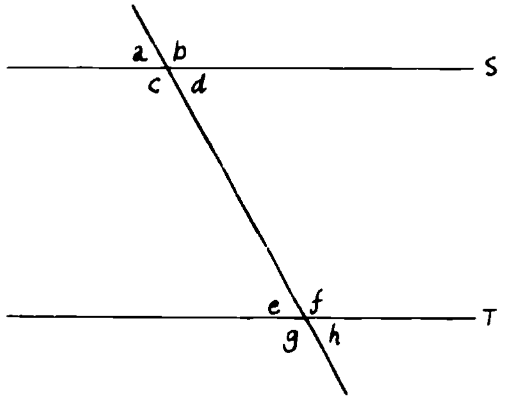
x and z

y and w

29.



Lines b and c are parallel. Line a intersects them both. In this circumstance, angle x equals angle w, and angle y equals angle z.

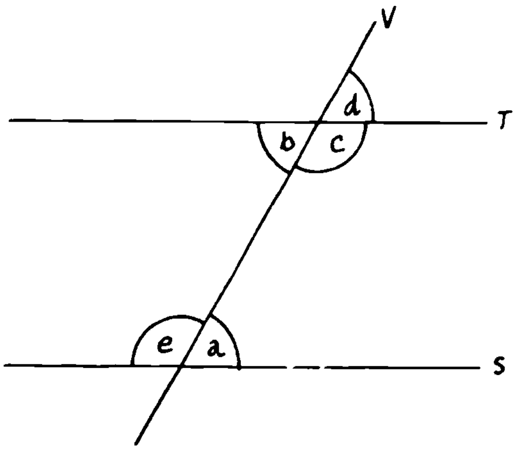


In the above figure, lines S and T are parallel. Which of the following is/are true?

☐ angle c = angle d
☐ angle c = angle e
☐ angle c = angle f
☐ angle e = angle g
☐ angle e = angle f
☐ angle e = angle d

angle c = angle f

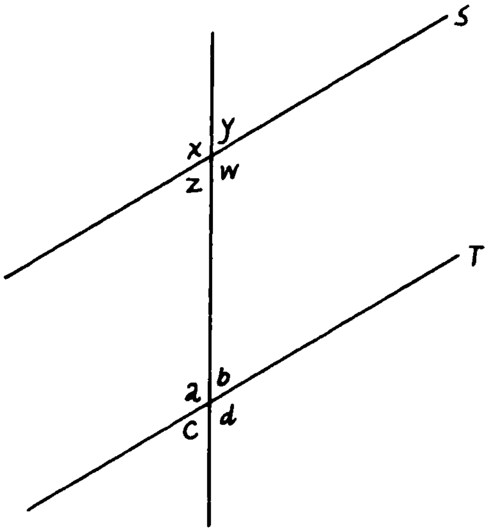
angle e = angle d

30.  In the above figure, lines S and T are parallel. You know from a preceding section that angle b equals: ☐ angle c ☐ angle d ☐ angle e	angle d
Because lines S and T are parallel, angle a equals angle b, and angle c equals angle e. From these facts you can deduce that angle a also equals: ☐ angle c ☐ angle d ☐ angle e	angle d

31.

Recall that when two straight lines intersect, the opposite angles are equal. Remember also that if angle a = angle b and angle b = angle c, then angle a = angle c.

In the figure below, lines S and T are parallel:

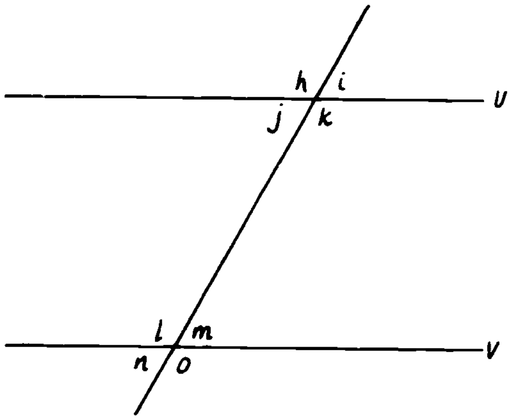


CHECK below the angle(s) which equal(s) angle w:

☐ a	a
☐ b	
☐ c	
☐ d	d
☐ x	x
☐ y	
☐ z	

32.

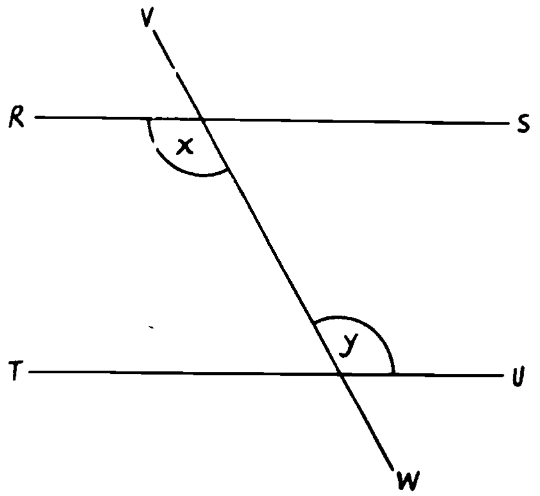
Lines U and V are parallel.



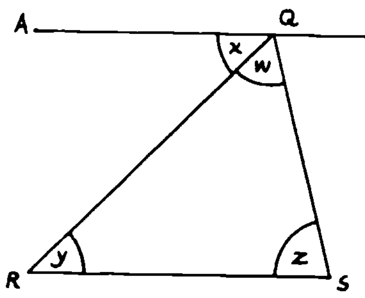
Which of the following is/are true?

☐ angle o equals angle i
☐ angle m equals angle i
☐ angle n equals angle i
☐ angle n equals angle h

angle m equals angle i
angle n equals angle i

33.  Given: straight lines RS, TU and VW in a plane. Angle x equals angle y. What relation, if any, exists between RS and TU? ☐ They are equal. ☐ They are parallel. ☐ They are perpendicular. ☐ No relation exists between them.	They are parallel.

34.



Straight line AB meets triangle QRS at the point Q. AB is parallel to side RS.

Angle x equals:

☐ angle y
☐ angle z
☐ angle w

angle y

35.

PREVIEW FRAME

You have learned some important facts about lines and angles lying in a flat surface. You will now learn many of the properties of various plane figures, beginning with triangles.

NO RESPONSE REQUIRED

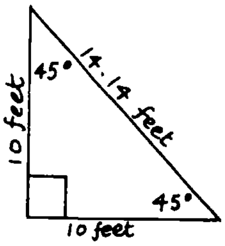
GO ON TO THE NEXT FRAME

36.

A right triangle is a triangle containing a right angle. A scalene triangle is a triangle with no two sides or angles equal. An isosceles triangle is a triangle with two equal sides and two equal angles. An equilateral triangle is a triangle with three equal sides and three equal angles.

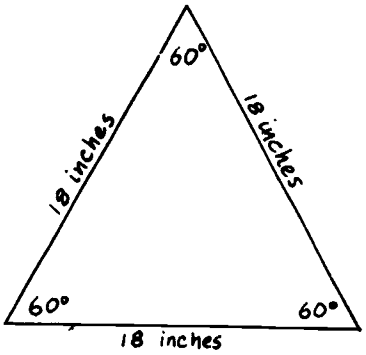
MATCH the terms below to the examples by writing one or more letters in each blank:

A. equilateral triangle
B. isosceles triangle
C. right triangle
D. scalene triangle



1. ____

1. B, C

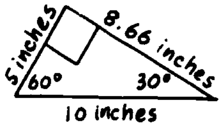


2. ____

2. A

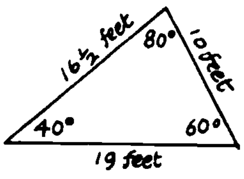
3. ____

3. C, D

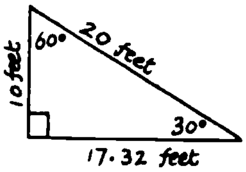


108

37.



Triangle A



Triangle B

Which of the following is/are true?

Triangle A is:

☐ a right triangle
☐ scalene
☐ isosceles
☐ equilateral
☐ equiangular

Triangle B is:

☐ a right triangle
☐ scalene
☐ isosceles
☐ equilateral
☐ equiangular

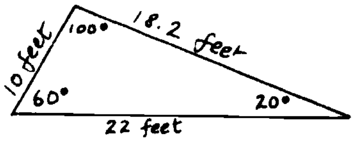
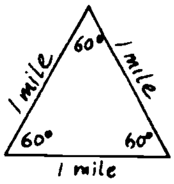
scalene

a right triangle
scalene

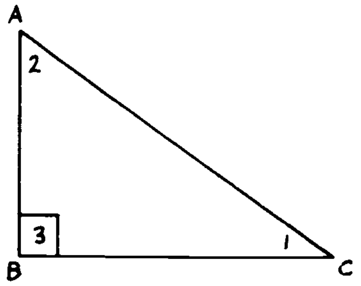
38.

MATCH the terms to the definitions and examples below by writing one or more letters in each blank:

A. equilateral triangle
B. isosceles triangle
C. right triangle
D. scalene triangle

1. ______ all sides are equal	1. A
2. ______ has two equal sides.	2. B
3. ______ has no two equal sides or angles	3. D
4. ______ contains a right angle	4. C
5. ______ 	5. D
6. ______ 	6. A

39.



In the figure above, ABC is a right triangle. Which of the following is true?

☐ Line AB = line BC
☐ Line AC = line AB
☐ Angle 1 = Angle 2
☐ Angle 1 = Angle 3
☐ None of these

None of these

40.

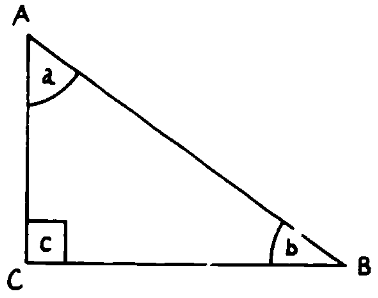
PREVIEW FRAME

Because it contains a right angle, the right triangle has several special properties which other triangles do not possess. For example, if you know the lengths of two sides of a right triangle, you can always find the length of the third side. In the next section you will learn how to do this.

NO RESPONSE REQUIRED

GO ON TO THE NEXT FRAME

41.

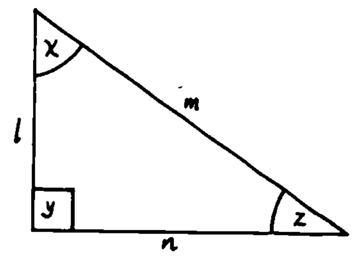


A right triangle is shown above.

The right angle is angle: ☐ a ☐ b ☐ c	c
The side opposite the right angle is side: ☐ AB ☐ AC ☐ BC	AB

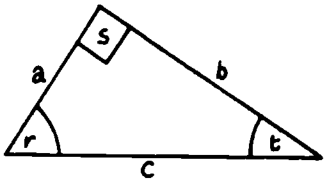
42.

The side opposite the right angle of a right triangle is called the hypotenuse. The other two sides are called the legs.



In the above right triangle, the hypotenuse is:	
☐ l	
☐ m	m
☐ n	
The legs are:	
☐ l	l
☐ m	
☐ n	n
The right angle is angle:	
☐ x	
☐ y	y
☐ z	

43.

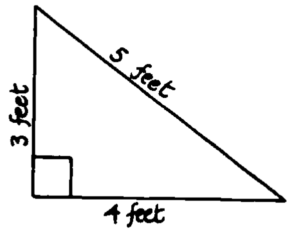


WRITE one or more letters from the above diagram in each blank below:

1. _____ legs
2. _____ hypotenuse
3. _____ right angle

1. a, b
2. c
3. s

44.



The lengths of the legs of the above right triangle are:

☐ 3 feet
☐ 4 feet
☐ 5 feet

CHECK below the squares of the lengths of the legs:

☐ 3
☐ 4
☐ 9
☐ 16
☐ 25

The sum of these two squares is:

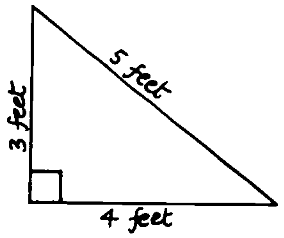
☐ 13
☐ 16
☐ 25
☐ 34

3 feet
4 feet

9
16

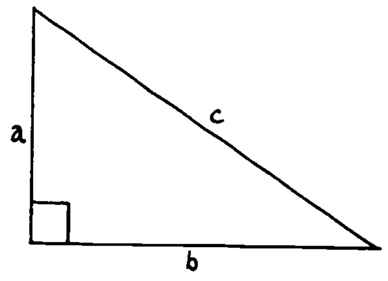
25

45.



The sum of the squares of the legs of the above triangle is: ☐ 7 ☐ 12 ☐ 16 ☐ 25	25
The hypotenuse squared is equal to: ☐ 5 ☐ 10 ☐ 16 ☐ 25 ☐ 50	25
The hypotenuse squared is therefore equal to: ☐ the sum of the legs ☐ the square of the sum of the legs ☐ the sum of the squares of the legs	the sum of the squares of the legs

46.

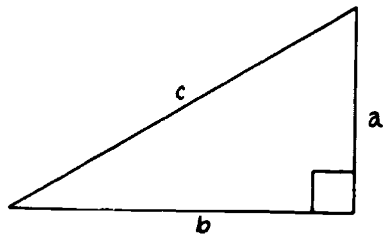


In the above right triangle, $a^2 + b^2$ equals:

- ☐ c
- ☐ c^2
- ☐ square root of c

c^2

47.



CHECK below the formula relating the hypotenuse and legs of the above right triangle:

- ☐ $c = a + b$
- ☐ $a^2 = b^2 + c^2$
- ☐ $a = b^2 + c$
- ☐ $c^2 = a + b$
- ☐ $c^2 = a^2 + b^2$
- ☐ $b^2 = c^2 + a^2$

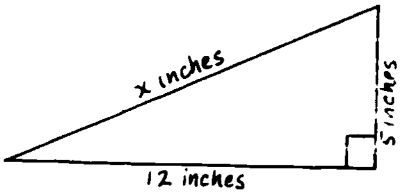
$c^2 = a^2 + b^2$

In this triangle, c is:

- ☐ a leg
- ☐ the hypotenuse

the hypotenuse

48.



PROBLEM: In the above right triangle, find the value of x.

You know that x^2 is equal to:

☐ 5 + 12
☐ the square root of 5 + 12
☐ $(5 + 12)^2$
☐ $(5)^2 + (12)^2$

x^2 is therefore equal to:

☐ 17
☐ the square root of 17
☐ $(17)^2$
☐ 25 + 144

Simplifying this, x^2 equals:

☐ 17
☐ about 4.1
☐ 289
☐ 169

$(5)^2 + (12)^2$

25 + 144

169

49.

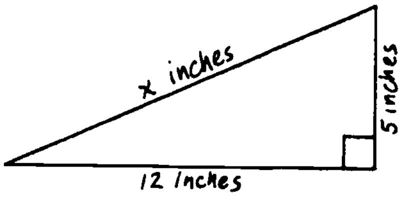


TABLE OF SQUARES

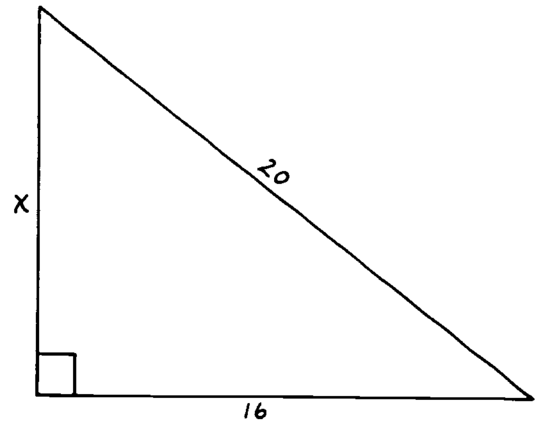
Number	Square of Number
11	121
12	144
13	169
14	196
15	225

In the preceding frame you found that the square of the hypotenuse equals 169.

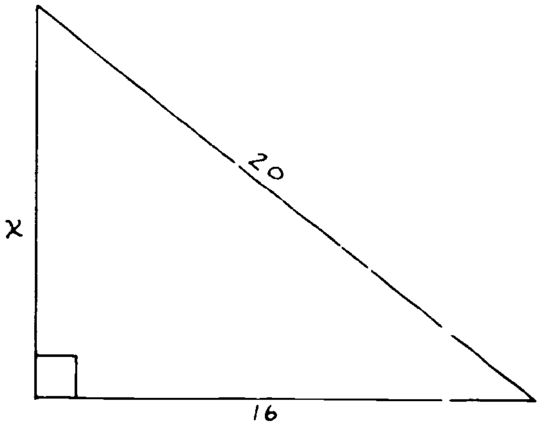
The value of x in the triangle is, therefore:

☐ 11
☐ 121
☐ 12
☐ 144
☐ 13
☐ 169
☐ 14
☐ 196
☐ 15
☐ 225

13

50. 	
In the above right triangle, the unknown value is: ☐ a leg ☐ the hypotenuse	a leg
Which of the following can be used to find x? ☐ $x^2 = (20)^2 + (16)^2$ ☐ $x = 20 + 16$ ☐ $(20)^2 = x^2 + (16)^2$ ☐ $(20)^2 = (x + 16)^2$	$(20)^2 = x^2 + (16)^2$

51.



If $(20)^2 = x^2 + (16)^2$, you can rearrange the equation (as you learned in a previous lesson) and obtain:

☐ $x^2 = (16)^2 - (20)^2$
☐ $x^2 = (16)^2 - (20)^2$
☐ $x^2 = (20)^2 - (16)^2$

$(20)^2 = 400$
$(16)^2 = 256$

Therefore, x^2 equals:

☐ 36
☐ 656
☐ 144
☐ -111

$x^2 = (20)^2 - (16)^2$

144

52.

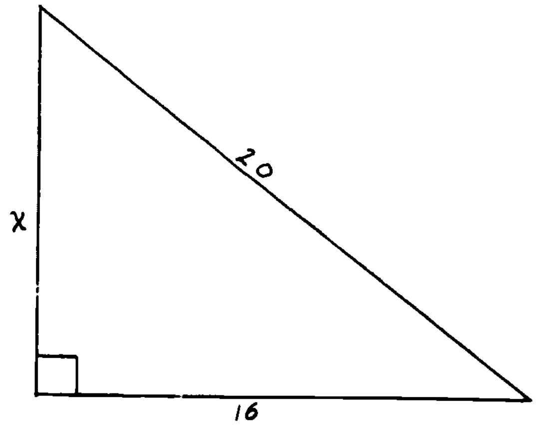


TABLE OF SQUARES

Number	Square of Number
11	121
12	144
13	169
14	196
15	225

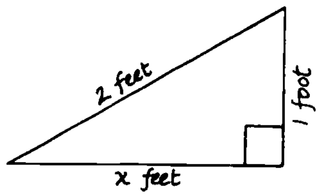
In the last frame you found that $x^2 = 144$. Therefore, the length of the unknown leg in the right triangle is:

☐ 11
☐ 12
☐ 13
☐ 14
☐ 15

12

122 - 1

53.



For a right triangle, $(\text{hypotenuse})^2$ equals $(\text{leg})^2 + (\text{leg})^2$.

In the above diagram:

☐ $x^2 = (2)^2 + (1)^2$
☐ $x = (2)^2 + (1)^2$
☐ $x = (2)^2 + (1)^2$
☐ $x^2 = (2)^2 - (1)^2$

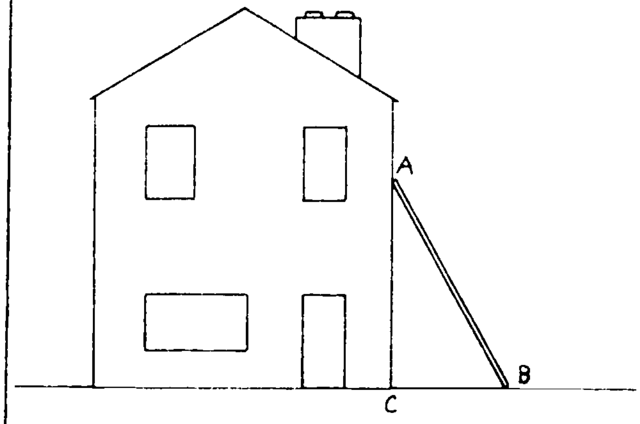
Therefore, x equals:

☐ $(5)^2$
☐ the square root of 5
☐ 5
☐ $(3)^2$
☐ the square root of 3
☐ 3

$x^2 = (2)^2 - (1)^2$

the square root of 3

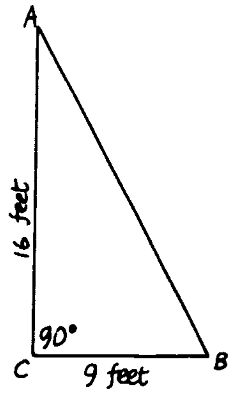
54.



In the above figure, a ladder is leaning against the wall of a house. The wall is perpendicular to the ground.

Angle ACB is equal to: ☐ 45° ☐ 90° ☐ 180°	90°
This triangle is a(n): ☐ equilateral triangle ☐ isosceles triangle ☐ right triangle	right triangle

55.



In the above diagram, AB is a ladder leaning against a wall. What is the length of the ladder? ($AB^2 = AC^2 + BC^2$)

☐ 18.36 feet
☐ 20.29 feet
☐ 24.56 feet
☐ 28.34 feet

18.36 feet

56.

PREVIEW FRAME

In a previous lesson you learned how to find the perimeter of various plane figures, such as the triangle and rectangle. It is also possible to calculate the <u>amount of space</u> enclosed by these plane figures. When you learn to do this, you will be able to find the amount of floor space in a room, or determine how much wood you need to make a box. The next section will teach you to make these calculations.

NO RESPONSE REQUIRED

GO ON TO THE NEXT FRAME

57.

A length, or a perimeter, is measured in feet or inches. The amount of space enclosed by a plane figure is measured in square feet or square inches.

In the above figure, the dotted lines could be measured in:

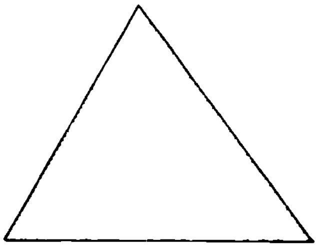
☐ feet
☐ inches
☐ square feet
☐ square inches

The shaded portion of the figure could be measured in:

☐ feet
☐ inches
☐ square feet
☐ square inches

feet
inches

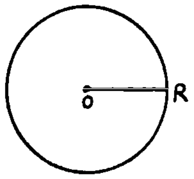
square feet
square inches

58.	
	
The shaded part of the above figure is called its area.	
Area can be measured in: ☐ feet ☐ square feet	square feet
The area is a measure of: ☐ the perimeter of the figure ☐ the space enclosed by the figure	the space enclosed by the figure

59.

"Feet" is often abbreviated "ft."
"Inches" is often abbreviated "in."

"Square feet" can be abbreviated "sq. ft."
"Square inches" can be abbreviated "sq. in."



The quantity 78.5 sq. in. would refer to:

☐ the radius OR of the circle
☐ the area of the circle

The quantity 5 in. would refer to:

☐ the radius OR of the circle
☐ the area of the circle

the area of the circle

the radius OR of the circle

PANEL 3

Figure 1

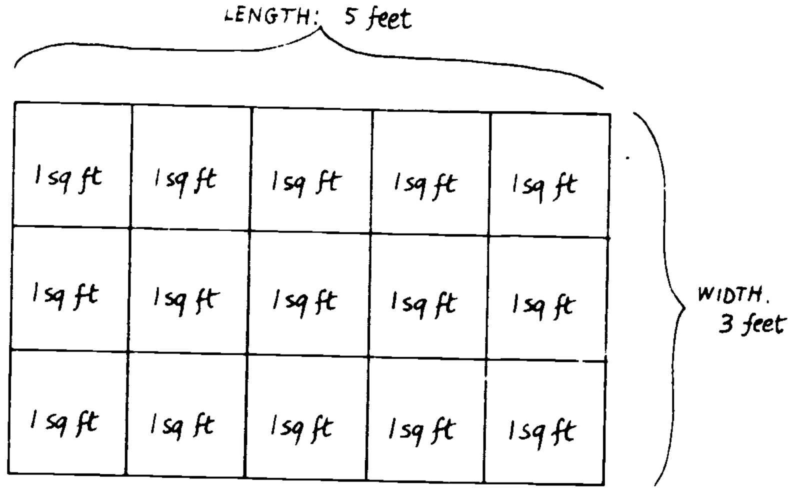
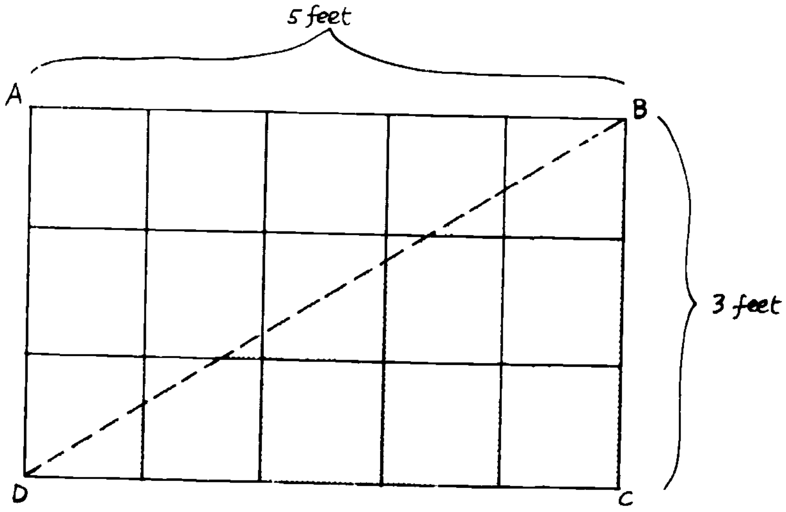
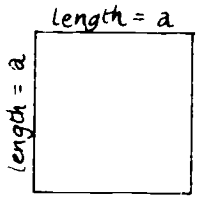


Figure 2



60.	
REFER TO PANEL III, Figure 1	
Figure 1 shows the floor of a small hallway, 3 feet by 5 feet in size.	
The hall floor is a: ☐ circle ☐ rectangle ☐ square ☐ triangle	rectangle
If you multiply its length times its width, you obtain: ☐ 3 square feet ☐ 5 square feet ☐ 15 square feet ☐ 18 square feet	15 square feet
In a rectangle, the length times the width: ☐ does not equal the area ☐ equals the area	equals the area
61.	
REFER TO PANEL III	
A man wishes to cover the floor space of the hallway with linoleum tiles. Each tile has an area of 1 square **foot**	
The area of the hallway is: ☐ 3 square feet ☐ 15 square feet ☐ 18 square feet	15 square feet
How many tiles does he need? ☐ 3 ☐ 5 ☐ 10 ☐ 15	15

n^2.	
A square is a rectangle in which the length equals the width.	
The area of a square equals: ☐ length times width ☐ perimeter ☐ square of hypotenuse	length times width
	
In the above diagram of a square, the letter a represents the value of: ☐ the length ☐ the width ☐ both of the above ☐ neither of the above	both of the above
The area of this square equals: ☐ a ☐ 2a ☐ a^2 ☐ a^3	a^2
This quantity should be measured in: ☐ inches ☐ square feet	square feet
What you have learned in this frame should tell you why the second power of a number is called its ____________.	square

63.

WRITE the letters A, B, and C where appropriate in the blanks below. You may write more than one letter in a blank:

A. area of a rectangle
B. area of a square
C. radius of a circle

______	equals length times width	A, B
______	equals width squared	B
______	can be measured in square feet or square inches	A, B
______	can be measured in feet or inches	C

64.

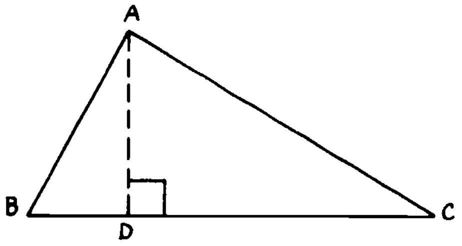
SOLVE the following problem:

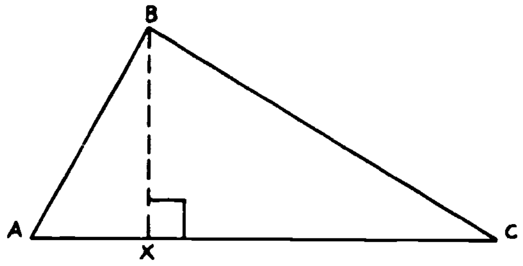
A rectangular park measures 100 feet by 75 feet. What is the area of the park?

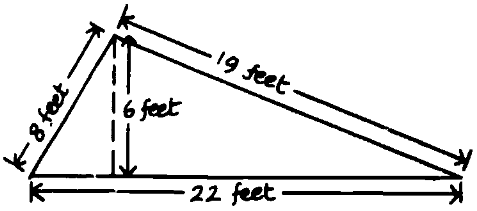
☐ 175 feet
☐ 175 square feet
☐ 1750 square feet
☐ 7500 feet
☐ 7500 square feet

7500 square feet

65.	
NOW REFER TO PANEL III, Figure 2	
The area of the rectangle ABCD in Figure 2 is:	
☐ larger than the area of the rectangle in Figure 1	
☐ smaller than the area of the rectangle in Figure 1	
☐ the same as the area of the rectangle in Figure 1	the same as the area of . . .
In Figure 2, the line DB divides the rectangle into:	
☐ two rectangles	
☐ two squares	
☐ two triangles	two triangles
From the Figure you can see that the area of triangle BCD is equal to:	
☐ half the area of the rectangle	half the area of the rectangle
☐ one third the area of the rectangle	
☐ the area of the rectangle	
The area of triangle BCD equals:	
☐ 5 square feet	
☐ 7.5 square feet	7.5 square feet
☐ 15 square feet	
☐ 30 square feet	

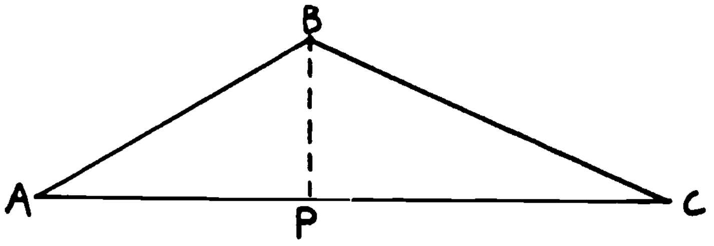
66.	
	
The above triangle "rests on" one side. This side is the line: ☐ AB ☐ AC ☐ AD ☐ BC	BC
The "highest" point in the triangle is point: ☐ A ☐ B ☐ C ☐ D	A
The dotted line AD is: ☐ parallel to the side the triangle rests on ☐ perpendicular to the side the triangle rests on	perpendicular to the side the . . .

67.	
	
The side a triangle "rests on" is called its <u>base</u>. The perpendicular from the base to the "highest point" of the triangle is called its <u>height</u>.	
The base of the above triangle is the line: ☐ AB ☐ BX ☐ AC ☐ BC	AC
The height of the triangle is the line: ☐ AB ☐ BX ☐ AC ☐ BC	BX

68.	
Let the area of a triangle be denoted by A. The base is denoted by b; the height is denoted by h.	
The formula for the area of a triangle is:	
$A = 1/2 (b \times h)$, or simply, $A = bh/2$	
	
In the above triangle, b equals:	
☐ 6 feet	
☐ 8 feet	
☐ 19 feet	
☐ 22 feet	22 feet
h equals:	
☐ 6 feet	6 feet
☐ 8 feet	
☐ 19 feet	
☐ 22 feet	
A equals:	
☐ 28 feet	
☐ 28 square feet	
☐ 132 square feet	
☐ 66 square feet	66 square feet

69.	
REFER AGAIN TO PANEL III, Figure 2	
Consider the triangle BCD.	
The base of this triangle measures:	
☐ 3 feet	
☐ 5 feet	5 feet
☐ 5.8 feet	
The height of this triangle measures:	
☐ 3 feet	3 feet
☐ 5 feet	
☐ 5.8 feet	
Using the formula A = bh/2 find the area of this triangle.	
The area of this triangle equals:	
☐ 7.5 sq.ft.	7.5 sq.ft.
☐ 8 feet	
☐ 8 sq.ft.	
☐ 15 feet	
☐ 15 sq.ft.	
The area of the rectangle ABCD equals:	
☐ 7.5 sq.ft.	
☐ 8 sq.ft.	
☐ 15 sq.ft.	15 sq.ft.
☐ 30 sq.ft.	

70.



In the above triangle, AB = 4 inches, BC = 5 inches, CA = 8 inches, and BP = 2 inches.

WRITE the formula for finding the area of a triangle.

____

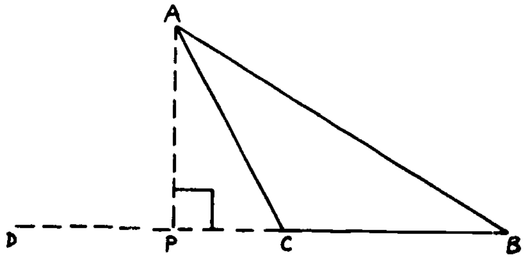
Now find the area of the triangle. The area of triangle ABC equals:

☐ 8 sq.in.
☐ 16 sq.in.
☐ 28 sq.in.
☐ 32 sq.in.

A = bh/2

8 sq.in.

71.



In the above figure, the dotted line DC is a continuation of the base of the triangle. The line AP is perpendicular to the continuation of CB.

The line AP is therefore:

☐ the base
☐ the height
☐ the hypotenuse
☐ a side

To find the area of triangle ABC, you would:

☐ find the area of triangle APC and APB, then subtract the area of APC from the area of APB to find your answer

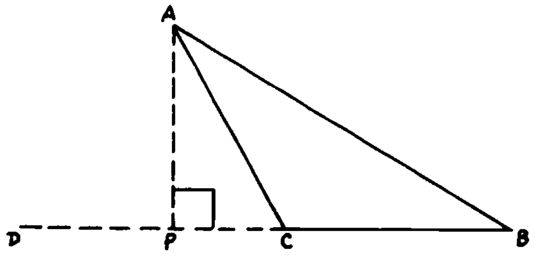
☐ find 1/2 of AC x AB

☐ find 1/2 of AP x CB

the height

find 1/2 of AP x CB

72.



Let AB = 15 feet, BC = 10 feet, CA = 8 feet, AP = 6 feet, and PC = 2 feet.

The area of triangle ABC equals:

☐ 6 sq.ft.
☐ 12 sq.ft.
☐ 30 sq.ft.
☐ 36 sq.ft.
☐ 48 sq.ft.

30 sq.ft.

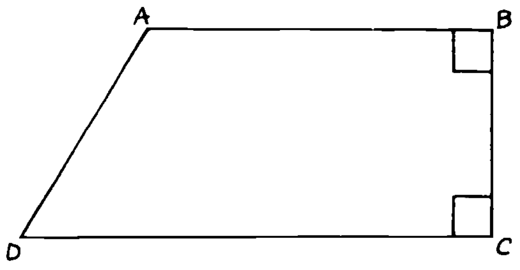
73.

PREVIEW FRAME

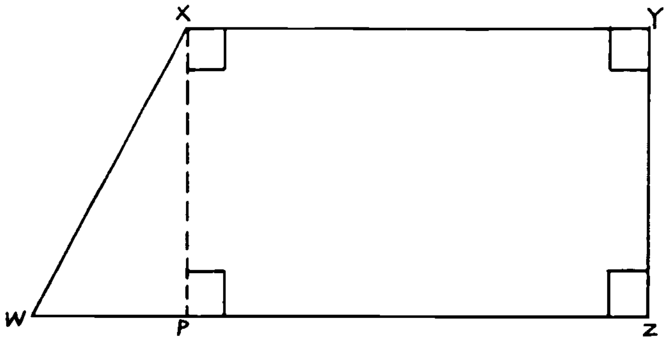
You have learned to find the area of triangles, rectangles and squares. In the next section you will learn to use these skills to determine the areas of irregular shapes which are composed of triangles, rectangles, and squares.

NO RESPONSE REQUIRED

GO ON TO THE NEXT FRAME

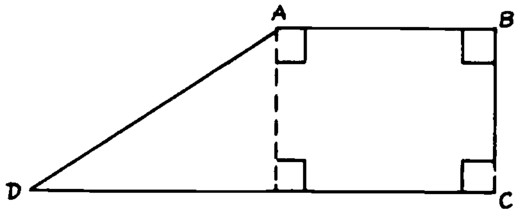
74.	
	
In the above figure, AB is parallel to DC, and BC is perpendicular to both. If a line perpendicular to DC were drawn downward from point A, the figure would be broken into two simpler figures. What would these figures be?	
☐ a rectangle and a square	
☐ a rectangle and a triangle	a rectangle and a triangle
☐ two triangles	
☐ two rectangles	
You can see that the area of the figure ABCD would be equal to:	
☐ the difference of the areas of the two simpler figures	
☐ the product of the areas of the two simpler figures	
☐ the sum of the areas of the two simpler figures	the sum of the areas of the . . .

75.



In the above figure, XY is parallel to WZ and YZ is perpendicular to both. The perpendicular XP has been drawn in to divide the figure.	
Let YZ = 10 feet, XY = 15 feet, and WZ = 20 feet.	
The line XP is the same length as: ☐ YZ ☐ XY ☐ WZ	YZ
The area of the rectangle XYZP equals: ☐ 75 sq.ft. ☐ 150 sq.ft. ☐ 200 sq.ft.	150 sq.ft.
The base of the triangle XPW equals: ☐ 5 feet ☐ 10 feet ☐ 15 feet	5 feet
The height of the triangle XPW equals: ☐ XY ☐ YZ ☐ WP	YZ
The area of the triangle XPW equals __________.	25 sq.ft.
The area of the whole figure equals __________.	175 sq.ft.

76.



Given: AB is parallel to CD and BC is perpendicular to both. AB = 14 in., DC = 30 in., BC = 10 in.

The perpendicular to DC from A has been drawn.

The area of the <u>rectangular</u> part equals:

- ☐ 70 sq.in.
- ☐ 140 sq.in.
- ☐ 30 sq.in.

140 sq.in.

The area of the <u>triangular</u> part equals:

- ☐ 80 sq.in.
- ☐ 150 sq.in.
- ☐ 160 sq.in.

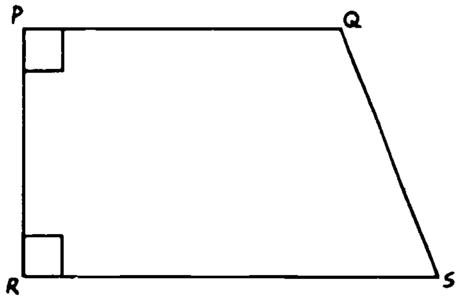
80 sq.in.

The area of the total figure ABCD equals:

- ☐ 150 sq.in.
- ☐ 220 sq.in.
- ☐ 300 sq.in.

220 sq.in.

77.



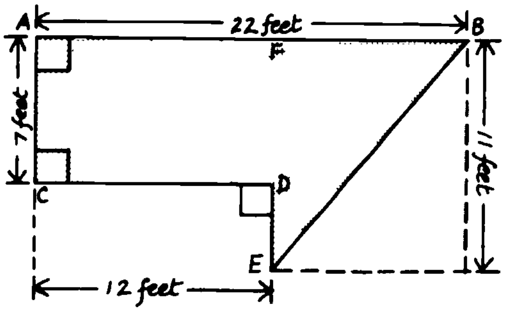
Given: PQ and RS are parallel, and PR is perpendicular to both. PQ = 16 feet, RS = 21 feet, and PR = 12 feet.

The area of PQSR equals:

☐ 126 sq.ft.
☐ 156 sq.ft.
☐ 222 sq.ft.
☐ 252 sq.ft.

222 sq.ft.

78.



To find the number of square feet in the shaded figure you would first draw in the perpendicular line:

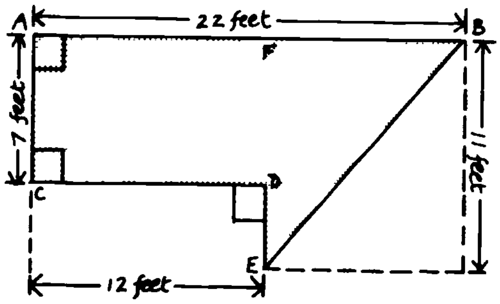
☐ AB
☐ BC
☐ CD
☐ DE
☐ DF
☐ EB
☐ ED

Then you would find the area of what rectangle ? (GIVE its letters).

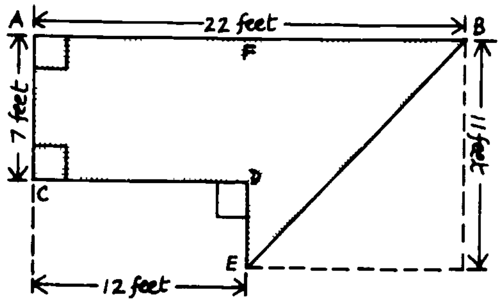
____

DF

ACFD
(any order)

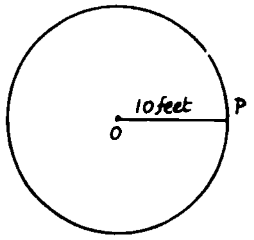
79.  	
Find the area of the rectangle ACFD. ________	84 sq.ft.
To find the area of the triangle FEB, you must find the length of what side(s) ? ☐ BE ☐ BF ☐ FE	BF FE

80.



You found the area of the rectangle ACFD to be 84 sq.ft. To find the length of side BF, you would subtract _____ (what number?) from _____ (what number?).	12 22
The length of side FE is __________.	11 ft.
Then the area (A = 1/2bh) of the triangle is equal to __________.	55 sq.ft.
The area of the entire shaded part is ___________.	139 sq.ft.

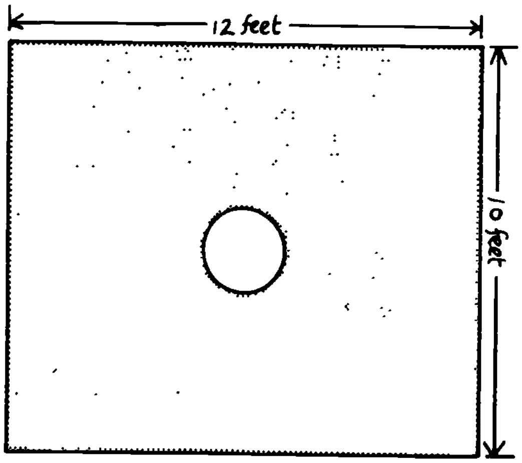
81. PROBLEM: A gardener has a square plot of ground which measures 8 feet on a side. If he uses one foot of space all the way around the plot for a hedge, how many square feet will he have left for a flower bed? (Hint: USE CAUTION! DRAW the plot before you calculate.) ☐ 24 1/2 sq.ft. ☐ 36 sq.ft. ☐ 49 sq.ft. ☐ 64 sq.ft.	36 sq.ft.
82. PREVIEW FRAME You can now find the areas of squares, rectangles, triangles, and combinations of these. In the next section you will learn how to find the area and circumference of a circle. NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME

83. The radius is the distance between the center and the outside of a circle. The diameter is the distance from one side to the other through the center of a circle.	
	
The radius of the above circle is line OP. If the radius is denoted by the letter r, the circumference (or distance around the outside) of the circle equals (3.14)(2r).* The area of the circle equals $(3.14)r^2$.	
For the above circle, the circumference equals: ☐ 3.14 feet ☐ 6.28 feet ☐ 31.4 feet ☐ 62.8 feet	62.8 feet
The area of the above circle equals: ☐ 3.14 sq.ft. ☐ 31.4 sq.ft. ☐ 62.8 sq.ft. ☐ 314 sq.ft.	314 sq.ft.
In which case do you square a number? ☐ to find feet ☐ to find square feet ☐ to find area ☐ to find circumference	to find square feet to find area
*You can test this: Mark the bottom of a wheel, and roll it forward until the mark comes to the bottom again. That distance equals the circumference of the wheel. Divide that by the diameter (twice the radius); the answer will be 22/7 or 3.14.	

84. The number 3.14 is often represented by the Greek letter π (pi).	
The formula $2\pi r$ gives: ☐ the area of a circle ☐ the circumference of a circle ☐ the radius of a circle	the circumference of a circle
The formula πr^2 gives: ☐ the area of a circle ☐ the circumference of a circle ☐ the radius of a circle	the area of a circle
If a circle has a radius of 3 inches, its circumference equals: ☐ 3π inches ☐ 6π inches ☐ 9π inches	6π inches
Its area equals: ☐ 6π sq.in. ☐ 9π sq.in. ☐ 18π sq.in. ☐ $9\pi^2$ sq.in.	9πsq.in.

85. $\pi = 3.14$ A circle has a radius of 4 inches. Its area equals: ☐ 12.56 sq.in. ☐ 25.12 sq.in. ☐ 50.24 sq.in. ☐ 100.48 sq.in.	50.24 sq.in.
86. A circle has a radius of 20 feet. Its circumference equals: ☐ 40 feet ☐ 62.8 feet ☐ 125.6 feet ☐ 1256 feet Its area equals: ☐ 400 sq.ft. ☐ 628 sq.ft. ☐ 1256 sq.ft. ☐ 2512 sq.ft.	125.6 feet 1256 sq.ft.

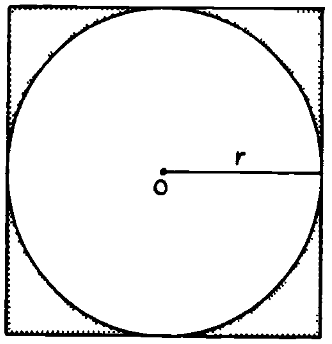
87.



PROBLEM: The diagram shows a circular window in a wooden wall. The wooden wall is shaded. The radius of the circle is 1 foot. $\pi = 3.14$. Answer the following questions to find the area of the wood.

The area of the entire rectangle equals: ☐ 12 sq.ft. ☐ 120 sq.ft. ☐ 144 sq.ft.	120 sq.ft.
The area of the circular window equals: ☐ 3.14 sq.ft. ☐ 6.28 sq.ft. ☐ 12.56 sq.ft.	3.14 sq.ft.
The area of the shaded part equals: ☐ 116.86 sq.ft. ☐ 120 sq.ft. ☐ 123.14 sq.ft. ☐ 126.28 sq.ft.	116.86 sq.ft.

88.



The figure shows a circle of radius r inside a square. Which of the following formulas should be used to find the shaded area?

☐ $(2r)^2 - \pi r^2$

☐ $r^2 - \pi r^2$

☐ $r^2 - 2\pi r$

☐ $r^2 - \pi r$

☐ none of the above

$(2r)^2 - \pi r^2 = r^2 (4 - \pi)$

89.

PREVIEW FRAME

You have learned to calculate the two-dimensional areas of many plane figures. In the next section you will learn how to find the amount of three-dimensional space enclosed by certain solid objects.

Distance can be measured in feet and inches. Area can be measured in sq.ft. and sq.in. Volume can be measured in cubic feet and cubic inches.

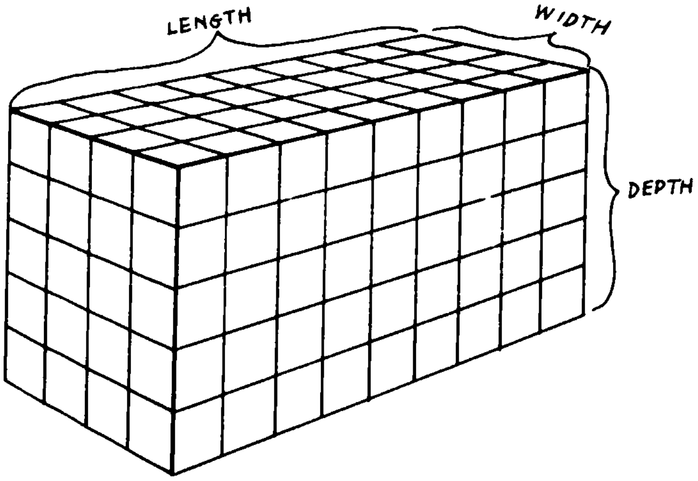
These expressions are often abbreviated cu.ft. and cu. in.

NO RESPONSE REQUIRED

GO ON TO THE NEXT FRAME

90.	
Cu.ft. is an abbreviation for: ☐ feet ☐ square feet ☐ cubic feet	cubic feet
Cu.in. is an abbreviation for: ☐ inches ☐ square inches ☐ cubic inches	cubic inches
Cu.ft. measures: ☐ area ☐ length ☐ volume	volume

PANEL 4



91. REFER TO PANEL 4 The panel shows a rectangular block of wood which has been sawed into many small blocks. The small blocks are one foot long, one foot wide, and one foot deep. Consider the top surface of the whole rectangular box. The length of the top surface is: ☐ 4 feet ☐ 5 feet ☐ 9 feet ☐ 36 feet	9 feet
The width of the top surface is: ☐ 4 feet ☐ 5 feet ☐ 9 feet ☐ 36 feet	4 feet
The area of the top surface is: ☐ 4 sq.ft. ☐ 5 sq.ft. ☐ 9 sq.ft. ☐ 36 sq.ft.	36 sq.ft.
How many small blocks make up the <u>top surface</u> of the large block? ☐ 4 ☐ 5 ☐ 9 ☐ 36	36

92.	
REFER TO PANEL IV	
The large rectangular block may be thought of as having layers, like a layer cake.	
How many layers does it have? ☐ 4 ☐ 5 ☐ 9 ☐ 36	5
Each layer contains: ☐ 4 small blocks ☐ 5 small blocks ☐ 9 small blocks ☐ 36 small blocks	36 small blocks
The large rectangular block must contain in all: ☐ 36 small blocks ☐ 144 small blocks ☐ 180 small blocks	180 small blocks
The area of each layer is 36 sq.ft. The depth of the large block is 5 feet. The total amount of space enclosed within the large block equals the area of each layer times the depth, or: ☐ 41 cu.ft. ☐ 144 cu.ft. ☐ 180 cu.ft. ☐ 360 cu.ft.	180 cu.ft.

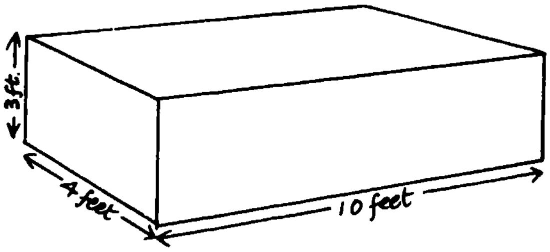
93.

REFER TO PANEL IV

The total amount of space enclosed by a solid object is called the volume of the object. The volume of a rectangular block equals length x width x depth. The volume of the large rectangular block in the panel equals:

- ☐ 41 cu.ft.
- ☐ 144 cu.ft.
- ☐ 180 cu.ft.

180 cu.ft.

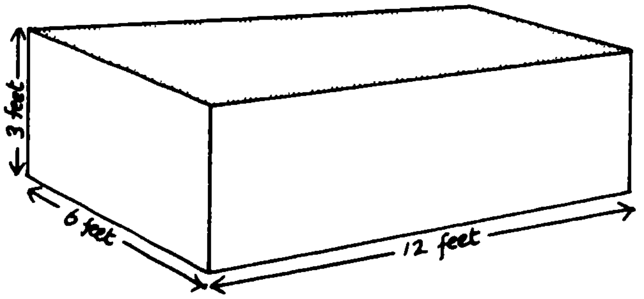


The volume of the rectangular box shown above equals:

- ☐ 30 cu.ft.
- ☐ 40 cu.ft.
- ☐ 120 cu.ft.
- ☐ 400 cu.ft.

120 cu.ft.

94.



Above is shown a rectangular box. The area of the top equals:

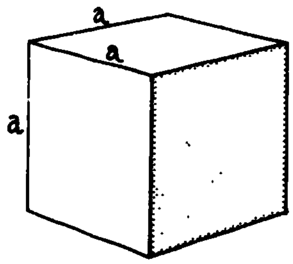
☐ 18 sq.ft.
☐ 72 cu.ft.
☐ 72 sq.ft.
☐ 216 cu.ft.

The volume of the box equals:

☐ 18 sq.ft.
☐ 72 sq.ft.
☐ 72 cu.ft.
☐ 216 sq.ft.
☐ 216 cu.ft.

72 sq.ft.

216 cu.ft.

95.	
A rectangular box whose length, width, and depth are all equal, is called a cube. The volume of a rectangular box equals length x width x depth. 	
The above cube has length, width, and depth equal to a. Its volume equals: ☐ a ☐ 3a ☐ a^2 ☐ a^3 ☐ $3a^3$	a^3
A box has a length of 2 ft., a depth of 3 ft. and a width of 5 ft. Is it a cube? ☐ yes ☐ no	no
Why? ______________	The three dimension are not equal. (or equivalent response)
The top surface of a cube is a: ☐ rectangle ☐ square ☐ triangle	square
The shaded side in the above cube is a: ☐ rectangle ☐ square ☐ triangle	square
The second power of a number is its ______________.	square
The third power of a number is its ______________.	cube

96.	
Every edge of a cube measures 3 inches. The volume of the cube equals: ☐ 3 cu.ft. ☐ 3 cu.in. ☐ 9 cu.in. ☐ 18 cu.in. ☐ 27 cu.in. ☐ 81 cu.in.	27 cu.in.
The area of any side of the cube is: ☐ 3 cu.ft. ☐ 9 cu.in. ☐ 9 sq.in. ☐ 27 cu.in.	9 sq.in.

97.

PROBLEM: Suppose a rectangular box measuring 6 feet by 8 feet by 10 feet is filled with cubes measuring 1/2 foot on a side. Answer the following questions to see how many cubes the box holds.

The volume of the box is:

- ☐ 48 cu.ft.
- ☐ 80 cu.ft.
- ☐ 480 cu.ft.

480 cu.ft.

The volume of each cube is:

- ☐ 1/2 cu.ft.
- ☐ 1/4 cu.ft.
- ☐ 1/8 cu.ft.
- ☐ 3/2 cu.ft.

1/8 cu.ft.

The rectangular box will therefore hold:

- ☐ 60 cubes
- ☐ 480 cubes
- ☐ 1920 cubes
- ☐ 3840 cubes

3840 cubes

98.

TERMINAL FRAME

A rectangular shipping carton measures 6 feet by 7 feet by 9 feet. How many boxes will it hold if each box measures 3 inches by 6 inches by 2 feet?

- ☐ 63 boxes
- ☐ 378 boxes
- ☐ 756 boxes
- ☐ 1512 boxes

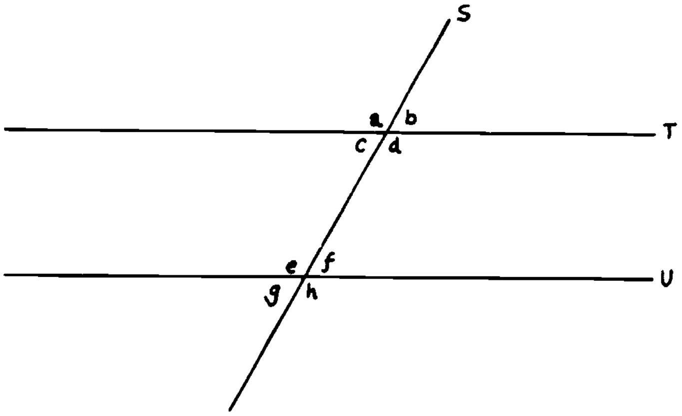
1512 boxes

Time completed __________

YOU HAVE NOW FINISHED THE FIRST PART OF THIS LESSON. WRITE DOWN THE TIME. THEN, AFTER YOU HAVE REVIEWED THE MAIN IDEAS IN THE FOREGOING LESSON, TAKE THE MASTERY TEST AT THE END OF THE BOOKLET.

MASTERY TEST

Time started ______________



1. In the above figure, lines T and U are parallel. Angle a is equal to which other angle(s)?

   a. ☐ b

   b. ☐ c

   c. ☐ d

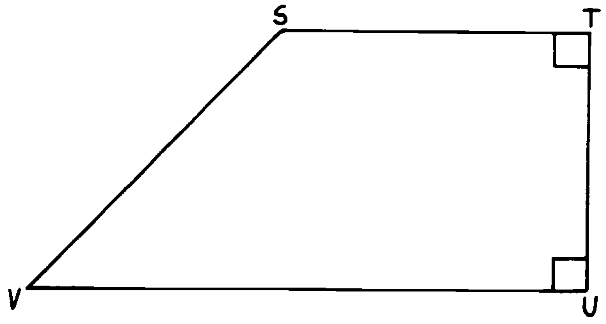
   d. ☐ e

   e. ☐ f

   f. ☐ g

   g. ☐ h

164



2. In the above figure, ST and UV are parallel and TU is perpendicular to both. ST = 5 inches, TU = 4 inches, and UV = 9 inches.

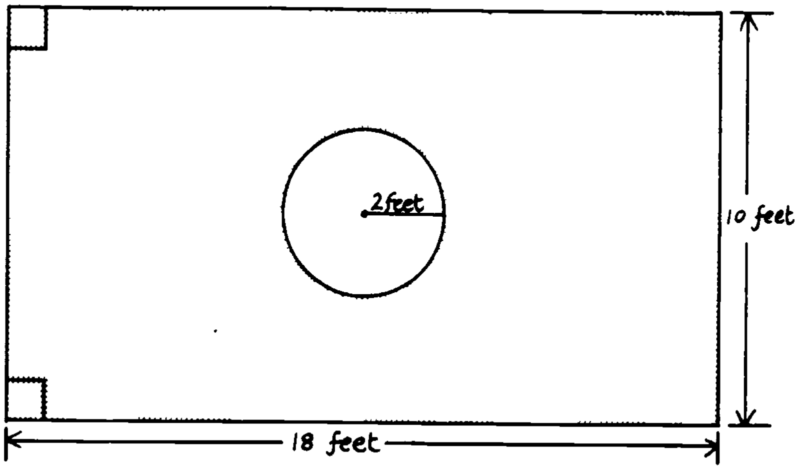
   What is the area of STUV?

   a. ☐ 12 sq. in.

   b. ☐ 20 sq. in.

   c. ☐ 28 sq. in.

   d. ☐ 36 sq. in.



3. The radius of the circle inside the rectangle is 2 feet. What is the area of the shaded part of the rectangle ? ($\pi = 3.14$)

   a. ☐ 12.56 sq. ft.

   b. ☐ 154.88 sq. ft.

   c. ☐ 167.44 sq. ft.

   d. ☐ 180.00 sq. ft.

   e. ☐ 192.56 sq. ft.

NOTE NOTE NOTE NOTE NOTE

Skip one(1) page to find page 88 and continue with question 4.

4. A rectangular box measures 3 feet by 2 feet by 2 feet. It is filled with cubes 6 inches on a side. How many cubes can it hold?

a. ☐ 12

b. ☐ 24

c. ☐ 48

d. ☐ 96

e. ☐ 192

Time completed ________________

WHEN YOU HAVE FINISHED THIS TEST, WRITE DOWN THE TIME. THEN TAKE THE LESSON TO YOUR INSTRUCTOR OR HIS ASSISTANT FOR CHECKING. WAIT UNTIL THE LESSON IS APPROVED BEFORE GOING ON TO THE NEXT LESSON.

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ADVANCED GENERAL EDUCATION PROGRAM

A HIGH SCHOOL SELF-STUDY PROGRAM

NUMBER SERIES
LEVEL: II
UNIT: 8
LESSON: 4

U.S. DEPARTMENT OF LABOR
MANPOWER ADMINISTRATION, JOB CORPS
NOVEMBER 1969

U.S. DEPARTMENT OF LABOR
MANPOWER ADMINISTRATION, JOB CORPS
NOVEMBER 1969

1.

PREVIEW FRAME

In mathematics we often deal with a sequence of numbers, such as, 2, 7, 12, 17, 22, 27.

In many such sequences the numbers are related to each other by a rule or law.

In the above example, each number is 5 greater than the preceding number.

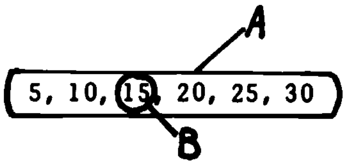
There are many different types of sequences of numbers, with various interesting properties. In this lesson you will learn about some of these sequences.

NO RESPONSE REQUIRED

GO ON TO THE NEXT FRAME

2. EXAMPLE: 2, 4, 6, 8, 10, 12, 14.	
In the above example, the difference between the first number and the second number is: ☐ 1 ☐ 2 ☐ 3	2
The difference between the second number in the sequence and the third number is: ☐ 1 ☐ 2 ☐ 3	2
The difference between the last number and the next-to-last number is: ☐ 1 ☐ 2 ☐ 3 ☐ 12	2
The difference between only two adjoining terms is: ☐ 1 ☐ 2 ☐ 12	2
If the sequence continued past 14, the next number in the sequence would be: ☐ 15 ☐ 16 ☐ 17	16

3.	
A. $7,\overset{4}{\frown}11,\overset{4}{\frown}15,\overset{4}{\frown}19,\overset{4}{\frown}23$	
B. $12,\overset{2}{\frown}14,\overset{3}{\frown}17,\overset{4}{\frown}21$	
In which example above is the difference between two adjoining terms always the same?	
☐ A ☐ B	A
Such a sequence of numbers is called an arithmetic progression.	
CHECK the arithmetic progression(s) below:	
☐ 1, 5, 17, 18, 54, 12, 8	
☐ 4, 7, 10, 13, 16, 19, 22	4, 7, 10, 13, 16, 19, 22
☐ 3, 9, 27, 81, 243	
☐ 21, 18, 15, 12, 9, 6, 3, 0	21, 18, 15, 12, 9, 6, 3, 0

4. EXAMPLE A: 7, 11, 15, 19, 23 EXAMPLE B: 3, 109, 0, 42, 5 If there is any rule for obtaining a number in a sequence from the preceding number, the sequence is called a <u>number series</u>. LOOK CAREFULLY at the examples. Example A: ☐ is a number series ☐ is not a number series	is a number series
Example B: ☐ is a number series ☐ is not a number series	is not a number series
An arithmetic progression: ☐ is a number series ☐ is not a number series	is a number series
5. EXAMPLE:  The example shows a number series which is an arithmetic progression. The expressions "number series" and "arithmetic progression" refer to the entire sequence of numbers. A single number within the series (or progression) is called a <u>term</u>. IDENTIFY the expressions below with respect to the above example by writing an A or a B in each blank:	
1. _____ arithmetic progression	1. A
2. _____ number series	2. A
3. _____ term	3. B

6.	
EXAMPLE: 1, 4, 16, 64, 256, 1024	
In the example, the second term (4) is equal to the first term (1) multiplied by: ☐ 1 ☐ 2 ☐ 4 ☐ 8	4
The third term is equal to the second term multiplied by: ☐ 2 ☐ 4 ☐ 8 ☐ 12	4
The last term is equal to the next-to-last term multiplied by: ☐ 2 ☐ 4 ☐ 10 ☐ 100	4
If this series continued past 1024, the next term would be: ☐ 1048 ☐ 2048 ☐ 4096 ☐ 9192	4096
Any term in the series is equal to the preceding term multiplied by: ☐ 2 ☐ 4 ☐ 8 ☐ 10	4

7.	
EXAMPLE: 1, 3, 9, 27, 81	
This example is called a geometric progression.	
In a geometric progression, any term can be obtained by:	
☐ adding a fixed number to the preceding term	
☐ multiplying the preceding term by a fixed quantity	multiplying the . . .
WRITE an A beside the arithmetic progressions, and a G beside the geometric progressions:	
1. _____ 1, 6, 11, 16, 21, 26	A
2. _____ 1, 5, 25, 125, 625	G
3. _____ 4096, 2048, 1024, 512	G
4. _____ -11, 0, 11, 22, 33, 44	A
8.	
MATCH the following:	
A. arithmetic progression B. geometric progression	
1. _____ A new term is obtained by adding a fixed quantity to the immediately preceding term.	1. A
2. _____ A new term is obtained by multiplying the immediately preceding term by a fixed quantity.	2. B

9.	
Some number series are neither arithmetic nor geometric progressions.	
Consider the following:	
6, 11, 17, 24, 32, 41, 51	
What is the difference	
between 6 and 11 ? _____	5
between 11 and 17 ? _____	6
between 17 and 24 ? _____	7
between 24 and 32 ? _____	8
between 32 and 41 ? _____	9
between 41 and 51 ? _____	10
Do the differences between the terms follow a pattern ?	
☐ yes ☐ no	yes
EXPLAIN why this is not an arithmetic progression:	Because the differences between terms are not the same. (or equivalent response)

10.	
WRITE the next term in each of the following series:	
5, 10, 15, 20, _____	25
24, 22, 20, 18, _____	16
2, 8, 32, 128, _____	512
2, 5, 9, 14, 20, _____	27
99, 88, 79, 72, 67, _____	64
11.	
2, 6, 18, 54, 162	
The above is a(n)	
☐ arithmetic progression ☐ geometric progression	geometric progression
Each term is equal to the preceding term:	
☐ plus a constant quantity ☐ multiplied by a constant quantity	multiplied by a constant . . .
What is the constant quantity in this case? _____	3

12.

In a geometric progression, each term is equal to the preceding term multiplied by a constant quantity.

This constant quantity is called the <u>common ratio</u>.

Example 1:

1, 7, 49, 343, 2401.

In this geometric progression the common ratio is:

☐ 1
☐ 6
☐ 7
☐ 14

7

Example 2:

1, 2, 4, 8.

In this geometric progression the common ratio is:

☐ 1
☐ 2
☐ 3
☐ 4

2

In a geometric progression, to get any term you multiply the preceding term by:

☐ the number series
☐ the first term
☐ the common ratio

the common ratio

13.

A geometrical progression can move backwards as for example: 16, 8, 4, 2, 1.

To get the second term, 8, you multiply the preceding term, 16, by 1/2.

The common ratio is therefore 1/2.

EXAMPLE: 27, 9, 3, 1.

The common ratio of this geometrical progression is:

☐ 1
☐ 3
☐ 1/3
☐ 9

1/3

14.

PREVIEW FRAME

There is an important formula which governs all geometrical progressions. In the next section you will derive this formula. Later, you will learn to use it to find unknown terms within a geometrical progression.

NO RESPONSE REQUIRED

GO ON TO THE NEXT FRAME

15.	
EXAMPLE: 2, 4, 8.	
This geometric progression contains: ☐ 1 term ☐ 2 terms ☐ 3 terms	3 terms
The first term is: ☐ 1 ☐ 2 ☐ 8	2
The last term is: ☐ 2 ☐ 3 ☐ 8	8
EXAMPLE: 4, 16, 64, 256.	
In a geometric progression we will denote the first term by the letter a, the last term by the letter ℓ, and the number of terms by the letter n.	
In the example directly above, a equals: ☐ 4 ☐ 16 ☐ 256	4
ℓ equals: ☐ 4 ☐ 16 ☐ 256	256
n equals: ☐ 4 ☐ 16 ☐ 256	4

16.

In a geometric progression, let:

a = the first term
ℓ = the last term
r = the common ratio
n = the number of terms

FILL IN the table below, giving a, ℓ, r, and n for each geometric progression:

Progression	a	ℓ	r	n
2, 6, 18, 54, 162	2	162	3	5
5, 10, 20, 40				
81, 27, 9, 3, 1, 1/3				
3, -3, 3, -3, 3				
-2, -8, -32				

a	ℓ	r	n
2	162	3	5
5	40	2	4
81	1/3	1/3	6
3	3	-1	5
-2	-32	4	3

17.

WRITE the letters a, ℓ, r, and n beside the appropriate definitions in the list below:

1. _____ the common ratio of a geometric progression
2. _____ the first term of a geometric progression
3. _____ the fourth term of a geometric progression
4. _____ the last term in a geometric progression
5. _____ the number of terms in a geometric progression
6. _____ the square root of a geometric progression

1. r
2. a
4. ℓ
5. n

16.	
The first term of a geometric progression is symbolized by the letter: ☐ a ☐ ℓ ☐ r ☐ n	a
The second term is found by multiplying the first term times the common ratio. This product can be written as: ☐ a ☐ an ☐ ar	ar
The third term is found by multiplying the second term times the common ratio. The product is: ☐ a ☐ ar ☐ ar^2	ar^2
The fourth term would be: ☐ a ☐ ar ☐ ar^2 ☐ ar^3	ar^3
A geometric progression of four terms can be represented as: ☐ a, an, 2an, 3an ☐ a, ar, 2ar, 3ar ☐ a, ar, ar^2, ar^3 ☐ a, a , a^2, a^3	a, ar, ar^2, ar^3
19.	
a, ar, ar^2, ar^3, ar^4	
The above geometric progression has _____ (how many) terms.	5
LOOK AT the second term. The power of r is _____ (number).	1
In the fifth term, the power of r is _____ (number).	4

21. $\begin{array}{cccccc} a, & ar, & ar^2, & ar^3, \ldots, & & ar^{n-1} \\ \text{1st term} & \text{2nd term} & \text{3rd term} & \text{4th term} & & \text{nth term} \end{array}$	
In a geometric progression the power of r is: ☐ equal to the number of the term in the progression ☐ twice the number of the term in the progression ☐ one less than the number of the term in the progression	one less than the number . . .
Therefore, if the number of the term is n, the power of r for that term is: ☐ n ☐ n + 1 ☐ n - 1 ☐ n^2 ☐ 2n	n - 1

20.

Geometric Progression	a	ar	ar^2	ar^3	ar^4	ar^5
Power of r	0	1	2	3	4	5
Number of Term	1	2	3	4	5	6

The above table describes the geometric progression in a general way. It applies to all geometric progressions.

The third term of the progression is written as:

☐ a
☐ ar
☐ ar^2
☐ ar^3

The power of r in the third term is:

☐ 1
☐ 2
☐ 3
☐ 4

You can see that in any geometric progression, the number of the term is:

☐ one less than the power of r
☐ equal to the power of r
☐ one greater than the power of r
☐ twice the power of r

ar^2

2

one greater than the power of r

22.

EXAMPLE:

Geometric Progression	a	ar	ar^2	ar^3
Power of r	0	1	2	3
Number of term	1	2	3	4

The value of the last term in a geometric progression is denoted by the letter:

☐ a
☐ ℓ
☐ r
☐ n

ℓ

The above example has:

☐ 1 term
☐ 2 terms
☐ 3 terms
☐ 4 terms
☐ 5 terms

4 terms

The number of the last term is:

☐ 1
☐ 2
☐ 4
☐ 5

4

If a geometric progression has n terms, the <u>number of the last term</u> is:

☐ a
☐ ℓ
☐ n
☐ n^2

n

23.

EXAMPLE:

Geometric Progression	a	ar	ar^2	ar^3	ar^4
Power of r	0	1	2	3	4
Number of term	1	2	3	4	5

The number of the last term above is:

☐ 1
☐ 3
☐ 5

5

The number of terms in the above example is:

☐ 1
☐ 5
☐ 6

5

The <u>number of the last term</u> is equal to the <u>number of terms</u>, that is:

☐ a
☐ ℓ
☐ r
☐ n

n

The power of r in the last term equals:

☐ n + 1
☐ n - 1
☐ n
☐ 1

n-1

The value of the last term, , is therefore:

☐ $\ell = ar$
☐ $\ell = ar^n$
☐ $\ell = ar^{n-1}$
☐ $\ell = ar^{n+1}$

$\ell = ar^{n-1}$

24.	
EXAMPLE: a, ar, ar^2, ar^3, ..., ar^{n-1}	
COMPLETE the equation below, using one of the terms in the above example:	
ℓ = ________	$\ell = ar^{n-1}$
25.	
EXAMPLE: 2, 4, 8.	
WRITE down the values for the above geometric progression:	
a = ____ ℓ = ____ r = ____ n = ____	$a = 2$ $\ell = 8$ $r = 2$ $n = 3$
$n - 1$ = ____	2
r^{n-1} = ____	4
ar^{n-1} = ____	8
Does this last value equal ℓ? ☐ yes ☐ no	yes
This example: ☐ proves the equation completed above ☐ does not prove the equation completed above	proves the equation completed . . .

26. PREVIEW FRAME You have developed the basic formula for geometric progressions: $\ell = ar^{n-1}$. In the next frames you will learn how to use this formula to obtain unknown terms within a geometric progression. NO RESPONSE REQUIRED	GO ON TO THE NEXT FRAME
27.	
The terms between the first and last terms of a geometric progression are called the geometric means.	
CIRCLE the geometric means in the progression below:	
9, 18, 36, 72, 144	9, [✓ 18, 36, 72,] 144
Consider the progression: 4, 20, 100.	
a = _____	4
ℓ = _____	100
geometric mean = _____	20
A geometric progression has six terms.	
How many geometric means will it have?	
☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6	4 (all terms but the first and last)

28.	
Consider the geometric progression: 96, 48, 24, 12.	
48 is: ☐ the first term ☐ the common ratio ☐ a geometric mean ☐ the last term	a geometric mean
12 is: ☐ the first term ☐ the common ratio ☐ a geometric mean ☐ the last term	the last term
What is 24 in this progression? ☐ the first term ☐ the common ratio ☐ a geometric mean ☐ the last term	a geometric mean

29.	
PROBLEM: Find the five geometric means between 3 and 192.	
TO SOLVE: First, we must decide what <u>n</u> is. If the progression contains $a = 3$ and $\ell = 192$, in addition to five intermediate terms, then n equals: ☐ 2 ☐ 3 ☐ 7 ☐ 8	7
Therefore, n-1 equals: ☐ 1 ☐ 2 ☐ 6 ☐ 7	6
We know that $\ell = ar^{n-1}$. We also know that $a = 3$, $\ell = 192$, and $n-1 = 6$. Which letter in the above formula do we <u>not</u> know? ☐ a ☐ r ☐ ℓ ☐ n	r

30. PROBLEM: Find the five geometric means between 3 and 192. Our formula is: $\ell = ar^{n-1}$. $a = 3$, $\ell = 192$, and $n-1 = 6$.	
We must solve the formula to find: ☐ a ☐ ℓ ☐ r ☐ n	r
ar^{n-1} is equal to: ☐ $6r^3$ ☐ $3r^6$ ☐ $5r^2$	$3r^6$
We must set this expression equal to: ☐ a ☐ ℓ ☐ r ☐ n	ℓ
The equation becomes: ☐ $6 = 3r^6$ ☐ $3 = 3r^6$ ☐ $192 = 3r^6$ ☐ $6 = 2r^3$	$192 = 3r^6$

31. PROBLEM: Find the five geometric means between 3 and 192. We have obtained the equation $3r^6 = 192$. We must solve it for: ☐ a ☐ ℓ ☐ r ☐ n	r
Dividing both sides of the equation by 3, we obtain: ☐ $\frac{3r^6}{3} = \frac{192}{3}$ ☐ $r^6 = 3\ (192)$ ☐ $\frac{3r}{3} = \frac{192}{6}$	$\frac{3r^6}{3} = \frac{192}{3}$
Solving this for r^6, we obtain: $\frac{3r^6}{3} = \frac{192}{3}$ ☐ $r^6 = 64$ $r^6 = 3\ (192)$ ☐ $r^6 = 486$ $\frac{3r}{3} = \frac{192}{6}$ ☐ $r = 32$	$r^6 = 64$
The sixth power of 2 equals 64, that is, $2^6 = 64$. Therefore, r equals: ☐ 2 ☐ 4 ☐ 6	2
Now, $(-2)^6$ also equals 64. This means that there are really two values for r, (+2) and (-2), and also two sets of five geometric means between 3 and 192. We will use only the positive values of r.	

32.

PROBLEM: Find the five geometric means between 3 and 192.

Using the formula, $\ell = ar^{n-1}$, we have found r and now know the values of a, ℓ, r, and n: $a = 3$, $\ell = 192$, $r = 2$, $n = 7$.

In a geometric progression, each term equals the preceding term multiplied by the common ratio, r.

The <u>second</u> term is therefore:

- ☐ 3
- ☐ 6
- ☐ 12
- ☐ 192

The <u>third</u> term is:

- ☐ 3
- ☐ 6
- ☐ 12
- ☐ 24

The <u>fourth</u> term is:

- ☐ 12
- ☐ 24
- ☐ 48
- ☐ 192

6

12

24

33.

PROBLEM: Find the five geometric means between 3 and 192 by using the formula $l = ar^{n-1}$.

$a = 3, l = 192, r = 2, n = 7.$

COMPLETE the table below:

Number of Term	1st	2nd	3rd	4th	5th	6th	7th
Progression	3	6	12	24			192

CHECK the terms below which are geometric means:

- ☐ 1st
- ☐ 2nd
- ☐ 3rd
- ☐ 4th
- ☐ 5th
- ☐ 6th
- ☐ 7th

Now, WRITE OUT the five geometric means between 3 and 192:

____

5th	6th
48	96

2nd
3rd
4th
5th
6th

6, 12, 24, 48, 96

(any order)

34.	
PROBLEM: Find the four geometric means between 2 and 64.	
SOLUTION: First find n.	
n equals: ☐ 2 ☐ 4 ☐ 6 ☐ 8	6
Next find r from the formula $\ell = ar^{n-1}$: ☐ $32 = 2r^3$ ☐ $64 = r^5$ ☐ $64 = 2r^5$	$64 = 2r^5$
Solving the above for r^5, we obtain: ☐ $r^5 = 16$ ☐ $r^5 = 32$ ☐ $r^5 = 64$	$r^5 = 32$
We know that $(2)^5 = 32$, Therefore, r equals: ☐ 1 ☐ 2 ☐ 3 ☐ 5 ☐ 32	2
Now WRITE down the four geometric means between 2 and 64: ____________________	4, 8, 16, 32
Note that $(-2)^5 = -32 = -\ell$, not $+32 = \ell$. When r has an odd power there is only one set of geometric means which gives ℓ its proper sign.	

35.	
PROBLEM: Find the single geometric mean between 4 and 36.	
To solve, first find n.	
n equals: ☐ 2 ☐ 3 ☐ 4	3
$l = ar^{n-1}$. Therefore, 36 equals: ☐ $2r^3$ ☐ $4r^3$ ☐ $4r^2$	$4r^2$
r^2 equals: ☐ 4 ☐ 9 ☐ 36	9
r equals: ☐ 2 ☐ 3 ☐ 9	3
The single geometric mean equals ar, that is: ☐ 6 ☐ 12 ☐ 16 ☐ 20	12
36.	
FIND the three geometric means between 6 and 7776: ____________________	36, 216, 1296

37.

TABLE:

x	2	4	6	8	. . . .	← x-row
y	6	12	18	24	. . . .	← y-row

Suppose that x and y are related so that when x takes any value in the x-row of the table, y takes the value in the y-row just below the x-value.

Thus, when x is 4, y = _____	12
And when y = 18, x = _____	6
The y-values are always exactly _____ (number) times as great as the corresponding x-values.	3
Thus, if x = 10, y = _____ .	30

38.

PROBLEM:

x	5	10	15	20 . . .
y	10	20	30	40 . . .

The above table shows how two number series are related. What is y when x = 30?

(Hint: Find the <u>relation</u> between any x-value and its corresponding y-value.)

For instance, when x = 5, y = _____.

10

Therefore, y-values are always _____ (how many) times as large as x-values.

2

Now DETERMINE the value of y when x = 30:

y = _____

60

39.

x	1	4	7	10	13	. . .
y	2	8	14	20	26	. . .

The above table shows how two number series are related.

When x = 19, y = _____

38

Time completed _______________

YOU HAVE NOW FINISHED THE FIRST PART OF THIS LESSON. WRITE DOWN THE TIME. THEN, AFTER YOU HAVE REVIEWED THE MAIN IDEAS IN THE FOLLOWING SUMMARY, TAKE THE MASTERY TEST AT THE END OF THE BOOKLET.

MASTERY TEST

Time started ________________

1. What is the next term in each of the following number series?

   a. 1, 4, 9, 16, 25, _____

   b. 7, 9, 13, 19, 27, _____

   c. 1, 2, 4, 8, 16, _____

   d. 99, 88, 79, 72, 67, _____

2. What are the four geometric means between 3 and 729?

3. The following table shows how two number series are related to each other. What will y equal when x is equal to 12?

x	1	2	3	4
y	2	4	6	8

Time completed ______________

WHEN YOU HAVE FINISHED THIS TEST, WRITE DOWN THE TIME. THEN TAKE THE LESSON TO YOUR INSTRUCTOR OR HIS ASSISTANT FOR CHECKING. WAIT UNTIL THE LESSON IS APPROVED BEFORE GOING ON TO THE NEXT LESSON.

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