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ABSTRACT

The use of a mathematical model supported by empirical findings had developed a method of cost effectiveness that can be used in evaluations between educational objectives and goals. Educational time allocation can be studied and developed into a micro-level economic theory of decision. Learning has been defined as increments which can be quantified by successive criterion tests of cognitive achievement. Using an input-output concept of the educational process, a marginal cost model was developed. A programing model was devised based on a multistep approach of learning, specifically using the Swedish IMU (Swedish acronym) Mathematics System for 7th Grade. The most important contribution of the programing model to an economic theory of education seems to be the identification of marginal product of student time with learning rate and a shadow pricing of this time resource that makes price proportional to learning rate, hence inversely proportional to time spent in learning. (MC)

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THE ECONOMICS OF TIME IN LEARNING

Nils-Olof Christoffersson

In the economics of education a main concern is with the study of relationships between educational objectives or goals and the scarce resources used as means towards those ends. This study is focused on the cost side; it treats student time in learning as one of those scarce resources and relates costs to time in education. When all students, faced with a specified learning task, are given the same amount of time for learning they will differ with respect to level of performance attained. If, on the other hand, time spent is allowed to vary and a specified criterion level of performance is set, all or most students will eventually reach that level. In the present study a model of time in learning has been developed and partially tested. The empirical data were taken in a learning situation, using the IMU system, a method of teaching mathematics, developed at the Malmö School of Education. The study was undertaken and completed as a Ph. D. dissertation at the Comparative Education Center, University of Chicago.

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Outside my dissertation committee I have received a great deal of help and encouragement from Professor Björn Thalberg, University of Lund, Sweden, and Professor Benjamin Bloom, University of Chicago. Special thanks go to my dissertation committee, in particular to its chairman Professor Mary Jean Bowman without whose patience, enthusiasm, and careful criticism this research would never have been started, carried through, or finished; in addition Professors C. Arnold Anderson and J. Alan Thomas have both contributed significantly to this dissertation by their never failing efforts and friendly advice.

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INTRODUCTION

Economic theory is concerned with human behavior as related to choice between alternative uses of limited resources. Neither the resources nor the ends need to be "economic" in the narrow sense of being quantifiable in money equivalents, although it is helpful in the analysis if that can be done.

In the economics of education we therefore study educational objectives or goals as related to resources which have competing uses outside the educational sector and also alternative potential uses within that sector itself. The main interest can be either in the study of the optimizing behavior of individuals faced with the problem of making decisions regarding their own education, or in finding an optimum over a set of feasible alternatives in the supplying of educational services and the conditions of access thereto.

The present study will center around problems of resource allocation in education especially in schools or school systems. More specifically the approach will be to examine the allocation of students' and teachers' time to the goal of skill learning.

The foundations of a theoretical analysis of investments by individuals in themselves and human investment by private firms are laid out in Becker's Human Capital.[1] Bowles' Planning Educational

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[1]Gary S. Becker, Human Capital (New York: Columbia University Press, 1964).

Systems for Economic Growth[1] exemplifies a macro societal optimization approach to educational decision making, whereas Thomas in his The Productive School[2] provides a systematic framework for analysis of resource allocation within a school or a school district relating both to intra school and extra school measures of "output."

The increasing interest of economists in the educational process itself is often justified on the grounds that school administrators, operating as they do in a largely non-market context, have no economic incentives to strive for efficiency in production[3] in contrast to the profit maximizing drives of business firms in the competitive pressures to efficiency.

The usual assumption of a given technology, using optimal available techniques is thus not applicable to education. On the contrary it is up to educators and educational researchers to compare different techniques of education, established or experimental, not only by applying learning psychology but also by using other tools of social science as well, such as those of economics.

Although many of the productivity studies that have been undertaken so far are useful in mapping out some general features of the internal economics of educational systems, they have as a rule been

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[1] Samuel Bowles, Planning Educational Systems for Economic Growth (Cambridge, Mass.: Harvard University Press, 1969).

[2] J. Alan Thomas, The Productive School. A Systems Analysis Approach to Educational Administration (New York: John Wiley and Sons, Inc., 1971).

[3] See e.g., Jesse Burkhead, Input and Output in Large City High Schools (Syracuse: Syracuse University Press, 1967), pp. 4-8.

somewhat less than satisfactory when it comes to infering policy measures from the findings. In part this is because the level of analysis has been semi-micro rather than micro in most cases (apart from cost-efficiency analysis of special projects, such as busing plans, and school lunches). Most productivity studies have been semi-micro in that they look to particular segments of education but use a fairly high level of aggregation in the categorization of inputs and outputs; at the best they consider, say, the high school as one single activity or possibly a few activities, producing a few likewise highly aggregated sorts of skills.

A procedure often used in this kind of study is to try to identify some important inputs and outputs, to get some satisfactory measures of those quantities, possibly validate the measures by statistical procedures, and then proceed to relate inputs to outputs.

Any possible intervening variables are then ignored on the grounds that whatever happens within the system is too complex to be analyzed, and also of less importance for the economic problem. It will be argued that this neglect is at least partly to blame for the shortcomings in the derivation of optimal procedures from the studies so far. It will also be argued that time is a critically important intervening variable among those frequently by-passed (relegated to the "black box" between observed inputs and outputs). Furthermore, time is a very important economic variable, both in general and in the economics of education in particular. Indeed student time has been shown to be one of the most significant of educational costs, measured as a rule as "students' foregone earnings."[1] A more general economic theory of time was presented by Becker in his article: "A Theory of the Allocation of Time."[2]

Efforts to secure productive efficiency in a business firm are usually based on technical knowledge. In education the equivalent "engineering know-how" must be largely built on learning theory, the state of which, however, is such that a direct application of resource allocation analysis, drawing on a relevant body of knowledge from learning theory, seems at present hardly feasible.[3]

There are, however, attempts in the direction of developing such a learning theory. Of particular interest for the economic analysis of resource use and development of production functions within the school are recent studies by educational psychologists who direct special attention to examination of "time to criterion-performance." Bloom's Mastery Learning Theory is an outstanding example.[4]

Time, being an important variable in economic theory and lately also in learning theory, should be worthy of an educational productivity analysis, especially if that could bring us any closer to a relevant decision theory for education. It is the aim of the

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[1]That this is truly a cost both for the individual and for the society was first emphasized by Theodore W. Schultz in The Economic Value of Education (New York: Columbia University Press, 1963).

[2]Gary S. Becker, "A Theory of the Allocation of Time," Economic Journal, LXXV, No. 299 (September, 1965), 493-517.

[3]For a discussion of input-output analysis in relation to learning theory see Burkhead, Input and Output in Large City High Schools, pp. 21-23, and Thomas, The Productive School, pp. 13-21.

[4]Benjamin S. Bloom, "Learning for Mastery," Evaluation Comment, Vol. I, No. 2 (Los Angeles: Center for Study of Evaluation of Instructional Programs, University of California, 1968).

present study to make an attempt to fill in some of the gaps mentioned above, by analyzing the use of time in educational processes, thereby, hopefully, contributing to the development of a micro-level economic theory of decision, pertinent to resource allocation within education. The main thrust of the dissertation will be theoretical but included is an empirical experiment applying part of the theoretical construct.

In the model that will be developed the benefits of education will be taken as given; that is, it will be assumed that in one way or another, there has been established a preference system with regard to educational outcomes, both in relation to production outside the educational system and concerning interrelations among the different educational "products." Most of the latter are taken as given but will remain unspecified. The "outputs" studied will be defined as successful performance on successive criterion tests of cognitive achievement. "Educational product" therefore, is defined as increments in learning, here called "steps." One step is determined by two successive, carefully defined achievement levels, and the step itself is the distance that the student has to cover in order to reach mastery of the learning task thus defined. The output, produced by a system, can thus be quantified by counting the number of students taking the step from one achievement level to another. In addition, resources are taken to be given from outside the system, and the problem will hence become one of optimizing subject to constraints.

Such a limited decision model is not unlike that of the business firm trying to maximize production, given prices and internal resources. In the case of education market prices play a much more limited role, since the "outputs" may not be priced in market terms. Inputs can and will in this model be priced, which makes it possible, in principle at least, to undertake a cost-benefit analysis. In a programming model, furthermore, even the "inputs" may be thought of as resources without market prices. Such a model will be developed, and shadow prices will be theoretically derived by solving for the dual variables in the problem.

We will first discuss in some detail the meaning of input-output in education and the problems of measurement. Learning theory as related to the model will be considered next, whereupon we are ready for the presentation of the model. To begin with we will show a one-step model of a very simple type and discuss various aspects of it. Then the model will be "dynamized" to sequences of steps, upon which a programming input-output model will be developed. Finally, some empirical tests of the model will be presented. We will not attempt, however, to make any actual evaluations of educational programs; the aim is rather to develop a method that can be used in such evaluations.

It is hoped that these explorations will yield theoretical implications concerning efficient production and optimal resource allocation within education, with practical applications in actual tests of the productive efficiency in an educational system. The approach presented here seems to be especially suited to the evaluation of educational projects and experiments involving instruction individualized by allowing learners to proceed at their own pace through a program.

CHAPTER I

OUTPUTS AND INPUTS IN A SCHOOL SYSTEM

Learning as Output

"Learning" is perhaps the most comprehensive description we can find for school outputs, although it is not all inclusive. We infer learning from changes in the behavior of the learner. According to Gagné:

> Learning is a change in human disposition or capability, which can be retained, and which is not simply ascribable to the process of growth. The kind of change called learning exhibits itself as a change in behavior, and the inference of learning is made by comparing what behavior was possible before the individual was placed in a 'learning situation' and what behavior can be exhibited after such treatment. The change may be, and often is, an increased capability for some performance. It may also be an altered disposition of the sort called 'attitude,' or 'interest,' or 'value.' The change must have more than momentary permanence; it must be capable of being retained over some period of time. Finally, it must be distinguishable from the kind of change that is attributable to growth, such as a change in height or the development of muscles through exercise.[1]

The products of learning include both cognitive and affective behavioral changes. Note that learning is defined as increments in human characteristics! However, in order to remove eventual causes of misunderstanding, we will sometimes speak of increments in

[1]Robert M. Gagné, The Conditions of Learning (New York: Holt, Rinehart and Winston, 1967), p. 5.

learning," or "incremental learning," although the expressions can be seen to have redundant elements.[1]

It should also be clear, from the above, that social interactions are learned behavior, and "socialization" is thus a product of learning. In fact, all the outcomes that we usually associate with education in schools can be thought of as products of learning. It is also obvious that not all learning in schools is deliberately planned; learning may be unintended, as is much of the learning from one's peers. Although this latter kind of learning can be thought of as sometimes adding to and sometimes diminishing the mass of learning "desired" by society, it will not be considered here.

Learning, of course, is not limited to schools - it is a lifelong process. Students come to schools (even to nursery schools for that matter) with a long history of learning already covered; they continue out-of-school learning over the school years, and when they enter post-school life they face a long succession of new learning experiences among which is on-the-job learning over the school years, and when they enter postschool life they face a long succession of new learning experiences among which on-the-job learning is of great importance in

[1]Are there any outputs from education that cannot be labeled learning? There are only a few, if we restrict ourselves to the immediate products of education (not taking into account outcomes where learned skills are necessarily mediating, such as future productivity and earning ability of students, etc.). Not directly associated with learning, however, are such incidental byproducts of schooling as quiet neighborhoods during school hours, schools' "baby-sitting" and the like. Those effects will not be considered in the present study; we are entirely going to focus on learning as the output of education.

a dynamic society.[1] This study, however, will be concerned only with learning in a school or a school system.

It is perhaps a less meaningful question to ask where "most learning" is accumulated. However, following Benjamin Bloom among others, one may point out the immense importance of early childhood, up until perhaps some of the first few grades in elementary school, for the acquisition of verbal skills, aptitudes and other characteristics that will enhance learning.[2]

This suggests one of the difficulties in measuring learning increments due to schooling, since the students are already on different points of their learning curves because of their respective backgrounds, with perhaps large differences in the learning climate of their home. Furthermore, differential home environments continue to play a role even after school entry, since the students spend a significant amount of time also in their homes. In economic terms: We have a "value added" problem and an "identification" problem.

Abilities and Skills as Inputs and Outputs

Before we proceed to discuss some measures of school learning output we shall take up some conceptual issues. The traditional approach in psychology to the study of learning has been in terms of the stimulus-response paradigm. One has typically focused on treatment

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[1]Becker, Human Capital, pp. 8-29. Jacob Mincer, "On-the-Job Training: Costs, Returns, and Some Implications," The Journal of Political Economy, LXX, No. 5, part 2 (Supplement, October, 1962), 9-49 and 50-79.

[2]Benjamin S. Bloom, Stability and Change in Human Characteristics (New York: John Wiley and Sons, Inc., 1964).

variables and the resulting output behavior of the learner without paying much attention to intervening variables. Recently, however, there has been an increasing interest in the "mediators" between stimuli and response within the learner himself. Fleishman identifies mediating processes with human abilities, making a distinction between basic ability and skill.[1] We shall use his definitions and follow closely his way of relating ability and skill to learning. According to Fleishman a basic ability is a general trait of an individual, inferred from response consistencies in learning tasks. These traits are fairly enduring, and difficult, if not impossible, to change in adults. Some of them are themselves products of learning, whereas others such as colorvision are dependent on genetic factors.

Basic abilities, in Fleishmans terminology, are general traits that the individual brings to a new learning task, influencing his performance in a variety of human tasks. Skill, in contrast, pertains to performance on a specific task or a limited group of closely related tasks. Basic abilities typically develop by what is called "over-learning," that is they have been learned, relearned, and practiced a great many times and therefore become very stable and enduring traits. Skill learning takes place throughout life, but very little if any improvement of abilities is believed to occur after the age of, say, 17.[2]

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[1]Edwin A. Fleishman, "Individual Differences and Motor Learning," in Learning and Individual Differences, edited by Robert M. Gagné (Columbus, Ohio: C. E. Merrill, 1967), pp. 165-191.

[2]Bloom, Stability and Change in Human Characteristics, pp. 52-94.

It seems likely then that basic abilities thus conceptualized are important determinants of both the final level of skill reached by any individual and his rate of learning. Basic abilities should be good predictors of individual performance. Verbal abilities are considered to be especially important for academic success in school. Numerical, spatial, and quantitative abilities are important in mathematics and engineering; physical abilities influence athletic skill levels; and so on.

The students' resources in the form of basic abilities are thus important as determinants of school "products" relating both to a stock concept of output - "mass of learning" acquirable for given students - and to a flow concept - rate of learning; both are central to this study. School factors interact with abilities and other student characteristics to "produce learning." From the theory of basic abilities as products of learning one might however, infer that those abilities may also be a school output, especially since the elementary school age covers a substantial part of the time in life when such abilities are developed. We will now investigate any eventual support of this suggestion.

Basic abilities are measured by IQ-tests and aptitude tests. There is considerable disagreement among learning psychologists as to what IQ-tests really measure, and as to their accuracy. Is there any evidence that schooling makes a difference to IQ-scores? Ample evidence shows that environment does influence intelligence measured as IQ. Bloom suggests that a conservative estimate of possible IQ-differences due to extreme environments would be about 20 points.[1] That,

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[1] Ibid., p. 89.

furthermore, these IQ-differences are strongly related to environment as measured by educational advantage has also been shown.[1]

A discussion of the old nature-nature problem is not intended here. Anne Anastasi points out that a more fruitful question to investigate would be: "How do environmental factors interact with hereditary in behavior development?" As examples of that kind of research she listed: effects of prenatal nutritional factors on IQ, cultural differences in patterns of child rearing related to intellectual and emotional development and influences of early experiences.[2]

Conceptually there should not be much objection to looking upon basic abilities as in part outputs of the schools, provided of course, that we realize that the school's contribution is marginal over contributions from the home and other environments, and probably also smaller. We should not, however, confuse what schools now do with what they possibly could do. The large variability in IQ due to environment seems to indicate the possibilities.

It is, however, very difficult to measure growth in basic abilities due to schooling. The inadequacy of IQ-tests is already mentioned; they can at the best serve as an indication that "schooling makes a difference." Aptitude tests measure a more limited range of basic abilities, and are more predictive of school success in subjects where they apply. Tests of verbal ability especially predict academic

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[1] See e.g., Anne Anastasi, Differential Psychology. Individual and Group Differences in Behavior (New York: MacMillan, 1958), p. 299.

[2] Anne Anastasi, "Heredity, Environment and the Question 'How?'," Psychological Review, LXXII (1958), 197-208.

performance in most subjects more accurately than IQ-tests do. But, again, it is not prediction that we want but a measure of output that could be related to input. Aptitude tests are less useful for that purpose, since they seem to capture not only basic abilities but also specific skills to some extent.

Learning Steps as Inputs and Outputs

In the above section of this chapter development of "intelligence" was discussed as an output of schools in a broad sense. Important as this aspect may be this study will be more concerned with outputs of learning as defined in a more narrow way. What we want to do is to look at the production process in some detail. The units we will use are called steps and consist of incremental learning in going from one performance level to the next. The time required for that may be as small as 2-3 hours. Obviously over a single step the gain, if any, in basic abilities must be infinitisemal. Those abilities will, therefore, be taken as given, when we analyze learning in terms of the step measure. In other words, basic abilities will be viewed as important, fixed determinants of <u>how far</u> a student will be able to go in achievement and <u>how fast</u>. There are other determinants of learning and learning rate, some of which are under control as "policy variables" in a school system; those are the ones, that we can manipulate in a resource allocation approach, when trying in some specified sense to "optimize" learning. Organization of learning content, use of teacher time, and use of instructional materials are examples of such variables that could be changed by decision makers.

The measurement of basic abilities as school outputs is difficult, to say the least. When it comes to performance, or task-specific achievement, we are much more fortunate. There is general agreement today in educational theory that the objectives of learning must be defined in terms of observable change in behavior. We must be able to state the desired terminal behavior of the learner and identify that behavior once it has been realized. Furthermore, programmed instruction, regardless of models of presentation, presumes the possibility of dividing up a learning sequence in very small steps and of defining in much detail the composition of each step.[1] We will assume that it is possible to use achievement scales to describe output as steps, that is steps are increments in learning occuring between chosen successive measures on the scale.

Basic abilities could as mentioned, be viewed as to some degree outputs resulting from learning as well as inputs in the acquisition of skills. This quality of being both an output of learning and an important input to learning is, however, equally characteristic of skill learning, where mastery of one unit may be crucial for learning the next one. Learning, therefore, may be viewed as a dynamic sequential process, in which output at one point becomes input at later stages. Educational theories, modern and old alike, have often and strongly stressed these interdependencies in learning. It is often advocated, for example, that learning content should be hierarchically organized in order to ensure that the necessary skills are present

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[1]See e.g., Michael Scriven, "The Case for and Use of Programmed Texts," in Programmed Instruction, ed. by Allen D. Calvin (Bloomington: Indiana University Press, 1969), pp. 3-36.

when needed. This points to the necessity of investigating costs associated with a succession of learning tasks.

Furthermore, research in learning theory by Fleishman (motor skill learning)[1] and Gagné (intellectual skill learning)[2] among others begins to suggest that in sequential learning basic abilities, while very important determinants of achievement and learning rate at early stages, are less predictive as learning continues; they suggest in contrast, habits and skills acquired in the learning sequence itself will become progressively more predictive of achievement and learning rate.

What all this seems to lead up to of relevance for the present study, is the immense importance of the organization of learning over time, or in terms of economics, the allocation of students' time in learning. It should not be argued carelessly, as is sometimes done, that school administrators and teachers have neglected this problem. In fact, one might argue that this oversight has been mainly among the economists who only recently (and somewhat reluctantly) have taken up this approach to resource allocation. It seems reasonable to argue, however, that the problem of how to make the best use of student time is a rather complex one, and, therefore, less likely to be solved satisfactorily by intuitive and "ad hoc" methods. In addition, it seems to

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[1]Fleishman, "Individual Differences in Motor Learning," in Learning and Individual Differences, ed. by Gagné, pp. 165-191.

[2]Robert M. Gagné and Noel E. Paradise, "Abilities and Learning Sets in Knowledge Acquisition," in Psychological Monographs, Vol. LXXV, No. 14 (Whole No. 518, 1961), and Robert M. Gagné, "Basic Studies of Learning Hierarchies in School Subjects," Final Report, Project No. 6-2949 Contract No. OEC-4-062949-3066 (Berkeley, Calif.: University of Calif. Berkeley, 1970).

be very likely that most schools have been neglecting to some extent individual differences among students when allocating their time in learning.

When the desired changes in behavior are affective rather than cognitive or skill related, the problem of measuring output is considerably more complicated, although not impossible. Such outcomes of education will not be considered here.

Other Inputs

The student clearly, is himself contributing highly important inputs in the production of learning; he is investing his own time, something of major interest for this study, and he brings to each learning task the whole history of his prior learning. Output at one stage will become input at some later stage or stages. This sequential phenomenon will be taken into account in developing the programming model.

The student also brings his individual set of all the numerous personal characteristics that facilitate or impair learning such as motivation, perseverance, anxiety, physical strength, and neural functions. Such characteristics, including the whole history of preschool learning, have to be taken as given when the students start school. In the course of schooling some traits that foster or impede learning can be modified, more or less, whereas others are essentially fixed, resisting change by any available treatment.

The problem of educational research is to find the treatment or instructional methods that are in some sense optimal when interacting with the individual characteristics in various learning situations. The economic problem then is to test the methods against constraining factors, especially various limitations of resources. When several feasible methods are judged to be equally good by educational standards, clearly the one should be chosen that minimizes cost in terms of resource input.

To clarify the point: we can for example, very well imagine that giving each student individual tutoring all the time in school might result in a large increase in learning compared to other methods. However, the resource limitations would no doubt be far exceeded by such a solution, which of course does not mean that tutoring could not be even justified economically if used on a more restricted basis.

Students differ so much in the characteristics of relevance for their performance in learning situations, that the question of improving instruction becomes in part one of finding methods adapted to those differences. Cronbach suggested, for example, that one might develop a special course of instruction in mathematics for students who are comparatively high in spatial ability, a second course for those who are higher on certain numerical or logical measures, and maybe a third course for students weak on both but high on a third aptitude.[1]

It can be shown, using tests measuring Thurstone's seven different aptitudes, that more than 50 per cent of the students are on the highest decile in at least one aptitude, and that a finer division

[1]Lee J. Cronbach, "How Can Instruction be Adapted to Individual Differences," in Learning and Individual Differences, ed. by Gagné, pp. 23-39.

of aptitudes will give the same result for perhaps 85 per cent of the students. We don't know yet how to utilize this fact, but it seems likely that by offering a greater diversity of teaching methods we should be able to economize on the students' human resources much better than we now do,

However, the important point for the subsequent discussion to be made here is that by "method" we do not necessarily mean same instruction to all students. A method, as we use the term, can consist of a whole bundle of differential treatments, with the instructional objectives as unifying elements. We thereby take into account the possibility that instruction may sometimes be successful to the degree that it provides for instructional diversity. In programmed instruction, for example, there may be many different paths leading to the same goal.

Various input measures have been adopted for input-output analysis of education, such as "quality" indices; for example, student teacher ratio, number of crowded classrooms, teacher education and experience, per student expenditure for materials, equipment, and building.[1]

It would not be necessary for this study to specify in detail either the inputs or their measure, since we will assume on the one hand that they can either be bought in necessary quantities on a market or that they exist in known and limited quantities, and on the other

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[1]Among numerous discussions of these measures we may refer to for example Burkhead, Input and Output in Large-City High Schools, especially pp. 4-17. Martin T. Katzman, "Distribution and Production in a Big City Elementary School System," Yale Economic Essays, Vol. VIII, No. 1 (Spring, 1968). Herbert J. Kiesling, "Measuring a Local Government Service: A Study of School Districts in New York State," The Review of Economics and Statistics, XLIX, No. 3 (August, 1967), 350-358. Thomas, The Productive School, especially pp. 11-30.

a new task, and the learning will be slowed down to the degree that the prerequisites are missing. If on the other hand aptitudes do in fact include specific skills, we would for our purposes nevertheless want to make the distinction and hence include "history of prior learning" in the set of an individual learner's disposition for learning.

The important feature of the Carroll model for our purposes is that the learning parameters determine time and learning simultaneously; in other words if learning, learner and method are given then time will be uniquely determined, provided that time is not predetermined by the system. Conversely, if time, learner and method are given, then the amount of learning will be uniquely determined. Furthermore, the time spent in learning by a given learner will also be time spent in using educational inputs, such as teacher time, school space time, and obviously the learner's own time. The student is, therefore, using up scarce resources, either having a market price or a potential shadow price. This links learning with costs of education, and time, as is obvious, plays a crucial role in this linkage, thus providing a basis for the development of a cost model of timing in education.

Summary

In this chapter we have discussed frame and process variables in education and especially the role of time in learning. It was suggested that time is an important variable directly affecting learning, and this assertion was illustrated by Carroll's model of school learning. The essential feature of this model is to postulate hand that there exist given technologies, using up inputs as specified by production functions. Broadly these inputs are students' and teachers' time, school space and equipment, materials and books, administrative and other personnel providing non teaching services.

Summary

In this chapter outputs and inputs in education have been examined against some background from learning theory. The learning output was said to be specific testable skills; outputs in the form of increases in basic abilities and those within the affective domain are not to be investigated here.

Both skills and abilities acquired through prior learning are major inputs, since learning tasks are often related to each other so that certain steps will be required as prerequisites for other steps. Both are treated as inputs in the present study.

a functional relationship between degree of learning as dependent and time spent and needed by the student as independent variables, so that

$$\text{Degree of Learning} = f\left(\frac{\text{time actuall spent in learning}}{\text{time needed.}}\right)$$

CHAPTER II

TIME AND LEARNING IN THE EDUCATIONAL PROCESS

Frame Variables, Process Variables and Timing in Education

This is a study of the economics involved in the use of time in education, especially students' and teachers' time but also the per-time-unit services provided, for example, by school space and equipment. In this chapter inputs of student time will be related to other inputs into education; student time clearly is a key process variable taking part directly and without mediation in the educational process.

Relationships in the social sciences are more often than not such that they can be described as functionals, that is, functions of an infinite number of variables. When we say that learning is dependent on the results of prior learning, we are really saying that learning is a functional, since it depends on the state at any particular point in time of the subject's learning history. From our previous discussion it should be clear that numerous other variables also enter into this functional.

For this reason it is impossible to get anything like a complete analysis. It is necessary to simplify drastically when developing a model of learning by using a few variables only (thus getting ordinary functions). The assumption is then that the variables chosen

CHAPTER III

A MARGINAL COST MODEL OF LEARNING: ANALYSIS OF ONE STEP

What Shall be Maximized?

So far we have discussed the concepts of input and output as related to learning, and in particular to learning in schools. We have also indicated that this research will delineate a model for resolution of an optimization problem under constraints imposed from outside the system.

What then shall be maximized? The goal is to maximize some sort of measure of skill learning, taking other kinds of outputs as given or taken for granted. Unfortunately, in thus limiting ourselves to the investigation of efficient production of skill learning we may introduce unidentified distortions in results more broadly conceived, since "affective products" of education, such as tolerance, and willingness to cooperate may be indirectly enhanced or discouraged as by-products of one or another pattern or method of inducing skill-learning. We will proceed, however, as if we could separate out the production of skill and treat it in isolation. Indeed, every investigation of human interaction processes inevitably entails some distortion.

Even if the product in which we are interested is in the cognitive or skill domain only, we still face a multi-product situation, since the learning tasks are so numerous.

will give a good enough approximation of reality. The problem is how to arrive at the best possible variables to use in the model. Without attempting to answer that obviously very difficult question in general it can be pointed out that the kind of variables most likely to do the job are dependent upon what kind of job we want the model to do.

If it is only prediction that we want to obtain from an investigation, then it may often be possible to use rather broadly defined categories and the causal interrelations between variables need not concern us very much. For decision making however, merely to be able to predict is not enough.

Thus when we want to organize the learning process in order to improve results, it may not be very helpful to know that school success is highly correlated with social background variables, like family income, parents' occupation and their education. One must know how social backgrounds enter into school success or lack thereof. What are the processes at work and in what sorts of interaction with schooling variables? Furthermore, social background variables are only approximately measuring inputs in the learning process, since there is always a variance around what is typical of any social class, for example in language habits and number of books in the home. We need to know therefore, which those variables are that enter <u>directly</u> into the learning process whether characteristics of the learner himself or of the learning environment.

Variables may conveniently be classified in two categories, as <u>frame variables</u> and as <u>interaction</u> or <u>process variables</u>. A frame variable specifies one or more attributes of the pupils or their

To illustrate: After the end of a course the participating students are given grades. Suppose, unrealistically, that the grades are objectively correct measures of student performance. How do we measure the "mass of learning" embodied in the students? Is the grade A twice as much as a B? And is F (failing) the same as zero learning? Against the last assumption one could argue that even the failing student may have learned something, although not enough for passing.

It is all too clear that a grading scale normally possesses no absolute zero point, nor are units on the scale equidistant from each other. In other words, it gives nothing more than a rough ordering of the students' performance. A good achievement test will give a more reliable and finer ordering of students, but the scale has the same shortcomings as the grading scale, something that is occasionally forgotten in educational discussions.

Carroll's model provides a cardinal scale only by expressing degree of learning in terms of time measures of inputs, which says nothing independently about the <u>output</u> value or extent or importance of the degrees of skill acquired. Carroll's "degree of learning," conceived as a function of actual time spent divided by time needed for learning is useful for some purposes but cannot as such be applied to a production function model.

Instead we shall try to identify "products" at a stage where they are more homogeneous, and then to assess the costs of such "products." Bloom's mastery learning theory brings up as one of its most important ideas, that the school product—student learning in a sense that will environment that measures inputs into learning only indirectly; such variables may sometimes, therefore, be poor indicators. The process variable, on the other hand, interacts directly with other variables in a concrete process affecting outcome (dependent variable).[1]

Examples of frame variables with regard to school achievement are: socio-economic status, school and class size, per-pupil expenditure, teacher training and experience. Process variables are, for example, students' broad learning capabilities whether acquired genetically or through prior learning, their particular aptitudes and skills, the expectations and motivations children bring to school, and the personality prejudices and intellectual capabilities of the teachers. It can be seen that home environment can influence achievement via several of these process variables. Student time in learning is the process variable that is of particular interest for this study.

A model aiming at prediction or at a mere description of the present state of affairs may very well make use of frame variables only. When one has in mind to construct a model for decision making, however, he must choose the variables such that they describe interactions in an essential way. Needless to say, the limit between the two classes of variables is unclear, and the same variable may in one context be a frame variable and in another context a process variable.

[1]This way of classifying variables in education seems to have been introduced by Dahllöf, who in a reevaluation of Swedish research concerning ability grouping argued that the time factor was a neglected one in this research. Urban S. Dahllöf, Skoldifferentiering och undervisningsförlopp. Komparativa mal och processanalyser av skolsystem I. Göteborg Studies in Educational Sciences 2 (Stockholm: Amqvist och Wiksell, 1967). Also in English (condensed form): Ability Grouping, Content Validity and Curriculum Process Analysis (New York: Teachers College Press, 1971).

be explained shortly—can indeed be made more homogeneous.[1] The present study may be looked upon as an economic analysis of the implications of the mastery learning theory. It focuses on the cost side in a comparative "cost-effectiveness" assessment of alternative educational programs operating within particular sets of constraints. It exemplifies some of the "converging concerns of educators and economists," to cite M. J. Bowman.[2]

Let us take a brief look at the mastery learning theory and see how it can be used in our own model. Mastery of a specified learning task is defined as a predetermined level of achievement that the student should reach. Bloom estimates that perhaps over 90 per cent of the students can reach such a standard even if it is set as high as what is now expected of, say, 1/3 of the students only. It is the task of educators to determine what should be meant by mastery and to organize instruction and find methods and materials such that the largest possible proportion of the students will be able to perform at that level.

Carroll's view that aptitudes are predictive of learning rates is assumed to hold. It is important that students be given enough time to reach the mastery level. Some students will need more time than others. Furthermore some students may need more help and effort; that is, the per time-unit cost for the slower students may be higher

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[1] Bloom, "Mastery Learning."

[2] Mary Jean Bowman, "Converging Concerns of Educators and Economists," in Comparative Education Review, VI, No. 2 (October, 1962), 111-119.

A typical quality of a frame variable is that when stating its impact we must, explicitly or implicitly, insert some sort of mediation between independent and dependent variables. Socioeconomic status tends to go together with school success because of something else, that is associated with SES and acts more directly on learning, such as better opportunities for verbal development, healthier food, etc. The use of process variables make it possible to go beyond existing practices to some extent, in search of optimal solutions. To take one example: most investigations of effects of class size on achievement seem to agree that those effects are negligible, with correlation close to zero, some have found a very small positive correlation, and some report even a small negative association with the larger classes doing better.[1] It would be a mistake, however, to derive from the findings the policy implication that classes should be made larger or be kept at their present size. It is conceivable, even probable, that instructional methods could be found such that smaller (or for that matter larger) classes would raise achievement, whether or not they would also pay off economically. What probably accounts for the findings is that teachers tend to teach in the same way in all classes, regardless of class size. Theoretically then one would expect to find a U-shaped curve. That has indeed sometimes been the case. Marklund found that classes of 26-30 students tend to do somewhat better than either classes of 21-25 or 31-25.[2] One might suspect that teaching was

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[1]Sixten Marklund, Skolklassens storlek och struktur (Stockholm: Almqvist och Wiksell, 1962).

[2]Ibid., p. 131.

than for the fast learners. Variable time and variable per time-unit cost will therefore be built into the present model.

We assume that the learning embodied in students who have successfully completed a learning task to mastery will be the same for each student, provided of course that only learning for that particular task is considered. With respect to <u>one single step</u>, then, the learning product becomes more homogeneous. Since we are allowing students to progress at their own speed the fast learners will naturally be accumulating <u>more</u> learning in the sense of taking more steps within any time period. The assumption of the same amount of learning for each student who has successfully completed the step will be only approximately justified in fact, since some students may achieve above the standard. This however, is a problem that we will abstract from in the theoretical model; in practical applications it must somehow be counted for. When discussing this point we shall refer to achievement above the standard as "overperformance," which should be carefully distinguished from the concept of "overlearning" as discussed in Chapter I.

It would seem, at this point, that to maximize educational product could mean, in view of the above discussion, either for maximize the number of students reaching a given achievement level or to maximize the number of successfully completed achievement levels for a given student.

We need a short, convenient term for the output thus to be maximized, and will, therefore, as we have already indicated, refer to it as number of steps. <u>One step, therefore, is the amount of learning, or incremental learning, that the learner accumulates when progressing</u>

somehow adapted for that class size, and that divergences in either direction would lead to somewhat lower achievement.

Another classification of variables would distinguish between policy variables under control by the decision maker, and exogeneously given variables that cannot be changed at will. Among the policy variables are some frame variables, and that may create a problem, because a change in such a variable will not necessarily have the expected effect, due to the intervening process variables. In a large number of studies it has, for example, been shown that per-pupil expenditure has no effect on output measured as achievement scores;[1] but no one has concluded from that finding that schools should be given less, or at least not more, money. Since the findings are inconclusive because of missing information, such a policy recommendation would certainly not be warranted.[2]

Time as a Process Variable in Educational Production

This study will concentrate on timing as an economic factor in educational production. To justify this objective, we have to show that time is involved in an essential way in educational production activities and that it thereby has economic significance. Since we

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[1]E.g. Burkhead, Input and Output in Large-City High Schools, Martin T. Katzman, "Distribution and Production in a Big City Elementary School System." (unpublished Ph.D. dissertation, Yale University, 1967), and Kiesling, "Measuring a Local Government Service."

[2]A tentative explanation could be that schools in fact do not operate on their efficiency surface; educational techniques exist that could produce more learning out of the given resources. We recall here our earlier remark that there are no economic forces that push schools towards maximum efficiency.

from one defined performance level to the next, say from level j to level k. The required change in performance level, the kth step, may be described as the shift between two successive measures at j and k on an achievement scale. As an aggregative cross section measure of output we can therefore use the number of students who have demonstrated the prescribed performance change.

This measure of output is additive as long as we add students over the same step, and we may treat the number of students having taken the same step as a cardinal variable. Longitudinally, however, the aggregate number of steps an individual takes will be shifts over a series of sequential performance levels that are specified by non-comparable measures; the jth, the kth and the lth steps cannot always be expressed in ratio or additive terms vis-a-vis each other. Two students may therefore have taken exactly the same number of steps but if their starting points differ we cannot assert that they have done the same amount of incremental learning. Only if they have taken an identical sequence of steps (say the jth and the kth) can we say that they have experienced or accomplished the same amount of new learning. To be precise: output may be described by the vector $(s_1, s_2, \ldots, s_n)$, where s_i is the number of students taking step i.

We will assume that it is possible to subdivide a step i into a number of smaller steps to such an extent that it will justify the use of differential calculus in the analysis. This is only approximately possible of course, since steps certainly cannot be made infinitesimal, but some types of programmed instruction demonstrate clearly how far such a subdivision can be carried. It is consistent, finally, as a are talking about a production process, it would indeed be surprising if time would not be an important factor.

Educational as well as other production takes time; if somehow the required time for a specified output can be lowered, other things equal, costs will be cut. To say that time is important in economics is true by definition almost. Most measures in economics are flows, hence quantities per time unit.

In fact time is an indispensible ingredient in any process - as is space - and other variables evaporate when abstracted from time. In this sense time is not a variable at all but a "medium" in which the process takes place. Time as an input variable, however, is always <u>someone's</u> time, for example students' time in learning and teachers' time in teaching, or the time of other services, such as those provided by school space and equipment.

In education and learning psychology, however, time spent in learning has very seldom been explicitly analyzed. In experimental psychology typical procedure is rather to count number of "runs" or number of "trials," which is of course approximately equivalent to time. Furthermore, the main purpose has been to explain typical or average behavior; at the most, individual differences are reported in the form of standard deviations. The present analysis requires, however, examination of the whole shape of the time distribution over individuals completing a specified task.

Numerous studies show that students do differ in the time they consequence to assume that any number of successive steps can be aggregated to form a larger step.

Dependent and Independent Variables

We shall now take a look at the variables that we have adopted and—with Carroll's and Bloom's models in mind—discuss these variables in terms of dependence and independence. We will also consider how they might be used as implemental variables and as target variables, where in the usual manner the implemental variables are such that they can be manipulated and the target variables are those we want to influence by changing the values of the implemental variables.

Since each variable can be either dependent or independent (with exceptions, in our case, to be mentioned shortly), we may conveniently picture the possibilities in the following table:

	Time	Learning	Method	Disposition of Learner
Dependent	1)	1)		
Independent	2)	2)	2)	3)

Figure 1.—Dependent and Independent Variables.

1) target variables
2) implemental variables
3) given at the start of each step

Method and disposition can only be independent variables. In the long run the disposition of the learner can be thought of as dependent will use to complete a given learning task to a specified criterion.[1] Nobody would of course, expect anything else; the extent of individual differences in time, however, may seem unexpectedly high, usually in the neighborhood of one to five when comparing the five per cent most rapid with the five per cent slowest students. In economic terms, there is a non-zero elasticity of substitution between time and aptitude in the acquisition of a skill, so that lack of ability can, within limits, be compensated by the use of more time in learning.

In "A Model of School Learning," Carroll develops a theory essentially centering around the use of time in learning.[2] Since that model is so closely related to the one used in this research, we will discuss some of its qualities. Carroll defines a learner's task as being one of proceeding from ignorance to knowledge or understanding of some specified fact. He does not claim the model to be a "learning theory" but rather a description of the "economics of the school learning process." Five factors are examined: Three of these are said to determine time needed for learning, 1) a set of aptitudes, 2) ability to understand instruction, 3) quality of instruction. The other two factors, 4) time allowed for learning, and 5) the time that the learner is willing to spend in learning ("perseverance"), are assumed to be determinants of time actually spent in learning.

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[1]See e.g., Richard C. Atkinson, "Computer based Instruction in Initial Reading," in Proceedings of the 1967 Invitational Conference on Testing Problems (Princeton: Educational Testing Service, 1967), pp. 55-67. Block, ed., Mastery Learning Theory and Practice. p. 92 and 95. John B. Carroll, "A Model of School Learning," in Teachers College Record, LXIV (May, 1963), 723-33.

[2]Carroll, "A Model of School Learning."

on method in prior exposures to learning but we are here considering a given learner at the moment, when he is entering a particular step.

It can be seen that there are four combinations, one of which will be ruled out as impossible, however:

1) Time and learning are both dependent on method and learner. This is the case in the Carroll model, as we have already pointed out.

2) Time and learning are both independent. This possibility must be ruled out, since that would mean the absence of dependent variables.

3) Time is independent and learning is dependent. This characteristic of a school, where time is completely rigid for each particular task, and learning hence a variable outcome of fixed time, a given method, and the learner's disposition. Even in a traditional schooling one has, however, usually allowed for some time flexibility by, for example, use of homework and extra help to low ability pupils outside the scheduled periods.

4) Time is dependent and learning independent. A special case under this category is that of mastery learning at a single step i, where learning becomes a constant across all individuals and time is a resultant variable.

As indicated in Figure 1 all variables but learner's disposition could be thought of as implemental variables. Time is an implemental variable in case 3), learning in case 4). Method is of course an

Aptitudes are defined in terms of the amount of time that the learner would need under optimal conditions for learning specified tasks to specified criteria of success. By definition, if each learner were allowed to proceed at his own rate, the high "aptitude" learner would complete a given task faster than the one with less aptitude. Given a fixed amount of time, learners with different aptitudes would end up on different levels of achievement, which is of course a more familiar picture.

It is notable that aptitudes according to this definition—that is amounts of time needed in learning—vary not only with the learners but also with the task involved. The aptitude-time measure is thus not a unitary one. There are several relatively independent learning rates, and an individual student may conceivably be a fast learner in one type of task and not in some other. Given enough time (opportunity), and given that he is willing to engage in the task as long as will be necessary (perseverance), then any learner would eventually complete a learning task to a specified performance level. Carroll does, however, allow for the possibility that the task is beyond the learner's capacity altogether. Then his aptitude for that particular task would approach zero as time approaches infinity.

Carroll formulated his model in the following way:

$$\text{Degree of Learning} = f\left(\frac{\text{time actually spent in learning}}{\text{time needed}}\right)$$

where time actually spent is equal to the smallest of 1) time allowed, 2) perseverance as amount of time the learner is willing to spend on implemental variable in all three possible cases, whereas learner's disposition is neither implemental nor target variable in the short run but is a target in the longer run. Case 4) is the one that will be investigated here—with an eye occasionally to the others also. It will be noted that in case 4) time is an outcome variable of the process. Since learning is specified in advance and learner's dispositions are given, the resulting average time for individuals with given dispositional characteristics may be thought of as a quality index of the method used in application to such individuals; the less the time to mastery the more successful the instruction. This circumstance will be used in later development of the model.

Researchers in the social sciences, especially economists and educators, have for the last decade increasingly paid attention to interrelations between the production of learning, human ability and subsequent income - individual as well as national.[1] In the present study, however, we will be interested only in relating ability to the immediate outcome of education, hence not to future income. Ability differences as can be seen, are translated into time

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[1]An excellent overview of the field and a major contribution to it is Mary Jean Bowman and C. Arnold Anderson, "Relationships among Schooling, Ability and Income in Industrialized Societies," in Bildungsökonomie-Eine Zwischenbilanz. Economics of Education in Transition, ed. by Klaus Hufner and Jens Nauman (Stuttgart: Ernst Klett Verlag, 1970). Some of the major studies are: Becker, Human Capital. Mary Jean Bowman, "Schultz, Denison and the Contribution of "Eds" to National Income Growth," Journal of Political Economy, LXXII, No. 5 (October, 1964), 450-464. Edward F. Denison, The Sources of Economic Growth in the United States and the Alternatives Before Us (New York: Committee for Economic Development, 1962). Edward F. Denison, Why Growth Rates Differ. Postwar Experience in Nine Western Countries (Washington, D.C.: Brookings Institution, 1967). Torsten Husén, Talent Opportunity and Career (Stockholm: Almqvist och Wiksell, 1969).

the task, and 3) time needed (i.e., aptitude), eventually increased by a time adjustment factor for poor quality of instruction.

As can be seen, amount of learning is here quantified as degrees of learning, a function with time units in both numerator and denominator. This quantification of "learning mass," however apt it may be as a psychometric-educational measure, is definitely not very helpful when one tries to use it in economic analysis. Not only will we have to somehow compare the outcome of often very disparate learning tasks, but we must also be able to compare different degrees of learning a given task, since there is no reason to believe that the scale is a quotient scale, where, for example, 20 degrees of learning would be twice as much as 10 degrees. The latter shortcoming can be removed by applying mastery learning theory, as will be seen; the first one (difficulty of comparing the outcomes of different learning tasks) we will have to live with.

In a study based on Carroll's model Kim investigated the problem of how to predict individual learning rates from relevant aptitude measures.[1] Three learning tasks were constructed: 1) beginning German, 2) statistical concepts, and 3) logical reasoning. The tasks were taught to 5th and 6th graders with no prior learning in the fields. Kim related obtained learning rates, defined as time to mastery of a specified task, to IQ and verbal ability (thought to be a close approximation to Carroll's ability to understand instruction), several

[1]Hogwon Kim, "Learning Rates, Aptitudes and Achievements" (unpublished Ph.D. dissertation, University of Chicago, 1968), pp. 53-54.

differences, keeping learning constant over students, step by step, regardless of their ability. In all probability some students will accumulate more learning by taking more steps, a problem that in this study will be handled by a sequence analysis.

We sum up the discussion in this section by stating what may be called the "mastery learning assumptions," supposed to hold for the theoretical models to be developed.

1) All the students in the target population are capable of reaching the predetermined criteria of performance on a defined step, subject to a time constraint as specified below.

2) Some students need more time than others in completing any specified step, that is, in going from one performance level to the next. Time needed may approach infinity, in which case the student's aptitude for that particular step is approaching zero.

In addition we assume:

3) Time actually spent in learning is measurable in an unambiguous way, and

4) When students have reached criterion performance (no less, no more) they go on to the next step.

A growing body of knowledge is accumulating especially at the University of Chicago, concerning the theory and application of mastery learning strategies.[1] We will at various points have reason to discuss

[1]See "An Annotated Bibliography," in Mastery Learning, ed. by Block, pp. 89-147.

aptitude factors, and achievement. He found correlations from about -.3 to about -.6 as can be seen from the following tables:

TABLE 1

SIMPLE AND MULTIPLE CORRELATIONS BETWEEN TIME NEEDED IN LEARNING AND APTITUDES[a]

Task	Primary Mental Abilities						Multiple R
	VM	NF	R	SR	PS	M	
German	-.455	-.461	-.363	-.364	-.448	-.526	.668
Statistics	-.321	-.371	-.414	-.306	-.318	-.299	.454
Logic	-.334	-.520	-.500	-.678	-.453	-.194	.739

[a]Hogwon Kim, "Learning Rates, Aptitudes and Achievements" (unpublished Ph.D. dissertation, University of Chicago, 1968), p. 41. The test used was "Primary Mental Abilities Test," Grades 6-9, 1962 ed. VM=Verbal Meaning, NF=Number Facility, R=Reasoning, SR=Spatial Relations, PS=Perceptual Speed. The coefficients are significantly different from zero ($p < .01$).

TABLE 2

INTERCORRELATIONS AMONG OTIS IQ, PMA VERBAL MEANING, AND TIME NEEDED IN LEARNING[a]

Variable	German		Statistics		Logic	
	Otis	PMA:VM	Otis	PMA:VM	Otis	PMA:VM
Otis IQ						
PMA: VM	.779[b]		.755[b]		.775[b]	
Needed time	-.417[b]	-.455[b]	-.302[c]	-.321[c]	-.457[b]	-.334[c]

[a]*Ibid.*, p. 45. Otis=Otis Quick-Scoring Mental Ability Test, PMA=Primary Mental Abilities Test.

[b]Significantly different from zero ($p < .01$).

[c]Significantly different from zero ($p < .05$).

these strategies. It should be noted however, that our main interest in mastery learning is what has just been stated as assumptions 1) and 2) above.

A Note on Cost-Benefit Analysis

After having examined cost-benefit analyses in the literature Prest and Turvey suggested that the best general description of those studies would be: "the aim is to maximize the present value of all benefits less that of all costs, subject to specified constraints," and they went on to state the following as main problems in cost-benefit analysis:

1) Which costs and which benefits are to be included?

2) How are they to be valued?

3) At what interest rate are they to be discounted?

4) What are the relevant constraints?[1]

The aim of this study of the use of time in education is to develop methods that could be used in a cost-benefit evaluation of educational production. The purpose is not to present a complete theory covering all the aspects of such an analysis but rather to contribute towards this by presenting a theory of cost evaluation in education with particular reference to the use of time as an input. Benefits, therefore, will be taken as given in one form or another, and when they are entered into the analysis we will assume that the

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[1]A. R. Prest and R. Turvey, "Cost-Benefit Analysis: A Survey" in Surveys of Economic Theory, Volume III, Resource Allocation, American Economic Association, Royal Economic Society (London: MacMillan and Company Limited, 1966), p. 158.

Kim also found that final achievement correlated highly with achievement at the end of each time period (usually above .75) and concluded:

1) "The amount of time needed in learning to mastery, in general, can be predicted significantly by relevant aptitude factors."
2) "Different learning tasks require distinctively different sets of aptitude factors in predicting the time needed in learning."
3) "The time needed in learning show high correlations with final achievement levels."
4) "General intelligence or verbal ability is correlated moderately with the time needed in learning in all three learning tasks used in the present study. This is interpreted as an indication that the ability to understand instruction (as measured by general intelligence or verbal ability) is different from the specific aptitudes needed to master a given task."[1]

Learning rate can also be defined as achievement gain in a given period of time. Using longitudinal achievement data Kim obtained essentially the equivalent result. This is a fact that will be of some importance later, as we will see.

Time needed in learning thus seems to be determined by some characteristics within the learner himself. These characteristics are aptitudes and ability to understand instruction. If they can be improved, as we suggested in Chapter I, we would have a resulting decrease in time needed for learning. That time actually spent in

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[1]Ibid., p. 53.

problem of how they should have been ordered or valued has already been solved.

There are several reasons for thus limiting the problem: 1) By taking an arbitrary given measure of benefits and concentrating on the cost side we are breaking the problem down into manageable pieces. 2) As has been stated, ours is a micro approach; decision's concerning societal benefits are essentially made at the macro level and the micro entities are expected to produce a given output at the least possible cost or produce a maximum output out of given resources as a means towards the broader goals. 3) A focus on analysis of the use of time in education is necessarily cost oriented in any case, since it centers on the input side.

Discounting will be explicitly carried out only when analyzing step sequences, and the problem of what interest rate should be applied will not be taken up. The relevant constraints, finally, will be our main concern, when developing the programming model. Time of course is the constraining factor of most interest in this study.

Before developing our cost model, we will need a somewhat more precise statement of the "benefits given" assumption. We may, for example, assume that social benefits related to different possible or conceivable levels of output are reflected in a social demand schedule for education. "Benefits given" may then be interpreted as meaning that we know at least over a relevant range social demand for the educational product as measured by number of students taking a specified step. Needless to say this knowledge may indeed be learning can be limited by school practices is self-evident. One of the important arguments in Bloom's mastery learning theory is that this habit of limiting time in uniform ways for all learners may have a detrimental effect on subsequent learning.[1]

Would it be possible finally to influence time spent in learning by manipulating instruction as Carroll postulates? Unfortunately, for our purposes, there are no studies that can give us a direct affirmative answer. Quality of instruction has been found to account for a rather high proportion of the variance in achievement, however.[2] It would seem then that for less powerful instructional treatment to produce the same achievement levels among students as a more powerful one, students would have to spend a longer time learning.

It is not quite clear whether Carroll's aptitudes and ability to understand instruction should be thought of as including both basic abilities and specific skills in Fleischman's terminology. If skills are not included—and this was apparently Kim's interpretation—then clearly one determinant of learning and time is missing. If learning tasks are sequential, which is very often the case, students are required to possess one or several specific skills in order to learn

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[1] Bloom, "Learning for Mastery."

[2] See e.g., Bobbie C. M. Anthony, "The Identification and Measurement of Classroom Environmental Process Variables Related to Academic Achievement" (unpublished Ph.D. dissertation, University of Chicago, 1967). Merlyn J. Behr, "A Study of Interactions between 'Structure-of-Intellect' Factors and Job Methods of Presenting Concepts of Modulus Seven Arithmetic" (unpublished Ph.D. dissertation, Florida State University, 1967). Lee J. Cronbach and R. E. Snow, Individual Differences in Learning Ability as a Function of Instructional Variables, Final Report, USOE, Contract No. OEC 4-6-061269-1217 (Stanford, Calif.: Stanford University, School of Education, 1969).

expected only to give a rough approximation of a conceived "true" relation. An optimum point would be determined by the intersection of a marginal cost curve and the demand curve.

Excluding the more unusual shapes of demand curves, such as positively sloping and backward bending curves, there are two limiting forms: 1) A horizontal curve, meaning that output should be expanded as long as the cost of one additional unit of output does not exceed a predetermined level. 2) A vertical demand curve, implying that a predetermined output should be produced regardless of costs.

In the first case decision makers are of the opinion that the increased benefit added by each additional unit is the same no matter how much output would be increased; in the second case the associated output is considered to be of such an importance that it should be embodied in all students in a specified group, say, an age-cohort. Intermediate forms include the downward sloping demand curves, giving the hypothetical demand for given cost levels. In economic theory these are supposed to be most common. In a micro model, however, such a downward sloping demand curve is of less interest, since a micro unit is usually assumed to have no influence on the value of benefits within its possibilities of expanding or contracting output. A business firm faced with a given price in a competitive market is parallel.

The concept of demand for a single step, using our own output measure avoids the confounding of demands with supply or costs since output is defined independently of costs. The demand can further be thought of as derived from the demand for a more complex entirety of which the step is a part.

We may, finally, interpret benefits given as meaning that a budget restriction is imposed from the macro level. The decision should ideally be based on some notion of society's total resources. An optimal point in the production space of an economy (including goods and services in the broadest sense) is generally conceived of as the highest feasible point of tangency between a production possibility (or transformation) surface (or rather hypersurface) and a utility (hyper-) surface, where feasibility is determined by the restrictions imposed by the limited resources of the economy. This point need not be unique.

The important thing to note here is that the budget restrictions faced by the production plants (schools) may or may not be compatible with the "true" resource limitations of the society; the optimal point of educational production may or may not be feasible within a given budget. These relations could and should be investigated by economists, but as seen from the micro level such an analysis is not necessary, since in either case optimality conditions have to be met also "from below." If the budget is more than necessarily restrictive the "second best" point must be aimed at, and even when "enough" money is granted, it does not necessarily follow that procedures will be optimal at the micro level.

Finally it should be noted that at least one important resource, namely student time, exists outside the conventional monetary budget frame altogether. Clearly ignoring student time as a cost, or making too low an estimate of this cost, will tend to bring forth a less than optimal organization of educational production. This fact alone would, seemingly, justify a rigorous cost analysis of educational production, especially if bearing on timing in education. To sum up then: "benefits given" can be interpreted as meaning that a societal demand curve is known or that budget restrictions and possibly also other forms of restrictions are imposed. Both lines will be pursued in the following analysis.

The Model

In this section will be developed a simple one step marginal cost model, where costs will be derived from students' time to completion. Given factors are the group of individuals from which students are recruited and those individuals' characteristics of importance for learning, the step to be learned the method of teaching and finally costs as specified below. The presumptive students may be thought of as, for example, a particular age cohort in a school district.

The learning situation is assumed to be the one described under paragraph 4, page 40. Learning is predetermined and time a resultant variable. The students may start as well as finish at different points in time; the important thing is that we know exactly the time it will take for each student from entering to finishing the step.

Let t_i be time needed to complete the step by student i. Furthermore let the students be arrayed so that i gives a rank ordering of time to completion, $i = 1, \cdots, n$, where 1 is the designation of the fastest and n of the slowest student. If two or more students take exactly the same time their ordering relative to each other is immaterial. It follows that

$t_i \leqslant t_j$ for $i < j$

Costs are:

C = total costs of taking x of the students through the defined step. $x = 1, \cdots, n$.

A = per student cost determined per time unit, including such items as cost of teacher and student hours (or whatever the time unit) but excluding all items entered into S below.

S = per student cost of supplies (books, materials) for the whole step from entering to completion; the supplies are such that each student need a fixed amount per the defined step. An example would be a textbook covering the content of the step.

MC = marginal cost

AC = average cost

"Overhead costs" are assumed to be zero or negligible. If we take students in order of achievement as measured by time to completion of the specified step we may trace a total cost curve for x students out of the whole population (age cohort) of n individuals.

$$(1) \quad C = Sx + A\,(t_1 + t_2 + \cdots + t_x)$$

$$= Sx + A \sum_{i=1}^{x} t_i$$

By the ordering of the students we may, however, express t_i as a function of x

(2) $t_i = f(x)$

$\sum_i t_i$ will then also be a function of x,

(3) $\sum_i t_i = F(x)$

which together with (1) will yield

(4) $C = Sx + A\,[F(x)]$

(5) $MC = S + A\,[F'(x)]$

The $F'(x)$ function in (5) can readily be identified with (2) since

$$\sum_i t_i \approx \int_0^x f\ (x)\ dx = F(x) \text{ and}$$

$$S + A\,[F'(x)]\ dx = Sx + A\ .\ F(x) + K$$

where K is a constant equal to zero by our assumption of negligible "overheads." This is consistent with the fact that C is by definition a primitive function of MC, which is trivial. The significance, however, of the present analysis lies in the fact that by construction of an ordered array of students we have been able to derive MC from an independent source, namely the function $t_i = f(x)$ distributing time over students. From now on we shall write (5) as

(5') $MC = S + A\,[f(x)]$

We can also calculate average cost by:

$$(6)\ AC = S + \frac{A\,[F(x)]}{x}$$

In Figure 2) the model is graphically exposed. The horizontal axis measures the output as number of students brought through the given step and the vertical axis measures both time units and cost units which is possible by the implicit proportionality of costs and time to completion. S is a constant function and the time scale has its zero point not in origo but at the intersection of the vertical axis with the S-curve. At each point in the range of x, total cost, C is equal to the area (in cost units) under the MC-curve up to that point.

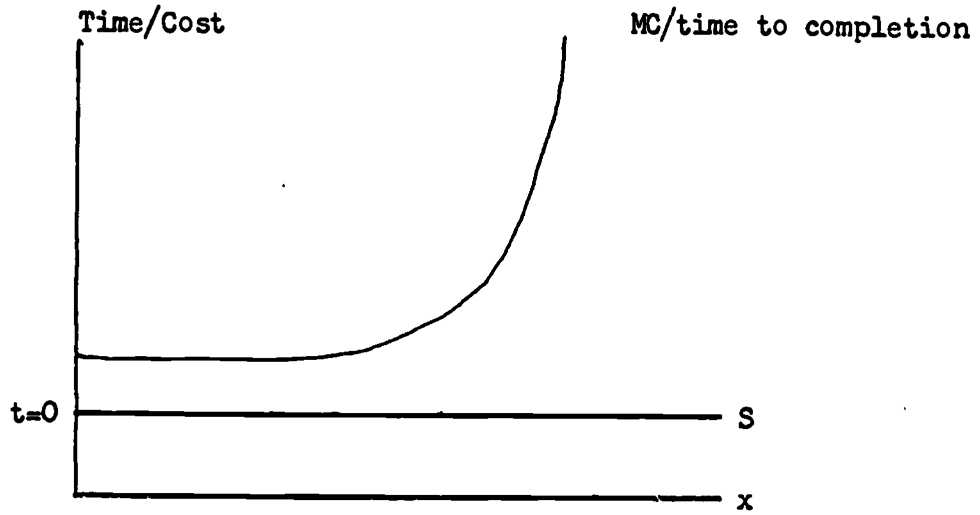

Figure 2.—Marginal cost and time to completion of a learning task. x = number of students out of an age cohort completing the step.

So far we have assumed that marginal cost, as we defined it, minus S, is portional to time to completion, which is a rather special kind of relationship. Before we proceed to a more general case we may point out, though, that this proportionality probably is a good approximation of <u>today's</u> practices in education. As we will see, there may be reasons for wanting to change that. As it is now, however, students in a particular school, taking the same course, will with high probability use up roughly equivalent amount of teacher time, occupy very nearly same amount of space, and use equally many books. Indeed, the way we usually conceive of equality of opportunity, would urge us to aim at such an equalization of educational resources. However, in the next chapter discuss a different approach to "equalization" within the scope of this model will be discussed.

If marginal costs are not proportional to the amount of time used by each student, then we will have to rewrite our equations in order to get a more general model than the above.

We start with the following relation:

$$(7)\ C = Sx + A \sum_i t_i + \sum_i H_i t_i$$

where A has the same meaning as before. $A \sum_i t_i$ is therefore to be interpreted as the part of the costs that is proportional to time. H_i is the per time unit cost, not counted in S or A, of student i. It is different from A in that it is variable over the students, whereas A is equal for all students. Since it is nonproportional to time it will change with x, number of students out of the age cohort. Both H_i and t_i are functions of x. Therefore the product $H_i t_i$ must also be a function of x, say $g(x) = H_i t_i$. Using finite differences, letting student j be the last one brought into the system, thus having the cost of that student represent marginal cost, we get

$$(8)\ MC = S + A\, t_i + H_i\, t_i \quad i = 0, \cdots, n$$

Using differential calculus in "idealizing" (7) and (8) will yield

$$(7')\ C = Sx + A\, F(x) + G(x)$$

$$(8')\ MC = S + A\, f(x) + g(x)$$

where $g(x)$ is a primitive function of $G(x)$. It is evident that $A\,F(x) + G(x)$, as well as its derivative $A\,f(x) + g(x)$ could be represented as a single function of x. We will however use this separation in a later development of the model and prefer for that reason the present notation.

The Marginal Cost Model and Educational Decision Making

Before we analyze the relevance of the model for making decisions in education we will discuss briefly some qualities of our model that needs some clarification. The marginal cost curve was derived from a time to completion curve which means that time is not uniform as we move along the x-axis. This is not, as a rule, the case in an economic analysis of this type, where the output axis usually measures quantity per time unit. It is interesting to note, though, that not all elementary textbooks on economic theory explicitly state output to be a flow. In fact some treatments are such that it is not at all clear, whether a stock or a flow concept is being communicated to the reader.

We will have to admit that x in our case seems to be a measure of a stock. This would not be totally damaging to the present study, however, since many of the conclusions could be valid nevertheless. Still, a flow measure is to be preferred.

Luckily there is one important quality of the step measure that makes it possible to interpret x as a flow. Each individual student will produce one and only one unit of a particular step. This means in turn that out of an age cohort a certain number of students will complete that learning task, say x students, and this x also measures our output. The implication is that, since the production period of schools is one year, this number x would measure output per year, being by virtue of that a flow, as we desired. Shorter or longer production cycles would of course not change anything in principle - semester, quarter or some other time period could equally well serve as the production span. It can now be seen that we actually deal with two time measures: 1) time units (in some specified use) per calendar time period, for example, number of student hours per week used in studying math, 2) elapsed or calendar time.

There are some qualifications to be sure. If individualization of instruction by allowing differences in time to completion is carried to the extreme, students of an age cohort may actually take a particular step in different academic years, such as would be the case in nongraded schools. When one considers a situation where a system is operating at a steady state of output,[1] this could not essentially change the

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[1]When using the model for comparing, for example, different methods of teaching a step we are, of course, comparing different "steady states."

analysis, however. A number of students of one age cohort may not complete a specified task during the same year as would most of their age mates, but their number would be offset by the number of students from the preceding age cohort, who are now completing the task instead of doing so the year before when they missed it, and so forth for the more complicated deviations from "normalcy."[1]

When a student has completed a task, he would normally be assigned or choose a new one, and this new step could of course also be described by the model outlined above. However, what the student is doing after completion is irrelevant for a one step analysis as long as he is productive elsewhere, which could also be outside school.

These qualifications create problems normally considered to be part of a dynamic analysis, which we do not claim to have undertaken, however. What we want to do at this stage is to investigate a constrained or limited system at a steady state of output, disregarding any eventual disrupting outside or inside shocks that could change the steady state. Furthermore, among all the different outputs the system may produce, we are for the moment only considering number of students completing one particular step.

Another way of looking at the flow-stock issue would be to consider the total output of a school system as a set of all possible steps. Then the output per production period could no longer be represented by a single number, x, but would have to be a vector

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[1]The steady state may be disrupted for many reasons; age cohorts could differ in size; school districts may change their boundaries used hence their populations; successive age cohorts may, although probably within very narrow limits, embody different ability distributions.

$x = (x_1, x_2, \cdots, x_n)$ where n is the number of steps that could be taken in the system, and x_i number of students taking step i per production period. There can be no doubt about x being a flow. What we are doing, at this stage of course, is studying one single element of this output vector and we will not get the full advantage of our approach until we find some way of analyzing simultaneously a whole sequence of steps, especially if those steps are related somehow, for example by some being preparatory to others.[1]

The model clearly assumes that costs may be assessed in a straightforward way and in monetary units. Since in the programming model to be developed we will treat some resources as existing in given quantities and derive shadow prices for those resources from their availabilities in the system, we have no reason at this point to try to go beyond market pricing. We will assume, hence, that for the marginal cost model costs can be derived from market prices and that those prices in effect will give us, at least to a near enough approximation, the social opportunity cost.

The most problematic part in this procedure is to estimate the cost of student time. An external measure would be estimated earnings foregone, taking the value of a student's time as equal to the income he could have obtained if gainfully employed. Empirically it is

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[1]In the theoretical development of the model we will make the assumption that costs are discounted, i.e., put in the form of constant dollars at present values. In reality this discounting has to be effected only if we are considering very large steps. Normally the steps we are contemplating are small enough to justify an approximation by use of current dollars and momentary instead of present values: benefits are being instantaneously paid for, as it were. This will be the approach of our empirical investigation.

common practice simply to use a mean value for a relevant age or age group (adjusted for unemployment), but this makes the value of student time the same for all students. That this procedure is unsatisfactory will be argued from theoretical considerations of the programming model in Chapter VI; it will be accepted for the time being, however.

From the point of view of educational decision making there are mainly two problems that the presented marginal cost model relates to: 1) Cost-benefit analysis concerning the student flow through an educational system. 2) Cost-benefit analysis of different methods of teaching one or more steps to a given number of students.

In the first case we may either think in terms of a rather large step, such as a college education, or in terms of optional steps within a larger framework. In either case it is quite clear that students should be admitted or encouraged to enter, whichever be the policy, as long as the marginal cost is below the marginal benefit, however defined. It may, of course, so happen that we run out of students before reaching that point, whether they are under compulsion to take the step or applying for the opportunity to do so.

When evaluating different teaching methods, two instances are distinguishable with distinctly different decision rules. First we may be faced with an all or nothing situation; all students have to be taught by one and only one of the methods under consideration. Then the method should be chosen that yields the lowest average cost as calculated by equation (6). Secondly we may be in such a position that we can offer more than one method to the students. Then a point by point comparison of marginal cost curves would disclose how students should be divided among methods, provided that we don't run into difficulties with "indivisibilities."

Summary

In this chapter we discussed educational production from an economic point of view, that of maximizing in the case of the "educational firm," especially when related to the Carroll model and mastery learning theory. We noticed that mastery learning helped us greatly in defining and measuring output as number of steps, or increments in learning occurring between carefully defined performance levels. The variables involved were discussed as dependent and independent, and implemental, target and interaction variables.

The various possible configurations of the variables were considered; for our purposes, and in mastery learning in general, the most useful way of treating time and learning, seemingly, is to deal with time as being dependent and learning as independent ie., the latter is predetermined, whereas the former is a resultant variable.

Cost-benefit analysis was reviewed and related to our model. By assuming benefits to be given we could concentrate on the cost side, either by minimizing costs of producing required output or maximizing output under cost conditions, set by macro level decision makers. Output was defined as number of students out of an age cohort taking a given step from one performance level to the next.

After these preliminaries the mathematical model was introduced, the basic characteristic of which is the time-cost relationship, generating a marginal cost curve.

The stock-flow difficulty was overcome by pointing to the fact that students go on to something else after finishing one step, and do not again reenter that step, so that our output measure could, in fact, be interpreted as a flow, "so many students" taking a particular step per academic year, or whatever may be the production period.

CHAPTER IV

MASTERY AND NON-MASTERY LEARNING - EQUALIZATION AS AN EDUCATIONAL POLICY GOAL

A basic idea behind this study is the use of what we have called "the mastery learning assumptions" in order to arrive at an unambiguous output measure. Those assumptions stated that all students could reach a specified criterion performance, given enough time to do so. We also assumed that time needed to reach criterion could be accurately measured, and that when criterion is reached students go on to the next step. It will be possible therefore, to compare the learning of different students, since each is assumed to embody similar amounts of learning, given that all have taken and reached mastery on the same steps. The economic problem however, may very well be to compare mastery and non-mastery learning, a point that will be discussed briefly although it is not a main concern of this study. It will not be possible totally to avoid discussing benefits, but they will still be taken as somehow given in value terms. One educational objective often considered to be non-economic will receive special attention, namely equalization, and an attempt will be made to define it and to analyze its costs.

Mastery and Non-Mastery Learning

It is generally accepted among scholars in the field that mastery learning strategies should be designed so that they remedy certain conceived deficiencies in today's practices in education. The assumption, hence, is that methods could be developed to utilize students' human resources better than schools do at present. Bloom suggests that schools now provide successful learning experiences only for at the most one-third of the students.[1] Mastery learning is a theory of how to obtain optimal learning conditions for the lower two-thirds of the students.

To the degree, therefore, that mastery learning strategies are successful, the end product will be "better" than in traditional schooling in the sense of being "finished" for, say, 90 per cent of the students as compared to one-third or less. If its goals are met mastery learning will produce a measurably larger output in stock terms, measured as percentage of successful students out of those entering a given step. Whether this will hold also when looking at flows (total learning accumulated per time unit) will be discussed later.

Whether or not in a particular situation mastery learning would be worth introducing depends on the value we attach to this "upgrading" of the product and on the additional cost, if any, of doing so. If no additional costs are associated with a change to mastery learning, as Block suggests,[2] (and if no one learns less in total, the mastery units aside), then the conclusion must be that today's school practices are indeed very inefficient. However, as yet there is very incomplete

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[1] Bloom, "Learning for Mastery," p. 2. A statement such as this must build on prior value judgements. To go into a discussion of those is however, outside the scope of this study.

[2] James H. Block, "Introduction to Mastery Learning: Theory and Practice," in Mastery Learning, ed. by Block, p. 10.

evidence to show the cost effects of a change to mastery learning. Even if costs for teachers' time, space and materials will remain largely the same, which is an open question, there is always the possibility that mastery learning will use more of student time as input, hence be more expensive, provided that student time is not considered a free good. Since it exists in a limited supply only it should, however, not be thus treated. Block's study on the effects of performance levels on cognitive, affective, and time variables[1] shows that in a dynamic setting, that is, a sequence of steps where each step builds upon the preceding one, the time needed by each individual for review and correction in order to obtain mastery may, under certain conditions, tend to decrease with each additional step taken. Therefore, the cost in terms of student time to completion may be considerable at early stages of a sequence but decreasing as students proceed. When comparing mastery and non-mastery learning great care must be taken in order to avoid an eventual bias produced by a neglect of sequential effects. This applies also to a comparison of situations where mastery is assumed; we will make an attempt to formalize the sequential or "dynamic" effects later on.

If we find when comparing non-mastery and mastery learning that costs are in fact not higher with mastery learning and that output is larger (strategies are successful in the mastery learning situation) then obviously it is to be preferred. The problem will arise when both

[1]James H. Block, "The Effects of Various Levels of Performance on Selected Cognitive, Affective, and Time Variables" (unpublished Ph.D. dissertation, University of Chicago, 1970), reviewed in Mastery Learning, ed. by Block, pp. 104-106.

costs and output increase; then cost minimization and output maximization are conflicting, and we must decide how much we would be willing to pay for the additional product.

Equalization and the Costs of Timing in Education

Economics is concerned not only with the production of goods and services, by means that are in some sense optimal, but also with the distribution of this product. In an economic analysis education may be viewed not only as a productive industry but also as a means to affect income distribution or the allocation of occupational roles within a society. In fact, the booming interest in education in most countries today (whether developed or not) seems to be at least as much directed towards equality goals as towards national income maximization.

In the U.S.A. one goal is for education to solve problems of cultural deprivation.[1] In Sweden and England, as in many other countries, far reaching educational reforms are designed to replace an elitist, selective school system with a comprehensive one, the motives being more on the equality side than the productive. Non-developed countries are often disturbed by the fact that equity is expensive in terms of alternative uses of the very limited resources. "Can we afford to introduce universal elementary education?" is indeed a realistic question in some countries today.

When the focus is on learning, as in this study, the question becomes: Who gets what learning and to what effect? Education can be

[1]For an overview of those problems and an annotated bibliography see Benjamin S. Bloom, Allison Davis and Robert Hess, Compensatory Education for Cultural Deprivation (New York: Holt, Rinehart and Winston, Inc., 1968).

viewed as for immediate or future consumption, or as an intermediate good (input into other processes). The consumer benefits of the learning is captured by the recipient of education, but also to some extent by other people ("neighborhood effects"). In the investment case the returns may be captured, wholly or partially, by the individual in the form of more income to higher educated people. In addition there may be spill-over effects to the incomes of others.[1]

Recently there has been much interest in and controversy about the issue of the incidence of benefits and burdens with respect to education. Windham investigated the system of financing higher education in Florida by examining the net incidence of the costs and benefits.[2] He calculated the net incidence of costs and benefits from estimated distributions of tax payments by income class and percentages of students at universities and junior colleges from each income class. Benefits were measured as present expenditure per student in each type of institution, which may have biased the estimate of the net incidence against the low income groups by ignoring a possible higher rate of return on education at higher levels. All the more striking, therefore, are the findings that the net gain is negative for all income groups up to $10,000 and positive thereafter. The system of financing higher

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[1] Mary Jean Bowman, "Social Returns to Education," in International Social Science Journal, XIV, No. 4 (1962), 647-659.

[2] Douglas M. Windham, Education, Equality and Income Redistribution, (Lexington, Ma: D. C. Heath and Company, 1970).

education in Florida is such that it produces a considerable redistribution of income in favor of high income groups.[1]

The problem of how to equalize educational opportunity is a question of how a society views ways and means of directing school efforts so as to compensate those who are considered to be disadvantaged.[2] It may be argued, for example, that the most effective way of equalizing educational opportunity is to improve pre-school and elementary education for the poor. Even when the focus is limited to higher education to change the system of financing would not suffice. The distribution of students in higher education depends also on prior learning and effects of earlier disadvantage on high school completion. Unquestionably there are many students who lack adequate learning experiences due to disadvantages both at home and in school. The question is then one of how

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[1]Hansen and Weisbrod presented findings similar to Windham's from California, by using a different method. This investigation aroused a lively debate in The Journal of Human Resources through the year 1970. The disagreements have been considerable both on the questions of whether the system of higher education in California is in fact regressive or progressive with respect to benefits and burdens and on questions of what indeed are the appropriate ways to assess distributive effects in one context versus another, and on policy implications. W. Lee Hansen and Burton A. Weisbrod, "The Distribution of Costs and Direct Benefits of Public Higher Education: The Case of California," Journal of Human Resources, IV, No. 4 (Spring, 1969), 176-191 and W. Lee Hansen and Burton A. Weisbrod, Benefits, Costs and Finance of Public Higher Education (Chicago: Markham Publishing Co., 1969). "Benefits and Costs of Higher Education and Income Redistribution: Three Comments. I. Elchanan Cohn, II. Adam Gifford, III. Ira Sharkansky" in The Journal of Human Resources, V. No. 2 (Spring, 1970), 222-236. Joseph A. Pechman, "The Distributional Effects of Public Higher Education in California," in The Journal of Human Resources, V, No. 3 (Summer, 1970), 361-370. Robert W. Hartman, "A Comment on the Pechman-Hansen-Weisbrod Controversy" in The Journal of Human Resources, V, No. 4 (Fall, 1970), 519-523.

[2]Mary Jean Bowman and C. Arnold Anderson, "Distributional Effects of Educational Programs," in Income Distribution Analysis, report from Conference on Income Distribution Analysis with Special Reference to Problems of Rural People (Raleigh, N.C.: North Carolina State University June, 1966), pp. 177-214.

far these inadequacies, and the frequently associated negative attitudes towards schooling could be remedied or prevented. It is this aspect of economic equalization strategies, not manipulation of school financing, that is of interest for the present study.

Equality in education usually is interpreted as meaning equality of educational opportunity, since total egalitarianism in the use of such opportunity is held to be nonfeasible and maybe not even desirable. However, the concept of equality of opportunity is itself ambiguous. We may ask with Anderson and Bowman: Does equality of opportunity mean: "a) An equal amount of education for everyone . . . b) Schooling sufficient to bring every child to a given standard . . . c) Education sufficient to permit each person to reach his potential . . . d) Continued opportunities for schooling so long as gains in learning per input of teaching match some agreed norm . . ."[1] Each interpretation raises new questions.

Komisar and Coombs suggest that the principle of equality in education is a secondary one, meaningful only dependent upon prior ethical judgement.[2] That is consistent with, or at least not conflicting with the idea that we are going to adopt here; equality could derive its meaning from a social preference function. A society may want equality of a specified kind, described by its context, its pragmatic

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[1] C. Arnold Anderson and Mary Jean Bowman, "Theoretical Considerations in Educational Planning," in Economics of Education I, ed. by Mark Blaug (Penguin Modern Economic Readings, Harmondsworth, England: Penguin Books, 1968), pp. 359-360.

[2] B. Paul Komisar and Jerrold R. Coombs, "The Concept of Equality in Education," in Studies in Philosophy of Education, III, No. 3 (Fall, 1964), 223-244.

applications to be guided by certain rules. This equality may not be desired at any cost, and realization of specified goals may prove to be more difficult than was anticipated; this will be the case when there has been an overoptimistic assessment of feasibility, as a result of disregarding resource constraints in the inevitable pulls and tugs against other preferences and competing, even directly incompatible, goals.

In this study a deliberately limited concept of equality is used; we define it in terms of the specified goals of mastery learning. Since we start with the answer, as it were, we have to search for the questions. In other words: Could some other approach to the specification of equality conceivably lead us to mastery learning? It would be helpful to start the search by investigating the possible meaning of equalization in the mastery learning context.

What, if anything, is equalized in mastery learning? It is evident that we are just about to ask Gardner's famous question. "Can we be equal and excellent too?"[1] In the introduction to his book Excellence Gardner tells us about an incident from the 1930's when he was a young professor. On the day of a final exam one of his students had written on the blackboard "Every Man an A-Student!" and "Share the Grades!"[2] Joking though he certainly was, this student almost stumbled upon mastery learning theory. If it were possible to teach every student up to an "A-level" of performance by giving him time and help this would

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[1]The quotation is the subtitle of John W. Gardner, Excellence (New York: Harper and Row, 1961).

[2]Ibid., p. xi.

certainly make students more, if not totally, equal with respect to measures on an achievement scale constructed for this particular course.

On this or any other criterion level chosen it might, however, be expected that some students overshoot the goal set for them (in other words they show overperformance) and some may not reach the criterion even with the time and help we are willing to supply. But even so there seems to be turned out a more uniform product in a mastery learning process than would be the case traditionally. Therefore, the students constituting the product must, in a sense, be more equal than those in the latter situation.

The proposition follows directly from the theory and does not have to be shown empirically. Another question is whether it is possible to create a mastery learning strategy and apply it in the real world. That, however, has been reported in several studies.[1] When focusing on one step only, and on students who complete this particular step, we may conclude, therefore, that mastery learning is equalizing. There are however, dimensions where mastery learning seem to lead towards less equality and this makes the problem an economic one; we have to pay a price for the obtained equality.

Time is in traditional schooling usually a dimension in which students are equal. In mastery learning some students, however, may take a longer time to completion than others, and thus become more unequal in this particular respect. Similarly some students may need more help from teachers than others or need more expensive materials.

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[1]For a review of relevant research see Mastery Learning, ed. by Block, pp. 89-147.

Therefore, mastery learning implies "unequal treatment" of students (which may be for very good reasons of course).

It is easy to see that all this in economic terms may be described as unequal distribution of costs. The fact that part of this cost is born by the students themselves, since they invest their own time, is immaterial in this context. The differential time to completion is perhaps the most serious source of inequality involved here, since it is accumulating with each step taken so that students may end up with vastly different learning embodiment in terms of number of steps taken. With respect to any particular step each student is equal to any other who has actually taken that step. Gross inequalities could, however, easily arise in the form of differentials in number of successive steps taken by different students. Bloom suggests that "One basic problem for a mastery learning strategy is to find ways of reducing the amount of time required for the slower student to a point where it is no longer a prohibitively long and difficult task for these less able students."[1]

Our task is not, however, to investigate what equality could mean in the context of mastery learning but rather to inquire whether some societal value judgement could lead us in that direction. In other words: Why would a society want to equalize by redistribution of educational costs and especially why would it want to do so by influencing time-to-completion patterns?

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[1]Bloom, "Learning for Mastery," p. 2.

There are, largely, two reasons for societal intervention in order to counteract perceived inequalities:

1) By so doing it may be possible to avoid later economic outlays, such as welfare payments; the object of equalizing measures is considered to be capable of supporting himself if certain remedial or preventive measures are taken. In other words, there is a pay-off involved where the direct costs of action are estimated to be at the most equal to the calculated opportunity cost of not interfering.[1]

2) In addition the society may have developed a philosophy of equality, integrated to its value system, according to which some inequalities are simply intolerable, whether or not some action to counteract them will pay off.

Both reasons may simultaneously apply in some cases, such as educating seriously handicapped children; they may in the end be able to take care of themselves to some degree, hence lowering societal costs, but that does not have to be the major reason for providing such an education, and the pay-off may be considerably smaller than the opportunity costs involved. Actions may be aimed at reducing or totally overcoming certain existing inequalities, such as social barriers to higher education, or they may be taken in order to counterbalance those which cannot be removed, such as permanent invalidity.

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[1]This could be argued also on other grounds, for example GNP maximization, and is not exclusively a redistribution argument.

It can now be seen that mastery learning may be adopted for the following equalizing reasons:

1) Increase relative future earning capacity of the slow learner by making his learning more adequate as an input into other activities.

2) When viewing education as a consumption good, increase of quality and possibly quantity consumed immediately and in the future by the slow learner.

3) In a dynamic setting possibly speeding up the learning of the slow learner by making prior learning a more adequate input in subsequent learning. (This, incidently, is objective 1) with "subsequent learning" substituted for "other activities.")

The last objective may be the most important one and we will have to return to this possibility later when discussing sequences of steps. In pursuing these three purposes we may or may not be able to capture a societal pay-off depending upon whether the gain for the slow learners minus a possible loss for the quick learners, is larger than or less than the costs that can be attributed solely to the upgrading.

We recall from equation (8') in the last chapter that $MC = S + A\ f(x) + g(x)$ was the general expression for marginal cost, $A\ f(x)$ being the proportional part of MC with A the proportionality constant. In order for costs to be equally distributed over all students <u>per step</u> $g(x)$ must offset $A\ f(x)$ so that $A\ f(x) + g(x) = C$, a constant for all x. We may, therefore, define equity in cost distribution in a mastery learning situation in such a way as to require that $A\ f(x) + g(x)$ is constant for all students.

Thus we have a marginal measure of the redistribution of costs. Over ranges where $A\ f(x_1) + g(x_1) > A\ f(x_2) + g(x_2)$, $x_1 < x_2$, the costs are distributed in such a way as to favor the rapid learner and vice versa.

If we want our decisions to be rational, then the relative value of $A\ f(x) + g(x)$ over ranges of x should reflect our willingness to invest differentially in slow or rapid learners. Over ranges where $A\ f(x_i) + g(x_i) < A\ f(x_j) + g(x_j)$, $x_i < x_j$, the implication must be that society either expects a high enough pay-off on compensatory education and/or undertakes this differential investment in order to counteract perceived inequalities. If the rapid learner is favored the implication is that society is expecting a higher pay-off on this investment.

This way of defining dividing points between equal and unequal cost distribution is of course not the only one possible; it could be argued that per time unit cost should be judged equal. That would require $g(x) + S$ to be zero for all x, and the cost distribution would be considered in favor of those x where $g(x) + S > 0$. In either case a measure of unequal cost distribution could be derived from the expression for marginal cost.

Another equalization objective may be added, given that mastery learning is already introduced:

4) Minimize differentials in time to completion in order to counteract the inequalities arising from mastery learning itself.

Objective 4) is indeterminate and must be pursued dependent upon some other condition. If time differentials are measured as time variance, minimization can occur around indefinitely many means of time to completion. Zero variance corresponds to a horizontal time to completion curve in our diagram (Figure 3, p. 73), but indefinitely many such horizontal lines exist. The necessary additional condition could, for example, be given as an "ideal" time to completion around which to minimize variance. We could require only positive time differentials to be minimized, if we would not want to risk that quick learners actually slow down their pace.

Our student population has to be specified so that we will not be able to minimize variance simply by selecting the students. Suppose that we pick the time of the fastest student as "ideal" time and ask ourselves: What would it cost to teach all students so that they would obtain mastery in that time? We would then, no doubt, arrive at a rising marginal cost curve, something like the one shown in Figure 3.

Furthermore, the MC-curve may at some point be vertical, implying that from hence on total equalization of time cannot be obtained at any cost. Empirical derivations from educational experiments of such marginal cost curves could be very useful in determining the limits to following objective 4). One would expect that this objective would not, usually, be pursued up to its extreme limit, except maybe in cases of trivially easy learning tasks.

By allowing time to vary somewhat we would be able to lower the MC-curve, but now we have two conflicting goals, one of minimizing

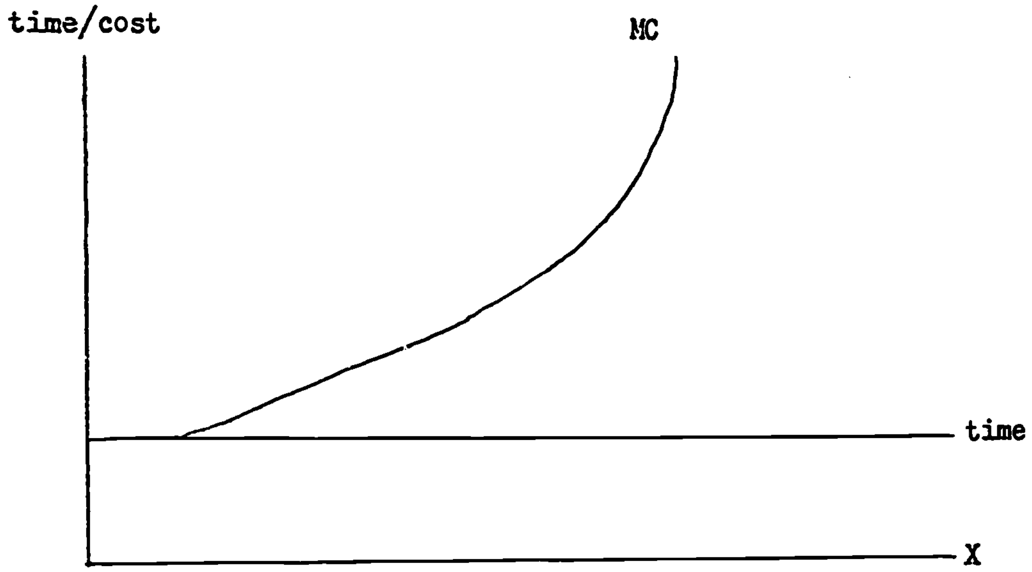

Figure 3.—Marginal cost for bringing students to mastery in same time.

costs and one of minimizing variance, and unless we can somehow attach weights to time variance versus costs we will not be able to get an unambiguous solution. If either costs or time variance could be put in the form of restrictions, the problem could be formulated as one of programming, that is, we would want to either minimize variance in time to completion subject to a budget restriction or minimize costs with the side condition that variance should not be allowed to surpass a given value.

Summary

In this chapter we discussed the costs of non-mastery versus mastery learning, and in the absence of any empirical evidence we had to be rather speculative and tentative about the matter. We concluded that mastery learning, under certain stipulations of goals and learning results, could be used to speed up the learning process by improving learning as an input into subsequent learning, and as a means towards equalizing income and consumption of education by improving learning as an input into other activities.

If, finally, mastery learning is adopted an important goal may be the minimization of the variance among students in time for completion.

CHAPTER V

PRODUCTION FUNCTIONS IN EDUCATION

The economics of education is concerned with production functions from at least two viewpoints. 1) The purpose is to study how education enters as an input into physical production functions. 2) The investigation is directed towards the production of education itself, relating the various inputs needed to obtain a specified learning output. Both approaches are necessary if one wants to understand the economics of the functioning of education in a society but it is somewhat impractical to cover both in one study. For our purposes, of course, the second approach is the interesting one.

Empirical Estimates of Production Functions in Education

Only recently have attempts been made to estimate comprehensive educational production functions. Basically, however, the same methods have been used as in earlier work aiming at relating certain characteristics of schools and students to achievement and to other measures believed to capture the essential outputs of schooling. The standard method, with rather small variations, has been multiple regression techniques, using cross-sectional and occasionally longitudinal data.

Thomas related a large number of social background, community and school variables to student performance using data from 206 high schools throughout the country.[1] Beginning teachers' salaries, teachers' experience, and number of volumes in the school library were found to influence students' test scores significantly when controlling for home and community variables.

The best known of these "quasi" productivity studies is the Coleman Report.[2] In this investigation the independent variables included per student expenditure on instruction, pupil-teacher ratio, teacher experience and training, curriculum variables, facilities (books in library, science laboratory facilities), school services like counseling, health, athletics, and social composition of the student body. Dependent variables were achievement in basic skills, general information, and verbal and non verbal ability tests. In general the report found that when social background was controlled, school inputs accounted for very little of the achievement variation. The study thereby triggered understandably, a debate on the efficiency of American schools. Bowles and Levin in a reanalysis of the Coleman data argued that conclusions regarding among other things "the ineffectiveness of school resources" were not "substantiated by evidence," but pointed out that they did not want to suggest the opposite to be true.[3]

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[1] J. Alan Thomas, "Efficiency in Education: A Study of the Relationship Between Selected Inputs and Mean Test Scores in a Sample of Senior High Schools" (unpublished Ph.D. dissertation, Stanford University: School of Education, 1962).

[2] James S. Coleman, et al., Equality of Educational Opportunity (Washington: U.S. Office of Education, 1966).

[3] Samuel S. Bowles, and Henry M. Levin, "The Determinants of Scholastic Achievement: An Appraisal of Some Recent Findings," Journal of Human Resources, III, No. 1 (Winter, 1968), 3-24.

Burkhead's study of inputs and outputs in large city high school systems has already been mentioned.[1] The major conclusions were that socio-economic factors, not per student expenditures as such, explain most of the variation in output, and that age of school buildings and teacher characteristics (experience, turnover rate, and salary) are the most important in-school determinants of output.

Kiesling, in a study of school districts in New York, found the relationship between per student expenditure and school district performance (mean achievement scores) to be rather weak, except in large urban school districts.[2] A logarithmic function fitted the data better than a linear one suggesting that there is a diminishing marginal return on dollar expenditure. He found no evidence of economies of scale. In fact, the data seemed to imply an overall diseconomies of scale; small schools were performing better. This effect disappeared, however, when a geographical categorization into rural, village, urban, and large city schools was introduced. In sharp contrast to most other studies Kiesling found a statistically significant negative relation between class size (measured by average daily attendance) and achievement.

In a study of Boston elementary school studies Katzman used as output measures, school attendance, schools' holding power, students' reading achievement, and proportion of students a) taking and b) passing entrance examinations to the high status, elitist Latin High

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[1] Burkhead, Input and Output in Large City High Schools.

[2] Herbert Kiesling, "Measuring A Local Government Service: A Study of School Districts in New York State," Review of Economics and Statistics, Vol. IL, No. 3 (August, 1967), and (Unpublished Ph.D. dissertation, Harvard University, 1965).

School institutions.[1] Pupils per classroom, ratio of students to staff, size of district and teacher characteristics (employment status, experience, degree level, turnover ratio) were significantly related to one or more of the output measures. Scale economies were mixed; large districts tended to do better in reading and to have greater holding power, but to do less well on Latin High School entrance examinations.

Production functions for black and white students were investigated by Hanushek.[2] Among in-school variables in his study especially those relating to teacher quality (teachers' verbal facility, experience, and race) were shown to have a substantial impact on achievement. Hanushek also found these variables to have different effects on white and black students.

Bowles estimated production functions from data collected for the Equal Educational Opportunity Survey (partly published in the Coleman Report).[3] Surrounding his findings with much caution about the present state of the art, said to be typically characterized by

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[1]Theodore Martin Katzman, "Distribution and Production in a Big City Elementary School System," Yale Economic Essays, VIII No. 1 (Spring, 1968), 201-256, and (unpublished Ph.D. dissertation, Yale University, 1967).

[2]Eric Alan Hanushek, "The Education of Negroes and Whites" (unpublished Ph.D. dissertation, Harvard University, 1968).

[3]Samuel Bowles, "Towards an Educational Production Function," in Education, Income, and Human Capital, ed. by W. Lee Hansen, Studies in Income and Wealth, No. 35 (New York: Columbia University Press, 1970), pp. 11-61. Coleman, et al., Equality of Educational Opportunity.

crude measurement and presence of specification errors, as well as by gross ignorance of the conditions for learning, he tentatively concludes that a ten per cent uniform improvement in all school inputs included in his equation would raise achievement (test score) by 5.7 per cent. Because of the linear regression technique used this would apply only in the neighborhood of the mean for the observed inputs.

Empirical Production Functions and Policy Making in Education

A general expression for a production function is

$$(1)\ O = f\ (x)$$

where O = output is a function of a number of inputs, $x = (x_1, \cdots, x_n)$. Theoretically and conceptually there is no difference between such production functions in education and in production for the market. In practical applications, however, the measurement both of input and output is usually considerably more complex for educational production and proxy measures have, therefore, been used to a large extent, especially for inputs. Schools of course are multiproduct enterprises as we have already pointed out; most productivity analyses concentrate on achievement as the only output, as we do in this study.

Even when output is defined in terms of measured achievement, however, there is some doubt as to the accuracy of the measure, since the tests used are often biased towards "general ability;" this may leave little individual variability to be explained by differential school inputs.[1] Equally serious, perhaps, is the use of less than

[1]According to Bloom an individual has normally reached about 50 per cent of his potential IQ development at the age of four and 80 per

satisfactory input proxies. The student brings to the learning situation his prior learning, his motivation, and his ability to learn. When social background, community, and peer group influence are used as inputs a measurement error is inevitably introduced. For instance, not all high income families use their money to buy a lot of books, and those who do may not allow their children to use them. On the other hand even in a low income and low social status family parents may, for example, be "good" models of language for their children, something that is thought to be very influential on school performance. Similar examples of possible exceptions to the general picture are simply too numerous to be lightly dismissed.

When the input measures relate to school characteristics they can be equally misleading. The number of books in a school library is as such less important than the frequency and efficiency by which those books are used, and teachers do not necessarily learn from experience, just to mention a few of the measures of in-school variables that have been used.

If the reason for establishing production functions is to get a general picture of present relationships in education, they may still be worth the effort, even though the term "production function" may somewhat exaggerate what is in fact being done. It is when policy implications are drawn that one must be especially aware of the limitations. A student from a "low" social class background may be highly motivated, high in verbal and other skills, and also have

cent at the age of eight. Furthermore, 50 per cent of the variation in intelligence is already accounted for at the age of four. Bloom: Stability and Change in Human Characteristics, p. 68.

adequate prior learning, when coming to school. If his school treatment is automatically differentiated on the basis of his social background it is almost bound to be inappropriate. Furthermore, the expectation of his potential achievement will be low and may become a self-fulfilling prophesy, producing sub-optimal learning.[1]

In "A New Model of School Effectiveness" Levin raises the question of whether it is realistic to assume that the explanatory variables are truly exogeneous, that is, determined outside the system as is implied by a one equation model of educational production.[2] Instead of a one equation model Levin suggested a simultaneous equation system.

To sum up, there seem to be at least two major measurement problems involved in educational productivity research so far: 1) Use of proxy variables that are blunted measures of the underlying variables they are intended to represent. 2) Interrelations among the explanatory variables are as a rule quite high, hence introducing serious bias in the estimates and precluding observations of separate effects or specified interaction terms. On both accounts the variance "explained" by variables included in the equations has been biased downward, giving very low coefficients of determination (R^2 often = .2 or less).

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[1]Experiments on the impact of teacher expectations are reported in Robert Rosenthal and Lenore Jacobson, Pygmalion in the Classroom (New York: Holt, Rinehart and Winston, 1968).

[2]Henry M. Levin, "A New Model of School Effectiveness," in Do Teachers Make a Difference? A Report on Recent Research on Pupil Achievement, U.S. Department of Health, Education and Welfare (Washington, D.C.: U.S. Government Printing Office, 1970), pp. 55-78.

There are many other baffling problems to be overcome when estimating production functions in education. Thus, even setting other problems of measuring output aside, it is difficult to get the crucial measures of schooling effects in value-added terms because so much learning takes place outside the school and prior to entering school. And aggregation problems are difficult to surmount in the absence of weights (prices) for the various products. The list could be extended.

Assuming now that all these obstacles could by some ingeneous statistical methods and refined measurement be surmounted, so that we would have reasonably good estimates of our inputs and outputs, and of the structural parameters by which they are linked, where would that leave us? Some of the variables x_i, $i = 1, \cdots, n$, in the production function $O = f(\bar{x})$ are policy variables that could be changed by decision makers. Ideally we would want the following conditions to hold

$$(2) \quad \frac{MP_i}{MP_j} = \frac{\partial f/\partial x_i}{\partial f/\partial x_j} = \frac{P_i}{P_j}$$

for all i and j.

MP_i = marginal product of input and i and p_i = social opportunity cost of input i. This of course is the well-known conditions for product optimization: the ratios of marginal products should be equal to the ratios of their prices, if we want to make the best possible use of scarce resources in production. In the case of a multiple or joint product the analysis will be more complicated. We would need weights (prices) for the different outputs, but let us by-pass these complications, since we have enough trouble already, as will be shown. The conditions (2) hold only under the assumption that the production function from which they are derived describes technically efficient points.[1] In other words: each input combination is assumed to yield highest possible output. <u>If and only if</u> schools are at least close to producing efficiently (on the efficiency frontier in the case of more than one output) it would make sense empirically to derive a production function in order to combine inputs by their social opportunity cost ratios. In all fairness it must be admitted that researchers in the field seem to be aware of this fact and usually include it among the reasons for using their findings with extreme care.

Even if technological efficiency is not present, estimates may, however, have value as a sociological investigation into prevailing relationships in educational production, but that is not our concern in the present study.

When making policy recommendations from productivity studies a very crucial question must be whether the observed production is efficient or not. At present we don't know the answer, but it seems fairly obvious that the evidence is against assuming technological efficiency in today's educational practices. Schools do not sell their products in a market and for both this and other reasons they are not forced towards efficiency by an economic incentives.

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[1]Strictly speaking they may hold even when efficient production is not assumed but then only by a prone accidence and hence without significance for decision making.

There exist pressures, to be sure, due to the fact that some inputs have to be bought on the market, but any eventual resulting cost minimization could be effected without regard to constraints in the form of required outputs; that is, costs and outputs could be lowered simultaneously without serious countereffects.

Production Functions and Cost Analysis

The problem of technical efficiency is a serious one in education but by no means is it unique for this field of economic application. The same is true in varying degrees for other public sectors as well; for example, health, transportation, defence. In fact, there can be raised some doubt as to whether efficiency in production is at all a valid assumption for empirical research. The value of this concept as a basis for theoretical derivation of optimal conditions on the other hand is beyond question. Operations research is a set of methods used increasingly by big private concerns to help identify technically available options and to price or cost these to estimate the economically most advantageous input mix and output level.

The equivalent in education would seem to be the application of an experimental approach to determine substitutabilities among pedagogic variables for given student populations, assuming that schools cannot determine the quality of student inputs. The concept of mastery offers a possibility of experimentation by keeping output constant at specified levels (number of students taking specified steps) while varying school inputs over ranges thought to be relevant, hence arriving at iso-product curves (or rather hyper-surfaces). If the social opportunity costs of the inputs are known it would be possible to select the optimal strategies.

Starting with a budget the problem becomes one of output maximization, and taking instead a given product we have to search for the minimum cost of producing this output. But with given resource constraints maximizing output would minimize costs per unit of output. Hence a cost minimizing and an output maximizing procedure under these constraints, are mirror images. More formally this can be shown by stating the maximization problem as a programming one and interpret its dual. This is the approach that we will use in the next chapter where we will try to "put things together." In brief

$$(3)\ \text{Max. } 0 = f(x),\ x = (x_i, \cdots, x_n)$$

subject to $g^i(x) \leqslant 0$, $x \geqslant 0$, $i = 1, \cdots, m$ where g^i functions express the availability of the m resources used in producing O.

If $\hat{x}$ is the solution to (3) then there exist λ_i; $i = 1, \cdots, m$, such that the Lagrangian

$$(4)\ L(\hat{x}, \lambda) = f(\hat{x}) - \sum_{i=1}^{m} \lambda_i\, g^i(\hat{x})$$

is a minimum with respect to λ_i, where the dual variables λ_i are to be interpreted as the shadow prices of the resources, implied by their availabilities. Therefore, implicit in the production maximization problem, is a cost minimization problem which is simultaneously solved.[1]

This, however, brings into the open the fact that the cost minimization approach is plagued by the same limitation as production function analysis; technological efficiency must be assumed. This is because cost minimization, naturally, must be based on some explicit or implicit formulation of productivity conditions and hence, indirectly, on some assumption about technical efficiency. This, incidently is consistent with the normal conception of a cost curve as the minimum rand of possible costs of producing different output levels. In a theoretical economic discussion it is customary to assume efficiency in production; in practical application it becomes crucial whether or not such an assumption is valid.

The cost function used in our model has a quality, however, that may be extremely important in that it is derived from a time-to-completion curve. Productive efficiency would require that for all combinations of inputs, and specified levels of output time to completion should be a minimum (given the present technical knowledge in education). This is consistent with Carroll's way of defining optimal educational procedures.[2] The time that a given student will need under ideal conditions to attain mastery of a

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[1]See e.g., Kelvin Lancaster, Mathematical Economics (New York: The MacMillan Company, 1968), pp. 61-76.

[2]Carroll, "A Model of School Learning and Problems of Measurement Related to the Concept of Learning for Mastery" in Mastery Learning, ed., by Block, pp. 31-32.

learning task is a limit set by conditions within the learner himself. This limit corresponds to the psychological concept of "aptitude." School variables, such as curriculum and the organization of learning situations, for example, in hierarchies, quality of instruction, and opportunity to learn offered to the students, influence directly or indirectly (in part by affecting student motivation) the amount of time that the student will actually need. Hence, given that we know mastery to have been attained, time needed will give us an index of the technological efficiency of the learning situation. To assume efficiency in production is equivalent, therefore, to assuming time minimization in the educational procedures in the technical sense, and an economic optimum could be described as the minimum of time to completion that is obtainable under given resource limitations.

It is unfortunate for the purposes of this study that very little is known about substitutabilities between inputs into the educational process. Traditionally schools seem to have been operating under the assumption of rather fixed production coefficients with close to zero elasticity of substitution. Examples of this are the policies of fixed teacher-pupil ratio, rigid timing of learning processes and very little variation in methods of teaching to allow for individual differences among students. Even the traditional school was not totally inflexible, however. Grade repetition and homework allowed for some substitutability of student time for other inputs and teachers could pay more attention to some pupils than to others, thereby allocating inputs according to some notion of "learning optimization."

In traditional schools there were nevertheless, fairly strict institutional limits to substitution of inputs. There is a trend in modern pedagogy to break up those rigidities and the question then becomes how and to what extent this is possible. A complicating factor in education, as opposed to production for the market, is that we have to deal with substitutability in the production function of one individual, and in the aggregate production function of say, a school, where substitution may occur across individual pupils.

Of special interest for this study is the elasticity of substitution between students' time and other inputs. Would it be possible, for example, to decrease student time by using more expensive materials, like programmed books, or by using more teacher time, and would it pay off by lowering costs? Investigations into educational production functions in order to determine elasticities of substitution between inputs would seem to be a very desirable research project.

If we instead look at the related problem of marginal returns to an isolated change in one of the inputs, we are equally ignorant, although common sense would lead us to believe that those returns are diminishing. It would, for example, seem to be plausible enough that ultimately there is diminishing marginal returns to the student time input for students of any given ability (or other initial traits). It seems to be safe to assume that other kinds of inputs, like teacher time, space, equipment, and materials also show diminishing returns with respect to individual students as well as in the aggregate. In fact some of them, such, for example, as school space will very rapidly reach a limit where the marginal product in all probability is zero if not negative.[1]

Summary

In this chapter we reviewed briefly some attempts to arrive at estimates of educational production functions. We discussed measurement difficulties and in particular those caused by use of proxies for inputs and problems of collinearities among variables. A case was made against the assumption of technological efficiency and it was concluded that policy actions cannot be based on estimates of educational production functions until we know this assumption to be at least approximately valid.

We related production maximization to cost minimization and found that by the underlying assumptions regarding production the latter approach, in fact, also takes technological efficiency for granted. However, we were able to push the analysis somewhat forward by tying efficiency to time to completion, asserting that procedures are efficient to the extent that time to "mastery" is minimized given the learner and the state of technological knowledge. The introduction of resource limitations makes the problem an economic one.

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[1]For a thorough discussion of marginal returns to inputs into education see Thomas, The Productive School, pp. 63-78.

CHAPTER VI

SEQUENCE ANALYSIS

If in a sequence the learning of one step affects the learning of subsequent steps, and in particular if this changes time to completion in later steps, it is evident that costs of steps thus related must also be interrelated. In this chapter we will incorporate into the theory the dynamic elements introduced by a multi-step analysis. This is all the more necessary since learning tasks in schools are very seldom isolated from each other. In fact, the interrelations are often quite intricate, including both branching and simple chains, sometimes crossing over subject boundaries not to mention even more complicated patterns.

However, for simplicity we will discuss only simple chains of steps, one and only one step at each level being dependent on prior equally unique steps; in other words, no parallel steps exist within the sequence. First we will introduce an additive cost model and then a programming model will be presented.

The Cost of a Seouence of Steps

When for any given input mix the time to completion of a specified learning task is minimized for a particular student, we have obtained efficiency in the technical sense, something which is the concern of research in educational methodology. We will assume this obvious condition for an economic optimum to be satisfied; in other words, all cost measures in the following analysis are supposed to be minimum under given input conditions. There are a number of questions that could be asked, and hopefully answered, by an economic analysis. One such question is to determine the most economical input combination. Another, which we have already discussed in connection with the basic cost model, is the determination of how much output should be produced.

When the problem is to bring a given number of students through a defined step sequence a new implemental variable is introduced, since by influencing learning at one step we also have some control over learning as an input into later steps.

Assuming for simplicity that all students achieve on the same level when entering the sequence, we may define output as mastery to a specified criterion on a test covering the whole sequence. We are now interested in finding the least cost "treatment conditions" that would lead us to the specified goal. There are many ways of varying the inputs in order to achieve this such as organizing the content of the steps and the order of sequencing, choosing among different methods of teaching, and trying out different kinds of equipment and materials. As long as students reach the specified final performance level, even mastery levels for prior steps may be thought of as variable. Variation in those mastery levels may be of some importance; Block has shown that varying mastery level requirements may influence time-to-completion patterns cumulatively over a sequence of steps. Maintaining a high level of mastery, he found, could increase students' efficiency of learning, so that time required in subsequent steps for feedback and correction procedures would tend to go down as compared to time used for those purposes by students who were required to attain a lower mastery level.[1]

The sequencing of mastery level criteria should, therefore, be treated as a policy variable, since it may have a significant influence on time-to-completion patterns, and probably also on the cost of materials that have to be used. Even though clear empirical evidence is lacking, one certainly cannot exclude the possibility that imposing a very high mastery level for certain key steps in a sequence actually could lower time to completion over the whole sequence for some or all students.

We assume now that a given student body has to be brought through a given sequence of steps (n students and m steps). Let the cost of bringing the i^{th} student through m consecutive steps in a sequence be

$$(1)\ C_i = S_{i1} + A_{i1}t_{i1} + \ldots + S_{im} + A_{im}t_{im}$$

$$= \sum_{j=1}^{m} [S_{ij} + A_{ij}\, t_{ij}]$$

S_{ij} = Cost of supplies (books, etc.) for student i in step j.

A_{ij} = Per time-unit cost of student i for step j (not counted in S_{ij}). This includes costs of teacher and student time, of school space, and so on.

[1]Block, "The Effects of Various Levels of Performance on Selected Cognitive, Affective, and Time Variables," pp. 69-75.

t_{ij} = Number of time units used by student i to complete step j. Summing over n student yields:

$$(2)\quad \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \sum_{j=1}^{m} [S_{ij} + A_{ij}\, t_{ij}]$$

Since sequences may take considerable time to complete it is no longer satisfactory to ignore discounting, and we therefore introduce a positive interest rate r by which costs can be discounted to present values. For simplicity r is assumed to be constant for all time periods. The present value of C_i is taken as of the start of the sequence.

We assume, as usual, complete knowledge of time distributions, but knowing the ordering of the students is no longer necessary. This knowledge may be thought of as arrived at from experience or by educational experiments. In practical applications of the theory the parameters will simply be ex post measures from teaching methods to be evaluated. Present value of C_i as per the start of the sequence would be

$$(3)\quad \hat{C}_i = \sum_{j=1}^{m} (S_{ij} + A_{ij}\, t_{ij})\ (1 + r)^{-q_j}$$

where q_j is elapsed time from the start of the sequence up til step j is finished. This time measure should not be confused with t, which refers to number of time periods actually spent at work in school. It should also be noted, however, that q normally depends on t; that is, q = f(t). If, for example a subject is taught in school 5 periods each week and a particular student completes his task in t = 50 periods, then q = 10 weeks or $\frac{10}{52}$ year. Therefore, if some ways are found by which time to completion for some or all students could be shortened the returns would depend both on the decrease in t and the associated decrease in q; there is a direct effect on outlays for time inputs and an indirect one on their calculated present values.

We now sum over students to get

$$(4)\quad \sum_{i=1}^{n} \hat{C}_i = \sum_{i=1}^{n} \sum_{j=1}^{m} (S_{ij} + A_{ij}\, t_{ij})\ (1 + r)^{-q_{ij}}$$

where q_{ij} obviously is elapsed calendar time from the start through step j for student i.

A change in the input mix or in mastery requirements may result in parameter changes ΔS_{ij}, ΔA_{ij}, Δt_{ij} and Δq_{ij}. After such a change (or such changes) (4) may be written

$$(5)\quad \sum_{i=1}^{n} \hat{C}_i + \Delta\hat{C}_i = \sum_{i=1}^{n} \sum_{j=1}^{m} [S_{ij} + \Delta S_{ij} + $$

$$+ (A_{ij} + \Delta A_{ij})\ (t_{ij} + \Delta t_{ij})]\ \ (1 + r)^{-(q_{ij} + \Delta q_{ij})}$$

For costs to be minimized over the sequence, meaning that $\sum_{i=1}^{n} \hat{C}_i$ is a minimum, would require that

$$(6)\quad \sum_{i=1}^{n} \Delta C_i \geqslant 0$$ for all possible changes in input mix or

mastery level requirements given that the goal is to have all or a specified fraction of the students pass the whole sequence and reach the final mastery performance level.

To exemplify the procedure: Take an increase in mastery requirements over one or more steps. Assuming no change in S (cost of books, etc.) or in A (costs per unit of time), this change would pay off if changes in t_{ij} are such that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (S_{ij} + A_{ij}\, t_{ij})\ (1 + r)^{-q_{ij}} >$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} [S_{ij} + A_{ij}\, (t_{ij} + \Delta t_{ij})]\ (1 + r)^{-(q_{ij} + \Delta q_{ij})}$$

There may, of course, be a cost increase ΔC and/or ΔA, associated with a change in mastery level requirements, in which case a positive pay off would be obtained if

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (S_{ij} + A_{ij}\, t_{ij})\ (1 + r)^{-q_{ij}} >$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} [S_{ij} + \Delta S_{ij} + (A_{ij} + \Delta A_{ij})\ (t_{ij} + \Delta t_{ij})]$$

$$(1 + r)^{-(q_{ij} + \Delta q_{ij})}$$

In either case (6) is not satisfied; there exists a change (mastery level increase) that would make $\sum_{i=1}^{n} \Delta \hat{C}_i < 0$, and the costs summed up over the sequence could be lowered.

It would seem that the effect of decreasing costs by increasing mastery standards, when at all possible, could most easily be secured by imposing an amount of overperformance early in the sequence, because then the pay off period will be longer. There may be some negative side effects to watch out for unfortunately. Block found in his study that although maximal cognitive learning was produced (in his sample) by demanding a 95 per cent mastery level, students required to maintain an 85 per cent level scored somewhat higher on tests of interest and attitude.[1] The possibility of requiring a higher level of mastery on some key steps (most likely at the beginning of the sequence rather than towards the end) is an interesting alternative that has not yet been investigated.

In Chapter I we discussed overlearning as a source of the development of basic abilities.[2] What Block observed was not an increase in learning originating from a gain in some of those abilities; that possibility could be safely excluded, since his data originated from a three step sequence taught altogether during one week in school. Nevertheless the analysis of an extended sequence, where those abilities are indeed developed, would be perfectly parallel.

When evaluating educational projects the above relatively simple cost accounting should be quite easy to apply. It is important,

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[1] Ibid., pp. 80-86.

[2] Basic abilities, it will be recalled, were defined as general traits of an individual, that are fairly enduring and relating to a variety of human tasks, in contrast to skills that pertains to performance on a specific task or a limited group of related tasks (Chapter I, pp. 10-11).

though, to note carefully the logic involved in condition (6). Knowing $\sum_{i=1}^{n} \Delta \hat{C}_i$ to be negative (costs have been lowered) is sufficient for concluding that an improvement has been achieved only if the benefits from taking the sequence have not thereby been decreased.

The mastery learning assumptions, however, make it possible for us to determine direction, if not the size, of an eventual benefit change. For any student whose time to completion over the sequence has been diminished benefits have actually been increased provided that the gain in time could be utilized. The reason for this is that he has made an opportunity gain; his accumulation of learning is the same, which is the signification of the mastery learning assumptions, at the same time as by finishing earlier he will have the opportunity to increase his benefits either by being gainfully employed at an earlier date or by being able to spend time on some other school activity, such as another subject or entering a new sequence in the same subject. If we are indifferent as to which students decrease their time the same holds also in the aggregate—in other words if time summed over students has gone down. Finally, knowing that over the full sequence $\sum_{i=1}^{n} \Delta \hat{C}_i \geq 0$ is not enough to conclude that we must be worse off to incur the extra C, since we could view this sequence as an early step in a still larger sequence—and so on and on. Thus far, indeed, we have not specified the full conditions for optimization.

Sketch of a Programming Model

To develop a programming model of timing in education is a study in its own right and quite an extensive one at that. Nevertheless we will in this section make an attempt to outline the major features of such a model, since it will provide us with some additional information of considerable value. The development of a detailed programming model of timing in education, in my opinion a desirable piece of educational research, is not intended.

We have already pointed out the different reasons for wanting to interfere with the distribution of costs over the students. The MC model assumes time distributions to be given and determines costs as a function of these distributions. A more general model would allow us to have time implicitly determined.

There are, as we recall, two major possible determinants of distributing costs: 1) Higher investment in some students may be preferable for economic reasons, because a larger pay off might be expected from this investment. These students are not necessarily those who show the most rapid learning. 2) Society may want to undertake compensatory education even if not economically profitable. So far we have not described how optimal conditions could be found when these two requirements are entered. Only the first one will be thoroughly analyzed, since the necessary extensions for the second are rather obvious.

In order to proceed in that direction we must make explicit several of the implicit functional relationships in the MC model.

The S parameter is a summation of costs of books, supplies, and similar cost figures. A and $g(x)$[1] include costs of time inputs, such as teacher and student time, space, equipment, administration. In a programming model these quantities will be explicitly included in a simultaneous equation system. The value of students' time in the MC model has to be predetermined by using foregone earnings. This is an approximation of a hypothetical market value of student time. It is less satisfactory for several reasons; if a mean value is used, which seems inescapable, it may grossly overstate some while underestimating other students' time. Maybe even more important: it could be argued that the value of student time should reflect what the students could do in school, not what they could possibly do outside school. In the programming model student time will be formulated as a constraint and thus evaluated at its shadow price.

One of the problems that the MC model was said to be capable of contributing towards solving, was the question of how many students should be admitted into an education at various steps. This may however turn out to be of less importance in a situation where this decision is largely left to the individuals. (The society may, however, still want to have some guidelines as to whether to encourage students to take, or discourage them from taking a certain education.) Often it seems to be more realistic to treat the number of students and their entering characteristics, relevant for learning, as given

[1]In equation (8'), p. 49.

factors and "optimize" learning over this student population. For this a programming model would seem to be more appropriate.

The MC model assumes resources to be completely acquirable in necessary and predetermined proportions. This may be less realistic in education; the programming model will distinguish between purchased inputs and inputs that exist in given amounts; in addition the amount of money that could be spent is restricted by a budget constraint. The measure of MC is an ex post one; solely by assuming a special kind of self-selection where students present themselves in the proper order or by using instruments predictive of learning rates, could one expect students to be introduced into the system approximately by order of their time to completion of future learning tasks. At present such prediction is not very accurate; moreover serious questions could be raised against using such measures for selective admittance, even if the predictive power were very high.

The important time-cost relationship, is sufficient reason, nevertheless, for developing such prediction instruments and of investigating further the economic significance of timing in education. In the following programming model time to completion is one among many other inputs in the "production of learning," and thus independent of the ordering of the students with respect to time to completion of steps.

For the programming model we assume, as we did for the cost accounting present value model, that students have to be taken through a sequence of ordered steps with no steps left out. The problem for the educational "production plant" is to bring as many students as possible as far as possible in some specified sense. On a scale appropriate to the macro point of view benefits may depend on the size of the output, but it is assumed that the range of possible outputs envisaged by the micro unit will be too small perceptibly to influence unit benefit values. Our earlier assumption that benefits are given may, therefore, be interpreted to mean that weights are given to each step in the sequence; in other words, there exist "relative prices" telling the school what "as many as possible as far as possible" would mean. The problem would then be to allocate existing resources in such a way as to maximize "total revenue."

For example, the weights may be signaling to the schools that efforts should be directed towards bringing as many students as possible through the lower steps, or they may imply that schools should invest heavily in a few students, bringing them very far even at some sacrifice for the slow learners.

It may, however, not be very realistic to assume that such an evaluation of steps has been effected, and especially not if we are dealing with rather small steps, as will usually be the case. Benefits may be attached to larger units than we are considering, and we will have to assume that steps within the sequence are homogeneous with respect to benefits, meaning that we are indifferent as to where on the scale an increase takes place. We arrive at our benefit measure by interpolation, as it were. In order to justify the procedure, however, we must make the steps equivalent in some sense, so that they can be added and compared. What we need, in other words, is a cardinal scale; by assigning benefit weights (prices) to the steps we had such a scale. If we do not have prices, it will be critical how we measure the step units in terms of their learning content.

Suppose that a benefit value is given to a sequence of n steps. We then try to approximate the unknown benefit measure for each step by subdividing the value for the whole sequence in such a way that to each step will be attached a value that is proportional to its learning content, somehow measured. Unfortunately there seems to be no proper yardstick for this measurement task. We simply have to assume that there exists such a thing as <u>amount</u> of learning, and that this quantity can be subdivided in learning tasks of equal or comparable length.

Carroll points out that in the case of programmed instruction learning rate as measured by number of frames per time period, covered by a particular student, is remarkably constant from lesson to lesson, and that the same holds for increments in new vocabulary and grammar points in foreign language learning.[1] In some cases, then, physical units seem, at least approximately, to be sufficient. Carroll suggests also that the amount of learning measured in physical units need not always be a linear function of time. The learning curve for new words to be spelled increases at a decreasing rate, and "insightful" learning curves may show sudden, sharp increases after periods of very slow improvement.

The problem may, in fact, be more easy to solve in practice than in theory in that we have the possiblity of making steps equal by

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[1]Carroll, "Problems of Measurement," in Mastery Learning, ed. by Block, pp. 37-41.

"construction," applying whatever measure of equality seems best to fit the situation at hand. Steps are equal because we decide that they are. It may even be possible to turn around Carroll's discussion of the possible shapes of the function relating learning and time and ask what scale would make learning a linear function of time. We construct in other words, a scale, such that the probability is maximized that a given learner under the same learning conditions will use the same amount of time for any two successive steps.

In the programming model to be developed, we assume that we have a "learning content" scale so that steps are either equal or measurable in equivalent units, hence comparable and additive. We, furthermore, assume that we are indifferent as to where on the scale an increase takes place. The introduction of benefit weights (prices) applicable directly to the step measure would not change anything in principle with respect to optimal conditions or conclusions drawn, but it would make the mathematical notation considerably more complex.

The objective function in the programming model to be developed is assumed to be continuous and differentiable. Otherwise no restrictions on its shape are assumed except the seemingly very reasonable ones that it shows diminishing or at the most constant returns with respect to isolated changes in any one of the inputs and constant returns to scale. The function, therefore, is assumed to be concave over the set of values in which we are interested.

The characteristics of the learners are given. The schools, in other words, have no way of selecting their student body and thereby increasing output. Measure of output will be the added number of student-steps taken in a given sequence.[1] No steps can be skipped but students may enter anywhere in the sequence, except that it is assumed that there are always new steps to be taken, no matter how fast or how advanced a particular student may be.

Outputs and inputs are flows, that is, amounts per production period, for example, teacher hours or student hours per week. This is, of course, the usual approach, but it seems worth pointing out nevertheless since some of the inputs are time inputs, and therefore to be interpreted as number of time units (e.g., hours) per production period (e.g., semester) hence time units per time period, which may be somewhat confusing if not made explicit.

In the following, subscripts will be used to refer to individuals, whereas superscripts will refer to inputs (resources). For example: t_i^j means the amount of time resource j used up by student i.

In the hope that the exposition will be easier to follow we start with the hypothetical case in which we have only one student in the system, or students of one type only, always taking the same amount of time to complete a step. Even from this oversimplified version of the model some implications may be drawn, but it is, of course, the generalization to n students that will yield the most interesting conclusions.

Inputs are:

$t^j = (t^1, \cdots, t^l)$ time inputs of which one, say t^1, is student time. $c^k = (c^1, \cdots, c^m)$ purchased inputs like books, and materials.

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[1] If steps are equal in learning content; otherwise they have to be weighted by learning content.

Among time inputs are space, equipment, teacher time, and other school services made available to the students in certain amounts per time unit. They all exist in fixed amounts and cannot be increased in the short run. Depending upon existing conditions such as the time span of the analysis and conditions in the labor markets (especially for teachers) one or more of these inputs may be considered in the c-vector, as being purchaseable. One input can never be purchased, however, namely student time; the t-vector, therefore, always includes at least the element t^1. All c^k, $k = 1, \cdots, m$, are subject to a budget contraint; a limited amount of money is made available per production period.

Suppose now that as a starting point for the analysis, we make the assumption that all school services, whether purchaseable or not, are made available to the students in the same amounts per production period for each student. Suppose further that for a given student the production function to be maximized is:

$$(7)\ \text{Max } O = f(t^1, \cdots, t^l, c^1, \cdots, c^m) = f(t, c)$$

Time inputs are available such that

$$(8)\ g^j(t) = t^j - \hat{t}^j \leqslant 0 \quad j = 1, \cdots, l.$$

That is, used up resources of service t^j must be at the most equal to the available amount $\hat{t}^j$.

Purchased inputs are restricted by a budget function such that the costs for all c^k must not exceed $\hat{b}$, which gives us the constraint

$$(9)\ h(c) = pc - \hat{b} \leqslant 0$$

where $p = (p^1, \cdots, p^m)$ is the price vector for the purchased resources $c = (c^1, \cdots, c^m)$. In addition we have the non-negativity constraints

$$t \geqslant 0, \quad c \geqslant 0$$

Necessary conditions for the problem to have a solution can be derived from the Langrangian function:[1]

$$(10)\ L = f\ (t, c) - \lambda^j\ g^j(t) - \mu\ h(c), \quad j = 1, \cdots, l$$

The general procedure is to take the partial derivative of L with respect to one variable at a time and set the resulting expression equal to zero. We thereby arrive at a set of equations, giving us the necessary conditions for $0 = f\ (t, c)$ to be a maximum under the given constraints. In addition, some other conditions must be fulfilled as specified below. The following conditions are of interest here: Either $t^j = 0$ or

$$(11)\ \frac{\partial L}{\partial t^j} = \frac{\partial f}{\partial t^j} - \lambda^j\ \frac{\partial g^j}{\partial t^j} = \frac{\partial f}{\partial t^j} - \lambda^j = 0 \quad j = 1, \cdots, l$$

and either $c^k = 0$ or

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[1]We apply here the so called Kuhn-Tucker conditions. For an exposition of the theory, see, for example: G. Hadley, Nonlinear and Dynamic Programming (Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1964), pp. 185-211.

$$(12)\ \frac{\partial L}{\partial c^k} = \frac{\partial f}{\partial c^k} - \mu \frac{\partial h}{\partial c^k} = \frac{\partial f}{\partial c^k} - \mu p^k = 0 \quad k = 1, \cdots, m$$

In addition the following must be true

$$(13)\ g^j(t) < 0 \Rightarrow \lambda^j = 0$$

$$(14)\ h(c) < 0 \Rightarrow \mu = 0$$

Necessary conditions are, however, in this case also sufficient, since the objective function (7) $O = f(t, c)$ is concave and all the constraints linear. By using the conditions

$$\frac{\partial L}{\partial \lambda^j} = 0,\ j = 1, \cdots, l, \text{ and } \frac{\partial L}{\partial \mu} = 0$$

we can actually solve for all variables t and c, and the Lagrangian multiplyers λ and μ.

From (11) we derive $\lambda^j = \frac{\partial f}{\partial t^j}$ for all $t^j \neq 0$, $j = 1, \cdots, l$, and interpret λ^j as the marginal product (in steps per time unit) of time input j. Hence, λ^1, which in some cases may be the only λ^j, is the marginal product of this particular student's time, in other words <u>his learning rate</u> at the margin or simply learning rate if amount of learning is a linear function of time.

From (12) we have that for all $c^k \neq 0$, $\mu = \frac{\partial f / \partial c^k}{p^k}$ which is the marginal product of purchased input k per money unit. We may call this the marginal product of money, and it must evidently be equal in all uses, that is, in all uses of the given budget $\hat{b}$. It is now possible to derive the wellknown condition

$$(15)\quad \frac{\partial f/\partial c^i}{\partial f/\partial c^j} = \frac{p^i}{p^j},$$

The ratio of the marginal products of any two inputs, i and j, should be equal to their price ratios.

$$(13)\quad g^j(t) < 0 \Rightarrow \lambda^j = 0$$

states that if an excess supply of time inputs exists they are used up to the point where the marginal product is zero. If that happens to the student's time input, the reasons may be that he is facing a limit set by his ability. This, of course, indicates a social misuse of student time. If some other time input turns out to have zero marginal product, there is an excess supply of that resource, which has to be removed in the long run in order to avoid a social misuse of resources. Generally of course such a misuse will be present long before the marginal product is zero. In the long run one must move in the direction of

$$\frac{\partial f/\partial t^i}{\partial f/\partial t^j} = \frac{p^i}{p^j}$$

where p^i/p^j is the ratio of social opportunity costs for the two time inputs i and j, and the easiest way of securing this is simply to move these inputs over to the c-vector, if one seeks a long run solution. (This does not mean, however, that t^j's are freely movable once the problem is set up in one form or antoher. Also the budget b will differ for each setup).

If money is supplied in excess so that $\lambda = 0$, money will be used up to the point where the last dollar no longer yields any extra output. Somehow this seems unlikely to occur in real life.

We may now ask, what would be the shadow price of the time inputs implied by their availabilities, in other words, which prices would make

$$(15)\quad \frac{\partial f/\partial c^i}{\partial f/\partial c^j} = \frac{p^i}{p^j}$$

true also for the resources that have no market price? There exists, following the above analysis, at least one such resource, namely student time.

Let p^1 be the shadow price of student time; from (15) we then have

$$(16)\quad p^1 = \frac{\partial f/\partial t^1}{\partial f/\partial c^k} \cdot p^k = \frac{\partial f/\partial t^1}{\lambda} = \frac{\lambda^1}{\lambda}, \text{ for all}$$

$$c^k \neq 0,\ k = 1, \cdots, m$$

which is the ratio of the MP of student time to MP of money. Similarly some other time input t^j would be shadow-priced

$$(16')\quad p^j = \frac{\partial f/\partial t^j}{\lambda} = \frac{\lambda^j}{\lambda},\quad j = 2, \cdots, 1$$

So far we have only one student in the system, and the implications are quite straightforward; the optimal allocation of the available resources can be read off from a system of equations and a shadow price of student time may be calculated, but the results are of no significance, other than maybe for descriptive purposes.

We are now going to generalize to n students, and also make possible a differential use of resources by the students, that is, it will be possible to let some students take more of the resources than others, if that should increase product. First of all the objective function has to be rewritten

$$(17)\ \text{Max} \sum_{i=1}^{n} O_i = \sum_{i=1}^{n} f_i(t, c) = F\ (t, c) \qquad i = 1, \cdots, n$$

where the summation is performed over n students. Note that the subscripts, now to be introduced, refer to individuals.

Since student time is a resource that cannot be distributed in more than one way—each student is using up his own student time—we have n constraints

$$(18)\ g_i^1\ (t^1) = t_i^1 - \hat{t}_i^1 \leqslant 0 \qquad i = 1, \cdots, n$$

We disregard here, obviously, the possibility of exchanging one student's time for another's, which could be done by using some of the students as teachers. Usually $t_i = t_j$ for all i and j, implying that all students get the same amount of time per production period. However, nothing will change in principle, if students are given differing amounts of time at their disposal, that is, optimum conditions could be arrived at also from individual "time budgets."

We write the equivalent to (8) and (9) as

$$(19) \sum_{i=1}^{n} g_i^j (t_i^j) = \sum_{i=1}^{n} t_i^j - T^j \leqslant 0, \quad j = 2, \cdots, 1$$

$$(20) \sum_{k=1}^{m} \sum_{i=1}^{n} h_i^k (c_i^k) = \sum_{k=1}^{m} \sum_{i=1}^{n} p^k \; c_i^k - B \leqslant 0$$

T is the vector of time resources (teacher time, space time, and so on) for all n students (excepting t^1, their own time) and B is the total budget for purchaseable resources.

The procedure we now have to follow is analogous to the one described above. We have a Lagrangian expression

$$(21) \; L = F\,(t, c) - \lambda_i^j \; g_i^j \,(t_i^j) - \mu \sum_{k=1}^{m} \sum_{i=1}^{n} h_i^k \,(c_i^k), \; j = 1, \cdots, 1$$

all partial derivates of which should be set equal to zero, except for the possibility of a solution where one or more variables are zero. Starting with the time of student i we get (for $t_i^1 \neq 0$)[1]

$$(22) \; \frac{\partial L}{\partial t_i^1} = \frac{\partial F}{\partial t_i^1} - \lambda_i^1 = 0 \quad i = 1, \cdots, n$$

and derive the marginal product or marginal learning rate of the i^{th} student

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[1] That t_i^1 in an optimal solution would come out as zero is highly unlikely, however, since it would be equivalent to saying that student i cannot learn anything at all, or, more precisely, cannot learn even the first step in the sequence.

$$\lambda_i^1 = \frac{\partial F}{\partial t_i^1} \qquad i = 1, \cdots, n$$

We also have (for $t_i^j \neq 0$)

$$(23)\quad \frac{\partial L}{\partial t_i^j} = \frac{\partial F}{\partial t_i^j} - \lambda_i^j = 0 \qquad \begin{array}{l} j = 2, \cdots, l \\ i = 1, \cdots, n \end{array}$$

hence, marginal product of some time input used by student i other than student time

$$\lambda_i^j = \frac{\partial F}{\partial t_i^j}$$

Similarly (for $c_i^k \neq 0$)

$$(24)\quad \frac{\partial L}{\partial c_i^k} = \frac{\partial F}{\partial c_i^k} - \mu \cdot p^h = 0 \qquad \begin{array}{l} k = 1, \cdots, m \\ i = 1, \cdots, n \end{array}$$

Marginal product of money, then, would be

$$\mu = \frac{\partial F}{\partial c_i^k \cdot p^k} \qquad \begin{array}{l} k = 1, \cdots, m \\ i = 1, \cdots, n \end{array}$$

We also have the conditions, equivalent to (13) and (14) stating that if there exists excess supply of any inputs, those will be used up to the point where MP = 0.

What do the optimal conditions tell us in this case? For the purchaseable resources it must hold that (for all $c_i^k \neq 0$)

$$(25)\quad \frac{\partial F/\partial c_i^h}{\partial F/\partial c_i^k} = \frac{p^h}{p^k} \quad \text{for all h, k and i}$$

The ratio of the marginal products in other words, of any two purchaseable resources must be equal to the ratio of their prices, and <u>this must hold for each student in the program</u>, assuming perfect distributility among students of all inputs other than student time. If h = k (25) reduces to

$$\frac{\partial F/\partial c_k}{\partial F/\partial c_i} = 1 \Leftrightarrow \frac{\partial F}{\partial c_i} = \frac{\partial F}{\partial c_k}$$

for any two students i and k; in other words, the MP of any purchaseable input must be equal for all students.

If we consider two time inputs (other than student time), say time inputs h and k, we have

$$(26)\quad \frac{\partial F/\partial t_i^h}{\partial F/\partial t_i^k} = \frac{p^h}{p^k}, \quad i = 1, \cdots, n$$

where p^h and p^k are shadow prices that can be uniquely determined by the optimal solution, hence by the availabilities of the resources. If we are contemplating only one such time input, say teacher time, it should be distributed so that

$$\frac{\partial F}{\partial t_j^{teach}} = \frac{\partial F}{\partial t_k^{teach}} \quad \text{for any two students j and k.}$$

In other words the marginal productivity of teacher time should be equal in all uses, that is for all students.

Summing up, the MP_i of any one resource, except student time should be the same in all uses over all students in order to obtain an optimal output for a given number of students. This is what might be intuitively expected, since given that for some resources $MP_j > MP_k$ for two students j and k, it should be possible to increase product by reallocating the resource away from student k, thereby increasing MP_k, and towards student j, with a resulting decrease in MP_j, a process which would be possible until $MP_j = MP_k$. We also have that ratios of marginal products should be equal to the ratios of their prices.

We finally compute the shadow price of student time as follows:

$$(27)\quad p_i^l = \frac{\partial F/\partial t_i^l}{\partial F/\partial c_i^k} \cdot p^k = \frac{\lambda_i^l}{\mu} \qquad \begin{array}{l} i = 1, \cdots, n \\ k = 1, \cdots, m \end{array}$$

Since λ^l generally varies over the students, whereas μ in an optimal solution is the same for all students, it can be seen that the shadow price implied by the availability of resources could be expected to be somewhat different for each student. Furthermore, a student's MP with respect to his own time, as pointed out earlier, may be identified as his learning rate at the margin, so that if one students' marginal learning rate is α times another one's the shadow price of his time will also be α times as high, or in terms of time to completion, if one student takes $\frac{1}{\alpha}$ times as long to complete his marginal task, his time will have to be shadowpriced at α times that of the student with whom he is compared. Shadow price then is proportional to learning rate at the margin and inversely proportional to time to completion of the marginal task. If learning is a linear function of time as suggested by Carroll, then learning rate for a given student is the same over the whole sequence, and MP is simply equal to that rate of learning.

Since time to completion of a learning task usually varies in such a way that the top five per cent of the students learn about five times as fast as the low five per cent, the shadow prices of their time, assuming stable learning rates, should also have the proportions of five to one, the fast student's time being five times as valuable as the slow student's. This seems to make sense intuitively. It must be pointed out that learning rate, however, is <u>determined in the system by the amount of resources devoted to each student</u>. It may very well turn out that an optimal solution to the programming problem will either reduce or increase the range of time to completion. The average productivity of the fast learner is probably higher than that of the slow learner with respect to any of the inputs, and certainly with respect to his own time. It is, however, the marginal productivities that determine resource allocation, and for that reason it may very well turn out that an optimal solution to the programming problem would allocate resources more in favor of slow learners than is being done in schools today.

It may finally be pointed out that the solution presented above, if all variables are different from zero and no excess supply exists, simply is a "general equilibrium" solution, except that student time is restricted to each particular student, thus allowing marginal products of student time to vary over student.

Summary

In this chapter the analysis was extended to incorporate sequences of steps related in such a way that earlier steps are required for later ones. Time to completion in later steps, it was concluded, depends partly on the allocation of resources on earlier steps. This was formalized in a present value cost model. A programming model was developed next, the major features of which was

1) Derivation of optimal conditions for maximizing output over a sequence when students and their entry characteristics are given.
2) Identification of MP of student time with learning rate at the margin.
3) Shadow pricing of resources that have no market price, especially student time.

The relation between the shadow price of student time and learning rate was examined, and the shadow price of a particular student's time turned out to be proportional to his learning rate at the margin.

CHAPTER VII

A STUDY OF EMPIRICAL COST CURVES IN TIME DIFFERENTIATED EDUCATION

In this chapter some empirical findings will be reported using time to completion data taken in a non-experimental learning sequence in mathematics. The student were allowed to take the time they needed to complete a step before going on to the next level. Söderkullaskolan in Malmö, Sweden, was chosen because this school has a comparatively long experience in teaching of this kind. Schools in Sweden are required to individualize instruction to meet the needs of the students; time differentiation, however, has normally not been used for this purpose until very recently.

Data on time required for learning a sequence of mathematics tasks, on mathematics ability, intelligence, marks in school subjects, and family background were taken for all students in grade 7 (13 year olds mostly), this is the first year of the upper department of the Swedish comprehensive nine year elementary school.[1]

The purpose of the empirical study was twofold:

1) To estimate empirical MC curves in order to illustrate the basic cost model developed in Chapter III.

2) To throw some light on the possible determinants of students' time to completion.

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[1]The most radical change for the students in going from grade six to grade seven is perhaps the alteration from class teachers to subject matter teachers.

The first task is quite straightforward, involving nothing more complicated than the plugging in of economic data concerning teacher cost, costs of books and materials, and so on—even though, as might be expected, there were some difficulties in arriving at accurate cost figures for some of the items and especially for the opportunity cost of students' time.

Some of the most powerful of the determinants of individual variation in time spent in learning evidently were not captured by the data collected, but such determination was not the main purpose of this study. The time distributions are important in themselves, regardless of what places individuals at various positions in those distributions.

The Students and the Data

Since our sample consists of all students on one grade level, the seventh, in a single school, and therefore is not a random sample, it will be necessary to consider the possibilities of bias in the selection as compared to the population of all seventh graders in Sweden.

There were 171 students in the sample, divided up in six classes, 86 were boys and 85 girls. Most of them (155 or 90.6 per cent) were 13 years old in 1969; this is the normal age in Sweden for 7th graders, the school starting age being seven. Thirteen (7.6 per cent) were 12; in exceptional cases students who by a test and on other grounds are judged to be mature enough may start school at the age of six. The three remaining students were over-aged one year, hence 14 years old in 1969, which may reflect either grade repetition or delayed school entry. Some students are permitted to start school at eight instead of at seven, because of poor health or for some other reason. This age mix is normal for the grade level.

It is usually believed that schools in Sweden are quite similar in terms of characteristics of their student bodies. Only recently has this assumption of similarity between schools been challenged. In a study (1969) of school segregation in the Malmö school system Swedner and Edstrand found that because of a certain segregation in residential areas, there does in fact also exist a social segregation of schools to the effect that some school districts have disproportionately large enrolments of students from either "high" or "low" social background.[1] The neighborhood rule is with rare exceptions a firmly established enrolment principle for elementary schools in Sweden, so it may be seen that some social imbalance in school enrolments could easily result from residential segregation.

Swedner's and Edstrand's investigation is of interest for the present study because Söderkullaskolan was among the schools investigated; all elementary schools in Malmö were in fact included. Using four categories of father's occupation, the same that have been used in this study, they calculated a segregation number for each elementary school in Malmö. This index was computed by the formula:

$$A = \frac{4a + 3b + 2c + d}{a + b + c + d}$$

[1]Harald Swedner and Gisela Edstrand, "Skolsegregation i Malmö," (School Segregation in Malmö), mimeographed (Lund: Department of Sociology, University of Lund, 1969).

where a is number of pupils with father (or guardian) in the "highest" social group, b is the number of pupils with father from the next to the highest, and so on. Theoretically, therefore, the quotient could vary from four (all in the highest category) to one (all in the lowest category): actually the range goes from 3.05 to 1.30. With an index of 1.74 Söderkullaskolan ranks in the middle of the elementary schools in Malmö; 36 schools have a higher index and 24 have a lower. By using the data of the present study an index of 1.85 was computed.[1] The expected value for Malmö computed from Svedner's whole sample would be 1.93,[2] implying that Söderkullaskolan has some overrepresentation of students from "lower" social strata in the Swedner-Edstrand sample; our sample has even less of that bias. This would be expected knowing the type of residential area from which the school recruits its students, being a "mixed" one consisting of rather large multi family dwelling blocks, interspersed with a few small blocks of one family townhouses.

Since the social class grouping for the purpose of the present study used the same classification, it is of interest to summarize briefly the rules that were used. The Central Bureau of Statistics 1952 worked out a scale consisting of three groups: I, II and III.

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[1]The slightly higher value could - except for a possible year to year variation - depend on the fact that so called special classes were excluded from the present study. (Special classes are designed for low ability students and students with other school problems, not likely to benefit from the teaching in usual classes.)

[2]*Ibid.*, p. 26.

Gustafsson and Swedner at the Department of Sociology, University of Lund, 1962 developed a new scale, mainly by subdividing group II into IIA and IIB, and their scale has since then been frequently used for sociological investigations. The following is a summary from "Skolsegregation i Malmö" (translated into English by the present author).[1]

To social group I belong:

1) Owners and leaders of large corporations
2) High ranking civil servants
3) University graduates employed by private enterprises
4) Self-employed university graduates (e.g., lawyers, physicians)
5) Other university graduates

To social group IIA belong:

1) Civil servants of medium rank (e.g., elementary school teachers, postal functionaries)
2) Employers of private enterprises on a medium level (e.g., accountants, engineers)
3) Owners and leaders of small enterprises

To social group IIB belong:

1) Low ranking civil servants (e.g., petty and non-commissioned officers, postal servants)
2) Low ranking employees of private enterprises (e.g., clerks, traveling salesmen, shop assistents)

[1] *Ibid.*, p. 8.

To social group III belong:

1) Low ranking government employees

2) Skilled workers

3) Non-skilled workers.[1]

The distribution of students with respect to father's occupation is displayed in Figure 4 (p. 153). School records were used as a source of information. The missing cases are those who could not be categorized, either for lack of information or because titles were not included in the list used. "Expected values" are the distribution from the Swedner-Edstrand investigation.[2]

A comparison with these expected values gives us the same information as the segregation index, and in addition also tells us that the highest occupation group is underrepresented and that the lowest is slightly underrepresented. Figure 5 (p. 154) gives the distribution of family income before tax for our sample, obtained from the local tax authorities in Malmö regarding the parents' income. For several reasons it does not give a totally accurate picture of a family's disposable income; for example, taxes are included and government transfers are excluded, which gives an underestimate of disposable income towards the lower end of the distribution and overestimate of it towards the higher. Income before tax is, however, the most accurate measure obtainable, and is in Sweden accessible upon request.[3]

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[1]A complete list of occupations, with the Swedish titles, available from the Department of Sociology, University of Lund.

[2]Ibid., p. 26.

[3]"Offentlighetsprincipen," the publicity principle, is applicable

The bias towards the middle is here more obvious than in the distribution of social groups. This is perhaps what might be expected from the type of residential area we are dealing with, since the very low income people cannot afford the rents in these relatively new houses and the very high income people are more likely to chose their dwellings in other residential areas. It may be of some interest also to note that family income correlates in the expected direction, but not very highly, with occupation ($r = -.32$, significant with $p < .01$).[1]

It is interesting and important to note that the bias towards the middle is very slight indeed, if at all present, when it comes to a measure of pupils' intelligence. Table IV (p. 150) gives the distribution and expected values.[2] Marks in Swedish, English, and Mathematics, and the mean of marks in the orientation subjects also provide some information on the nature of our sample. Teachers in the elementary schools of Sweden use a five point scale (one being the lowest and five the highest grade), and they are expected to distribute marks in

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here, saying that no information, with explicitly stated exceptions, could be hidden from the public by central and local government officials.

[1]A higher correlation may have been obtained by using logarithms of income. There are also factors depressing income, not showing in social status, for example illness, deceased fathers and divorces.

[2]WIT III was used, a group test of general intelligence, developed by Per Anders Westrin at the Department of Educational Psychology, University of Lund. It consists of four parts: Analogies, Contrasts, Number Combinations, and Puzzle (geometrical figures). The two first parts together yield a measure of verbal intelligence and the third and fourth non verbal intelligence. Split half reliability coefficients for 13 year olds is .93 on total score, .91 for verbal and .85 for non-verbal intelligence. Test-retest reliability is .84. For this and other information on the test see WIT III Manual, (Stockholm: Skandinaviska Testförlaget AB, 1969).

such a way that for all students in the whole country at the same level and taking the same courses an approximately normal distribution will occur. To help teachers approach this goal, standardized tests are given to the students' every third year. If this goal is achieved for the students in Söderkullaskolan, the distribution of marks seems to demonstrate that we are dealing with a sample that does not deviate appreciably from a random one. (Tables 5 and 6, pp. 150-151).

For our purposes, then, using grade seven in one school rather than a random sample of students seems to be preferable, since random sampling would introduce an uncontrolled variation in the treatment conditions of the learning situation. In mathematics the six classes were taught in two large groups, consisting of three classes each, by two teachers and one teaching assistant, the two large groups both having the same teachers and teaching assistant. This fact also, it seems, should contribute to ensuring that treatment conditions were roughly the same for all students involved, at least as far as would be possible in a non-experimental setting.

Summing up, then, the distribution of students with respect to fathers' occupation and parents' income, to their own intelligence, and to marks in school subjects seem to indicate that some tentative generalizations should be possible from our sample.

The Mathematics Materials Used

Having reported the characteristics of the learners, their physical environment, and social background, we now turn to the learning tasks involved.

As mentioned no attempt was made experimentally to control the learning situation; to the contrary, great care was taken not to change in any significant way the teaching and learning situation from what it would presumably have been were no data collected. Since the teaching assistent's job to a large extent consists in keeping track of students' progress through the course, it was natural to ask her to make notes of time taken by every student for each task; only minor changes of her usual routines in class were in fact necessary. Net time in each task could be reported since it is also the teacher assistant's duty to register students' absence.

Very fortunate for our purposes was the fact that the "new math" is now taught in grade seven in Swedish schools. Through the whole sequence selected for our study, the subject was elementary set theory. Therefore, it may be concluded with close to certainty that the students in taking the sequence were confronted with something of which they knew nothing beforehand, and that learning outside school was kept to a minimum. For the first part of the sequence, during which no homework was permitted, learning outside of school was virtually eliminated, since only in very exceptional cases would parents be able to tutor their children in set theory.

The material that was used, called IMU (short for the Swedish for individual mathematics instruction, "individuell matematik-undervisning") has been developed at the Department of Educational and Psychological Research, School of Education, Malmö, commissioned by the Central Board of Education in Sweden, and is being distributed by Hermods, a correspondence school and a producer of school materials.[1]

Most important for our purposes is the fact that each student is allowed to progress at his own individual speed. Teaching in the form of lecturing is kept at a minimum level and mainly concerned with instruction in the use of the materials; some group instruction and also individual instruction, when called for, are provided by the teachers. The material is partly differentiated with respect to difficulty, so that the most able students can choose more advanced and the less able easier work. The same content is taught to all students, however, although to differing degrees of depth. In addition the students are brought back go the "zero level" at different points of the program, where they learn exactly the same, basic mathematics. The upper department - grade levels seven through nine of the Swedish Comprehensive School - is covered by nine so called modules, with no grade level labels. Should some of the students finish all nine modules before the end of the comprehensive school, they will be provided with materials from the "gymnasium" (senior high school, roughly).

Each module consists of three components: the A component is common for all students, and thus of special interest to us, the B and C components are each divided up, into three versions of different

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[1] IMU Högstadiet Ett undervisningssystem i matematik utarbetat av Skolöverstyrelsen. (IMU, Upper Department. A Teaching System in Mathematics developed by the Central Board of Education; Malmö: Hermods 1970.) The system consists of a number of booklets, tapes for tape recorders, materials for laboratory work (not for the first sequence, the one used in the present study), and transparents for overhead projection.

levels of difficulty. The students discuss with their teachers which version should be chosen, after having taken a diagnostic test following the A and B component respectively. Figure 6 (p. 155) shows the student flow through a module.

There is no required mastery level in the strict sense; rather, the student is encouraged to reach the mastery level that corresponds to his qualifications. A minimum level exists, however, for each component which the student must reach before being allowed to go on. The amount of learning over and above this will determine the choice of B and C versions.

For each module there is one booklet for the A component, taken by all students, and one for each of the three B and three C versions. In the first module, the one used in this study, there are, however, only two B versions, B1 and B2-3. Each module also has additional review and correction materials, assigned as needed after each diagnostic test; when time was used on such materials it was added as appropriate to the time used in the A, B and/or C components. Some students were also sent back in the "program," in which cases this time also was added. After having proceeded through the whole module the student takes an evaluative test and goes on to the next module. Hence we have data on time to completion of the first module: time in component A for all the students, and time in each of the two B versions and the three C versions according to the students' assignments after the diagnostic tests.

During the time they studied the A component the students in our sample were not allowed to take their mathematics materials home with them, which contributes towards keeping outside learning for this component at a minimum. From the B component and on they were required or perhaps rather permitted, to do some homework. The time was reported by the students for the purpose of this study - usually they are not required to do that. Great care was taken in order to make the students understand that this report was not intended for the teachers and that whatever was reported should not be revealed. This procedure, it was believed, should minimize the risk that students used the report for the purpose of impressing their teachers. It is probably best, nevertheless, to look upon these reports of time used at home with some suspicion; there seems to exist no reliable source of information for time spent in learning outside the school. The teachers were not willing to extend the period during which homework was prohibited beyond the A component.

Analysis of Time to Completion Data and Derivation of Empirical Marginal Cost Curves.

In this section we will attempt to describe the time curves and add economic data in order to obtain MC curves. The exercise must be seen as an example of how this can be done. No comparisons are made with other teaching situations—once again no controlled experiment was intended—and the statistical analysis is only preliminary, mainly consisting of curve plotting.

Table 7 (p. 152) gives the statistics for the time distributions over the whole sequence. One student did not complete any of the steps, and another one completed only step A. Looking at the distribution of time for the A step, which gives the most reliable information as far as the sum total of time used is concerned, we can see that there is a difference of about one to four between the slowest and the fastest student. Not counting the top and bottom two per cent would still make the difference one of about one to three. This is somewhat lower than the difference of one to five or six that is usually reported. As can be seen, however, the difference tends to increase somewhat on the higher levels, in spite of the fact that those levels are differentiated with respect to difficulty. (The slow student tends to take the easy version.)

With the exception of B1, with only ten students, and time in school for C2, all distributions are positively skewed, which means that there is a long tail to the right of the distribution; a small number of students take a very long time for completion.

The cumulative frequency distributions of sum of time in school and at home for all steps are shown in Figures 7 through 12 (pp. 156-161). The positive skewness appears as a comparatively long and flat upper part of the curves. The distribution of C2, however, seems to be approximately normal. In Figure 13 (p. 162) this curve is plotted on a normal probability paper, and comes out as can be seen, rather close to a straight line. The most probable reason for C2 being normal and not skewed is that, by selection, the extreme students at both ends of the total sample had been sorted out; they are in C1 or C3.

It would be premature to conclude from this study, using only one curricular subject and just a few time distributions, that ability as measured by time to completion is positively skewed. Motivational factors may enter into this and will be taken up for discussion later on. There is, however, something in the nature of time as an ability scale that makes this positive skewness highly plausible. As will be seen, this has important economic implications. When using an arithmetic time scale three hours, for example, is exactly three times as much as one and six is three times two. But when used as a measure of human ability arithmetic equidistance in the units of the scale may be inappropriate. If, for example, one person could run the 100 yards dash in, say, ten seconds and another one in twenty, we would affirm that the first one is twice as fast. We normally, however, would not conclude that he is twice as able an athlete; the difference, intuitively, would seem to be much larger than that. Improving one's capacity from 20 to 19 seconds would for most people be quite easy, whereas improving the world record on the 100 yards dash by even one tenth of a second is an extreme and rare achievement indeed.

A difference of one arithmetic time unit at the upper end of the scale, therefore, seems to carry less weight than a difference of one time unit at the lower end of the scale. In other words when going from the slowest pupils, taking comparatively long times, to ever shorter time, each arithmetic reduction in time entails a progressively greater rate of decrease in physical time units. It is of some interest, therefore, to examine what happens to the skewness of the time distributions when those distributions are scaled logarithmically.

Human ability is often thought of as being normally distributed. Some statistical-theoretical foundation for this can perhaps be found in the "law of the large numbers." That measures of ability, such as IQ and aptitude scales, often do come out very nicely as approximate normal curves, may on the other hand be a construct, since items not supporting normalcy are sorted out when tests are developed. If we assume, nevertheless, that ability is normally distributed when "correctly" measured, whatever that means, then ability as measured by time to completion would be normally distributed in the logarithms, if indeed the time scale as an ability measure is logarithmic.

To explore this possibility the cumulative time to completion distributions of two steps (A and B2-3 where most of the students were included) were plotted on a semi-log paper (Figure 14, p. 163) and logarithms of the same distribution were plotted on a normal probability paper (Figure 15, p. 164). As can be seen, those log distributions do in fact (by inspection) come close to normalcy in the logarithms. The plots in Figure 11 (p. 160) have, approximately, the shape of normal and non-skewed, cumulative distributions, moreover, the same distributions on probability paper come out as almost straight lines, showing the same thing.

The importance of the cumulative frequency distribution becomes clear when we consider its relation to an MC-curve. Assume a case where costs are perfectly proportional to time in learning. Then the MC function is simply the inverse of the cumulative time frequency distribution with the time scale converted to a cost scale by the appropriate weight. The more complicated cases of MC curves will be transformations of this inverse. Introducing costs of supplies, equal for all students per step (S), would for example simply mean adding a constant to the curve.

Cost curves, as plotted in Figure 16 through 21 (pp. 165-170), were computed from the empirical data. Three different measures of student time were used: 1) student time valued at zero, 2) student time valued at 5.15 Swedish kronor (1969) and 3) student time valued at 3.00 Swedish kronor per hour. In the second case wage per hour before tax in the age group 14-19 was used, and in the third case the value of student time was assumed to develop linearly, from age at school entry when the value was assumed to be zero to the midpoint of earnings in the age group 14-19.[1] This procedure is less satisfactory than the shadow pricing of the programming model; it treats all students' time as of equal value whereas the shadow price derived in a programming model would be inversely related to the time the individual takes. Shadow prices thus tend to flatten the curve as compared to the use of mean values of foregone earnings.

Other costs were derived from the (ex post) budget of the Malmö school system. Since the breakdown of this budget is rather detailed and the principle is to credit costs to "real" rather than merely "accountant" entries it is believed that figures are fairly accurate. The method is not flawless it must be admitted; school space costs are especially difficult to estimate. The costs can be itemized as follows:

Teachers and other school personnel

Free school materials (books, stationery, and so on)

Equipment

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[1]Source: Svenska folkets inkomster. Betänkande avgivet av låginkomstutredningen, SOU 1970:34 (The Swedish Income Distribution, Report from the Low Income Committee, Publications from Government Committees, 1970, No. 34), (Stockholm: Inrikesdepartementet, 1970), p. 101.

School Space

Administration

School lunches

School health

Busing

AV materials

Experts (e.g., consultants, educational psychologists, curators)

"Other"

The per student cost was calculated to 3.90 Swedish kronor per period in school and adjusted downwards by subtracting costs of books and supplies (to be replaced by the costs of the IMU material) and the difference between the cost of using a teacher assistant and the cost of a third teacher. The per student cost exclusive of materials and student time thus arrived at was 3.35 Swedish kronor per period in school. As can be seen, even the lower of the two estimates of student time (excluding zero) is almost as high. The IMU material has a market price from which the cost per step was calculated to be 5.00 Swedish kronor for each student.

It can be seen from the graphs that the skewness of the time distribution, as may be expected, has resulted in a sharp rise of the MC curve (approximately for the ten per cent slowest students in this sample). Quite aside from the obvious "social" arguments that can be mobilized for finding ways and means of reducing time differentials among students, there apparently are also purely economic reasons for wanting to do so. Depending upon the costs of reducing the time differentials, this may or may not, however, lower the MC curve. If the MC-curve is not lowered when time differentials are diminished, then in turn any eventual benefit change has to be taken into account when making decisions concerning measures aimed at reducing the time to completion for the slowest learners.

Relationships among Variables

In order to explore somewhat how the variables in the sample relate to each other, a correlation analysis was used. Table 3 (p. 149) shows the zero order coefficients for time to completion of the first step, IMU A, with final achievement on the whole module; ability in math;[1] verbal, non-verbal and total intelligence;[2] father's or guardian's occupation;[3] family income; marks in selected school subjects in grade 6 (spring semester) and grade 7 (fall semester). The table also displays interrelations among the "independent" variables.

As can be seen we have the usual correlation in the neighborhood of .4 to .7 among measures of school achievement. Remarkable is the absence of a significant correlation between either family income or father's occupation and any of the other measures (with two exceptions where r is quite low and not significant, with $p < .01$). There is a correlation between the two family background variables but smaller than perhaps might be expected.

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[1]as measured by an instrument developed for the evaluation of the IMU project by the Department of Educational and Psychological Research, School of Education, Malmö.

[2]as measured by WIT III.

[3]as measured by the scale described on pp. 120-122.

Time to completion of IMU A correlates moderately and in the expected direction ($r < 0$) with intelligence and with ability and achievement measures—which is what Kim also found. Again noteworthy is the absence of correlation with social background variables. When time to completion of steps B2-3, C1, C2, and C3 was related to the other measures it turned out, however, that the correlations mostly were near zero.[1] For all versions of the C-step r was never significantly different from zero ($p < .05$) whereas time to completion of B2-3 correlates in the neighborhood of -.2 with most of the other variables, as can be seen from the correlation matrix.

The structure of the correlation matrix raises several questions. Why is it that ability as measured by time to completion of a learning task does not correlate more highly with other ability measures? And why does this already moderate correlation go down as new steps are added. This decline and finally disappearance of a significant correlation may have several possible explanations:

1) measurement errors
2) when students choose between the versions of B and C they are thereby divided up in ability groups to some extent, thus becoming more homogeneous within the chosen groups
3) when homework is introduced, r becomes lower (homework, as we recall, being permitted from step B and on)

[1] B1 was not included in the correlation study since only 10 students chose this version.

4) explanation offered by the economic theory of utility maximization in time allocation.

One obvious source of measurement error is the possibility, not to say probability that some students over-perform relative to the criterion; in other words, we were not able to measure exactly when they reached criterion and no more. From the correlation between time to completion of A with the final test ($r = -.46$, see Table 3, p. 149) it seems plausible that the slow learner show the least overperformance. That homework could reduce r may entail (partly at least) an aggravated measurement error, since students' reports on homework probably are quite unreliable. Only 117 out of 171 students reported any homework at all, but this is not conspicuously low, since the policy is to reduce homework to a minimum in Swedish elementary schools. One cannot be sure, however, that none of the other 59 students learned anything at home. Also it is very probable that reports in some cases of time used at home in studying mathematics were guesswork rather than accurate knowledge.

Time at home in both B and C has a low but significant positive correlation with the result on the final test, which perhaps supports the "overperformance" explanation. Time spent at home in C, but not in B, is also (positively and significantly) correlated with marks.

With all these explanations, it is still true, however, that the moderate measure of r for the first, and more controlled, step is in very good agreement with what others have found. Kim's study, for example, was a more carefully controlled experiment in educational measurement (which this study was not intended to be), yet his correlations are only slightly higher than those reported here for step A. It would be satisfactory to get a more powerful explanation than the above 1) to 3), and we will now turn to economic theory and see if it can provide one.

In the theory of timing in education, as laid out especially in Chapters II and VI of this study, we have treated learning in schools as a production problem using either a cost-minimization or product-maximization approach. Student time has been treated as a societal resource, and we have completely ignored the fact that students may want to use their time in a different way. Following Becker's time allocation theory[1] we may instead treat the students as utility maximizers, restricted by a time budget. This was the approach of Adelman and Parti when developing a theory of student motivation upon which they based a study of student verbal achievement.[2] They viewed the students' use of time in school as directed towards maximizing a stream of present and future rewards, among which may be parental and teacher approval, expected future earnings, peer acceptance, and leisure. For the following analysis we also add out-of-school time and rewards associated with activities for which such time is an input.

Students are assumed to maximize utility functions:

(1) Max U (t) subject to

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[1]Becker, "A Theory of the Allocation of Time."

[2]Irma Adelman and Michael Parti, "The Determinants of Student Achievement: A Simultaneous Equation Approach" (unpublished manuscript, Chicago: Northwestern University, 1971), pp. 2-8.

$$(2) \sum_{i=1}^{n} t_i - \hat{t} = 0$$

where $t = t_1 \cdots t_n$ is time spent in activities 1 to n, and $\hat{t}$ is available time. We assume for simplicity that physical inputs in those activities are either freely available or not necessary. The student's utility, in other words, depends only on how he spends his time. If free to divide his time between school and non-school activities the student would, therefore, choose the time input combination that gives him maximum satisfaction in very much the same way as a consumer maximizes utility by allocating his given budget on different market goods and services.

Let $t_s = t_1 \cdots t_m$ be school activities, and $t_h = t_{m+1} \cdots t_n$ be non school activities. Then a necessary condition for utility maximization would be

$$(3) \quad \frac{\partial U}{\partial t_i} - \lambda = 0 \qquad i = 1 \cdots n$$

where λ is a Lagrange multiplyer. It follows that

$\lambda = \dfrac{\partial U}{\partial t_i} = \dfrac{\partial U}{\partial t_j}$ all i and j (independently of whether they belong to the t_s or the t_h vector). In other words, marginal utility should be the same in all uses of time, whether in-school or out-of-school time.

Usually, however, the student is <u>not</u> free thus to divide his time. He is restricted to use a certain specified amount of time in school, which introduces two new constraints instead of constraint 2) above.

$$(4) \quad \sum_{i=1}^{m} t_{s_i} - \hat{t}_s = 0$$

leaving the rest of total time for non-school activities, which may be expressed:

$$(5) \sum_{j=m+1}^{n} t_{h_j} - \hat{t}_h = 0$$

where t_h is time available for out of school activities. We have now the new necessary conditions, introducing Lagrange multiplyers λ and μ:

$$(6) \quad \frac{\partial U}{\partial t_{s_i}} - \lambda = 0 \qquad i = 1 \cdots m$$

$$(7) \quad \frac{\partial U}{\partial t_{h_j}} - \mu = 0 \qquad j=m+1 \cdots n$$

When these constraints are introduced, marginal utility may be (probably is), different for time used as inputs in school activities and in non-school activities. We can assert, however, that

$\dfrac{\partial U}{\partial t_{s_i}} = \dfrac{\partial U}{\partial t_{s_j}}$ for all i and j, in other words marginal utility will be equal in all uses of <u>school time</u> (and equivalently for marginal utilities of non-school time as inputs in non school activities).

School activities do, however, as was pointed out by Adelman and Parti[1] include also "non-approved" activities. Daydreaming, looking

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[1] *Ibid.*, pp. 2-4.

out of the window, playing tricks on the teacher, and similar non-academic uses of <u>school time</u> may not <u>per se</u> seem to be very rewarding as we see it, but the proportion of time used by a student in non-approved activities must be seen in relation to the alternatives open to him, which may <u>for him</u> not seem to be very rewarding either.

From this theory we would predict, or at least find very plausible, that students do spend some time in school in pursuit of non-academic goals and that this time proportion will vary among individuals. The empirical time measure in the present study is, therefore, a sum of time inputs used in academic pursuits and time inputs used in non-approved activities. Suppose now that the "true" relation between time to complete a learning task (excluding, that is, all time spent in school in non academic activities) and intelligence is perfect and linear, that is, the "true r" is unity. If proportion of time spent in non academic activities is unrelated to intelligence, we would expect the measured r to be lower than the "true r" because of the introduced spread around the regression line. More likely, perhaps, is the possibility that highly intelligent students, as result of the prevalent reward system in schools, would tend to devote a higher proportion of their time to academic activities. Then non-linearities would depress the measured r, which therefore will tend to go down also in this case. If the proportion of time spent in learning ("true time") as compared to that spent in non-approved activities will tend to be larger for high ability students, we may here have an explanation also of the skewness of the time curves, hence of the sharp upward turn of marginal cost curves. Measured time, in other words, would tend to depart from time actually used as input in learning in such a way that it will tend to overestimate time spent in learning more and more as we move from fast and able students towards slow low ability students.

If we now introduce homework, it can be seen from equations 6 and 7 that measured r in all probability will departure even more from a "true" relation, the reason being that the time input to be used in homework has to be taken out of a different "time budget" altogether. Let $\frac{\partial U}{\partial t_s}_{\text{math}}$ be the MU of time from "study math in school" and $\frac{\partial U}{\partial t_h}_{\text{math}}$ MU of time spent studying math at home. In general we will have

$$\frac{\partial U}{\partial t_s}_{\text{math}} \neq \frac{\partial U}{\partial t_h}_{\text{math}}$$

(with equality only by coincidence, in which case (4) would no longer constrain the student). It can be seen that the student would add time in studying math at home only as long as the MU of doing so is less than the MU of the alternatives open to him. In "equilibrium" one would expect

$$\frac{\partial U}{\partial t_s}_{\text{math}} < \frac{\partial U}{\partial t_h}_{\text{math}}$$

The alternatives open to the student, when out of school (teasing sisters and brothers, playing softball, eating ice cream, and so on) are much more powerful competitors for the use of his time than alternatives to academic work within school (daydreaming, scribbling in textbooks, and so forth). Again his decision as to what to do, may or may not relate to his intelligence, but in either case the result of introducing homework probably would be to further depress the r. This is so since time in studying reported by the student would include, in the same way as time spent in school, some time used in non-academic activities. Because of the much more powerful "distractors" at home, this proportion would probably be higher than in school for most students.

At this point one might well wonder whether this complicated economic apparatus is at all necessary or useful in explaining students' use of time, in school or at home. Why not simply say that students are more or less motivated, for example, to study mathematics. This is, however, to overlook a very significant part of the economic argument. Besides, there seems to be some research evidence to the effect that student interest is only a weak determinant of learning.[1] The economic point is that the amount of time a student is willing to spend on mathematics is dependent not only on his subjective evaluation of the rewards of doing so, but also on his simultaneous evaluation of the alternatives open to him. According to this theory, a student may well answer a questionnaire in such a way as to indicate a very high interest in mathematics; yet he may not be willing to spend very much time studying it, since he has an even higher interest in getting peer approval by trying to bring the teacher close to a nervous breakdown (or in less obvious and therefore less noticed ways).

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[1]Carroll, "Problems of Measurement," in Mastery Learning, ed. by Block, p. 32.

Summing up then, it would seem highly likely that a hypothesized relation between time in learning as an ability measure and other ability measures would be clouded as long as we cannot measure the exact amount of time used as an input in learning. As can be seen this is in accord with Carroll's treatment of time as "time needed" and "time spent" in learning, which in the above analysis would be equivalent to time actually used as an input in learning and time spent in school, regardless of its use. Hopefully, the economic analysis could also contribute by throwing some light on what would be involved in trying to make time spent in school more nearly like time needed "under ideal conditions,"—ideal, that is, in maximizing the efficiency of in-school time with respect to the production of learning. The above mentioned study by Adelman and Parti was addressed to exactly this problem.

Predictability of Time to Completion Patterns

For the purpose of educational decision making there are two problems involved in predicting time to completion of learning tasks:

1) We wish to know whether observed data will be approximately repeated under similar conditions, and
2) We wish to know, in addition to the above, approximately where on the MC curves a particular student (or perhaps rather a particular type of students) will be located.

Knowing that time-to-completion patterns are largely repetitive for successive age cohorts would be a sufficient basis for many educational decisions. Economic evaluation of teaching methods, where we are faced with an "all-or-nothing" problem, is an example; in other words when only one of the evaluated methods may be chosen, it will not be necessary to predict individual students' learning rate. Determining student flow through a system would also be an example, if a "proper" self selection by the students could be assumed. If we want to target certain measures to a particular group of students within the distribution, say the 10 per cent slowest, without being especially interested in who those students are, it would also be enough to know that a particular time curve could be expected approximately to repeat itself.

Although the characteristics of the student body in the present investigation are such as to make it very plausible that the time patterns obtained are not merely coincidental, we of course have no basis for concluding anything about the stability of the time pattern from the present study alone. The negative correlation between time to completion and ability (as well as achievement measures) has been shown in several studies, however, and should be regarded as established, together with the fact that this correlation is rather moderate. Repeated studies are necessary in order to investigate further the shape of the time distributions, to see, for example, if the skewness is a characteristic typical of such distributions.

Turning now to the second prediction problem, queries involving allocation of "marginal" students within a system or between systems would require an approximate knowledge of who those students are. From the last section of this chapter and from other research it seems that the basis for such predictions is not very good. To illustrate the point a plot was made of the cumulative frequency distribution of time in IMU A and of the means for different intelligence groups (Figure 22, p. 171). We can now compare what would have been the result of "prediction" of the time to completion distribution by perfect knowledge and of using intelligence as a prediction instrument. As can be seen, the agreement is not overwhelming.

Summary

In this chapter some empirical findings were presented. A sequence of learning tasks in mathematics, taken by students in the upper department (grade 7) of a Swedish elementary school, was investigated with respect to time to completion and these measures were used for constructing MC cost curves for different steps in the sequence. The skewed time distribution was shown to result in sharply rising MC curves for the slowest ten per cent of the students.

Time in learning was related to intelligence, ability and achievement measures and time in step A showed a moderate, negative correlation with these, whereas the correlation for later steps got closer to zero and finally disappeared. A tentative explanation was offered by applying an Adelman-Parti specification in time allocation theory. Students were said to maximize utility by allocating some time to studying mathematics and some to non approved activities, measured time in school being a sum of time in those different uses.

Predictability was said to be low as far as individual time to completion goes. Not all decision making, however, requires such predictability—only a reasonable assurance of approximate stability in time patterns from year to year for successive age cohorts would be necessary. Research has yet to establish the existence of such stability.

APPENDIX

TABLES AND ILLUSTRATIONS TO CHAPTER VII

TABLE 3

INTERCORRELATIONS AMONG VARIABLES

(zero order correlation matrix)

		1	2	3	4	5	6	7
1. Time to completion, IMU A			.28	(.07)	(.03)	-.46	-.53	-.29
	N=		159	117	116	169	170	162
2. Time to completion, IMU B2-3				.26	.21	-.14*	-.13*	(-.02)
	N=			117	116	159	159	159
3. IMU B Time at home					.52	(.11)	(.04)	(-.01)
	N=				116	117	117	112
4. IMU C Time at home						.21*	(.13)	(.10)
	N=					116	116	111
5. Final test, first modul							.68	.61
	N=						169	162
6. Ability in math, IMU								.54
	N=							162
7. Intelligence verbal								
	N=							
8. Intelligence non-verbal								
	N=							
9. Intelligence total								
	N=							
10. Father's occupation								
	N=							
11. Family income								
	N=							
Marks Grade Level 6								
12. Swedish								
	N=							
13. Mathematics	N=							
14. English								
	N=							
15. Average of orientation subjects								
	N=							
Marks Grade Level 7								
16. Swedish								
	N=							
17. Mathematics, general course								
	N=							
18. Mathematics, special course								
	N=							
19. English, general course								
	N=							
20. English, special course								
	N=							
21. Average of orientation subjects								
	N=							

( ) Coefficients within parenthesis are not significantly different from zero with $p < .05$.

*Starred coefficients significantly different from zero with $p < .05$.
All other coefficients are significantly different from zero with $p < .01$.

8	9	10	11	12	13	14	15	16	17	18	19	20	21
-.43 162	-.40 162	(.11) 164	(.03) 170	-.37 170	-.50 170	-.34 170	-.36 170	-.35 170	-.44 45	-.37 125	(-.03) 26	-.39 143	-.46 170
(.03) 154	(.02) 154	(-.06) 155	(.00) 159	-.15 159	-.18 159	(-.06) 159	-.14* 159	-.16* 159	-.29* 35	(-.07) 124	-.40* 20	(-.03) 139	(-.19) 159
(.02) 112	(.03) 112	(.04) 114	(-.04) 117	(-.02) 117	(-.02) 117	(.09) 117	(.00) 117	(.12) 117	(.20) 34	(.04) 83	(.15) 21	(.02) 96	(.05) 117
(.13) 111	.18* 111	(.11) 112	(-.06) 116	(.10) 116	.19* 116	.27 116	(.14) 116	.29 116	(.18) 36	.25* 80	.58 19	.18* 97	.25 116
.58 162	.67 162	(-.06) 164	(-.06) 169	.60 169	.77 169	.60 169	.65 169	.59 169	.72 44	.91 125	(.26) 27	.60 143	.70 169
.58 162	.64 162	(-.06) 165	(.09) 171	.60 171	.68 171	.53 171	.54 171	.52 171	.39 46	.55 125	(.26) 27	.50 143	.57 169
.43 162	.83 162	(-.07) 157	(.07) 162	.64 162	.52 162	.62 162	.56 162	.58 162	(.25) 40	.47 122	(.30) 24	.58 138	.53 162
	.82 162	(-.10) 157	(.09) 162	.38 162	.50 162	.41 162	.37 162	.36 162	.41 40	.48* 122	(.20) 24	.41 138	.47 162
		(-.09) 157	(.10) 162	.60 162	.58 162	.61 162	.56 162	.54 162	.46 40	.54 122	(.25) 24	.61 138	.59 162
			.32 165	(-.06) 165	-.14* 165	(-.10) 165	(-.10) 165	(-.08) 165	(-.12) 44	(-.03) 121	(-.04) 26	(-.05) 139	(-.12) 165
				(.11) 171	(.03) 171	.16* 171	(.06) 171	(.06) 171	(.03) 46	(.10) 125	(-.04) 27	(.06) 143	(.05) 171
					.59 171	.74 171	.73 171	.75 171	.48 46	.47 125	(.24) 27	.75 143	.66 171
						.60 171	.70 171	.54 171	.55 46	.78 125	(.16) 27	.53 143	.67 171
							.64 171	.65 171	.45 46	.42 125	.68 27	.75 143	.60 171
								.65 171	.43 46	.58 125	(-.05) 27	.68 143	.74 171
									.42 46	.45 125	.34* 27	.70 143	.71 171
										0	(.28) 24	.47* 21	.63 46
											(.50) 3	.44 122	.61 125
												0	.39* 27
													.67 143

TABLE 4

PERCENTAGE OF STUDENTS IN EACH INTELLIGENCE GROUP

N=162

Stanine points	1	2	3	4	5	6	7	8	9	Total	Mean stanine
Verbal Part											
Sample	.6	4.9	13.6	17.9	12.3	22.8	17.9	6.8	3.1	100	5.3
Expected	4.0	7.0	12.0	17.0	20.0	17.0	12.0	7.0	4.0	100	5.0
Non Verbal Part											
Sample	6.2	7.4	15.4	21.0	16.7	17.9	12.3	2.5	0.6	100	4.5
Expected	4.0	7.0	12.0	17.0	20.0	17.0	12.0	7.0	4.0	100	5.0
Total											
Sample	3.1	6.8	11.1	18.5	22.2	21.6	11.7	3.7	1.2	100	4.9
Expected	4.0	7.0	12.0	17.0	22.0	17.0	12.0	7.0	4.0	100	5.0

Source of expected values: WIT III Manual

TABLE 5

PERCENTAGE DISTRIBUTION OF MARKS AT THE END OF THE SPRING SEMESTER, GRADE LEVEL 6

N=171

Marks	1	2	3	4	5	Total	Mean
Swedish	2.9	20.5	42.7	28.7	5.3	100	3.1
Mathematics	5.8	23.4	45.0	20.5	5.3	100	3.0
English	1.2	21.6	39.2	31.6	6.4	100	3.2
Expected	7.0	24.0	38.0	24.0	7.0	100	3.0

Source for expected values: Läroplan för grundskolan (Curriculum for Comprehensive School), p. 90.

TABLE 6

PERCENTAGE DISTRIBUTION OF MARKS AT THE END OF THE FALL SEMESTER, GRADE 7.

N=171

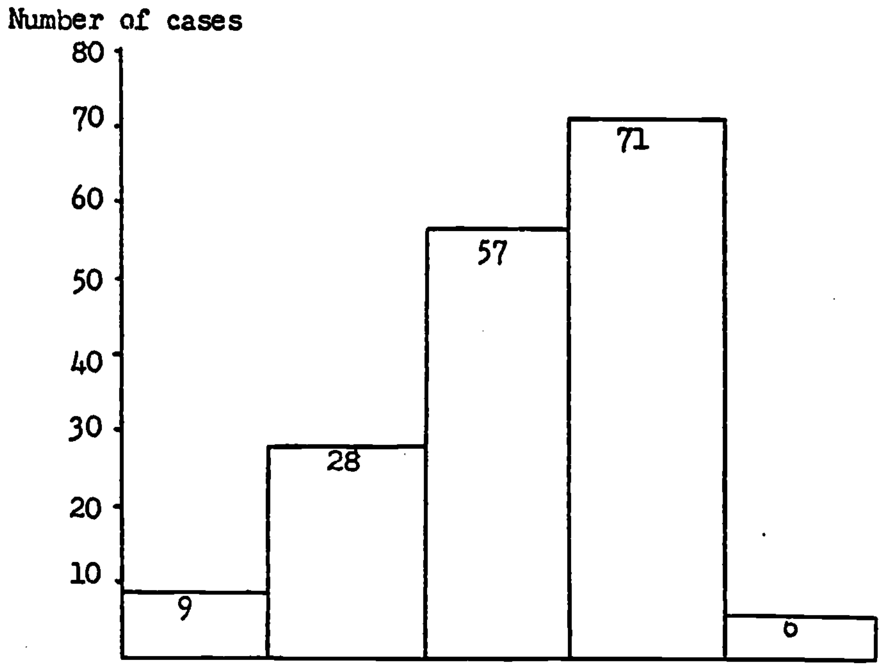
	1	2	3	4	5	Total	Mean
Swedish	1.8	26.9	45.6	23.4	2.3	100	3.0
Math G (N=46)	28.3	23.9	34.8	10.9	2.2	100	2.3
Math S (N=125)	7.2	24.0	37.6	25.6	5.6	100	3.0
English G (N=27)	7.4	25.9	59.3	7.4	0	100	2.7
English S (N=143)	7.0	21.0	42.7	23.8	5.6	100	3.0
Expected	7.0	24.0	38.0	24.0	7.0	100	3.0

In mathematics and in English students study either a "general" course with a more practical orientation, G in the table, or a "special" course, which is more theoretical, S in the table. (With the introduction of the IMU material, see pp. 120-124, this division has in math become a mere formality.) Source of expected values: Läroplan för grundskolan (Curriculum for the Comprehensive School), p. 90.

TABLE 7

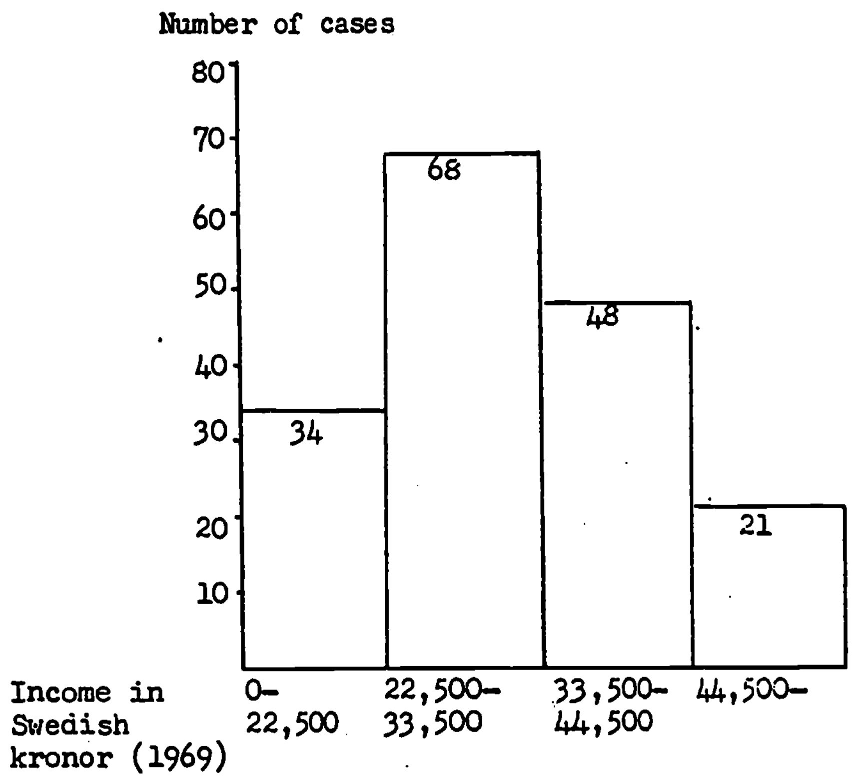
DISTRIBUTION OF TIME FOR COMPLETION OF LEARNING TASKS IN MATHEMATICS. TIME IN MINUTES.

	N	Minimum	Mean	Maximum	Standard Deviation	Skewness
Time in school						
A	170	240	532	1040	141	.7
B1	10	320	624	880	168	-.3
B2-3	159	240	629	1440	173	.9
C1	16	320	672	2000	449	1.8
C2	47	160	374	640	118	.0
C3	106	240	434	960	135	1.0
Time at home						
B	117	8	104	535	99	2.0
C	116	5	106	340	79	.9
Sum of time						
B1	10	328	629	915	173	-.8
B2-3	159	375	701	1515	169	1.1
C1	16	320	712	2075	461	1.9
C2	47	442	722	109	109	.1
C3	106	240	514	960	140	.7



Social Group	I	IIA	IIB	III	Missing
Per cent	5.3	16.4	33.3	41.5	3.5
Adjusted Per Cent	5.5	17.0	34.5	43.0	—
Expected Per Cent	11.5	16.3	26.7	45.5	—

Figure 4.—Number of pupils with fathers in occupation groups I to III. Expected per cent from Swedner and Edstrand "Skolsegregation i Malmö," p. 26, showing the distribution for their whole sample of all elementary school children in Malmö 1968 (grades 1 through 9).



Per cent	20	40	28	12
Expected Per cent	27	30	21.5	21.5

Figure 5.—Family income distribution. Expected values computed from a sample by "Låginkomst-utredningen" (Low Income Committee), "Svenska folkets inkomster," SOU 1970:34 (The Swedish Income Distribution, Report from the Low Income Committee, Publication from Government Committees, 1970 No. 34) p. 221. Values in 1969 Swedish kronor.

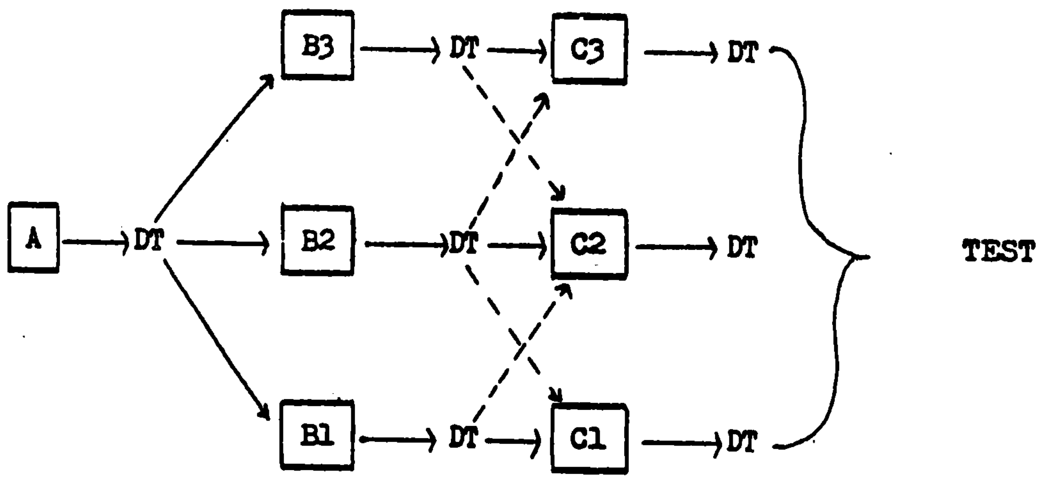


Figure 6.—Student flow through a module of the IMU material. DT = Diagnostic test. The whole module is completed by taking an evaluative test.

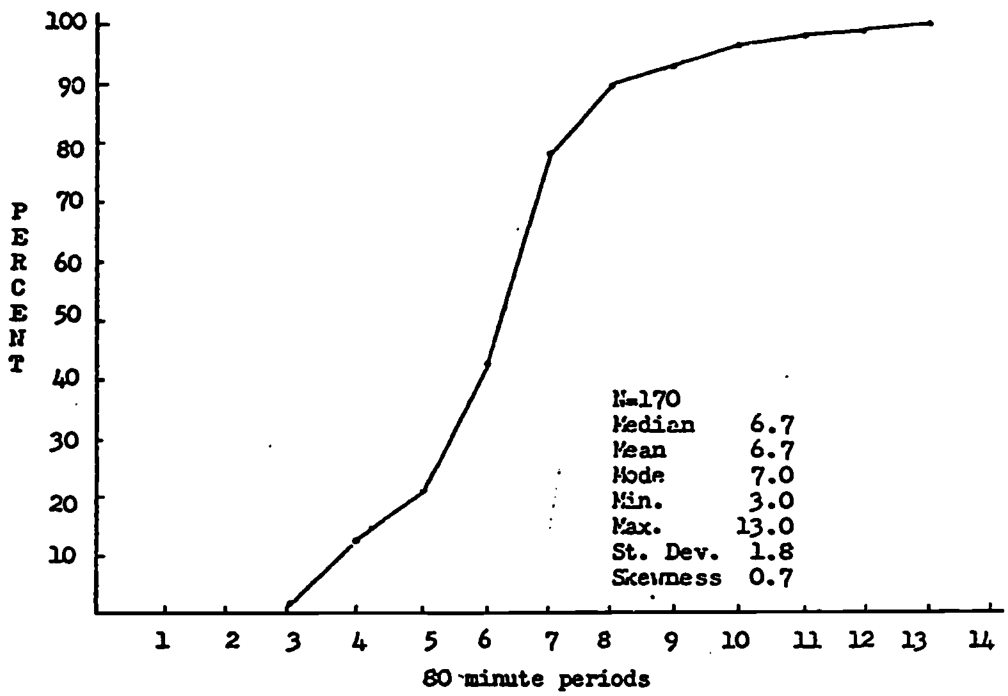


Figure 7.—Cumulative frequency distribution of time to completion, IMU A.

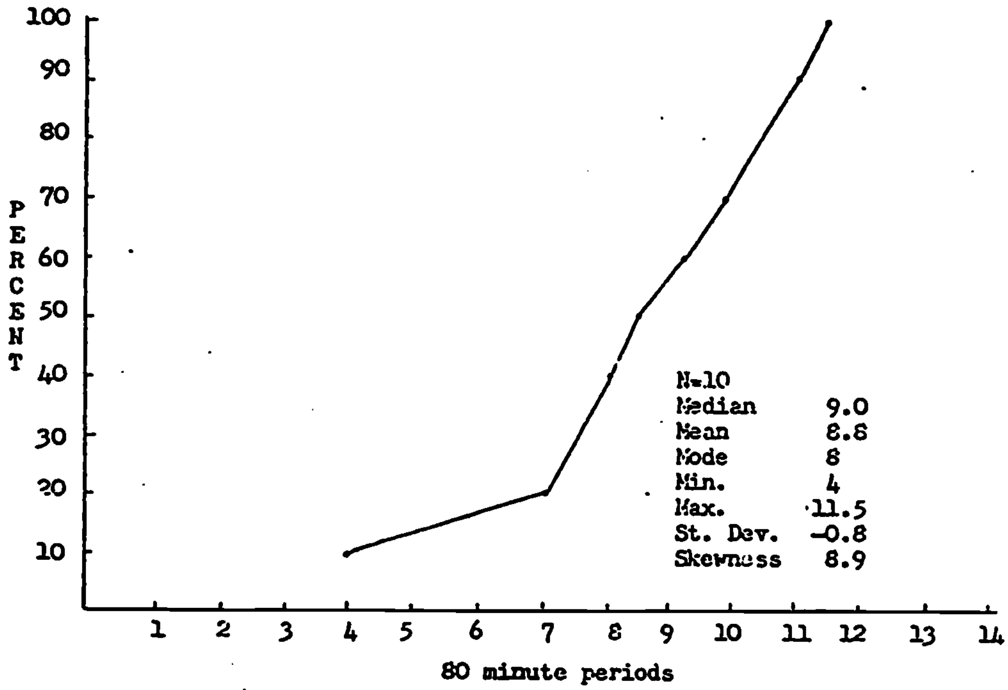


Figure 8.—Cumulative frequency distribution of time to completion, IMU B1.

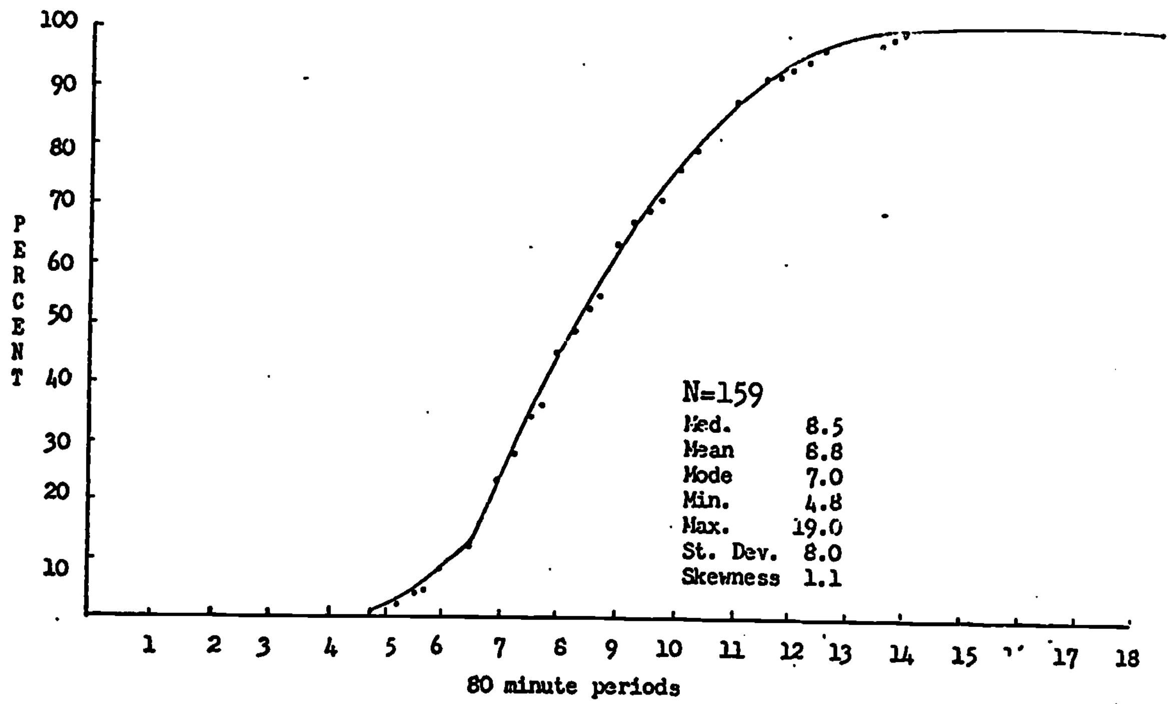


Figure 9.—Cumulative frequency distribution of time to completion, IMU B2-3.

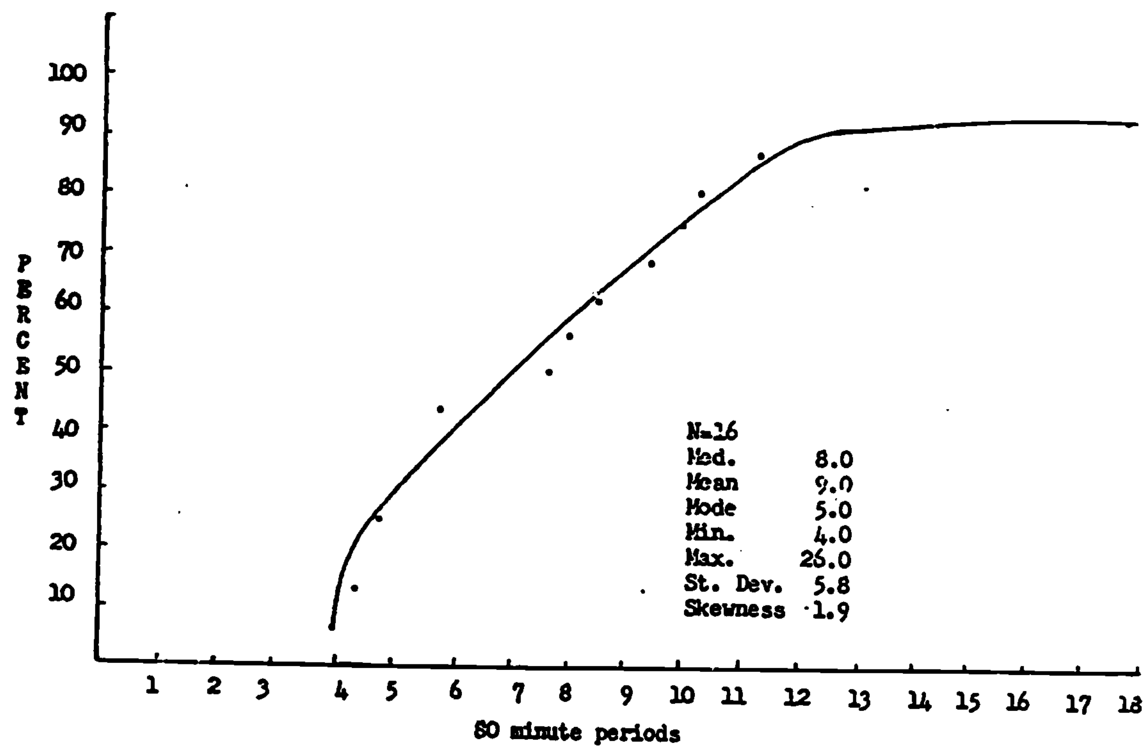


Figure 10.—Cumulative frequency distribution of time to completion, IMU C1.

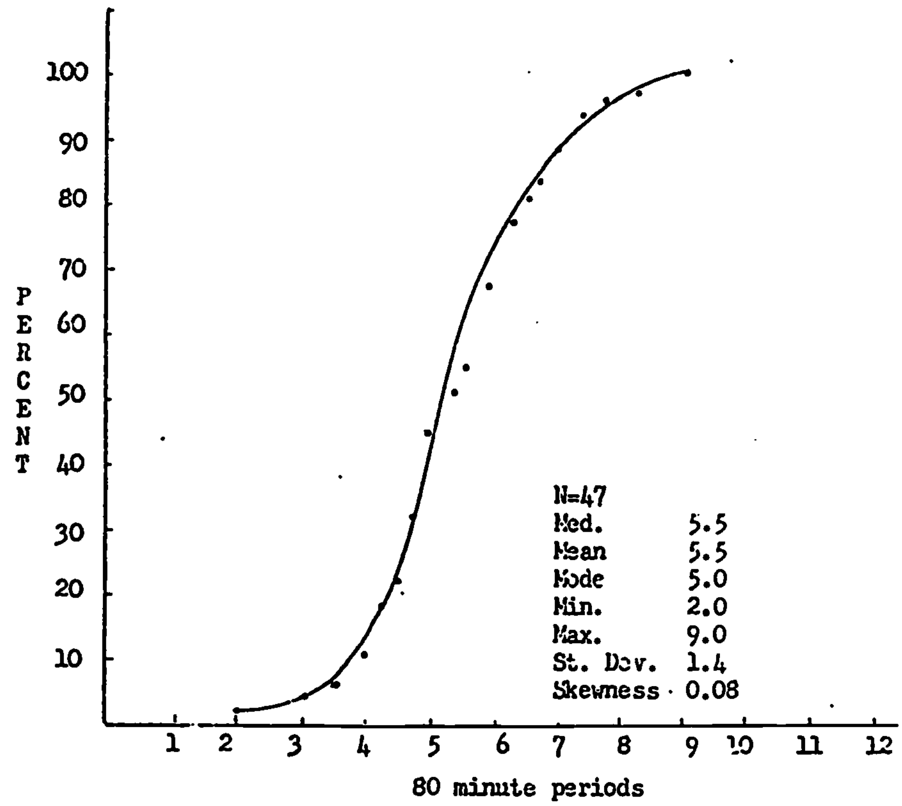


Figure 11.—Cumulative frequency distribution of time to completion, IMU C2.

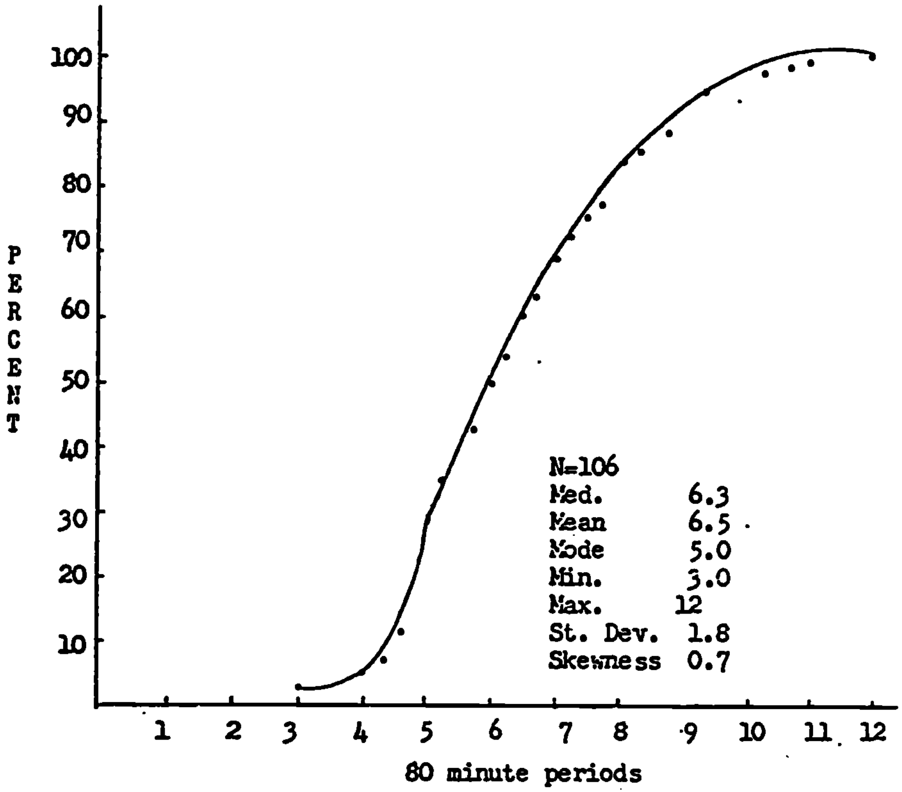


Figure 12.—Cumulative frequency distribution of time to completion, IMU C3.

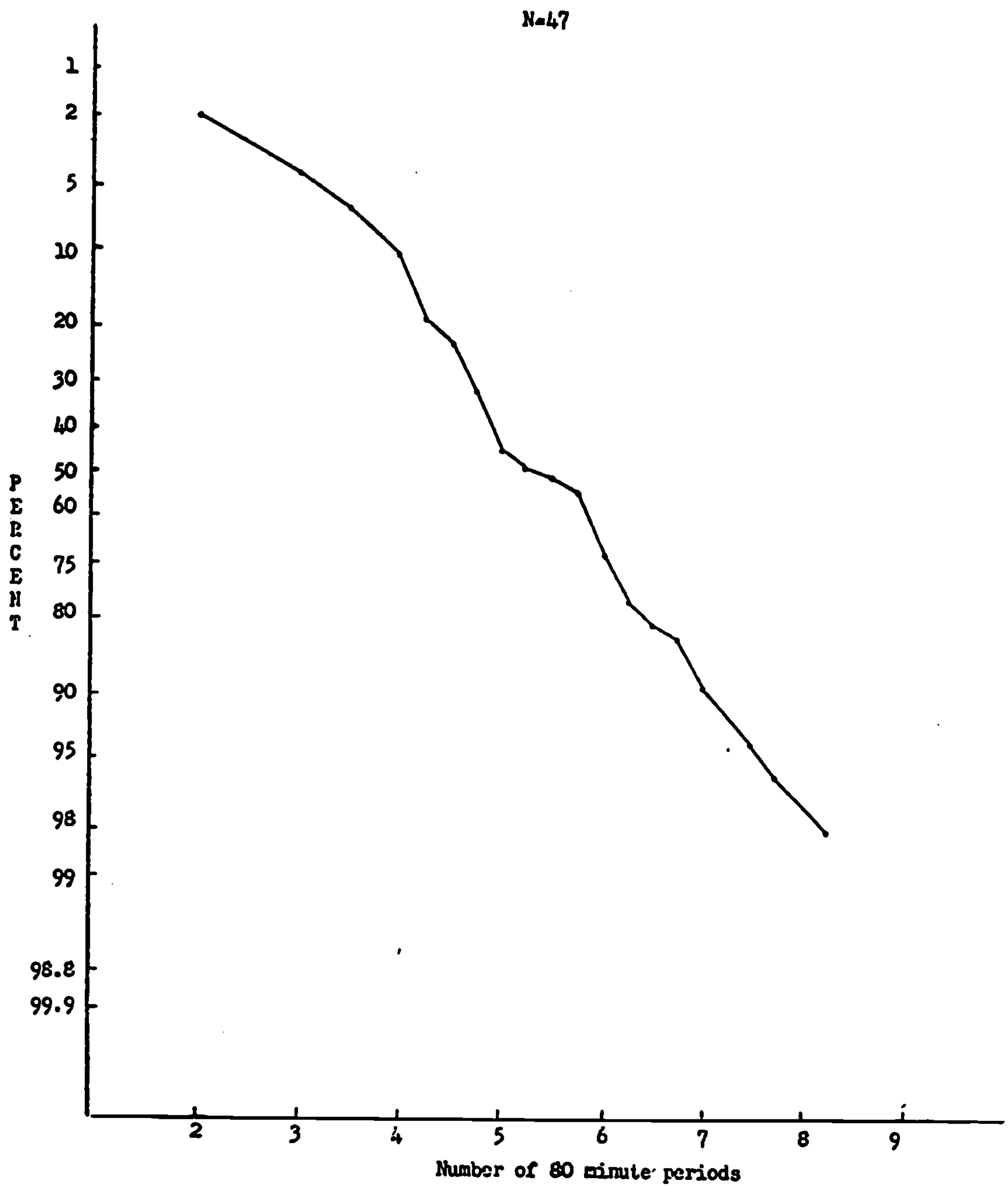


Figure 13.—Cumulative frequency distribution of time to completion, IHU C2, on a normal probability scale.

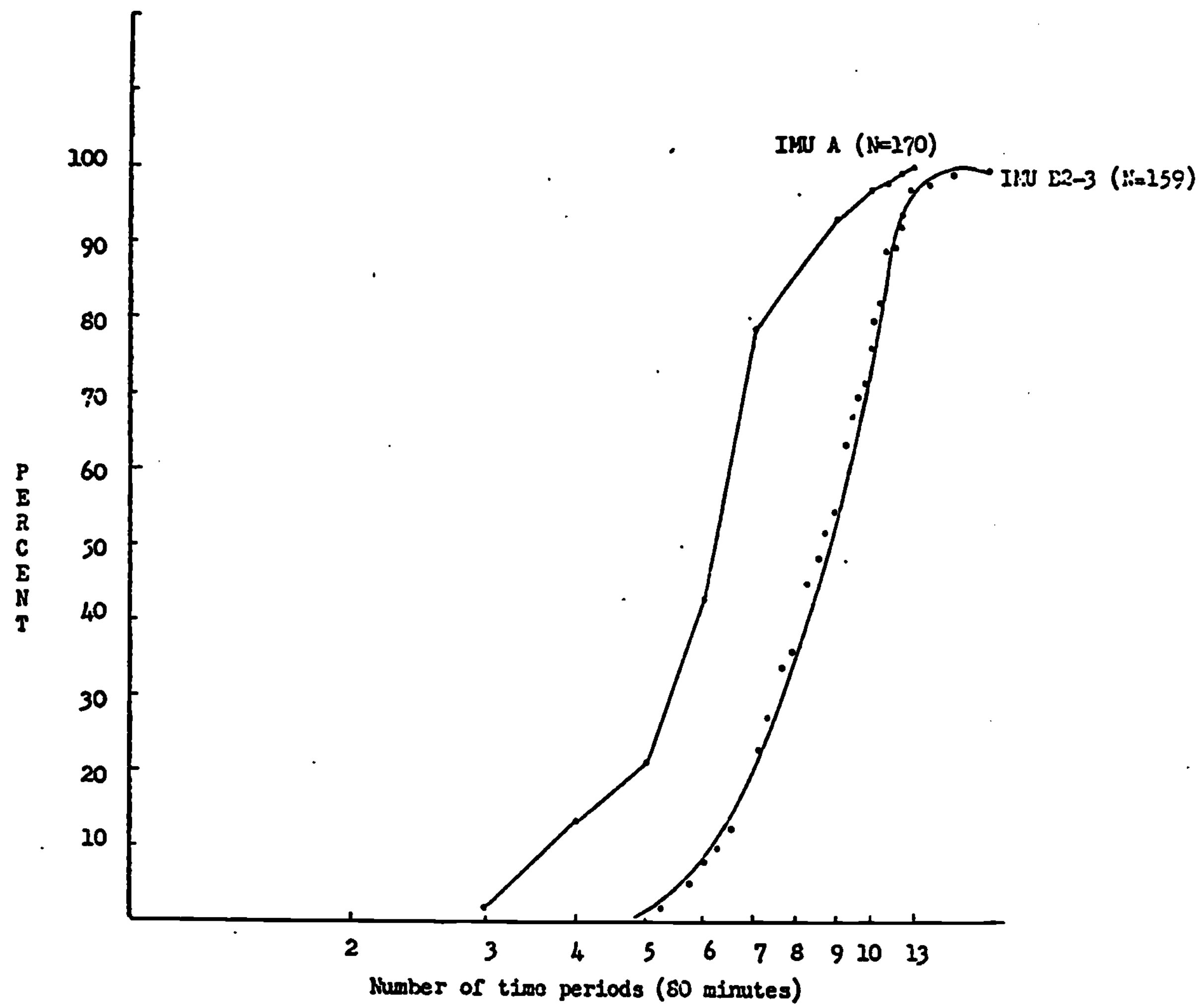


Figure 14.—Cumulative frequency distribution of time to completion, IMU A and IMU B2-3, on a semilog scale.

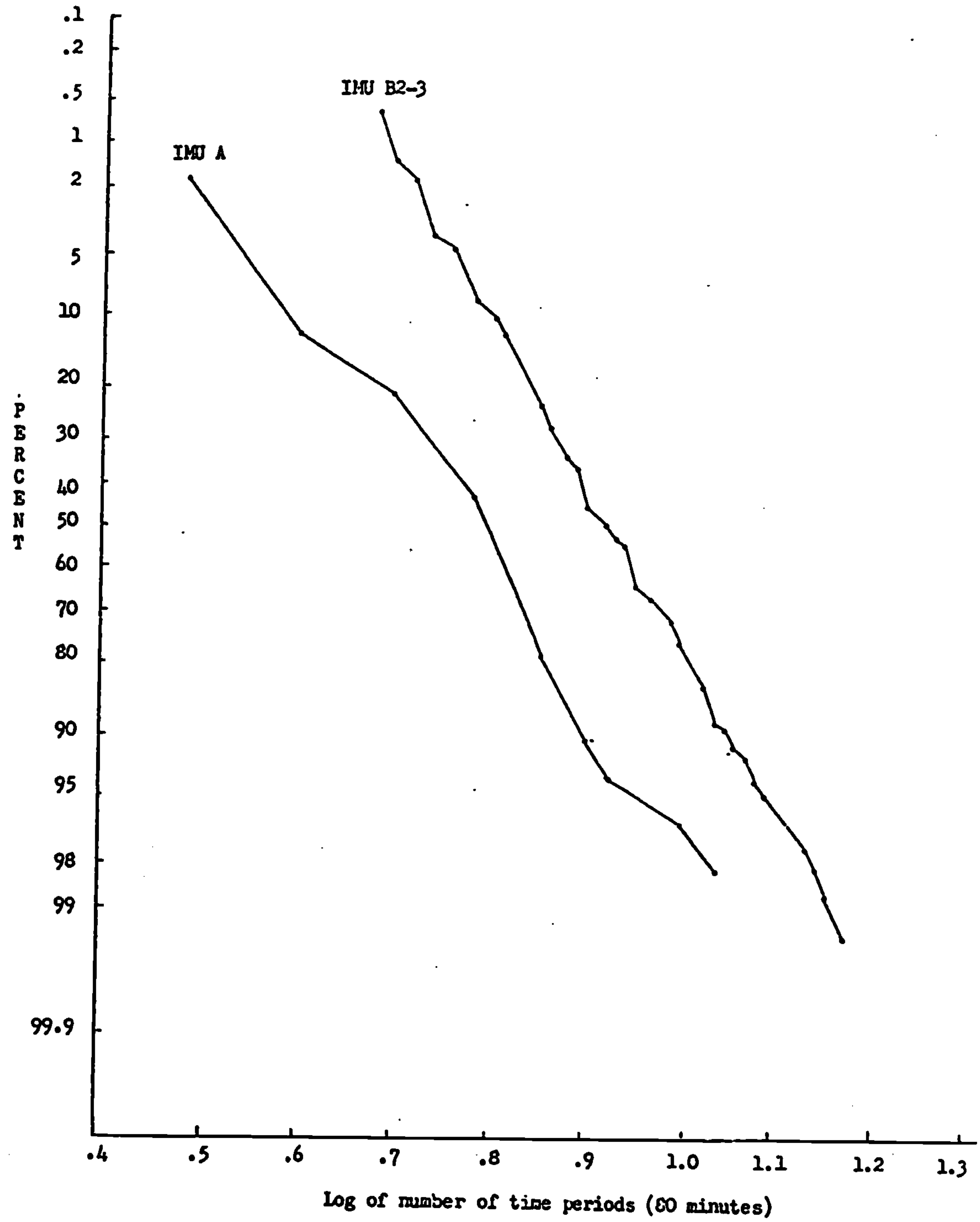


Figure 15.—Cumulative frequency distribution of time to completion, IMU A and IMU B2-3, on a normal probability scale.

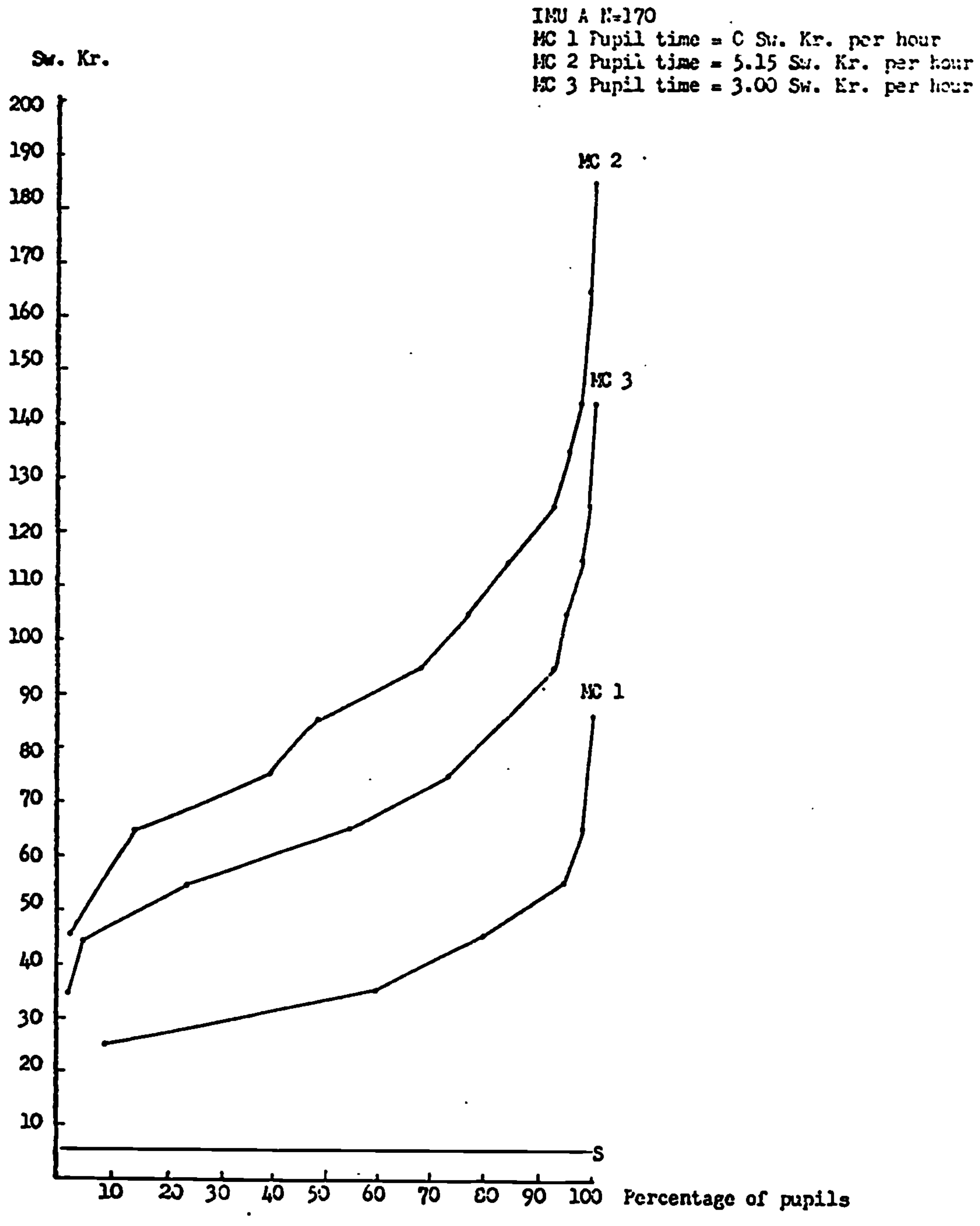


Figure 16.—Marginal cost of step IMU A, S = cost of materials.

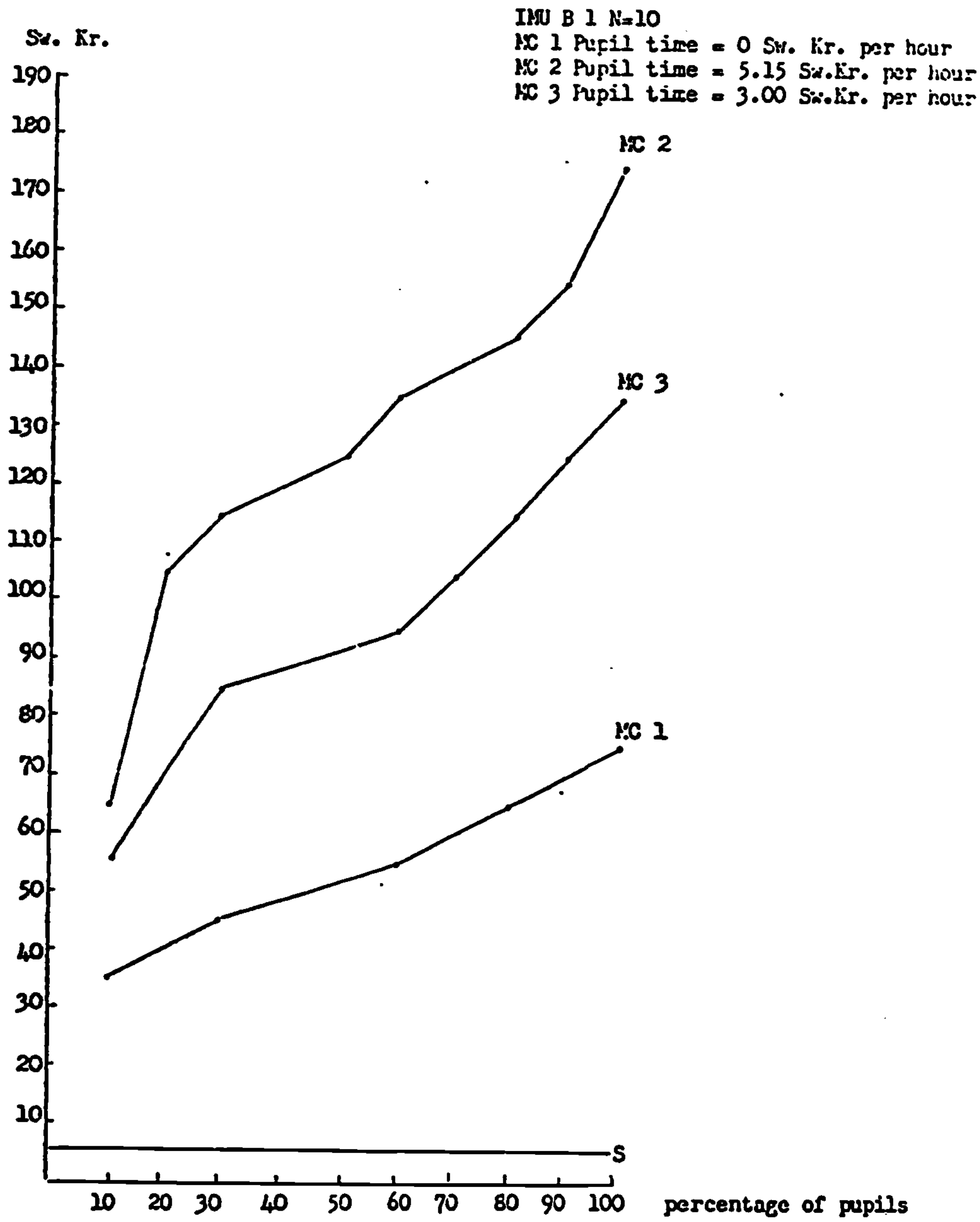


Figure 17.—Marginal cost of step IMU B1, S = cost of materials.

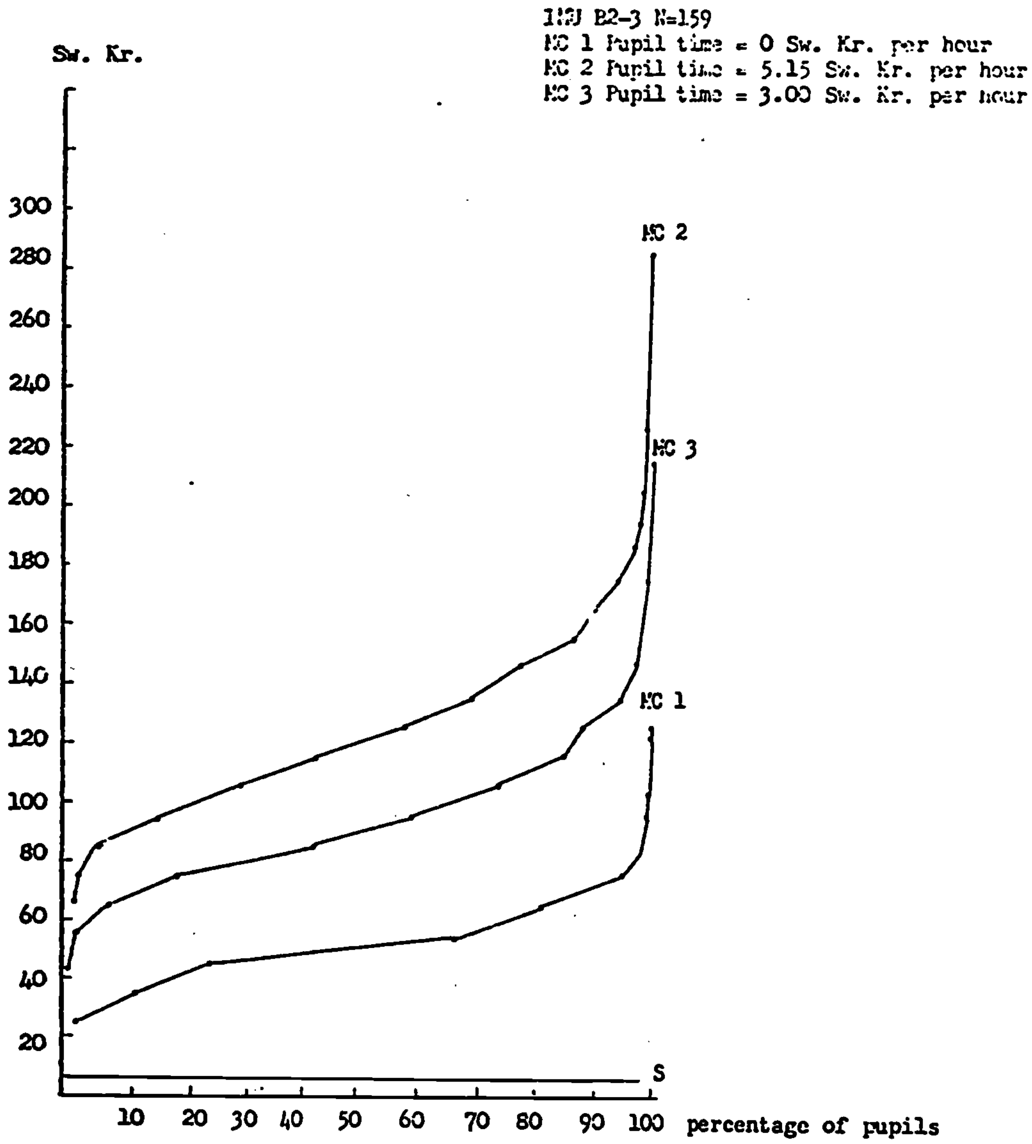


Figure 18.—Marginal cost of step IMU B2-3, S = cost of materials.

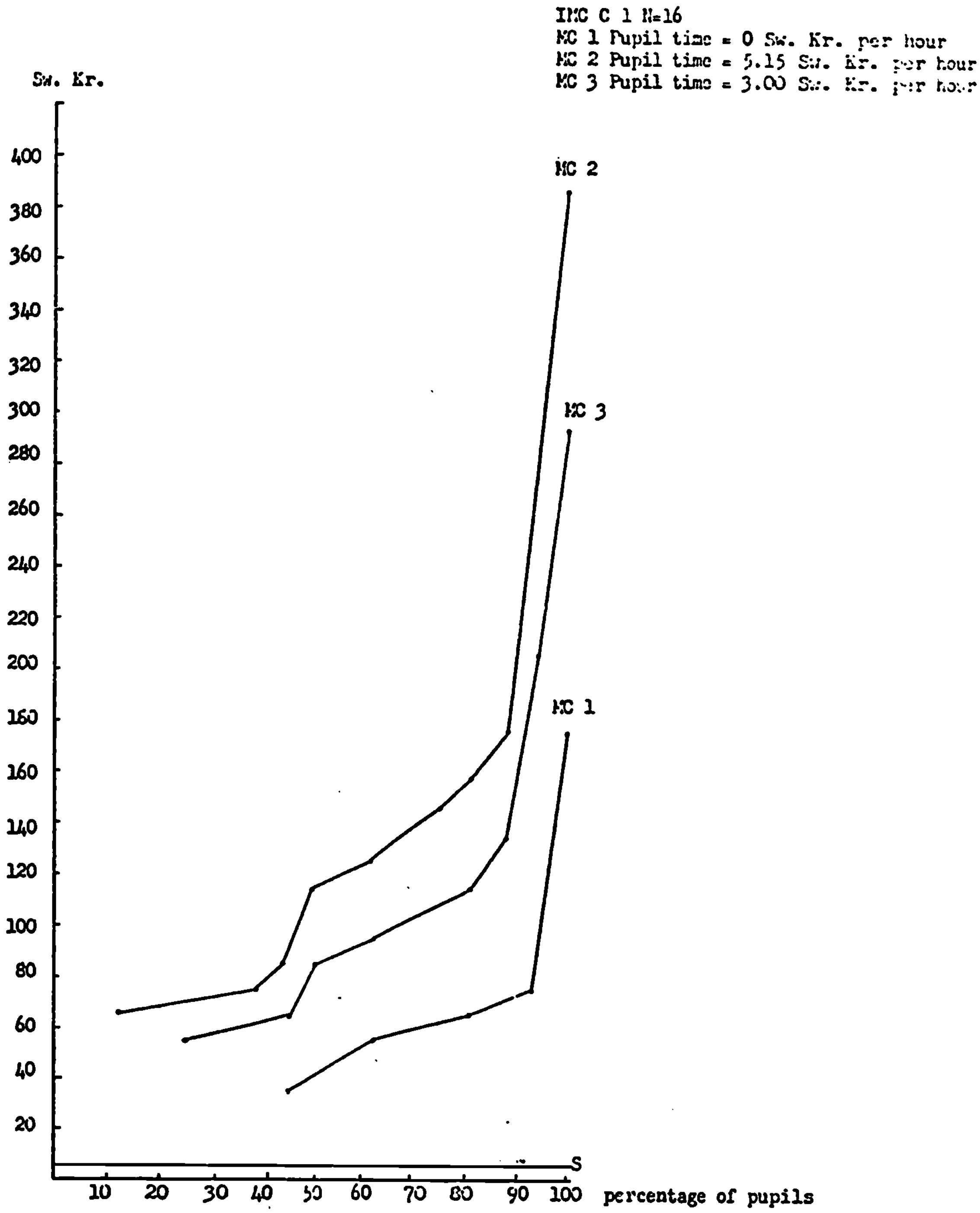


Figure 19.—Marginal cost of step IMU Cl, S = cost of materials.

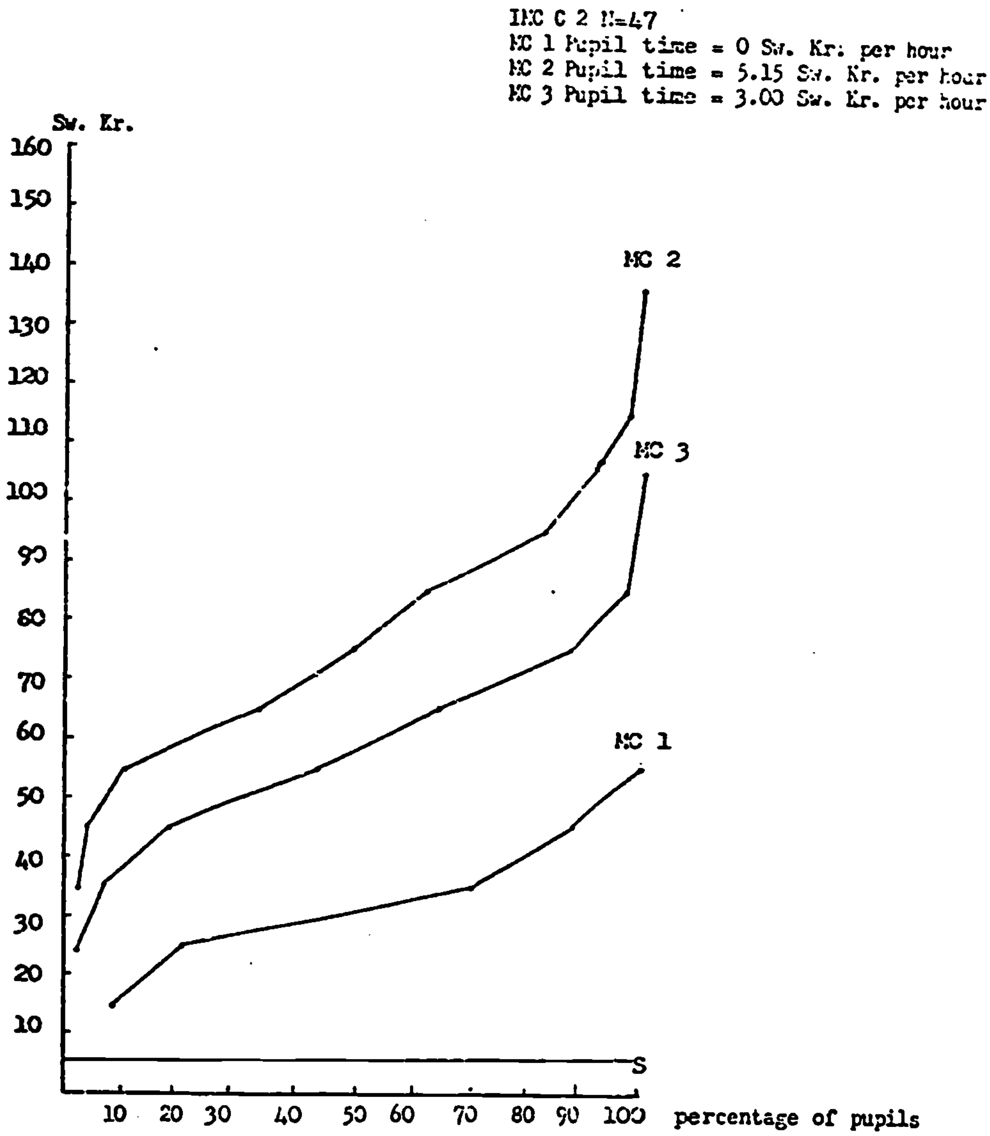


Figure 20.—Marginal cost of step IMU C2, S = cost of materials.

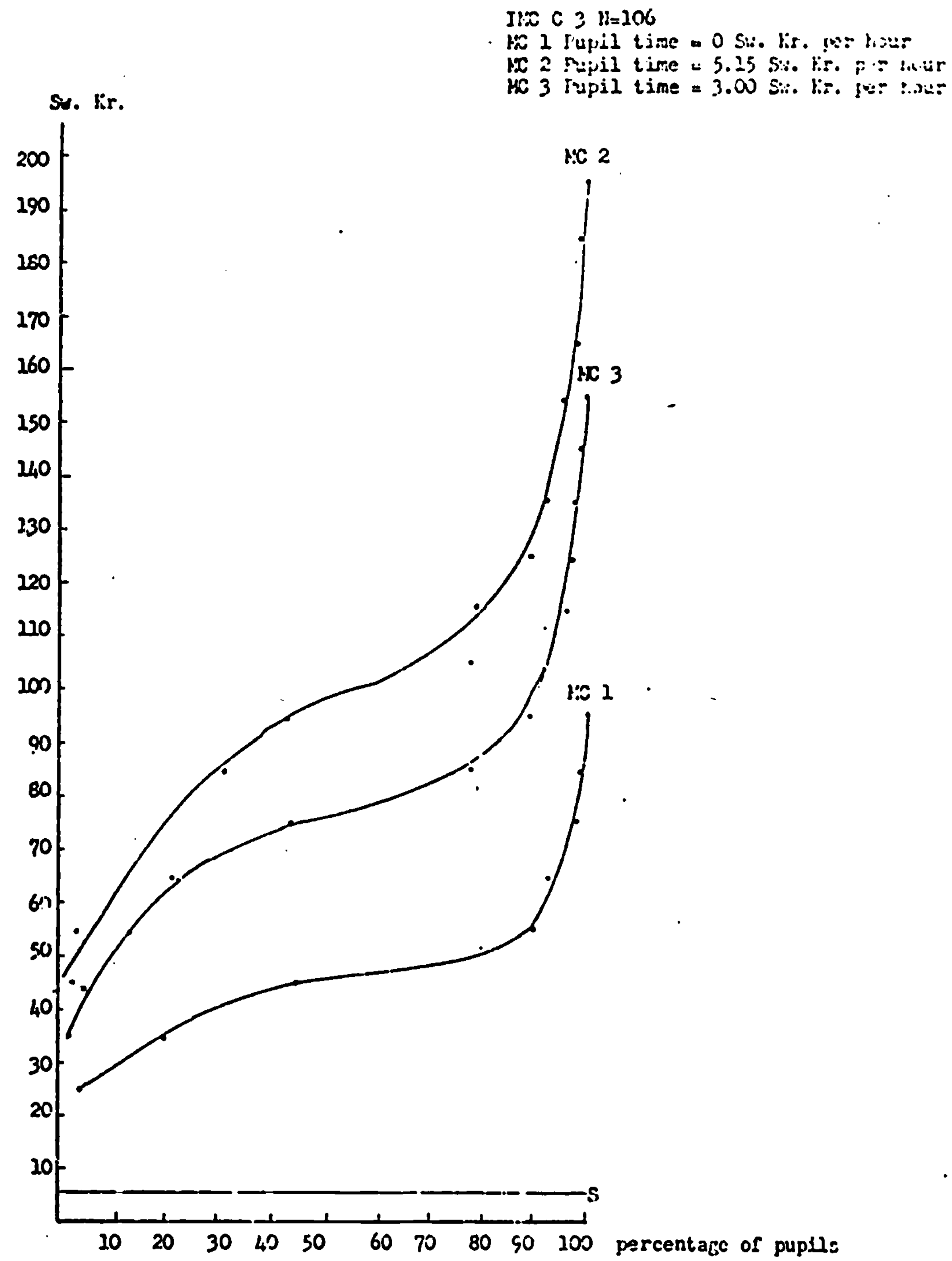


Figure 21.—Marginal cost of step IMU C3, S = cost of materials.

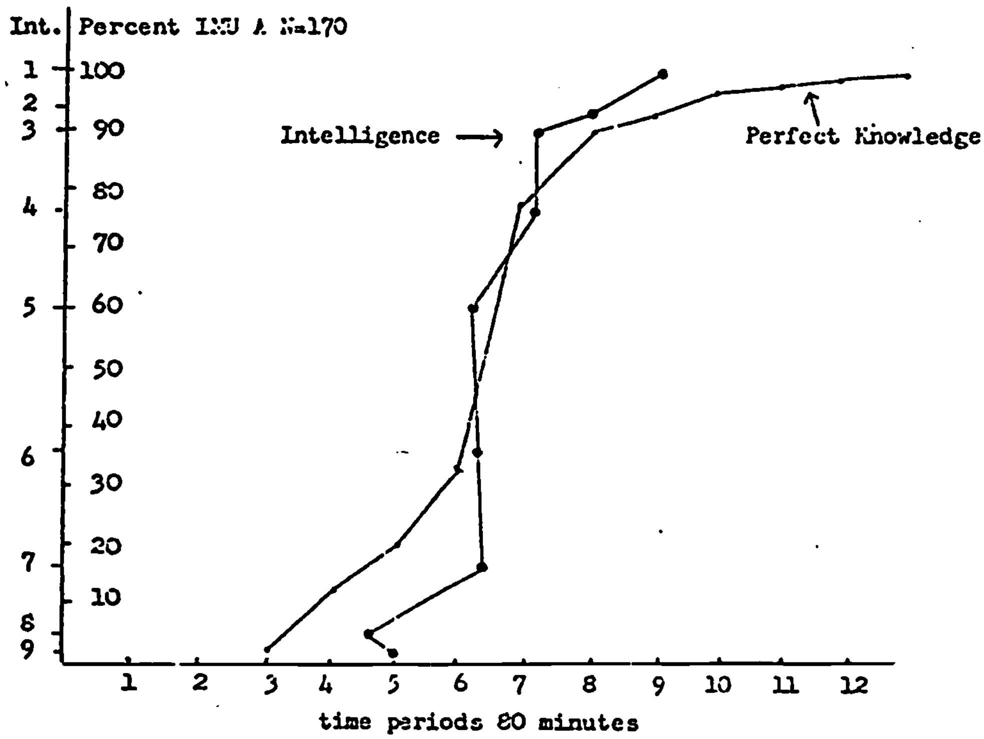


Figure 22.—Cumulative frequency distribution as predicted by "perfect knowledge," and by intelligence.

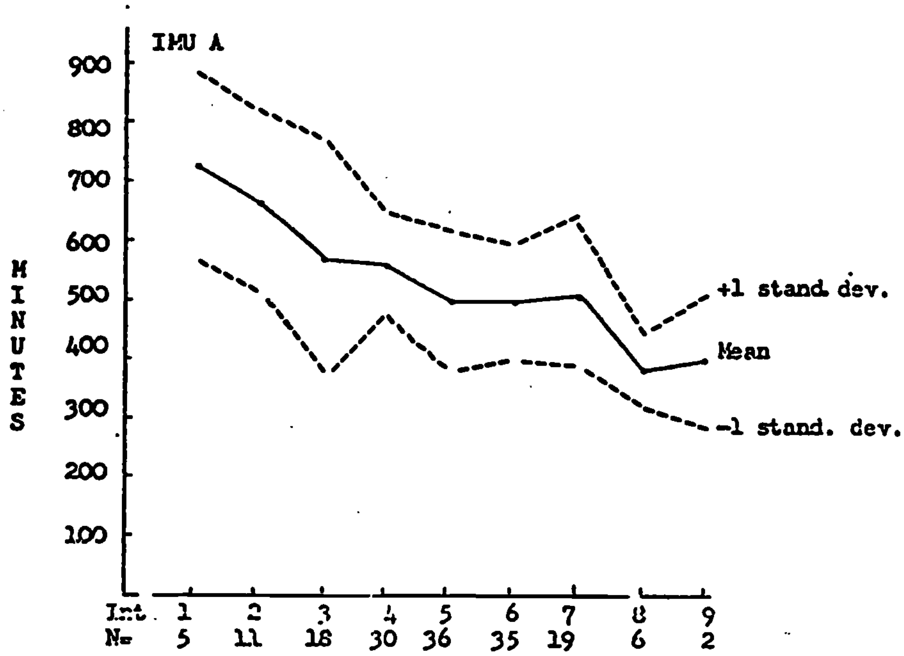


Figure 23.—Mean of time to completion, IMU A, for different intelligence groups (1 low - 9 high)

CHAPTER VIII

CONCLUSION

The major aim of this study was to develop cost-benefit methods relevant to timing in education and applicable to educational decision making. Output to be maximized was defined as skill learning, although it was fully acknowledged that schools are in fact expected to "produce" other kinds of output as well.

After a review of relevant learning theory and a discussion of input and output variables in education, a marginal cost model was developed (Chapter III), tracing costs from differential time that students need in order to reach mastery on a specified learning task. Various aspects of this model were then discussed and related to mastery learning theory. In particular we were trying to find out what some broadly stated societal goals of "equalizing" would mean within the limits of the presented cost model.

A multistep approach was then introduced (Chapter VI); by analyzing a simple sequence of steps, one step building upon the preceding ones, we were able to draw some implications with respect to the costs of successive steps. The amount of time spent in learning at one level, it was assumed, could influence time needed at later levels. The programming model, which was laid out in the same chapter, generalized the idea of time allocation so that we could analyze conditions for optimal allocation of not only student time, but also, and simultaneously, the optimal use of other resources as well, such as teacher and school space time. The objective function was a learning-production function expressed as number of students taking specified steps, that were either constructed or weighted to be equivalent and hence comparable. Restrictions on production were resources at disposal, expressed in terms of "time budgets" and a monetary budget.

Finally the theoretical construct was put to a partial test. The empirical analysis was based on time-to-completion data taken in a non-experimental learning sequence in mathematics. The students were allowed to take the time they needed to complete a step before going on to the next level. Data on time required for learning a sequence of mathematics tasks, and on mathematics ability, intelligence, marks in school subjects, and family background were obtained for all students in grade 7 in one school (13 year olds mostly; grade 7 is the first year of the upper department of the Swedish comprehensive nine-year elementary school.) The purpose of the empirical study was twofold:

1) to estimate empirical MC curves in order to illustrate the basic cost model developed in Chapter III;
2) to throw some light on the possible determinants of students' time to completion.

The first task is quite straightforward, involving nothing more complicated than the plugging in of economic data concerning teacher cost, costs of books and materials, and so on, even though, as might be expected, there were some difficulties in arriving at accurate figures for some items, this being especially true for the opportunity cost of students' time.

Time to complete the first step (IMU A) was shown to correlate moderately with other measures of school achievement and with intelligence but not with family background (family income and father's occupation). For the second step (IMU B) the correlation with measures of intelligence and achievement was low and for the third step (IMU C) it had disappeared. Two factors may contribute to this decline in correlations; first homework is introduced with the second step, and second, students select among options after the first step and those options differ in difficulty. Some of the most powerful of the determinants of individual variation in time spent in learning evidently were not captured by the data collected, but such determination was not the main purpose of this study. The time distributions are important in themselves, regardless of what places individuals at various positions in those distributions. A major finding of the empirical research was the positive skewness of time distributions, resulting in a sharp upturn of the MC-curves, approximately for the ten per cent slowest students. Whether this is a general trait or typical of this particular sample only, or maybe typical of learning mathematics, one cannot tell.

This and other studies show clearly the dilemma when trying to use the time variable in order to "equalize" learning, whatever meaning we would like to attach to this equalization. If the tentative conclusion of this research is correct, that typically time distributions are positively skewed, this fact would seem to aggravate the problem. The slow learner will be left more and more behind, that is the important fact of the matter; and those students who are in the skewed part of the distribution will be especially outstanding in this negative sense. Often it is asserted that in learning situations where time individualization is provided for, competition between students would be minimized and the students would presumably "compete with themselves" or with a fixed standard. Is it not possible, though, that the slow learner will feel "out-competed" by being left behind, with the same detrimental effects as are assumed from competition in more traditional learning situations.

A possible consequence, therefore, of introducing mastery learning or other types of instruction allowing students to take differential time, might be that the society would want to minimize time differences, and thus aim certain measures towards the slow students. In an economic analysis of such measures the marginal cost curve of this study should be of some value, since we here have to compare two forces working on the margin and in opposite directions: a downward push since time for the slow students will be lowered (if measures are successful) and an upward pull by increased costs for those students. If present day school practices are technologically inefficient, as is often assumed, this cost increase might be damped, but insofar as equalization is the goal increase in cost could hardly be avoided. The possibility of over-all increases in efficiency (at all levels and for all, or most, students) is another matter, which introduces the much larger question of how to use resources "freed" by increased efficiency.

As pointed out in Chapter I, learning is a lifelong process beginning before the child enters school and extending beyond the end of formal schooling. Applying our concept of step sequences to this broader context, we may speculate on possible out-of-school determinants of time-to-completion patterns in school subjects, and on any further effects beyond school. By widening the scope of the analysis we do, in effect, examine the efficient use of time over a whole lifecycle, placing school learning in a context of what precedes, accompanies, and comes after schooling. It is evident, then, that one possible explanation of the large variability between individuals in their time to complete a learning task in school may be that for some individuals the learning of pre-school steps is inadequate as an input in later steps, taken in school. This lack of preparation may, furthermore, accumulate with each step taken, if no remedy is provided. In this sense the present analysis ties in with the whole complex of problems related to deprived living, and the costs of minimizing time differentials may be looked upon as compensatory outlays.

Similarly we may extend the analysis forward and speculate about the possible results of learning in school when this learning becomes an input in post-school activities. The adequacy of this input will no doubt have a significant influence not only directly on the efficiency of an individual's use of time at work and his possibilities of enjoying leisure, but also on further learning through experience, whether informal or formalized on-the-job training. So once again we touch upon the problem of deprivation. In this broad context the opportunity cost of not taking remedial measures, if they exist, could be very high indeed.[1]

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[1]For a thorough discussion of the concept of opportunity costs in

The programming model gives some clues as to how resources may be reallocated in order to be used more effectively. A good teacher may behave as if maximizing output in a way that comes close to what the programming model describes, allocating her time and other inputs under her command so as to equalize marginal products. Other teachers may have to be instructed to do so.

Intuitively it might seem that the able students, who are in most cases also the fast learners are the ones who would benefit mostly from an additional unit of any input. Intuition may, however, be very misleading; whereas the fast and able learners certainly show higher <u>average</u> learning products with respect to inputs, this may not be true for <u>marginal</u> products. Some of the slow learners are, no doubt, limited by poor genetic endowment, others, however, may be hindered by lack of relevant prior learning, at home and/or in earlier schooling. In economic terms: prior investment in the human capital of this category of slow learners has been low relative to other students. In this latter category we might get relatively high marginal returns to additional investment, at least if measures to speed up their learning are taken early enough during the course of their schooling.

The programming model, although operational in the conceptual sense was not empirically tested and is, furthermore, not even testable as it is now formulated. This study seems to suggest that future

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general and as related to investment in human capital in particular, see: Mary Jean Bowman, "The Costing of Human Resource Development," in <u>The Economics of Education</u>, ed. by E.A.G. Robinson and J.E. Vaizey (New York: St. Martin's Press, 1966), pp. 421-450.

research be directed towards this problem. That would, incidentally, relate very neatly to work, now being done in economics of education, directed towards estimation of school production functions. It is clear that among the empirical problems associated with the programming model is a probable, non-linearity of the objective function. The constraints are linear and hence in a form that is easily applicable empirically. In order for the objective function is to be estimated, we must, in the present "state of the art," be able to put it in a linear or quadratic form.

The theoretical conclusions that were derived from the programming model would, however, seem to justify its existence, even though it might not be fully explored empirically. This, after all, is a characteristic the model shares with other economic theory. The most important contribution of the programming model to an economic theory of education seem to be the identification of marginal product of student time with learning rate and a shadow pricing of this time resource that makes price proportional to learning rate, hence inversely proportional to time spent in learning.

The immediate practical value of the present study seems to be that it offers a tool for economic evaluation of different teaching methods by an analysis of the costs of time in learning. We have already, during the course of exposition, pointed out some considerations for educational decision making when applying the marginal cost model, of which some of the most important can be summarized:

1) We must have a thorough knowledge of the distribution of time-to-completion for the target population of students, who may or may not comprise a whole age cohort.

2) In addition to knowing the time curve we must in some applications of the model also be able to identify individual students, or rather types of students, along this curve.

3) Some students will, inevitably, overperform with respect to a criterion. This must somehow be accounted for in a final analysis.

4) We also assume that we have accurate cost data. Difficulties of measurement and the cost-benefit approach were reviewed in Chapter III.

It is to be expected that when going from this oversimplified model of the world to reality itself, we will have to make some adjustments. We assumed, for example, that all students would eventually reach mastery on a given step. In real life we will have to deal with some failing students, no doubt, and the costs of those must also count in an economic evaluation. They contribute to costs but not to benefits.

The present-value-model of the costs of sequential steps has the double virtue of being directly applicable in a straightforward way and of supplying the decision maker with a large amount of information. Especially valuable is the possibility we have of using the model to control long term effects. These are, of course, the ones we usually strive for in education; yet all too often it turns out that spectacular new teaching methods do not have long lasting effects. When we are using education to counteract cultural deprivation, for example, such a neglect may be tragic and run counter to the long-ranging goals society has set for itself with respect to welfare.

An ever growing proportion of the population in most countries, technologically developed or not, are engaged in formal education—whether as students, teachers, or other school personnel. The concern of the present study is not with the very complicated problem of allocation in the large of societal resources to education. The question is rather, put in one brief sentence: How do we make the best use of those resources once they are entrusted to the educational sector? It was in the spirit of perhaps being able to throw some light on some of the problems involved here, that this study was undertaken.

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Abstract card

Christoffersson, N.-O. The economics of time in learning. *Didakometry* (Malmö, Sweden: School of Education), No. 34, 1971.

In the economics of education a main concern is with the study of relationships between educational objectives or goals and the scarce resources used as means towards those ends. This study is focused on the cost side; it treats student time in learning as one of those scarce resources and relates costs to time in education. The empirical data were taken in a learning situation, using the IMU system a method of teaching mathematics, developed at the Malmö School of Education. The study was undertaken and completed as a Ph.D. dissertation at the Comparative Education Center, University of Chicago.

Indexed:
1. Comparative education
2. Economics of education
3. Individualized instruction

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