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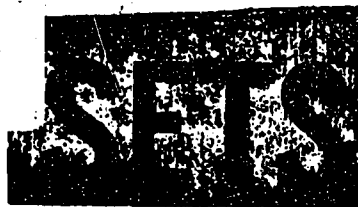
## ABSTRACT

A set of ten teacher-prepared Learning Activity Packages (LAPs) in beginning algebra and nine in intermediate algebra, these units cover sets, properties of operations, number systems, open expressions, solution sets of equations and inequalities in one and two variables, exponents, factoring and polynomials, relations and functions, radicals, rational expressions, coordinate geometry, quadratic equations and inequalities, quadratic functions, and systems of equations and inequalities. Each unit contains a rationale for the material; a list of behavioral objectives; a list of resources including texts (with reading assignments and problem sets specified), tape recordings, commercial games, filmstrips, and transparencies; a problem set for student self-evaluation; suggestions for advanced study; and references. For other documents in this series, see SE 015 193, SE 015 194, SE 015 196, and SE 015 197. (DT)

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**L**EARNING  
**A**CTIVITY  
**P**ACKAGE



ALGEBRA 93-94

REVIEWED BY

*JAR*

LAP NUMBER

~~1~~ 1

WRITTEN BY Diane Evans

*OK JS*

## Instructions

- I. Read Rationale
- II. Read BEHAVIORAL OBJECTIVES
- III. Resources
  - A. All work must be done in your notebook with pencil only.
  - B. Keep your notebook up to date. Your teacher may ask for it at any time without warning.
  - C. Work all the Exercises in at least ONE text for each goal. Always check your exercises in your notebook (see your teacher).
- IV. Self-Evaluation
  - A. Must be taken at completion of activities for each section.
  - B. Does not affect your grade in any way.
- V. Advanced Study
  - A. To be done only after all previous work has been satisfactorily completed.
  - B. Must be approved by teacher.
- VI. Progress Test and LAP Test
  - A. Teacher graded
  - B. Recycling may take place at this time if test is not satisfactory.

DO NOT LOSE YOUR LAP. If you do, you must buy another one.

## Rationale (The LAP's Purpose)

A natural place to begin the study of any branch of mathematics is in the theory of sets. Sets can be thought of as the building blocks of mathematics. Before a student can proceed to any form of higher mathematics, he must be familiar with what a set is, operations with sets, and how sets can be used.

In this LAP you will be given a systematic study of the subject of sets, including the basic notation associated with sets. Other concepts you will study are subsets, equality and matching sets, operations on sets, infinite sets, and graphing sets.



## Section 1

BEHAVIORAL OBJECTIVES : At the completion of your prescribed course of study, you will be able to:

1. Given any set stated in words, write and/or identify it in set notation.
2. Given any set written in the description method, rewrite or identify it in the listing or roster method.
3. Given any set written in the listing method, rewrite or identify it in the description method.
4. Given a particular set and a list of elements, decide which are elements of that set and which are not.
5. Given any sets, tell which are infinite and which are finite.
6. Given a rule for a set (descriptive form), denote whether or not the result is the empty set -  $\emptyset$ ,  $\{\}$ .
7. Given a list of numbers, be able to name those that are prime & those that are composite.
8. Given any two sets, denote whether or not they are matching sets (one to one correspondence).

Resources

Objective 1

Vanatta, read pp. 8-12, Ex. 1 page 12  
Nichols, read pp. 1-3, Ex. 6 page 1 and 8 page 29  
Wollensak tape C-3451 Introduction to Sets

Objectives 2,3

Dolciani, read pp. 10-14, Ex. 1-6 and 7-26 even 6 page 14; and 1,2,8,9, 12, 13, 16 written (roster only) page 14.

Objectives 4,5,6

Payne, read pages 1-4 and the 2 definitions on page 12 Ex. 1-10 and 38-40 pages 4-6.  
Nichols, read pages 1-3, Ex. 1-4, 6, 9-11 pages 3 and 4  
Dolciani, read pages 13-14 Ex. 2-12 even (omit roster) page 14; Ex. 15, 17, 18, 19, page 27  
Wooton, read page 16-18, ex. 31-36 page 19  
Pearson, read pages 1-4, Ex. 1-10 and 38-40 pages 4-6  
Introduction to Sets, frames 1-57  
Wollensak C-3010, Prime Numbers

Objective 7

Payne, read problems 16, 17 on page 5, Ex. 15, 16 page 5; 3 page 48.  
Nichols, read 4-6, Ex. 2-6 page 5  
Wollensak C-3010, Prime Numbers

Objective 8

Payne, read pages 12-14, Ex. 14,15 pages 15, 16  
Nichols, read pages 7, 8, Ex. 1 page 829  
Dolciani, read pages 13-14 Ex. 9-16 and page 14  
Wooton, read pages 20-23, Ex. 27,28 page 25  
Introduction to Sets frames 226-255

Self-Evaluation

OBJECTIVE

1 I. Write the following sets in roster notation:

- \_\_\_\_\_ 1. The set of vowels in the word algebra.
- \_\_\_\_\_ 2. The months of the year whose names begin with D.
- \_\_\_\_\_ 3. The odd numbers between 5 and 11.
- \_\_\_\_\_ 4. The states that have a boundary in common with S.C.
- \_\_\_\_\_ 5. The even numbers between 8 and 12.
- \_\_\_\_\_ 6. All numbers divisible by 25 less than 100.

2 II. Rewrite these sets using the listing method.

- \_\_\_\_\_ 7. {all whole numbers less than 4}
- \_\_\_\_\_ 8. {capitals of S. C., Ga., N.C.}
- \_\_\_\_\_ 9. {colors in the American flag}
- \_\_\_\_\_ 10. {even num. less than 10}
- \_\_\_\_\_ 11. {num. less than 20 that are divisible by 6}

3 III. Rewrite these sets using the description method.

- \_\_\_\_\_ 12. {2, 4, 6}
- \_\_\_\_\_ 13. {Washington, D. C.}
- \_\_\_\_\_ 14. {Lyndon Johnson, Richard Nixon}
- \_\_\_\_\_ 15. {5, 10, 15, 20, 25}
- \_\_\_\_\_ 16. {1, 3, 5, 7, 9}

4 IV. Match the set on the left with an element of the right not in the set.

- \_\_\_\_\_ 17. {1, 1/2, 3, 4, 9}

- a. 1
- b. 1/2
- c. 3
- d. 4
- e. 9

18.  $\{1, 2, 3, \dots\}$

19.  $\{ \text{all the numbers larger than five and less than } 12 \}$

5, 6 V. Classify the following sets as being either infinite, finite, or empty ( $\emptyset$ ).

- 20. the set of odd natural numbers
- 21. the set of even natural numbers
- 22. the set of prime numbers
- 23. the set of prime numbers greater than 10 and divisible by 2
- 24. the set of even prime numbers
- 25. the set of odd prime numbers
- 26. the set of even numbers that are odd

7 VI. Classify the following numbers as being either prime or composite.

- 27. 13
- 28. 51
- 29.  $9^2$
- 30. 6
- 31. 101

8 VII. Given the following pairs, decide whether or not they are matching (one to one correspondence).

- 32.  $A = \{2, 5\}$   
 $B = \{1, 3, 8\}$



SELF-EVALUATION (continued)

33

34

35

A =

B =

A =

B =

DO YOU HAVE ANY OTHER EDUCATIONAL OBJECTIVES, TIME  
THE PROGRESS TEST

BEHAVIORAL OBJECTIVES: At the conclusion of your prescribed course, you should be able to:

9. Determine whether or not a set is a subset of a given set.
10. Given a set list all the subsets, proper subsets which are proper subsets of the given set.
11. Given a pair of sets, determine whether or not they are disjoint.
12. Given a pair of sets, determine if it is disjoint.
13. Given any pair of sets, name their union and intersection in the indicated notation.
14. Given a universal set with any number of elements, determine its number of subsets.
15. Given any set which is a subset of a universal set, name the set which is its complement.
16. Given a Venn diagram, choose from a list the operation(s) being illustrated.





OBJECTIVES

11,12 I. Indicate which of the following statements are true or false.

1. The set of all even natural numbers and the set of all prime numbers are disjoint sets.

2. If  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  and  $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ , then  $A \cap B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ .

3. The set of all natural numbers and the set of all integers are disjoint sets.

4. If  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  and  $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ , then  $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ .

13 II. Give the union and intersection of the following pairs of sets.

1.  $A = \{2, 5\}$ ,  $B = \{1, 3, 8\}$

2.  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

3.  $A = \{1, 2, 3, 4, \dots\}$ ,  $B = \{1, 3, 5, 7, \dots\}$

4.  $A = \{1, 2, 3, 4, \dots\}$ ,  $B = \{1, 3, 5, 7, \dots\}$

5.  $A = \{50, 51, 52, \dots\}$ ,  $B = \{50, 51, \dots, 99, 100\}$

6.  $A = \{4, 5, 6, 8\}$ ,  $B = \{2, 5, 6, 10\}$

13,14,15 III. Indicate whether the following sentences are true or false.

12. If  $N$  = the set of all natural numbers and  $P$  = the set of all prime numbers, then  $P \cap N = P$ .

13. If  $E$  = the set of all even natural numbers,  $O$  = the set of all odd natural numbers, and  $N$  = the set of all natural numbers, then  $E \cup O = N$ .

14. If  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ , then  $A$  has eight subsets.

15. If  $B = \{1\}$ , then  $B$  has two subsets.



16. For any set  $A$ ,  $A \cap \bar{A} = A$

17. For any sets  $A$  and  $B$ ,  $A \cup \bar{A} = B$

18. The set  $\{0, 1\}$  has 8 subsets.

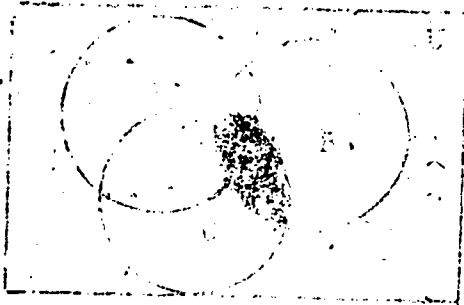
19. The intersection of the set of prime numbers and the set of even numbers is the empty set.

20. Given a set of 3 elements, there are exactly 7 proper subsets of that set.

21. Given any infinite set, there exists only a finite number of subsets.

16. In each of the following Venn Diagrams, indicate the name for the set represented by the shaded AREA.

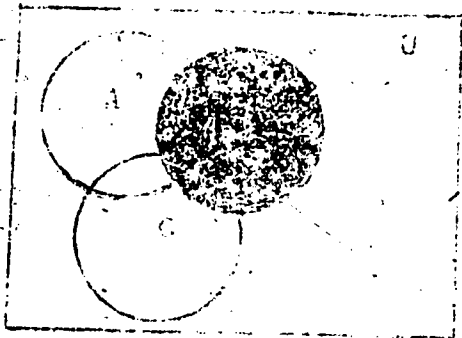
22.



The Shaded Area is

- a.  $A \cup B$
- b.  $B \cap C$
- c.  $A \cup C$
- d.  $B \cap C$
- e.  $A \cap C$

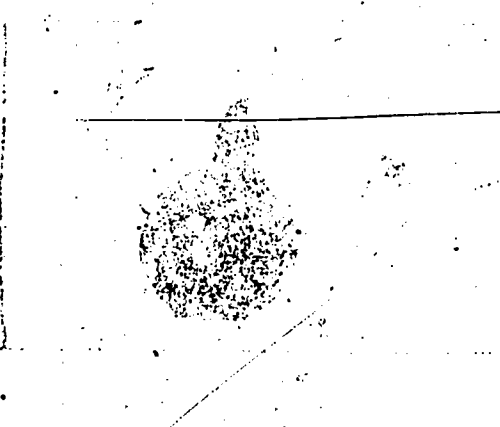
23.



The Shaded Area is

- a.  $B \cap A$
- b.  $A \cap B$
- c.  $B \cup A$
- d.  $(A \cup B) \cap C$
- e. none of these

24.



The shaded area is

A.  $(A \cup B) \cap C$

B.  $(A \cap B) \cap C$

C.  $A \cap B$

D.  $(A \cup B) \cup C$

E. none of these

25.



The shaded area is

A.  $(A \cup B) \cap C$

B.  $A \cap (B \cup C)$

C.  $(A \cap B) \cup C$

9. 10 VIII. Which of the following statements are true or false?

Every member of B is also a member of A.

21. Every member of A is also a member of B.

22. Every member of A is also a member of C.

23. There is only one subset of a set with two elements.

24. The set of natural numbers is a subset of the set of integers.

25. The set of natural numbers is a subset of the set of real numbers.

IF YOU HAVE PASSED THE TEST, YOU MAY TAKE THE LAP TEST. CONSULT YOUR INSTRUCTOR.

Advanced Study.

The following is an application of set theory.

1- Take either kingdom and draw a Venn diagram showing proper subsets. Do the same with one particular species. The most interesting may be Sapiens.

2- Color TV is based on the idea that there are three primary colors, each transmitted separately and then blended in your set circuits at home. Use a Venn diagram of the three primary colors to show how the colors are blended. Use set notation to describe the different color obtained.

3- Given the following quadrilaterals construct a Venn Diagram showing their proper relationships.

Quadrilaterals

Rectangle

Square

Parallelogram

Isosceles

Trapezoid

4- Find the prime factorization of 10,001.

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Audio Visual

Wollersak Teaching Tapes: G-3451 G-3505 G-3010 G-3452



**L**ARNING  
**A**CTIVITY:  
**P**ACKAGE

**RULES OF GAME**

1.  $a + b = b + a$
2.  $ab = ba$
3.  $(a + b) + c = a + (b + c)$
4.  $(ab)c = a(bc)$
5.  $a(b + c) = ab + ac$
6.  $a - (b - c) = a - b + c$
7.  $a - (b + c) = a - b - c$

PROPERTIES  
OF  
OPERATIONS



ALGEBRA 93 - 94

REVIEWED BY

*DS*

LAP NUMBER

~~1~~ 2

WRITTEN BY Diane Evans

### RATIONALE (The LAP's Purpose)

Mathematics can be thought of as a game in which you perform moves by specific rules. For example:

WHAT NUMBER IS EQUAL TO  $4 + 3 \times 5$ ?

You may work it out this way: =

$$4 + 3 = 7 \times 5 = 35$$

or you may compute it this way:

$$3 \times 5 = 15 \text{ and } 4 + 15 = 19$$

As you can see there are two possible answers!!

Obviously, both ways cannot be correct because

35 is not equal to 19! The expression  $4 + 3 \times 5$

must have only one meaning! It is customary to

use parentheses, which are mathematical punctu-

ation marks, to make the meaning of such phrases

clear.

In this LAP you will not only learn how parentheses

are used but also about the "order of operations" -

that is the order in which certain operations are

to be performed. Though you have studied the

properties of operations in the past, you will review

them again because of their importance in

learning mathematics.

## Behavioral Objectives

Upon completion of your prescribed course of study, you will be able to:

1. Write the simplest name for any numerical expression which involves the use of grouping symbols and order of operation.
2. Given any mathematical sentence, identify which of the following properties (if any) are being illustrated.
  - a) The Commutative Property of Addition (GPA)
  - b) The Commutative Property of Multiplication (CPM)
  - c) The Associative Property of Addition (APA)
  - d) The Associative Property of Multiplication (APM)
  - e) "Symmetric Property of Equality" (SPE)
  - f) "Distributive Property of Multiplication over Addition" (DPMA)
3. Given any set, determine if it is closed with respect to a given operation.
4. Given any mathematical sentence, identify which of these properties (if any) are being illustrated.
  - a) Multiplicative identity
  - b) Property of one for Division (P1D)
  - c) Additive identity
  - d) Property of Zero for Multiplication (PZM)
5. Given any mathematical sentence, identify which of the following properties is being illustrated:
  - a. multiplicative inverse
  - b. additive inverse
6. Given any mathematical sentence involving one operation, write an equivalent sentence using the inverse operation.
7. Given any word phrase, of the type in Appendix I, translate it into an equivalent mathematical phrase.
8. Correctly write a mathematical sentence of the type in Appendix II which would be used to solve a given word problem.

## RESOURCES

NOTE: (EOL means every other letter.)

### Objective 1

Vanatta, read pp. 47-48, Ex. 1-4 page 47.

Dolciani, read p. 23, Ex. 18-20, 30-34 page 24.

Nichols, read pp. 31-33, Ex. 1-25 odd pages 33-34; 2 EOL page 37.

Payne, read page 18, Ex. 1-5 page 18.

Wooton, read pp. 10-16, Ex. 1-10, 25-32 pages 13-14; 1-10 odd, 17-29 odd pages 15-16; 1-10 page 51.

Pearson, read pages 52-59, Ex. 1-25 odd page 55; 1-3 EOL pages 56-57; 1-5 EOL page 58.

### Objective 2

Vanatta, read pp. 27-33; Ex. 2, 7, 10 page 29; 13, 18, 19, 24 page 34; 14, 15 page 37; 14 page 30; 6, 20 page 34.

Dolciani, read page 69, 73-76; Ex. 1-26 page 74; 15, 16 pages 100-101; 1, 6, 7 page 70; 2 page 98.

Nichols, read pages 34-37, 40-46; Ex. 1 page 37; 1 EOL, 2 pages 40-41; 1, 2 EOL, 3 pages 42-43.

Payne, read pages 31-38, Ex. 1, 3 page 33; 1-51 odd pages 35-37.

Wooton, read pages 48-52, 55-59, 71-76; Ex. 11-28 pages 50-51; 1-6, 7-23 odd pages 57-58.

Pearson, read pp. 139-145, 166-169; Ex. 1-14 pages 141-142; 1-3, 4 EOL, 5, 6 pages 144-145; 1-4 EOL, 5-7 pages 168-169.

Wollensak C-3453 The Commutative Property  
C-3454 The Associative Property  
C-3455 The Distributive Property

### Objective 3

Dolciani, read pages 70-71, Ex. 1-14 oral p. 72; 1-12 written page 73.

Nichols, read pages 37-39, Ex. odds page 39; 4 a, c, e page 51; 8 (state why or why not) page 53.

Payne, read page 31; Ex. 62-73 pages 37-38.

Wooton, read page 47, Ex. 1-10 pages 49-50.

Pearson, read pages 135-137, Ex. 1-12 pages 136-137.

Wollensak C-3456 The Closure Property

### Objective 4, 5

Vanatta, read page 27 and study 2-5; page 28 study 2-5; Ex. 1, 4, 8, 9 page 29; 12, 13 page 30; 15-17, 21, 22 page 34; 17-19 page 37.

Dolciani, read pages 77, 121, 138; Ex. 33-42 page 141; 1-10 oral page 122.

Pearson, read pages 162-166, 178-180, 204; Ex. 1, 2 EOL, 3, 5-7 EOL, 9 pages 164-166; 2 a, b, e, f, g, 5 a-h, page 165, 3 a-e, 4 a-d, g-i page 296.

Wollensak C-3459 Identity Element  
C-3451 The Inverse Element

RESOURCES (cont')

Objective 6

Nichols, read pages 46-48, Ex. 1-3 EOL pages 47-48.

Objective 7

Dolciani, read p. 51, Ex. 1-22 pages 52-53; 1-25 even page 54; 3, 6, 13, 14 page 55.

Nichols, read pages 48-49, Ex. 1-21 page 49.

Wooton, Ex. 11-24 page 13; 11-16 page 16.

Pearson, read pages 145-147, Ex. 10-13 page 147.

\* Appendix I

Objective 8

Vanatta, read pages 154-156, Ex. 1-10, 12-15 pages 156-157.

Dolciani, read pages 57-58, Ex. (write equation only) 1-14 odd

pages 57-58; 1-14 page 18.

Payne, read pages 65-66, Ex. 21-23 pages 65-66.

Wooton, read pages 29-33, Ex. 31-38 page 33 (equation only).

Pearson, read pages 157-159, Ex. (write equations only) 1-19 odd

pages 158-159; 5-15 odd pages 176-177.

\* Appendix II

Wollensak C-3801 Open Phrase, Open Sentence

C-3803 Open Sentence: Solution

C-3809 Reading Written Problems

Games

Equations by Layman Allen

\* required

APPENDIX I

1. What is the cost of  $n$  pencils at 3 cents each?
2. What is the cost of  $x$  articles at  $y$  dollars each?
3. How far can a boy run in  $h$  hours at the rate of 6 miles per hour?
4. The sum of two numbers is 7 and one of them is  $x$ ; what is the other number?
5. Represent in terms of  $x$  two numbers that have the ratio 3:4.
6. If  $x$  represents the sum of two numbers and one of them is 5, what is the other?
7. What is the total weight of  $n$  boys weighing  $y$  pounds each?
8. The sum of two numbers is  $x$  and one of the numbers is 5. What is the other number?
9. How many cents are there in  $d$  dollars?
10. Mary is  $n$  years old now. How old was she 3 years ago?
11. How many inches are there in  $x$  feet and five inches?
12. A man had  $x$  dollars and spent  $y$  dollars. How much did he have left?
13. If  $n$  represents a certain number, represent the next larger consecutive number.
14. A parcel weighs  $t$  pounds and a smaller parcel weighs  $\frac{2}{3}$  as much. What is the weight of the smaller parcel?
15. What is the average weight of two boys who weigh  $x$  pounds and  $y$  pounds each respectively?
16. How much salt remains when  $x$  pounds have been used from a bag containing  $a$  pounds?
17. What is the perimeter of a square one side of which is  $s$ ?
18. The difference between two numbers is 2 and the smaller number is  $n$ . What is the other number?
19. One part of  $t$  is  $w$ . What is the other part?
20. Elizabeth's age is now 7 years. How old will she be in  $n$  years?
21.  $x = 5y$ . Upon what does the value of  $x$  depend?
22. What is the perimeter of a triangle whose sides are  $a$ ,  $b$ , and  $c$ ?
23. What is the perimeter of a rectangle whose length is  $l$  and whose width is  $w$ ?
24. What is the area of a rectangle whose base is  $b$  and whose height is  $h$ ?
25. How many inches are there in  $y$  yards,  $f$  feet and  $l$  inches?



APPENDIX 2

Write the mathematical sentence which would be used to solve each word problem.

1. Five times a certain number is 105.
2. The greater of two numbers is twice the smaller and their sum is 48.
3. Mary is 5 times as old as her brother and their combined ages total 18 years.
4. One number is 4 times another and their difference is 24.
5. A man walks 6 hours at a certain rate and then proceeds 3 hours at twice his former rate. If he walked 24 miles in all, at what rate did he start walking?
6. The length of a rectangle is three times its width and its perimeter is 56 feet.
7. The sum of three numbers is 56. The second number is 3 times the first and the third number is 4 times the first. What are the numbers?
8. The sum of the three angles of any triangle is 180 degrees. In a certain triangle ABC, angle A is twice as large as angle C and angle B is three times as large as angle A.
9. Two numbers have the ratio 5:6 and their sum is 88. Find the numbers.
10. Separate 92 into two parts such that one part is three times the other.
11. One number is three times another. Six times the larger diminished by twice the smaller is 48.
12. When a quart of cream cost four times as much as a quart of milk, 5 quarts of milk and 3 quarts of cream cost \$2.72, what is the cost of each per quart?

C. Applications

1. If you mix 4 grams of salt and 84 grams of water, what percent of the total solution is salt?
2. Compound X is composed of elements Y and Z in the ratio of 3:2. If you had 50 grams of element Y, how many grams of element Z would you need to utilize all of Y into making compound X?

Self-Evaluation Test

Behavioral Objectives

1 I. Write the simplest answer for each of the following:

- \_\_\_ 1.  $56 \div 2 \times 3 \div \frac{1}{2}$
- \_\_\_ 2.  $(\frac{3}{4} - \frac{1}{3})(3 + 9) + 3 [(5 - 3) \div 6]$
- \_\_\_ 3.  $15 - 3 \times 2 + (18 - 5) \cdot 5$
- \_\_\_ 4.  $3 \cdot 4 + 2 + 5 \cdot 3 - 3 \cdot 7$
- \_\_\_ 5.  $5 \cdot 0 + 4 - 2 \cdot 2$
- \_\_\_ 6.  $[5 \cdot (2) + 11] \div 7$
- \_\_\_ 7.  $4 \times \{78 - [5 \cdot 4 + (15 \div 3)] \times 3\} \cdot 2$
- \_\_\_ 8.  $28 \div 2 - 2 \times 6 \div 3 + 3$
- \_\_\_ 9.  $(7 + 5)^2 - 6$
- \_\_\_ 10.  $7 \times 8 - 6$

2 II. For each given sentence, write the name of the property illustrated. (Use abbreviations such as CPA, etc.)

- \_\_\_ 11.  $(46 + 21) + 7 = 46 + (21 + 7)$
- \_\_\_ 12.  $16 \cdot (5 + 7) = (5 + 7) \cdot 16$
- \_\_\_ 13.  $4 \cdot (7 + 13) = 4 \cdot (13 + 7)$
- \_\_\_ 14.  $15 \cdot (6 \cdot 2) = (15 \cdot 6) \cdot 2$
- \_\_\_ 15. If  $a + b = c$  then  $c = a + b$
- \_\_\_ 16.  $4 \cdot [x + (2 + 5)] = 4 \cdot x + 4 \cdot (2 + 5)$
- \_\_\_ 17.  $ax + ay = a \cdot (x + y)$

3 III. Each of the following statements is either true or false.

- \_\_\_ 18. The set  $\{0, \frac{1}{2}, 1\}$  is closed under multiplication.
- \_\_\_ 19. The set of odd natural numbers is closed under the operation of multiplying by the factor 2.
- \_\_\_ 20. The set of even natural numbers is closed under addition.
- \_\_\_ 21. The set  $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots\}$  is closed under addition.
- \_\_\_ 22. The set of all prime numbers is closed under addition.

4

IV. For each given sentence, write the name of the property illustrated. (Use abbreviations such as P.I.M., etc.)

- \_\_\_\_\_ 23. If  $3 + 7 = n$  then  $n$  is a natural number.
- \_\_\_\_\_ 24.  $y \cdot (5 - 4) = y$
- \_\_\_\_\_ 25.  $5 \cdot (x - x) = 0$
- \_\_\_\_\_ 26.  $x + 0 = x$
- \_\_\_\_\_ 27.  $7 + 0 = 7$
- \_\_\_\_\_ 28.  $6 \cdot 1 = 6$

6

V. For each statement, write a correct related problem using the inverse operation.

- \_\_\_\_\_ 29.  $11 - 4 = 7$
- \_\_\_\_\_ 30.  $\frac{1}{10} + \frac{1}{2} = \frac{3}{5}$
- \_\_\_\_\_ 31.  $51 \div 3 = 17$
- \_\_\_\_\_ 32.  $\frac{15}{23} \times \frac{7}{5} = \frac{3}{4}$

7

VI. For each phrase write a correct mathematical phrase.

- \_\_\_\_\_ 33. The sum of four times  $n$  and 7.
- \_\_\_\_\_ 34.  $15m$  decreased by one-half  $x$ .
- \_\_\_\_\_ 35. Four  $y$  divided by the sum 5 and  $x$ .
- \_\_\_\_\_ 36. The quotient of seven  $x$  and fifteen.
- \_\_\_\_\_ 37. Three times the difference of 2 and  $q$ .
- \_\_\_\_\_ 38. The product of 5 and  $x$  increased by  $t$ .
- \_\_\_\_\_ 39. Seven less than four  $p$ .
- \_\_\_\_\_ 40. Four times the sum of  $n$  and 7.

8

VII. Translate each word sentence into an equivalent open mathematical sentence. (Do not solve)

- \_\_\_\_\_ 41. The difference of three times a number ( $w$ ) and six is the number increased by four.
- \_\_\_\_\_ 42. The quotient of seven and the sum of a number ( $x$ ) and eight plus five is twenty-three.
- \_\_\_\_\_ 43. One-fifth of a certain man's lifetime ( $y$ ) spent in childhood, plus one-third of his life served in the armed services, totals the eight years he wasted as a bum and the two-fifths of his life as a missionary.

5

VIII. For each given sentence, write the name of the property illustrated.

- \_\_\_\_\_ 44.  $2 \cdot \frac{1}{2} = 1$
- \_\_\_\_\_ 45.  $6 + (-6) = 0$
- \_\_\_\_\_ 46.  $\frac{2}{3} \cdot \frac{3}{2} = 1$
- \_\_\_\_\_ 47.  $(-7) + 7 = 0$
- \_\_\_\_\_ 48.  $8 \cdot \frac{1}{8} = 1$

ADVANCED STUDY

1. Dolciani, Modern Algebra, Page 90 nos. 21-31. (Work at least six problems)
2. Dolciani, Modern Algebra, page 95 nos. 17-21. (Work at least 4 problems)
3. Dolciani, Modern School Mathematics, page 52 nos. 17-20. (Work all problems)
4. Research the concept of field - Select a system of numbers and determine if the system is a field. Write a report on your findings, giving reasons for your conclusions.
5. Prepare a bulletin board showing all the properties and their relationships to the following sets of numbers: naturals, wholes, integers, and rationals.

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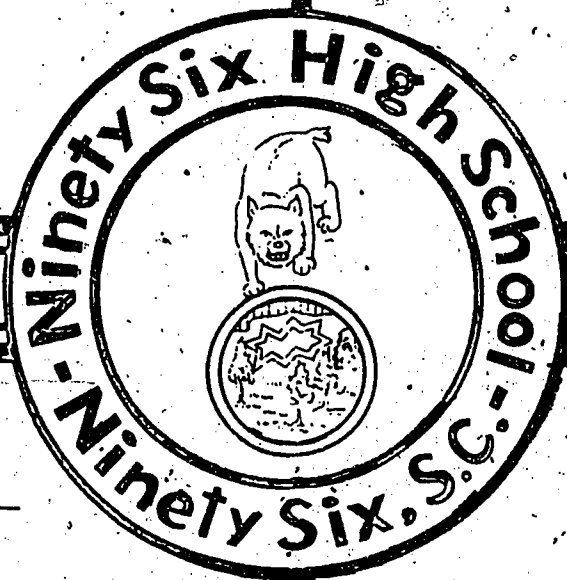
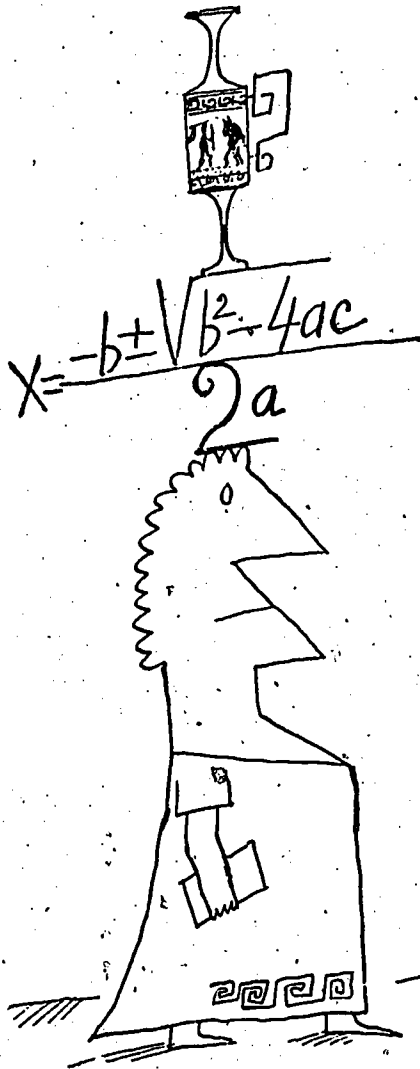
1. Vanatta, Glen D. and Goodwin, A. Wilson, Algebra One: A Modern Course, Charles E. Merrill Publishing Co., 1966.
2. Dolciani, Mary D., Berman, Simon L., and Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.
3. Nichols, Eugene D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.
4. Pearson, Helen R., and Allen, Frank B., Modern Algebra A Logical Approach, Ginn and Company, 1964.
5. Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis G., Algebra One, Harcourt, Brace and World, Inc., 1969.
6. Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics Algebra 1, Houghton, Mifflin Company, 1967.
7. Wollensak Teaching Tapes

Wollensak C-3453 The Commutative Property  
C-3454 The Associative Property  
C-3455 The Distributive Property  
C-3456 The Closure Property  
C-3459 Identity Element  
C-3451 The Inverse Element  
C-3801 Open Phrase, Open Sentence  
C-3803 Open Sentence: Solution  
C-3809 Reading Written Problems

8. Equations by Layman Allen

**L**ARNING  
**A**CTIVITY  
**P**ACKAGE

RATIONAL NUMBERS



ALGEBRA 93-94

REVIEWED BY

*al*

LAP NUMBER

3

WRITTEN BY

*Diane Evans*



### Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Mantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

### RATIONALE:

You have studied many sets of numbers through your mathematical career. The first set you discussed was the set of "NATURAL NUMBERS" (1, 2, 3, . . .). You then added zero and the set became the "WHOLE NUMBERS", after which you extended the set to include additive inverses and the set became the "INTEGERS". When you finally added the multiplicative inverses and arrived at the "RATIONAL" numbers, it appeared as though you were finished!

In this LAP you will extend the set of "RATIONAL" numbers. We will call the numbers that we ADD, the "IRRATIONAL" numbers. The set then becomes the "REAL" number system. Once you have at your disposal knowledge of the complete set of REAL numbers you will be equipped to investigate the basic concepts of elementary Algebra!

## SECTION 1

### Behavioral Objective

After having completed your prescribed course of study, you will be able to:

1. Write or identify the definition of the sets of natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.
2. Given any number, determine if it is a member of the set of
  - a. natural numbers
  - b. whole numbers
  - c. integers
  - d. rational numbers
  - e. irrational numbers
  - f. real numbers
3. Given any statement involving relationships among the sets of natural, whole, integer, rational, irrational, and real numbers, determine if it is true or false.
4. Determine if each of the sets of natural, whole, integer, and rational numbers is a field. If a set is not a field, state the properties that do not apply. Appendix I will be completed and turned in to the teacher.
5. Given a pair of integers  $a$  and  $b$ , determine whether  $a < b$ ,  $a = b$ , or  $a > b$ .
6. Given two or more rational numbers, compute their sum, difference, quotient, and/or product.

### RESOURCES

#### I. Reading and Problems. NOTE: EOL means every other letter.

1. Vanatta, Algebra One, #1, pp. 22, 80-82, Ex. 1-5, p. 83: #2, pp. 79-83, Ex. 14 p. 84: #3, pp. 79-83, Ex. 13 p. 83: #4, pp. 28-29, 85, Ex. 2, 3, 5 p. 86: #5: #6 pp. 90-106, Ex. 1-13 pp. 92-93, 1-24, p. 95, 1-16 p. 99, 1-20 odd (bottom) p. 100-101, 1-26 p. 104, 1-18 p. 106, 21-25 p. 107.
2. Dolciani, Modern Algebra, Book 1, #1, 2, 3, 4, 5: #6 pp. 125-126, 128-130, 133-135, 138-140, Ex. 1-16 p. 126, 1-12 p. 130, 1-11 p. 135, 1-12 p. 140, 1-8 p. 141.
3. Nichols, Modern Elementary Algebra, #1, 2, 3: #4 pp. 104-105, Ex. 1-3 p. 105: #5 pp. 55-59, Ex. 1-5 EOL pp. 59-61, 1-2 EOL p. 62: #6 pp. 62-63, 85-88, 92-94, 96-98, Ex. 1-2 EOL p. 64-65, 6 a - n p. 68, 3a-z p. 89, 1 a-y p. 95, 2 all p. 99.
4. Dolciani, Modern School Mathematics, Algebra I, #a pp. 22, 316, 425, Ex. #: #2, 3, 4: #5 pp. 5-8, Ex. 1-49 odd pp. 9-10: #6.

## RESOURCES(cont')

5. Payne, Algebra One, #1 pp. 2, 52-54, Ex. \_\_\_\_; #2 pp. 52-54, Ex. 1-12 p. 54, 1-15, 32-41 p. 55; #3 pp. 52-54, Ex. 13-20 p. 54, 16-30 p. 55; #4 pp. 535, 528, Ex. \_\_\_\_; #5 pp. 56-57, Ex. 1-61 odd pp. 57-58; #6 pp. 66-69, 73-74, 75-77, 79-80, 82-83, Ex. 1-51 even p. 69, 1-34 p. 74, 1-30 pp. 77-78, 10-24 p. 81, 1-12 p. 83.
6. Pearson, Modern Algebra, Book One, #1,2,3,4 \_\_\_\_; #5 pp. 37-38, 1-6 EOL pp. 38-39; #6 \_\_\_\_.
7. Arithmetic of Directed Numbers, A programmed unit, #6.
8. Supplementary LAPs-Integers, Rational Numbers, D. Evans; Ninety Six High School.

### II. Audio

Wollensak C-3458 The Real Number System  
C-3331 Directed Numbers: Addition  
C-3332 Directed Numbers: Subtraction  
C-3333 Directed Numbers: Multiplication  
C-3334 Directed Numbers: Division

### III. Visual - filmstrips

Comparing Fractions: Adding and Subtracting

Multiplying Fractions

Multiplication of Signed Numbers

Dividing Fractions

### IV. Games

The Conversion Game by Evans

Equations by Layman Allen

Crossword Puzzle - "A Short Review of Fractions"

SELF-EVALUATION 1

1 I. Write the definition or set for each of the following:

1. natural numbers
2. whole numbers
3. integers
4. rational numbers
5. irrational numbers
6. real numbers

2 II. Identify the following numbers as elements of naturals (N), wholes (W), integers (I), rationals (Q), irrationals (Z), or Reals (R). List all the sets that contain each number.

\_\_\_\_\_ 7. 28

\_\_\_\_\_ 8.  $\sqrt{9}$

\_\_\_\_\_ 9.  $\frac{3}{4}$

\_\_\_\_\_ 10.  $\sqrt{7}$

\_\_\_\_\_ 11.  $1.\bar{3}$

\_\_\_\_\_ 12. .010010001...

\_\_\_\_\_ 13. 0

\_\_\_\_\_ 14. -18

\_\_\_\_\_ 15.  $-\frac{1}{8}$

3 III. True or False.

\_\_\_\_\_ 16. The natural numbers are a subset of the whole numbers.

\_\_\_\_\_ 17. The integers are a subset of the natural numbers.

\_\_\_\_\_ 18. The whole numbers are a subset of the rationals.

\_\_\_\_\_ 19. The rational numbers contain the integers and the fractions.

\_\_\_\_\_ 20. The natural numbers are a subset of the rational numbers.

\_\_\_\_\_ 21. The integers are not a subset of the rational numbers.

Self-Evaluation (cont')

3 IV. True or False.

- N = set of natural numbers  
 W = set of whole numbers  
 I = set of integers  
 Q = set of rational numbers  
 Z = set of irrational numbers  
 R = set of real numbers

\_\_\_\_\_ 22.  $Q \subset R$

\_\_\_\_\_ 23.  $Q \cup Z = R$

\_\_\_\_\_ 24.  $W \subset N$

\_\_\_\_\_ 25.  $Z \subset R$

\_\_\_\_\_ 26.  $N \subset R$

4 V. Is each of the following sets a field? If no, write the properties necessary to make it a field.

\_\_\_\_\_ 27. whole numbers

\_\_\_\_\_ 28. integers

\_\_\_\_\_ 29. natural numbers

\_\_\_\_\_ 30. rational numbers

5 VI. In each blank write  $<$ ,  $>$ , or  $=$  to make a true statement.

31.  $7$  \_\_\_\_\_  $2$

36.  $-2$  \_\_\_\_\_  $2$

32.  $-10$  \_\_\_\_\_  $5$

37.  $3 \cdot 5$  \_\_\_\_\_  $-15$

33.  $0$  \_\_\_\_\_  $18$

38.  $-5$  \_\_\_\_\_  $3$

34.  $8 + 1$  \_\_\_\_\_  $9$

39.  $-7$  \_\_\_\_\_  $-9$

35.  $0$  \_\_\_\_\_  $-17$

40.  $10$  \_\_\_\_\_  $-10$

6 VII. Work the following:

\_\_\_\_\_ 41.  $-\frac{2}{3} + \frac{4}{5} =$

\_\_\_\_\_ 42.  $-\frac{2}{3} \cdot \frac{4}{5}$

\_\_\_\_\_ 43.  $-\frac{4}{5} + \frac{2}{3}$

\_\_\_\_\_ 44.  $-\frac{6}{5} + \frac{3}{4}$

Self-Evaluation (cont')

\_\_\_\_\_ 45.  $-\frac{2}{3} + \frac{4}{5}$

\_\_\_\_\_ 46.  $-\frac{4}{7} + \frac{3}{4}$

\_\_\_\_\_ 47.  $-\frac{2}{5} \cdot \frac{3}{4}$

\_\_\_\_\_ 48.  $-\frac{1}{6} + -\frac{2}{3}$

\_\_\_\_\_ 49.  $\frac{1}{2} - \frac{1}{7}$

\_\_\_\_\_ 50.  $\frac{2}{5} + -\frac{4}{5}$

\_\_\_\_\_ 51.  $-8 \times -7$

\_\_\_\_\_ 52.  $9 + -3$

\_\_\_\_\_ 53.  $-6 - 9$

\_\_\_\_\_ 54.  $28 + -7$

\_\_\_\_\_ 55.  $-18 + -9$

\_\_\_\_\_ 56.  $36 \cdot -2$

\_\_\_\_\_ 57.  $-6 + -12$

\_\_\_\_\_ 58.  $3 - -7$

\_\_\_\_\_ 59.  $-18 - -2$

\_\_\_\_\_ 60.  $12 + -13$



## SECTION 2

### Behavioral Objective

After having completed your prescribed course of study, you will be able to:

7. Given any rational number of the form  $\frac{a}{b}$ , express it in decimal form and state whether it is a terminating or repeating decimal.
8. Given any rational number expressed in decimal form, write it in the form  $\frac{a}{b}$  where  $a$  is a whole number and  $b$  is a natural number.
9. Given any pair of rational numbers, name the number midway between them.
10. Given any word phrase like the ones in Appendix I, translate it into an equivalent mathematical phrase.

### RESOURCES

#### I. Reading and Problems.

1. Vanatta, Algebra One, #7,8,9,10 \_\_\_\_.
2. Dolciani, Modern Algebra, #7 pp. 400-402, Ex. 1-12 even p. 403: #8 pp. 400-402, Ex. 13-20 p. 403: #9, p. 398, Ex. 15-20 p. 400: #10 \_\_\_\_.
3. Nichols, Modern Elementary Algebra, #7 pp. 30-31, 65-77, Ex. 1-4, 5-6 EOL, p. 67-78: #8 pp. 68-70, 1-2 EOL p. 70: #9 p. 71, Ex. 1 p. 73: #10 p. 79, Ex. 1-19, pp. 79-80.
4. Dolciani, Modern School Mathematics, Algebra I, #7 pp. 422-423, Ex. 1-8 p. 426: #8 pp. 424-425, Ex. 9-19 odd p. 426: #9 p. 2, Ex. 21-22 p. 3, 19-22 p. 115: #10 p. 10, Ex. 11-23 odd p. 13, 31-38 p. 33.
5. Payne, Algebra One, #7 p. 30, Ex. \_\_\_\_: #8 p. 30, Ex. 64-69 p. 30: #9 \_\_\_\_: #10 p. 128, Ex. 1-35 odd pp. 128-130.
6. Pearson, Modern Algebra, Book One, #8 pp. 268-269, Ex. \_\_\_\_: #9 pp. 35-36, Ex. 1-4 p. 36: #10 p. 145, Ex. 10, 12-14 p. 147.

#### II. Audio

Wollensak C-3801 Open Phrase, Open Sentence

## APPENDIX I

IN THIS SECTION you will learn to translate from a word phrase to a mathematical expression. This will help you when you later solve word problems. You are to fill EACH blank below with a mathematical expression. YOU ARE NOT TO DO ANY COMPUTING. (The first two problems have been completed for you as examples.)

1. The sum of 3 and the product of 2 and 6 is  $3 + 2(6)$   
(You should not write  $15$ , or  $3 + 12$ , since that requires computing!)
2. Three more than the square of  $x$  is  $x^2 + 3$
3. The sum of 5 and 9 is
4. The sum of  $\frac{2}{3}$  and  $-6$  is
5. The sum of 17 and  $x$  is
6. 5 more than 7 is
7. 18 increased by 12 is
8.  $x$  more than 10 is
9. 3 more than  $x$  is
10. The sum of  $2x$  and  $5 + 3x$  is
11.  $5t$  increased by  $5 - x$  is
12. The square of the sum of 3 and 4 is
13. The sum of the squares of 3 and 4 is
14. The square of the sum of  $2x$  and  $3y$  is
15. The sum of the squares of 5 and  $m$  is
16. Three times the square of  $x$  is
17. The square of the product of 3 and  $x$  is
18. The quotient of 17 divided by 6 is
19. The quotient of  $x$  divided by 3 is

20. The square of the opposite of 5 is \_\_\_\_\_
21. The opposite of the square of 5 is \_\_\_\_\_
22. The square of the opposite of  $x$  is \_\_\_\_\_
23. The opposite of the square of  $x$  is \_\_\_\_\_
24. Monday and \_\_\_\_\_ are consecutive days of the week.
25. Tuesday, \_\_\_\_\_ and Thursday are consecutive days of the week.
26. 1, 23, -62, and -14 are integers. -15, -16, -17, \_\_\_\_\_, and -19 are consecutive integers.
27. If  $x$  is an integer, then  $x$ ,  $x + 1$ , and \_\_\_\_\_ are consecutive integers.
28. If  $y$  is an integer, then  $y - 2$ ,  $y + 1$ ,  $y$ ,  $y + 1$ , \_\_\_\_\_, and  $y + 3$  are consecutive integers.
29. If  $t$  is an integer, then  $3t$  is an integer. Also,  $3t$ ,  $3t + 1$ , \_\_\_\_\_, \_\_\_\_\_, and  $3t + 4$  are consecutive integers.
30. -3, 0, 5, 7, and 212 are integers. -8, 0, 2, 16, -40, and 18 are even integers. If  $k$  is an even integer, then  $k + 8$  is an \_\_\_\_\_ integer. 18, 20, \_\_\_\_\_, and 24 are consecutive even integers. If  $x$  is an even integer,  $x$  and \_\_\_\_\_ are consecutive even integers. If  $t$  is an even integer, then  $t - 2$ ,  $t$ ,  $t + 2$ , \_\_\_\_\_, and  $t + 6$  are consecutive even integers.
31. 7 is an odd integer. 3, 5, 7, \_\_\_\_\_, and 11 are consecutive odd integers.
32. If  $m$  is an odd integer, then  $m$  and \_\_\_\_\_ are consecutive odd integers.
33. If  $r$  is an odd integer, then  $r$ ,  $r + 2$ , \_\_\_\_\_, and  $r + 6$  are consecutive odd integers.
34. The average of 6 and 4 is: \_\_\_\_\_
- The average of 5, 82, 16, 93, and 74 is: \_\_\_\_\_
- The average of  $a$ ,  $b$ ,  $c$ , and  $d$  is: \_\_\_\_\_

SELF-EVALUATION 2

7 I. Express the following fractions as decimals and state if they are repeating or terminating.

1.  $\frac{4}{9}$

2.  $\frac{3}{8}$

3.  $\frac{2}{11}$

4.  $\frac{2}{7}$

8 II. Express the following decimals as fractions.

5. .12

6.  $\overline{.274}$

7.  $\overline{.53}$

8. .684

9.  $\overline{.73}$

10. .82

9 III. Find the rational number midway between the following:

11.  $9\frac{1}{2}$  and  $11\frac{1}{3}$

12. 2.19 and 1.11

13. -3.12 and 3.76

14.  $\frac{1}{6}$  and  $\frac{3}{24}$

15.  $\frac{3}{4}$  and  $\frac{15}{16}$

Self-Evaluation (cont')

10 IV. Write the mathematical phrase of each word phrase.

\_\_\_\_\_ 16. sum of 17 and  $x$

\_\_\_\_\_ 17. 3 more than  $x$

\_\_\_\_\_ 18. the square of the sum of 3 and  $x$  is

\_\_\_\_\_ 19. three times the square of  $x$

\_\_\_\_\_ 20. the quotient of  $x$  divided by  $2y$

\_\_\_\_\_ 21. \_\_\_\_\_ is the next consecutive odd integer after  $x$

\_\_\_\_\_ 22.  $t$  is an integer, give the next three consecutive integers

\_\_\_\_\_ 23. number of feet in  $7t$  yards.

\_\_\_\_\_ 24. number of quarts in  $(a + 3t)$  gallons

\_\_\_\_\_ 25. worth in cents of  $y$  eight-cent stamps

APPENDIX I (Obj. 4)

Put an x by each property that holds for the given sets of numbers.  
Put a circle (0) by each property that does not hold. Do not leave a blank.

PROPERTY	NATURALS	WHOLE	INTEGERS	RATIONALS	REALS
Closure for +					
Closure for x					
Commutative +					
Commutative x					
Associative +					
Associative x					
Distributive					
Add. Identity					
Mult. Identity					
Add. Inverses					
Mult. Inverses					

#### REFERENCES

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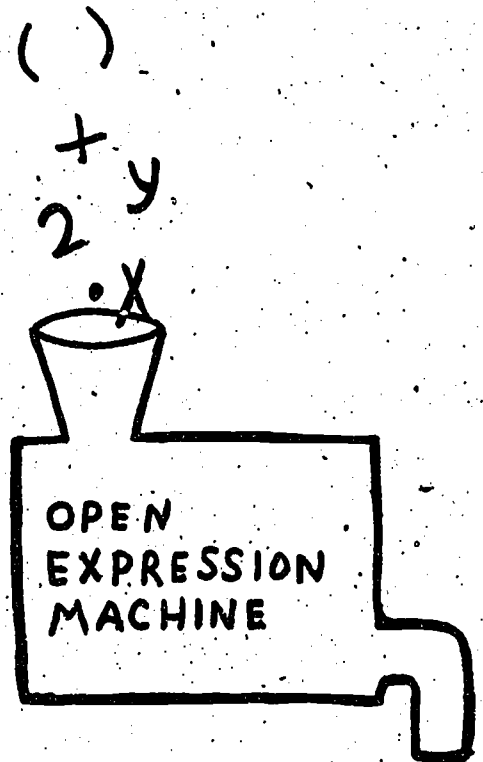
Nichols, Eugene D., Arithmetic of Directed Numbers, Holt, Rinehart and Winston, 1962.

Wollensak teaching tape C-3801.

Equations, a game by Layman Allen.



**L** EARNING  
**A** CTIVITY  
**P** ACKAGE



OPEN  
EXPRESSIONS

$$2 \cdot (x + y) = 2x + 2y$$



ALGEBRA 93-94

VIEWED BY

*J. Ketcher*

LAP NUMBER 4

WRITTEN BY Diane Evans

### RATIONALE (The LAP's Purpose)

One of the most important concepts in the study of Algebra is that of a variable. In this LAP you will study terms and expressions, most of which contain one or more variables. Using many of the previously introduced properties and definitions, you will learn to apply new theorems, listed on the next page, which are concerned with equivalent expressions.

You will develop the ability to judge whether two given expressions are equivalent. This skill is necessary in solving equations. While you are developing this skill, you will begin to learn how to prove theorems.

## PROPERTIES AND THEOREMS

For every number  $x$ ,  $y$ , and  $z$ , the following apply:

Distributive property of multiplication over addition

$$x(y + z) = xy + xz$$

$$xy + xz = x(y + z)$$

$$(y + z)x = yx + zx$$

Distributive property multiplication over subtraction

$$x(y - z) = xy - xz$$

$$(y - z)x = yx - zx$$

$$xy - xz = x(y - z)$$

Multiplication by  $-1$        $x(-1) = -x$

Division by  $-1$        $\frac{x}{-1} = -x$

Opposite of  $x - y$        $-(x - y) = y - x$

Opposite of  $x + y$        $-(x + y) = -x - y$

$$(-x)y = -(xy)$$

Some additional theorems to be covered in this LAP

$$-(-x) = x$$

$$-(-x)y = xy$$

$$(-x)(-y) = xy$$

THEOREMS TO BE DEVELOPED IN THIS LAP

$$\forall x \forall y \neq 0 \forall z \quad \frac{xz}{y} = \left(\frac{x}{y}\right) (z)$$

$$\forall x \forall y \neq 0 \forall z \quad \frac{xz}{y} = (x) \left(\frac{z}{y}\right)$$

$$\forall x \forall y \neq 0 \forall z \quad \frac{xz}{y} = \left(\frac{1}{y}\right) (xz)$$

$$\forall x \neq 0 \forall y \neq 0 \quad \left(\frac{1}{x}\right) \left(\frac{1}{y}\right) = \frac{1}{xy}$$

$$\forall x \forall y \neq 0 \forall r \forall s \neq 0 \quad \left(\frac{x}{y}\right) \left(\frac{r}{s}\right) = \frac{xr}{ys}$$

(Multiplicative Identity Theorem)  $\forall x \forall y \neq 0 \forall r \neq 0 \quad \frac{xy}{yr} = \frac{x}{y}$

(Addition Theorem)  $\forall x \forall y \neq 0 \forall r \forall s \neq 0 \quad \frac{x}{y} + \frac{r}{s} = \frac{xs + ry}{ys}$

(Subtraction Theorem)  $\forall x \forall y \neq 0 \forall r \forall s \neq 0 \quad \frac{x}{y} - \frac{r}{s} = \frac{xs - ry}{ys}$

(Division Theorem)  $\forall x \forall y \neq 0 \forall r \neq 0 \forall s \neq 0 \quad \frac{\frac{x}{y}}{\frac{r}{s}} = \frac{x}{y} \cdot \frac{s}{r}$

$$\forall x \forall y \neq 0 \quad \frac{-x}{y} = -\frac{x}{y}$$

$$\forall x \forall y \neq 0 \quad \frac{x}{-y} = -\frac{x}{y}$$

$$\forall x \forall y \neq 0 \quad \frac{-x}{-y} = \frac{x}{y}$$

$$\forall x \forall y \neq 0 \quad -\left(\frac{x}{-y}\right) = \frac{x}{y}$$

$$\forall x \forall y \neq 0 \quad -\left(\frac{-x}{y}\right) = \frac{x}{y}$$

$$\forall x \forall y \neq 0 \forall r \neq 0 \forall s \neq 0 \quad \frac{x \left(\frac{-y}{r}\right)}{r \left(\frac{-s}{y}\right)} = \frac{xy}{rs}$$

## SECTION 1

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given any polynomial, classify it as
  - a. monomial
  - b. binomial
  - c. trinomial
2. Given any open expression and replacements for the variables, compute the value of the expression.
3. Given a pair of expressions, determine whether or not they are equivalent.
4. Using the appropriate properties, definitions, and theorems, write equivalent expressions for any given expression.
5. Given a pair of rational expressions, write a single equivalent expression that names their product.
6. Given a rational expression, use the multiplicative identity theorem to write a single equivalent expression where the numerator and denominator have no common factors.
7. Given a pair of rational expressions, write a single equivalent expression that names their sum.

### RESOURCES

Obj. 1, 2

Vanatta, #1 read p. 67-71, Ex. 4 p. 71; #2 read p. 72, Ex. 1-27 p. 73.

Dolciani, Modern Algebra, Bk. 1, #1 \_\_\_\_: #2 read pp. 36-37, Ex. 1-9 written p. 37, 37-46 p. 43.

Nichols, #1 and 2, read pp. 119-122, Ex. 1, 2 pp. 121-124.

Wooton, #1 and 2, read pp. 52-54, Ex. 1-19 oral p. 54.

Payne, #1 and 2, read pp. 307-309, Ex. 1-10, and 18-27, pp 309-310.

Pearson, #1 and 2, read pp. 240-241, Ex. 1, 2 p. 241, and 3 p. 243.

Obj. 3

Nichols, read pp. 124-132, Ex. 6, 7 pp. 127-128; 1-12 pp. 130-131; 4 p. 132.

Payne, read pp. 86-87, Ex. 1-19 pp. 87-88.

RESOURCES (Cont')

Obj. 4

Vanatta, read pp. 67-71, Ex. 5 p. 71, nos. 1-20 even p. 125,  
no. 3 p. 75.

Dolciani, pp. \_\_\_\_, Ex. 1-30 even p. 79.

Nichols, read pp. 133-137, Ex. 2-6 pp. 135-136.

Wooton, read pp. 55-57, Ex. 7-24 p. 58, 1-28 p. 59.

Payne, read pp. 88-90, Ex. 1-20 p. 90, 39-43 p. 92.

Pearson, read pp. 242-243, Ex. 1-27 p. 242, nos. 1, 2 p. 243

Obj. 5

Vanatta, read pages 320-321, Ex. 1-5, 7-10 page 322.

Dolciani, MA, read pages 292, Ex. 1-20 even (oral) page 293.

Nichols, read pages 146-150, Ex. 1,2 p. 148, 3a,c,d,f,g,i page 148;  
3a,c,e,g,i, 4 page 150.

Payne, read pp. 386-394, Ex. 1-9, 11-31 odd, pages 387-388; 1-11,15,  
17,19 pages 394-395.

Wooton, MSM, read pp. 320-323, 328-330, Ex. 1-10(written) pp. 323-324;  
1-10 p. 330; 1-7,9,16 p. 331.

Pearson, read pp. 397-400, 401-402; Ex. 1,2,4abdfhkmq page 398; ladfhkio;  
page 400; 1-4, 7, 8, 10, 12, 16, 20-22 pages 402-403.

Obj. 6

Nichols, read page 151, Ex. lacdeg, 2 page 152.

Payne, read pp. 405-408, Ex. 1-3, 5-21 odd, 22, 25 page 408.

Obj. 7

Vanatta, read pp. 236-330, 331-332, 333-335, Ex. 1,2,3,6,8 page 328;  
1,4,5,7,9,10,11,12,16 page 330; 1,3,4,6,8,9,10,11,13,15 page 333;  
1,3,5,6,8,9,10,12,14,15,23,24 page 335.

Dolciani, MA, read pp. 298-300, Ex. 1-14 even pages 298-299; 1-16 even,  
20 p. 301.

Nichols, read pp. 152-153, Ex. 1a,c,e,g,i,j,1 page 153.

Wooton, MSM, read pp. 332-336, Ex. 7,12 page 334; 1,3,5,7,10,11,13,15  
pages 337-338.

Payne, read pp. 398-403, Ex. 1,4,5,7,12,13,15,16,18,20,23,27,28,32,35,  
36,39,48,50,54,60,62,64,66 pages 403-405.

Pearson, read pp. 403-405, Ex. 2,4,7,8,12 page 405.

SELF-EVALUATION

OBJ. 1

Classify the following as monomials, binomials, or trinomials.

- \_\_\_\_\_ 1.  $x^5$
- \_\_\_\_\_ 2.  $-x^4 + 6x^2$
- \_\_\_\_\_ 3.  $5 - 4x + 2y$
- \_\_\_\_\_ 4.  $456x^4y^5$
- \_\_\_\_\_ 5.  $4x^3y^2x^5 + 6$

OBJ. 2

Determine the value of each expression if the replacement for  $x$  is 5 and the replacement for  $y$  is -3.

- \_\_\_\_\_ 6.  $5x^2$
- \_\_\_\_\_ 7.  $x + y$
- \_\_\_\_\_ 8.  $4x^2 + 3y$
- \_\_\_\_\_ 9.  $\frac{x^2 + y}{x}$
- \_\_\_\_\_ 10.  $\frac{x^3 - y^3}{y}$

OBJ. 3

Given the following pair of expressions, are they equivalent? Write Yes or No.

- \_\_\_\_\_ 11.  $x(-y)$  and  $(-x)y$
- \_\_\_\_\_ 12.  $4 - x$  and  $x - 4$
- \_\_\_\_\_ 13.  $-(a - b)$  and  $b - a$
- \_\_\_\_\_ 14.  $-(a + b)$  and  $b + a$
- \_\_\_\_\_ 15.  $(a + b)^3$  and  $a^3 + b^3$



SELF-EVALUATION (cont.)

Obj. 4

Change each of the following expressions to equivalent expressions with the least number of terms (simplify)

- |           |   |           |                           |
|-----------|---|-----------|---------------------------|
| _____ 16. | $4x + 3x$   | _____ 22. | $4xy - xy + 3yx + 4y$     |
| _____ 17. | $7y - 8y$   | _____ 23. | $19m + m - 3n + 7n + 2$   |
| _____ 18. | $-5(x - 4y) + 5(x - y)$                               | _____ 24. | $3r + 5x - 6 - 7r + 4$    |
| _____ 19. | $c - 3d - c + 4$                                      | _____ 25. | $x - 2y - x + y$          |
| _____ 20. | $-\frac{3}{4}x - 7 + \frac{2}{5}y + \frac{5}{8}x + 5$ | _____ 26. | $-(3x + y) + (2x - 3y)$   |
| _____ 21. | $4(2x - 5z) - 2(3x - 10z)$                            | _____ 27. | $-2(4a + 3b) - (5a - 2b)$ |

Obj. 5

For each of the following, write its equivalent in a single expression.

- |     |                                   |     |                                     |     |   |
|-----|-----------------------------------|-----|-------------------------------------|-----|---|
| 28. | $\frac{1}{3} \cdot \frac{2}{7}$   | 29. | $-\frac{1}{2} \cdot \frac{5}{3}$    | 30. | $-\frac{2}{3} \cdot \frac{5}{7}$  |
| 31. | $\frac{6}{y^2} \cdot \frac{y}{2}$ | 32. | $\frac{a-b}{2} \cdot \frac{8}{a-b}$ | 33. | $\frac{5ab^2}{21c^2} \cdot \frac{3bc^2}{8a^2} \cdot \frac{7a^2}{30c^4}$ |

Obj. 6

Simplify.

- |     |                                     |     |                                     |     |                                       |
|-----|-------------------------------------|-----|-------------------------------------|-----|---------------------------------------|
| 35. | $\frac{2-a}{a} \cdot \frac{a}{3-a}$ | 36. | $\frac{7}{6} \cdot \frac{-6}{6}$    | 37. | $\frac{1}{xy} \cdot \frac{x(y-1)}{2}$ |
| 38. | $\frac{2a}{a} \cdot \frac{b}{4}$    | 39. | $\frac{4+a}{5} \cdot \frac{5}{4-a}$ | 40. | $\frac{3xy}{7} \cdot \frac{7}{2y}$    |

Obj. 7

Compute the following:

- |     |                             |     |                                 |     |                               |
|-----|-----------------------------|-----|---------------------------------|-----|-------------------------------|
| 41. | $\frac{2}{3} + \frac{1}{7}$ | 42. | $\frac{a}{b} \cdot \frac{x}{y}$ | 43. | $\frac{-3}{4} + \frac{-1}{2}$ |
| 44. | $\frac{a}{7} + \frac{a}{3}$ | 45. | $\frac{2x}{5y} + \frac{x}{2y}$  | 46. | $\frac{3}{x} + \frac{4}{y}$   |

SELF-EVALUATION (cont')

Answer true or false to the following:

bj.  
5

\_\_\_\_\_ 47.  $\frac{3}{5} \cdot \frac{4}{5} = \frac{12}{5}$

\_\_\_\_\_ 48.  $(-\frac{3}{4})(-2) = \frac{(-3)(-2)}{4}$

\_\_\_\_\_ 49.  $\frac{3}{x} \cdot \frac{2}{y} = \frac{3y}{2x}$

\_\_\_\_\_ 50.  $(-x) \cdot y \cdot (-\frac{1}{y}) = x$

6

\_\_\_\_\_ 51.  $\frac{x-3}{x-2} = \frac{3}{2}$

\_\_\_\_\_ 52.  $\frac{2}{-3} \cdot \frac{-3}{5} = \frac{2}{5}$

\_\_\_\_\_ 53.  $\frac{3m}{7+m} = \frac{3}{7}$

6

\_\_\_\_\_ 54.  $\frac{4y}{5y} = \frac{4}{5}$

\_\_\_\_\_ 55.  $\frac{(-7)x2}{5x(-7)} = \frac{2}{5}$

\_\_\_\_\_ 56.  $\frac{3-x}{4-x} = \frac{3}{4}$

7

\_\_\_\_\_ 57.  $\frac{2}{4} + \frac{5}{3} = \frac{7}{7} = 1$

\_\_\_\_\_ 58.  $\frac{3}{2x} + \frac{4}{3x} = \frac{17}{6x}$

\_\_\_\_\_ 59.  $\frac{2}{x} + \frac{1}{y} = \frac{3}{xy}$

\_\_\_\_\_ 60.  $\frac{x}{3} + \frac{k}{4} = \frac{4x+3k}{12}$

\_\_\_\_\_ 61.  $\frac{-2}{3} + \frac{4}{7} = \frac{2}{21}$

\_\_\_\_\_ 62.  $\frac{-3k}{2} + \frac{-k}{4} = \frac{-7k}{4}$

If you have satisfactorily completed your work, take the Progress Test.  
Consult your teacher first.

## Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Given a pair of rational expressions, write a single equivalent expression that names their difference.
9. Given a pair of rational expressions, write a single equivalent expression that names their quotient.
10. Given a pair of rational expressions which involves the additive inverse of an expression, write a single expression equivalent to it.
11. Given any complex rational expression, use the appropriate properties, theorems, and definitions to write a single expression-equivalent to it.
12. Given a word phrase, change it to an equivalent mathematical phrase.

## RESOURCES

### Obj. 3.

Vanatta, read pp. 326-335, Ex. 4,5,7,9-12, page 328; 2,3,6,8,13,14,15, 17,18 page 330; 2,5,7,12,14 page 333; 2,4,7,11,13,16,18,20,21,25 page 335.

Dolciani, MA, read pp. 298-300, Ex. 1-14 odd pages 298-299; 1-16 odd, 19, 21, 22, page 301.

Wooton, MSM, read pp. 332-336, Ex. 1-6,9,11 page 334; 4,8,9,12,14, 16,17,18 pages 337-338.

Payne, read pp. 398-403, Ex. 8,9,11,21,22,24,29,30,31,33,38,41,44, 56,60,61,63,65 pages 403-405.

Pearson, read pp. 403-405, Ex. 3,5,6,9-11 page 405.

### Obj. 9

Vanatta, read pp. 323-324, Ex. 1-15 pp. 324-325.

Dolciani, MA, read p. 295, Ex. 1-12 p. 295; 1-4 pp. 296-297.

Nichols, read pp. 154-155, Ex. 1 a,b,d,e,g,i,k,l, and 2 a,b,e,f,h, i,j,l,m pages 156-160.

Payne, read pp. 396-397, Ex. 1-22 even page 397.

Pearson, read pp. 401-402, Ex. 5,9,13,14,15,17,18 pages 402-403.

### Obj. 10

Nichols, read pp. 154-155, Ex. 7 a,c,d,f,g,j,l; 8 a,b,c,d,g,h,i,l, m,o,p,q,s,u,v; 10 a,c,d,g,j,k,o,q, pages 158-160.

RESOURCES (cont.)

Obj. 11

Vanatta, read pp. 341-342, Ex. 1-7, 10, 12, 13 page 343.

Dolciani, MA, read p. 304, ex. 1-8, 11, 12, 15, 16, 23, page 305.

Nichols, read pp. 161-163, Ex. 9 a,b,c,e,f,g,h,i,j,k,l,m,n,q,r,t;  
11 a,c,d,e,f,g,n,p,q,r,s,i,l,m; 12 a,c,e,f, pages 159-161.

Ex. 1 a,c,f,h,j,m,n,p; 2 a,c,d,f,g,h,k,l,m,o,p,q,s,t,v,x,y,z,  
a',b',c',e',g',i',j',k',l', pages 163-164.

Payne, read pages 409, Ex. 1-11, 19, 21, 23, 25, 27, 29, 35 pages  
410-411.

Pearson, read pages 406-407 (examples 7 and 8), Ex. 46 a,c,e,f,h,  
j page 407.

Obj. 12

Vanatta, pp. \_\_\_\_, Ex. 1-6 p. 74, 1 p. 75, 1 p. 77.

Dolciani, read pp. \_\_\_\_, Ex. 1-24 p. 42.

Nichols, read page 165, Ex. 1 all parts; 2 a, c, d, f, g, k, n, o;  
3 a, c, f, h, i, l, m, n, q, r, u, w, x; z, a', c' pages 165-167.

Wollensak Teaching Tapes C-3801: Open Phrase  
C-3802: Open Sentence

\* Appendix

\* Required (turn in to teacher)

SELF-EVALUATION 2

OBJ. I. For each of the following expressions, write its equivalent in a single expression:

8

(1)  $\frac{5}{8} - \frac{2}{3}$

(2)  $\frac{1}{2} - \frac{2a-1}{a}$

(3)  $\frac{x}{y} - \frac{-2}{3}$

(4)  $\frac{5b}{3y} - \frac{3b}{4y}$

(5)  $\frac{2}{x} - 5$

(6)  $\frac{-3}{x} - \frac{-2}{y}$

9

(7)  $x + \frac{2}{3}$

(8)  $\frac{a}{b} + \frac{1}{2b}$

(9)  $\frac{-3}{4} + \frac{m}{n}$

(10)  $\frac{2}{3} + \frac{5}{6}$

(11)  $\frac{-a}{b} + \frac{c}{-d}$

(12)  $\frac{3}{2} + \frac{x-y}{2}$

10

(13)  $-\frac{-5}{-3}$

(14)  $-\frac{-5x}{3x}$

(15)  $-\frac{4w}{-r}$

11

(16)  $\frac{12}{\frac{6}{x}}$

(17)  $\frac{\frac{x}{y} + 1}{\frac{x}{y} - 1}$

(18)  $\frac{\frac{x+y}{x-y}}{\frac{x-y}{x+y}}$

(19)  $\frac{\frac{a}{x} + \frac{a}{y}}{\frac{b}{x} - \frac{b}{y}}$

(20)  $\frac{2 - \frac{1}{x}}{4 - \frac{1}{x}}$

(21)  $\frac{x}{1 + \frac{1}{x}}$

II. True or False.

8

22.  $\frac{1}{y} - 2 = \frac{1-2}{y} = \frac{-1}{y}$

23.  $\frac{a}{2} - \frac{b}{8} = \frac{a-b}{6}$

24.  $\frac{-x}{3} - \frac{y}{6} = \frac{-2x-y}{6}$

25.  $\frac{3a}{2b} - \frac{5b}{6a} = \frac{9a^2-5b^2}{6ab}$

26.  $\frac{a}{2} - \frac{-a}{3} = \frac{5a}{6}$

SELF-EVALUATION 2 (cont')

- \_\_\_\_\_ 27.  $\frac{-2}{3} - \frac{4}{2} = \frac{-6}{5}$
- \_\_\_\_\_ 28.  $\frac{3}{4} - \frac{-2}{3} = \frac{17}{12}$
- 9 \_\_\_\_\_ 29.  $\frac{3}{4} + \frac{a}{b} = \frac{3a}{4b}$
- \_\_\_\_\_ 30.  $\frac{x}{y} + \frac{a}{b} = \frac{xb}{ya}$
- \_\_\_\_\_ 31.  $\frac{3}{5} + \frac{x}{y} = \frac{5x}{3y}$
- 10 \_\_\_\_\_ 32.  $\frac{(-x)3}{y(-7)} = \frac{-3x}{7y}$
- \_\_\_\_\_ 33.  $-\frac{1}{r-s} = \frac{1}{s-r}$
- \_\_\_\_\_ 34.  $-\frac{1}{x+y} = \frac{1}{-x-y}$
- \_\_\_\_\_ 35.  $-\frac{-3x}{-2y} = \frac{3x}{2y}$
- 11 \_\_\_\_\_ 36.  $\frac{\frac{1}{2}}{3} = \frac{3}{4}$
- \_\_\_\_\_ 37.  $\frac{\frac{a-b}{a+b}}{\frac{a-b}{a+b}} = 1$
- \_\_\_\_\_ 38.  $\frac{\frac{a+b}{2}}{6} = \frac{1}{3}$
- \_\_\_\_\_ 39.  $\frac{\frac{a+b}{x} - \frac{3}{2}}{\frac{x}{y} + \frac{3}{2}} = \frac{2x - 3y}{2x + 3y}$
- \_\_\_\_\_ 40.  $\frac{1}{1 + \frac{1}{x}} = \frac{x}{x+1}$

12 III. Change the following word phrases to equivalent mathematical phrase.

- \_\_\_\_\_ 41. The product of seven and the sum of some number and five.
- \_\_\_\_\_ 42. The sum of x and twice y.

VALUATION 2 (cont.)

- \_\_\_\_\_ 43. Two-hundred dollars less than one-third of last year's salary.
- \_\_\_\_\_ 44. The difference of  $4t$  and  $5$  multiplied by the sum of  $4t$  and  $6$ .
- \_\_\_\_\_ 45. Number of feet in  $18y$  inches.
- \_\_\_\_\_ 46. The number of pints in  $3y$  quarts.
- \_\_\_\_\_ 47. Worth in cents of  $(4y - 1)$  nickels.
- \_\_\_\_\_ 48. Number of inches in the perimeter of a square with  $x$  feet for the length of the side.

If you have satisfactorily completed your work, you may take your  
LAP TEST. Consult your teacher first.



APPENDIX  
(to be turned in to the teacher)

1. 3 dimes are worth \_\_\_\_\_ cents.
2.  $x$  dimes are worth \_\_\_\_\_ cents.
3.  $7x$  dimes are worth \_\_\_\_\_ cents.
4.  $\frac{x}{2}$  dimes are worth \_\_\_\_\_ cents.
5.  $x - 4$  dimes are worth \_\_\_\_\_ cents.
6. 7 3-cent stamps are worth \_\_\_\_\_ cents.
7.  $k + 4$  3-cent stamps are worth \_\_\_\_\_ cents.
8. 50 7-cent stamps are worth \_\_\_\_\_ cents.
9. If I have 3 nickels, and 4 dimes and 2 quarters, then I have \_\_\_\_\_  
coins worth \_\_\_\_\_ cents.
10. If I have 4 nickels and  $x$  dimes and 3 quarters, then I have \_\_\_\_\_  
coins worth \_\_\_\_\_ cents.
11. If I have  $x$  nickels and  $3x$  dimes and  $x + 2$  quarters, then I \_\_\_\_\_  
have coins worth \_\_\_\_\_ cents.
12. Al is 12 years old. 5 years ago he was \_\_\_\_\_ years old and  
8 years from now he will be \_\_\_\_\_ years old. 3 times his present  
age is \_\_\_\_\_. Bill is 4 years younger than Al. Bill  
is \_\_\_\_\_ years old.
13. Ed is  $x$  years old. 3 years from now he will be \_\_\_\_\_ years  
old and 2 years ago he was \_\_\_\_\_ years old. Dave is 4 times  
as old as Ed is now. Dave is \_\_\_\_\_ years old. 2 years ago  
he was \_\_\_\_\_ years old. Hal is 2 years younger than Ed.  
Hal is \_\_\_\_\_ years old. In 5 years he will be \_\_\_\_\_  
years old. Sam is 6 years older than Dave. Sam is \_\_\_\_\_ years old.

ADVANCED STUDY

I. Complete the following proofs by writing the correct reason in the blank space provided.

(1) Prove:  $\forall x, y \neq 0, \forall r, s \neq 0, \left(\frac{x}{y}\right) \left(\frac{r}{s}\right) = \frac{xr}{ys}$

Proof: Statements	Reasons
a. $\left(\frac{x}{y}\right) \left(\frac{r}{s}\right) = \left[x \left(\frac{1}{y}\right)\right] \left[r \left(\frac{1}{s}\right)\right]$	_____
b. $= x \left[\left(\frac{1}{y}\right)r\right] \left(\frac{1}{s}\right)$	_____
c. $= x \left[r \left(\frac{1}{y}\right)\right] \left(\frac{1}{s}\right)$	_____
d. $= (xr) \left(\frac{1}{y} \cdot \frac{1}{s}\right)$	_____
e. $= (xr) \left(\frac{1}{ys}\right)$	_____
f. $= \frac{xr}{ys}$	_____

(2) Prove:  $\forall x, r, y \neq 0, \forall s \neq 0, \frac{x}{y} + \frac{r}{s} = \frac{xs + ry}{ys}$

Proof: Statements	Reasons
a. $\frac{x}{y} + \frac{r}{s} = \frac{xs}{ys} + \frac{ry}{sy}$	_____
b. $= \frac{xs}{ys} + \frac{ry}{ys}$	_____
c. $= (xs) \left(\frac{1}{ys}\right) + (ry) \left(\frac{1}{ys}\right)$	_____
d. $= (xs + ry) \left(\frac{1}{ys}\right)$	_____
e. $= \frac{xs + ry}{ys}$	_____

(3) Prove:  $\forall x, y \neq 0, \forall r, s \neq 0, \frac{x}{y} - \frac{r}{s} = \frac{xs - ry}{ys}$

Proof: Statements	Reasons
a. $\frac{x}{y} - \frac{r}{s} = \frac{xs}{ys} - \frac{ry}{sy}$	_____
b. $= \frac{xs}{ys} + \left(-\frac{rs}{sy}\right)$	_____
c. $= \frac{xs}{ys} + \left(-\frac{rs}{sy}\right)$	_____
d. $= \frac{xs}{ys} + \left(-\frac{rs}{ys}\right)$	_____
e. $= \frac{xs + (-rs)}{ys}$	_____
f. $= \frac{xs - rs}{ys}$	_____

ADVANCED STUDY (cont')

II. Work Problems 1-16 page 325, Vanatta.

III. Dolciani, p. 319, Just. for Fun.

IV. Dolciani, pp. 328-330, Extra for Experts.

V. Work the following:

$$(1) \quad 5x - \frac{3}{5x - \frac{3}{5x}}$$

$$(2) \quad \left[ \frac{2x+1}{x} - \frac{x}{2x+1} \right] \cdot \left[ \frac{5x-1}{x} + \frac{x}{5x-1} \right]$$

$$(3) \quad \left[ \frac{4}{x+1} + \frac{3}{x-2} \right] \cdot \frac{x+3}{7x-5}$$

$$(4) \quad \frac{2 + \frac{5a}{a+2b}}{\frac{3a}{a+2b}}$$

VI. Prepare a chart using a Venn diagram showing the relationships among polynomials, monomials, binomials, and trinomials.

VII. Dolciani, Modern Algebra, Bk. 1, work any ten problems from 1-15 on pages 43-44.

VIII. Nichols, page 132, number 7.

IX. Nichols, page 141, number 3.

## REFERENCES

### Nichols (abbreviation)

Nichols, Eugene, D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

### Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Ginn and Company, 1964.

### Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis, Algebra One, Harcourt, Brace and World, Inc., 1969.

### Wooton, MSM (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics, Algebra 1.

### Dolciani, MA (abbreviation)\*

Dolciani, Mary P., Berman, Simon L., Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

### Vanatta (abbreviation)

Vanatta, Glen D., Goodwin, A. Wilson, Algebra One, A Modern Course, Charles E. Merrill Publishing Inc., 1966.

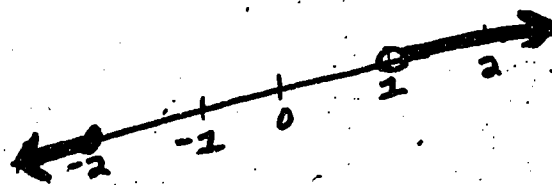
Wollensak teaching tape C-3801 - Open Phrase .  
C-3802 - Open Sentence.

**L** EARNING

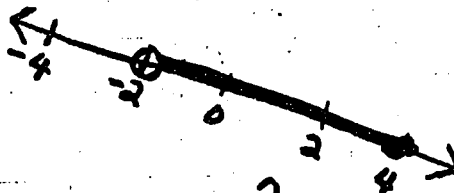
$$x + 2 = 5$$

**A** CTIVITY

**P** ACKAGE

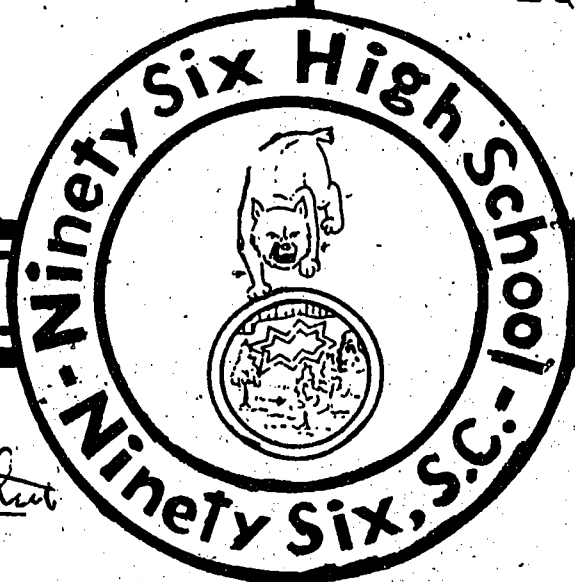


$$2x + 6 \leq 12$$



$$-3x + 4 = 6x - 2$$

SOLUTION SETS OF  
EQUATIONS AND  
INEQUALITIES



Algebra 93-94

REVIEWED BY

*J. R. [Signature]*

LAP NUMBER

7

WRITTEN BY

*Marian Evans*

### Acknowledgement

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its original form.

## RATIONALE

In daily life you most often express yourself in English sentences. Because of the importance of clean and effective communication, a great deal of time in school is spent studying the English language.

In mathematics, ideas are expressed in a combination of English sentences and special mathematical sentences. Mathematical sentences consist of mathematical symbols rather than WORDS. An example of a mathematical sentence is  $3x + 5 = 9$ . An understanding of the types and properties of mathematical sentences is essential to your advancing in mathematics.

## SECTION I.

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given any mathematical sentence, classify it as being true, false, or neither.
2. Given any linear equation where the solution is dependent on the addition and/or the subtraction property determine the solution set showing all steps and giving reasons.
3. Given any linear equation where the solution is dependent on the multiplication and/or division property, determine the solution set showing all steps and giving reasons.
4. Given any linear equation whose solution is dependent upon a combined use of addition, subtraction, multiplication, and/or division properties; determine the solution set showing all steps and giving reasons. Appendix I will be completed and turned in to the teacher.

### RESOURCES

#### Obj. 1

\* Nichols, read pp. 174-176, Ex. 1-21 odd pages 175-176.

Payne, read pp. 20-22, 101-104, 110-112, 114-116, Ex. 1-12 page 22; 1-10 page 104; 11-20 pages 111-112; 1-18 page 116.

Wooton, read pp. 29-33, Ex. 1-9 page 33.

Pearson, read pp. 57-59, Ex. 3 page 58.

#### Obj. 2

Vanatta, read pp. 47-52, Ex. 1, 2, 6, 7, 13, 14, 17, 18, 19, page 53.

Dolciani, read pp. 80-82, Ex. 1-30 odd p. 83.

Nichols, read pp. 188-191, 177, Ex. 1-48 every 4th problem pages 190-191; 1-10 page 177.

Payne, read pp. 101-104, 110-112, Ex. 11-20 pages 104-105; 1-10 page 111.

Wooton, read pp. 116-118, Ex. 1-10 page 119.

Pearson, read pages 151-152, Ex. 1 a, f, h page 152; 3 d, e, f, g, h, i, m, n, o, s, t page 152.

\* required



RESOURCES 1 (cont')

Transparency: Properties of Equality (3M)

Games: Equations

Obj. 3

Vanatta, read pp. 47-52, Ex. 3,4,5,8-12,15,16, and 20 page 53.

Dolciani, read pp. 83-84, Ex. 1-20 even pages 84-85.

Nichols, read pp. 191-192, Ex. 1-41 odd p. 192; 11-29 odd pages 177-178.

Wooton, read pp. 116-120, Ex. 11-26(written) pages 119-120.

Payne, read pp. 105-107, Ex. 1-23 odd p. 106.

Pearson, read pp. 150-153, Ex. 1 b,c,d,e,i,j; 2 and 3 a,b,c,j,k, l,p,q,r; and 4 pages 152-153.

Games: Equations

Obj. 4

Vanatta, read pp. 53-54, Ex. 5-20 p. 55.

Dolciani, read pp. 86-87, 91-93, Ex. 1-4, 11-15, 29-33 page 88; 1-26 odd (written) p. 93.

Nichols, read pp. 193-196, 178-180; Ex. 1-45 (every fourth one) pp. 195-196; 1-39 every fourth one pp. 179-180.

Wooton, read pp. 116-120; 131-134, Ex. 27-55 odd page 120; 1-39 odd page 134.

Payne, read pp. 107-110, pp. 112-114, Ex. 1-55 every 4th one pages 109-110; 1-39 every fourth one pages 113-114.

Pearson, read pp. 154-156, pp. 174-175, Ex. 1 EOL, 3,6,9 pages 155-156; 1 a,c,e,g,i,h; 2 a,c,e,g; 3 a,c,e,g; 4 a,c,e,g,i page 175.

\* Appendix I

Audio Tapes: C-3801 Open Phrase, Open Sentence

C-3803 Open Sentence: Solution

Filmstrip: Proof in Algebra: Solving Equations

Games: Equations

\* REQUIRED

SELF-EVALUATION 1

Obj.

1. I. Classify each sentence into one of the following categories:

T if the sentence is true.

F if the sentence is false.

R if at least one replacement, but not every replacement, for the variable or variables will result in a true statement.

N if the sentence is neither true or false and there is no replacement for the variable or variables which will result in a true statement.

E if the sentence is neither true or false and every replacement for the variable or variables will result in a true sentence.

\_\_\_ a.  $x + 1 = 2x$

\_\_\_ j.  $d^2 = -4$

\_\_\_ b.  $2 = 1.4$

\_\_\_ k.  $\frac{1}{8} = .125$

\_\_\_ c.  $9 = 3 \times 3$

\_\_\_ l.  $\frac{1}{3} = 33\frac{1}{3}\%$

\_\_\_ d.  $12 = 6 \times 6$

\_\_\_ m.  $\frac{2n}{n} = 3$

\_\_\_ e.  $1y + 11 = 10$

\_\_\_ n.  $-(x - y) = y - x$

\_\_\_ f.  $1\% = .1$

\_\_\_ o.  $-1(x - 4) = 4 - x$

\_\_\_ g.  $a = 2a$

\_\_\_ p.  $x - y = y - x$

\_\_\_ h.  $c + 1 = c$

\_\_\_ q.  $-2(x - y) = (y - x) \cdot 2$

\_\_\_ i.  $7r - r = 6r$

2. II. Determine the solution sets. Show all steps and give reasons for parts b and c.

a.  $x - 3 = 6$

b.  $x + 2 = 5$

c.  $y - 14 = 4$

d.  $12 = b - 1$

e.  $26 = x + 16$

f.  $-t + \frac{1}{5} = 3\frac{3}{5}$

g.  $2.5 = r - 1.5$

h.  $.05 + x = 3.5$

SELF-EVALUATION 1 (cont')

i.  $x + 2.34 = 3.06$

3. III. Determine the solution sets. No denominator is zero. Show all steps and give reasons for parts a and g.

a.  $2a = 22$

f.  $\frac{1}{2}y = 60$

b.  $3x = 5$

g.  $\frac{2}{3}a = 4$

c.  $.3m - 3$

h.  $\frac{3}{a} = 4$

d.  $\frac{1}{4}r = 25$

i.  $\frac{22}{7} = \frac{3y}{7}$

e.  $4 = \frac{m}{4}$

4. IV. Determine the solution sets if the universal set is the set of real numbers. No denominator is zero. Show all steps and give reasons for parts a and c only.

a.  $3u + 5 = 1$

h.  $7(z - 1) - 2(2z - 3) = 0$

b.  $2w + 3 = 5$

i.  $3x - 7 = -(7 - 3x)$

c.  $\frac{3x}{2} - 6 = 7$

j.  $\frac{4x + 7}{3} = \frac{4}{3}x + 7$

d.  $18x + 11 = 9x - 70$

k.  $\frac{x - 3}{x} = \frac{1}{4}$

e.  $\frac{1}{2} + \frac{1}{3}x = 1$

l.  $\frac{7}{y + 2} = \frac{11}{y}$

f.  $3n + 50 = -10$

m.  $\frac{2x + 11}{4} = \frac{3x - 7}{5}$

g.  $\frac{p + 1}{2} = 2$

n.  $8x + 91 = -5x - 17$

4. V. Write the reason for each step in the following:

(1)  $6x + 1 = 9$

$6x + 1 - 1 = 9 - 1$

$6x + 0 = 9 - 1$

$6x = 9 - 1$

$6x = 8$

$\frac{6x}{6} = \frac{8}{6}$

$1 \cdot x = \frac{8}{6}$

$x = \frac{8}{6}$

$x = 1\frac{1}{3}$

equation

a.

b.

c.

d.

e.

f.

g.

h.

SELF-EVALUATION 1 (cont')

(2)  $\frac{3x}{2} - 1 = 4$

$\frac{3x}{2} - 1 + 1 = 4 + 1$

$\frac{3x}{2} + 0 = 4 + 1$

$\frac{3x}{2} = 4 + 1$

$\frac{3x}{2} = 5$

$\frac{3x}{2} \cdot 2 = 5 \cdot 2$

$3x \cdot 1 = 5 \cdot 2$

$3x = 5 \cdot 2$

$3x = 10$

$\frac{3x}{3} = \frac{10}{3}$

$1 \cdot x = \frac{10}{3}$

$x = \frac{10}{3}$

$x = 2 \frac{1}{3}$

equation

- a. \_\_\_\_\_
- b. \_\_\_\_\_
- c. \_\_\_\_\_
- d. \_\_\_\_\_
- e. \_\_\_\_\_
- f. \_\_\_\_\_
- g. \_\_\_\_\_
- h. \_\_\_\_\_
- i. \_\_\_\_\_
- j. \_\_\_\_\_
- l. \_\_\_\_\_
- m. \_\_\_\_\_

If you have satisfactorily completed your work, you may take the Progress Test. CONSULT YOUR TEACHER FIRST.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

5. Given any mathematical sentence involving absolute value, determine the solution set.
6. Given any verbal problem, translate it into an equivalent mathematical sentence and find its solution set.
7. Given any statement using the properties of inequalities, determine if it is true or false. Appendix II will be completed and turned in to the teacher.
8. Given any inequality whose Universal set is the set of real numbers, determine and/or graph the solution set on the number line.
9. Given any pair of polynomials, write their product.

### RESOURCES

#### Obj. 5

Nichols, read pp. 186-188; Ex. 1-29 odd page 188.

Payne, read pp. 148-151; Ex. 1-23 odd page 149.

Wooton, read pp. 165-168, Ex. 1-23 odd page 168.

Pearson, read pp. 217-219, page 224 Ex. 3, Ex. 6 page 219; 13 page 226.

#### Obj. 6

Vanatta, read \_\_\_\_, Ex. 1-14 page 74.

Dolciani, read page 92, Ex. 1-8 page 94.

Nichols, read pp. 180-182, pp. 214-217, pp. 221-223, Ex. 1-10 pages 181-182; 1-9 odd pages 216-217; 1-8 odd pages 222-223.

Wooton, read pp. 120-123, pp. 124-130, pp. 131-133, Ex. 1-6 pages 122-123; 1-27 odd pages 128-130; 1-21 odd pages 134-136.

Payne, read pp. 132-136; Ex. 1-9 odd pp. 133-134; 1-15 odd pages 134-135; 1-8 odd pages 136-137.

Pearson, read pp. 157-159, Ex. 1-19 odd pages 158-159; 5-17 odd pages 176-178.

Audio Tapes: C - 3809 Reading Written Problems.

RESOURCES 2 (cont')

Obj. 7

Vanatta, read pp. 55-58, Ex. \_\_\_\_.

Dolciani, read pp. 159-163, Ex. \_\_\_\_.

\* Appendix II

Transparencies: Properties of Inequality

(\* required)

Obj. 8

Vanatta, read pp. 55-58, Ex. 1-10 page 58.

Dolciani, read pp. 159-162, Ex. 1-10, 14-16 page 163.

Nichols, read pp. 182-185, pp. 241-246, Ex. 1 a,c,e,g,i,k pages 183-185; 2 a,c,e,g and 3 a,c,e,g pages 183-185; 1 a,c,e,g,i,k,m,o,q,r and 2 a,c,e,g and 3 a,c,e page 244.

Wooton, read pp. 157-159, Ex. 1-10 page 159.

Payne, read pp. 117-122, p. 124 exercise 5; Ex. 1-19 odd pp. 119-122, 23-29 odd pages 119-122; 11-25 odd page 126.

Pearson, read pp. 72-74, Ex. 1 c,d,h,i,j and 4 a,c,d,g,i pages 73-74.

Audio Tapes: C-3805 The Compound Sentence  
C-3806 Inequality and Equality Sentences

Filmstrip: Graphs of Inequalities in One Variable

Transparencies: Properties of Inequality

Obj. 9

Vanatta, read pp. 289-290, 293-294, Ex. 1-20 even p. 290; 1-30 odd pp. 294-295.

Dolciani, read pp. 206-207, 209, Ex. 1-6 oral page 207; 1-20 even page 210.

Nichols, read pp. 199-201, Ex. 1 a,c,e,g,i,k and 2 a,c,e,g,i,k,m,o,q,s,u,w,y page 201.

Wooton, read 272-275, 277-279, Ex. 1-23 odd pages 274-275; 13-39 odd page 279.

Payne, read pp. 313-317, Ex. 1-29 odd p. 315.

Pearson, read pp. 170-173, Ex. 1, 5, 7 pages 172-173.

SELF-EVALUATION 2

Obj. I. Solve the following:

5

(a)  $|x| = 8$

(d)  $|6 - 2x| = 2$

(b)  $|x - 3| = 4$

(e)  $|\frac{1}{2}x + 4| - 2 = 3$

(c)  $|3t + 1| = 7$

(f)  $|\frac{2 + x}{3}| = 2$

6

II. Write the equation used to solve each verbal problem and solve the problem. Show your work.

(a) The sum of a number and 1 is equal to the product of 3 and the number. What is the number?

(b) Multiplying a no. by 3 gives the same result as adding 4 to the number. What is the no.?

(c) Taking one-half of a number gives the same result as adding 5 to the number. What is the no.?

(d) How long is a rectangular plot if its length is 9 ft. longer than its width, and its perimeter is 94 ft.?

(e) The difference between the length and the width of a rectangle is 11 inches. What is the length and the width of the rectangle if its perimeter is equal to 26 inches?

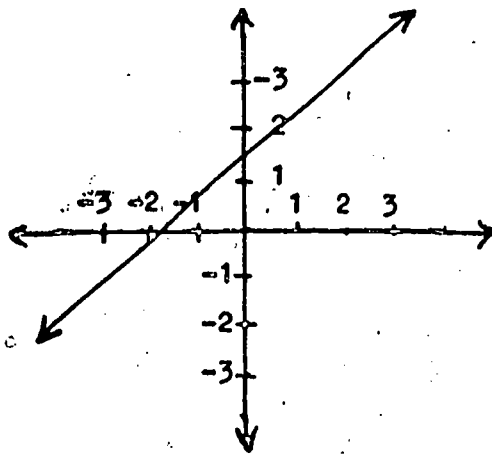
7

III. TRUE OR FALSE.

- \_\_\_\_\_ 1. If  $x < 6$ , then  $x + 2 > 6 + 2$ .
- \_\_\_\_\_ 2. If  $x < 5$  and  $c < 0$ , then  $x \cdot c < 5 \cdot c$ .
- \_\_\_\_\_ 3. If  $K > 6$  and  $c > 0$ , then  $K \cdot c > 6 \cdot c$ .
- \_\_\_\_\_ 4. If  $4 < 12$  and  $-2 < 0$ , then  $4 \div -2 > 12 \div -2$ .
- \_\_\_\_\_ 5. If  $7 < K$ , then  $7 - 6 < K - 6$ .
- \_\_\_\_\_ 6. If  $8 > m$ , then  $8 - 7 < m - 7$ .
- \_\_\_\_\_ 7. If  $T < 4$  and  $c < 0$ , then  $Tc < 4c$ .
- \_\_\_\_\_ 8. If  $k > 7$  and  $c < 0$ , then  $k \cdot c < 7 \cdot c$ .

SELF-EVALUATION 2 (cont')

\_\_\_\_\_ 9.



9 III. Graph the following systems. Use the graph paper provided.

10.  $3x = 2 - y$   
 $3y + 3x = 0$

11.  $2x > y$   
 $3x + 5y = y$

12.  $2x + y = 6$   
 $x + y = y + 3$

13.  $3x < 2 - y$   
 $3y + 3x > 0$

10 IV. Work the following problems. SHOW YOUR WORK.

14. Two men start out from the same city and travel in opposite directions. One travels north at an average rate of 35 mph and the other man travels south at 40 mph. In how many hours will they be 250 miles apart?

15. The sum of four consecutive odd integers is 152. What are the integers?

16. Jim and Joe ride their motorbikes in opposite directions from Joe's house on the highway. They start at the same time. We find them 19 miles apart 15 minutes later. The average speed of Joe's bike is 8 miles per hour less than the average speed of Jim's bike. Determine the average speed of Joe's bike.

17. In Sue's bank she has some dimes and some nickels. She has two more dimes than she has nickels. In all she has \$1.10. How many dimes and how many nickels does she have?

18. How much water must be added to 16 pounds of a 25% salt solution to reduce it to a 15% solution?

If you have satisfactorily completed your work, take the LAP Test. CONSULT YOUR TEACHER FIRST.



SELF-EVALUATION 2 (cont')

9

VI. For each of the following expressions, write an expression which is equivalent to it and which does not contain parenthesis.

a.  $-3(2a - 5m + 4n)$

b.  $(a + 4n)(a - n)$

c.  $(4 + 5b)(5b - 4)$

d.  $-(x - 1)(1 - x)$

e.  $(2a + 3)(2a + 3)$

f.  $(2a + 3)(2a - 3)$

If you have satisfactorily completed your work, take the LAP Test.  
CONSULT YOUR TEACHER FIRST.

APPENDIX 1

1. Write an explanation of the addition property of equality.
2. Write an explanation of the subtraction property of equality.
3. Write an explanation of the multiplication property of equality.
4. Write an explanation of the division property of equality.
5. Write the reason for each in the following.

(1)  $2x = 10$

$$\frac{2x}{2} = \frac{10}{2}$$

$$1 \cdot x = \frac{10}{2}$$

$$x = \frac{10}{2}$$

$$x = 5$$

equation

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(2)  $\frac{x}{3} = 10$

$$\frac{x}{3} \cdot 3 = 10 \cdot 3$$

$$x \cdot 1 = 10 \cdot 3$$

$$x = 10 \cdot 3$$

$$x = 30$$

equation

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(3)  $x + 2 = 9$

$$x + 2 - 2 = 9 - 2$$

$$x + 0 = 9 - 2$$

$$x = 9 - 2$$

$$x = 7$$

equation

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APPENDIX 1 (cont')

$$(4) \quad y - 3 = 7$$

$$y - 3 + 3 = 7 + 3$$

$$y + 0 = 7 + 3$$

$$y = 7 + 3$$

$$y = 10$$

equation

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$$(5) \quad 3x + 6 = 33$$

$$3x + 6 - 6 = 33 - 6$$

$$3x + 0 = 33 - 6$$

$$3x = 33 - 6$$

$$3x = 27$$

$$\frac{3x}{3} = \frac{27}{3}$$

$$1 \cdot x = \frac{27}{3}$$

$$x = \frac{27}{3}$$

$$x = 9$$

equation

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$$(6) \quad \frac{-3n}{4} - 2 = 4$$

$$\frac{-3n}{4} - 2 + 2 = 4 + 2$$

$$\frac{-3n}{4} + 0 = 4 + 2$$

$$\frac{-3n}{4} = 4 + 2$$

$$\frac{-3n}{4} = 6$$

$$\frac{-3n}{4} \cdot 4 = 6 \cdot 4$$

$$-3n \cdot 1 = 6 \cdot 4$$

$$-3n = 6 \cdot 4$$

$$-3n = 24$$

$$\frac{-3n}{-3} = \frac{24}{-3}$$

$$1 \cdot n = \frac{24}{-3}$$

$$n = \frac{24}{-3}$$

$$n = -8$$

equation

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APPENDIX 2

1. Explain the following:

A. If  $a < b$ , then  $a + c < b + c$ ; and  $a > b$ , then  $a + c > b + c$ .

B. If  $a > b$ , then  $a - c > b - c$ ; and  $a < b$ , then  $a - c < b - c$ .

C. If  $a > b$  and  $c > 0$ , then  $ac > bc$ ; and  $a < b$  and  $c > 0$ , then  $ac < bc$ .

D. If  $a > b$  and  $c < 0$ , then  $ac < bc$ ; and  $a < b$  and  $c < 0$ , then  $ac > bc$ .

E. If  $a < b$  and  $c > 0$ , then  $a + c < b + c$ .

F. If  $a > b$  and  $c < 0$ , then  $a + c < b + c$ .

2. True or False.

\_\_\_\_\_ 1. If  $-2x < 8$ , then  $x < -4$ .

\_\_\_\_\_ 2.  $3x < 9$ , then  $x > 3$ .

\_\_\_\_\_ 3.  $x + 3 < 6$ , then  $x < 3$ .

\_\_\_\_\_ 4. If  $-\frac{x}{6} > 2$ , then  $x < -12$ .

\_\_\_\_\_ 5. If  $x - 3 < 5$ , then  $x > 8$ .

\_\_\_\_\_ 6. If  $-6x > 12$ , then  $x > -2$ .

\_\_\_\_\_ 7. If  $\frac{x}{3} < 9$ , then  $x > 27$ .

\_\_\_\_\_ 8. If  $6x < 18$ , then  $x < 3$ .

\_\_\_\_\_ 9. If  $x - 8 < 28$ , then  $x < 20$ .

\_\_\_\_\_ 10. If  $2x + 9 < 19$ , then  $x < 5$ .

ADVANCED STUDY

I. Work the following. Show your work.

1. A car starts out at a point 3 miles outside of town at a rate of 50 mph. How long will it take for the car to be 353 miles from the town? ( $d = rt$ )

Total distance  $d_t =$  rate times time ( $rt$ ) plus the 3 miles out of the town.

$$353 = 50(t) + 3$$

solve for  $t$

2. An airplane travels 702 miles from its point of origin to its destination, it made one stop to pick up passengers and then flew at a rate of 310 mph. for 2 hours to arrive at its terminal point.

Given the equation  $d_t = d_1 + d_2$

But  $d_2 = R \cdot t = 310(2)$  So:

$$702 = d_1 + 310(2) \quad \text{solve for } d_1$$

II. Work the following, showing your work.

1. The total electrical resistance in a series circuit is equal to the sum of the individual resistances. If the first resistor has a rating of two ohms (a measure of resistance), and the second resistor's rating is 7 ohms, then what is the ohm rating of the third resistor, given the total circuit resistance is 10 ohms?

Write the equation.

Solve the ohm rating of the third resistor.

2. The total electrical resistance in a parallel circuit may be found by equating the reciprocal of the total resistance to the sum of the reciprocals of the individual resistances.

$$r_1 = 3, r_2 = 6, r_3 = ? \quad \text{Total resistance is 1 ohm}$$

Write the equation, then solve for  $r_3$ .

3. The focal length of a lens can be found by dividing the product of the image distance ( $d_i$ ) and the object distance ( $d_o$ ), by their sum.

Write the equation, if the focal distance is 1 and the object distance is 2.

Solve the equation for the image distance.

ADVANCED STUDY (cont')

III. Wooton, read pp. 160-163, Ex. 1-26 even (written) pp. 164-165.

IV. Wooton, Ex. 10 18 page 174 any 4 problems.

V. Dolciani, Ex. 23-32 page 163 any 5 problems.

VI. Dolciani, read 164-165, Ex. 1-20 any 8 problems.

VII. Dolciani, page 168 any 5 problems.

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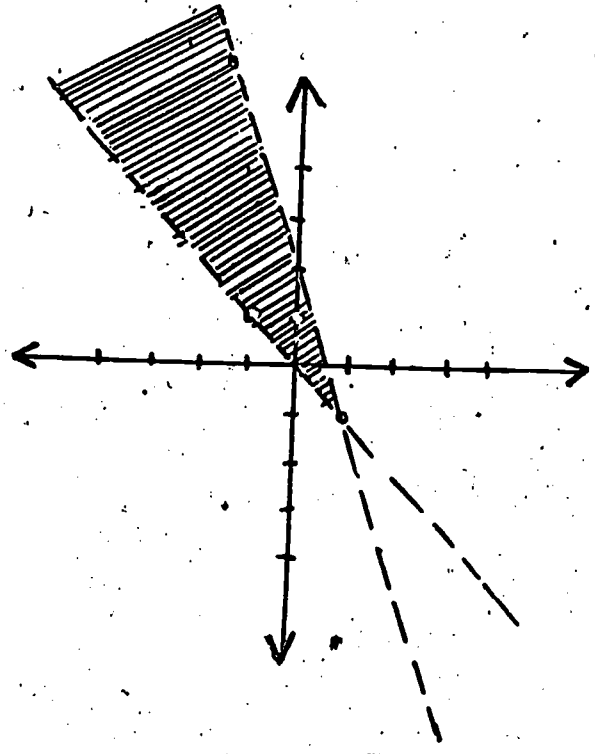
Wollensak teaching tapes C-3801, C-3803, C-3805, C-3806, and C-3809

Transparencies: 3M Properties of Equality  
Properties of Inequality

Filmstrips: Proof in Algebra: Solving Equations  
Graphs of Inequalities in One Variable

Games: Equations by Layman Allen

**L** |  
**L** EARNING  
**A** CTIVITY  
**P** ACKAGE



EQUATIONS AND INEQUALITIES  
WITH TWO VARIABLES



Algebra 93-94

WRITTEN BY  
*Diane Evans*

LAP NUMBER 8

WRITTEN BY Diane Evans



### Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Wantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

RATIONALE  
EQUATIONS OF TWO VARIABLES

Graphs are not new to you. In your study of history, geography, and science, many relationships were made clear by graphing. For example, temperatures in relation to altitude might be indicated by means of a graph. Here the temperature depends upon the altitude. When a quantity depends upon another so that corresponding values can be determined, a graph of their corresponding values can be made.

You have learned how some physical problems can be translated into equations and inequalities. You will continue to learn about word problems in this IAP. You will also learn how to set some of these ideas in a pictorial manner. Their notions should be more meaningful to you through graphing.

Since an equation or inequality represents a relationship of variables, we can associate a point with each pair of values, of this relationship.

We shall study graphs, which will help us gain insight into relationships described by equations and inequalities.

## SECTION 1

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given an equation in two variables and an ordered pair of numbers for replacements of those variables, tell whether the resulting equation is true or false.
2. Identify or define the following:
  - A. Cartesian coordinate system
  - B. Descartes
  - C. abscissa
  - D. ordinate
  - E. origin
3. Given an ordered pair of real numbers, locate the point on a coordinate system corresponding to that ordered pair.
4. Given an equation of two variables put it in standard form.
5. Given an equation in two variables, name at least three ordered pairs of real numbers that are members of the solution set.
6. Given an equation or inequality of two variables, graph it.

### RESOURCES

#### Obj. 1

Dolciani, read pp. 333-335, Ex. 1-10 oral p. 335.

Nichols, read pp. 259-260, Ex. 1-12 pages 260-261.

Wooton, read pp. 189-192, Ex. 1-10 page 192.

Pearson, read pp. 428-431, Ex. 1-3 page 431.

C. Algebra (programmed) Frames 65-90.

C. Geometry (programmed) Frames 1-83.

#### Obj. 2

Dolciani, read pp. 365, 337-338, Ex. 1-32 page 339.

Vanatta, read pp. 191-195, write definitions in Obj. 2.

Wooton, read pp. 194-195, Ex. 1-6 oral pages 195-196; write definitions in Obj. 2.

RESOURCES 1 (cont')

Obj. 3

- Vanatta, read pp. 193-195, Ex. 1 page 196.
- Dolciani, read pp. 337-338, Ex. 1-12 page 340.
- Nichols, read pp. 263-264, Ex. 1-6 pages 264-266.
- Wooton, read pp. 194-195, Ex. 1-14 (written) pages 196-197.
- Payne, read pp. 167-170, Ex. 1-14 pages 170-172.
- Pearson, read pp. 431-432, Ex. 1-4 page 432; 1-5 pages 436-437.
- C. Algebra (programmed) read Unit 1 - Book 3, Ex; Frames 1-65.
- C. Geometry (programmed) Frames 128-181; 329-338.

Obj. 4

- Vanatta, read pp. 246-248, Ex. \_\_\_\_.
- Nichols, read pp. 261-262, Ex. 1-16 page 262.
- Wooton, read pp. 197-200, Ex. 1-6 page 201.

Obj. 5

- Nichols, read pp. 266-267, problems assigned in next objective.
- Wooton, read pp. 197-200, Ex. 7-12 page 201.
- C. Algebra (programmed) read Unit 1, Book 3, Ex - Frames 66-98.

Obj. 6.

- Nichols, read pp. 266-267, 276-280, Ex. 1-12 pages 267-268; 1-4 pages 280-281.

\* Payne, read pp. 178-180, 205-207, Ex. 14-21 page 181; 1-10 even page 207.

Pearson, read pp. 448-451, 487-489, Ex. 1-2 page 452; 1-2 page 488; 1-7 parts a and b only page 490.

C. Algebra (programmed) read Unit 1, Book 3, Ex. frames 99-132 Unit 1; frames 1-124 Unit 2.

C. Geometry (programmed) read Unit 2, Book 3, Ex. frames 182-291 and 324-379.

\* required

SELF-EVALUATION 1

j.

1. I. Next to the equations listed below, there are 3 ordered pairs of numbers; tell whether or not it satisfies the equations.

1.  $x + y = 7$  ; (6, 1) ; (-10, 3) ; (6.99, .01)

2.  $2x + 3y = 6$  ; (0, 2) ; (2, 0) ; (1,  $\frac{4}{3}$ )

3.  $3m = 2n + 4$  ; (0, -2) ; (-2, 0) ; (-5, -2)

4.  $\frac{1}{2} |x + y| = \frac{1}{3} |x - y|$  ; (0, 0) ; (6,  $-\frac{6}{5}$ ) ; (6, -30)

5.  $2a = 3 |b| - 1$  ; (0,  $\frac{1}{3}$ ) ; (0,  $-\frac{1}{3}$ ) ; (-4, -3)

II. Graph the following ordered pairs on the coordinate system to the right.

6. (5, 3)

7. (-2,  $3\frac{1}{2}$ )

8. (-4, -6)

9. ( $2\frac{4}{5}$ , -3)

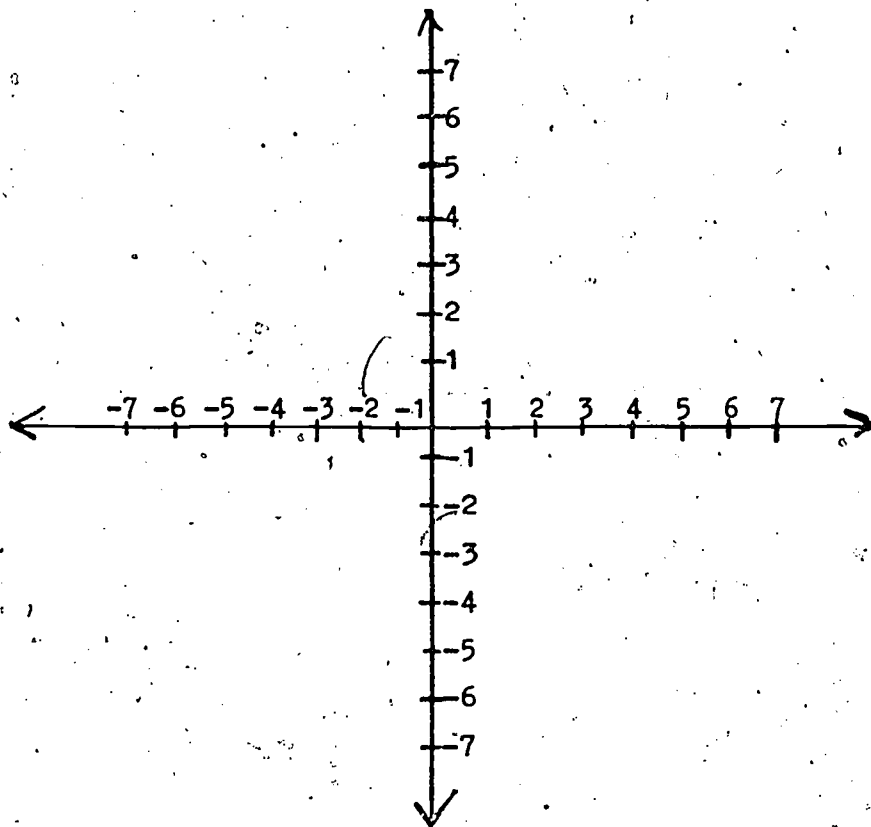
10. (0, -4)

11. (-2, 0)

12. (0, 2)

13. (0, 0)

14. (5, 0)



III. Define the following terms.

15. abscissa

16. origin

17. Descartes

SELF-EVALUATION 1 (cont')

18. Cartesian coordinate system.

19. ordinate

4 IV. For each equation below, find an equivalent equation in standard form:

20.  $2x = -3 - 8y$

21.  $\frac{x - y}{x + y} = 3$

22.  $3x - 2y + (-3) = 2(3y - 6x) + 4$

23.  $\frac{4x + 2}{6} = \frac{-3x + 6y}{-2}$

24.  $\frac{-2}{3x + 7y} = \frac{-4}{5x - 2}$

5 V. Which of the ordered pairs listed to the right are members of the solution set of the equations on the left. (There may be more than one answer for each equation).

25.  $2x - 3y = 12$

a) (3, -1)

26.  $2x + 3y - 1 = x + y$

b)  $(\frac{3}{2}, -3)$

27.  $\frac{x + y}{3} = 2x$

c) (0, 0)

d) (6, 0)

e) (1, 5)

f) (1, 0)

6 VI. Graph each of the following sentences. The universal set in each case is the set of real numbers. (Use the graph paper provided)

28.  $y = 2x + 6$

29.  $2x + 3y > 1$

30.  $\frac{2(3 - 3x)}{y + 1} = -3$

31.  $2x - y = 4$

32.  $x - y \leq 3$

33.  $2x - y > -4$

If you have satisfactorily completed your work, take the Progress Test. CONSULT YOUR TEACHER FIRST.

SELF-EVALUATION: 1 (cont')

A large grid of graph paper, approximately 30 columns wide and 40 rows high, intended for a self-evaluation. The grid is mostly empty, with a few scattered handwritten marks and small scribbles. There are some faint, illegible markings in the center and lower right areas of the grid.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

7. Given a system of equations in two variables, graph their solution set.
8. Given the graph of a pair of equations in two variables, tell whether they are:
  - A. dependent
  - B. inconsistent
  - C. independent
  - D. inconsistentand if they are independent, name the point of intersection.
9. Given a system of equations and/or inequalities in two variables, graph their solution set.
10. Given a word problem, translate it into an open mathematical sentence and solve for the unknown.

### RESOURCES

#### Obj. 7

Vanatta, read pp. 233-234, Ex. 1-8 page 241.

Dolciani, read pp. 267-269, Ex. 1-8 page 369.

Nichols, read pp. 268-270, Ex. 1-13 pages 270-271.

Wooton, read pp. 223-225, Ex. 7-15 page 228.

Payne, read pp. 219-221, Ex. 1-10 pages 222-223.

Pearson, read pp. 465-466, Ex. 1-3 page 466.

C. Algebra (programmed) read Unit 3, Book 3 (includes obj. 9)  
Ex. frames 8-105.

C. Geometry (programmed) frames 181-191; 324-279.

#### Obj. 8

Vanatta, read pp. 233-234, Ex. given for obj. 7.

Dolciani, read pp. 367-369, Ex. given in obj. 7.

Nichols, read pp. 271-275, Ex. 1-10 page 276.

Payne, read pp. 234-235, Ex. 1-18 odd page 243 (do not use slope; use graph method)



RESOURCES 2 (cont')

Pearson, read pp. 480-481, Ex. 1-9 page 482.

Obj. 9

Dolciani, read pp. 350-352, 379-380, Ex. 1-12 even written page 352; 9-11 page 370; 5-14 even page 380.

Nichols, read pp. 281-284, Ex. 1-3 page 284.

Wooton, read pp. 252, Ex. 1-12 page 253.

Payne, read pp. 214-222, 244-245, Ex. 11-14 page 223; 1-10 page 246.

Pearson, read pp. 490-491, Ex. 1-2 page 491.

C. Geometry (programmed) frames 324-477.

C. Algebra (programmed) (same as obj. 7)

Obj. 10

Vanatta, read pp. 154-158, 164-166, 169-170, 214-216, 235-237, Ex. 1, 2, 8, 10, 14 page 156; 1-9 page 159; 1, 2 pages 166-167; 17 page 177; 11 page 175; 6 page 178; 1, 2, 4, 5, 10 page 170.

Dolciani, read pp. 166-171, 172-175, 178-180, 182-183, 310, Ex. 1, 2, 4, 10, 19 page 168; 6, 7 page 167; 1, 4, 7, 10, 13 bottom page 171; 4, 5, 6 page 177; 1-3 pages 180-181; 2-5 page 183; 1, 3-5 page 311.

Nichols, read pp. 223-228, Ex. 1-14 even page 224-225; 1-9 even, 11 pages 227-228.

Wooton, read pp. 169-172, 353-355, Ex. 1-36 even pages 173-176; 1-30 even pages 356-358.

Payne, read pp. 128-136, 139, Ex. 1-9 pp. 133-134; 1-15 even pages 134-135; 1, 3, 5 page 136; 1-4 pages 139-140.

Pearson, read pp. 251-252, 296-299, Ex. 1-30 even pages 252-253.

SELF-EVALUATION 2

Obj.

7 I. Graph each system of equations and name the point intersection.  
(approximately) Use the graph paper provided.

1.  $2x - y = 0$   
 $2x + y = -4$

2.  $3x + y = 10$   
 $2x - y = 1$

3.  $4x = 2y$   
 $2x - y = 2$

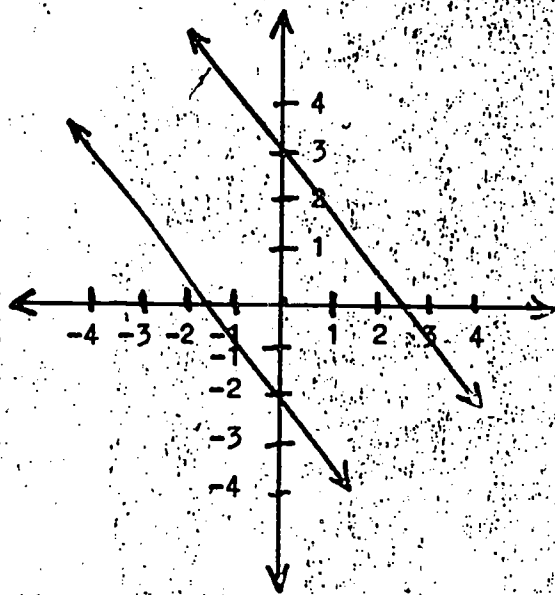
4.  $3x + 5y = 4$   
 $12 - 9x = 5y$

5.  $2x + 3y = 8$   
 $x + y = 3$

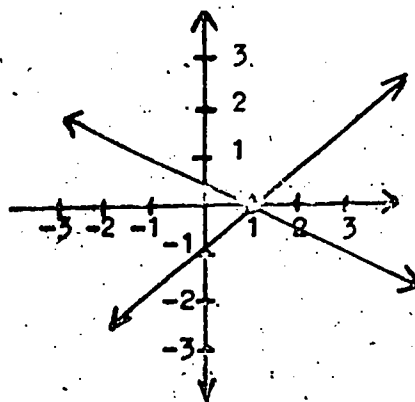
6.  $x + y = 1$   
 $y = -x$

8 II. Categorize the following graphs of pairs of equations as being  
(a) dependent (b) inconsistent (c) independent and if they  
are indepdnt, name the point of intersection.

\_\_\_\_\_ 7.

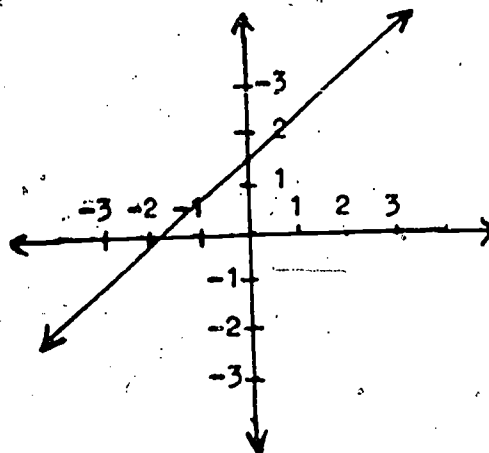


\_\_\_\_\_ 8.



SELF-EVALUATION 2 (cont')

9.



9 III. Graph the following systems. Use the graph paper provided.

10.  $3x = 2 - y$   
 $3y + 3x = 0$

11.  $2x > y$   
 $3x + 5y = y$

12.  $2x + y = 6$   
 $x + y = y + 3$

13.  $3x < 2 - y$   
 $3y + 3x > 0$

10 IV. Work the following problems. SHOW YOUR WORK.

14. Two men start out from the same city and travel in opposite directions. One travels north at an average rate of 35 mph and the other man south at 40 mph. In how many hours will they be 250 miles apart?

15. The sum of four consecutive odd integers is 152. What are the integers?

16. Jim and Joe ride their motorbikes in opposite directions from Joe's house on the highway. They start at the same time. We find them 19 miles apart 15 minutes later. The average speed of Joe's bike is 8 miles per hour less than the average speed of Jim's bike. Determine the average speed of Joe's bike.

17. In Sue's bank she has some dimes and some nickels. She has two more dimes than she has nickels. In all she has \$1.10. How many dimes and how many nickels does she have?

18. How much water must be added to 16 pounds of a 25% salt solution to reduce it to a 15% solution?

If you have satisfactorily completed your work, take the LAP Test. CONSULT YOUR TEACHER FIRST.

SELF-EVALUATION '2 (cont')

A large grid of graph paper, consisting of approximately 30 columns and 40 rows of small squares. The grid is mostly empty, with a few faint, scattered marks and a small handwritten mark resembling a checkmark or the number '7' in the upper right quadrant.

SELF-EVALUATION 2 (CONT.)

A large grid of graph paper, consisting of approximately 30 columns and 40 rows of small squares. The grid is mostly empty, with a few faint, illegible marks scattered across it.

APPENDIX

Work the following problems. Show your work and turn it in to your teacher with this sheet.

1. Tony broke his bank and found he had \$5.35 in nickels and dimes. The bank contained ten more dimes than nickels. How many nickels and how many dimes did he have?

nickels \_\_\_\_\_ dimes \_\_\_\_\_

2. Mr. James weighs 30 pounds more than his son. His son weighs twice as much as Mrs. James. Their combined weight is 495 pounds. How much does Mr. James weigh?

\_\_\_\_\_

3. Jim and John went hunting and shot 21 rabbits in all. John shot three less rabbits than Jim. How many did each boy shoot?

4. A man purchases some three-cent stamps and some one-cent stamps for \$3.05. There are 19 more three-cent stamps than one-cent stamps. How many of each kind does he buy?

number of 3¢ \_\_\_\_\_

number of 1¢ \_\_\_\_\_

5. At a certain time two airplanes start from the same airport and travel in opposite directions at 300 miles an hour and 250 miles an hour respectively. In how many hours will they be 1375 miles apart?

6. At a certain time a train leaves New York going to Albany traveling at 75 mph. At the same time a train leaves Albany going to New York traveling at 50 mph. In how many hours will they meet if New York is 375 miles from Albany?

\_\_\_\_\_

7. John left Greenville traveling to Atlanta driving 40 mph. At the same time Sam left Atlanta traveling to Greenville driving 55 mph. In how many hours will they meet if Greenville is 190 miles from Atlanta?

\_\_\_\_\_

8. How much water must be added to a barrel containing 48 pounds of a 10% brine to obtain a 6% brine?

\_\_\_\_\_

APPENDIX (cont')

9. How many ounces of water must be added to 80 ounces of a 5% acid solution to produce a 2% acid solution?
-

ADVANCED STUDY

1. Mixture problem from chemistry:

What quantities of gold 80% and 20% pure should be mixed to give 12 grams of 70% pure gold?

Let  $x = 80\%$  pure gold  
 $y = 20\%$  pure gold

$$.4x = \text{gm of gold in } 80\%$$

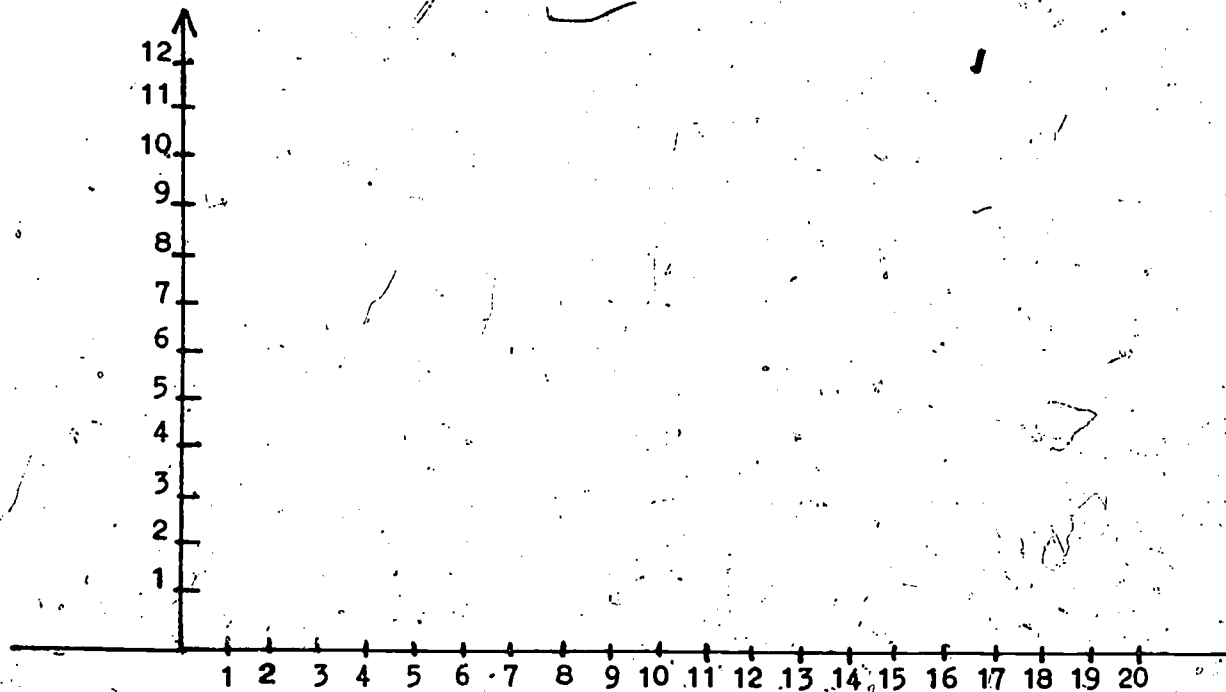
$$.6y = \text{gm of gold in } 20\%$$

So the two equations are

$$x + y = 12$$

$$.8x + .2y = 12 \quad (.7)$$

Graph to find  $x$  and  $y$

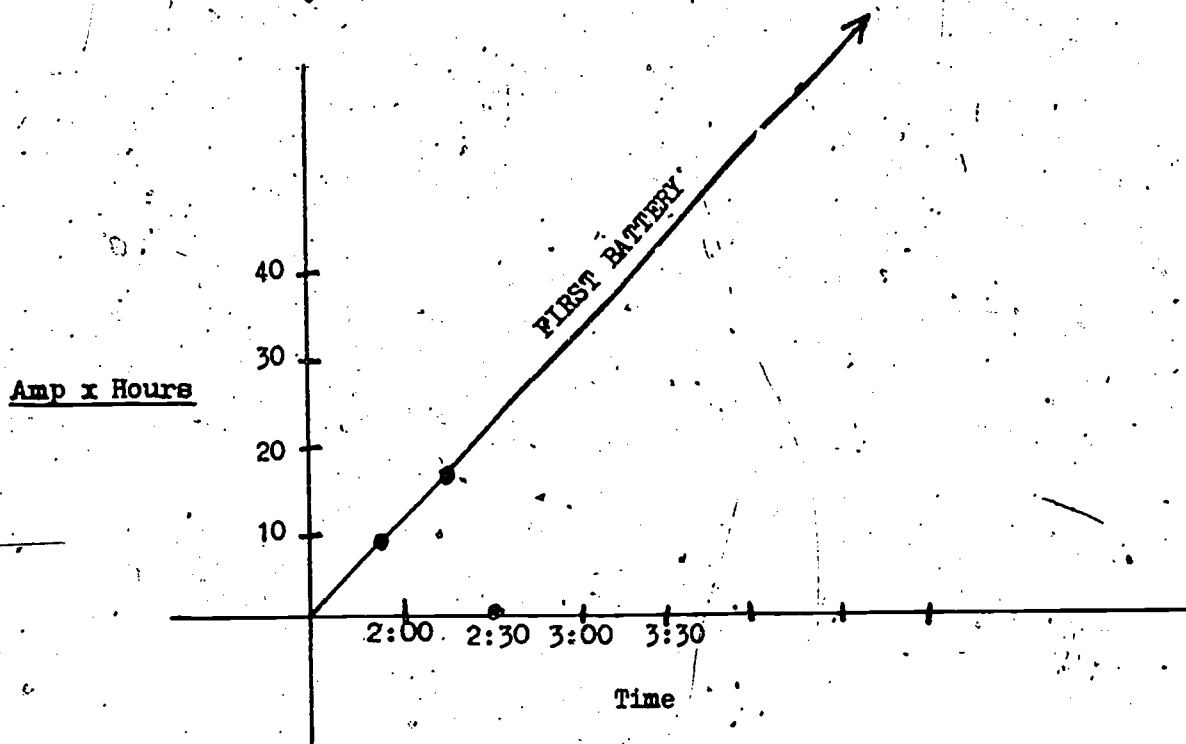




ADVANCED STUDY (cont')

A 12 volt D.C. generator can charge a battery at the rate of 20 amperes which is 20 coulombs of charge per second. It starts charging a new battery at 1:30 P.M.

Another D.C. 12 volt generator can charge a battery at the rate of 40 amperes. It starts charging a similar battery at 2:30 P.M. When will both batteries have the same charge? What will the charge be?



Let (0,0) be time to = 1:30 with 30 min. intervals. Plot the second battery and find the time of equal charge, and the amount of charge?

ADVANCED STUDY (cont')

III. Work any 5 of the following problems:

A. Dolciani, page 172, numbers 14-18.

B. Nichols, page 227, nos. 7-9.

IV. Work any 5 of the following:

A. Dolciani, page 182, numbers 13,14,16; page 191, numbers 56,58.

B. Nichols, page 176, number 14; page 177, number 4.

V. Work any 6 of the following:

A. Dolciani, page 184, numbers 13,14; page 311, numbers 9,10;  
page 318, numbers 1-5.

B. Vanatta, page 176, numbers 9, 20; page 178, number 7.

VI. Payne, page 244, numbers 23-26.

Payne, page 247, numbers 1-6.

VII. Payne, read pp. 250-252, Ex. 1-4 pages 252-253.

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Nichols, Eugene D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

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Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis, Algebra One, Harcourt, Brace and World, Inc., 1969.

Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenback, Edwin Fr., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics, Algebra 1, Houghton, Mifflin Company, 1967.

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Dolciani, Mary P., Berman, Simon L., Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

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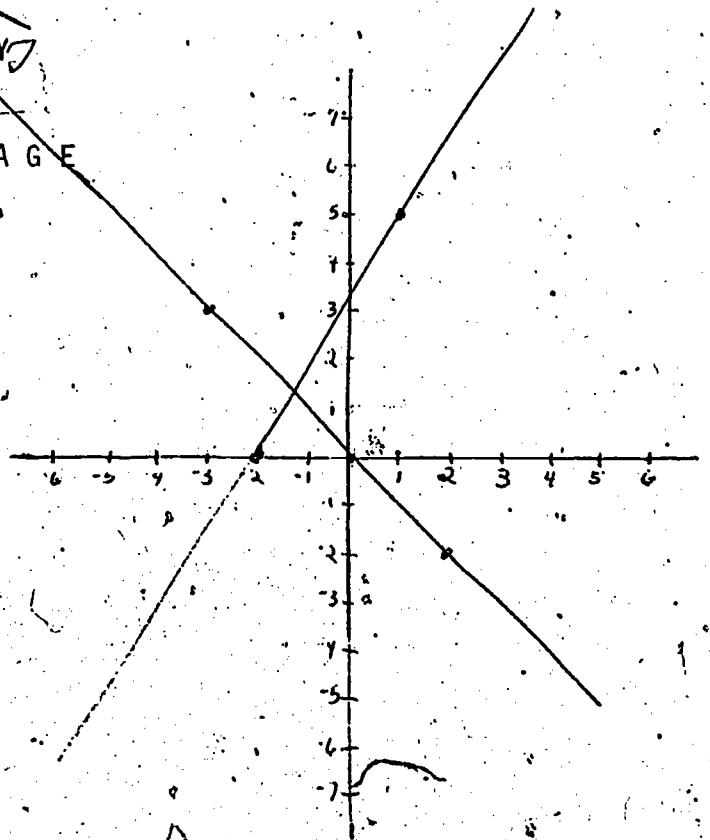
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Helmer, Ralph T., Kocher, Frank., and Lottea, John, J., A Program in Contemporary Algebra, Book 3, Equations and Inequalities in Two Variables, Holt, Rinehart and Winston, Inc. New York, N. Y., 1963

Programed Geometry (abbreviation)

Nichols, Eugene D., Kalin, Robert., Garland, Henry., Introduction to Coordinate Geometry, Holt, Rinehart and Winston., 1963.

**L** EARNING  
**A** CTIVITY  
**P** ACKAGE



SOLUTION SETS OF EQUATIONS  
WITH TWO VARIABLES



Algebra 93-94

REVIEWED BY

*H. H. Hester*

LAP NUMBER

9

WRITTEN BY

*Marianne B. Evans*

## RATIONALÉ

In your previous LAFs you have been solving applied problems by using linear equations and inequalities involving one variable. Actually, most applied problems can be solved in this manner, depending upon your ingenuity! There are instances where it is preferable to use two variables rather than one. This requires that you be able to solve systems of linear equations and inequalities.

In LAF 8, you had some experience in finding solution sets to systems of linear equations and inequalities, through the use of graphing. You also found that the "graphing technique" was of limited value since your results were only approximations of the correct solutions.

In this LAF, you will be learning more precise techniques of computing the solution sets for systems of linear equations and inequalities. This will enable you to solve applied problems, using two variables rather than one!

## SECTION 1

### Behavioral Objectives.

At the completion of your prescribed course of study, you will be able to:

1. Given a pair of linear equations in two variables, compute their solution set using the COMPARISON method.
2. Given a pair of linear equations in two variables, compute their solution set using the SUBSTITUTION method.
3. Given a pair of linear equations in two variables, compute their solution set using the ADDITION method.
4. Given a word problem, TRANSLATE it into an OPEN mathematical sentence (or sentences) and SOLVE for the UNKNOWN (or unknowns).

### RESOURCES

#### Obj. 1

Nichols, read pp. 293-297, Ex. 1 all parts pages 297-298.

Games: Graphing Pictures nos. 11, 6, 22

#### Obj. 2

Vanatta, read pp. 249-250, Ex. 1-8 pages 250-251.

Dolciani, read pp. 378, Ex. 1-18 even page 378.

Nichols, read pp. 299-300, Ex. 1,2 page 300.

Payne, read p. 235, Ex. 1-18 even page 236.

Wooton, read pp. 240-242, Ex. 1-18 even page 243.

Pearson, read pp. 477, Ex. 1,2 page 478.

Games: Graphing Pictures: nos. 11, 6, 22.

#### Obj. 3

Vanatta, read pp. 246-248, Ex. 1-4 page 245; 5-10 page 246; 1-4 page 248.

Dolciani, read pp. 370-371, 374-375; Ex. 1-18 even page 372; 1-18 even page 376.

RESOURCES 1 (cont')

Nichols, read pp. 301-303, Ex. 1,2 page 303.

Payne, read pp. 227-229, Ex. 1-10 p. 230.

Wooton, read pp. 236-238, 1-31 even page 239.

Obj. 4

Vanatta, read p. 253, Ex. 1-7, 11-15 pages 253-254.

\* Dolciani, read pp. 372-373, Ex. 1,3,5,6 page 373; 1-4 page 374;  
2,3,4 p. 379; 1-3 page 311; 1-3 page 385.

Wooton, read pp. 244, 247-248, Ex. 1-20 p. 245; 1-22 even pages  
250-251.

Pearson, read pp. 482-483, Ex. 1-3, 6, 17, 21 pp. 484-486.

Wollensak tape C-3809

Nichols, read pp. 228-232, 305; Ex. 1-18 even pages 232-234; Ex.  
11,12 p. 238; 4 p. 408.

\* required

## SELF EVALUATION

OBJECTIVE 1. Solve by the COMPARISON method.

1.  $3x - 10 = y; y = 4x$
2.  $x = 12 + 2y; x = 3 + 3y$
3.  $x + y = 10; 2x + 2y = 20$
4.  $\frac{1}{2}x = y; y + \frac{3}{15}x = 7$
5.  $x + y = \frac{25}{2}; \frac{x}{2} + \frac{y}{2} = 5$

OBJECTIVE 2. Solve by SUBSTITUTION method.

6.  $7x + 9y = 16; x + y = 2$
7.  $x + y = 20; y = 2x + 5$
8.  $3(x + 2) + 3y = 21; x + 2y = 8$
9.  $3x + 2y = 15; -x = 2 - 4y$
10.  $\frac{1}{3}x + \frac{2}{9}y = 2; \frac{7}{3}x + \frac{1}{27}y = \frac{43}{6}$

OBJECTIVE 3. Solve by ADDITION method. Check by substituting your solution in each problem.

11.  $x + y = 5; 2x - y = 7$
12.  $8x - 3y = 15; 13x - 3y = 15$
13.  $4x + 3y = 14; 9x - 2y = 14$
14.  $2x + 3y = 12.4; 4x + -6y + 3y = -5.8$
15.  $3sx + 2ty = -5st; 4sx + -5ty = 24st$

OBJECTIVE 4. Find the EQUATIONS and solve.

(One Variable)

16. John has twice as many nickels as quarters and 3 fewer dimes than quarters. The sum of the values of the coins is \$1.95. Find how many of each kind of coin he has.



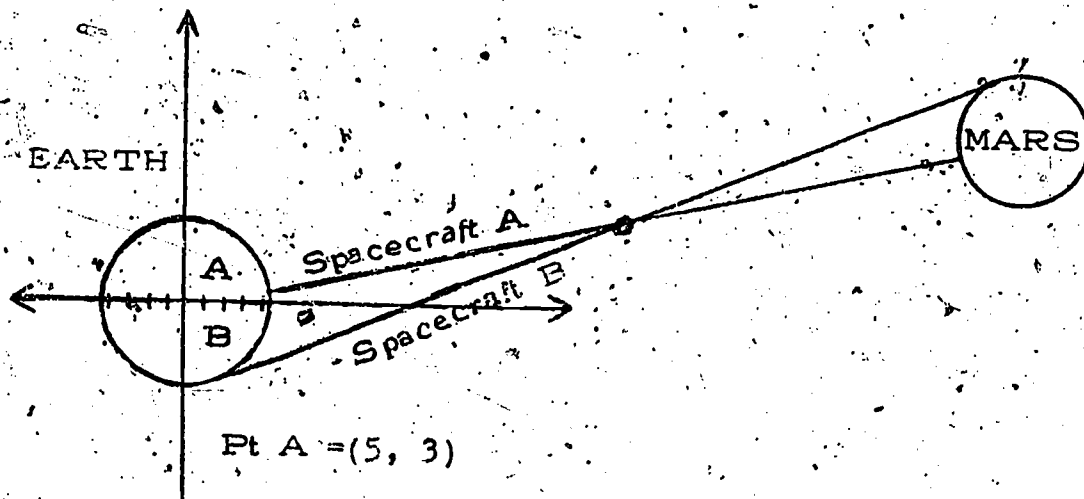
SELF - EVALUATION (cont')

17. A solution of salt and water weighs 100 lbs and it is 10% salt. How much water must evaporate to leave the solution at 30% salt concentration?
18. Two cars start together in opposite directions, one at 40 mph the other at 50 mph. How long will it be before they are 300 miles apart?

Solve 19-23, using two variables.

19. The sum of two numbers is 19. Their difference is 1.
20. The sum of two numbers is 20. Twice one is 3 times the other.
21. A rectangle is twice as long as it is wide. The sum of length and width is 9.
22. John's age now is 2 less than twice his sister Sue's age. In five years John's age will equal 3 times Sue's age now.
23. A plane flies 360 mph with the wind and 270 mph against. What is the speed of the plane in still air? What is the wind speed?

ADVANCED STUDY



Pt A = (5, 3)

Pt B = (4, -4)

Units are in thousands of kilometers.

The spacecraft have only ONE opportunity for a mid-course rendezvous for supplies and fueling.

- Two spacecraft A and B are going to Mars. Spacecraft A contains men and light equipment. Spacecraft B contains fuel, heavy equipment and life-support supplies.

Question: At what point do the ships rendezvous?

Note: Spacecraft A path:  $x - 2y = -1$

Spacecraft B path:  $3x - 4y = 28$

- A man exerting a force of 150 pounds and using a lever 6 feet long would be able to lift a weight of how many pounds if he placed the fulcrum 2 feet from the weight?
- A man in an automobile is traveling 5 times as fast as a boy on a bicycle. The time required by the boy in going 40 miles is 3 hours greater than that required by the man going 50 miles. What is the rate of travel of the boy?..of the man?
- Job Problems (These problems are best done on a one-day-total-cost job basis) e.g.

If John does the job in 3 days and Sue in 4 days, how long will it take them to do it together?

- Vanatta, page 252, nos. 9, 10.
- Dolciani, page 377, nos. 19, 20.

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Vanatta (abbreviation)

Vanatta, Glen D., Goodwin A. Wilson, Algebra One, A Modern Course, Charles E. Merrill Publishing Co., 1966.

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Wooton

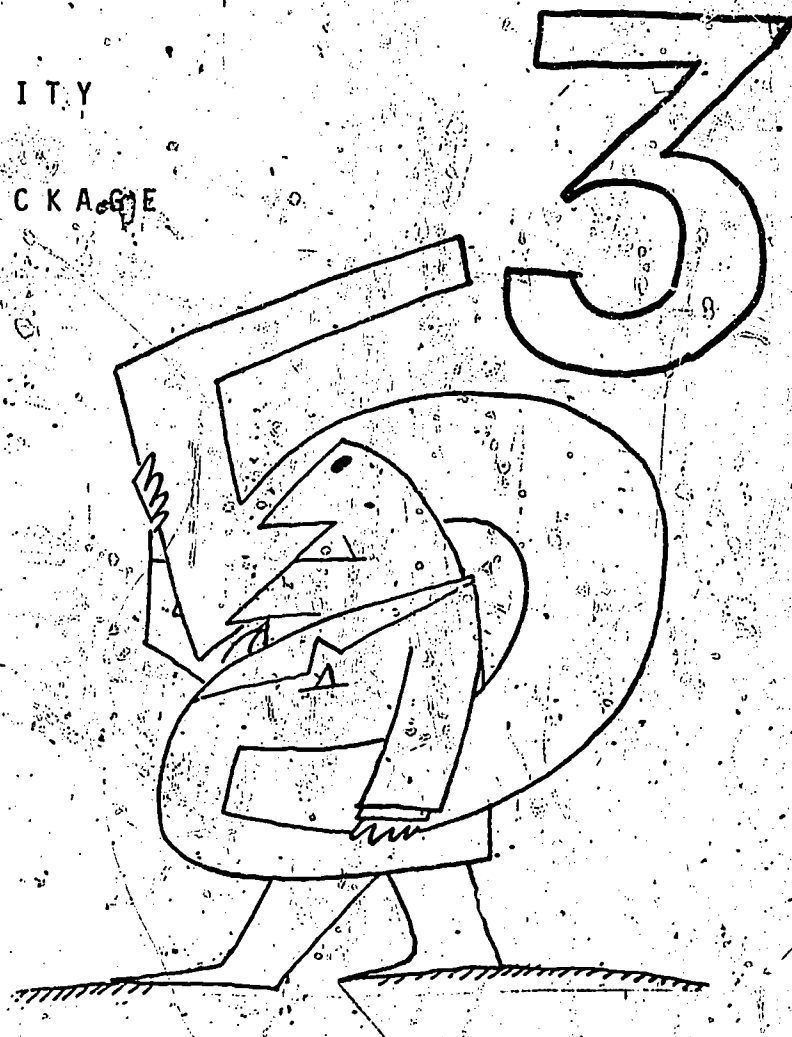
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Wollensak Teaching Tapes: C-3809

L  
LEARNING  
A  
ACTIVITY  
P  
PACKAGE



EXPONENTS

Algebra 93-94



VIEWED BY

*A. R. R. R.*

LAP NUMBER 10

WRITTEN BY Diane Evans

## RATIONALE

The purpose of this LAP is to introduce the fundamental theorems of exponents and radicals. At this stage you will not be expected to prove these laws, but will discover them through observing existing patterns. Familiarity with these theorems is an important prerequisite for the learning of factoring, extension of the concept of function, and in using scientific notation.

A wide variety of experiences will be provided to enable you to associate the basic theorems of exponents to their application either in mathematics or science. The laws will be extended from natural number exponents to negative exponents. Rational and real exponents will be left to a later date. Scientific notation will be used in demonstrating application of exponents.

## SECTION 1

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given any number written in exponential form, write it as a product where the factors are alike.
2. Given any number expressed in exponential form, name the base and the exponent.
3. Use the product of powers property ( $\forall x \in \mathbb{R} \forall m, n \in \mathbb{N} X^m \cdot X^n = X^{m+n}$ ) to rename any given product of powers so that no base is used more than once. (Simplify).
4. Given any rational number, and a base, write the rational number in exponential form using the given base.
5. Given any number expressed in exponential form (power of a base), write it as a decimal numeral.
6. Given two or more monomial expressions of the form  $a^m \cdot a^n$ ,  $(a^m)^n$ ,  $\frac{a^m}{a^n}$ ,  $(\frac{a}{b})^n$ , or  $(ab)^m$ , use the laws of exponents together with the associative and commutative properties of multiplication to rename it as an equivalent expression.

### RESOURCES

#### Objectives 1, 2, 3, 4

Nichols, read pp. 312-314, Ex. 2, 4 every other letter, 3 a, c pp. 314-315.

Vanatta, read pp. 69, 113-114, Ex. 1-18 page 114.

Dolciani, read p. 203, Ex. 1-24 page 204.

Payne, read pp. 257-259, Ex. 1-25 odd, 33-41 odd pages 259-260.

Pearson, read pp. 340, Ex. 1-5, 6 a,b,c,j,k,l, 7 a,b,c, 8 a,b,i,j,l, 10 page 341.



RESOURCES (cont')

Introduction to Exponents frames 27-29 (Obj. 1)  
frames 10-26, 30-32 (Obj. 2)  
frames 1-9 (Obj. 3)  
frames 33-35 (Obj. 4)

\* Appendix I

Objective 5

Nichols, read pp. 312-313, Ex. 1 a-j page 314.

Objective 6

Nichols, read pp. 315-319, Ex. 1-3 every other letter page 316;  
1,2,5,6 every other letter page 317;  
1,2 every other letter page 318;  
1-10 page 319.

Vanatta, read pp. 114-116, Ex. 1-16 page 116;  
1-12 top page 117;  
1-23 even page 117;

Dolciani, read pp. 204-205, 215-217, Ex. 1-16 even page 205; 1-10  
page 206; 1-24 even top page 218; 1-10 pages 218-219.

Payne, read pp. 260-265, Ex. pages 262-264 every number divisible  
by 4; 1-41 odd page 266.

Wooton, read pp. 315-319, Ex. 1-45 odd page 271.

Pearson, read pp. 342-343, 347-349, Ex. 1 every other letter, 2  
page 342; 1, 2, 3 every other letter page 343; 1,2,3 every other  
letter, 4,5,6 pages 345-346.

Introduction to Exponents Frames 80-141  
197-205  
218-220

\* required

SELF-EVALUATION 1

OBJECTIVE

1 I. Match each exponential form on the left with its equivalent product on the right.

- |                |   |
|----------------|---|
| _____ 1. $6^4$ | A. $2 \times 2 \times 2$                            |
| _____ 2. $3^2$ | B. $4 \times 6$                                     |
| _____ 3. $2^3$ | C. $3 \times 3$                                     |
| _____ 4. $4^6$ | D. $6 \times 6 \times 6 \times 6$                   |
|                | E. $4 \times 4 \times 4 \times 4 \times 4 \times 4$ |

2 II. In each of the following, circle the exponent and underline the base.

(5)  $6^4$

(6)  $a^4$

(7)  $b^x$

(8)  $x^2$

4 III.A. Write each of the following in exponential form using 2 as the base.

\_\_\_\_\_ 9. 64

\_\_\_\_\_ 10. 32

\_\_\_\_\_ 11. 4

\_\_\_\_\_ 12. 16

B. Write the following in exponential form using 4 as the base.

\_\_\_\_\_ 13. 16

\_\_\_\_\_ 14. 4

\_\_\_\_\_ 15. 64

5 IV. Write the following as decimal numerals.

\_\_\_\_\_ 16.  $3^4$

\_\_\_\_\_ 17.  $-5^2$

\_\_\_\_\_ 18.  $7^2$

\_\_\_\_\_ 19.  $(\frac{2}{3})^4$

\_\_\_\_\_ 20.  $(-5)^2$

\_\_\_\_\_ 21.  $-4^3$

\_\_\_\_\_ 22.  $(-4)^3$



SELF-EVALUATION 1 (cont')

V. Simplify the following.

3

\_\_\_\_\_ 23.  $3^4 \cdot 3^2$

\_\_\_\_\_ 24.  $x^3 \cdot x^4 \cdot x$

\_\_\_\_\_ 25.  $a^2 b^3 ab^4$

\_\_\_\_\_ 26.  $x^2 y^3 x^4 y^2$

6

\_\_\_\_\_ 27.  $(r^2)^3$

\_\_\_\_\_ 28.  $(a^5)^3$

\_\_\_\_\_ 29.  $(3ab^3)^2$

\_\_\_\_\_ 30.  $(4xy)^2$

\_\_\_\_\_ 31.  $(3a^2 m^3)^2$

\_\_\_\_\_ 32.  $\left(\frac{a^2}{b^3}\right)^2$

\_\_\_\_\_ 33.  $\frac{c^2 m^3}{cm^4}$

\_\_\_\_\_ 34.  $\frac{18 x^3 y^2}{3xy}$

\_\_\_\_\_ 35.  $\left(\frac{a^3}{b^4}\right)^2$

\_\_\_\_\_ 36.  $\left(\frac{-x}{y}\right)^3$

\_\_\_\_\_ 37.  $\frac{-48r^5 s^7}{-4r^2 s^4}$

\_\_\_\_\_ 38.  $(r^5)^3$

\_\_\_\_\_ 39.  $(3 r^2 s^3)^4$

\_\_\_\_\_ 40.  $\frac{-15rs^4}{3rs}$

If you have satisfactorily completed your work, take the Progress Test. Consult your teacher first.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

7. Given any non-zero rational expression involving exponents, write equivalent expressions using only positive exponents.
8. Use the distributive property to name the product of a monomial and a polynomial.
9. Given any positive number, express it in scientific notation.
10. Given a number expressed in scientific notation, express it as a decimal numeral.
11. Given two or more numbers expressed in scientific notation, find the indicated sum, difference, product or quotient.
12. Given a verbal problem involving very large or very small numbers, express the numbers in scientific notation and find the solution of the problem.

### RESOURCES

#### Objective 7

Nichols, read pp. 320-324; Ex. 1-15 even page 322; 1-11 even top page 323; 1, 2 every other letter bottom page 323; 3 a,b,d,f, g,h,j,n,p,w,r, 6 a,b,e,i,j pages 335-336.

Vanatta, read pp. 117-119, Ex. 1-40 even pages 119-120.

Dolciani, read pp. 232-233, Ex. 1-14 page 233.

Payne, read pp. 267-268, 273-274, Ex. 1-9 page 269.

Wooton, read pp. 324-326, Ex. 1-45 odd pages 327-328.

Pearson, read pp. 347-349, Ex. 1 page 349; 3-7 page 350.

Introduction to Exponents frames 142-196.

#### Objective 8

Vanatta, read pp. 120-121, 137, Ex. 1-20 even page 121; 1-20 even pages 137-138.

RESOURCES 2 (cont')

Objective 8 (cont')

Dolciani, read pp. 206-207, Ex. 1-14 page 207.

Wooton, read pp. 272-274, Ex. 1-18 oral page 274 ALSO,

(a)  $2x(3x^2 + 2x - 5)$

(b)  $2a(a^2 - 3a + 2)$

(c)  $3y(2y^2 + y - 3)$

(d)  $5d(6 - d + 2d^2)$

(e)  $xy(x - 2xy + y^2)$

Pearson, read page 350, Ex. 8 page 350.

Objectives 9, 10, 11, 12

Nichols, read pages 339-341, Ex. 1-3 pages 340-341.

Dolciani, read pp. 376-377, Ex. 1-17 page 278.

Payne, read p. 268, Ex. 25-55 page 269; 27-32 page 260.

Pearson, read pages 350-351, Ex. 1-10 pages 351-353.

Introduction to Exponents    Frames 56-78 (Obj. 9)  
  Frames 79        (Obj. 12)

\* Appendix 2

\* Nichols Ex. 4 pages 340-341.

\* required

SELF-EVALUATION 2

Objective

7 I. Write the following using only positive exponents and simplify.

1.  $5^9 \cdot 5^{-4}$

7.  $x^{-2} \cdot y^4$

2.  $\frac{x^3 y}{x^4}$

8.  $\frac{x^3 y^4}{x^{-4} y^{-2}}$

3.  $2x^{-3}$

9.  $\frac{5x^2}{r^{-3}}$

4.  $\frac{3x^{-2}}{a^3 b^2}$

10.  $3x^{-4}$

5.  $\frac{2x^{-6}}{8y^4}$

11.  $\frac{6xy}{-2^{-1}}$

6.  $\frac{2x^{-6}}{-8x^4}$

12.  $\frac{3b}{a^{-2} c^4}$

8 II. Simplify the following:

13.  $3x(2x - 3y + 4c)$

14.  $a^2(3a - 2b + c)$

15.  $(3xy)(2x^2y^3)$

16.  $(2x^3y^4)(3xy^4)$

17.  $3x^2y(2x + 3y + 4xy)$

18.  $-2a^3b(a^4b - a^3b^2 + 2a^2b^4 - b^5)$

19.  $3x^2y(5 - 2xy^4 + 3x^2y^3 - y^5)$

9 III. Express each in scientific notation.

20.  $68.5 =$  \_\_\_\_\_

21.  $.205 =$  \_\_\_\_\_

22.  $.0024 =$  \_\_\_\_\_

23.  $186,000,000,000 =$  \_\_\_\_\_

24.  $.0000000612 =$  \_\_\_\_\_

SELF-EVALUATION 2 (cont')

10

IV. Express each as a decimal numeral.

25.  $3.2 \times 10^4 =$  \_\_\_\_\_

26.  $2.9 \times 10^{-4} =$  \_\_\_\_\_

27.  $3.1 \times 10^2 =$  \_\_\_\_\_

28.  $6.7 \times 10^{-8} =$  \_\_\_\_\_

11

V. Simplify, leaving the answer in scientific notation.

29.  $(4.5 \times 10^2) + (3.6 \times 10^3) =$  \_\_\_\_\_

30.  $(3.7 \times 10^4) - (2.3 \times 10^2) =$  \_\_\_\_\_

31.  $(6.2 \times 10^5) \cdot (2.1 \times 10^3) =$  \_\_\_\_\_

32.  $\frac{3.4 \times 10^3}{.2 \times 10^2} =$  \_\_\_\_\_

33.  $\frac{14 \times 10^4 \times 2 \times 10^{-6}}{7 \times 10^{-2}} =$  \_\_\_\_\_

34.  $\frac{3 \times 10^{-6} \times 21 \times 10^4}{9 \times 10^{-4}} =$  \_\_\_\_\_

12

VI. Solve each problem.

35. Give, in scientific notation, the number of minutes in a year.  
(1 year = 365 days)

36. The speed of sound at sea level is 760 mph. Give this speed in feet per second written in scientific notation.

SELF-EVALUATION 2 (cont')

37. Spaceships travel at speeds of 18,000 mph. How many miles per second is this?

38. The sun is 93,000,000 miles away from earth. How far is this in feet? Express in scientific notation.

IF you have satisfactorily completed your work, take the LAP TEST. Consult your teacher first.

APPENDIX I

Objective

1. I. Write the following as a product where the factors are alike.

A.  $7^4$

B.  $10^2$

C.  $8^6$

D.  $9^3$

E. 6

2. II. In each of the following, name the base and exponent.

A.  $7^4$  base \_\_\_\_\_ exponent \_\_\_\_\_

B.  $a^9$  base \_\_\_\_\_ exponent \_\_\_\_\_

C. 2 base \_\_\_\_\_ exponent \_\_\_\_\_

D.  $x^3$  base \_\_\_\_\_ exponent \_\_\_\_\_

E.  $5^2$  base \_\_\_\_\_ exponent \_\_\_\_\_

4. III. Write each number on the left in exponential form using the number on the right as the base. Example  $27 = 3 \times 3 \times 3 = 3^3$

A. 16 Use 2 as base \_\_\_\_\_

B. 9 Use 3 as base \_\_\_\_\_

C. 64 Use 4 as base \_\_\_\_\_

D. 64 Use 2 as base \_\_\_\_\_

E. 64 Use 8 as base \_\_\_\_\_

APPENDIX II

OBJECTIVE

9 I. Express the following in scientific notation.

1.  $3,000,000,000 =$

2.  $463,000,000,000 =$

3.  $.049 =$

4.  $.0000000000061 =$

10 II. Write the following as decimal numerals.

1.  $3.42 \times 10^6$

2.  $6.12 \times 10^{-4} =$

3.  $7.412 \times 10^6 =$

4.  $3.216 \times 10^{-7} =$

5.  $6.014 \times 10^4 =$

11 III. Compute the following:

1.  $3 \times 10^4 \times 6 \times 10^6$

2.  $\frac{6.8 \times 10^4}{3.4 \times 10^6}$

3.  $\frac{3 \times 10^7 \times 15 \times 10^{-2}}{9 \times 10^4}$

4.  $\frac{10 \times 10^4 \times 2 \times 10^7}{5 \times 10^8 \times 2 \times 10}$

5.  $(4.5 \times 10^2) + (3.6 \times 10^3)$

6.  $(3.7 \times 10^4) - (2.3 \times 10^2)$

7.  $(4.1 \times 10^6) + (2.4 \times 10^4)$

8.  $(6.1 \times 10^3) - (5.3 \times 10^2)$

9.  $(7.6 \times 10^{10}) + (5.6 \times 10^8)$

10.  $(4.3 \times 10^8) - (3.2 \times 10^6)$



ADVANCED STUDY

- I. Payne, read pp. 257-259, Ex. 43, 45, 47 pages 259-260.
- II. Nichols, read pp. 332-335, Ex. 1, 2, 3 c,e,i,m,p, 4 pages 335-336.
- III. Write a mathematical formula for the volume of a cube of edge  $X$ . The volume of a cube is equal to the product of the length, width, and height. Given a cube, write the formula for its volume using exponents. What happens to volume if you should double the length of the edge?
- IV.
  1. Light travels at a speed of three hundred million meters per second. How far is the sun from the earth (meters) if it takes 8 minutes for light to travel from the sun to the earth? Express in scientific notation.
  2. A Radar beam is directed toward the moon and the reflected beam is received  $2.6 \times 10^0$  seconds later. The beam travels at  $1.86 \times 10$  mile per sec. How far is the moon from the earth? Express in scientific notation.

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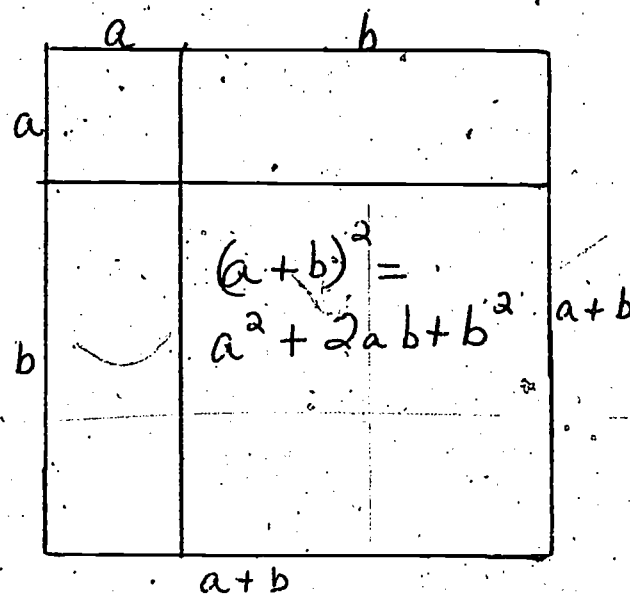
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**L** EARNING  
**A** CTIVITY  
**P** ACKAGE



FACTORING AND POLYNOMIALS



Algebra 93-94

DEVELOPED BY

*Shelby*

LAP NUMBER

11

WRITTEN BY

Diane Evans

### Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Wantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

## RATIONALE

In arithmetic, before you could solve practical problems, you had to be able to perform the fundamental operations with numbers. You needed to know the addition combinations before you could find the total cost of a number of items. In order to find the cost of several pounds of an item at a given price per pound, you needed to know how to multiply. Before you could work problems containing fractions and decimals, you had to learn the operations with those special types of numbers.

In algebra we will be dealing largely with polynomials. You must learn to perform the basic operations with polynomials before you can use them in applications. In this LAP you will learn to use polynomials in addition, subtraction, multiplication, and division. You will also learn to solve equations involving polynomials.

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given an algebraic phrase, identify the coefficients, factors, terms and degree of the phrase.
2. Given a polynomial of 1, 2, or 3 terms, determine if it is a monomial, binomial, or trinomial.
3. Given a polynomial, write it in descending or ascending order.
4. Given any pair of polynomials compute their
  - a. sum
  - b. difference
  - c. product
  - d. quotient

RESOURCES

OBJECTIVES 1,2

Nichols, read pp. 119-120, 199-200, 384-385, Ex. 1 page 201.

Vanatta, read pp. 67-71, Ex. 2,3,4 page 71.

Payne, read pp. 307-310, Ex. 1-27 pages 309-310.

Wooton, read pp. 52-53, Ex. 1-19 odd oral page 54; 19-24 page 55.

\* Appendix I parts I-III

OBJECTIVES 3,4

Vanatta, read pp. 133-135, 137-139, 140-143; Ex. 1-15 even p. 134; 1-12, 13, 15, 16, 19 page 136; 1-24 even p. 137; 1-24 even p. 139; 1-20 every 4th problem p. 140; 1-30 even p. 141; 1-20 even p. 144.

Dolciani, read pp. 198, 200-201, 2-3, 2-9, 219-222; Ex. 1-20 odd written p. 199; 1-8 p. 201; 1-9 page 202; 1-20 even p. 210; 1-14 even p. 207; 25-28, 33-40 page 210; 9-27 even p. 220; 1,3,6,8,12,15,18,24,25,29 p. 223.

Wooton, read pp. 310-323, Ex. 19-28,33,35 page 59; 17-49 odd pages 86-87; 1-41 odd pages 105-106; 1-23 odd pages 274-275; 1-39 odd p. 279; 1-17 even bottom p. 314; 1-29 odd pp. 319-320.

\* Appendix I parts IV, V

\* required

SELF-EVALUATION 1

OBJ.

- 1 I. For the polynomial  $2a^3 + 4a^2b^3 + 9a^2c^4$  state each of the following:
- The degree of polynomial
  - The degree of the polynomial with respect to a
  - The degree of the polynomial with respect to b
  - The degree of the polynomial with respect to c
  - The coefficient of  $a^3$
  - The number of terms in the polynomial

- 2 II. Classify each of the following on either a monomial, a binomial, or a trinomial (all letters are variables).

- |                      |   |
|----------------------|---|
| 1. $x + y$           | 5. 26   |
| 2. $4m$              | 6. $4.5a - 1.2b + 3.6c$                       |
| 3. $-\frac{1}{2}mry$ | 7. $\frac{1}{3} - \frac{1}{4} + \frac{1}{5}s$ |
| 4. $2x - y - z$      | 8. $3xyz - 2ab$                               |

- 3 III. Express the polynomial in ascending order of b and then in descending order of a

$$3a^2b^2 - 4a^3b^4 + 2ab^3 + 5a^4b$$


---



---

3,4

- IV. Find each sum and arrange in order of decreasing degree in n.

1.  $(3n^3 + 5 - 2n) + (n^2 - 6n - 8)$

2.  $(2m^3n - 3m^2n^2) + (4m^2n^2 - mn^3) + (-mn^3 - 7m^3n)$

OBJ.

SELF-EVALUATION 1 (cont')

4a V. Add

$$\begin{array}{r} (1) \quad 3x^2+6x+4 \\ \underline{-2x^2-6x+1} \end{array}$$

$$\begin{array}{r} (2) \quad 3xy-6x^2+3y \\ \underline{4xy+2x^2-4y} \end{array}$$

$$\begin{array}{r} (3) \quad 5x^2-3x+1 \\ \underline{2x^2-6x-4} \end{array}$$

$$(4) \quad (7x^2+6x+1)+(-4x^2-3x-6) =$$

$$(5) \quad (-3x+6y-3)+(4x-2y-7) =$$

4b VI. Subtract

$$\begin{array}{r} (1) \quad 3x^2+6x-1 \\ \underline{2x^2-4x+6} \end{array}$$

$$\begin{array}{r} (2) \quad 7x+3y-6 \\ \underline{-2x-4y+1} \end{array}$$

$$\begin{array}{r} (3) \quad 8x+6y-7 \\ \underline{-2x+6y+7} \end{array}$$

$$(4) \quad (6x+7y-2) - (8x+6y-7) =$$

$$(5) \quad (2x^2+7x-3) - (6x^2+3x+1) =$$

4a,b VII. Simplify

$$1. \quad (3x+2y-1) + (4x+6y) - (2x+3y+2) =$$

$$2. \quad (5x^2-6x+1) - (4x^2+2x+1) + (6x^2+9x-2) =$$

$$3. \quad (4xy-6x+7) + (2xy+6x-2) - (4xy+3x-7) =$$

4c VIII. Multiply

$$\underline{\hspace{2cm}} \quad 1. \quad (x+1)(x+5)$$

$$\underline{\hspace{2cm}} \quad 2. \quad (2x-6)(2x+7)$$

$$\underline{\hspace{2cm}} \quad 3. \quad (5y+8)(4y-3)$$

$$\underline{\hspace{2cm}} \quad 4. \quad (7z-3)(6z+2)$$

$$\underline{\hspace{2cm}} \quad 5. \quad (8x+4)(8x-4)$$

$$\underline{\hspace{2cm}} \quad 6. \quad (2x^2+1)(3x^2-5)$$

$$\underline{\hspace{2cm}} \quad 7. \quad (4x^2+1)(2x^2-9)$$



## Self-Eval. -1 con't

## IX. Multiply

1.  $\frac{x+6}{x-9}$

2.  $\frac{2x-3}{x+1}$

3.  $\frac{7x-7}{2x+3}$

4.  $\frac{4x^2+1}{2x^2-6}$

5.  $\frac{6x^2-7}{2x^2+1}$

6.  $3x^2+6x-9$

7.  $2xy+3x-1$

8.  $-3x^2+2x-6$

$\frac{2x+3}{2x+3}$

$\frac{2x+4}{2x+4}$

$\frac{x+4}{x+4}$

9.  $6x^2-2x+1)(2x^2+4)$

10.  $(2x^2-3xy+2)(3x^2+4x+3)$

## X. Divide

1.  $x^2-7x+12 \div x-4$

2.  $6x^3-x^2+3x-20 \div 3x+4$

3.  $30x^2-28x+8 \div 5x-3$

4.  $\begin{array}{r} 2x+4 \overline{) 2x^2-8x-34} \end{array}$

5.  $x^5+32 \div x+2$

6.  $\begin{array}{r} 7x-2 \overline{) 14x^3+38x^2-5x+9} \end{array}$

If you have satisfactorily completed your prescribed course of study,  
take the PROGRESS TEST. CONSULT YOUR TEACHER FIRST.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

5. Write the prime factorization of any given composite number.
6. Given any polynomial, express it in factored form when the polynomial:
  - a. has a common monomial factor
  - b. is written as the difference of two squares
  - c. is a perfect square trinomial
  - d. is of the form  $x^2 + (a + b)x + ab$ .
7. Given a polynomial of the form  $ax^2 + bx + c$ , express it in factored form.
8. Given a quadratic equation, determine the solution set by factoring.

### RESOURCES

#### OBJECTIVE 5

Vanatta, read pp. 286-289, Ex: 1-10 page 289.

Payne, read pp. 324-325, Ex. 22-41 odd page 325.

Wooton, read pp. 280-282, Ex. 1-33 odd page 283.

#### OBJECTIVE 6

Vanatta read pp. 289-292, 293-297, 299-300, 301-305; Ex. 1-5 p. 290; 1-22 even, page 292; 1-30 even p. 297; 1-14 even page 305.

Nichols, read pp. 359-360, 355-356, 381-384, Ex. 1-2 every other letter pages 360-361; 1-59 odd p. 356; 1-45 odd p. 384.

Payne, read pp. 326-335, 338; Ex. 1-15 even, 31-45 even pages 326-328; 1-18 odd pages 329-330; 1-40 odd pages 331-333; 1-24 odd pp. 334-335; 1-12 odd pages 338-339.

Wooton, read pp. 282-287, Ex. 1-53 odd page 286; 1-50 odd p. 287.

Pearson, read pp. 247-249; 373-386, 392-393; Ex. 1-4 every other letter p. 248-249, 4,6,10,12 EOL pages 246-247; 1-10 odd p. 274; 2-4 EOL p. 377; 1,2,4 EOL pp. 380-381; 1-4 EOL p. 382; 1-12 EOL pp. 385-386; 1-31 EOL p. 393.

(CONT')

RESOURCES 2 (cont.)

OBJECTIVE 7

Vanatta, read pp. 297-300, Ex. 1-16 even p. 300.

Nichols, read pp. 357-359, Ex. 1-59 odd pp. 358-359.

Payne, read pp. 335-339, Ex. 1-35 odd p. 337.

Pearson, read pp. 387-388, 395-396, Ex. 1-7 even, 8 every other letter, page 388.

OBJECTIVE 8

Nichols, read pp. 361-363, Ex. 1-25 odd page 363.

Payne, page 340, 358-359; Ex. 1-19 odd pages 340-341; 1-15 odd pages 360-361; 1-26 odd page 368.

Pearson, read pp. 389-395, 592-593, Ex. 1, 2 every other letter pages 390-391; 1-2 EOL page 593.

SELF-EVALUATION 2

OBJ.  
5

I. Find the prime factors of the following:

1.  $78 =$  \_\_\_\_\_

3.  $833 =$  \_\_\_\_\_

2.  $143 =$  \_\_\_\_\_

4.  $180 =$  \_\_\_\_\_

6

II. Express in factored form.

1.  $7x + 14y$

2.  $-2x^2 + 4y^2$

3.  $6xy - 3ax + 9xb$

4.  $9d^2 - 1$

5.  $4r^2 - 9s^2$

6.  $m^2 - 4n^2$

7.  $9a^2 - 81b^2$

8.  $9x^2 - 42x + 49$

9.  $x^2 - 12x + 36$

10.  $16x^2 + 32xy + 4y^2$

7

III. Find the factors.

1.  $x^2 - 3x - 10$

2.  $c^4 - 2c^2 - 63$

3.  $18 + 3x - 10x^2$

4.  $6y^2 - 17y + 12$

5.  $8x^2 - 10xy + 3y^2$

6.  $6x^2 - 5x - 21$

7.  $45x^2 + 320x + 35$

8.  $9x^2 + 6x - 8$

9.  $15y^2 - y - 2$

10.  $30x^2 + 39x - 9$

SELF-EVALUATION 2 (cont')

8 IV. Solve the following by factoring.

1.  $x^2 - 25 = 0$

2.  $x^2 - 5x + 6 = 0$

3.  $x^2 - 2x = 15$

4.  $x^2 - 8 = 7x$

5.  $2x^2 - 5x + 3 = 0$

6.  $6t^2 - 5t + 1 = 0$

7.  $6y^2 - 25y + 25 = 0$

8.  $9x^2 - 49 = 0$

9.  $a^2 - 3a = 10$

10.  $x^2 = 3x$

If you have satisfactorily completed your prescribed course of study,  
take the LAP TEST. CONSULT YOUR TEACHER FIRST.

APPENDIX I

I. Write the definition for each of the following.

1. polynomial
2. monomial
3. binomial
4. trinomial

II. Tell if each of the following is a monomial, binomial, or trinomial.

\_\_\_\_\_ 1.  $2x+3y$

\_\_\_\_\_ 4.  $8x-7xyz$

\_\_\_\_\_ 2.  $6xyz$

\_\_\_\_\_ 5.  $3xyz+2kl+l$

\_\_\_\_\_ 3.  $4x+9y+2$

\_\_\_\_\_ 6.  $3xyzl$

III. Give the degree of each of the following and identify the coefficients

\_\_\_\_\_ 1.  $3x^2y^5+3xy^3+2x^4y^2$

\_\_\_\_\_ 2.  $6x^2+7x^3+9y^4$

\_\_\_\_\_ 3.  $2xy-3x^2+6x^2y^3$

\_\_\_\_\_ 4.  $5x^2+3xy+9xy^2$

\_\_\_\_\_ 5.  $3x+7y+9xy$

IV. Rewrite the following in descending order of powers of x.

\_\_\_\_\_ 1.  $3xy+2x^2+6x^4-3x^3y$

\_\_\_\_\_ 2.  $7x^2y-2x^5y+3x^4+x$

\_\_\_\_\_ 3.  $x^3-6x^6+6x^9-2x$

\_\_\_\_\_ 4.  $3xy^2+4x^2y+7x^3y^4$

\_\_\_\_\_ 5.  $x^5y-6xy^3+2x^3y^2$

\_\_\_\_\_ 6.  $x^7y^5-3x^4y^2+8x^2y^3-2xy$

V. Write each of the polynomials in part IV in ascending order of powers of x.

ADVANCED STUDY

I. Work the following:

1.  $2x^5 + 9x^2 - 2x^3 - 5x^4 - 7x + 3 + 2x^2 - 3x + 1$
2. multiply:  $(3x^3 - 6x^2 + 9x - 6)(2x^2 - 3x - 9)$
3. divide:  $9x^N + 2 - 6x^{N+1} + 24x^{N+1} - 3x^N$

II. Show how synthetic division works and work the following using synthetic division.

1.  $x^3 - 3x^2 + 5x - 6 \div x - 2$
2.  $x^3 - 7x - 100 \div x - 5$
3.  $x^3 + 25 \div x + 5$

III. Solve any three of the following:

1.  $2x - (3x + 7) + 5(2x - 2) = 8x + 1$
2.  $(3x + 1)(2x - 6) - 3x(-2x + 4) + 6$
3.  $\frac{3x}{2} + \frac{6x - 4}{5} = \frac{11 + 2x}{3}$
4.  $\frac{3x - 4}{10} - \frac{6x + 2}{5} = \frac{x - 2}{2} + \frac{2x + 3}{4}$

IV. Nichols, read pp. 361-373, Ex. 1,2 EOL page 364; 1 p. 373; 1,2 EOL page 366.

V. Payne, read pp. 358-373, Ex. 1-27 odd p. 363; 1-25 odd page 369-370.

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### Vanatta (abbreviation)

Vanatta, Glen D., Algebra One: A Modern Course, Charles E. Merrill Publishing Co., 1966.

### Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., and Wooton, William; Modern Algebra, Book Two, Houghton Mifflin Co., 1965.

### Nichols (abbreviation)

Nichols, Eugene D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

### Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra I, Houghton Mifflin Co., 1967.

### Payne (abbreviation)

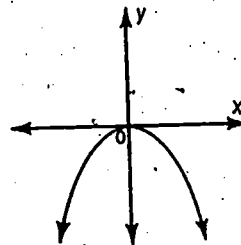
Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis G., Algebra One, Harcourt, Brace and World, 1969.

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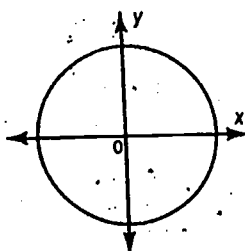
Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, 1964.



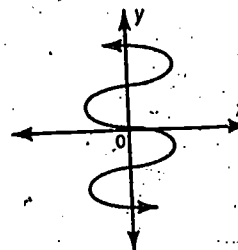
**L** EARNING  
**A** CTIVITY  
**P** ACKAGE



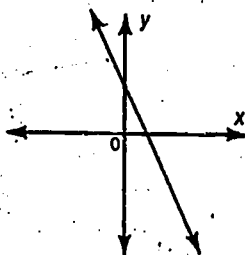
RELATIONS



AND



FUNCTIONS



Algebra 93-94

VIEWED BY

*D. P. Lewis*

LAP NUMBER

12

WRITTEN BY

*Sharon Evans*

### Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Wantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

## RATIONALE

The words RELATION and FUNCTION in mathematics are probably new to you! Consider the OPERATION of ADDITION with which you have worked for most of your school years . . . It is not only a RELATION but also a FUNCTION! There are many things which you have studied and which you will study in your future mathematics courses that are relations and functions.

Throughout mathematics we pair numbers and obtain set of ordered pairs which are relations. These ordered pairs and graphing (which you have previously studied) will serve as a basis for the study of relations and functions.

In this LAP you will be concerned primarily with the meaning of relation and function. Graphing relations and functions will be stressed in order to give you experience in actually working with these ideas which are basic to future courses in mathematics and science!

## SECTION 1

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given two finite sets, list the ordered pairs which belong to their Cartesian Set (Cartesian Product).
2. Given two subsets of the real numbers, graph their Cartesian Set on the coordinate plane.
3. Given a relation defined by a rule of correspondence, name the ordered pairs which belong to this relation.
4. Given a relation, name:
  - a. its domain
  - b. its range
5. Given a relation, determine whether or not that relation is a function where the relation is defined by:
  - a. a set of ordered pairs
  - b. a graph
  - c. a rule or correspondence
6. Given a relation, name its inverse.

### RESOURCES

#### Objectives 1, 2

Nichols, read pp. 393-395, Ex. 1, 3 page 394; 1, 3, 5, 6 page 395.

Wooton, read pp. 377-379, Ex. 19, 20 page 381.

Pearson, read pp. 431-435, 441-442, Ex. 1, 3, 5 page 432;  
1 a, c, f pages 436-437; 1, 3, 5 pages 442-443.

RESOURCES (cont')

Objective 3

Nichols, read pp. 396-397, Ex. page 398 1 - a, c, d, f  
2 - a, c, d, f  
3 - a, c, d, f

Payne, read pp. 174-175, Ex. 4, 5 page 176; 1, 4, 5, 8, 10,  
21 pages 180-181.

Pearson, read page 542-543 Ex. 1, 2, 3, 5, 8a,b,c page 544.

Objectives 4, 5

Nichols, read pp. 398-399, Ex. pages 399-404 1 - a,b,d,f,g,h,j,k  
3 - a,b,d,f,g,h,j,k  
4 - a,b,d,f,g,h,j,k  
5, 8a,c,e,g  
9 a thru n  
10 a thru j

\*\* Payne, read pp. 175-179, 466-467; Ex. 1, 2, 10, 11, 12, 15,  
17, 18-21 pages 176-177; 28 page 181; 1, 2, 4, 5, 7, 9-12,  
13, 14, 21-30, 33, 35 page 467.

Pearson, read pp. 545-550, Ex. 1a,b, 2a,b, 5a, 6a pages 545-  
547; 1-20 pages 550-551.

\* Appendix I

Objective 6

Nichols, read pp. 404-407, Ex. 1 a, f, 1, 2 a, c, e, h, i  
pages 407-408.

Payne, read pp. 481-483, Ex. 1, 2, 6, 7, 11-14, 15-17, 19,  
21, 24 pages 483-484.

\* required

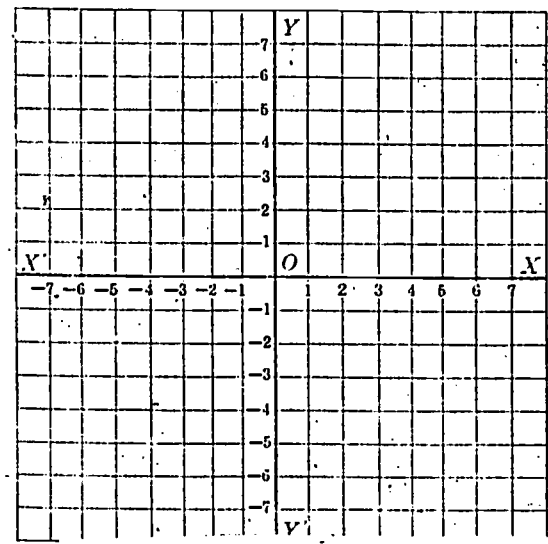
\*\* recommended

SELF-EVALUATION I

Q1.

1. I. Given  $A = \{2, 3, 5\}$  and  $B = \{3, 5\}$
1. Find  $A \times B$
  2. Find  $B \times A$

2. II. Graph  $A \times B$  from Example I



3. III. 1. If the Universal set is the set of real numbers, which one of the following ordered pairs belong to the solution set of the relation  $y = 2x - 1$ .

- (a)  $(0, -1)$
- (b)  $(\frac{3}{4}, \frac{1}{2})$
- (c)  $(10, 19)$
- (d)  $(\frac{5}{8}, \frac{1}{4})$
- (e) all of these

2. If the Universal set is the set of real numbers, which of the following belongs to the solution set of the relation  $2x - 3y = 1$ ?

- (a) all ordered pairs in the coordinate plane
- (b)  $(5, 3)$
- (c)  $(10, \frac{19}{3})$
- (d)  $(4, 2)$
- (e) none of these



SELF-EVALUATION 1 (cont')

4 IV. List the domain and range of the following.

1.  $(2,1) (2,3) (3,4) (5,6) (7,6)$

DOMAIN

RANGE

2.  $y = 2x + 1$

3. 
$$\begin{array}{c|cccc} x & 3 & -2 & -7 & 0 & -7 \\ \hline y & 4 & 6 & 8 & 2 & 4 \end{array}$$

4.  $y = x^2$

5a V. Which of the following relations are functions?

(a)  $\{(-1,1) (0,0) (0,1) (1,2) (1,3)\}$

(b)  $\{(1,3) (3,17) (2,3) (3,2)\}$

(c)  $\{(-2,1) (-1,2) (0,0) (1,2) (2,1)\}$

(d)  $\{(1,1) (1,2) (1,3) (1,4)\}$

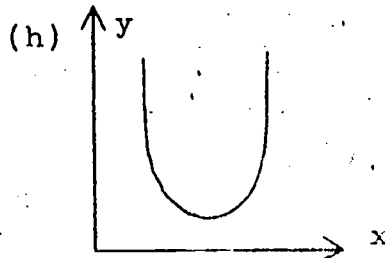
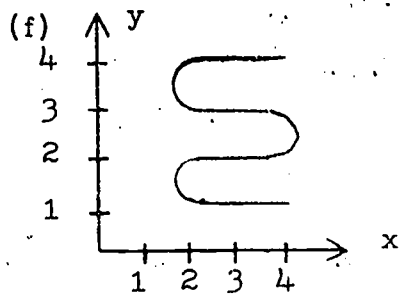
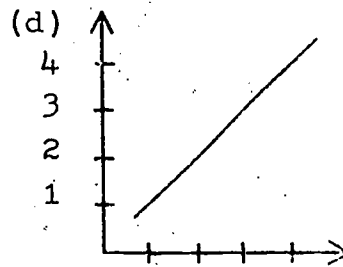
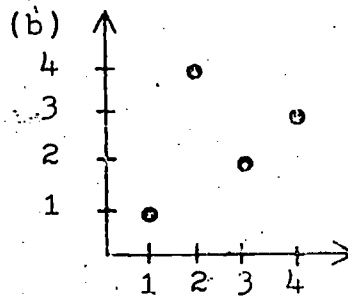
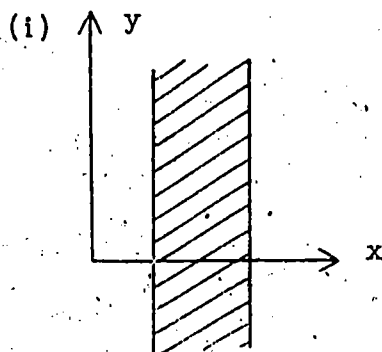
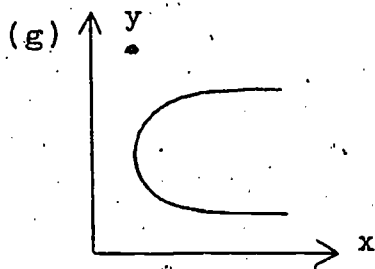
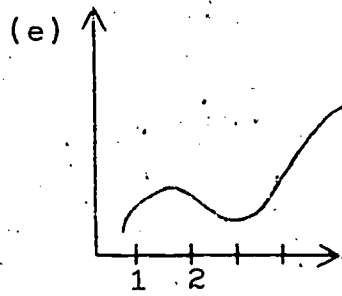
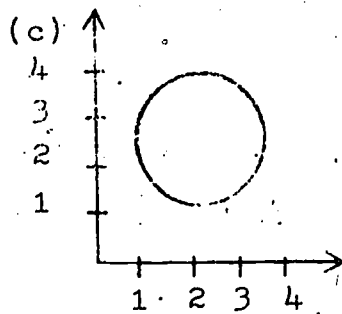
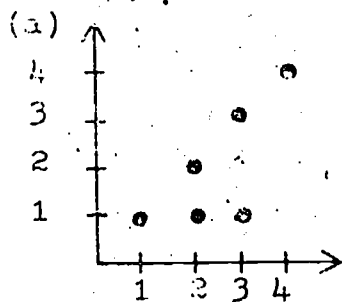
(e) none of these

Continued on the following page.

SELF-EVALUATION 1 (cont')

5b

VI. Below are graphs of some relations, tell which of them are functions by answering YES and NO for those which are not functions.





SELF-EVALUATION 1 (cont')

5c VII. Which of the following relations is a function?

(a)  $y \leq 2x + 1$

(b)  $x = -3$

(c)  $y = 2x + 1$

(d)  $y = x$

(e)  $y > x + 2$

(f)  $y = -2$

6 VIII. Write the inverse of each of the following.

(1)  $\{(-1,2), (2,1), (3,2), (4,7)\} =$  \_\_\_\_\_

(2)  $y = x + 5$  \_\_\_\_\_

(3)  $2x + 3y = -1$  \_\_\_\_\_

(4)  $y = 2x^2$  \_\_\_\_\_

(5)  $(3,1) (-2,4) (-6,8) (4,-2)$  \_\_\_\_\_

(6)  $5x = 1 - y$  \_\_\_\_\_

If you have satisfactorily completed your work, take the Progress test. Consult your teacher first.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

7. Given a function and a real number, compute the value of the function at the given number.
8. Given a function, name whether it is a linear function or a quadratic function.
9. Given a linear function, construct its graph.
10. Given a linear function, determine its slope.
11. Given a quadratic function of the form  $f(x) = ax^2 + bx + c$  where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ , construct its graph.

### RESOURCES

#### Objectives 7, 8

Nichols, read pp. 408-409, Ex. 1 a,b,e,f,h,i, 2 top p. 410.

Payne, read pp. 179-181, 349-353, 471-472; Ex. 11-13 page 181; 1-5, 7, 10 page 473; 1-10 middle page 353; 5 page 183; 1, 2 checkpoint page 183.

Pearson, read pp. 554-555, Ex. 1 a,b,c,d,g, 3 a,b,c - page 555.

\* Appendix II

#### Objectives 9, 10

Vanatta, read pp. 199-204, Ex. 1-5 pages 204-205.

Nichols, read pp. 410-412, Ex. 1 a,b,c,f, 2; 3 a,b,d,e,g, 4 pages 410-411; 1 a,b,c,d,f,g, 2 a,c,e,f,i, 1 page 412.

Payne, read pp. 178-180, 185-188; 471-472, Ex. 14,16,18,20 page 181; 1,2,5,8,12,27 page 189; 13,15 page 473.

Dolciani, read pp. 346-348, Ex. 11-18 page 348.

#### Objective 11

Nichols, read page 413, Ex. 1 a-f, 2 a,d,g, 3a, 5c, 9a, d' pp. 414-415.

Payne, read pp. 349-353, Ex. 1-3, 7, 8, 11, 21 pages 353-354.

Wooton, read pages 394-399, Ex. 1-12 page 399.

SELF-EVALUATION 2

OBJ.

7

I. For each of the following functions, find the value indicated.

\_\_\_\_\_ 1. Find  $f(2)$  for  $f(x) = 6x + 1$

\_\_\_\_\_ 2. Find  $f(-3)$  for  $f(x) = 2x - 1$

\_\_\_\_\_ 3. Find  $f(0)$  for  $f(x) = \frac{x+1}{6x}$

\_\_\_\_\_ 4. Find  $f(30)$  for  $f(x) = x^2 - x$

\_\_\_\_\_ 5. Find  $f(-10)$  for  $f(x) = \frac{8-2x}{4}$

8

II. Determine if each of the following equations is linear or quadratic.

\_\_\_\_\_ 1.  $y = x$

\_\_\_\_\_ 2.  $y = 2x + 1$

\_\_\_\_\_ 3.  $y^3 = x^4 + 2 + 4x^2$

\_\_\_\_\_ 4.  $y = x^2 + 2$

\_\_\_\_\_ 5.  $3x + 2y = 6$

9

III. Graph the following linear functions. Use the graph paper that follows.

1.  $y = 2x$

2.  $y - 2 = \frac{1}{3}x + 3$

3.  $y = -3$

4.  $2x + 4y = 8$

5.  $3y = -2x + 6$

6.  $x = 5$

10

IV. Give the slope of each of the following linear functions.

\_\_\_\_\_ 1.  $2x + 3y = 4$

\_\_\_\_\_ 2.  $y = 6x - 1$

\_\_\_\_\_ 3.  $y - 2 = \frac{1}{3}x + 3$

\_\_\_\_\_ 4.  $y = 6$

\_\_\_\_\_ 5.  $x = -2$

SELF-EVALUATION 2 (cont')

11 V. Graph the following quadratic functions. Use the graph paper that follows.

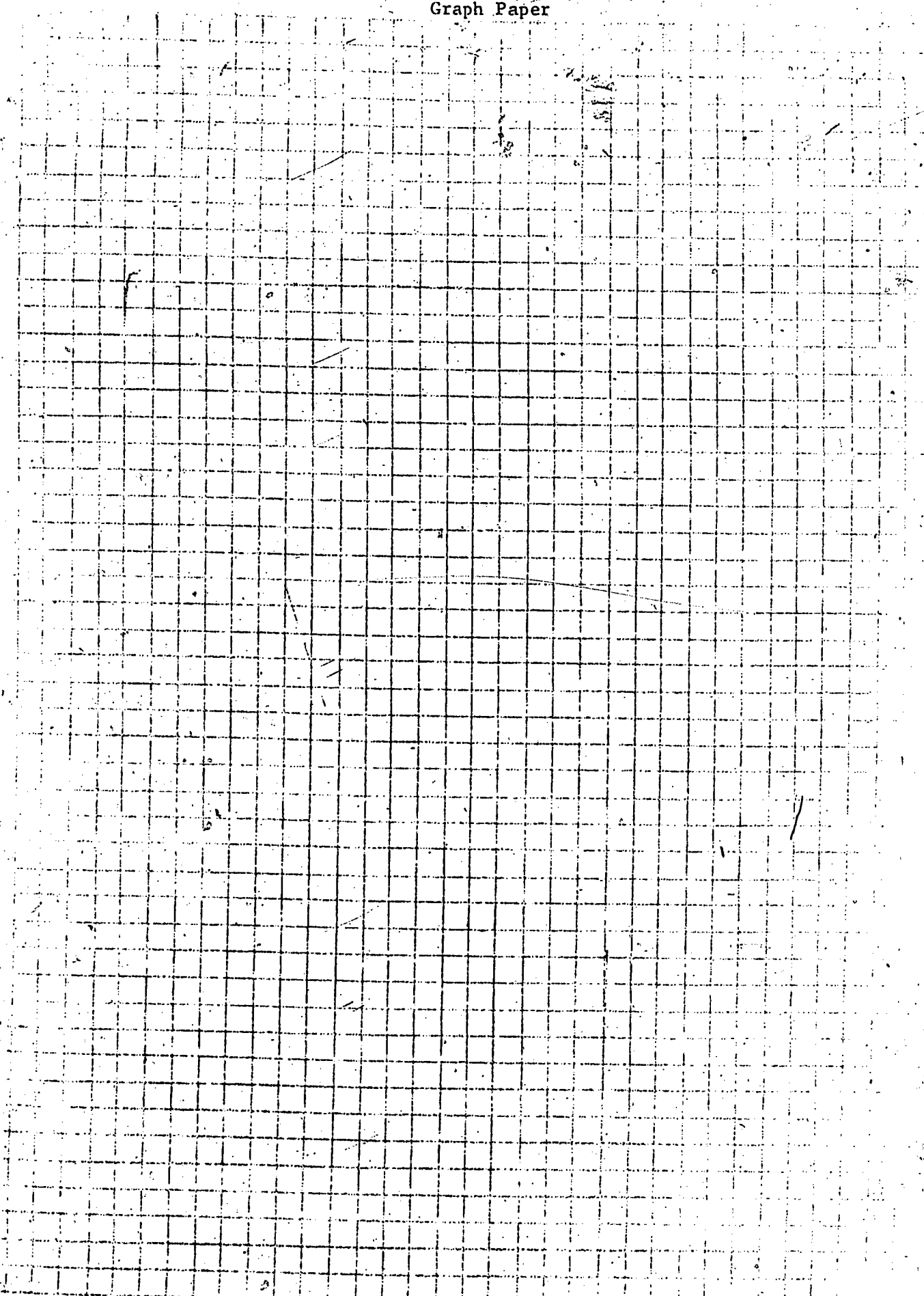
1.  $f(x) = x^2 - 2x + 1$

2.  $f(x) = x^2 - 2$

3.  $f(x) = x^2 + x - 6$

If you have satisfactorily completed your work, take the IAP TEST. Consult your teacher first.

SELF-EVALUATION 2 (cont')  
Graph Paper



ADVANCED STUDY

1. Payne, Ex. 39, 40 page 184.
2. Payne, read pp. 193-195, Ex. 1-20 even  
page 195.
3. Dolciani, read pp. 346-348, Ex. 1-10  
page 348.
4. Pearson, read pp. 559-560, Ex. 1-5 page  
560.

APPENDIX I

I. Write the domain and range of the following.

DOMAIN

RANGE

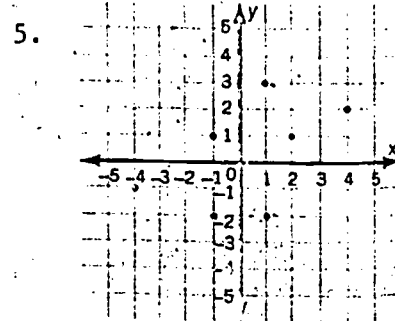
1.  $(8,1)(7,2)(-3,1)(7,-6)$

2.  $y = x^2$

3.

x	y
-8	5
0	2
-6	7
2	5

4. y is equal to twice x



5 II. Determine if the following is a function. Write F if it is a function. If it is not a function, write R for relation only.

\_\_\_\_\_ 1.  $(3,6)(2,4)(-4,2)(-6,4)$

\_\_\_\_\_ 2.

x	-6	-2	-4	6
y	8	1	3	7

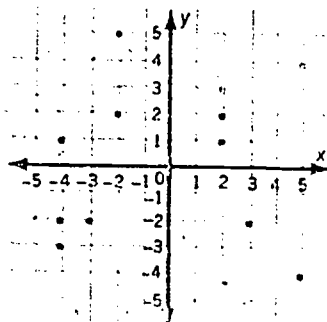
\_\_\_\_\_ 3.  $y = x + 1$

\_\_\_\_\_ 4.  $(3,-3)(4,-4)(5,-6)(7,-8)(3,-9)$

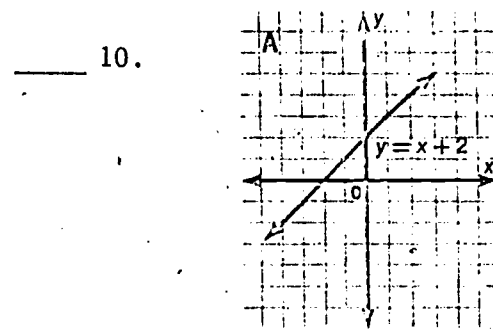
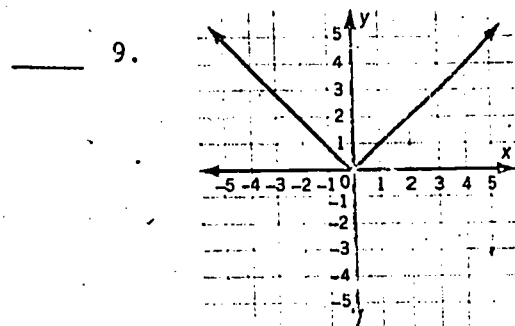
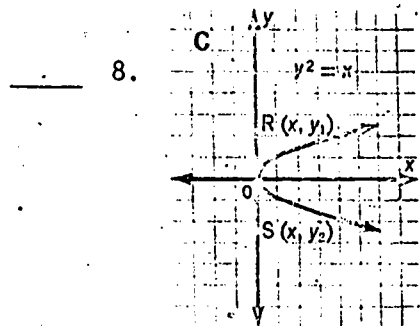
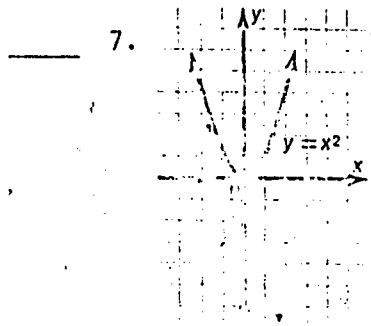
\_\_\_\_\_ 5.

x	y
0	7
7	0
3	5
9	4
5	3

\_\_\_\_\_ 6.



APPENDIX I (cont')





APPENDIX II

1. Define linear function.

2. Define quadratic function.

3. Determine if the following are linear or quadratic.

\_\_\_\_\_ a.  $y = x$

\_\_\_\_\_ b.  $y^2 = 4x + 2$

\_\_\_\_\_ c.  $y = 3x^2 + 2x + 1$

\_\_\_\_\_ d.  $y = 3x + 2$

\_\_\_\_\_ e.  $x^2 + y^2 = 25$

\_\_\_\_\_ f.  $2x + 3y = 7$

APPENDIX III

For each of the following linear functions, rewrite each in slope-intercept form, state the slope, and y-intercept, and graph each. (USE THE GRAPH PAPER THAT FOLLOWS.)

(1)  $2x + 3y = -6$  \_\_\_\_\_ Slope = \_\_\_\_\_ y-int. \_\_\_\_\_

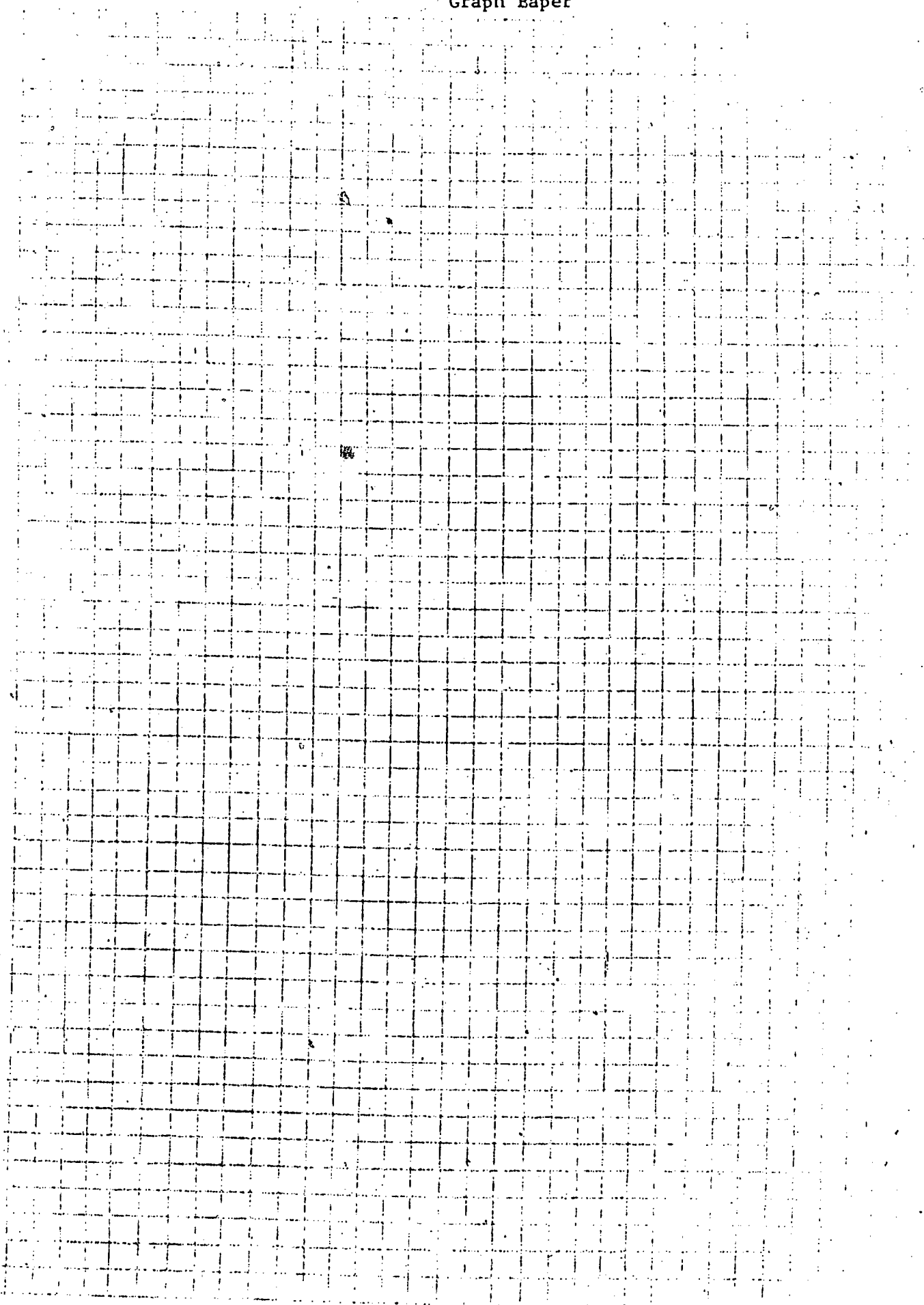
(2)  $2y = -4x + 8$  \_\_\_\_\_ Slope = \_\_\_\_\_ y-int. \_\_\_\_\_

(3)  $y = -x$  \_\_\_\_\_ Slope = \_\_\_\_\_ y-int. \_\_\_\_\_

(4)  $-18x - 6y = 18$  \_\_\_\_\_ Slope = \_\_\_\_\_ y-int. \_\_\_\_\_

(5)  $3x = 6y - 12$  \_\_\_\_\_ Slope = \_\_\_\_\_ y-int. \_\_\_\_\_

APPENDIX III (cont.)  
Graph Paper



## REFERENCES

Vanatta (abbreviation)

Vanatta, Glen D., Algebra One: A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965.

Nichols (abbreviation)

Nichols, Eugene D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Wooton (abbreviation)

Dolciani, Mary P., Wooton, William, Beckenbach, Edwi F., Sharron, Sidney, Modern School Mathematics, Algebra 1, Houghton Mifflin Co., 1967.

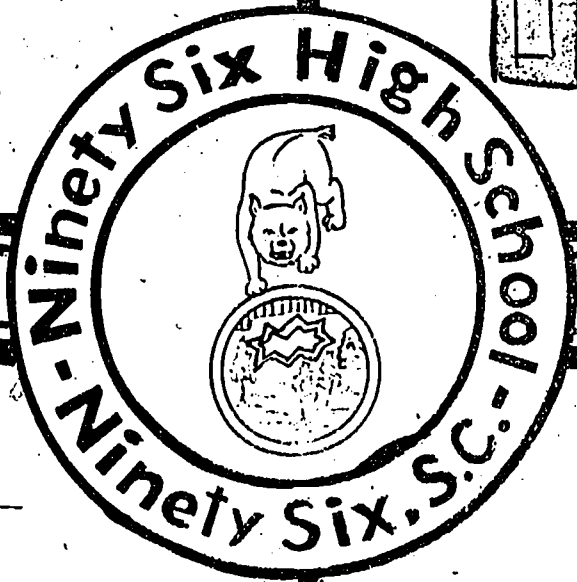
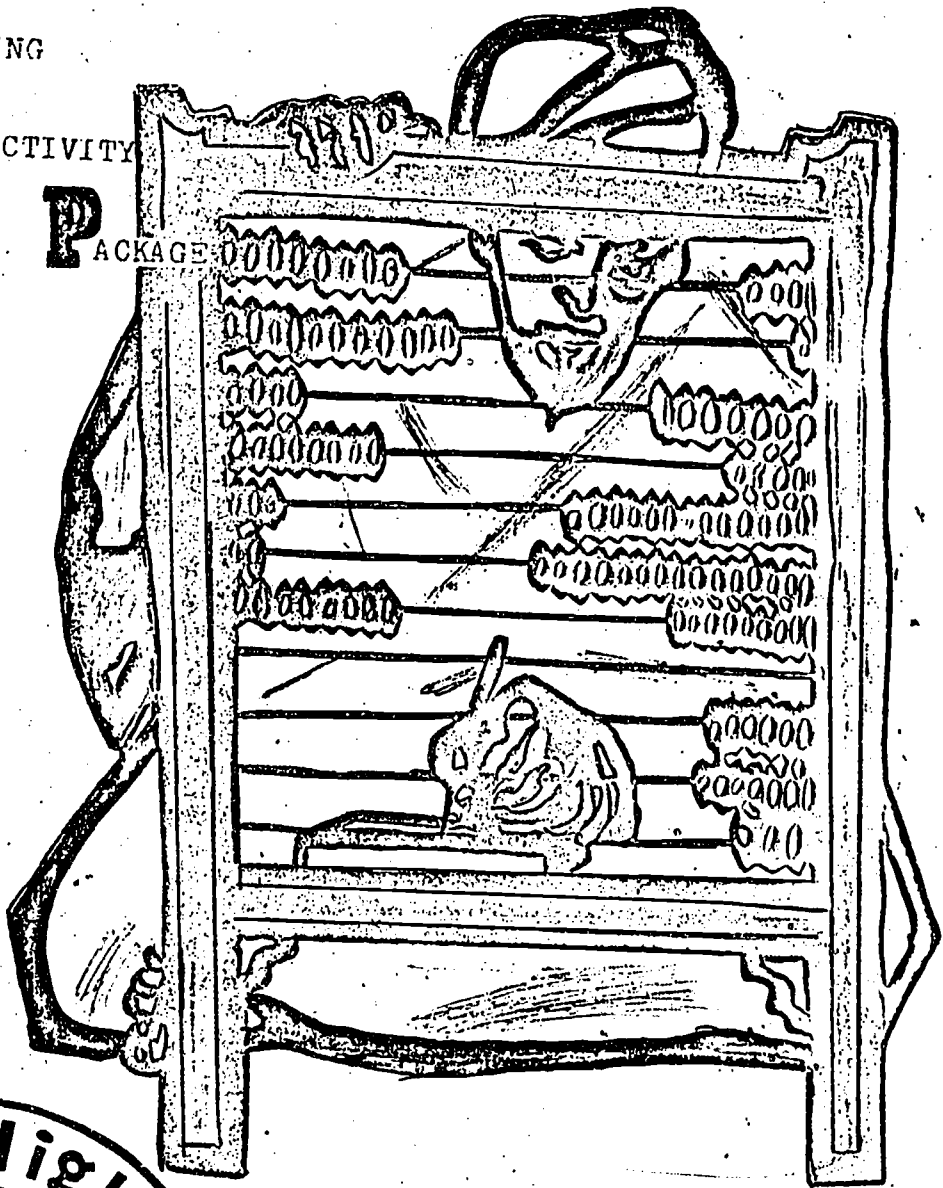
Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F.; Lankford, Jr., Francis G., Algebra One, Harcourt, Brace and World, 1969.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, 1964.

L  
EARNING  
A  
CTIVITY  
P  
ACKAGE



NUMBER SYSTEMS

ALGEBRA 103-104

REVIEWED BY

*J.R.*

LAP NUMBER

14

WRITTEN BY

*Evans*

*EK JS*

## Instructions

- I. Read Rationale
- II. Read Behavioral Objectives
- III. Resources
  - A. All work must be done in math notebook with pencil only.
  - B. Keep your notebook up to date. Your teacher may ask for it at any time (without warning).
  - C. Work all the Exercises in at least two text for each objective.

Always check your exercises (see your teacher)

- IV. Self-Evaluation
  - A. Must be taken at completion of activities for each section.
  - B. Does not affect your grade in any way.
- V. Advanced Study
  - A. To be done only after all previous work has been satisfactorily completed.
  - B. Must be approved by teacher.
- VI. Progress Test and LAP Test
  - A. Teacher graded
  - B. Recycling may take place at this time if test is not satisfactory.

DO NOT LOSE YOUR LAP. If you do, you must buy another one.

### Rationale (The LAP's Purpose)

You have studied many mathematical systems in the past. When you first learned to count, you used the set of natural numbers. In your early study of arithmetic, you learned how to add, subtract, multiply, and divide. You soon found that some division and subtraction problems had no answers. To handle such situations, the set of integers was developed for closure over subtraction, and the set of rational numbers was developed for closure over division.

Irrational numbers such as  $\sqrt{5}$  and  $\pi$  are not included in the number sets as yet developed. The set of real numbers is the union of the rational and irrational numbers. It is the most complete number system to be developed.

In this LAP you will study the set of real numbers, its subsets, and properties.

Later, you will extend the field properties of the set of real numbers into the field of complex numbers which give meaning to numerals such as  $\sqrt{-2}$ .

## Section I

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Identify the following sets: natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers when written
  - a) in set notation
  - b) as definitions
2. Determine the relationships between the sets of natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.
3. Given the following sets: natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers, identify the properties of each:
  - a) by completing the chart in Appendix I
  - b) by answering True or False questions
  - c) by answering multiple choice questions
4. Given any number system, state whether or not it is a field. If it is not, list the missing properties.
5. Given two or more real numbers, compute their sum, difference, product, and/or quotient.
6. Write or select from several definitions, that which defines "density" for a given number system.



## Resources

### I. Reading and Problems.

1. Vanatta, Algebra Two. #1 pp. 10-13, 29-34, Ex. \_\_\_\_: #2 pp. 29-35, Ex. \_\_\_\_: #3 pp. 5-11, 16-17, 29-37, Ex. 1-16 p. 6-7, 1-11 p. 15, 1-10 p. 26, 1-10 p. 53: #4 \_\_\_\_: #5 pp. 10-11, 22-23, Ex. 5-14 p. 11-12, 1, 3, 6, 8, 9 p. 25: #6 pp. 17-18, Ex. 1, 2, 4-8 p. 19, 8 p. 35.
2. Nichols, Modern Intermediate Algebra. #1 pp. 1, 14, 21, 33, 34, Ex. 5-8 p. 18; 15, 19 p. 42: #2 pp. 1, 4, 21, 33, 34, Ex. 11-13 p. 41, 6 p. 35: #3 pp. 2-3, 14-17, 21-23, Ex. 1-4 p. 2, 3 p. 5, 3 p. 17; 1, 2, 4-10 p. 23; 1, 4, 5, p. 35; 1, 5, 30-39 p. 37-43: #4 p. 1, 14, 23, 34, Ex. 1 p. 17: #5 pp. 30-31, Ex. 11, 12 p. 23-24, 7 p. 39, 1 a, b, c, g, h, i p. 32: #6 p. 27-28, Ex. 1, 3, 4, 5 p. 28.
3. Dolciani, Modern Algebra, Bk Two. #1 pp. 2, 8, 157, 251, Ex. \_\_\_\_: #2 \_\_\_\_: #3 pp. 11-15, Ex. 1-16 p. 12, 1-20 even p. 16: #4 \_\_\_\_: #5 pp. 22-24, 26-29, Ex. 1-24, even p. 26, 1-12 p. 29: #6 p. 256, Ex. 5-16 p. 257.

### II. Activities

Complete the chart in Appendix 1.

### III. Audio

For each Wollensak you use, you must secure a work sheet from your teacher, complete it, and turn in to teacher.

Wollensak C-3453 The Commutative Property  
C-3454 The Associative Property  
C-3455 The Distributive Property  
C-3456 The Closure Property  
C-3457 The Inverse Elements  
C-3458 The Real Number System  
C-3459 Identity Elements

### IV. Video

Filmstrip: Rational and Irrational Numbers  
Transparency: T-510, Math, Visual No. 8, Complex numbers  
Transparencies: T-510, Math, Visual Nos. 13, 14 Properties of Real Numbers

### V. Games

Cross Number Puzzle - Review of Fractions (II)  
Cross Number Puzzle - Things to Know About Fractions  
The Conversion Game (Bingo game on operations with real numbers)  
"Propo" (Bingo game on real number system and properties)

Self-Evaluation I

- 1 I. Identify the following sets of numbers. Write the name of the set in the blank.

- \_\_\_\_\_ 1. {... -2, -1, 0, 1, 2 ...}
- \_\_\_\_\_ 2. union of rationals and irrationals
- \_\_\_\_\_ 3. the natural numbers and zero
- \_\_\_\_\_ 4. the set of numbers starting with one and formed by successively adding one
- \_\_\_\_\_ 5. all numbers of the form  $\frac{a}{b}$  where  $a, b \in I$  and  $b \neq 0$ .
- \_\_\_\_\_ 6. non-repeating decimals cannot be written as  $\frac{a}{b}$

- 2 II. True or False.

- \_\_\_ 7. The real numbers equal the union of the rationals and the integers.
- \_\_\_ 8. The real numbers are a subset of the rational numbers.
- \_\_\_ 9. The integers are a subset of the real numbers.
- \_\_\_ 10. The natural numbers are a subset of the integers.
- \_\_\_ 11. The intersection of the integers and the rationals is  $\emptyset$ .
- \_\_\_ 12. The rationals are a subset of the irrationals.
- \_\_\_ 13. The natural nos. are a subset of the whole numbers.
- \_\_\_ 14. The intersection of the rationals and the irrationals is the set of real numbers.

- 3 III. Multiple choice: for each of the following write the letter(s) for the correct answer.

- \_\_\_ 15. Which of these properties do not hold for the reals?
- a) closure property for addition
- b) associative property for multiplication
- c) distributive property of multiplication over addition
- d) additive identity
- e) none of these
- \_\_\_ 16. Which of these items holds for the natural numbers?
- a) multiplicative inverses
- b) additive inverses
- c) multiplicative identity
- d) additive identity
- e) none of these

Self-Evaluation (cont')

\_\_\_ 17. Which of the following does not hold for the rationals?

- a) associative property of multiplication
- b) closure for addition
- c) multiplicative inverses
- d) commutative for multiplication
- e) none of these

\_\_\_ 18. Which of these holds for the irrationals?

- a) multiplicative identity
- b) additive identity
- c) multiplicative inverses
- d) closure for multiplication
- e) none of these

\_\_\_ 19. Which of these does not hold for the integers?

- a) additive inverses
- b) multiplicative inverses
- c) closure for addition
- d) multiplicative identity
- e) none of these

\_\_\_ 20. Which of these properties hold for the rationals?

- a) additive inverses
- b) multiplicative identity
- c) additive identity
- d) multiplicative inverses
- e) all of these

\_\_\_ 21. Which of these holds for the real numbers but does not hold for the irrational numbers?

- a) multiplicative inverses
- b) closure for multiplication
- c) additive inverses
- d) closure for addition
- e) none of these

4 IV. State if each of the following is true or false. If false, state the property(s) to make the statement true.

\_\_\_ 22. The set of integers is a field. \_\_\_\_\_

\_\_\_ 23. The set of real numbers is a field. \_\_\_\_\_

\_\_\_ 24. The set of irrational numbers is a field. \_\_\_\_\_

\_\_\_ 25. The set of natural numbers is a field. \_\_\_\_\_

Self-Evaluation (cont')

5 V. Multiple Choice: Simplify the following choose the letter of the correct answer.

\_\_\_ 26.  $\frac{3}{3} + \frac{1}{2} =$  a)  $\frac{3}{5}$  b)  $\frac{3}{6}$  c)  $\frac{7}{6}$  d) none of these

\_\_\_ 27.  $-3 - (-6) =$  a) -9 b) 3 c) -3 d) -9 e) none of these

\_\_\_ 28.  $\frac{-4}{3} + \frac{-2}{5} =$  a) 1 b)  $\frac{-26}{15}$  c)  $\frac{-2}{15}$  d) none of these

\_\_\_ 29.  $\frac{3}{4} \times \frac{5}{3} =$  a)  $\frac{8}{12}$  b)  $\frac{8}{7}$  c)  $\frac{5}{4}$  d) none of these

\_\_\_ 30.  $-\frac{1}{2} \div \frac{3}{5} =$  a)  $\frac{-5}{6}$  b)  $\frac{-3}{10}$  c)  $\frac{2}{10}$  d) none of these

\_\_\_ 31.  $(-8) + (-6) =$  a) 14 b) -2 c) -14 d) none of these

\_\_\_ 32.  $-9 \times -7 =$  a) -63 b) -16 c) 63 d) 16 e) none of these

\_\_\_ 33.  $-81 \div 9 =$  a) 9 b) -9 c) 3 d) none of these

\_\_\_ 34.  $\frac{4}{9} =$  (a)  $4.\overline{9}$  b)  $\overline{.49}$  c)  $\overline{.4}$  d) none of these

\_\_\_ 35.  $\frac{7}{25} =$  a) .28 b) .725 c) 7.25 d) none of these

6 VI. Write the definition of density.

APPENDIX 1

Write an X by each property that holds for the given set.  
 Write a circle (0) by each property that does not hold.  
 Do not leave a blank.

PROPERTIES	NATURAL	WHOLE	INTEGERS	RATIONALS	IRRATIONALS	REALS
Closure for +						
Closure for x						
Commutative for +						
Commutative for x						
Associative for +						
Associative for x						
Distributive						
Additive identity						
Additive inverse						
Multiplicative identity						
Multiplicative inverses						

If you have mastered your Behavioral Objectives take the LAP TEST.

ADVANCED STUDY

1. Draw a Venn diagram showing the set of real numbers and its subsets.
2. Prove: The sum of two even numbers is an even number.
3. Prove: The product of two odd numbers is an odd number.
4. Prove: The product of an even and odd number is an even number.
5. Prove:  $\forall a, b$  if  $a < b$ , then  $a < \frac{2a+b}{3} < b$ .
6. Find a number for which  $n^2 - n + 41$  is not a prime number.
7. Prove  $\sqrt{2}$  irrational.

### References

1. Vanatta, Glen C. and Goodwin, A. Wilson, Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.
2. Dolciani, Mary P., Berman, Simon L., Wooton, William, Modern Algebra Book Two,
3. Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.
4. Transparency T-510 Visual No. 8, Complex Numbers
5. Transparencies T-510 Visuals 13,14 Properties of Real Numbers
6. Audio Tapes

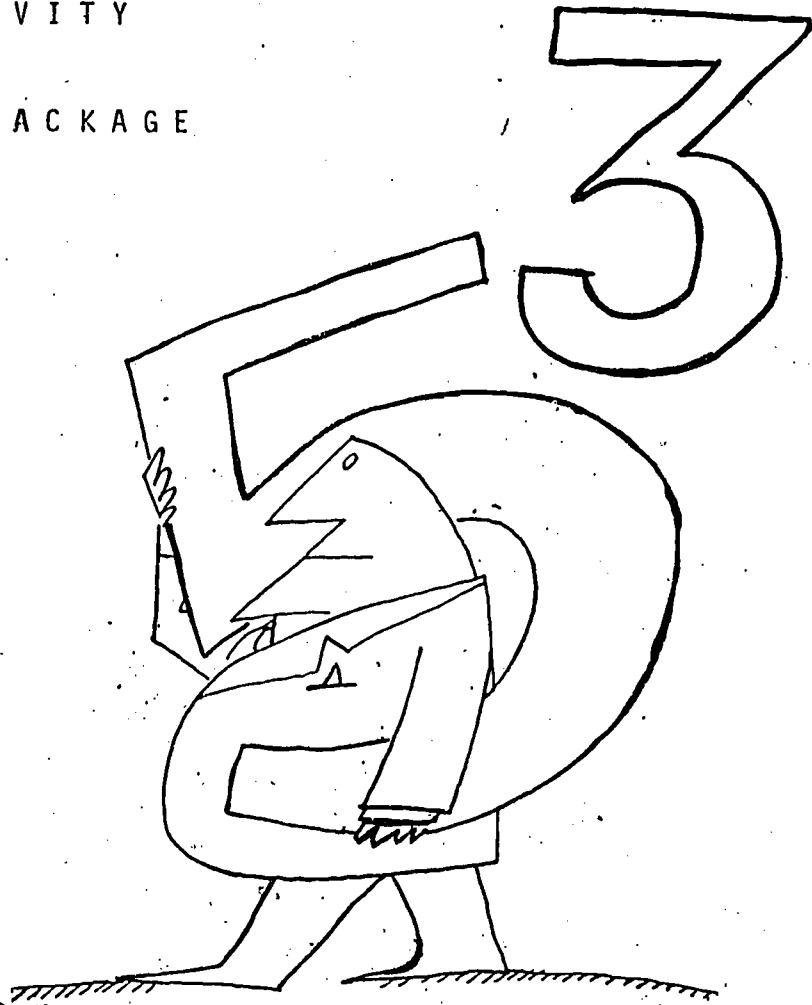
Wollensak Teaching Tapes: C-3453  
C-3454  
C-3455  
C-3456  
C-3457  
C-3458  
C-3459

**L** EARNING

EXPONENTS AND RADICALS

**A** CTIVITY

**P** ACKAGE



Algebra 103-104

REVIEWED BY

LAP NUMBER 15

WRITTEN BY Diane Evans



## RATIONALE (The LAP's Purpose)

Have you ever wondered why exponents are added when powers with the same base are multiplied? Why do we subtract exponents when dividing? You may have learned how to manipulate exponents and radicals, but do you really understand what you are doing? If some of these rules have been forgotten, we will not only recall them in this LAP, but will also learn why we operate with them as we do.

We will examine operations with exponents and radicals more formally than in previous studies. Some of the topics covered will be the laws of exponents, square root, rational number exponents, radical equations, and scientific notation. Upon completion of this LAP, you should have a good foundation for future studies in mathematics.

In later LAPS, concepts involving exponents and radicals will be extended to such important topics as logarithms, rational expressions, quadratic functions, and complex numbers. Exponents and radicals are the building blocks for many topics in mathematics and in the applications of mathematical concepts to scientific studies.

## Section 1

### Behavioral Objectives

Upon completion of your prescribed course of study, you will be able to

1. Identify and apply any of the following in simplifying expressions\* involving integral exponents.
  - a. Definition of Integral Exponent
  - b. Definition of Zero Exponent
  - c. Definition of Negative Exponent
  - d. Product of Powers Property
  - e. Power of a Power Property
  - f. Power of a Product Property
  - g. Power of a Quotient Property
  - h. Quotient of Powers Property
  - i. Negative Exponent Property
  - j. Order of Operations Agreements
2. Given the expression  $\sqrt[n]{x}$ , identify:
  - a. the radical
  - b. the radical sign
  - c. the radicand
  - d. the index
3. Given any expression involving radicals, write the simplest form\* of any of the following:
  - a. any radical
  - b. a product or quotient of radicals
  - c. a sum or difference of radicals
  - d. a product or quotient of sums or differences of radicals
  - e. an expression having a radical as its denominator
4. Simplify radical expressions for which the radicand is a rational number.

\* NOTE: To simplify a radical or exponential expression is to write the expression so that:

1. there is no radical in the denominator
2. each exponent of the radicand is a natural number less than the index
3. each exponent is written as a positive exponent
4. no real number is expressed in exponential notation
5. there are no unnecessary parenthesis

## RESOURCES

### I. Reading and Problems.

Vanatta, Algebra Two. #1 pp. 8, 243-247, Ex. 1-24 p. 244, 1-10 p. 247, 24-28 p. 27: #2 p. 246, Ex. \_\_\_\_: #3 pp. 248-250, 252-254, 256-257, Ex. 1-9 at top and bottom p. 251, 1-8, 10-15 top p. 255, 1-9, 12-16 bottom p. 255-256, 1-18 odd p. 253, 20, 21, 25-27 top p. 255, 1-8 p. 257: #4 pp. 248-250, Ex. 13, 14, 15, 17, 18, 21 top p. 251, 10, 13-17 bottom p. 251.

Dolciani, Modern Algebra, Bk. 2. #1 pp. 117-118, 153-154, 29, Ex. 1-28 even oral p. 120, 1-20 odd written pp. 120-121, 1-18 even p. 154, 17-24 p. 156: #2 p. 247, Ex. \_\_\_\_: #3 pp. 258-260, 263-266, Ex. 1-14 even p. 261, 15-40 odd p. 261, 1-12, 16, 19 p. 264, 1-12, 36 pp. 266-267: #4 p. 259, Ex. \_\_\_\_.

Nichols, Modern Intermediate Algebra, #1 pp. 44-45, Ex. 1-3, 5-7 pp. 45-47: #2, 3, 4 pp. 47-48, 50 Ex. 1-30 pp. 48-49, 4 p. 50.

Dolciani, Modern School Mathematics, Book 2. #1 pp. 325-327, 330-333, 399, Ex. 1-60 pp. 328-329, 1-26 pp. 331-332, 1-48 pp. 333-334: #2, 3, 4 pp. 325-326, 300-301, 332-333, Ex. 1-36 p. 326, 1-20 p. 331, 1-22 p. 333.

Payne, Algebra Two. #1 pp. 57-58, 72-76, 348-349, 353-354, Ex. 1-48 pp. 58-59, 1-12 p. 73, 1-35 pp. 76-77, 1-9 p. 349, 1-50 p. 350, 1-21 pp. 354-355: #2, 3, 4 pp. 5-10, 12-15, 17-18, Ex. 1-32 p. 6, 1-40 p. 9, 1-84 pp. 10-11, 1-62 pp. 13-14, 1-51 pp. 15-16, 1-51 p. 18-19.

Pearson, Modern Algebra, A Logical Approach. #1 pp. 390-395, Ex. 1-8 pp. 391-392, 1-4 p. 396.

Introduction to Exponents (Programmed) #1a frames 1-14, 22-26, 39: #1b frames 105-117: #1d frames 80-96: #1e 118-128: #1h frames 142-152: #1i frames 178-196: #1g frames 197-204.

### II. Games

Equations by Layman E. Allen

## Self-Evaluation I

1 I. Give a quantified statement for each of the following.

1. The definition of the zero exponent.
2. The definition of the negative exponent.
3. Product of powers property.
4. Powers of a power property.
5. Power of a product property.
6. Power of a quotient property.
7. Quotient of powers property.

II. Write in simplest form without negative exponents:

$$8. \left( \frac{-2s^2}{3r^2s^4} \right)^{-2}$$

$$9. \frac{a^{-2} \cdot b^3}{a^6 \cdot b^{-3}} =$$

$$10. 2 \cdot 3 + 7 \cdot 4 \div 2 - 9 =$$

$$11. \frac{(x^2y)^3}{x^3y^2}$$

$$12. \frac{(6a)^0}{6a^0}$$

$$13. \frac{b^{-2}}{a^{-5}c^{-6}}$$

$$14. \frac{(2x^2y^3)^4}{8x^8y^{14}}$$

2 III. For  $\sqrt[n]{x}$ , complete the following.

15.  $\sqrt[n]{x}$  is called \_\_\_\_\_

16.  $\sqrt{\quad}$  is called \_\_\_\_\_

17. For  $\sqrt[n]{x}$ , the  $x$  is called \_\_\_\_\_

18. For  $\sqrt[n]{x}$ , the  $n$  is called \_\_\_\_\_

Self-Evaluation (cont')

3 IV. Write the simplest name for each of the following.

19.  $\sqrt[3]{54a^3b^2c^4} =$

20.  $\sqrt{128}$

21.  $\sqrt[3]{50} \cdot \sqrt[3]{5}$

22.  $\sqrt[4]{a^3b^2} \cdot \sqrt[4]{a^3b} =$

23.  $10\sqrt{40} \div 5\sqrt{10} =$

24.  $\sqrt[5]{64x^7} \div \sqrt{2x^4} =$

25.  $\sqrt{48} + \sqrt{12} =$

26.  $3\sqrt{18} + \sqrt{200} =$

27.  $\sqrt[3]{54} - \sqrt[3]{16} =$

28.  $5\sqrt{27} - \sqrt{12} =$

29.  $\frac{1 + \sqrt{2}}{1 - \sqrt{2}} =$

30.  $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$

4 V. Simplify the following.

31.  $\sqrt[3]{\frac{8}{216}}$

32.  $\sqrt{\frac{18}{49}}$

33.  $\sqrt{\frac{3}{2}}$

34.  $\sqrt[3]{\frac{8}{2a^2}}$

35.  $\sqrt[5]{\frac{3x^6}{32}}$

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE THE PROGRESS TEST

## SECTION 2

### Behavioral Objectives

Upon completing your prescribed course of study, you will be able to:

5. Given a radical expression, write the simplest equivalent expression using fractional exponents.
6. Given an expression with fractional exponents, write the simplest equivalent radical expression.
7. Given a real number raised to the power of any rational number, write the simplest name for the number.
8. Given an expression of the form  $\sqrt[n]{b^m}$ , write it in simplest form using the theorem,  $\sqrt[n]{b^m} = (\sqrt[n]{b})^m$ .
9. Given any expression involving rational exponents, simplify indices, write their product in simplest radical form.
10. Given two or more radical expressions with different indices, write their product in simplest radical form.
11. Given a radical equation, find its solution set.
12. Given a real number, write it in scientific notation.

## RESOURCES

### I. Reading and Problems

Vanatta, Algebra Two, #5 \_\_\_\_: #6 pp. 245-246, Ex. 25-30 p. 248:  
#7 pp. 245-246; Ex. 22-30 p. 247, 7-15, 24, p. 248: #8 \_\_\_\_: #9  
pp. 245-246, 13-21 p. 247: #10 p. 254, Ex. 5,9,16,17,18 top p. 255,  
10, 17-21, 23 p. 256: #11 pp. 258-260, Ex. 1-9 p. 260: #12 \_\_\_\_.

Dolciani, Modern Algebra, #5 pp. 333-334, Ex. 1-6 p. 335: #6 \_\_\_\_:  
#7 pp. 334-335, Ex. 3-10 p. 334, 15-22 p. 335: #8 p. 258, Ex. \_\_\_\_:  
#9 \_\_\_\_: #10 \_\_\_\_: #11 pp. 334-335, Ex. \_\_\_\_: #12 \_\_\_\_.

Nichols, Modern Intermediate Algebra, #5,6,7,8 pp. 49-50, Ex. 1-3, 5-13,  
pp. 50-51: #11 pp. 51-52, Ex. 1-5 pp. 52-53: #12 p. 53 Ex. 1,2,3 p. 54.

Dolciani, Modern School Mathematics, Book 2, #5,6,7,8 pp. 399-400, 402,  
Ex. 7-40, 47-49 p. 401: #11 pp. 402-403, 334-336, Ex. 27-36 p. 332, 1-36  
pp. 336-337: #9 41-46 p. 401: #12 pp. 412-413, Ex. 1-38, pp. 413-414: #11  
pp. 334-336, Ex. 1-28 even p. 336.

Payne, Algebra Two, #5,6,7,8 pp. 355-356, Ex. 1-92 pp. 357-358: #12 p.  
351, Ex. 1-16 p. 352.

Pearson, Modern Algebra, A Logical Approach, #5,6,7,8 pp. 398-401,  
403-404, Ex. 1-9 p. 402, 1-5 pp. 405-406: #9 p. 405, Ex. 6-12 p. 406:  
#10 pp. 396-397, Ex. 1-5 p. 398.

INTRODUCTION TO EXPONENTS(programed) #5,6 frames 15,16,206-228,  
234-235,250-252,259-261: #8 frames 262-271 #12 frames 51-79.

### II. Games

Equations by Layman Allen.

Self-Evaluation II

5 I. For each of the following write the simplest equivalent expression using fractional exponents.

\_\_\_\_\_ 1.  $\sqrt{x}$

\_\_\_\_\_ 2.  $\sqrt[3]{m^2n}$

\_\_\_\_\_ 3.  $\sqrt[4]{y^5}$

\_\_\_\_\_ 4.  $\sqrt[3]{2x^2y^3}$

6 II. For each of the following write the simplest equivalent expression using radicals.

\_\_\_\_\_ 5.  $(4ab)^{\frac{1}{3}}$

\_\_\_\_\_ 6.  $7^{\frac{2}{5}}$

\_\_\_\_\_ 7.  $(3x)^{\frac{1}{2}}$

\_\_\_\_\_ 8.  $a^{\frac{3}{4}}$

7 III. Write the simplest name for the following.

\_\_\_\_\_ 9.  $25^{\frac{3}{2}}$

\_\_\_\_\_ 10.  $16^{\frac{1}{4}} 8^{\frac{1}{3}} 9^{\frac{1}{2}}$

\_\_\_\_\_ 11.  $\frac{5^0 16^{\frac{3}{4}}}{8^{\frac{1}{3}}}$

\_\_\_\_\_ 12.  $16^{-\frac{3}{2}}$

\_\_\_\_\_ 13.  $81^{\frac{3}{4}}$

8 IV. Write the following in simplest form.

\_\_\_\_\_ 14.  $\sqrt[3]{27^2}$



Self-Evaluation (cont')

\_\_\_\_\_ 15.  $\sqrt[4]{81^2}$

\_\_\_\_\_ 16.  $\sqrt[3]{8^4}$

\_\_\_\_\_ 17.  $\sqrt[5]{32^4}$

9 V. Write the following in simplest form.

\_\_\_\_\_ 18.  $a^{\frac{2}{3}} a^{\frac{3}{4}}$

\_\_\_\_\_ 19.  $y^{\frac{1}{4}} + y^{-\frac{3}{4}}$

\_\_\_\_\_ 20.  $(64x^4)^{\frac{1}{8}}$

\_\_\_\_\_ 21.  $(-8x^6)^{\frac{1}{3}}$

\_\_\_\_\_ 22.  $x^{\frac{2}{3}} y^{\frac{2}{3}}$

10 VI. Write the following in simplest radical form.

\_\_\_\_\_ 23.  $(5\sqrt{2})(2\sqrt[3]{2})$

\_\_\_\_\_ 24.  $(4\sqrt[4]{2})(\sqrt[6]{4})$

\_\_\_\_\_ 25.  $\sqrt{2} \cdot \sqrt[3]{3}$

\_\_\_\_\_ 26.  $\sqrt[3]{2} \cdot \sqrt[4]{2}$

11 VII. Solve the following radical equations.

\_\_\_\_\_ 27.  $\sqrt{x} - 3 = 1$

\_\_\_\_\_ 28.  $4\sqrt{x} + 1 = 25$

\_\_\_\_\_ 29.  $\sqrt{2x + 1} = \sqrt{4x - 23}$

Self-Evaluation (cont')

\_\_\_\_\_ 30.  $9 - \sqrt{x+2} = 5$

\_\_\_\_\_ 31.  $3\sqrt{x} - 1 = \sqrt{x} + 1$

12. VIII. Write each of the following in scientific notation.

\_\_\_\_\_ 32. 2.61

\_\_\_\_\_ 33. 2,000,600

\_\_\_\_\_ 34. .002712

\_\_\_\_\_ 35.  $30.61 \times 10^{-2}$

\_\_\_\_\_ 36.  $.00027 \times 10^{-6}$

IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, TAKE THE LAP TEST.

ADVANCED STUDY

I. The following are in depth problems. To get advanced study credit, you must do at least one set and 80% of that set.

1. Dolciani, Modern Algebra. p. 156, numbers 29-38.
2. Dolciani, Modern Algebra. p. 261, numbers 43-50.
3. Dolciani, Modern Algebra. p. 265, numbers 21, 22, 24  
p. 267, numbers 27-30
4. Vanatta, Algebra Two. p. 252, numbers 23-30.
5. Vanatta, Algebra Two. p. 255, numbers 28-30  
p. 256, number 24  
p. 257, numbers 11, 12
6. Vanatta, Algebra Two. p. 260, numbers 13-15  
p. 262, numbers 21, 22, 39, 40

II. Develop a game involving exponents.

III. Develop a set of rules to adapt the Equation game to operating with exponents.

IV. Can YOU do this??

The earth gravitational pull on an object is directly proportional to its mass ( $m$ ) and inversely proportioned to the square of the distance of the object from the center of the earth ( $d^{-2}$ )

Mathematically this may be written:

$$F = km d^{-2}$$

If an object experiences a force (weight) of 120 lbs. at the earth's surface ( $4 \times 10^3$  miles from the center of the earth), what pull or force would the same object experience at  $8 \times 10^3$  miles from the center of the earth.

V. EXTRA FOR EXPERTS - Dolciani, Algebra One, pp. 276-278.

VI. Modern School Mathematics, p. 332 nos. 21-26, page 234 nos. 29-36.

#### REFERENCES

Vanatta, Glen D., and Goodwin, Wilson A., Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

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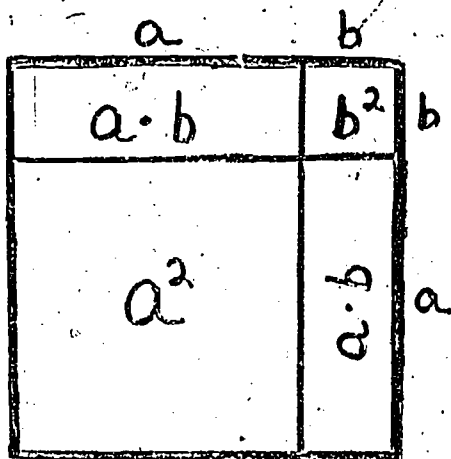
### Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Mantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

L  
LEARNING

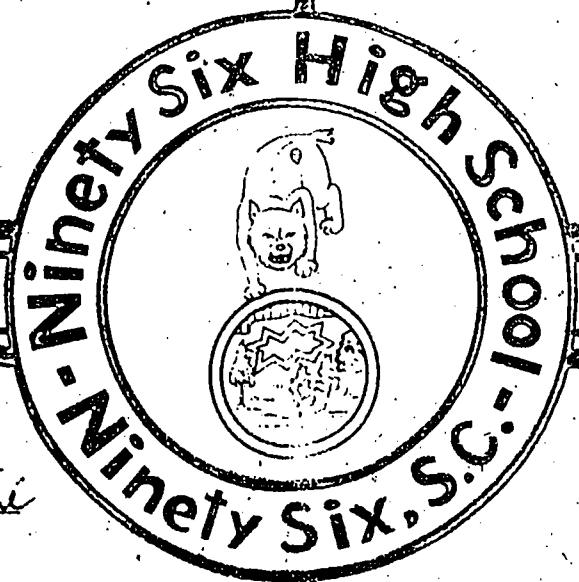
A  
ACTIVITY

P  
PACKAGE



$$(a+b)^2 = a^2 + 2ab + b^2$$

POLYNOMIALS AND  
FACTORING



Algebra 103 - 104

REVIEWED BY

*J. Kitchi*

LAP NUMBER

16

WRITTEN BY Diane Evans

### Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Santuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

### RATIONALE (The LAP's Purpose)

We will now extend our concept of a mathematical system to include the set of polynomials. You will learn how to add, subtract, multiply, divide, and factor polynomials. The field properties are used extensively in applying these operations on polynomials.

You probably will recognize many of these skills from your previous work in Algebra. They are presented again only in more depth because of their importance when working with rational expressions, polynomial functions, and solving equations which will be developed in your future study.



## SECTION 1

### Behavioral Objectives

After completing your prescribed course of study, you will be able to:

1. Given an algebraic expression, state whether or not it is a polynomial.\*
2. Given a polynomial:
  - a. State the degree of the polynomial.
  - b. State if it is a polynomial over the integers, rationals or reals. \*
  - c. State if it is monomial, binomial, trinomial or perfect square trinomial.
  - d. State if it is in two or more variables.
3. Given a polynomial and replacements for the variables, determine the value of the polynomial.
4. Given any two polynomials, name the polynomial in its simplified form which represents their:
  - a. sum
  - b. difference
  - c. product.\*\*
  - d. quotient (where the polynomial of higher degree is divided by the polynomial of lesser degree).
5. Given any polynomial, name its additive inverse.
6. Given any polynomial, factor it over the integers if possible. \*\*
7. Given any polynomial that is factorable over the rationals, write it as a factorization over the integers times some rational number.  
(The examples and exercises in Appendix 3 are highly recommended for all students as a review of factorization of polynomials.)

\* Check definition in Appendix 1.

\*\* Apply the full patterns listed in Appendix 2 where applicable.

## RESOURCES

### I. Reading and Problems.

1. Vanatta, Algebra One, #1, 2 pp. 56-58, Ex. \_\_\_\_: #3 pp. 56-58, Ex. 41-46 p. 60: #4 pp. 61-62, 63-68, 73, Ex: 1-4, 6, 8, 17, 20, 21, 30 pp. 62-63, 2, 5, 8, 14, 16, 20, 21, 24, 27, 29 p. 65, 1, 2, 5, 8, 11, 16, 19 pp. 68-69, 1, 3, 4, 9, 11, 12, 16, 20, 21, 25 p. 73: #5 \_\_\_\_: #6 pp. 74-76, 77-79, Ex. 5, 6, 8, 11, 13, 15, 16, 19, 25, 27, 34, 42 p. 76, 1, 2, 8, 10, 12, 14, 16, 17, 18, 20, 21, 23, 25 p. 79, 1, 5, 7, 13, 16, 18 p. 80: #7 \_\_\_\_.
2. Dolciani, Modern Algebra, Book 2, #1, 2, 3, \_\_\_\_: #4a, b, c, d, pp. 121-122, 128-129, 140-141, Ex. 1, 9, 15, 16, 20, 21, 22, p. 123, 8, 9, 14, 17, 21, 26, 31, 32 p. 142: #5, 6, 7 \_\_\_\_.
3. Nichols, Modern Intermediate Algebra, #1, 2, pp. 59-62, Ex. 1, 2 p. 60, 1-4 p. 62, 4-6 p. 93: #3 p. 63, 1-18 p. 63, 1 a-h p. 91: #4, 5 pp. 64-68, 70-78, Ex. 1, 2 p. 65, 1-3 p. 67, 1-3 pp. 69-70, 1-3 p. 72, 1-2 p. 74-75, 1-20 p. 78, 2-3 p. 91-92: 36, 7 pp. 78-90, Ex. 1-2 p. 80, 1-2 p. 82, 1-2 pp. 84-85, 1-2 pp. 86-87, 1-2 p. 88, 1-2 pp. 89-90, 1-8 p. 90.
4. Dolciani, Modern School Mathematics, Book 2, #1, 2 pp. 56-57, Ex. 1-2 p. 60: #3 \_\_\_\_: #4, 5, pp. 58-59, 251-254, 256-258, 272-275, Ex. 5-46 p. 60-61, 1-40 pp. 255-256, 1-46 pp. 258-259, 1-35 p. 76: #6, 7 pp. 259-263, Ex. 1-8 p. 263, 1-66 pp. 264-265.
5. Payne, Algebra Two, #1, 2, pp 49-51, Ex. 1-20, 26-35 p. 52: #3 pp. 56-57, Ex. 21-26 p. 66: #4, 5 pp. 52-53, 57-58, 72, 73, 74-76, Ex. 1-7 pp. 54-55, 10, 11, 13-17, 23-28, 30, 31 p. 54-55, 43-48 p. 56, 1-26 (omit ones on complex numbers) p. 59; 1-12 p. 73, 1-35 p. 76: #6, 7 pp. 65, 68, 70; 80-81, Ex. 1-54 pp. 66-67, 1-38 p. 69; 1-30 p. 71, 33-58, p. 72, 1-31, pp. 81-82.
6. Pearson, Modern Algebra II, L.A. #1, 2 pp. 148-149, 153-154, Ex. 1-7 p. 150, 1-5 p. 155: #3 pp. 148-149, Ex. 8-9 p. 150: #4, 5 pp. 156-161, Ex. 1-6 p. 157, 1-15 pp. 161-163: #6, 7, pp. 164-166, 172-184, Ex. 1-5 p. 165, 1-5 p. 166, 1-18 pp. 174-175, 1-20 p. 176, 1-3 p. 178, 1 a-p, p. 179, 1-20, p. 182, 1-18 p. 183, 1-2 p. 184.

### II. Games

Equations by Layman Allen.

APPENDIX 1

Behavioral Objective 1

I. Definition: A polynomial in the variable  $x$  is the set of all symbols  $a_0 + a_1x + a_2x^2 \dots + a_nx^n$  where  $n$  can be any non negative integer and the coefficients  $a_0, a_1, \dots, a_n$  are members of specified set.

Some examples of polynomials in one variable are:

1.  $4 - 3x + 6x^2$  which can be written  $6x^2 - 3x + 4$ . The specified set is the integers. We, therefore, say this is a polynomial over the integers.

2.  $\frac{1}{2} + 4x + \frac{2x^2}{3} - 5x^3 + 4x^4$

This is a polynomial over the \_\_\_\_\_

3.  $5y^2 - 3y$

This is a polynomial over the \_\_\_\_\_

4.  $\frac{1}{2}$

This is a polynomial over the \_\_\_\_\_

5.  $2x$

This is a polynomial over the \_\_\_\_\_

Nichols

Refer to page 60 of the text for the definition of a polynomial

in  $n$  variables. In this definition, the symbols  $x_1, x_2, x_3,$

$\dots, x_n$  represent  $n$  different variables such as  $x, y, z, w,$  etc.

The following are examples of expressions which are not polynomials:

1)  $\frac{4}{x}$       2)  $5xy^{-1}$       3)  $\frac{4x^2y+2}{y}$       4)  $\sqrt{x} + 2x^{-3}$

QUESTION: Is the number "0" a polynomial? Careful, does it conform to the definition. ?

ANSWER: Yes, it certainly does. Think about it.....!

## APPENDIX 2

The following are examples of different types of factoring you will work in in Appendix 3. Study these examples and then complete Appendix 3.

I. Apply the following patterns where applicable in computing the

product of two binomials:

a.  $(x + y)^2 = x^2 + 2xy + y^2$

b.  $(x - y)^2 = x^2 - 2xy + y^2$

c.  $(x + y)(x - y) = x^2 - y^2$

(d)  $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

(e)  $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

II. Use the following patterns where applicable to factor polynomials

over the integers:

a.  $acx^2 + (bc + ad)x + bd = (ax + b)(cx + d)$

b. the distributive property to remove common factors

c. grouping-by-pairs

d.  $x^2 + 2(xy) + y^2 = (x + y)(x + y)$

e.  $x^2 - a^2 = (x + a)(x - a)$

f.  $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

g.  $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

h.  $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$

i.  $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

APPENDIX 3

PRACTICE SHEET ON FACTORING

I: REMOVING MONOMIAL FACTORS

EXAMPLE:

$$2x^2y + 4x^3y^2 + 2xy^3 =$$

$$2xy(x + 2x^2y + y^2)$$

\* EXERCISES:

1.  $-3ax + 6a^2xy$

2.  $16x^2 - 12xy + 8xy$

II: TRINOMIALS OF FORM  $ax^2 + bx + c$

EXAMPLE:

$$2x^2 + 5x + 3 =$$

$$(2x + 3)(x + 1)$$

EXERCISES:

1.  $a^2 - 2a - 15$

2.  $10x^2 - 11x - 6$

3.  $9a^2 - 24a + 16$

III: THE DIFFERENCE OF TWO SQUARES

EXAMPLE:

a.  $25x^2 - 1 =$   
 $(5x - 1)(5x + 1)$

b.  $t^2 - b^2 =$   
 $(t - b)(t + b)$

c.  $(a + b)^2 - 49 =$   
 $(a + b)^2 - 7^2 =$

$$[(a + b) + 7][(a + b) - 7] =$$

$$(a + b + 7)(a + b - 7)$$

d.  $16t^4 - 1 =$

$$(4t^2)^2 - 1^2 =$$

$$(4t^2 - 1)(4t^2 + 1) =$$

$$[(2t)^2 - 1^2][(4t^2 + 1)] =$$

$$(2t - 1)(2t + 1)(4t^2 + 1)$$

III: THE DIFFERENCE OF TWO SQUARES (continued).

EXERCISES:

1.  $x^2 - y^2$

7.  $9x^2 - 12xy + 4y^2 - 16$

2.  $4a^2 - 25b^2$

8.  $x^2 + y^2 + 2xy - 9$

3.  $9x^2 - 49$

9.  $x^4 - y^4$

4.  $x^2 - (y + z)^2$

10.  $16 - a^4b^4$

5.  $(2a - b)^2 - 25$

11.  $81 - a^4$

6.  $(x^2 + 2x + 1) - y^2$

12.  $b^4 - 16a^4$

IV: FOUR TERMS - COMMON FACTORS IN EACH PAIR

EXAMPLE:

$xy + x + y^2 + y =$

$(xy + x) + (y^2 + y) =$

$x(y + 1) + y(y + 1) =$

$(x + y)(y + 1)$

EXERCISES:

1.  $xy - y + 2x - 2$

2.  $rs + 2r + 3s + 6$

3.  $2xy + y - 6x - 3$

4.  $5a + 3ab - 3b - 5$

5.  $3ax + 5bx - 3ay - 5by$

V: TRINOMIALS OF THE FORM  $ax^2 + bxy + cy^2$  AND TRINOMIAL SQUARES

EXAMPLE:

a.  $4a^2 + 12ab + 9b^2 =$

$(2a + 3b)(2a + 3b) =$

$(2a + 3b)^2$

b.  $6x^2 + 11xy + 4y^2 =$

$(2x + y)(3x + 4y)$

EXERCISES:

1.  $9x^2 + 24xy + 16y^2$

2.  $4t^2 + 4t + 1$

3.  $4k^2 + 16km + 16m^2$

4.  $36m^2 + 84pm + 49p^2$

5.  $6x^2 - xy - 12y^2$

6.  $20t^2 - 9tq - 20q^2$

VI: SUM OF CUBES

EXAMPLE:

a.  $a^3 + b^3$   
 $(a + b)(a^2 - ab + b^2)$

b.  $t^3 + 8 =$   
 $t^3 + 2^3 =$   
 $(t + 2)(t^2 - 2t + 4)$

EXERCISES:

1.  $x^3 + 125$   
2.  $27x^3 + 1$   
3.  $64 + y^3$   
4.  $x^6 + y^6$   
5.  $27x^3 + 64y^3$

VII: DIFFERENCE OF CUBES

EXAMPLE:

a.  $a^3 - b^3 =$   
 $(a - b)(a^2 + ab + b^2)$

b.  $x^3 - 27 =$   
 $x^3 - 3^3 =$   
 $(x - 3)(x^2 + 3x + 9)$

EXERCISES:

1.  $64 - 27c^3$   
2.  $x^3 - 125$   
3.  $x^6 - y^6$   
4.  $343x^3 - 8y^3$   
5.  $8x^3 - 125y^3$

VIII: OTHER TYPES - COMBINATIONS - FACTOR USING THE DIFFERENT METHODS

a. COMPLETE FACTORING

EXERCISES:

1.  $ma^2 - mb^2$   
2.  $a^4 - 1$

c. GROUPING

1.  $ax + ay - bx - by$   
2.  $a^3 - a - a^2 + 1$

b. COMMON BINOMIAL FACTORS

EXERCISES:

1.  $3(a - b) - 4x(a - b)$   
2.  $(x^2 - y^2) - 5(x + y)$

d. POLYNOMIALS OR DIFFERENCE OF SQUARES

1.  $4a^2 + 9b^2 - 25c^2 - 12ab$   
2.  $25c^2 - 4a^2 - 9b^2 - 12ab$

SELF-EVALUATION

I. Identify which of the following is not a polynomial. Write YES or NO. If NO, explain your answer.

1. 5

2.  $x^2 + 2x + 4$

3.  $\frac{x+1}{x}$

4.  $\frac{x^2}{5} - 2x + \frac{1}{6}$

5.  $\frac{5}{x} + 7$

II. Consider the following polynomials:

6.  $2xy$

7.  $\frac{1}{5}xy^2 + 3x^2 + 2$

8.  $\sqrt{2}x - x^2y$

9.  $x^2 + 2xy + y^2$

10.  $\sqrt{3}x^2 + 7x^3 + 3xy + 7$

a. Give the degree of each of the above polynomials.

b. Which of the above polynomials are over the reals, over the rationals, over the integers?

c. Which of the above are monomials, binomials, trinomials, perfect square trinomials?

III. Given the polynomials and replacements below, compute the value of the polynomial.

11.  $x^2 - x$ ;  $x = 2$

12.  $2(xy - 3)$ ;  $x = -3$  and  $y = 1$

13.  $\frac{(1xy^2)}{3} + (x - y) + 7$ ;  $x = y$

14.  $xy + \frac{c}{d}$ ;  $x = 1$ ,  $y = 2$ ,  $c = -1$ ,  $d = 4$



IV. Perform the indicated operation.

15.  $(az^3 + bz^2 - z) + (-az^3 + z - 4)$

16.  $(2-x) - (x^2 + 3x + 5)$

17.  $(4y^2 + y + 3) \cdot (y-2)$

18.  $(5z^2 - 16z + 3) \div (z - 3)$

19.  $(2a - 3)^2$

20.  $(a + 2)^2$

21.  $(a + b) \cdot (a - b)$

22.  $(x^5 + x^3 - 7x^2 + 2x) + (3 + 4x - 6x^3) =$

23.  $(19a^2 + 26ab - 5ac) - (16ac + 15a^2 - 2ab)$

24.  $(2x^2 - 3x + 5)(3x^2 + 4x - 2) =$

25.  $(6x^3 - 19x^2 + 21x - 10) \div (3x - 5)$

26.  $(3x^2 - 26)(3x^2 + 2y)$

V. True or False?

27.  $-(2y)^2 = -4y^2$

28.  $-(2x^2 - 3x + 4) = -2x^2 - 3x - 4$

29.  $-(x-y) = y - x$

30.  $-(-4a^2 - 4ab + b^2 - 9) = 4a^2 + 4ab - b^2 + 9$

VI. Factor over the integers if possible.

31.  $3x^2 + 17x + 10$

38.  $-x^2 + 1$

32.  $a^2 + 4a + 4$

39.  $x^2 - y^2 - 6x + 9$

33.  $33x^3 - 121x^2$

40.  $x^4 - 16$

34.  $xy + 3y - 2x - 6$

41.  $x^6 - y^6$

35.  $a^2 - b^2$

42.  $a^{10} - b^{10}$

36.  $8x^3 - a^3$

43.  $x^4 - 4ax^3 - 3a^2x^2$

37.  $x^3 + 27a^3$

VII. Factor over the integers with a rational monomial factor.

44.  $y^2 - y + \frac{1}{4}$

46.  $x^2 - \frac{1}{4}$

45.  $\frac{1x^2}{4} + \frac{1x}{3} + \frac{1}{9}$

Grade your own test. If you have satisfactorily completed your work you may take the LAP TEST. CONSULT YOUR TEACHER FIRST.

ADVANCED STUDY

1. Research and learn to use synthetic division. Demonstrate your knowledge by completing either of the following groups of exercises:

- a. Vanatta, Algebra Two, pp. 71-72, numbers 1, 3, 6, 8, and 10.
- b. Dolciani, Modern Algebra, Book Two, p. 523, numbers 1, 2, 3, 4, 6.

2. Dolciani, Modern Algebra Two, p. 145 numbers 23-26.

3. Write a paper on Blaise Pascal. Tell how his famous triangle is used in expanding a binomial like  $(x + y)^8$  and factor the following problems using the information you have found.

(a)  $(x - y)^{15}$

(b)  $(a - b)^9$

(c)  $(243x + 32y)^5$

(d)  $(x + 2y)^4$

## REFERENCES

1. Vanatta, Glen D., Algebra Two: A Model Course, Charles E. Merrill Publishing Co., 1966.
2. Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965.
3. Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.
4. Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Co., 1968.
5. Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis G., Algebra Two, Harcourt, Brace and World, 1969.
6. Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Book Two, Ginn and Company, 1966.
7. Equations by Layman, Allen

**L** LEARNING

**A** ACTIVITY

**P** PACKAGE

$$2x - \frac{5}{2x - \frac{5}{2x}}$$

RATIONAL  
EXPRESSIONS



REVIEWED BY

*J. Ketcher*

ALGEBRA 103-104

LAP NUMBER 17

WRITTEN BY Diane Evans

### Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Mantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

### RATIONALE (The LAP'S Purpose)

In the past you have learned to solve equations. You have also learned that equations are not always in a form which is easy to work. Many times it is necessary to simplify equations to get them in a workable form. This LAP is essential to your future work in solving equations.

In this LAP "algebraic fractions" will also be reviewed along with the study of the set of rational expressions under addition and multiplication. The treatment of the operations is based on the notion of the quantification of variables over the set of real numbers and the resulting availability of field properties. Addition and multiplication of rational expressions is done first by strict application of definitions so that the underlying principles, when short-cuts are used, will be understood. The concluding section will make use of rational expressions in problem solving.

## Behavioral Objectives

After the completion of your prescribed course of study, you will be able to:

1. Given an expression, determine whether or not it is a rational expression.
2. Given a rational expression, express it in simplified form; i.e., express it so that the numerator and denominator have no common factors.
3. Given a rational expression (or a sum, difference, product or quotient of two rational expressions), find all replacements for the variables for which the expression(s) is undefined.
4. Given two or more rational expressions, determine the least common denominator for these expressions.
5. Given a rational expression, find
  - a) its multiplicative inverse (if it exists)
  - b) its additive inverse
6. Given two rational expressions, express any of the following in simplified form:
  - a) the product of the two expressions
  - b) the quotient of the two expressions
  - c) the sum of the two expressions
  - d) the difference of the two expressions
7. Given a complex rational expression, write its simplified form.
8. Solve equations involving rational expressions.
9. Solve any given word problem involving rational expressions.

## RESOURCES

### I. Reading and Problems

1. Vanatta, Algebra Two, #1 \_\_\_\_\_: #2 pp. 81-83, Ex. 1-18 even pp. 83-84: #3 pp. 21, Ex. 2 p. 22: #4 pp. 84-86, Ex. \_\_\_\_\_: #5 \_\_\_\_\_: #6 a b, pp. 87-90, Ex. 2, 3, 4, 8, 9, 11, 13, 14 p. 80, 2, 4, 5, 6, 7, 8, 10 p. 90, 5, 9, 11; 12 p. 91; 6 c d, pp. 81-85, Ex. 2, 3, 5, 13, 14, 16, 19, 22, 23, p. 86-84, 15, 16 p. 96: #7 pp. 91-92, Ex. 1, 3, 4, 9, 11 pp. 92-93, 18 p. 96: #8 \_\_\_\_\_: #9 \_\_\_\_\_, Ex. 1, 2, 5, 6, 8, 9, 12, 14, 16, 17 pp. 138-139, 2-4 p. 141.
2. Dolciani, Modern Algebra, Book 2, #1 \_\_\_\_\_: #2 pp. 158-160, Ex. 1, 3, 5, 11, 15, 20, 22, 26, 29-32 pp. 160-161: #3 pp. 157-158, Ex. 1-24 odd p. 158: #4 pp. 164-165, Ex. \_\_\_\_\_: #5 \_\_\_\_\_: #6 a b, pp. 161-162, Ex. 3-6, 8, 11, 15, 16, 19-21, 23, 24, 27, 28 pp. 162-163, 19, 32 p. 169: #6cd pp. 164-165, Ex. 5, 8, 10, 11, 12, 15, 18, 25, 26, 28, 31, 32, 35, 38, 42 pp. 166-167: #7 pp. 167-168, Ex. 1, 7, 9, 10, 13, 14, 25, 26 pp. 168-169: #8 pp. 169-170, Ex. 2, 5, 6, 15, 17, 18 p. 171: #9 pp. 173-174, Ex. 1, 3-9, 13, 16 pp. 174-176.
3. Nichols, Modern Intermediate Algebra, #1 pp. 95-96, Ex. 1-9 oral, p. 96: #2 pp. 96-98, Ex. 1-9, p. 98: #3, 4, 5, 6 pp. 98-106, Ex. 1 (a-j) pp. 99-100, 1 (a-j), 2(a-j) p. 102, 1 (a-j), 2(a-j) pp. 104-105, 1(a-j), 2(a-j) pp. 107-108: #7 pp. 108-109, Ex. 1-9, p. 109: #8, 9 \_\_\_\_\_.
4. Dolciani, Modern School Mathematics, Book 2, #1 \_\_\_\_\_: #2 pp. 277-279, Ex. 1-36 pp. 279-280: #3, 4, 5, 6 pp. 280-282, 284-286, Ex. 1-28 pp. 282-283, 1-62 even pp. 284-286: #7 pp. 284-286, Ex. 1-62 odd p. 286: #8, 9 \_\_\_\_\_.
5. Payne, Algebra Two, #1, pp. 95-96, Ex. 1-12 p. 97: #2 pp. 98-99, 102-103, Ex. 1-12, p. 100, 1-10, p. 102, 1-39, p. 103: #3, 4, 5, 6 p. 104, 106, 110, 111, Ex. 13-33 p. 97, 1-32, p. 105, 1-24, p. 107, 1-40, p. 112: #7, p. 113, Ex. 1-20 p. 114: #8, 9 \_\_\_\_\_.
6. Pearson, Modern Algebra II, A Logical Approach, #1 p. 187, Ex. \_\_\_\_\_: #2 p. 187; Ex. 20, p. 53, 1 (a-f) p. 191, 28 a p. 196: #3, 4, 5, 6, pp. 187-189, Ex. 12, p. 53, 2(a-h) p. 191, 4 (a-h) p. 192, 28(b,c) p. 196: #7, pp. 188-189, Ex. 3(a-f), p. 191, 28(d-f) p. 196: #8, 9 \_\_\_\_\_.

### II. Audio

Wollensak C-3809 Reading Written Problems

### III. Games

Equations by Layman Allen



SELF-EVALUATION

OBJ.

I. Which of the following are rational expressions? Circle the number by each rational expression.

1.  $\frac{1}{\sqrt{x}} + 2$

5.  $\frac{y(y+3)}{2}$

2.  $\frac{1}{\sqrt{2}} \cdot x^2 + 2$

6.  $\frac{\sqrt{x}}{\sqrt{x}} + 2 \quad (x \neq 0)$

3.  $\frac{1}{\sqrt{x^2+2}}$

7.  $\frac{x^2+y^2}{z^2}$

4. 2

8.  $\frac{3}{x}$

2 II. Simplify the following rational expressions:

9.  $\frac{2}{4}$

12.  $\frac{r^2 - s^2}{r^2 + 3rs + 2s^2}$

10.  $\frac{x^2}{xy}$

13.  $\frac{x^2 + xy - 2x}{x^2 + 2xy + y^2 - 4}$

11.  $\frac{x^2 - z^2}{yx + yz}$

14.  $\frac{u^3 - v^3}{u^2 + uv + v^2}$

Self-Evaluation (cont')

- 3 III. Decide on the replacements for the variables for which the following expressions are undefined:

15.  $\frac{3}{x+2}$

18.  $\frac{12x+2}{x^2-4}$

16.  $\frac{t}{2-t}$

19.  $\frac{3t+3}{4t-t^2-4}$

17.  $\frac{5+x}{3y-4}$

- 4 IV. What is the Least Common Denominator of each pair of expressions?

20.  $\frac{x}{y}; \frac{x+2}{y \cdot 2}$

21.  $\frac{2}{x^2-y^2}; \frac{6}{(x-y)}$

22.  $\frac{2x}{x^4-16}; \frac{3y}{(x^2+4)(x-2)}$

23.  $\frac{2}{xyz}; \frac{5}{rxy}$

24.  $\frac{6x+2}{x(x-y)}; \frac{3x-y}{x^3-y^3}$

Self-Evaluation (cont')

V. Find the multiplicative and additive inverse of each of the following expressions; then simplify.

25.  $\frac{1}{2}$

29.  $y - x$

26.  $\frac{x}{y}$

30.  $\frac{x}{x+1}$

27.  $\frac{x}{x+2}$

31.  $\frac{y}{-x}$

28.  $\frac{\frac{9}{x}}{\frac{1}{4}}$

32.  $\frac{\frac{1}{4}}{\frac{1}{4}}$

VI. Perform the indicated operation.

33.  $\frac{x}{y} + \frac{x+2}{2y}$

39.  $\frac{y^2}{4x} \div \frac{y}{x}$

34.  $\frac{x}{y} - \frac{(x+2)}{2y}$

40.  $\frac{8x^3 + 27}{3x^2 - 3} \cdot \frac{x^4 - 1}{2x^2 - x - 6}$

35.  $\frac{3s^2 + 8r + s - r}{9s^2 - r^2} + \frac{s - r}{r - 3s}$

41.  $\frac{4x^2 - 6x + 9}{2x - 3} \div \frac{8x^3 + 27}{4x^2 - 12x + 9}$

36.  $\frac{y^2}{4x} \cdot \frac{x}{y}$

42.  $\frac{5x-x^2}{x^2+8x+12} \cdot \frac{x^2-2x-8}{x^2-4x-5}$

37.  $\frac{2x}{x^2+3x+2} - \frac{x}{x^2-1}$

43.  $\frac{xy+x}{5y} + \frac{x^2+y^2}{3y}$

38.  $\frac{3x+4}{x^2-16} + \frac{x-3}{x^2+8x+16}$

Self-Evaluation (cont')

7 VII. Simplify the following complex rational expressions:

$$44. \frac{\frac{4x}{y^2} + \frac{x}{y}}{y - \frac{y^2 - xy}{y}}$$

$$46. \frac{\frac{2}{x} + \frac{3}{y}}{\frac{x}{y} + \frac{y}{x}}$$

$$45. \frac{t - u - \frac{t^2 + tu}{t}}{\frac{3u}{t} - \frac{u}{t^2}}$$

$$47. \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}}$$

8 VIII. Solve the following:

$$48. \frac{2x}{3} + 8 = 0$$

$$49. \frac{2x}{3} - \frac{2x+10}{5} = \frac{1}{3}$$

$$50. \frac{3-2x}{8} - \frac{x-3}{6} - 1 = \frac{x+4}{3} + \frac{1}{24}$$

9 IX. Solve the following.

51. The average of two numbers is 15. Find the numbers if the smaller is two-thirds of the larger.

52. One card-sorter can process a deck of punched cards in 30 minutes, while another can sort the deck in 45 minutes. How long would it take the two sorters together to process the cards?

53. A solution of silver nitrate in water is 12% silver nitrate. How many ounces of the compound must be added to 23 ounces of this solution to produce a 20% solution?

54. Three men receive together \$1285 from a business venture. If A's share is \$25 more than  $\frac{2}{3}$  of B's share, and C's share is  $\frac{4}{15}$  of B's share, find the amount of money each should receive.

55. The length of a rectangle is two feet longer than its width. Find the width if the perimeter of the rectangle is 144 feet.

GRADE YOUR OWN TEST. If you have satisfactorily completed your work, you may take the LAP TEST. CONSULT YOUR TEACHER FIRST.

ADVANCED STUDY

1. Make up a game using rational expressions.

At least 80% of any set of the following problems  
MUST BE COMPLETED for credit.

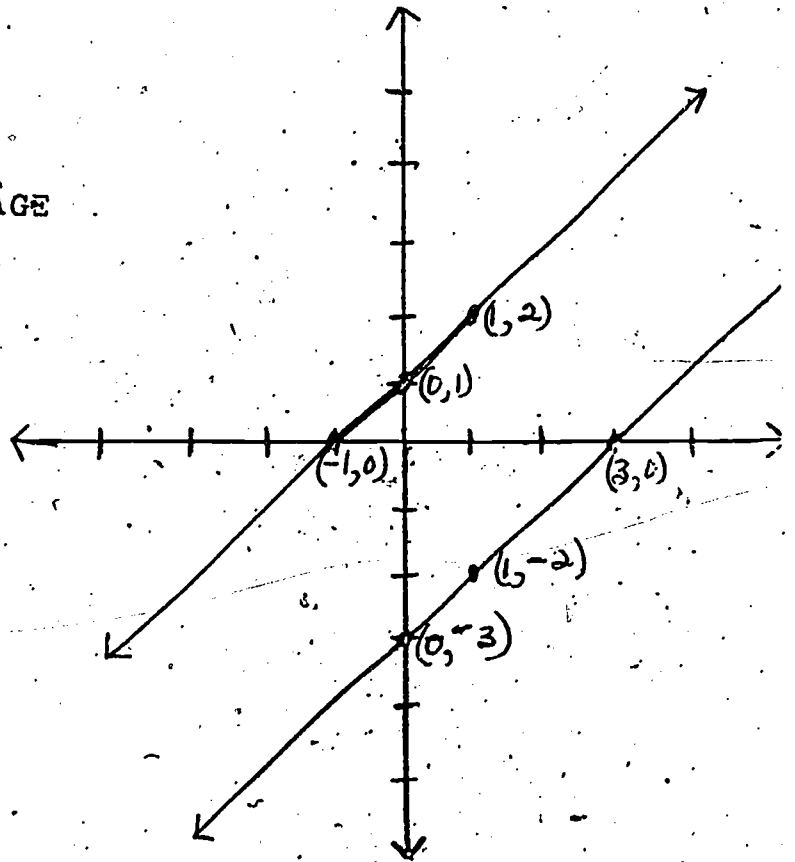
2. Allendoerfer, Fundamentals of Freshman Mathematics,  
Ex. 1 -20 p. 74.
3. Allendoerfer, Ex. 17-20 p. 78, 15-22 p. 80.
4. Allendoerfer, Ex. 10-20 p. 80.
5. Dolciani, Modern Algebra Two, Ex. 26, 31, 33, 34,  
p. 177, and 9, 10, 15, 19 p. 182.

## REFERENCES

1. Vanatta, Glen D., Algebra Two: A Modern Course, Charles E. Merrill Publishing Co., 1966.
2. Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra, Book Two, Houghton Mifflin Co., 1965.
3. Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.
4. Dolciani, Mary P., Wooton, William, Beckenbach, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Co., 1968.
5. Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis G., Algebra Two, Harcourt, Brace and World, 1969.
6. Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach; Book Two, Ginn and Company, 1966.
7. Allendoerfer, C. B.; Oakley, C. O., Fundamentals of Freshman Mathematics, McGraw-Hill Book Company, Inc., 1959.
8. Wollensak Teaching Tape C-3809 Reading Written Problems.
9. Equations (game) by Layman Allen.

**L** LEARNING  
**A** ACTIVITY  
**P** PACKAGE

INTRODUCTION  
TO  
COORDINATE  
GEOMETRY



ALGEBRA 103-104

REVIEWED BY

*J. P. Ricketts*

LAP NUMBER 18

WRITTEN BY *Ms. Deane Egan*



## RATIONALE (The LAP's Purpose)

In 1600 European mathematicians worked with two branches of mathematics - geometry and algebra. However, there was no link between these two branches. René Descartes, a French philosopher and mathematician, provided the connection in his Geométrie, published in 1637, by devising a scheme for locating points by using numbers. From this idea the whole subject of analytic geometry or coordinate geometry has developed.

In this LAP you will begin an introduction to coordinate geometry. You will study the most basic coordinate figure, the straight line. You will also investigate the concepts of slope, intercepts, distance, midpoint, parallelism, and perpendicularity.



## SECTION 1

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Identify or define the following:
  - a. cartesian coordinate system
  - b. Descartes
  - c. abscissa
  - d. ordinate
  - e. origin
2. Given a coordinate system for a line:
  - a. Find the coordinate of any given point.
  - b. Given two points, find the coordinate of any point of the segment joining the two points.
  - c. Given the coordinates of two points, determine the distance between them.
3. Given a coordinate system for a plane:
  - a. Given a point, identify its coordinates.
  - b. Given an ordered pair, graph the corresponding point.
  - c. Given a point, identify the quadrant or the axis which contains the point.
  - d. Given the lengths of two sides of a right triangle, use the Pythagorean Theorem to find the length of the third side.
  - e. Given a geometric figure, one or more of whose sides lie along a horizontal or a vertical line, find lengths of segments or coordinates of points for this figure.
4. Given the coordinates of two points in a plane:
  - a. Use distance formula to determine the distance between them.
  - b. Find the coordinates of the midpoint of the segment joining them.
  - c. Find the slope of the line containing them.
5. Given the slope of a line or sufficient information to determine this slope, decide whether:
  - a. the line "rises to the right".
  - b. the line "falls to the right".
  - c. the line is horizontal.
  - d. the line is vertical.

RESOURCES 1

Obj. 1: Vanatta, Algebra Two, read p. 115, Ex. Define the terms in  
Obj. One.  
Dolciani, Modern Algebra, read p. 81, Ex. Define the terms in  
Obj. One.

Obj. 2: Nichols, read pp. 115-117, 122, ex. 1, 2 page 116; 1-4 pages  
117-118, 1-6 page 122.  
Wooton, read pp. 154-155, 160, ex. 5-8 page 159.  
Payne, read p. 136, ex. 1-22, pp. 138-139.  
Pearson, read p. 204, ex. 1-20, p. 205.

Obj. 3: Vanatta (a) \_\_\_\_; (b) p. 115-116, ex. 1-4 page 117; (c)-(e) \_\_\_\_.  
Nichols, read pp. 118-119, ex. 1-9 pages 119-121.  
Wooton, read pp. 155-157, 160-161, ex. 9-24 page 159.  
Pearson, read pp. 205-210, ex. 1-12 pages 210-211.  
Wollensak Tape C-3852 Graphing Linear Functions  
Games: Graphing Pictures  
An Ordered Pair Code,

Obj.s 4,5: Vanatta (#4a) read pp. 152-153, ex. 1, 2, 4 page 154;  
(b) read pp. 154-155, ex. 1-3 pages 155-156;  
(c) read pp. 142-143, ex. 1-12 odd page 145.  
(#4) \_\_\_\_.

Dolciani, (#4a,b) read page 294, ex. 1-6 page 295;  
(c) read pages 84-88, Ex. 1-12 even page 89.  
(#5) \_\_\_\_.

Nichols, #4,5 read pages 122-126, 129-133, ex. 1-14 even  
pages 122-123; 1-8 pages 124-125; 1-14  
even page 131; 1-14 even page 133.

Wooton, #4,5 read pages 168-172, 433-437, ex. 1-18 bottom  
page 173; 1-30 pages 437-438.

Payne, #4,5 read pages 140-146, 148-151, ex. 1-14 page 143;  
1-12 pages 146-147; 18-23 page 153.

Pearson, #4,5 read pages 212-218, ex. 1-4, 12, 13, pages  
216-217; 1-14 pages 218-219.

Wollensak C-3854 The Slope of a Line

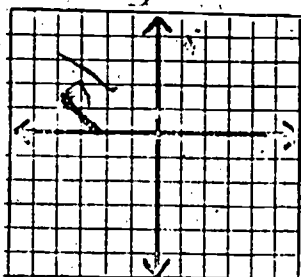
SELF-EVALUATION 1

Obj.

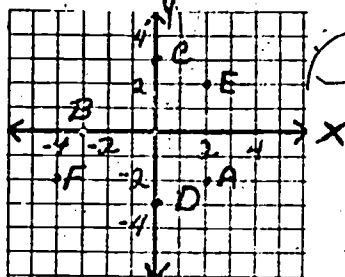
1. I. a. Define Cartesian coordinate system.

b. For whom is the Cartesian coordinate system named?

c. On the following coordinate plane, label (1) the abscissa, (2) the ordinate, (3) the origin.



2. II. Use this coordinate system to answer the following questions.



(1) Give the coordinate for each of the following points.

A \_\_\_\_\_

B \_\_\_\_\_

B \_\_\_\_\_

F \_\_\_\_\_

C \_\_\_\_\_

D \_\_\_\_\_

(2) Give the distance between the following pairs of points.

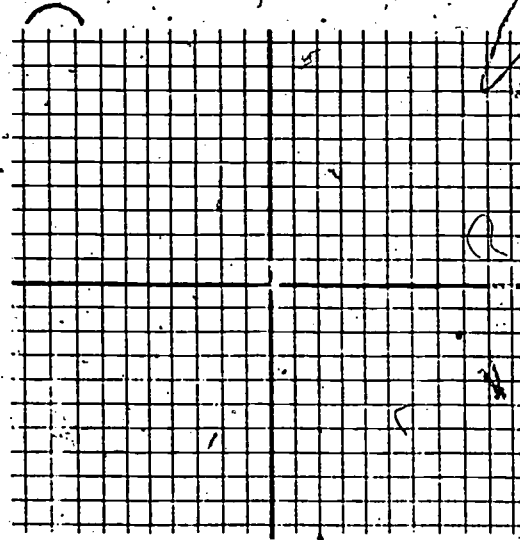
A and E \_\_\_\_\_

C and D \_\_\_\_\_

SELF-EVALUATION 1 (cont.)

3b III. (1) Use the following graph and plot these points.

- A (2, 4)
- B (-6, -4)
- C (-1, -1)
- D (0, 5)
- E (-3, 4)
- F (4, -2)
- G (5, 0)



3c (2) Identify the quadrant or axis which contains each of the following points.

\_\_\_\_\_ A (4, -1)

\_\_\_\_\_ E (-6, 8)

\_\_\_\_\_ B (2, 8)

\_\_\_\_\_ F (7, -1)

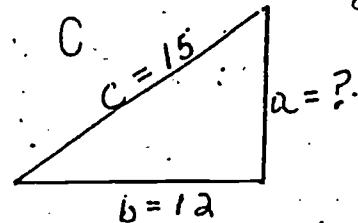
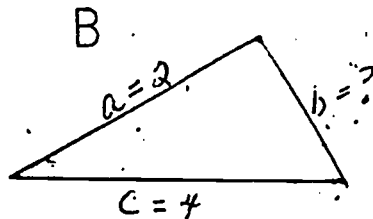
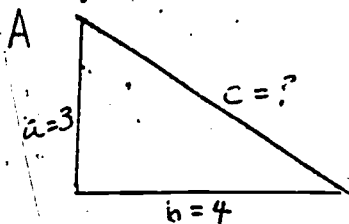
\_\_\_\_\_ C (-7, -2)

\_\_\_\_\_ G (0, -3)

\_\_\_\_\_ D (-3, 0)

\_\_\_\_\_ H (-1, -4)

3d (3) Find the length of the third side in each of the following triangles.

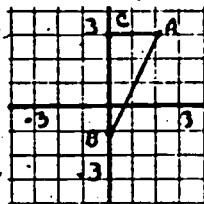


C = \_\_\_\_\_

B = \_\_\_\_\_

A = \_\_\_\_\_

(4) Find the length of each of the sides of the following triangle.



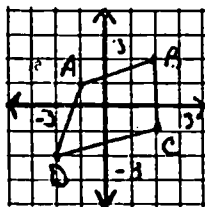
AC = \_\_\_\_\_

CB = \_\_\_\_\_

AB = \_\_\_\_\_

SELF-EVALUATION 1 (cont')

(5) Write the coordinates for the points of the vertices of the following figure.



- A. \_\_\_\_\_  
 B. \_\_\_\_\_  
 C. \_\_\_\_\_  
 D. \_\_\_\_\_

4) IV. For each of the following pairs of points, find:

- (a) the distance between them  
 (b) the coordinate of the midpoint of the segment joining each pair  
 (c) the slope of the line containing each pair

	DISTANCE	MIDPOINT	SLOPE
(1) (4, 0) (0, -3)	_____	_____	_____
(2) (0, 5) (-2, -2)	_____	_____	_____
(3) (4, 3) (8, 7)	_____	_____	_____
(4) (-2, 8) (5, -3)	_____	_____	_____
(5) (8, -2) (-3, 9)	_____	_____	_____

5) V. Given two ordered pairs: (a) determine the slope of the line joining them, and (b) determine if the line: (1) rises to the right (2) falls to the right (3) is horizontal (4) is vertical

	A. SLOPE	B. DIRECTION
1. (8, 2) (3, 7)		
2. (-2, -5) (-4, -9)		
3. (2, 7) (9, 7)		
4. (-5, 3) (-5, -4)		
5. (8, 0) (0, -2)		

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK ON SECTION 1, CONSULT YOUR TEACHER, THEN TAKE THE PROGRESS TEST ON SECTION 1.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

6. Given an equation of a line:
  - a. write an equivalent equation in slope intercept form.
  - b. determine the slope of the line.
  - c. determine the y - intercept of the line.
  - d. sketch and/or identify the graph of the line.
7. Write the equation of a given line, when given any one of the following:
  - a. the slope of the line and the y - intercept of the line.
  - b. the coordinates of a point on the line and the slope of the line.
  - c. the coordinates of two points on the line.
8. Given the coordinates of a point or information sufficient to find such coordinates, write:
  - a. the equation of the horizontal line containing this point.
  - b. the equation of the vertical line containing the point.

### RESOURCES 2

- Obj. 6: Vanatta, read pages 146-148, ex. page 148, work any 10 problems.  
Nichols, read page 144, ex. 2 a - j page 144.  
Wooton, read pp. 161-162, 175-177, ex. 1-12 oral page 162, 5-18 page 172.  
Payne, read pages 162-165, ex. 22-27 page 166.  
Pearson, read pages 161-162, 175-177, ex. 1-7 page 222, 4 page 227.  
Wollensak Tape C-3852 Graphing Linear Functions  
C-3855 Slope Intercept Form  
Transparency 3M The Straight Line  
Games: Graphing Pictures  
An Equation Code
- Obj. 7: Dolciani, 7a, read pages 90-93, ex. 19-26 page 93: 7b read pages 90-93, ex: 1-8 page 93: 7c \_\_\_\_.  
Nichols, 7a read pages 142-143, ex: 1 page 144: 7b read pp. 142-143, ex. even numbers bottom page 143: 7c 2, 4, 6, 8 top page 143.  
Wooton, read pages 175-177, Ex. 1-6 and 13-18 orals page 178, 1-12 written page 178, 13-24 page 179.  
Payne, read pages 162-165, ex. 7a \_\_\_\_: 7b, 9-17 page 166: 7c, 1-8 page 165, 48 page 167.  
Pearson, read pages 224-226, Ex. 1 page 228, 1-17 pages 233-236.
- Obj. 8: Nichols read pp. 139-140, Ex. 2a,c,e,g,i page 140.  
Payne, read pp. 162-165, ex. 1-55 even pages 165-167.  
Pearson, read pp. 223-226, Ex. 1-4 pages 226-227.

SELF-EVALUATION 2

Obj.

6 I. Rewrite the following in slope-intercept form, state the slope and y-intercept, and graph each (use graph paper on next page).

(1)  $2x + 3y = -6$  \_\_\_\_\_ m = \_\_\_\_\_ b = \_\_\_\_\_

(2)  $2y = -4x + 8$  \_\_\_\_\_ m = \_\_\_\_\_ b = \_\_\_\_\_

(3)  $y = -x$  \_\_\_\_\_ m = \_\_\_\_\_ b = \_\_\_\_\_

(4)  $-18x - 6y = 18$  \_\_\_\_\_ m = \_\_\_\_\_ b = \_\_\_\_\_

(5)  $3x = 6y - 12$  \_\_\_\_\_ m = \_\_\_\_\_ b = \_\_\_\_\_

7 II. In each problem below, use the given information to write the equation for a line.

(1)  $m = 5, b = 2$  \_\_\_\_\_

(2)  $m = -2, P_1(-3, 4)$  \_\_\_\_\_

(3)  $P_1(1, -1), P_2(-1, -1)$  \_\_\_\_\_

(4)  $m = -9, b = 0$  \_\_\_\_\_

(5)  $m = \frac{4}{5}, P_1(3, -1)$  \_\_\_\_\_

(6)  $P_1(2, 3), P_2(-1, -4)$  \_\_\_\_\_

(7)  $m = -\frac{2}{3}, b = 3$  \_\_\_\_\_

(8)  $m = 5, P_1(-2, -4)$  \_\_\_\_\_

(9)  $P_1(4, -3), P_2(-6, 2)$  \_\_\_\_\_

8 III. A) Write the equation for: (1) the vertical line, and (2) the horizontal line through  $(-2, 3)$ .

(1) vertical \_\_\_\_\_

(2) horizontal \_\_\_\_\_

IV. Multiple Choice.

9 1. If a line is vertical and passes through the point  $(-2, -3)$  then its equation is:

(a)  $x = -3$

(b)  $y = -3$

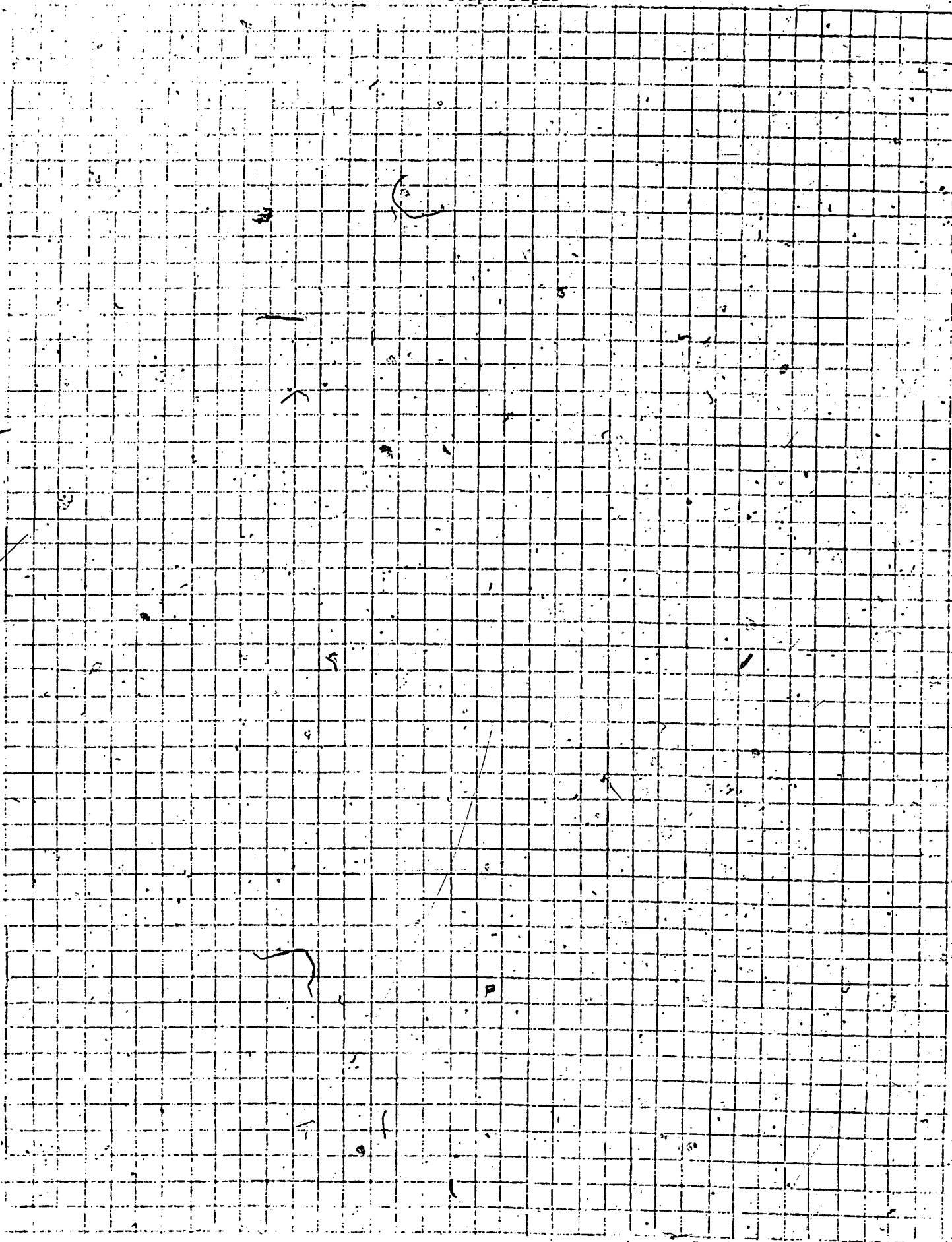
(c)  $x = -2$

(d)  $y = -2$

(e) none of these



SELF-EVALUATION 2 (cont')  
Graph Paper





SELF-EVALUATION 2 (cont')

9. 2. If a line is horizontal and passes through the point  $(a,b)$ , then its equation is:

- a)  $x = a$                       d)  $y = b$   
 b)  $x = b$                       e) none of these  
 c)  $y = a$

Questions 3 - 5 refer to the line with equation  $3x - y = 2$ .

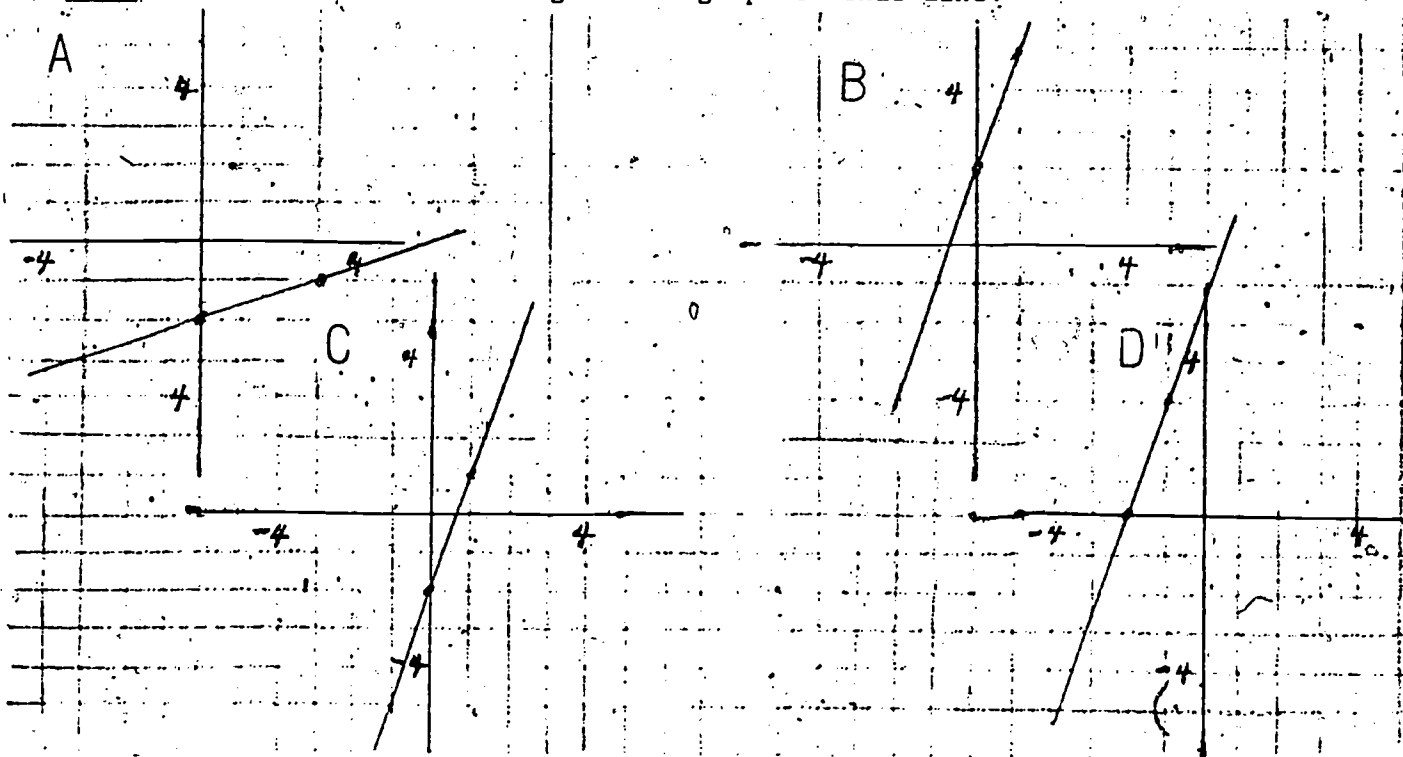
6. 3. The slope - intercept form of the equation of this line is

- (a)  $-y = -3x + 2$                       (d)  $3x + -y = 2$   
 (b)  $y = 3x + 2$                       (e)  $y = 3(x - 2)$   
 (c)  $y = 3x - 2$

6. 4. The slope and y - intercept of this line are:

- (a)  $-3, (0,2)$                       (d)  $3, (0,-2)$   
 (b)  $-3, (0, -2)$                       (e) none of these  
 (c)  $3, (0,2)$

6. 5. Which of the following is the graph of this line?



(e) none of these

7. V. Choose the correct equation in Column B for each item in Column A:

COLUMN A

COLUMN B

1. The line contains  $(1,1)$  and  $(2,2)$       A.  $2x + y = -2$

SELF-EVALUATION 2 (cont')

- \_\_\_ 2. The line contains  $(-2, -3)$  and has slope  $\frac{1}{2}$ .      B.  $x - 2y = 4$
- \_\_\_ 3. The line has slope  $-2$  and y-intercept  $(0, -2)$ .      C.  $y = x$
- \_\_\_ 4. The line contains  $(1, -1)$  and has slope  $-1$ .      D.  $x + y = 0$
- \_\_\_ 5. The line has slope  $\frac{1}{2}$  and contains  $(2, -1)$ .      E. none of these

WHEN YOU HAVE COMPLETED YOUR RESOURCES AND SELF-EVALUATION, CONSULT YOUR TEACHER. IF YOU HAVE DONE SATISFACTORY WORK, YOU MAY TAKE YOUR PROGRESS TEST ON SECTION 2.

### SECTION 3

#### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

9. Given the slopes of two lines or sufficient information for finding slopes, determine if,
  - a. the two lines are parallel
  - b. the two lines are perpendicular
  - c. the two lines are neither parallel nor perpendicular
10. Given the coordinates of points on two lines, such that certain of the coordinates are variables, determine the value of the missing numbers when the lines are:
  - a. parallel
  - b. perpendicular
11. Given the equation of a line and a point not on the line, write and/or identify the equation of a line through the given point and parallel and/or perpendicular to the given line.

#### RESOURCES 2

- Obj. 9: Vanatta, read pp. 150-151, Ex. 3,4 p. 152.  
Nichols, read pp. 133-135, Ex. 1-3 p. 136.  
Wooton, read pp. 175-177, 439-440, Ex. 1-8 and 14-18 page 441, 43-44 page 179.  
Payne, read pages 155-156, Ex. 1-16 pages 157-158.
- Obj. 10: Nichols, read pp. 133-135, Ex. 4-9 page 136.  
Wooton, read pp. 175-177, 439-440, Ex. 41-42 page 179.
- Obj. 11: Vanatta, read pp. 150-151, Ex. 6-7 p. 152.  
Wooton, read pp. 175-177, 439-440, Ex. 25-36 page 179, 9-12 page 441.

SELF-EVALUATION 3

9 I. Given the following pairs of slopes, determine if the lines with these slopes are parallel, perpendicular, or neither.

\_\_\_\_\_ 1.  $\frac{2}{5}, \frac{5}{2}$

\_\_\_\_\_ 2.  $\frac{1}{2}, \frac{3}{6}$

\_\_\_\_\_ 3.  $\frac{1}{3}, -3$

\_\_\_\_\_ 4.  $-5, \frac{-10}{2}$

\_\_\_\_\_ 5.  $\frac{4}{3}, \frac{-3}{4}$

\_\_\_\_\_ 6.  $-6, 6$

9 II. Given the following pairs of linear equations, determine if their graphs are parallel, perpendicular, or neither.

\_\_\_\_\_ 7.  $y = \frac{7}{4}x - 1$   
 $y = \frac{7}{4}x + 6$

\_\_\_\_\_ 8.  $3x - 6y = 9$   
 $2x + y = 4$

\_\_\_\_\_ 9.  $2y = 3x - 12$   
 $2x + 3y = 3$

\_\_\_\_\_ 10.  $7x + y = 7$   
 $2y = -14x + 4$

10 III. \_\_\_\_\_ 11. Determine  $x$  such that the line through  $P_1(x, 3)$  and  $P_2(-2, 1)$  is parallel to the line through  $P_3(5, -2)$  and  $P_4(1, 4)$ .

\_\_\_\_\_ 12. Determine  $x$  such that the line through  $A(x, 3)$  and  $B(-2, 1)$  is perpendicular to the line through  $C(5, -2)$ , and  $D(1, 4)$ .

\_\_\_\_\_ 13. Determine  $m$  such that  $y = mx + 5$  is perpendicular to  $y = 2x + 5$ .

\_\_\_\_\_ 14. Determine  $C$  such that  $Cx + y = -2$  is parallel to  $2x + y = 6$ .

SELF-EVALUATION 3 (cont')

- 11 IV. Write the equation of a line parallel to each given line and through the given points.

\_\_\_\_\_ 1. (2,3)  $y = -3x + 1$

\_\_\_\_\_ 2. (-3,2)  $4x - y = 6$

\_\_\_\_\_ 3. (4,1)  $y = \frac{-3}{4}x + 6$

\_\_\_\_\_ 4. (-2,-3)  $5x + 2y = 3$

- 11 V. Write the equation of a line perpendicular to each given line & through the given points.

\_\_\_\_\_ 1. (3,1)  $y = -x + 2$

\_\_\_\_\_ 2. (-1,3)  $2x - y = 4$

\_\_\_\_\_ 3. (-3,-2)  $3x - 2y = 4$

\_\_\_\_\_ 4. (7,-2)  $4x - 3y = 5$

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, CONSULT YOUR TEACHER.  
THEN TAKE THE LAP TEST.

ADVANCED STUDY

- I. Nichols, read pages 137-138, Ex. work any 4 of 1-11, pp. 138-139.
- II. Wooton, Ex. 19, 20 page 441.
- III. Nichols, Ex. 7, 8 page 129.
- IV. Nichols, Ex. 5, 7, 8 page 125.
- V. Vanatta, work any 5 of the following: page 152 nos. 8, 9, 10;  
page 154 nos. 3, 6, 10;  
page 156 no. 5
- VI. Work any 4 of the following:

1. Determine an equation of the line satisfying the stated conditions.
  - a. Through  $(-3, 2)$  and parallel to the line joining  $(2, 3)$  and  $(1, -2)$ .
  - b. With  $x$  - intercept 2 and  $y$  - intercept 3.
  - c. Through  $(b, -2b)$  with slope  $\frac{2}{b}$ .
2. Show that the figure whose vertices are  $(2, 1)$ ,  $(4, 2)$ ,  $(5, 2)$ , and  $(7, 3)$  is a parallelogram.
3. If a line has  $x$  - intercept  $a$  ( $a \neq 0$ ) and  $y$  - intercept  $b$  ( $b \neq 0$ ), show that an equation of the line (called the intercept form of the equation) is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

4. If  $(x_1, y_1)$  and  $(x_2, y_2)$  are two points of a line and if  $x_1 \neq x_2$ , then an equation of the line (called the two-point form of the equation) is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

5. Show that if the graphs of the linear equations  $A_1x + B_1y = C_1$  and  $A_2x + B_2y = C_2$  are parallel, then  $A_1B_2 = A_2B_1$ .
6. Show that if  $A_1B_2 = A_2B_1$ , then the graphs of the linear equations  $A_1x + B_1y = C_1$  and  $A_2x + B_2y = C_2$  are either the same line or parallel lines.

## REFERENCES

Vanatta, Glen D., and Goodwin, Wilson A., Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra and Trigonometry, Book Two, Houghton Mifflin Co., 1965.

Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson, Helen R. and Allen, Frank B., Modern Algebra, A Logical Approach, Book Two, Ginn and Company, 1966.

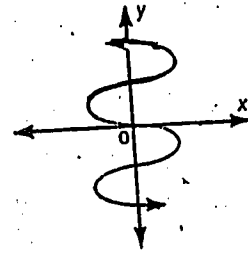
Payne, Joseph N., Zambone, Floyd F., Lankford, Jr., Frances G., Algebra Two, Harcourt, Brace and World, 1969.

Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Company, 1968.

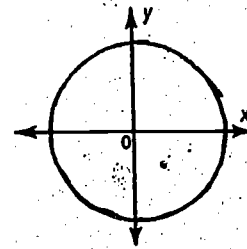
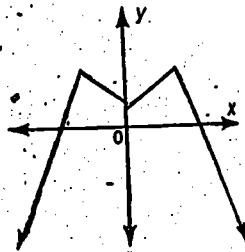
Wollensak Teaching Tapes C-3852  
C-3855  
C-3854

3M Transparencies - The Straight Line

**L** EARNING  
**A** CTIVITY  
**P** ACKAGE



RELATIONS,  
FUNCTIONS,  
AND  
INEQUALITIES



Algebra 103-104

VIEWED BY

*A. Kitchin*

LAP NUMBER

19

WRITTEN BY

*Diare Evans*



## RATIONALE

In mathematics, the concept of a function is very important and extremely useful. It appears in almost every branch of the subject. The concept in mathematics, however, has a slightly different meaning than in ordinary language. We use the word function to denote a certain specific type of correspondence between the elements of two sets. But previous to any discussion on functions one must initially be concerned with the idea of a relation.

In this LAP emphasis is placed on review and extension of concepts which are basic to the study of relations, functions, and inequalities. The basic definitions important to the study of functions will be studied more formally than in previous units. You will have enough experience with graphing and analysis of graphs so that you should be able to transfer your knowledge of functions to future studies in mathematics and to situations in other academic fields or occupational endeavors.

## SECTION 1

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given a pair of sets:
  - a. Determine the product set ( $A \times B$ )
  - b. Construct the graph of the determined product set.
2. Given a set of ordered pairs and a product set, determine whether or not that set of ordered pairs is a subset of that product set.
3. Write and/or identify the definition of these terms: relation, domain, range, and function.
4. Given a relation as a set of ordered pairs, name its range and domain.
5. Write and/or identify any or all of the five ways to express a relation.
6. Given a relation, designate it by
  - (a) a statement
  - (b) an equation
  - (c) the roster method (ordered pairs)
  - (d) constructing a table
  - (e) displaying its graphand name its domain and range. Appendix I will be completed and turned in to the teacher.
7. Given a relation, determine its inverse.

## RESOURCES I

### Obj. 1

Nichols, read pp. 155-157, Ex. 1-16 pp. 157-158.

Pearson, read pp. 31-34, Ex. 1-8 pp. 34-35.

### Obj. 2

Nichols, read pp. 158-160, Ex. 1 page 160.

### Obj. 3

Vanatta, read pp. 99-103, Ex. write the definitions of  
the words in this goal.

Dolciani, MA, read pp. 203-204, Ex. \_\_\_\_.

Filmstrip: Relations and Functions

Transparency: 3M - Functions

### Obj. 4

Vanatta, Read pp. 99-103, Ex. 10 page 105.

Dolciani, MA, read pp. 203-204, Ex. 1-8 oral page 205.

Nichols, read pp. 158-160, Ex. 2 page 160.

Wooton, MSM, read pp. 149-151, Ex. 1-10 oral page 152.

Payne, read pp. 196-198, Ex. 1-6 page 197; 1-11, 18-22

pages 198-199.

### Obj. 5

Vanatta, read pp. 99-103, Ex. write the five ways to express  
a relation.

Dolciani, MA, read pp. 203-204, Ex. \_\_\_\_.

### Obj. 6

Vanatta, read pp. 99-103, Ex. 1-5 pp. 103-105.

Nichols, read pp. 160-164, Ex. 1-3 pp. 163-164.

Wooton, MSM, read pp. 154-158, Ex. 1-24 pp. 158-159.

RESOURCES I (cont')

Payne, read pp. 194-195, Ex. 1-6, 11, 12, 16-20 page 195.

Wollensak teaching tapes - C-3852: Graphing Linear Functions

C-3855: Slope Intercept Form

Goal 7

Nichols, read pp. 164-169, Ex. 1-3 page 168.

Wooton, MSM, read pp. 404-407, -Ex. 1-7 (state if inverse  
and draw graphs) page 407.

Payne, read pp. 220-222, Ex. 1-5 page 222.

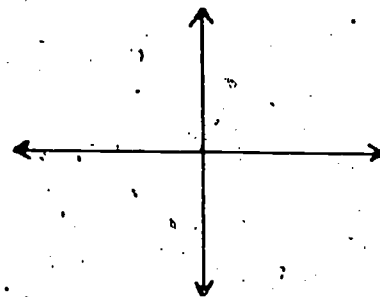
Pearson, read pp. 293-299, Ex. 1-7 pp. 300-301; 3 a - h page 306.

SELF-EVALUATION 1

Obj. 1 I. Given  $A = \{2, 3, 5\}$  and  $B = \{3, 5\}$ .

1. Find  $A \times B$ .

2. Graph  $A \times B$ .



2 II. Given  $N = \{1, 2, 3, 4, \dots\}$ . Consider  $N \times N$ . Which of the sets below is a relation in  $N \times N$ ?

3.  $\{(0, 1), (1, 0), (2, 3)\}$

4.  $\{(1, 1), (1, 2), (1, 3)\}$

5.  $\{(\frac{1}{2}, 2), (2, \frac{1}{2}), (5, 5), (6, 1), (7, 7)\}$

6.  $\{(-2, 2), (-6, 2), (5, 5), (6, 6)\}$

4 III. Identify the domain and range of each of the sets in Part II. Assume they are relations in  $R \times R$ .

7.  $D =$

$R =$

8.  $D =$

$R =$

9.  $D =$

$R =$

10.  $D =$

$R =$

SELF-EVALUATION 1 (cont')

3 IV. Write the definitions of the following words:

1. relation

2. domain

3. range

4. function

5 V. List the 5 ways to express a relation.

1.

2.

3.

4.

5.

6 VI.A.GIVEN: the relation  $3x + 1 = y$ ,

1. write it in words

2. write 5 ordered pairs

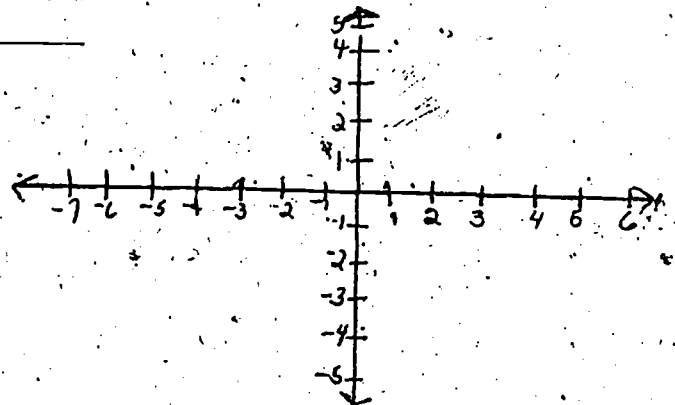
3. make a table using these ordered pairs

x	y

4. graph the ordered pairs

5. write the domain \_\_\_\_\_

range \_\_\_\_\_



SELF-EVALUATION 1 (cont')

6 B. Given the relation:  $(3,9)(-1,-3)(2,6)(0,0)(-3,-9)$

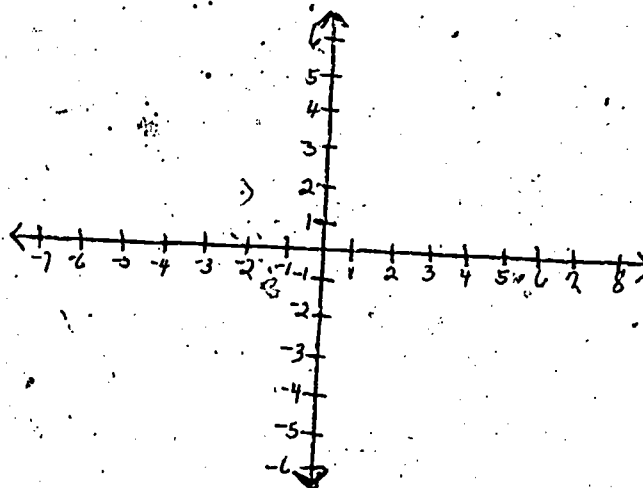
1. Write an equation \_\_\_\_\_

2. Write the relation in words \_\_\_\_\_

3. Construct a table

$x$	$y$

4. Graph the relation



5. Write the domain \_\_\_\_\_

range \_\_\_\_\_

4 VII. Write the domain and the range of each of the following:

DOMAIN

RANGE

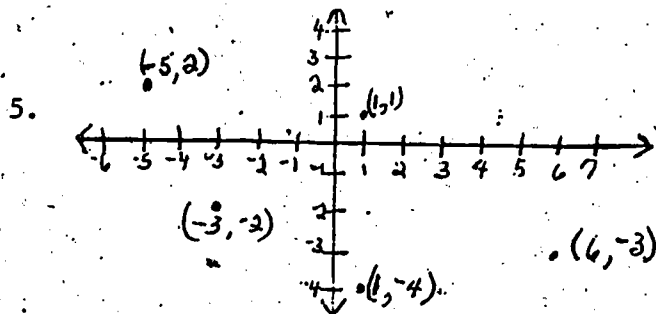
1.  $(8,1)(7,2)(-3,1)(7,-6)$

2.  $y = x^2$

3.

$x$	$y$
-8	5
0	2
-6	7
2	5

4.  $y$  is equal to twice  $x$



SELF-EVALUATION 1 (cont')

7 VIII. Write the inverse of the following relations:

(a)  $\{(2,3), (4,4), (6,1)\}$  \_\_\_\_\_

(b)  $x + 2 = y$  \_\_\_\_\_

(c)  $y = x^2$  \_\_\_\_\_

(d)  $\{(-1,4), (5,-7), (2,-3), (4,-6)\}$  \_\_\_\_\_

If you have satisfactorily completed your work, take the PROGRESS TEST.



SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Given a relation, decide if that relation is a function or not.
9. Given any function, determine the value of the function for any given number.
10. Given a function, name its inverse and decide if its inverse is itself a function.
11. Apply the vertical line test to determine if a relation is a function.
12. Given two functions  $f$  and  $g$  over the reals and a real number " $a$ " determine the following:
  - (a)  $a \cdot f(x)$
  - (b)  $f(a \cdot x)$
  - (c)  $f(x) + g(x)$
  - (d)  $f(x) \cdot g(x)$
  - (e)  $f(g(x))$
  - (f)  $g(f(x))$
  - (g)  $f(a)$
13. Given a function  $f$ , be able to identify it as:
  - (a) a constant function
  - (b) the identity function
  - (c) the greatest integer function
  - (d) a linear function
  - (e) a non-linear function
14. Given a proportion function:
  - (a) Identify it as a direct proportion function or as an inverse proportion function.
  - (b) Find its constant of proportionality (constant of variation)
15. Construct the graph of an inequality of degree 1 (i.e. a relation which is a subset of the product set  $R \times R$ ).

RESOURCES 2

Obj. 8

Vanatta, read 105-106, Ex. 3, 5, 7, 8 page 107.

Dolciani, Modern Algebra, read pages 207-208, Ex. 1-24 even pages 208-209.

Nichols, read pages 169-173, Ex. 1 page 171.

Payne, read pp. 199-201, 220-222, Ex. 1-3 (checkpoint) and 1-9 page 101; 11 page 206; 1-5 page 222.

Wooton, MSM, read pages 154-157, Ex. 5-16 pages 157-158; 13-16 page 153.

Pearson, read pages 273-275, Ex. 1, 2, 5 page 276.

Filmstrip: Relations and Functions

Transparencies 3M: Functions

Obj. 9

Vanatta, read pages 108-110, Ex. 1-4, 12-15, 20, 26, 28 pages 110-111.

Dolciani, MA, read pages 207-208, Ex. 1-16 page 209.

Payne, read pages 199-201, Ex. 1-5 page 201, 16-21 page 203.

Wooton, MSM, read pages 149-151, Ex. 11-18 oral page 152, 1-8 written page 152.

Pearson, read pp. 277-280, Ex. 1-3 pages 280-281.

Obj. 10

Vanatta, read pages 112-113, Ex. 1, 3-8 page 114; 9 page 126.

Nichols, read pp. 169-173, Ex. 3 page 173; 2, 3 page 268.

Payne, read pages 220-222, Ex. 1-20 even page 223; 28-40 pages 224-225; 8-17 and 21-23 page 226.

Pearson, read pp. 293-299, Ex. 1-7 page 300.

Obj. 11

Nichols, read pages 169-173, Ex. 2 pages 172-173.

Resources 2 (cont')

Obj. 12

Nichols, read pp. 173-176, Ex. 1-6 pages 175-176.

Wooton, MSM, read pp. 152-153, Ex. 1-34 pages 152-153.

Payne, read pp: 215-217, Ex. 1-25 pages 218-219.

Pearson, read pp. 277-280, 288-291, 316-319, Ex. 1-10 pages 280-281;  
\* 1-7 page 292; 1-10 pages 319-321.

Obj. 13

Nichols, read pp. 176-179, Ex. 1-7 page 178.

Wooton, MSM, read pp. 183-183, 187-188, Ex. 1-24 pages 184-185;  
1-15 page 188.

Payne, read pp. 206-208, Ex. 1-46 pp. 209-212.

Pearson, read pp. 302-305, Ex. 1-2 page 305.

Obj. 14

Nichols, read pp. 179-185, Ex. 1-5 pages 185-186.

Wooton, MSM, read pp. 180-183, Ex. 1-24 pages 184-185.

Payne, read pp. 206-208, Ex. 1-46 pages 209-212.

Pearson, read pp. 307-308, 310, Ex. 1-14 pages 308-309; 1-13 page 311.

Filmstrip: Direct Variation

Obj. 15

Vanatta, read pp. 121-123, Ex. 2, 3, 6, 7 page 124.

Nichols, read pp. 186-190, Ex. 1-2 pages 189-190.

Wooton, MSM, read pp. 164-167, Ex. 1-28 page 167; 1-16 pages 188-189.

Payne, read 441-442, Ex. 1-14 page 442.

Pearson, read pp. 314-315, Ex. 1-7 pages 315-316.

Wollensak teaching tape, C-3806: Inequality and Equality Sentences

Filmstrip: Graphs of Inequalities in One Variable

SELF-EVALUATION 2

Obj.

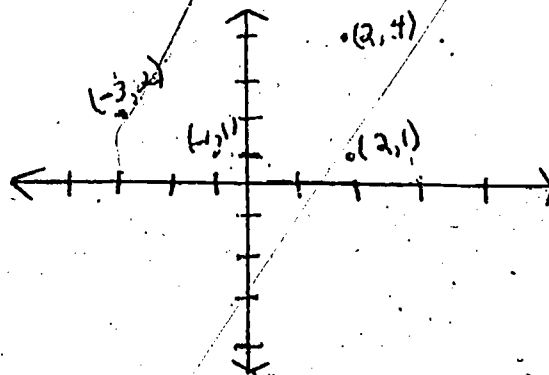
8. I. Determine if each of the following is a function. Write F for function if it is a function. If it is not a function, write R for relation only.

\_\_\_\_\_ 1.  $(4,1)(6,8)(2,1)(-4,3)$

\_\_\_\_\_ 2. 

$x$	-6	-2	-4	-6
$y$	8	1	3	7

\_\_\_\_\_ 3.



\_\_\_\_\_ 4.  $y = x + 1$

\_\_\_\_\_ 5.  $(3,-3)(4,-4)(5,-6)(7,-8)(3,-9)$

\_\_\_\_\_ 6. 

$x$	$y$
0	7
7	0
3	5
9	4
5	3

9. II. For each of the following functions, find the value indicated.

\_\_\_\_\_ 1. Find  $f(2)$  for  $f(x) = 6x + 1$

\_\_\_\_\_ 2. Find  $f(-3)$  for  $f(x) = 2x - 1$

\_\_\_\_\_ 3. Find  $f(0)$  for  $f(x) = \frac{x+1}{6x}$

\_\_\_\_\_ 4. Find  $f(30)$  for  $f(x) = x^2 - x$

\_\_\_\_\_ 5. Find  $f(-10)$  for  $f(x) = \frac{8-2x}{4}$

10. III. Write the inverse of each of the following. State whether the inverse is a function or only a relation. Circle F or R.

\_\_\_\_\_ F or R 1.  $y = 7x + 6$

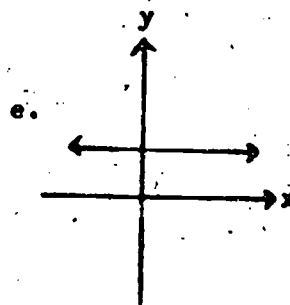
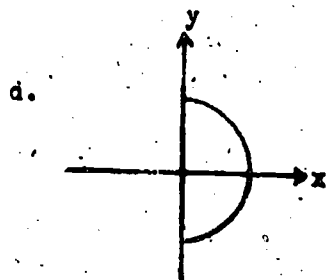
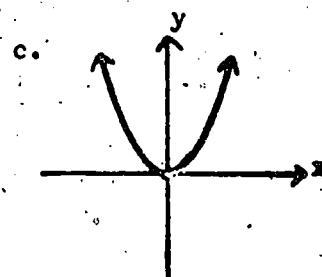
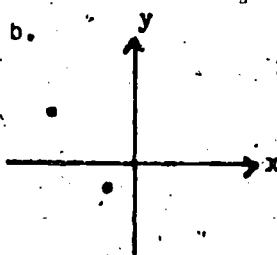
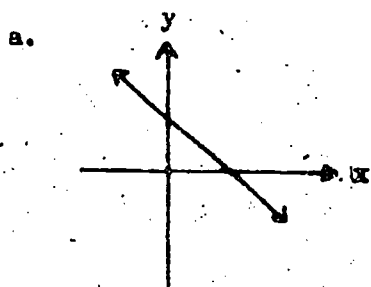
\_\_\_\_\_ F or R 2. 

$x$	-6	-6	-6	-6	-6
$y$	4	2	8	9	7

SELF-EVALUATION 2 (cont')

- \_\_\_\_\_ F or R 3.  $y = x$
- \_\_\_\_\_ F or R 4.  $y = -6 - 2x$
- \_\_\_\_\_ F or R 5.  $(4,7) (-3,2) (9,8) (2,-4) (4,8)$
- \_\_\_\_\_ F or R 6. 
$$\begin{array}{c|cccccc} x & -2 & -4 & 0 & 10 & -3 & -2 \\ y & 7 & 3 & 10 & 0 & 8 & 11 \end{array}$$

11 IV. Which of the following is not the graph of a function? (Circle the correct answer)



12 V. Given  $f(x) = x^2$  and  $g(x) = x + 2$ , find the following:

1)  $f(x) \cdot g(x) =$

2)  $f(g(x)) =$

3)  $g(f(x)) =$

4)  $2 \cdot g(x) =$

5)  $g(2x) =$

6)  $f(x) + g(x) =$

7)  $f(100) =$

SELF-EVALUATION 2 (cont')

13 VI. Classify the functions below as linear, non-linear, constant, greatest integer, or identity. A function may have two such classifications.

- \_\_\_\_\_ 1.  $f(x) = x^2$
- \_\_\_\_\_ 2.  $f(x) = a$  where  $a$  is a real number
- \_\_\_\_\_ 3.  $f(x) = 2x$
- \_\_\_\_\_ 4.  $f(x) = [x]$
- \_\_\_\_\_ 5.  $f(x) = x$

14 VII. Identify each of the following functions as direct or inverse proportional functions; then find the constant of proportionality.

- \_\_\_\_\_ 1.  $f(x) = \frac{2x}{4}$
- \_\_\_\_\_ 2.  $f(x) = \frac{1}{2}x$

13 VIII. Which of the following when in  $R \times R$ , are not linear functions?

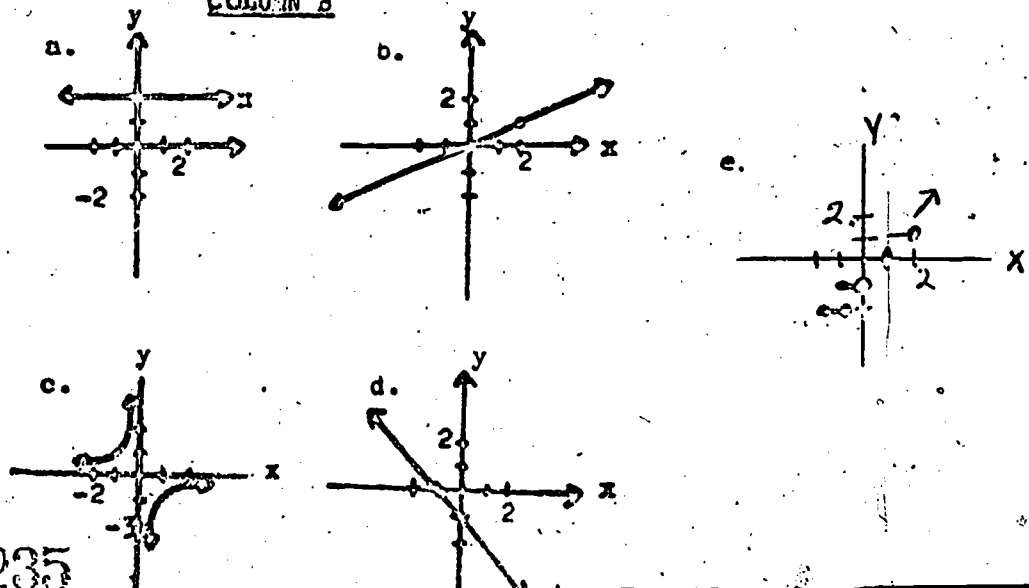
- a. a direct proportion function
- b. the greatest integer function
- c. the identity function
- d. the function defined by  $y = 2x + 3$

13,14 IX. Choose from Column B a graph of the type of function given in Column A.

COLUMN A

- \_\_\_\_\_ 32. Inverse proportion function
- \_\_\_\_\_ 33. Constant function
- \_\_\_\_\_ 34. Direct proportion function
- \_\_\_\_\_ 35. Greatest integer function

COLUMN B



SELF-EVALUATION 2 (cont')

15 X. Graph the following on the graph paper included. (next page)

(1)  $y > 3x + 1$

(2)  $y \geq -2x$

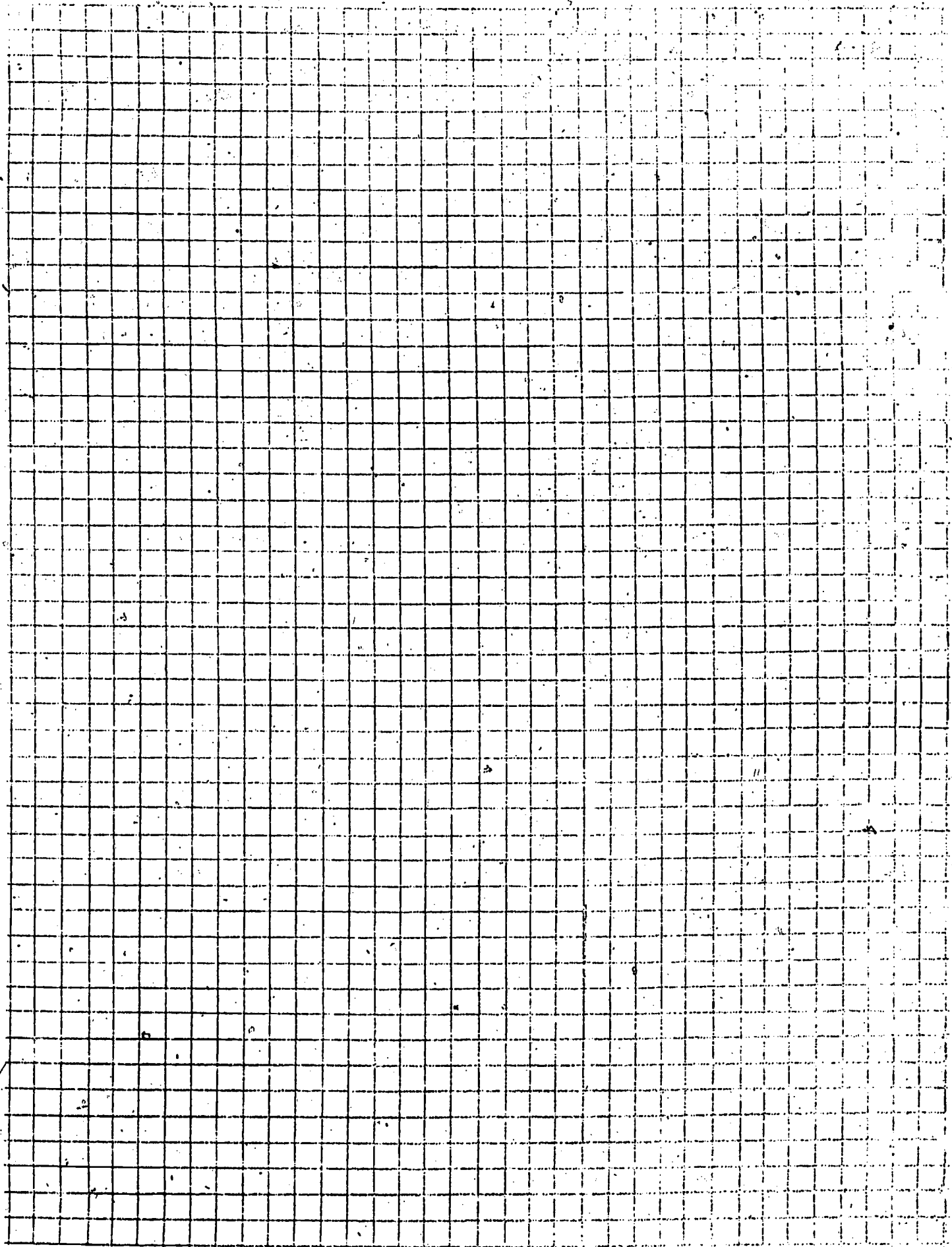
(3)  $y < -4x + 1$

(4)  $y \leq -x - 3$

If you have satisfactorily completed your work, take the LAP test.  
CONSULT YOUR TEACHER FIRST.

SELF-EVALUATION 2 (cont')

Graph Paper





APPENDIX 1

I. Given the relation: 5 more than twice x is equal to y

a. write an equation \_\_\_\_\_

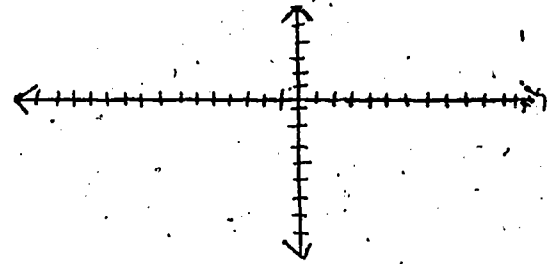
b. write 5 ordered pairs \_\_\_\_\_

c. construct a table using the 5 ordered pairs 

x
y

 \_\_\_\_\_

d. graph the ordered pairs



II. Given the relation : (4,8)(5,10)(2,4)(-1,-2)(-2,-4)

a. write an equation \_\_\_\_\_

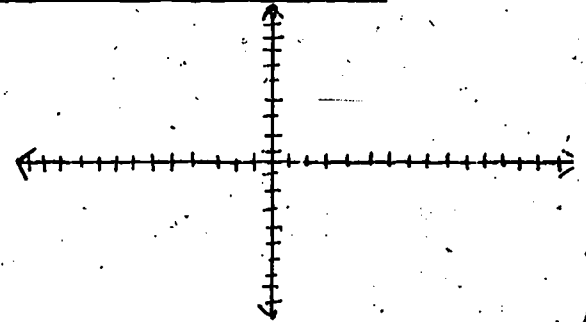
b. write the relation in words \_\_\_\_\_

c. construct a table 

x	y
---	---

 \_\_\_\_\_

d. graph the relation



III. Given the relation :

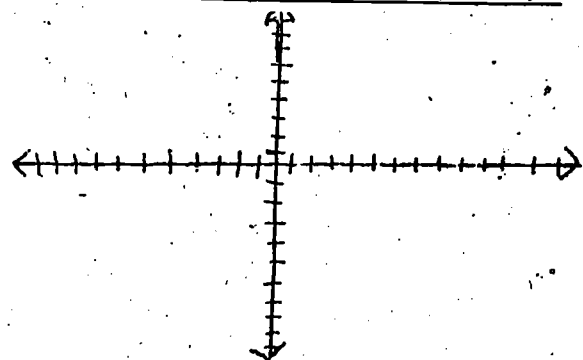
x	y
-2	0
-1	1
2	4
3	5
6	8

a. write an equation \_\_\_\_\_

b. write the relation in words \_\_\_\_\_

c. write the table in ordered pairs ( ) ( ) ( ) ( ) ( )

d. graph the relation



## ADVANCED STUDY

- I. Newton's Law of Universal Gravitation is expressed as follows:

$F_{(grav)} = \frac{G M m}{R^2}$  where  $F_{(grav)}$  is the attractive force in newtons between two masses ( $M$ ) and ( $m$ ). These masses are expressed in kilograms.  $R$  is the distance between the two centers of mass and is expressed in meters.  $G$  is the proportionality constant with a value of

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2.$$

Find the force ( $F$ ) that the earth with mass  $M = 5.98 \times 10^{24}$  kg exerts on a body of mass  $m = 10$  kg located at its surface. Radius of earth

$$R = 6.38 \times 10^6 \text{ meters}$$

- II. Payne, page 225, no. 41
- III. Dolciani, page 210, nos. 31-40
- IV. Dolciani, read p. 218, Ex. 1-16 any 6 page 219
- V. Vanatta, page 121 no. 8, page 112 no. 30, page 104 no. 8.

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### Wooton (Abbreviation)

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### Dolciani (abbreviation)

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### Vanatta (abbreviation)

Vanatta, Glen D., Goodwin, A. Wilson, Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Wollensak teaching tapes: C-3806 Inequality and Equality Sentences

C-3852 Graphing Linear Functions

C-3855 Slope-Intercept Form

Filmstrips: 1. Relations and Functions

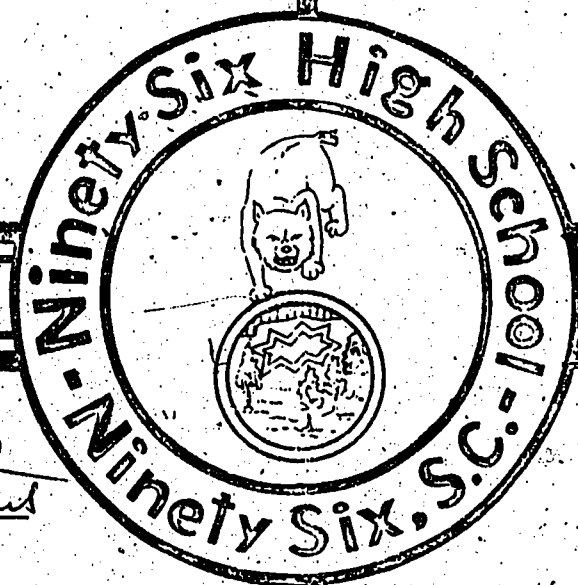
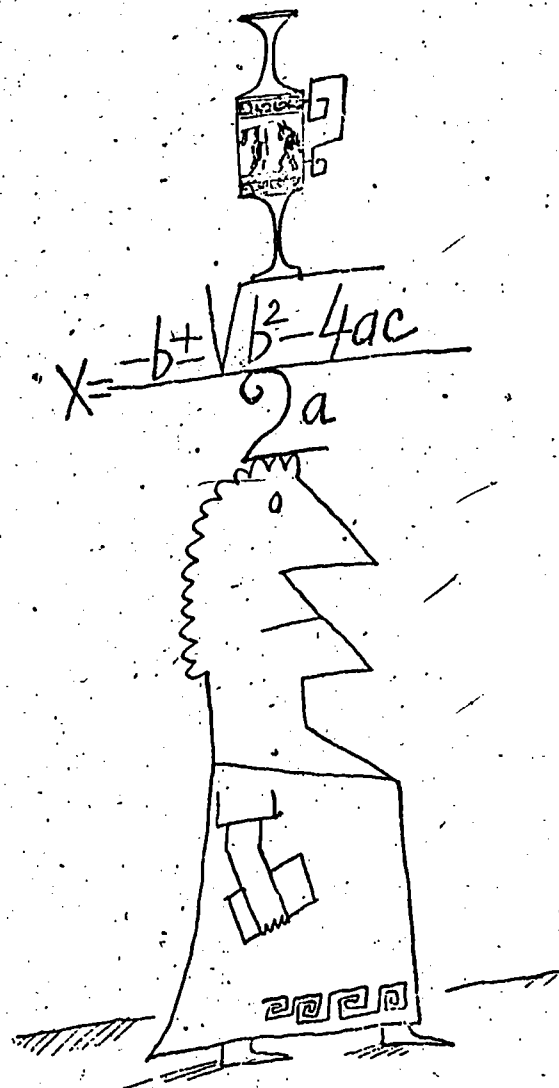
2. Direct Variation

3. Graph of Inequalities in One Variable

Transparencies 3M - Functions

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LEARNING  
A  
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PACKAGE

QUADRATIC  
EQUATIONS  
AND  
INEQUALITIES



Algebra 103-104

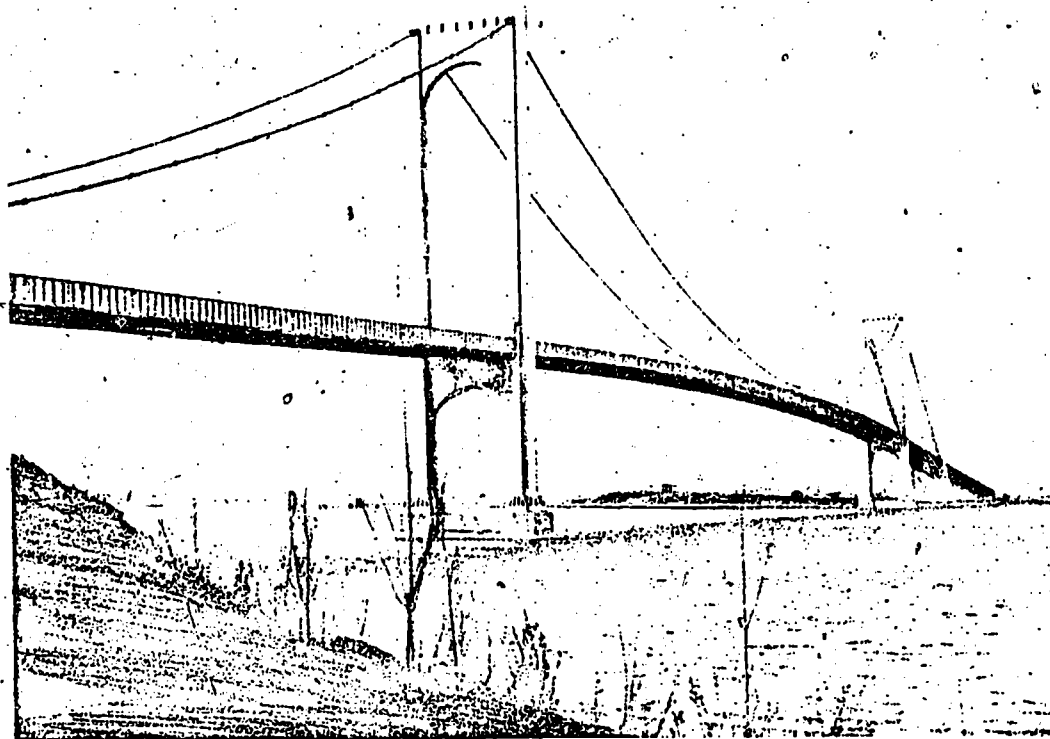
REVIEWED BY

*J. Schuch*

LAP NUMBER 20

WRITTEN BY Diane Evans

## RATIONALE



In the preceding LAP, you studied linear functions which were defined by linear equations in two variables. Unfortunately, nature was not so kind; so in your study of science you will need a working knowledge of all forms of quadratic equations and inequalities. For example, the cable of the bridge in the picture above forms a parabolic curve and can be reduced to a quadratic equation.

In this LAP we will study all quadratic equations and inequalities and some other kinds of equations which are expressible as quadratic equations and inequalities.

5

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Define quadratic equation or quadratic function.
2. Sketch or identify the graph of a quadratic function.
3. Given any factorable quadratic function, determine its roots.
4. Given any quadratic function that is not factorable, determine its roots in simplest form by completing the square.
5. State and/or identify the quadratic formula.
6. Given the equation  $Ax^2 + Bx + C = 0$ , derive the quadratic formula.
7. Given any equation that is not factorable, determine its solutions by substituting into the quadratic formula.

RESOURCES

Obj. 1

Vanatta, read p. 163, write the definition of quadratic equation.

Nichols, read p. 221, write the definition of quadratic equation.

Dolciani, read p. 220, write the definition.

Payne, read p. 251, write the definition.

Pearson, read p. 337, write the definition.

Obj. 2

Vanatta, read pp. 163-164, ex. 1-10 even page 164.

Dolciani, read p. 220, ex. 5-10 page 234.

Payne, read pp. 251-253, ex. 1-6 page 153.

Pearson, read pp. 337-340; ex. 1 a b, 2, 3 b c e (draw graphs only)  
p. 340.

Obj. 3

Vanatta read pp. 165-166, Ex. 2, 5, 6, 7, 10, 12, 14, 17, 18 page 167.

Dolciani, \_\_\_\_\_, ex. 1-8 even, 15, 18, 22, 28. page 136.

Resources 1 (cont')

Wooton, read pp. 265-268, ex. 1-26 every fourth problem.

Pearson, read pp. 176-177, ex. 3 page 178.

Obj. 4

Vanatta, read pp. 167-170, ex. 1, 3, 7, 10, 14 page 172.

Dolciani, read pp. 268-270, 1, 3, 5, 6, 8 written pages 270-271.

Wooton, read pp. 337-340, ex. 1-8 page 340.

Payne, read pp. 257-259, ex. 1-10 even p. 259; 19-32 odd p. 260.

Pearson, read pp. 355-357, ex. 1-12 even page 183 (solve by completing the square)

Obj. 5

Vanatta, read p. 170, state the quadratic formula.

Dolciani, read p. 269, state the quadratic formula.

Nichols, read pp. 224-225, state the quadratic formula.

Wooton, read p. 339, state the formula.

Payne, read pp. 260-261, state the formula.

Obj. 6

Exercise for all books: Derive the quadratic formula.

Vanatta, read pp. 269-170, ex. above.

Dolciani, read p. 268, ex. above.

Nichols, read pp. 224-225, ex. above.

Payne, read p. 260, ex. above.

S-M Transparency 6M - The Quadratic Formula

Obj. 7

Vanatta, read pp. 170-171, ex. 2, 4, 5, 8, 11 p. 172 solve by using quadratic formula.

Dolciani, read pp. 268-270, ex. 10, 13, 15, 18, 20 written p. 270.

Nichols, read pp. 224-226, ex. 2 every other letter page 227.

Wooton, read pp. 337-340, ex. 9-20 even page 341.

Payne, read pp. 260-261, ex. 1-20 odd page 262.

Pearson, read pp. 355-357, ex. 1, a,b,c,e,g, 3 a,b,c,d page 358.

SELF-EVALUATION 1

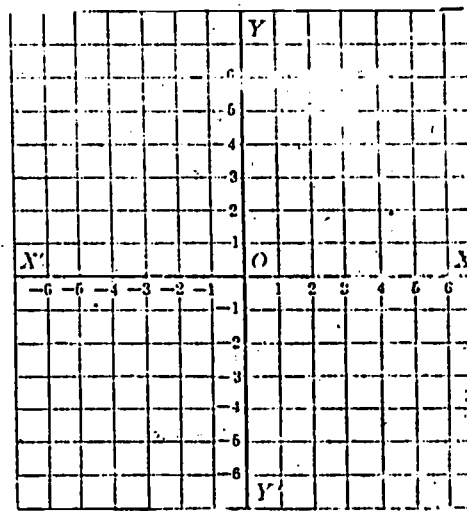
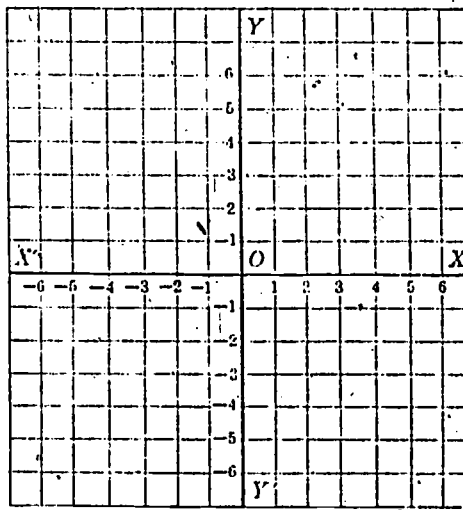
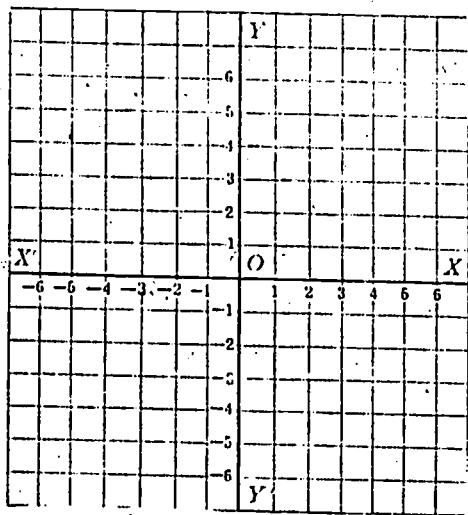
I. Define: quadratic equation.

II. Graph the following.

(A)  $x^2 - 3x - 4 = y$

(b)  $y = 3x^2 + 17x + 20$

(c)  $y = -2x^2 - 3x + 2$



III. Solve the following by factoring.

\_\_\_\_\_ 1.  $x^2 - 3x + 2 = 0$

\_\_\_\_\_ 3.  $x^2 + 4bx - 12b^2 = 0$

\_\_\_\_\_ 2.  $6x^2 - 5x + 1 = 0$

\_\_\_\_\_ 4.  $\frac{x^2 - 12}{3} = \frac{x^2 - 4}{4}$

IV. Find the roots of each of the following by completing the square. Show your work.

\_\_\_\_\_ 1.  $x^2 + 11x + 24 = 0$

\_\_\_\_\_ 2.  $x + \frac{1}{x-1} = \frac{9}{2}$

\_\_\_\_\_ 3.  $x^2 + 5x - 7 = 0$

\_\_\_\_\_ 4.  $3x^2 + 8x + 2 = 0$



SELF-EVALUATION 1 (cont')

V. State the quadratic formula.

VI. Derive the quadratic formula. Begin with  $Ax^2 + Bx + C = 0$ .

VII. Find the roots of the following by substituting into the quadratic formula. SHOW YOUR WORK!

\_\_\_\_\_ (1)  $3r^2 + r - 1 = 0$

\_\_\_\_\_ (2)  $x^2 - 2x - 1 = 0$

\_\_\_\_\_ (3)  $6x^2 + 10x + 3 = 0$

\_\_\_\_\_ (4)  $5x^2 + x + 1 = 0$

\_\_\_\_\_ 5.  $\frac{1}{2} - \frac{7}{6}x = x^2$

If you have satisfactorily completed your work, you may take the progress test. Consult your teacher first.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Write and/or identify the definition of imaginary numbers.
9. Given any square root you will be able to determine whether it is imaginary or real. If it is real, you will be able to determine if it is rational or irrational.
10. Given a quadratic equation  $ax^2 + bx + c = 0$ ,  $a \neq 0$ :
  - a. Find the sum of the roots of the equation.
  - b. Find the product of the roots of the equation.
11. Given a quadratic equation of the form  $ax^2 + bx + c = 0$ ,  $a \neq 0$ :
  - a. Name the discriminant of the equation.
  - b. Specify the number of real roots of the equation.
  - c. Give the nature of the roots by determining:
    1. If the roots are real or imaginary.
    2. If the roots are real determine if they are rational or irrational.
    3. If the roots are equal or unequal.
    4. How many times the graph will touch the x axis.
12. Write a quadratic equation  $ax^2 + bx + c = 0$ :
  - a. Having a given set  $\{r, s\}$  as its solution set.
  - b. When given the sum and product of its roots.
  - c. When certain coefficients are unknown and sufficient information about the roots of the equation is given to find these coefficients.
13. Given a fractional equation:
  - a. Write the corresponding quadratic equation. (assuming one exists)
  - b. Find the solution set of the quadratic equation.

RESOURCES 2

Obj. 8

Vanatta, read page 166, ex. write the definition of imaginary numbers.

Payne, read pp. 24-25, ex. write the definition of imaginary numbers.

Obj. 9

Vanatta, read pp. 166, 30-32, ex. Appendix I.

Payne, read pp. 24-25, ex. 1-27 even page 26.

Filmstrip: Rational and Irrational Numbers.

Obj. 10

Vanatta, read pp. 172-173, ex. 1-10 even pages 173-174.

Dolciani, read page 273, ex. 1-15 odd (oral) page 274.

Nichols, read pp. 228, ex. 2 pages 229-230.

Payne, read pp. 267-268, ex. 1-6 page 268.

Wooton, read pp. 343-344, ex. 1-9 page 344.

Pearson, read pp. 359-362, ex. 2 page 362.

Obj. 11

Vanatta, read p. 175-177, ex. write the discriminant of the equation  $ax^2 + bx + c = 0$ ; ex. 1-10 page 177.

Dolciani, read pp. 275-278, ex. write the discriminant of  $ax^2 + bx + c = 0$ ; 1-10 page 278.

Payne, read pp. 264-266, ex. 1-10 page 266.

Wooton, read pp. 374-376, ex. 1-14 page 377.

Obj. 12

Vanatta, read pp. 172-173, ex. 11, 14, 16, 18, 20 page 173.

\* Dolciani, read pp. 273-274, 16-23 even page 274 top; ex. 1, 2, 6, 13, 15, 16, 18, 21, 23 bottom pages 274-275.

Nichols, read pp. 228-229, ex. 1, 3-6 pages 229-230.

Payne, read pp. 246-271, ex. 1-14 pp. 266-267; 1-14 p. 269; 1-18 p. 271.

Wooton, read pp. 343-344, ex. 16-23 p. 345; 1-14 even, 19-24 pages 345-346.

Obj. 13

Nichols, read pp. 230-232, ex. 1, 2 pages 232-233

Pearson, read pp. 365, ex. 4, 6 pages 368-369.

\* required

SELF-EVALUATION 2

Obj.

8 I. Write the definition of imaginary numbers.

9 II. State whether each of the following is real or imaginary. If real, state if it is rational or irrational.

\_\_\_\_\_ 1.  $\sqrt{18}$

\_\_\_\_\_ 2.  $\sqrt{-4}$

\_\_\_\_\_ 3.  $\sqrt{24}$

\_\_\_\_\_ 4.  $\sqrt{25}$

\_\_\_\_\_ 5.  $\sqrt{-100}$

10 III. Write the sum of the roots of the following:

\_\_\_\_\_ 6.  $y^2 - 2y - 9 = 0$

\_\_\_\_\_ 7.  $2x^2 + 3x = 0$

\_\_\_\_\_ 8.  $2x^2 - 8x - 1 = 0$

\_\_\_\_\_ 9.  $3x^2 + 15x + 2 = 0$

10 IV. Write the product of the roots of the following:

\_\_\_\_\_ 10.  $y^2 - 2y - 9 = 0$

\_\_\_\_\_ 11.  $2x^2 + 3x = 0$

\_\_\_\_\_ 12.  $2x^2 - 8x - 1 = 0$

\_\_\_\_\_ 13.  $3x^2 + 15x + 2 = 0$

11 V. In each of the following equations:

(a) determine the value of the discriminant.

(b) specify the number of real roots of the equation.

(c) determine if the roots are real or imaginary.

(d) if the roots are real, determine if they are rational or irrational.

(e) state if the roots are equal or unequal.

(f) state how many times the graph touches the x-axis.

14.  $x^2 + 4x + 3 = 0$

SELF-EVALUATION 2 (cont')

15.  $x^2 - 5x + 7 = 0$

16.  $3x^2 - 2x = 4$

17.  $-5x^2 - x - 3 = 0$

18.  $-3x^2 + 4x + 1 = 0$

19.  $3x^2 - 4x + \frac{4}{3} = 0$

12a VI. Write an equation for each of the following solution sets.

\_\_\_\_\_ 20.  $\{5, -7\}$

\_\_\_\_\_ 21.  $\{0, \frac{1}{3}\}$

\_\_\_\_\_ 22.  $\{-\frac{1}{7}, -\frac{1}{9}\}$

\_\_\_\_\_ 23.  $\{\sqrt{2} + \sqrt{3}, \sqrt{2} - \sqrt{3}\}$

\_\_\_\_\_ 24.  $\left\{\frac{-1 + \sqrt{2}}{2}, \frac{-1 - \sqrt{2}}{2}\right\}$

12b VII. Given the following sum and product of roots, write an equation.

\_\_\_\_\_ 25. sum = -7                      product = 3

\_\_\_\_\_ 26. sum = 9                              product = -2

\_\_\_\_\_ 27. sum =  $\sqrt{3}$                               product = 0

SELF-EVALUATION 2 (cont.)

- \_\_\_\_\_ 28. sum = 0                      product = -7
- 12c VIII. Find the real values to satisfy the conditions given.
- \_\_\_\_\_ 29. For what value(s) of "a" will the sum of the roots be 8?
- $$x^2 - (a^2 - 2a)x + 3 = 0$$
- \_\_\_\_\_ 30. For what value(s) of "b" will the equation  $2x^2 + 4x + (2 - b - b^2) = 0$  have exactly one root?

- 13 IX. For each of the following fractional equations:
- (a) Write the corresponding quadratic equation (assuming one exists).  
 (b) Find the solution set of the quadratic equation.

31.  $\frac{1}{x} + \frac{x-1}{x(x+2)} = \frac{-x}{x+2}$

32.  $x - 4 = \frac{-1}{x}$

If you have satisfactorily completed your work, you may take the PROGRESS TEST. Consult your teacher first.

### SECTION 3

#### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

14. Given a radical equation:
  - a. Write the corresponding quadratic equation. (assuming one exists)
  - b. Find the solution set of the quadratic equation.
  - c. Find the solution set of the radical equation.
  - d. State whether the two equations are equivalent.
  - e. Name the roots of the quadratic equation which are not permissible roots of the radical equation.
  
15. Given any quadratic inequality:
  - a. Find the solution set of the inequality.
  - b. Graph the solution set on a number line.
  
16. Given a word problem solvable by means of a quadratic equation:
  - a. Translate the problem into a quadratic equation.
  - b. Solve the problem.
  
17. Given a word problem solvable by means of a fractional equation:
  - a. Translate the problem into a quadratic equation.
  - b. Solve the problem.

RESOURCES 3

Obj. 14

Work one set of problems.

Vanatta, read pp. 258-260, ex. 1-14 page 260.

Nichols, read pp. 233-235, ex. 1, 2, pages 235-236.

Payne, read pp. 360-363, ex. 1-26 even pages 361-362; 1-25 even pp. 363-364.

Pearson, read pp. 370-371, ex. 1-3 page 371.

Wooton, read pp. 334-336, ex. 1-36 even page 336.

Dolciani, read pp. 281-282, ex. 1, 3, 13, 15, 17, 24, 26 pages 282-283.

Obj. 15

Vanatta, read pp. 205-206, ex. 1-8 page 208.

Nichols, read pp. 239-243, ex. 1-3 pages 243-245.

Payne, read pp. 276-278, ex. 3-8 (bottom) page 278.

Wooton, read pp. 362-363, ex. 1-16 even page 364.

Dolciani, read pp. 279-280, ex. 1-8 page 280.

Obj. 16

Vanatta, read pp. 179-181, ex. 1, 2, 4, 7, 15, 19 pages 181-183; 13, 14, page 211.

Nichols, read pp. 221-226, ex. 4-12 pages 227.

Wooton, read pp. 337-340, ex. 1, 2, 7 page 342; 1, 2, 4, 7, 13 pages 269-270.

Dolciani, \_\_\_\_, ex. 32, 38 page 201; 1-4, 8-13 pages 137-138.

Goal 17

Vanatta, read pp. 179-181, ex. 5, 6 page 182; no. 18 page 160.

Nichols, read pp. 230-232, ex. 1, 2 pages 232-233.

Dolciani, read pp. 178-179, ex. 10, 19, page 182.



SELF-EVALUATION 3

bj.

14 I. For each of the following radical equations:

- Write the corresponding quadratic equation (assuming one exists).
- Find the solution set of the quadratic equation.
- Find the solution set of the radical equation.
- State whether the two equations are equivalent.
- Name the roots of the quadratic equation which are not permissible roots of the radical equation.

(1)  $x - 3 = \sqrt{2x - 3}$

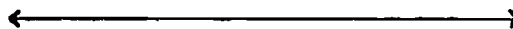
(2)  $\sqrt{3x + 2} = 3\sqrt{x} - \sqrt{2}$

(3)  $5x - \sqrt{2x + 1} = 4x + 1$

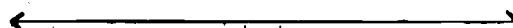
(4)  $\sqrt{x + 4} + \sqrt{x - 3} = 7$

15 II. Solve the following inequalities and graph their solution sets on the real number line.

(5)  $3x^2 - 5x - 4 \leq 0$

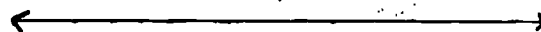


(6)  $2x^2 + 5x < 3$

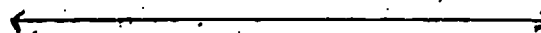


SELF-EVALUATION 3 (cont')

(7)  $x + 3x \leq 10$



(8)  $3x - x^2 \geq 0$



16 III. Write the equation and solve the following word problems.

(9) Find the 2 consecutive positive integers whose product is 756.

(10) Find the length of a side of a square if the length of a diagonal is 5 units greater than the length of a side.

(11) The length of a rectangle is 4 feet more than twice the width. If the area of the rectangle is 30 square feet, find the length and width.

(12) The rug in a bedroom is 9 feet by 12 feet. If the area of the rug is 154 sq. feet, how wide is the strip of bare floor around the rug if the bare strip is of uniform width?

17 IV. Write the equation and solve the following.

(13) A certain integer increased by 4 times its reciprocal equals  $8\frac{1}{2}$ . Find the number.

SELF-EVALUATION 3 (cont')

- (15) Jim can pick a bushel of apples in 25 minutes. Sam can pick a bushel in 15 minutes. How long will it take the boys to pick a bushel together?



If you have satisfactorily completed your work, you may take the LAP test. Consult your teacher first.

APPENDIX I

State whether each of the following is real or imaginary. If real, state if it is rational or irrational. Write each in simplest form.

1.  $\sqrt{144}$

2.  $\sqrt{-81}$

3.  $\sqrt{24}$

4.  $\sqrt{-100}$

5.  $\sqrt{70}$

6.  $\sqrt{36}$

7.  $\sqrt{-1}$

8.  $\sqrt{-126}$

9.  $\sqrt{25}$

10.  $\sqrt{-25}$

ADVANCED STUDY

1. Wooton, page 342 nos. 49, 50
2. Payne p. 264 nos. 55-62
3. Dolciani, p. 271 nos. 49, 50
4. Wooton, page 346, nos. 26-28
5. Wooton, page 342 nos. 49, 50
6. Pearson, page 363, nos. 9-14
7. Dolciani, pages 283-284, nos. 43-46 any 3 problems.

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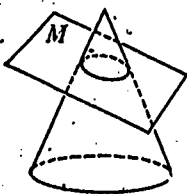
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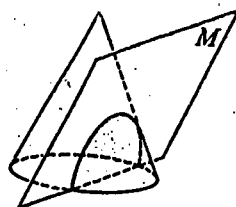
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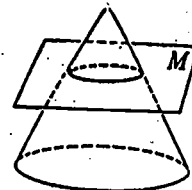
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P  
ACKAGE



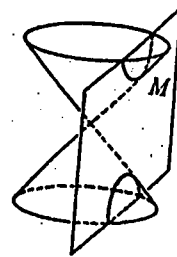
Ellipse



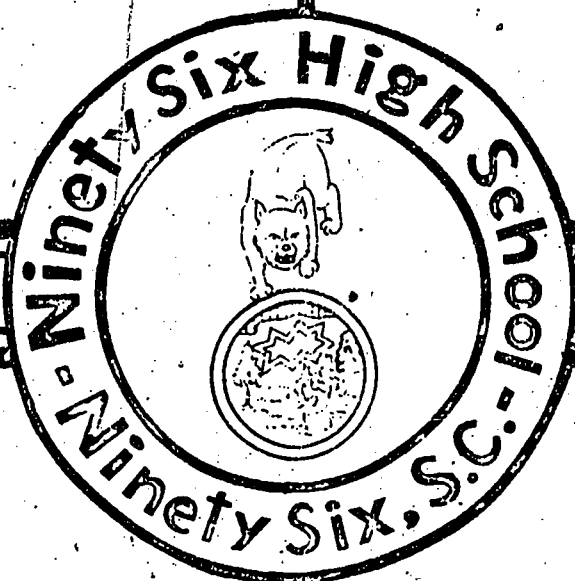
Parabola



Circle



Hyperbola



QUADRATIC FUNCTIONS

Algebra 103-104

REVIEWED BY

*J. Ritchie*

LAP NUMBER 21

WRITTEN BY Diane Evans

## RATIONALE

In preceding LAPs you studied functions in general, and more specifically, the straight line. Recall that Descartes is credited with originating the Cartesian coordinate system. In the concept of coordinates, Descartes gave mathematicians a new way to look at mathematical information. Not only did he show that first degree, or linear, equations can be graphed as straight lines, but he also showed that all second degree, or quadratic equations can be graphed to become circles, ellipses, parabolas, or hyperbolas. These quadratic functions are collectively referred to as conic sections.

Conics appear frequently in nature and in numerous applications; for example, the orbits of planets about the sun are ellipses. The supporting cables of a suspension bridge form a parabola. The hyperbola appears as the edges of the shadow cast on a wall by a lampshade. In this LAP we will investigate the graphs of these quadratic functions in some detail. In addition, we will study quadratic inequalities.



## SECTION 1

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given a relation in  $R \times R$ , determine whether or not it is a quadratic function.
2. Write and/or identify the definition of conic section.
3. List and/or identify the four conic sections.
4. Describe and/or identify the descriptions of the following terms as they relate to a cone:
  - a. element
  - b. axis
  - c. circle
  - d. ellipse
  - e. parabola
  - f. hyperbola
5. Write and/or identify:
  - a. the definition of a circle
  - b. the standard form of the equation of a circle with radius  $r$  and center at the origin.
  - c. The standard form of the equation of a circle with radius  $r$  and center  $(h,k)$ .
6. Given the equation of a circle, write and/or identify:
  - a. the center
  - b. the radius
  - c. the graph of the circle
7. Given the center and radius of a circle,
  - a. graph and/or identify the curve
  - b. write and/or identify the equation in standard form

## RESOURCES 1

### Objective 1

\* Nichols, read pp. 195-199, Ex. 1 a-d page 199-200.

Pearson, read pp. 337-339, Ex. 2 page 340.

### Objectives 2, 3, 4

Nichols, read pp. 312-313, Exercise Appendix I.

Wooton, read pp. 456-457, Exercise Appendix I.

Dolciani, read pp. 330-331, Ex. Appendix I.

Vanatta, read pp. 183-185, Ex. Appendix I.

Pearson, read pp. 697, Ex. Appendix I.

Payne, read page 417, Ex. Appendix I.

### Objective 5

Exercise for all books: Write the definition and equations in Objective 5.

\* Vanatta, read pp. 191-192, Ex. above.

Dolciani, read pp. 300-302, Ex. above.

Wooton, read pp. 442-443, Ex. above.

Payne; read pp. 418-419, Ex. above.

3M Transparency: Circle

### Objectives 6, 7

\* Vanatta, read pp. 191-193, Ex. 1, 3, 5, 6, 8, 10 page 194; 11-16 page 194.

Dolciani, read pp. 300-302, Ex. 11-16 page 302; 1-4, 9, 10 pp. 300-302 (graph and write equations).

Wooton, read pp. 442-443, Ex. 13-17, 20 page 443; 1-8 page 443 (graph and write equations).

Payne, read pp. 418-419, Ex. 1-6 page 419; 3, 4, 6, 9-12 page 420.

3M Transparency: Circle

\* required

SELF-EVALUATION 1

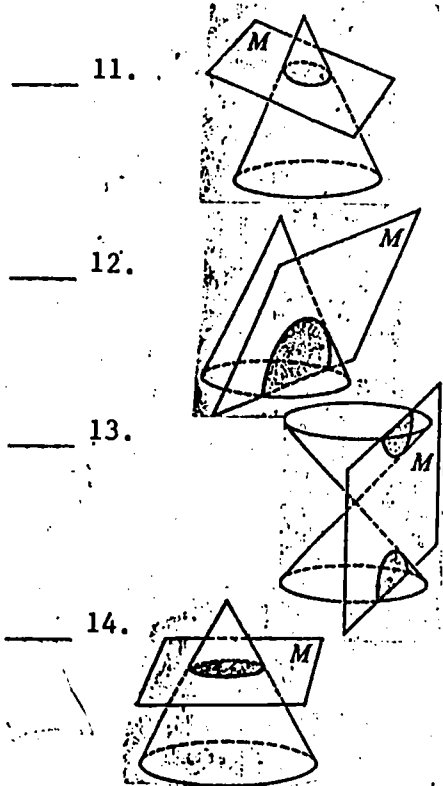
I. True or False.

- \_\_\_\_\_ 1. A conic section is the set of points determined by a plane intersecting a cone.
- \_\_\_\_\_ 2. An axis is a straight line that lies wholly within the surface of a cone.
- \_\_\_\_\_ 3. A parabola is the section of a cone formed by a plane that is perpendicular to one element.
- \_\_\_\_\_ 4. A hyperbola is the section of a cone formed by a plane that intersects the cone so that the plane is parallel to one element.
- \_\_\_\_\_ 5. An element is a line that joins the vertex of a cone with the center of the circle that is its base.
- \_\_\_\_\_ 6. An ellipse is the section of a cone formed by a plane that cuts completely through the cone perpendicular to the axis. A circle is a special kind of an ellipse.

II. Which of the following are quadratic functions?

- \_\_\_\_\_ 7.  $x^2 + y = 1$
- \_\_\_\_\_ 8.  $y = x + 2$
- \_\_\_\_\_ 9.  $3x - 2y^2 = 7$
- \_\_\_\_\_ 10.  $y = \frac{1}{2}x^2 - 3$

III. Match each figure on the left with its name on the right.



- A. ellipse
- B. circle
- C. hyperbola
- D. parabola

SELF-EVALUATION 1 (cont')

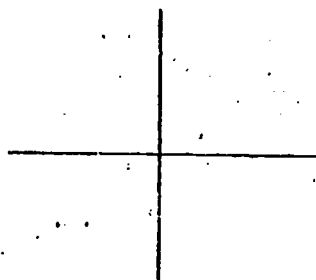
IV. 15. A. Define circle.

B. Write the equation of the circle with radius  $r$  and center  $(0,0)$ .

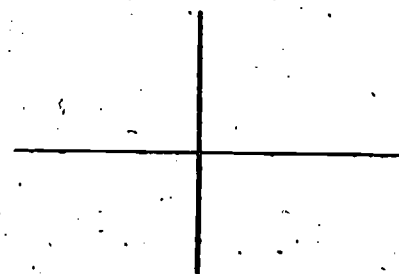
C. Write the equation of the circle with radius  $r$  and center  $(h,k)$ .

V. Give the center and radius of the following circles and graph each.

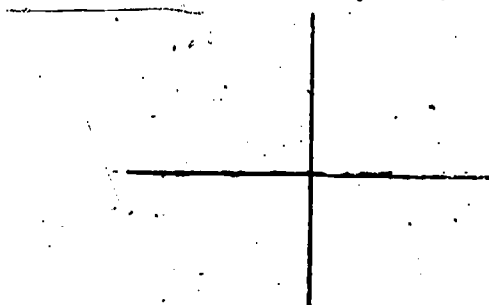
16.  $x^2 + y^2 = 36$



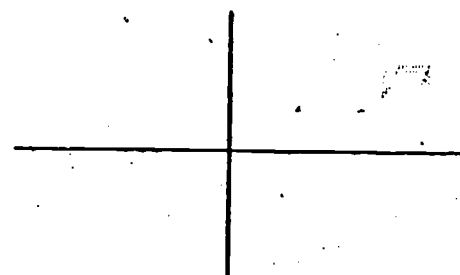
18.  $x^2 + y^2 + 12x + 11 = 0$



17.  $(x + 5)^2 + (y - 8)^2 = 4$

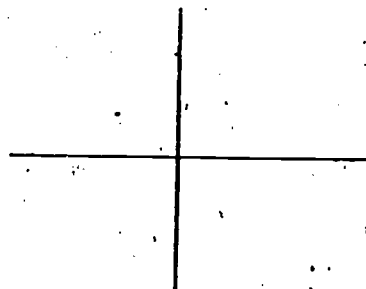


19.  $x^2 + y^2 - 10x + 4y + 20 = 0$

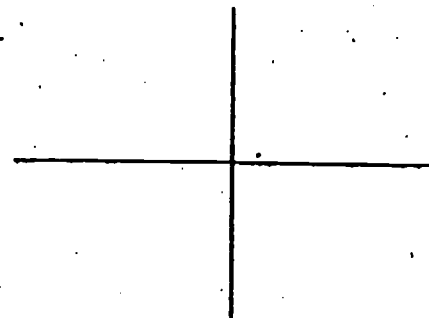


VI. For each given center and radius (1) graph the curve, and (2) write the equation in standard form.

20.  $C(2,1), r = 3$

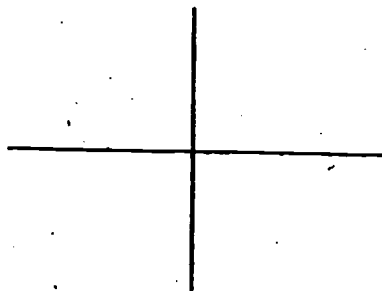


21.  $C(0,0), r = 6$

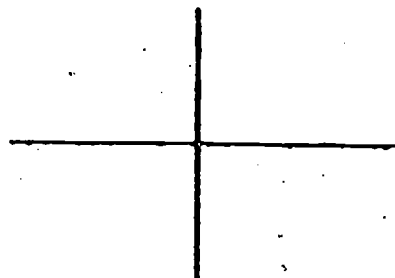


SELF-EVALUATION 1 (cont')

22.  $C(4, -2), r = \sqrt{7}$



23.  $C(-8, -1), r = 3\sqrt{2}$



If you have satisfactorily completed your work, take the Progress Test. Consult your teacher first.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Write and/or identify the definitions of the following terms:
  - a. parabola
  - b. axis of symmetry
  - c. the value of  $p$
  - d. focus ( $F$ )
  - e. directrix
  - f. vertex ( $V$ )
9. Write and/or identify a description of the equations  $x^2 = 4py$  and  $y^2 = 4px$ .
10. Given any equation of the form  $x^2 = 4py$  and/or  $y^2 = 4px$ ;
  - A. determine the value of  $p$
  - B. determine the focus
  - C. determine the equation of the directrix
  - D. graph the curve, focus, and directrix
11. Given a focus and an equation of a directrix,
  - a. graph the curve
  - b. write the equation of the parabola
12. Given an equation of the form  $(y - k)^2 = 4p(x - h)$  or  $(x - h)^2 = 4p(y - k)$ , determine the vertex, focus, directrix, and sketch the graph.

## RESOURCES 2

### Objective 8

Exercise for all texts: Write the definitions of the terms in Objective 8.

Vanatta, read pp. 146, 196, Ex. above.

Nichols, read p. 146, Ex. above.

Wooton, read pp. 444-445, Ex. above.

Payne, read pp. 425-426, Ex. above.

3M Transparency: Parabola

### Objective 9

Vanatta, read p. 188, Ex. describe the equations  $x^2 = 4py$  and  $y^2 = 4px$ .

### Objective 10

Vanatta, read pp. 188-190, Ex. 1, 2, 5, 7, 9, 10 pages 190-191.

3M Transparency: Parabola

### Objective 11

\* Vanatta, read pp. 188-190, Ex. 11-16 page 191.

\* Dolciani, read page 306, Ex. 15-18 page 306.

### Objective 12

Vanatta, read pp. 203-204, Ex. 4, 7, 12, 13 page 204.

Wooton, read pp. 444-448, Ex. 1-6 page 448.

\* required

SELF-EVALUATION 2

OBJ.

8 I. Define the following:

a. parabola

b. axis of symmetry

c. the value of p

d. focus (F)

e. directrix

f. vertex (V)

9 II. Describe the graph of each of the following:

1.  $x^2 = 4py$

2.  $y^2 = 4px$

10 III. Write the value of p for each of these.

\_\_\_\_\_ 1.  $x^2 = -16y$

\_\_\_\_\_ 2.  $y^2 = 100x$

\_\_\_\_\_ 3.  $x^2 = -6y$

\_\_\_\_\_ 4.  $y^2 = -2x$

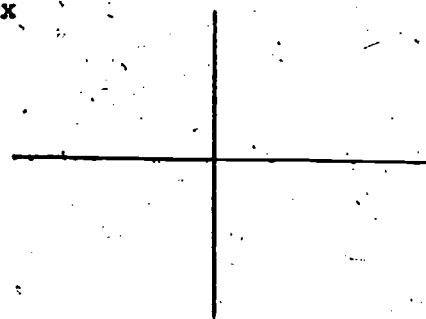
\_\_\_\_\_ 5.  $x^2 = 10y$

10 IV. For each of the following: (1) determine the value of p, (2) determine the focus, (3) determine the equation of the directrix, (4) locate two points other than the vertex and graph the curve, focus, and directrix.

1.  $x^2 = 16y$



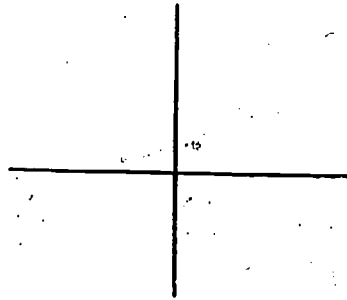
2.  $y^2 = -20x$



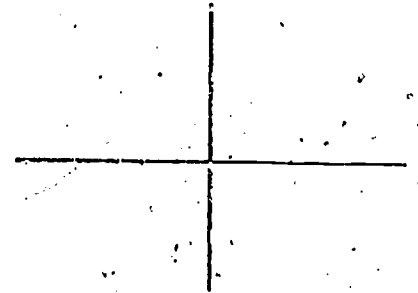


SELF-EVALUATION 2 (cont')

3.  $y^2 = 2x$

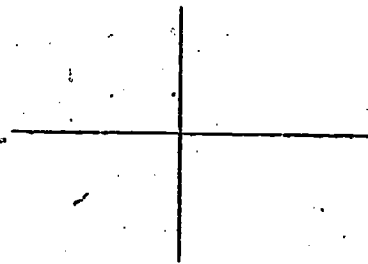


4.  $x^2 = -8y$

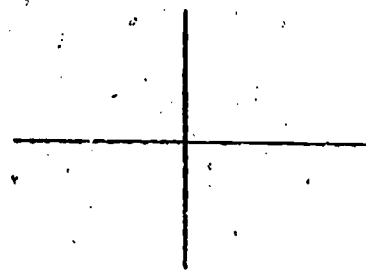


11 V. Given the following foci and directrices, graph each curve formed by them.

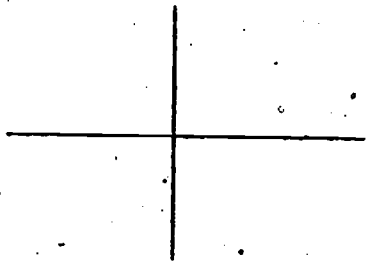
1. F (2,0) x = -2



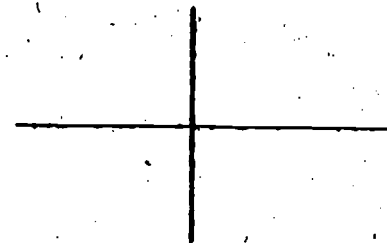
2. F (0,-4) y = 4



3. F  $(-\frac{3}{2}, 0)$  x =  $\frac{3}{2}$



4. F  $(0, \frac{1}{3})$  y =  $-\frac{1}{3}$



11 VI. Write an equation for each parabola in example V above.

1. \_\_\_\_\_

2. \_\_\_\_\_

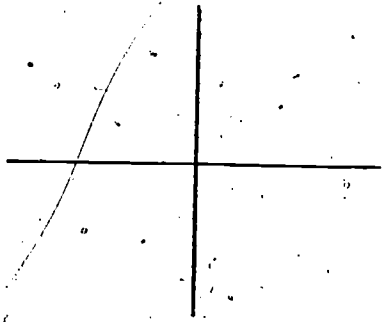
3. \_\_\_\_\_

4. \_\_\_\_\_

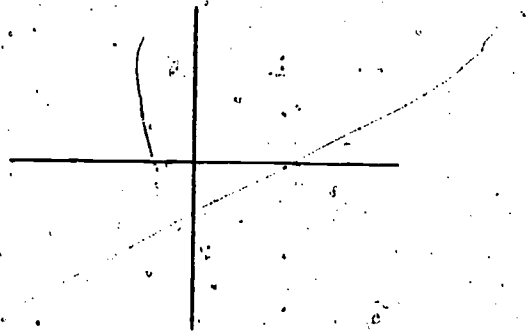
SELF-EVALUATION 2 (cont.)

- 12 VII. For each of the following (1) give the vertex, (2) give the focus, (3) give the directrix, (4) plot the graph, vertex, focus, and directrix.

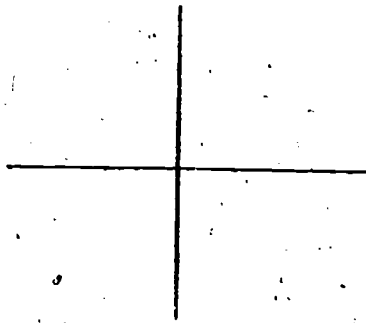
1.  $(y - 2)^2 = 16(x + 2)$



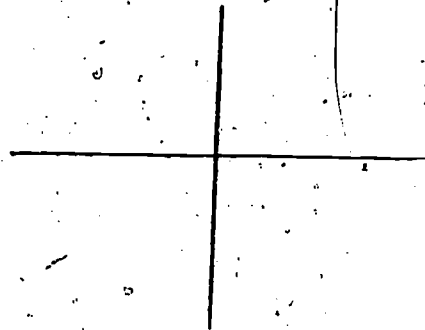
2.  $(x + 3)^2 = -8(y - 1)$



3.  $(y + 2)^2 = -2(x + 1)$



4.  $x^2 + 8x + 8y + 8 = 0$



If you have satisfactorily completed your work, take the PROGRESS TEST.  
Consult your teacher first.

SECTION 3

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

13. Write and/or identify the definitions of the following terms:
  - A. ellipse
  - B. foci
  - C. vertices
  - D. major axis, length of major axis
  - E. minor axis, length of minor axis
14. Given a drawing of an ellipse, identify the following parts:
  - A. major axis
  - B. minor axis
  - C. foci
  - D. vertices
  - E. center
15. Write and/or identify the standard form of both an ellipse with its center at the origin and major axis on the x-axis and an ellipse with its center at the origin and major axis on the y-axis.
16. Given any equation of an ellipse, determine by looking at the equation if the major and minor axes are on x or y and/or determine the length.
17. Given any equation of an ellipse, determine
  - A. the semi-major (a) and semi-minor (b) axes
  - B. the foci
  - C. graph the ellipse, plot the foci and vertices
18. Given the major and/or minor axes, the length of the semi-major (a) and semi-minor (b) axes and the center at the origin, determine the equation of the ellipse.

OBJECTIVES 3 (cont')

19. Given the vertices and foci,
  - A. determine the equation of the curve
  - B. sketch the curve
20. Given the equation of an ellipse whose center is not at the origin, determine
  - A. the center
  - B. the foci
  - C. whether the major axis is parallel to the x or y axis
  - D. sketch the graph, plot the center, vertices, and foci

RESOURCES

Objectives 13, 14, 15, 16

Exercise for all texts: Appendix 2 parts I-IV

Vanatta, read pp. 194-196, Ex. above.

Payne, read pp. 421-423, Ex. above.

Wooton, read pp. 449-452, Ex. above.

3M Transparency: The Ellipse

Objective 17

\* Appendix II part V

Vanatta, read pp. 195-197, Ex. 1, 3, 4, 7, 9 page 197.

Dolciani, read \_\_\_\_, Ex. 1, 2, 6, 7 page 308; plot foci and vertices.

Wooton, read pp. 449-452, Ex. 1-8 even page 452.

3M Transparency: The Ellipse.

Objective 18

Vanatta, read pp. 194-197, Ex. 11, 12 page 198.

Objective 19

Vanatta, read pp. 194-197, Ex. 17, 18 page 198.

Objective 20 (work all exercises)

Dolciani, read \_\_\_\_, Ex. 23, 24 page 309.

Vanatta, read pp. 203-204, Ex. 2, 8, 10, 15 page 204.

Pearson, \_\_\_\_, Ex. 7 b, e, g page 691.

\* required

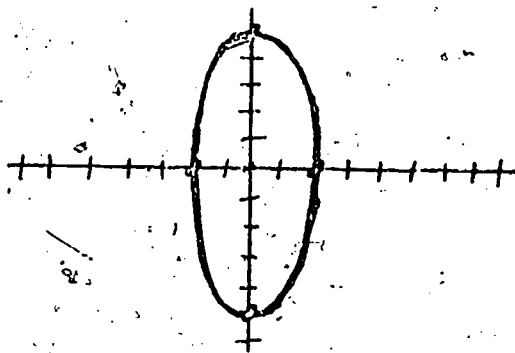
SELF-EVALUATION 3

OBJ.

13 I. Define the following terms:

- a. ellipse
- b. foci
- c. vertices
- d. major axis
- e. length of major axis
- f. minor axis
- g. center

14 II. Using the following graph identify these parts: (a) major axis, (b) minor axis, (c) foci, (d) vertices, (e) center.



15 III. 1. Write the equation of the ellipse with center at the origin and major axis on x-axis.

2. Write the equation of the ellipse with center at the origin and major axis on y-axis.

SELF-EVALUATION 3(cont')

- 16 IV. Determine in each of the following if the major axis is on x or y and give its length.

\_\_\_\_\_ 1.  $\frac{x^2}{15} + \frac{y^2}{9} = 1$

\_\_\_\_\_ 2.  $\frac{x^2}{100} + \frac{y^2}{64} = 1$

\_\_\_\_\_ 3.  $\frac{x^2}{121} + \frac{y^2}{144} = 1$

\_\_\_\_\_ 4.  $\frac{x^2}{9} + \frac{y^2}{49} = 1$

- 16 V. Determine in each of the examples in problem V if the minor axis is on x or y and give its length.

\_\_\_\_\_ 1. \_\_\_\_\_ 3.

\_\_\_\_\_ 2. \_\_\_\_\_ 4.

- 17 VI. In each of these find the value of a (the semi-major axis) and the value of b (the semi-minor axis).

\_\_\_\_\_ 1.  $\frac{x^2}{25} + \frac{y^2}{9} = 1$  \_\_\_\_\_ 3.  $\frac{x^2}{81} + \frac{y^2}{121} = 1$

\_\_\_\_\_ 2.  $\frac{x^2}{100} + \frac{y^2}{36} = 1$  \_\_\_\_\_ 4.  $\frac{x^2}{16} + \frac{y^2}{49} = 1$

- 17 VII. Find the foci for each of the following.

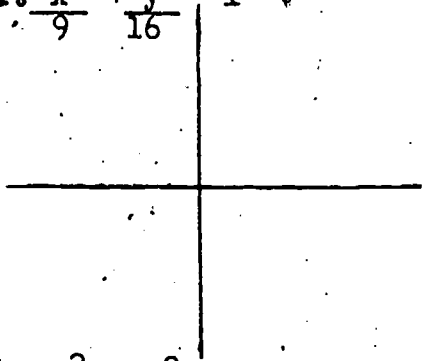
\_\_\_\_\_ 1.  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  \_\_\_\_\_ 3.  $\frac{x^2}{100} + \frac{y^2}{64} = 1$

\_\_\_\_\_ 2.  $\frac{x^2}{36} + \frac{y^2}{4} = 1$  \_\_\_\_\_ 4.  $\frac{x^2}{9} + \frac{y^2}{81} = 1$

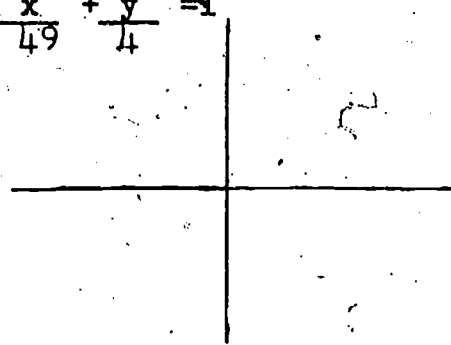
SELF-EVALUATION 3 (cont')

17 VIII. Graph the following ellipses, plot the foci, and vertices of each.

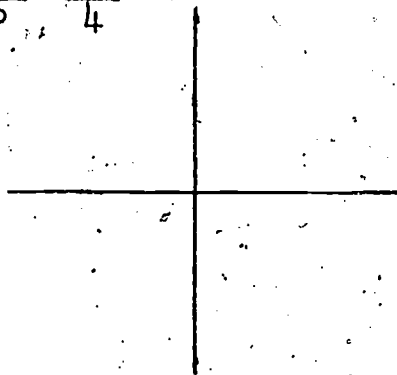
1.  $\frac{x^2}{9} + \frac{y^2}{16} = 1$



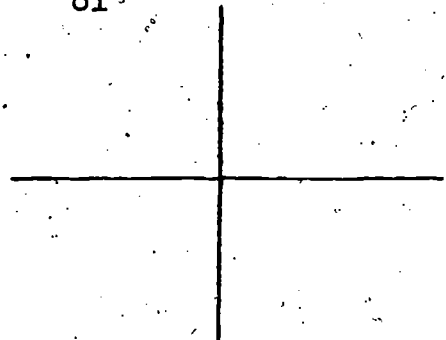
2.  $\frac{x^2}{49} + \frac{y^2}{4} = 1$



3.  $\frac{x^2}{36} + \frac{y^2}{4} = 1$



4.  $\frac{x^2}{9} + \frac{y^2}{81} = 1$



18 IX. Given the following (centers and values of a and b, write an equation for each ellipse.

\_\_\_\_\_ 1. a = 10  
b = 3  
C(4,1)  
Major axis parallel to x

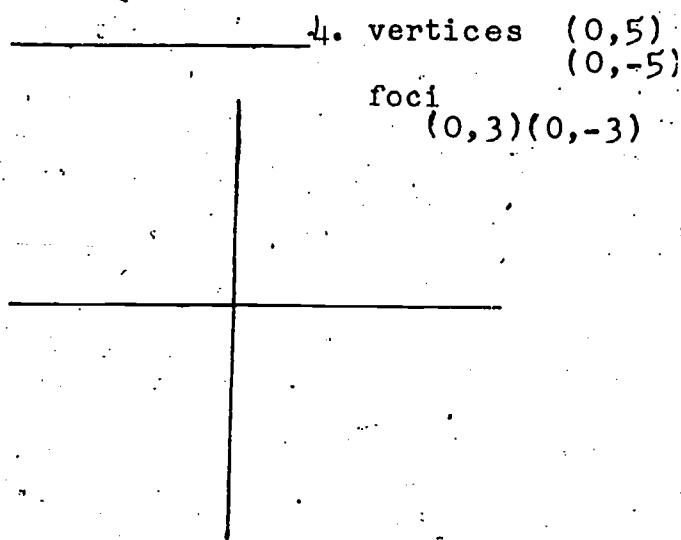
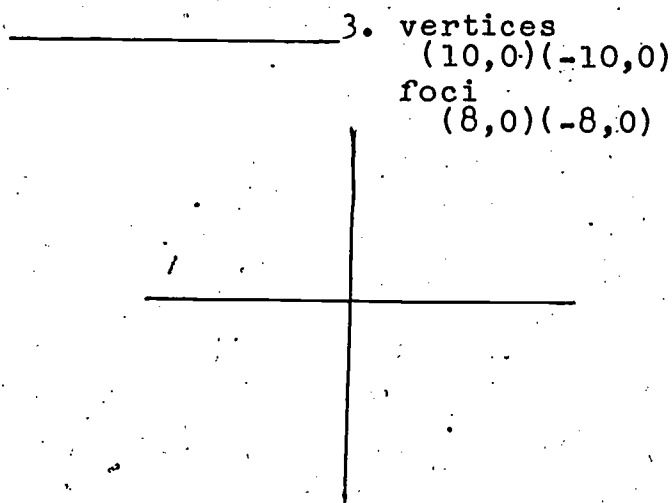
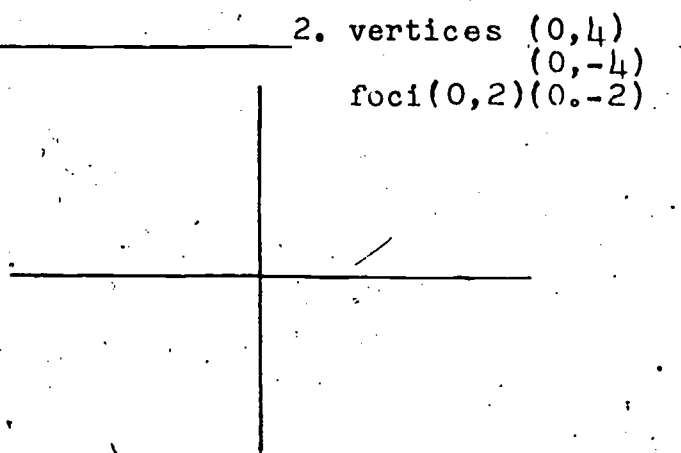
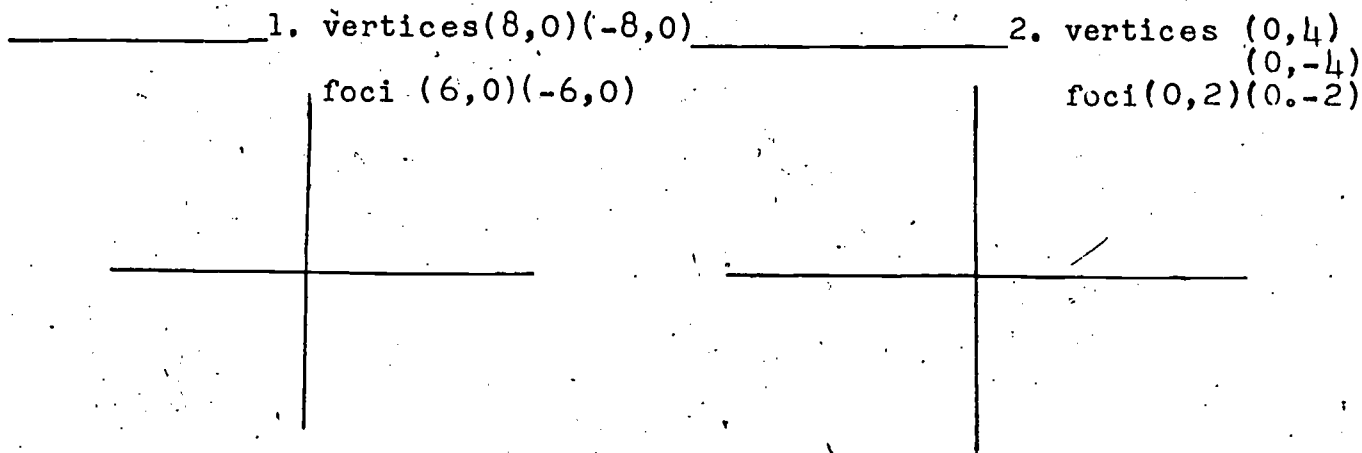
\_\_\_\_\_ 2. a = 4  
b = 2  
C(-1,3)  
Major axis parallel to y

\_\_\_\_\_ 3. a = 12  
b = 9  
C(0,7)  
Major axis parallel to x

\_\_\_\_\_ 4. a = 12  
b = 8  
C(3,-2)  
Major axis parallel to y

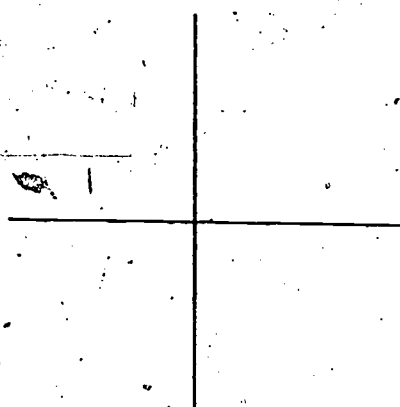
SELF-EVALUATION 3 (cont')

- 19 X. Given the following vertices and foci, write an equation for each and graph the curve, centers are at (0,0).

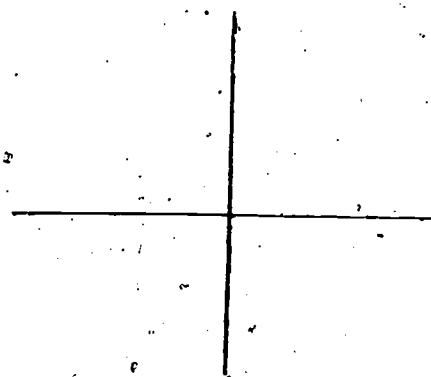


- 20 XI. For each of the following give (1) the center, (2) the foci, (3) tell if the major axis is parallel to x or y, (4) sketch the curve and plot the center, vertices, and foci.

1.  $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{25} = 1$



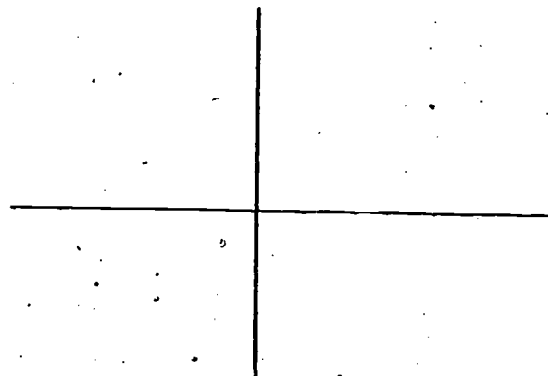
2.  $\frac{(x-3)^2}{100} + \frac{(y+4)^2}{36} = 1$





SELF-EVALUATION 3 (cont')

3.  $25x^2 + 9y^2 - 100x - 36y - 89 = 0$



If you have satisfactorily completed your work, take the PROGRESS TEST. Consult your teacher first.

SECTION 4

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

21. Write and/or identify the definition of
  - A. hyperbola
  - B. transverse axis and its length
  - C. conjugate axis and its length
22. Given a drawing of a hyperbola, identify the following parts:
  - A. transverse axis
  - B. asymptotes
  - C. conjugate axis
  - D. foci
  - E. vertices
  - F. center
23. Identify the standard form of
  - A. the ellipse whose center is at the origin and transverse axis on x
  - B. the ellipse whose center is at the origin and transverse axis on y
24. Given the equation of a hyperbola, determine
  - A. the length of the transverse axis, the length of the conjugate axis, draw the asymptotes, and sketch the curve.
  - B. the coordinates of the foci and plot them on the graph
25. Determine the equation of a hyperbola when given
  - A. the transverse axis and the length of a and b
  - B. the foci and vertices
26. Given any equation of a hyperbola whose center is not the origin, determine
  - A. the center

SECTION 4  
BEHAVIORAL OBJECTIVES (cont')

- B. the length of the transverse and conjugate axes
- C. the vertices
- D. plot the asymptotes
- E. the foci
- F. draw and/or identify the sketch, plot the center, vertices, and foci

RESOURCES

OBJ. 21, 22, 23

Exercise for all resources: Appendix 3

\* Vanatta, read pp. 198-201, Ex. above.

Payne, read pp. 427-430, Ex. above.

Wooton, read pp. 453-457, Ex. above.

3M Transparencies: The Hyperbola.

OBJ. 24

\* Vanatta, read pp. 198-201, Ex. 1, 2, 4, 7, 9 page 202.

Dolciani, read pp. 311-312, Ex. 1, 2, 4, 7, 10 pages 311-312 (follow directions in Obj. 24).

Payne, read pp. 427-340, Ex. 3-8 page 430 (follow directions in Obj. 24).

Wooton, read pp. 453-457, Ex. 1-10 even page 457 (follow directions in Obj. 24).

3M Transparencies: The Hyperbola.

OBJ. 25

\* Vanatta, read page 201, Ex. 11-14 page 202.

Payne, read pp. 427-430, Ex. 9-12 page 430.

3M Transparencies: The Hyperbola.

SECTION 4  
RESOURCES (cont')

OBJ. 26

\* Vanatta, read pp. 203-204, Ex. 3, 6, 11, 14 page 204.

Wooton, read \_\_\_, Ex. 21, 22 page 458.

Pearson, read \_\_\_, Ex. 7 page 695, (follow directions in Obj. 26).

3M Transparencies: The Hyperbola.

\* required

SELF-EVALUATION 4

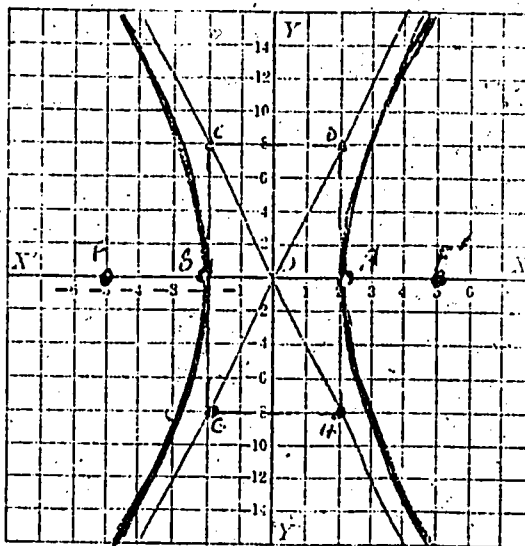
OBJ.

21 I. Write the definition of hyperbola.

21 II. a. Write the definition of transverse axis, give its length.

b. Write the definition of conjugate axis, give its length.

22 III. Identify these parts of the following graph: (1) transverse axis (2) asymptotes (3) conjugate axis (4) foci (5) vertices and (6) center.



23 IV. (1) Write the equation of the ellipse whose center is at the origin and transverse axis on x.

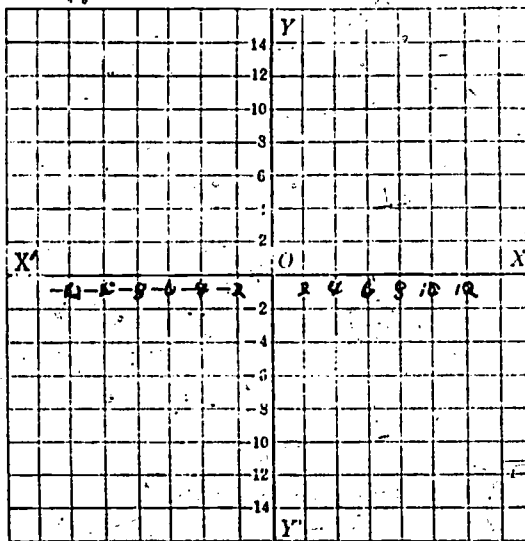
(2) Write the equation of the ellipse whose center is at the origin and transverse axis on y.

24 V. For each of the following (1) determine the length of the transverse axis and conjugate axis, (2) draw the asymptotes, (3) sketch the curve, (4) determine the foci and plot them on the graph.

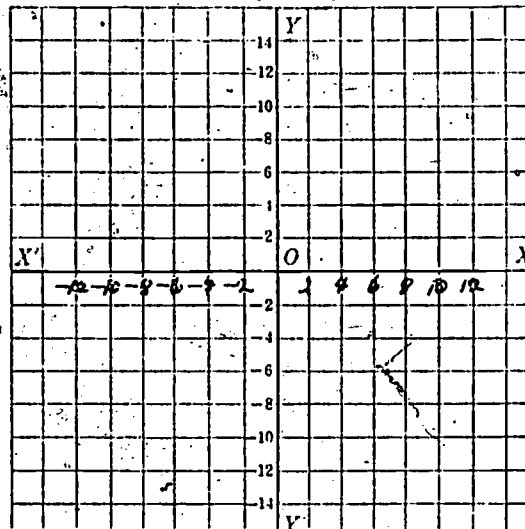
1.  $\frac{x^2}{100} - \frac{y^2}{64} = 1$

(graph is on the following page)

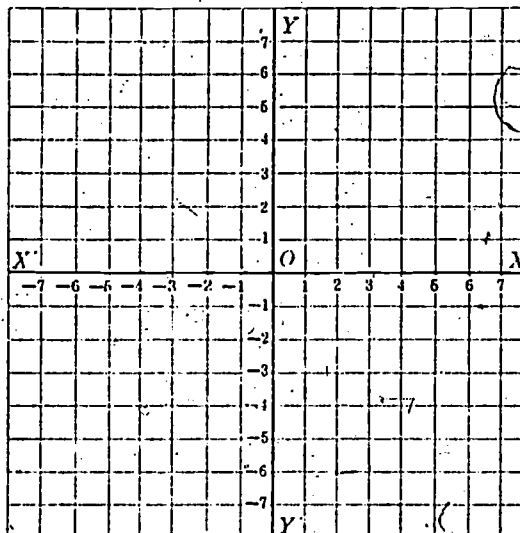
SELF-EVALUATION 4 (cont')



2.  $\frac{y^2}{49} - \frac{x^2}{36} = 1$

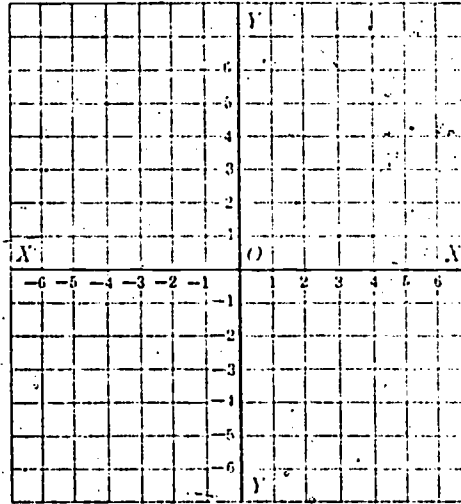


3.  $\frac{y^2}{1} - \frac{x^2}{16} = 1$



SELF-EVALUATION 4 (cont')

4.  $\frac{x^2}{1} - \frac{y^2}{4} = 1$



25 VI. Given the following values of a and b and transverse axis, write an equation for each.

1. a = 2, b = 3, transverse axis on x \_\_\_\_\_
2. a = 4, b = 1, transverse axis on y. \_\_\_\_\_
3. a = 2, b = 7, transverse axis on x \_\_\_\_\_
4. a = 9, b = 12, transverse axis on y \_\_\_\_\_

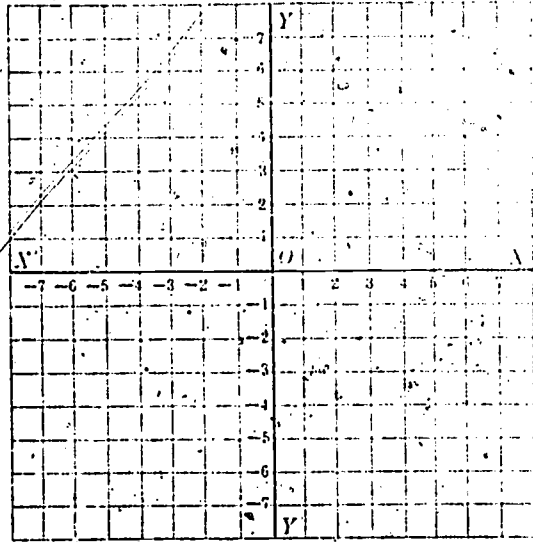
25 VII. Given the following foci and vertices, write an equation for each hyperbola.

1. F(12,0)(-12,0)  
V(8,0)(-8,0) \_\_\_\_\_
2. F(0,6)(0,-6)  
V(0,3)(0,-3) \_\_\_\_\_
3. F(0,2)(0,-2)  
V(0,1)(0,-1) \_\_\_\_\_

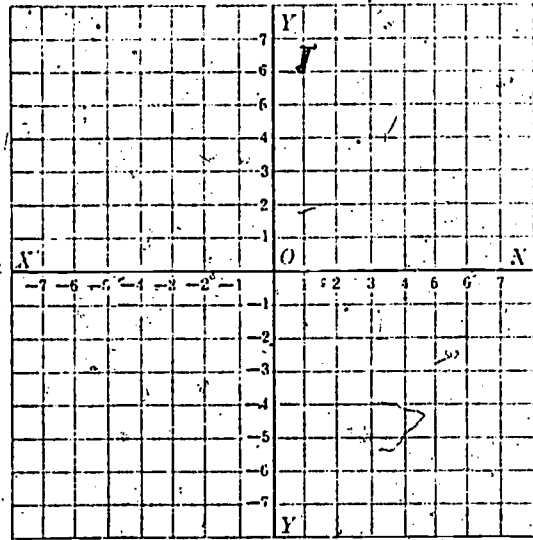
SELF-EVALUATION 4 (cont')

- 26 VIII. For each of the following (1) give the center, (2) give the length of the transverse and conjugate axes, (3) vertices, (4) foci, (5) plot the asymptotes, plot the curve, center, vertices and foci.

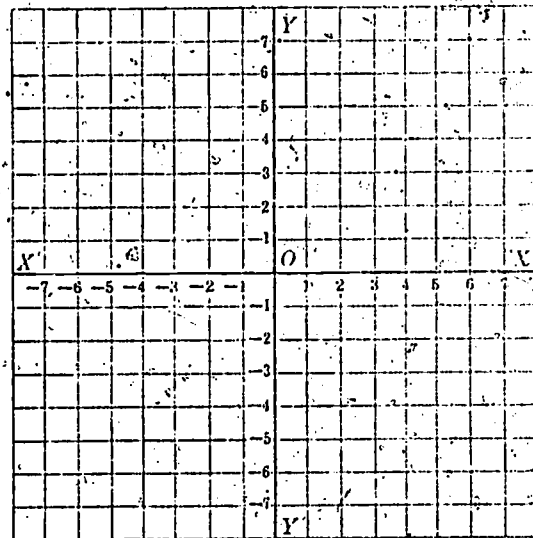
(1)  $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{16} = 1$



(2)  $\frac{(y+3)^2}{64} - \frac{(x-5)^2}{36} = 1$



(3)  $25x^2 - y^2 + 150x + 125 = 0$



If you have satisfactorily completed  
Your work, take the PROGRESS TEST.  
Consult your teacher first.

205



SECTION 5

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

27. Given any quadratic inequality, determine its graph.
28. Given any word problem, determine its equation and determine its solution.

RESOURCES

Obj. 27 (both are required)

Vanatta, read pp. 205-208, Ex. 9-12; 16 page 208.

Dolciani, read \_\_\_\_, Ex. 10, 12 page 309.

Obj. 28

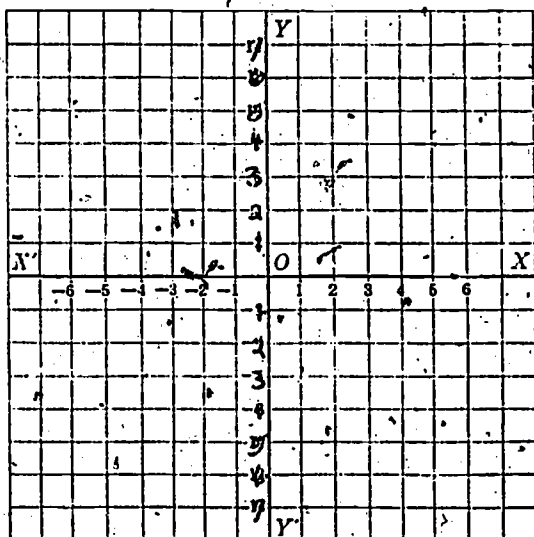
Vanatta, read pp. 179-181, Ex. 1-4, 6-8, 16, 19 pages 181-182; 13-14  
page 211.

SELF-EVALUATION 5

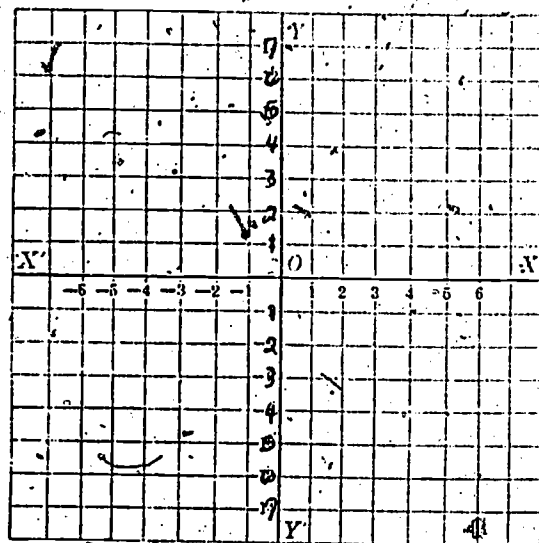
OBJ.

27. I. GRAPH the following inequalities.

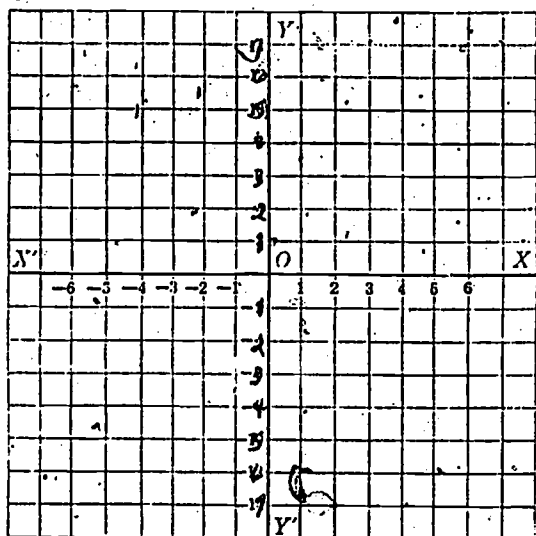
(1)  $x^2 + y^2 < 36$



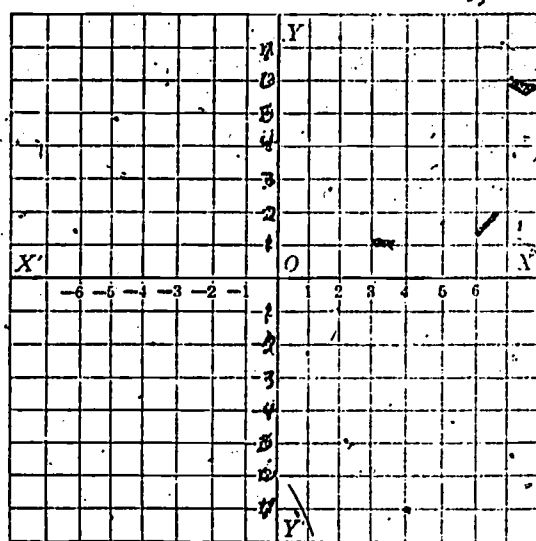
(2)  $9x^2 + 25y^2 > 225$



(3)  $9y^2 - x^2 < 9$



(4)  $x^2 \geq 16y$



SELF-EVALUATION 5 (cont')

28 II. Solve the following problems.

1. Three times the square of a positive integer, decreased by twice the product of the number and the next smaller integer, is 143. Find the number.
2. Find two consecutive integers such that, if twice the larger is added to three times the square of the smaller, the sum will be 58.
3. The base of a triangle is 4 feet less than the altitude, and the area of the triangle is 48 square feet. Find the length of the base.
4. A rectangular lot is surrounded on all sides by a driveway 5 yards wide. The lot is twice as long as it is wide. If the area of the lot and driveway together is 6600 square yards, find the dimensions of the lot.
5. One leg of a given right triangle exceeds the other by 2 feet. If the hypotenuse is 10 feet, find the legs of the triangle.
6. A field of tomatoes contains 3825 plants. The number of plants in each row is 5 less than twice the number of rows. Find the number of plants in each row.

If you have satisfactorily completed your work, take the LAP TEST.  
Consult your teacher first.

ADVANCED STUDY

I. Work any 4 of the following:

Vanatta, page 194 nos. 17-20.

page 211 no. 5; page 214 no. 40.

Dolciani, page 326 no. 11.

II. Work any 4.

Vanatta page 198 nos. 15, 16, 19, 20

page 214 no. 41

Dolciani page 326 no. 13.

III. Work any 4 from no. 1 and one from no. 2.

(1) Vanatta page 202 nos. 15-20

page 212 no. 18, page 214 no. 43

Dolciani page 326 no. 12, 14

(2) Dolciani page 312 no. 19, 20

IV. Work any one of these three.

(1) Graph. (A)  $xy = 36$   
(B)  $xy = -10$

(2) Vanatta page 205 nos. 16, 19, 20

(3) Write a report on LORAN, a system of navigation. Tell how it uses the concept of hyperbola (at least 500 words).

V. Work any 5 of the following.

Vanatta page 212, no. 20; page 214 no. 44, 45.

Dolciani, page 237, nos. 22, 25, 27, 28

page 309, nos. 13, 14.

APPENDIX I

I. Write the definition of conic section.

II. Name the four conic sections.

(1)

(2)

(3)

(4)

III. Write a description or definition of each of the following terms as they relate to a cone.

1. element

2. axis

3. circle

4. ellipse

5. parabola

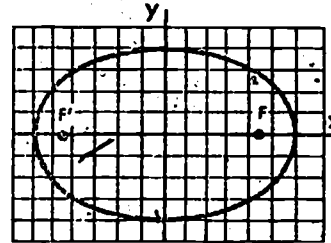
6. Hyperbola

APPENDIX 2

I. Define the following:

- a. ellipse
- b. foci
- c. vertices
- d. major axis
- e. minor axis

II. Identify the following parts of the ellipse in the figure:  
 (a) major axis, (b) minor axis, (c) foci, (d) vertices (e) center.



III. Describe the ellipses with the following equations:

(a)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  where  $a > b$

(b)  $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$  where  $a > b$

IV. In each of the following, determine (1) if the major axis is on x or y and give its length, and (2) if the minor axis is on x or y and give its length.

(a)  $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(b)  $\frac{x^2}{25} + \frac{y^2}{100} = 1$

(c)  $\frac{x^2}{49} + \frac{y^2}{81} = 1$

(d)  $\frac{x^2}{49} + \frac{y^2}{9} = 1$

APPENDIX 2 (cont')

V. In each equation in IV, determine the value of a and b.

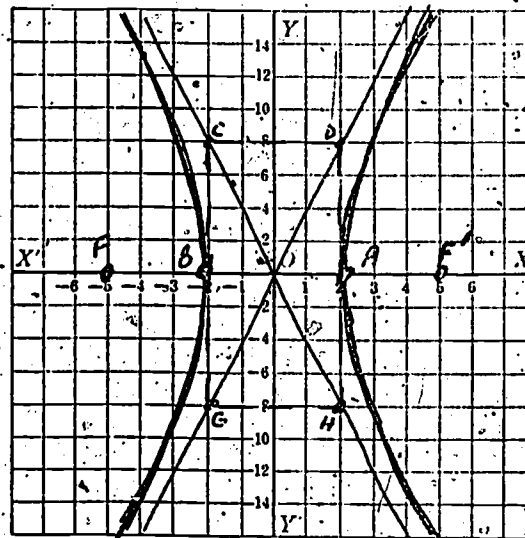
APPENDIX 3

I. Define the following:

- a. hyperbola
- b. transverse axis - give its length
- c. conjugate axis - give its length

II. Identify the parts of the following graph:

- a. conjugate axis
- b. transverse axis
- c. asymptotes
- d. foci
- e. vertices
- f. center



- III. a. Write the equation of the ellipse whose center is at the origin and transverse axis on the  $x$  axis.
- b. Write the equation of the ellipse whose center is at the origin and transverse axis on the  $y$  axis.



REFERENCES

Vanatta, Glen D., and Goodwin, Wilson A., Algebra Two, A Modern Course, Charles E. Merrill Publishing Co., 1966.

Dolciani, Mary P., Berman, Simon L., and Wooton, William, Modern Algebra and Trigonometry, Book Two, Houghton Mifflin Co., 1965.

Nichols, Eugene D., Modern Intermediate Algebra, Holt, Rinehart and Winston, Inc., 1965.

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Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Frances G., Algebra Two, Harcourt, Brace and World, 1969.

Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Sharron, Sidney, Modern School Mathematics, Algebra 2, Houghton Mifflin Company, 1968.

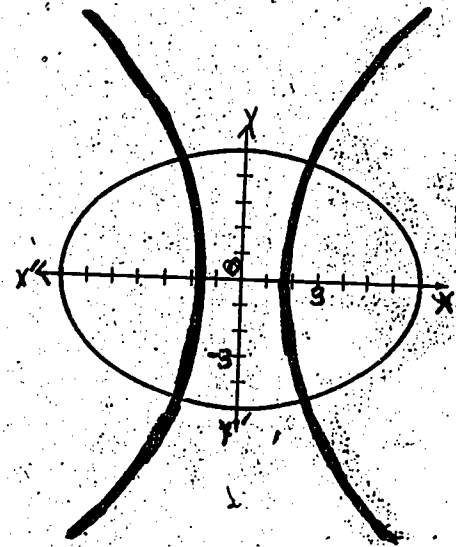
3M Transparencies - The Circle

The Parabola

The Ellipse

The Hyperbola

**L** |  
**L** EARNING  
**A** CTIVITY  
**P** ACKAGE



SYSTEMS OF EQUATIONS  
AND INEQUALITIES



Algebra 103-104

VIEWED BY

*J. Ritchie*

LAP NUMBER 22

WRITTEN BY Diane Evans

## RATIONALE

Many problems in mathematics result in mathematical models involving more than one sentence, yet the problem requires a single solution. Two beams of light across the sky might each be described as the graph of a linear equation. If they were to cross one another, you might then have a point of intersection, the single solution.

This LAP should help you gain an understanding of systems of equations or of inequalities. The use of previously learned concepts about functions will be used in developing methods for solving these systems. Equations of conic sections will also be continued and studied with systems of first and second-degree equations so that you should be able to solve these systems, and relate the solutions to their graphs.

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given an equation, or system of linear equations, and ordered pairs of real numbers, determine if the ordered pairs satisfy the equation or system of equations.
2. Given a system of two linear equations:
  - a. graph the system  $R \times R$
  - b. find the solution set of the system
3. From a system of linear equations, determine if the system is independent, inconsistent, dependent, or consistent when given:
  - a. the equations of the system
  - b. the graph of the equations of the system
  - c. the solution set of the system
  - d. the number of ordered pairs of the solution set of the system
4. Given two systems of equations, determine if they are equivalent.
5. Given a system of two linear equations, rewrite each equation in the form  $Ax + By + C = 0$ .
6. Given a system of two linear equations, solve the system by any one of the following methods:
  - a. the comparison method
  - b. the substitution method
  - c. the addition method

SECTION 1 (cont')

Behavioral Objectives (cont')

7. Given a word problem to be solved by the use of a system of two linear equations:
- write a system of equations which fits the problem.
  - solve the resulting system.
  - give the solution of the original problem.

RESOURCES

Objective 1

Nichols, read pp. 289, 290-294, Ex. 1, 3, 5, 7 page 290; 1, 3, 4 page 292.

Objective 2, 3

Vanatta, read pp. 215-218, Ex. 1, 3, 4, 9, 10, 13 page 219.

Dolciani, read pp. 95, Ex. 1, 2, 5, 6, 15 page 95.

Nichols, read pp. 290-294, Ex. 1-3, 5, 7, 8 page 294.

Payne, read pp. 291-293, 303-306, Ex. 1-10 even page 294; 1-4, 6, 11-13 pages 305-306.

\* Appendix I

Objectives 4, 5

Nichols, read pp. 295, Ex. 1 a, b, 2 a, c, e pages 295-296.

Objective 6

Vanatta, read pp. 219-222, Exercises

6(a) [for explanation read Nichols 296-300] Ex. 6, 8, 9 solve by comparison.

6(b) Ex. 1, 4, 9, 10 solve by substitution

6(c) 2, 3, 5, 7 solve by addition

Dolciani, read pp. 95-98, Exercises

6(a) 1, 2, 5 pages 99-100 (see Nichols reading for explanation) solve by comparison

RESOURCES (cont')

6(b) 3, 4, 7, 8, 10 pages 99-100 solve by substitution

6(c) 6, 11, 13, 14, 17, pages 99-100 solve by addition

Nichols, read pp. 296-300, Exercises

6(a) 1-7, 9-11 pages 297-298 solve by comparison

6(b) 1-11 odd, 12 page 299 solve by substitution

6(c) 1 a, c, e, g, j, k, l; 2 a, c, e, g, h, j pages 300-301  
solve by addition.

Payne, read pp. 294-301, 309-310, Exercises

6(a) 2, 3, 4, 7, 8 page 298-299 (solve by comparison)

6(b) 2, 4, 5, 6, 8 page 300 (solve by substitution)

6(c) 1-12 even pages 298-299 (solve by addition)

Objective 7 (all problems are required)

\* Nichols, read pp. 296-300, Ex. 3 a, c, e, g, i, k pages 301-302.

Payne, read pp. 294-301, 309-310, ex. 27, 28 pages 306.

Wooton, read pp. 210-212, Ex. 3-5, 9, 12, pages 213-214.

Dolciani, read p. 102, Ex. 6, 11, 12, 13 page 103.

\* required



SELF-EVALUATION 1

Obj.

- 1 I. Match each equation or system of equations with the ordered pair that belongs to its solution set.

- |          |                |    |           |
|----------|----------------|----|-----------|
| _____ 1. | $4y - x = 1$   | A. | $(4,1)$   |
| _____ 2. | $y = 2x + 3$   | B. | $(-5,11)$ |
|          | $y = x - 1$    | C. | $(7,2)$   |
| _____ 3. | $3x - 4y = 8$  | D. | $(-4,-5)$ |
| _____ 4. | $x + y = 6$    |    |           |
|          | $3x + 3y = 18$ |    |           |

- 2 II. Graph the following systems. (Use the graph paper that follows the self-evaluation.) Determine if each system is consistent or inconsistent, dependent or independent.

5.  $x + y = 7$   
 $3x - 2y = 6$

6.  $3x - 2y = 2$   
 $6x - 4y = -8$

7.  $x - 4y = 24$   
 $4x + y = 2$

III. True or False.

- 3 \_\_\_\_\_ 8. The system  $2(x - y) = 5$  and  $4x - 4y = 10$  is dependent and inconsistent.

- \_\_\_\_\_ 9. If a system's solution set is the set of all ordered pairs, then the system is independent.

- \_\_\_\_\_ 10. The solution set of the system  $x = 4$  and  $y = 2$  is the set containing the ordered pair  $(4,2)$ .

- \_\_\_\_\_ 11. Parallel lines are independent and inconsistent.

- \_\_\_\_\_ 12. A system of equations that has only one point in its solution set is dependent and inconsistent.

- 4 \_\_\_\_\_ 13. The system  $\begin{cases} 3 + y = 4 + 2x \\ 2x = y \end{cases}$  is equivalent to  $\begin{cases} y = 2x \\ y - 2x = -1 \end{cases}$

- 5 \_\_\_\_\_ 14.  $3 + y = 4 + 2x$  is in standard form.

SELF-EVALUATION (cont')

- 4 \_\_\_\_\_ 15. The system  $\begin{cases} x + 1 = y + 2 \\ 3(x + 1) = 2(y - 1) \end{cases}$  is equivalent to  $\begin{cases} x - y = 1 \\ x + 1 = y - 1 \end{cases}$
- 5 \_\_\_\_\_ 16. Standard form for  $\frac{3}{x + 2} = \frac{4}{y - 6}$  is  $3y - 18 = 4x + 8$ .

6 IV. Solve the following by the specified method.

COMPARISON

17.  $\begin{cases} 3x + y = 9 \\ x - y = 16 \end{cases}$

18.  $\begin{cases} 7x + 7y = 14 \\ 3y = x + 3 \end{cases}$

SUBSTITUTION

19.  $\begin{cases} 3x + 2y = 6 \\ x - 2y = 4 \end{cases}$

20.  $\begin{cases} y = 7x + 2 \\ 2x - 4y = 5 \end{cases}$

ADDITION

21.  $\begin{cases} 8x + 2y = 1 \\ 2x + 3y = 4 \end{cases}$

22.  $\begin{cases} 2x - 3y = 4 \\ 3x + 2y = 5 \end{cases}$

7 V. Write an equation for each of the following and solve.

23. The sum of two numbers is 59. If the sum of 12 and twice the first number is 4 more than the second number, what are the numbers?

24. A collection of nickels and dimes has a total value of \$2.40 and contains 35 coins. How many of each kind of coin are there in the collection?



SELF-EVALUATION 1 (cont')

25. The perimeter of a rectangle is 44 inches. If its length is decreased twice its width, the result is 17. Find the length and width.

If you have satisfactorily completed your work, take the PROGRESS TEST. Consult your teacher first.

SELF-EVALUATION 1 (cont')

A large grid of graph paper, consisting of approximately 30 columns and 40 rows of small squares. The grid is mostly empty, with some faint, scattered marks and smudges. There are a few small, dark ink-like marks scattered across the grid, including a small scribble in the center and some faint lines.

## SECTION 2

### Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Given a system of a linear equation and a second-degree equation, or a system of two second degree equations:
  - a. graph the equation or equations of the system
  - b. solve the system algebraically
  
9. Given a word problem to be solved by a system of one linear and one second-degree equation, or two second-degree equations:
  - a. write a system of equations which fit the problem.
  - b. solve the resulting system.
  - c. give the solution set of the original problem.
  
10. Graph in  $R \times R$  any of the following systems:
  - a. a system of two linear inequalities.
  - b. a system of one linear inequality and one-second degree inequality.
  - c. a system of two second-degree inequalities.
  
11. Given a graph of a system of inequalities as listed in Objective 10, identify the solution set of that system.

### RESOURCES

#### Objective 8

Vanatta, read pp. 231-237; Exercises

Solve by graphing: 1, 4, 5, 9 page 234 and 1, 4, 6, 7 page 237.

Solve algebraically: 3, 6, 8 page 234 and 2, 3, 7 page 237.

Dolciani, read pp. 320-321; Exercises

Solve by graphing: 3, 5, 7, 9 page 321, and 5, 14, 15 page 325.

Solve algebraically: 1, 4, 6, 9 page 321, and 4, 6, 14, 15 page 325.

Nichols, read pp: 312-322, Exercises 1-6 page 314; 1 a, c, e, g, i, k, m, o, q and 2 a, c, e, g, i pages 316-317; 1-23 odd page 321.

RESOURCES (cont')

Wooton, read pp. 463-469, Exercises  
Solve by graphing: 2, 6, 7, 9, 13 page 464;  
Solve by algebra: 1, 3, 8, 9 pages 466-467.

Pearson, read pp. 687-697, 698-705, Exercises  
Solve by graphing 3, 4, 7, 8, 9 page 699;  
Solve by algebra 1, 4, 7, 8 page 699.

Objective 9 (all problems are required)

Nichols, read pp. 312-322, Ex. 2 a, c, d, page 317; 2 a, c, e page 323.

Dolciani, read pp. 320-321, Ex. 1, 5, 8 page 322; 34 page 325.

Wooton, read pp. 466-469, Ex. 1, 2, 4, 5 pages 467-468.

Objectives 10, 11

\* Vanatta, read pp. 229-230 and 238-239, Ex. 1, 3, 6, 7 page 230; 2, 3, 6, 8 page 239.

Nichols, read pp. 324-327, Ex. 1 a, c, i, k, m, o, q, 2 a, c, 3 d, 4 a, c, 5 a, c, e pages 327-328.

Wooton, read pp. 216-219, Ex. 19, 21, 22 page 465.

Pearson, read pp. 642-644, 706-708, Ex. 5 EOL, 6f, 8 d, e, f page 645;  
1 a, b, d, 2 a, b, d, e page 708.

SELF-EVALUATION 2

Objective

8 I. Solve the following by graphing. (Use the graph paper provided)

$$1. \begin{cases} x^2 + y^2 = 36 \\ x - y = 5 \end{cases}$$

$$2. \begin{cases} y^2 = 4x \\ \frac{x^2}{4} + \frac{y^2}{9} = 1 \end{cases}$$

$$3. \begin{cases} 4x^2 - 9y^2 = 36 \\ \frac{x^2}{9} + \frac{y^2}{4} = 1 \end{cases}$$

$$4. \begin{cases} xy = 8 \\ x + y = 4 \end{cases}$$

8 II. Solve algebraically. Show your work.

$$5. \begin{cases} x^2 + 4y^2 = 32 \\ x - 2y = -8 \end{cases}$$

$$6. \begin{cases} x^2 + y^2 = 16 \\ 2y = x - 10 \end{cases}$$

$$7. \begin{cases} x^2 = 12y \\ x = y + 1 \end{cases}$$

$$8. \begin{cases} \frac{x^2}{4} + \frac{y^2}{9} = 1 \\ \frac{x^2}{4} - \frac{y^2}{9} = 1 \end{cases}$$

9, 11 III. Solve the following by graphing. (Use the graph paper provided).

$$9. \begin{cases} 2x + y < 4 \\ x - 3y > -6 \end{cases}$$

$$10. \begin{cases} x^2 + y^2 \leq 16 \\ 2x - y > 1 \end{cases}$$

$$11. \begin{cases} x^2 + y^2 < 9 \\ \frac{x^2}{4} + \frac{y^2}{9} \geq 1 \end{cases}$$

$$12. \begin{cases} \frac{x^2}{9} - \frac{y^2}{16} < 1 \\ x^2 + y^2 = 49 \end{cases}$$

SELF-EVALUATION 2 (cont')

13.  $x^2 \geq 16y$

$3x - y \leq 2$

14.  $x + y \leq 2$

$\frac{x^2}{9} + \frac{y^2}{25} > 1$

- 9 IV. Write an equation for each of the following and solve. Show your work.
15. The perimeter of a rectangle is 26 inches. Its area is 12 square inches. Find the dimensions of the rectangle.
16. The product of two numbers is 8. The sum of their reciprocals is  $\frac{3}{4}$ . What are the numbers?
17. The area of a right triangle is 24 sq. in. The measure of the hypotenuse is 10 in. Find the measure of the two legs.
18. Find two numbers such that the square of their sum is 20 more than the square of their difference, and the difference of their squares is 24.
19. Find two numbers whose difference is 2 and whose product is 2.

SELF-EVALUATION 2 (Cont')

20. Find two numbers such that the sum of their squares is 170 and the difference of their squares is 72.

If you have satisfactorily completed your work take the LAP TEST.  
Consult your teacher first.

SELF-EVALUATION 2 (cont')

A large grid of graph paper, approximately 30 squares wide and 40 squares high, intended for a self-evaluation. The grid is mostly empty, with a few scattered small marks and faint lines.



APPENDIX 1

Objective

- 3c I. Given the following solutions to the systems of linear equations, determine if each system is inconsistent, consistent, dependent, or independent.
- a.  $(3, 8)$  in common
  - b. no points in common
  - c. all points in common
  - d.  $(-3, 2)$  in common
  - e. equations are parallel
  - f. equations have same graph
- 3d II. Given the following numbers of ordered pairs of solution sets of linear equations, determine if each system is inconsistent, consistent, dependent, or independent.
- a. no points are in the solution set
  - b. one point is in the solution set
  - c. an infinite number of points are in the solution set

ADVANCED STUDY

I. Dolciani, read pp. 100A to 101, ex. 1-6, 12, 13, 14, 15 page 101.

II. Work any one of the following:

1. Nichols, read pp. 302-310, ex. 1, 2 page 306;  
1, 2a,c,d,e,h,j,l,m,o pages 310-311.
2. Payne, read pp. 327-329, 334-336, ex. 1-10 page 329; 21 page 337.
3. Wooton, read pp. 649-652, 657-661, ex. 1-12, 21-26 pages 652-653; 1-8 page 661.

Dolciani, ex. 11-14 page 326.

III. Wooton, read pp. 223-226, ex. 1-20 oral page 226.

IV. Vanatta, read pp. 224-227, ex. 1-4, 6 pages 227-228.

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