#### DOCUMENT RESUME

ED 068 573

TM 002 082

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TITLE An Examination of Differential Item Response

Characteristics for Six ATGSB Candidate Groups.

INSTITUTION Educational Testing Service, Princeton, N.J. REPORT NO PR-72-4

PUB DATE Aug 72

NOTE 34p.; Revised edition

EDRS PRICE MF-\$0.65 HC-\$3.29

DESCRIPTORS Caucasian Race; Comparative Analysis; Degree

Requirements; \*Educational Research; \*Graduate Study; Higher Education; \*Item Analysis; Masters Degrees;

Negroes; \*Racial Factors; \*Sex Differences;

Statistical Analysis

IDENTIFIERS \*Admission Test for Graduate Study in Business

#### **ABSTRACT**

Differences in test response characteristics between black and white, male and female groups who took the Admission Test for Graduate Study in Business (ATGSB) in February, 1971, were examined. Three studies were covered: 1) a study of the interactions between the difficulty level of test items and group membership for the above mentioned groups, 2) a comparison of the mean criterion scores for those candidates who omit items, and 3) a comparison of the response randomness of the subgroups involved. Six groups of examinees were studied: black females, black males, fee-free females, fee-free males, white females, and white males. Significant interactions were found for every subsection of the ATGSB. Other conclusions include: more significant results from the race within female comparisons than within male; neither sex was favored over the other in a uniform fashion; neither race was favored uniformly over the other; generally, the white group exhibited the lowest mean criterion scores for those who omit particular items, and the fee-free group exhibited the highest; and there appears to be more randomness of response in the fee-free group than in either of the regular center groups. (LH)



# AN EXAMINATION OF DIFFERENTIAL ITEM RESPONSE CHARACTERISTICS FOR SIX ATGSB CANDIDATE GROUPS

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August 1972

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## AN EXAMINATION OF DIFFERENTIAL ITEM RESPONSE CHARACTERISTICS FOR SIX ATGSB CANDIDATE GROUPS

The purpose of this study was to examine differences in test response characteristics between black and white, male and female groups who took the Admission Test for Graduate Study in Business (ATGSB) during February, 1971. More specifically, this report covers three studies: (1) a study of the interactions between the difficulty level of test items and group membership for the above mentioned groups, (2) a comparison of the mean criterion scores for those candiates who omit items, and (3) a comparison of the response randomness for the subgroups involved. This study considers six groups of examinees: black females, black males, fee-free females, fee-free males, white females, and white males. The nature of these examinee groups will be discussed later.

### Characteristics of the ATGSB

The test used in this study was the regularly scheduled ATGSB administered during February, 1971. The test has five separately timed sections. Two of these sections are identical in content, format, and difficulty. These two sections require the examinee to read three excerpts from the current business literature. After reading these three passages, the examinee is asked a series of questions about the content of each of the passages. In each case the examinee cannot return to the passages for reference. These sections are termed Reading Recall I and II, respectively.

A second verbal section contains discrete verbal items consisting of antonyms, analogies, and sentence completion type items in that order. For the purpose of the present study, this section was considered as though it contained three separate subsections, each subsection characterized by its item type.

A third section consists of rather traditional mathematical type items: questions involving graphs and charts along with a few questions which require knowledge of simple algebra. A final section called "Data Sufficiency" presents a mathematical problem and provides data in the form of a series of statements. The examinee is then required to judge whether these data are sufficient to solve the problem stated.

In summary, for research purposes this study separated the ATGSB into seven separate tests: Reading Recall I, Reading Recall II, Antonyms, Analogies, Sentence Completion, Mathematics, and Data Sufficiency.

#### The Student Population

The February administration was conducted in two types of settings. One setting was the regular test center where examinees paid a fee to take the test, the other was a free testing given at locations termed fee-free centers. These fee-free centers were predominantly black colleges located primarily in the south. Although the majority of examinees in the fee-free centers were black, other minority and a few white examinees took the test in the fee-free centers. In this study only black fee-free candidates were sampled from the fee-free centers.



In the regular center group, an overwhelming majority of examinees were taking the ATGSB for the first time. The modal age for each candidate group defined by sex and race (black and white) was 22, whereas the median ages were for black and white females being 23, for black males, 26, and for white males, 24. Similar descriptive statistics were not available for the feefree group.

The geographic distribution of students in each candidate group, classified by the undergraduate college attended, is given in Table 1. The regional classification used is that of the Census Bureau. Chi-square statistics were calculated using the proportions given in the 1970 census for the total population group as the expected figure. Significance at the .05 level was obtained, and it was concluded that relatively more black students in the candidate group attend institutions in the west than would be expected from the census data.

Table 1

Percentage of Candidate Groups Attending Undergraduate Institutions in Four Regions and Foreign Areas (Census Bureau Classification)

Candidate Group	West	North Central	South	Northeast	Foreign
Black - Female	15	17	53 <sup>*</sup>	16	
White - Female	18	28	21	33	
Black - Male	15	27	37 <sup>***</sup>	21	1
White - Male	16	31	22	30	2

<sup>\*71%</sup> of the Black - Females attending institutions in the south attended predominantly Black institutions.

<sup>\*\*\*74%</sup> of the Black - Males attending institutions in the south attended predominantly Black institutions.

The geographic location of the center where the ATGSB was administered was similar to the candidates' undergraduate institution. The percentage of the examinee group taking the test in each census region is reported in Table 2.

Table 2

Percentage of Candidate Groups Taking the ATGSB in Centers in Four Regions and Foreign Areas

Candidate Group	<u>West</u>	North Central	South	Northeast	Foreign
Black - Female	16	22	7171	18	
White - Female	21	24	20	33	2
Black - Male	20	29	26	24	2
White - Male	19	29	22	26	5

It should be noted that these percentages differ only slightly from those reported for the attending colleges. The difference is due in part to relocation by those out of college and to general student mobility.

In this study a total of 2930 candidates were sampled for each research question. The individual group sample sizes were: fee-free males, 485; fee-free females, 370; regular center black males, 630; regular center black females, 150; white males, 995; and white females, 300. Each of the latter samples were drawn randomly from the total examinee population in the regular centers, whereas all fee-free candidates were included in the study.

These samples differed significantly in terms of mean scores. It was decided not to match the samples in terms of total score though, as that would make the group labels misleading, i.e., a low scoring subgroup would be compared with another complete group. The use of a low scoring subgroup would require a redefinition of the groupings that would be contrary to the aims of this study.



Study One: Statistically Defined Test Bias

The problem of defining what is meant by test bias has received considerable attention by Cardall and Coffman (1964), Cleary and Hilton (1968), and Potthoff (1966) among others. Basically, two approaches have been taken: with or without a criterion variable present.

The case of defining test bias with a criterion is most straightforward and logically most appealing. When a criterion variable is present, the definition of test bias simply says that a test is not biased if individuals from different groups who have the same test scores have the same expected criterion scores. Some further difficulties exist if the test is not perfectly valid, (see, for example, Thorndike, 1971) but most researchers have continued to pursue the problem by considering homogeneity of regressions. In some cases it is not feasible to use a criterion variable to investigate test bias. For example, suitable criteria are difficult to define, especially when the tests are admissions tests and the criteria are variables that reflect some notion of successful performance. Assuming an appropriate criterion variable can be identified, it is sometimes a very expensive proposition to collect criterion data, and the collection often renders such research studies to be not feasible due to high project costs.

It naturally follows that a number of research studies have tried to attack test bias questions without resorting to collecting any criterion variables. This is logically a more difficult task because one immediately encounters a difficulty in trying to define

test bias. Although there seems to be no generally suitable means to define test bias in the absence of a criterion, several attempts have been made to answer such questions by examining a concept which seems closely related — that of item—group interaction (Potthoff 1966).

The problem of defining item-group interaction is, in itself, difficult. One can say that there is not bias present in a test if the difference in p values (the proportion who answer an item correctly) is identical for all items in the test for any two groups. If multi-variate statistical tests are made of this hypothesis, difficulties can arise if the variance matrices for each group are not homogeneous or if the p values are not close to 1/2. Potthoff (1966) has given a number of different techniques for handling such situations.

#### Method for Study One

In this study, a method of estimating bias was needed that was both inexpensive and readily available from the standard item analysis procedures now in use. The standard item analysis calculates p values for each item in the test for each group under study. Since p values can vary from only 0 to 1, and one often concludes the existence of bias when items are taken with p values close to one of these extremes, a transformation of the p values, commonly used by ETS, termed delta, was used as the unit being studied. Delta is defined as the value  $\Delta$  satisfying the equation,



$$p = \frac{1}{4\sqrt{2\pi}} \int_{\Lambda}^{\infty} e^{-1/2} \left(\frac{x-13}{4}\right)^{2} du$$

and is calculated for every item in each item analysis. The delta scale is approximately normal with a mean of 13 and a standard deviation of 4. Thus, a p value of .5 is associated with a delta of 13.

The definition of test bias used in this study was that of itemgroup interaction. It was hypothesized that in the absence of an
item-group interaction, the differences in item deltas for these
two groups would be distributed as a normal distribution with some
unknown mean and some unknown variance. If the differences did not
form a normal distribution, bias, as defined by item-group
interaction, would be concluded.

The method used to determine whether the differences in deltas were normally distributed was to plot these differences on normal probability paper and estimate whether these plots formed a straight line as would be found had there been no item-group interaction; i.e., item delta differences constant subject only to an error term associated with items. Since there were six groups of examinees from which item delta differences were to be calculated, only five group pairs were independent. That is, the item delta differences for any pair of groups could be obtained by knowing the differences for just five independent pairings. The problem then was to select the independent pairings. Since racial bias was considered to be of most importance, it was decided to examine

racial bias within sex, and then make an overall comparison between sexes. Therefore, the group pairings were: (1) white male vs. regular center black male, (2) white male vs. fee-free male, (3) white female vs. regular center black female, (4) white female vs. fee-free female, and (5) male (pooled over race) vs. female (pooled over race). Each of these comparisons was independent.

The general method of determining whether the points fall on a straight line is a generalization of the Kolomogorov-Smirnov technique for testing for normality. The generalization involves estimating the hypothesized normal distribution parameters with the sample parameters. The hypothetical normal distribution is plotted as a straight line, and confidence bands are drawn for a given significance level and number of items. If any point falls outside the band, the normality hypothesis is rejected. The significance level used in this study is the .05 level. Items (points) falling outside the band are noted.

In addition to the plots, repeated measures analysis of variance was run for each subtest with race and sex as factors and items as repeated measures. This analysis was not performed to test for the appropriate item-group interaction effects since group statistics were being used and no appropriate error term could be used in a significance test. The analysis of variance was performed to provide an overall picture of the proportion of the sum of squares that accounted for each line in the analysis of variance table, which would provide a lead as to the magnitude of the item-group interaction with respect to the other factors in the analysis.

#### Results of Study One

The results of Study One could be divided into seven subsections, each subsection dealing with a specific subtest. The repeated measures analysis of variance results are presented in the last part of this section. The following presentations indicate where significant non-normality has been concluded, and try to provide some help in identifying items that contribute to the item-group interaction by noting items that fall outside the confidence band in the analysis. It may be somewhat misleading that the noted items are items where delta differences differ significantly from the normal distribution specified by the sample estimates. If the differences deviate greatly from a normal distribution, the sample specifications may also form a less than desirable criterion. This possibility should be kept in mind in reading the results. A selection of the plots appears in the appendix.

Items which fell outside the confidence band for one or more group comparisons are listed in Table 3. The column labeled Frequency denotes the number of times (5 is the maximum) that the particular item was found to lie outside the confidence band—a degree of bias figure. The Groups column indicates the respective group pairs where this deviation was found. The final column indicates the direction of the deviation. "Difference Less" means that the difference between the two groups should be less if the item—group interaction were eliminated, while "Difference More" implies the opposite. In a sense, the last two columns

indicate the group favored by the interaction. Difference Less indications show interactions which favor the white group, while Difference More indications show interactions which favor the regular center black or fee-free group.

Table 3

Deviate Items Found in Reading Recall I

Item No.	Frequency	Groups	Difference Less	Difference More
1	2	2-4, 2-6		2
3	1	<b>2-</b> 4	1	_
5	1	M-F		1
6	1	2~4	1	
7	1 2	2 <b>-</b> 6	1	
8		2-6, 2-4	1	1
9	1	2 <b>-</b> 6	1	
10	1	2 <b>-</b> 6	1	•
11	1	2 <b>-</b> 6	1	
14	2	2-6, M-F	2	
15	1	1 <b>-</b> 5	1	
17	1	1 <b>-</b> 5	1	
19	1	2-6		1
22	1	2-6, M-F		2
23	1	2 <b>-</b> 6	1	-
25	1	2-4, M-F	ī	1
27	1	2-6	_	ī
29	2	1-5, 2-6	2	<b>-</b>
30	2	1-5, 2-6	2	

Codes are used in the Groups column for convenience in presentation. The codes are designated as follows: 1 = white male; 2 = white female; 3 = regular center black male; 4 = regular center black female; 5 = fee-free male; 6 = fee-free female; M = male (pooled data); F = female (pooled data). This notation is used throughout this section.

As can be seen by examining Table 3, 19 of the 30 items showed differences that fell outside the confidence bands at least once. Half of these items involved the white female vs. fee-free female comparison (13 of 26). The items noted were found to be deviates in only one or two of the five comparisons. Four of the 26 noted item differences were attributable to the white female vs. regular center black female comparison. The same was true for both the white male vs. fee-free male comparisons and the male vs. female comparisons. No evidence of racial bias was found in the white male vs. regular center black male comparisons.

In the Reading Recall II section, interactions similar to that found previously in Reading Recall I were found. The results are summarized in Table 4. As before, most of the interactions involve the female comparisons (18 of 20 racial comparisons), and a substantial number involve fee-free candidates (11 of 21). Six of the items had large deviations in 2 of the 5 comparisons. Item 23 tend distinctly to favor the white group. An item by sex interaction was also found to be significant in the pooled male vs. pooled female comparison. There were no interactions found for the white male vs. fee-free male comparison.

The Antonym section (Table 5) shows proportionately more deviate items than do either of the Reading Recall sections. Only 2 of 14 items were not classified as deviates. For this section, deviate items within the male group were much more frequent than in the past two sections. Only two items were classified as

Table 4

Deviate Items Found in Reading Recall II

Item No.	Frequency	Groups	Difference Less	Difference More
1	2	1-3, 2-6	1	1
2	1	2 <b>-</b> 6		1
4	1	2 <b>-</b> 6	1	
4 5 8	2	2-6, M-F	2	
	1	2-4	1	
10	1 .	2-4		. 1
11	1	2 <b>-</b> 6	1	
16	1	2-6		1
18	2	2 <b>-</b> 6, 2-4	2	_
20	2	2-6, M-F		2
23	2	2-4, 2-6	2	_
24	2	1-3, 2-6	ī	1
26	1	2-4	ī	
27	1	2-4	ī	
28	2	2-4, 2 <b>-</b> 6	2	
30	1	M-F	_	1

Table 5

Deviate Items Found in Antonyms

Item No.	Frequency	Groups	Difference Less	Difference More
1	3	1-3, 1-5, M-F	2	1
2	2	1-5, M-F	1	1
3	3	1-3, 1-5, M-F	1	2
4	1	M-F		้ำ
6	2	1-5, M-F	. 2	-
7	2	1-3, 1-5	2	
8	1	M-F	_	1
9	2	1-5, M-F	٦	7
10	ī	1-5	i	_
11	ī	ī <b>-</b> 5	_	7
12	ī	2-6		1
13	ī	2 <b>-</b> 6	1	. 1
			<del></del>	



deviates within the female group, and these indicators both involved the fee-free group. Also, more deviate items were found in the male-female comparions than in previous subtests. Actually, the major source of the item by sex interaction in this section involved item 2, and that item heavily favored the female group.

Table 6 shows that relatively few deviate items were noted for the Analogies section. Both deviate items involved in the male-female comparison indicate a favoring of the male group, while the three items involved in the race interaction favor the regular center black and fee-free candidates. No evidence of item-group was found in the white male vs. fee-free male comparison.

Table 6

Deviate Items Found in Analogies

Item No.	Frequency	Groups	Difference Less	Differency <u>More</u>
1	2	M-F, 2-4	1	1
2	1	M-F	1	
10	1	1-3		1
13	1	2-6		1

Only one comparison showed significant interaction for the Sentence Completion section. That comparison was the white female vs. fee-free female, as can be seen by examining Table 7. Items 3 and 8 tended to favor white females, while items 4 and 12 tended to favor fee-free females. All other comparisons showed no significant deviation from the normality hypothesis.

Table 7

Deviate Items Found in Sentence Completion

Item No.	Frequency	Groups	Difference Less	Difference <u>More</u>
3	1	2-6	1	
4	1	2 <b>-</b> 6	_	1
8	1	2 <b>-</b> 6	1	
12	1	2-6		1

The results for the Data Sufficiency section appear in Table 8. As with the previous two sections, the extent of the interaction present in the section is less than the first three sections. Only four deviate items were found. Each of these items showed a deviation favoring the white group within sex. There were no significant results for either the white male vs. regular center black male or the male vs. female comparisons.

Table 8

Deviate Items Found in Data Sufficiency

Item No.	Frequency	<u>Groups</u>	Difference Less	Difference More
7	1	2-4	1	
9	1	2-6	1	
10	1	2-4	± 1	
12	<u>.</u> 1	1 <b>-</b> 5	1	

By far the most noted extent of item-group interaction occurred in the Mathematics section (Table 9). Of the 54 items in the section, 33 were identified as deviate in one or more of the comparisons. A large proportion of the deviations favors the lower



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scoring group, as indicated by its relatively frequent occurrence in the Difference Less column. In each case where the male vs. female difference was significant, the difference favored the female group. In the within sex comparisons for racial group differences, 30 out of 40 noted items involved the fee-free group, 9 in the male group, 21 in the female group. Item 45 was the only item noted in each group comparison as deviating from the hypothesized distribution.

Table 9

Deviate Items found in Mathematics

	20		and of CS	
Item No.	Frequency	Groups	Difference Less	Difference <u>More</u>
1	1	2-4		7
2	ī	2-6		1 1
3	ī	1-5		i
J,	ī	2-4	-	Ţ
7	i	2-4	1	
1 2 3 4 6 8 9 12	i		1	
0	2	2-6	1	
7	2	2-6, M-F		2
75	Ţ	2-6		1
15	Ţ	2-6	1	
19	3	1-3, 1-5, 2-6	2	1
20	1	1-5	1	
22	1	M-F		1
24	1	1-5	l	_
25	1	2-6	ī	
26	1	1-5	ī	
25 26 28	1 3 1 1 1 1	2-6	ī	
32	1	M-F	-	٦
33	2	1-5, 2-6	1	1 1
37	ī	2-6	± ;	ĺ
38	้ำ	2-6		1
40	3	1-5, 2-6, M-F	1	
41	1		1	2 1 5
45	<u> </u>	1-3		1
45	2	1-3, 2-4, 1-5, 2-6, M-F	_	5
ДO 1 2	1	2-6	1	
47	2	1-3, 2-6		2
48	2	2-6, M-F	1	2 1 2
49	2	1-3, M-F		2
50	1	2-6	1	
50 51 52 53	2 1 3 1 5 1 2 2 1 1 2 3 3	2-6		1
52	2	2-6, M-F		1 2 3 3
53	3	2-4, 2 <b>-</b> 6, M-F		- 3
54	3	1-3, 1-5, 2-6		3
	-	-,, <del>-</del> -		)

In summarizing the results of Study One one clear fact stands out. If the notion of item-group interaction is accepted as a definition of test bias, then each section of the ATGSB is biased in some way. Of the 35 comparisons made (7 subsections x 5 comparisons per subsection), 25 were found to be significant.

Items were noted whose item delta difference fell outside the confidence bands for the set of item delta differences. These were items that differed from the hypothetical normal distribution where the parameters for that distribution were taken to be the sample estimates. They were noted merely to provide a clue as to the nature of the deviation present.

Most of the indications of race by item interaction within sex seemed to occur in the female groupings (12 of the 20 significant within sex results) and in the fee-free comparisons (11 of 20). Generally, the interaction present favored no one particular race. Four of the 7 male vs. female comparisons were significant. Neither men nor women seemed to be favored over the other.

The item deltas for the six groups were also analyzed by analysis of variance. The structure of the analysis was conceived to be a 2 x 3 factorial (sex and race as factors) with repeated measures (items). Since there were no error terms available for a significance test, only the percentage of the total sum of squares attributable to the various factors in the analysis were given. These were given in order to display the importance of each factor



in relation to the others; e.g., determining whether a sex interaction (sex x item interaction) or a race interaction (race x item interaction) seems more eminent. The percentages appear in Table 10.

The differences in means among the racial groupings (white, black regular center, and fee-free) and the variability in item deltas is more apparent from the high percentages of sums of squares attributable to race and items respectively. It appears that race by item interaction is contributing more than the sex by item interaction in each of the subtests. The degree of sex x race interaction and sex x item interaction appears minimal in comparison to the remaining factors. The three factor interaction also appears to be minimal.

Table 10

Percentage of Total Sum of Squares

Attributable to Factors in ANOVA

#### Factor

	Sex	Race	Sex x Race	Items	Sex x Items	Race x Items	Sex x Race x Items
Reading Recall I	1.59	25.79	.62	64.62	.90	5.75	•73
Reading Recall II	•53	32.45	.71	56.52	.80	7.87	1.12
Antonyms	.02	16.62	•54	75.24	1.35	5.41	.82
Analogies	.02	17.96	•92	75.95	.66	4.05	.45
Sentence Completion	.12	23.29	.68	72.88	•33	1.78	.91
Mathematics	2.07	30.14	.01	61.33	•91	4.47	1.07
Data Sufficiency	•47	21.98	.15	71.54	•52	4.43	•92

Study Two: Omit Behavior

In a study of culturally deprived youth, Flaugher and Pike (1970) determined that, because of the inappropriate difficulty level of the test being studied, higher scoring students in a low scoring group omitted large numbers of items, the opposite of the pattern in a middle-scoring group. This study attempted to determine whether the mean criterion scores (section scores) for those who omit differed among the three groups: white, regular black, and fee-free black. In each case, only within-sex differences were considered.

#### <u>Method</u>

The standard item analysis program calculates an index of the average ability level, mean criterion score, for the group of examinees choosing each option, including omitting. The mean criterion score is on a scale with a mean of 13.0 and a standard deviation of 4.0, corresponding to the delta scale for item difficulty value.

For example, if the criterion used was the score on the total test, the mean score of the total sample would be assigned a value of 13.0. If the average score for the group choosing a particular option was above the sample mean, the group's mean criterion score would be greater than 13.0; if their average was below the sample average, it would be less than 13.0.

If we consider all possible ranking patterns constructed in such a way that the first digit indicates the ranking, in terms of



mean criterion score, of the white group, the second digit indicating the ranking of the regular center black group, and the third digit indicating the fee-free group ranking, there are six possible patterns.

For example in the first pattern, denoted 1, 2, 3, the criterion score for the white group was lowest and for the feefree group, it was the highest. Under null conditions of a random pattern in omit mean criterion scores, approximately 1/6 of the items of any given section should fall in each category defined by these six orderings. If evidence can be provided to show that this is not the case, we can conclude that there is some systematic difference in mean criterion scores for the three groups under study.

#### Results

In this study the criterion scores were ranked from low to high; i.e., the lowest mean criterion score received a rank of one. The number of items in each ranking pattern for both males and females appear in Table 11. In cases where ties were found, the ties were broken by using random digits from a table of random numbers.

From an examination of the frequency of occurrence for the various patterns in Table 11 it appears quite obvious that these patterns are occurring in a nonrandom fashion. For example, the fee-free group almost never has the lowest mean criterion score, as indicated by a one in the third digit of the patterns. On the other hand, the white group mean criterion score was the lowest among the three groups in the majority of cases. There



appears to be some doubt whether the regular black or white group has the lowest mean criterion score for the analogy and sentence completion type items. The frequencies of pattern occurrence is the same for both males and females, with women tending to show a slightly wider distribution of patterns.

Table 11
Frequency of Ranking Pattern of
Omit Mean Criterion Scores

•			Patte	ern		
Test	123	132	213	231	<u>312</u>	<u>321</u>
Reading Recall I Male Female	16 14	6 6	5 5	0	1 3	1 0
Reading Recall II Male Female	18 10	4 13	6 2	1 1	0 3	. 0
Antonyms Male Female	12 9	0 3	0	0	0	0
Analogies Male Female	6	1	4 1	0 0	2	0
Sentence Completion Male Female	<u>4</u>	0 1	6 3	0	1	1 2
Mathematics Male Female	45 29	2 13	5 5	0 4	1 2	0 C
Data Sufficiency Male Female	10 4	1 7	2 2	0	1 0	0· 1

In addition to obtaining counts of the ranking pattern of mean criterion scores for those who omit, three-way analyses of variance were performed for each type of test with sex, race, and items as factors and the respective criterion scores as observations. Although the items were correlated to an extent, it was believed that if any race effects were found, some strength could be added to the above argument. Using this type of analysis, the group averages were tested for equality over the other two factors.

In each case a significant race effect was found, and a sex x race interaction was found for analogies and antonyms. The third order interaction was used as the error term and the tabled results appear in Appendix II. These results should be taken with extreme caution and are presented only as supplementary evidence (weak as it is) of rejecting the null hypothesis of random differences in mean criterion score for those who omit.

Study Three: Randomness of Response

When exploring the differences in performance on a multiplechoice test for two groups of students, a third possible source
of difference is varying test-taking strategies. Flaugher and
Pike (1970) reported one such study, which investigated the
randomness of response that was evident in a group of lowscoring high school students. Through the use of an index
of randomness in responding (Pike & Flaugher, 1970) they found
that less randomness was characteristic of a particular item type
within the test, that of the verbal analogy item. The present
study was conducted to replicate and extend these particular
findings on a new population.

#### Method

In many kinds of paper and pencil tests an examinee is presented a list of items each followed by the possible answers, or alternatives. The examinee may use a number of strategies in answering an item. He may know the answer (or think he knows the answer) and mark that alternative, he might be able to eliminate some of the alternatives and guess among the remaining alternatives, he may guess completely, or he may omit the item. The ability to eliminate some of the alternatives as not being plausible and then responding may be referred to as the amount of nonrandomness present in the responses, the more alternatives eliminated, the less the randomness in that response.



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In order to measure the amount of randomness present, the proportion of examinees responding to each of the distractors was studied. If the distribution of responses to the various distractors was flat, that is, evenly distributed, then there was evidence of complete guessing on the part of the group of examinees who did not choose the correct response. To the extent that the distribution differed from this, a non-random guessing pattern was assumed. In order to measure the flatness of a group of probabilities for a finite set of categories, Shannon (1949) developed a quantity denoted H , termed entropy, where:

$$H = \sum_{i=1}^{r} p_{1} \log p_{i}$$

The concept of entropy was first developed in thermo-dynamics but has become the subject of information theory and has been applied in the mathematical theory of communication by Shannon and Weaver (1949).

For the purpose of this study the definition of entropy used was

$$H = (P - \sum_{i=1}^{r} p_i \log p_i/P)/\log r ,$$

where P is the proportion of examinees answering the item incorrectly, p<sub>i</sub> is the proportion of examinees who chose distractor i , and r is the number of distractors. This form was used so that when all distractors were chosen with equal frequency,



the entropy value was one, and when all examinees who missed the item responded with the same distractor, the entropy value was zero. Results

The item entropy was calculated for each item on each subtest for each of the six groups under study. The mean of these item entropies was then calculated for each of the six groups. These means appear in Table 12. The racial and sex differences are significant at the .05 level for each section in question, the analyses appearing in Appendix III. The patterns seem similar for each section and both races within those sections. That is, the entropy for the regular center black students tends to be slightly lower than that for the white group, while the entropy for the fee-free group tends to be relatively high for each section. Basically, this says that the randomness is choosing distractors is greatest in the fee-free group. Apparently all distractors appear equally likely to fee-free candidates who do not know the correct answer. The randomness for the remaining two groups appears to be less. In these groups, students tend to discriminate more among the distractors, a fact which suggests that they are able to eliminate some alternatives as being implausible or that they are especially attracted to a "strong" distractor.

In examining sex differences within race, there appears to be more randomness in the female group than in the male groups. This conclusion holds for 19 of the 21 within-race comparisons made over the 7 sections.



Table 12

## Mean Group Values of Entropy

Test	White	Regular Center Black	Fee-Free
Reading Recall I Male Female	• 795 • 843	.798 .850	.869 .890
Reading Recall II Male Female	• 753 •813	.727 .790	.820 .834
Antonyms Male Female	.863 .886	.843 .883	.920 .930
Analogies Male Female	.851 .879	.825 .859	.900 .893
Sentence Completion Male Female	.876 .923	.855 .883	.915 .903
Mathematics Male Female	.753 .813	•727 •790	.820 .834
Data Sufficiency Male Female	.675 .754	.651 .724	.803 .800

#### Summary and Conclusions

- Significant interactions were found for every subsection of the ATGSB.
- More significant results were obtained from the race within female comparisons than from the race within male comparisons.
- 3. In comparing male and female samples, pooled over the different racial groupings, neither sex was favored over the other in a uniform fashion.
- 4. In the race within sex comparisons, neither race was favored uniformly over the other.
- 5. In general, the white group exhibited the lowest mean criterion scores for those who omit particular items, and the fee-free group exhibited the highest mean criterion scores for those who omit particular items. This pattern appears less frequently in the case of analogies and sentence completion, with the results being similar for both male and female groups.
- 6. There appears to be more randomness of response in the feefree group than in either of the regular center groups.



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Appendix

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