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ABSTRACT

This workbook for elementary teachers presents basic definitions and examples in the areas of set theory, numeration systems, fundamental operations and geometry. Text organization does not use programming frames. Answers to examples are included in a separate section. (DT)

A PROGRAMMED INTRODUCTION TO MODERN MATHEMATICS

for

ELEMENTARY SCHOOL TEACHERS

First Printing September, 1972

Richard E. Werstler, Author

Richard Immers, Editor

ADRIAN COLLEGE
ADRIAN, MICHIGAN

FORWARD

The release of A Programmed Introduction to Modern Mathematics for Elementary School Teachers marks the third edition of an original manuscript developed in 1964 by the author. Subsequently, detailed new formats were designed by Miss Carol Seager in 1968 and Mr. Richard Immers in 1972 as part of research projects through Advanced Studies in Education. A self programmed text format was used in the 1968 edition while in the 1972 edition the program more closely resembles the traditional problem solving approach with answers to accompany the text.

In addition, new sections of study were developed by Mr. Immers. The author is indebted to both editors for their insight and professional competence in relating Elementary School Mathematics teaching and learning processes to previous educational experiences of pre-service students who plan to become teachers.

REW

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SET THEORY

DEFINING SETS

A group of objects or symbols is called a set. The concept of set is synonymous with the words collection, group, class, or aggregate.

1. The windows in a house make up a _____ of windows.
2. The houses on one city block form a _____ of houses.

A set may further be defined as a collection of objects or symbols in which it is possible to determine whether any given object or symbol does or does not belong to the set. Each object or symbol in a set is called an element or a member of that set. The symbol ϵ means is an element or member of. In Chart I below, three different sets are represented.

CHART I

SET A	SET B	SET C
apple	hammer	cake
cupcake	pliers	glass
fork	ruler	ruler
glass	saw	spoon
sandwich	screwdriver	teacup

3. The fork is an element of Set _____ ?
4. The teacup is an element of Set _____ ?
5. The ruler is an element of Sets _____ and _____ ?
6. Which element of Set A is also an element of Set C ?
7. The knife is an element of Set _____ ?

SET NOTATION

Generally capital letters (A,B,C) are used to denote specific sets and lower case letters (a,b,c) are used to denote the elements or members in a set. Braces { } are used to enclose sets. Commas are used to separate elements within braces. The order in which the elements are listed is unimportant unless one is considering ordered sets. For example, Set A, the set of vowels, would be denoted:

$$A = \{a,e,i,o,u\}$$

Denote the following sets:

8. Set X is the set of letters of the alphabet used to spell "*and*."
9. Set Y is the set of counting numbers 1 to 5.
10. Set Z is the set of letters of the alphabet used to spell "*was*."

EMPTY SET

A set containing no elements or members is called an empty set or the null set. The empty set is denoted by the symbols Ø or { } (nothing between the braces).

11. Set P is the set of all whales in Lake Michigan. Set P = ______?
12. Indicate which of the following are empty sets:

    (A) The set of even counting numbers.

    (B) The set of women presidents of the U.S.

    (C) The set of odd counting numbers.

    (D) The set of all cats that fly.

    (E) The set of all men ten feet tall.

SUBSETS

Set A is said to be a subset of Set B if and only if each element of Set A is an element of Set B. This is denoted by $A \subseteq B$. The empty set (Ø) is also a subset of every set. The maximum number of subset combinations in any set is equal to 2 *times the power* determined by the number of elements in the set. The number of subsets of a given set is found by raising 2 to a power equal to the number of elements in the set.

Elements	# of Subsets	Mathematical Formula
0	1	2^0
1	2	2^1
2	4	2^2

For example, if Set T = {hat, coat}, the maximum possible number of subset combinations would be 2^2 or 4. The subsets of Set T are:

{ hat}
{ coat }
{ hat, coat }
Ø

13. List all the possible subsets of {1, 3, 5}.
14. List ten subsets of {2, 6, 10, 14}.
15. List all the possible subsets of {a, b}.

PROPER SUBSETS

Set A is said to be a proper subset of Set B if and only if each element of Set A is an element of Set B and there is at least one element of Set B that is not an element of Set A. This is denoted by $A \subset B$.

For example, the set {a, b, c} has seven proper subsets:

{a}
{b}
{c}
{a, b}
{a, c}
{b, c}
Ø

16. List all the possible proper subsets of {1, 2}.

17. List all the possible proper subsets of {m, n, o, p}.

FINITE SETS

A finite set is a set containing either no elements or members or a definite number of elements or members. For example, {#, $, &} is a finite set with three elements.

18. List three finite sets.

INFINITE SETS

An infinite set is a set containing an unlimited number of elements. The set of all even counting numbers is an infinite set and may be denoted {2, 4, 6, ...}. The ellipsis means "*and so on*" and is used to indicate omissions. Indicate whether the following sets are finite or infinite:

19. The population of Michigan.

20. The stars in the universe.

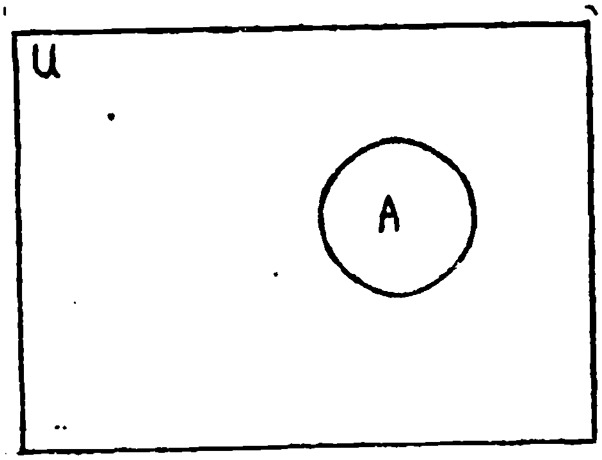
21. The seconds in a day.

22. The odd counting numbers.

23. All fractions with even denominators.

UNIVERSAL SET

The universal set or universe, denoted as Set U, is an all-inclusive set containing all the elements under discussion. The set may change from one discussion to another. The universal set may be illustrated by use of the Venn diagram. The universal set is represented by a rectangle and subsets by smaller circles. For example:



In the above example illustrating the universal set, Set U represents the set of all animals in the world and Subset A represents the set of all cats and dogs.

COMPLEMENT

The complement of Set A, denoted $\bar{A}$, is the set of elements in the universal set that are not elements of Set A. For example, if Set U = {1, 2, 3, 4, 5} and Set A = {1, 3, 5}, then $\bar{A}$ = {2, 4}.

Let U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. State the complement of each of the following sets:

24. A = {1, 3, 5, 7, 9}

25. B = {0, 2, 4, 6, 8, 10}

26. C = { }

27. D = {1, 5, 10}

28. E = {4, 8}

INTERSECTION

The set intersection of Set A and Set B is the set consisting of all the elements which belong to both Set A and Set B and is denoted by A ∩ B. For example, if Set A = {a, b, c, d, e} and Set B = {b, d}, then A∩B = {b, d}.

DISJOINT SETS

Set A and Set B are said to be disjoint if A∩B = ∅. For example, if A = {1, 2, 3, 4} and B = {5}, then A∩B = ∅ and Set's A and B are said to be disjoint.

Compute the intersection of the following sets:

29. A = {1, 2, 3}, B = {3, 4, 5}

30. R = {0}, S = {0, 2, 4}

31. G = {1, 3, 5, ...}, H = {2, 4, 6, ...}

32. L = {5, 10, 15, 20}, M = {1, 5, 10}

UNION

The union of Set A and Set B is the set consisting of all the elements in Set A or in Set B or in both Set's A and B and is denoted A ∪ B. For example, if Set A = {1, 2, 3, 4} and Set B = {3, 4, 5, 6}, then A ∪ B = {1, 2, 3, 4, 5, 6}.

Find the union of the following sets:

33. D = {0, 5, 10}, E = {10, 15, 20}

34. X = {a, e}, Y = {a, b, c, d, e}

35. P = {#, $, +, @}, Q = {*, !, +, &}

36. C = {1, 3, 5}, D = {1, 2, 3, ...}

NUMERATION SYSTEMS

HINDU-ARABIC SYSTEM

The Hindu-Arabic numeration system consists of ten basic number symbols (0,1,2,3,4,5,6,7,8,9) called digits. This system uses as a base the set whose number is ten and is thus called the decimal system or the base 10 number system. Each digit in a numeral has a place value determined by its position. These place values are consecutive powers of 10. The place value of any digit (except the ones or units digit) is equal to 10 times the place value of the digit immediately to the right. For example, the digits of the Hindu-Arabic numeral 1,346 have the following place values:

1	3	4	6
1000's	100's	10's	1's
10^3	10^2	10^1	10^0

Write the Hindu-Arabic numeral for each of the following numbers:

1. Eight
2. Six hundred eighty-one
3. Ten thousand three hundred five
4. Seventeen
5. Two hundred twenty-five
6. One million four hundred one

Express each of the following numerals in terms of units, tens, hundreds, etc.:

7. 23

8. 471

9. 35

10. 10,872

11. 1,567

12. 4

EXPANDED NOTATION

To indicate the total place value of a numeral, expanded notation is used. For example:

$$54 = (5 \times 10) + (4 \times 1) = 50 + 4$$

meaning expanded notation

Write the meaning and expanded notation for the following numerals:

13. 73

14. 587

15. 3,279

NON-DECIMAL SYSTEMS OF NUMERATION

A system of writing numerals is a numeration system. The decimal system is a base 10 place value system. However, the base 10 is not essential to a place value system. To indicate another base, the base is written in English as a subscript. For example, 23_{four} (note subscript) represents 2 bases and 3 ones where the base is four.

In a base 4 numeration system, the digits 0, 1, 2, and 3 are needed. The symbols for the number 4 and for any number greater than 4 are formed by a combination of these digits. For example, the first

15 numerals in the base 10 numeration system are as follow in a base 4 numeration system:

BASE 10	BASE 4
1	1
2	2
3	3
4	10
5	11
6	12
7	13
8	20
9	21
10	22
11	23
12	30
13	31
14	32
15	33

The place value of each digit in the numeral 10,231 in a base 4 numeration system is show below:

1	0	2	3	1
256's	64's	16's	4's	1's
4^4	4^3	4^2	4^1	4^0

The place value of any digit (except the ones or units digit) is equal to 4 times the place value of the digit immediately to the right. In non-decimal numeration systems, the base given determines the consecutive place value powers.

Write the first 15 numerals in the base 10 numeration system using the following bases:

16. 7

17. 3

18. 5

19. 6

20. 2

21. 8

22. 9

CHANGING BASES

To change from base 10 to another base continue dividing the base 10 numeral by the new base using the remainders as the new numeral. For example:

71_{ten} = ______ five

71/5 =	14	R1
14/5 =	2	R4
2/5 =	0	R2

Read remainder from bottom up

Thus 71_{ten} = 241 five

Change the following base 10 numerals to the base indicated:

23. 247 to base 9

24. 81 to base 4

25. 19 to base 2

26. 49 to base 6

27. 143 to base 7

To change a numeral from another base to base 10, first determine the consecutive place value powers and then multiply each numeral in the number by its place value. For example:

$12021_{three} = \_\_\_\_\_ \text{ ten}$

81	27	9	3	1
1	2	0	2	1

$1(81) + 2(27) + 0(9) + 2(3) + 1(1) = 142$

Thus $12021_{three} = \underline{142}$ ten

Write each of the following numerals as a base 10 numeral:

28. 571_{nine}

29. 100101_{two}

30. 403_{eight}

31. 3102_{four}

32. 565_{seven}

33. 251_{six}

ADDITION OF BASES

Addition in other bases is exactly the same as in base ten, however, one must remember the digits needed for that base. For example:

$$\begin{array}{rl} 365 & \text{seven} \\ + \underline{432} & \text{seven} \\ 1130 & \text{seven} \end{array}$$

Perform the following additions:

34. $112_{five} + 301_{five}$

35. $3_{four} + 13_{four}$

36. $100101_{two} + 1001_{two}$

37. $24_{six} + 42_{six}$

38. $32_{eight} + 25_{eight}$

SUBTRACTION OF BASES

Subtraction in other bases is exactly the same as in base ten, however, we must again remember the digits we can use for that base. For example:

$$\begin{array}{r l} 6105 & \text{eight} \\ -\ \underline{5437} & \text{eight} \\ 446 & \text{eight} \end{array}$$

Perform the following subtractions:

39. $621_{seven} - 540_{seven}$

40. $421_{five} - 132_{five}$

41. $101001_{two} - 1101_{two}$

42. $2120_{three} - 102_{three}$

43. $321_{four} - 123_{four}$

FUNDAMENTAL OPERATIONS

LANGUAGE OF ADDITION:

In the mathematical expression A + B = C, A and B are called addends and C the sum. If the sum of two addends is equal to one of the addends, then the other addend is zero.

Compute the following sums:

1. 23 + 87
2. 252 + 0
3. 73,541 + 26,108
4. 203 + 172
5. 60,016 + 201

LANGUAGE OF SUBTRACTION

The operation of subtraction is the inverse of that of addition. In the expression A - B = C, A is called the minuend, B the subtrahend, and C the difference. Subtraction is not always performable with whole numbers and performance is dependent on the order of the numbers. In order to find the difference, the minuend must be larger than the subtrahend.

Compute the following differences:

6. 353 - 225
7. 1,872 - 627
8. 231 - 9
9. 4,554 - 4,385
10. 2,697 - 598

LANGUAGE OF MULTIPLICATION

The operation of multiplication is simply repeated addition. In the mathematical expression A x B = C both A and B are called factors and C the product. When two even numbers are multiplied, the product is even. For example 4 x 4 = 16. When two odd numbers are multiplied, the product is odd. For example 7 x 5 = 35. When one even and one odd number are multiplied, the product is even. For example 3 x 4 = 12. Changing the order of two factors does not change the product.

Compute the following products:

11. 584 x 37
12. 61 x 43
13. 17 x 17
14. 4,004 x 107
15. 928 x 20

Predict whether the product will be odd or even:

16. 26 x 141
17. 8,327 x 93
18. 42 x 56
19. 19 x 2,860
20. 7 x 35

LANGUAGE OF DIVISION

The operation of division is simply repeated subtraction. In the expression A/B = Q, r, A is called the dividend, B the divisor, Q the quotient, and r the remainder. Division is not always performable with whole numbers and performance is dependent on the order of the numbers.

In order to find the quotient, the dividend must be larger than the divisor.

Compute the following quotients:

21. 128/35

22. 54/17

23. 225/15

24. 6,229/801

25. 64/9

TRICHOTOMY PROPERTY

If A and B are any two whole numbers, the trichotomy property states:

A is greater than B (denoted $A > B$)

A is equal to B (denoted $A = B$)

A is less than B (denoted $A < B$)

Only one of these relations is possible between 2 numbers.

Complete the following according to the trichotomy property:

26. 54 ____ 27

27. 17 ____ 19

28. 62 ____ 542

29. 27 ____ 27

30. 84 ____ 73

COMMUTATIVE PROPERTY OF ADDITION

The commutative property of addition is also known as the law of order. The commutative property states that if A and B are any two whole numbers, then A + B = B + A. Thus the order in which we add whole numbers is insignificant. For example:

3 + 2 = 5

2 + 3 + 5

Therefore 3 + 2 = 2 + 3

However, the commutative property does not hold true for subtraction since 3 - 2 does not equal 2 - 3.

ASSOCIATIVE PROPERTY OF ADDITION

The associative property of addition is also known as the law of grouping. The associative property states that for any whole numbers A, B, and C, (A + B) + C = A + (B + C). Thus the order in which we group whole numbers for addition is insignificant. For example:

3 + 2 + 8 = (3 + 2) + 8 = 5 + 8 = 13

3 + 2 + 8 = 3 + (2 + 8) = 3 + 10 = 13

Therefore (3 + 2) + 8 = 3 + (2 + 8)

ADDITIVE IDENTITY

The identity element for addition is zero. The additive identity states that if A is any whole number, then 0 + A = A and A + 0 = A. Zero added to any number gives a sum that is identical to that number. For example:

0 + 7 = 0

COMMUTATIVE PROPERTY OF MULTIPLICATION

The commutative property of multiplication is also known as the law of order. The commutative property states that if A and B are any two whole numbers, then A x B = B x A. Thus the order in which we multiply whole numbers is insignificant. For example:

$$3 \times 4 = 12$$

$$4 \times 3 = 12$$

$$\text{Therefore } 3 \times 4 = 4 \times 3$$

However, the commutative property does not hold true for division since 4/2 does not equal 2/4.

ASSOCIATIVE PROPERTY OF MULTIPLICATION

The associative property of multiplication is also known as the law of grouping. The associative property states that for any whole numbers A, B, and C, (A x B) x C = A x (B x C). Thus the order in which we group whole numbers for multiplication is insignificant. For example:

$$4 \times 8 \times 2 = (4 \times 8) \times 2 = 32 \times 2 = 64$$

$$4 \times 8 \times 2 = 4 \times (8 \times 2) = 4 \times 16 = 64$$

$$\text{Therefore } (4 \times 8) \times 2 = 4 \times (8 \times 2)$$

MULTIPLICATIVE IDENTITY

The identity element for multiplication is one. The multiplicative identity states that if A is any whole number, then 1 x A = A and A x 1 = A. One times any number gives a product that is identical to that number. For example:

$$1 \times 5 = 5$$

DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION

The distributive property of multiplication over addition states that if A, B, and C are any whole numbers, then A(B + C) = AB + AC and (B + C)A = BA + CA. For example:

$$7(2 + 1) = 7(2) + 7(1) = 14 + 7 = 21$$

Multiplication takes precedence over addition. There is no distributive property for addition over multiplication since we cannot assert that A + (BC) = (A + B)(A + C).

Fill in the blanks to demonstrate the distributive property:

31. ____ = 5(2) + 5(7)

32. ____(3) = (8)____ + (1)____

33. 4____ = ____(3) + ____(2)

34. 6(1) + 6(7) = ____

35. ____(2 + 5) = 7____ + 7____

State which properties justify the following:

36. 9 × 1 = 9

37. 8 + 3 = 3 + 8

38. (4 × 1) × 7 = 4 × (1 × 7)

39. (2 + 3) + 1 = 2 + (3 + 1)

40. 7(4 + 9) = 7(4) + 7(9)

41. 8 + 0 = 8

42. 9 × 5 = 5 × 9

43. (8 + 2) + 6 = 8 + (2 + 6)

44. 0 + 2 = 2

45. 4 + 1 = 1 + 4

46. 1 x 3 = 3

47. (5 x 3) x 7 = 5 x (3 x 7)

PRIME AND COMPOSITE NUMBERS

A prime number may be defined as a number divisible only by itself and 1. The smallest prime is 2 and 2 is the only prime that is an even number. A composite number may be defined as a number divisible by a number other than itself and 1. The smallest composite number is 4 which is divisible by 2. The sieve of Eratosthenes was developed around 194 B.C. by the Greek scholar Eratosthenes to show the prime and composite numbers between 1 and 100. Follow these steps to apply the sieve of Eratosthenes to the first one hundred numbers. Cross out the 1. Next, cross out all the multiples of 2 that follow 2 but do not cross out the 2. Do the same for all the multiples of 3, 5, and 7 but do not cross out the 3, 5, or 7. The numerals not crossed out will represent the prime numbers.

48. Apply the sieve of Eratosthenes to Table I below to find the prime numbers less than or equal to 100.

TABLE I

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

49. How many prime numbers are there less than or equal to 100?

50. List the prime numbers less than 100.

51. How many composite numbers are there less than or equal to 100?

52. List the first 25 composite numbers less than 100.

GEOMETRY

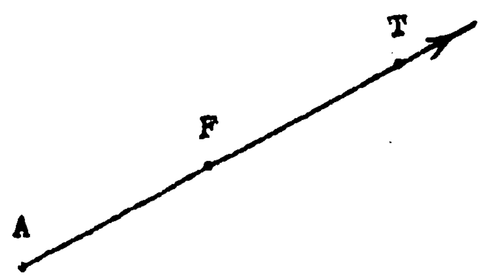
DEFINING GEOMETRY

Geometry is that part of mathematics which studies sets of points. A point may be defined as a location in space having position but no magnitude. Space is the set of all points. The union of all points which are in a row is a set of points joined to form a line. Only a single line can be drawn to connect two points.

1. Draw a line containing both points C and D below.

RAY

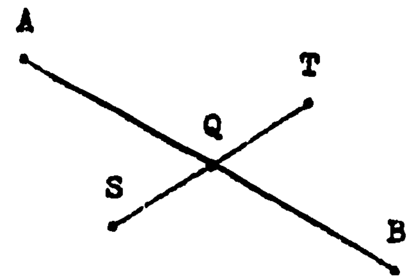
A ray is a particular kind of subset of a line consisting of any one point on the line extending in one direction from the point. For example AT is a ray:



2. What point occurs on ray AT?

VERTEX

All lines intersect even though they may not cross. The point where two lines intersect is called the vertex and is given a letter name. For example:

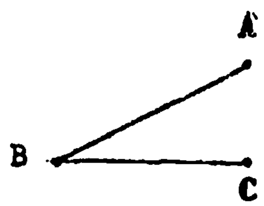


If two lines do not cross, their intersection is the empty set, denoted Ø.

3. In the example above, the vertex of lines AB and ST is point _____?

ANGLE

An angle is formed by two separate rays with a common end point. For example:



The common end point is called the vertex of the angle and the two rays are called the sides of the angle. The angle above is denoted by the symbol ∠ ABC or by the symbol ∠ CBA. The point which is the vertex occupies the central position of the name and is the name of the angle.

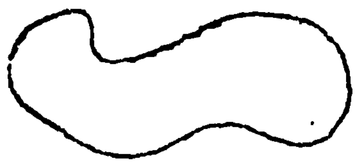
4. The vertex of the above angle is the common end point ________?

5. The above angle is also known simply as ∠ _______?

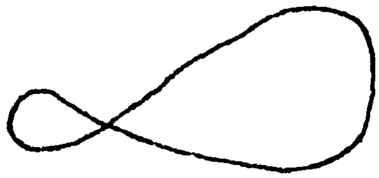
6. The sides of the above angle are the rays ______ and ______ ?

CLOSED CURVES

A closed curve may be drawn by placing a pencil on any point and making a path that returns to the starting point. A simply closed curve has no distinctive points and does not cross itself. For example:



The most common example of a simple closed surve is a circle. A non-simple closed curve crosses itself. For example:

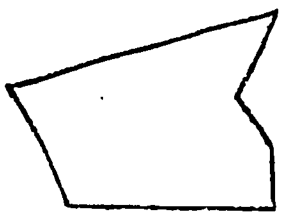


The most common example of a nonsimple closed curve is a figure eight.

7. Draw three examples of a simple closed curve.
8. Draw three examples of a nonsimple closed curve.

POLYGONS

A polygon is a simple closed curve made up of closed broken line segments. For example:



Triangles, rectangles, and squares are the most common forms of polygons.

9. Draw three examples of a polygon.

ANSWERS TO SET THEORY PROBLEMS

1. set
2. set
3. A
4. C
5. B, C
6. glass
7. By definition the knife does not belong to either Set A, Set B, or Set C.
8. X = {a, n, d}
9. Y = {1, 2, 3, 4, 5}
10. Z = {w, a, s}
11. Ø
12. B, D, E
13. {1}, {3}, {5}, {1, 3}, {1, 5}, {3, 5}, {1, 3, 5}, Ø
14. {2}, {6}, {10}, {14}, {2, 6}, {2, 10}, {2, 14}, {6, 10}, {6, 14}, {10, 14}
15. {a}, {b}, {a, b}, Ø
16. {1}, {2}, Ø
17. {m}, {n}, {o}, {p}, {m, n}, {m, o}, {m, p}, {n, o}, {n, p}, {o, p}, {m, n, o}, {n, o, p}, {m, o, p}, {m, n, p}, Ø
18. {a, b, c, d, e}, {1, 3, 5}, {¢, @, +, ¼}
19. finite
20. infinite
21. finite
22. infinite
23. infinite

24. $\overline{A}$ = {0, 2, 4, 6, 8, 10}

25. $\overline{B}$ = {1, 3, 5, 7, 9}

26. $\overline{C}$ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

27. $\overline{D}$ = {0, 2, 3, 4, 6, 7, 8, 9}

28. $\overline{E}$ = {0, 1, 2, 3, 5, 6, 7, 9, 10}

29. $A \cap B$ = {3}

30. $R \cap S$ = {0}

31. $G \cap H = \emptyset$

32. $L \cap M$ = {5, 10}

33. $D \cup E$ = {0, 5, 10, 15, 20}

34. $X \cup Y$ = {a, b, c, d, e}

35. $P \cup Q$ = {#, $, +, @, *, !, &}

36. $C \cup D$ = {1, 2, 3, 4, 5, ...}

ANSWERS TO NUMERATION SYSTEMS PROBLEMS

1. 8

2. 681

3. 10,305

4. 17

5. 225

6. 1,000,401

7. 2 10's, 3 1's

8. 4 100's, 7 10's, 1 1's

9. 3 10's, 5 1's

10. 1 10,000's, 0 1,000's, 8 100's, 7 10's, 2 1's

11. 1 1,000's, 5 100's, 6 10's, 7 1's

12. 4 1's

13. 73 = (7 x 10) + (3 x 1) = 70 + 3

14. 587 = (5 x 100) + (8 x 10) + (7 x 1) = 500 + 80 + 7

15. 3,279 = (3 x 1,000) + (2 x 100) + (7 x 10) + (9 x 1) = 3,000 + 200 + 70 + 9

16. 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 20, 21

17. 1, 2, 10, 11, 12, 20, 21, 22, 100, 101, 102, 110, 111, 112, 120

18. 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 30

19. 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23

20. 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111

21. 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17

22. 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16

23. 304
    nine

24. 1101
    four

25. 10011
    two

26. 121
    six

27. 263
    seven

28. 469

29. 37

30. 259

31. 210

32. 292

33. 103

34. 413
    five

35. 22
    four

36. 101110
    two

37. 110
    six

38. 57
    eight

39. 51
    seven

40. 234
    five

41. 11100
    two

42. 2011
    three

43. 132
    four

ANSWERS TO FUNDAMENTAL OPERATIONS PROBLEMS

1. 110
2. 252
3. 99,649
4. 375
5. 60,217
6. 128
7. 1,245
8. 222
9. 169
10. 2,099
11. 21,608
12. 2,623
13. 289
14. 428,428
15. 18,560
16. even
17. odd
18. even
19. odd
20. odd
21. 3, r. 23
22. 3, r. 3
23. 15
24. 7, r. 622

25. 7, r. 1

26. >

27. <

28. <

29. =

30. >

31. 5(2 + 7)

32. (8 + 1)3 = 8(3) + 1(3)

33. 4(3 + 2) = (4(3) + 4(2)

34. 6(1 + 7)

35. 7(2 + 5) = 7(2) + 7(5)

36. multiplicative identity

37. commutative property of addition

38. associative property of multiplication

39. associative property of addition

40. distributive property of multiplication over addition

41. additive identity

42. commutative property of multiplication

43. associative property of addition

44. additive identity

45. commutative property of addition

46. multiplicative identity

47. associative property of multiplication

48.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

49. 25

50. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

51. 74

52. 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38

ANSWERS TO GEOMETRY PROBLEMS

1.

2. F

3. Q

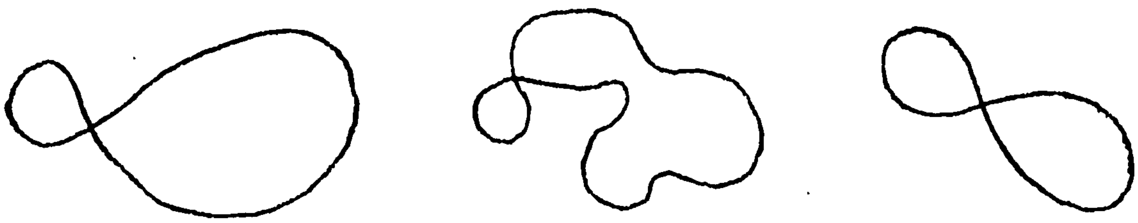
4. B

5. B

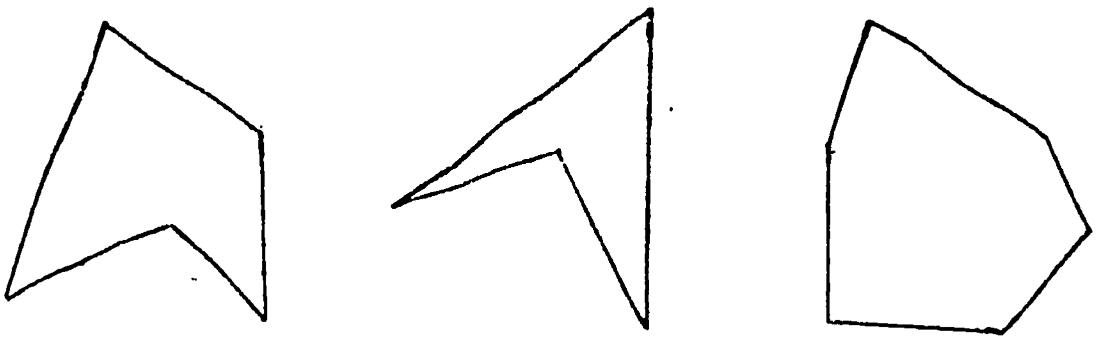
6. BA, BC

7.

8.



9.



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