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ABSTRACT

Presented is the basis for an integrated approach to teaching reading skills and mathematics concepts at the elementary school level. A general explanation of concept formation, of oral and written language, and of mathematics symbols, with specific suggestions as to their application in mathematics, is included in the first section of the pamphlet. The second section deals with the specialized skills needed for reading and thinking in mathematics. These skills include decoding words and math symbols, understanding the processes of mathematics, and applying the decoding and comprehension skills to problem solving. A list of eight suggestions and two references are given to help the teacher and students in developing their mathematics vocabulary. Reading comprehension skills are detailed, with activities specified for helping students with story problems, graphs, and charts. The final section deals with the role of the teacher as one of management and includes a discussion of objectives, evaluation, diagnosis, and organization of materials and experiences. (DT)

IMPROVING

READING-STUDY SKILLS IN MATHEMATICS K-6

The University of the State of New York / THE STATE EDUCATION DEPARTMENT
Bureau of Elementary Curriculum Development / Albany 12224 / 1972

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THE UNIVERSITY OF THE STATE OF NEW YORK

FOREWORD

One of the more serious problems at the elementary school level concerns teaching students to read mathematics independently. Students must learn to make their own interpretations as they read a variety of mathematics materials independently, write their own mathematical statements, and learn how and where to find information about mathematical topics. These tasks require specialized skills more complex than a mastery of word attack skills and acquisition of sight vocabulary. They require an extensive background of math concepts. Therefore, a deliberate plan for teaching these reading skills and math concepts should be the basis for an integrated approach to learning. The ideas in this booklet can form the basis for such a plan. By reading this booklet, a teacher may gain ideas, confidence, and inspiration to do the things he knows should be done. In fact, the value of this publication depends on its use by the classroom teacher at the local level in a way that will make a difference to learners. But reading about things does not, in itself, institute change. To accomplish this, carefully planned curriculum and staff development procedures must actively involve many people who have expertise in areas of curriculum design and evaluation, classroom teaching, mathematics, reading, learning theory, and administration.

It is hoped that the organization of this publication, emphasizing an integrated approach to the theme of reading for math, will serve to clarify and extend teachers' concepts and provide a rational basis for curriculum development and implementation locally.

The implementation stage is indeed the critical one. It's like the little boy, of tremendous energy and imagination who was telling his parents about his marvelous summer with his grandfather. Finally he reached the most exciting part of his vacation - where his grandfather had taken him out to the middle of the lake and let him swim back. His parents gasped with amazement and said, "My goodness! That lake is 4 miles wide! How did you ever manage to swim to shore?" Whereupon the boy answered, "Oh, I can swim all right. The tough part was getting out of the bag!" That's implementation--the tough part for curriculum development!

An original manuscript was prepared by Olivia Hill, irector of Reading, and Norman Michaels, Director of mathematics of the Bedford Central School System. John J. Sullivan and Fredric Paul, associates, Bureau of Mathematics Education, and Alberta C. Patch, associate, Bureau of Reading, gave direction and suggestions for improvement, and contributed additional materials for the final manuscript. Frank S. Hawthorne, Chief, Bureau of Mathematics Education, and Jane B. Algozzine, Chief, Bureau of Reading Education, also reviewed the manuscript.

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I. LEARNING-------------------------------------a lifetime affair

Many educators feel that, in one sense, there is not much that can be directly taught - that we can only provide opportunities for the learner and guidance as he organizes his experiences in his continuous quest for knowledge.

This quest is individual in pace and style. In order to capitalize on children's enthusiasm for learning, the classroom environment must be rich in resources. Piaget and others have pointed out that young children grasp abstract concepts slowly, in a pattern that builds on layers of direct experience with a variety of materials and events. Why stress direct experience? An ancient Chinese proverb gives a partial answer:

I hear ... and I forget.
I see ... and I remember.
I do ... and I understand.

A critical change lies in the teacher's willingness to move from an instructional model which emphasizes listening, reading, and drill as the primary means of providing learning experiences, to one which guides the learner from a base of concrete experience, through increasingly abstract materials. In this manner, he facilitates acquisition of both the content of math and the processes of communication in a meaningful pattern. This process occurs in an atmosphere which creates and supports an attitude that learning is often exciting and sometimes frustrating, may occur during "fun time" as well as during times of quiet listening or reading, and that a "teacher" could be an adult, a friend of his own age group, something printed, a machine, manipulative materials, demonstrations, or games.

In the Prado Museum in Madrid there is a drawing by Goya which portrays a very old man hobbling along on two canes. Below the picture, the legend reads: *Aún a prendo* - "I am still learning." It's a lifetime affair.

A. Concept formation........... it's a matter of experience

What is a concept? Which math concepts are essential to K-6 learners? Let's agree that a concept is a broad generalization which enables an individual to interpret his environment. The concrete reality of facts gain meaning in a framework of organizing concepts. Conceptual learning demands use of the processes of analysis, synthesis, and evaluation to understand an infinite variety of combinations and permutations of facts and events. These processes are fundamental to thinking and, with organizing concepts, are the focus of instruction in math.

How can concepts be taught? The approach may be inductive or deductive, individual or large group, but it is critical that the emphasis be on the learner's needs in terms of his own background and in terms of math objectives appropriate to his level.

If the learner is being introduced to a concept which is new to him, it is best approached through the use of direct, concrete experiences which allow him to work with real objects and events. Examples of such activities are -

- Counting the students who are present and subtracting that number from the number possible;
- Using real money to buy things;
- Constructing models of geometric figures; or
- Planning a bus trip to learn about distance time.

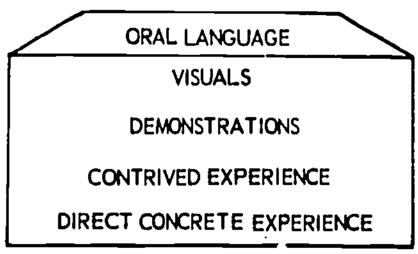
If such activities are not realistically possible for learners, then representations of real life may be achieved. Visuals, including films, filmstrips, transparencies, and pictures, represent a higher level of abstraction but they do form a means of linking the concrete to the abstract. Learning is enhanced more when manipulative materials are used by children engaged in oral discussion.

B. Oral Language .... an abstraction

As the learner interprets and analyzes his concrete experiences again and again while relating them to the oral language he hears at the same time, he gradually develops a summarizing idea or concept which is symbolized by the spoken word. The concrete experiences can be decreased when the learner is able to use vocabulary, definitions, and procedures successfully.

New content has required the introduction of new vocabulary. Vocabulary, however, is second in importance to the development of the concept at the experience level. For example, the names of properties of number systems - commutative, associative, distributive - should not be used orally by the teacher until the student has some concept of those ideas.

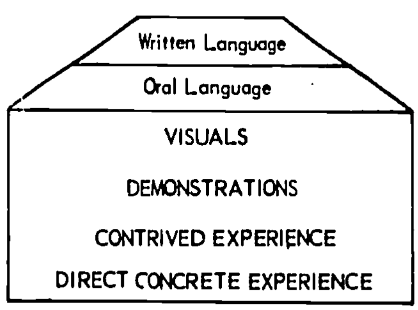
A verbal symbol is highly abstract. For this reason a symbol which grows out of the learner's firsthand experience while accompanied by oral language acquires lasting meaning. Contrived experiences with manipulative objects are useful substitutes, while demonstrations by the teacher and visuals are more abstract and removed from the learner's real world of experience.



The spoken word, which summarizes an idea or concept

C. Written language .... an abstraction of an abstraction

Oral language is symbolic and therefore removed from the learner's own world of firsthand experiences. Written words, such as product, dividend, plus and pi, and math symbols such as > , +, : , are even further removed. First, each has to be turned back into oral language, or decoded. For maximum understanding, the oral symbol must be firmly based on a foundation of relevant experience. Thus, the written or visual symbol can be shown as an extension of oral language.



It is now clear that three questions may arise when a student "reads math."

1. Are his math concepts sufficiently broad and clear?
2. Does he understand and use the verbal symbols?
3. Can he decode the written symbol?

This requires mastery of word attack skills and a sight vocabulary, in addition to math experiences.

D. Math symbols .... a kind of shorthand

Symbols may represent either single words, such as the symbol for a triangle (△) or the symbol for seven (7) or a concept, ∅, null set. The introduction of new topics into the content of elementary school math has made necessary the use of new symbols as well as new vocabulary. Success with reading math symbols is critical since they provide a kind of shorthand invaluable even to very young learners. Math symbols are indispensable. Their meanings and relationships are acquired as children experiment, discover, explore and manipulate, while they say, hear, and discuss the ideas and processes which lie behind it. Memorizing definitions will not result in the understanding necessary to successful manipulation of symbols. Since math symbols are numerous, they should be introduced gradually and consistently with the appropriate learning objectives.

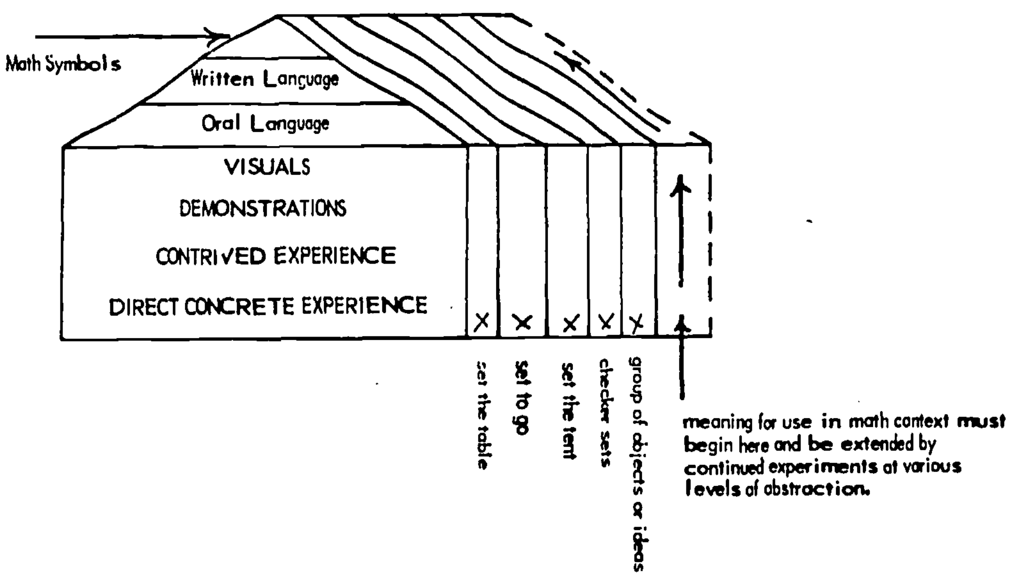
It is essential that a depth of true understanding of math symbols be created at every level of instruction from kindergarten through grade 6 by using relevant experiences and varied manipulative materials. Greater understanding insures longer retention, better application of new ideas, and increased power in reasoning and problem solving. Intermediate grade level teachers may feel it inappropriate to use manipulative materials, but if learners are to be able to "think math," such use is an essential prerequisite.

E. Summary .... concepts keep growing

In addition to a developmental process of concept development which begins with direct, concrete experience and moves to increased abstraction, there is the dimension of depth, or range. Depth is gained by acquiring many different experiences which are related to a concept. An example is the meaning of "set."

The child probably has this word well established in his speaking/listening vocabulary when he comes to school. He may have "set" the table, been all "set" to go, helped his father "set up" the tent, asked his mother if the jello had "set" or lost his checker "set." When he is in math class and the teacher uses the word "set," he may wonder which of his stock of experiences applies. It is the math teacher's task to explore the learner's previous conceptual learnings and make precise, definite plans to extend these to include the math meaning, which may be new for him.

The two dimensions of concept formation are summarized by the following visual model.



II. READING AND THINKING IN MATH... subsets of specialized skills

The ability to read depends upon the successful comprehension of symbols. The decoding skills necessary to turn written representation into oral representation are numbered by the hundreds. There are many comprehension skills necessary also. While relatively few apply specifically to math, these need to be taught systematically in developmental sequence as an integral part of every math class at every level.

A. Decoding .... words and math symbols

Vocabulary in math is often technical and specific. A single word such as commutative, percent, or ratio symbolizes a complex concept. Many other words such as factor, prime, principal, and set may be common in social usage but have precise mathematical meanings. A third group of words is comprised of "little words" such as saw, when, and every. These words are among the most difficult of all those which a beginning reader must learn, because they have no referents. The child cannot draw a picture of the word "then." His concept of its meaning forms slowly, and recognition of both the oral and the visual symbol comes at an even slower rate. Certain "little words" used constantly by the math teacher and in math textbooks include: however, thus, and, or, but, if, and then. These are particularly associated with reasoning and proof as in the following example:

Prove: $\frac{3}{7} \neq \frac{5}{12}$

Suppose $\frac{3}{7} = \frac{5}{12}$

then $3 \times 12 = 7 \times 5$
and thus $36 = 35$

But this is not true.
Therefore $\frac{3}{7} \neq \frac{5}{12}$.

Plans for teaching or reviewing these words need to be included.

Vocabulary development and decoding skills need to be taught by the math teacher for three categories of words:

1. The technical vocabulary and specialized symbols for math.
2. Vocabulary which has wide usage generally but has specific meanings in a math context; and
3. The "little words" which have no referents but are found in 75 percent of all reading material.

A pattern for learning math symbols has been described as being most effective when the student has experience with concrete objects concurrent with a use of oral language. The visual symbol or written word is presented last. It must be remembered that learners need many experiences at each level before they can be expected to bring meaning to a symbol or word.

A learning pattern may include the following steps:

1. Involve the children in a concrete experience with objects and events within their own setting or involve them in a manipulative experience with counters or other objects.

2. Use the appropriate words or phrases during their involvement.

3. Provide many opportunities for children to use the words orally.

4. Write words on the chalkboard or overhead projector, or put them on the bulletin board with an appropriate visual.

Here is an example of a math teacher's plan for teaching the meaning and use of the word "subset" based on this pattern:

DEVELOPMENTAL STAGES	TEACHING ACTIVITIES
	A. Review concept of set
1. Concrete experience plus oral language	1. Have children demonstrate with concrete examples from the room, such as pencils, straws, or books.
2. Contrived experience and demonstration plus language	2. Have children use flannel board objects, while the teacher demonstrates.
3. Visual recall plus language	3. Have children use their imaginations and describe sets orally.
4. Written symbol recognition	4. Write the word "set" on the board and have children read it.
	B. Underneath "set" write "subset"
1. Presentation of the written word.	1. Divide the word and discuss each part.
2. Concept formation, the critical stage	2. Go through steps 1, 2, and 3 of A above with many opportunities for children to use the term in both oral and written work.
3. Teacher analysis of learner needs	3. Give practice worksheets, to be done in pairs.

4. Reteach	4. Plan to reteach where necessary

A plan for teaching the meaning and use of the symbol would include such notations as these:

DEVELOPMENTAL STAGES	TEACHING ACTIVITIES
	A. Review and extend the concepts children already hold by using immediately available experiences.
1. Concrete experience level with oral language - a three dimensional level	1. Exploit these opportunities to illustrate "greater than," "less than": More children than chairs in cafeteria More weight on one side of a seesaw Two few children for a game of basketball Too much noise!
2. Contrived experiences with manipulative materials which represent people, books, chairs, etc.	2. Use any sort of counters. Arrange in rows: o o o o (row 1) o o o o (row 2) Then ask, "Which row has more?" o o o o (row 1) o o o o (row 2) Again,"Which row has more?" This approach directs students from what they already know about equals to what may be a new concept - greater, less than.

Continue with unequal rows:

o o o o o (row 1)
o o o o (row 2)

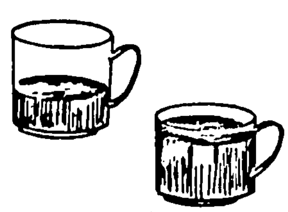
Ask, "Which row has the greater number of counters?"

Continue to develop the meaning by using water, sand, and various other concrete materials.

B. <u>Move to a higher level of abstraction by using pictures.</u>

3. Two dimensional materials, such as film and filmstrips

1. The development of understanding of > and < includes a use of pictures in the same way as concrete objects were used:



Which cup has more water?

4. The symbolic stage

C. <u>Demonstrate the use of the symbols > and < .</u>

Say, "How can we show with symbols, that 7 minus 3 is equal to 4?"

$$7 - 3 = 4$$

"How can we show with symbols that 7 is greater than 4?"

"How can we show with symbols that 4 is less than 7?"

Learning the language of math symbols is a recurring task for students in math classes. It is important to point out that although the idea that seven is greater than four might be shown in many ways, people all over the world have agreed to use $7 > 4$.

5. The application stage

D. Provide many opportunities for children to use the written symbol.

Helping children learn to read and manipulate number sentences which contain math symbols is one of the primary goals of teachers. Students must be able to read these sentences with the same understanding and facility as they read word sentences. This requires much practice and translation of single number phrases such as the following:

$3 + 4$ (three plus four)

$5 - 3$ (five minus three)

$3 + 2 = \square$ says: If we have three, and to that add two, what will the sum be?

In concrete terms, it means push together three things and two things and count!

$3 + \square = 5$ says: If we have three and then add something, the sum will be five.

$\square - 2 = 3$ says: We have some number, such that when we subtract two, we have three.

Such sentences may be illustrated in many ways using concrete materials, pictures, overhead projections, or the chalkboard. An example of number sentence composition using manipulative materials follows:

The teacher starts with five counters on the overhead projector.
A sheet of paper is placed over them.
We now have a representation of $\square$.
The teacher reaches under without looking and takes two out.
The paper is lifted up and the remainder counted.

Question: How many were there before we took out two?

In order to decide, we put back the two and count.

The next level is to operate without counters. When we have this situation (/__/ - 2 = 3), we would add the two to the three to determine the solution.

Let's take a look at the learning process leading to mastery of reading number sentences:

Level I

A. The teacher takes three counters and places them on left of the work space on overhead.

B. Next, two counters are placed in the center of the work space.

C. The counters are pushed to the right of the work space.

D. The counters are counted orally.

Level 2

A. While repeating Level 1-A, 3 is written on the overhead.

B. While repeating 1-B, +2 is written on the overhead.

C. While repeating 1-C, = is written on the overhead.

D. While repeating 1-D, the /_/ is written on the overhead and a numeral is written in the frame to represent the sum.

Level 3

A. The 2-A, B,and C actions are repeated silently. No verbal directions are given.

Level 4

A. Students perform similar operations with concrete materials,modeling the teacher.

B. Students perform similar operations from prepared worksheets. The student is now "reading" the number sentences and following directions. The teacher is indirectly involved,moving from group to group.

Level 5

A. Students perform similar processes manipulating visual math symbols.

The following are examples of different kinds of number sentences:

1. An open sentence is one which contains one or more place holders: $\square + \Delta = 5$

2. A closed sentence is one without place holders: $2 + 3 = 5$.

3. A true sentence is $7 + 4 = 11$

4. A false sentence is $2 + 3 = 7$

Difficulties may arise when a change in the position of a symbol may or may not change the meaning. Some children become conditioned by a format to which they attribute special properties. When the format is changed, they become confused, as with

$$4 + 3 = \square \text{ and } \begin{array}{r} 4 \\ \underline{+3} \end{array}$$

In this case the process is the same in both examples but the arrangement is different. Other examples are:

1. (-) subtraction as <u>take away</u>

   If we have seven objects and take away four, three <u>remain</u>. It is important to note that <u>seven</u> objects were used to set up a model for the situation.

$$\begin{array}{r} 7 \\ \underline{-\ 4} \end{array} \qquad 7 - 4 = 3$$

2. (-) subtraction as <u>comparison</u>

   If I have seven objects and my brother has four, how many <u>more</u> do I have?

We now see that the difference is three. Eleven objects are required to set up the model for this situation though the problem is written the same way.

$$\begin{array}{r} 7 \\ \underline{-\ 4} \end{array} \qquad 7 - 4 = \square$$

3. (-) the multipurpose bar. Students have to "read" the situation from the context of the problem.

A B

a. (-) line segment .____. is indicated $\overline{AB}$

b. (-) continuing pattern .343434 ...is indicated $.\overline{34}$

c. (=) two bars, one over the other, form an equal sign.

4. (-) in fractions

a. $\frac{3}{4}$ as a fraction means that a whole has been divided into 4 equal parts and we are considering 3 of them.

b. $\frac{3}{4}$ as a division pair means that three is to be divided by 4. The bar represents the division operation.

c. $\frac{3}{4}$ as a ratio is sometimes written 3:4. In this situation we are concerned with the number of objects to be compared, as in the following example:

In each room there are three desks and four chairs. The ratio is $\frac{3}{4}$ or 3:4

d. Most important, $\frac{3}{4}$ is a rational number.

Mastery of the concept of a given symbol includes development of the learner's ability to manipulate the symbol in a variety of contexts. This is the routine task of mathematics. For example, the concept of multiplication eventually includes recognition of the process in each of the following instances:

$\begin{array}{r} 7 \\ \underline{\times\ 4} \end{array}$ 7 x 4 7:4 7(4) (7)(4)

The power of a number sentence is that so little tells so much. A number sentence can contain a complete concept or principle, such as, the commutative principle (3 + 4 = 4 + 3) or it can be expanded to represent a complex story problem. Oral practice and much chalkboard work should be preliminary to a student working independently from the text or devising his own number sentences. His success is based on a thorough knowledge of mathematical symbols.

Learners meet many words during math which have not been taught as part of a developmental program in either math or reading. Therefore, when students read orally during math and meet words which are unknown, it is best if the teacher simply tells the word, rather than presenting a minilesson for a specific word attack skill at that time. A note should be made to plan a separate vocabulary development program for such learners which is independent of the "regular" reading or math program.

Of course, each student is taught at his appropriate level in math as well as in reading. If his instructional level in reading is lower than that in math, efforts should be made to decrease the learner's reliance on reading as a means of achieving in math. If he doesn't become frustrated by an inability to deal successfully with the written word in math he will be able to continue his growth in that subject area and at the same time acquire the necessary reading skills during language arts. Incidentally it is of dubious value to form ability groups based on reading scores alone.

Options for implementing a vocabulary development program follow:

- Make sure that the learner is at his appropriate level in math.
- Teach and review the math reading vocabulary systematically, using a variety of means. Remember, an average learner must see, hear, and use a word approximately 40 times before he can be expected to decode it and apply it successfully.
- Teach the math symbols with equal thoroughness.
- Have a poor reader sit beside a good reader where he can ask the pronunciation of words.
- Make cassette tapes to accompany pages of story problems.
- Use "language master" type machines for teaching and reinforcement of vocabulary and symbols.
- On your worksheets or boardwork, use, to the extent possible, those math symbols and vocabulary which have been taught in math class.
- When material to be read contains vocabulary in addition to that which has been taught, read the pages with the children. They can then perform the math processes alone. If such material must be used outside of the classroom, provide parents with information about the child's reading problem and encourage them to read the words for the child - but not work the problem.

At the primary level, a learner may be at low instructional levels in reading and math and be developmentally sound. Therefore, the number of sight words which he meets in math should be minimal and should be mostly those previously developed by the math teacher.

Activities which may be helpful in teaching these skills are detailed on pages 1-7 in the State Education Department publication "English Language Arts, Reading Section K-12."

At the conclusion of grade 6, most learners have acquired proficiency in these word attack skills:

- Using generalizations governing sounds in attacking new words;
- Attacking words with common variations in pronunciation;
- Identifying and using inflectional endings;
- Identifying and using common prefixes and suffixes;
- Using syllabication and accent in identifying new words; and
- Identifying and using compound words.

Activities which may be applicable to teaching these decoding skills in math class are detailed on pages 8-11 in the State Education Department Publication, "English Language Arts, Reading K-12."

Instant recognition of the "little words" and application of the word attack skills may continue to be major problems for the poor reader at the intermediate grade level. Remedial instruction in reading is strongly urged for students reading at a level more than 2 years below expectancy level.

B. Comprehension .... story problems, graphs, and charts

Skill in reading comprehension is directly related to success in math problem solving. This relationship is more easily assessed at the primary level. Even when the learner's general comprehension may be adequate for other reading, skills for reading math need to be specifically taught, practiced, and reinforced by the math teacher.

Story Problems

Story problems in math are extremely compact and involve complex and abstract relationships. There may be a heavy load of facts and concepts which students may find uninteresting when unrelated to their own world of experience. A background of experience with concrete math materials is often assumed, leading to further problems. Variations in typographical arrangements, specialized abbreviations, and a readability level that is often higher than that of the basal reader create additional problems for the reader of math. Understanding is achieved by slow, careful reading and rereading. A clear understanding of the words and phrases is necessary to permit intensive concentration on the math processess.

Some of the reading and study skills which require specific, direct, and planned instruction are:

Restate the problems in the learner's words;
Identify the relevant facts and numerical values;
Recall information;
Use aids for retention such as notes and diagrams;
Recheck the notes and diagrams against the problem;
Organize facts into a general understanding of the problem;
Recognize relationships;
Organize processes to find the solution;
Adjust the rate of reading.

Reading problems may become obvious when the learner is asked to apply these skills to specific material. The average or high achiever may have difficulties in one or two subskills, but these may not be recognized until he has problems with reading in math class.

These skills do not appear as byproducts of generalized reading skill development or even as a result of training in the broad areas of study skills or reading in the content areas. Their effective use is the result of direct, systematic teaching and practice in the reading tasks that demand the requisite skills.

In the primary grades, teachers may use classroom situations, the daily needs of the children themselves, and real people in the real world to act out or role play problems with the emphasis on stating the problem, that is, what it is we have to find out.

Examples include:

1. There are 25 children in the class. Twelve of them go to Art Class. How many children remain?

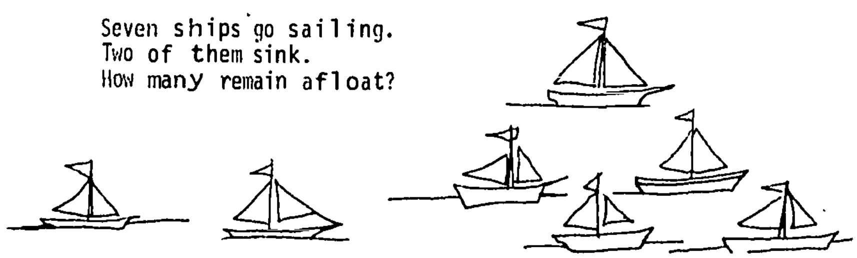
2. We are having a party. There will be 10 boys and 10 girls. How many chairs will we need so that everyone may sit?

3. There are 16 people in the group. We have seven pencils. How many more pencils do we need for everyone to have a pencil?

The next step is still at the concrete level but pictures, counters, tallies, or cut-outs are substituted for real objects or people. The teacher will still emphasize recognition of the problem by asking orally, "What do we want to know?" "What do we have to find out?" and by making sure that the children see the problem. Children will learn to solve problems when they see relationships among the facts.

Children should be encouraged to make a booklet of their own story problems, using pictures, cartoons, or whatever form is easiest for them to show the problem and their solution of it. An example of a child's story problem follows:

Seven ships go sailing.
Two of them sink.
How many remain afloat?



When written problems are introduced some students will need help. The teacher may read the problem orally to them, emphasizing key words and phrases, cueing the students to such phrases as how many, how many more, what percent of. etc. The teacher may ask several students to put the problem into their own words or to rephrase it in picture form or number sentence form. This translates the problem into symbolic form and tests their understanding of the terms of the problem. It also helps the poor reader by allowing him to concentrate on the math processes rather than on decoding the written word.

Students at the intermediate levels can be encouraged to develop their own story problems from daily situations and from the daily news which is a rich resource for problems. They tend to create problems with extraneous details and will need specific and extra practice in deleting extraneous details. This is an extremely valuable evaluative activity which teaches them to read critically for absurdities, irrelevancies, and ambiguities in story problems.

Example: 1. My brother John, who is 2 years older than I, weighs 100 pounds. I weigh 80. How much more does he weigh than I?

2. Twenty-three adults and 14 children got on a 50-foot ferry. How many passengers were on board?

3. We bought our car, a 1968 Ford, for $2400. Within a short time we had to sell it for $2000. How much did we lose?

   This is a typical ambiguous problem since, if the car was used at all, its value depreciated substantially and the answer cannot be $400. Consideration of such ambiguities, however, can be valuable.

Students will also need extra practice with problems which do not give enough information, such as these:

1. Mary had several dolls. She got two more for her birthday. How many does she have now?

2. John had 25 marbles. His brother gave him his marbles. How many does he have now?

3. Dad went fishing. Yesterday he caught seven fish. He caught more today. How many more did he catch?

Practice with given situations in which students design the problem question, gives them insights into the need to answer the specific question. It also allows them to see the various possibilities for other questions.

1. Mary collected six eggs; Helen collected eight eggs.

2. One quart is spilled from a gallon container.

3. There are five boys and three times as many girls.

When students write their own problems they become familiar with the phraseology and the general form of the story problem. Using their own terms they can more readily translate them into the symbolic number sentence.

> A Boy Scout troop went on a hike. If the boys traveled for 5 hours at an average speed of 3 miles an hour, what is the total distance they hiked?
>
> $5 \times 3 = \square$

Unless students are given much practice in reducing story problems to the essentials, they will have difficulties writing the problem in terms of what the situation is and what the question is.

It is profitable for students to write a story problem from a number sentence.

$$16 + 7 = \square$$

My brother who is 16, is 7 years younger than his friend Jim. How old is Jim?

The number sentence is the math solution to a great many stories or verbal problems in all grades. This may be the crucial point in the development of problem solving techniques.

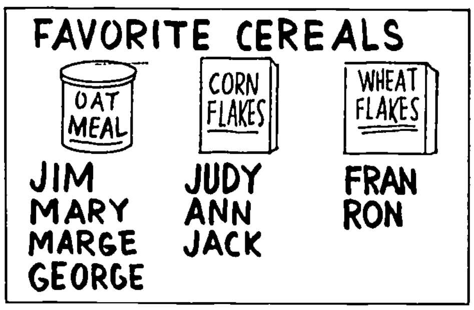
The student must note the technical terms, separate facts given from facts required for solution, sense the sequence of ideas, and see the relationships among the number quantities. The student having difficulty in reading may need to reread each phrase to make sure he understands the vocabulary. Unless students have clear, accurate meanings for the words in the problem, interpretation will be faulty. Finally, he will need to reread the entire paragraph to get the whole picture. The following is a summary which provides options for providing help for students. As proficiency increases, fewer of the activities or steps would be used, until the learner is able to solve problems efficiently.

1. Preteach the vocabulary.
2. Evaluate textbook story problems to insure that none are ambiguous.
3. Read the passage through to get the general idea.
4. Reread more slowly to clarify statements, detect relevant or irrelevant facts, or to group the ideas more specifically.
5. Help learners visualize the problem by drawing sketches or diagrams.
6. Use concrete objects to test or demonstrate processes.
7. In oral discussion, go through the steps necessary for solving the problem.
8. Help children write their own number sentences and solve each other's problems.
9. Have students reread to see if the solution is reasonable.
10. Help them rewrite problems in simpler terms.
11. Use problems without numbers.
12. Provide answer keys which can be used to correct problems instantly.

Stating problems simply and and clearly is not a strong feature of every textbook. Teachers as well as students become annoyed when they meet a problem that reads, "Determine whether or not the set of hot dogs is equal in number to the set of boys on the campout." What is meant is, "Find out if there are enough hot dogs for each boy to have one." Teachers must take care not to reproduce this nonsense in their own work.

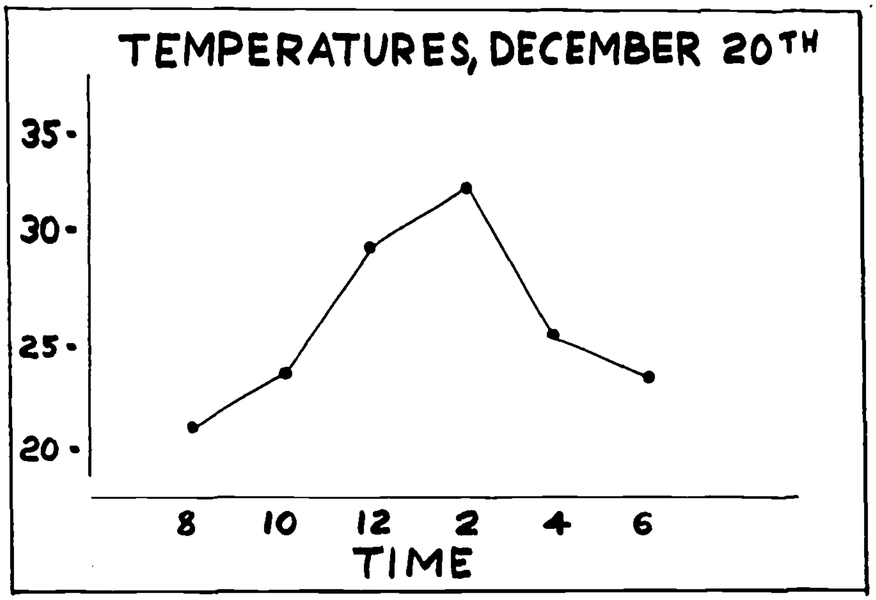
GRAPHS AND TABLES

GRAPHS are pictorial representations for comparing data and for simplifying and communicating relationships. At as early an age as 5, when children have reached the collecting and sorting stage in their development, they can express their understanding in this form.



DAYS FOR MUSIC LESSONS	
MONDAY	JILL, HENRY, MARK
TUESDAY	MARY, ANN
WEDNESDAY	GEORGE, ALLAN, PAMELA
THURSDAY	JOHN, WILL
FRIDAY	JUDY, JOAN, WALT

Graphs help children of all abilities, expecially slow learners, to see relationships which otherwise may not have much significance for them. Children can find graphs in newspapers and periodicals which they can share with the class. They should be encouraged to create their own graphs with a simple written explanation to go with them. These graphs need not be an end in themselves but should lead to gathering further information, computation, or discussion.

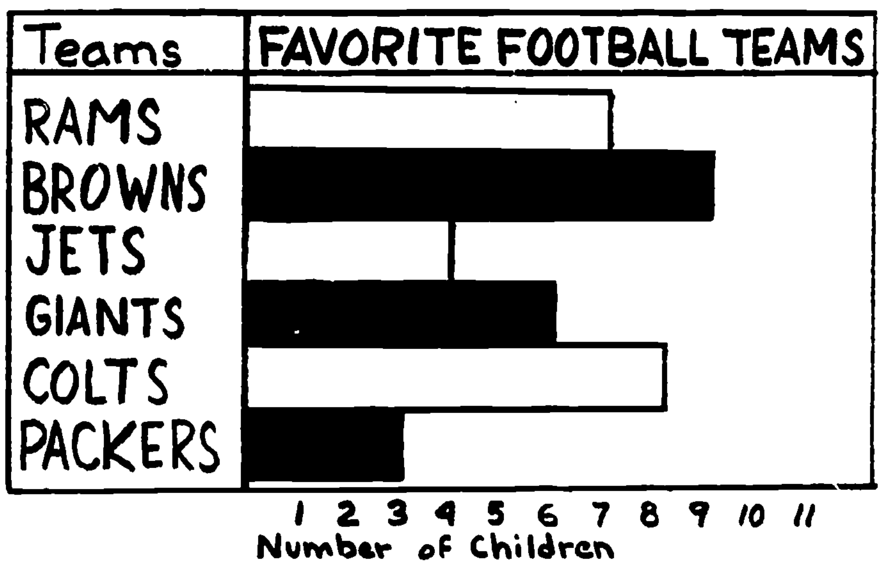


By comparing themselves with their peers and identifying equalities and inequalities, children soon become familiar with the phrases longer than, shorter than, the same length, more than, and less then. In early primary grades the children should use themselves to record data.

Example: How many are wearing red - line up;
How many are wearing blue - line up;
How many are wearing both blue and red?

There should be an all-other-colors line to complete the classifying experience. The length of the lines can be recorded by using a camera.

For the next level, students bring with them a square of stiff paper. After they line up they place the paper on the floor to form a bar graph. Then have the students step away to "read" the graph and color square for square on their graph paper to match the floor graph. The new graph is read to determine if the conclusions are the same.



It should be stressed that we are using the word "graph" to mean any picture, sketch, or diagram which helps explain or illustrate ideas. This is important because practically every operation, every concept, in mathematics can be illustrated. This is an enormous aid to understanding; children should be encouraged to "sketch a picture" so that it becomes second nature.

Mathematics data are often presented in tables which must read functionally as train schedules, tax tables, and calendars.

Tables are also used to discover patterns in data as in the following:

Find the sum of the first n natural numbers.

It is helpful to construct a table: (n represents number of natural numbers; s represents sum.)

n	1	2	3	4	5	6	7	...	n
s	1	3	6	10	15	21	28	....	s

The table helps us discover a rule for finding the sum of the first "n" natural numbers.

III. THE ROLE OF THE TEACHER................. a catalytic one

Much is written today about the changing role of teachers. They are urged to stimulate and guide learners through an organized multimedia environment which encourages choices of materials and experiences. As managers of highly individualized programs with many options, teachers must plan and organize materials and experiences which will provide for different learning rates, styles, and developmental sequences. Planning and organization result from evaluation of each student's conceptual development, skills, and abilities in terms of instructional objectives which have been defined.

A. Objectives .... prerequisite to evaluation

Evaluation of the outcomes of teaching reading skills for math must give attention to instructional objectives. Many objectives are contained in this publication but they must be rewritten in terms of expected behavior. The concept of reading as the interpretation of symbols should be introduced and emphasized from beginning levels by asking children to "read" what they have discovered, constructed, or represented in their mathematical experiences. Students at the intermediate levels should be made aware of the difference between narrative and technical reading and know the requirements and adjustments they must make as to purposes, rate, format, vocabulary, and content.

Standardized tests and teacher-made paper and pencil tests may be used to assess reading skills. Equally important is the continual use of observations and oral questions as students manipulate concrete materials, work independently on practice material, or read story problems. Interviews or discussions with individuals and small groups may reveal attitudes toward the reading aspects of math. The teacher assesses progress while in the classroom by watching, checking, and analyzing questions and responses while the children are working. Evaluation, thus becomes individualized.

B. Diagnosis .... a basis for planning

We know that students in elementary school vary greatly in their decoding and reading comprehension skill levels as well as in their math computational and problem solving skills. Diagnosis must be a continuous process on both a daily basis and a long-range basis. Pretesting before the introduction of new topics will indicate if the prerequisite skills have been mastered and if review or reteaching is necessary for concepts, processes, or vocabulary. Standardized test scores may be useful for long-range planning. Those students seriously handicapped in reading or math should be referred for specific remedial services. A class profile chart which shows standardized test data, diagnostic information from teacher-made tests, and daily observations of individual progress facilitates planning for the varied needs of students.

An excellent way to accommodate diversity is to use a laboratory approach such as is outlined in the State Education Department's, "Teaching Elementary Mathematics Using Laboratory Approaches," a publication available from the Bureau of Elementary Curriculum Development. Use of such materials and experiences is based on systematic analysis of diagnostic information.

C. Organizing .... materials and experiences

The organization of materials involves the deliberate creation of a stimulating environment for reading in math at each grade level. Some steps to insure that appropriate materials and experiences are available may include the following:

1. Make a written inventory of all the materials available at each level for teaching math. List them according to the following categories, indicating supply and accessibility:

   a. Concrete materials which provide direct first-hand experiences of a three-dimensional nature for learners, such as balance scales and stop watches.

b. Items which provide representations of experiences, such as counters, beads, rods, and scales.

c. Materials which combine media such as tapes, records, and filmstrips accompanied by workbooks, flashcards, with audio tape, and 16-mm. film.

d. Visual material such as overhead projections, flat pictures, charts, filmstrips, and silent 8-mm. film loops.

e. Audio materials such as tapes and records.

f. Resource materials such as games, puzzles.

g. Textbooks, workbooks, library books.

2. Compare the inventory against the diagram on page 5 Are materials available in sufficient quantities at <u>each</u> level of abstraction?

Analysis of the inventory may show a high reliance on concrete materials for learning while a lack of such materials may indicate overreliance on the textbook. The major criteria should be the needs of the learners at each level and the role materials and experiences fulfill in realizing those needs.

Teachers need to know the materials which comprise the math programs thoroughly, the format, style, the level of reading, the strengths, and weaknesses. No one set of materials will be a complete program and teachers need to know when to supplement and what is available. A mathematics program should not be limited to one set of any materials but should include a variety for review, extension, and enrichment.

Teachers should capitalize on children's natural tendency to express situations graphically and extend their pictographs to more sophisticated graphs and tables. The vocabulary of mathematics should be taught as any other language, through word structure, through words in context, and through the use of a glossary or a mathematics dictionary. Students needing continual support and encouragement should have ample time for oral discussion of mathematical concepts with opportunities to relate them to their own experience and to use concrete materials for clarification and understanding.

A program is more than the organization of materials and experiences, however. The classroom teacher is responsible for providing motivation, guidance, and assistance to learners as they test ideas and solve problems in a stimulating environment. The personal inspiration which the teacher communicates to learners contributes greatly to their acceptance of responsibility for their own learning. Thus, the role of the teacher includes creative leadership in addition to the separate and disparate functions of setting objectives, diagnosing needs, and planning and organizing the environment. Together these create a learning system in which the teacher manages available resources for learners effectively and efficiently.

IV. CURRICULUM DEVELOPMENT .... a new ball game for the 70's

Since there is great diversity in the needs and interests of communities in the State, it is expected that each school district may wish to respond to this publication in a different manner. Responses will be determined in part by the district's philosophy as well as by financial considerations. However, instant add-on programs, piecemeal approaches, or additional personnel are the least desirable. If a curriculum development project is to be undertaken at a district or regional level, long-range planning is a prerequisite. The characteristics and needs of the learners, including the specifics of their performance in reading, writing, speaking and listening, must be identified before goals and instructional objectives are established. Human and technical resources must be identified and evaluated against established criteria. These tasks require leadership as long range plans become the specific tasks of determining instructional objectives, selecting materials, writing illustrative activities, choosing a format, designing inservice modules, devising procedures for implementation and evaluation, and establishing means for redesign of the materials after trial use. Inherent in the new emphasis on curriculum design and development at the district and regional level is acceptance of increased responsibility for these activities.

Initiation of a curriculum project for reading and math requires that priorities be established locally and that allocation of specific resources of time, personnel, and finances be made. It is highly significant that the ways in which strategies for planning, organization, implementation, and evaluation are initiated and carried out are closely related to acceptance of the materials by those who will be using them most - the classroom teacher. Their involvement at each stage of a curriculum project is of critical importance in terms of building personal interests and increasing professional capabilities. Representation is also necessary from administrators, curriculum designers, parents, and the community if implementation is to be effective.

V. SUMMARY .... Chapters I through IV

An environment must be created at the elementary level, which will allow learners to gain knowledge, skills, and attitudes for reading in math, at their own rate, in their own style. Learning builds from the concrete level of firsthand experience to the abstractions of language and symbols through a systematic sequence of activities designed to involve learners with specific materials and processes. Such interaction results in the conceptual learnings necessary for reading math.

Reading in math has subsets of specialized skills which must be learned. These include decoding words and math symbols, understanding the processes of math, and application of decoding and comprehension skills to problem solving. Each of these skills is developmental in nature and must be presented at the learner's instructional level. The teacher's role, therefore, becomes one of management.

As manager of programs which must be individualized in terms of levels of conceptual development, skill acquisition and ability to apply the skills as well as the learner's style and pace, the teacher must plan and organize carefully and extensively. Such planning and organization begin with the diagnosis of learner needs, the development of objectives, and then moves to the assessment and organization of materials and experiences to meet needs, and finally to the evaluation of the program.

In addition to the managerial component, the teacher supplies the intangible ingredient that personally stimulates and motivates learners toward higher levels of achievement.

Each district is expected to respond to this publication according to its own needs and priorities. Development of curriculum at the local level is a necessary and valuable activity. State Education Department personnel are prepared to assist in this effort.