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ABSTRACT

Proposed curriculum topics for elementary school mathematics have been arranged in topic form. Six areas are covered: numbers and numeration, sets, whole numbers, fractions, problem solving, and geometry and measurement. The first section briefly explains the six areas and also includes a short discussion of teaching strategies, number sentences, problem solving, and developmental algorisms. The second section presents details of a curriculum for each of the six areas in kindergarten through grade six. (DT)

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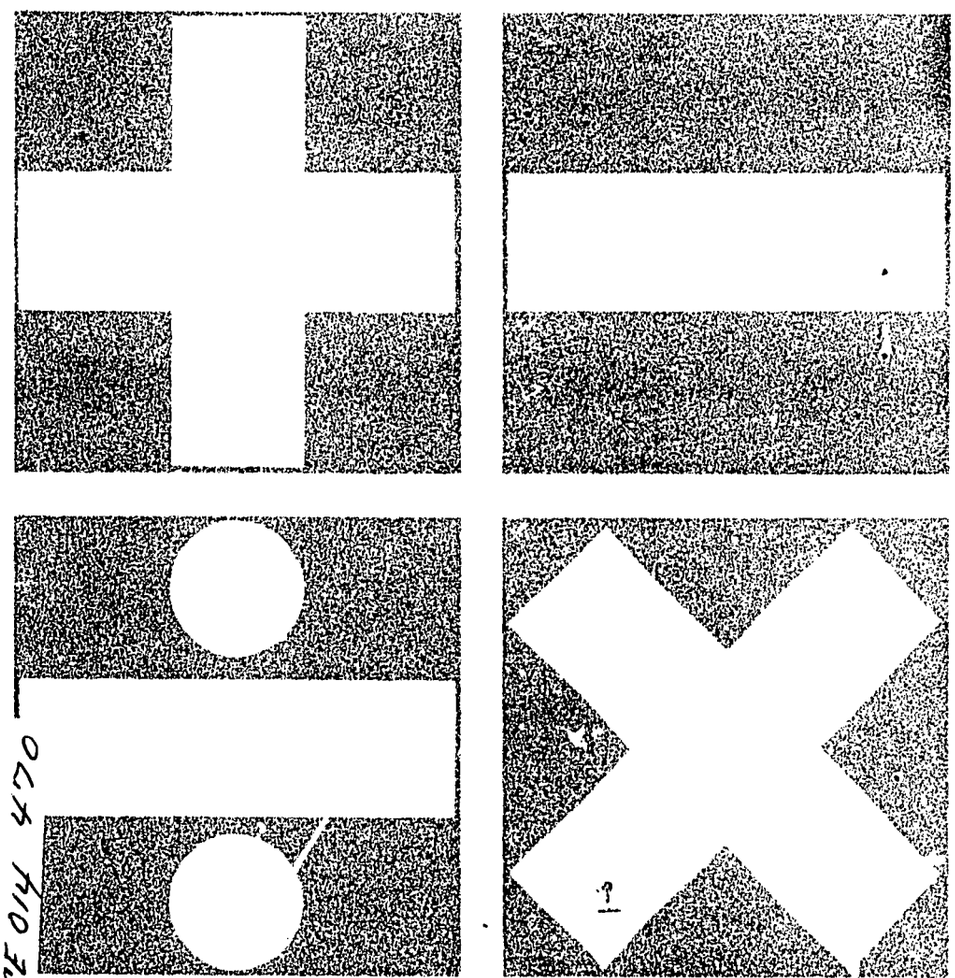
Mathematics K-6

a recommended program



A recommended program

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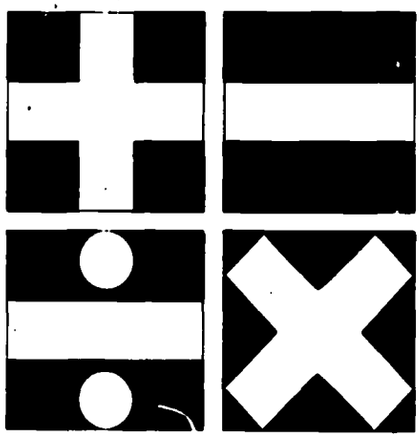

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Contents

Overview

The Curriculum

The elementary mathematics curriculum has been presented in topic form for purposes of aiding the teacher in preparing lesson plans and in understanding the sequential development of specific concepts. The curriculum proposals have been arranged in six areas entitled:

- **Number and Numeration: Counting, Reading, and Writing**
- **Sets: Language and Symbols**
- **Operations on Whole Numbers; Processing of Numerals**
- **Operations on Fractions: Processing of Fractions**
- **Problem Solving**
- **Geometry and Measurement**

The **Number and Numeration; Counting, Reading, and Writing** area develops the basic ideas built into our number system. Ways of signifying number, cardinality, ordinality, and concepts for place-value are developed. Number and numeral patterns are carefully explored. Much attention is given to the development of a quantitative vocabulary and symbols to aid the pupils in making comparisons that involve the concepts of "between-ness," "greater than," and "less than," as well as "equality." The pupils are guided to precision in reading and writing numerals.

The **Sets; Language and Symbols** area provides us with tools to look at numbers in another way. This area also provides us with some ways to refine our understanding of what is done when we manipulate or process numerals. As this part of the curriculum is more fully developed in the intermediate grades, a pattern for encouraging logical thinking emerges from these activities. Intuitive reasoning skills are provided within the framework of the ideas relating to sets.

The **Operations on Whole Numbers; Processing of Numerals** area relates to addition, subtraction, multiplication, and division of whole numbers. Addition and subtraction are coupled as "doing-undoing" operations or processes. Multiplication and division are treated with the same relatedness. The use of many and varied experiences with the manipulation of different objects is stressed for the purpose of developing meanings. This is followed with the examination of pictures having elements of quantity. These pictures may take the form of printed, prepared materials, or hand-drawn representations made by the teacher and the pupils. They may be realistic or abstract. The number property of, say, "fiveness" exists just as much in a picture of five dots as it does in a picture of five trees. These explorations, essentially intuitive in nature, lead into actual use of numerals and algorisms. Meanings and understandings should be developed before proceeding to the use of algorisms.

The **Operations on Fractions; Processing of Fractions** area treats fractions as equal parts of one whole as well as equal parts of one group. The pairing of addition-subtraction, and multiplication-division as "doing-undoing" operations is encouraged. The same sequential form of development of understanding through objects, pictures, and numerals is suggested. Care has been given to develop concepts of "cardinality," "ordinality," and "between-ness," when dealing with common fractions, decimal fractions, and percentages.

The **Problem Solving** area gives stress to estimating answers before and after computations. Pupils are directed to look over the problems to be solved before going on to the computations. Nonreading, problem-solving approaches are used in the primary grades. During the intermediate grades, as the child gains skill in reading abilities, reading is introduced into problem-solving situations. Attention is given to the refinement of reading abilities as they relate to mathematical activities.

The **Geometry and Measurement** area presents an intuitive study of geometrical structures. Rules and proofs are not emphasized. Plane and solid geometrical figures are explored in terms of their properties. The children are urged to create nonstandard figures and examine and describe them. Rather than begin with standard units of measure, the children are given an opportunity to create units of measure. The youngsters are led to the use of the metric system of measures after they have gained facility in using the measures which are in common use.

Throughout these proposals, the stress is to begin where the children demonstrate their levels of attainment. Basic ideas are built through intuitive exploration. Introduction of new ideas usually begins with the use of objects and the physical manipulation of them. Many and varied experiences with differing materials are usually necessary to understanding before going to the abstract use of numerals to represent the number properties of the objects. The transition from the use of objects to the use of numerals should be made as quickly as possible; the use of objects should be neither bypassed nor prolonged. Teachers should not hesitate to reintroduce objects whenever the need becomes apparent.

The Teaching Process

Elementary mathematics offers opportunities for the development of important teaching techniques. The following presents the ideas underlying the formulation of the elementary mathematics curriculum.

Explorative Activities

As stated earlier, work with objects is encouraged before abstract work with numerals. These activities draw upon the previous mathematical experiences of children and provide materials for the children to manipulate with meaning and understanding. After manipulation of objects, which may be wooden blocks, sticks, or discs, the children are encouraged to work on the pictorial level. These pictorializations may first be presented by the teacher for the children to examine and study various number relationships. They may take the form of actual pictures of childlike activities, or they may be simple dots to illustrate a certain number property. The numbers should represent both the counting and measurement uses of number. "Five dogs" would be a counting use of number. "Five pounds of sugar" would be a measurement use of number. The former is exact; the latter is never exact. The children should be encouraged to pictorialize concepts of quantities with their own work in drawing. During such activities they will be led to refining their own ideas about number. Working with numerals is the last step in this explorational process.

The teacher will find that the sequence of working with objects, pictures, and numerals is stressed throughout these proposals. A variety of materials on the physical level will help the child develop meanings for numbers and operations before being guided to the abstract or symbolic level where he works with numerals and symbols that represent physical situations.

The Role of Discovery in Teaching and Learning

Improving the teaching and learning of elementary school mathematics should be viewed in terms of method (how we teach) as well as content (what we teach). It is important that the teacher does not rely wholly upon *telling* as the chief or only method of instruction, and does make increased use of *guided discovery*. Note the importance of guidance in the process of discovery.

Pure discovery as the one and only teaching method would be wasteful of time. The use of guided discovery, however, will ensure more powerful learning than the use of telling or didactic methods. The teacher should maintain a healthy balance between telling and guided discovery if maximum learning is to be achieved and an attitude of inquiry is to be fostered.

Number Sentences: Closed and Open; Direct and Indirect

The use of number sentences, although not entirely new to the elementary school mathematics program, does receive greater emphasis in this program than in the past programs.

Number sentences can be classified as closed and open sentences, and as direct and indirect sentences.

Closed and Open Sentences

Number sentences such as $3 + 8 = 11$; $43 = (6 \times 7) + 1$; and $12 \times 3 > 25$ are closed sentences. They can be thought of as true mathematical statements, true relationships among numbers. (Or they can be false, as $3 + 8 = 12$; $12 \times 3 < 35$.)

Number sentences such as $3 + n = 11$; $\square = (6 \times 7) + 1$; and $12 \times 3 > ?$ are open number sentences. The task for the child is to find a number to replace the variable (n, $\square$, or ?) to make the sentence true.

Direct and Indirect Sentences

Number sentences such as $7 + 5 = \square$; $13 - 8 = n$; $3 \times 17 = ?$; and $17 \div 4 =$ ..... are direct sentences. The operation to be performed is directly indicated in the sentences and will provide the answer.

Number sentences such as $17 + \square = 124$; $n - 118 = 73$; $437 = 23 \times ?$; and ..... $\div 13 = 72$ are indirect sentences. The operation symbol shown in the sentences does not indicate the operation to be used to obtain the answer.

In the first of these, $17 + \square = 124$, the child thinks, "I have a sum (124) and an addend (17). I must find the unknown addend, so I subtract: $124 - 17 = \square$." He then has a direct sentence and can proceed to find the answer (solve the number sentence).

In the second of these, $n - 118 = 73$, he thinks, "I have two addends (118 and 73). I must find the sum, so I add: $118 + 73 = n$." This is a direct sentence and the + sign tells the operation to use.

In the third of these, $437 = 23 \times ?$, the child thinks, "I have a product (437) and a factor (23). I must find an unknown factor, so I divide: $437 \div 23 = ?$."

Now he has a direct sentence and can proceed with the processing of the numerals.

In the last of these, ... $\div$ 13 = 72, he thinks, "I have two factors (13 and 72), I must find a product, so I multiply: 13 $\times$ 72 = ..." Again, he has rewritten an indirect sentence into direct sentence form and can proceed to solve the equation.

The purpose of emphasizing number sentences in the new program of study is to provide much-needed understanding and practice in expressing number relationships. It should be clearly noted by the teacher that the working with open sentences to find correct answers is not, and should not be, a guessing game. The child should not guess at the answer; rather, he is guided in the use of verbal thought patterns that will *logically* lead him to the correct answer.

Using Direct and Indirect Number Sentences in Problem Solving

The work with direct and indirect number sentences is not an end in itself in this program of study. Rather it is a means to a more important end — better problem-solving ability.

Current research indicates that the use of *direct number sentences* only in the initial stages of working with verbal problem solving is markedly superior to the use of *indirect number sentences*. In a problem such as:

John had some money. He spent 17c and had 25c left. How much money did he have to begin with?

the child should think directly: "I have two addends. I must find a sum or a total, so I will add: 17 + 25 = n."

In the initial stages, grades 3 and 4, research seems to indicate that he should not think in an indirect or roundabout way and write an indirect sentence, n — 17 = 25. This will not show the correct operation to use. The same direct thinking to develop problem-solving ability applies to all four operations in the early grades. Beginning with the fourth grade, the child can be guided in his ability to solve problems by using both direct and indirect sentences.

Problem-Solving Sequences

Many opportunities are created for the children to develop problem-solving procedures. They develop problems from their own activities in and out of school.

In the primary grades, the work is carried on orally. The problems are developed with objects, pictures, and numerals, sequentially. At the abstract numerical level, the direct number sentence is used as an aid. The pupils

are guided toward expressing relationships with a direct number sentence. The creation of problems at the oral level is coupled with the number sentence method of expression in order to place a minimum reliance upon reading.

During the upper elementary grades, reading skills are introduced and developed. The need to analyze written materials enters into the mathematical scheme. Here, the use of both direct and indirect number sentences to express the meaning of the task is developed. The child is guided to make an analysis of written material.

The work is carried forward with the use of true and false number sentences. The child is encouraged to seek many true answers, compare some false answers, supply missing information in a logical manner, and detect illogical material. The intent is to have children think about the data, what needs to be known, and the operations to be performed.

Problem solving in this proposed program of study has moved from nonreading activities to refined mathematical reading skills. Less stress is put upon reading in problem-solving activities while the pupil is in the early stages of gaining reading competency and facility with number sentences in abstract form.

Developmental Algorisms

Readiness plays an important role in the instructional process. It is carefully assessed by the teacher and put into the instructional program to ensure that the readiness not provided for by the previous experiences of the child is effectively made part of the planning for teaching.

The introduction of each new idea begins with a review of previous experience and learning. Out of this there should emerge an understanding of a problem or a situation that is yet unsolved or unresolved. The use of manipulative materials is encouraged after understanding of the problem is developed.

After the child has developed meaning and understanding at the physical and pictorial levels, there is much attention given to exploring ways of arriving at a solution. These may take the form of nonstandard algorisms which may be less mature and not as efficient as the standard algorism in terms of time, but which may provide insights into the rationale of the processing of the numerals. As with the use of objects mentioned earlier, the use of nonstandard, immature algorisms is coupled with the study of simple algorisms from the past history of elementary mathematics. Gradually, the use of the standard algorism is reached.

This development of conceptual algorisms is directed to assist in providing more meaning to the standard algorisms. Instead of *starting* with the standard algorism that leaves out much of the meaning in the hope of stressing efficiency and speed in reaching a conclusion, the standard algorism becomes an ***end point*** in the teaching process.

Individual Differences

The manner in which the areas of study in these proposals are developed places much emphasis on individual differences among groups of children and indi-

vidual children. Meaning is always developed at the onset of any topic in a specific part of the curriculum. The sequence — objects, pictures, and numeral manipulation and processing — is a continuum that allows the teacher to return to previous parts of the learning sequence whenever the need is apparent. Children will vary in the amount of time needed to work with objects for the purpose of gaining meaning. Some children may need to return to the physical level when they are having difficulty in understanding certain algorisms. The plan of the proposals encourages this flexibility. Individual differences are provided for in the proposals with this basic method of developing understandings and meanings.

The syllabus is not directed for use with the extremely slow or extremely fast learners. Many of the skills that are developed must be mastered by all children. The developmental sequence and the open-ended nature of mathematical topics suggest ways of adapting these proposals for slow learners as well as for children who would profit from extensions of the abstract ideas contained herein. Teachers should not consider the grade level stratifications inflexible. Teachers are urged to modify these grade levels in terms of their pupils and their past experience.

Inventorying Previous Learnings

Beginning a new school year has always necessitated devoting some time to the assessment of the pupils' understanding and skill in the mathematical competencies developed during the previous years. Important ideas are reviewed and basic computational skills are reinforced. The cumulative nature of mathematical learnings makes it important to find out where the children are, and to begin from the present level of development. The amount of time spent in these activities will vary, but no new extensions of previous topics should be attempted until a thorough inventory and reteaching of needed levels of achievement are reached.

There must always be provision in the course of study for the maintenance of understandings and skills already attained. Practice in the maintenance and use of important mathematical information must be provided for at all grade levels. Using acquired understandings and skills reinforces their importance and maintains efficiency in their use.

Diagnostic and Remedial Activities

The inventory of pupils' competencies will reveal some children who need specific diagnosis and remediation. The diagnostic work most often takes the form of close observation of the individual child while he is actually going through some specific thought process. The child is encouraged to *think out loud* while working. In this manner, the teacher is able to locate where the difficulty is and give direct aid to the child. *Integrative* practice to develop meanings and *repetitive* practice to develop facility now enter into the instructional process for aiding the individual child. Drill activities geared toward automatic responses of the basic number facts also provide many clues to the

weaknesses of certain children. Formal, periodic tests will also help to point out areas that need more work. These tests may be wholly devoted to a single arithmetical operation for diagnostic purposes.

Remedial activities in mathematics require an adjustment somewhere in the sequence of developing meanings. At times, premature imposition of formal language patterns (rules) may get in the way of understanding. It is advisable at such times to return briefly to the actual use of manipulative objects or devices to explain some ideas. Perhaps a restudy of the rationale of the algorisms might suffice rather than going to work with objects. The teacher must feel free to use objects on any grade level for the remedial work. These proposals in elementary mathematics have an established sequence of learning activities which become remedial activities when we backtrack on the sequence of objects, pictures, and numerals.

Desired Outcomes

In formulating these proposals for elementary school mathematics education, K–6, the intent has been to provide understanding of the important ideas relevant to mathematical thinking. Continuity has been emphasized for the total mathematical experience of the children throughout all of their school mathematics program. Definitions have been presented that are valid in elementary as well as advanced mathematical work.

The need for a basic store of information and understanding relating to the operations is given importance. The information will enable the students to compute with efficiency and competency and to solve problems, both in school and out of school, thoughtfully and correctly. Automatic recall of basic number facts is one of the desired outcomes.

The refining of reasoning abilities in exploring logical and illogical relationships is also given much attention. Within the rigor of mathematical thinking processes, much attention is given to the precision in which ideas are presented and conclusions are reached.

Accompanying the high degree of competency sought is the confidence and positive attitude toward mathematics that is gained when the child fully understands his mathematical actions because of his knowledge of our number system.

The Curriculum

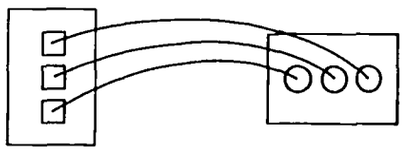
Kindergarten

Number and Numeration; Counting, Reading, and Writing

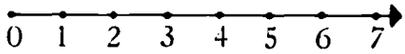
Explore the cardinal and ordinal ideas of numbers up to 10.
Encourage counting to 10.
Develop the idea that the last number counted tells how many (number property in the group).
Introduce the reading and writing of numerals through 10.
Develop the idea of "one-to-one correspondence."

The elements of the two sets are matched in one-to-one correspondence.

The two sets have a common property which is the cardinal number, 3.



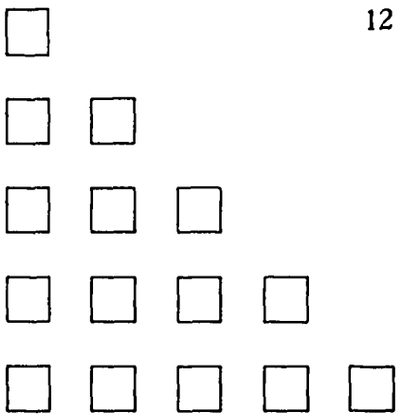
Explore and compare the number properties of groups of objects and pictorial representations of objects.
Combine simple sets of objects to produce new sets (sums less than 10).
Compare the number properties of sets of objects with the number property of a primary ruler, or a number line.



Sets; Language and Symbols

Build meaning for the term "set" by using it synonymously with "group."
Explore the idea of "sets"' (groupings, collections) with various objects.
Encourage the children to make and name sets of similar and dissimilar objects (physical level).
Place sets of objects, up to 10 objects, into order relationships (ordinal).

The physical objects explored in the manner illustrated depict some of the structural aspects of our number system. The "one more" idea is carefully developed.



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Identify, through counting, the number property of various sets.

Operations on Whole Numbers; Processing of Numerals

Develop familiarity with the counting numbers.
Explore the ideas of "greater than," "less than;" "bigger than," "smaller than;" "more than," "fewer than;" etc.
Develop familiarity with the numerals used for the counting numbers.
Explore the idea of "between-ness."

Operations on Fractions; Processing of Fractions

Develop the meaning of fractions as equal parts of a whole and equal parts of a group. Begin with objects, and then move in pictorializations.
Explore the idea that "wholes may be divided into many equal parts."
Introduce the meaning of *half* with objects.

Problem Solving

Encourage oral exploration of numerical problems that arise in school (attendance, cookies, milk, etc.).
Begin to develop relationships of size, number, and value.
Encourage comparing activities in the nontechnical language of the children.

Geometry and Measurement

Develop activities offering the children opportunity to create various ways of measuring (distance, weight, capacity, etc.).
Introduce common units of measure such as half-pint, quart, inch, foot, etc.
Explore the need and use of the calendar, clock, thermometer.
Work for recognition of likenesses and differences of various shapes (circle, ball, rectangle, triangle, square, wheel, etc.).

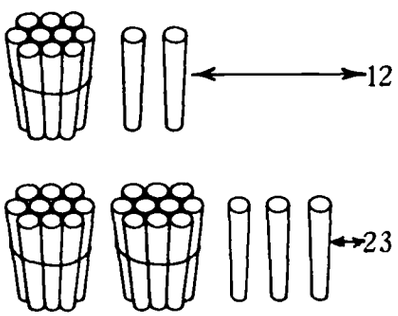
Develop familiarity with money used in classroom activities.
Stress comparative activities such as "bigger than," "smaller than"; "heavier than," "lighter than"; etc.

Grade 1

Number and Numeration; Counting, Reading, and Writing

Review the "cardinal" and "ordinal" ideas of numbers up to 10.
Develop cardinal and ordinal ideas of numbers up to 10 with sets, groups of objects, matching sets, and pictorial representations.
Develop counting through 100.
Represent larger numbers (11–100) with sets of 10's and 1's in packets of manipulative materials to show grouping in our number system.

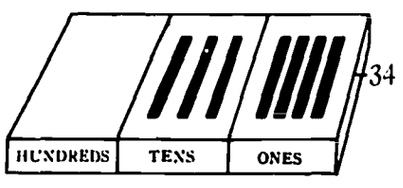
We have bundled together 10 sticks to indicate a group of 10. These activities with objects need careful development.



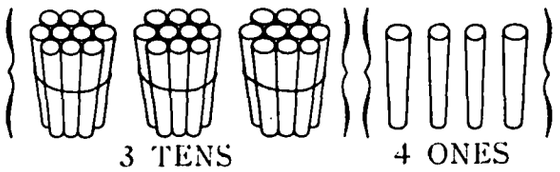
Develop the use of place-value devices with numbers 11–100 (do not use packets here).

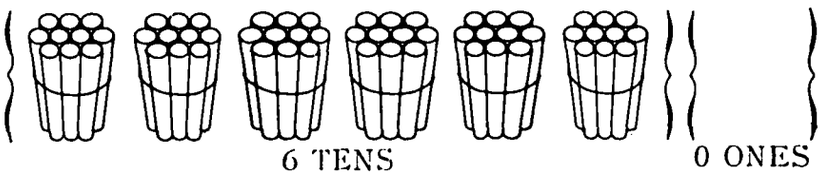
Place Value

The place-value box is an example of one-to-one correspondence when one object in the tens column is used to indicate a grouping of 10 objects.



At this point refer to the set of 10's and the set of 1's. The use of the numeral, 34, indicates there are 3 members in the set of 10's and 4 members in the set of 1's. In 60, there are 6 members in the set of 10's and the set of 1's is empty.

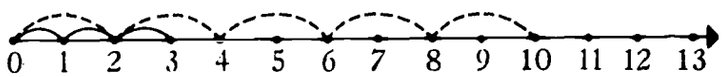




Children become familiar with the basic structure of our decimal system of numeration as they group by tens and learn the names of numbers through 100.

Stress the idea of "one-to-one correspondence" when you substitute a single object to represent a group of 10 objects.
Read and write numerals through 100.
Explore counting by ones on a number line or primary ruler that begins at zero. Then have the children count by twos on the same number line or ruler.

A Number Line



Sets;
Language and Symbols

Match sets for "one-to-one correspondence."
Review the ordinal nature of sets of whole numbers.
Introduce the idea that numerals such as 29 indicate there are 2 members in the set of 10, 9 members in the set of 1.
Review the idea of sets with children using objects and pictorial representations.
Explore the number properties of differing sets (cardinality answers "how many" are in the set).

Operations on Whole Numbers;
Processing of Numerals

Study the addition and subtraction facts with sums through 10.

Have children use objects as they begin quantitative explorations. Have them proceed from there to pictorializations, and then to abstract computations with numerals as the final step.
Explore sets of objects, pictorializations, and number lines to show the inverse or "undoing" idea.
Develop the notion that addition and subtraction are "doing-undoing" operations and are related.

"Doing–Undoing"

$3 + 2 = 5$
three plus two equals five
$5 - 2 = 3$
$5 - 3 = 2$

or

$(3 + 2) - 2 = 3$
$(2 + 3) - 3 = 2$

Introduce the use of the equal sign (=).
Encourage use of horizontal and vertical algorisms in addition with a maximum of 3 addends. Stress commutativity showing that the order of the addends does not change the sum.
Develop the commutative property of addition (order property).

Commutativity

$3 + 2 = 5$
$2 + 3 = 5$
$3 + 2 = 2 + 3$

Develop the associative (grouping) property of addition.

Associativity

$(4 + 3) + 2 = 4 + (3 + 2)$
$7 + 2 = 4 + 5$
$9 = 9$

The order in which 3 or more addends are added does not affect the sum.

Addition and subtraction through sums of 18 may begin at the latter part of the school year.

Operations on Fractions; Processing of Fractions

Develop with physical materials the meaning of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
Explore the notion of a unit fraction as being one of several equal parts of one, or parts of one group using objects, pictures, and a number line.

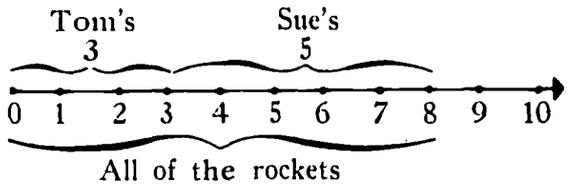
Problem Solving

Continue oral exploration of numerical problems which arise in the classroom.

Combine and separate sets of physical objects.
Create simple word problems that are suggested with groupings of physical materials and pictorializations.
Develop understanding of "larger than," "smaller than," "more than," "less than," etc.
Encourage the creation and use of patterns with objects and pictorializations.
Introduce the use of direct number sentences in adding and subtracting with answers less than 10. A direct number sentence shows the operation to be used.

Example
Tom has 3 rockets and Sue has 5 rockets. When they play together with all their rockets, how many do they have?

$3 + 5 = ?$ or



Geometry and Measurement

Continue to stress comparative activities, **greater than, less than,** and **same as, more than, many more than, a few more than.**
Explore patterns for likenesses, differences.
Work for recognition of squares, rectangles, circles, triangles, cubes, rectangular prisms, spheres, and pyramids.
Develop common units of measure such as time to the quarter hour, temperature, inch, foot, cup, pint, quart, gallon, pound, hour, day, month, year.
Work with money equivalents such as pennies, nickles, and dimes.
Stress building of a quantitative vocabulary.
Develop the ability to recognize 2 (twoness), 3 (threeness) at a physical level without counting.
Continue the use of pupil-created measurements (steps to cross the room, etc.).

Grade 2

Number and Numeration; Counting, Reading, and Writing

Develop the cardinal idea of numbers up to 100; review previous work.
Explore and develop the ordinal ideas of numbers up to the 31st.
Redevelop the place-value idea of representing numbers.
Count, read, and write numerals up to 500.

Emphasize

342 means 3 hundreds, 4 tens, 2 ones.
Read: three hundred forty-two.

With set terminology

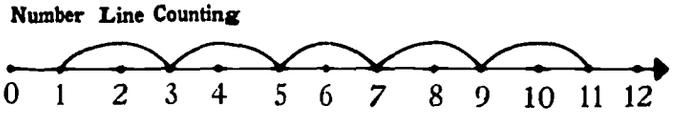
342 means 3 members in the set of hundreds,
4 members in the set of tens,
2 members in the set of ones.

Explore the various patterns inherent in number sequences (repeating numerals, sequencing, etc.).

Have the children look for patterns as in
1, 3, 5, 7, 9, 11, 15, 17, 19 ...
3, 5, 8, 12, 17, 23 ...

Count by 2's, 5's, 10's, progressing from objects to pictorializations, and finally to the number line.

Number Line Counting



from 1 by 2's

Explore Roman numerals through XII.

Sets; Language and Symbols

Combine and separate sets of 10 or less objects.
Explore commutativity with sets that are combined through addition and multiplication.
Develop the use of brackets for set notation {}.
Explore multiplicative aspects of sets (2 sets of 3 are equal to 1 set of 6).
Introduce numerals such as 450 as containing 4 members or elements in the set of 100's, etc.
Match sets of objects for comparisons of differences in the number properties.
Pictorially represent operations with sets using brackets {}.

Operations on Whole Numbers; Processing of Numerals

Addition and Subtraction

Inventory and reteach work of the previous grade; diagnose weaknesses, provide remedial work.
Build up through work with physical objects, pictorializations, and numerals, the addition and subtraction facts of sums through 18. Use direct number sentences.
Redevelop the commutative property for addition.
Redevelop the associative property for addition.
Add two 2-digit numerals renaming (without carrying) the sums using the vertical and horizontal algorisms.
Develop the idea of adding two 2-digit numerals with renaming ones (carrying).

Renaming

$$\begin{aligned} 12 + 15 &= (10 + 2) + (10 + 5) \\ &= 10 + (2 + 10) + 5 \\ &= 10 + (10 + 2) + 5 \\ &= (10 + 10) + (2 + 5) \\ &= 20 + 7 \\ &= 27 \end{aligned}$$

Subtract 2-digit numerals without renaming (no taking away; no "borrowing").

Without Renaming

$27 - 12 = 15$

$$\begin{array}{r} 19 \\ -8 \\ \hline 11 \end{array}$$

$38 - 21 = 17$

Develop subtraction from 2-digit numerals with renaming.

With Renaming

$$\begin{aligned} 27 - 8 &= (20 + 7) - 8 \\ 8 &= (7 + 1) \end{aligned}$$

$27 - 8 = 20 + 7 - 7 - 1$
$= 20 - 1$
$= 19$

Renaming is done in terms of 10's and 1's.
Many variations may be suggested by children exploring this process.

Explore inequalities in number sentences; introduce use of symbols (>) greater than, (<) less than.

Inequalities

$3 + 4$ and $3 + 2$ do not name the same number.
$3 + 4$ signifies a greater quantity than $3 + 2$,
or $3 + 4 > 3 + 2$.
$3 + 2$ is less than $3 + 4$ and may be symbolized.
$3 + 2 < 3 + 4$.

Encourage use of horizontal and vertical algorisms when subtracting.

Multiplication and Division

Relate multiplication to counting activities when counting by 2's, 5's, 10's.
Explore multiplication facts through products of 100 using the physical, pictorial, and numerical levels sequentially.
Explore the idea of commutativity with respect to multiplication.

Commutativity

$2 \times 3 = 6$
$3 \times 2 = 6$
$2 \times 3 = 3 \times 2$

The order of factors does not change the product.

Introduce the operation of division as the inverse (undoing) of multiplication.

Inverse Operations

When you multiply you have two factors with which to obtain the unknown product.

Factor		Factor		Product
3	×	2	=	?

When you divide you have one factor and the product. You seek an unknown factor.

Factor		Factor		Product
3	×	?	=	6
?	×	2	=	6

or

As a direct number sentence:

Product		Factor		Factor
6	÷	3	=	?
6	÷	2	=	?

Develop division facts having no remainders by directly relating them to the multiplication work.

Operations on Fractions; Processing of Fractions

Re-explore equal parts of one with physical materials.
Review the meanings of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
Introduce $\frac{1}{5}$, $\frac{1}{6}$ with physical materials, pictorial materials, and numerals sequentially.
Compare the various unit fractions relating to one certain whole (pie) and one certain group (blocks).
Encourage the recognition and naming of the unit fractions introduced.
Locate halves on a number line or primary ruler.
Explore *more than, less than,* and *between-ness.*

Problem Solving

Continue exploration of comparative relationships.
Continue to encourage the creating of simple word problems.
Continue the use of patterning with objects and pictures and drawings.
Develop the use of number sentences (direct) and correlate with computational activities.
Introduce number line as a device for solving problems.
Encourage estimating answers before computing. Make use of knowledge of place value.
Encourage looking at reasonableness of answers as a method of checking.
Mental computation should be encouraged in problem-solving and computational activities.
Encourage use of properties of order and grouping in numerical activities.

Geometry and Measurement

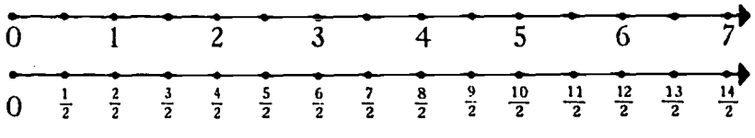
Inventory and reteach previous work if necessary.
Recognize geometric figures and how they are used.
Compare and create common geometric figures and describe their properties.
Use equalities and inequalities in comparing the various measures introduced in the previous grade.
Introduce seconds, minutes, 5-minute intervals, yard, dozen, week, teaspoon, tablespoon, ounces, cup.
Review the ability to recognize **twoness, threeness,** and introduce explorative activities to build up ability to recognize **fourness.**
Continue to build facility with quantitative terms.
Introduce money equivalents with a quarter and half dollar.
Develop use of a primary ruler.

Grade 3

Number and Numeration; Counting, Reading, and Writing

Inventory and reteach if necessary the ordinal and cardinal ideas of number; diagnose weaknesses, provide remedial work.
Extend work in cardinality and ordinality to 1,000.
Develop counting from any number to 10,000.
Develop reading and writing numerals to 10,000.
Extend counting by 2's, 5's, and 10's to include 3's, 4's, 6's, 7's, 8's, and 9's.
Explore the various number patterns caused by the varied counting activities.
Develop the concepts relating to the use of Roman numerals through C (100).
Encourage counting on the number line with halves and fourths.

Halves on a Number Line



Practice counting by halves from any designated point on the number line.

Sets; Language and Symbols

Illustrate commutativity and associativity with the use of actual objects, then follow with pictorial representations.
Provide varied explorations of the various kinds and purposes of sets of actual objects.
Continue the use of brackets for set notation.

Develop further explorations of the number properties of various sets.
Explore the non-numerical aspects of the various sets children organize.

Operations on Whole Numbers; Processing of Numerals

Addition and Subtraction

Inventory, reteach if necessary, the previous grade level's addition and subtraction work; diagnose weaknesses and provide remedial work.
Use direct and indirect number sentences; open and closed.
Present activities that continue to explore equalities and inequalities.
Add two 3-digit numerals without and with renaming (use of expanded notation).

Expanded Notation

$$
\begin{array}{rl}
345 & \text{3 hundreds, 4 tens, 5 ones} \\
+146 & \text{1 hundred, 4 tens, 6 ones} \\
\hline
 & \text{4 hundreds, 8 tens, 11 ones} \\
 & \text{or} \\
 & \text{4 hundreds, 8 tens, 1 ten and 1 one} \\
 & \text{or} \\
491 = & \text{4 hundreds, 9 tens, 1 one}
\end{array}
$$

Subtract two 3-digit numerals with and without renaming.

Multiplication and Division

Inventory previous grade level's multiplication and division work; reteach if necessary.
Develop multiplication and division facts with products through 81 (9×9).
Introduce the distributive law and begin to explore multiplication of 2- and 3-digit numerals by a 1-digit numeral (9 as limit).

Explore the division ideas such as:

a. How many times? b. What part of?

Stress the distributive law as "dividing a number by parts."

$$
\begin{aligned}
36 \div 3 &= (30 + 6) \div 3 \\
&= (30 \div 3) + (6 \div 3) \\
&= 10 + 2 \\
&= 12
\end{aligned}
$$

$$
3\overline{)108} \quad \text{or} \quad \begin{array}{r} 30 + 6 = 36 \\ 3\overline{)90 + 18} \end{array}
$$

$$
\begin{aligned}
&\frac{36}{3} \text{ or } \frac{30 + 6}{3} \\
&= \frac{30}{3} + \frac{6}{3} \\
&= 10 + 2 \\
&= 12
\end{aligned}
$$

Develop division of 2- and 3-digit numerals by a 1-digit numeral with the

answers having and not having remainders. (Use nonzero numerals.)
Use vertical and horizontal algorisms.
Explore short algorism for division.
Develop automatic recall of addition, subtraction, multiplication, and division facts.

Operations and Fractions; Processing of Fractions

Review the unit fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.
Explore **greater than, less than, between-ness, more than, smaller than.**
Re-examine the ideas of fractions being parts of a whole or parts of a group.
Explore the concept of *more than* or *less than* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
Use varied materials in the development of meanings of fractions.
Introduce eighths, tenths, twelfths, sixteenths meaningfully.
Develop meanings and encourage use of the terms **numerator** and **denominator:**

$\frac{5}{6}$ **numerator**
denominator

Develop meanings for fractions other than unit fractions ($\frac{2}{3}$, $\frac{3}{4}$).
Relate common fractions to parts of a dollar ($\frac{1}{4}$, $\frac{1}{2}$).
Relate these activities to the number line.
Explore the idea of equivalence as it pertains to fractions ($\frac{1}{2}$ and $\frac{2}{4}$ are names for the same number).

Problem Solving

Explore comparative relationships further.
Continue development of 1-step problems with the use of direct number sentences, such as:

26 children from our room and 28 from the room across the hall are going on a field trip. What is the total number of children going on the trip?

Encourage the children to think aloud as they work on problems.

26 + 28 = ?
(written as a direct number sentence)

Encourage mental computation and estimations before and after computing, using knowledge of place value.

49 + 32 =
49 is about 5 tens
32 is about 3 tens

Answer is about 8 tens, or 80.

Use number line as well as physical and pictorial materials to solve problems.
Start to build logical thought patterns with respect to numerical activities.
Build problems from the computational activities of this grade level.

"If I know parts (addends) of a number, I add."
"If I know a sum and an addend, I subtract to find an unknown addend."

Introduce simple, 2-step problems. Continue to use the direct number sentence as a way of expressing the problem.

Two-Step Problems

1. There are 29 children in our class. On Monday, 3 boys and 2 girls were absent. How many children were present on Monday?
2. How many children in all were absent?
   $3 + 2 =$
   $5 =$
3. How many were present?
   $29 - 5 =$
   $24 =$

Introduce use of verbal problems involving reading.

Geometry and Measurement

Inventory, reteach if necessary, recognition of common geometric figures.
Intuitively develop the ideas of points, lines, line segment, ray, triangles, polygon, square corners, rectangles, squares, and circles having common centers.
Review the visualization of twoness, threeness, fourness.
Develop the idea of fiveness without counting. Encourage the children to visualize larger groups of objects in terms of smaller groups.
Continue to stress the development of a quantitative vocabulary.
Review common measures in terms of equalities and inequalities.
Practice using and reading weight, temperature, ruler scales.
Review time telling to minutes.
Extend use and understanding of money and money equivalents up to 1 dollar.
Relate common fraction work to common measures.
Introduce addition with some common measures.

Adding Common Measures

2 quarts + 2 quarts = 4 quarts
4 quarts = 1 gallon

4 feet + 2 feet = 6 feet
3 feet = 1 yard
6 feet = 2 yards

Provide many opportunities to estimate in terms of common measures.

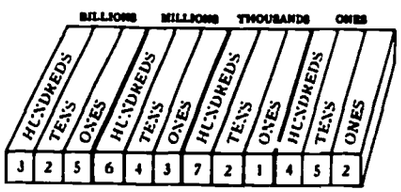
Grade 4

Number and Numeration; Counting, Reading, and Writing

Review the ideas of "cardinality" and "ordinality" previously developed.
Explore and compare the positioning of numerals in Roman and Arabic systems of numeration. 1.....10.....100
Practice counting by 2's, 3's, 4's, 10's.
Introduce the idea of odd and even numbers with physical objects leading to the exploration of numeral patterns. Children discover that even numbered sets of objects can be grouped by 2's without a remainder.
Reteach Roman numerals through C (100).
Read and write Roman numerals through MM (2,000).
Develop familiarity with large numbers. Stress that such place values are grouped by 3's.

Place Value

Groups of three digits are separated by commas.



325,643,721,452

Read and write numerals to 1,000,000.
Utilize number line, abacus, and other counting devices in numerical work after preliminary work with physical objects and pictures.

Sets;
Language and Symbols

Continue to provide activities for children, having them apply the term "set."

Provide the children with practice in using the term *set* through such activities as identifying the set of children in the room with blond hair:

$$\left\{ \text{Mary, Jane, Tom, Mark, Susan} \right\}$$

Identify the set of children responsible for the bulletin boards:

$$\left\{ \text{Barbara, Jane, Charles} \right\}$$

Point out that different sets may have common elements or members (such as Jane).

Illustrate physically and pictorially that differing sets may have common elements or members.
Continue the use of brackets.
Introduce the use of commas to separate the elements (members) of a set.
Employ set language to explore the concepts of odd and even numbers.

$$\text{Set A} = \left\{ 2, 4, 6, 8 \right\},$$

symbolizing the even numbers between 1 and 10.

$$\text{Set B} = \left\{ 2, 3, 4, 5, 6, 7, 8, 9 \right\},$$

symbolizing the whole numbers between 1 and 10.

Have the children note the elements common to both sets.

Introduce sets of numerals with the same number of numerals
a: alike except for the order they are in (equal sets)
b. not alike (equivalent sets)

Operations on Whole Numbers;
Processing of Numerals

Addition and Subtraction

Inventory, reteach if necessary, the previous grade level's addition and subtraction work; diagnose weaknesses, provide remedial work.
Stress application of properties of distributivity, commutativity, and associativity when adding.

"Right Hand" Addition

```
  342
  441
+632
-----
    5 sum of ones column
  110 sum of tens column
1,300 sum of hundreds column
-----
1,415 total
```

"Left Hand" Addition

```
  342
  441
+632
-----
1,300 sum of hundreds column
  110 sum of tens column
    5 sum of ones column
-----
1,415 total
```

Provide practice in column addition (5 or more addends).
Continue work in subtraction of 3-digit numerals.
Encourage the renaming of numbers as an aid to understanding and mental computation.
Encourage checking of work through examining the reasonableness of the answer.
Verify answers through the use of standard, computational, checking procedures.

Multiplication and Division

Inventory, reteach if necessary, the previous grade level's multiplication and division work; diagnose weaknesses, provide remedial work.
Complete the mastery of multiplication and division facts with products to 81 (9×9).
Gradually develop the standard multiplication algorism while introducing 2-digit factors; stress commutativity.

Commutativity

$3 \times 15 = 45$
$15 \times 3 = 45$
$3 \times 15 = 15 \times 3$

Distributivity

$$\begin{aligned} 3 \times 15 &= 3 \times (10 + 5) \\ &= (3 \times 10) + (3 \times 5) \\ &= 30 + 15 \\ &= 45 \end{aligned}$$

Demonstrate and explore the factor $\times$ factor $=$ product relationships in multiplication and division.
Develop the multiplication of 3-digit numerals by 2-digit numerals.
Basic division facts should be reviewed along with multiplication; extend facts to include products up to 144.

Introduce multiplication, mentally, by 10, 100, 1,000, and 20, 30, 40.....90.
Review and continue work with division of 2- and 3-digit numerals by 1-digit numerals, first without, then with remainders.
Introduce division with 2-digit divisors.

Developmental Algorism

```
   1
  20
23/489
 460   (20 × 23)
  29
  23   ( 1 × 23)
   6
```

489 = (460) + (23) + 6

Check: (21 × 23) + 6 = 489

Standard Algorism

```
   21
23/489
 46   tens (2 tens × 23)
 29
 23   ones (1 one  × 23)
  6
```

Check: (21 × 23) + 6 = 489

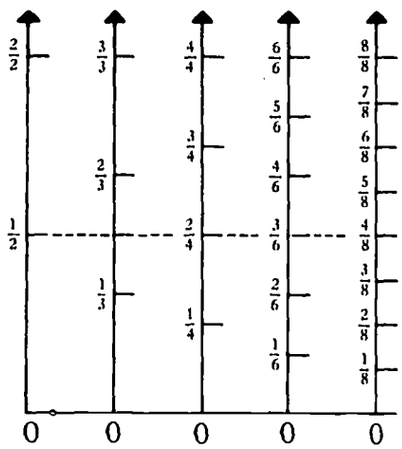
```
  23
× 21
---
   3    1 × 3
  20    1 × 20
  60   20 × 3
 400   20 × 20
 ---
 483
+ 6
 ---
 489
```

Encourage estimating as a way of checking and encourage judging the reasonableness of an answer. Verify answers through the use of standard, computational, checking procedures. Develop division skills gradually to build up to examples with two zeros in the quotient (product); then include examples with medial zero in the answer.

Operations on Fractions; Processing of Fractions

Review understanding of fractions through sixteenths as representing equal parts of a whole and equal parts of a group.
Utilize number line to show sequences of fractional numbers.
Use varied objects and devices to compare equivalent fractions. Compare differences among various fractions.
Review the comparisons among common fractions and parts of a dollar.
Develop the idea that common denominator means "common-sized part."
Introduce the idea of adding and subtracting common fractions having common denominators; use physical materials, pictorializations, and numerals in sequence to develop meaning of addition and subtraction of common fractions.

Compare Fractions



Develop methods of determining common denominators for fractions with unlike denominators.

$$\frac{1}{2} + \frac{1}{4} = ?$$

$$\frac{1}{2} = \frac{2}{4}$$

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{2+1}{4}$$

$$= \frac{3}{4}$$

Relate search for common denominators to the number line and parts of a circular figure.
Begin to develop the addition and subtraction of fractions with unlike denominators.

Decimals

Introduce decimal notation for tenths and hundredths.
Name tenths with decimal and fractional notation.
Explore the order relationships among common fractions and decimals ("greater than," "less than," "between-ness").
Develop addition and subtraction with tenths in decimal form.
After understanding of decimals to hundredths is developed, compare this type of notation to that of our monetary system.
Relate decimal activities to measurement work.

Problem Solving

Translate work with 1- and 2-step problems into *direct* number sentences.
Continue to develop comparative relationships.

Explore the idea of **average** and relate temperatures, test marks, etc.

Find the average spelling mark for a girl who had the following weekly marks: 70, 85, 90, 60, 90. A guess of 75 could be checked as in:

70	5 below		
85		10 above	
90		15 above	
60	15 below		
90		15 above	
	20 below	40 above	

75 is not great enough for the average. Perhaps 80 would be better:

70	10 below		
85		5 above	
90		10 above	
60	20 below		
90		10 above	
	30 below	25 above	

80 is closer to the average. It may be 78 or 79.

Finding Averages

```
  70
  85
  90   5)395
  60    35   tens (7 tens × 5)
+ 90    45
 395    45   ones (9 ones × 5)
```

79 is the average that was computed after estimating.

Introduce the making of pictorial representations of actual quantities and lead into graphing techniques.
Examine use of scale drawings as used on maps and room measurements.
Develop idea of making tabular arrangements of numerical data.
Continue use of verbal problems involving reading; develop mathematical reading skills:

1. Solving problems with no numbers,
2. Solving problems that contain unneeded numbers,
3. Exploring problems with not enough information.

Encourage estimations before and after computations.
Guide the children through analytic problem-solving activities.
Continue to look for patterns.

Geometry and Measurement

Review previous work.
Develop the concept of a *closed* plane figure.
Extend measurements of familiar objects, using common fractions to sixteenths and decimal fractions to tenths.
Continue intuitive explorations of common plane and solid geometrical figures. Use such words as **point, line, line segment, ray, angle, chord, diameter, edge, face, radius, side.** It would be appropriate to talk of sets of angles, sets of triangles, etc.

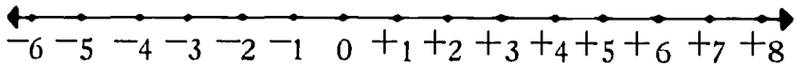
Utilize positions of lines in a plane and in space to develop intuitive understanding of such terms as **intersecting, parallel, perpendicular,** etc.
Review addition with common measures. Stress equivalents and nonequivalents. Introduce subtraction with common measures.
Develop idea of perimeter in common plane figures without the use of formulas, and using objects found in the classroom.
Begin to introduce the metric system of measurements with comparisons to units of measure already learned (linear measure).
Continue to stress the building of a quantitative vocabulary.
Explore idea of "between-ness" among common measures.
Provide many opportunities to estimate with common measures.

Grade 5

Number and Numeration;
Counting, Reading, and Writing

Develop ability to read and write up to 1,000,000,000.
Continue practice counting from any number by 2's, 3's, 4's ... through 10's.
Use a number line to aid in introducing negative whole numbers.

Use a number line that shows negative whole numbers (—5 is read as "negative five").



Introduce exponents, using powers of 10, as another way of naming numbers.

$10^1 = 10$
$10^2 = 10 \times 10 = 100$
$10^3 = 10 \times 10 \times 10 = 1{,}000$
$10^4 = 10{,}000$
$10^5 = 100{,}000$
$10^6 = 1{,}000{,}000$

Demonstrate and develop the idea that exponents may be utilized for naming large numbers.

Develop Exponential Notation

$$\begin{aligned} 1492 &= 1000 + 400 + 90 + 2 \\ &= (1 \times 1000) + (4 \times 100) + (9 \times 10) + 2 \\ &= (1 \times 10^3) + (4 \times 10^2) + (9 \times 10^1) + 2 \end{aligned}$$

(The last form is exponential.)

Introduce prime numbers and factors, and limit exploration to 100 as the upper limit.

Prime numbers are those whole numbers greater than one which can be divided only by one and themselves with no remainders.

Develop the idea that numbers other than one and the primes are composite numbers.
Introduce the use of a bar over Roman numerals($\overline{\text{XXIII}} = 23{,}000$) to denote *thousands*, while reviewing Roman numerals through M-M (2,000).

Sets; Language and Symbols

Review the ideas of equal sets and equivalent sets.
Continue with the use of brackets and commas in set notation.
Develop first with the use of objects, then pictorial materials, and finally, with set notations, the operations of union and intersection of sets.
Introduce the use of $\cup$ for symbolizing the union of two or more sets; the use of $\cap$ for symbolizing the intersection of two or more sets; the use of capital letters to symbolize specific sets.

Union of Sets

Let Set A = the set of all the boys in the class with blonde hair,
Set B = the set of all the girls in the class with blonde hair,

or

$$\text{Set A} = \{\text{John, George, Mark, Jim}\}$$

$$\text{Set B} = \{\text{Sally, Jane, Susan}\}$$

Then

$$\text{Set A} \cup \text{B} = \{\text{John, George, Mark, Jim, Sally, Jane, Susan}\}.$$

(The *union* of two sets contains all of the members of either set or both sets.)

Intersection of Sets

Let Set D = the set of all the children in the front seats
Set E = the set of all the children in the first row of seats

or

$$\text{Set D} = \{\text{Alice, Hilda, Carol, Tom}\}$$

$$\text{Set E} = \{\text{Grace, Harold, Terry, Bob, Alice}\}$$

(Note that Alice is a member of both sets.)

Then

$$\text{D} \cap \text{E} = \{\text{Alice}\}$$

(The *intersection* of two sets contains the members common to both sets.)

Operations on Whole Numbers; Processing of Numerals

Ensure recall and mastery of basic addition, subtraction, multiplication, and division facts to sums and products to 144 (12×12). If necessary, reteach and provide further practice. Diagnose weaknesses; provide remedial work. Continue maintenance of skills with column addition, subtraction, multiplication, and division.

Increase checking consciousness and facility through the exploration of differing methods of verifying answers.

Teach punctuation of number sentences using parentheses and processing the sentence. In $(3 \times 4) + 5 = n$, we do the work indicated by parentheses first. Teach the order of operations when working with a number sentence having more than two differing operations.

$3 + 6 \times 5 = 33$
$(3 + 6) \times 5 = 45$

If there are no parentheses in a number sentence, multiplication takes precedence over addition. If there are parentheses, do the work in the parentheses first.

$3 + 2 \times 5 \div 2 - 4 = ?$ Multiply (2×5) first
$3 + 10 \div 2 - 4 = ?$ Then divide $(10 \div 2)$
$3 + 5 - 4 = ?$ Add $(3 + 5)$ next
$8 - 4 = 4$ Subtract $(8 - 4)$ finally
4

Introduce work with factors in multiplication and division.

Sets of Factors

24

24×1
12×2
8×3
6×4

$\{1,2,3,4,6,8,12,24\}$ = set of all whole number factors for 24

29

29×1

$\{1, 29\}$ = set of all whole number factors of 29

Explore prime and composite numbers.

23 is a prime number
21 is a composite number, $3 \times 7 = 21$
3 and 7 are both primes, thus they are called the *prime factors* of 21.

Review mental multiplication by multiples of 10, 20, 100, 1,000.
Complete work with 2-digit divisors (factors) and 2- and 3-digit quotients (products).
Introduce 3-digit divisors (factors).
Provide for diagnosis of difficulties and remedial work.
Maintain skills previously acquired.

Operations on Fractions; Processing of Fractions

Review previous work done with fractions.
Explore **greater than, less than,** and **between-ness** with fractions.
Encourage the children to write sets of equivalent fractions.

$$\left\{\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16} \ldots\ldots\ldots\ldots\ldots\right\}$$

Reteach meaning of common denominator.
Review methods of determining common denominators . . . then develop algorisms for determining common denominator.
Introduce least common denominators.
Develop meanings for equivalents between improper fractions and mixed numbers.

$$\frac{3}{2} = \frac{2}{2} + \frac{1}{2} = 1 + \frac{1}{2} = 1\frac{1}{2}$$

$$\frac{15}{8} = \frac{8}{8} + \frac{7}{8} = 1 + \frac{7}{8} = 1\frac{7}{8}$$

and

$$1\frac{3}{4} = 1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}$$

$$2\frac{2}{3} = 2 + \frac{2}{3} = \frac{6}{3} + \frac{2}{3} = \frac{8}{3}$$

Introduce addition and subtraction of fractions with unlike denominators.
Introduce improper fractions and develop meanings with varied materials.
Use the physical, pictorial, and numerical sequence in introducing the multiplication of fractions by a whole number; fractions by fractions.
Develop factor $\times$ factor = product relationship on intuitive and pictorial levels.
Practice addition and subtraction of like fractions with horizontal and vertical algorisms; also use unlike and unrelated denominators.
Continue using parentheses in abstract number sentences. $(6 \times \frac{1}{2}) + 4 =$
Intuitively develop the operation of dividing whole numbers by fractions.

Decimals

Review addition and subtraction with tenths in decimal form.
Divide by 10's to explore ideas about decimals.
Stress equivalents with decimal and common fractions.

$\frac{1}{10} = .1$

$\frac{2}{10} = .2 \qquad \frac{1}{5} = .2$

$\frac{5}{10} = .5 \qquad \frac{1}{2} = .5$

Develop the *law for one* (principle of compensation) in the division of decimals.

Law for One

When you multiply the numerator and denominator by the same number, excluding zero, the relationship between the numerator and denominator remains unchanged. We may think of the dividend as the numerator and the divisor as the denominator in a division example. Actually we are multiplying by 1, the identity element for multiplication in our number system, and the relative values are unchanged.

$10 \div 2 = \frac{10}{2} = \frac{10}{2} \times 1$

$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} \ldots\ldots\ldots$

Applying the "law for one"

Example	*Multiplied by 1*		*Answer*
$\frac{10}{2}$	$\times \frac{2}{2}$	$= \frac{20}{4}$ or	5
$\frac{10}{2}$	$\times \frac{5}{5}$	$= \frac{50}{10}$ or	5
$\frac{10}{2}$	$\times \frac{7}{7}$	$= \frac{70}{14}$ or	5

With Decimal Fractions

$\frac{1.0}{.2} \times \frac{10}{10} = \frac{10}{2}$ or 5

The **law for one** applies to common as well as decimal fractions.

Introduce rounding of decimal fractions.
Develop decimal fractions to thousandths.

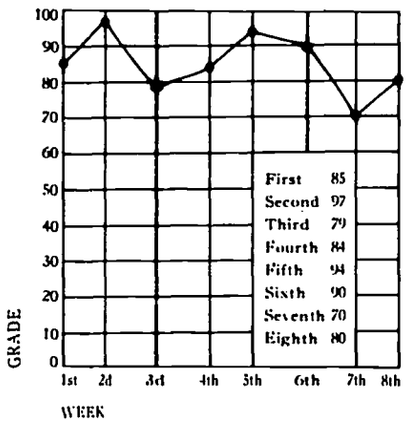
Introduce addition, subtraction, multiplication, and division to hundredths.

$$\begin{array}{r} 5.75 \\ +2.68 \\ \hline \end{array} \qquad \begin{array}{r} 10.00 \\ -3.92 \\ \hline \end{array} \qquad \begin{array}{r} 2.50 \\ \times 3 \\ \hline \end{array} \qquad 5\overline{)10.50}$$

Problem Solving

Discuss and state problems during problem-solving activities.
Encourage thinking about work before starting it.
Introduce the *indirect* number sentence (missing parts other than the answer), $4 + ? = 8$, $\square - 2 = 5$, ... $\times 9 = 7$_, $9 = 9 \div n$. Whenever possible make more than one number sentence (direct and indirect) to fit a given problem.
Continue work with 2-step verbal and nonverbal (number sentences) problems.
Develop use of parentheses in number sentence to fit 2-step problems.
Continue averages. Have children plot data on line and bar graphs.

My Weekly Spelling Grades



Present unorganized data for the children to organize and make tables.
Develop further the use of pictorial graphing techniques.
Estimations and mental computations should be encouraged.
Provide sets of problems with:

a. 2 or more true number sentences
b. 1 true, 1 false number sentence
c. all false number sentences

Write several number sentences for each problem.
Provide sets of problems with:

a. extraneous information
b. not enough information
c. reasonable data supplied by student
d. unrealistic information

Continue seeking numeral patterns.
Provide for reading of simple scale drawings, graphs, charts, and tabular data (timetables).

Geometry and Measurement

Review previous work.
Extend understanding and use of units of length, liquid measure, dry measure, time, weight, and money in common and metric measures.
Explore comparisons of **greater than, less than, equal to,** and **between-ness** among differing measures.
Begin to emphasize terminology related to common geometric figures (set terminology).
Relate pupil-derived formulas to previous exploratory work with perimeters.
Intuitively develop ways of measuring area of closed plane figures such as squares, rectangles, triangles, circles. Develop the idea of square measures as standard units (square inch, square foot, square yard, square mile).
Explore the ideas of **equality, similarity, congruency,** and **symmetry** with respect to plane figures; use set terminology.
Extend the use of decimal notation in measures to hundredths.
Continue to build and refine quantitative terminology.
Emphasize familiar renaming procedures in the addition and subtraction of denominate numbers.
Present open and incomplete geometric figures for intuitive exploration.
Introduce the use of the compass and protractor.
Work at estimating various quantitative measures.

Grade 6

Number and Numeration; Counting, Reading, and Writing

Review the reading and writing of numerals up to 1,000,000,000.
Develop the meaning of cardinal numbers through 1,000,000,000.
Introduce rounding numbers to the nearest 10, 100, and 1,000.

Rounding Numbers

When the last digit is 1, 2, 3, or 4, the ones, tens, or hundreds place is shown as zero.

51 becomes 50 rounded to the nearest ten
63 becomes 60 rounded to the nearest ten
740 becomes 700 rounded to the nearest hundred

When the last digit is 5, 6, 7, 8, or 9, the numeral in the tens place (hundred's place, etc.) is increased by one.

76 becomes 80 rounded to the nearest ten
98 becomes 100 rounded to the nearest ten
170 becomes 200 rounded to the nearest hundred

Relate this to the work of estimation and approximate computation.
Redevelop the use of exponents with powers of 10.

$5{,}682 = (5 \times 10^3) + (6 \times 10^2) + (8 \times 10^1) + (2 \times 1)$

Introduce the exponential form of powers of 2, 3, 5, and 8.

$5^3 = 5 \times 5 \times 5 = 125$

$4^4 = 4 \times 4 \times 4 \times 4 = 256$

Explore counting in bases other than ten with emphasis on the place-value idea (base two, three, four, five, and eight).

Base 8 and the Place-Value Idea

base 10	*base 8*	*read for base 8*
1	1	one
2	2	two
3	3	three
4	4	four
5	5	five
6	6	six
7	7	seven
8	10	one zero, which means one of the base and the absence of ones
9	11	one one, which means one of the base and one unit
10	12	one two
11	13	one three
12	14	one four
13	15	one five
14	16	one six
15	17	one seven
16	20	two zero, which means two of the base and the absence of ones
.	.	
.	.	
.	.	
.	.	
.	.	
63	77	seven seven
64	100	one zero zero, which means one base times base, the absence of bases and the absence of ones
.	.	
.	.	

Compare the number patterns and number meanings of these other bases with base ten.

base 2 1, 10, 11, 100, 101, 110, 111, 1,000....

base 3 1, 2, 10, 11, 12, 20, 21, 22, 100....

base 4 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30....

base 5 1, 2, 3, 4, 10, 11, 12, 13, 14, 20....

Review Roman numerals through MM and multiplicative bar ($\overline{XX} = 20{,}000$).

Sets; Language and Symbols

Review the developmental work with set terminology and notation.
Review the concepts of union and intersection.
Develop notation for the empty set (null set) as brackets { } or ϕ.

Empty Sets

E = the set of live elephants on teacher's desk = { } or ϕ
C = the set of children in the room with orange eyes = { } or ϕ

Explore the notion of subsets, use the symbol $\subset$ to designate " subset of."

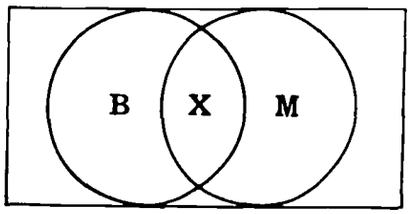
Set A = children in the classroom
Set B = boys in the classroom
Set G = girls in the classroom

All members of Set B are also members of Set A. Set B is a subset of Set A, expressed as $B \subset A$.

All members of Set G are also members of Set A. Set G is a subset of Set A, or $G \subset A$.

Introduce the use of diagrams (circular, rectangular, etc.) to demonstrate the relationships between sets.

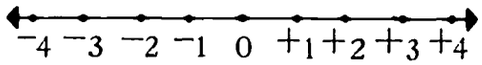
Set B = boys in the sixth grade
Set M = boys in the school band
Set X = sixth-grade boys in the school band



$B \cap M = X$

$X \subset B$ and $X \subset M$

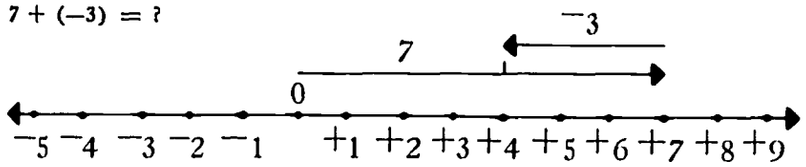
Explore set of integers as steps to the left and right of zero on the number line.



Operations on Whole Numbers; Processing of Numerals

Work toward mastery of basic addition, subtraction, multiplication, and division facts. Diagnose weaknesses, provide remedial work.
Further develop working with factors and products in multiplication and division.
Stress the order of operations when working with different operations in the same number sentence, using parentheses as punctuation marks.
Introduce the addition of integers as steps on a number line. Show use of number line in the operation of the addition of integers.

$7 + (-3) = ?$



$7 + (-3) = 4$

Stress applications of associativity and commutativity with integers.
Introduce prime factors by developing meaning.
Provide for the maintenance of skills previously acquired.
Maintain checking consciousness through varied techniques of verifying results.

Find the set of factors of 36.

36
36 × 1
18 × 2
12 × 3
9 × 4
6 × 6

The set of factors of 36 { 1,2,3,4,6,9,12,18,36 }
Unique prime factors (each only divisible by 1 and itself) of 36 are 2,2,3,3.
Note two different prime factors, 2 and 3.

Operations on Fractions; Processing of Fractions

Review previously learned fraction work.
Review addition and subtraction of common fractions, mixed numbers, and improper fractions.
Continue to compare common fractions and decimal fractions and to recognize relative position on a number line.
Review and drill with equivalent fractions.
Develop the idea that a rational number is a whole number or a quotient of whole numbers such as $\frac{2}{3}$, and introduce ratios in the form of fractional notation.
Develop **between-ness, greater than,** and **less than** with fractions.
Develop further the factor-product relationship among fractions.
Introduce the algorisms for multiplication of fractions, encourage children to pictorialize their answers, and demonstrate commutativity.

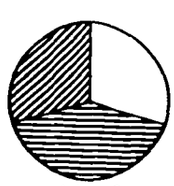
$$\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6} \text{ or } \frac{1}{3}$$

Commute

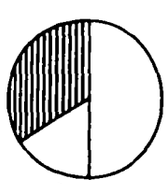
$$\frac{2}{3} \times \frac{1}{2} = \frac{2 \times 1}{3 \times 2} = \frac{2}{6} \text{ or } \frac{1}{3}$$

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{3} \times \frac{1}{2} = \frac{2}{6} \text{ or } \frac{1}{3}$$

Using Diagrams



$\frac{1}{2} \times \frac{2}{3}$



$\frac{2}{3} \times \frac{1}{2}$

Use the distributive property in the multiplication of mixed numbers.

$$2\frac{1}{2} \times 3\frac{2}{3} = 2\frac{1}{2} \times \left(3 + \frac{2}{3}\right)$$

$$= \left(2\frac{1}{2} \times 3\right) + \left(2\frac{1}{2} \times \frac{2}{3}\right)$$

$$= \left(\frac{5}{2} \times 3\right) + \left(\frac{5}{2} \times \frac{2}{3}\right)$$

$$= \frac{15}{2} + \frac{10}{6}$$

$$= \frac{15}{2} + \frac{5}{3}$$

$$= 7\frac{1}{2} + 1\frac{2}{3}$$

$$= 7\frac{3}{6} + 1\frac{4}{6}$$

$$= 8\frac{7}{6}$$

$$= 9\frac{1}{6}$$

In some cases it may be simpler to express mixed numbers as improper fractions.

$$2\frac{1}{2} \times 3\frac{2}{3} = \frac{5}{2} \times \frac{11}{3} = \frac{55}{6} = 9\frac{1}{6}$$

Provide practice in multiplying fractions and fractions; whole numbers and fractions; whole numbers and mixed numbers; etc.

Redevelop meaning of division of fractions stressing the factor-product relationship; utilize common denominator method; relate division to multiplication through the unknown factor approach.

Product ÷ Factor = Unknown Factor

$$\frac{3}{5} \div \frac{2}{3} = ?$$

Common Denominator Method

$$\frac{3}{5} \div \frac{2}{3} = \frac{9}{15} \div \frac{10}{15} = 9 \div 10 = \frac{9}{10}$$

Teach the meaning and use of the reciprocal when dividing by a fraction. Relate the **law for one** to reciprocals (multiplicative identity element; the neutral element).

The Reciprocal

In order to divide by a fraction, it should be determined how many times that fraction is contained in 1. The reciprocal of any fraction indicates the number of times that fraction is contained in 1. A fraction multiplied by its reciprocal equals 1.

Fraction Reciprocal Yields

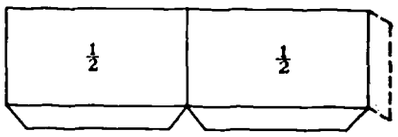
$\frac{1}{2} \times \frac{2}{1} = 1$

$\frac{2}{3} \times \frac{3}{2} = 1$

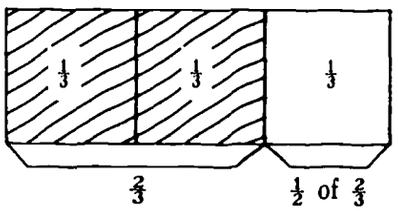
$\frac{8}{7} \times \frac{7}{8} = 1$

With Diagrams

There are two $\frac{1}{2}$'s in 1.



$\frac{2}{3}$ is contained in one $\frac{3}{2}$ or $1\frac{1}{2}$ times.



After understanding of the process has been developed, teach division involving fractions by using the **law for one** (reciprocal idea).

Law for One

Product Factor Unknown Factor

$\frac{3}{4} \div \frac{1}{2} = n$

How many $\frac{1}{2}$'s are there in $\frac{3}{4}$?

Multiply the known factor and product by the reciprocal of the known factor, applying the **law for one.**

$\frac{3}{4} \times \frac{2}{1} \div \frac{1}{2} \times \frac{2}{1} = ?$

$\frac{3}{4} \times \frac{2}{1} \div 1 = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

Decimals

Develop the principle of **law for one** in the division of decimal fractions.

$.24\overline{)25.48}$ or $25.48 \div .24 = ?$

Applying the **law for one,** multiply the factor and the product by 100.

$.24 \times 100 / \overline{25.48 \times 100}$

$24\overline{)2548}$ or $2548 \div 24 = ?$

Review addition, subtraction, multiplication, and division of decimals through hundredths.

$$\begin{array}{lll} 3.62 & 4.85 & 56.55 \\ +5.80 & \times\ .03 & -8.31 \end{array} \qquad 2.4\overline{)352.68}$$

Introduce the operations with decimals to thousandths.
Explore decimal **equivalence, less than, more than,** and **between-ness.**

Per Cent

Move from decimal notation to the concept of per cent; develop idea of per cent as a comparing process.
Compare fractions and decimal fractions to per cent.

$\frac{3}{100} = 3\%$ $\qquad$ $.06 = 6\%$
$.60 = 60\%$

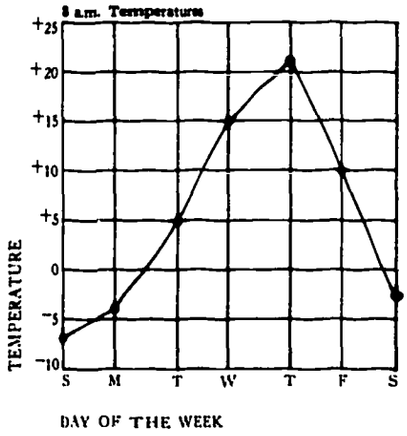
$\frac{100}{100} = 100\%$ $\qquad$ $\frac{1}{2} = \frac{50}{100} = 50\%$

Develop ideas of per cent with the object, pictorialization, and numeral sequence.
Work with simple examples, always stressing the hundredths aspect of per cent.
Explore cardinal relationships among per cents and common fractions.
Develop idea of rounding percentages ($33\frac{1}{3}\%$, $66\frac{2}{3}\%$, $8\frac{1}{3}\%$).
Compare ordinal relationships with whole numbers, decimal fractions, and percentages.
Relate finding a per cent of a number to factor $\times$ factor $=$ product idea. Also, relate to previous work with fractions and decimals. $\frac{1}{4}$ of $24 = n$ may be expressed as $\frac{1}{4} \times 24 = n$, or $0.25 \times 24 = n$.

Problem Solving

Review work of the previous grade level.
Refine graphing, scale drawing, tabulating, and organizing numerical data.
Continue problems and number sentences; direct and indirect, open and closed.
Develop 3-step problems using number sentences with parentheses.
Continue with analyzing various problem types.
Refine mental computation skill and estimating ability.
Review direct and indirect number sentences in abstract form.
Introduce ratios in problems.
Utilize problems involving the set of integers.
Explore typical business relationships in mathematical terms.
Continue to stress numeral patterns.

The temperature at the airport weather station, taken at 8 a.m., was —7 on Sunday, —4 on Monday, +5 on Tuesday, +15 on Wednesday, +21 on Thursday, +10 on Friday, and —3 on Saturday. Plot these figures on a graph.



Read and develop timetables.

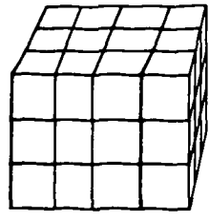
Geometry and Measurement

Inventory previous work. Stress estimating common measures.
Review pupil-derived formulas for finding perimeters and areas.
Stress properties (physical) of geometric figures such as squares, triangles, circles. Introduce the regular polygon.
Use objects and pictorializations to explore volumes of rectangular solids (rectangular prisms). Think in terms of layers.

12 cubic units in bottom layer
3 layers of 12 cubic units each
36 cubic units in all

or

3 × 4 (cubic units) = 12 (cubic units)
3 × 12 (cubic units) = 36 (cubic units)



Develop the use of cubic inch, cubic centimeter, cubic foot, and cubic yard when exploring the volume of rectangular solids.
Extend work with triangles to include the idea of measuring angles with a protractor.
Include construction activities with a compass, protractor, and straightedge (copying a triangle, bisecting a line, bisecting an angle).
Provide practice in adding, subtracting, multiplying, and dividing numbers that are measures of common objects.
Continue to explore the ideas of **equality**, **inequality**, and **between-ness**.
Continue to extend meanings for a quantitative vocabulary.
Continue to explore and create open and incomplete geometric figures by describing their physical properties.

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