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ABSTRACT

This paper summarizes the developmental activities in mathematics conducted at the first grade level. Two instructional units of "Developing Mathematical Processes" were taught. The instructional units are described through a description of the activities that were tried, an explanation of evaluation procedures, and a report of the results of each evaluation. The purpose of the pilot activities was to verify the appropriateness of a mathematics curriculum for first grade children; the data displayed are exploratory data which assess the feasibility of the instructional procedures. (Author/DT)

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Developing Mathematical Processes: Prototypic Tryout of Materials for First Grade Children, Huegel School 1968-69, Madison, Wisconsin

Report from the Project on Individually Guided Elementary Mathematics, Phase 2: Analysis Of Mathematics Instruction

Wisconsin Research and Development
CENTER FOR COGNITIVE LEARNING

THE UNIVERSITY OF WISCONSIN
Madison, Wisconsin

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DEVELOPING MATHEMATICAL PROCESSES: PROTOTYPIC TRYOUT OF MATERIALS FOR FIRST GRADE CHILDREN, HUEGEL SCHOOL 1968-69, MADISON, WISCONSIN

by

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Report from the Project on

Individually Guided Elementary Mathematics

Phase Two, Analysis of Mathematics Instruction

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This Working Paper is from Phase 2 of the Project on Individually Guided Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include indentifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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ABSTRACT

This paper summarizes the developmental activities in mathematics conducted in First Grade at Ray W. Huegel School, Madison, Wisconsin, during the 1968-69 school year. Two instructional units of Developing Mathematical Processes were taught. The instructional units are described through a description of the activities that were tried, an explanation of evaluation procedures, and a report of the results of each evaluation.

I

INTRODUCTION

The purpose of the pilot activities described in this paper was to verify the appropriateness of a mathematics curriculum for First Grade children. Effective instructional procedures previously developed were incorporated into a program titled Developing Mathematical Processes (DMP) (Romberg & Harvey, 1969). The reader should be cautioned that this document reports the activities of a pilot formative evaluation of the material; the data displayed are exploratory data which assess the feasibility of the instructional procedures, not the comparative effectiveness of the procedures.

OVERVIEW OF DEVELOPMENTAL PROCEDURES

Harvey, Romberg, and Fletcher (1969) have reported the steps involved in developing the instructional materials and procedures used in the study. Briefly, the content is described and then a task analysis is made. This involves a listing of the prerequisite behaviors needed to reach specified terminal objectives. Sets of behaviors are organized into sequential topics and then materials and activities are suggested. These are developed with the teachers and members of the project. Estimates are made for instructional time needed to complete each topic; however, classes were allowed to progress at their own rate. Table 1 indicates the staff estimate of number of days and actual number of days spent by three

Table 1

Instructional Time Spent Per Topic (Days)

Topic	Class B_1	Class A_2	Class B_2	Staff Estimate of Days Needed*
1	2	5	5	4
2	10	10	8	
3	4	4	4	3
4	3	3	2	3
5	2	2	5	2
6	7	6	4	4
8	5	2	2	1
9		2	5	3
10			6	1
11	2	4	5	1
12		4	4	3
7 & 13	11	16	12	
14	5	9	3	4
15	8	13	10	6
Orientation of Numerals	5	2	2	
16	5	9	2	4
17	2	2	5	
18	4	9	4	
19	4	3	4	
20	1	3	1	
21	27			

* Blanks indicate an estimate was not given for that topic.

B_1 - High Ability Group, No DMP A_2 - Low Ability Group, DMP
B_2 - Low Ability Group, no DMP

teachers on each topic.

During the school year 1968-69 these materials were tried out at the Ray W. Huegel Elementary School, a Madison school, located on the outskirts of Madison, Wisconsin. The staff, headed by Principal Jerry Johnson, is organized in the Multiunit plan (Klausmeier, Morrow, & Walter, 1968) with three units operating: Unit I, Initial Skills for Kindergarten and first and second years; Unit II, Intermediate Skills, for third and fourth years; Unit III, Independent skills, for fifth and sixth years. The materials were tried out with the five teachers of the Huegel staff teaching Level 1 (First Grade) students.

STUDENTS

Seventy-two children from four first-year level classrooms participated in the study. The students met for mathematics instruction 30 minutes each morning.

All of the students who participated in the study attended one of two Kindergarten classes at Huegel School the year prior to this. Of these two groups, one class had participated in this math program the previous year and the other had not (Romberg & Roweton, 1969). Therefore, each Kindergarten class was divided into high- and low-ability groups. Students were placed in the First Grade according to that grouping. Therefore, there were four First Grade groups, two high ability and two lower ability, with one of each having had the math program. Such an arrangement was agreed to in order to provide information on the program's applicability to both ability levels and would demonstrate the effectiveness of the Kindergarten math program.

TEACHERS

Five women teachers were involved in teaching the material. They had taught school an average of 2 years each. The teachers instructed the math classes and administered the individual evaluations for their own group. However, they discussed their problems with and were aided by staff members of the Research and Development Center (Miss Moscovitch and Mrs. Marilyn Roweton).

In January of the year, one of the classrooms (the high-ability group that did not have DMP) had a change of teachers. The classroom and the new teacher seemed to adjust well to the change.

With one of the teachers there was some disagreement over instruction. The teacher of the high-ability group that had the math program disagreed with the DMP staff on pacing and enrichment. She felt students needed more activities on new topics (vertical acceleration) while the staff felt the students needed expanded and enriched activities (horizontal acceleration). Therefore, her group was allowed to progress at a much faster rate than the other classes. Because of this pacing, the staff was not always able to keep ahead of her and provide her with materials. However, being an experienced teacher, she was able to provide for her class without a great deal of assistance. There were many occasions that the learning experiences and the items on evaluations differed from those used in the other classes. Therefore, data from that class are treated separately when appropriate. Also, data on instructional time spent per topic and from the California Achievement Test were not obtained for that class.

MATERIALS

The teachers were provided with a teachers' manual, physical teaching

aids, and work sheets. The teachers' manual included for each topic a set of objectives, a list of materials needed (physical aids), instructions on introducing the topic, and a list of planned activities.

FORMATIVE EVALUATION PROCEDURES

The main reason for gathering formative data is to revise materials. Three primary sources of information were used for that purpose. First, weekly meetings were held with the teachers. Each meeting was recorded and typescripts prepared. Second, teachers recorded systematic information in their manuals. The third souce of information was the evaluations of the children.

PRETEST INFORMATION

In September of 1968, a random sample of First Graders were given a pretest* to determine their mathematical background (see Table 2). Ten of the children were from the Kindergarten class (A.M.) that used the math program; eight were from the Kindergarten class (P.M.) that did not use the math program; and eight were new students. In most classes, the A.M. group did much better than the other two groups. As had been expected, the A.M. group had the highest percentage of total correct responses, the P.M. group was next, and the new students performed the worst. It was felt that since the same person taught both the A.M. and P.M. classes, some of the math material from the A.M. class was unintentionally presented to the afternoon (P.M.) class. This would account for the P.M. group obtaining scores in between the A.M. and new student groups.

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* The seven items for this pretest were selected from the set of items used to evaluate the Kindergarten program during the previous year. For details see Working Paper No. 24 (1969).

Table 2

Random Sample of 1st Graders, Pre-test

September 5, 1968

Classes	A.M.		P.M.		New	
Questions	#	Percent	#	Percent	#	Percent
1. Properties Classification	6	60	5	62.50	3	37.50
2. Ordering by Length	10	100	7	87.50	8	100.00
3. Compare Representation of Length	9	90	7	87.50	5	62.50
4a. Compare Sets of Objects	7	70	3	37.50	1	12.50
4b. Equalize Sets of Objects	7	70	3	37.50	1	12.50
5. Tally Sets of Objects	9	90	7	87.50	4	50.00
6. Compare Sets of Tally Marks	7	70	2	25.00	4	50.00
7. Compare Representations of Sets and Objects	5	50	2	25.00	0	0

A.M. class - Had Math Program in Kindergarten (10 students)
P.M. class - Didn't have DMP in Kindergarten (8 students)
New class - No Math Program (8 students)

II

INSTRUCTION

ARITHMETIC UNIT I: Assigning Numbers to Objects and Sets

TOPIC 1: IDENTIFYING PROPERTIES OF OBJECTS. After completing this topic, the student was expected to name objects, state their common properties, and understand and use the words "object," "property," "alike," and "different." Several objects were presented to the class for examination and manipulation to determine the properties each possessed. The student was expected to verbalize the properties the various objects had in common, as well as recognize those objects that had nothing in common. Various objects such as a ball, a box, a flannel board, felt cut-outs, and items located in the room, including the children themselves, were used in this manner. Teachers reported this to be a good lesson.

The felt cut-outs varied in color, width, length, and presence of a dot. Chidren were first asked to describe the properties of individual cut-outs, and later to state their likenesses and differences. A group game was also used in which one child, upon thinking of an object, would ask the class "What object is ___, ___, ___?" The class would then attempt to identify the object.

TOPIC 2: CLASSIFYING OBJECTS. The main objectives for this topic were

to teach children to classify objects according to like properties. A flannel board with felt cut-outs and envelopes with various paper cut-outs were among the materials used.

The teacher introduced classification to the children by arranging felt cut-outs into groups and telling the children what classification rule she was using. Later, the teacher classified the children, but rather than stating the rule, she had the children determine the classification rule themselves. This was found to be an interesting game for the children. However, difficulty was encountered when the children were allowed to group classmates according to their own classification rules because they were not consistent in what rule they were using. One teacher reported it was a very difficult section for the children and that there was not enough variety in the cut-outs.

EVALUATION

Children were individually tested over the objectives of Topics 1 and 2. These tasks were presented orally to each child. The testing time was approximately 5-10 minutes per child.

On the first task, the child was required to name the classification rule used in grouping plastic forms (varying in size, shape, and color). Second, the child was asked to group the forms according to another rule and to state his classification rule. On the third task, the forms were grouped using the dimension not previously used and the child was again asked to state the classification rule. The results are summarized for all four classes in Table 3. All but two children were able to answer the questions correctly. However, several children had difficulty in classifying and stating their own classification rule. It was suggested that the words "size" and "shape" needed better clarification. The two children that missed questions all came from the same classroom.

Table 3

Evaluation of Topics 1 & 2, First Grade, By Class

Task	A_1 n = 16		B_1 n = 22		A_2 n = 15		B_2 n = 19	
	#	%	#	%	#	%	#	%
1. State Teacher's Classification Rule (Color)	16	100	22	100	15	100	19	100
2. Classify and State Own Rule (shape or size)	16	100	22	100	15	100	17	89.47
3. State Teacher's Classification Rule (Color or size)	16	100	22	100	15	100	18	94.73

\# - Number of correct responses

A_1 - High ability, DMP

B_1 - High ability, No DMP

A_2 - Low ability, DMP

B_2 - Low ability, No DMP

TOPIC 3: COMPARING OBJECTS ON LENGTH. This section was devoted to teaching the child to compare objects on length by placing the objects side by side, and stating the relationship that exists, e.g., equality, greater than, less than. The materials consisted of small objects in the room and Numbars.[1]

The teacher introduced the topic by showing the children two objects of different lengths. It was emphasized that before comparing objects, the property to be used must be specified and it must be common to both objects. Next, the objects were placed side by side to determine which was longer.

In one activity, partners worked together and compared each other on length of feet, hands, fingers, arms, etc. Teachers reported that the children enjoyed this activity immensely.

TOPIC 4: EQUALIZING LENGTH. In these lessons, the children were taught to equalize objects that were unequal in length by "taking away" from the longer one or "adding to" the shorter one. Numbars, Lots-A-Links,[2] and Unifix Cubes[3] were used in the lessons.

In one activity, Lots-A-Links of assorted colors were distributed to each child. After classifying them by color, they were told to compare

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[1] Numbars, modified from Unit Blocks, Stern, and Gould Structural Arithmetic Materials (Boston: Houghton Mifflin). Description: wooden sticks in 10 different lengths, each a multiple of the smallest and each distinctively colored.

[2] Lots-A-Links, Amsco Industries, Inc. Description: colored, plastic ovals, that can be connected.

[3] Unifix Cubes, #TN 42-10 (London: Philograph Publications, Ltd.). Description: plastic cubes of assorted colors, same size, connect together to make bars of various lengths.

the length of the red chain with the length of the blue chain. After determining which was longer, they had to make the two chains equal in length. Unifix Cubes were also used in this manner. Teachers reported that Numbars were more difficult to work with since they are of varying lengths and cannot be separated into uniform parts.

TOPIC 5: ORDERING LENGTH. Teaching the students to arrange more than two objects from smallest to largest and from largest to smallest was the objective of this topic. Among the materials used were Numbars, Lots-A-Links, and various small objects.

Several Numbars were placed in random order on the board. The children were told they were going to make stairsteps with them. A child was asked to go to the board and pick out the longest of the Numbars and to place it off to one side of the board. (It was demonstrated to be the longest by individually comparing it with all the others.) Another child was selected to choose the largest of the remaining Numbars and to place it next to the first. The process continued until all Numbars had been chosen and ordered. Later the children were able to equalize the Numbars by putting others with the shorter Numbars.

TOPIC 6: REPRESENTING LENGTH. In this topic, it was pointed out that not all objects can be directly compared (side by side). After discussing this problem, the children were shown that a third object such as Lots-A-Links chain, covered meter sticks, or carpenter rulers could be used to represent one or both of the immovable objects.

The children were asked to compare such things as tables, the sizes of their waists, necks, etc., by using some of the materials previously mentioned. After measuring their waists, they then ordered the class from

largest to smallest. The teachers reported that the children were interested in length the entire time.

EVALUATION

At the end of the sixth topic, the children were evaluated on the objectives taught in the third through the sixth topics. Again the children were evaluated individually and questioned orally on four tasks. Unifix Cubes (all the same color), Lots-A-Links, and a sheet of construction paper with two white strips were used in the evaluation.

In the first task, bars of Unifix Cubes were presented to the child who was asked to order them by length. Task Two consisted of presenting the child with two bars of cubes and asking him first which is shorter and, second, having him demonstrate how he could make them equal in length. Next, the sheet of construction paper with two white strips was displayed in front of the child, who was asked to determine which strip was longer (by using Lots-A-Links chains). Equalizing by "taking away" was found to be more difficult than "putting with." It should also be noted that Class B_2 (slow, no DMP) was the only class that had children answering incorrectly. In that group, 11 children made the 13 errors. The data from this evaluation are summarized in Table 4.

TOPIC 7: TALLYING LENGTH. This topic will be discussed with Topic 13. Since the teachers reported that the children were having difficulty understanding the topic at this point, it was discontinued until after the children had been introduced to numerosity of sets.

TOPIC 8: IDENTIFYING MANYNESS OF SETS. The children were taught to recognize that sets may differ in number and that this is another property of the set. With a flannel board and felt cut-outs, the teacher displayed

Table 4

Evaluation of Topics 3-6, By Class

Task	A_1 N = 16		B_1 N = 22		A_2 N = 15		B_2 N = 19	
	#	%	#	%	#	%	#	%
1. Ordering Objects on Length	16	100	22	100	15	100	19	100
2. a) Comparing Length	16	100	22	100	15	100	19	100
b) Equalize by Putting With	16	100	22	100	15	100	15	78.94
c) Equalize by Taking Away			22	100	15	100	10	52.63
3. Comparing Representations of Length	16	100	22	100	15	100	19	100
4. Tally Number of Links in a Chain	16	100						

\# - Number of correct responses

A_1 - High ability, DMP

B_1 - High ability, No DMP

A_2 - Low ability, DMP

B_2 - Low Ability, No DMP

two groups of stars, unequal in number. Next the children were asked to name the properties of the group. Afterwards the teacher pointed out the significance of the numerousness of property. Numerosity or manyness of a set is not an obvious property of a set to most young children. The teachers indicated that the children liked using the word "numerousness."

TOPIC 9: COMPARING SETS ON MANYNESS. In comparing the manyness of sets, initially all objects used were uniform in size to prevent confusion with varying lengths. Felt cut-outs, beans, and discs were used in the lessons. The children were taught to compare sets by using one-to-one matching. If the first group (A) has objects left over, then it has more objects than the second group (B).

In the first lesson, two unequal sets were displayed on the flannel board; children were asked to match the objects of the sets one to one and then state one of the following relationships: $A = B$, $A > B$, or $A < B$.

The teachers reported that children enjoyed creating their own sets of objects to depict the relationship of one set to another. For example, one activity grouped all children in the class with red hair and another grouping consisted of those with brown hair. The class then compared the two groups to discover which group was larger or smaller.

TOPIC 10: EQUALIZING SETS ON MANYNESS. After the children were taught to compare sets, they were asked to equalize two sets by either putting more objects with the smaller group or taking objects from the larger group. Felt cut-outs, beans, and discs were used. Activities for demonstrating the topic followed the same procedure used with Topic 9. That is, two unequal sets were displayed. After the class determined what the relationship was between the two sets, they equalized the sets by "putting with"

or "taking from." Next, the children were required to check their accuracy by re-matching the objects in the two groups.

TOPIC 11: ORDERING SETS ON MANYNESS. Once the children were able to make comparisons between two sets on the property of manyness, they had no trouble ordering two or more sets from largest number to smallest number (or smallest to largest). Materials used in the activities were a flannel board, felt cut-outs, beans, and discs.

Four sets of 2, 3, 4, and 5 objects were placed on the flannel board. The children were asked to select the set with the greatest number of objects. (The question was repeated for the remaining sets.) The first set was placed to the far left and each additional set was placed next to it in descending order (largest to smallest). This procedure was also followed to order sets from smallest to largest. Individual and small group activities followed.

TOPIC 12: REPRESENTING MANYNESS OF SETS. This topic involved physically representing the number of objects in a set using felt cut-outs, beans, discs, etc. After this was accomplished, the children were required to compare, equalize, and order the physical representations of two or more sets that could not be matched one-to-one.

The class was read the Minnemast story of Ugboo's Big Problem (Minnemast, 1967b) as an introduction to the topic. The class discussed how Ugboo solved the problem using stones to represent the number of sheep he took out to the field and comparing this set with the number of sheep he brought back at night. In one of the activities, a group of children, represented with discs, was asked to leave the room. When they returned, the class was to determine if any were "missing" (by matching one to one).

One of the teachers reported that the children enjoyed the story, while another felt that the children could not relate to such an ancient era.

TOPIC 13: TALLYING NUMEROUSNESS OF SETS. In Topic 13, tally marks were substituted for the beans, discs, etc., used in Topic 12 to represent the number of objects in a set. The children were again asked to compare, equalize and order the representations of the sets. The Minnemast story of Tal's Aching Back (Minnemast, 1967a) was read to the class. The children were then asked to tally the numerousness property of the sets of beans, discs, etc. Teachers reported that children liked working in pairs to solve problems.

TOPIC 7: TALLYING LENGTH. Topic 7 was re-introduced at this time. Tally marks were used to represent the length of objects. A unit of length was chosen, such as Lots-A-Links, covered meter sticks, covered carpenter rulers, etc., and the children were to represent with tally marks the number of times the unit was read. The teacher then read the story Inch by Inch by Leo Lionni (Lionni, 1962) which relates how a worm "measures" objects by the length of his body. This story was then acted out by having one of the children pretend he was a worm. He then measured the length of two walls. The class then compared the tally marks to decide which wall was longer or shorter. The teachers reported that it was a problem getting the children to evenly space the tally marks in order to aid them in their comparisons.

EVALUATION

After completing Topic 7, the children were evaluated on the objectives of Topics 7 and 13. The children were individually evaluated and the test was administered orally. The testing took 10-15 minutes per child. Materials included beans, 8 felt stars, 15 paper clips, 2 red sheets of premarked construction paper in a plastic cover, and an evaluation card for each student. Separate data were recorded on six tasks.

For Task One, the child was asked to compare and equalize two sets of paper clips and in the second task, he had to order three sets of paper clips. The child had to represent and compare the numerousness of two sets of stars using beans for the third task. Tallying was the subject of the fourth task. The student was asked to represent the numerousness of two sets of stars using tally marks and then to determine which set was larger. Task Five involved measuring a sheet of paper with a pencil, and tallying the number of pencil lengths required to equal the length of the paper. To cover Topic 7, the child was presented with a chain of Lots-A-Links, and was asked to tally the number of links in the chain.

The children had difficulty in ordering sets. It was felt they needed more instruction on tallying length. As can be seen by Table 5, all children eventually answered the questions correctly. However, several needed help with the questions on tallying sets, comparing representations, and tallying length.

UNIT II: RECOGNIZING AND WRITING NUMERALS

TOPIC 14: COUNTING. At the end of these lessons, the students were required to verbally count up to ten objects in a group. Beans, discs, a flannel board, felt cut-outs, as well as the Minnemast story of Nat's Numbers (Minnemast, 1967c) were used in the lessons.

Activities included having the children verbally count the number of objects in a given set. "Zero" was introduced as the numeral which represents the empty set (contains no objects). Several finger games were used; e.g., Johnny Works With One Hammer, Beehive, How Many Times?

Table 5

Evaluation of Topics 7-13, First Grade, By Class

Task	A_1 n = 17			B_1 n = 20			A_2 n = 18			B_2 n = 16		
	#	%	Helped	#	%	Helped	#	%	Helped	#	%	Helped
1. Compare and Equalize Sets	17	100	0	20	100	0	18	100	1	16	100	0
2. Order Sets	17	100	0	20	100	0	18	100	3	16	100	1
3. Represent and Compare Sets	17	100	0	20	100	3	18	100	0	16	100	0
4. Compare Tallies	17	100	0	20	100	2	18	100	0	16	100	0
5. Tally Length of Paper	17	100	0	20	100	3	18	100	1	16	100	0
6. Tally Number of Links	17	100	0	20	100	0	18	100	0	16	100	0

A_1 - High ability, DMP

B_1 - High ability, No DMP

A_2 - Low ability, DMP

B_2 - Low ability, No DMP

Helped - Number of students helped in order to understand the question

\# - Number of correct responses

(ball bouncing), etc. The children enjoyed counting another child's hops, or the number of times he touched his toes. They also liked an activity involving partners; one child would tell the second child a number and the second child had to make a set containing that number of objects; they both checked the results.

TOPIC 15: RECOGNIZING WRITTEN NUMERALS 0-10. The objectives of this topic were as follows: the children were required to verbally state the number when presented with the written symbol, select the number of objects corresponding to the written number, select the correct written number corresponding to a given number of objects, and select the correct written numeral corresponding to a number word. Materials used included beaded numeral cards, blindfolds, Numbars, and a flannel board.

The beaded numeral cards were used for tactile sensation. One activity involved placing a certain number of objects on the flannel board and asking the children how many were there. Next the teacher wrote the written symbol on the board and also had them feel the beaded card. [The beaded numeral cards were used with blindfolds; the children in this case tried to identify the numeral by feeling its shape.] An activity similar to one used in Topic 14 was also used. The children were divided into small groups, each having a large rubber ball and a set of beaded numeral cards. One child bounced the ball a number of times and then selected another child to choose the card that corresponded with the number of times he bounced the ball. One of the teachers reported that the children also enjoyed using musical instruments (drums, cymbals, and triangles) in place of bouncing the ball. All agreed that the children enjoyed this section.

ORIENTATION OF NUMERALS. Between Topics 15 and 16, a section on orientation of numerals was presented. The main purpose of the section was

to help the children recognize the correct direction or positioning of the numerals, since children have a tendency to reverse the direction of some numbers when first learning to write them. The children were taught that direction is a "property" of shapes. Activities included having the children group a number of objects on their direction property. As another aid in learning direction of numerals, paper clips were placed on the beaded numeral cards to indicate the top. Some of the teachers reported that their children did very well on this section.

TOPIC 16: WRITING THE NUMERALS 0-10. Topic 16 was devoted to teaching the students to write the numerals 0-10 without using guidelines. Beaded numeral cards were used to review the numeral symbols. Numeral practice sheets containing a numeral in solid and then dotted lines which gradually diminished to no guide lines were given to the children for individual writing practice. Another activity again made use of a rubber ball; the teacher bounced the ball and the children wrote the numeral representing the number of times it was bounced. If a child wrote the correct response, the teacher gave him two taps on the head. If the numeral was written backwards, he received only one tap. The children were reported to enjoy a game in which they were to divide into three teams; each child was given a beaded numeral card face down and discs. On the word "go," they had to look at the card, decide which team was theirs, and arrange themselves in order. The first team to arrange itself or the team with the least number of errors was the winner. Another game reportedly enjoyed by the children involved beaded cards, pencils, and paper. The teacher held a beaded card

up for one row to see. The row clapped their hands the number of times shown on the card, while the other children wrote the numeral to match the number of times the row clapped their hands.

TOPIC 17: REPRESENTING NUMBERS USING WRITTEN NUMERALS. After completing this topic, the children were expected to be able to represent the number of objects in a set by using the correct numeral. Unifix Cubes beans, discs, felt cut-outs, or Lots-A-Links were used in these lessons. The child was required to write the numerals himself; he was not given any cards to use.

An exercise used in connection with Topic 17 made use of the flannel board. Objects were shown on the flannel board and one of the children was asked to write the corresponding numeral. The child then created a new set and called on another classmate to write the number. In another activity, the teacher gave the children paper and had them fold it into squares. The teacher wrote a numeral in each square on the chalkboard and told the children to draw the number of objects in the corresponding squares on their paper to match the numerals on the board.

EVALUATION

After completing Topics 14 through 17, an evaluation was given. The evaluation was divided into two parts.

In Part I, which was group administered, a pad of paper was distributed to each child. A list of numerals was read to the group in a given random sequence and the children were asked to write the numeric symbol. Each page was to contain only one numeral and each number was read three times. Each response was scored for correct shape (the form or contour of the numeral) and for correct orientation (the direction the numeral was facing).

Results of Part I indicate that the children had little difficulty producing correct shapes or correct orientations. (See Table 6.) Those numerals most often reversed were the 2 and 3. Some children also had more difficulty with orientation than with shape (86.7% of the children correctly shaped the numbers, whereas only 70.7% correctly oriented the numbers). Of the total mistakes made, 2.9% were reversals and .8% were errors in shape.

There was a very high increase in correct responses over last year's results. Last year, 73% of the children made correct shape responses to all of the numerals, compared to this year's 86.7%. Last year only 15% of the children had the correct orientation on all of the numerals, whereas this year 70.7% made no orientation errors.

Part II was administered individually. Testing time per child was approximately 10 minutes. Tasks One through Three involved placing a certain number of objects on the table and asking the child to state the number and write the appropriate numeral. Tasks Four through Six consisted of writing a numeral and asking the student to display a set of beans or discs which represented that numeral.

Results on Part II indicate that the children experienced no difficulty translating the numerals; i.e., equating the numeral to objects. (See Table 7.)

UNIT III - WRITING SENTENCES

TOPIC 18: WRITING THE SYMBOLIC MATHEMATICAL VERBS > AND < . The objectives of this topic were to teach the children to compare two objects, sets, or numerals and to write a mathematical statement of their numerical

Table 6

Evaluation of Topics 14-17, Part I, First Grade, By Class

Numeral Written	Correct Shape								Correct Orientation							
	A_1 n = 16		B_1 n = 23		A_2 n = 19		B_2 n = 16		A_1 n = 17		B_1 n = 23		A_2 n = 19		B_2 n = 16	
	#	%	#	%	#	%	#	%	#	%	#	%	#	%	#	%
1	51	100	69	100	57	100	48	100	51	100	69	100	57	100	48	100
2	51	100	68	98.55	57	100	48	100	49	96.07	69	100	49	85.96	45	93.75
3	51	100	69	100	57	100	48	100	48	94.11	64	92.75	53	92.98	46	95.83
4	51	100	69	100	56	98.24	48	100	51	100	69	100	57	100	45	93.75
5	51	100	69	100	57	100	47	97.91	51	100	67	97.10	54	94.73	45	93.75
6	51	100	69	100	56	98.24	47	97.91	48	94.11	67	97.10	55	96.49	43	89.58
7	51	100	67	97.10	57	100	48	100	51	100	69	100	54	94.73	44	91.66
8	49	96.07	66	95.65	57	100	46	95.83	51	100	69	100	57	100	48	100
9	51	100	67	97.10	56	98.24	48	100	51	100	68	98.55	54	94.73	48	100
10	51	100	68	98.55	57	100	47	97.91	51	100	69	100	56	98.24	44	91.66
Total	508	99.6	681	98.69	567	99.47	475	98.95	502	98.43	680	98.55	546	95.78	456	95.00

- Number of correct responses

A_1 - High ability, DMP

B_1 - High ability, No DMP

A_2 - Low ability, DMP

B_2 - Low ability, No DMP

Table 7

Evaluation of Topics 14-17 Part II, First Grade

		A_1		B_1		A_2		B_2	
Write Numerals Represented With Physical Objects		n = 17		n = 23		n = 19		n = 16	
		#	%	#	%	#	%	#	%
1	7	17	100	23	100	18(R)	94.7	15(R)	93.8
2	9	16(R)	94.1	23	100	17(R)	89.5	15(IS)	93.8
3	4	17	100	23	100	19	100	15(R)	93.8
Represent Written Numeral With Physical Objects									
4	3	17	100	23	100	19	100	16	100
5	8	17	100	23	100	19	100	16	100
6	0	17	100	23	100	19	100	16	100
Totals		101	99.01	138	100	111	97.36	93	96.87

R - Reversals
IS - Incorrect Shape

relationship ("less than" <, "more than" >). To introduce the symbols, the teacher gave background information on the symbols and how their meaning was developed. Materials that were used to illustrate the meaning of the symbols were a flannel board, felt cut-outs, Numbars, Lots-A-Links, discs, sign cards, etc. Worksheets were used which gave lists of two numerals but omitted the symbol. The children were to decide the relationship and then write in the missing symbol. After the children had mastered the symbols, story problems were introduced to them. They used discs, etc., to represent the numbers involved in the problem and to solve the problem. The teachers suggested that better activities were needed for this topic.

TOPIC 19: WRITING THE MATHEMATICAL RELATIONS = and ≠. The objectives of Topic 19 were to teach the child to compare two sets or two lengths, to decide whether they are equal or unequal in number or length, and to write the corresponding mathematical statement correctly. He should be able to use the symbols =, ≠, <, and > correctly. A flannel board, a card with the equal sign on it, a card with the unequal sign on it, Numbars, and discs were used in the activities. In one exercise, the teacher placed two sets of objects on the flannel board and asked the class to raise the equal card if the sets were equal in number or the unequal card if they were not. Later, the students took turns at making the sets. After they were able to use these signs correctly, the unequal sign was replaced by the "more than" (>) and "less than" (<) signs. One teacher complained that the >, < symbol cards could be replaced by one card, or that the top and bottom of the cards should be differentiated with a paper clip.

TOPIC 20: WRITING THE OPERATIONS + and −. For Topic 20, the student

was required to complete the following sentences correctly:

$a + \boxed{?} = b$ and $a - \boxed{?} = b$

Numbars, discs, beans, a flannel board, felt numerals (including + and - signs), and symbol cards were used in the lessons. Two sets of objects, unequal in number, were placed on the flannel board. The students were asked to compare the sets and to answer how many objects had to be "put with" or "taken away" from the other set in order for them to be equal in number. The process was also presented using numerals, with the + and - signs being introduced. For practice, the students were asked to construct unequal sets and lengths, to compare and equalize them, and then to write a mathematical sentence describing the process. Weight scales were also used to demonstrate the process of equalizing. These scales were reported to be very successful in the + and - equations.

TOPIC 21: WRITING A COMPLETE MATHEMATICAL SENTENCE EXPRESSING AN EQUALITY. At the conclusion of Topic 21, students were to be able to use the symbols +, -, and = to write a complete mathematical sentence. They were also to demonstrate the validity of the statement using objects, sets, or tally marks (i.e., demonstrate that the equation was true). Materials used included Numbars, beans, discs, worksheets, Unifix Cubes, and sign cards. After comparing and equalizing sets, the students used sign cards to symbolically express a mathematical equation. Next they were required to "prove" the statement was true by manipulating objects or sets, and finally to write the sentence with paper and pencil. Worksheets containing mathematical statements with either a missing sign or a missing numeral were given to the children as practice material. They could use objects and sign cards to demonstrate the correctness of their statements.

The teachers admitted they were amazed with the ability of the students to write mathematical equations. Some of the teachers used story problems in their lessons. Children enjoyed making their own story problems. One class was taught to write both left- and right-handed sentences simultaneously. However, since the children became somewhat confused, it was decided that right-handed sentences (6 = 3 + ___, operation occurs on the right side of the equality symbol) would be taught first and left-handed sentences later. One problem encountered was when a numeral was to be "taken away" from the left side; the children had a tendency to "take away" the wrong number, e.g., given 6 = 3, the children would write 3 - 6 = 3.

EVALUATION

After completing Topics 18 through 21, an oral evaluation was given to each child individually. Testing time per student was approximately 15 minutes. Summary data are presented in Table 8. However, due to the number of errors, more detailed data are presented in Table 9. Although there were only four kinds of questions, 23 separate responses were coded.

In the first task, the child was given paper and pencil and asked to write the symbols =, +, >, -, ≠, < in that order. In Task Two and Three the child was asked to equalize Numbars and sets of discs and then write the corresponding equations. In Tasks Four and Five, the student was presented with a written statement; e.g., 8 = 6, and asked to complete it and demonstrate its validity using Numbars and discs. In Task Six, the child was presented with a story problem. He was given porcelain objects, paper, and pencil to use in solving it.

Table 8

Evaluation of Topics 18-21, First Grade, By Class

Task	A_1 n = 17			B_1 n = 24			A_2 n = 19			B_2 n = 19		
	#	%	H	#	%	H	#	%	H	#	%	H
1 Writing Symbols	136	100	0	172	89.58	4	140	92.10	3	122	80.26	4
2 Completing Equations	102	100	3	239	99.58	5	183	96.31	13	179	94.21	11
3 Demonstrations	102	100	0	144	100.00	0	114	100.00	1	111	97.36	2
4 Story Problem	51	100	0	72	100.00	0	57	100.00	6	57	100.00	1
Total	391	100	3	627	96.75	9	494	96.29	23	469	91.42	18

H - Helped
A_1 - High Ability, DMP
B_1 - High Ability, No DMP
A_2 - Low Ability, DMP
B_2 - Low Ability, No DMP

Table 9

Evaluation 5 Detailed Data

n = 79

Task	Last Year's %'s	Children Answering Correctly (No.)	Percent	Extra Help
I. Writing Symbols				
=	85	78	98.73	0
+	78	77	97.46	1
>	59	56	70.38	3
−	81	79	100.00	0
≠	67	68	86.07	1
<	56	57	72.15	2
Comparing Objects and Selecting Correct Symbol				
1. ≠ or >		77	97.46	4
2. ≠ or <		78	98.73	3
II. Completing Equations				
1. With Objects				
a. 7 = 3 △□ } bars	100	79	100	2
b. 7 △□ = 3 } bars	85	79	100	1
c. 1 = 4 △□ } discs	100	79	100	1
d. 1 △□ = 4 } discs	96	79	100	1
2. With Sentences (n = 62)				
a. 7 = 3 △□		59	95.2	13
b. 7 △□ = 3		59	95.2	13
c. 1 = 4 △□		57	91.9	12
d. 1 △□ = 4		59	95.2	16
3. Written Sentences				
a. 8 = b	85	76	96.20	3
b. 2 = 3	89	77	97.46	8
III. Demonstrating Truth of Completed Equation				
1. With Objects				
a. 7 = 3 △□ } bars	96	79	100	1
b. 7 △□ = 3 } bars	63	79	100	1
c. 1 = 4 △□ } discs	100	79	100	1
d. 1 △□ = 4 } discs	96	79	100	2
2. Written Sentences				
a. 8 = 6	81	78	98.73	0
b. 2 = 3	81	77	97.46	2
IV. Story Problem				
1. Writing Equation (10 = 7△□ , 7 = 10△□)		79	100	7
2. Verbal Response		79	100	1
3. Demonstration		79	100	0

The results indicated that the children especially had difficulty writing the symbols > (62.9% wrote it correctly), < (64.5% wrote it correctly), and ≠ (82.3% wrote it correctly). However, when they had to use two of these symbols in comparing objects, they did much better (96.8% and 98.4% wrote the correct symbol). The children seemed to have no trouble working with objects. However, when they had to write sentence equations, their accuracy decreased.

Comparing the percentages (see Table 9) with last year's group (1967-1968), one can see that the children performed much better this year. Although it was not a great increase, the percentages of correctly written symbols (>, ≠, and <) did increase. Also, there was a percentage increase in completing left-handed equations correctly and in writing sentence equations. Additional Topics (22, 23, 24, and 25) were prepared, but these were not used in all the classes because of insufficient time.

BRIEF DESCRIPTION OF UNIT IV: Grouping and Place Value

Of those that advanced to this point, one teacher felt it was a rather dull unit as compared with the others. Topic 22 was mainly concerned with grouping objects by tens. The children were to be given a large number of objects to group into tens and were to verbally describe the grouping in terms of tens and ones; e.g., 4 groups of ten and 7 ones. In Topic 23, the students were to learn how to express numbers up to 100 in expanded notation; e.g., m(10) + n(1). The activities are the same for both Topics 22 and 23; however, in 23 the student was to record the groups in expanded notation. Topic 24 dealt mainly with expressing numbers in compact notation and converting numerals from one form of notation to another.

In Topic 25, students were to group objects into hundreds and thousands and to write the numerals up to 9999 in expanded and compact notation.

Expanded notation, since it describes precisely what is seen when grouping by tens, will hopefully help children better understand compact notation.

FINAL EVALUATION

The addition section of the California Achievement Test (Test 4, Section C) was given to the children in June of 1969. (See Table 10).

In comparing the two higher ability classes, one can see that the group (A_1) that had the DMP math program in Kindergarten generally did better. The same can be said of the lower ability groups; A_2 did somewhat better than B_2. However, the higher ability children maintained their standing when compared with those children of lower ability that had the math program in Kindergarten (B_1 did better than A_2). The mean number of correct responses for the total group was 21.68 (highest score possible was 25). This score gave them a grade placement of 2.5; i.e., second year, fifth month. Comparing them with the norm group (1963) of high ability First Grade children, they placed within the 94th percentile. This was a substantial increase over the pilot group's (1967-1968) percentile ranking of 85 (Romberg & Roweton, 1969). This performance of the groups on the standardized test was very pleasing.

Table 10

California Achievement Test (Test 4 - Section C)

Mean Number of Correct Responses

	Mean Number Correct (25 Problems)	Number Facts (20 Problems)	Two-Place Addition (5 Problems) (21-25)	Adding Zeros (4 Problems) (6, 8, 11, 18)	Carrying (2 Problems) (22, 25)
B_1 n=20	23.0	20.0	3.0	4.0	.35
A_2 n=18	21.06	18.56	2.50	3.72	.22
B_2 n=18	20.83	18.56	2.28	3.78	.22
Total Mean	21.68 94th Percentile Rank	19.07	2.61	3.84	.27

Given June 3, 1969
Norms - 1963

A_1 - High ability, DMP
B_1 - High ability, No DMP

A_2 - Low ability, DMP
B_2 - Low ability, No DMP

III

SUMMARY

The intent of this report is to summarize the developmental work associated with the use of DMP prototype materials constructed and pilot tested the previous year in the same school (Romberg & Roweton, 1969). The formative evaluation supports the contention that teachers using the materials and procedures outlines were successful in getting students to acquire most of the stated objectives. Most important information was used to improve the materials--teachers' guides, pupil materials, and evaluations. The revised materials include a wider variety of activities, the reordering of some topics (like tallying) and the preparation of more comprehensive evaluations.

Only by trying materials out in schools with regular teachers and students can valid data be gathered to provide information about the utility of the materials and to suggest changes in order to construct improved materials.

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