DOCUMENT RESUME

ED 066 311 SE 014 455

TITLE	Suggestions on the Teaching of College Mathematics.
INSTITUTION	Mathematical Association of America, Berkeley, Calif. Committee on the Undergraduate Program in Mathematics.
PUB DATE	Jan 72
NOTE	39p.
AVAILABLE FROM	CUPM, P. O. Box 1024, Berkeley, California 94701
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	*College Mathematics; *Course Organization; Effective Teaching; *Instruction; *Mathematics Education; Teacher Education; Teaching Methods; Teaching Quality
IDENTIFIERS	CUPM

ABSTRACT

This handbook on effective college mathematics teaching offers suggestions on course planning; lecturing; using the blackboard, textbooks, and visual aids; making assignments; scheduling, composing, and grading tests; and giving grades. It also discusses feedback and instructor evaluation and includes a sample questionnaire that might be given to students. A section on the role of teaching assistants suggests that a program of regular visitation, evaluation, orientation, and guidance by regular faculty members be followed. The final section lists professional journals and periodicals concerned with higher education, and gives a bibliography of books on mathematics, mathematics history, and mathematics teaching. (DT)

COMMITTEE ON THE UNDERGRADUATE
PROGRAM IN MATHEMATICS

SUGGESTIONS

ON THE

TEACHING OF COLLEGE MATHEMATICS

January, 1972

MATHEMATICAL ASSOCIATION OF AMERICA

SUGGESTIONS ON THE TEACHING OF COLLEGE MATHEMATICS

A Report of the Panel on College Teacher Preparation

COMMITTEE ON THE UNDERGRADUATE PROGRAM
IN MATHEMATICS

January, 1972

The Committee on the Undergraduate Program in Mathematics is a committee of the Mathematical Association of America charged with making recommendations for the improvement of college and university mathematics curricula at all levels and in all educational areas. Financial support for CUPM has been provided by the National Science Foundation.

Additional copies of this report may be obtained without charge from CUPM, P.O. Box 1024, Berkeley, California 94701.

COMMITTEE ON THE UNDERGRADUATE PROGRAM
IN MATHEMATICS

Alex Rosenberg, Chairman
Cornell University

Grace E. Bates
Mount Holyoke College

Ralph P. Boas
Northwestern University

D. W. Bushaw
Washington State University

Llayron L. Clarkson
Texas Southern University

Earl A. Coddington
University of California,
Los Angeles

Ronald L. Graham
Bell Telephone Laboratories

I. N. Herstein
University of Chicago

John W. Jewett
Oklahoma State University

John L. Kelley
University of California,
Berkeley

Donald L. Kreider
Dartmouth College

William J. LeVeque
Claremont Graduate School

Werner C. Rheinboldt
University of Maryland

Arnold E. Ross
Ohio State University

Dorothy Stone
University of Rochester

Maynard Thompson
Indiana University

James H. Wells
University of Kentucky

June Wood
South Texas Junior College

E. G. Begle
School Mathematics Study Group
(Ex Officio)

Victor L. Klee, President
Mathematical Association of
America
(Ex Officio)

Andrew Sterrett, Denison University
Executive Director

Paul J. Knopp, University of Houston
Associate Director

William E. Mastrocola, Colgate University
Associate Director

Katherine B. Magann
Administrative Assistant

PANEL ON COLLEGE TEACHER PREPARATION

D. W. Bushaw
Washington State University
Chairman

Gerald Freilich
Queens College

Malcolm W. Pownall
Colgate University

William J. LeVeque
Claremont Graduate School

Donald R. Sherbert
University of Illinois

Dorothy Stone
University of Rochester

CONTENTS

FOREWORD*

Nobody should deny that it is important that the teacher teach effectively; however, some may well ask why it is necessary to choose this moment to emphasize the importance of the effective teaching of mathematics. Are we not extraordinarily effective in producing young mathematicians? Indeed, are we not embarrassingly successful? Do we not have a surfeit of young Ph.D.s in mathematics today, who are unable to find jobs suiting their talent, training and appetite? Of course the answer to these rhetorical questions is that the community of academic mathematicians performs satisfactorily in reproducing its own kind, but this is by no means the only criterion of success in teaching, certainly not the only task we have as teachers of mathematics. One should add, parenthetically, that it is even doubtful whether we should always credit ourselves with actually producing young mathematicians. Nevertheless, we may fairly claim that we do encourage intelligent young people to become mathematicians--once they have succeeded in overcoming the many and varied obstacles that lie between elementary school and graduate mathematics.

However, the main point to be made is that we are far less successful in teaching effectively those who are not destined to become professional mathematicians; and these, of course, constitute the vast majority of our clientele as teachers of undergraduate mathematics. We certainly must take into consideration the potential users of mathematics, since the main argument for the importance of mathematics today is precisely the ubiquity of its many applications. Thus future biologists, economists, architects, town planners, ecologists,...(as well as physicists, chemists, statisticians) need to understand mathematics and to be able to apply mathematics to their own disciplines. This means they must learn to think mathematically--a very different matter from solving mathematical problems of a familiar form by the use of a specified technique--and must know how to set up mathematical models of non-mathematical situations, and how to judge the appropriateness of those models.

We also have a responsibility, as teachers of mathematics, to cater for the future citizen, the future adult. Mathematics, we should not be ashamed to proclaim, is part of our great cultural heritage. A person is deprived by being blind to it--as most people are today. We cannot begin to justify our own privileged position as scholars--which means that we are concerned with creating mathematics, learning mathematics, and teaching mathematics--unless we strive to awaken in as many people as possible, irrespective of their chosen vocation, an awareness of the nature of our science, and its significance for our civilization, material and spiritual.

---

* The Panel on College Teacher Preparation is grateful to Professor Peter J. Hilton, Battelle Seattle Research Center and University of Washington, for this provocative, personal statement of his views on the importance of effective teaching.

Thus, we see that it is a matter of great social importance that we should teach as well as we possibly can. Moreover, when considering the problem of teaching those who are not professionally oriented towards pure mathematics, our task becomes more complicated because we must prepare ourselves to answer broader questions about the significance of the particular topics under discussion. This clearly calls for deeper understanding, on the part of the teacher, of the subject matter itself than that characterized by the ability to get the right answer to questions posed by others and to teach students a similar skill. It also calls, of course, for an understanding of the nature of the students' difficulties, actual and potential, and the ability to encourage the student to express his difficulties without shyness or fear of ridicule. Further, it is no easy task, even if one understands why the student fails to understand, to find a way to improve his understanding. Simple repetition, for example, is very unlikely to be effective! Thus, to sum up, it would seem that all the most difficult problems of teaching mathematics are, really to be found in the task of teaching undergraduates who, while they may very well plan to use mathematics in their special field of interest, are not going to be professional mathematicians.

So far we have been giving what might be called "unselfish" reasons for teaching mathematics effectively. However, it is very easy to give "selfish" reasons too! It is a common experience of those who have spent much of their professional life in teaching that the necessity to give an exposition of a particular topic leads to a substantial clarification of their own ideas. This can happen even when the topic is relatively elementary. In particular, we often see the significance of a mathematical idea with which we are currently concerned in our research far better when we fully comprehend the origins from which it springs. Those origins may, of course, be within mathematics itself or outside mathematics. We may also argue that a selfish reason for teaching mathematics well is that, in our own enlightened self-interest, we should do so in order to keep our jobs! Moreover, and less cynically, there is enormous enjoyment in good teaching, and ineffective teaching is a notorious source of severe frustration.

Granted that the teaching of mathematics is important, it follows that it must be seen by the future mathematicians to be important. This implies that the faculty must set a good example to the graduate student by the quality of their own teaching and the attention which they pay to improving it. For the graduate student will usually model himself on some outstanding faculty member, so that the attitude towards teaching of the most influential members of the faculty will largely determine the attitude of the student. If, for example, a senior professor of conspicuous research talent shows an impatience, or even disdain, for undergraduate teaching, or, perhaps, the teaching of future biologists, then this attitude is liable to be transmitted to his graduate students. On the other hand, enthusiastic and effective teaching not only generate in the student joy in mathematics, they also are likely to help him to become a fine teacher in turn. Thus both graduate and undergraduate

students will derive healthy, positive attitudes to teaching from the example of an outstanding scholar who himself responds with imagination and enthusiasm to the challenge of undergraduate teaching.

By a similar argument it follows that the faculty must give attention to the nature and quality of the teaching done by the graduate students in their capacity as TAs. It must often seem to a TA that the teaching which he does is simply part of a formal device whereby he becomes entitled to a bare subsistence wage, and that the only really important activity for him is his research. This view should not be inculcated in the graduate student. He must realize that his period as a graduate student is a period of apprenticeship in his craft, and that his craft comprises, in essential fashion, both research and teaching. His teaching, and the interest of the faculty in his teaching, must neither of them be perfunctory. The faculty must help him in very many ways to lay the groundwork of a good teaching technique and to recognize that teaching constitutes an integral part of his professional competence. It is thus of importance to try to set down procedures and criteria of effectiveness which are likely to lead to good teaching. It is to such a task that the authors of this pamphlet have addressed themselves.

INTRODUCTION

The purpose of this booklet is to disseminate some ideas about practices that are believed to have contributed to successful teaching of mathematics in colleges and universities.

In recent years many graduate departments of mathematics have been distributing sets of suggestions for effective teaching to their teaching assistants. The validity of most of these suggestions is not limited to one university or to teaching assistants; for some time it has been clear that it would be useful, if only as a device for avoiding unnecessary duplication of effort, to make generally available a compilation of these suggestions.

In the fall of 1970 the Panel on College Teacher Preparation of the Committee on the Undergraduate Program in Mathematics (CUPM) asked graduate departments for copies of materials of the kind described above. More than seventy-five documents were received in response to this request, and we gratefully acknowledge the importance of those materials for the preparation of this booklet.

Most of the suggestions included here are widely practiced. Thus, it would be misleading to ascribe them to any particular persons or institutions. Special mention should nevertheless be made of the efforts of Leroy J. Dickey and Kenneth M. Hunter. As graduate students at the University of Wisconsin they prepared a document entitled Some Helpful Hints to Good Teaching (1966). These Hints have been widely used and emulated, especially after their existence was made known to a wider public by CUPM in its Newsletter Number 3 (August, 1968), and their effects are clearly visible in many of the materials that the Panel received. We, too, have borrowed freely from the Hints, as well as from similar documents prepared elsewhere. Materials from the following graduate departments have been especially helpful: University of California, Berkeley (physics as well as mathematics), Rensselaer Polytechnic Institute, Massachusetts Institute of Technology, University of Michigan, University of Colorado, Indiana University, Washington State University, and Stanford University. An additional source of suggestions appearing in this booklet is You and Your Students, a pamphlet prepared by a faculty committee at M.I.T.

The ideas offered here are suggestions only. We would not presume to propose any specific prescription for good teaching. Also, most of the advice given in this booklet is based upon the traditional class-lecture format for teaching mathematics, for this is the format within which most prospective readers will be expected to operate, and their opportunities for trying other systems may be limited. This does not mean that we believe the traditional way to be the best way. It is unlikely that any optimal way of teaching mathematics has yet been found and even more unlikely that, if found, it has been widely adopted. In fact, many colleges and universities are now experimenting with new styles and patterns of teaching

undergraduate mathematics, and although there has not been time to complete most of these experiments or to evaluate the results, they are worth watching. [Brief accounts of several of them appear in CUPM Newsletter Number 7, New Methods for Teaching Elementary Courses and for the Orientation of Teaching Assistants (1972).] Other new ideas are sure to appear. In general, any teacher of mathematics who is serious about his craft and alive to its possibilities will always be searching for ways of improving his methods.

Finally, the fact that so large a part of this booklet is concerned with everyday teaching practices should not be taken to mean that they are more important than everything else. For one thing, there is no substitute for the instructor's mastery of the subject he is trying to teach. For another, his attitudes toward his students and his subject set the tone for his total teaching activity, and are often as influential on the subsequent mathematical development of his students as anything he does. Maintenance of technical proficiency and productive attitudes is essential; the goal of this book, however, is to deal with other matters.

I. CONDUCTING THE COURSE

Planning

"And gladly wolde he lerne, and gladly teche."
Chaucer, The Canterbury Tales

To be effective in the classroom, a teacher must ordinarily spend some time thinking about every course he will teach long before the onset of classes. He should know the goals of the course and make some key decisions affecting the likelihood that those goals will be attained.

It is important to consider how the course fits into the curriculum. For instance, if the course is prerequisite to other courses, there will be a certain minimal body of material that the instructor must cover thoroughly lest his students experience unnecessary hardship when they move on. A teacher should decide in advance on his approach to the major topics in the course and on the level at which he will present them. This requires that he know something about the interests and preparation of the students he can expect in the class. A class consisting entirely of mathematics majors should be treated quite differently from a class consisting, say, of business students.

It is good to have an overall conception of the course, and the text should be examined with this in mind. Before classes begin, one should note sections of the book deserving emphasis as well as sections that can be omitted without serious loss if time pressures develop.

The preparation of any supplementary material and location of source books are also best done in advance. The teacher may wish to enrich his own background for the course: not only the mathematical content but also historical background, interesting applications, and recent developments. Students like to hear about these matters, but few instructors are ready to present supplementary material of this kind without some advance preparation.

If there are useful reference books available to the students, an annotated bibliography should be prepared and distributed to them. It may be desirable to put some books on reserve in the library or to notify local bookstores that certain books are being recommended.

Time spent in advance planning is amply rewarded by a more smoothly running, more consistently paced course and by heightened student respect for the seriousness of the enterprise. On the other hand, planning should not be too rigid or detailed. Classes, like individuals, differ from one another and should be treated accordingly.

Good Beginnings

"No bubble is so iridescent or floats longer than that blown by the successful teacher."
Sir William Osler, quoted in Harvey Cushing, Life of Sir William Osler, volume II

There are several housekeeping details that should be attended to at the first class meeting. The title and number of the course should be displayed in order to ensure the proper placement of the students. Then your name, office number, phone number, and office hours should be displayed. Your policies on examinations, homework, and grading should be explained. Students have a right to know the bases on which they will be judged, and they are more likely to meet your expectations if they know what those expectations are. Indeed, you may wish to give the students a significant voice in setting these policies for their class.

Office hours are important. Some of your most effective moments as a teacher may occur when you are helping students in your office. Leaving these encounters on a hit-or-miss basis can cause frustration for the students when they do not find you in your office and irritation for you when they do. Scheduling office hours at staggered times may make the opportunity available to more students; it should also be made clear that you can be seen by appointment if the office hours are inadequate, and you should remind students frequently of your availability. If nothing else, these reminders reassure the students of your interest in helping them learn.

Students are often in doubt about whether to try to take verbatim notes at the risk of not understanding each point as it is presented, to take no notes at all, or to use a strategy somewhere between. Advice appropriate to your style of teaching will be welcome.

Students are impressed by an instructor who quickly gets to know them. Taking roll, at least until you know everyone, is one way of identifying names with faces. Returning assignments to individual students at the beginning of each class period, although time-consuming, is another. When roll is taken on the first or second day of class, you may wish to note where each student is seated. Most of the students will be in approximately the same locations all term.

The title and author(s) of the text should be announced, together with any basic reference books students may want to buy or consult in the library. Students will have a better perspective on the role and objectives of the course if some time is spent outlining the topics to be covered and briefly discussing how the course relates

to other courses in mathematics or to other disciplines. Then the first topic can be introduced and the course is under way.

Students tend to show an initial burst of enthusiasm. The first two weeks may be best used to develop a topic that is new or useful to most of them. What is learned first is often learned well, so a topic that will reappear frequently through the course should probably be taught first.

Some teachers think that beginnings must be slow. However, the first few weeks should set the pace for the entire course. One good reason for this is that some students must decide on the basis of this period whether to stay in the class or move up, down, or out, and should not be misled by an atypical pace.

The Lecture

"The ideal condition
Would be, I admit, that men should be
right by instinct;
But since we are all likely to go astray,
The reasonable thing is to learn from those
who can teach."

Sophocles, Antigone, Ode II

General remarks on preparation and meeting the class

Prepare before every class meeting! This does not mean that you should write out a complete lecture or even a detailed outline beforehand. It does mean that before each session with your students you should have a clear idea of what you wish to cover and how you wish to cover it. Don't try to lecture off the top of your head. The results can be disastrous. However, do not be an inflexible slave to your plan. If an interesting point arises unexpectedly, you should be able to grant it a short digression.

While preparing for class, look several sections ahead in the text for the sake of perspective. Prepare more material than you expect to need in an hour, but do not feel obliged to cover everything you have prepared.

Try to begin each class with a short discussion of material with which the students already feel relatively comfortable, rather than plunging directly into new territory. A brief résumé of the previous lecture may help to provide continuity. By inviting questions on recent material at the start of the period, you may find yourself provided with a natural springboard for the topic of the day. Associating new material with the old helps students to enter the new on a firmer footing.

When presenting a new idea, explain how it fits into the general scheme of the course. A common complaint among students is that although they may understand this or that piece of information, they are not just sure what it "means." Sometimes they express this by saying that the idea is unmotivated: it isn't clear why they are studying it, how it impinges on other ideas, or how it will be used. It may sometimes be necessary to expect students to learn on faith, but this should not be allowed to become standard policy.

It is often useful to allocate blocks of time for discussion of the fundamental concepts of the course, how they relate to one another, and their applications in other disciplines.

It is useful to ask frequent questions of individual students. One of the many advantages of this technique is that students are given a chance to participate in the development of new concepts. When the questions bear on material already covered, they can remind students of what they should know. Questioning also gives the teacher an opportunity to rework concepts when students show that they are unsure about them. Problems exposed in this way are probably shared by several students, if not a majority of the class, so time spent on them is not wasted. Another advantage is that it helps to keep students alert. You may recall from your own experience how hard it was to daydream in a foreign language class when the professor might call on you at any moment. Call on all of the students. If you call on just the bright ones, the poorer students will be discouraged; if you call on just the slow ones, the better students will be bored and may develop a false sense of security.

Avoid lecturing for more than a short period if you possibly can. Students get many long lectures, but they seldom have the appetite or powers of concentration to follow them properly. In particular, it defeats the purpose of having a small number of students in a class to lecture them as though they were a vast multitude.

It makes for a much livelier class to introduce new material on the basis of answers and conjectures drawn from the students. The best balance between this "discovery" method of teaching and more conventional approaches will depend on the circumstances. You should in any case avoid giving the class the impression that they are being talked down to.

Every good teacher wants rapport with his class, but it is amazing how many instructors give their lucid explanations to the blackboard, the walls, a window, or a point about one foot over the students' heads. LOOK THEM IN THE EYE! It helps to pick out one or two students and address yourself directly to them, watching their faces for signs of understanding. Of course, the same students should not always be chosen for this role.

Even well-prepared students will often not ask questions for fear of appearing foolish before their classmates. It is not <u>you</u> they are worried about, but their peers. The problem is a serious one. There are several things that can help.

* Encourage students to speak up by asking many questions and having as many students as possible recite. When there are many questions, a timid student will feel that his own question will attract little notice.

* Encourage conjectures and do not ridicule inept questions or wrong answers. Give the students the feeling that they are all on an equal footing in your esteem. You can learn from their mistakes. Above all, avoid sarcasm in any form. Nothing can damage your relationship with a class faster than sarcasm, however warranted it might seem.

* LISTEN TO YOUR STUDENTS. When someone volunteers an answer to one of your questions, you may realize as he begins to talk that he is on the wrong track. Resist the urge to quiet him. Instead, try to understand what he is saying, acknowledge any merit in it, determine his misconceptions, and tactfully point them out to him. Then let him try again, or give someone else a chance. Many instructors misinterpret a question before it is completely formulated. After you have tried to answer a question, give the student who asked it a chance to say whether he is satisfied.

In general, strive for as much informality in the classroom as your own personality and the circumstances will allow. Studies have shown that students usually perceive teachers who conduct an informal class as actually smarter and more knowledgeable in the subject than those who are stiffer and more formal. This impression is certainly often erroneous, but it has to be reckoned with. At any rate, informal give-and-take can be more pleasing for both you and your students.

Don't be defensive when you make a mistake. No one is perfect, and an impression of integrity is more important than an impression of omniscience. Request help from the students and correct the error together. If someone asks a question you cannot answer, do not dissemble to cover your ignorance. Try to work out the answer with the help of the class, or promise to bring it to the next meeting.

An obvious but important consideration is that your classroom technique must be adapted to the class you are teaching. A calculus class of 300 engineers, a small honors section in calculus, a remedial class in sub-freshman mathematics, and an advanced undergraduate course for majors must all be handled differently. The pointers mentioned above obviously apply mainly, but not entirely, to smaller classes and to less mature and less well-motivated students. For instance, student participation is not generally feasible in a large class. In this case the instructor should make an even greater point of reviewing the material covered in the previous lecture, of presenting examples in detail, and of putting material in perspective. In an honors class or a class for majors there is temptation to cover

the ground faster by lecturing with relatively little class participation, but even here some class participation may well be worth the time.

Essential blackboard techniques

It is most important that you begin writing at the top of one panel of the blackboard, move down that panel, and go on to the next panel. Do not erase the first panel until all available panels have been used. You should not skip around the blackboard, placing equations haphazardly here and there. Talk while writing.

If you are right-handed, consider beginning at the right-hand panel (as seen by the class). When this panel is full, move to the next panel to the left. In this way you will not always be standing in front of a panel already filled (and from which some students may still be taking notes) while you are writing on the next panel.

If your room is equipped with a triple-layer blackboard, use the board sandwiched between the front and back boards first. When finished, push it up and pull the front board down, so that what was just written is still in sight. After using the front board, push it up and use the back board, leaving the front board visible above. Then move to the next panel if there is one.

If you have more than two rows of students in the class and the room has a level floor, do not write all the way to the bottom of the board, because those farther back will not be able to see. If there is a desk in front of the class, keep it clear of such large objects as lecterns and briefcases.

Some teachers work with eraser in hand, using it to simplify and correct as they go along. This is guaranteed to irritate students who are taking notes. Instead of revising by erasure, draw a distinct line through the offending terms and write the improved forms above them.

You should put complete statements on the board, not just a few symbols. Your students' notes will consist almost entirely of what you have written on the blackboard; if you have omitted some crucial "ifs" and "thens," then that is the way the student will probably remember the statement. His notes should fill, not reinforce, gaps in his recollections of a lecture.

When you solve a problem at the blackboard, you should write a complete statement of the problem or give a precise reference. Define in writing any variables used in the solution. Unless the routine computations are prohibitively long, work the problem to completion in order to give your students practice in recognizing a solution and to give yourself an opportunity to illustrate checking and interpreting results.

Important items that you want your students to memorize (formulas for the derivatives of the trigonometric functions, for example) may be emphasized by outlining them in chalk. (Although some things may need to be learned cold--this is particularly true of definitions--students should always be encouraged to think problems through from first principles or basic results instead of immediately appealing to some memorized rule.)

Textbooks

Although textbooks are a nearly indispensable part of teaching, they can also be sources of difficulty. The temptation to condemn the chosen text and hand out an alternate set of notes should be strongly resisted. The text can be changed for later classes, but once it has been designated for a term it is best to live with it for that term and compensate for its deficiencies by lecturing especially well. Frequent criticism of the text before a class undermines its authority, persuades the students that they have been victimized in being asked to buy the book, and generally erodes morale. Of course, specific errors in the text should be corrected, but general carping should be avoided.

There is also the possibility that a text is so well written that it seems to leave little for the teacher to do. A teacher can always clarify and augment the text with additional examples, alternative derivations and proofs, and applications. The teacher should take advantage of the text's comprehensiveness to give more attention to the individual problems of his students.

Students, especially in the more elementary courses, usually have difficulty reading mathematics, even well-written mathematics; actually reading a page or two aloud in class, with comments on important ideas, details that need to be filled in, the structure of the argument, etc., may be of great educational value. This is teaching students to read mathematics--one of the most valuable services a mathematics teacher can perform.

In any case, the text and syllabus provide cohesion, and extensive deviations are not wise. This is particularly true in multi-section classes. Uniformity among sections is essential if students are not to have extra trouble in the next course.

Visual aids

Even the teacher who relies mainly on blackboards may occasionally be able to make good use of an overhead projector. Complicated drawings can be made in advance, and for some elementary courses professionally made projectuals are available.

Modern devices for the inexpensive reproduction of printed and written material offer many opportunities. Material complementary

to the text, model answers to homework or test questions, lists of references, and special tables can now be put into students' hands quickly, neatly, and at very low cost. There is less justification than ever for wasting class time on the laborious copying down of such things.

There now exists a wide range of movies on mathematical topics, and these might be used to enrich your lectures or, from time to time, to substitute for them. For example, the film "Mathematical Induction" by Leon Henkin could be shown in an off-period for students who need a review of that principle. Films of this kind are regularly reviewed in the American Mathematical Monthly. A large number of films produced by The Mathematical Association of America may be rented or purchased from Modern Learning Aids, 1212 Avenue of the Americas, New York, New York 10036. If there is an office of audio-visual services on your campus, help in arranging to show films may be obtained there.

Assignments

"There is no royal road to geometry."
Euclid, quoted by Proclus,
Commentary on Euclid, prologue

The rationale

Some instructors give no assignments at all on the assumption that students will work on their own and ask questions about points which they do not understand. Instructors like this may have forgotten their own ways as undergraduates. Courses compete with one another for student time, and courses in which assignments are not given regularly are very likely to suffer. After a while the students reach a point where they are so far behind that they cannot ask even the most basic questions, and recovery becomes almost impossible.

Thus there should be frequent assignments, with a reasonable selection of routine exercises and more difficult problems. Many practices are possible. You can require that the assignments be handed in, and grade one or two problems without telling the students in advance which ones those will be. Another possibility is not to ask that all assignments be handed in, but to collect two or three serious problems a week, which should then be graded and returned. Some teachers ask their students to accumulate homework in a notebook which is called in for checking from time to time.

One basic approach is to give short assignments daily, due the next class meeting, and returned by you at the following meeting. This method has certain advantages:

* You have continual opportunities to see how your students are doing.

* Short assignments can be corrected quickly and are less likely to discourage students from studying for the course.

* Students' correct ideas are reinforced while they are still fresh, and their wrong ones are corrected before they have had time to become habitual.

* The student becomes accustomed to regular and systematic study and will not wait until the day before a test to begin studying.

* Returning assignments frequently helps you to learn students' names and to maintain person-to-person relationships with them.

* Each student acquires a clear idea of what sorts of problems he should be able to do and builds up a set of solved problems to use for review.

Another possibility is to give weekly assignments. If you prepare and distribute a dittoed assignment sheet, the student can keep track of which problems he has finished and which are giving him trouble. Some students prefer to study in large units of time instead of a short period each day. Assignment sheets allow students to work ahead and to budget study time in their own ways. Problem sets announced a week before they are due can include more difficult problems which could not reasonably be assigned overnight.

Part of an instructor's task is to encourage students to talk about mathematics as well as to listen and to read about it. One way to do this is to permit, indeed to encourage, students to discuss their assignments with one another while they are working on them. A good ground rule is that any amount of discussion is allowable, but each student should write up his own solutions independently.

On correcting papers

If a student is not serious, you can hardly afford to spend much time with him. If he is, you are obliged to read his papers carefully and to comment appropriately, either orally or by way of notes written on the papers before they are returned to him. If he worked hard and produced a well-constructed answer, he deserves more than a mere checkmark. Tell him briefly where his work is especially good. If he went wrong, find out where and point out the error or set him on the right track. In some cases ask him to try again and to submit a new solution. Of course, you may not be able to treat all the students in this ideal way all the time, but you can try.

It is well known that most problems can be correctly solved in various ways. If a student chooses a method different from the one you would have chosen, be certain that his "right answer" doesn't conceal fallacies or compensating errors. Even when the student's method is correct, it is not out of order to call his attention to easier or more powerful methods when they exist. This does not mean that he should be told that his method is wrong just because there is a better one.

Sometimes problem solutions are posted or are reproduced and distributed to the students. This may be very helpful, but is not an adequate substitute for appropriate comments on the work of each individual student.

You undermine the student's developing responsibility and his respect for himself and for the subject if you do not expect him to submit his work on time in decently organized, legible form. Students should be required to write in complete sentences, with careful attention to logical connectives, and their writings should show steady progress toward the attainment of an acceptable mathematical style.

Homework should probably not count very heavily in a student's grade. The homework record can be used to bolster the grade of a student who unaccountably fell down in one or two tests, or to help settle borderline grade cases, or in pass-fail decisions. It should be made clear to the students that they have not really finished a homework assignment until they understand how to do the problems on it. One can reinforce this point by including on each test some problems taken directly from the homework.

Tests

> "The whole art of teaching is only the art of awakening the natural curiosity of young minds for the purpose of satisfying it afterwards."
>
> Anatole France,
> The Crime of Sylvestre Bonnard

Attitudes and objectives

From the standpoint of student-instructor relationships and the cultivation of a cooperative team spirit, the usual examination system is a potential obstacle. Just when the instructor has established himself as the student's staunch ally, he is obliged to shift into the role of judge and jury and, it may be, executioner. At this stage particularly, the instructor should seek means of preventing his own metamorphosis into a hatchetman in the eyes of his students.

It is essential that both the instructor and the students be fully aware of the objectives of the examination and of the interpretation to be put on the results. A test is merely a quantitative observation on the process of learning. Both the instructor and the student are being observed. The examination is an observation on their progress as a team.

The principal objectives of examinations are:

* To evaluate each student's achievement relative to that of his colleagues and predecessors.

* To evaluate the instructor's teaching.

* To provide a learning experience in the examination itself and in the review for it.

If the examination is to be a valid assessment of the student and the instructor, it must be very carefully formulated. It is the instructor's responsibility to compose the examination, but much can be accomplished by involving the students in the process. One reasonably successful device is to ask each student to submit one question (including his own solution) about a week before the date of the examination. The instructor can then assemble an examination by making a suitable selection from the questions received. Sometimes the problems submitted will be much too difficult for use on an examination and will need to be revised by the instructor. This type of student involvement sometimes helps, especially in more advanced courses, to neutralize a potentially antagonistic atmosphere between examiner and examinee. It also provides some variety and stimulation, while helping the student to direct his review work toward definite objectives.

It is a useful practice to select some test questions from homework assignments and from questions that caused difficulty in previous tests. This encourages students to keep previously covered topics fresh in their minds and not to sweep old difficulties under the rug. Of course, if this practice is used the students should be told!

There will be some situations in which the instructor may call in a colleague to be the examiner. This method has the advantage of preserving the solidarity of the instructor-student team, but it has the disadvantage of placing a heavy responsibility on a guest examiner whose viewpoint, knowledge, and priorities may differ markedly from those of the instructor.

Examinations can be instruments for teaching even while they measure what students learn. A well-posed, invigorating test question may prompt students to do some serious thinking; even if they do not answer it successfully they may be eager to hear the answer when it is subsequently discussed in class.

If take-home examinations are feasible, the students can be led, especially in advanced courses, to carry out some important parts of the subject on their own. Material learned in this way is sometimes learned better than it is by other methods.

Types of tests

Tests may be divided into two very broad and obvious classifications: open-book and closed-book. When the student is permitted to use his text, his notes, and reference material, the questions can be more authentically representative of problems met in practice. Memory for details--spontaneous recall--will play a secondary role in this type of test. The question is whether he can produce when he has access to the resources that would normally be at his disposal. This applies not only to the time-restricted open-book examination but also to the take-home examination. The take-home examination, for which a week or more may be allowed, tests the student's ability to arrive at a solution at his own pace. However, the open-book examination is not always the prototype of professional practice. There are numerous professional situations in which the engineer, scientist, economist, or mathematician must respond and act without the benefit of his library. Closed-book examinations are usually more appropriate in elementary courses since these courses consist primarily of basic mathematical skills which must be thoroughly mastered if the student is to progress satisfactorily to concepts and techniques of a more complicated nature.

Scheduling tests

Individual preferences run from many short quizzes to a few comprehensive examinations. Frequent tests encourage students by reducing the course content to more manageable segments and by providing more nearly continuous assessment of progress. They also tend (as do frequent homework assignments) to even out students' study efforts over time. On the other hand, less frequent tests compel students to integrate larger segments of material in their minds and to retain the material over longer periods; they thus reward relative scholarly maturity. Also, they divert less time from other class activities. If asked to set its own examination schedule, a class will usually choose a reasonable plan well between the extremes.

In any case, it should not be possible for one bad test to ruin a student's grade for the course; the simplest protection against this is to schedule enough tests.

Give at least one test early in the term so that people who are in difficulty in your course will recognize their predicament and be able to do something about it before it is too late.

Composing tests

When you sit down to write a test, begin by listing the main ideas, theorems, and methods for which the student is held responsible, and try to cover most of them. Don't despair if a few are missed, but it may be unfair to some students to accent one part of the material while completely neglecting others. The learning and review functions of a test should be given at least as much attention as the evaluative function.

Try to maintain a reasonable balance between "plug" and "grind" problems, applications, and theory. Give an easy question or two first to allow less confident students to get off to a good start. The first few questions can be used to identify failing students by having those questions cover what you feel is absolutely minimum knowledge. Problems should range fairly uniformly from easy to difficult; otherwise all the students except the very best and the very worst may get grades that do not differ significantly from one another. For the same reason, it is wise not to weight problems by difficulty. If you do intend to assign different weights to questions, this information should appear on the examination.

Don't give students unnecessary difficulties (and excuses)--be sure that the copy is readable and problems are clearly stated. Encourage questions during the examination about the interpretation of the problems, but reserve the right not to answer some questions.

Making test questions significantly interdependent is a form of double jeopardy and should be avoided. This is as important for the convenience of the grader as for fairness to the student. For example, if problem 1 consists of determining a function satisfying prescribed conditions, then problem 2 should not involve explicit use of that function, such as calculation of its derivative or integral. The student who fails to do problem 1 might be unjustly penalized on problem 2. If a student arrives at an incorrect solution to problem 1, he might encounter great difficulty in using it in problem 2. This could make the fair assignment of partial credit difficult and could adversely affect a student's performance on the remaining problems because of time limitations.

Similarly, repetition should be avoided. Each problem should have its own purpose, and no purpose is served by having two problems test the same concept.

Try to avoid problems in which success depends too heavily on mastery of skills which it is not the purpose of the test to evaluate. Before a test is used, work through it carefully, using the actual form that the student will use, not your preliminary notes (or, preferably, have someone else do it). It is unpleasant to discover afterward that a problem was defective, thus upsetting the students and making meaningful evaluation difficult.

Don't ask for proofs unless students know exactly what they may assume. Give "story problems." Make students apply definitions and work examples instead of merely stating general definitions and theorems; the important thing is whether the students understand and can use the general ideas, not whether these ideas have been learned by rote. Expect students to show their work and give reasons for what they do. Don't give full credit for unsupported correct answers unless the problem is trivial.

In determining the length of a test remember that you have certain advantages over the student: general experience, previous exposure to the problems, and probably a more relaxed state of mind. A test that takes you more than fifteen minutes to do is almost certainly too long for a fifty-minute period.

A time by which examination papers must be handed in should be clearly announced and firmly observed. It is unfair to allow some students to go on working when others must leave, for example to go to another class.

Grading tests

* Grade and return examination papers promptly. Students conclude that their class has a low priority among your activities if more than one or two periods go by without having the papers returned. The grades, your written comments on the papers, and classroom discussion of the examination will all mean more when the students' memories of the examination and of what they were thinking when they took it are still fresh.

* Take reasonable steps to increase the objectivity and uniformity of grading. Ask students to sign papers where you will not see their names every time you pick up the papers. Grade one problem at a time. Shuffle papers between problems in order to offset possible tendencies to grade the first papers differently from later ones. To some extent, partial credit on problems can and should be decided in advance. Certain common mistakes can be anticipated, and if penalties for these mistakes are predetermined, grading will be more consistent.

* When you return the papers, list the grade ranges and the median grade on the blackboard. Without this information, a student with a low score may believe that almost no one in the class performed any better on the examination than he did. If the class is small, you may want to list all grades on the board. If the examination covered some important concepts and perhaps unified several ideas, this is an opportunity to stress these matters again. Moreover, an examination often exposes ideas which are not yet clear to enough students. Your last good opportunity

to dispel a particular misconception may come when the papers are returned, and the opportunity should not be lost. This is also a time when particularly elegant solutions by students may be shown to the class. If time does not permit discussion of the examination at length, at least distribute a set of solutions when you return the papers. Sets of solutions should not be distributed immediately after the examination is taken, however; this practice tends to cut off worthwhile discussion of the examination questions among the students themselves.

Good Endings

"To teach is to learn twice."
Joseph Joubert, Pensées

Proper planning before the course begins and minor adjustments during the term should avert any necessity for covering the second half of the course in the last week. The student profits little from attempts to cover material at breakneck speed toward the end of the term. More than likely, he will not be enthusiastic to begin another course in mathematics following one which does not have a satisfactory ending.

It is a revealing and sometimes embarrassing experiment to ask a student to name the most important theorem he knows. Even mathematics majors often come up with distressing answers. Not all of the blame for a student's inability to judge the relative importance of ideas should be put on his shoulders. The end of the term is an ideal time for the instructor to give his students one last comprehensive view of the course, distinguishing the peaks from the hills and the hills from the plains--and explaining why the peaks are peaks.

The Giving of Grades

"A teacher affects eternity; he can never tell where his influence stops."
Henry Adams,
The Education of Henry Adams

The assigning of grades at the end of the term is a task that deserves serious thought and considerable care. The final grade a student receives in a course normally becomes part of his permanent record and may have far-reaching effects on his career.

The instructor can prepare for this critical phase of the term's work by constructing and grading his tests with care and by keeping thorough and well-organized records of his students' performances. The results of each examination and each set of homework should be clearly recorded, together with such information as the date, the class average, the weight it carries in the final average, and perhaps a brief note indicating what material was covered. Grade books are available for precisely this purpose. Besides facilitating the determination of valid final grades, care in record keeping can be a valuable aid in diagnosing problems of individual students along the way. It also enables a colleague to understand the basis for the grade you have assigned if some question arises when you cannot be reached.

When final averages are computed according to the policy you have chosen, the resulting numbers will generally fall into reasonably well-defined groups with a few scattered intermediate cases. Once you have reached your decision about the clusters, the intermediate cases should be considered individually. Such factors as diligence in doing the homework, attendance, participation in class discussions, whether the quality of work improved or declined during the term, evidences of understanding that do not show in the record, and so on, can guide decision-making in these cases. The reasons for your ultimate decisions should be clear enough in your mind or in your records so that a student can be convinced of the justice of his grade if he comes in to discuss it.

If a student does question his grade, he deserves a fair hearing and a courteous review of his performance over the term. If your evaluation of a student's work has been based on careful deliberation, he will usually be satisfied by such an analysis. Do not dismiss the student peremptorily, but consider the possibility that the student did in fact receive an inappropriate grade.

There are times when a student will insist that he knows the material, but for various reasons his record does not show it. At such times you should remind the student that the letter grade reflects his actual performance and not what he might have done if...

II. FEEDBACK AND EVALUATION

"The decent docent doesn't doze:
He teaches standing on his toes.
His student dassn't doze--and does,
And that's what teaching is and was."
David McCord, What Cheer

All of us are evaluated by our students, whether we like it or not. The question is how these evaluations can be communicated in a timely and productive fashion. Open communication between faculty and students serves not merely to let the professor know whether students like his course, but also to enable him to present his case to the students. More importantly, it indicates to the instructor the areas where he can improve his performance.

Written evaluations made by students at the end of the term can be valuable to the instructor but they are of no help to a student unless he happens to have the same instructor again. More or less immediate oral evaluation or feedback, for instance in the form of questions asked, probably has much greater potential value. Instantaneous feedback, of course, need not even be oral; the perceptive teacher who looks at students while he talks can hardly miss signs of puzzlement, boredom, or pleasure on their faces. In a perfect class, all the students understand everything that is being done all the time. Perfect classes do not exist, but a good teacher wants his class to approximate this ideal as closely as possible. If, in spite of your standing invitation to ask questions, you sense that you have "lost" a student, you might pause and ask him if something needs further explanation. But a word of caution: impatience on your part with the nature of the student's question may result in an impassive and unreadable face on that student for the rest of the term.

Another type of feedback that may be especially useful to the inexperienced teacher is obtained by spending the last few minutes of each period discussing what went wrong and what went right that day. LISTEN CAREFULLY to what the students have to say, even if it seems unreasonable. It might be a good idea to take notes. Feel free to tell the students how you feel about the way things went. If the class was dead, say so. Make it clear that the students share responsibility if the class is a drag.

Several times each term (more frequently near the beginning of the course) some time might be reserved when students are invited to discuss more global aspects of the course, such as its pace, the usefulness of the text, quantity and quality of assignments, use of class time, the grading of homework, etc. Information of this sort can also be picked up by coming to the classroom early and chatting informally with your students.

Depending on local conditions, more formal types of evaluation can occur in several ways.

* TAs and other instructors often exchange visits to one another's classes.

* In some institutions, equipment and operators are available for videotaping an instructor at work so that he can have the opportunity to see himself as others see him.

* A senior colleague who is respected as an excellent teacher may be invited (e.g., by the instructor himself) to visit a class and afterward frankly discuss his reactions with the younger teacher, and perhaps also note them down on a form made available for the purpose. A form similar to that given at the end of this section might be useful in this situation.

* At the end of the term many faculty members who teach a large class hold a meeting with their assistants to discuss and evaluate the course they have just finished.

* Any teacher should consider asking all the students in his classes to fill out a questionnaire at the end of a course. The sample evaluation form which follows was prepared by members of the Mathematics Graduate Student Association of the University of California, Berkeley.

Instructor Evaluation

Name of Instructor_____

Course_____

Term and Year_____

1. What characteristics of your present instructor's teaching do you consider his strengths? (Teaching refers to his complete behavior as a teacher, not just in class.)

2. What characteristics of his teaching do you consider his weaknesses?

3. What were the good and bad characteristics of how this instructor taught the problem session in relation to the teaching in the lecture section?

4. There are many skills and techniques that a teacher can use to reach his goals. There are also many attitudes that affect his teaching ability. The following items refer to specific topics, some of which you may consider irrelevant or unimportant. Please comment in as much detail as possible about the good or bad aspects of any of these topics on the instructor's teaching that you consider important.

   i) Materials covered--the choice of topics, the organization, the pace, the text and other references, etc.

   ii) Class format--the style of interaction between teacher and student, i.e., formality, lecture (monologue), lecture (dialogue), group discussion, individual student presentations, etc.

   iii) Skills used in the classroom--speaking, boardwork, preparation for class, drawing, etc.

   iv) Treatment of topics--motivation of definitions, theorems, and proofs; relative emphasis on problems, examples, and theory.

   v) Help outside the class--office hours, availability, friendliness, helpfulness, etc.

   vi) Homework--number of problems, relevance to course topics, difficulty, reader's good and bad points, etc.

vii) Exams and quizzes--relevance to course work, difficulty, quality as evaluators, quality as teaching devices, number, etc.

viii) Grading policy--fairness of the evaluation system he uses, etc.

ix) Attitude toward student--friendly, open to questions, honest, sensitive to your understanding of the material, to your needs, to your desires, to your problems, etc.

x) Attitude toward teaching--his desire to teach well, to find out whether he is teaching well, to adjust his techniques to fit the teaching situation, etc.

5. List any suggestions you have about any skills or techniques he should have used.

6. List any suggestions you have about how the course itself could be changed for the better.

7. Rate the teaching of this instructor on the seven point scale given below: (Circle the number)

7 = Excellent
6 =
5 =
4 = Average
3 =
2 =
1 = Terrible

8. Any other comments.

University of California, Berkeley

III. MATTERS OF SPECIAL INTEREST TO FACULTY AND TEACHING ASSISTANTS WHO COOPERATE IN TEACHING LARGE CLASSES

The teaching assistant can be used in a program which is the backbone of a successful and creative system of instruction for large numbers of university students. This is fortunate, because many university faculties are forever caught in a dilemma between their responsibility to process hordes of students and their natural inclination to minister to the unique needs of individual students. The proper use of the TA's talents is not only a key to the problem of numbers, but when handled creatively may help to solve the problem of individual needs as well.

To the Professor

There are many instances of frustration among TAs who wish for more creative outlets for their abilities than they are given. Faculty members have a responsibility to the students in their classes, but they should also be actively involved in the development of TAs as teachers. This means engaging TAs in the planning and conduct of courses in a more than perfunctory way, possibly at some cost in time and autonomy to the faculty member. Where it is the role of TAs to assist a senior lecturer by conducting problem or recitation sections, efforts of this kind will surely be repaid by better communication and morale among all concerned.

Every lecturer in the lecture-recitation system should take over a problem session from time to time to learn firsthand how the rank and file of students are doing. Lecturers should also visit the sessions led by their TAs at intervals and discuss their performances with them afterward. This is another occasion where an evaluation form of the type shown on pages 19-20 may be useful. In some large departments several faculty members have released time specifically for working with TAs.

Often for good reason, graduate students believe that when faculty members are trying to help them find jobs, they can evaluate their talent for mathematics and research potential, but they cannot evaluate them as prospective teachers. Often the best that a faculty member can honestly say about an applicant's teaching is that no complaints have been received. This situation does not impress TAs (or prospective employers) with the importance attached to teaching at the university. This is an important if secondary reason for close interaction between senior faculty members and TAs, and in particular for class visitations.

The dangers of faculty members' remaining ignorant of the strengths and weaknesses of their junior colleagues are even greater, of course, when classes of moderate size are handled more or less

independently by TAs, with no direct communication between the students in those classes and regular faculty members. Correspondingly, there is an even greater need for a program of regular visitation and evaluation and for a program to provide definite orientation and guidance to inexperienced teachers.

TAs should not be expected to have the same grasp of what should be emphasized as an experienced lecturer. Some explicit advice about this normally leads to greater satisfaction for all concerned. The following extract from a commentary distributed at Rensselaer Polytechnic Institute is an example of the kind of information that helps inexperienced teachers.

Mathematics I Comments for Instructors

SECTION	COMMENT
1 - 9	Emphasize Sections 1, 2, 4, 5, 7. (Fisher and Ziebur)
	Stress that $\sqrt{a^2} = \lvert a \rvert$.
	Emphasize the correspondence of real numbers and points on the line.
	Sets are used throughout the book, so be sure to go over the notation.
	Mention set-valued functions, as they are used in Chapter 5 in connection with the integral.
	Do not spend much time on the notion of completeness.
	Section 3 will be a review for most students.
	Include examples showing that a graph need not be a curve.
	Do some problems involving the [x] function.
	Indicate that given the five following basic identities and the definitions of the trigonometric functions, other identities can be quickly derived.
	(1) $\sin^2 x + \cos^2 x = 1$ (2) $\sin (x + y) = \sin x \cos y + \cos x \sin y$ (3) $\cos (x + y) = \cos x \cos y - \sin x \sin y$ (4) $\sin (-x) = -\sin x$ (5) $\cos (-x) = \cos x$
	A list of basic identities is in Appendix A, page 743.
	Students should know the sine and cosine of 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, π , $3\pi/2$, 2π, and multiples.
	Theorem 8.3 bears discussion since it generalizes later.

Rensselaer Polytechnic Institute

To the Teaching Assistant

The lecture-recitation system functions most effectively when the TAs attend the faculty lectures regularly. Without some fairly regular direct contact with what goes on in the lecture it is hardly possible for a teaching assistant to keep abreast of the progress of the course and be up-to-date on what the professor is emphasizing and attempting to do. In particular, you need always to be aware of the principal aims of the course and to keep them in mind as you plan for and carry out discussions with your own students. We recommend, therefore, that you make a point of listening to as many of the lectures as possible.

It is probably expected that you will meet with the professor regularly to plan and discuss the work to be accomplished in the discussion sections. You should find out just what needs to be accomplished and whether you are expected to use any particular methods of accomplishing it. These meetings should also serve as occasions for reporting on the progress of the students, calling attention to points that seem to be causing general difficulty, and making suggestions about the conduct of the course in future weeks or years. You are a valuable communication link between the students and the professor.

You may sometimes regard the professor's instructions as unduly restrictive. If the course is built around his lectures, other activities must complement them effectively, and it should be assumed that the professor is a good judge of how this can be brought about. You will rarely find, however, that all the details of handling every section are prescribed for you. You will have ample latitude for trying out various ways of presenting material, conducting discussions, and dealing with individual students within the framework set by the professor in charge.

By the way, you should not resent occasional visits to the meetings of your class. This is a reasonable way for the professor to keep in touch with the students and to help you develop your teaching proficiency. In fact, if he does not visit your class on his own initiative you might invite him to visit and to offer suggestions based on his visits. You might learn something from him, and he might even learn something from you.

IV. READING AND SEMINARS RELATED TO TEACHING

To the extent that you are involved in it, teaching mathematics is your profession; it is therefore natural that you take a professional interest in it. This implies a continuing vital involvement in mathematics itself (about this we shall have nothing further to say here*), but also leads naturally in various other directions: to the history of mathematics; to the philosophical, cultural, scientific, and even political environment of mathematics; to trends and new ideas in mathematics education; to the activities of certain professional organizations; and so on. More generally, it is useful to know something about educational theories and movements and about the history, philosophy, and current problems of higher education as a whole. It is possible, of course, to devote so much time to such matters that your development as a mathematician and your effectiveness as a teacher may suffer. A steady but controlled interest in them can nevertheless enhance your teaching not only by way of specific improvements but also by strengthening that intangible quality called perspective.

A simple way to cultivate this kind of professional interest is to browse through appropriate journals from time to time. These include:

The American Mathematical Monthly
The Two-Year College Mathematics Journal
The Mathematical Gazette
Educational Studies in Mathematics
The International Journal of Mathematical Education in Science and Technology
Newsletter, Conference Board of the Mathematical Sciences (CBMS)
Notices of the American Mathematical Society

Some of the leading periodicals concerned with higher education in general are:

The Journal of Higher Education
College and University
The Chronicle of Higher Education
Improving College and University Teaching
Bulletin, American Association of University Professors

As might be expected, at least from the mathematical standpoint, the contents of these periodicals are very uneven in interest and value, but some sifting can yield useful material.

---

*On this subject, see CUPM Newsletter Number 5 (September, 1970), "Maintaining Mathematical Momentum," and D. E. Christie and J. H. Wells, "Alternatives to Research," American Mathematical Monthly, Vol. 74, No. 8 (October, 1967), pp. 1001 - 1004.

It is easier to make a habit of scanning publications of these kinds if recent issues are kept in a separate, conveniently located collection--perhaps in a departmental lounge or in a corner of the departmental library. If your department does not already have such a collection, you might propose that one be established.

The same collection might also contain a selection of suitable books. The following items merely illustrate some of the possibilities.

Apostol, T. M., et al., editors. Selected Papers on Calculus. Belmont, California, Dickenson Publishing Company, 1969, $9.00.

Bell, E. T. Men of Mathematics. New York, Simon and Schuster, 1961, $7.95, paper $2.95.

Boyer, C. B. A History of Mathematics. New York, John Wiley and Sons, 1968, $12.95.

Courant, R. and Robbins, H. What is Mathematics? New York, Oxford University Press, 1941, $10.95.

Dubisch, Roy. The Teaching of Mathematics, 2nd ed. New York, John Wiley and Sons, 1963, paper $3.95.

Eble, Kenneth E. The Recognition and Evaluation of Teaching. Project to Improve College Teaching, American Association of University Professors, Washington, D.C., $1.00.

Eves, H. Introduction to the History of Mathematics, 3rd ed. New York, Holt, Rinehart and Winston, 1969, $12.95.

Hadamard, J. Psychology of Invention in the Mathematical Field. New York, Dover Publications, 1945, paper $1.35.

Hofstadter, R. and Metzger, W. P. Development of Academic Freedom in the United States. New York, Columbia University Press, 1955, $10.00.

Khinchin, A. Ya. Teaching of Mathematics. New York, American Elsevier, 1968, $9.50.

Kline, M. Mathematics in Western Culture. New York, Oxford University Press, 1953, $9.50, paper $2.95.

McKeachie, Wilbert J. Teaching Tips: A Guide for the Beginning College Teacher, 6th ed. Lexington, Massachusetts, D. C. Heath and Company, 1969, paper $3.95.

Morris, W. H., ed. Effective College Teaching. Washington, D.C., American Council on Education, 1970, paper $3.50.

Newman, J. R., ed. The World of Mathematics. New York, Simon and Schuster, 1956, paper $14.95.

Noble, Ben. Applications of Undergraduate Mathematics in Engineering. New York, The Macmillan Company, 1967, $9.95.

Nowlis, V., Clark, K. E., and Rock, M. The Graduate Student as Teacher. Washington, D.C., American Council on Education, 1968, paper $2.00.

Pólya, G. Mathematical Discovery. New York, John Wiley and Sons, 1962, $15.00.

Rothwell, C. E. The Importance of Teaching: A Memorandum to the New College Teacher. New Haven, Connecticut, The Hazen Foundation, n.d., paper, free.

Rudolph, F. The American College and University. New York, Alfred A. Knopf, Inc., 1962, $6.95, paper $2.95.

Steinhaus, Hugo. Mathematical Snapshots, 2nd ed. New York, Oxford University Press, 1969, $7.50.

Struik, D. J. A Source Book of Mathematics. Cambridge, Massachusetts, Harvard University Press, 1969, $11.95.

Van der Waerden, B. L. Science Awakening. New York, Oxford University Press, 1961, $7.50; John Wiley and Sons, paper $2.65.

(Of course, it is worthwhile to familiarize oneself with such books whether or not they are kept in a separate collection!) The collection would also be the logical place to keep a set of CUPM publications, selected Yearbooks of the National Council of Teachers of Mathematics, a file of reprints or photocopies of appropriate articles, and perhaps an assortment of sample textbooks.

In some mathematics departments seminars have been set up for the study of issues and problems related to mathematics instruction. (Seminars and classes on the history and philosophy of mathematics are nothing new, of course.) The success of these efforts seems to depend greatly on the spirit in which they are led. One seven-week program, designed for teaching assistants, deals successively with: the role of mathematics in the physical, biological, and social sciences; curriculum planning; course planning; measurement and evaluation of student achievement and instructor effectiveness; teaching aids; teaching as a profession. Another style is exemplified by a year-long program of discussions based on a reading list

drawn from sources like those listed above. Guest speakers can add interest and depth to these activities. Probably one of the best ways to cultivate a professional orientation towards mathematics teaching is to participate in a program of this general kind if it is available or to start one if it is not.