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ABSTRACT

Using a broad definition of research to include evaluative studies of large curriculum projects or teacher-training programs, Investigations in Mathematics Education (IME) is a quarterly journal of expanded abstracts and critical analyses of recent research in mathematics education. The 16 abstracts in this issue were done especially for IME by professionals in the field in the United States and Canada. Each abstract states the purpose, rationale, research design and procedure, findings, and interpretations of the study as well as notes on the study from the abstractor. A wide range of topics is covered, including mathematics language, problem-solving, sequencing, structured learning, attitudes, and calculus with computers. Availability and ordering information for ERIC documents is included where applicable. (JM)

Vol. 5, No. 3, Summer, 1972

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INVESTIGATIONS IN MATHEMATICS EDUCATION

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Summer, 1972

NOTES . . .

from the Editor

What constitutes a research study? This question becomes crucial in selecting articles and documents to be abstracted for Investigations in Mathematics Education. We believe that readers of this journal will be best served by a broad interpretation of that issue rather than a narrow definition. We have abstracted evaluative studies of large curriculum projects or teacher-training programs even though these projects and programs did not show evidence of the careful control of variables usually associated with research studies. Evaluations of mathematics education programs call for practical applications of many research techniques in settings not particularly amenable to classic research design. Analyses of studies where research techniques have been applied to evaluate existing programs can provide important insight into these special problems.

The first abstract in this issue discusses a study which looks carefully at a particular research technique (item-sampling) as applied to formative curriculum evaluation. As our abstracter points out, this particular study is not experimental. But it does provide guidelines for the use of the item-sampling technique in other formative evaluation studies. The application of this particular technique is important to all researchers. We believe it is a good example of the advantage to be gained by adopting a broad view of research when selecting articles for this journal.

We appreciate comments from our readers concerning the coverage of articles abstracted in this journal. Readers who would like to see specific reports or documents abstracted by Investigations in Mathematics Education are encouraged to write the editor.

Jon L. Higgins
Editor

ED 040 877 24 SE 008 991
THE FORMATIVE EVALUATION OF PATTERNS IN ARITHMETIC GRADE 6 USING ITEM SAMPLING, PHASE 2: ANALYSIS OF MATHEMATICS INSTRUCTION (PARTS 1, 2, AND 3). Braswell, James. Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning. Spons Agency--Office of Education (DHEW), Washington, D.C. Bureau of Research. Pub Date Mar 70, Note--253p. EDRS Price MF-$0.65 HC-$9.87
Descriptors--*Curriculum Evaluation, Doctoral Theses, Educational Television, *Elementary School Mathematics, *Evaluation, Grade 6, *Instruction, Mathematical Concepts, Televised Instruction.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Arthur F. Coxford, University of Michigan.

1. Purpose

To investigate the application of the technique of item-sampling to formative curriculum evaluation in mathematics.

2. Rationale

In the item-sampling technique, a set of n items is randomly partitioned into r subsets. The r subsets of items are randomly assigned to s subjects so that each subject responds to only a subset of the items. Theoretically the descriptive statistics obtained for an item, a subset of items or the entire set of n items are estimates of the respective population descriptive statistics. There is evidence which suggests that means obtained by item sampling techniques may be significantly greater than means obtained by conventional procedures. Even so, when subjects are exposed only to item sampling techniques, the conditions which influence performance are assumed uniform and inflated means are much less important.

In mathematics curriculum development a great number of objectives are sought over a year. In formative evaluation of a mathematics program, information on all these objectives is desirable at several times during the year so that the curriculum developer may identify weaknesses and institute correctional procedures. These needs cannot be met by conventional testing procedures. They may be satisfied by the item-sampling technique because all objectives can be measured several times during the year without having every subject respond to every test item.

In applying an item-sampling technique to formative curriculum evaluation in mathematics several interrelated questions need to be answered.

1) What sample size is needed?
2) What accuracy of the sampling estimates is desirable and obtainable?
3) How many items need to be used?
4) How many items per test are desirable and feasible?

3. Research Design and Procedure

The mathematics program which was formatively evaluated was the sixth grade portion of Patterns in Arithmetic (PIA) which is made up of 64 fifteen minute TV sessions with pre and post activities, Teacher Notes and Pupil Exercises. Two TV lessons were viewed each week by the students in 62 participating classrooms within a fifty mile radius of Madison, Wisconsin.

Twelve 20 item tests were developed for use in the evaluation. Testing was done four times during the 1968-69 school year. The first administration (T_1) followed program 5 (September) of PIA, T_2 followed program 20 (November), T_3 followed program 41 (February), and T_4 followed program 63 (May). At each administration every student completed a test, all tests were administered and a student completed a different test at each administration.

Each evaluative instrument contained twenty items. There were thirteen multiple choice items and seven free response (work out answer and record it) items. Each test had the same directions for administration. In strict item-sampling situation, items are randomly assigned to tests. In the present study a pool of items was constructed in June 1968, partitioned into homogenous (by content area) subsets, and each test was constructed by selecting items from a variety of content areas and a variety of difficulty levels. The aim was an interesting, informative, and balanced test. Thirty-five minutes was hypothesized to be sufficient for test completion by almost all students.

The first administration of the twelve tests was prescribed by the researcher. Participating teachers were required to follow directions carefully so that each test was taken by two or three students in each class. Upon receipt of the student rosters along with indication of the test completed by each student, the investigator assigned tests to be completed by each student at T_2, T_3

and T_4. The assignment was random within the restriction that no student should take the same test twice.

In order to provide a basis for making judgments concerning the effectiveness of PIA, each item measuring a program objective was classified into one of 5 categories. These categories were (1) Mastery level 1--Item easy for most students at the end of the year; (2) Mastery level 2--Area received strong emphasis during the year yet high level mastery is not expected; (3) Mastery level 3--Items represent more complicated aspects of content in PIA as well as problems which are conceptually difficult and computationally complicated; (4) Transfer level 1--Items involve minor extensions of concepts; and (5) Transfer level 2--Items are usually conceptually difficult, represent extension of program content and contain difficult computations. A lower bound of acceptable end of year performance was arbitrarily set for each of these categories.

4. Findings

1) The average class time needed to complete the tests at T_1, T_2, T_3 and T_4 was 23.8 min., 24.8 min., 23.9 min., and 23.8 min.
2) Four items appeared on two different tests. Analysis of the response rates for these four items demonstrated that 120 random responses produced a reasonably stable estimate of item difficulty.
3) A growth profile of correct response rates was constructed for each item.
4) The 240 items administered at each testing period were partitioned by content area to aid interpretation. The content area results formed the basis for decisions made regarding changes in PIA.
5) On the basis of the test results
   a) Major revisions in two TV programs of PIA were deemed necessary.
   b) Several areas of weakness were identified, i.e. measurement, problem solving, long division, and ratio.
6) The item pool was weak in that it did not include items representative of all major objectives of PIA.

5. Interpretations

1) Large variations in responses to single items should not be considered as firm evidence that program

change is needed. Rather, a set of related items should be examined if single items suggest the possibility of a problem.

2) The item pool used in item-sampling must include only items which provide information relative to the objectives of the program being evaluated.

3) Test instruments should be constructed so that every pupil will have the opportunity to respond to each item.

4) Test items should be classified prior to their use in evaluation. A minumum number of categories should be used and the final level of performance for each category indicated by setting a lower bound criterion for each category.

5) An aid in making the testing results useful is to group items into content areas and to indicate, via a code, the type of instructional emphasis on each bit of content occurring between testing periods. It is recommended that two symbols be used--one for extensive coverage in the program and one for significant review.

Abstracter's Notes

This is not a report of an experimental study. It is a careful discussion of the technique of item-sampling as applied to formative evaluation. The author has provided well reasoned arguments for variations from the completely random assignment of items to tests and tests to subjects. He has provided good guidelines to other curriculum developers for the successful use of the item-sampling technique in formative evaluation.

Arthur F. Coxford
University of Michigan

EJ 032 208 270 SE 502 697
ISOLATION OF FACTORS THAT INFLUENCE THE ABILITY OF YOUNG CHILDREN TO ASSOCIATE A SOLID WITH A REPRESENTATION OF THAT SOLID Brumbaugh, Douglas K., Arithmetic Teacher, v18 n1, pp49-52, Jan '71

Descriptors--*Classification, *Geometric Concepts, *Instruction, *Mathematical Concepts, Association (Psychological), Preschool Children, Readiness

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jerry P. Becker, Staff Associate (Mathematics), National Science Foundation Science Liaison Staff, New Delhi/India (on leave during 1971-72 from Rutgers University, New Brunswick, N.J.)

1. Purpose

To investigate the ability of young children to comprehend the relationships that exist between a solid and its representations expressed in the form of sketches or photographs.

2. Rationale

Various ideas, concepts, and topics are regarded as important in the intellectual development of a child. An important question, in this regard, is: What is the best time at which these ideas and topics can be comfortably acquired? Also, are there factors in the perceptual development of some young children which enhance or inhibit the growth of their ability to associate different size solids with representations of a given size?

3. Research Design and Procedure

Five solid shapes were used in this investigation: right circular cylinder, sphere, ellipsoid, square based rectangular parallel piped, and cube. These shapes were used because the possibility exists that one of them might be mistaken, on the basis of only some of its defining characteristics, for another solid in the set. As an example, failure to consider height could result in selection of a rectangular solid as a cube, or vice-versa. To introduce an additional factor--that of size--four solids of each shape were used: one small, two medium, and one large. Solids were constructed so that the height of the small rectangular solid and the length of the base of the middle sized one were each the same as the length of an edge of the middle sized cube. Similarly, the height of the middle sized rectangular solid and the length of the base of the large rectangular solid were each the same as the length of an edge of the large cube.

A color photograph and a black line sketch of a middle sized solid of each shape were used as classification stimuli. The sketch and photograph of each solid were fastened together, back to back, and hung on a string fastened to a container in which the solids could be placed. (This procedure facilitated switching from one representation to another half way thru the test.) A randomized order was used in presenting solids to the children for classification.

Seventy one three-year-olds and 58 four-year-olds were tested, all individually. Children were to find the representation which they felt depicted the solid they were holding and then place the solid in the box with the chosen representation. Selections were recorded, compiled, and later analyzed in three ways: item analysis, Paired T Test, and multivariate analysis.

4. Findings

An item analysis showed that correct classification of cubes and rectangular solids were most difficult for the children, whereas, cylinders, spheres, and ellipsoids were relatively easy to classify.

Out of a possible 20 items, the mean score for subjects was 17.43, indicating that the test was easy for many of the children. The distribution was negatively skewed and exhibited a strong ceiling effect.

The Paired T Test was used to compare several partial scores for each child, with results showing that three- and four-year-olds associate a solid equally well with its photographic or sketch representation. Children could more easily classify solids that were the same size as the solid in the sketch or photograph. Although both large and small size solids were difficult for children to classify, neither was significantly more difficult than the other.

A multivariate analysis was carried out, with the following factors considered: (1) Teacher and experience (whether or not the student had formal lessons pertaining to shape), (2) Order of presentation (sketches or photographs first), (3) Socio-economic level, (4) IQ. Results showed that the Teacher and the Experience of the child with an instructional unit dealing with shape were the most influential factors in the child's performance. It was not possible to differentiate between Teacher and Experience or to ascertain which, if either, made the more significant contribution to the child's performance. Results showed, however, that there is high potential effectiveness in teaching three- and four-year-olds to identify shapes and that the child's performance in the classification of any medium or large solids and of cubes of any size will serve as a good indicator of the experience (or teacher) with which the child was associated.

IQ also affected performance of the children. Small solids of any shape as well as any cylinder can be used to discriminate between IQ groups. Medium IQ children (between 90 and 110) and high IQ children (above 110) scored significantly better than children with low IQs (below 90).

When order of presentation was significant, higher scores were achieved by children who first observed photographs for the placement of the 10 solids, and then observed sketches for the remaining 10 solids; but only rectangular solids provided a basis for discrimination between children.

Children were classified into one of five socio-economic levels using the Hollingshead Two Factor Index; however, socioeconomic level was not a significant factor in any of the comparisons.

5. Interpretations

The study showed that three- and four-year-old children are able to classify solids by either pictures or sketches of the solids. Their ability to identify solids correctly appears to be influenced by a reasonable training program and the IQ level of the children. Further, placement of solids appears to be dependent on physical characteristics of the solids being classified and data show that the children had the most difficulty differentiating between cubes and rectangular solids.

Abstracter's Notes

Studies such as this, in which a child's acquisition of mathematical ideas and relationships is studied, are always of interest to mathematics educators. However, a clear cut rationale for the study is not stated, nor does it seem obvious. For example, it is not clear what the results of such research can tell us about the intellectual development of a child. Nor is it clear how such results fit into the picture of a child's mathematical thinking development. And what are the implications for curriculum development? These are questions that might be discussed so that mathematics educators get a better understanding of how the research can help us and be used.

A mean of 17.43 out of a possible 20 items seems quite high. Accordingly, it is not clear why an extensive analysis of the data is made, leading to the conclusion that the test was easy for many of the children.

It was found that three- and four-year-olds associate solids equally well with their photographic or sketch representations. But what might be the characteristics of the photographic and sketch representations which might lead us to hypothesize otherwise? Also, it was found that the Teacher and the Experience of the child with an instructional unit dealing with shape was the most influential factor in a child's performance. While not being able to differentiate between these factors, the author goes on to mention that a child's performance in the classification of any medium or large solids and of cubes of any size will serve as a good indicator of the experience (or teacher) with which the child was associated. I am not sure of the value of this observation nor of the degree to which it might be generalized.

IQ is found to affect the performance of a child. However, IQ is a general kind of construct, which may have many components. In particular, spatial ability is a part of the more general concept of IQ and may have played a role in the performance of children on the test. For example, might not children with "high" spatial ability be able to perform consistently better than children with "lower" spatial ability? In general, examination of the role of particular aspects of mental ability in performance on mathematical tasks may be more revealing than by examining the role played by more general IQ measures.

Jerry P. Becker
Rutgers University

ED 047 958 SE 010 796
MEASURING VOCABULARY AND SYMBOL FAMILIARITY IN THE LANGUAGE OF MATHEMATICS, Byrne, Mary Ann; Kane, Robert B., Georgia Univ., Athens,; Purdue Univ., Lafeyette, Ind. Pub Date Feb 71, Note--11p.; Paper presented at the Annual Meeting of the American Educational Research Association (Feb. 4-7, 1971, New York City, N.Y.) EDRS Price MF-$0.65 HC-$3.29
Descriptors--*Concept Formation, *Elementary School Mathematics, *Mathematical Vocabulary, Mathematics Education, *Reading Research, *Secondary School Mathematics.

Expanded Abstract Prepared Especially for I.M.E. by L. D. Nelson, University of Alberta.

1. Purpose

To develop a difficulty measure of mathematical terms and mathematical symbols as a step in the development of readability formulas appropriate for mathematical materials.

2. Rationale

The level of vocabulary difficulty in reading material is usually determined by comparing the words in the material with a list of words having a certain familiarity or frequency of use. Quite serious problems arise, however, when the material contains a large proportion of specialized vocabulary such as is found in mathematics textbooks. This vocabulary is made up of words which may have general meaning but are used in mathematics in specific contexts (e.g. set); which may have mathematical meanings different from ordinary meanings (e.g. field); which may have meaning only in mathematics (e.g. perimeter); and the like. There is also a complex system of symbols (e.g. the square root sign). It was to determine a measure of difficulty for such mathematical terms and symbols that the authors conducted this study.

3. Research Design and Procedure

It was proposed to obtain measures of familiarity of mathematical terms and mathematical symbols of seventh and eighth grade students in the United States. To do this the authors proceeded to develop two measuring instruments - one for mathematical terms and one for mathematical symbols. If the student could remember a definition, give an example, or give an explanation in his own words he was deemed to be familiar with a term. The student was the judge and would check either "know" or "don't know" for the term. The familiarity score for a term was the percentage of students who indicated they knew the term. A similar procedure was used to get a familiarity score for a mathematical symbol.

A sampling frame or list of mathematical terms to be tested (1165 terms in all) was compiled from pre-calculus mathematics books. Pre-calculus texts were used because the authors were primarily interested in the readability for seventh and eighth graders of elementary and secondary materials. The phase "additive identity" and the words "additive" and "identity" would all be included in the frame. If familiarity of a word which appeared in a phrase was wanted, such as "acute" in "acute triangle", the word whose familiarity was to be determined was set between asterisks (*acute* triangle). From the list of 1165 terms approximately 5000 unique tests of 100 items each were made up using a randomization program.

The 154 symbols were obtained from the same textbooks and 9 tests of 36 symbols each were generated. Symbols were randomly selected and each symbol appeared in at least two of the tests.

Seventh and eighth grade students from the United States were selected by what the authors call a proportionate stratified random sampling. From the sample of students approximately 350 responses were obtained for each term and 250 responses for each symbol. Measures of stability, level of agreement of scores, and other checks into the precision of the results were obtained.

4. Findings

Frequency distributions of mathematical terms and mathematical symbols according to intervals of familiarity were presented. Lists of mathematical terms whose degree of familiarity were found to be between 90% and 100% and between 80% and 90% were given. A list of symbols known by at least 70% of the students was also given.

5. Interpretations

The following observations were made.

1. Students tended to distinguish between word forms with some precision.

e.g.

Word Form	Familiarity
equal	92%
equation	83%
equality	72%
equate	24%

2. Consistency of student responses and differentiation according to form in which they appear can be noted in the following example.

Word Form	Familiarity
commutative	71%
associative	76%
distributive	67%
commutativity	44%
associativity	39%
distributivity	38%

Some rules for different word forms may have to be established.

3. Students respond discriminatively to mathematical words used in different contexts.

e.g. degree of an angle - 77%

degree of a polynomial - 26%

4. Familiarity measures for mathematics vocabulary (terms and symbols) now exist.

Abstracter's Notes

The authors leave us with the question, "Will a measure of vocabulary difficulty have predictive power in a readability formula?" Most readability formulas do contain vocabulary difficulty as a predictor variable and the care with which this research was carried out would indicate that the results will prove very useful in this connection. However, as the authors point out, the validity of the measures they have developed is yet to be established. In any event it is almost certain that these measures will provide a useful guide for those involved in producing mathematics material for junior high school pupils at least.

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EJ 033 809 310 SE 502 713

A TECHNIQUE FOR STUDYING CONCEPT FORMATION IN MATHEMATICS

Collis, K. F., Journal for Research in Mathematics Education, v2 n1, pp12-22, Jan '71

Descriptors--*Cognitive Processes, *Concept Formation, *Learning, *Mathematical Concepts, *Secondary School Mathematics, Grade 8

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Richard J. Shumway, The Ohio State University

1. Purpose

Is there a potential usefulness for a particular card sorting task in the study of mathematical concept formation?

2. Rationale

Card sorting tasks have long been used by psychologists to study "artificial" concept formation. It is proposed here that such tasks can be designed to study the formation of mathematical concepts taught in school mathematics. It is implied that the card sorting task suggested helps to meet "A fundamental methodological requirement...that the researcher be able to observe, record, and quantify the child's mathematical thinking without too much disturbing, disrupting, or distorting it."

3. Research Design and Procedure

Based on an examination of Hubbard, Numbers in Relationship (Academy Press, 1964) and discussions with grade 8 teachers using the text, 56 items similar to the following were chosen and printed on 3x4 cards:

1.	$3 \times 4 = 4 \times 3$	7.	$w - m = m - w$
19.	$y = ax + b$	20.	$b = y - ax$
23.	$5 - 5$	25.	0×7
28.	$2x = 8$	31.	$3x - 7 = 12 - 7$
40.	$2b = a$	42.	$b/a = 1/2$
48.	x is integral $3x = 7$	50.	$\frac{b}{0} =$

According to the experimenter and a panel of four expert teachers, the items fell into the six categories: (1) commutative principle with contrasting examples, (2) equivalent formula, (3) zero (with one contrast), (4) equation with one variable

$x = 4$, (5) ratio, and (6) impossible statements for these students.

Two items were rejected as redundant. For the card sorting task each subject was given a pack of the 54 cards, instructed to lay them all face up on the table, and arrange the cards in any groups which seemed to go together. Cards which did not fit were kept separate. There was no time limit. The subject was not shown any category system and was free to make as many groups as she wanted. The experimenter recorded the items placed in each group and the number of groups for each subject. For a pilot study, five subjects were randomly selected from one grade 8 class in each of four convent girls schools. Each of the 20 subjects was given the card sorting task six times over a period of seven months in 1965. According to the experimenter, the student records were examined in two ways,

> (a) the development of what may be termed "pure" categories, that is, categories containing two or more cards but with no misconceptions or irrelevant cards included and (b) the patterns of category development that showed up upon inspection of the individual protocols.

The experimenter was interested in the number of "pure" categories at each administration (graph suggests that the number of pure categories at each administration ranged from about 60 to 200) and the pattern of category development. The data were summarized descriptively.

4. Findings

The number of "pure" categories formed "increased" from first administration to last administration. Smaller categories were integrated in later administrations "in order to associate the cards concerned with a higher level principle." Based on the first three administrations the experimenter was "able to" predict the emergence of new categories as distinct groups in later administrations. No statistical tests of significance were used.

5. Interpretation

The investigator concludes that

a) designing, administering, and interpreting the results of the card sorting task are "skills already within the competence of the classroom mathematics teacher."

b) "the card-sorting technique would be of assistance to a teacher in tracing a child's conceptualization of the various principles in a particular mathematics course."

c) the technique would offer teachers and psychologists in remedial education "an aid to determining the adequacy of child's cognitive functioning level... without intervention of...reading and written expression..." variables.

d) "the possibilities of the technique for use in educational research have been enhanced by developments in the field of factor analysis that enable data gathered by the means described above to be analyzed more objectively."

Abstracter's Notes

It is important to emphasize that this is a feasibility study for a particular research technique. Some questions come to mind:

1. Can this card-sorting task be analyzed in the experimental psychologist's terms? Is it in the reception or selection paradigm? Are the concepts conjunctive, disjunctive, conditional, biconditional, or some more complicated combination? Is the task simply concept formation when it involves multiple concepts? What about relevant and irrelevant attributes?
2. Did the experimenter entertain the hypothesis that the changes he observed could be caused by the training provided by the task itself? What is the reliability of the instrument?
3. Why were there so many (200) distinct pure categories at the sixth administration? What were they?
4. What types of statistical analysis could be applied to the data available from the card-sorting tasks? Could they have been illustrated with the pilot data? The experimenter's advice would be welcome.
5. What limitations does the experimenter see for the card-sorting task?

Since this is a feasibility study we assume the findings reported in the pilot study are simply illustrative of some of the potential questions which could be asked. The value of the report is in the description of the card-sorting task and the suggestion that it may be useful in educational research.

Richard J. Shumway
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ED 040 878 24 SE 008 994
EFFECTS OF NUMBER OF INSTANCES AND EMPHASIS OF RELEVANT ATTRIBUTE VALUES ON MASTERY OF GEOMETRIC CONCEPTS BY FOURTH- AND SIXTH-GRADE CHILDREN (PARTS 1 AND 2). Frayer, Dorothy Ann, Wisconsin University, Madison. Research and Development Center for Cognitive Learning. Spons Agency--Office of Education (DHEW), Washington, D.C. Bureau of Research. Pub. Date, Mar. '70, Note--125p. EDRS Price MF-$0.65 HC-$6.58.

Descriptors--*Elementary School Mathematics, *Geometric Concepts, Grade 4, Grade 6, *Instruction, *Learning, Mathematical Concepts, *Mathematics Education, Research.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Shirley A. Hill, University of Missouri, Kansas City.

1. Purpose

To devise a set of prototypic tasks which would test various aspects of concept learning. To determine the effect of two instructional variables, number of instances and emphasis of relevant attribute values, on the performance of these tasks.

Two hypotheses were tested: 1) that level of concept mastery would increase as a function of the increase in number of instances presented and 2) that emphasis of relevant attribute values would facilitate concept learning.

2. Rationale

Concept learning research should be extended in several ways: 1) a wider range of concepts should be examined, with careful specification of the essential characteristics of the concepts; 2) various instructional procedures should be utilized, verbal as well as nonverbal strategies; 3) a set of differentiated response measures should be employed to assess both short-term and long-term retention; and 4) performance of subjects of different ages on the same task should be compared.

The study attempts to deal with these four needs within the following framework. Fourth- and sixth-grade children were taught geometric concepts which bore complex interrelationships to one another. A strategy for characterizing the concepts was developed. This consisted of determining the attributes relevant and irrelevant to the concept and of determining the relationships of each concept to the other. Concepts were taught by a combination of definitions and examples, with variation in number of examples and relative emphasis of relevant attribute values. Eleven tasks were identified as test items to measure attainment of each concept.

The effect of number of instances presented on concept attainment has not been clearly established in previous research.

Emphasis of relevant attribute values has been shown to improve concept learning but research has been on inductive tasks only. This study investigates the effect for deductive tasks.

3. Research Design and Procedure

The subjects were 154 fourth-grade and 126 sixth-grade children. The fourth-grade children comprised the entire fourth-grade population of one school in a midwestern suburban community. All sixth-grade children were from a middle school in the same community. These classes were selected from the total sixth-grade classes for convenience of scheduling.

Instructional materials and tests were constructed and refined in a pilot study. They were developed from a set of behavioral objectives based on an analysis of cognitive processes in concept learning. Concepts used were the geometric concepts, quadrilateral, trapezoid, parallelogram, rectangle, rhombus, square and kite.

Lessons designed to teach the concepts were similar to the usual school lessons but controlled the particular variables of interest, number of examples and emphasis of relevant attribute values. Combinations of these variables and counterbalancing resulted in eight different treatments. Within each of the classes subjects were randomly assigned to one of the eight treatment groups. Each group had four lessons, one on background, one or two on attributes, one or two on concepts.

Each concept lesson had two positive and two negative instances. Thus half the groups got 4 instances, half 8 instances. For half the groups the concept lessons also had questions directing attention to the relevant attribute values and a review of these relevant values.

Tests were a multiple-choice test and a completion test developed from the pilot study tests.

Experimenters were two graduate students, familiar with materials and procedure.

The design was a treatments X blocks design with subjects nested within class and treatments crossed with class. A two-way fixed effects analysis of variance model was assumed with the mean square error term as the denominator of the F-ratio for both main effects and interaction. Independent variables were number of concept examples (4 or 8) and emphasis on relevant attribute values (presence or absence of emphasis).

4. Findings

Reliability estimates (Hoyt) for the total multiple-choice test were .81 for grade 4 and .86 for grade 6; for the total completion test were .87 for grade 4 and .87 for grade 6.

Multivariate analyses of covariance were carried out for each grade level. Dependent variables were total score on the multiple-choice test (MT) and total score on the completion test (CT). The covariate was the raw score on the Paragraph Meaning test of the SAT, in order to reduce variability due to differences in reading ability. The analysis revealed that the covariate had a highly significant correlation with the dependent variables.

There was a significant variation among mean vectors over the six class groups. A t test indicated differences were not due to the different experimenters but to differences among class groups.

The variation in mean vectors due to emphasis of relevant attribute values was highly significant for Grade 4 ($p<0.0085$ on MT, $p<0.0076$ on CT). The effect of number of instances and the interaction between number of instances and emphasis of relevant attribute values were not significant.

For Grade 6, there was no significant variation among mean vectors for any of the main effects or interactions.

5. Interpretations

While item difficulties increased from task level to task level on the test, there is not sufficient evidence to suggest a hierarchy of task complexity. Refinements in tests and expansion of subtests might permit more analytical differentiation of levels of concept mastery.

Interference in tasks may have been due to different meanings previously associated with concept labels and to similarity of concept labels themselves.

The general lack of effect of the variable of number of concept instances may be related to the fact that all groups received both positive and negative instances. Greater effects might have resulted from use of positive instances only.

The significant effect of emphasis of relevant attribute values at Grade 4 and lack of effect at Grade 6 suggest that the greatest effect of such emphasis is on the ability to correctly label attribute values.

Abstracter's Notes

This is a very well-designed and well-conceived study, focusing upon important variables in concept learning in mathematics. As is so often the case, however, the results leave us with continuing uncertainties about the effects of the variables. The limits of time and other constraints imposed on research in the classroom make definitive answers extremely illusive.

A factor which must have some effect on the results is the students' background and exposure to the concepts taught. Even though they were not a part of the formal program previously, many of the concepts (squares, rectangles, etc.) were those to which nearly everyone is exposed informally in varying degree prior to Grade 4. This cannot be fully accounted for or controlled in experiments of this type.

The question of an effect of number of instances is complicated or clouded by the equating of positive and negative instances. Previous experimentation in concept learning suggests we might look more closely at the particular sequence, combination (perhaps ratio) of positive and negative instances before we can determine clearly the comparative effects of number of instances.

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ED 047 960 SE 010 799
TEMPORAL POSITION OF REVIEWS AND ITS EFFECT ON THE RETENTION OF MATHEMATICAL RULES. Gay, Lorraine R., Florida State Univ., Tallahassee. Pub Date Nov 70. Note--85p.; Paper presented at the Annual Meeting of the American Educational Research Association (Feb. 4-7, 1971, New York, N.Y.)
EDRS Price MF-$0.65 HC-$3.29.
Descriptors--*Achievement, Algebra, Doctoral Theses, *Elementary School Mathematics, Geometry, *Instruction, *Learning, Retention, *Time Factors (Learning)

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Roland F. Gray, University of British Columbia

1. Purpose

The purpose of this study was to investigate the effectiveness of spaced reviews in terms of retention of mathematical rule learning. Specifically, the investigator sought to determine

a) the effects of one review on rule retention
b) the effects of temporal position of one review on rule retention
c) the effects of two reviews on rule retention (regardless of temporal position)
d) the effects of temporal position of two reviews on rule retention.

Three hypotheses were developed from a review of previous research. They are in summary:

a) One review will significantly enhance retention of rule learning
b) Temporal position of one review will not have a significant effect on retention of rule learning
c) One early and one late review will be more effective than either two early reviews or two late reviews in strengthening retention of rule learning.

A number of subsidiary questions were also investigated.

2. Rationale

An extensive literature search was made of studies of the relationship of review to retention of both non-meaningful and meaningful learning. The latter studies usually employed reading passages. These studies of retention of meaningful learning tended to support the following generalizations:

a) both early and late reviews seem to affect retention equally, but for different reasons. An early review appears to promote consolidation of what has been learned; while a late review appears to promote relearning of what has been forgotten. Therefore, one would not expect differences in retention scores when comparing a group receiving an early review with a group receiving a late review.
b) Retention does not vary when degree of original amount of learning is the same for all subjects.
c) Spaced reviews or distributed practices are more effective than massed practice.
d) One review will produce greater retention than no review and two reviews will produce nearly three times as much retention as one review.

However, none of the earlier researchers had investigated the effects of review on retention of intellectual skill or concept learning such as mathematical rule learning. Nor had any previous investigation been made of the effects of temporal position of reviews on such learning.

The author derived the purposes and hypotheses from implications of these previous studies.

3. <u>Research Design and Procedure</u>

This study consisted of two separate experiments as described below.

Experiment I was designed to examine the temporal position effects of one review on retention of mathematical rule learning. The sample was composed of 53 grade eight subjects randomly assigned to four groups.

Group 1 received one review one day after original learning
Group 2 received one review one week after original learning
Group 3 received one review two weeks after original learning
Group 4 received no reviews

Experiment II was designed to examine the temporal position effects of two reviews on retention of mathematical rule learning. The sample consisted of 67 grade seven subjects randomly assigned to four groups.

Group 1 received a review one day and two days after original learning
Group 2 received a review one day and seven days after original learning
Group 3 received a review six and seven days after original learning
Group 4 received no reviews.

All subjects in both experiments were taught four mathematical rules by a C.A.I. program. Two were algebraic rules: raising an algebraic expression to an indicated power; and determining the exponent of the product of indicated factors. Two were geometric rules: finding the measure of a third angle of a triangle when two are given; and finding a geometric mean.

All subjects in both experiments were given the rules, shown examples, then asked to practice until they attained a success criterion of two successive correct solutions for each rule.

Each review group in both experiments practiced until the same criterion level of success was attained. A delayed retention test was given to all subjects 21 days after initial learning.

The following experimenter constructed tests consisting of eight items, two for each rule, were used.

a) Pre-learning and post learning test
b) Pre-review one and post review one test
c) Pre-review two and post review two test
d) Delayed retention test

A record was made of the number of examples and the time required to reach mastery at each session.

Data were analyzed by analysis of variance and covariance techniques.

4. Findings

(a) For Experiment I

(i) All review groups scored significantly higher than the no review group on the delayed retention test ($p < .05$)
(ii) The effect of temporal position of one review was not significant

(b) For Experiment II

(i) All review groups scored significantly higher than the no review group on the delayed retention test ($p. < .01$)

(ii) The group with an early and late review scored significantly higher than the group with two early reviews (p. < .01)

(iii) The group with an early and late review scored higher than the group with two late reviews. Differences were not significant.

(c) Additional findings

(i) The number of examples required to reach the criterion of mastery at the first review were approximately the same as the number required at the time of original learning. However students took about 50% less time to reach mastery at review one.

(ii) One review improved retention by up to 45%.

(iii) A second review required approximately 50% fewer examples to reach mastery and in approximately 95% less time.

(iv) A second review improved retention up to 150%.

5. Conclusions and Interpretations

The experimentor concluded, "...that with respect to the retention of mathematical rules, one review is more effective than no review, regardless of temporal position. Optimal retention over a three week interval, however, is obtained with two reviews, one early and one delayed."

Abstractor's Notes

This is an excellent example of a carefully conducted piece of research designed to explore a limited and quite specific aspect of mathematical learning. Within the confines of the purposes and design one can be reasonably confident that the findings are no more than minimally affected by extraneous or non-experimental variables. This study could well serve as an exemplar in graduate training programs. However, some additional comment should probably be made.

1. One is impelled to note that throughout the written report there are numerous editorial lapses which at times result in comprehension difficulties.
2. The C.A.I. instructional procedure though possibly somewhat mechanical and narrow in approach certainly could serve well to control the teacher variable so critical in most educational studies.
3. Finally students should be cautioned that the findings from this study relate only to the kind of review which is characterized by repetition of original learning. There are, of course, other

methods of review such as those which call for re-learning and application of previous learning to new situations. Such other approaches could well prove to be more efficient than the type of review investigated in this study, though they could present design challenges not posed in the present study.

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ED 040 881 SE 009 056

THE CLOZE PROCEDURE AS A MEASURE OF THE READING COMPREHENSIBILITY AND DIFFICULTY OF MATHEMATICAL ENGLISH. Hater, Mary Ann; Kane, Robert B., Purdue University, Lafeyette, Ind., Pub. Date '70, Note--25p., EDRS Price MF-$0.65 HC-$3.29.

Descriptors--*Cloze Procedure, Mathematical Vocabulary, *Mathematics Education, *Reading Comprehension, *Reading Difficulty, Research, Secondary School Mathematics.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Donald J. Dessart, The University of Tennessee, Knoxville.

1. Purpose

To validate the cloze technique as a measure of the comprehensibility and difficulty of mathematical English (ME).

2. Rationale

In the cloze technique certain words or symbols are deleted from written passages and replaced with blanks which students attempt to complete during the reading of the passages. The score for each passage is the number of responses which match the deleted words.

This technique had been shown to produce reliable and valid measures of reading comprehensibility and difficulty for passages of ordinary English (OE), but the procedure had not been validated for use with ME. In this study, the technique was adapted for ME by using definitions of word-tokens, math-tokens, and an ordering of these tokens as provided by Hater.

3. Research Design and Procedure

Five ME passages were selected: P(1), Matrices, using a discovery approach; P(2), the Metric System; P(3), Matrices; P(4), Statistics; and P(5), Logic. Each passage was lengthened or shortened to approximately 700 tokens, which included questions, pictures, and graphs, but did not include exercises.

For each of the ME passages, five cloze tests were constructed (designated Form i, where i= 1,2,3,4,5) by deleting the ith token from the passage for Form i and then every fifth token thereafter until 130 tokens had been deleted. A total of 25 cloze tests were constructed and were designated, <u>Cloze Test</u> <u>P(j)</u>, <u>Form (i)</u>, where j= 1,2,3,4,5.

Multiple choice tests of twenty eight items with five choices for each item were prepared to cover each of the five passages. Care was taken to insure that the directions and test items did not contain math-tokens that were not included in the ME passages or in elementary mathematics textbooks nor word-tokens that were not included in the ME passages or in certain defined sections of <u>The Teacher's Word Book of 30,000 Words</u> (1944). Each test was designated Comprehension Test P(k), where k= 1,2,3,4,5.

During three days, data was collected on 1,717 students enrolled in grades 7 through 10 of schools in Cincinnati, Dayton, Springfield, and Lincoln Heights, Ohio. On the first day, a cloze test was administered, six days later an ME passage was studied, and on the following day the ME passage was reviewed for ten minutes and a comprehension test was taken.

The design of the experiment with Comprehension Test P(1) shown as an illustrative example is given below. Similar designs were used for the remaining tests.

	Day 1	Day 2	Day 3
Group 1 :	Cloze Test P(1), Form (1)	Read P(1)	All Take Compre- hension Test P(1)
Group 2 :	Cloze Test P(1), Form (4)	Read P(1)	
Group 3 :	Cloze Test P(1), Form (5)	Read P(1)	
Group 4 :	Cloze Test P(i)	Read P(1)	
Group 5 :	Cloze Test P(j)	Read P(j)	

$$i \neq j \neq 1$$

The numbers of students which had been randomly assigned to each of the 25 groups are shown below in the first five lines (Groups 1-5). In addition, special subgroups were selected as indicated in the remaining lines. Subgroup 1* consisted of students randomly selected from Group 1. Validation Group 1' and Cross Validation Group 1" were formed by randomly assigning one-half of Group 1 to each group.

	P(1)	P(2)	P(3)	P(4)	P(5)
Group 1 :	211	225	209	225	220
Group 2 :	31	28	33	32	31
Group 3 :	32	32	29	31	32
Group 4 :	30	31	35	34	33
Group 5 :	28	30	35	31	29
Group 1*:	28	31	27	30	30
Group 1':	111	111	109	115	106
Group 1":	100	114	100	110	114

4. Findings

The following conclusions concerning the experimental design of the study were reached primarily through the use of the analysis of variance.

(a) There was no reason to believe that the means and variances of scores of all cloze forms over the same passage were unequal. Therefore, it appeared justifiable to use only one cloze form to test the validity of the cloze tests as measures of reading comprehensibility.

(b) There were no significant differences in the means of scores on comprehension tests taken by groups of students who were administered a cloze test over a passage different from the one tested by the comprehension test. Consequently, it was felt that the taking of a cloze test did not "unduly sensitize" students and cause them to respond "differentially" to comprehension tests.

(c) In comparing the means of the results of comprehension tests for groups of students who read and studied the passages with those who had not, significant differences were found for each of the five passages. It was concluded that the comprehension tests were, in fact, measures of reading comprehension and not merely former knowledge possessed by the students.

Conclusions related to four major hypotheses of the study were:

(a) Since reliability coefficients as provided by the Kuder-Richardson Formula 20 (K-R 20) were greater than .90 for all cloze tests, it was concluded that the cloze tests were highly reliable; and furthermore, this conclusion was valid for small groups of 30 as well as for large groups of 200 students. In addition, reliability coefficients for the comprehension tests, again using K-R 20, were greater than .77, and it was therefore felt that the comprehension tests also represented reliable measures.

(b) Both linear and quadratic models were investigated to describe the relationship between the cloze test score, x, and the comprehension test score, y, of the students. It was found that the model, $y= ax^2 + c$, accounted for more variance than the model, $y= bx + c$; however, it was felt that if certain limitations due primarily to the length of the comprehension tests were eliminated, the linear model would be as useful as the quadratic model in predicting comprehensibility from cloze rest results. Furthermore, an average correlation coefficient of 0.69 was found between the cloze test and comprehension test scores over all five passages for the validation groups.

(c) The product-moment coefficient of correlation between the means of cloze test scores for the five validation and the five cross-validation groups was .99. The coefficient for the means of comprehension test scores between the same groups was approximately 1.00. On the basis of these findings, it was concluded that rankings by means of cloze tests and comprehension tests are reliable rankings.

(d) The product-moment coefficients of correlation between the means of cloze tests and comprehension tests were .54 for the validation groups, .51 for the cross validation groups, and .83 for the combined groups. Although these coefficients were not found to be statistically significantly different from zero, it was felt that the relationship between cloze test and comprehension test scores was probably great enough (particularly in the combined groups, when all forms were used in the analysis) to warrant further study of the relationship.

5. Interpretations

The investigators felt that the results of the study supported a conclusion that cloze tests over passages of ME provide reliable measures and valid predictors of reading comprehensibility by the students included in the study.

Furthermore, it was felt that there existed sufficient evidence to support the probable conclusion that cloze tests are reliable and valid predictors of the reading difficulty of ME passages. This conclusion was based primarily on the consistency of rankings of cloze test and comprehension test means of all samples of the population.

Abstracter's Notes

Teachers have always felt that difficulties associated with reading have a very decided effect upon learning in mathematics, but relatively few researchers have turned their attention to problems of this area. The authors of this study have not only delved into this fascinating subject but have done so with a well conceived, a well designed, and a well executed investigation.

This study demonstrated that the cloze technique is certainly worthy of further investigation, in spite of a key disappointment in the finding that the correlations between the means of cloze tests and comprehension tests did not attain desired levels of statistical significance. Perhaps, if in a future study more extensive ME passages could be used, a stronger relationship between the two measures will be determined.

The reliabilities for the cloze tests as found by use of Kuder-Richardson Formula 20 were very high, indeed! The authors observed that certain intrinsic characteristics of the tests contributed to these high reliabilities, but a serious problem arose due to the fact that a number of students quit before completing all items. Since attempting to complete all of a test is probably one of the most fundamental assumptions in using K-R 20, a thorough investigation of the reliability of cloze tests using other techniques as well as K-R 20 would seem most appropriate.

Since earlier investigations had shown the cloze technique to be both reliable and valid for OE, one might speculate whether the value of the technique decreases as ME departs from OE. A simple measure of this might be given by observing the ratio of math-tokens to work-tokens in passages. Presumably, as this "math ratio" decreases from one passage to another, one might observe that the passages are becoming more similar to OE types. It would seem that a series of studies of reading comprehensibility utilizing passages with varying "math ratios" could be quite revealing.

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EJ 044 724 310 SE 504 000
SOME EXPERIMENTS ON STRUCTURED LEARNING. Jeeves, M.A., Educational Studies in Mathematics, v3 n3/4, pp454-476, Jun 71
Descriptors--*Cognitive Development, *Learning Processes, *Learning Theories, *Mathematics Education, *Research, Child Development, Conference Reports, Developmental Tasks, Task Performance, [Group Theory]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Merlyn J. Behr, Northern Illinois University

1. Purpose

To seek information concerning questions about structural learning to include the following: i) How can structural learning be studied with a sufficient degree of control to allow some quantification of performance? ii) Is it possible to examine the more marked individual differences in performance in terms of strategies employed in structural learning? iii) How can transfer effects be maximized not only in terms of levels of performance attained on latter tasks, but also in terms of degree of efficiency and sophistication of strategies which are developed? iv) Can limiting factors in human information processing which affect the rate of structural learning be identified?

2. Rationale

The widespread interest in and enthusiasm for 'new mathematics' has brought about the inevitable and desirable involvement of the educational psychologist whose concern is with the gross effects of what different materials achieve and therefore concern about questions related to success and failure rates and rates of progress in developing mathematical skills. Additional involvement on the part of the experimental psychologist brings with it a concern more with the question of how complex learning and thinking takes place. Thus while the educational psychologist is especially concerned with what is achieved, the experimental psychologist is more concerned with how and why it is achieved. The need to subject the innovations of mathematics education to the kind of investigation which gives answers to the questions of what factors are responsible for success or failure and not just whether subjects succeed or fail is argued. The approach used in these investigations parallels that used by Sir Frederic Bartlett in studying perceptual-motor skills; the work of Bartlett repeatedly brought out that it is more important to know how the various components of a complex skill were built up and integrated into a whole, than to know whether the skill was successful or not. This time-tested approach, it is argued, should at least be given a fair trial in the study of cognitive skills. This provides the framework of the reported investigations. It is argued that there is sufficient evidence to suggest that the apparent dichotomy between the stimulus-response and cognitive psychologists' theories of learning may be unreal, and that cognitive processes may in fact use as building blocks smaller chunks of behavior built upon stimulus-response type principles.

3. Research Design and Procedures

Pilot studies using mathematical groups of small order made it clear that it was possible to gain considerable insight into the process assumed to underlie structured thinking by using embodiments of such groups. In early experiments the actual embodiments of the structures was accomplished by having the subject (S) sit facing a window in which cards containing appropriate symbols (such as coloured shapes) could be displayed by E. S was provided with a duplicate set of cards. The dependent variable consisted of the showing of a card by E. The two independent variables were the card appearing in the window previous to the card appearing as the experimental event and the card played by S. S was told that he would play a game, controlled by E, and that he was to guess the rules of the game. S was told that the card to appear would depend solely on what was displayed by E and on the card he played. S was informed that E's only interest was to find out how S went about discovering the rules of the game. Records were kept of the cards exposed by E and the cards played and predictions made by S. After four preliminary trials S was directed to say which card he expected to see exposed next and that this would continue until he was correct every time. In early experiments, only groups of order two and four were used. In later experiments the game was completely automated and group and non-group structures of up to ten elements were used. Ss played each game until a predetermined criterion of success of correct number of predictions was reached or were given a predetermined number of plays and then tested on all possible state and play combinations. Measures taken were as follows: number of instances to reach criterion, number of erroneous predictions made to criterion or in a predetermined number of plays, the number of erroneous predictions made in the test phase of the game, equation scores. In all experiments Ss attempted at least two tasks; later experiments required four tasks on consecutive days. Each experiment was conducted with first-year university students as Ss and also with 10- and 11-year-old children.

4. Findings

Subjects' evaluations of what they thought the games were about fell into three categories: the operator evaluation regarded the card played by S as an operator acting on the card displayed by E; the pattern evaluation split the number of different possible combinations into distinct groups and dealt with such groups as subwholes of the whole game; the memory evaluation found no rules or simply memorized the results of the combinations separately. Combinations of these "pure" types made six distinguishable evaluations. To each type of evaluation there corresponded a game strategy; for example, the operator strategy consisted of continued playing of the same card to determine its effect on the displayed card.

Information from one experiment supported the conjecture that the strategies could be considered to be ordered from high to low corresponding to the order of the listing of evaluations above. The data suggested that "higher order" strategies reflected deeper insights into the structure of certain mathematical groups.

A consistent trend for higher order evaluations was found among subjects who had their tasks in the order four-group followed by two-group, than in reverse order. It seems therefore that the practice of "throwing subjects in at the deep end" paid off as reflected by the criterion.

The effects of structural relations was investigated in an experiment in which all subjects were given four different groups on four successive days. Various structural relationships existed between the groups such as order of the group, embeddedness, recursion, and overlap. The sequencing of the groups to subjects provided for various combinations of the structural relationships. It was found that children (10-11) particularize (go to a group of lesser order) more easily than generalize, and have more difficulty in generalizing than adults; there were apparent interaction effects between age and various combinations of the group relationships.

In one experiment one group of Ss was free to select their strategy and for another group a strategy was imposed on the Ss. A sex by group interaction among adult subjects was obtained. The selection of strategy favored males and the reception of strategy favored females.

5. Interpretations

Three pieces of evidence seem to support the view that a reduced load on short term memory facilitates structural learning: 1) Operator-type (the highest order) strategies were correlated with high performance. 2) Experimentation in which Ss were provided with the opportunity to modify the task according to their choice (otherwise controlled by E) and thus to follow through immediately on a conjecture had a dramatic affect on Ss rate of learning. 3) It was found that groups with different kinds of sub-group structures affected the rate of learning the group structure. The 2 x 4 eight group was learned less easily than the 2 x 2 x 2 eight group. That is, apparently Ss were able to learn more easily when the task could be broken down into smaller chunks.

Concerning the question of how the subjects learned: it was evident from a specially designed experiment in which the sub-group structure of the 3 x 3 nine group was brought out by a perceptual aid that S's learning went beyond learning stimulus-response combinations; the author called this "structural learning."

After analysis of S's performance on a variety of tasks it was concluded that, in this experimental setting, learning could be more elegantly handled by a model which assumes that structural learning is taking place in addition to simple S-R learning.

Concerning the methodology of studying structural learning, it was concluded that if measures had been confined to simple measures of success or failure, instead of more refined submeasures, the temptation to conclude that an explanation of the results in terms of an S-R model would have been greater than justifiable. It was only by analyzing the component parts of subjects' performances that an understanding of the process of the learning taking place began to unfold.

Abstractor's Notes

The research reported makes a definite contribution to the study of mathematical learning. The point that the innovative programs in mathematics education need to be subjected to the careful kind of research which answer, in addition to the question of whether students learn, the question of how and why students learn is well taken. In this regard this work makes a significant contribution in identifying certain aspects of learning and provides a possible model for further research. Numerous researchable questions are raised; for example, the question of whether interactions will be obtained between learning strategy and intelligence, age, personality factors, etc.

An operational definition and/or discussion of what structural learning is would have been helpful. The reader is given some help with this question through the delineation of some criteria which experimental tasks should satisfy in order to be used in the study of structural learning. However, a later statement in the paper: "...our data...supported the view that in addition to learning stimulus-response-outcome combinations in a rote manner, something more was being learned which we called "structural learning," increased this reader's concern about a definition of the term.

Some questions about the experimental design, for example, how were the samples selected, was random assignment to treatment groups accomplished, exactly what were the experimental tasks, caused serious restrictions to be placed on any attempt to generalize the results to other situations, and especially to a classroom setting.

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EJ 038 736 010 SE 503 271
SOME STRATEGIES FOR SOLVING FOR SOLVING SIMPLE MULTIPLICATION COMBINATIONS Jerman, Max, Journal for Research in Mathematics Education, v1 n2, pp95-128, Mar 70
Descriptors--*Computer Oriented Programs, *Elementary School Mathematics, *Multiplication, *Mathematical Models, Computers, Grade 3, Grade 6, Learning, Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by John G. Harvey, University of Wisconsin, Madison.

1. Purpose

The purpose of the studies reported in this paper is not clearly delineated. In the introductory remarks the author states that no one has attempted to "analyze in-depth the strategies used by children to find the product of simple combinations," and it could be inferred that this is the purpose of the studies. However, the studies concern themselves with the power of linear regression models to predict the success latency of finding the product of simple combinations and using those results to place the simple combinations in serial order from easiest to most difficult.

2. Rationale

An explicit rationale is not developed within the paper. However, it seems possible to abstract from the whole paper the following justification for the studies reported:

a) If the 100 simple multiplication combinations could be serially ordered from easiest to most difficult the design of instructional sequences designed to teach them would be influenced; at least the design of drill-and-practice exercises would be facilitated.

b) The problem is logically parallel to the research conducted by Suppes et al. on addition and subtraction combinations and an extension of the work initiated by Suppes et al. on multiplication combinations.

c) Other mathematics educators have investigated the processes by which children solve multiplication problems.

The abstractor wishes to thank Mr. Clyde A. Wiles for his assistance in the preparation of this report.

3. Research Design and Procedure

Two studies are reported in this paper, the Preliminary Study and the Follow-up Study. In the Preliminary Study the linear regression models are tested for their ability to predict success latency, and in the Follow-up Study the same ten models are again investigated together with one additional model which was postulated as a result of the Preliminary Study. The first eight models assume that the simple products are formed by successive addition and thus these models are formed by taking into account the ways in which this can be done. The variable used in the linear regression analysis for each of these models is called "NSTEPS"; it is calculated by counting the number of operations required to complete the additive computation, the number of partial sums stored in the child's memory, and the number of iterations which must be made. The ninth model postulates that the children have mastered the combinations, have arranged them in some sort of internal array or matrix and look up the solution when the combination is presented. The variables used in the linear regression analysis for this model are the number of the row and the number of the column in which the required product appears, for example, for 4 x 5 the numbers 4 and 5 are entered. The tenth model is one previously tested by Suppes et al. and uses two variables; the weights are calculated for the number which is smaller and the one which is larger. The eleventh model is a modification of Model 6; instead of using the value NSTEPS when the product n x n, n x (n-1) or n x (n+1) is considered, the value 0 is used for n x n, and the value n - 1 or n + 1 for n x (n-1) or n x (n+1), respectively. Otherwise the eleventh model and the sixth model are exactly the same.

In each of the studies using a CAI program format every subject was presented with all 100 of the multiplication combinations over a period of two days; the sequence in which these combinations were presented was determined by two computer-generated randomizations of the combinations each of which was broken into two parts of 50 problems each. In each experiment intact classes were used, but the subjects from those classes were randomly assigned, by student number, to a treatment group. For the Preliminary Study the response times of 24 students in grade three, 56 students in grade four, 20 students in grade five and 32 students from grade six from Grant, Walter Hays, Oak Knoll and Clifford Schools were used. In the Follow-up Study, the response times of 132 grade four students, 126 grade five students and 67 grade six students from those schools were used.

The analysis of the data is the same for each model in the studies and was executed as follows:

a) Regression coefficients were obtained for each of the identified dependent variables.
b) A stepwise, multiple linear-regression analysis program was used to obtain regression coefficients, multiple correlation R, and R^2 for each of the models.
c) Another measure of goodness of fit S^2 based upon the mean predicted success latency was computed.

4. Findings

In both the Preliminary Study and the Follow-up Study Model Six (the smaller member is added the larger member's number of times) gave the best overall prediction of success latency. This model accounted for approximately 69 to 80 percent of the variance observed in the first study and from 73 to 78 percent of the variance in the second study. However, Model Six did not have the highest number of best predictions for individual combinations; a variety of models gave the best prediction for individual combinations. The only developmental trend which can be observed in the first eight models is a monotonic decreasing constant value; the weights assigned to NSTEPS did not decrease monotonically though for each of these models the weights determined using the grade six data is lower than those for the other grade levels. The weights assigned to the variables in Models Nine, Ten and Eleven are monotonically decreasing.

In the Follow-up Study Model Eleven gave overall predictions nearly equal to that of Model Six, and it fit specific combinations much more often than did other models.

5. Interpretations

In all cases the variables chosen give good accounts of the data. The lack of monotonicity was tentatively ascribed to the feeling that children use different strategies for different combinations.

The investigator concludes that these studies "seem to support the findings of Brownell and Carper." Further he states "it appears that once a student uses a strategy to find the solution of a particular combination, he may not change the strategy as he grows older." In addition, he concludes that the combinations n x 0, n x 5, n x n, n x(n - 1) and n x (n + 1) appear to have heuristics associated with them that facilitate their retention.

Abstractor's Notes

1) Even though the title of the paper indicates that strategies for finding simple products are studied, the author states "There has been no claim that the actual

strategies used by students in this study were those presented in this paper." The abstractor feels that the author is to be scored for his deceptive title, but he does find attractive the ability to predict success latency from a measure of structural complexity.

2) The conclusions are properly restrained.

3) The face validity of the first eight models is debatable since they are constructed after interviews with only two third grade classes. If the investigator had randomly sampled a larger group, the face validity of these models would have been greatly improved.

4) The calculation of linear regression equations by grade level does not seem most reasonable. Grouping of students by other factors such as prior achievement, IQ, or error rate might have yielded better results.

5) The author states that Models One, Two and Five were expected to show consistently better fits for each grade level; he does not justify this speculation.

6) The paper is not well organized. As mentioned earlier neither the purpose nor the rationale are clearly stated.

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EJ 033 610 270 SE 502 710
AN INVESTIGATION OF STRUCTURE IN ELEMENTARY SCHOOL MATHEMATICS: ISOMORPHISM Lamon, William E; Scott, Lloyd F., Educational Studies in Mathematics, v3 n1, pp95-110, Sept 70.
Descriptors--*Elementary School Mathematics, *Instruction, *Mathematical Concepts, Grade 4, Grade 6, Mathematics, Modern Mathematics

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Nicholas A. Branca, Stanford University.

1. Purpose

The major question addressed by this study was whether young children can be expected to gain an appreciation of mathematical structure. One aspect of structure, the concept of isomorphism, was isolated for particular attention.

2. Rationale

Although structure is a component of modern school mathematics, theorists (such as Russel, Gagne, Bruner and Piaget) dealing with mathematical conceptual development provide little support for the expectation that elementary school children can gain much appreciation of mathematical systems. The hypothesis that is given is that concepts embodied in a true appreciation of mathematical structure are beyond the reach of most elementary school youngsters.

3. Research Design and Procedure

The subjects were 74 elementary school children from Quebec (32 fourth graders and 42 sixth graders) chosen according to their availability and mathematical background (26 in the Dienes experimental program, 24 in the Cuisenaire program, and 24 in a traditional (non-modern) program). Each child was directly and individually taught by an experimenter in a sequence of two 50-minute lessons a week over a period of five weeks. Within the framework of a study of abstract structures, a special approach to the teaching of isomorphism was used. The subjects were led at their own speed through a series of 58 mathematical tasks which were associated with six levels of development of the concept of the mathematical group. The use of concrete examples was maximized with the mathematical game used as the motivational agent. Two embodiments of the mathematical Klien group were presented, one dealing with color-shape representation and the other with a displacement representation. The subjects were expected to identify the moves or changes in the first structure which corresponded to moves or changes in the second one. Children were encouraged through whatever prompting seemed appropriate at the time, but were not lead through concept revelations or hints dealing with the mathematics involved.

Records were kept of the time required to complete the 58 correspondence tasks, the number of correct responses made up to the attainment of the criterion, and the extent to which isomorphism had been learned, and three scores were computed for each subject. Children were deemed to have identified the isomorphism of a particular group if their use of task cards and their commentary revealed that they had discovered that the identity element in one game always had to be mapped with the identity element of the other game and if operating on A by B resulted in C in the first game, then operating on the isomorph of A by the isomorph of B in the second game resulted in the isomorph of C. An analysis of variance of the effects of grade and program differences was performed on both the time scores and the number of correct responses. A percentage distibution of the scores based on the extent to which isomorphism had been learned was constructed. It was assumed that the percent of subjects abstracting the isomorphic concept would be less than or equal to 15%. Test statistics were computed by grade level.

4. Findings

The analysis of variance on the time scores indicated a significant interaction effect ($P < 0.05$) between programs and grades and a significant effect ($P < 0.01$) of the mathematical program. The children with traditional mathematical background required significantly less time than children with Dienes or Cuisenaire backgrounds. No significant differences were found between grades and programs on number of correct responses. The hypothesis that no more than 15% of the total experimental sample as well as that of the fourth or sixth graders would conceptualize the isomorphism was accepted.

5. Interpretations

The results on performance scores, combined with classroom observation seem to provide evidence that fourth and sixth grade subjects generally cannot abstract the concept of a mathematically defined isomorphism following an extensive learning sequence such as that employed in this investigation. Results, including objective data as well as subjective impressions accumulated during the teaching phase, should serve to prompt an uneasiness about the impregnability of the structure objective in modern elementary school mathematics.

Abstracter's Notes

The lack of description of the teaching materials and criterion measures used in the study raises some serious questions regarding the nature of generalizability of the results to all elementary school children and to all aspects of the structure objective in modern school mathematics. In this study, an understanding of the concept isomorphism presupposes an understanding of the Klein group structure. Questions which are raised include the following:

> What were the nature and objectives of the series of tasks used in the study?
>
> What were the six levels of development of the concept of mathematical group associated with the tasks?
>
> How well did the subjects do in learning the structure of the group?
>
> Why was a structure whose embodiments can be isomorphically mapped to each other in more than one way used as the underlying model? (There exist six possible isomorphic mappings of one embodiment of the Klein group structure to a second embodiment.)

Questions regarding the scoring system and the analyses used can also be asked. What is the relationship between the measures used? Are they independent? Although the subjects from the traditional mathematics program required significantly less time to complete the tasks, they also had fewer correct responses.

Finally, the analysis used to test the hypothesis that no more than 15% of the subjects would conceptualize the isomorphism does not justify its acceptance. Although the test statistic used is not described, the table presented indicates that the confidence interval for the percent of subjects conceptualizing the isomorphism ranges from approximately 4.5 to 25.5 percent.

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EJ 031 713 040 SE 502 645
ATTITUDES TOWARD MATHEMATICS OF FACULTY AND STUDENTS IN THREE HIGH SCHOOLS Roberts, Fannie M., School Science and Mathematics, v70 n9, pp785-793, Dec '70
Descriptors--*Secondary School Mathematics, *Student Attitudes, *Teacher Attitudes, Secondary School Students, Secondary School Teachers, Teacher Characteristics, [Torsten Husen Attitude Scales]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by R. E. Pingry, University of Illinois.

1. Purpose

To determine what attitudes toward mathematics are held by high school students and their teachers. In particular an attempt was made to measure

(a) attitude toward mathematics as a process,
(b) attitude about the difficulties of learning mathematics, and
(c) attitudes toward the place of mathematics in society.

2. Rationale

Attitudes are an important consideration in relationship to cognitive learning. Favorable attitudes not only promote learning but may lead the student to continue his study even after leaving the influence of the teacher. Teachers play some part in the imparting of attitudes about mathematics, even if it is a poor attitude toward the subject. If attitude has such an important place in the learning of mathematics, then basic information is needed concerning the attitudes toward mathematics teachers and students have.

The instrument used for measuring attitudes used scales that had been used in the International Study of Mathematics Achievement. These scales were based on an assumption of a continuum of attitude toward mathematics.

The first scale assumed that attitudes toward mathematics as a process would range from those that mathematics processes are governed by fixed, inflexible rules to those that process in mathematics is flexible, permitting latitude in problem solving, and is still developing. The other two scales had similar constructs.

3. Research Design and Procedure

An attitude toward mathematics inventory was given to a sample of high school mathematics students and junior high school mathematics students. The same instrument was administered to the students' teachers. In addition, questions were asked of the high school seniors concerning the number of years they had studied mathematics. It was possible to identify those junior high school pupils who were taking general mathematics. At the senior high level it was possible to identify those students who had given some indication they were college-bound and those who were not college-bound. It was also possible to categorize the teachers into the mathematics-science faculty and the non-mathematics-science faculty.

The attitude scales were assigned numerical values, two for each favorable response, zero for an unfavorable response, and one for an uncertain response. It was possible for the score to be anywhere in the range from zero to 46 since the scores on the items were added to get the total score.

The mean scores of the total scores and each of the three part scores were compared between various categories of students and teachers. The three parts of the attitude score were

Scale I, mathematics as a process,
Scale II, difficulties of learning mathematics,
Scale III, place of mathematics in society.

Differences were tested for significance at the 1% level, and in some cases at the 5% level.

4. Findings

In all, 323 students and 112 teachers completed the attitude inventory. By comparing the mean attitude scores for different groups the following findings were made.

(a) For the students studied in this sample the college-bound seniors scored higher on the scale which measured attitude toward mathematics as a process. For the eighth and ninth grades, however, the general mathematics students considered mathematics processes more flexible than did the algebra students.

(b) Attitudes toward mathematics by students and teachers were similar. On the scale measuring attitude toward mathematics as a process the teachers scored higher. However, on the scale measuring attitude toward the place of mathematics in society the students scored higher.

(c) The attitudes toward mathematics of the mathematics-science group of teachers are higher than for teachers in the non-mathematics-science group.

(d) Both students and teachers chose neutral positions in their attitude toward the difficulty of mathematics and the place of mathematics in society.

5. Interpretations

The author concluded that "the attitude scores suggested that both students and teachers view mathematics as a system interlaced and hedged about with fixed rules and strictures and with little score for flexibility." The author also suggested that "The rather small disparity between student scores and teacher scores suggests that attitudes toward mathematics, once adopted, may be relatively stable over the years."

Abstracter's Notes

Most who have taught mathematics have become aware of the fact that the students' and teachers' attitudes toward mathematics are extremely important in mathematics learning. Despite this fact there is very little known about the attitudes now held by students and teachers and the actions that a teacher can take to help shape attitudes. In this research the author tried to measure objectively some attitudes as held by teachers in students in three schools. Some points were not clear in the report, however, and these questions arise.

1) Were the many comparisons made on differences of means, pre-planned comparisons, or were they post-facto comparisons?

2) What statistical method did the author use to compare means?

3) Were samples chosen appropriately for the comparisons made?

4) Were samples chosen from all the mathematics students in the schools, or were all the mathematics students surveyed?

5) Were all the teachers in the three schools surveyed, or was a sample selected?

6) How did the attitudes of a particular teacher compare to the attitudes of the students in his class?

7) What were the attitudes of the students and teachers? The scores are given, but it would be interesting to have more description of what these scores meant in terms of attitudes.

8) Did the analysis of comparing means tend to hide interesting information in the data? For example, is it possible that a study of the extreme cases in particular situations may not contribute more to classroom practice?

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EJ 013 674 310 SE 500 960
A STUDY OF THE POSSIBLE IMPROVEMENT OF PROBLEM SOLVING ABILITY IN MIGRANT CHILDREN Schnur, James O., School Science and Mathematics v69 n9, pp821-826, Dec 69.
Descriptors--*Elementary School Science, *Learning, *Migrant Children, *Problem Solving, *Student Ability, *Student Characteristics, Psychological Characteristics, (Kagan's Matching Familiar Figures Test).

Expanded Abstract and Analysis Prepared Especially for I.M.E. by E. Glenadine Gibb, The University of Texas at Austin.

1. Purpose

The purpose of the study was to determine the effect of A-Blocks treatment (a subunit of the Elementary Science Study Attribute Games and Problems) has upon enhancing reflectivity through the modification of an impulsive conceptual tempo of migrant children. More specifically, the study proposed to answer the question, "Will A-Blocks treatment modify impulsive tempo?"

2. Rationale

In efforts to understand human psychological make-up which has come to be known as conceptual tempo, Kagan has generated three conceptual tempos: reflective, impulsive, and neutral and has developed instruments by which these traits can be classified and measured by latency time (time lapse from stimulus to first response) and by errors (number of errors made). Also, Kagan has elaborated on the relationships between problem solving and impulsive-reflectivity, namely, that (1) reflection is critical to ease of solution and success in problem solving; (2) that if alternatives are not reflected upon, the student is apt to implement the first idea that occurs to him; (3) that impulsivity has a higher probability of failure, and (4) that incorrect patterns of problem solving may cause withdrawal from problem solving involvement and hostility toward intellectual situations. Thus the study was based on work done by Kagan on information processing and claims made in the Elementary Science Study that experiences with Attrubute Games and Problems can help provide familiarity and skill necessary for solving problems involving classification and dealing with relations between classes.

3. Research Design and Procedure

The sample was composed of eighteen children (eleven male and seven female) of migrating, seasonal crop harvesters and who were enrolled in the 1968 Summer Workshop at the New York State Center for Migrant Studies, State University College, Geneseo, New York. The subjects ranged in chronological age from approximately 4.5 to 14.2 years.

Nine subjects were assigned to each of the two groups (control or treatment). An examiner-teacher was assigned to each subject. Prior to treatment each subject was given Form 1 of the Kagan Matching Familiar Figures Test. Subjects assigned to the control group were then allowed to color, draw, or paint for a thirty minute period for each of six sessions. The examiner-teacher was allowed to converse with the subject but was asked not to direct the activity during each period. The examiner-teacher for each subject in the treatment group guided his subject through one A-Blocks procedural card during each of the six thirty minute periods, interacting and stimulating the subject's investigation within the scope of the given card. Upon completion of the six periods, Form 2 of the Kagan Matching Familiar Figures Test was administered to all subjects.

4. Findings

The t-test was used to test significance of differences between the pre-test and the post-test of each group and between post-test results of two groups when compared to each other. The results of the null hypotheses were as follows:

Hypotheses	Decision *
There is no significant difference:	
1) between pre- and post test for control group on measure of latency time	Not rejected
2) between pre- and post tests for treatment group on measure of latency time	Not rejected
3) between pre- and post tests for control group measure of errors	Not rejected
4) between pre- and post tests for treatment group on measure of errors	Not rejected

*.05 level of significance was used for decision making

5) between two groups on post measure of latency time — Not rejected

6) between two groups on post-test measure of errors — Rejected

The Fisher Exact Test was used to analyze the conceptual tempos generated by the post test (Form 2 Matching Familiar Figures Test). Results did not support significant association between tempos and treatments.

5. Interpretations

The general conclusion generated by this study was "The A-Blocks portion of the Elementary Science Study's Attribute Games and Problems unit does not enhance the reflectivity of migrant children as measured by latency and number of errors."

Findings did suggest to the investigator that "if one's goal is to increase the latency time between stimulus and initial response in migrant children, the method used with the control group generates the most positive trend." Also, the investigator hypothesized that "a more therapeutic approach to working with migrant children tends toward making them slightly more reflective, at least with regard to latency time."

Limitations of the study as identified by the investigator noted the possibility of the more negative, stressful situation created by the A-Blocks treatment, that the treatment may not have been of long enough duration, and that "the figures in the Matching Familiar Figures Test were less well known and thus less familiar to migrant children than to children of a more 'average' experiential background."

Abstractor's Notes

One of the educational goals is that of enabling individuals to solve problems. Awareness and indeed identification of psychological traits which are related to the attainment of this goal are necessary. If reflection is critical to success in problem solving, then one is confronted with the task of seeking ways to develop reflective behavior on the part of individuals who do not demonstrate this trait. Whether or not this modification can be effected equally well by the same tasks at different age levels and/or from diversified backgrounds is questionable. Also instruments to identify behavior and change in behavior must be sensitive to the subjects being measured. As noted by the investigator, figures as identified as being familiar to one population may not necessarily be familiar to another population with different experiential background.

Although this study provided an answer to the questions asked with respect to the treatments used when applied to a small sample scattered across a wide range of ages, little, if anything, could be said to be learned. The abstractor suggests that a larger sample, stratified according to age level, might provide further insight into behavior modification. Also, some effort should be made to verify the assumptions that the instruments used are appropriate for the experiential background of migrant children and that the treatment does in fact involve a valid procedure for such modification.

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ED 040 859 SE 008 821

THE EFFECTS OF DIFFERING PRESENTATIONS OF MATHEMATICAL WORD PROBLEMS UPON THE ACHIEVEMENT OF TENTH GRADE STUDENTS.

Sherrill, James M., Texas Univ., Austin. Pub Date 1 Jul 70,
Note-- 25p., EDRS Price MF-$0.65,HC-$3.29.
Descriptors--*Achievement, *Instruction, *Learning, *Problem Solving, *Secondary School Mathematics, Visual Stimuli.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jeremy Kilpatrick, Teachers College, Columbia University.

1. Purpose

To study the effects of providing accurate and inaccurate pictorial representations on tenth grader's achievement in solving mathematical word problems.

2. Rationale

Drawings and diagrams have been advocated as aids both to problem solving and to instruction in problem solving. Some writers on methodology have argued against the purposeful introduction of errors into the problem-solving situation; others have suggested that errors can be used to advantage. Regarding the mode of presenting printed mathematical word problems, there seems to be some disagreement as to whether pictorial representations should be used and, if so, whether they should be accurate.

3. Research Design and Procedure

Early in the fall, one of three test forms containing 20 word problems in a five-choice format was randomly given to each of 322 tenth graders. In Form A, each item stem consisted only of a verbal statement of the problem. In Form B, the items were identical to those in Form A except that each was accompanied by an accurate diagram. In Form C, the items were identical to those in Form A except that each was accompanied by an inaccurate diagram. Moreover, in each case the distortion made one of the four distractors appear to be the correct choice.

The problems had been chosen for their discriminating ability from a pool of 40 items administered in a pilot study. Items were selected from algebra and geometry textbooks and from scales used in the National Longitudinal Study of Mathematical Abilities.

School records were used to obtain information concerning the students' IQs (California Mental Maturity Test, N = 236), reading achievement (California Achievement Test, N = 244), and grade averages (ninth grade mathematics, N = 249). The average IQ was 108; the average reading score was 10.1 (presumably a grade equivalent score).

4. Findings

Descriptive data for the three forms are shown in Table 1.

TABLE 1

Performance on the Three Forms of the Problem Test

Form	N	Mean	S.D.	Alpha
A	114	4.94	2.12	.40
B	96	8.56	3.31	.70
C	112	3.79	2.13	.53

The Form B group's performance was significantly ($p < .005$) higher than that of the Form A group, and the Form A group's performance was significantly ($p < .005$) higher than that of the Form C group (ANOVA followed by a modification of Duncan's New Multiple Range Test). On 17 of the 20 items, the Form B group had the highest percent getting the item correct. The Form C group had the lowest percent on 15 of the items. For the subjects taking Form C, the distractor suggested by the incorrect diagram was favored over the correct answer on 16 of the items.

When subjects were divided into three (presumably equal) groups on the basis of IQ and a two-way ANOVA performed, IQ level was significantly ($p < .005$) associated with performance on the test. A significant ($p < .005$) ordinal interaction between IQ and test form indicated that the difference between forms was greater in the high IQ group than in the middle and low IQ groups. Similar main effects and ordinal interactions were found when subjects were divided into three reading achievement groups and into three grade average groups.

5. Interpretations

The subjects "tended to place emphasis on the pictorial representation rather than the prose description of the problem situation." The effect of presenting a diagram, particularly an inaccurate one, in a testing situation was very strong. Teachers should emphasize the importance of separating the logical from the illogical in solving problems, since students tend to assume that the information they are given, at least on tests, is correct.

Abstracter's Notes

Additional descriptive information would have been helpful. In particular, from how many classes, schools, and districts did the subjects come? Were they all enrolled in a geometry course?

There is a puzzling discrepancy between the total number of subjects and the total degrees of freedom in each two-way ANOVA, but this flaw is minor in view of the clear pattern of the results.

More disturbing are some of Sherrill's readings of his results. He sees in the significant interaction between IQ and test form a demonstration "that the variables [test form] had more effect in the high IQ group than in the other two IQ groups." In his graph of the group means, however, the differences between forms for the middle and high IQ groups are almost identical; the contrast is between them and the low IQ group. Sherrill concludes that the effect of presenting an inaccurate diagram "was so strong that the high reading group taking Form C scored lower than the low reading group taking either Form A or Form B." But surely this result could have occurred in the absence of any main effect due to test form.

Sherrill does not discuss the differences between the groups in the number of items omitted. I calculated the average number of items omitted to be .43, .14, and .06 for Forms A, B, and C, respectively. Since most of the omitted items came at the end of the scale, one can only conclude that, on the average, the items took longer to solve when no drawing was given.

If the sample problem Sherrill gives is any indication, perhaps one should interpret his results as follows: given a test composed of multiple-choice items describing a mathematical situation for which a diagram is appropriate, students are more likely to get an item right if they are also given a correct scale drawing. The internal consistency of the test is improved, and the items take less time to work. Furthermore, the better the student (in terms of IQ, reading achievement, or mathematics grade), the more the correct drawings help. If an incorrect diagram is given that appears to make one of the distrac rs correct, then students are likely to choose that distractor, thereby getting the item wrong. The internal consistency of the test is improved somewhat by giving incorrect diagrams, and the items take less time to work. But the test appears to lose validity, since performance on the test is no longer positively correlated with IQ, reading achievement, or mathematics grade.

In his rationale, Sherrill cites several authors who favor capitalizing on errors in problem solving. These people, however, were referring to the sort of errors students generate as they solve problems, not to the sort of errors a teacher might introduce into the statement of a problem. Sherrill's study is interesting and nicely done, but as he appears to acknowledge in his discussion, it really speaks more to issues of achievement testing than to issues of problem-solving instruction.

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EJ 033 608 270 SE 502 706
A CALCULUS-WITH-COMPUTERS EXPERIMENT Smith, David A.
Educational Studies in Mathematics v3 n1, pp 1-11, Sep 70
Descriptors--*Calculus, *College Mathematics, *Instruction, Computer Assisted Instruction, Computer Oriented Programs, Mathematics

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James M. Sherrill, The University of British Columbia.

1. Purpose

To enhance the study of the concepts of calculus by interjecting selected problems, to be solved using the computer, into a freshman honors-level calculus course.

2. Rationale

Young (1968) suggested two extremes for possible approaches to the use of computers with elementary calculus: "...to use the computer only in the places where the course at present uses numerical methods [or] ... to rethink the course from the beginning." Smith's approach, purposely between the two extremes, is based on two assumptions, "...the use of the computer...in the mathematics curriculum is inevitable" and the changes needed for the restructuring of the curriculum to make best use of the computer will be many years in coming due to "intertia in our educational system." While the time is passing, Smith feels there should be many "experiments tampering with the standard curriculum...to gain the necessary experience to make the changes wisely."

3. Research Design and Procedure

Fourteen problems, to be solved on the computer, were designed for a one year long freshman honors-level calculus course. The problems were coordinated with the course to "enhance the study of the concepts of calculus." The 14 problems started with questions simple in both mathematics and programming, but increased in difficulty very quickly. Several of the problems were paired, i.e., a problem would be repeated to be solved using a different technique.

Seventeen freshmen participated in the study. The students were chosen by the usual selective process for honors-level calculus (scores on College Board tests and high school class rank) and most had no experience with either calculus or programming. The students were told on the first day of class about the experiment and that the problems to be solved using the computer would not be a required part of the course. The students used Modern University Calculus (Bell, Blum, Lewis, and Rosenblatt, 1966) as a textbook and a handout, "Introduction to Computer Programming", prepared for the study. The handout covered flowcharting and the parts of FORTRAN language deemed necessary to solve the compter problems. At the end of the course, each student filled in a questionnaire concerning all the aspects of the study.

The third to fifth class sessions were spent considering programming and the mechanics of getting a program run on the computer. As originally conceived, the experiment was to use the WATFOR system of FORTRAN programming on an IBM 360/Model 75 via telephone lines to the local IBM 360/Model 30. Due to technical problems the experiment had to use the FORTRAN compiler on the local Model 30. Loss of the WATFOR system meant the loss of free-form input and output, much more detailed diagnostics and other useful, but less crucial features.

A graduate student was hired to aid in the study by testing the feasibility of the computer problems, giving assistance in programming, assisting in the evaluation of the experiment, and anticipating difficulties.

4. Findings

Of the 17 students in the study, only one did substantially more than one-half of the problems. The student that completed the most problems had no previous programming experience, but he was the only one in the study "who became enthused with the opportunity to use the computer." The data gathered by the questionnaire showed that 1/3 to 2/3 of the problems should have been required and the class was split on whether they had been given enough programming preparation to program the problems for the computer.

The lack of productivity was also affected by the following problems: (1) turnaround time ranged from 10 minutes to over a week; (2) the keypunching service could not give an accurate prediction of day and time of completion; (3) due to (2) the students decided to keypunch their own decks resulting in failure due to grammatical errors; and (4) the time and place of graduate student assistance was not convenient.

Two other findings Smith reports are that the one student who completed substantially more than half of the problems "benefitted mathematically from the experience" and that Problem III and the corresponding chapter in the text contributed to the students' understanding of the epsilon-delta concept.

5. Interpretations

The following conclusions were reported by Smith in his article: (1) the problems form a reasonable, and for the most part useful, set of computer exercises for such a course (2) the problems should be tailored to the content and the pace of the course (3) students should be guided through the harder problems and shielded from significant problems in numerical analysis (4) freshmen are not sufficiently well motivated to work hard at optional material solely for what they can learn from it (5) some portion of the assigned work should be required and made part of the grade (6) the type and quantity of preparation in programming given the students was barely adequate (7) easier access to the computer and consistently fast turnaround is needed (8) an in-class terminal is needed for demonstrations (9) better personal assistance is needed as soon as the problem arises (10) the"new generation" of programmable electronic calculators could serve as an alternative to the digital computer and (11) the experiment was a partial success with respect to the principal goal of using numerical computations to enhance the study of the calculus; Smith observed "some solid indications that this goal is reasonable and desirable." Failure was with respect to getting "the students to complete an adequate amount of work on the computer to be very confident of our conclusions or for them to get the maximum benefit from the experience."

Abstractor's Notes

Since the purpose of the study was to "enhance" the study of calculus by using the computer to solve selected problems, two types of questions arise--ones of control and ones of comparison.

With no control over the minimal amount of work to be done and no control over the means by which the work completed was done, Smith's doubts are well founded concerning the conclusion that the problem set forms a "reasonable, and for the most part useful, et of computer exercises for the calculus course."

Even overlooking the novelty effect of initial use of the computer, in the absence of a comparison group for the study, questions are raised concerning such findings as (1) one student "benefitted mathematically from the experience," (2) problem III did "contribute to their [the participants] understanding of the epsilon-delta concept", and concluding that the study was a partial success in attaining the principal goal of "using numerical computations to enhance the study of calculus."

It is agreed that the problems form the nucleus of the material presented in the study, but it would have been helpful if the author had spent less time discussing the problems and more time discussing the bases for the conclusions.

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EJ 033 811 310 SE 502 717
PROBLEM-SOLVING PERFORMANCES OF FIRST-GRADE CHILDREN Steffe, Leslie P.; Johnson, David C., Journal for Research in Mathematics Education, v2 n1, pp50-64, Jan '71
Descriptors--*Arithmetic, *Elementary School Mathematics, *Learning, *Problem Solving, *Student Ability, Grade 1

Expanded Abstract and Analysis Prepared Especially for I.M.E. by C. Alan Riedesel, Georgia State University

1. Purpose

The purpose of the study was to investigate differential performances among categories of first grade children when solving several types of arithmetical word problems.

2. Rationale

Previous research relating Piaget's theory in the school curriculum points up a marked relationship between performance on conservation of number tasks and addition and subtraction achievement. It was determined that children who exhibit the ability to make quantitative comparisons involving a forward transformation perform better on addition and subtraction problems than those who do not display such an ability.

A forward transformation is a process of set comparison characterized by the rearrangement, either physically or mentally, of one or both of two sets of objects which were presented initially in different arrangements. On the other hand, a reverse transformation involves returning two sets of objects to their original configurations after observing both the initial states and object movement.

Furthermore, problems involving a described action are easier than those involving no described action, and some subjects perform better on a problem solving test whenever manipulative objects are present. The investigators assert the need for further study concerning the relationship of levels of conservation of number with performance on a problem solving test involving a greater variety of problem types which do or do not involve a described action and which may or may not be solved using manipulative objects.

3. Research Design and Procedure

Four schools from among the elementary schools in Walton County, Georgia, comprised the population. The first grade children of all four schools were given the test of quantitative comparisons and the Lorge-Thorndike I.Q. Tests, Level 1, Form A. In all 192 children received both tests. Children were categorized into tow I.Q. groups ranging from 80-97 or 103-120. The remaining 111 children (several students were lost to the study through absence) were separated into four ability groups: (a) quantitative comparison scores 0-7 and I.Q. scores 80-97, (b) quantitative comparison scores 0-7 and I.Q. scores 103-120, (c) quantitative comparison scores 10-15 and I.Q. 80-97, and (d) quantitative comparison scores 10-15 and I.Q. scores 103-120. Then a problem solving test was administered to 108 of these 111 children. The problem solving test was composed of 48 items with six problems of each of eight types. The eight problem types call for the following mathematical sentences: a + b = n, a - b = n, a + n = b, and n + a = b and involved or did not involve described action. Measures of internal consistency reliability coefficients, and a principal component analysis were computed for each test. The quantitative comparison test achieved a KR-20 of .89 while the eight six item sub-tests of the problem solving tests had KR-20 ranging from .59 to .69. A multivariate analysis of variance was then performed on each six item sub-test involving the eight different problem types.

4. Findings

1. On the test of quantitative comparisons, those items involving unequal numbers of objects in the set tended to load on different factors than those involving equal numbers of objects.
2. The mean scores for a + b: A (described action) and a + b : N (no described action) were appreciably greater than other types.
3. Significant positive skewness was found for a + b : A and a + b : N and significant kurtosis (platykurtic) for all other types.
4. Across the four basic structural types of problems the achievement on described action problems was not appreciably different from the problems with no action described.
5. There was no significant difference between measured I.Q. levels on the problem solving tests.
6. From MANVOVA it was determined that for the effect of Q (levels of quantitative comparison) and C (levels of problem conditions) and the main effect due to C significant differences occurred.
7. For a + b: A and a + b: N, the main effect Q and the main effect C were significant.
8. There was a significant interaction for a + b: N for Q and C. That is, aids tend to help low conservers more than high conservers.

9. Forward and reverse transformation items appear equally difficult.

5. <u>Interpretations</u>

Following their findings, the authors provide a section called "discussion and implications." The key points (in the view of the abstractor) are listed below:

1. The test of quantitative comparisons had good psychometric properties.
2. For children who met criterion on the test used in this study, it is not at all clear that those children were able to use properties or consequences of the relations involved or whether they perceived of the relations as quantitative relations or not.
3. It appears that forward transformation may be basic to solution of arithmetical word problems for which relevant solution strategies are available.
4. The variable, described action, operated differently in this study (no differences between described action items and no described action items) than in previous studies. However, in the previous studies it was suggested that the significance of the variable might have been the result of instruction.
5. Further experimentation needs to be conducted in which Problem Structure and Problem Conditions are systematically varied and outcomes assessed both by direct achievement tests and transfer tests.

Abstracter's Notes

When one reads any research report he finds that a number of questions have been answered by the report and also that the study raises a number of other questions and also produces a number of reactions. The questions and reactions of the abstracter are listed below:

1. The size of the N was very good for this type of study.
2. This is a carefully designed study. However, the reporting was confusing to the abstractor. While all the information was given it was difficult for the abstractor to "dig it out." This may have been due to the number of variables involved.
3. The children were not given the choice of using or not using objects. What would have happened if they had done so?
4. The researchers used problems involving the "take-away" idea and the "how many more are needed" idea (a - b = n and a + n = b or n + a = b). Were the comparison type "subtraction" problems such as "Tom has 8 model cars and Tim has 5 model cars. Which boy has more cars?" used in the study? If so how were they classified?

5. Should teachers use the quantitative comparisons test as a "readiness" measure for first grade work with verbal problem solving?

6. How are the problem types a + n = b and n + a = b different? One might hypothesize them to be of this type: Joe has six cents. He wants to buy an eight-cent candy bar. How much more money does he need to buy the candy bar, and how much more money will Joe need to buy a candy bar if he has six cents and the candy bar costs eight cents?

7. In this study conservation as measured by the quantitative comparisons test appears to be a better predictor of problem solving success for some problem solving types than IQ. What is the correlation between the test and various IQ measures? Is quantitative comparison a measure of I.Q.?

8. "Forward Transformation" appears to be basic to the solution of arithmetical word problems for which relevant solution strategies are available. What is the relationship between forward transformation and word problem solving? That is, would instruction concerned with word problems have a measurable effect upon forward transformation?

9. There seems to be some confusion in the use of the terms Quantitative Comparisons and Forward Transformations. At one point it appears that Forward Transformations are a portion of Quantitative Comparisons, while at another point, the Forward Transformations are used as a variable in themselves.

10. The study is representative of the type of careful study that should be conducted to ascertain the developmental thought patterns of children.

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THE EFFECT OF SEQUENCE IN THE ACQUISITION OF THREE SET RELATIONS: AN EXPERIMENT WITH PRESCHOOLERS Uprichard, A. Edward, Arithmetic Teacher, v17 n7, pp597-604, Nov '70
Descriptors--*Elementary School Mathematics, *Instruction, *Preschool Learning, *Set Theory, Learning, Mathematical Concepts, Sequential Approach

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Leslie P. Steffe, The University of Georgia

1. Purpose

Uprichard arranged the three set relations "equivalence," "greater than," and "less than" into six possible permutations from which he generated six instructional sequences, one for each permutation. His purpose was to determine which of these six sequences would be most efficient in terms of the time it took to learn the three set relations and in terms of the vertical transfer to a related task involving the set relations.

2. Rationale

Uprichard identified two strategies for studying effects of instructional sequence on learning. The first is that in which a logical or ordered sequence of instruction is compared with a random or scrambled sequence. The second is that in which two or three concepts of a given domain of knowledge are singled out for study and then sequences of the concepts are considered as a fixed effect in a factorial design. Uprichard used the second strategy.

3. Research Design and Procedure

A random sample of thirty-two preschool children from the Liverpool, New York area were randomly assigned to eight treatment groups. The experimental groups were formed by the six instructional sequences. Two control groups were also formed, one of which was administered criterion tests and a posttest at the end of instruction and one of which was administered only the posttest. Each experimental group received instruction on the set relations in the prescribed sequence for that group. Instruction was administered in modules where a module consisted of three twenty-five minute instructional sessions. A criterion test was administered following each module. Within each module, instruction was given on only one relation. If three out of four subjects in a group scored at least three out of four correct responses on the criterion test, that group met criterion for that relation and was administered instruction on the next relation in their sequence in the next module. Instruction ended for all groups when any one group met criterion on all three relations. Instructional sessions as well as the tests focused on the numbers "three," "four," "five," and "six." The criterion tests each consisted of four items, one for each of "three," "four," "five," and "six." In case of equivalence, a child was directed to select one board from three choice boards that had the same number of holes in it as a sample board. In the case of "greater than," the child had to select a choice board that had one more hole in it than the sample board (likewise for "less than"). A vertical transfer test consisted of six items (three for each relation) each of which contained directions analogous to the criterion tests. The only crucial difference in the vertical transfer test and any criterion test was that a conscious attempt was made to include misleading perceptual cues in the transfer test. A posttest was formed from the last criterion test and the vertical transfer test combined.

4. Findings

1) The group that met criterion on all three relations first was the group with the sequence equivalence - greater than - less than (EGL). They were administered one module on "equivalence" and three on "greater than" after which they met criterion on all three relations.

2) The ELG (equivalence - less than - greater than) group had been administered one module on "equivalence" and three on "less than" when instruction was terminated. They had reached criterion on "equivalence" and "less than" but missed meeting criterion on "greater than" by one correct response.

3) The GEL and GLE groups had been administered three and four modules, respectively, on "greater than" and the GEL group had been administered one module on equivalence when instruction was terminated. Each group reached criterion on "equivalence" and "greater than."

4) the LEG and LGE groups had been administered four modules on "less than" when instruction was terminated and reached criterion only on "equivalence."

5) A one-way ANOVA using the posttest as a dependent variable and the six experimental groups and the two control groups as the independent variable revealed differences among the groups ($p < .01$). A post-hoc analysis utilizing the Sheffé method revealed differences between only (a) the EGL group and each one of the two controls, and (b) the average of EGL and ELG and the average of the two controls.

6) The control groups scored an average of approximately 28% on the vertical transfer test and approximately 26% on the last criterion test. The EGL group scored an average

of approximately 61% on the vertical transfer test and approximately 79% on the last criterion test.

5. <u>Interpretation</u>

Uprichard interpreted his data in terms of trends suggested and/or indicated. The following were the most interesting:

1) "Less than" is not a difficult set relation for preschoolers to learn after they have acquired "equivalence" and "greater than." However, the LEG and LGE groups did not reach criterion on "less than" even though they had received 300 minutes (four modules) of instruction on "less than."

2) Preschoolers can be trained to conserve the one-to-one correspondence between equivalent sets containing three, four, five, or six elements.

3) There appears to be a hierarchal relationship among the set relations "equivalence," "greater than," and "less than."

4) Preschoolers have the capacity to acquire set relations although the study did not yield direct evidence to support the notion that systematic instruction on set relations ought to begin at this age level.

Abstractor's Notes

Whether or not this study will act as a catalyst in generating new ideas for exploring the effect of instructional sequence on learning, as Uprichard hopes, will only be answered in retrospect. It does, however, raise questions. First, "greater than" and "less than" are closely related in that A is greater than B if and only if B is less than A. It thereby is not too surprising that no instruction had to be given on "less than" in the EGL group for children to meet criterion on "less than." Also, in the ELG group, criterion was met on "greater than" for all practical purposes. These two facts suggest that Uprichard may not have generated the most efficient (in his terms) instructional sequence by just taking permutations of the three relations. Another <u>natural</u> sequence is to give instruction on "equivalence" and then on "greater than" and "less than" simultaneously. Such an instructional strategy is not inconsistent with Piaget's notion of reversibility. Second, if the domain of knowledge of concern is expanded, what concept or set of concepts should come next in the sequence EGL? Not only is it important to ask what comes next but also to ask whether structural properties of the set relations could be done simultaneously. That is, if one is concerned with the structure of the relations (transitivity, symmetric, and asymmetric properties of the relation, etc.), is it profitable to give instruction on these properties at the same time as such instruction as Uprichard administered is given?

A collection of questions and concerns specific to the study need to be raised because they are not satisfactorily answered in the article. The nature of the instruction in the instructional sessions is not elucidated. Such knowledge is essential in interpreting the results. For example (a) were counting procedures employed?, (b) were matching procedures employed?, (c) was instruction given on arranging the numbers three through six in sequence? (d) was instruction given on "one more than," and "one less than?", and a host of others. In the criterion test example given, set equivalence is tested in terms of "the same number," "greater than" is tested in terms of "one more than" and "less than" is tested in terms of "one fewer than." Granted, all three set relations are included in the tasks set, but the tasks go beyond essential aspects of set relations. It would seem that on one hand the children in Uprichard's study acquired more knowledge than just knowledge of set relations (knowledge of three through six and "one more than" and "one fewer than") while on the other hand evidence was not compelling that children had acquired set relations per se. In the latter case, knowledge of the properties of the relations need to be considered as well as collections with more than six members which had not been used in instruction.

The tasks in the vertical transfer test come closer to conservation tasks than any others. That they are conservation tasks is equivocal because (1) the control children scored, on the average, as well on these tasks as on the criterion tasks, (2) the experimental group did not demonstrate clear-cut superior performance on the criterion tasks as on the vertical transfer tasks, and (3) the criterion tasks are not conservation tasks.

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