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ABSTRACT

In this report, the author suggests changes in the treatment of overhead costs by hypothesizing that "the effectiveness of standard costing in planning and controlling overhead costs can be increased through the use of probability theory and associated statistical techniques." To test the hypothesis, the author (1) presents an overview of the problem, (2) reviews probability theory, (3) discusses the selection of the proper empirical probability distribution to be used in the study, (4) uses the normal distribution to develop a cost control chart for dollar costs of manufacturing supplies, and (5) discusses the use of simple linear and multilinear regression and correlation analysis for budgeting overhead costs. (Author)

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THE USE OF PROBABILITY THEORY AS A BASIS FOR PLANNING AND CONTROLLING OVERHEAD COSTS IN EDUCATION AND INDUSTRY

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TABLE OF CONTENTS

I. SUMMARY

In surveying current literature and evaluating the standard cost systems in use today, we find a need for changes to remedy the weaknesses of the present system. Seemingly, the most appropriate means of making such changes is through the use of probability theory and associated statistical techniques using computer technology.

Thus, the writer in suggesting changes in one area, the treatment of overhead costs, has hypothesized that "the effectiveness of standard costing in planning and controlling overhead costs can be increased through the use of probability theory and associated statistical techniques."

In order to test the hypothesis, the writer (1) presented an overview of the problem, (2) did an extensive review of current literature (the results of this review are not included per se in this report), (3) reviewed probability theory, (4) discussed the selection of the proper empirical probability distribution to be used in the study, (5) used the normal distribution to develop a cost control chart for $ costs of manufacturing supplies, and (6) discussed the use of simple linear and multilinear regression and correlation analysis for budgeting overhead costs.

The methods employed can be used, with minor adjustments, for any type of operational costs in industry and other areas such as government and education.

The major problem of using this approach in industry is obtaining the cooperation of operational personnel. A similiar problem exists in

government and education, but to a larger degree. In these areas the use of appropriation budgets with the feeling of need to spend the entire amount is present. Also, there is no backlog of standard cost data upon which to base the approach.

The approach suggested can be used in government and education after such problems are overcome.

The methods suggested in this paper will be tested with case studies as the concluding portion of a doctoral dissertation.

II. THE PROBLEM

Throughout the entire history of the accounting profession, accountants have been seeking new and more efficient methods which will provide more extensive quantitative data to be used in managerial decisions. The greatest break-throughs are occurring at an accelerated rate, especially since the advent of modern computer technology.

The field of cost accounting, because of its very nature, lends itself to the use of modern computer technology coupled with methods derived from the areas of statistical analysis and operations research. We have merely begun to utilize these areas in the solution of cost accounting problems. However, there are many problems yet to be solved.

This paper is concerned with one such problem area. This problem area is that of one phase of the standard cost system.

<u>The role and responsibilities of cost accounting in regard to overhead costs</u>. The duties of the accountant are necessarily interwoven with managerial planning and control. This statement is especially appropriate in relating the role of the cost accountant, since he is

being called upon more and more to provide the quantitative accounting data which is necessary to sound managerial decisions in our complex business world. This is true in dealing with the planning and control of overhead costs, as well as, materials costs, labor costs, distribution costs, administrative costs, etc. This basic role remains unchanged whether you are using a standard cost system or some other cost accounting system.

The role or roles that standard costs are supposed to perform must be clearly distinguished before any attempt is made to discuss present-day standard costs and their effectiveness in these roles.

Gillespie[1] indicates that standard costs are called upon in the following areas: (1) cost control and reduction, (2) price determination, (3) inventory valuation, and (4) clerical economy. Vance[2] agrees with these four and adds the forcing of effective managerial review of costs as a prime function of using standard costs.

Pate, in discussing the functions of standard costs, made the following comments:

> The highlighting of "out-of-line" costs through variances is inherent in the standard cost system, facilitating management by exception. Standard costs based upon engineered standards embodying a practical low-cost operating plan to produce normal volume of salable product, provide the basis for developing budgets to be used in measuring performance.[3]

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[1]Cecil Gillespie, Cost Accounting and Control (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1957), pp. 403-5.

[2]Lawrence L. Vance, Theory and Technique of Cost Accounting (Rev. Ed.; New York: Henry Holt and Company, 1958), p. 371.

[3]D. L. Pate, "'Standard" Features of Standard Costs," N.A.A. Bulletin, XL (August, 1959), p. 57.

Anderson[4] made basically the same points in his article. In referring to the standard cost system in operation in his company, Allen summarized the functioning of a good standard cost system in the following terms:

> It seems that standards play some part in almost every major procedure, system, or decision which is reviewed by our management. This applies to special cost studies, investigation concerning new products, production forecasts, selling price calculations, interplant cost comparisons, and accounting procedures of all types.[5]

The Standard Cost System. The present standard cost system requires the development of a standard cost for each cost element. In the case of overhead this standard cost is generally in the form of dollars per hour or unit or a percentage of materials cost or labor cost. Certainly these standard costs can be developed by departments or by processes to be more closely related to responsibility accounting.

These standard costs are used as the basis for costing production while actual costs are accumulated and compared periodically with the costs charged to production. This periodic comparison gives rise to variances which must be analyzed for cause. This variance analysis will show the need for corrective action where the variances are material in amount. In this periodic analysis of variances, we suspect that some variances have offset other variances.

The Standard Cost System Evaluated. The standard cost system, as we know it today, is a very elaborate approach to the problems involved in the management and control of the modern industrial enterprise.

4G. A. Anderson, "Budgeted and Standard Costs-Different and Alike," N.A.A. Bulletin, XL (February, 1959), pp. 91-2.

5David H. Allen, "How We Developed and Use Standard Costs," N.A.A. Bulletin, XXXVI (November, 1954), p. 372.

However, the typical standard cost system of today has several inherent weaknesses.

It is the opinion of the writer and others that this system can be modified to: (1) do a more efficient job in its present setting of the industrial complex, and (2) serve the needs of other institutions such as educational institutions and non-industrial companies. In order to point out the validity of this opinion, an evaluation of the typical standard cost system is necessary at this point.

The amount of material that has been written pointing out the advantages of present standard cost systems is almost overwhelming. Taylor[6] points out three justifications for the present type of standard cost system; these are:

1. The timeliness of information
2. The greater amount of information provided
3. The operating cost of the system is often lower than for any other method of producing cost information.

These points are a good summary of the material dealing with the advantages of the standard cost system in present use. These are advantages, to be sure, of the present system over previously formulated systems. However, each of these advantages may become disadvantages if if a modified system can be formulated, which will:

1. Give more timely information
2. Provide a greater amount of information
3. Provide such information at a reduction in operating costs.

The Need For Change. It is interesting to note that a consider-

[6]Paul C. Taylor, "Functioning of Standards in Cost Control," N.A.C.A. Bulletin, Vol. 32, sect. 1, (March, 1951), pp. 795-801.

able number of articles have been written in the past few years suggesting that such a system is not only needed but also feasible. Many of these articles have suggested that the use of tools supplied by the fields of statistics and operations research should be included in such a system.

Typical of such suggestions is the following quotation:

> The heart of managerial use of standard costs is the analysis of variances. Variances signal the need for managerial investigation so that control may be kept effective and better ways of doing things may be discovered. But there is a danger here. Which variances are significant enough to warrant investigation? Are significant variances over-looked because they offset one another to show an apparently satisfactory condition?
>
> Management tends to use judgement in deciding whether or not a variance on a given item deserves investigation. For some items, any tiny variance from standard may spark scrutiny. For other items, 5, 10, or 25 per cent variances from standard may be necessary to spur follow-ups. These judgements generally grow from the experience and know-how of the executives involved. Guesses or hunches are fundamental parts of managerial behavior; yet these subjective methods often engender management disagreements and barren investigations.
>
> Another difficulty is that the accounting system often complies variances for a period of time. A cost-conscious management will follow up variances quickly--sometimes daily or even hourly. But delayed reports and everyday busy work often allow variances to accumulate so that it becomes too late to find out what caused the variances. Further, favorable and unfavorable variances are frequently combined, so that significant variances may be offset in accounts and in management reports. Each overtime authorization, for example, is an incremental decision and should not be related to average rates of overtime allowances. This combination of delayed reporting and cost accumulations that represent a conglomeration of different operations makes it difficult to find causes for variance and to trace causes for them below the foreman level to individual machines, men and materials.
>
> In summary, the accountant often grinds out variances without any indication of their significance. When should management be concerned about a given variance? Frequently the answer to such a question is based on subjective judgements, guesses, or hunches. The field of statistics offers

tools to help reduce these subjective features of variance analysis.[7]

The use of such tools in providing a greater amount of information for managerial use at a lower operating cost will necessitate the use of statistical sampling and other statistical technqiues which are based on the probability concept. This poses somewhat of a problem because many people have an aversion to the use of sampling techniques and insist upon a complete detailed inspection. However, statistical sampling can achieve equivalent or better accuracy than complete inspection. This is explained in the following quotation:

> The greater accuracy is possible because sample surveys can often be more carefully designed, personnel can be better trained and supervised, and special problems can be followed up more easily with a sample survey than with a census.[8]

One other quotation seems pertinent to the present discussion:

> The usual approach to cost control, which proceeds from the accumulation of dollar variances which are then to be eliminated, places the variance reports of flexible budgeting and standard costing in a position of preeminence in the field of cost control. Cost control is assumed to proceed or start from the variance reports, which are viewed as a form of watch-dog or blinking light that is supposed to snap at or issue a warning upon the occurrence of waste, which must then be eliminated or "shut off." Thus, it is seen that the traditional apporach to cost control is limited in the sense that it does not give rise to control where it really counts, that is, before conditions get out-of-line or as conditions are getting out-of-line. Traditional cost control is really after-the-fact cost control; waste is assumed to occur before cost control can perform its function of eliminating the continuance of waste.[9]

The preceeding discussion points out (1) the need for changes

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[7]Charles T. Horngren, Cost Accounting: A Managerial Emphasis (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962), p. 748.

[8]William L. Ferrera, "An Integrated Approach To Control of Production Costs," N.A.A. Bulletin XLI (May, 1960), p. 53.

[9]Horngren, op. cit., p. 749.

in our present concept of the standard cost system, (2) the feasibility of such changes, and (3) the directions which these changes should take. However, it would be impossible within the practical limits of this paper, to discuss all of the changes which are needed in all of the areas of standard costing. Therefore, the writer will confine his investigation to the area of planning and controlling overhead costs. If the application of statistical methods is successful in the area of overhead costs, such an application could also be made in the areas of material costs, labor costs, distribution costs, administrative costs, and etc.

A proposal. The previous discussions leads the writer to the formulation of the following hypothesis:

> The effectiveness of standard costing in planning and controlling overhead costs can be increased through the use of probability theory and associated statistical techniques.

Delimitations. This study will be concerned with determining the validity of the foregoing hypothesis and will be limited to the use of standard costing concepts as applied to overhead costs to the exclusion of other types of costs.

III. PROBABILITY THEORY

In the previous section, the writer called attention to several weaknesses in the typical standard cost system in use at the present time. The writer also suggested that probability theory and associated statistical techniques might provide a basis for modifications to the present standard cost system aimed at the elimination of these weaknesses.

The purposes of this section are: (1) to review the basic concepts of probability theory, (2) to investigate the seemingly most appropriate probability distributions, (3) to select the most feasible distribution to be used as a basis for modifications to the standard cost system, and (4) to suggest which statistical techniques are most applicable in carrying out the purposes of this study.

Definition. Probability theory is a branch of mathematics which deals with the measurement and evaluation of problems concerning uncertain events. Such problems arise when there is doubt as to whether an event will occur or where there is doubt as to the form in which the event will occur.

Probability theory provides the mathematical tools for the construction of a model describing such a problem under uncertainty. The model would take into consideration all possible outcomes to the problem and assign a probability factor to each outcome.

Probability Factor. The assignment of a probability factor may be based on either of two approaches to the definition of probability; these approaches are: (1) the classical or "a priori" approach and (2) the empirical or relative frequency approach. The classical definition of the probability of an event is stated as follows:

> If an event can occur in N mutually exclusive and equally likely ways, and if n of these outcomes have an attribute A, then the probability of an outcome with the attribute A is the fraction n/N.

Thus, the probability of A would be denoted: $P(A) = n/N$. This definition is limited by its dependence upon events being both mutually exclusive and equally likely. Either or both of these conditions may be absent in a given problem.

The empirical or relative frequency definition of the probability of an event is as follows:

> If in N trials, where the value of N is very large, an event having the attribute A occurs n times, then the probability of an outcome with the attribute A is the fraction n/N.

Again, we have P(A) = n/N, but without the limitations of the classical definition. Therefore, the empirical definition will be intended throughout this study unless otherwise indicated.

All probabilities are fractions by definition and as such must fall within the range of proper fractions, which includes the end points 0 and 1, where 0 indicates impossibility and 1 indicates absolute certainty. This condition is denoted mathematically as follows:

$$0 \leq P(A) \leq 1$$

Since all probabilities are fractions, limited to the range 0 to 1, the sum of the probabilities assigned to all of the different outcomes which are possible must be one:

$$\sum_{i=1}^{n} p(a_i) = 1$$

Probability rules. In the assignment of probabilities to either single events or combinations of events, certain rules must be followed. These rules are listed below:

1. The probability of event A is designated: $P(A) = \frac{n}{N} = p$ and the probability of the non-occurrence of event A is designated: $Q(A) = \frac{N-n}{N} = 1 - p$.

2. The probability of the occurrence of any one of several desirable events, which are mutually exclusive, is equal to the sum of the individual probabilities of these events. Therefore,

$$P(A \text{ or } B) = P(A) + P(B)$$

The addative law for mutually exclusive events.

3. The probability of the joint occurrence of two or more independent events is equal to the product of the individual probabilities of these events. This multiplication rule is expressed:

$$P(A + B) = P(A)\ P(B)$$

4. Rule number 2 can be modified to eliminate the mutually exclusive restriction by subtracting the joint probability of A + B from the sum of the individual probabilities. This general addative law is expressed:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \,\&\, B)$$

5. Rule number four can be generalized to eliminate the restriction of independence of the variables and this will allow for conditional probabilities. This rule can be expressed:

$P(A \,\&\, B) = P(A)\ P(B|A)$ where $P(B|A) =$ probability of B when A is known to have occurred.

Sample Space and Random Variables. The term "sample space" indicates a set of variables or values which represent all possible outcomes of an experiment. As an example, the sample space of an experiment based on rolling a pair of dice is presented below:

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

A random variable is a numerically-valued variable which may vary over a definite range of values. This range of values is defined by the sample space of the experiment and each value has an assigned probability of occurrence. A random variable may be either discrete or continuous.

Returning to the dice rolling experiment, the random variable (number of points showing) may take on integer values from the range 2-12 inclusive. These values and their assigned probabilities are:

V	P	V	P	V	P
2	1/36	6	5/36	10	3/36
3	2/36	7	6/36	11	2/36
4	3/36	8	5/36	12	1/36
5	4/36	9	4/36		

Probability Distributions: A Definition. The set of all possible values, the range of these values, and the associated probabilities of these values constitute a probability distribution. The preceeding list of such values and probabilities constitute a discrete probability distribution for rolling a pair of dice.

If the random variable is continuous, probabilities must be computed for intervals rather than specific values. The probability for such an interval can be calculated from the following formula:

$$P(i<x<j) = \int_i^j f(x)dx$$

The use of the formula is dependent upon knowledge of the distribution function F(X). If this function is not known, the relative frequency approach based upon a large number of observations can be used in the assignment of probabilities. In some cases, judgement values of probabilities have proved to be quite acceptable.

Use of Probability Distribution. In the use of probability theory, it is necessary to realize that the concept of the probability distribution is at the very heart of statistical analysis, estimation, and hypothesis testing. However, the probability distribution of the universe is generally unknown. Thus, we are usually forced to use the probability distribution of a sample or samples to approximate the universe probability distribution.

The probability distribution derived from the sample or samples should be compared with the formal distributions included in statistical literature for which tables of values are readily available. This comparison could significantly affect the methodology used and the results obtained in applying probability theory.

Such a comparison avoids the criticism of our hypothetically assuming normality within the universe probability distribution. A comparison of this type could also be quite time consuming unless a chi-square test for normality is successful.

Another approach to be used, instead of a continued comparison following an unsuccessful chi-square test, is to assume normality based upon the Central Limit Theory, larger samples and Chebyshev's Inequality.

The Central Limit Theorem states that. as the size of the sample increases, the sampling distribution of means approaches normality and the single restriction is the requirement of finite population variance. This restriction relates to the prevailing preponderance of practical situations. This provides a fairly good basis for the use of the normal distribution in our analysis.

The Chebyshev Inequality Theorem states that if X is a random variable with a mean μ and a finite variance δ^2, the probability of X taking on a value outside the control interval $\mu-j\delta$ to $\mu+j\delta$ can be expressed:

$$P(|X-\mu|>j\delta) \leq \frac{1}{j^2}$$

or

$$P(|X-\mu|>3\delta) \leq \frac{1}{9} \quad \text{where } j = 3$$

This theorem applies to any probability distribution. Thus, if our probability distribution from samples shows the probability of random variables within the limits of $\mu \pm 3\delta$ to be greater than 8/9, we can use the normal distribution as a good approximation of the universe probability distribution.

If the chi-square test fails and the test using Chebyshev's Inequality fails, further comparison of the probability distribution drawn from samples with the other formal distributions should be conducted even though relatively large samples were used.

Once a dicision is made as to the best approximation of the form of the universe probability distribution, we are able to make inferences and test hypotheses about the universe based upon sample analysis.

Expected Value. While knowledge of the universe probability or a good approximation thereof is necessary, one of the most useful measures for our purposes is the measure of expected value. This measure of expected value denotes the average value of the random variable. It is computed by summing the products of all values of the random variable with their respective probability factors. Therefore this measure is dependent upon the probability distribution. While the measure of expected value does denote the mean or average value of the universe of random variables, it may or may not be equal to any of the possible values of the random variable.

We may also compute a standard deviation for the probability distribution by using the expected value measure as the mean of the distribution. This is done by (1) weighting the differences between values of the random variable and the measure of expected value by the probability factors for the values of the random variable, (2) summing these weighted differences, and (3) taking the square root of the resulting sum.

The mathematical expressions used to denote (1) expected value or mean and (2) standard deviation are:

A. Discrete functions

1. $E(X) = \sum_{i=1}^{n} x_i \cdot P(x_i) = \mu$

2. $\sigma = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 P(x_i)}$

B. Continuous functions

1. $E(X) = \int_{-\infty}^{\infty} x F(x) d_x = \mu$

2. $\sigma = \sqrt{\int_{-\infty}^{\infty} (x_i - \mu)^2 f(x)dx}$

The mathematical expressions for the continuous variable more clearly indicates the dependence of these concepts on some degree of knowledge about the underlying distribution function.

<u>The Normal Distribution as a Basis for Standard Costs</u>. Based on the concepts and methodology previously discussed, the writer hypothesizes that the normal distribution will provide a very good approximation of the underlying probability distribution in a very large majority of cases of application. This statement does not negate the necessity of testing the pertinency of the normal distribution in each case under consideration.

Using the normal distribution, a continuous function, and its associated probability values; the concept of expected value; and the procedure for computing a standard deviation; we can determine a mean value and standard deviation for an overhead cost element and any associated probability interval desired. We could then adopt the mean value as a standard cost for the particular overhead cost element and the probability interval could be viewed as a control device. With the probability interval corresponding to the desired level of confidence, any mean value outside the interval would be investigated for assignable cause while those means within the interval would be assumed to be deviating from the universe mean or standard cost due to chance.

The standard cost would be considered an interval cost with a given probability factor and with the expected value or mean as the value to use for journal entries. Such an approach using intervals for control purposes and single values for recording purposes seems highly feasible.

Such an approach is not limited to treatment of overhead costs or even to production costs for that matter. The approach can be used for production costs, distribution costs, administrative costs, and service costs. The approach can be used for cost planning and control in business, industry, government and education. The major difficulty to be encountered in using this approach is the separation and definition of cost elements.

IV. A MODIFIED STANDARD COST TREATMENT OF OVERHEAD COSTS

The basic concepts and methods for developing probabilistic standard costs as intervals of cost with mean values were presented in the discussion in the previous section. In this section, the writer will show the application of these concepts and methods to establish standard costs and probability intervals and discuss (1) the use of these costs and intervals in the operation of a standard cost system, (2) their effect upon interval cost control, (3) their effect upon performance reports, and (4) the application of these concepts and methods in other areas of cost control such as education.

Application of Method. The cost of manufacturing supplies are known to be quite significant in the operation of process 4 in a particular manufacturing company. Therefore, management is concerned with exercising effective control over this overhead cost element.

The underlying probability distribution for the universe is unknown for this cost element. The universe variance is defined as finite based upon past experience.

Fifty samples of four observations each were chosen at random over a period of time while technology remained unchanged. Each observation showed the cost of manufacturing supplies used during a one hour period. The means of these samples are listed in the following table.

TABLE 1

FREQUENCY TABLE OF 50 SAMPLE MEANS OF SAMPLES OF 4 TAKEN FROM PROCESS 4 DURING TWO MONTHS OF STUDY.

Class Limits	Frequencies f	Interval Deviation d	fd	fd^2
0.00 - 40.00	0	-4	0	0
40.00 - 44.00	4	-3	-12	36
44.00 - 48.00	8	-2	-16	32
48.00 - 52.00	10	-1	-10	10
52.00 - 56.00	10	0	0	0
56.00 - 60.00	9	+1	+ 9	9
60.00 - 64.00	4	+2	+ 8	16
64.00 - 68.00	5	+3	+15	45
68.00 - -----	0	+4	0	0
	50		- 6	148

The mean of means, the standard deviation of this distribution, and the standard error of the mean are computed below:

$$\bar{X} = Md + \frac{fd}{n} i = 54.00 + \frac{-6}{50} 4 = 53.52$$

$$\bar{s} = i \sqrt{\frac{\Sigma f(d)^2}{n} - \frac{(\Sigma fd)^2}{N}} = 4\sqrt{2.96 - .72}$$

$$= 4(1.5) = 6.00$$

$$\sigma_{\bar{x}} = \sqrt{s^2 \frac{n}{n-1}} = \sqrt{36 \cdot \frac{100}{99}} = 6.03$$

The following table represents a goodness-of-fit test using the normal distribution in a chi-square analysis.

TABLE 2

A CHI-SQUARE GOODNESS-OF-FIT TEST USING THE NORMAL DISTRIBUTION WITH TH: RESULTS OF TABLE 1

Class Limits	Probability (Normal)	Expected Frequency fe	Observed Frequency fo	f_o^2/fe
0 - 40	.01255	.63	0	0.00
40 - 44	.04566	2.28	4	7.02
44 - 48	.12058	6.03	8	10.61
48 - 52	.22250	11.12	10	8.99
52 - 56	.25781	12.89	10	7.76
56 - 60	.19859	9.93	9	8.16
60 - 64	.10138	5.07	4	3.16
64 - 68	.03273	1.64	5	15.24
68 - --	.00820	.41	0	0.00
	1.00000	50.00	50	60.94

$$X^2 = 60.94 - 50 = 10.94$$

$$X^2_{.01} = 13.277 \text{ with 4 degrees of freedom}$$

From the results of Table 2, we concluded that the normal distribution is a good approximation of the underlying population distribution.

Using Chebyshev's inequality for 95% confidence we computed:

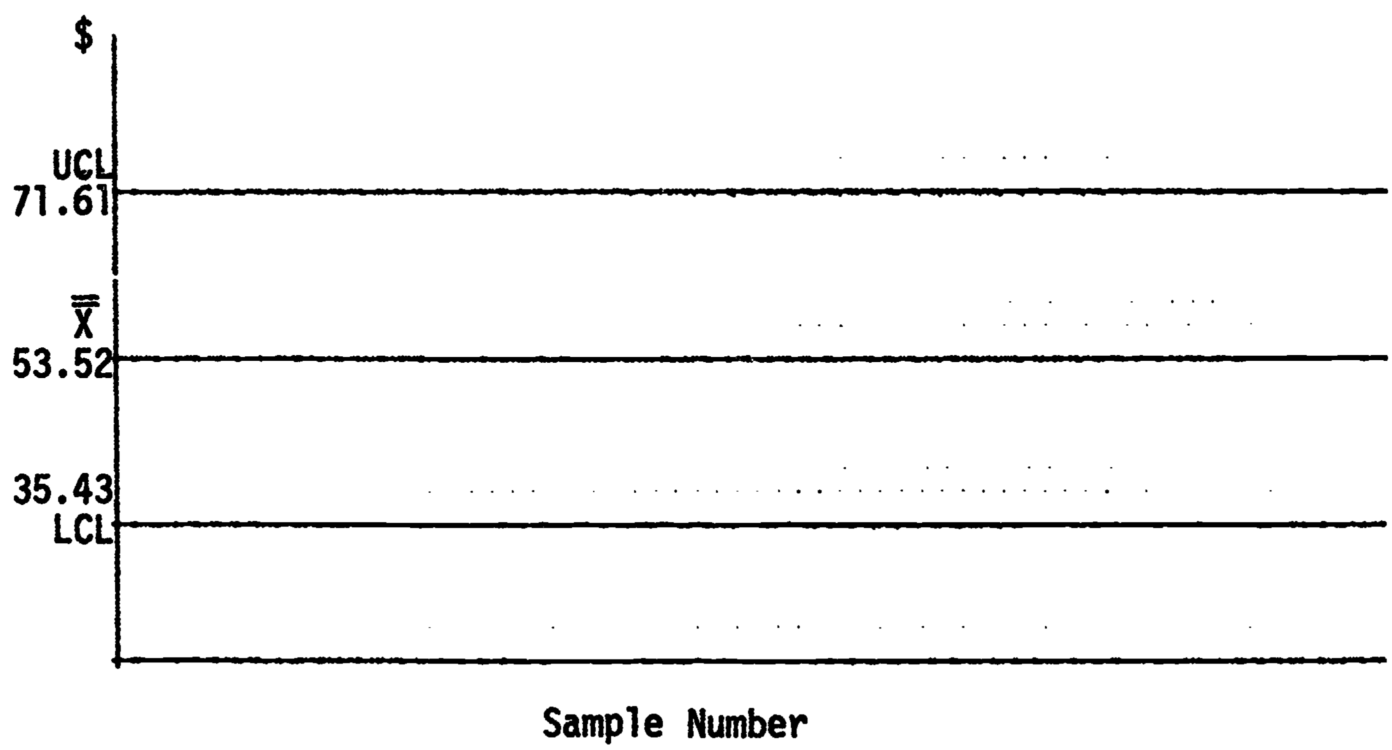
$$P(|\overline{X} - \mu| > j\, \sigma_{\overline{x}}) \leq \frac{1}{j^2} = \frac{1}{(1.96)^2} = \frac{1}{3.84} = .26$$

Thus, the probability of a sample mean being within the interval of $\mu \pm 1.96\, \sigma_{\overline{X}}$ would be greater than 1 - .26 or .74. In our test study of fifty samples, there were approximately 45 observations or 90% of the observations within the interval $\mu \pm 1.96\, \sigma_{\overline{X}}$. Thus, the inequality approach would cause an acceptance of the use of the normal distribution.

The mean of the sample means, $\overline{\overline{X}}$, would be used as an approximation of μ and the standard error of the mean, $\sigma_{\overline{X}}$, would be used in setting control limits around μ. Generally, we would use the interval $\mu \pm 3\sigma_{\overline{X}}$ or $53.52 \pm 3(6.03)$. Our interval would then have control limits of 35.43 to 71.61.

FIGURE 1

CONTROL CHARTS FOR $ COSTS OF MANUFACTURING SUPPLIES



Each of the sample means which fall outside these limits should be eliminated and other samples substituted until all fifty sample means are within the control limits. In our study, all means were within the control limits and the probability of selecting a sample whose mean was outside these limits would be less than .02 (see probability factors for extreme classes in Table 2).

Use of Probability Concepts in the Standard Cost System. The expected value of the cost element, which is the mean of means in our study, will be used as the standard cost of manufacturing supplies per hour of operation in process 4. This standard cost would be the cost applied to production on a per hour of operation basis and would also be the cost figure to be used in the preparation of operational budgets.

The control limits established around the standard cost would be the basis of cost control over actual manufacturing supplies expense in process 4. Random samples of manufacturing supplies cost on an hourly basis would be selected daily and their means would be checked for compliance with the control limits (see Fig. 1). Any sample outside the limits or any consistent movement of means toward a control limit would cause an investigation for assignable cause. Thus, effecting timely cost control with appropriate action taken.

Summary. The development of standard costs through the use of statistical procedures and the establishment of a cost control chart (Fig. 1) will increase cost control. This is accomplished by means of daily samples being selected at random and plotted on the control charts; thus, indicating any need for investigation for assignable cause of variance. The use of daily samples will provide a means for more timely action and will improve performance reports. The cost of such a procedure is not great when compared to the possible savings. The use of computer programs will also help to lower the cost factor.

V. THE OVERHEAD BUDGET AND VARIANCE ANALYSIS BASED ON THE PROBABILITY CONCEPT

In the previous section the use of the expected value or the mean of sample means as a standard cost was discussed. It was also suggested that this same value could be used as the budgeted costs.

While the use of this value as the budgeted cost may be feasible, an alternate approach can be obtained through the use of correlation and regression analysis.

Simple Linear Regression and Correlation Analysis. Simple linear regression and correlation analysis has been used for some time to select the best basis for overhead application. It is a very natural extention to use this same technique in the preparation of budgets.

The following table shows the cost of manufacturing supplies used (Y) and the number of direct labor hours used (X) for twenty observations of 2 hours operation in process 4.

TABLE 3

COST OF MANUFACTURING SUPPLIES AND DIRECT LABOR HOURS FOR TWENTY OBSERVATIONS OF 2 HOURS OF OPERATION OF PROCESS 4

Observation	DLH X	$MS Y
1	70	101
2	74	89
3	72	90
4	71	98
5	73	91
6	74	97
7	64	80
8	82	93
9	76	108
10	85	99
11	75	108
12	72	106
13	67	75
14	70	88
15	80	93
16	74	96
17	72	91
18	71	88
19	72	96
20	74	100
	1468	1887

$\Sigma X^2 = 108{,}186$

$\Sigma Y^2 = 179{,}405$

$\Sigma XY = 138{,}857$

Calculating regression line and coefficient of correlation, we obtain:

$b = .809$

$a = 34.964$

$y' = a + bx = 34.964 + .809X$

$$s_{y.x} = \sqrt{\frac{\Sigma(Y - Y')^2}{n - 2}} = \sqrt{\frac{1082.8773}{18}} = 7.756$$

$$r_{yx} = \sqrt{\frac{[\Sigma XY - \Sigma X \, \Sigma Y]^2}{[\Sigma X^2 - \frac{(\Sigma X)^2}{n}] \; [Y^2 - \frac{(\Sigma Y)^2}{n}]}}$$

$$= \sqrt{\frac{[138{,}857 - \frac{(1468)\,(1887)}{20}]^2}{[108{,}186 - \frac{(1486)^2}{20}] \; [179{,}405 - \frac{(1887)^2}{20}]}} = .4556$$

Thus, when using the regression equation: $Y' = 34.964 \pm .809X$; our estimate $Y' \pm 1.96\, s_{y.x}$ or $Y' \pm 15.21$ would include the observed Y value with a confidence of 95%. The interval, $Y' \pm 15.21$, may be wider than we might desire and the coefficient of correlation between the cost of manufacturing supplies and direct labor hours may be lower than desired. However, simple regression and correlation methods comparing the costs of manufacturing supplies with (1) units of materials used and (2) machine hours produced even less desirable results in this case.

Using present methods, the use of direct labor hours as a basis for budgeting and applying this overhead cost element would be used because of a higher coefficient of correlation.

Multiple Linear Regression and Correlation Analysis. The possibility exists that multiple linear regression and correlation analysis might improve our predictive ability by increasing the coefficient of correlation and reducing the size of the standard errors. The table which follows is the basis for a multiple linear regression and correlation analysis.

TABLE 4

Cost of Manufacturing Supplies (Y), Direct Labor Hours (X_1), Units of Materials Used (X_2) and Machine Hours (X_3) For 20 Observations of Two Hours of Operation of Process 4.

Observation #	$S.C. Y	DLH X_1	M.U. X_2	M.H. X_3
1	101	70	83	39
2	89	74	71	44
3	90	72	72	37
4	98	71	80	29
5	91	73	71	44
6	97	74	72	44
7	80	64	79	38
8	93	82	78	42
9	108	76	83	43
10	99	85	76	44
11	108	75	81	38
12	106	72	78	34
13	75	67	76	50
14	88	70	73	37
15	93	80	72	45
16	96	74	69	46
17	91	72	78	48
18	88	71	69	42
19	96	72	83	40
20	100	74	78	31
	1887	1468	1522	815

Using standard statistical procedures, a multiple regression and correlation analysis yielded the following results:

1. Simple linear coefficients of correlation

r_{y1} = .4556 s

r_{y2} = .4318 ns

r_{y3} = −.3787 ns

r_{12} = −.0793 ns

r_{13} = +.1905 ns

r_{23} = −.34844 ns

where s means significant.

2. βi's

β_1 = .55233 s

β_2 = .35144 s

β_3 = −.36222 s

3. Multiple Coefficient of Determination

$R^2_{y.123}$ = .54059 s

4. Coefficients of multilinear determination eliminating one independent variable.

$R^2_{y.12}$ = .42796 s

$R^2_{y.13}$ = .43247 s

$R^2_{y.23}$ = .22304 ns

5. Coefficients partial determination.

$r^2_{y1.23}$ = .40871 s

$r^2_{y2.13}$ = .19051 ns

$r^2_{y3.12}$ = .19689 ns

$r^2_{y12.3}$ = .46366 s

$r^2_{y13.2}$ = .43528 s

$r^2_{y23.1}$ = .42024 s

6. Unbiased standard deviations

$\hat{s}_y = 8.48076$

$\hat{s}_{x_1} = 4.78373$

$\hat{s}_{x_2} = 4.68928$

$\hat{s}_{x_3} = 5.42775$

7. b_i's, a, and $Y' = a + bx$

$b_1 = .97919$ <u>s</u>

$b_2 = .63559$ <u>ns</u>

$b_3 = -.56596$ <u>ns</u>

$a = -2.82808$

$Y' = -2.82808 + .97919X_1 + .63559X_2 - .5696X_3$

8. Separate $R^2_{y.123}$ into its component parts and $R_{y.123}$

$R^2_{y.123} = .2516 + .1518 + .1372 = .5406$

$R_{y.123} = .735$

The results of our analysis indicate that, while we have increased the amount of variance of the dependent variable which can be explained, we have not significantly improved our predictive ability since only b_1, the coefficient associated with direct labor hours is statistically significant in this case.

The cost predictions of manufacturing supplies based upon the preceeding multilinear analysis are not statistically superior to the predictions based on the simple linear analysis in this case. However, it is highly probable that the multilinear approach will improve predictions in other cases.

<u>Variance Analysis</u>. Just as in the use of control charts in section IV, we would investigate any observations which were outside the limits of our probability interval. This would involve comparing the actual costs of manufacturing supplies with budgeted or predicted costs

and checking the variance against $\pm$ 1.96 sy.x for 95% confidence. Any variances larger than $\pm$ 1.96 sy.x would be investigated for probable cause.

Summary. The use of correlation and regression analysis will provide more realistic budgets for overhead costs with greater predictive value than budgets prepared without their use. In many cases, simple linear analysis will suffice while multilinear analysis may be necessary in other cases. The decision to use multilinear analysis is based upon significantly improved predictive ability.

APPENDIX

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