DOCUMENT RESUME

ED 064 147 24 SE 014 126

AUTHOR Taranto, Maria; Mermelstein, Egon
TITLE A Study of Number Conservation With Tasks Which Vary in Length, Area and Volume. Final Report.
INSTITUTION Hofstra Univ., Hempstead, N.Y.
SPONS AGENCY National Center for Educational Research and Development (DHEW/OE), Washington, D.C.
BUREAU NO BR-1-0426
PUB DATE Jun 72
GRANT OEG-2-71-0426B
NOTE 56p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS Cognitive Development; *Conservation (Concept); Elementary School Mathematics; *Learning; Mathematical Concepts; *Mathematics Education; Number Concepts; Preschool Learning; *Research
IDENTIFIERS *Piaget (Jean)

ABSTRACT

The objective of this study was an attempt to clarify the nature of number conservation with number conservation tasks using variations in length, area, and volume. According to Piagetian theory, conservation is attained successively for number, length, area, and, finally, volume. It was hypothesized that success on the number conservation tasks involving length, area, and volume would follow the same order suggested by Piaget's research. To test the hypothesis, a constructed probability distribution derived from Piagetian theory and a scalogram analysis were used. Eighty children from four to seven years old composed the sample. The hypothesis was not confirmed. In particular, relative to the constructed probability distribution analysis, the order of development of number conservation involving length, area, and volume is individual in nature, while the scalogram analysis suggested no one general order of difficulty of the tasks. (Author/JM)

Final Report

Project No. 1-0426B

Grant No. OEG- 2- 71- 0426B

A Study of Number Conservation with Tasks which Vary in Length, Area and Volume

Maria Taranto - Egon Mermelstein

Hofstra University

Hempstead, New York

June 1972

The research reported herein was performed pursuant to a grant with the Office of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgement in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
National Center for Educational Research and Development

Abstract

The objective of this study was an attempt to clarify the nature of number conservation.

The number conservation task was examined with variations in length, area and volume. Since, according to Piagetian theory, number conservation precedes length conservation which precedes area conservation which precedes volume conservation, it was hypothesized that success on the number conservation tasks involving length, area and volume would follow the same order that Piaget's research suggests for conservation in general.

In order to test this hypothesis, a constructed probability distribution derived from Piagetian theory and a scalogram analysis were utilized. A sample of eighty children ranging in age from four to seven years was utilized.

The results from both analyses suggest that the hypothesized hierarchy relative to the number conservation tasks was not confirmed. In particular, relative to the constructed probability distribution analysis, the order of development of number conservation involving length, area and volume is individual in nature, while the scalogram analysis suggested no one general order of difficulty of the tasks.

Preface

The theoretical and experimental development of the study was supported by a university grant. We wish to acknowledge the assistance of Miss Maglie Bovet and Mrs. (Dr.) Hermione Sinclair, University of Geneva, Geneva, Switzerland, in the development of the experimental tasks. In addition, we wish to express our appreciation to Miss Bovet for assisting us with the clinical method of questioning.

The successful completion of the project could not have been accomplished without the cooperation of the Hofstra Nursery School and Homestead Elementary School. In particular, we express our gratitude to Dr. Bruce Grossman, Mrs. Renee Kaplan, Assistant Director, and to the Nursery School teachers, Mrs. Renie Powers, Mrs. Beverly Caldwell. Mrs. Betty Vasquez, Teaching Principal of the Homestead Elementary School, provided a most congenial atmosphere in which to conduct the experiments. The teachers, Mrs. Batters, Mrs. Burgress, Mrs. Lawrence, Miss Lombardi, Mr. Salt, and Mrs. Tiernan were most helpful in adapting their schedule to our needs. Mr. Howard Bass, the custodian, was most helpful in assisting us set up the experiments.

The commitment and dedication to the project of the research assistants is most noteworthy. Our sincere appreciation is expressed to: Miss Helen Blackman, Miss Annette Cohen, Miss Debra Ebert, Miss Valerie Mannebach, Miss Norma Pedecine, Miss Toby Resnick, Miss Mindy Schmidt and Mrs. Judith Waitz. It is also worthy of mention that Mrs. Waitz's prior work in conservation with us enabled her to serve as a constructive critic on the theory as well as the experimentation.

The task of preparing the final report, as well as the arduous task of transcribing the tapes, and organizing the data were ably performed by the Secretary for the project, Miss Susan Paolillo.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

A. Problems and Objectives

1. Literature

Study of the development of number conservation in the child has been attempted through different approaches. Among these approaches scalogram techniques have become quite popular over the past decade (Wohlwill, 1960; Goldschmid, 1968; D'Mello and Willemson, 1969; Siegelman and Block, 1969; and Schwartz and Scholnick, 1970). However, where Goldschmid (1968) and Siegelman and Block (1969) have concluded that number conservation is the first conservation ability of a scale of abilities which develop in the concrete operational stage, Wohlwill (1960), D'Mello and Willemson (1969) and Schwartz and Scholnick (1970) have utilized the scalogram technique to assess the order of development of a series of abilities, hypothesized to be prerequisites to the development of the number conservation concept.

Although the scalogram technique in part unites the above mentioned studies, there are significant discrepancies among them. First, a basic assumption of the scalogram is that it describes a unitary developmental process (Wohlwill, 1960; Schwartz and Scholnick, 1970). One of the shortcomings of most scalogram based studies is a function of lack of theoretical underpinning of the tasks selected for scaling, since temporal sequence of development is not in and of itself a sufficient condition for defining a unitary developmental process (Wohlwill, 1960). Wohlwill (1960) and D'Mello and Willemson, (1969) rely on the notion of levels of abstractness to predict the difficulty level of their tasks, but the selection of tasks is founded on an intuitive speculation of the prerequisite abilities for success on the number conservation task. Schwartz and Scholnick (1970) utilizing Elkind's (1967) analysis of the logical judgments necessary to solve conservation problems begin with a prescribed sequence of logical steps. Nonetheless, that these steps follow an order of difficulty, or that they are sequentially developed does not appear in the study to follow directly from Elkind or any other theoretical structure. Finally, Siegelman and Block (1969) utilize the tasks studies by Smedslund (1964) but offer no theoretical reason for this selection of tasks.

The attempt here is not to discount the above findings but to caution that a scale of events are not necessarily related to a unitary psychological process. Further, it is suggested by Wohlwill (1960) that the strength of the assumption that a scaled set of tasks does, in fact, describe a unitary psychological process rests on a theoretical basis for the selection of tasks.

A second note of caution regarding the above findings and suggested by Wohlwill (1960) and Schwartz and Scholnick (1970) is that the Guttman type scale only describes order of difficulty of items for a group, and the results can not be generalized to describe the order of difficulty of items for individuals. In fact, it is suggested by Griffiths; Shantz and Sigel (1967) and Wohlwill (1960) that there is sufficient need to study item difficulty within individuals and further, as we see it, as a test of one of the basic hypotheses of Piagetian theory, namely that there is a relationship between the development of conservation among individuals and within a given individual.

A second popular approach to the study of the development of number conservation, which also involves a concern with scaling a series of tasks on order of difficulty, focuses on variations of the conservation task itself. The development of number conservation is studied through an analysis of what is commonly termed "stimulus characteristics" and related to a theory of concept learning explained largely by cue discrimination (Zimiles, 1966; Rothenberg, 1969; Rothenberg and Orost, 1969; Peters, 1967, 1970; Gottfried, 1969; and Halford and Fullerton, 1970).

However, where Zimiles (1966), Rothenberg (1969) and Rothenberg and Orost (1969) suggest a whole series of cues which vary in order of difficulty, Peters (1970) Gottfried (1969) and Halford and Fullerton (1970) focus on the relative difficulty of discrimination of number cues from length cues. In fact, where Peters (1967, 1970) suggests, as is more popular, a concern with the discrimination of the relevant cue (number) over the irrelevant cue (length), both Gottfried (1969) and Halford and Fullerton (1970) stress that the discrimination of number vs. length involves the understanding that both are relevant cues in the solution of the number conservation task. One may even question if the assumptions of learning levels are the same in these two previously mentioned approaches to the study of discriminative cues.

It might be speculated that where the first group, involving scalogram analysis, is interested in a study of the prerequisites to an initial understanding of number conservation, the second group, involving analysis of stimulus characteristics, is interested in a study of the development of a more generalized concept of number conservation. However, the scalogram group says nothing explicitly of the relationship between the particular task of number conservation utilized in their studies to the development of the concept itself. Further, none of the researchers of the "stimulus characteristics" group offers a conceptual framework consistent with Piagetian theory for the selected choice of task variations utilized in their studies.

There are, however, certain commonalities among both above mentioned approaches to which we would also like to add a note of caution.

Primarily, although the traditional number conservation task was utilized by Zimiles (1966), Peters (1970), Rothenberg (1969) and Rothenberg and Orost (1969), Halford and Fullerton (1970) and Schwartz and Scholnick (1970), it is not utilized by Wohlwill (1960), Gottfried (1969) or D'Mello and Willemson (1969). Secondly, where Wohlwill (1960), Gottfried (1969) and D'Mello and Willemson (1969) did not use the clinical method of questioning, at all, the other researchers mentioned have only used a variety of abridged versions of the clinical method. This reflects a problem of differences in conception of the concept of number conservation between the Genevan school (Piaget, Inhelder, Bovet, Sinclair, et al.) and those researchers mentioned above. Gruen (1966), Mermelstein, et al. (1967), Mermelstein and Meyer (1969) have pointed out the possibility that the concept of conservation as measured in such cases is other than the Piagetian concept of conservation.

Although creativity in the definition and measurement of the concept of number conservation as a function of various operational terms is certainly not to be inhibited, it has been suggested by Goldschmid (1968) that the use of the term conservation in studies of conservation has become rather arbitrary in view of the lack of consistency in conservation tasks used, as well as the differences in testing procedures. We would speculate, further, a certain arbitrariness of the use of the term conservation not only among the researchers as reported above but also regarding them and the Genevan school. Indeed, when one notes in the studies reported above, both those concerned with "scalogram analysis" and those concerned with "stimulus characteristics," one finds a behavioral interpretation of the concept of conservation rather than a Piagetian interpretation of conservation. As stated above, the behavioral interpretation is clearly reflected in the particular mode of operational definition of the concept, generally based on an emphasis on the principles of learning theory, and in the particular methodology underlining the form of assessment of the presence or absence of conservation in a child. Finally, it is reflected in the lack of a general theoretical framework for the selection of tasks and the prediction of order of difficulty of tasks, and the reliance on empirical findings to dictate a difficulty order.

It must be stressed that our intention, here, is to clarify distinctions in point of view, and especially where historically there has been a neglect of and need for clarification. It seems clear to us that in order to avoid further confusions regarding the nature of the concept of conservation and to maintain the integrity of it as viewed within a Piagetian framework, we are obligated to employ not only the Genevan "operational" definition of the concept of conservation, but the unabridged clinical method in assessing the presence or absence of the concept in a child. Mermelstein and Schulman (1967), Mermelstein (1967), Pratoomraj and Johnson (1966), and Smedslund (1966) have all suggested the efficacy of using the clinical method to assess intellectual development in children.

Putting aside the limitations of the preceding studies, one finds that they have served an interesting complementary function to the Genevan research on number conservation. The "stimulus characteristics" group have highlighted a difficult to ignore finding; that a child may conserve number on one task with one set of elements or one transformation and not on another task with a different set of elements or a different transformation. In other words, the early notion that the concept of conservation of number is independent of the perceptual aspects of the situation, i.e., the elements and transformations utilized in the test, does not seem to be borne out, at least not in all children, or, that is, at all levels of development of the concept. Zimiles (1966) speaks of Piaget's reference to "true" conservation where the child recognizes conservation as holding throughout any and all transformations or displacements of the objects whose number is conserved. The Genevan school has also recognized this discrepancy, but the theoretical explanation differs.

The concept of decalage (gap) can be employed to describe the same phenomena. However, the emphasis with "decalage" is not on the characteristics of the stimulus but on the characteristics of the intellect. Inhelder (1943, p. 31) defines decalage as ". . . a downward dropping movement from one plane to another and is used to refer to aspects of cognitive development which appear at a state subsequent to the one at which they are normally expected." Vertical decalage refers to the movement say from the plane of activity to the plane of representation for a given concept, for example, the concept of number conservation in the concrete operational stage and in the formal operational stage. Horizontal decalage refers to the movement within a common level of development, say, the concrete operational stage, but among various systems of action, for example, conservation of quantity and conservation of weight. Within the development of a given concept, say, number conservation, there also appears to occur decalages in the expression of the cognitive structure. In plain language, a child may conserve number in one task and not in another. However, it is speculated that the development of number conservation even of a generalized concept covers a much shorter period than the whole period of the concrete operational stage.

In view of the preceding discussion of literature, it is the purpose of this study to investigate a series of variations of conservation of number tasks where both the selection of tasks and the proposed order of difficulty follows from Piagetian theory. Secondly, we wish to examine this proposed relationship of order of difficulty of this set of number conservation tasks both in groups and within individuals. Finally, in an attempt to maintain the integrity of the Genevan notion of conservation, we wish to utilize Piagetian-type tasks and the full unabridged Genevan clinical method of assessing the presence or absence of the concept in a child.

2. Theory

As mentioned earlier, the concept of number conservation has been researched in the United States for well over a decade with many modifications of the original "classic" task to measure number conservation, modifications both in task as well as procedure. Research utilizing the "classic" task of number conservation began almost three decades ago in Geneva, Switzerland. The findings from this body of research are mammoth. Perhaps, some of the most significant findings of the Genevan research regarding the development of number conservation are: (1) that number conservation is but the first concept of conservation developed by the child in a whole series of kinds of conservation including conservation of substance, conservation of continuous quantity, conservation of length, conservation of area, conservation of weight, and conservation of volume (Piaget, Inhelder and Szeminska, 1960; Lovell, 1962); (2) that the various kinds of conservation as measured across children adhere to the aforementioned order of development; (3) that conservation of discontinuous (discrete) quantity develops before conservation of continuous quantity; (4) that within the development of the concrete operational stage there are decalages or gaps in time between success as measured on one task and success as measured on other tasks representing the same operatory system, (Inhelder, 1968 /French, 1943, 1963/).

The development of the various kinds of conservation, in terms of age has been empirically demonstrated (Piaget, Inhelder and Szeminska, 1960; Lovell, 1962) as follows: number conservation (after approximately 5 years of age); conservation of length (after approximately 7 years of age); conservation of area (after approximately 7 years of age); conservation of volume (after approximately 12 years of age). These findings have been derived from cross-sectional studies rather than longitudinal studies on individuals. Therefore, the order of development as stated above may not be identical to the order of development within an individual. The age difference between number on the one hand and length and area on the other is clearly evident. Similarly, the age difference between length and area on the one hand and volume, on the other hand, is clearly evident. However, both length and area are generally acquired by the same time, after approximately seven years of age. Since these findings have been demonstrated only in groups, it is plausible to suggest that within an individual with clearer basis for discrimination we should also find an order of development between conservation of length and conservation of area. Further, based on the fact that the definition of area "includes" length, it seems reasonable to suggest that within an individual the development of conservation of length precedes the development of conservation of area.

Based on the above findings, it seems plausible to suggest three possibilities: (1) that where differences have occurred in a child's performance on variations of the number conservation task the concept of decalage may be extended to account for these differences in performance; (2) that the development of the various kinds of conservation, and their order of development might give some clue to a unified approach to the study of the development of number conservation; (3) that within the development of conservation of quantity, the order of development of discontinuous quantity (number conservation) and continuous quantity (conservation of amount) might give some further clue on the nature of conservation of number.

To be specific, if one keeps in mind that the various kinds of conservation develop in a fixed order as measured in groups of children of increasing age, then we may ask if there is an order of difficulty in the success on a series of number conservation tasks that differ on the dimensions in which the various kinds of conservation differ, i.e., length, area, weight, and volume.

<u>Proposed theoretical relationship between number conservation tasks involving length, area and volume and length conservation, area conservation and volume conservation</u>: Piaget as early as 1937, (Piaget, 1937) and more recently others, Conant (1951), Kuhn (1962), Campbell (1969) have suggested parallel trends in the development of scientific concepts or theories in the history of the individual and the history of science. The development of scientific theories or particular models of reality,

or as Kuhn terms them, "Paradigms," appear yet to have more specific parallels to the development of scientific theories or concepts in the individual. For example, Kuhn (1962, pp. 103, 104) persuasively suggests that the seventeenth-century commitment to the mechanico-corpuscular explanation or interpretation, proved immensely fruitful for a number of sciences, ridding them of problems that had defied generally accepted solutions and suggested others to replace them. For example, according to Kuhn, in dynamics, Newton's laws of motion were reinterpretations of well-known observations in terms of interactions of primary neutral corpuscles. Since primary neutral corpuscles could act on each other by contact, the mechanico-corpuscular view directed scientific attention to a brand new subject of study, the alteration of particulate motions by collisions. Huyghens, Wren and Wallis carried this further by experimenting with colliding pendulum balls. According to Kuhn, Newton embedded the results of Huyghens, Wren and Wallis in his laws of motion. The equal "action" and "reaction" of the third law of motion are the changes in quantity of motion experienced by the two parties to a collision. Thus, we see how a particular point of view or a model of reality (the mechanico-corpuscular) was developed.

Piaget, in his study of intellectual development of children, has employed the conservation model (the notion of invariance over transformation) in explaining the growth of scientific concepts in children. Undoubtedly, the conservation view of studying scientific concepts in children was influenced by twentieth-century physics (Einstein's theory of Relativity) and nineteenth and twentieth-century mathematics, (in particular, Felix Klein, the interpretation of the various Geometries, Euclidean, Projective, and Topology as sets of transformations under which certain properties remain invariant).

In the same sense as the mechanico-corpuscular point of view directed scientific attention to a brand new subject of study, the notion of conservation, when applied to intellectual development as Piaget did, focuses scientific attention to a brand new subject of study, the development of conservation in the child. Since number conservation, although not the first concept of conservation to develop in the child (object permanence according to Piaget develops in the first two years of life) nevertheless, is the first conservation in which logical reasoning according to Piaget seems in evidence. The question we may ask of number conservation is similar to the one one may ask of the third law of motion as interpreted by the mechanico-corpuscular point of view. That is, what is the relationship between Newton's third law of motion and the mechanico-corpuscular point of view? And more generally, one may ask, how is Newton's third law of motion related to Huyghen, Wren and Wallis' colliding pendulum experiments, relative to the mechanico-corpuscular point of view?

Accordingly, with respect to the number conservation task, one may ask, how is number conservation related to length conservation, area conservation, weight conservation, and volume conservation, relative to the conservation model as a point of view? As discussed previously in the literature, there is sufficient empirical evidence to suggest a relationship in terms of order of development of the various kinds of conservation in children. Namely, number conservation precedes length conservation and area conservation, which precede conservation of volume. Given this order of development of the various conservations it seems reasonable to suggest that on any given set of number conservation tasks involving the dimensions of the other conservations, the order of development will be similar. Therefore, in the development of number conservation, given a set of number conservation tasks that differ on the above dimensions, length, area and volume*, it is hypothesized that the order of development of ability to pass these tasks will parallel the order of development of the kinds of conservation where these tasks find their analogue. In other words, relative to number conservation and given this particular set of tasks, it is hypothesized that the same order of development in children should hold as that which holds in the development of conservation in general; specifically, conservation of length, conservation of area, and conservation of volume.

Further, in the development of number conservation, given a variety of tasks that differ on the above dimensions, length, area and volume, it is hypothesized that where the elements of the task form discontinuous collection vs. where the elements of the task form continuous collections within each dimension, number conservation will be mastered by a child on the task involving discontinuous collections prior to mastery of number conservation on the task involving continuous collections. And finally, given a number conservation task that involves transforming discontinuous quantity to a continuous quantity, it seems reasonable to suppose that in order to pass such a task the child must be facile not only with the concept of number conservation but also conservation of amount. This complexity is further reflected in the mode of questioning involved in the task where the child first is asked about the numerical equivalence of glasses in the two sets of different sized glasses, then questioned as to whether the amounts of water in the two beakers would refill the same number of glasses, (see task II, under the procedure section, description of the tasks).

*We have intentionally omitted the dimension of weight as it does not seem to be integrally related to the concept of number as are length, area and volume.

HYPOTHESIS OF THE STUDY

It is hypothesized that success on the classic number conservation task precedes success on the number conservation task involving length for discontinuous substances, and

that success on the number conservation task involving length for discontinuous substances precedes success on the number conservation task involving length for continuous substances, and

that success on the number conservation task involving length for continuous substances precedes success on the number conservation task involving area for discontinuous substances, and

that success on the number conservation task involving area for discontinuous substances precedes the number conservation task involving area for continuous substances, and

that success on the number conservation task involving area for continuous substances precedes success on the number conservation task involving volume for discontinuous substances, and

that success on the number conservation task involving volume for discontinuous substances precedes success on the number conservation task involving volume for continuous substances, and

that success on the number conservation task involving volume for continuous substances precedes the number conservation task involving a "change" from discontinuous substances to a continuous substance.

MATHEMATICAL DESCRIPTION OF HYPOTHESIS OF THE STUDY

1. Let A, B, C, D, E, F, G, H, represent the "classic" number conservation task, the number conservation task involving discontinuous length . . ., respectively, as described above.

2. Let r_a, r_b, r_c, r_d, r_e, r_f, r_g, r_h represent the response scored 1, 2, or 3 corresponding to stage 1, stage 2, or stage 3 respectively on tasks A, B, C, D, E, F, G, H then, according to the hypothesis:

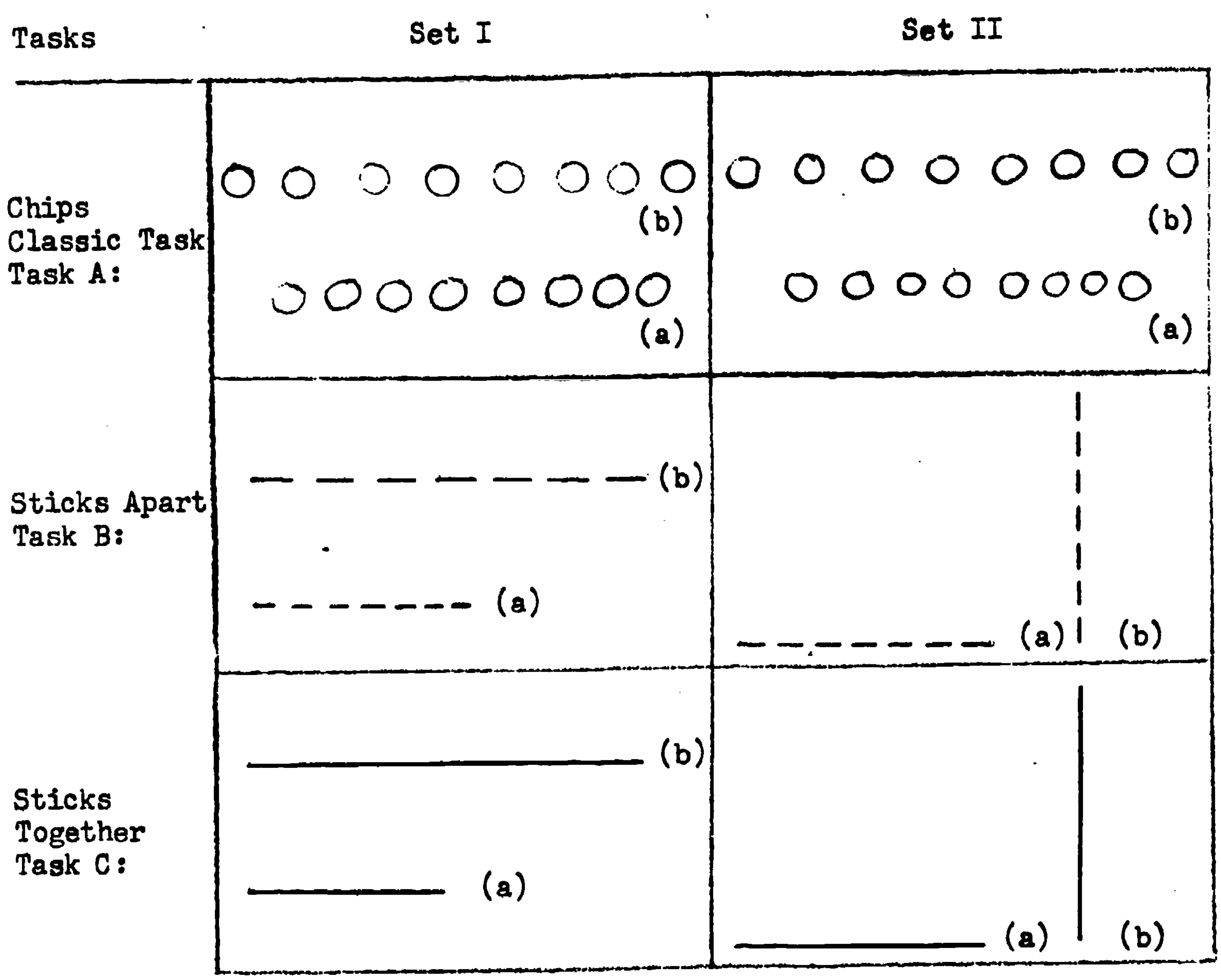
$$r_a \geq r_b \geq r_c \geq r_d \geq r_e \geq r_f \geq r_g \geq r_h$$

B. Description of Activities and Procedures

1. Number Conservation Tasks
   a. Rationale for the Configurations (Size and Shape) and Number of Elements Utilized in the Tasks
      (1) Configuration

Two sets of tasks, I and II, will be utilized to test the hypothesis. There are eight tasks within each set. Each set of tasks is conceptually the same. That is, in both sets of tasks, I and II, the elements of the two collections within each task of sets I and II involve the dimension of length, area and volume. The elements of the two collections within each task also form either two discontinuous or two continuous collections. The difference between Set I and Set II, however, is that in Set I the difference between the two collections in each task is based on size whereas in Set II the difference between the two collections is based on shape.

Final State of Collections (a) and (b) for Each Task of Set I and Set II



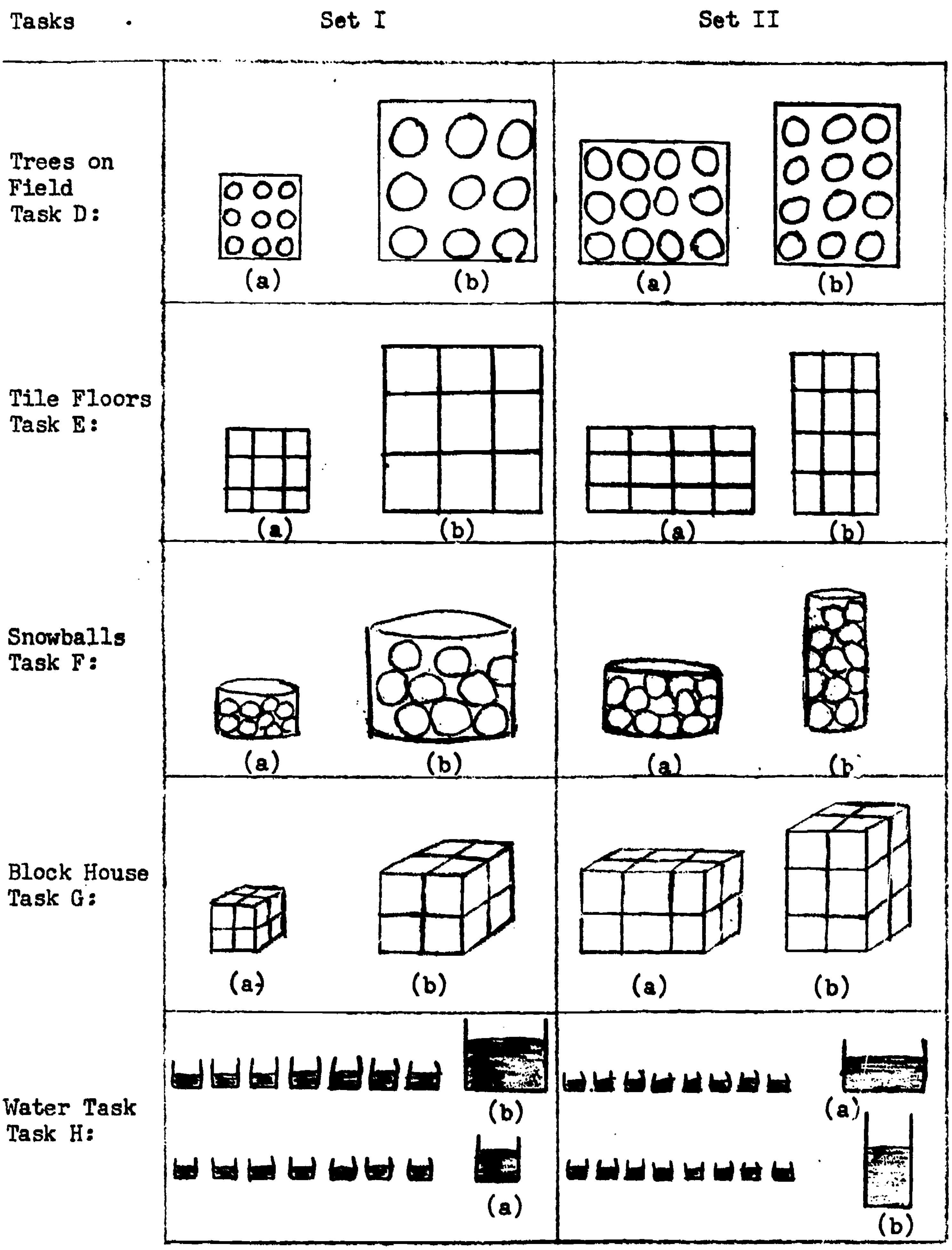

Tasks
Set I
Set II
Trees on Field Task D:
Tile Floors Task E:
Snowballs Task F:
Block House Task G:
Water Task Task H:
(a)
(b)

Figure 1

As shown in Figure 1, for Set I, in the initial and final states the two collections within each task differ in configuration on the basis of a size difference in the elements of each collection. For Set I, the size ratio of the elements from one collection to the other collection within each task is a constant 1:2. Thus, the 1:2 ratio is invariant across the eight tasks of the Set in the initial and final state of the collections. The shape of the two collections within each task is the same within each task of the set.

In Set II, the elements of the two collections within each task are identical in size, in the initial and in the final state. In Set II, the final states of the two collections within each task differ in configuration on the basis of the shapes of the final states of the two collections. The final states of the two collections effect a difference between the two collections on at least one of the linear dimensions of one collection relative to the other. Thus in Set I, with the elements within each of two collections in a fixed ratio of 1:2, shape remains invariant over the transformation (from the initial state to the final state) whereas in Set II, with the elements within each of the two collections in a fixed ratio of 1:1 (equality of size), apparent shape does not remain invariant over the transformation.

Because all of the studies on conservation reported here involve either a change in apparent shape after a transformation between the two collections of the task or a difference in size between the elements of the two collections, there is sufficient empirical basis for Set I and Set II. On a theoretical basis, as mentioned earlier, the conservation model of reality was influenced by nineteenth-century conservation models in physics and mathematics. Thus, the configurations of size and shape in Sets I and II may be related to the invariants under a set of transformations (namely rigid motions--i.e., rotation, translation, reflections) as described by Kline (1964). Kline (1964) states that Euclidean space (space as we know it) in contrast to Projective and Topological space, is characterized by the invariance of shape and size under the set of rigid motions.

(2) Number of Elements

The number of elements utilized across the collections within each task in the present study is constant as is the case with traditional Piagetian tasks. However, the choice of the particular number of elements used in each task within and across Sets I and II varies as a function of practicality regarding the particular elements of each task and the form of their total configuration. The number of elements also varies from task to task in an effort to reduce memorization of a particular number by the child, should he count. The particular range of number of elements utilized in this study, however, varies as a function of the findings from studies related to perception and memory. It is generally known that the perception of a small number of elements is a task which is done largely through the process of visual memory and is readily mastered by very young children (Potter and Levy, 1968). In fact, many of the "so called" successes on number conservation by very young children have been based on findings from studies using small aggregates of elements, in particular, five or fewer elements in each collection. Piaget, on the contrary, has always used over six elements in each of the two collections on his tasks. Further, George Miller has cautioned about the magical quality of the number seven, in that, across most adults approximately seven unrelated words or facts can be retained in immediate memory. The number of elements utilized in this study, based on the above findings, therefore, ranges from seven to ten.

b. Description of Tasks

Set I

Task A: Classic Number Conservation Task
Materials: One Collection of Blue Chips
One Collection of Red Chips
(same size elements)
Number of Elements in Each Collection: 8

An optical one-to-one correspondence* is established between the two rows of discontinuous elements (chips). An equal space is left between each chip within each row. One row is formed above the other. The child is questioned as to his recognition of the numerical equivalence. The experimenter then extends one of the rows of chips beyond the length of the other and the child is requestioned as to the numerical equivalence of the chips in the two rows. (See sample procedure and method of questioning, p. for further detail.)

Task B: Sticks Apart (Discontinuous Length)
Materials: Two Collections of Sticks
(size difference 1:2 ratio)
Number of Elements in Each Collection: 7

The child is questioned as to any difference in the sticks of the two collections until he notes the difference in size. As above, an optical correspondence is established between the two rows of discontinuous elements (sticks). An equal space is left between each stick within each row. The child's recognition of numerical equivalence is established. By simultaneous placing, the experimenter and child then build a row of sticks where the sticks in each row are equidistant from one another, but due to the size difference, one row is much longer than the other. The child is then requestioned as to the numerical equivalence of the sticks in the rows. (See sample procedure and method of questioning, p. for further detail)

Task C: Sticks Together (Continuous Length)
Materials: Two Collections of Sticks (as above)
Number of Elements in Each Collection: 8

*The one-to-one correspondence is perceptually evident in that the elements of the two rows are perfectly aligned one above the other in each row.

The procedure is similar to that in Task B except that after an optical one-to-one correspondence and numerical equivalence are established, the experimenter and child each build, by simultaneous placing of sticks, two continuous roads (that is, the sticks are aligned end to end in each road). The procedure then follows, as above, Task B.

Task D: Fir Trees on Fields (Discontinuous Area)
Materials: Two Collections of Fir Trees and Two Fields
(size difference 1:2 ratio)
Number of Elements in Each Collection: 9

The procedure is similar to that in Task B except that after an optical one-to-one correspondence and numerical equivalence are established, the experimenter and child each cover, by simultaneous placing of fir trees, their own field (the smaller trees on the smaller field and the larger trees on the larger field). The child is then requestioned as to the numerical equivalence of the fir trees in the two fields.

Task E: Square Tiles (Continuous Area)
Materials: Two Collections of Square Tiles
(size difference 1:2 ratio)
Number of Elements in Each Collection: 9

The procedure is similar to those above except that after an optical one-to-one correspondence and numerical equivalence are established, the experimenter and child each make a floor by simultaneous placing of their tiles. The child is then requestioned as to the numerical equivalence of tiles in the two floors.

Task F: Snowballs in Jars (Discontinuous Volume)
Materials: Two Collections of Snowballs and Two Jars
(size difference 1:2 ratio in snowballs and jars)
Number of Elements in Each Collection: 9

The procedure is similar to the above except that the experimenter and child, through simultaneous placing, drop their snowballs into two jars, one for the experimenter and one for the child (the smaller snowballs into the smaller jar and the larger snowballs into the larger jar). The child is then requestioned as to the numerical equivalence of the snowballs in the jars.

Task G: Block House (Continuous Volume)
Materials: Two Collections of Block Cubes
(size difference 1:2 ratio)
Number of Elements in Each Collection: 8

The procedure is similar to the above except that the experimenter and child, through simultaneous placing of blocks, construct a house, respectively, out of their own blocks. The child is then requestioned as to the numerical equivalence of the blocks in the two houses.

Task H: · Glasses of Water (Discontinuous to Continuous)
Materials: Two Collections of Glasses of Water; Two Glass Containers (size difference of the two collection of glasses 1:2; size difference of the two glass containers 1:2)
Number of Elements: 7 glasses in each collection

An optical one-to-one correspondence is formed between the two collections of glasses, and numerical equivalence is established. Through simultaneous pouring of glasses, one by one, the child and experimenter pour the water in their collection of glasses into two containers (the smaller container for the smaller glasses; the larger container for the larger glasses). The child is then questioned if the two beakers would refill the same number of little glasses if they were poured back.

Set II

The same tasks are utilized in Set II except that where size of elements differs from one collection to the other in the tasks of Set I and shape of collections remains the same, in Set II, size of the elements in each task remains the same across the collections while "shape" of the two collections in each task varies.

2. Mode of Assessment: Clinical Method of Questioning

As stated in the literature section, the method of assessing conservation employed in this study was the clinical method of questioning as utilized by the Genevan school. The clinical method refers to a general approach to questioning about a given concept, for example, causality, conservation, class inclusion, etc. It is, therefore, a general approach or attitude toward questioning which varies in format from one concept, say causality, to another concept, say, conservation, etc.; and even varies, specifically, from child to child. What is in common across various specific applications of the clinical method is the commitment to a flexible set of questions, which evolve, in part, from the theoretical assumptions about the nature of the cognitive structure, hypothesized to underlie the knowledge of a given concept, and in part from the actual responses of the child to a given task. A general discussion of the clinical method, as first formulated by Piaget, can be found in The Child's Conception of the World. A more specific account of the technique as employed in a particular concept, in this case, number conservation can be found in The Child's Concept of Number.

In line with the Genevan school, then, we shall follow the traditional format of the clinical method. First, a one-to-one correspondence will be experimentally established and the child will be questioned as to his recognition of the numerical equivalence. Second, a transformation will be effected. Third, the child will be requestioned as to the numerical equivalence of the two collections. The format of questioning in this case will be, "Do we still have the same number of (blocks, trees, tiles, etc.) in our (houses, field, floor, etc.) or do you have more (blocks, trees, tiles, etc.) or do I have more (blocks, trees, tiles, etc.)? What do you think?" The three possible choices will be permuted from task to task and from child to child. Fourth, based on his response, the child will be questioned as to why he thinks either that (1) the numerical equivalence still exists, or (2) the numerical equivalence no longer exists. We consider this fourth step crucial to the maintenance of integrity to the Piagetian notion of number conservation. It is important for a qualitative as well as a quantitative assessment of both conservers and non-conservers. Regarding conservers, it is asserted by the Genevan school that knowledge of the concept of number conservation implies the existence of concrete operational reasoning which is characterized by the properties of a group structure, namely, in this case reversibility, compensation and identity. Accordingly, it is important in assessing the child's knowledge of the concept of number conservation to assess that, in fact, he can support this assertion of the concept of conservation by the arguments of reversibility, compensation and identity. To the extent that children do use just these arguments to assert conservation, we have a test of whether the knowledge of number conservation, or that is, the cognitive structure underlying this knowledge is correspondent with the group structure.

Furthermore, even among these group properties, Piaget asserts that reversibility is the crucial determinant of an internalized operational structure. In fact, it may be suggested that perceptually different material and the natural array of the material, for example, water in jars, as opposed to chips in a row, may lend itself in an argument for conservation to be explained more readily by one argument than another. Both in our experiments and in the vast amount of data gathered by the Genevan school this seems to be the case. For example, it is well documented that children, when presented with conservation of amount (water in jars), tend to argue by compensation ("This one's taller and thinner but that one is shorter and wider.") and reversibility ("I can show you because if I pour it back it's the same.") while arguing by identity ("We put down the same number and you haven't added or subtracted any.") and by reversibility ("They are the same because if I push them back I can show you.") with chips in a row. Therefore, while the knowledge of conservation of quantity is readily seen to elicit arguments of compensation, reversibility and identity, it seems that conservation of continuous quantity (amount) lends itself more to the arguments of compensation and reversibility while conservation of discontinuous quantity (number) lends itself more to the arguments of identity and compensation. This does not mean to say that some children do not use all three arguments in each case. It only suggests variation in trends regarding different materials and their natural physical array. Recognizing the possibility that children of this age do not have a great deal of flexibility moving from one explanation of an event to suggest alternative explanations, we still intend to assess if children can argue by all three explanations.

Regarding non-conservers, there are two considerations. First, non-conservers do not assert numerical equivalence and, therefore, they do not reason by compensation, reversibility or identity. Instead, as shown by the Genevan research and countless replications involving our own work, non-conservers do not relate the various aspects of the situation presented. A transformation does not link the various states of the material, and the non-conserver's thought centers on states of the material, or aspects of these states as separate and unrelated bits of information. Given, that in this study the child will be presented with various tasks, we should like to assess qualitatively what the different centrations of his thought are from task to task. Second, a non-conserver, who asserts that the number no longer remains the same between the two collections, can be asked if there is any way that he can make the number the same. Children who are, supposedly, close to developing the concept of conservation of number assert that, when one collection of chips has been spread out, if you move the chips back where they were, the number will again be the same. This is, in fact, "empirical" reversibility. Unlike the conserver, the child does not argue that the number is the same because you can move the chips back as a proof. Instead, he asserts that the number will be the same if you move the chips back (thus reconstituting the original optical correspondence).

Given this general outline, the procedure regarding our particular tasks was as follows:

In both sets of tasks, one varying in size between the two collections where the "shape" of the collections is held constant, and the other varying in shape between the two collections where the size of the elements in the two collections is held constant, an optical one-to-one correspondence will be established between the two collections for each successive task, and numerical equivalence will be established. The child and experimenter, then, engage in a game (building houses, putting fir trees on fields, making tile floors) where each thus transforms his own collection. In one set of tasks, the transformation is one of shape of the collection, where the experimenter's collection differs in "shape" from the child's, but the size of objects is the same. In the other set of tasks, the transformation is inherent in the difference of size in the elements of the two collections, but the shape of the collections is made to be the same.

For each set of tasks, the method of questioning, as described above, remains the same. A sample outline of this procedure and method is given on the next page.

Training of Experimenters in the Clinical Method: Four of the eight research assistants were given training in the use of the clinical method of questioning and conducted a pilot study last year. All of the eight research assistants were given training in the clinical method of questioning for a period of approximately two months in the Fall, 1971. In the initial phase of the study, either one or both principal investigators provided direction if needed to the research assistants in the testing situations.

SAMPLE OF CLINICAL METHOD OF QUESTIONING

Task G, Set I (Continuous Volume) Variable: Size; Material: Blocks

Trial I

I. An optical one-to-one correspondence is established between the two collections.

E. (Puts down a row of blocks) "Can you put down just as many blocks?"

S. (Child puts one block in front of each block in row)

E. "Now, do we have exactly the same number of blocks?"

S. "Yes."

II. Both collections are displaced.

E. "O.K. Let's play a game of building houses. You build a house with your row of blocks and I'll build a house with mine. But let's each of us put each of our blocks down at exactly the same time.

(E and S each build a house)

E. "Now I want you to look very carefully at each of our houses, and tell me, do we have the same number of blocks in our two houses? or do you have more blocks? or do I have more blocks? What do you think?"

III. The child is requestioned as to the numerical equivalence of the elements of the two collections.

A. Conserver

S. "We still have the same number of blocks."

E. "Why do you think we have the same number?"

S. "Because we put them down at the same time." (identity)

E. "But my house is so much bigger than yours; doesn't that mean I have more blocks?"

S. "No. We still have the same number, but your blocks are bigger."

E. "What does that mean; my blocks are bigger?"

S. "Your blocks are bigger so you have a big house; my blocks are little so I have a little house." (compensation)

E. "Do you think if we put the blocks back in the lines, that for every block of mine there will be a block of yours?" (reversibility)

S. "Yes."

B. Non-Conserver

S. " Now you have more blocks than I do."

E. "Why do you think so?" (checking for centrations)

S. "Because your house goes all the way up to there. And mine only goes up to here."

E. "Did we put down our blocks at the same time?" (identity)

S. "Yes."

E. "And now I have more?"

S. "Yes."

E. "Is there any way you can make it so that we both have the same number of blocks in each of our houses?" (check for reversibility)

S. (Removes top layer of blocks of my house. Height of both houses is now the same.)

E. "And now we both have exactly the same number of blocks in our two houses?"

S. "Yes."

E. "If we have the same number, do you think for every block I have you have a block?"

S. "Yes."

E. "You mean, if you put each of your blocks with each end of mine, I wouldn't have any left over and you wouldn't have any left over?"

S. "Yes."

(Verification: Both E. and S. put down blocks one by one in parallel matching rows.)

E. "What Happened?"

S. "I have some left over."

E. "Why do you think that happened?"

S. "Because you took some of yours away."

Trial II

IV. Reverse collections and repeat entire procedure, i.e., if experimenter has collection of large objects and child has collection of small objects, give collection of large objects to child and take small objects for yourself.

3. Scoring of Responses for the Tasks

Responses of the children for each task were scored Stage 1 (non-conserver), Stage 2 (transitional), and Stage 3 (conserver). A child's response was given a score of 1 if on one trial or both trials of a task he asserted non-conservation of number. A child's response was given a score of 3 (conservation) if on both trials of a task he asserted conservation of number and was able to argue by reversibility, compensation or identity. A child's response was given a score of 2 (transitional) if: (a) on either trial 1 or trial 2 the child vacillated between asserting conservation and asserting non-conservation; (b) the child's answer vacillated between trial 1 and trial 2; and (c) the child asserted conservation but could not give any reason for his answer.

Scoring of the responses was done by four groups of three raters meeting three times a week. To determine reliability of scoring the three raters in each group independently rated a random set of ten Ss and interrater reliability was determined by amount of consentual agreement of ratings. Across the four groups agreement of scoring was practically unanimous and it was, therefore, inferred that the raters were using the same criteria in judgment. The procedure for scoring then went as follows: for each set of raters, one of the raters read a protocol while each rater independently rated the protocol and noted her reason for judgment. The three raters then stated their judgments and in cases of disagreement, the discrepancy was discussed. In a few cases where a complete agreement among the raters could not be attained, the matter was discussed with one of the principal investigators, and a judgment was reached. In most cases, however, the group of three raters was composed of two students and one principal investigator. On the whole, universal rating agreement was very high, well over 90 percent.

The rating procedure was executed by grade with twenty tasks on ten children being rated by each of the four sets of raters at each sitting. Since the order of tasks was randomized in testing, the tasks were also randomized in the scoring process. Given the design for testing then, the first set of raters rotated in reading the first protocol for each of the ten children in the nursery school. They then repeated the procedure for the second protocol for each of the ten children, thus avoiding any set for grading a certain child as a "conserver" etc. across tasks. They then rated the first and second set of protocols respectively for the ten children in the kindergarten, first grade, etc. Thus each of four testers rated an equal number of protocols and children across the four grades. This procedure was carried out for both Set I and Set II.

4. Population Sample

From the population of Hofstra University Nursery School, and the Homestead Elementary School, a sample of 80 children, 40 female, 40 male children ranging in age from 4 years to 7 years of age were selected for Sets I and II. The foregoing age range, as indicated by the work in the area, covers the total period of the development of number conservation from non-conservation to full conservation of number.

Forty children, 10 four year olds, 10 five year olds, 10 six year olds, and 10 seven year olds were administered Set I, while another group of 40 children, 10 four year olds, 10 five year olds, 10 six year olds, and 10 seven year olds were administered Set II. Three children in Set I and three children in Set II were ultimately dropped from the sample due to the acquisition of incomplete data. The reason for administering Set I and Set II to a different sample of children is suggested by the number of tasks in each set (8). Sixteen tasks we believe are too many tasks to administer to a child within a few weeks without effecting some form of learning.

5. Procedure

In order to test out the hypothesis, each child was given a series of eight number conservation tasks, two trials, where necessary*, on each task. The trials are not cumulative, but the second served as a check on the first. Each task (two trials) took approximately ten minutes to administer. Because the judgment on attention span was based on our pilot study, usually around twenty minutes for these children, on our tasks, only two tasks were administered per sitting. Further, since the tasks are somewhat similar and the data might reflect a learning set rather than a "true" difficulty order, the order of tasks were varied from child to child. In other words, each child was given a different random order of tasks as selected from a table of random numbers. In the current study, each child was tested only once whenever possible by each experimenter. Experimenters worked in pairs where one tested a child while the other recorded. The "recorder" tested the same child on

*It was clear in many instances that two trials were not always necessary. In the nursery school especially, only one trial was given when it was clear that the child could not attend anymore. In other instances, where the child completely satisfied the criteria for conservation and boredom was a distinct possibility, the second trial was not given.

**The recorded operated a tape recorder, as well as noting the activities of the child.

another task while the "tester" then recorded. Each pair of experimenters administered four tasks per sitting. Each child was tested over a period of one week with two sittings per child per day, and two tasks approximately per sitting. The total testing time for the present study was three months.

6. Design

a. Organization of Testing Procedure for Study

Figure 2

RANDOMIZED ORDERS OF 8 TASKS FOR 10 CHILDREN, NURSERY SCHOOL, AND DESIGN FOR THE ADMINISTRATION OF TASKS

Set I	Tuesday A	Tuesday B	Wednesday C	Wednesday D	Thursday E	Thursday F	Friday G	Friday H
Children	*MN	NM	TA	AT	DV	VD	HJ	JH
I ______	**7	8	5	6	1	2	3	4
II ______	5	4	1	2	3	8	6	7
III ______	5	2	7	1	3	4	6	8
IV ______	7	2	6	1	4	3	5	8
V ______	3	8	6	1	5	2	4	7
	DV	VD	HJ	JH	MN	NM	TA	AT
VI ______	8	2	3	1	5	4	7	6
VII ______	5	7	6	1	8	4	2	3
VIII ______	5	3	6	2	4	8	7	1
IX ______	6	3	5	7	4	2	1	8
X ______	3	6	2	1	8	4	7	5

CODE

A. Chips (8)***
B. Sticks Apart (7)
C. Sticks Together (8)
D. Trees on Fields (9)
E. Tiles (9)
F. Snowballs (9)
G. Blocks (8)
H. Water (7)

*Refers to initials of Experimenters.

**Numbers 1, 2 . . . 8 in the randomized orders above represent tasks A, B . . . H respectively.

***Refers to number of elements used in each collection.

Figure 2 describes the organization of the administration of tasks designed to test the hypothesis:

$$r_a \geq \quad r_b \geq \quad r_c \geq \quad r_d \geq \quad r_e \geq \quad r_f \geq \quad r_g \geq \quad r_h$$

(the response score on task A is greater than or equal to the response score on task B which is greater than or equal to the response score on task C, etc.)

Reading vertically, the order of children tested for any given set of Experimenters, for example, MN is Tuesday, Children I, II, III, IV, V, on two tasks and Thursday, VI, VII, VIII, IX, X on two tasks.

The design described for the Nursery School, Set I, was the format for the Kindergarten, First Grade and Second Grade children for Set I as well as the children for Set II.

Because the experimenters had most of their training experience with Nursery School children on Set I, and because of other practical considerations, the order of testing began with Nursery School children on Set I, and then proceeded to Kindergarten, First Grade, and Second Grade. Relative to Set II, primarily because of the lack of pilot study data on Set II and the problems of age encountered in testing the Nursery School for Set I, the Kindergarten, First and Second Grade children were tested first and finally the Nursery School children were tested.

While the testing of the children generally conformed to the design, children absences, experimenter absences and particular plans of the teacher necessitated minor adjustments of the design.

b. Probability Distribution of Confirming Instances of the Hypothesis

The probability distribution constructed on the basis of a confirmation of the hypothesis is outlined in Table 2. The numbers 1, 2, 3 in Table 1 correspond to response scores Stage 1, Stage 2, and Stage 3, respectively, for each task. Reading horizontally each order of the response scores for the eight tasks, A, B, C, D, E, F, G, H is a confirming instance of the hypothesis.

Probability Distribution of Confirming Instances of the Hypothesis

TASKS

A	B	C	D	E	F	G	H
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	1
2	2	2	2	2	2	1	1
2	2	2	2	2	1	1	1
2	2	2	2	1	1	1	1
2	2	2	1	1	1	1	1
2	2	1	1	1	1	1	1
2	1	1	1	1	1	1	1
3	3	3	3	3	3	3	2
3	3	3	3	3	3	2	2
3	3	3	3	3	2	2	2
3	3	3	3	2	2	2	2
3	3	3	2	2	2	2	2
3	3	2	2	2	2	2	2
3	2	2	2	2	2	2	2
3	3	3	3	3	3	3	1
3	3	3	3	3	3	1	1
3	3	3	3	3	1	1	1
3	3	3	3	1	1	1	1
3	3	3	1	1	1	1	1
3	3	1	1	1	1	1	1
3	1	1	1	1	1	1	1
3	3	3	3	3	3	2	1
3	3	3	3	3	2	2	1
3	3	3	3	3	2	1	1
3	3	3	3	2	2	2	1
3	3	3	3	2	2	1	1
3	3	3	3	2	1	1	1
3	3	3	2	2	2	2	1
3	3	3	2	2	2	1	1
3	3	3	2	2	1	1	1
3	3	3	2	1	1	1	1
3	3	2	2	2	2	2	1
3	3	2	2	2	2	1	1
3	3	2	2	2	1	1	1
3	3	2	2	1	1	1	1
3	3	2	1	1	1	1	1
3	2	2	2	2	2	2	1
3	2	2	2	2	2	1	1
3	2	2	2	2	1	1	1
3	2	2	2	1	1	1	1
3	2	2	1	1	1	1	1
3	2	2	1	1	1	1	1

Table 1

The total number of possible orders of responses 1, 2 or 3 on 8 tasks is 3^n or $3^8 = 6501$. The number of "orders" of the 8 tasks which confirms the hypothesis as described is 45. Assuming then from probability theory, the equal likelihood of occurrence of events, or "orders," then the probability of selecting any one of the confirming events or "orders" is $\frac{45}{6501}$ or .007. Thus, if it is the case that a subject obtains any one of the orders in which the hypothesis confirmed, because of its probability, we may safely conclude that the occurrence is due to "factors" other than chance. Thus, we have described a set of probability statements concerning each subject. Clearly the fixing of a confidence level at .007 based on a theoretical position is in line with Johnson (1967, p. 6) in which he points to the employment in research of "arbitrary" levels of significance usually .05 or .01.

In addition, it seems appropriate to analyze the data in terms of a scàlogram analysis similar to the one suggested by D'Mello and Willemson (1969) and others. Thus, while our first analysis originates from individual probability statements across a given set of tasks, based on a theoretical model, the scalogram analysis examines the "empirical" data not across a set of tasks for any individual, but analyzes a task across a given set of individuals, and compares the percent of subjects who "passed" a given task (Stage 3), who were "transitional" (Stage 2) and who "failed" a given task (Stage 1) relative to the other tasks. Thus, according to the hypothesis forwarded, we expect that the largest percentage will pass Task A, a smaller percentage will pass Task B, etc. and the smallest percentage of the sample will pass Task H.

Utilizing the scalogram analysis and the probability distribution based on the theoretical model, we will be able to make statements about the population, from the sample, as well as make statements about individuals in the population.

C. Results

1. Analysis of the Data Relative to the Hypothesized Hierarchy of Tasks

Two general analyses of the data were performed, one by individual instances of confirmation of the hypothesized hierarchy of tasks for Set I and Set II respectively, and one by a scalogram analysis to determine the scalability of items across the sample of 37 children for Set I and 37 children for Set II, respectively.

Analysis of empirical distribution of scores across the set of tasks "within" an individual relative to the hypothesized (theoretical) distribution: Within the analysis for individual confirmations of the hierarchy, there were four separate analyses. First, the data were examined for each individual across all eight tasks. Tables 2 and 3 depict the empirical distribution of scores for each individual across the eight tasks for Set I and Set II respectively.

EMPIRICAL DISTRIBUTION OF SCORES FOR EACH INDIVIDUAL ACROSS THE EIGHT TASKS: SET I, SET II

Table 2

Children	Tasks A	B	C	D	E	F	G	H
Nursery								
1	3	2	3	3	2	3	3	2
2	1	2	1	3	3	3	2	3
3	1	3	1	1	3	1	1	1
4	1	1	1	1	1	1	1	1
5	1	1	1	2	1	1	1	1
6	1	2	2	3	1	1	3	1
7	1	1	1	1	1	1	1	1
8	3	1	3	2	3	1	2	2
Kindergarten								
9	1	2	2	3	2	3	3	3
10	3	2	3	3	3	3	2	3
11	2	3	3	2	3	2	2	3
12	2	2	2	3	3	2	2	1

Set I

Table 3

Children	Tasks A	B	C	D	E	F	G	H
Nursery								
1	1	1	3	1	2	1	2	1
2	2	2	3	3	1	3	3	1
3	1	1	1	1	1	2	1	1
4	1	1	2	1	1	1	1	1
5	2	1	1	1	2	1	2	1
6	1	2	2	1	2	2	1	1
7	1	1	1	1	1	1	1	1
8	3	1	3	3	3	1	3	1
9	1	1	1	1	1	1	1	1
10	2	3	3	3	3	3	3	2
Kindergarten								
11	1	2	1	1	1	1	1	1
12	2	2	2	2	2	1	1	1

Set II

Table 2 (Continued)

Children	A	B	C	D	E	F	G	H
13	3	3	3	3	3	3	3	1
14	1	1	1	1	1	1	1	1
15	3	3	3	3	3	3	3	3
16	3	2	3	3	3	3	3	3
17	2	2	2	3	2	1	2	1
1st Gr.								
18	3	3	3	3	2	3	3	3
19	3	3	2	3	3	3	3	2
20	3	3	3	3	3	3	3	3
21	3	3	3	3	3	3	3	2
22	3	3	3	3	3	3	3	3
23	3	2	3	2	2	3	2	3
24	1	1	3	3	3	3	2	3
25	3	3	2	2	3	3	3	2
26	2	2	3	2	2	2	3	2
27	3	3	3	3	3	3	3	2
2nd Gr.								
28	3	3	3	3	3	3	3	3
29	3	3	3	3	3	3	3	3
30	2	3	3	3	2	3	3	2
31	3	2	2	3	3	3	3	3
32	3	3	3	3	3	3	3	3
33	3	3	3	3	3	3	3	3
34	3	2	3	3	3	2	3	1
35	3	2	3	3	3	3	3	3
36	3	3	1	3	3	3	3	3
37	3	3	2	3	3	3	2	3

Tasks — Set I

Table 3 (Continued)

Children	A	B	C	D	E	F	G	H
13	1	1	1	2	1	3	2	1
14	1	2	3	2	2	2	1	3
15	3	2	3	3	2	1	2	1
16	3	3	3	3	3	3	3	3
17	1	3	3	2	3	1	2	2
18	1	1	2	3	1	3	1	2
19	2	2	2	2	2	3	2	1
1st Gr.								
20	3	2	2	3	2	3	1	1
21	3	3	3	3	3	3	3	3
22	3	2	3	3	2	3	2	3
23	3	3	3	3	3	3	3	3
24	3	3	3	3	3	3	3	2
25	3	3	3	3	2	3	2	3
26	3	3	3	3	3	3	3	3
27	3	3	2	3	3	3	3	3
28	3	3	3	3	3	3	3	3
2nd Gr.								
29	3	3	3	3	3	3	3	3
30	3	3	3	3	3	3	3	3
31	3	3	3	3	3	3	3	3
32	3	3	3	3	3	3	3	1
33	3	3	3	3	3	3	3	3
34	3	3	3	3	2	3	3	3
35	3	2	3	2	3	3	3	3
36	3	3	3	3	3	3	3	3
37	3	3	3	3	3	3	3	3

Tasks — Set II

In Set I, 13 individuals manifested confirmation of the hypothesis, while 24 individuals did not. Of the 13 confirming instances, 10 were unitary patterns, i.e., either all 1's, 2's, or 3's. In Set II, 16 individuals manifested confirming instances of the hypothesis, while 21 did not. Of the 16 confirming instances, 13 were unitary patterns.

Second, the hypothesis was examined for two subsets of the eight tasks, only the discrete tasks i.e., tasks B, D, and F and only the continuous tasks i.e., C, E, and G for Set I and Set II respectively. Tables 4, 5, 6 and 7 depict the empirical distribution of scores of individuals for these subsets.

EMPIRICAL DISTRIBUTION FOR TWO SUBSETS OF EIGHT TASKS, DISCRETE AND CONTINUOUS, FOR EACH INDIVIDUAL: SET I

Set I

Table 4

Children	Discrete Tasks B	D	F
Nursery			
1	2	3	3
2	2	3	3
3	3	1	1
4	1	1	1
5	1	2	1
6	2	3	1
7	1	1	1
8	1	2	1
Kinderg.			
9	2	3	3
10	2	3	3
11	3	2	2
12	2	3	2
13	2	3	3
14	1	1	1
15	3	3	3
16	2	3	3
17	2	2	1
1st Gr.			
18	3	3	3
19	3	3	3
20	3	3	3
21	3	3	3
22	3	3	3
23	2	2	3
24	1	3	3
25	3	2	3
26	2	2	2
27	3	3	3

Table 5

Children	Continuous Tasks C	E	G
Nursery			
1	3	2	3
2	1	3	2
3	1	3	1
4	1	1	1
5	1	1	1
6	2	1	3
7	1	1	1
8	3	3	2
Kinderg.			
9	2	2	3
10	3	3	2
11	3	3	2
12	2	3	2
13	3	3	3
14	1	1	1
15	3	3	3
16	3	3	3
17	2	2	2
1st Gr.			
18	3	2	3
19	2	3	3
20	3	3	3
21	3	3	3
22	3	3	3
23	3	2	2
24	3	3	2
25	2	3	3
26	3	3	3
27	3	3	3

Table 4 (Continued) Set I Table 5 (Continued)

Children	Discrete Tasks B	D	F
2nd Gr. 28	3	3	3
29	3	3	3
30	3	3	3
31	2	3	3
32	3	3	3
33	3	3	3
34	2	3	2
35	2	3	3
36	3	3	3
37	3	3	3

Children	Continuous Tasks C	E	G
2nd Gr. 28	3	3	3
29	3	3	3
30	3	2	3
31	2	3	3
32	3	3	3
33	3	3	3
34	3	3	3
35	3	3	3
36	1	3	3
37	2	3	2

EMPIRICAL DISTRIBUTION FOR TWO SUBSETS OF EIGHT TASKS, DISCRETE AND CONTINUOUS, FOR EACH INDIVIDUAL; SET II

Table 6 Set II Table 7

Children	Discrete Tasks B	D	F
Nursery 1	1	1	1
2	2	3	3
3	1	1	2
4	1	1	1
5	1	1	1
6	2	1	2
7	1	1	1
8	1	3	1
9	1	1	1
10	3	3	3
Kinderg. 11	2	1	1
12	2	2	1
13	1	2	3
14	2	2	2
15	2	3	1
16	3	3	3
17	3	2	1
18	1	3	3
19	2	2	3

Children	Continuous Tasks C	E	G
Nursery 1	3	2	2
2	3	1	3
3	1	1	1
4	2	1	1
5	1	2	2
6	2	2	1
7	1	1	1
8	3	3	3
9	1	1	1
10	3	3	3
Kinderg. 11	1	1	1
12	2	2	1
13	1	1	2
14	3	2	1
15	3	2	2
16	3	3	3
17	3	3	2
18	2	1	1
19	2	2	2

Table 6 (Continued) Set II

Children	Discrete Tasks B	D	F
1st Gr.			
20	2	3	3
21	3	3	3
22	2	3	3
23	3	3	3
24	3	3	3
25	3	3	3
26	3	3	3
27	3	3	3
28	3	3	3
2nd Gr.			
29	3	3	3
30	3	3	3
31	3	3	3
32	3	3	3
33	3	3	3
34	3	3	3
35	2	2	3
36	3	3	3
37	3	3	3

Table 7 (Continued)

Children	Continuous Tasks C	E	G
1st Gr.			
20	2	2	1
21	3	3	3
22	3	2	2
23	3	3	3
24	3	3	3
25	3	2	2
26	3	3	3
27	2	3	3
28	3	3	3
2nd Gr.			
29	3	3	3
30	3	3	3
31	3	3	3
32	3	3	3
33	3	3	3
34	3	2	3
35	3	3	3
36	3	3	3
37	3	3	3

In Set I, 21 individuals manifested confirming instances of the hypothesis for discrete tasks only while 16 did not. Of the 21 confirming instances 18 were unitary patterns. For continuous tasks only, 24 individuals manifested confirming instances, while 13 did not. Nineteen of the 24 confirming instances were unitary patterns. In Set II for discrete tasks, only 26 individuals confirmed the hypothesis, 11 did not. Of the 26 confirmations, 23 were unitary patterns. For continuous tasks only, 32 individuals manifested confirmations of the hypothesis, while 5 did not. Of the 32 confirming instances, 21 were unitary patterns.

Third, the data was examined across another subset of tasks, length, area and volume, disregarding the discrete, continuous dimension for Set I and Set II respectively. Tables 8 and 9 depict the empirical distribution of scores for each individual across this subset. For each individual, the highest scores attained on the length, area and volume tasks (either discrete or continuous) were utilized to construct the distribution.

EMPIRICAL DISTRIBUTION OF SCORES FOR THE SUBSET
LENGTH, AREA, VOLUME TASKS FOR EACH INDIVIDUAL

Table 8

Children	Tasks: Length B or C	Area D or E	Volume F or G
Nursery			
1	3	3	3
2	2	3	3
3	3	3	1
4	1	1	1
5	1	2	1
6	2	3	3
7	1	1	1
8	3	3	2
Kinderg.			
9	2	3	3
10	3	3	3
11	3	3	2
12	2	3	2
13	3	3	3
14	1	1	1
15	3	3	3
16	3	3	3
17	2	3	2
1st Gr.			
18	3	3	3
19	3	3	3
20	3	3	3
21	3	3	3
22	3	3	3
23	3	2	3
24	3	3	3
25	3	3	3
26	3	2	3
27	3	3	3
2nd Gr.			
28	3	3	3
29	3	3	3
30	3	3	3
31	2	3	3
32	3	3	3
33	3	3	3
34	3	3	3
35	3	3	3
36	3	3	3
37	3	3	3

Set I

Table 9

Children	Tasks: Length B or C	Area D or E	Volume F or G
Nursery			
1	3	2	2
2	3	3	3
3	1	1	2
4	2	1	1
5	1	2	2
6	2	2	2
7	1	1	1
8	3	3	3
9	1	1	1
10	3	3	3
Kinderg.			
11	2	1	1
12	2	2	1
13	1	2	3
14	3	2	2
15	3	3	2
16	3	3	3
17	3	3	2
18	2	3	3
19	2	2	3
1st Gr.			
20	2	3	3
21	3	3	3
22	3	3	3
23	3	3	3
24	3	3	3
25	3	3	3
26	3	3	3
27	3	3	3
28	3	3	3
2nd Gr.			
29	3	3	3
30	3	3	3
31	3	3	3
32	3	3	3
33	3	3	3
34	3	3	3
35	3	3	3
36	3	3	3
37	3	3	3

Set II

In Set I, there were 28 individuals who manifested confirming instances of the hypothesis, 25 of those individuals manifested unitary patterns. Nine individuals did not manifest confirming instances. In Set II, there were 31 confirming instances of the hypothesis, 24 of which were unitary patterns. There were 6 instances of patterns which did not confirm the hypothesis.

Finally, the hypothesis was examined for the discrete, continuous dimension each of the two length, area and volume tasks for Set I and Set II respectively. Tables 10, 11 and 12 depict the empirical distributions of scores on these subsets for each individual in Set I. Tables 13, 14 and 15 depict the empirical distributions of scores in Set II.

EMPIRICAL DISTRIBUTION OF SCORES FOR THE SUBSETS, LENGTH AREA, AND VOLUME RESPECTIVELY, RELATIVE TO DISCRETE-CONTINUOUS DIMENSION FOR EACH INDIVIDUAL; SET I

	Table 10		Table 11		Table 12	
Children	Length		Area		Volume	
	Dis-crete	Contin-uous	Dis-crete	Contin-uous	Dis-crete	Contin-uous
Nursery						
1	2	3	3	2	3	3
2	2	1	3	3	3	2
3	3	1	1	3	1	1
4	1	1	1	1	1	1
5	1	1	2	1	1	1
6	2	2	3	1	1	3
7	1	1	1	1	1	1
8	1	3	2	3	1	2
Kinderg.						
9	2	2	3	2	3	3
10	2	3	3	3	3	2
11	3	3	2	3	2	2
12	2	2	3	3	2	2
13	3	3	3	3	3	3
14	1	1	1	1	1	1
15	3	3	3	3	3	3
16	2	3	3	3	3	3
17	2	2	3	2	1	2
1st Gr.						
18	3	3	3	2	3	3
19	3	2	3	3	3	3
20	3	3	3	3	3	3
21	3	3	3	3	3	3
22	3	3	3	3	3	3
23	2	3	2	2	3	2

(Continued)

Children	Table 10: Length — Discrete	Table 10: Length — Continuous	Table 11: Area — Discrete	Table 11: Area — Continuous	Table 12: Volume — Discrete	Table 12: Volume — Continuous
1st Gr.						
24	1	3	3	3	3	2
25	3	2	2	3	3	3
26	2	3	2	2	2	3
27	3	3	3	3	3	3
2nd Gr.						
28	3	3	3	3	3	3
29	3	3	3	3	3	3
30	3	3	3	2	3	3
31	2	2	3	3	3	3
32	3	3	3	3	3	3
33	3	3	3	3	3	3
34	2	3	3	3	2	3
35	2	3	3	3	3	3
36	3	1	3	3	3	3
37	3	2	3	3	3	2

EMPIRICAL DISTRIBUTION OF SCORES FOR THE SUBSETS, LENGTH AREA, AND VOLUME RESPECTIVELY, RELATIVE TO DISCRETE-CONTINUOUS DIMENSION FOR EACH INDIVIDUAL: SET II

Children	Table 13: Length — Discrete	Table 13: Length — Continuous	Table 14: Area — Discrete	Table 14: Area — Continuous	Table 15: Volume — Discrete	Table 15: Volume — Continuous
Nursery						
1	1	3	1	2	1	2
2	2	3	3	1	3	3
3	1	1	1	1	2	1
4	1	2	1	1	1	1
5	1	1	1	2	1	2
6	2	2	1	2	2	1
7	1	1	1	1	1	1
8	1	3	3	3	1	3
9	1	1	1	1	1	1
10	3	3	3	3	3	3
Kinderg.						
11	2	1	1	1	1	1
12	2	2	2	2	1	1
13	1	1	2	1	3	2

(Continued) Table 13 | Table 14 | Table 15

Children	Length Dis-crete	Length Contin-uous	Area Dis-crete	Area Contin-uous	Volume Dis-crete	Volume Contin-uous
Kinderg. 14	2	3	2	2	2	1
15	2	3	3	2	1	2
16	3	3	3	3	3	3
17	3	3	2	3	1	2
18	1	2	3	1	3	1
19	2	2	2	2	3	2
1st Gr. 20	2	2	3	2	3	1
21	3	3	3	3	3	3
22	2	3	3	2	3	2
23	3	3	3	3	3	3
24	3	3	3	3	3	3
25	3	3	3	2	3	2
26	3	3	3	3	3	3
27	3	2	3	3	3	3
28	3	3	3	3	3	3
2nd Gr. 29	3	3	3	3	3	3
30	3	3	3	3	3	3
31	3	3	3	3	3	3
32	3	3	3	3	3	3
33	3	3	3	3	3	3
34	3	3	3	2	3	3
35	2	3	2	3	3	3
36	3	3	3	3	3	3
37	3	3	3	3	3	3

In Set I, for the length tasks, 28 individuals manifested confirming instances of the hypothesis while 9 did not. Of the 28 confirming instances, 22 were unitary patterns. For the area tasks, 32 individuals manifested confirming instances of the hypothesis, while 5 did not. Of the 32 confirming instances, 27 were unitary patterns. For the volume tasks, 32 individuals manifested confirming instances of the hypothesis, while 5 did not. Of the 32 confirming instances, 27 were unitary patterns.

In Set II for the length tasks, 28 individuals manifested confirming instances of the hypothesis while 9 did not. Of the 28 confirmations, 26 were unitary patterns. For the area tasks, 32 individuals confirmed the hypothesis while 5 did not. Of the 32 confirmations, 24 were unitary. For the volume tasks, 32 individuals also confirmed the hypothesis while 5 did not. Of the 32 confirmations, 23 were unitary patterns.

Relative to Set I and Set II across the eight tasks, the number of confirmations of the hypotheses are approximately the same, in particular, the confirmations relative to the strict hierarchy interpretation are identical. The patterns for the confirmations of the hypotheses relative to the various subsets for Set I and Set II are similiar, with the exception of the subset of involving length, area and volume relative to the continuous dimension.

2. Scalogram Analysis across Individuals Relative to the Hypothesized Ranking and the Empirical Ranking of the Tasks.

Green's (1956) method of scalogram analysis was utilized to assess the scalability of the eight tasks relative to a) the theoretical ranking of tasks (hypothesized hierarchy) and b) relative to the empirical ranking of tasks based upon their observed popularities for both Set I and Set II. Since Green's scalogram analysis utilizes dichotomous data, and our data was scored 1, 2 and 3 i.e., non-conservers, transitionals and conservers, the scores were reassigned to fit the dichotomous requirement of pass/fail. Since transitionals, Stage 2 responses could either be considered as conservers, Stage 3 or non-conservers, Stage 1, two separate analyses were performed. In one case, the 2's were treated as 1's, thus as failures. In the second case, the 2's were treated as 3's, thus passes. In Set I there were approximately 18% Stage 2 responses. In Set II there were approximately 17% Stage 2 responses.

Tables 16, 17, 18 and 19 depict the summary of results of Green's scalogram analysis for the theoretical and the empirical ranking of tasks for Set I and Set II under the two above mentioned conditions. The four tables depict the theoretical and empirical ranking of the eight tasks and the coefficient of reproducibility (Rep_A) and the index of consistency (I) for Set I and Set II, under each of the two conditions.

SUMMARY OF RESULTS UTILIZING GREEN'S SCALOGRAM ANALYSIS FOR THE THEORETICAL AND EMPIRICAL RANKING OF THE TASKS IN SETS I AND II

Table 16

Order	A	B	C	D	E	F	G	H	Rep_A	I
Theoretical	1	2	3	4	5	6	7	8	.86	.36*
Empirical	4	5	6	7	1	3	8	2	.89	.50

Set I 2's as 1's

*An index of consistency (I) above .50 is considered to be a good indication of scalability of the set of items. (Green, 1956)

Table 17

Order	A	B	C	D	E	F	G	H	Rep$_A$	I
Theoretical	1	2	3	4	5	6	7	8	.95	.58
Empirical	4	5	7	2	3	6	1	8	.96	.67

Set I 2's as 3's

Table 18

Order	A	B	C	D	E	F	G	H	Rep$_A$	I
Theoretical	1	2	3	4	5	6	7	8	.91	.53
Empirical	6	3	4	1	7	2	5	8	.93	.70

Set II 2's as 1's

Table 19

Order	A	B	C	D	E	F	G	H	Rep$_A$	I
Theoretical	1	2	3	4	5	6	7	8	.92	.53
Empirical	3	4	5	2	6	7	1	8	.93	.53

Set II 2's as 3's

Whereas the tasks under the theoretical ranking for Set I, 2's as 1's are not scalable (I = .36), the tasks under the theoretical ranking in both Set I and II under the other conditions suggests scalability. The empirical rankings based upon observed popularity suggest scalability under both conditions for Set I and Set II. It should be noted, however, the theoretical order of tasks differs from the empirical order under both conditions for Set I and Set II, and the empirical orders differ from one another under both conditions for both Set I and Set II.

3. Analysis of Spontaneous Conserver Explanations and Non-Conserver Centrations

Both Set I and Set II were qualitatively analyzed with an interest in examining the possible differential effects of the different tasks on types of spontaneous conserver explanations and possible differential centrations per task of non-conservers.

a. Conserver Explanations

Tables 20 and 21 depict the distribution of different types of spontaneous conserver explanations relative to each of the eight tasks of Set I and Set II. More than one response type may have been given by some children.

TYPES OF SPONTANEOUS CONSERVER EXPLANATIONS PER TASK

Table 20

Tasks	Identity	Comp.	Revers.	Counting	Gestalt	No. of Responses
A	7	6	3	4	0	20
B	3	3	5	6	0	17
C	5	5	3	8	0	21
D	7	1	1	8	6	23
E	7	3	2	6	6	24
F	9	8	1	6	0	24
G	4	2	2	6	3	17
H	10	6	1	3	0	20
Totals	52	34	18	47	15	166

Set I

Table 21

Tasks	Identity	Comp.	Revers.	Counting	Gestalt	No. of Responses
A	10	3	7	1	0	21
B	9	1	6	4	0	20
C	20	1	9	8	0	38
D	10	11	1	8	0	30
E	8	5	1	6	1	21
F	9	8	0	5	0	22
G	8	8	0	5	2	23
H	3	9	3	0	0	15
Totals	67	46	27	37	3	190

Set II

Along with the typical identity, compensation, reversibility and explanations, a category for explanations relative to counting and one relative to an appeal to Gestalt type configurations reflected the data. Considering the spontaneous explanations of identity, compensation and reversibility only in Set I for tasks A and C, the one-dimensional tasks, there is a higher percentage of both the identity and compensation types of explanation than of the reversibility type of explanation although this pattern is reversed in task B. For tasks D and E, the two-dimensional tasks and for tasks F, G and H, the three-dimensional tasks, there is a consistent rank order preference for identity, compensation and reversibility with identity being the most preferred type and reversibility being the least preferred type.

Considering the same explanations for Set II for tasks A, B and C, the order is also consistent but slightly different with identity the most preferred explanation, next reversibility and finally compensation. For tasks D and E, and tasks F, G and H, with the exception of task H, the pattern is similar to that of those tasks in Set I, in other words, identity is most preferred, next compensation and finally reversibility.

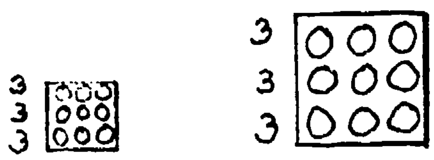
Considering the identity, compensation and reversibility explanations across the eight tasks of Set I and Set II respectively, there is a consistent pattern of explanation preference with identity the most preferred type, next compensation, and finally reversibility. This pattern seems largely to be a function of the two and three-dimensional tasks for both Set I and Set II.

As might be expected, a higher number of compensation explanations are consistently utilized for the two and three-dimensional tasks. However, the overwhelming explanation preference across all tasks of both Set I and Set II is the identity explanation.

Finally, relative to the two remaining categories, counting and Gestalt configurations, there is an extremely high proportion of explanations for conservation by counting i.e., "I know they're the same because I counted." And in answer to the question, "How can you prove that we have the same number?" the child says, "I can count them." Finally in Set I there is a fairly strong concentration of configurational explanations* for tasks D and E and a somewhat smaller number of configurational explanations for task G. In Set II, although not in task D, task E and G also reflect this type of explanation although in a somewhat reduced amount.

---

*Configurational explanations appealed to the structure of the two sets part to whole in each. For example, in task D children said, 3x3x3 here and 3x3x3 here, or 3 and 3 and 3, and 3 and 3 and 3.



In order to see if there was a preferential use of one type of explanation by individual conservers across tasks, the original data sheets were scanned. There appeared no perseverence whatsoever in the conservers explanation preference. These results further support our findings on task difficulty. Just as task difficulty appears to be a solely individual phenomenon, so too, relative to conservers, does explanation preference per task.

b. Centrations of Non-Conservers

The relevant spatial dimensions of each task were noted for Set I and Set II and non-conserver explanations (centrations) were categorized accordingly. Tables 22 and 23 depict the centrations of non-conservers relative to the relevant spatial dimensions of each task. Although the three dimensional tasks F, G and H for Set I and Set II suggest width, height, and depth as relevant attributes none of the children referred to depth as a reason for non-conservation, thus depth was not included in the tables.

NUMBER AND TYPE OF CENTRATIONS OF NON-CONSERVERS: SET I, II ON THE EIGHT TASKS

Table 22

Task	Length	Density	Size	N
A	5	0	-	5
B	3	0	2	5
C	5	-	2	7
Task	Width	Height	Size	N
D	0	0	1	1
E	0	0	3	3
Task	Width	Height	Size	N
F	0	3	3	6
G	0	1	2	3
H	0	1	3	4

Set I

Table 23

Task	Length	Height	"Shape"	N
A	8	-	-	8
B	4	2	0	6
C	3	0	1	4
Task	Width	Height	"Shape"	N
D	1	3	0	4
E	4	3	0	7
Task	Width	Height	"Shape"	N
F	3	6	0	7
G	3	9	2	14
H	4	12	0	16

Set II

While a small number of children centered on height (the vertical dimension) in task B only one child centered on shape in task C, apparently referring to the difference in dimensions between the two collections. For tasks D and E, the two-dimensional tasks, the centrations are distributed between width (length) and height with more centrations on height in task D and almost the same number of centrations on height and width in task E. No children centered on shape. Tasks F, G and H, the three-dimensional tasks, much resembled tasks D and E in being distributed between centrations on width and height. However, there is complete consistency here in a much higher proportion of centrations on height rather than width. A very small number of children in task G centered on shape.

Considering comparable tasks across Set I and Set II, tasks A, B and C, the length tasks, yield a much higher number of centrations on length than density, size, height or shape. Tasks D and E across Set I and Set II yield somewhat unsimilar results. In Set I where there are no centrations on width or height and centrations relate solely to the set variable, size in Set II tasks D and E yield no centrations on the set variable, "shape" and are distributed between width and height. For tasks F, G and H in Set I where the centrations are exclusively on size and height, in Set II for tasks F, G and H, the centrations are almost exclusively on width and height with very few centrations on "shape." Considering all eight tasks for Set I, there is a fairly consistent occurrence of centrations on size, the set dimension. For Set II, however, there are very few centrations on shape, the set dimension.

D. Discussion

1. Hierarchy Hypothesis

In line with Elkind's position (Chapter 2, 1971) that the Piagetian developmental approach and the psychometric approach to intelligence or intellectual development may be viewed as complementary and not contradictory, the hierarchy hypothesis about the development of number conservation and the scalogram analysis relative to the number conservation tasks will be discussed.

What seems clear from the data relative to the hierarchy hypothesis of the development of number conservation is that within any individual across the eight tasks, the strict hierarchy interpretation (">" rather "=") is confirmed in few instances in Set I and II. While a unitary pattern across the eight tasks is confirmed for considerably more individuals, there is, thus, no evidence for any general order of acquisition of these tasks across the children. This fact becomes more evident when we examine subsets of the eight tasks, length, area, volume, and discrete versus continuous. As the number of confirmations increase so do the unitary patterns. While the lack of confirmation of the hierarchy hypothesis within individuals does not suggest any general order of development, support for the developmental nature of number conservation is evident. Few nursery school and kindergarten children conserve on all eight tasks while many first graders and almost all second graders conserve on all the tasks. The order of acquisition, however, appears relative to particular individuals.

One notes that the number of confirmations of hypothesis increases as we move from the eight tasks considered as a unit to the various subsets of the tasks. The number of confirmations in various subsets of the eight tasks are particularly significant when one considers the nursery school and kindergarten children. Clearly, if one expects a general order of development, then such an order would manifest itself during this period. However, the increasing number of confirmations of the hypothesis, relative to these subjects are primarily those of unitary patterns of success on the tasks with the number of strict hierarchical order confirmations remaining essentially the same as the number of hierarchical patterns across all eight tasks. This pattern holds with the exception of the length, area, and volume subset, relative to the continuous tasks, Set II, which evidences considerably more confirming instances of the strict hierarchy, but nevertheless considerably fewer than the confirmations by a unitary pattern.

The generally consistent occurrence of unitary patterns in the subsets relative to the nursery school and kindergarten children provides more evidence that the tasks are not dependent upon each other for acquisition.

While the developmental approach suggests that number conservation develops over time and that the order is relative to each individual, from the psychometric or statistical point of view, relative to the tasks, this suggests that variations in responses across the tasks within an individual are unique or from the point of a group of individuals are random or attributable to chance. Thus we may expect that across the group of individuals no one discernable order should be evident. Or, in other words, the tasks are not "generally" scalable for **any** one order of difficulty.

Green's scalogram analysis of the data utilizing both the theoretical ranking as well as the empirical ranking of tasks confirms the scalability of the items in all instances except one. In other words, in the theoretical ranking as well as the empirical ranking, both different rankings were found to be scalable. While at first glance such findings seem contradictory with the findings on individual instances of confirmations, they suggest that either order is scalable for that particular population. Or in other words, no one general ranking of difficulty is evident. In support of these findings, Goldschmid and Bentler (1968) in their discussion of the rank orders of task difficulty relative to conservation, state, "The rank orders of difficulties of various tasks are not identical from scale to scale, or from sample to sample, so that conclusions regarding relative difficulty of tasks cannot be drawn from the data. There appears to be no one sequence of task difficulties, making it impossible to establish the Guttman sequence." (p. 801)

The finding relative to no general order or scalability of Piagetian items is also in agreement with Tuddenbaum (p. 29, 1971) who states: Within a given stage we fail to discover smooth ordinal scales . . . Instead we find "declages" or as a statistician would say relative independence of different cognitive tasks. And further, Tuddenbaum (p. 80, 1971) relative to his data states: "I think what our data may show is that before they become consistent across tasks, the task that a particular child is able to master doesn't give you much notion of whether or not he will be able to master other ones. In the stages where they are just getting these things, it seems very much a matter of chance which ones they get first. And then later, presumably, they have them all."

The rejection of the hypothesis utilizing the developmental approach suggests that the development of number conservation does not conform to a general ordinal dimension, which further suggests that the tasks across groups of individuals would not conform to one general rank order of difficulty which was confirmed empirically. Thus from the developmental approach (the analysis of individual orders), the data suggests that the development of number conservation, relative to our given tasks, is not ordinal in nature, and from the psychometric point of view, the data also suggests that the tasks do not conform to any one rank order of difficulty. In other words, the developmental approach and the psychometric approach complement each other relative to the data.

While the data has suggested that the development of number conservation does not conform to an ordinal dimension, the question now arises whether the development of the various stages of intellect, sensori-motor, pre-operational, concrete-operational and formal operation conform to an ordinal dimension.

While Nivette (p. 55, 1971) and Evans (p. 61, 1971) seem to believe that the stages of development differ qualitatively and hence do not conform to an ordinal scale, Tuddenbaum (p. 74, 1971) and Beilin (P. 186, 1971) seem to believe that the stages of development manifest qualitative as well as quantitative differences.

However, our data relative to the hierarchial development of number conservation does not support the notion of an ordinal dimension. While it may be suggested that the results of our study do not indicate an ordinal dimension relative to the development of number conservation across the dimensions of length, area and volume, and thus indicate the possibility that the development of the various conservations, length, area and volume do not conform to an ordinal dimension, our study has focused on children at particular ages rather than particular children across age. Both Nivette (1971) and Tuddenbaum (1970), and Goldschmid and Bentler (1968), sensitive to this problem, indicate the need for longitudinal research to demonstrate whether Piaget's hierarchy of stages conform to an ordinal dimension.

2. Method of Questioning as a Variable in the Testing for Item Scalability

Tuddenbaum (1971), Nivette (1971), and several others have suggested that while the clinical method of questioning may be appropriate for the development of theory, it is too non-standardized, not rigorous enough, and too qualitative, as well as, unsuitable for the construction of intelligence scales which indicate individual differences among children.

Relative to the construction of intelligence scales based on Piagetian tasks, Tuddenbaum states (1971, p. 75): "A crucial consideration is whether or not our items assess the cognitive structures which the original experiments were intended to demonstrate. I think they do." Relative to the training studies, Mermelstein and Meyer (1969), Mermelstein (et al) (1967) contended that the successes reported relative to the training of number conservation were a function of the different criteria utilized to measure the presence of conservation. It was argued that the criteria utilized by the investigators did not emerge from Piagetian theory and consequently, the "conservation" measured was not the Piagetian type of conservation.

Equally as potent as establishing criteria derived from Piagetian theory in establishing the presence of a particular Piagetian concept is the mode of assessment one utilizes. What seems clear is that if we are to speak of a Piagetian concept, we must utilize the same mode of assessment. Simply put, changing the mode of assessment changes the concept. Mermelstein has argued elsewhere on both a theoretical as well as an empirical basis for the advantage of utilizing the clinical method of questioning when assessing the intellectual development of the child. Mermelstein and Shulman (1967); Mermelstein (1967).

Tuddenbaum (P.66, 1971) contends, however, that when one's purpose is not to substantiate theory but to compare different children under identical conditions, the method of inquiry must not introduce variability and a standardized approach with a reduction of language seems appropriate.

It is our contention that by attempting to control for the apparent variability, hence "error," introduced by the clinical method of questioning, Tuddenbaum has introduced other sources of variance which may be more significant than the one for which he is attempting to control. Tuddenbaum suggests that the utilization of the clinical method implies a qualitative judgment, from which a numerical score is given, but he fails to acknowledge that the standardized approach even with its uniform questions also requires a qualitative judgment from which a numerical score is assigned. Even though in the standard approach the use of non-verbal responses in the form of the child's activities is often employed, still a qualitative judgment relative to the child's actions must be made and from that a quantitative score is assigned. Thus both modes of assessment initially rely on qualitative judgments. However, the clinical method of questioning by virtue of the rephrasing of questions may be viewed as taking into account individual variation among children while the standardized approach does not. The need for eliminating variation as a means of control may well suggest standardization when the variation is completely unknowable, but to the extent that variations are predictable, it seems a better form of control is by accounting for the variation through systematic exploration of it.

Further, a standardized approach which attempts to check on the child's understanding of directions while serving as a control for consistency of responses, may equally serve the purpose of teaching the child to respond the way the experimenter wishes to have him respond and not the way he would naturally respond.

Thus, in our opinion, the use of the standardized approach in assessing the intellectual development of children on Piagetian tasks may introduce more variability than the use of the clinical method.

3. Types of Conserver Explanations

The preponderence of use of the identity explanation by conservers across all eight tasks of Set I and Set II may be interpreted within Piagetian theory. Use of the identity explanation reflects an understanding that certain operations or activities on a collection of objects do not modify or alter the collection of objects relative to a particular property--in this case number. One could speak of such activities as null activities since they have no consequence on the property in question. The identity operation seems to be a direct extension of object-identity or permanence which is mastered in the preceeding stage of development. It's the invariance under a set of actions in the preoperational stage that the object acquires its existence and permanence. It is the invariance under a set of operations in the concrete operational stage that number achieves its existence and permanence. In each case a form of conservation develops out of the recognition that certain operations or activities on objects do not alter a basic property which remains identical before and after the transformation--in the first case, the object itself, in the second, the number of objects.

The greater use of the identity explanation over the compensation explanation, even in the two and three dimensional tasks, seems to support the notion of the primacy of activity over perception in the development of operational logic. Even while the compensation explanation reflects knowledge of the results of activities, as well as of perceptual states, it appears to be a less basic understanding than simply that any action performed on a collection (whatsoever) except for addition and subtraction doesn't change the number in the collection of objects. In one sense, this explanation is both more general and more vague than that of compensation or even reversibility. All activities other than addition and subtraction are irrelevant but no concern is given to the particular results of these actions as is the case in the compensation explanation.

Although the data is nowhere near conclusive, it might be suggested that the fairly consistent order of identity, compensation, and reversibility across Set I and Set II may reflect a developmental trend in conserver explanations. On the other hand, one may add a note of caution that certain explanations seem linked to certain forms of questioning. For example, children tend to use the reversibility explanation as a "proof" argument and the identity explanation as an answer to the question, "How do you 'know' we have the same number?"

A final suggestion follows from the high number of counting explanations across Set I and Set II and the concentrations of Gestalt type configuration explanations for tasks D, E and G. The strong reliance on counting techniques as an explanation for conservation may suggest at most the possibility that visual correspondence is not a universal means by which children establish equality among two sets, at least, the possibility that the relationship between counting and conservation may be more significant than was previously believed.

The fact that a fairly consistent concentration of Gestalt configurational explanations were elicited by tasks D, E and G in Set I may have contributed to their high empirical rankings. Configurational explanations tended to relate parts to whole in each of the two collections, and since shape was constant, it perhaps became a heuristic method for establishing a form of pictorial-numerical correspondence. "This one has 3 and 3 and 3, and this one has 3 and 3 and 3." In the equivalent tasks of Set II, it was necessary to imagine rotation of the configuration before such a correspondence could be established, but in a few cases, children still utilized this heuristic method.

It is not clear whether conservation of this sort is a function of child or task. Since the cases were not overwhelming, it seems more a function of individual children than task. Yet, since the occurrences were concentrated on a few tasks only, and the same tasks for Set I and Set II, it seems quite possible that the tasks themselves were eliciting these certain responses in certain children.

4. Types of Non-Conserver Centrations

Relative to one, two and three dimensional tasks, types of centrations differed little between the two and three dimensional tasks.

Children seem to disregard the dimension of depth at this age focusing only on differing aspects of height, width, or size. In the one dimensional tasks, children centered on length vs. the other aspect, density. If size may be considered a global way of speaking of either length or height or width, or in the two and three dimensional tasks, both height and width, then the data consistently reflect the Genevan suggestions that centrations on aspects such as length, height, width and size reflect the characteristic of perception to focus on the boundaries of a collection in reference to another collection, i.e. centering on the increase of length in one collection over the other, or the increase in height or width.

It is interesting to note that "territory covered" for the one dimensional tasks, whether the collections are in a vertical or horizontal direction elicits centrations almost exclusively on the horizontal dimension, whereas in the two and three dimensional tasks where each collection covers territory in both a vertical and horizontal direction there seems to be much more of a trend to focus on the vertical dimension. This is particulary true in the three dimensional tasks. One might suggest that general experience has "taught" one to judge amount by the terminal boundary (upper limit) of a container.

These findings seem quite consistent with traditional findings of centrations of tasks involving conservations of length, area and volume, and suggests a possibility that centrations in these tasks on number conservation with length, area and volume dimensions, do in fact reflect the centrations of older children in tasks of length, area and volume conservation.

5. Implications for Education

The general lack of support for the hypothesized hierarchical order of tasks within individuals, across all eight tasks as well as the various subsets, and the findings from the scalogram analyses that very different orders of the tasks "appeared" scalable, suggests that no one order of acquisition of development of number conservation is discernable across different children. The general trend from non-conservation in all the tasks at age 5 to full conservation of all the tasks at age 7, however, does suggest that the number conservation concept as applied to collections of objects whose configurations vary in length, area and volume, respectively, does have a developmental period of approximately one to two years.

The absence of any general order of development relative to the given number conservation tasks supports the general experience argument espoused in our earlier studies (Mermelstein and Shulman, 1967; Mermelstein, et al, 1967; and Mermelstein and Meyer, 1969). More specifically, the present findings support and articulate our earlier contention that if the child constructs his own reality then it does not follow that he will assimilate information in the "order" presented to him. (Mermelstein and Meyer, 1969). These findings also support the findings of Meyer and Mermelstein (1968) who demonstrated in a training study that relative to the prescribed order of acquisition in addition to three random orders, there were no significant differences in acquisition of number conservation relative to the various orders of presentation of tasks.

The facts that no one order of acquisition seems to describe the development of successive number conservation tasks suggests two relatable conclusions. First, there is the implication that variations on these tasks are a function of individual differences in children, not of any apriori order of difficulty of task attributes, such as length, area and volume. Presenting conservation tasks to children in any prescribed logical order, with the belief that they will assimilate them in that order seems questionable as a means of training or teaching. It seems rather that teaching should concentrate more on individual differences among children. Second, if as Piaget suggests, general experience is a significant factor in the development of operational logic, then, it seems one might suggest providing a wide range of experiences to children--particularly experiences where children can manipulate, operate on and observe the results of those activities.

As suggestions for curriculum modification the direction we are indicating is towards more variation of experiences to be offered to a class of children so that they may sort out their own strengths and weaknesses. Games that involve manipulating objects with length, area and volume properties, making, and comparing collections, unmaking them and comparing the number of elements seem to be much in need for this sort of development to occur.

6. Conclusions

It is concluded that relative to number conservation tasks involving length, area and volume, both the constructed probability distribution analysis and the scalogram analysis suggest that the order of acquisition of these tasks is individual.

References

Berlin, H. Developmental stages and developmental processes. In Green, Ford and Flamer (Eds.), Measurement and Piaget. New York: McGraw Hill, 1971.

Campbell, D. Objectivity and the social locus of scientific knowledge. Presidential Address to the Division of Social and Personality Psychology of American Psychological Association, Washington, D. C., September 1969.

Conant, J. Science and common sense. Yale University Press, 1951.

D'Mello, Sydney & Willemson, Eleanor. The development of the number concept: a scalogram analysis. Child Development, 1969, 40, pp. 681-688.

Elkind, D. Piaget's conservation problems. Child Development, 1967, 38, pp. 15-27.

Elkind, D. Two approaches to intelligence: Piagetian and psychometric. In Green, Ford and Flamer (Eds.), Measurement and Piaget. New York: McGraw Hill, 1971.

Evans, Glen. Open discussion: implications for assessment of cognitive growth. In Green, Ford and Flamer (Eds.), Measurement and Piaget. New York: McGraw Hill, 1971.

Goldschmid, M. L., & Bentler, P. M. The dimensions and measurement of conservation. Child Development, 1968, pp. 787-802.

Gottfried, Nathan. The relationship between concepts of conservation of length and number. Journal of Genetic Psychology, 1969, 114 (1), pp. 85-91.

Green, B. A method of scalogram analysis using summary statistics. Psychometrica, 1956, 21, pp. 79-88.

Griffiths, J. A., Shantz, C. A. & Sigel, I. E. A methodological problem in conservation studies: the use of relational terms. Child Development, 1967, 38, pp. 841-848.

Gruen, G. E. Note on conservation: methodological and definitional considerations. Child Development, 1966, 37, pp. 977-983.

Halford, G. S., & Fullerton, T. J. A discrimination task which induces conservation of number. Child Development, 1970, 41, pp. 205-213.

Inhelder, B. The diagnosis of reasoning in the mentally retarded. New York: The John Day Co., 1960 (first published in French, 1943).

Johnson, C. Some hypotheses about the null. Paper presented at Session 72, Statistical Method and Theory, Annual Meeting of the American Educational Research Association, New York, February, 1967.

Kline, M.. Geometry, mathematics in the modern world. Scientific American, September 1960.

Kuhn, T. The structure of scientific revolutions. Chicago: University of Chicago Press, 1962.

Lovell, K. The growth of basic mathematical and scientific concepts in children. New York: Philosophical Library, Inc., 1962.

Mermelstein, Egon. A note on Piaget's clinical method of questioning. Alberta Journal of Educational Research, 1967, 13.

Mermelstein, Egon; Carr, E.; Mills, D. & Schwartz, J. Training techniques for the concept of conservation. Alberta Journal of Educational Research, 1967, 13, pp. 185-200.

Mermelstein, Egon & Meyer, E. Conservation training techniques and their effects on different populations. Child Development, 1969, 40, pp. 471-490.

Mermelstein, Egon & Shulman, L. Lack of formal schooling and acquisition of conservation. Child Development, 1967, 38, pp. 39-52.

Meyer, E. & Mermelstein, E. Sequence and acquisition of conservation. Paper presented to American Educational Research Association, Chicago, February, 1968.

Nivette, J. Comments on Bentler's paper. In Green, Ford and Flamer (Eds.), Measurement and Piaget. New York: McGraw Hill, 1971.

Peters, Donald L. Task variations and individual differences in Piaget's conservation of number. Merril-Palmer Quarterly, 1967, 13 (4), pp. 295-308.

Peters, Donald. Verbal mediators and cue discrimination in the transition from non-conservation to conservation of number. Child Development, 1970, 41 (3), pp. 702-722.

Potter, Mary C. & Levy, Ellen I. Spatial enumeration without counting. Child Development, 1968, 39 (1), pp. 265-272.

Piaget, J.; Inhelder, B; & Szeminska, A. The child's conception of geometry. London: Routledge and Kegan Paul, 1960.

Pratoomraj, S. & Johnson, R. Kinds of questions and types of conservation tasks as related to children's conservation responses. Child Development, 1966, 37, pp. 343-353.

Rothenberg, Barbara. Conservation of number among four and five year old children: some methodological considerations. Child Development, 1969, 40 (2), pp. 383-406.

Rothenberg, B. & Orost, Jean H. The training of conservation of number in young children. Child Development, 1969, 40 (3), pp. 707-726.

Schwartz, Marilyn Miller, & Scholnick, Ellen Kofsy. Scalogram analysis of logical and perceptual components of conservation of discontinuous quantity. Child Development, 1970, 41 (3), pp. 695-706.

Siegelman, Ellen & Block, Jack. Two parallel scalable sets of Piagetian tasks. Child Development, 1969, 40 (3), pp. 951-956.

Tuddenham, R. Theoretical regularities and individual idiosyncrasies. In Green, Ford, and Flamer (Eds.), Measurement and Piaget. New York: McGraw Hill, 1971.

Smedslund, J. Concrete reasoning: a study of intellectual development. Monographs of the Society for Research in Child Development, 1964, 29 (2), Serial No. 93.

Smedslund, J. Microanalysis of concrete reasoning, I: The difficulty of some combinations of addition and subtraction of one unit, and II: The effect of number transformations and non-redundant elements and some variations in procedure. Scandanavian Journal of Psychology, 1966, 7, pp. 145-156, (a).

Wohlwill, J. F. A study of the development of the number concept by scalogram analysis. Journal of Genetic Psychology, 1960, 97, pp. 345-377.