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ABSTRACT

These three pamphlets by the Department of Education in Victoria, Australia, provide an overview of the elementary mathematics curriculum which is described in greater detail in SE 012 723 to 729. The aims and philosophy, and the content and structure of the course are outlined, and methods of classroom organization to allow for individual differences are discussed. The majority of the publications consists of an annotated listing of the major topics to be covered in each section of the course. Appendices summarize the basic themes emphasized in the curriculum. (MM)

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EDUCATION DEPARTMENT, VICTORIA

COURSE OF STUDY

FOR

PRIMARY SCHOOLS

MATHEMATICS, 1964

4299/69

INTRODUCTION

Aims

Mathematics, the language of order and size, is a vehicle of expression and a mode of thinking about the world as well as a useful tool for handling the varying quantitative situations of everyday life.

The narrower utilitarian training in the necessary skills and techniques of computation should contribute to this broader end, rather than become an end in itself. Approached from this viewpoint, the subject can and should make a valuable contribution to the child's education in its widest sense.

Stated briefly, the aims in the primary school should be:

(a) to foster and develop a clear understanding of the fundamental concepts of magnitude, order, time, and space;

(b) to develop speed and accuracy in the required computational skills;

(c) to enable the child to use these understandings and skills to think soundly about and deal effectively with the quantitative situations that arise in his expanding personal and social experience.

In attempting to achieve these aims the teacher must preserve the initial enthusiasm of the child by cultivating his interest and by permitting him to experience the pleasure and the satisfaction that the subject can provide.

The Importance of Understanding

The aims outlined above emphasize that the first and the most essential requirement is a thorough understanding of the concepts studied. To the extent to which the child has progressed in mathematics and within the limits of the child's intellectual development, each idea must be thoroughly understood before the pupil is asked to proceed to one more advanced. This is not to deny that understanding will become deeper and wider as the ideas are employed to attack problems and more advanced ideas.

Thus the development of mathematical thought can be improved by placing greater emphasis, in our teaching methods, on induction and abstraction rather than on deduction. The child needs adequate opportunity for experience and experimenting to obtain the necessary background for the principles and the generalizations required for efficient and economical mathematical thought and practice.

The nature of society today and its probable development in the future indicate that both children and adults will need a fuller understanding of mathematical ideas. The demand for mathematicians and the importance of mathematics in the sciences suggest that not only must our secondary schools produce students with greater mathematical skill, but they must also produce more students who have chosen to continue the study of mathematics. It is the task of the primary school to prepare children by preserving and developing their interest in and love of the subject and by providing them with the opportunity to fully understand mathematical ideas.

While mathematical ideas originate in the contemplation of concrete materials, and the findings of mathematics are often applied to practical situations or used to solve problems, it must not be forgotten that the abstracting of mathematical ideas may often be prevented by the tendency to limit the study of mathematics to concrete materials and real-life situations. This point is taken into account in the design of the course by providing two broad sections: **Pure Number and Applied Number.**

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Mathematical understanding and computational skills are not necessarily achieved together. In fact it is possible to have sound mathematical understanding with limited computational skill, and it is also possible to obtain a high degree of computational skill with little understanding of the ideas that are being manipulated. The aims of this course are to develop both understanding and computational skill, but the course is structured on the premise that understanding must first be soundly established and on this understanding a gradually increasing skill in computation will be built. Thus in assessing the results of the child's learning teachers must realize that "getting the right answer" may indicate only some facility with computation which may be achieved simply by rote learning. To discover whether the child understands or not, the teacher must question, probe, and discuss.

Individual Differences

Once understanding is accepted as being important and the controlling factor in determining the speed of progressing through mathematical ideas, teaching must be organized to allow time for understanding to develop. Not only does this involve the provision of many experiences and activities through which understanding may grow, but it requires a system of class-room organization that recognizes that understanding will be achieved by different children in different ways and at different speeds.

No longer can it be accepted that a certain level can be achieved at the same time by all children irrespective of ability. This course outlines a certain progression of mathematical ideas and skills. Where age or grade levels are indicated, or where it is suggested that a certain time is spent on any stage, it is accepted that this might be the standard achieved by a child of average ability. The more capable children will not only progress faster and further but will also be able to make a wider application of their understanding, and their work will have greater richness and variety. On the other hand, the less capable children will move more slowly and because their depth of understanding is less they will not achieve the same quality or breadth of mathematical appreciation.

In principle this course is an "ungraded" course, a ladder of graduated steps to be mastered at the rate at which understanding can be achieved by each child. To progress faster than at this rate means that understanding becomes less, and the child is then forced to rely on rote learning and the achieving of a mere mechanical skill.

From the earliest days at school children are not only different in intelligence but are different in the range and the type of mathematical experiences they have received. Many will have an apparent facility with numbers while others will have very limited mathematical ideas. It is important that, in the first school year, the teacher searches very carefully to establish what mathematical ideas are in fact understood clearly and provides for each child those experiences necessary to complement or build up the previously learned mathematical ideas.

While providing for individual rates of learning presents to the teachers many problems in class-room organization, and the emphasis on understanding demands a more thoughtful approach to testing, the results in children's improved learning and attitude are an adequate reward. Rejection of the belief that all children can reach the one level in any given year removes from teachers the pressures to force children to achieve an apparent artificial result and enables all children to achieve success at a level appropriate to their ability.

Principles of Method

The emphasis noted above on both understanding and individual differences indicates that these considerations are the controlling factors in determining the teaching method employed.

Although the class lesson is still one important form of presentation, and much valuable work may be done by letting the class work together on occasions to enable the brighter children to give the lead to others, the class will for most of the time work in groups based on ability and progress achieved. This form of organization does not imply that brighter children are allowed to coast at half pace, or that slower children will be continually harried to achieve some artificial class standard. Rather it insists that each child should be working to the maximum of his ability, at a level where he can gain sufficient experience in success to build up his confidence to attack new and challenging problems.

This approach places greater responsibility on the teacher, who will be required to work at a number of grade levels, some of these levels being higher and some lower than the nominal grade level of the class he is teaching. While progress can be planned on this basis, and experience can indicate the spread of abilities likely to be met, nevertheless each class will present its own problems. The organization of the school must be such as to allow children to move from grade level to grade level, and in the new grade to carry on from the level reached in the previous grade.

Again the emphasis on understanding and individual differences implies that testing techniques must be carefully considered. No longer can one single test of mathematical calculations be considered adequate. Several tests at different levels will be required to cater for the several different levels of achievement, and tests for "the right answer" must be supplemented by more verbal and individual tests that probe for the understanding of mathematical ideas. The results of such tests, together with the teacher's observation, should be recorded so that a picture of each child's progress may be built up.

No longer should a child be considered to be a complete failure at any grade level; rather each child should be considered to have made sound progress at an appropriate grade level. This does not ignore the fact that there are times when, for administrative purposes such as the transfer from Grade VI, a child must be compared with all other children in the same grade. It merely insists that, for teaching purposes, testing should assess what has been achieved and what is yet to be done, and it must be based soundly on what has in fact been taught.

Thus the class-room organization will provide for groups working at different levels, for groups doing different types of work, and for children producing work which, though different in quality, is consistent with the child's ability.

This course must not be interpreted rigidly. One stage need not in fact be fully mastered before other stages are commenced. Although certain stages need to be understood before the child is asked to attack more advanced ideas, it is not always necessary for full mastery to appear. There must be overlapping and parallel development, and new ideas must be developed as the child shows his readiness for them, irrespective of the teacher's initial planning.

Progress in learning mathematical ideas is not a steady, regular progress. While a child is reaching out for a new concept there will often be a period where there is no apparent progress. This is the time when the new idea should be presented in many different ways and in many different situations. During this period the child is organizing the idea in his mind, and this is usually followed by a flash of insight or understanding which results in a leap forward to the next difficulty. This progress will appear to be irregular. The teacher must exercise patience and plan carefully in the periods of apparent lack of progress, and act quickly to take advantage of this insight when it appears.

In presenting mathematical ideas to children, teachers should use all and any material that will help to clarify the ideas in the child's mind. In the development of ordinal work through counting, many types of material and devices will be used, with the concrete materials being continually replaced by abstract thought. Although in presenting the pure number section of the course many teachers will use one particular type of material, it must be remembered that the other materials may have an important use and that the material has but one purpose: to present mathematical ideas so that they may be understood with sufficient clarity to enable the child to handle them in the abstract. The Cuisenaire method and material have been successfully adopted by many teachers, and for their benefit a **Curriculum Guide** is available. It should be recognized, however, that this guide considers not only the Cuisenaire method, but incorporates, where it is appropriate, excellent techniques which have been used by teachers for many years. Other materials are also available: Colour Factor Material; Stern; Shaw (with "Structa" apparatus); Jones (with "Avon" apparatus); Montessori; Dienes (Multi Base Arithmetic Blocks); Lowenfeld (Poleidoblocs); and the Unifex material. Most of these materials employ "number pieces" depicting numbers 1 to 10. Some use colours merely to differentiate pieces; some do not use colour; while others place great emphasis on the use of colour. All these systems enable mathematical ideas to be introduced earlier than has been traditional practice; some give special emphasis to certain aspects of mathematics; while others simply claim to teach "conventional" mathematics in a new way. There are, however, similarities in theoretical viewpoints:

1. Children should learn not only to perform arithmetical and mathematical operations, but also to understand these operations.
2. Structured material affords a more systematic treatment of mathematics than "real life" situations can provide.
3. The material can be used as a diagnostic tool.
4. The representation of numbers by "number pieces" reduces the likelihood of the child's doing the operations by counting.
5. Children learn by discovery, and the material makes this kind of learning easier because it is "self corrective".
6. Learning should take place at the child's own pace.
7. The kind of activity engaged in when learning through insight and through discovery is intrinsically motivating.
8. In most systems the learning process is analysed into stages, in the belief that before a concept is formulated its components should be learned and that a period of practice should follow its formulation.

(For descriptions of these materials see *Education Research*—The National Foundation for Educational Research in England and Wales—Volume 4, No. 3—June 1962.)

The design of this course is strongly influenced by the philosophy of the Cuisenaire method. This does not imply any criticism of other materials, but accepts the fact that, after a number of years of experience with the Cuisenaire method, a high degree of improvement in mathematical understanding has been achieved, and techniques suitable for Victorian primary schools have been developed. Teachers will find that they can successfully achieve the requirements of the **pure number** section of this course by using the Cuisenaire material, the literature published by the Cuisenaire Company, and the **Curriculum Guide** prepared by the Victorian Education Department. Schools or departments of schools are equally free to use other materials provided they can achieve the same degree of success and subject to the demands for some degree of uniformity within each individual school.

In the **applied number** section of the course, concepts concerning such things as length, weight, time, capacity, movement, and direction are developed. The experiences and the materials used to develop these ideas are those that adequately demonstrate the concept. Therefore the emphasis is on much practical work in measuring, weighing, etc., first with informal units, and later with conventional units. The practical work should continue until the mathematical relationships between the units of measure are clearly understood by the child, and are handled capably and with confidence. Only after these relationships are firmly established should there be any attempts to link these ideas formally with the pure number work and to proceed to calculation within this field.

The key to efficient learning in this course is not to "teach", if by this we mean the filling of the minds of children with factual knowledge based on rote learning. Rather, "teaching" is achieved by forcing the child to think, to express his ideas, to draw conclusions, and to use these in mathematical expressions. The teacher should not simply tell, but should continually ask questions, ask "how", ask "why". This process of "leading children to learn" demands from the teacher more flexible planning, adaptability, and a sound knowledge of the subject.

At no stage in this course is the term "drill" advocated, if by this is meant the repetition of ideas, not understood, but eventually retained simply by the mere number of repetitions. If the term "drill" is used, it refers to the process where ideas, first clearly understood, are repeated in valid situations, in solving equations or problems. Rather than giving mere drill, the teacher is required to construct situations where the child will meet the particular ideas to be memorized in a number of different ways. Therefore automatic response in number facts will emerge gradually. During the early years, where the chief aim is to develop understanding, the response will be slow and may need to be supported by reference to the particular material used. As understanding develops and the ability to work in the abstract becomes evident, the response will increase in speed and become automatic with a minimum of drill.

It should be continually kept in mind that, at all times, whether the aim is understanding or automatic response, the child should be faced with situations that are mathematically valid and that challenge the intellect.

The Structure of the Course

Although in many respects it is artificial to divide the development of mathematical thought into separate compartments, it is convenient in presenting a course to group together certain aspects of mathematical ideas for the convenience of teachers. Thus this course has two broad divisions—**pure number** and **applied number.**

The **pure number** section deals with two basic concepts: ordinal number and cardinal number.

Ordinal number is concerned with the number sequence and encompasses those ideas normally described as counting and numeration and notation.

Cardinal number refers to the idea of a number as a whole or group, and leads to the study of the mathematical equation and the four operations within the equation. In fact, ordinal and cardinal ideas are complementary, and each is necessary for the understanding of the other. They are separated mainly because different types of material are used to demonstrate the ideas.

Ordinal number is best taught by using a material consisting of single units—beads, counters, chairs, or tables.

Cardinal number is best taught by a material specially structured to demonstrate the mathematical relationships of wholes. It is in this section of the work that number rods such as the Cuisenaire material have an important place.

The **applied number** section of the course relates to the practical experience work in studying the spatial relations in materials within the environment. This course demands a continuous program of practical experience before computational work is required.

NOTE.—For immediate consideration we are concerned only with the **pure number** section of the course and its revision from the beginning of the primary school to approximately the end of Grade III, which should be achieved by the average child of about nine years of age.

PURE NUMBER

SECTION A—INTRODUCTORY SECTION

This section of the course is designed to be mastered by the child before he proceeds to study the basic ideas of equality, addition, multiplication, division, and subtraction. The average child should be ready for these basic ideas at the age of approximately six years, which normally should be reached early in the Grade I year. The time spent on this introductory section will therefore depend on the time spent in the Preparatory Grade, the intelligence of the child, and the experiences he has had before starting school.

Although through their early life at home and early school experiences most children will have begun to learn the sequence of number names and to recognize some of them, they will know little of their true significance. What is essential at this stage is that a considerable variety of experiences is given in which counting seems useful to the child. The mastery of one-to-one correspondence between the number names and the objects to be counted is essential. Much practice needs to be given to ensure accuracy in this particular technique, but the practice must be associated with activities that are of interest to the child. This may extend beyond the number ten, since the child will accept twelve as a number in the same way as he accepts seven or eight, without any awareness of its particular properties.

The figures that represent the numbers may be familiar to children before they come to school; some will be known and named, but any gaps or inconsistencies can be filled in or corrected. A wealth of experience in recognizing the numbers should be given through games and activities. Aids to counting, such as beads and sticks, can be used to help children dissociate number from particular objects and can lead to early ideas of place value.

The beginnings of ideas of cardinal number lie in activities in grouping material. It is not simple for the child to see the last object in a group as "seven" and yet also see the group itself as "seven". While number pictures or groups of counters may help this concept with small numbers, the significance of "four" in relation to "five" is easily lost, and the comparative size of each group is not obvious. It is here that one of the structured mathematical materials mentioned in the introduction becomes important. Through relating numbers to materials of fixed relative size the cardinal concept of number can be clearly established.

However, in this introductory section it is too early to expect any clear development of the cardinal concept of number other than that discovered in grouping. It is important that, if a structured mathematical material is to be used in later sections, this introductory section should provide adequate time for the child to become completely familiar with the particular material.

For those using the Cuisenaire material, guidance on this preparatory work is covered in Section A of the *Curriculum Guide.*

DETAILS OF THE INTRODUCTORY SECTION

Topic	*Comments*
The Development of the Ability to Count	
1. The sequence of number names (to ten at least).	This is readily learned by the repetition of counting rhymes, most of which are traditional, e.g., "One, two, buckle my shoe". A list of activities to develop this skill is shown in Appendix 1 of the *Curriculum Guide.*
2. One-to-one-correspondence.	Children need to learn to touch the object as they say the number. Activities as suggested in the *Curriculum Guide*, Section A, are useful. Once simple, one-to-one correspondence is mastered, the child learns the more difficult task of stopping at a number before the end of a series, e.g., the child may stop at "seven" in a series of "ten". A list of activities is given in Appendix 1 of the *Curriculum Guide.*
The Development of the Concept of Ordinal Number	
1. Sequence of number names.	Here the number names "one", "two", etc., are related to their order —first, second, etc.
2. One-to-one correspondence.	Similarly the previous activities on one-to-one correspondence can be repeated, using the ordinal names "first", "second", and so on.
3. The ability to select an object from a series.	Such activities as :—"Find the third flower-pot on the window-sill."
4. The need for a specific point of reference.	Activities to show that the position of a number in a series depends firstly on the point from which counting started and secondly on the direction in which counting proceeds.
The Introduction to Cardinal Number	
1. Grouping objects.	Count blocks, etc., into groups of three. Count three blocks, three counters, and so on.
2. Re-arranging groups.	The counters are grouped in different patterns, using the same type of counter. The child should realize that "fiveness" is unaffected by the arrangement of the group.

DETAILS OF THE INTRODUCTORY SECTION—*continued*

Topic	*Comments*
3. Altering the constituents of a group.	Keeping the same pattern of grouping, make groups of five beads, five buttons, etc.
4. Constructing a number of groups of the same size.	Make three groups of four counters, two groups of five counters, etc.
5. Constructing groups of unrelated objects.	For example, a table, a ball, a pen, paper, and a flower make a group of five. The child has to abstract the idea of "fiveness".
	For further activities on the above topics, see Appendix 1 of the *Curriculum Guide.*
Vocabulary	
Big, small, bigger, smaller, long, short, longest, shortest, larger, same, different, difference, length, size.	All children should be familiar with a basic vocabulary of mathematical terms. Where a structured material is used there will be a vocabulary specific to the material, e.g., with Cuisenaire material such terms as:—end to end, side by side, blue rod, red rod, above, below.
Development of Skill in Handling a Structured Mathematical Material (if such is to be used) and a Knowledge of Its Properties	Those using the Cuisenaire material should see Section A of the *Curriculum Guide.* Those using other materials should refer to the appropriate literature for guidance.

NOTE.—Once the ability to count is developed, the other steps—development of the concept of ordinal number, of the concept of cardinal number, and of familiarity with the structured mathematical material—may proceed side by side. In addition, the ideas begun in this section will be continued and extended in the next section.

At the end of this section the child should be ready to commence a study of the basic mathematical ideas concerned with cardinal number. He has an awareness of numbers to ten, knows the ordinal sequence of these numbers, has begun a study of cardinal ideas through limiting and grouping, and is familiar with whatever material is to be used for extending the cardinal ideas. He has, in addition, prepared a vocabulary to be used in studying further ideas.

SECTION B—THE STUDY OF BASIC MATHEMATICAL IDEAS

This section of the course is inserted for those who are using the Cuisenaire method of teaching arithmetic. With this method approximately one half-year is spent studying the basic mathematical ideas, using the colour names of the Cuisenaire rods instead of the numerical values. A second half-year is spent studying the same ideas but expressing them in the numerical values one to ten.

Some of the other structured mathematical materials also follow this type of procedure, while some have their own variations. The provision of this section, however, permits space for the course to be adapted according to the materials used. For those not using one of the structured materials, this section will not apply, except that the understandings aimed at here must be achieved. The early part of Section C should be devoted to developing these understandings even though working in number.

Whichever approach is used, the standards proposed by the end of Section C should be achieved approximately one year after the completion of Section A.

To avoid repetition, the ordinal work appropriate to Sections B and C is listed only at the beginning of Section C

This section indicates that mathematical ideas can be studied and thoroughly understood without any reference to numerical values other than counting. The virtue of this type of work is that at this age the young child has a very limited idea of cardinal number, and this limitation normally makes it difficult for him to understand new and difficult mathematical ideas when they are expressed in a language which is itself a difficulty.

Studied without the problem of the numerical language these mathematical concepts can be thoroughly understood, and then a simple change of language enables the child to express the same ideas in numerical values.

Following is a summary of the stages of Section B. Full information on the details of this section may be found in Section B of the *Curriculum Guide.*

1. Equality.
2. The operation of addition.
3. The operation of multiplication.
4. The reading of equations combining addition and multiplication.
5. The operation of subtraction.
6. The reading of equations combining addition, multiplication, and subtraction.
7. The operation of division.
8. The reading of equations involving all four operations.

SECTION C—THE STUDY OF BASIC MATHEMATICAL IDEAS USING THE NUMBERS 1 TO 10

Section C should be completed by most children approximately one year after completing Section A.

In this section the child should not only understand the mathematical ideas but be able to work with them. He should therefore work on two levels—the level of **definition** where, for example, he knows what addition is although he may be very limited in his ability to add, and the level of **operation** where he can in fact add numbers together.

The ordinal work begun in Section A is continued and extended to prepare for the work of the following section. The concept of cardinal number is completed and the key ideas of equality, addition, multiplication, subtraction, and division are thoroughly understood. Equations should be created and manipulated, using the range of numbers to ten. A great part of the work should be oral and the emphasis is on the creation of equations by the child himself, at first with the aid of concrete materials and later as an abstract process. Although it is desirable that children should work accurately in creating equations and in solving uncompleted equations, the teacher should be continually searching to ensure that the children understand how the processes function and relate to each other and that they are becoming aware of the pattern and order in the number system.

DETAILS OF SECTION C

Topic	*Comments*
The Extension of Ordinal Ideas	
1. Counting at least to twenty.	The child needs to be able to count at least to twenty in order to attack the next section. There is, however, no reason why those who can do so without difficulty should not extend their counting skill further.
2. The extension of the study of ordinal number to twenty.	Activities to develop the following topics of Section A should be extended:—The sequence of number names; one-to-one correspondence; selecting from a series; need for a specific point of reference.
3. The ability to write the figures from 1 to 20.	For those using the Cuisenaire material the writing of figures to 10 can be taught in Section B and the writing of figures 11 to 20 done in this section. Suggested activities are outlined in Appendix 2 of the *Curriculum Guide.*
4. Recognition of the words for the numbers to ten.	The writing of the number words is not strictly a mathematical skill and may well be handled during reading or spelling activities. Progress in this skill should be measured in terms of reading skill rather than mathematical achievement.
The Development of Cardinal Number and the Study of Basic Mathematical Ideas NOTE.—The early grouping activities of Section A should be extended and completed in Section B or, if structured material is not being used, in the early part of this section.	
1. Equality.	From an understanding of equality (the idea that a whole or group is equal to another whole or group), the mathematical equation is developed. This idea is extended to show that each side of the equation may be broken up into various groups. (See Section B of the *Curriculum Guide.*)
2. The study of addition.	First the child must understand the nature of addition—the putting together of groups—and then be able to in fact add. Practice should be given in the manipulation of equations; e.g., $3 + 3 + 3 + 1 = 4 + 4 + 2$. By varying the order of the numbers, the child can create equations such as: $3 + 1 + 3 + 3 = 4 + 2 + 4$; $1 + 3 + 3 + 3 = 2 + 4 + 4$; $2 + 4 + 4 = 3 + 3 + 1 + 3$. This process is referred to in the *Curriculum Guide* as "manipulation of equations".
3. The study of multiplication.	Multiplication is seen as the process of adding equal wholes; e.g., $3 + 3 + 3 = 9$; $3 \times 3 = 9$. After numbers and factors are studied, e.g., $2 \times 2 = 4$, $3 \times 3 = 9$, $2 \times 4 = 8$, this can be extended to the study of factors together with addition; e.g., $10 = 3 \times 3 + 1$.
4. The study of subtraction.	Again the emphasis is on understanding the nature of subtraction. The idea of subtraction as complementary addition or as the difference between numbers is more useful than the "take away" idea. The children should create equations; e.g., $4 - 3 = 1$; $4 - 1 = 3$; $3 = 4 - 1$; $1 = 4 - 3$, and also create equations using the ideas of addition and multiplication; e.g., $4 + 3 - 2 \times 2 = 3$.
5. The study of division.	**Children should understand the different aspects of division:** **Quotition division—Size of group is known. Find how many groups.** **Partition division—Number of groups is known. Find size of group.** **Division with remainders may also be introduced.**

DETAILS OF SECTION C—*continued*

Topic	*Comments*
6. Development of the ability to record and interpret written equations.	When all the operations have been established orally, the child may then record his equations in writing. This involves the signs =, +, ×, −, ÷, (). [For explanation of brackets see *Curriculum Guide*, Section B—Subtraction, and Section C—Solving Equations.] When the child can record his own ideas he may then be led to interpret and solve equations written by the teacher. Although it is convenient and necessary for the child to express his equations in writing, the value of oral work should not be overlooked.
7. Development of the ability to create equations without using any concrete aids.	The ability to express mathematical ideas contained in the materials used is gradually extended to develop the ability to create and solve equations without any material. This skill involves a degree of memorization, and the response to number bonds may at first be rather slow. The frequent use of the number facts in equations is more profitable than mere drill.
8. The introduction of fractions.	When all the operations have been studied, ideas of fractions may be introduced. All fractions with denominators from 2 to 10 should be studied. Two concepts are involved: (*a*) The fraction as the relation between two numbers; i.e., one-fifth expresses the relationship of one to five. (*b*) The fraction as an operator, i.e., $\frac{1}{5}$ of 5, $\frac{3}{7}$ of 7. (For details see Section C of the *Curriculum Guide*.)

SECTION D—THE STUDY OF BASIC MATHEMATICAL IDEAS USING THE NUMBERS 1 TO 20

This section is simply an extension of the previous section into larger numbers, and no new mathematical ideas are introduced. It should be completed by a child of average ability in approximately one school term. Counting is extended to cover at least the numbers that will be studied in detail in the next section, and the emphasis is on discovering the pattern and order of the number system. As counting extends into larger numbers, place value may be taught, although it is not essential at this stage. Therefore, the teaching of place value as a requirement has been postponed to Section E.

While the study of the four operations and fractions within an equation involves no new ideas, there should be an increasing skill in handling more complex equations, in the ability to manipulate numbers, and in understanding the relationships between the operations. This is achieved by the extensive use of oral work, and by written work that involves mostly the creation of equations. The solving of uncompleted equations is a skill which takes longer to develop, and these equations should therefore still be of a relatively simple nature.

Although automatic response is still not a requirement of this section, the child will have developed fairly rapid recall of those number combinations he has met most frequently. By careful planning the teacher can determine the range of number facts experienced.

It should be noted that the increasing facility in handling numbers which appears in this section should not necessarily lead to the assumption that the child fully understands what he is doing. The teacher should continue to question and probe for understanding. The child should be able to explain verbally the meanings of the mathematical ideas he is using, but this explanation should be in the child's language, and not the parroting of definitions supplied by the teacher.

Although this section lists a number of different stages, it is important to realize that these run parallel rather than consecutively. Although the ordinal work is listed first, it runs throughout the whole section. Throughout the whole section there will still be considerable work with concrete materials. This will tend to be when the larger numbers are first met and when more difficult ideas are being developed. Mental work will be carried on throughout the whole section, although it will reach a higher standard towards the end. The length of time required to master each step and the dependence on the concrete materials will vary according to the increasing differences in the abilities of children.

In fact, the sections of the course tend to overlap. For example, when the child is working with concrete materials in the early part of this section he should be working competently in an abstract way with the ideas of Section C. Towards the end of this section, when he is working confidently with the numbers to twenty, he may have proceeded to the early part of Section E, using concrete materials to study larger numbers.

DETAILS OF SECTION D

Topic	***Comments***
The Extension of Ordinal Number 1. Counting to at least 144.	Both the sequence of the number names and the idea of order in a series should be treated. Counting to 144 is the minimum ability required before attacking Section E, but children should be permitted to go further if they can do so without difficulty. Counting should be done forwards and backwards, starting at any point within the range.

Topic	*Comments*
2. Counting to show the pattern and order of the number system.	Any counting activity that shows the sequence, repetition, or rhythm of numbers may be used. A start should be made in counting by twos, threes, etc., up to twelves. Counting by even numbers presents simple repetitions, e.g., counting by twos results in the last digits being 2, 4, 6, 8, 0. Counting by fours, sixes, and eights results in the same final digits but in different order. Counting by fives results in the last digit sequence 5, 0; 5, 0.
(*a*) Counting in groups	Simple exercises in doubling and halving may be introduced.
(*b*) Serial counting	Once established, serial counting should be developed, starting from any number. Counting in the tens sequence from any number is particularly important.
(*c*) Number charts	The visual presentation of number charts enables the sequences to be seen more clearly and shows horizontal, vertical, and diagonal sequences. (See *Curriculum Guide*, Section E.) In extending these ideas the child should use some visual or concrete aid before he is expected to recall the number sequences.
3. Recognition and writing of the figures to 144.	
4. Recognition and writing of words from one to twenty.	
The Further Study of Basic Mathematical Ideas	
1. The four operations and fractions in equations using numbers to twenty.	This is a simple extension of Section C to show that the ideas understood there still work when used with larger numbers. In the early stages, concrete aids or one of the structured mathematical materials may be used. (See *Curriculum Guide*, Section D.)
(*a*) Oral work	The reading of equations illustrated by the material used. The importance of oral work cannot be over-emphasized.
(*b*) Recording equations	Once the equations can be created orally and with confidence they may be written. The emphasis should still be on the creation of ideas by the child from the material in front of him. Care should be taken to see that the signs and brackets are used correctly and that the equations do in fact balance.
(*c*) Interpreting written equations and solving uncompleted equations	This is more difficult than the creation of his own equations by the child. Therefore these equations should be fairly simple and carefully graded. At this stage the child may use concrete materials to solve problems if necessary. The emphasis should be on how to attack an equation and on how to handle the operations, rather than on the numbers used. Therefore the degree of difficulty depends on how many operations are used in the equation and how these are combined. $9 + 7 = x$ may be a difficult number fact, but it is simple in that only one operation is used. $2 \times 2 + \frac{1}{2}$ of $8 - (3 + 2) = x$ uses simple number facts but is more difficult in terms of the operations used.
2. The manipulation of equations.	If the child understands the mathematical ideas studied, he should be able to rearrange the elements of an equation or substitute one idea for another and still maintain mathematical balance. The ability to manipulate equations with confidence and understanding is probably one of the best indications of progress.
(*a*) The rearrangement of elements of an equation	From any given equation the child should be able to produce a number of different equations using the same elements; e.g., $4 + 6 + 5 = 9 + 6$ $6 + 5 + 4 = 6 + 9$ $9 + 6 - (4 + 6) = 5$.
(*b*) The substitution of one or more elements in an equation by equivalent elements	This skill indicates that the child can see a number in many different ways; e.g., $4 + 6 + 5 = 9 + 6$ $2 \times 2 + 3 \times 2 + \frac{1}{2}$ of $10 = 3 \times 3 + (10 - 4)$ $3 + 1 + \frac{1}{2}$ of $12 + 10 \div 2 = 3 \times 3 + 2 \times 3$. It also gives practice in the basic number facts appropriate to each element. It further provides a skill which when reversed enables the child to solve uncompleted equations:— $\frac{1}{2}$ of $12 - (10 - 4) + 12 \div 6 = x$ $6 - 6 + 2 = 2$.

DETAILS OF SECTION D—*continued*

Topic	*Comments*
(c) The manipulation of equations involving both transposition and substitution	Both the above skills may be used in one equation; e.g., $4+6+5=9+6$ $3\times 3+\frac{1}{2}$ of $12-(2\times 2+3\times 2)=5$. NOTE.—This type of work not only requires an understanding of the operations and of equations, but provides a larger variety of experiences from which the child will eventually make generalizations which explain mathematical laws treated in later sections. These skills may first be demonstrated and practised using appropriate materials, but will be extended into the following stages where the child should work in an abstract way without concrete aids.
3. The abstract creation and manipulation of equations. (a) Oral work	This work extends the ideas in (1) and (2) above, but the child is required to depend on his ability to work mentally. Oral work enables the child to express more ideas unhindered by the task of writing. Oral work should be done confidently before too much work in the following steps is required.
(b) Recording of equations ..	This involves the recording of ideas similar to those expressed orally. It also involves taking a given equation and writing many equations using the techniques of transposition and substitution.
(c) Interpreting written equations and solving uncompleted equations	The interpretation of written equations involves knowing what to do to prove that the equation is balanced. It requires an understanding of the order in which the signs are used and may be understood with little knowledge of number facts. Solving uncompleted equations involves the above-mentioned skills and a knowledge of the number facts. These skills are developed through the use of very carefully graded examples.
4. The extension of the study of fractions. (a) The fraction as a relation between two numbers.	This is a simple extension of the ideas treated in Section C.
(b) The fraction as an operator.	As the child studies numbers to 20, his work with the fraction as an operator will include— (i) fractions where the denominator is the same as the number operated on, e.g., $\frac{9}{12}$ of 12, $\frac{13}{17}$ of 17, $\frac{11}{20}$ of 20. (ii) fractions (using small numbers at first) where the denominator and the number operated on are not the same, e.g., $\frac{1}{4}$ of 8, $\frac{3}{5}$ of 10. This work is facilitated by much practice in counting in groups referred to above. (See *Curriculum Guide*, Section D.)

SECTION E—THE INTRODUCTORY STUDY OF BASIC MATHEMATICAL IDEAS USING NUMBERS TO 144

Now that the basic mathematical ideas are thoroughly understood, they are exercised in studying selected large numbers. Choosing a limit of 144 is purely arbitrary, and there is no reason why there cannot be a selection of bigger numbers, though for practical purposes the intensive study of numbers to 144 embraces all the ideas found in the traditional tables to twelve. Although concrete materials may be used in the first look at each number, the emphasis is on abstract mental manipulation. Automatic response is still not a main aim, but through frequency of use, the response to the most commonly used number facts will be more rapid. As in the previous section, by carefully directing the child's creative work and by thoughtful structuring of the equations set before the child, the teacher can ensure that he meets frequently those number facts in which automatic response will later be required. The main aim is to ensure that the child gradually develops a sense of confidence and achievement with increasing mastery of the number system.

The distinction between Sections E and F is not clear-cut. Section E is characterized by a feeling out of ideas; it is rather a time of exploration and adventure into large numbers. Section F is characterized by a sense of mastery and depth of quality in the child's creative work. At this level the differences in the achievement of children become more marked, the bright child may exhibit in this section a quality of work and a depth of understanding which will not be achieved by the slow child even at the end of Section F. Thus any attempt to suggest how long should be spent in this stage must be rather vague. The average child might spend some two school terms in this area and perhaps reach an age of eight years before he shows the quality of work that indicates the transition to Section F.

In studying large numbers the emphasis tends to be on ideas of multiplication and division; therefore the teacher must take care to ensure that the earlier addition and subtraction ideas are not neglected. Again, this can be achieved by wise direction of the creative work and careful selection of equations for exercising the techniques of substitution developed in earlier sections.

Again the level of difficulty is not in the number studied but rather in terms of the type of equation used. In his first look at a new number the child tends to use simpler equations, then to extend the [illegible] in terms of those ideas with which he is [illegible]. The teacher needs to analyse the child's [illegible] it to remedy weaknesses and omissions.

DETAILS OF SECTION E

Topic	*Comments*
The Extension of Ordinal Ideas	
1. Counting to at least 1,000— (a) by ones (b) by twos, threes, fours . . . and all the numbers to twelve (c) serial counting starting from any number (d) by even numbers (e) by odd numbers (f) mixed counting	In all counting work the emphasis should be on the varied types of counting that bring out the patterns and the sequences of numbers. Thus the functions of the first three topics listed opposite are really interwoven. Counting work should be continued and extended throughout the whole section. (See *Curriculum Guide*, Section E.)
2. Studying patterns in number sequences—use of the number chart.	(See *Curriculum Guide*, Section E.)
3. Doubling and halving of numbers.	Numbers, including fractions, may be doubled and halved, to any limit to which the child can go. It is important that the child develops his own technique for doubling and halving and is not directed by the teacher. (See *Curriculum Guide*, Sections E and F.)
4. The ability to recognize and write the figures from 1 to 1,000.	
5. The ability to recognize and write the words from one to one hundred and forty-four.	The limit of 144 is suggested simply to keep in line with the range of numbers for this section. However, once the child can write the words to one hundred there is no reason why he cannot gradually progress to writing the words to nine hundred and ninety-nine, that is, in line with the limits for place value.
6. The study of place value to 999.	Place value may have been treated earlier but must at the latest be studied in detail during this section. An understanding of place value gives added depth to the child's counting ideas and is essential for the later work dealing with large numbers. (See *Curriculum Guide*, Section E.) The use of extended notation, e.g., $57 = 50 + 7$, not only assists the child to understand place value, but also lays an important basis for his understanding of the written processes developed in later sections.
The Application of the Basic Mathematical Ideas to Large Numbers	
1. The simple extension of the understandings of Section D to numbers to 144. (a) Oral work . . . . . . (b) Written work . . . . (c) Interpreting written equations . . (d) Solving uncompleted equations . .	The initial approach is to explore selected numbers using the techniques already well established in earlier sections. At first the child tends to use less complex equations, e.g., $4 \times 8 = 32$; $8 \times 4 = 32$; $32 \div 8 = 4$; $32 \div 4 = 8$; $8 + 8 + 8 + 8 = 32$. Gradually the complexity of the equations returns as confidence with this number increases, e.g., $32 - (8 + 8 + 8) = 8$; $32 - 8 - 8 - 8 = 8$; $32 - 2 \times 8 = 2 \times 8$; $\frac{1}{2}$ of $16 + \frac{1}{3}$ of $24 + \frac{1}{4}$ of $32 + \frac{1}{5}$ of $40 = 32$. This confidence is developed by the use of much oral work when a new number is attacked and by permitting the use of concrete materials until the new ideas are clearly established. The ideas, not the numbers, are important. To study multiplication ideas, numbers rich in these ideas may be used, with prime numbers serving as a contrast. (See *Curriculum Guide*, Section E.) When doubling and halving techniques are well established, then procedures may be those suggested in the *Curriculum Guide*, Section F.
2. The ability to manipulate equations using selected numbers from 1 to 144. (a) The transposition of elements of an equation	This is simply an extension of the ideas treated previously. (See Section D for details.) The size of the number treated gives the child more scope to use the knowledge gained and thus enables understanding to be deepened and made more secure.

DETAILS OF SECTION E—*continued*

Topic	*Comments*
(*b*) The substitution of one or more elements in an equation by equivalent elements (*c*) The manipulation of equations using both transposition and substitution	It is important for the teacher to plan carefully the type of equation presented to the child to ensure that all aspects are covered and weaknesses remedied. **The planned use of the substitution techniques** lays the foundation for the efficient automatic response to number facts demanded in later sections. For example, in using the equation $36 - 24 = 12$, if the teacher wishes to exercise multiplication ideas, then substitution produces: $9 \times 4 - 4 \times 6 = 3 \times 4$, $3 \times 12 - 8 \times 3 = 6 \times 2$, etc. Or, if the use of brackets is to be practised, then substitute for 24 using addition or subtraction facts:— $36 - (16 + 8) = 12$, $36 - (34 - 10) = 12$. Or, to practise division, substitute for twelve using division facts:— $36 - 24 = 36 \div 3$, $36 - 24 = 60 \div 5$. However, it must be noted that too much teacher direction may be dangerous. These techniques should be preceded by, associated with, and followed up by much free creative work. It is through careful observation of the child's creative work that the teacher determines how much directed work is required.
3. The abstract creation and manipulation of equations.	As in previous sections this skill develops parallel with the other topics. The study of a new idea or a new number should begin with and end with creative work. The skill demanded here is to be able to work with all the numbers studied, using all the operations to create and manipulate equations with confidence and accuracy.
(*a*) Oral work	Oral work provides scope for numerous examples to be studied. The teacher may write on the black-board the varied equations given by a group of children, and use them for discussion.
(*b*) Written creative work ..	Through free creative written work (say "write equations concerning the number 48"), the child is able to develop his equations using the patterns and sequences discovered in the ordinal work and using the techniques developed in the earlier stages. The following are actual examples of children's creative work:— *Doubling and halving*: $20 \times 5 = 100$; $40 \times 2\frac{1}{2} = 100$; $80 \times 1\frac{1}{4} = 100$; $160 \times \frac{5}{8} = 100$. *Subtraction, using equal additions*: $18 - 12 = 6$; $24 - 18 = 6$; $30 - 24 = 6$; $36 - 30 = 6$. *Division, where doubling of both elements gives a constant answer*: $18 \div 2 = 9$; $36 \div 4 = 9$; $72 \div 8 = 9$; $144 \div 16 = 9$. *Relation between denominator of fraction and whole number is constant*: $\frac{1}{3}$ of $27 = 9$; $\frac{1}{4}$ of $36 = 9$; $\frac{1}{5}$ of $45 = 9$; $\frac{1}{6}$ of $54 = 9$. *Multiplication ideas explored through fractions and translated into division ideas*: $\frac{1}{2}$ of $36 = 18$ $36 \div 18 = 2$ $\frac{1}{3}$ of $36 = 12$ $36 \div 12 = 3$ $\frac{1}{4}$ of $36 = 9$ $36 \div 9 = 4$ $\frac{1}{6}$ of $36 = 6$ $36 \div 6 = 6$ $\frac{1}{9}$ of $36 = 4$ $36 \div 4 = 9$ $\frac{1}{12}$ of $36 = 3$ $36 \div 3 = 12$ $\frac{1}{18}$ of $36 = 2$ $36 \div 2 = 18$ NOTE.—The same idea is repeated in the same order in each sequence.

Topic	*Comments*
(*c*) The interpretation of written equations	The emphasis here is not so much on getting an answer to an equation as on how to set about working it out; e.g., $6 \times 8 - (20 - 10) + 15 \div 3 - \frac{1}{2}$ of $12 = 37$. The child should understand the function of the brackets, and how operators tie numbers together, thus giving him a method of attacking the equation. The actual numbers used are not important.
(*d*) The ability to solve uncompleted equations	Here the ability developed in (*c*) above is used to find the solution, e.g.:— $4 \times 8 + \frac{1}{2}$ of $24 - 36 \div 12 = x$ The numbers used should be those with which the child has a fair degree of facility. The position of the unknown elements should be varied, e.g., $36 - (20 + x) = 6$ $x = 24 + \frac{1}{12}$ of 48 $4 \times 8 - \frac{1}{2}$ of $x = 24$ $48 - (x \times y) = 12$
4. The extension of the study of fractions.	(See *Curriculum Guide*, Section E.)
(*a*) The fraction as a relation between two numbers, e.g., $\frac{1}{24}$, $\frac{7}{36}$	The two topics (*a*) and (*b*) are an extension of the work of earlier sections. The actual fractions used will depend on the selection of numbers studied in this section.
(*b*) The fraction as an operator, e.g., $\frac{1}{24}$ of 24 $\frac{3}{5}$ of 5 $\frac{1}{8}$ of 48	
(*c*) The fraction as a number less than 1, e.g., $\frac{1}{2}$ of 1 $\frac{1}{3}$ of 1	This idea is associated with the doubling and halving work mentioned above, where through halving the child meets a great range of fractions between 1 and 0. Another concept introduced is that of the fraction as another ordinal number, e.g., 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$ . . .
(*d*) The equivalence of fractions, e.g., $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$ $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$	The understanding of equivalence is essential to any further work in fractions. It will have been met incidentally when the child works with, say, the fractions $\frac{1}{2}$ and $\frac{3}{6}$. There should not be at this stage any attempt to teach rules, but rather to present fractions together in situations where the child can see easily the relationships. The use of one of the structured mathematical materials is almost essential for showing this idea. This topic should be treated mainly by oral work. (See *Curriculum Guide*, Section E.)

SECTION F—THE EXTENDED STUDY OF BASIC MATHEMATICAL IDEAS USING THE NUMBERS TO 144

As indicated in Section E, the work of this section is distinguished by the child's increasing mastery of the number system and the quality of the equations he can create. In earlier sections the numbers were a means through which the mathematical ideas could be understood. In this section the mathematical ideas are used to study and explore larger numbers. Although the numbers to 144 are studied in some detail, at least a sample of numbers beyond this range should be investigated.

The child is now led to organize numbers and mathematical ideas of a size and complexity which will not only show him the need for formal written techniques of working with the four operations but will also enable him to realize that these are simply efficient ways of handling large numbers.

The parts of this section concerned with the development of techniques for handling larger numbers and the achievement of automatic response are essential prerequisites to understanding the formal setting out of the written processes and to success with the resultant calculations.

Thus while the greater part of one school year will be devoted to Section F, "the development of techniques for handling larger numbers" and "the activities leading to the formal processes" will, during the later part of this work, have reached a stage where there is a gradual transition to the formal setting out of processes.

For the child of average ability this level would probably be reached by the end of the Grade III year.

As this section relies on the child's ability to work abstractly with mathematical ideas there will be a decreasing use of concrete aids or of reliance on any of the structured mathematical materials.

DETAILS OF SECTION F

Topic	*Comments*
The Extension of Ordinal Ideas	
1. Counting any reasonable sequence of numbers within 1 million— (*a*) by ones	Counting forwards and backwards. The child's understanding of the order and values of numbers beyond 1000 is developed through the work in place value and numeration and notation, as well as through formal counting within a specified range, e.g., starting at 89,994 count to 90,008.
(*b*) by twos, threes, fours . . and all the numbers to 12	This should be done abstractly at least to 144 to prepare for the later work on multiplication tables. Extension beyond 144 would be solely for interest.
(*c*) serial counting starting from any number	
2. Studying patterns in number sequences—using number charts.	An extension of the work of Section E.
3. Doubling and halving numbers.	
(*a*) Doubling and halving beginning from any number	To any limit to which the child can go.
(*b*) Doubling and halving using multiplication ideas	This is a development of (*a*) to cover examples such as $12 = 12 \times 1$; 6×2; 4×3; 3×4; etc. $24 = 24 \times 1$; 12×2; 8×3; 6×4; etc. (See *Curriculum Guide*, Section F.)
4. Serial addition and serial subtraction.	Using simple number facts with the tens sequence, e.g., $8 + 7 = 15$; $18 + 7 = 25$; $28 + 7 = 35$; etc. $13 - 5 = 8$; $23 - 5 = 18$; $33 - 5 = 28$; etc.
5. The recognition and writing of figures to 1,000,000.	
6. The recognition and writing of words to one thousand.	
7. Place value to thousands. (*a*) Recognition of numbers to 1 million	The skill required in (5) above is simply to recognize and write figures, e.g., 756,476—Seven hundred and fifty-six thousand, four hundred and seventy-six. This is very little more difficult than the skill of reading the number 476.
(*b*) The values of figures to thousands	The more difficult task, involving a greater understanding of place value, is to tell that the number 7654 is made up of 4 units, 5 tens. 6 hundreds, 7 thousands.
(*c*) The use of extended notation to thousands	The work started in Section E is associated with the work in (*b*) above, e.g., $7654 = 7000 + 600 + 50 + 4$. This work leads to the ideas developed later in this section entitled "The development of techniques for handling larger numbers".
The Application of Mathematical Ideas to All Numbers to 144	
1. The manipulation of equations (*a*) Reading equations (*b*) The manipulation of equations using the techniques of transposition and substitution (*c*) Creative work with equations (*d*) The interpretation of equations (*e*) The solving of uncompleted equations	This is an extension of the work of Section E to cover all the numbers to 144 not previously studied. Although early reference may be made to concrete aids, the emphasis is on increasing the ability to work abstractly. During this work the teacher should carefully plan the equations studied and use the technique of substitution to assist in achieving automatic response.
2. The extension of the study of fractions. (*a*) The fraction as a relation between two numbers (*b*) The fraction as an operator (*c*) The fraction as a number less than 1	The ideas in (*a*), (*b*), and (*c*) are simply an extension of earlier ideas to the numbers being studied. At the same time there should be an improvement in quality and variety in the work with fractions using smaller numbers.

DETAILS OF SECTION F—*continued*

Topic	*Comments*
(*d*) The equivalence of fractions ..	Throughout Section E the child has developed a general understanding of the equivalence of fractions. Here he is led to discover the short cuts and to study the relationship between numerators and denominators, e.g., $\frac{2}{3}=\frac{x}{9}$. (*a*) The relationship between x and 9 is the same as that between 2 and 3; or (*b*) the relationship between x and 2 is the same as that between 9 and 3. In earlier sections the child has probably seen that it is possible to add or subtract fractions with the same denominators, e.g., $\frac{1}{7}+\frac{2}{7}=\frac{3}{7}$ For those children who thoroughly understand these relationships the addition and subtraction of any fractions are simple steps.
3. Simple decimal notation; to one decimal place.	Where children have thoroughly understood simple vulgar fractions, especially tenths, and have also understood place value, decimal notation as a form of writing numbers should be introduced, e.g., $1\frac{1}{10}=1{\cdot}1$; $2\frac{7}{10}=2{\cdot}7$.
4. The development of techniques for handling larger numbers.	This work should be commenced at the beginning of this section, then extended through the activities of section (6) below until it becomes the formal process work. The child has in fact used the ideas in sections (A) and (B) below in earlier manipulation, although perhaps unconsciously. The purpose now is to direct his attention to these principles and laws, so that he will not only understand how the processes work with large numbers but will also develop a variety of skills to use according to the combination of numbers met. This work may be started with the use of concrete materials but should soon be done abstractly. (For further details, see *Curriculum Guide*, Section F.)
(A) Place value	This work, commenced earlier, is now extended to large numbers, e.g., $200+70+3=\square$
(*a*) Extended notation	$70+200+3=\square$ 273 means □ hundreds □ tens □ units $6000+400+\square+5=6435$.
(*b*) Re-grouping	The re-grouping of units and tens in addition, e.g., $13+27=40$ $17+23=40$ $13+27=17+23$. Graded exercises lead to such examples as: $234+148=248+\square$
(B) Arithmetical laws	It is not expected that the child will be able to state these laws formally or know their names, but rather that he can use them and explain what happens.
(*a*) Commutative law of addition .	The order of addends in a two-addend equation may be changed without a change in the sum, e.g., $5+3=3+5$; $27+48=48+\square$
(*b*) Commutative law of multiplication	The order of two factors may be changed without a change in the product, e.g., $8\times7=7\times8$; $8\times24=24\times\square$
(*c*) Associative law of addition ..	Numbers can be grouped in different ways without altering the sum, e.g., $3+4+7$ may be grouped (*a*) $(3+4)+7=7+7$ or (*b*) $3+(4+7)=3+11$.
(*d*) Associative law of multiplication	We can multiply only two numbers at one time, but the associative law holds that the product of three or more numbers is the same no matter how these numbers are grouped, e.g., $3\times4\times6$ (*a*) $(3\times4)\times6$ $=12\times6$ $=72$ (*b*) $3\times(4\times6)$ $=3\times24$ $=72$.

DETAILS OF SECTION F—*continued*

Topic	*Comments*
(*e*) The combination of commutative and associative laws	Exercises such as:— (i) Re-arrange the order and group the numbers to make addition easier, e.g., $27 + 36 + 14 + 33 = (27 + 33) + (36 + 14) = 60 + 50$. (ii) Use substitution and re-grouping, e.g., $60 \times 4 = (10 \times 6) \times 4 = 10 \times (6 \times 4) = 10 \times 24 = 240$.
(*f*) The distributive law	The distributive property connects addition and subtraction with multiplication and division, e.g., in the expression 5×12 the multiplication property of the 5 may be distributive over the addition facts contained in 12, i.e., $5 \times 12 = 5 \times (8 + 4) = 5 \times 8 + 5 \times 4 = 40 + 20$. Similarly, division properties may be distributed over the dividend, e.g., $15 \div 3 = (9 + 6) \div 3 = 9 \div 3 + 6 \div 3 = 3 + 2$.
(*g*) Inverse relations	Exercises should be given to show that subtraction will undo addition, and addition will undo subtraction, e.g., $8 + 5 - 5 = 8$; $9 - 4 + 4 = 9$. Again, multiplication will undo division and vice versa, e.g., $18 \div 6 = 3$; $3 \times 6 = 18$.
(C) Free application of these techniques	(See *Curriculum Guide*, Section F.) The child should now be presented with examples in which the above techniques are used freely to find the answer. Whatever method suits the child, or the particular pattern of numbers, is acceptable. (*a*) $24 + 28 =$ (*b*) $5 \times 14 =$ (*c*) $36 - 19 =$ (*d*) $56 \div 15 =$ As ability develops in this skill the more directive work of (6) below will refine the techniques.
5. Mastery of, and automatic response in, all number facts and tables.	The ability to recall number combinations has begun to develop in earlier sections and been further developed especially through the technique of substitution. Reasonable rapidity of recall is essential before any serious attack is made on the written process work, but for efficient calculation this recall must be automatic. Throughout this section, games and activities to develop this automatic response should be going on, together with careful planning of equations presented to the child to direct attention to those combinations mastered. Although most children should achieve automatic response in all number combinations by the end of this section, the weaker children will take longer. Some may always have a response something less than automatic. Activities to assist in the mastery of number combinations are set out in the *Curriculum Guide*.
(*a*) Addition and subtraction facts to twenty	Complete mastery of all the basic addition and subtraction facts up to twenty is essential, and these, together with the skills of serial counting, serial addition, and serial subtraction, should enable the child to master addition and subtraction facts beyond twenty. At all times inverse relationships should be emphasized, e.g., $9 + 6 = 15$; $6 + 9 = 15$; $15 - 6 = 9$; $15 - 9 = 6$.
(*b*) Multiplication and division facts contained in the numbers to 144	All these combinations will have been met in earlier sections and in the first part of this section, in relation to the numbers studied. These combinations should be re-organized into useful groupings of tables and games, activities, and drill used to develop automatic response. Again the inverse relations should be emphasized, e.g., $3 \times 2 = 6$; $2 \times 3 = 6$; $6 \div 2 = 3$; $6 \div 3 = 2$; $\frac{1}{2}$ of $6 = 3$; $\frac{1}{3}$ of $6 = 2$. It should be emphasized that by the end of this section all the prescribed number facts should be known with both instant recall and accuracy.

Topic	*Comments*
6. Activities leading to the formal setting out of the processes.	The following activities utilize the skills developed in (4) above and lead gradually to the formal process work. Experience with all the activities is important, although which of them will be emphasized depends on the final decision as to which method is used to work each process. The numbers prescribed in the equation will determine which activities are most suitable. In both addition and subtraction, children can proceed to the traditional forms of setting out in stages. In examples that do not involve re-grouping, the traditional form of setting out may be reached while other methods (such as extended notation) are still being used for more difficult examples.
(*a*) Addition	Taking the expression 642 + 345, the following activities may be developed:—

(*a*)
$$
\begin{aligned}
&642 + 345\\
&= 600 + 40 + 2 + 300 + 40 + 5\\
&= 600 + 300 + 40 + 40 + 2 + 5\\
&= 900 + 80 + 7\\
&= 987.
\end{aligned}
$$

(*b*) Next the setting out may be re-arranged—

$$
\begin{array}{lll}
642 & 600 + 40 + 2 & 642\\
345 & 300 + 40 + 5 & 345\\
\hline
 & 900 + 80 + 7 = 987. & 987
\end{array}
$$

Further examples will involve re-grouping in units and tens only, involving two addends and a three-digit sum. No example involving re-grouping to be set in traditional form.

$$
\begin{array}{ll}
638 & 600 + 30 + \ 8\\
244 & 200 + 40 + \ 4\\
\hline
 & 800 + 70 + 12\\
 & = 800 + 80 + \ 2\\
 & = 882.
\end{array}
$$

(*b*) Subtraction — Using the equation 85 − 67 = x, several activities are useful:—

(*a*) The complementary approach:—

$$
\begin{aligned}
&67 + x = 85\\
&67 + (3 + 15) = 85\\
&67 + 18 = 85.
\end{aligned}
$$

(*b*) The addition of equals to unequals leaves their difference the same.

$$85 - 67 = 88 - 70 = 18$$

(*c*) Using extended notation:—

$$
\begin{aligned}
85 - 67 &= 80 + 5 - (60 + 7)\\
&= 80 + 5 - 60 - 7\\
&= 80 - 60 - 7 + 5\\
&= 20 - 7 + 5\\
&= 13 + 5\\
&= 18.
\end{aligned}
$$

(*d*) By extended notation and equal additions:—

$$
\begin{array}{lll}
85 & 80 + 5 & 80 + 15\\
-67 & 60 + 7 & 70 + \ 7\\
\hline
 & & 10 + \ 8 = 18.
\end{array}
$$

(*c*) Multiplication — The distributive law is useful in explaining the multiplication process One-digit multiplier with no re-grouping in final addition.

(*a*) $36 \times 4 = x$
$$
\begin{aligned}
36 \times 4 &= 4 \times 36\\
&= 4 \times (30 + 6)\\
&= 4 \times 30 + 4 \times 6\\
&= 120 + 24\\
&= 144.
\end{aligned}
$$

(*b*) $(30 + 6) \times 4$
$$
\begin{array}{r}
30 + \ 6\\
\times 4\\
\hline
120 + 24 = 144.
\end{array}
$$

DETAILS OF SECTION F—*continued*

Topic	Comments
(*c*) Multiplication—*continued*	(*c*) $\begin{array}{r} 36 \\ \times 4 \\ \hline 24 \\ 120 \\ \hline 144. \end{array}$
(*d*) Division	The distributive law again helps to give a fuller understanding of the division process. One-digit divisor, with dividend restricted to 144. (*a*) $48 \div 4 = (40 + 8) \div 4$ $= 40 \div 4 + 8 \div 4$ $= 10 + 2$ $= 12.$ (*b*) Introducing re-grouping:— $56 \div 4 = (50 + 6) \div 4$ $= (40 + 16) \div 4$ $= 40 \div 4 + 16 \div 4$ $= 10 + 4 = 14.$

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EDUCATION DEPARTMENT, VICTORIA

COURSE OF STUDY

FOR

PRIMARY SCHOOLS

MATHEMATICS, 1967

APPLIED NUMBER, SECTIONS A–F

1209/67.

INTRODUCTION

The Course of Study for Primary Schools in Mathematics appears in three parts. The first part, published in 1964, contained the pure number course, Sections A–F. It concerned the development of pure number ideas with the younger children. In this booklet appears the applied number course, Sections A–F. A separate booklet contains the mathematics course in pure and applied number, Sections G—I.

While the course at the Sections A to F levels has been published in two separate parts, the subject of mathematics should be seen to have unity. The material that follows is still concerned with number, but here the child sees number in the context of measurement. It is necessary that all parts of the course should be read in conjunction with one another. The development of Sections A—F is outlined in the flow chart appended to this booklet. The overall development of the complete primary school course can be seen in the flow chart placed in the *Course of Study for Primary Schools : Mathematics* (Sections G to I), 1967, published simultaneously with this booklet.

1209/67.—2

4

APPLIED NUMBER

VOCABULARY

Not all words listed will be introduced in the early sections, but, by the end of Section F, children should have a very good understanding of the vocabulary. Although some words apply specifically to one topic, many apply to more than one, and these have not been duplicated. Listening to children as they work at their various activities should enable the teacher to gauge the extent and the understanding of the vocabulary and to decide where specific lessons are needed. The comparative degree has not been included, but it is assumed that, as the child begins to compare, these words will be added. Names for formal units, for example, pound, hour, foot, pint, are not listed below, but appear in the course development in the sections where they are introduced.

Length : Long, short, high, low, tall, broad, narrow, wide, thick, thin, pair, span, height, measure.

Weight : Heavy, light, big, little, large, small, great, weight, balance.

Volume and Capacity : More, less, full, empty, nearly, almost, same, enough, whole, many, few, some.

Time : Fast, slow, quick, time, early, late, soon, before, after, hurry, morning, afternoon, day, night, clock, swift.

Money : Money, cost, change, buy, sell, dear, cheap, worth, value.

Spatial Relations : Beside, inside, outside, underneath, behind, front, between, middle, top, bottom, over, under, above, below, up, down, near, far, back, next, blunt, sharp, straight, curved, rough, smooth, pointed, round, flat, edge, border, boundary.

SECTION A

Topic	*Comments*
Length	
A. Development of vocabulary, and experience in handling various materials.	Free and directed play with a wide variety of materials.
B. Sorting, leading to comparison and equality.	
Volume and Capacity	
As for **Length**	As above.
Weight	
A. As for **Length A** and **B.**	As above.
B. Introduction of balances.	
Time	
Establishing a sense of routine through regular daily activities.	
Money	
A. Ideas of barter.	Playing shop, at first undirected, then exchanging one object for one coin.
B. The need for money.	
Spatial Relations	
A. Knowledge of own school area.	
B. Ideas of left and right.	At this stage limited to the child's own person.
C. Matching of simple regular and irregular shapes.	Use of inset boards and simple jigsaws. Free play with blocks and shapes of all kinds.
D. Recognition (but not definition) of circle, square, triangle, star.	Sorting shapes, at first undirected, later, according to specified attributes.

SECTION B

Topic	*Comments*
Length	
A. Comparison of lengths.	Measuring the same distance with many different units.
B. Measuring with informal units and selecting an appropriate unit.	Finding equality in length, for example, one stick equal to two sticks.
C. The necessity for a common unit.	
Volume and Capacity	
A. Ideas of conservation—finding differently shaped vessels of equal capacity.	
B. Comparing capacity of different vessels, using informal units.	
C. Need for a common unit.	
D. Selecting containers in which to pack materials.	Sufficient to have a large enough container, for example, a basket for shopping, a box for blocks.
Weight	
A. Finding objects of different weights and of equal weights by handling and by using balances.	For example, one block may equal six shells.
B. Size is not necessarily an indication of weight.	
Time	
A. Names of the days.	Not necessarily in sequence.
B. Awareness of the significant times of the day.	The bell rings at nine o'clock.
C. The clock as a device for measuring time.	
Money	
A. Recognition of coins—1 cent, 2 cent, 5 cent, and 10 cent.	
B. Simple relative values—limited to one-cent coins.	For example, 2 [illegible]ent coins = 1 two-cent coin. 5 one-cent coins = 1 five-cent coin.
C. Shopping to 10 cents—one object and no change involved.	
Spatial Relations	
A. Knowledge of school area and location of equipment.	
B. Ideas of left and right.	Gradual extension beyond the child's own person.
C. Sorting, arranging, matching, and joining shapes.	Jigsaw puzzles, including some involving shape only, for example, square cut into four triangles, circle cut into parts.
D. Recognition (but not definition) of oval, diamond, oblong.	
Statistics and Graphs	
Incidental work such as observation and comparison of the growth of plants provides experience for later development.	

SECTION C

Topic	*Comments*
Length	
A. Formal unit—the foot.	
B. Measuring, using the foot and informal units.	Some children may begin to estimate length.
Volume and Capacity	
A. Formal unit—the pint.	
B. Ideas of conservation related to the pint.	Vessels of different shape can hold one pint.
C. Finding capacity of different vessels, using the pint and other units in common use, and also using informal units.	Units could include cup, spoon, egg-cup, measuring-glass.
D. Selecting appropriate containers in which to pack materials.	Children begin to relate size and shape of material to size and shape of containers.
Weight	
A. Formal unit—the pound.	Materials of different density, for example, sand and sawdust, should be available.
B. Weighing, using the pound and informal units.	The following should be available—various materials; articles weighing one pound or multiples of one pound.
Time	
A. Names of days in sequence. 7 days = 1 week.	Use in weekly calendar.
B. The clock and telling the time in hours.	Explanation of " o'clock ".
C. Awareness of the duration of one hour.	
Money	
A. Recognition of twenty-cent coin. Symbol for cent.	
B. Relative values—limited to two coins.	For example, 2 five-cent coins = 1 ten-cent coin. 2 ten-cent coins = 1 twenty-cent coin.
C. Shopping to 10 cents—more than one object, no change.	2 books at 5 cents each. A book at 5 cents and 1 pencil at 3 cents.
Spatial Relations	
A. Extension of environmental knowledge to include way home.	
B. Left and right.	
C. Sorting, arranging, matching, and joining shapes continued.	Putting shapes together to make other shapes. Pattern work with shapes.
D. Recognition of known shapes by sig… nd by description.	
E. Finding shape in the world around.	
Statistics and Graphs	
Pictorial representation.	For example, growth of plants, recording of weather information.

SECTION D

Topic	*Comments*
Length	
A. Measuring with the foot and with informal units.	
B. Estimating and checking measurements.	
C. Need for the inch.	"Height" corner marked in feet could lead to recognition of the need for the inch.
D. Free ruling.	
Volume and Capacity	
A. Parts of the pint—½ pint and ¼ pint.	
B. Finding capacity of different vessels, using the pint, the ½ pint, and the ¼ pint.	Children should continue to use informal units.
C. Packing materials compactly in appropriate containers.	
Weight	
A. Introduction of the half-pound.	The half-pound may be found by halving the pound.
B. Weighing, using the pound and the half-pound.	Materials of different density should still be used.
Time	
A. Names of the months. Reading the date.	Not necessarily in sequence.
B. Telling the time to half an hour.	
C. 24 hours = 1 day. Noon, midday, midnight.	
Money	
A. Recognition of fifty-cent coin.	
B. Relative values to 20 cents.	
C. Shopping :	
(*a*) No change—up to 20 cents, more than one article.	For example, sandwich 10 cents, cake 5 cents. Present 15 cents ; no change.
(*b*) Change—up to 10 cents, one article only.	For example, apple 4 cents. Present 5 cents ; 1 cent change.
Spatial Relations	
A. Extension of environmental knowledge—location of prominent buildings and landmarks.	Street and road names and terms "left" and "right" should be used freely.
B. Activities with shapes continued.	Children's free ruling will often reveal known shapes.
C. Ideas of symmetry developed from folding.	
D. Recognizing and drawing straight and curved lines.	
Statistics and Graphs	
Pictorial representation.	Use of concrete aids (such as match-boxes) to represent, for example, numbers of boys and girls in a grade or bottles of milk used per day.

SECTION E

Topic	*Comments*
Length	
A. Formal unit—the inch.	
B. Measuring with the foot, the inch, and with informal units.	
C. Estimating and checking measurements.	Formal and informal units.
D. Need for a longer unit.	
E. Free and directed ruling.	For example, (*a*) ruling long and short lines, (*b*) joining dots and measuring the line, (*c*) ruling lines of stated lengths, (*d*) ruling a line on a line.
Volume and Capacity	
As for Section D.	Continuing experience enables children to pack more efficiently.
Weight	
A. Introduction of the quarter-pound.	The quarter-pound may be found by halving the half-pound.
B. Weighing, using the pound, the half-pound, and the quarter-pound.	
C. Symbols—1 lb., ½ lb., ¼ lb.	
D. Finding by estimation articles that weigh approximately 1 lb.	
Time	
A. Names of months. The calendar. Reading and writing the date.	Awareness of the sequence of the names of the months should be developing.
B. Telling the time—to quarter-hour ; to five minutes.	
C. Awareness of the minute.	
Money	
A. Recognition of dollar note. Symbol for dollar.	
B. Relative values to 20 cents.	
C. Shopping :	
(*a*) Up to 10 cents—more than one article.	For example, 1 toy 2 cents, 1 book 4 cents. Present 10 cents ; change 4 cents.
(*b*) Up to 20 cents—one article.	For example, 1 cake 12 cents. Present 15 cents ; change 3 cents.
Spatial Relations	
A. Continue the work of previous sections in environmental knowledge and shapes.	Exercises with paper-folding and with nail boards could be included.
B. Functions of various shapes.	Circle—for wheel.
C. Recognition (but not definition) of solids—ball, cylinder, cone, rectangular solid.	Children should realize that solids are bounded by surfaces. These surfaces should be discussed.
D. Lines—horizontal and vertical.	
Statistics and Graphs	
Extension of pictorial representation.	

SECTION F

Topic	*Comments*
Length	
A. Formal unit—the yard.	
B. Measuring with the yard, the foot, the inch, and with informal units.	Activities should continue to be practical and as far as possible concerned with the child's environment.
C. Estimating and checking measurements. Recording results.	
D. Relationships between the inch, the foot, and the yard. 12 inches = 1 foot. 3 feet = 1 yard.	
E. Ruling and measuring lines to one foot in length. Measurement accurate to $\frac{1}{4}$ inch.	
F. First ideas of perimeter.	Measuring the distance around an object, using string or a tape measure.
Volume and Capacity	
A. Formal unit—the gallon. 8 pints = 1 gallon.	
B. Finding capacity of different vessels, using all known units.	
C. Packing and stacking blocks of uniform size.	
Weight	
A. Formal unit—the ounce (symbol oz.). 1 pound = 16 ounces. $\frac{1}{2}$ lb. = 8 oz. $\frac{1}{4}$ lb. = 4 oz.	The ounce may be found by dividing the pound into sixteen equal parts by the process of progressive halving.
B. Weighing, using the pound, the half-pound, the quarter-pound, and the ounce.	Attention should be drawn to various types of scales
C. Finding by estimation articles that weigh approximately 1 lb., 1 oz.	
Time	
A. Names of the months in sequence. The calendar. 12 months = 1 year.	
B. Telling the time to minutes. 60 minutes = 1 hour.	
C. Awareness of the second.	
D. Estimation of the time taken for specified activities.	
Money	
A. Relative values to one dollar.	
B. Activities with money, limited to one dollar. Economical ways of putting out stated amounts.	
Shopping and giving change.	Change should be given by " counting on ".
Writing dollars and cents.	
Simple money operations (+, —, ×).	Within the limits of the child's pure number experience.
Spatial Relations	
A. Extension of environmental knowledge to include sketch maps (not to scale).	For example, schoolroom, school-ground, surrounding district.
B. Attributes and uses of shapes—pyramid, arch, wedge.	Children should handle, discuss, dissect, and draw both plane and solid figures.
C. Lines—parallel.	
Statistics and Graphs	
Extension of pictorial representation to simple column or bar graphs.	Strips of paper may be used to represent ages, heights, and school activities. Experience in other subjects will provide material suitable for graphing.

Appendix

TOPIC	SECTION A	SECTION B	SECTION C
PATTERN AND ORDER IN THE NUMBER SYSTEM	Sequence of number names ; one-to-one correspondence. Limiting—ordinal aspect to 10. Grouping ; rearrangement ; alteration of group constituents ; limiting number of groups.	Cardinal number (grouping activities continued). Counting.	Value relationships. Counting to 20 (at least). Ordinal aspects to 20.
PLACE VALUE NUMERATION and NOTATION	Recognition of numbers to 10.	Recognition of figures to 10. Writing figures to 10.	Recognition of and writing figures to 20. Recognition of the words to 10.
BASIC PROPERTIES	Equality ; difference.	Informal experiences.	Maintenance.
MATHEMATICAL SENTENCES			Oral reading (equality ; four operations) using the numbers to 10. Recording equations. Interpretation of equations. Creating equations. Solving equations. Manipulation—rearranging and substitution (renaming).
OPERATIONS PROCESSES (ALGORITHMS)	Familiarization with structured material. Vocabulary.	Oral reading (equality, four operations). Understanding the nature of equality and the basic operations (including inter-relations).	Study of operations and their relationships ; Equality and inequality. Addition, multiplication. Subtraction. Division.
FRACTIONS			Introduction of fractions : (*a*) as a relation between two numbers, (*b*) as an operator.
NUMBER FACTS			Experience with the number facts to 10.
PROBLEM SOLVING			Simple stories made up to fit an equation.
ESTIMATIONS and APPROXIMATIONS	Development of ability to make comparisons. Informal approach through size, colour, length, weight. Ideas of equality and difference.	Maintenance.	Estimation associated with the work on measurement. (All measurement should be associated with comparison and estimation.)
MEASUREMENT	Free and directed play with a wide variety of materials. Water and sand play ; clay ; activities using a balance. Routine of the day.	Length : Comparing lengths ; learning to measure using informal units. Volume and Capacity : Directed and free activities with water. Informal units. Packing objects. Weight : Balance activities. Time : Awareness of significant times of day ; the clock as a measure. Names of the days.	Comparison—Estimation and measurement. Informal units continued. Length : Introduction of the foot. Volume and Capacity : Introduction of the pint. Weight : Introduction of the pound. Time : The day. Days in sequence. 7 days = 1 week. Telling the time in hours.
MONEY	Playing shop—exchanging coins or tokens for objects.	Recognition of coins (to 10 cents). Shopping to 10 cents—one object and no change. Simple relative values—limited to one-cent coins.	Recognition of coins to 20 cents. Symbol for cent. Relative values—limited to two coins. Shopping to 10 cents—more than one object, no change.
SPATIAL KNOWLEDGE	Knowledge of own school area. Ideas of left and right. Informal play (blocks, jigsaws). Development of visual pattern appreciation. Recognition of simple shapes : Circle, square, triangle, star.	Knowledge of school area and location of equipment. Shapes : Oval, oblong, diamond. Sorting, arranging, matching, and joining shapes.	Environmental knowledge—way home. Shapes : Descriptive properties using every-day vocabulary. An awareness of attributes of size, shape, colour. Finding shapes.
STATISTICS and GRAPHS		Incidental activities.	Pictorial representation.

APPENDIX.—*continued*

SECTION D	SECTION E	SECTION F
Counting at least to 144. (Use of zero with confidence.) Counting to show pattern and order of the number system. Counting groups; serial counting. Use of visual/concrete aids.	Counting by ones, twos, threes, etc., up to twelves to at least 1,000. Serial counting starting from any number. Counting by odd and by even numbers. Pattern in number. Doubling and halving.	Counting range 1 to 1 million. Memorized group counting by ones, twos, threes, etc., up to twelves within the range of tables. Pattern in number. Doubling and halving. Serial addition and subtraction.
Recognition and writing of figures to 144. Recognition and writing of words to 20.	Recognition and writing of figures to 1,000. Recognition and writing of words to one hundred and forty-four. Place value to 999.	Recognition and writing of figures to 1,000,000 and of words to one thousand. Place value to thousands; the use of extended notation to thousands. Decimal notation to tenths.
Maintenance.	Maintenance.	Commutative property of addition and multiplication. Associative property of addition and multiplication. Inverse relations. Combination of these properties. The distributive property. Activities associated with axioms, for example, properties of constant difference.
Manipulation of equations—rearrangement and substitution of wide variety.	The abstract creation and manipulation of equations (selected numbers to 144). Consolidation and extension of the work of Section D, with lessening use of concrete aids. Equality and inequality (=, ≠, <, >).	Maintenance of skill in the abstract manipulation of equations. Solution of equations, each involving a single operation, by a variety of methods.
The four operations in equations using numbers to 20 and simple fractions.	Extension to include numbers to 144. Doubling and halving. Using all operations in the creation and the manipulation of equations.	Interrelations of the operations. Prerequisite skills and some stages of refinement in the formal setting out of the processes.
Extension of the study of fractions: (*a*) as a relation between two numbers, (*b*) as an operator. Fractions in creative work.	Fractions: As a relation between two numbers, as an operator, as a number less than one; equivalence of fractions.	Maintenance.
First experience with combinations to 20. Extension of experiences with combinations to 10.	Deeper experience with numbers to 20 through carefully directed creative work. Experience with selected numbers to 144. Preparation for automatic response with number facts to 10.	Automatic response in all tables to 144. Automatic response in addition and subtraction facts to 20.
Maintenance.	Maintenance.	Maintenance.
Maintenance.	Estimation and approximation as recommended in work on measurement.	Maintenance.
Comparison—estimation and measurement continued, using formal and informal units. Length: The foot. Free ruling. Volume and Capacity: Pint, ½ pint, and ¼ pint. Weight: The pound and half-pound. Time: Telling the time to the half-hour; noon, midday, midnight. Names of months; reading the date. 24 hours = 1 day.	Length: The inch. Free and directed ruling. Volume and Capacity: Maintenance. Weight: Quarter-pound. Time: Telling time to ¼ hour and then to 5 minutes. Names of the months. Writing the date. The minute.	Length: The yard. Relation between inch, foot, and yard. Ruling lines of specific lengths. Perimeter. Volume and Capacity: Introduction of the gallon and the quart. Weight: Introduction of the ounce; 16 oz. = 1 lb. Fractional parts of the pound. Time: Telling time to nearest minute. Estimating time. Awareness of the second.
Coins: 1c, 2c, 5c, 10c, 20c, 50c. Relative values to 20c. Shopping—no change, up to 20c, more than one article; change, up to 10c, one article.	Recognition of the dollar and of the symbol $. Shopping—up to 10c, more than one article; up to 20c, one article.	Recognition and relative value of all coins. Writing dollars and cents. Simple money operations (+, −, ×) through practical experience with shopping (limit $1). Economical ways of putting out amounts of money.
Environmental knowledge—location of prominent buildings and landmarks. Lines, shapes, surfaces, solids. Ideas of symmetry.	Recognition of solids: Ball, cylinder, cone, and rectangular solid. Lines: Horizontal and vertical.	Consolidation and extension. Simple sketch maps. Shapes and their identification in the child's environment. Lines—parallel.
Maintenance.	Maintenance.	Bar graphs.

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EDUCATION DEPARTMENT, VICTORIA

COURSE OF STUDY

FOR

PRIMARY SCHOOLS

MATHEMATICS, 1967

PURE AND APPLIED NUMBER, SECTIONS G, H, I

300/69.

INTRODUCTION

The Course of Study for Primary Schools in Mathematics appears in three parts. The first part, published in 1964, contained the pure number course, Sections A—F. It concerned the development of pure number ideas with the younger Children. The second part, published simultaneously with this booklet in 1967, contains the applied number course, Sections A—F. This booklet, the third part, completes the primary school course in mathematics. It contains the course in pure and applied number, Sections G—I. It is necessary that all parts of the course should be read in conjunction with one another. The overall development of the primary school course can be seen in the flow chart appended to this booklet.

The course as a whole has been influenced by data from both overseas and interstate sources. A further important factor in its development has been the information gathered from classroom trials in Victorian schools. It should be realized that additional research findings and further developments in teaching techniques will influence future modifications.

In implementing this course the following points should be borne in mind :

1. The aims and the discussion on methods presented in the introduction to the *Course of Study for Primary Schools : Mathematics*, 1964, apply to the course as a whole from Section A to Section I.
2. The two aspects of mathematics—pure and applied number—are not unrelated parts and should not be seen as such by children. The child's awareness of mathematics should involve more than the recall of number facts and the use of formal processes. He should understand how mathematics is related to the world about him and how mathematics can give him a better understanding of that world.
3. This course has been planned so that most children should complete Section H at the end of the Grade VI year. However, it is recognized that children learn at different rates ; hence Section I has been included to cater for the child of better than average ability. It provides for enrichment and acceleration. Enrichment is provided for by topics which are not essential to the basic mathematical ideas already mastered by the child, but which add greater depth to understanding. Teachers should make an appropriate selection from the topics listed. In addition to maintaining skills and understandings developed in earlier sections, this section provides the opportunity for the child to experience the basic topics at a more advanced level.
4. The development of the concept of number is a continuous process that extends from the early years of the primary school to the later years of secondary education. This course provides a suitable foundation for work undertaken in the secondary school.
5. Many of the topics included in this course correlate closely with work done in other subject areas. Every opportunity should be taken to make use of such links to aid the development of mathematical understandings, to practise skills, and to illustrate the practical nature of the subject.

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SECTION G

Topic	*Comments*
Pattern and Order in the Number System	
A. Counting forwards or backwards, any reasonable sequence of numbers within one million— by ones ; by twos, threes, . . . and all the numbers to twelves ; by tens, twenties, thirties, and so on to nineties ; by hundreds ; by thousands.	Reference should be made to the *Course of Study for Primary Schools : Mathematics*, 1964, Sections D, E, and F. Oral counting must be purposeful, and activities should be limited to a specified range of numbers. For example : (*a*) Count by ones from 899,986 to 900,005 (*b*) Count by thousands from 1,871 to 24,871 (*c*) Count by fifties from 938 to 1,938 (*d*) Complete the following sequences : (i) 2, 5, 8, *, *, *, 20 (ii) 56, 49, *, *, *, 21 (iii) 4, 11, *, *, 32 (iv) 51, 44, 37, *, *, *, 9
B. Further study of patterns in number sequences.	For example, complete the following sequences (*a*) 1, 4, 9, 16, *, *, * (*b*) ·7, ·8, ·9, *, *, * (*c*) $\frac{5}{8}$, $\frac{7}{8}$, *, *, *, *, $\frac{17}{8}$ (*d*) 0, 1, 3, 6, 10, *, *, *
C. Doubling and halving numbers.	For example, complete the following : (*a*) 3, 6, 12, 24, *, *, * (*b*) 10, 5, $2\frac{1}{2}$, $1\frac{1}{4}$, *, * (*c*) $\frac{1}{16}$, $\frac{1}{8}$, *, *, 1, *, *, *, * (*d*) $24 \times 16 = 12 \times \square = 6 \times \triangle = 3 \times \nabla = 1\frac{1}{2} \times \diamond$ (*e*) $1\frac{1}{2} \div 8 = \square \div 16 = \triangle$ (*f*) $81 \times 48 = \square \times 24 = \diamond \times 12 = \nabla \times 6 = \triangle \times 3$
D. Serial ideas— addition ; subtraction ; multiplication ; division.	(*a*) $7 + 6 = 13$, $17 + 6 = 23$, $27 + 6 = 33$, $60 + 70 = 130$, $600 + 700 = 1,300$ (*b*) $8 - 5 = 3$, $18 - 5 = 13$, $98 - 5 = 93$, $80 - 50 = 30$, $800 - 500 = 300$ (*c*) $9 \times 4 = 36$, $90 \times 4 = 360$, $90 \times 40 = 3,600$ (*d*) $18 \div 6 = 3$, $180 \div 6 = 30$, $180 \div 60 = 3$, $1,800 \div 60 = 30$
Place Value	
The study of place value extended to the limits— ·01 to 1,000,000.	(*a*) Reference should be made to *Course of Study for Primary Schools : Mathematics*, 1964, Section F, The Extension of Ordinal Ideas (7). (*b*) The introduction to the upper limit was made in Section F. (*c*) Counting activities, oral and recorded, should be regularly taken. Such activities should be limited in range. Ranges should be chosen to illustrate particular features. For example : Count the next twenty numbers beginning at 499,898. (*d*) Following the introduction of decimal notation to hundredths (see (*j*) below), activities such as suggested in (*c*) above should be taken throughout the extended range. For example : (i) Count by tenths from 1·3 to 4·1 (ii) Count by hundredths from ·9 to 1·13

Topic	Comments

(*e*) Although much counting will be by ones, some should be by tenths, hundredths, twos, threes (and so on to twelves), fifties, hundreds thousands.

(*f*) Attention should be drawn to the use of the comma in notation. Its value, both in terms of punctuation and for guidance in the immediate recognition of the periods (units, thousands, and millions), should be appreciated.

(*g*) Extended notation emphasizes the place value of each digit. For example :

(i) 9,134 = □ + ◇ + △ + ▽

(ii) 700 + 10,000 + 4 + 8,000 = □

(iii) □ = 2 × 10,000 + 4 × 1,000 + 8 × 100 + 5 × 10 + 3 × 1

(*h*) Relations within the notation system can be emphasized by techniques of renaming. For example :

(i) 426,315 = □ thousands △ hundreds ◇ units
= □ tens ▽ units
and so on

(ii) Eighteen ten-thousands, three hundreds, and five units = □

(iii) 60,000 can be renamed as : 6 × 10,000
or 60 × 1,000
or 600 × 100
or 6,000 × 10

(*i*) The progressive movement of digits to the left in relation to the decimal point is equivalent to multiplication of a number by 10, by 100, and so on. A similar movement to the right is equivalent to division by 10 by 100, and by other powers of 10.

Thus 113·00 × 10 = 1,130·0
113·00 × 100 = 11,300
1·13 × 10 = 11·3
113 ÷ 10 = 11·3
113 ÷ 100 = 1·13

(*j*) Extension of decimal notation to hundredths. In Section F, children became familiar with tenths expressed in decimal notation. In extending the notation to hundredths and in consolidating Section F work, counting exercises involving the use of structured aids, bead frames, the number line, and the abacus are valuable. Decimal currency (dollars and cents) can be a useful model for decimal notation. The following examples are intended as a guide in the development of this aspect of notation.

(i) Count by hundredths from 3·08 to 3·13. (This example should be read as "three and eight hundredths; three and nine hundredths; three and ten hundredths; three and eleven hundredths"; and so on at this stage).

(ii) Read (orally) the following :
1·19 = one and □ hundredths
1·20 = one and □ hundredths
1·20 = one and □ tenths
1·31 = one and □ tenths and △ hundredth
1·31 = one and □ hundredths
1·09 = one unit + □ tenths + ◇ hundredths

(iii) Write the following amounts as cents :
$3.08, $1.17, $0.15

(iv) Write as dollars :
233 cents, 105 cents, 26 cents

Topic	*Comments*

Basic Properties

The maintenance and the extension of understanding and skill with the basic properties.

Reference should be made to Section F of the course.

Work with the distributive property should be extended so that the child both understands and can utilize the distribution of multiplication over a number of addends,

e.g. $149 \times 7 = (100 + 40 + 9) \times 7$
$= (100 \times 7) + (40 \times 7) + (9 \times 7)$

The child should also be aware that multiplication and subtraction can be linked through the distributive property,

e.g. $394 \times 5 = (400 - 6) \times 5$
$= (400 \times 5) - (6 \times 5)$

The extension of work should also include the instances where the basic properties with whole numbers do not apply,

e.g. *Commutative Property*
$9 \div 3 \neq 3 \div 9$
$14 - 9 \neq 9 - 14$

Associative Property
$8 \div (4 \div 2) \neq (8 \div 4) \div 2$
$9 - (4 - 2) \neq (9 - 4) - 2$

Children should see these properties as a means of relating, co-ordinating, and making judgments about various aspects of the mathematics course rather than as isolated pieces of knowledge. Algebraic generalizations are unnecessary.

Equations

Maintenance of Section F in the following areas :

A. Reading and interpreting equations

B. The manipulation of equations using particularly the technique of substitution

C. Creative work with equations

D. The solving of incomplete equations

If there is likelihood of doubt about *the order of operations*, then brackets should be used,

e.g. $5 + (3 \times 2) = 5 + 6 = 11$, but
$(5 + 3) \times 2 = 8 \times 2 = 16$

If there are no brackets, then the convention is that multiplications and divisions precede additions and subtractions in order, as read from left to right.

e.g. $3 + 6 \div 2 \times 4 - 7 = 3 + 3 \times 4 - 7$
$= 3 + 12 - 7$
$= 8$

The use of a rule such as BODMAS or BOMDAS is to be discouraged.

Complex examples involving combinations of different operations should not be set by the teacher, although children should not be discouraged from using such examples in creating equations.

The use of "of". In a formal exercise involving several operations, "of" is to be interpreted as multiplication. Nevertheless, such types of exercises have little value for the purpose of the course and should not be set by teachers ; "of" should be restricted, in general, to single-operation exercises such as the following :

$\frac{1}{8}$ of $\square = 32$
$\square$ of $30 = 20$

Most cases of manipulation of equations in Section G will arise incidentally in the treatment of other aspects of the course, for example, interrelationship between operations and the recording of relations in spatial relations.

Interrelationships of Operations

A. Addition and multiplication (Multiplication as repeated addition)

Ideas developed in previous sections should be consolidated and extended. Following revision using numbers within the tables, larger numbers can be used.

Examples :

(i) $7 + 7 + 7 = \square \times \triangle$
(ii) $240 + 240 + 240 + 240 = \square \times \diamond$
(iii) $36 \times 4 = \square + \square + \square + \square$

Topic — *Comments*

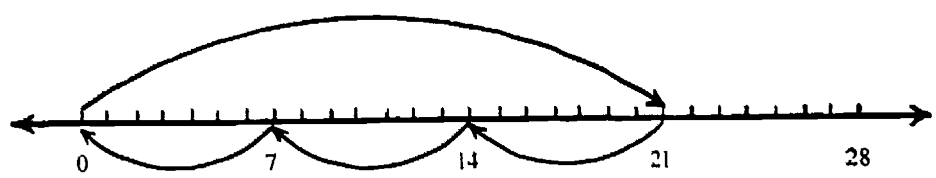
B. Subtraction and division
(Division as repeated subtraction)

Revision and extension of ideas already gained in earlier sections.

Examples :

(i) (*a*) 21 − 7 − 7 − 7 = □
(*b*) 21 − (3 × △) = 0
(*c*) Solve 21 ÷ 7 = □ by subtraction

(ii) Use the information shown in the diagram to write an equation using subtraction



(iii) 51 ÷ 17 = □

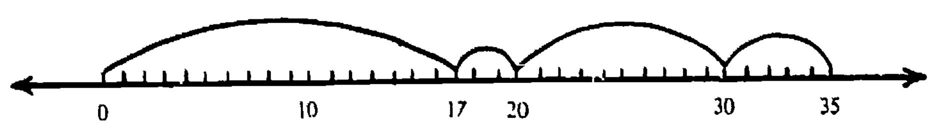
(iv) 28 ÷ 9 = □

C. Multiplication and division
(Inverse operations)

Example : If 13 × 12 = 156
then 156 ÷ □ = 13
and 156 ÷ 13 = □

D. Addition and subtraction
(Inverse operations)

Solve 35 − 17 = □ by addition



17 + (3 + 10 + 5) = 35
17 + 18 = 35
and 35 − 17 = 18

Formal Processes

In Section G, the formal processes of addition, subtraction, and multiplication are further developed. At any one time various stages of refinement may be in use for examples of varying difficulty. The formal process of division, based on successive subtractions, is introduced. The traditional form of long division need not be reached in this section.

Automatic Response Activities

As a necessary prerequisite, automatic response in addition and subtraction facts to 20 and multiplication and division facts to 144 should be reinforced with frequent exercises and variety of presentation. The decade facts (6 + 9 = 15, 16 + 9 = 25, 26 + 9 = 35, and so on) should be well known and used freely in computation.

A. Addition—to 4 addends with thousands

(*a*) Regrouping, 2 addends to 3 digits, in traditional form

Further stages of refinement :

(i)	(ii)
348	348
476	⁵⁸476
14	824
110	
700	
824	

(*b*) Extension of the above techniques to 2 addends of 4 digits

(*c*) Extension of the above techniques to addition involving more than 2 addends. Limit : 4 addends with thousands.

Topic	Comments

B. Subtraction—minuends to 9,999

Using the expression 642 − 164, the following activities may be developed:

(*a*) The complementary approach

$$164 + \square = 642$$
$$164 + (6 + 30 + 400 + 42) = 642$$
$$164 + 478 = 642$$

(*b*) The addition of equals to unequals leaves their difference the same (Also true for subtraction of equals from unequals)

$$\begin{aligned} 642 - 164 &= 648 - 170 \\ &= 678 - 200 \\ &= 478 \end{aligned}$$

(*c*) Some suggested stages of refinement for formal process. Using extended notation and equal additions:

(i)

$$\begin{array}{lrcrcr} 642 & 600 & + & \overset{140}{\not{4}0} & + & \overset{12}{\not{2}} \\ 164 & \overset{200}{\not{1}00} & + & \overset{70}{\not{6}0} & + & 4 \\ \hline & 400 & + & 70 & + & 8 \\ \hline \end{array}$$

(ii)

$$\begin{array}{ccc} 6 & \overset{14}{\not{4}} & \overset{12}{\not{2}} \\ \overset{2}{\not{1}} & \overset{7}{\not{6}} & 4 \\ \hline 4 & 7 & 8 \\ \hline \end{array}$$

(iii)

$$\begin{array}{r} 642 \\ 164 \\ \hline 478 \\ \hline \end{array}$$

(*d*) Extension of the above techniques to subtraction involving numbers to 9,999

C. Multiplication—multiplicands to 999, multipliers to 99

(*a*) Extension of the techniques given in Section F to multiplicands of 3 digits, e.g. 231 × 3

(*b*) Regrouping, multiplicands to 3 digits, multipliers to 9

Some suggested stages of refinement:

(i)

$$\begin{array}{l} 146 \times \;\; 9 \\ 100 + \;\; 40 + \;\; 6 \\ \qquad\qquad\quad \times \;\; 9 \\ \hline 900 + 360 + 54 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 900 \\ 360 \\ 54 \\ \hline 1{,}314 \\ \hline \end{array}$$

(ii)

$$\begin{array}{rl} 146 & \\ \times 9 & \\ \hline 54 & \ldots (9 \times 6) \\ 360 & \ldots (9 \times 40) \\ 900 & \ldots (9 \times 100) \\ \hline 1{,}314 & \\ \hline \end{array}$$

(iii)

$$\begin{array}{r} 146 \\ \times 9 \\ \hline 1{,}314 \\ \hline \end{array}$$

Topic	Comments
	(c) **Regrouping, multiplicands to 3 digits, multipliers to 99**
	Some suggested stages of refinement :
D. Division—dividends to 9,999, divisors to 12	(a) Relation between multiplication and division,

(c) **Regrouping, multiplicands to 3 digits, multipliers to 99**

Some suggested stages of refinement :

(i)
```
       53
   20 + 7
   ------
       21 . . . (7 × 3)
      350 . . . (7 × 50)
       60 . . . (20 × 3)
    1,000 . . . (20 × 50)
   ------
    1.431
```

(ii)
```
       53
     × 27
     ----
       21
      350
       60
    1,000
   ------
    1,431
```

(iii)
```
       53
     × 27
     ----
      371
    1,060
    -----
    1,431
```

D. Division—dividends to 9,999, divisors to 12

(a) Relation between multiplication and division,

e.g. since $12 \times 11 = 132$,
then $132 \div 12 = 11$
and $132 \div 11 = 12$

(b) Relation of division to repeated subtraction,

e.g. $24 - 8 - 8 - 8 = 0$
$24 - 3 \times 8 = 0$
$24 \div 8 = 3$

(c) *The division process.* The computational process of division suggested is based on estimations and successive subtraction. According to his ability, the child can confidently use several partial or separate quotients, which are added to produce the final quotient. This lessens the difficulty of calculation that the child encountered when an exact estimation was required by the traditional method.

With continued experience in division, the child progresses towards an exact estimation in thousands, hundreds, tens, and units respectively. The following activities will assist the child's development of this skill :

(i) $32 \div 8 = 4$
$320 \div 8 = 40$ — 32 tens $\div$ 8 = 4 tens
$3{,}200 \div 8 = 400$ — 320 tens $\div$ 8 = 40 tens

(ii) $3{,}200 \div 8 = 400$
$3{,}200 \div 80 = 40$ — 320 tens $\div$ 8 tens = 40
$3{,}200 \div 800 = 4$

(iii) What is the greatest multiple of 10 that is less than or equal to $246 \div 3$?

One child may develop division thus :

```
3 | 149 |
     30 | 10 . . . (10 × 3)
    --- |
    119 |
     30 | 10 . . . (10 × 3)
    --- |
     89 |
     30 | 10 . . . (10 × 3)
    --- |
     59 |
     30 | 10 . . . (10 × 3)
    --- |
     29 |
     27 |  9 . . . (9 × 3)
    --- | --
      2 | 49
```

Topic	*Comments*
	Another child may work thus :

```
3 | 149
     60 | 20
    ---
     89
     60 | 20
    ---
     29
     27 |  9
    ---------
      2 | 49
```

Some children with a high degree of efficiency in estimation may work thus :

```
3 | 149
    120 | 40
    ---
     29
     27 |  9
    ---------
      2 | 49
```

Fractions

Topic	Comments
A. A fraction as :	
(i) A number greater than 1	e.g. $\frac{7}{5}$, $3\frac{4}{5}$
(ii) A whole number	e.g. $\frac{3}{1}$, $\frac{8}{8}$, $\frac{12}{2}$
(iii) A remainder in division	It is not always possible to divide one whole number by another and get a whole number result, e.g. $15 \div 7 = \square$ The result of dividing 15 by 7 produces two whole numbers—quotient 2, with remainder 1. The remainder 1 is the uncompleted part of division and can be expressed as a fraction of the measure 7 : $15 \div 7 = 2\frac{1}{7}$
(iv) A variation in notation for division	e.g. $27 \div 9 = \frac{27}{9}$
B. Addition and subtraction of fractions :	
(i) Nature of the operations of addition and subtraction in relation to fractions	
(ii) The relationship between the operations of addition and subtraction extended to fractions	
(iii) The development of skill in adding and subtracting fractions—	
denominators up to and including sixteenths, but excluding elevenths and thirteenths ;	
answers need not be expressed in lowest terms ;	
unlike but related fractions using up to three addends ;	e.g. $\frac{5}{6} + \frac{1}{3} + \frac{1}{6} = \square$
examples involving regrouping.	e.g. $1\frac{3}{4} + 1\frac{1}{2} = \square$

Topic	Comments

This work builds upon that undertaken in earlier sections. For many children, concrete representations of the operations of addition and subtraction in relation to fractions will be required. This can be done with rods, with number lines, and with area diagrams.

In solving numerical examples the child may utilize any of a variety of approaches. The following are possibilities in subtraction. In solving the given example, the child will not necessarily record all the steps detailed or adopt the method of recording shown. However, he will be able to explain what he has done.

(*a*) $2\frac{1}{4} - 1\frac{5}{6} = 2\frac{3}{12} - 1\frac{10}{12}$
$= 2\frac{3}{12} - (1\frac{3}{12} + \frac{7}{12})$
$= 2\frac{3}{12} - 1\frac{3}{12} - \frac{7}{12}$
$= 1 - \frac{7}{12}$
$= \frac{5}{12}$

(*b*) $2\frac{1}{4} - 1\frac{5}{6} = 2\frac{3}{12} - 1\frac{10}{12}$
$= (2\frac{3}{12} + \frac{2}{12}) - (1\frac{10}{12} + \frac{2}{12})$
$= 2\frac{5}{12} - 2$
$= \frac{5}{12}$

(*c*) $2\frac{1}{4} - 1\frac{5}{6} = 2\frac{3}{12} - 1\frac{10}{12}$
$= (\frac{24}{12} + \frac{3}{12}) - (\frac{12}{12} + \frac{10}{12})$
$= \frac{27}{12} - \frac{22}{12}$
$= \frac{5}{12}$

(*d*) $2\frac{1}{4} - 1\frac{5}{6} = 2\frac{3}{12} - 1\frac{10}{12}$
$= (1 + 1 + \frac{3}{12}) - 1\frac{10}{12}$
$= (1 + \frac{12}{12} + \frac{3}{12}) - 1\frac{10}{12}$
$= 1\frac{15}{12} - 1\frac{10}{12}$
$= \frac{5}{12}$

(*e*) $2\frac{1}{4} - 1\frac{5}{6} = 2\frac{3}{12} - 1\frac{10}{12}$
$= (2 - 1) + \frac{3}{12} - \frac{10}{12}$
$= 1 + \frac{3}{12} - \frac{10}{12}$
$= \frac{12}{12} + \frac{3}{12} - \frac{10}{12}$
$= \frac{15}{12} - \frac{10}{12}$
$= \frac{5}{12}$

C. Multiplication of fractions

(i) The nature of the operation of multiplication and its relations to other operations

Multiplication of two numbers, one of which is a whole number, is the process of adding equal numbers,

e.g. $5 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
$= \frac{5}{4}$

(ii) The development of skill in multiplying fractions by whole numbers and whole numbers by fractions—

fractions up to and including twelfths ;

whole numbers to ten.

The child is familiar with a fraction operating upon a whole number,

e.g. $\frac{1}{4}$ of $8 = \square$

He must now be led to see that this equation can be written in a slightly different form without changing its meaning,

e.g. $\frac{1}{4} \times 8 = \square$

For " of " he can write " × " and for " × " he can read " of ".

Number lines, rods, and area diagrams can be used to give the child a concrete representation of the process,

e.g. $8 \times \frac{1}{7} = \square$

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Topic	Comments
D. Addition and subtraction of decimal fractions	Where necessary, the child should be provided with concrete experiences of the operations of addition and subtraction in relation to decimals.
(i) The development of understanding and skill in adding and subtracting decimal fractions	The child should be aware that decimals are added and subtracted in the same way as whole numbers are.
(ii) Examples should be limited to addends of units and tenths	The vertical setting out can be used after the equation form has been mastered.
(iii) Three addends can be utilized	Children should be encouraged to check the reasonableness of an answer by using estimation and by using equivalent common fractional forms of decimal fractions.
E. Multiplication of decimal fractions	
The development of understanding and skill in multiplying decimals by whole numbers and whole numbers by decimals	
Examples should be limited to the multiplication of units and tenths by whole numbers and of whole numbers by decimals. Generally, the multiplier should be one of the following : 1, 2, 3, and so on to 12 100, 1,000 ·1, ·2, ·3, and so on to ·9	Children should be guided to make the generalization that, when multiplying by 10 or multiples of 10, the digits in the multiplicand move one place to the left in relation to the decimal point for each zero in the multiplier.

Length

Topic	Comments
A. Individual experiences involving estimation and subsequent measurement of lengths in terms of— inches ; feet and inches to six feet ; and yards and feet to one chain	The use of personal, individual techniques for estimation and measurement should be encouraged. Estimation may, in the early stages, be assisted by (for example) hand spans or paces. Such devices should be replaced by mental comparisons with familiar lengths as soon as practicable. Ideas of "rounding off" should be encouraged and further developed. Answers such as "About 5 yards", "About one chain", or "About $2\frac{1}{2}$ feet" are desirable.
The significance of precision involved in the use of units of measurement should be appreciated	Activities should include the measurement of curves. Techniques used for this may include the use of string or the use of the wheel.
B. Introduce the chain and the mile	The relationship between the chain and the yard can be established by actual measurement.
C. Tables : 36 inches = 1 yard 22 yards = 1 chain 80 chains = 1 mile and relationships such as : 6 inches = $\frac{1}{2}$ of 1 foot 6 inches = $\frac{1}{6}$ of 1 yard	Relationships within the length tables should be developed after the child is familiar with each larger unit. Where possible, these relationships should be *discovered* through simple problems such as : "How many match-boxes ($1\frac{1}{2}$ inches wide) are needed to measure one foot ?"
D. Ruling and measuring lines to one foot in length. Measurement accurate to $\frac{1}{16}$ inch	Lines should be ruled with increasing accuracy of length, on both lined and unlined paper. Measurement should not be restricted to lines in horizontal planes, but should be extended to the measurement of lines in oblique and vertical planes.
E. Further experience in the estimation and the measurement of perimeter	Perimeters of rectangular and non-rectangular shapes should be examined. Methods employing actual measurement should be used.
F. Problems involving simple one-step reduction	Some problems should cater for actual measurement. Recorded work should be expressed in equation form where appropriate. Children's experience with basic laws can be enriched and applied in problems involving simple reduction. The number line is a most useful model in problem activities.

Topic	*Comments*
Area	
Planned activities to promote ideas of area : (i) A surface has two dimensions (ii) Two or more areas can be compared (iii) Different shapes can have the same area. (The area of a shape is not altered if the shape is cut and rearranged) (iv) The unit for measuring a region of a surface is an areal unit (v) Some unit shapes are suitable for measuring regions, others are not.	Activities may include colouring or covering areas, or producing new shapes (of equal area) by paper cutting or block rearrangement. Opportunity should be taken to involve the sense of touch in the appreciation of surfaces of different textures. Simple jigsaws, involving some regular geometric shapes, both with and without " picture " assistance, are useful activities. " Patterned " surfaces may be compared as regards area. Such surfaces may include dress materials, wall-papers, book covers, tiled areas, and linoleum. Shapes which are not " patterned " may be compared by methods such as— (*a*) superimposing one shape on another ; (*b*) superimposing a transparency, on which one shape is outlined, on a second shape ; (*c*) matching one shape with paper and, by cutting the paper, covering the second shape as completely as possible ; and (*d*) covering shapes with shapes of uniform size. A variety of shapes should be available for choice, for example, paper, wood, linoleum, and plastic squares and rectangles, small black-boards, newspaper sheets, blocks, and cut-outs in the shapes of circles, quadrants, triangles, parallelograms, and hexagons. Measurement should be made, using informal units. Children should select an appropriate unit for the task and name the area in terms of the unit chosen. Graticules, transparent sheets on which networks of regular shapes are constructed, are useful devices for comparison of areas and for informal measurement.
Volume	
A. Activities to develop further the idea of volume as a measure of space that can be occupied by air, liquids, or solids, or by a mixture of these.	Such activities may involve an extension of water and sand play of earlier sections. Here, activities may involve more than two vessels at one time. Vessels of various shapes should be used. Experiments in which rubber or clay moulds are used to prepare plaster casts will assist in the development of the "space filling" idea. In addition to activities noted above, the use of plastic materials will assist understanding. Children should come to realize that change of shape can occur without change of volume ; in fact, this is normally the case. " Plastic " here means having plasticity.
B. Activities relating volume and weight of substances.	Further experiences should reinforce the notion that volume does not necessarily indicate weight. That is : (*a*) Objects of the same " size " (i.e. volume) may have different weights (*b*) Objects having the same weight may be of different volumes.
C. Informal measurement : (i) Comparison involving relative capacities (ii) Comparison involving packing and counting or counting of already stacked units such as bricks, etc.	Capacity can be used as a reliable indicator of the volume of a vessel. Tipping water or sand from one vessel to another can assist in comparison of volumes. In certain cases, the packing of blocks of uniform size can assist in the comparison of volumes of rectangular shapes. Measurements of certain volumes should be made in Section G in terms of informal units. For example : "This shape contains 108 blocks." "This wall contains 272 bricks."
Capacity	
A. Tables : 2 pints = 1 quart 4 quarts = 1 gallon 8 pints = 1 gallon	The word " quart " is mainly intended for vocabulary use since the quantity is not widely used today.
B. Fractional forms	1 pint = $\frac{1}{8}$ gallon $\frac{1}{2}$ pint = $\frac{1}{16}$ gallon $\frac{1}{4}$ pint = $\frac{1}{32}$ gallon

Topic	*Comments*
C. Practical experience	Experience in comparison, estimation, and measurement, using suitable containers filled with liquids. This is an important aspect of the topic.
D. Reduction : Gallons to pints Pints to gallons	Simple one-step reduction.
E. Problems	These may involve practical experience, reduction, and fractional forms.
Weight	
A. Introduction of stone and ton. (Ton—vocabulary usage.)	Use of the units in everyday life, with some discussion of historical development.
B. Estimation of weight	Estimated weights should be checked by measurement. Children should be encouraged, when estimating, to choose the most appropriate unit for the purpose.
C. One-step reduction and fractional forms	(*a*) A typical development may be : 5 stones 8 lb. $= 5$ stones $+ 8$ lb. $= (5 \times 14)$ lb. $+ 8$ lb. $= (10 \times 7)$ lb. $+ 8$ lb. $= 70$ lb. $+ 8$ lb. $= 78$ lb. (*b*) 2 lb. 7 oz. $= 2\frac{7}{16}$ lb. 6 stones 4 lb. $= 6\frac{4}{14}$ stones $= 6\frac{2}{7}$ stones
D. Simple problems	These may involve one-step reduction, fractional forms, and practical activities.
Time	
A. (i) Tables : 60 seconds = 1 minute 60 minutes = 1 hour 24 hours = 1 day 7 days = 1 week 14 days = 1 fortnight 52 weeks and 1 day = 1 year 52 weeks and 2 days = 1 leap year (ii) Days in each month	Compound quantities should also be expressed in fractional form. e.g. 2 hours 15 minutes $= 2\frac{1}{4}$ hours $2\frac{2}{7}$ weeks $= 2$ weeks 2 days The second should be introduced as a small unit of time.
B. (i) Telling time correct to nearest minute	
(ii) Time and its duration	Time checks. A stop-watch should be used, if available. Estimating and timing actions. Simple time-tables concerned with daily life, for example, school, home (breakfast-time, bedtime, television times).
(iii) The calendar	Reading the calendar and recording the date.
C. Simple reduction activities—one-step	e.g. $2\frac{1}{4}$ hours $= 2\frac{1}{4} \times 60$ minutes $= (2 + \frac{1}{4}) \times 60$ minutes $= (2 \times 60 + \frac{1}{4} \times 60)$ minutes $= (120 + 15)$ minutes $= 135$ minutes
D. The story of time	

Topic	*Comments*
Money	
A. Recognition of all coins and notes. Writing amounts in dollars and cents.	
B. Money relationships :	
(i) Dollar–cent relationships	For example : $1.00 = 100 cents ; $4.63 = 463 cents ; 300 cents = $3.00 ; 296 cents = $2.96
(ii) In terms of various coins	For example : $2.35 = □ fifty-cent coins + △ ten-cent coins + ◇ five-cent coins
(iii) Fractional parts	For example : 7 cents / $0.07 } = $\frac{7}{100}$ dollar
C. Processes : +, −, ×, ÷ Oral to $1 Recorded to $10 Multipliers and divisors to 10	Processes should be carried out in dollars and cents, not as decimal operations at this stage. Children should be introduced to processes in money following their understanding of them in pure number, e.g. $3.54 ÷ 6. Rename $3.54 as 354 cents and divide $3.54 ÷ 6 = 354 cents ÷ 6 = 59 cents = $0.59 or 59c
D. Problems—oral and recorded	Shopping activities such as listing articles (to three items) and totalling prices.
Spatial Relations	
A. Techniques for, and practice in, the accurate ruling and measurement of lines.	Reference should be made to Sections E, F, and G, Length.
B. Some properties of common shapes. Shapes considered should include rectangles, triangles, circles, hexagons, ellipses, and a variety of irregular plane shapes ; rectangular and triangular pyramids and prisms, cylinders, and cones.	Descriptive properties only should be considered. For example : (*a*) Spheres have curved surfaces and no edges (*b*) A cube has twelve edges of equal length (*c*) A cone has one curved surface and one flat surface (*d*) All triangles have three sides Understanding of the properties of common shapes should come from examination and discussion of familiar things such as a chalk box, an ice-cream cone, a basket-ball, an ice-cream dispenser, a tin can, and so on.
C. The construction of shapes made up from triangles and rectangles.	Unit shapes may be cut from paper or card, or be plastic or wooden, for example, coloured plastic "inch" squares. Constructional activities form a desirable introduction to tessellation and the study of area. (See Section G, Area.)
D. The use of nets to assist with the discovery that relatively complex patterns and shapes may be composed of smaller basic shapes, and, also, that such patterns and shapes may be decomposed in some cases into smaller, simpler units.	(*a*) A net is an overall pattern of identical shapes such as squares, rectangles, diamonds, or triangles. (*b*) Nets may be prepared on paper or on transparent sheets. Such prepared nets should be used for activity work until the child has developed sufficient skill to prepare them, and frequently after this time to allow most of the activities to be directed to creative work and discovery.
E. Angles and their measurement in terms of turning through full turns, half turns, and quarter turns. The right angle	(*a*) The idea of an angle should come from the child's own experiences of changes of direction such as— (i) the changing direction of the sun during the day ; (ii) changes in direction of travelling to and from school. (*b*) A right turn is another name for " quarter turn ". Here a turn means a change in direction like that of the minute-hand of a clock during an hour—a complete rotation.

Topic	Comments
	(c) The idea of right angle can be developed from that of the right turn. Right angles can be formed by paper folding, and models so obtained can be used for comparison purposes.
	(d) It is not intended that relatively small units of angle measurement, for example, the degree, should be introduced to children at this time.
F. The ability to give accurate, simple directions to specific locations, to prepare simple maps, and to follow spoken or written directions.	(a) Orientation of the school grounds in terms of north, south, east, and west. The possibility of integration with social studies should be considered.
	(b) Locations should lie within the familiar environment of the child; for example, within the classroom, the school-ground, or in areas adjacent to the school.
	(c) Child-made, fictitious maps should be encouraged, and study and discussion of prepared maps such as street-directories should be carried out.
Statistics and Graphs	
A. Pictorial graphs	These graphs should be based on facts and figures arising from the pupils' own activities at school and at home,
B. Bar graphs	e.g. class statistics can provide much information for graphing—sex, height, weight, age, month of birth, and so on.
	Number lines can help in the construction of simple histograms. Simple graphs can be built up with crosses above the numeral to represent the data.

SECTION H

Topic	*Comments*
Pattern and Order in the Number System	
A. Further development of the child's ability to count backwards and forwards through any reasonable sequence of numbers to 10,000,000— by ones, twos, threes . . . twelves ; by tens, twenties, thirties . . . nineties ; by hundreds, two hundreds, . . . nine hundreds ; by thousands.	Refer to Section G, Pattern and Order in the Number System.
B. Further study of patterns in number sequences	In presenting sequences to children, sufficient terms should be given to suggest a pattern, and then a further term given to act as a check.

Sequences such as the following are suitable :

(*a*) 2, 3 ; 4, 6 ; 6, 9 ; *, * ; *, *; *, *; 14, 21

(*b*) 7 ; 16 ; 25 ; 34 ; * ; * ; * ; 70

(*c*) $7\frac{1}{2}$; * ; * ; 3 ; $1\frac{1}{2}$; 0

(*d*) 9 ; 90 ; * ; * ; * ; 900,000

(*e*) 7,500 ; 750 ; 75 ; * ; * ; ·075

(*f*) 169 ; 144 ; * ; * ; 81 ; * ; *

(*g*) 1 ; $\frac{3}{4}$; $\frac{5}{8}$; $\frac{7}{16}$; * ; * ; * ; [illegible]

(*h*) 1 ; 1 ; 2 ; 3 ; 5 ; 8 ; * ; * ; * ; 55

C. Pattern in measurement

Where measurement results in sets of numbers having a relationship, children should be encouraged to discover this relationship.

For example, the following table could represent measurements made of the circumferences and the diameters of various circular areas :

Circumference	*Diameter*
$22\frac{1}{4}$ inches	7 inches
11 inches	$3\frac{1}{2}$ inches
$37\frac{3}{4}$ inches	$12\frac{1}{4}$ inches
440 paces	140 paces

A reasonable discovery would be that in each case the circumference is about three times the length of the diameter.

D. Pattern and number facts

Magic squares and other similar devices employing one or more of the operations are valuable aids for the development of a sense of pattern and for the mastery of number facts.

As an enrichment activity Roman notation and some research into, and discussion of, Roman and Chinese number systems could be undertaken.

Place Value

The study of place value extended to the limits—·001 to 10,000,000

Although the pattern of decimal notation is quite familiar to the majority of children at this level, this should not be taken for granted. Exercises, as suggested for Section G, should be used and extended to develop a fuller understanding of the number system.

The abacus and the bead-frame are two aids which can assist in this development.

Topic	Comments

Basic Properties

Maintenance and extension of understanding and skill with the basic properties

In this section, the distributive property of division over addition receives emphasis. This property is important in the understanding of the short division algorism,

e.g. $342 \div 3 = (300 + 30 + 12) \div 3$
$= (300 \div 3) + (30 \div 3) + (12 \div 3)$

Work with the basic properties is extended to include rational numbers (see Section H, Fractions).

Application of the basic properties will be found useful in computational activities with compound quantities.

Equations

A. Reading and interpretation of equations

Comments made in preceding sections apply in Section H.

B. The manipulation of equations, using particularly the technique of substitution

The manipulation of equations will arise, generally, when treating other aspects of the course. This is shown, for example, in the comments made on the following topics :

Basic properties.

Multiplication and division of fractions.

Division of decimal fractions.

C. Creative work with equations

D. The solving of incomplete equations

Formal Processes

A. Maintenance and extension of the child's understanding and skill in the four processes :

(i) Addition—sums to 100,000
(ii) Subtraction—minuends to 100,000
(iii) Multiplication—
multiplicands to 10,000,
multipliers to 100
(iv) Division—
dividends to 10,000,
divisors to 100

In Section H, the formal processes of addition, subtraction, multiplication, and division, as these apply to whole numbers, should be mastered.

As a necessary prerequisite, automatic response in addition and subtraction facts to 20 and multiplication and division facts to 144 should be reinforced with frequent exercises and variety of presentation. The decade facts ($6 + 9 = 15$, $16 + 9 = 25$, $26 + 9 = 35$, and so on) should be well known and used freely in computation.

Children should be encouraged to check the reasonableness of an answer by "rounding off", e.g. 76×84. "My estimate is $80 \times 80 = 6,400$. After computation, does 6,384 seem a reasonable result ?"

The method of successive subtraction should be extended to divisors not greater than 100.

By the end of this section, the child's estimates of the partial divisions should be sufficiently exact to allow him to set out his working in the traditional form.

```
                              261
                              ---
                                1
                               60
                              200                 261
(a) 32 | 8357 |        (b) 32 | 8357      (c) 32 | 8357
         6400 | 200             6400               6400
         ----                   ----               ----
         1957 |                 1957               1957
         1920 |  60             1920               1920
         ----                   ----               ----
           37 |                   37                 37
           32 |   1               32                 32
         ----                   ----               ----
            5 | 261                5                  5
```

Topic	*Comments*
B. Introduction of the process of short division— dividends to 1,000, divisors to 12.	The method of short division utilizes— (*a*) the distributive property of division over addition (see Section H, Basic Properties), (*b*) renaming the dividend in suitable multiples of the divisor. Short division may be introduced through the use of both structured and concrete aids. Carefully graded examples are necessary, beginning with examples that require no regrouping, for example, $488 \div 4$.

$$4 \,\overline{)\, 400 + 80 + 8} \qquad\qquad 4 \,\overline{)\, 488}$$
$$100 + 20 + 2 \qquad\qquad 122$$
$$122$$

For more difficult examples involving regrouping, a suggested development may be as follows :

(*a*)
$$\begin{aligned} 756 \div 4 &= (400 + 356) \div 4 \\ &= (400 + 320 + 36) \div 4 \\ &= 100 + 80 + 9 \\ &= 189 \end{aligned}$$

(*b*)
$$4 \,\overline{)\, 400 + 320 + 36}$$
$$100 + 80 + 9$$
$$189$$

(*c*)
$$4 \,\overline{)\, 756}$$
$$189$$

Later examples should involve remainders.

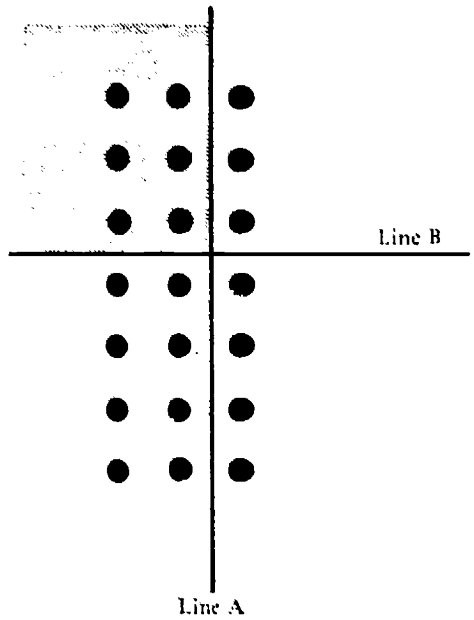
Topic	*Comments*
C. Problems within limits prescribed above (not more than two operations).	Problems should be carefully graded in difficulty of computation and difficulty of reading for understanding.

Fractions

Topic	*Comments*
A. The nature of a fraction	Maintenance of understanding as developed in earlier sections.
B. Addition and subtraction of vulgar fractions The computational skill developed to solve addition and subtraction examples should be maintained and extended to include— three addends, denominators up to thirty, the expression of answers in lowest terms.	For example, $\frac{3}{4} + \frac{1}{3} + \frac{5}{8} = \square$ $1\frac{4}{7} - \frac{3}{4} = \square$
C. Multiplication of vulgar fractions Maintenance and extension of skill in multiplying fractions, to the multiplication of a fraction by a fraction. The number of factors should be limited to two. Factors that are mixed numbers can be utilized.	This is an extension of work undertaken in Section G. The generalization that the child may derive from his experiences is that, when multiplying two fractions, he multiplies the two denominators to make a new denominator, and then multiplies the two numerators to make the new nume

Topic	*Comments*
Denominators of fractions used in examples need not be greater than twelve (with the exception of hundredths multiplied by whole numbers).	To aid the establishment of this generalization, the child could be presented with numerous examples which make use of number lines, rods, arrays, and area diagrams,

e.g. $\frac{3}{7} \times \frac{2}{3} = \square$



The unit of 21 pegs or dots is set, and the child is asked to use it in solving the given problem. Two-thirds of the array is established. A pencil could be used to indicate this (Line A). The child's attention is then directed to the two-thirds and he is asked to find three-sevenths of this section of the array. This can also be indicated with a pencil (Line B). The isolated section of the array is then related to the whole— $\frac{6}{21}$.

Working from a number of examples in which whole numbers are renamed as fractions may help some children to discover the relevant patterns.

e.g. $3 \times 2 = \square$

$3 \times 2 = 6$

↓ ↓

$\frac{9}{3} \times \frac{4}{2} = \frac{36}{6}$

$\frac{36}{6}$ is another name for 6

Techniques for shortening computation will not be required in this section.

D. Division of vulgar fractions

Division of a fraction by a whole number and of a whole number by a fraction. Denominators of fractions used in divisors should not be greater than twelve.

This topic draws upon a considerable number of mathematical understandings developed in other sections of the course.

Mechanical methods should be avoided when presenting this topic to a child. While the generalization "invert the divisor and multiply" may eventually be derived, it will be upon the basis of understanding rather than by rote learning.

When the child has completed this stage he should realize:

(*a*) When the divisor decreases the quotient increases

$8 \div \frac{1}{2} = 16$

$8 \div \frac{1}{4} = 32$

(*b*) If the divisor remains constant and the dividend decreases the quotient decreases

$\frac{1}{2} \div 2 = \frac{1}{4}$

$\frac{1}{4} \div 2 = \frac{1}{8}$

$\frac{1}{8} \div 2 = \frac{1}{16}$

Topic	*Comments*

(*c*) There are various notational ways of expressing a division situation
$3\overline{)12}$, $\frac{12}{3}$, and $12 \div 3$

(*d*) For every fractional number, $\frac{a}{b}$ (when neither a nor b equals zero), there is another fractional number $\frac{b}{a}$, so that their product is one.

This is the reciprocal property

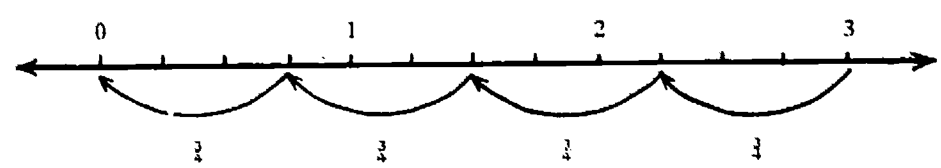
$\frac{3}{4} \times \frac{4}{3} = 1$ $\frac{2}{1} \times \frac{1}{2} = 1$

In developing the topic, use may be made of :

(*a*) Division as successive subtraction. This approach could be used to introduce division of whole numbers by fractions,

e.g. $3 \div \frac{3}{4} = \square$

can be seen as :



(*b*) Division as the inverse of multiplication,

e.g. $4 \div \frac{1}{8} = \square$

can be seen as

$\frac{1}{8} \times \square = 4$

As the problems increase in difficulty, this method of finding the correct solution becomes less satisfactory.

(*c*) An algorism for division of fractions :

$\frac{3}{4} \div 5 = \square$

$\frac{3}{4} \div 5 = \frac{\frac{3}{4}}{5}$ Change in notation

$= \frac{\frac{3}{4}}{5} \times 1$

$= \frac{\frac{3}{4}}{5} \times \frac{\frac{1}{5}}{\frac{1}{5}}$ Renaming 1 as $\frac{\frac{1}{5}}{\frac{1}{5}}$

$= \frac{\frac{3}{4} \times \frac{1}{5}}{5 \times \frac{1}{5}}$

$= \frac{\frac{3}{4} \times \frac{1}{5}}{1}$ Multiplication of the denominator, 5, by its reciprocal, $\frac{1}{5}$, gives a new denominator of 1

$= \frac{3}{4} \times \frac{1}{5}$

$= \frac{3}{20}$

The following is an alternative approach which embodies the same fundamental ideas used in the previous example :

$7 \div \frac{2}{5} = \square$

$\frac{2}{5} \times \square = 7$ Division re-phrased in terms of multiplication

$\frac{5}{2} \times \frac{2}{5} \times \square = \frac{5}{2} \times 7$ Both sides of the equation are multiplied by the reciprocal of $\frac{2}{5}$

$1 \times \square = \frac{5}{2} \times 7$

$\square = \frac{5}{2} \times 7$

$\square = \frac{35}{2}$

Topic	*Comments*
E. The basic properties related to vulgar fractions: (i) Commutative property of addition and multiplication (ii) Associative property of addition and multiplication (iii) Identity elements for addition and multiplication (iv) Distributive property of multiplication over addition	At this stage in his development, the child has achieved reasonable skill with fractions. It is now that the justification for his procedures should be questioned more carefully. The degree of awareness the child has about his manipulations with fractions should be made more explicit through an examination of the basic properties. In the main, the investigation of these properties will be done orally through an examination of numerical situations. Children will **not** be required to state these properties in formal or stereotyped definitions.
F. Addition and subtraction of decimal fractions Maintenance and extension of the child's understanding and skill in addition and subtraction of decimal fractions In this section the work is extended to include hundreds, tens, units, tenths, and hundredths. Three addends may be used in addition	This topic presents excellent opportunities for linkage with other aspects of the course. Previously, the child has been encouraged to see money in terms of dollars and cents. At this stage it is appropriate to consider cents as parts of a dollar. This knowledge can then be used to aid understanding of addition and subtraction of decimals in non-money situations.

G. Multiplication of decimal fractions

Maintenance and extension of the child's understanding and skills in multiplication of decimal fractions

In this section the multiplication of decimals by decimals is introduced. Difficulties in computation should be limited Products should be restricted to two decimal places.

Comments:

For those children who require a visual model to assist in building up understanding, graph paper and base ten blocks could be used.

Because of the child's background in vulgar fractions, it may be appropriate to use this knowledge and understanding in developing this topic,

e.g. $\cdot 4 \times \cdot 3 = \frac{4}{10} \times \frac{3}{10} = \frac{12}{100} = \cdot 12$

$7{\cdot}4 \times \cdot 2 = \frac{74}{10} \times \frac{2}{10} = \frac{148}{100} = 1{\cdot}48$

$\cdot 32 \times 7 = \frac{32}{100} \times \frac{7}{1} = \frac{224}{100} = 2{\cdot}24$

Some children may approach the topic through an examination of pattern,

e.g. ·8 of 700
·8 of 70
·8 of 7
·8 of ·7

As a result of his experiences, a child might discover the relationship that exists between the product and two factors. He may be able to explain in his own words that when two decimal fractions are multiplied there are as many decimal places in the product as there are in the sum of the decimal places in the separate factors.

H. Division of decimal fractions

To introduce division of a decimal fraction by a whole number and of a whole number by a decimal

Comments:

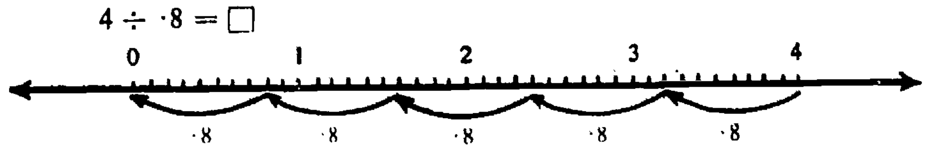
Difficulties in computation should be restricted. Quotients should be limited to two decimal places. All divisions should be exact within these limits. Decimal divisors should be limited to tenths, and whole numbers to known tables.

This topic can be developed in a variety of ways. For some children the starting point would be work with base ten material, graph paper, and number lines. Others may build upon their understanding of the nature of the process of division, work with decimal currency, operations with vulgar fractions, skill in renaming, or identification of pattern.

Some examples are presented below :

(*a*) Division as successive subtraction

4 ÷ ·8 = □



(*b*) $0.80 ÷ 5 = □

Rename $0.80 as 80c

80 cents ÷ 5 = □ cents

(*c*) ·54 ÷ 9 = □

54 hundredths ÷ 9 = 6 hundredths

·54 ÷ 9 = ·06

Topic	*Comments*

(*d*) Division as the inverse of multiplication

$12 \div \cdot 6 = \square$

$\cdot 6 \times \square = 12$

(*e*) Pattern

$300 \div 5 = 60$	$500 \div 100 = 5$
$30 \div 5 = 6$	$500 \div 10 = 50$
$3 \div 5 = \cdot 6$	$500 \div 1 = 500$
$\cdot 3 \div 5 = \cdot 06$	$500 \div \cdot 1 = 5000$

I. Basic properties related to decimals :
- (i) Commutative property of addition and multiplication
- (ii) Associative property of addition and multiplication
- (iii) Identity elements for addition and multiplication
- (iv) Distributive property of multiplication over addition

The extension of this topic to include hundredths presents few new difficulties, but it does provide the opportunity to re-emphasize important mathematical properties and maintain the skills involved in these processes. Children should have a variety of experiences through which they can verify that the mathematical properties previously encountered with whole numbers and vulgar fractions also apply to decimals.

J. Percentage :

Extension of the concept of vulgar and decimal fractions to percentage

No new mathematical understandings or skills beyond those already mastered with fractions are introduced. The new work lies mainly in terminology.

The child should know the notation for percentage, be able to express a percentage as both vulgar and decimal fractions, and be able to express simple vulgar and decimal fractions as percentages,

e.g. $\frac{75}{100} = 75\%$

$\cdot 25 = 25\%$

Length

A. Tables :

8 furlongs = 1 mile

220 yards = 1 furlong

10 chains = 1 furlong

Extend the use of fractional relationships in keeping with the child's understanding of both vulgar and decimal fractions.

Problems involving reduction should generally involve not more than two units.

B. Continued development of the child's ability to make estimations of lengths

Typical estimations may be :
- (i) About two chains
- (ii) About twenty yards
- (iii) About ten chains (after pacing)
- (iv) About one mile (after a time check)
- (v) About four feet
- (vi) About $7\frac{1}{2}$ inches

(Distances can be checked with a hodometer—a counter of revolutions)

C. Perimeter of circular and other shapes

The measurement of perimeter should be continued. Methods used should be appropriate to the shape considered. No formalization of formulæ is necessary.

D. Ruling and measuring lines with a maximum error of $\frac{1}{16}$ inch

E. Processes :

Addition, subtraction, multiplication, and division

Multipliers and divisors should be restricted to whole numbers to 12. The uses of both quotition and partition division are to be well understood.

Developmental steps could well follow those for processes in pure number (Section G).

Techniques such as doubling and halving and renaming could be employed where appropriate.

For example :

19 chains 19 yards × 8 = (20 chains − 3 yards) × 8

= 160 chains − 24 yards

= 158 chains 20 yards

In some cases exercises should be checked by actual measurement.

Topic	*Comments*
F. Problems	Problem situations should frequently be structured to involve processes. In division especially this is very important. Problems should be realistic and challenging to the child. In general, not more than two units should be employed in one exercise. Children should be encouraged to examine problems with a view to the discovery of the most economical method of solution.
Area	
A. Informal measurement	See Section G. Measurement employing informal units should continue both during the introduction to formal units and later in conjunction with formal units. Comparison between the size of regions should continue, and measurement may involve formal or informal units.
B. Formal units : The square inch The square foot The square yard 144 sq. in. = 1 sq. ft. 9 sq. ft. = 1 sq. yd.	The advantages of the establishment of formal units for ease in communication should be considered. The advantages contained in the choice of a " square " formal unit should be understood. Children should not confuse area measurement with shape. For example, a square foot is a unit of measurement and may take any shape. A region one foot square, however, is a specific shape and size—a square with sides one foot in length.
C. Computation of area Rectangles and right-angled triangles	Calculations for the measures of area, either rectangles or shapes made up from rectangles, should initially consist of counting square units. Later, it should be discovered that the measure can be calculated by multiplying the number of rows by the number of units per row. The area measure for the triangle can be calculated by considering a right-angled triangle as forming half of the corresponding rectangle.
D. Perimeter and the measure of area	Children should be aware that both of these measures may concern one shape. The distinction between the nature of these two units of measure should be quite clear—perimeter is a linear measure, area is not.
Volume	
A. Maintenance of understandings developed in Section G	
B. A formal unit : The cubic inch	As in the cases of length and area, the child must realize the need for a formal unit for measuring volume. By using cubes with one-inch edges, volume can be measured. Economical counting methods lead to volume being calculated as : "Number of units in a row × number of rows × number of layers".
C. Estimation activities	Estimation activities should involve simple three-dimensional shapes with rectangular faces, and which can be measured by— (*a*) packing with cubes, or (*b*) breaking the shape down to cubic-inch blocks.
Capacity	
A. Tables : 20 fluid ounces = 1 pint	
B. Fractional forms	For example, (*a*) 30 fl. oz. = $1\frac{1}{2}$ pt. = $1\cdot5$ pt. (*b*) 2 gallons 2 pints = $2\frac{1}{4}$ gallons = $2\cdot25$ gallons.
C. Practical experience	Experience in comparison, estimation, and measurement, using suitable containers filled with liquids, powders, or grains should be continued.
D. Reduction—one-step	Development of the formal setting out.

Topic	*Comments*
E. Processes : +, −, ×, ÷ Multipliers and divisors to 12 Two units	Activities leading to the formal setting out of the processes. Fractional forms may be used where appropriate. 9 gal. 4 pt. ÷ 4 = (9 gal. + 4 pt.) ÷ 4 = (8 gal. + 1 gal. + 4 pt.) ÷ 4 = (8 gal. + 12 pt.) ÷ 4 = 2 gal. + 3 pt. = **2 gal. 3 pt.**
F. Problems	Within the limits suggested in D and E.
Weight	
A. Tables. Introduction of tons, hundredweight, and quarters: 20 cwt. = 1 ton 4 qr. = 1 cwt. 28 lb. = 1 qr. or 2 stones 112 lb. = 1 cwt. 2,240 lb. = 1 ton	Use of the units in everyday life, with some discussion of historical development.
B. Fractional forms	2 tons 10 cwt. = $2\frac{10}{20}$ tons = $2\frac{1}{2}$ tons = 2·5 tons.
C. Reduction—one-step	Development of the formal setting out.
D. Processes : +, −, ×, ÷ Multipliers and divisors to 12 Two units	Activities leading to the formal setting out of the processes. Fractional form may be used where appropriate. 2 lb. 11 oz. + 1 lb. 7 oz. = 2 lb. + 11 oz. + 1 lb. + 7 oz. = (2 + 1) lb. + (11 + 7) oz. = 3 lb. + 18 oz. = 3 lb. + (1 lb. + 2 oz.) = 4 lb. + 2 oz. = 4 lb. 2 oz. 2 lb. 4 oz. × 8 = $2\frac{1}{4}$ lb. × 8 = $(2 + \frac{1}{4})$ lb. × 8 = $(2 \times 8 + \frac{1}{4} \times 8)$ lb. = (16 + 2) lb. = 18 lb. 4 tons 16 cwt. 3 tons 9 cwt. 2 tons 13 cwt. 9 tons 38 cwt. = 9 tons + 1 ton + 18 cwt. = 10 tons 18 cwt.
E. Problems	Within the limits suggested in C and D.
Time	
A. Vocabulary : (i) a.m. and p.m. (ii) Century	
B. Skills and understandings introduced in earlier sections should be maintained	Reading and interpretation of time-tables, for example, rail, bus, and airline time-tables.
C. Reduction—one-step	Development of the formal setting out.

Topic	*Comments*
D. Processes : +, −, ×, ÷ Multipliers and divisors to 12 Two units	Activities leading to the formal setting out of the processes. Fractional forms may be used where appropriate. For suggested activities see Section H, Weight, above.
E. Problems	Within the limits suggested in C and D. Enrichment activities such as "The Story of Time" or "History of Clocks"
Money	
A. Recognition of all coins and notes. Writing amounts in dollars and cents.	
B. Money relationships	As for Section G, with extension to include decimal fractions, e.g. 1 cent / \$0.01 } = $\frac{1}{100}$ dollar = 0·01 dollar
C. Processes : +, −, ×, ÷ Oral to \$5 Recorded to \$100 Multipliers to 100 Divisors to 20 Activities leading to short division of money with divisors to 10	In processes, money can be named in either dollars and cents, dollars and decimal fractions of a dollar, or cents. The method used for naming money will depend on individual preference and on the particular situation, e.g. \$25.12 ÷ 16 Rename \$25.12 as 2512 cents \$25.12 ÷ 16 = 2512 cents ÷ 16 = 157 cents = \$1.57 Remainders should be introduced only in real-life problems. An extension of division should include dividing an amount of money by another amount of money (exact division only).
D. Problems—oral and recorded	Shopping activities should include simple invoices to four items, and shopping problems (two operations) within \$100.
Spatial Relations	
Maintenance of Section G topics and their extension to include : (i) Description of angles as right angles, acute angles, or obtuse angles (ii) An understanding of north-east, north-west, south-east, south-west (iii) Pattern composition to involve the use of compass and ruler	Direct comparison using a model of a right angle formed by paper-folding is suggested.
Statistics and Graphs	
A. Construction and interpretation of : Pictorial graphs Bar graphs	The emphasis at this level should be upon interpretation rather than construction of graphs. Suitable activities for graphing should be closely related to the child's environment. (¼″ square graph paper is very useful).
B. Interpretation of : Line graphs	
C. Investigation of random events	Discovering long-term trends in the frequency of events. Activities could include the tossing of objects, the use of traffic counts, and a study of the frequency with which letters of the alphabet occur.

SECTION I

Topic	*Comments*
Pattern and Order in the Number System	
A. Maintenance of skill in counting	
B. Consolidation of the ability to identify patterns in number sequences	
Enrichment—	
History of number, for example, Egyptian and Arabian number systems	
Investigation of the patterns in number, for example, even and odd numbers. Square and triangular numbers. Rectangular and prime numbers.	
Place Value	
Maintenance of skill and understanding of place value developed in earlier sections	
Enrichment—	
Systems with bases other than 10	For example, the binary system.
Indices—as notation for repeated multiplication of factors	
Basic Properties	
The maintenance and extension of understanding and skill with the basic properties	Algebraic generalizations of the basic properties are not necessary.
Equations	
Maintenance of understandings and skills developed in Section H	
Formal Processes	
A. Number fact activities	Maintenance of number facts and tables through frequent exercises.
B. Maintenance and consolidation of the child's understanding of, and skill with, the four processes	Further attention should be given to short division. " Rounding off " to check the reasonableness of an answer should be encouraged.
C. Problems (usually not more than two operations)	
Fractions	
A. The nature of a fraction The child's understanding of a fraction should be extended to include the ideas of ratio and rate	A ratio is a pair of numbers used to express a comparison, e.g. " 5 is to 3 ", that is, $5:3$ or $\frac{5}{3}$. A rate expresses a relationship between a pair of quantities of different kinds, e.g. 25 miles per hour, that is, 25 *miles* in 1 *hour*.
B. Maintenance of understanding and skill in adding and subtracting vulgar fractions Computational limits should be identical with those of Section H, but examples which involve combinations of these two operations can be used.	

Topic	*Comments*

C. Multiplication of fractions

Maintenance and extension of skill in multiplying fractions to include procedures for shortening computation

In general, limits applying to Section H apply to this section. However, examples can be extended to include three factors.

Children should be able to utilize some shortened forms in computation.

The introduction to shortened forms should be made through the renaming of fractions and the basic properties,

$$\text{e.g. } \frac{6}{7} \times \frac{2}{3} = \frac{2 \times 3}{7} \times \frac{2}{3}$$
$$= \frac{2 \times 3 \times 2}{7 \times 3}$$
$$= \frac{2 \times 2}{7} \times \frac{3}{3}$$
$$= \frac{2 \times 2}{7} \times 1$$
$$= \frac{2 \times 2}{7}$$
$$= \frac{4}{7}$$

Later, after considerable experience in renaming and recording numbers, children will derive the traditional procedure for shortening computation when multiplying fractions.

D. Division of fractions

Work in this section is extended to include division of a fraction by a fraction

The extension of understanding and skill required to cover situations of fractions divided by fractions is small. Initial work may be through the common denominator method.

For example, $\frac{3}{4} \div \frac{1}{12} = \square$

becomes "how many twelfths in nine-twelfths ?"

$$\frac{9}{12} \div \frac{1}{12} = \square$$

An alternative approach is to see division in terms of multiplication. This is profitable, because division within the set of fractions is defined as the inverse operation of multiplication. See Section H for an identical approach with division of a whole number by a fraction. After considerable experience, children will discover that some of the steps can be excluded. Further simplification can be introduced through renaming and the use of the basic properties. This work is similar to that discussed under multiplication of fractions.

E. Addition and subtraction of decimals

Maintenance and extension of the child's understanding and skill with addition and subtraction of decimal fractions

Work can be extended to include thousandths. More than three addends can be considered. Examples involving a combination of the two processes should be considered.

F. Multiplication of decimals

Maintenance of understanding and extension of skill in multiplying decimal fractions

In computation examples can be increased in difficulty to include those which have products of three decimal places

G. Division of decimals

Understanding of, and skill with, division of decimals should be extended to include division of decimals by decimals

Computational difficulties should be restricted. All division should be exact and restricted to three decimal places. Divisors should be limited to two digits.

In addition to the activities presented in Section H, the following approach could be used :

Building upon the child's understanding of equivalence of fractions, the denominator can be renamed as a whole number,

$$\text{e.g. } 91{\cdot}8 \div 2{\cdot}7 = \square$$

$$\frac{91{\cdot}8}{2{\cdot}7} \times \frac{10}{10} = \frac{918}{27}$$

Topic	*Comments*
H. Percentage Maintenance of understanding developed in Section H and the extension of skills to include simple calculations of percentage with whole numbers	For example, 6% of 200 3% of 50 16% of $4
Length	
A. Units of length The metre and the centimetre	The arbitrary nature of units of measurement may be discussed. Advantages of both the British and the metric systems of length measurement should be considered. Through actual measurement an approximate relationship between (i) inches and centimetres (e.g. 1" ≈ 2·5 cm.) and (ii) metres and inches (e.g. 1 metre ≈ 39 in.) should be discovered. NOTE.—The idea "approximately equal to" may be symbolized by : ≈ or ≑ or ≃
B. Formal processes in length to include both whole numbers and decimal fractions	Reference should be made to Section H. Process work should, in general, involve not more than two units. Long, complicated exercises should generally be avoided. Process work should frequently result from problems taken from everyday life. Although formal processes are necessary, it is important that children be encouraged to use other techniques where these are suitable to the problem concerned. Such techniques will involve an understanding of the basic properties of arithmetic.
C. Relationships within the tables	Fractional relationships within the tables should be well understood and frequently used. The complexity of these relationships should be governed by the child's understanding of fractions at the Section H level. Problems involving these relationships should be employed frequently and should, in general, be restricted to two units.
D. Estimation and measurement	Activities involving estimation and subsequent measurement should be continued. Centimetres and metres should be used. "Rounding off" to a given precision should be practised, for example, to the nearest centimetre, to the nearest mile, to the nearest tenth of an inch.
E. Perimeter measurement	Perimeters examined should include those of regular and irregular shapes. Children should be encouraged to generalize when calculating the perimeters of rectangular shapes and when measuring perimeters of circles. Thus, a rule for perimeter of rectangular shapes is discovered, and the perimeter of a circle is seen to be approximately three times the diameter.
Area	
A. The measure of area of both rectangular and triangular regions	
B. Units of area and their relationships : 144 sq. in. = 1 sq. ft. 9 sq. ft. = 1 sq. yd.	
C. Perimeter and the measure of area	Children should be aware that various shapes equal in measure of area may have different perimeters. When a sheet of paper is cut into pieces, and these pieces are rearranged to form a different shape, its area remains unaltered, but it may have a perimeter different from that of the original shape.
D. Estimation and measurement	Estimation of the measures of area should continue. Regions such as floors, doors, sheets of paper, and sections of the school-ground can be used for this work. Children should decide on the appropriate unit to be used. Where irregular or circular shapes are used, a practical method, such as counting squares, will be needed for measurement.

Topic	*Comments*
Volume	
A. Key ideas that have been developed in earlier sections should be maintained	
B. The following formal units should be introduced : (i) The cubic foot (ii) The cubic yard (iii) The cubic centimetre	By the end of this section, children should be able to calculate the volumes of simple three-dimensional shapes with rectangular faces. Where possible and where practicable, such calculations should be preceded by estimation and followed by practical measurement.
C. The relationships : (i) 1,728 cubic inches = 1 cubic foot, and (ii) 27 cubic feet = 1 cubic yard should be understood	Children should be able to discover that 1,728 cubic inches equal the measure of one cubic foot and that 27 cubic feet equal the measure of one cubic yard.
Capacity	
A. Maintenance and extension of understandings and skills developed in previous sections	
B. (i) Reduction	Generally one-step.
(ii) Processes : +, −, ×, ÷ Multipliers and divisors to 100 Generally restricted to two units	Fractional forms should be encouraged where appropriate.
(iii) Problems	Within limits suggested in (i) and (ii).
Weight	
A. Tables : 1 short ton = 2,000 pounds	Weights of bushels of oats, wheat, pollard, and bran.
Weights of bushels of oats, wheat, pollard, and bran	Children should appreciate that a bushel is a measure of capacity.
B. (i) Reduction	Generally one-step.
(ii) Processes : +, −, ×, ÷ Multipliers and divisors to 100 Generally restricted to two units	Fractional forms should be encouraged where appropriate.
(iii) Problems	Within limits suggested in (i) and (ii).
Time	
A. Maintenance and extension of understandings and skills developed in previous sections to include : (i) A.D. and B.C. (ii) Leap Year	
(iii) Easter	A variable date in either March or April.
(iv) Time Zones	Greenwich. International Date Line.
B. (i) Reduction	Generally one-step.
(ii) Processes : +, −, ×, ÷ Multipliers and divisors to 100 Generally restricted to two units	Fractional forms should be encouraged where appropriate. Difficult working should be avoided.
(iii) Problems	Within limits suggested in (i) and (ii).
C. Directed projects and activities	Enrichment activities needing more accurate recording of time, as in foot races and team games.

Topic	*Comments*
Money	
Maintenance and extension of understandings and skills with money to include :	
(i) Processes : +, −, ×, ÷ Oral to $10 Recorded to $1,000 Multipliers and divisors to 100	Division should include dividing an amount of money by another amount of money. Approximations should include "rounding off" in decimal currency. For example, Cost of 17⅞ tons of sugar at $50.07 per ton ≈ cost of 18 tons at $50 per ton. So cost ≈ $900.
(ii) Problems (limited usually to two operations)	Shopping activities, within the limits suggested, should include both oral and recorded work. Activities may include simple invoices and business transactions, percentage, ratio, and hire purchase (simplest ideas only). In this context ratio is related to the comparison of prices on a unit basis.
Spatial Relations	
A. Maintenance and extension of skills and understandings developed in previous sections	
B. Angle measurement extended to include ½, ¼, ⅓, ⅙, and ⅛ turns, and multiples of these	Models for these angles can be made from paper-folding using paper or cardboard circles.
C. Consideration of symmetry in terms of natural objects and geometrical shapes	Examples of symmetry about a line should be considered.
D. Constructions and pattern work :	
(i) Rectangle	The rectangle (ruler and set square construction).
(ii) Triangle	The triangle (given the length of each side ; ruler and compass construction).
(iii) Hexagon	The regular hexagon (constructed in a circle of given diameter).
(iv) Octagon	
(v) Developments of common solids— rectangular prisms, triangular prisms, cylinders, cones.	Development nets may be cut and formed into three-dimensional models.
Statistics and Graphs	
A. Graphs Number lines and co-ordinates	Children should be encouraged to construct simple line graphs.
B. Tables and measures of central tendency	Children should recognize the need for organizing data and developing some skill in making and reading tables. They should also be aware of the terms *arithmetic mean*, *mode*, and *median* and of how these are useful in describing data. Examples should be simple and related to the children's interests.

APPENDIX

TOPIC	SECTION A	SECTION B	SECTION C
PATTERN AND ORDER IN THE NUMBER SYSTEM	Sequence of number names ; one-to-one correspondence. Limiting—ordinal aspect to 10. Grouping ; rearrangement ; alteration of group constituents ; limiting number of groups.	Cardinal number (grouping activities continued). Counting.	Value relationships. Counting to 20 (at least). Ordinal aspects to 20.
PLACE VALUE NUMERATION and NOTATION	Recognition of numbers to 10.	Recognition of figures to 10. Writing figures to 10.	Recognition of and writing figures to 20. Recognition of the words to 10.
BASIC PROPERTIES	Equality ; difference.	Informal experiences.	Maintenance.
MATHEMATICAL SENTENCES			Oral reading (equality ; four operations) using the numbers to 10. Recording equations. Interpretation of equations. Creating equations. Solving equations. Manipulation—rearranging and substitution (renaming).
OPERATIONS PROCESSES (ALGORITHMS)	Familiarization with structured material. Vocabulary.	Oral reading (equality, four operations). Understanding the nature of equality and the basic operations (including inter-relations).	Study of operations and their relationships : Equality and inequality. Addition, multiplication. Subtraction. Division.
FRACTIONS			Introduction of fractions : (*a*) as a relation between two numbers, (*b*) as an operator.
NUMBER FACTS			Experience with the number facts to 10.
PROBLEM SOLVING			Simple stories made up to fit an equation.
ESTIMATIONS and APPROXIMATIONS	Development of ability to make comparisons. Informal approach through size, colour, length, weight. Ideas of equality and difference.	Maintenance.	Estimation associated with the work on measurement. (All measurement should be associated with comparison and estimation.)
MEASUREMENT	Free and directed play with a wide variety of materials. Water and sand play ; clay ; activities using a balance. Routine of the day.	Length : Comparing lengths ; learning to measure using informal units. Volume and Capacity : Directed and free activities with water. Informal units. Packing objects. Weight : Balance activities. Time : Awareness of significant times of day ; the clock as a measure. Names of the days.	Comparison—Estimation and measurement. Informal units continued. Length : Introduction of the foot. Volume and Capacity : Introduction of the pint. Weight : Introduction of the pound. Time : The day. Days in sequence. 7 days = 1 week. Telling the time in hours.
MONEY	Playing shop—exchanging coins or tokens for objects.	Recognition of coins (to 10 cents). Shopping to 10 cents—one object and no change. Simple relative values—limited to one-cent coins.	Recognition of coins to 20 cents. Symbol for cent. Relative values—limited to two coins. Shopping to 10 cents—more than one object, no change.
SPATIAL KNOWLEDGE	Knowledge of own school area. Ideas of left and right. Informal play (blocks, jigsaws). Development of visual pattern appreciation. Recognition of simple shapes : Circle, square, triangle, star.	Knowledge of school area and location of equipment. Shapes : Oval, oblong, diamond. Sorting, arranging, matching, and joining shapes.	Environmental knowledge—way home. Shapes : Descriptive properties using every-day vocabulary. An awareness of attributes of size, shape, colour. Finding shapes.
STATISTICS and GRAPHS		Incidental activities.	Pictorial representation.

SECTION D	SECTION E	SECTION F
Counting at least to 144. (Use of zero with confidence.) Counting to show pattern and order of the number system. Counting groups ; serial counting. Use of visual/concrete aids.	Counting by ones, twos, threes, etc., up to twelves to at least 1,000. Serial counting starting from any number. Counting by odd and by even numbers. Pattern in number. Doubling and halving.	Counting range—1 to 1 million. Memorized group counting by ones, twos, threes, etc., up to twelves within the range of tables. Pattern in number. Doubling and halving. Serial addition and subtraction.
Recognition and writing of figures to 144. Recognition and writing of words to 20.	Recognition and writing of figures to 1,000. Recognition and writing of words to one hundred and forty-four. Place value to 999.	Recognition and writing of figures to 1,000,000 and of words to one thousand. Place value to thousands ; the use of extended notation to thousands. Decimal notation to tenths.
Maintenance.	Maintenance.	Commutative property of addition and multiplication. Associative property of addition and multiplication. Inverse relations. Combination of these properties. The distributive property. Activities associated with axioms, for example, properties of constant difference.
Manipulation of equations—rearrangement and substitution of wide variety.	The abstract creation and manipulation of equations (selected numbers to 144). Consolidation and extension of the work of Section D, with lessening use of concrete aids. Equality and inequality (=, ≠, <, >).	Maintenance of skill in the abstract manipulation of equations. Solution of equations, each involving a single operation, by a variety of methods.
The four operations in equations using numbers to 20 and simple fractions.	Extension to include numbers to 144. Doubling and halving. Using all operations in the creation and the manipulation of equations.	Interrelations of the operations. Prerequisite skills and some stages of refinement in the formal setting out of the processes.
Extension of the study of fractions : (*a*) as a relation between two numbers, (*b*) as an operator. Fractions in creative work.	Fractions : As a relation between two numbers, as an operator, as a number less than one ; equivalence of fractions.	Maintenance.
First experience with combinations to 20. Extension of experiences with combinations to 10.	Deeper experience with numbers to 20 through carefully directed creative work. Experience with selected numbers to 144. Preparation for automatic response with number facts to 10.	Automatic response in all tables to 144. Automatic response in addition and subtraction facts to 20.
Maintenance.	Maintenance.	Maintenance.
Maintenance.	Estimation and approximation as recommended in work on measurement.	Maintenance.
Comparison—estimation and measurement continued, using formal and informal units. Length : The foot. Free ruling. Volume and Capacity : Pint, ½ pint, and ¼ pint. Weight : The pound and half-pound. Time : Telling the time to the half-hour ; noon, midday, midnight. Names of months ; reading the date. 24 hours = 1 day.	Length : The inch. Free and directed ruling. Volume and Capacity : Maintenance. Weight : Quarter-pound. Time : Telling time to ¼ hour and then to 5 minutes. Names of the months. Writing the date. The minute.	Length : The yard. Relation between inch, foot, and yard. Ruling lines of specific lengths. Perimeter. Volume and Capacity : Introduction of the gallon and the quart. Weight : Introduction of the ounce ; 16 oz. = 1 lb. Fractional parts of the pound. Time : Telling time to nearest minute. Estimating time. Awareness of the second.
Coins : 1c, 2c, 5c, 10c, 20c, 50c. Relative values to 20c. Shopping—no change, up to 20c, more than one article ; change, up to 10c, one article.	Recognition of the dollar and of the symbol $. Shopping—up to 10c, more than one article ; up to 20c, one article.	Recognition and relative value of all coins. Writing dollars and cents. Simple money operations (+, −, ×) through practical experience with shopping (limit $1). Economical ways of putting out amounts of money.
Environmental knowledge—location of prominent buildings and landmarks. Lines, shapes, surfaces, solids. Ideas of symmetry.	Recognition of solids : Ball, cylinder, cone, and rectangular solid. Lines : Horizontal and vertical.	Consolidation and extension. Simple sketch maps. Shapes and their identification in the child's environment. Lines—parallel.
Maintenance.	Maintenance.	Bar graphs.

SECTION G	SECTION H
Extension of ordinal ideas through number lines, patterns, and charts. Counting forwards and backwards by ones, twos, threes, etc., up to twelves ; by hundreds ; by thousands. Seriation—order in sequences.	Consolidation of Section G. Pattern in number—seriation complicated to include squares. Enrichment : History of number—Roman and Chinese.
Numeration to 1,000,000. Place value to 1,000,000. Decimal notation to hundredths	Numeration and notation to 10,000,000. Decimal notation to thousandths. Writing numbers correctly to one, two, or three decimal places. Place value extended to cover the range of the above notation.
Further experience involving the basic properties—involving both pure number and applied number. In pure number the basic properties can supply alternative methods of attack in situations involving formal processes, as well as leading to the formal setting out of the processes.	Basic properties—as for Section G, with emphasis on their use with fractions and with applied number in activities in conjunction with, and leading to, the formal setting out of processes in applied number. The distributive law of division over addition.
Manipulation of equations—maintenance. The order of operations. Solution of equations to find unknown elements.	Maintenance of Section G, with extension to include equations involving vulgar or decimal fractions in simple form.
Continuation [illegible] leading to the formal setting out of the processes. Formal pr[illegible] Addition to 10,000 (four addends) ; subtraction (minuend [illegible]999) ; multiplication (multiplicands to 999 ; multipliers to 99) . [illegible]n (dividends to 9,999 ; divisors to 12).	Continued development of formal processes (formal development of short division to follow [illegible] understanding of long division). Long division (divisors to 100). Formal processes extended to decimals—multiplication of decimals by decimals (products to two decimal places) ; division of a decimal by a whole number and a whole number by a decimal.
Fractions as numbers equal to or greater than one. Addition and subtraction of vulgar fractions through equivalence. Multiplication of vulgar fractions by whole numbers, and whole numbers by fractions. Decimals (to tenths only)—addition, subtraction, multiplication (decimals by whole numbers and whole numbers by decimals)	Extension of the operations with vulgar fractions to include multiplication of fractions by fractions ; division of a fraction by a whole number, and a whole number by a fraction. Fractional equivalents of compound quantities ; decimalization (where exactly expressible in thousandths). Percentage introduced and linked with decimal and vulgar fractions expressed as hundredths.
Maintenance.	Maintenance.
Problem solving—associated with both pure number and applied number.	Maintenance.
Estimation associated with formal processes ; with measurement as in Section F. "Rounding off".	"Rounding off" at any point within the number range of this section. Estimations in conjunction with process work and measurement.
Length : The chain and the mile. The relationships within the tables. Problems and practical experience involving one-step reduction. Area : Development of basic ideas of area measurement (no formal units or formal rules). Volume : Counting blocks used to construct regular prisms. Capacity : Gallons, quarts, pints—fractional parts—one-step reduction. Weight : The stone ; the ton (vocabulary). One-step reduction. Time : Read the calendar.	Development of formal setting out of the four processes with measures of length, weight, time, and capacity. (Multipliers and divisors to 12 ; 2 units). Exercises involving reduction—generally one step. Length : The furlong. Extension of measurement and estimation techniques. Area : The square inch, square foot, square yard. Rectangular and right-angled triangular surfaces (count area units). Volume : The cubic inch. Rectangular prisms. Practical experience with volume. Capacity : The bushel. Fluid ounce. Weight : Tons, hundredweight, quarters. Time : Terms "a.m.", "p.m.", century.
Recognition and relative value of all coins ; equivalents of a dollar ; simple shopping (limit $1) ; four processes within $10 (multipliers and divisors to 10).	Simple shopping exercises to a suggested limit of $5. Operations within $100 (multipliers to 99 ; divisors to 20). Simple problems (one or two steps) on transactions within $100.
Lines : Accurate drawing and measurement ; measuring curved lines. Angles : Turns—full, half, and quarter. Right angles. Surfaces : Descriptive properties of shapes—rectangle, square, circle, triangle, ellipse, hexagon, and octagon. Solids : Cube, sphere, hemisphere, pyramids. Direction : North, south, east, west.	Compass and ruler activities—geometric pattern based on "networks". Descriptive properties—lines, simple geometrical shapes, common solids. Angles—extension to acute, obtuse. Compass points—N., N.E., S.E., etc.
Pictorial graphs. Number lines.	Pictorial and bar graphs—interpretation and construction. Line graphs—interpretation. Investigation of random events.

SECTION I	TOPIC
Consolidation and further extension of pattern in number. Enrichment : History of number—Egyptian and Arabian. Even and odd numbers. Square and triangular numbers. Rectangular and prime numbers.	PATTERN AND ORDER IN THE NUMBER SYSTEM
Numeration and notation to 10,000,000. Decimal notation to three decimal places. Enrichment : Number systems with bases other than 10.	PLACE VALUE NUMERATION and NOTATION
Maintenance.	BASIC PROPERTIES
Maintenance.	MATHEMATICAL SENTENCES
Maintenance and extension to include division of decimals by decimals.	OPERATIONS PROCESSES (ALGORITHMS)
Extension of the operations with vulgar fractions to include division of a fraction by a fraction. Simplification of fractions. Rate. Ratio. Extension of percentage to include simple computational activities.	FRACTIONS
Maintenance.	NUMBER FACTS
Problems should usually involve not more than two operations.	PROBLEM SOLVING
Further development of ideas expressed in Section H.	ESTIMATIONS and APPROXIMATIONS
Consolidation and extension of Section H process work, with occasional exercises involving three units (multipliers and divisors to 100). Variety in exercises using fractions, the basic laws, and equations. Length : Extension to centimetre and metre. Area : Rectangular surfaces (sq. in., sq. ft., sq. yd.). Volume : Rectangular prisms—counting blocks. Cubic foot, cubic yard, cubic centimetre. Capacity : Consolidation. Weight : Short ton ; bushel. Time : Consolidation of Section H.	MEASUREMENT
Consolidation of previous work. Shopping exercises to a suggested limit of $10. Four operations within $1,000 (multipliers and divisors limited to two digits). One-step and two-step problems to $1,000.	MONEY
Angles—measure of turning—full turn ; $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$ turns, and multiples of these. Constructions : Rectangle, triangle, hexagon, octagon. Pattern work. Development of common solids. Symmetry of natural objects and geometrical shapes.	SPATIAL KNOWLEDGE
Interpretation of circle graphs. Construction of line graphs. Number lines and co-ordinates. Simple measures of central tendency : median, mode, arithmetic mean.	STATISTICS and GRAPHS