ABSTRACT
        This paper examines the fiscal impacts on public
schools of some of the federal aid proposals under consideration by
Congress. The four major discussion sections focus on (1) an outline
of the overall analytic framework, including a conceptual model of
federal, State, and local relationships; (2) a brief presentation of
the theoretical analysis on which the statistical work is based; (3)
the calculation results of the impacts of various aid alternatives
pertaining to local school districts and States; and (4) a discussion
of the overall policy implications, especiall y with respect to the
two most widely discussed federal aid alternatives--general revenue
sharing and direct federal aid to school districts channeled through
the States. (RA)

THE IMPACT OF FEDERAL AID TO STATES

ON FINANCIAL SUPPORT FOR THE PUBLIC SCHOOLS

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INTRODUCTION

During the last few years, both the Congress and the Executive
Branch have been approaching a decision on the federal response to
the fiscal problems of state and local government. According to many
observers, some form of new, large scale federal financial aid to
states is imminent. Until very recently, it seemed that the vehicle
would be a variant of revenue sharing--either completely unres-
tricted aid or block grants for education and other major functions.
Now, since the President's State of the Union Address in January,
attention has shifted to the possibility of direct federal aid for
the public schools, to be financed, at least according to the initial
Administration announcements, by a new value-added tax. It is not
yet clear what the outcome will be as the Congress deals with the
variety of plans now before it. But whatever federal funding mechan-
ism ultimately emerges--whether it provides direct aid to school
districts or general aid to states--one thing is certain: Any form
of federal intergovernmental aid on the scale now being contemplated
will have major effects on financial support for the public schools.

The purpose of this paper is to examine the fiscal impacts on
public schools of some of the federal aid proposals now being dis-
cussed and debated in Washington. "Fiscal impact," in this context,
means changes in expenditures and taxes of school districts and
school-related expenditures and taxes of states that result from
the institution of new forms of federal aid. These fiscal conse-
quences of federal policy are, or should be, important to federal
policymakers and to various groups and segments of the population
who are affected by levels of support for the schools and by state
and local tax burdens.

Presently available information on the consequences of alterna-
tive federal funding schemes is not sufficient to meet the needs of
interested parties. With respect to revenue sharing, for example,
no one is able to say what percentage of shared revenues would be
used for state tax reduction or what percentage would be made
available to local school districts. Nor is it known how much of

the latter would be allocated by local officials to increased school spending and how much to local property tax relief. If that information were available, not only for revenue sharing, but for other alternatives, it might be instrumental in molding the attitudes of school administrators, teachers' organizations, and taxpayers' groups--not to mention state and local legislators--toward rival proposals. Moreover, the same information could well affect the perceptions and assessments of federal decisionmakers and thus have both a direct and indirect bearing on the final federal decision.

The main reason that information on fiscal impacts of aid is not available and is difficult to obtain is that so much depends on the behavior of the state or local aid recipient. Results can rarely, if ever, be predicted from specifications of the aid scheme along. If a state, for example, receives a federal grant "earmarked" for the improvement of education, but responds by reducing state taxes or not raising them as much as it would have otherwise, the net impact of aid on education outlays may be much less than the amount of the grant. In general, adaptive behavior by aid recipients can produce impacts that differ substantially from both the effects intended by the grantor and the nominal effects reported in the account books. Only by constraining aid recipients very tightly--in effect, dictating state-local fiscal behavior in detail-- would it be possible for the federal government to guarantee fiscal outcomes fully in line with federal objectives. Since such an alternative is neither feasible nor desirable, it becomes important to concentrate on aid formula *design*, using information about state-local responses to allocate funds in a way that is likely to accomplish the national purpose.

The central analytical question, therefore, is whether fiscal responses of state governments and local school districts can be predicted. Are those behaviors sufficiently systematic that stable relationships between spending and taxing and such external factors as federal aid can be discovered? So far, the answer from this study is a highly qualified "yes". As will be shown, the fiscal

behavior of local school districts, using statewide aggregates or averages as units of observation, is fairly well "explained" (in the statistical sense) by economic and demographic variables including state and federal aid. The statistical model that derives from that analysis can be used to make at least rough predictions of effects of infusions of outside aid on school spending and local school tax aggregates. Moreover, certain threats to the credibility of statistical expenditure determinant models--especially the lack of consistency that was found in earlier studies when the same model was applied to data for different years--have been partly eliminated by basing the models more closely on results of a theoretical analysis.

The fiscal behavior of state governments themselves is much less well explained. This is not surprising since, in studying the states, we are dealing with individual decisionmaking units rather than aggregates, multi-function rather than single-purpose governments, and governments that vary greatly in structure and responsibilities. Nevertheless, it is disappointing because it means that state aid to school districts, which is one of the important determinants of local educational spending, is itself not very well explained. This does not mean that consequences of federal policies cannot be analyzed, since it is still possible to examine implications of alternative assumptions about state responses, using the available results to delimit the relevant range of behavior, If anything, it points up the importance of aid formula design in assuring that state behavior will be at least generally compatiable with the objectives for which federal money is being provided.

The remainder of the paper is organized into four sections: first, an outline of the overall analytical framework; second, a condensed presentation of the theoretical analysis on which the statistical work is based; third, presentation of the results pertaining to local school districts and states; and finally, a discussion of the overall policy implications, especially with respect to the two most widely discussed federal aid alternatives, general revenue sharing and direct federal aid to school districts channeled through the states.

FRAMEWORK FOR ANALYSIS

STATE-LOCAL INTERACTIONS

The effects of any form of federal aid on financial support for the public schools will depend on fiscal decisions by both state governments and local school districts. The role of each level is most clearly seen in the case of unrestricted federal aid to states. Given complete freedom, each state would determine for itself how much federal aid will be channeled to school districts, how much will be expended by the state for other purposes (either directly or as aid to cities and counties), and how much will translate into reductions in state taxes. Local decisions would then determine how much of the additional money that does become available to districts will go for increased school spending, as opposed to local property tax relief.

It might seem from the above that the state is involved only when it is the direct recipient of federal funds; however, that is not the case. Suppose, for example, that the federal government aids districts directly, completely bypassing the states. It would be reasonable, in that case, to expect state officials, seeing the additional funds available to districts, to revise their own estimates of local needs downward and to respond to the influx of federal funds with a partially offsetting reduction in state aid. That response and its effect on the local districts would have to be considered in measuring the net impact of federal grants.

Moreover, there are other complexities. Even if a state allocated none of its shared revenues to education, local school districts could still benefit. For example, a decision to provide additional state aid to cities or counties could affect school finance by reducing horizontal competition among local governments for property tax revenues. Similarly, a reduction in state income or sales taxes could benefit school districts by making local taxpayers more able and willing to support higher school levies. Because of these multiple interactions, it is necessary to work with a two-sector model of government that is capable of dealing with the stimulus-response relationships in both directions.

SCHEMATIC VIEW OF THE MODEL

The federal-state-local system to be modeled is depicted in
Figure 1, which shows the main fiscal variables that enter into
an analysis of the impacts of federal aid. Note that "other local
governments" (cities, counties, etc.) are acknowledged, but not
treated as a separate sector as they would be in a larger model.
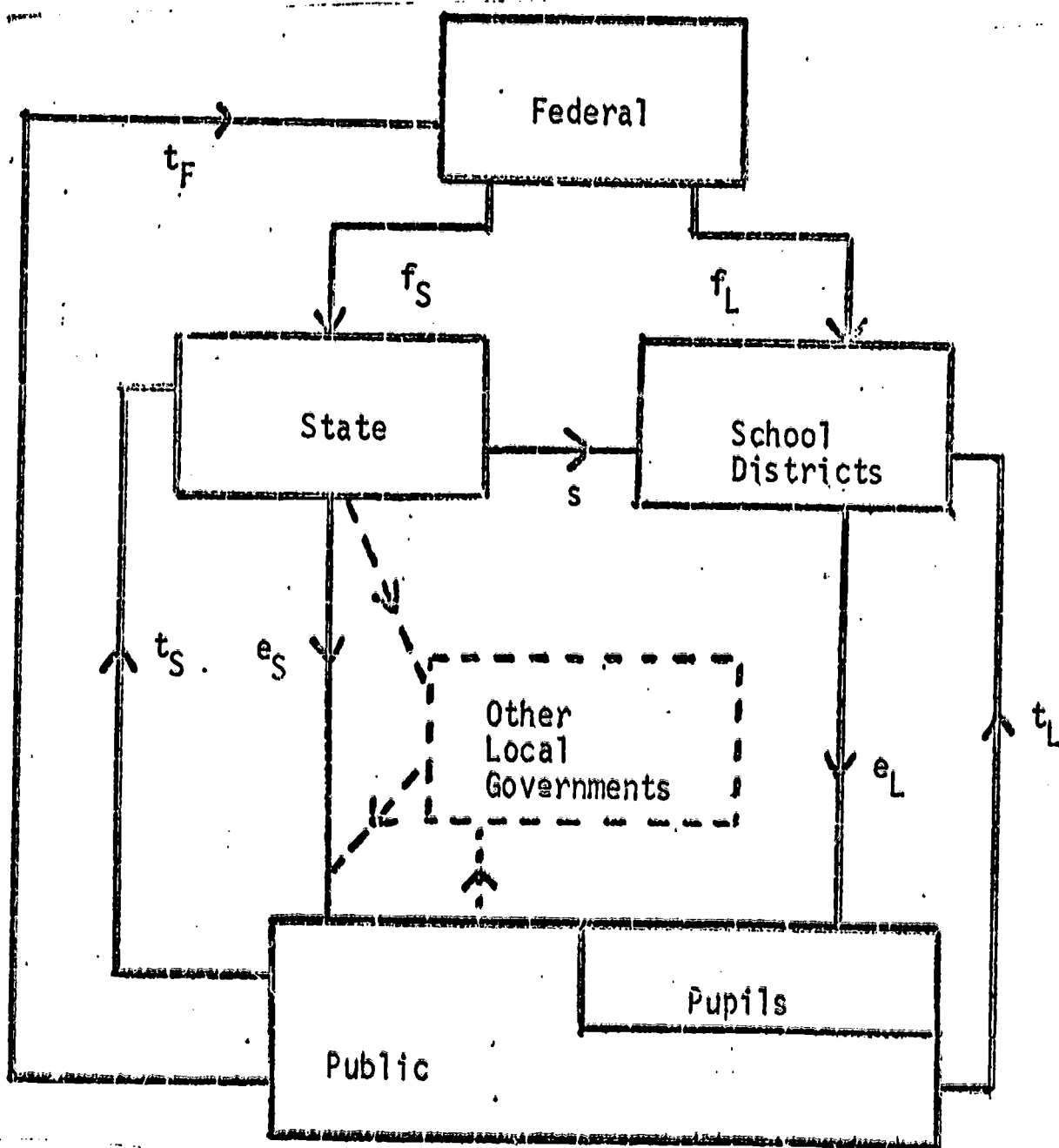Instead, their expenditures and taxes are aggregated with those

Figure 1. Schematic Diagram of the Federal-State-Local System

of the state. This means that all references to "state" expenditures and taxes should be understood as applying to the whole state-local sector, except for local school districts.

Definitions of the fiscal variables represented in the diagram are as follows:

e_L = real expenditure per pupil by local school districts. This is used as the measure of the level of educational services. It is obtained by deflating the dollar amount of expenditure per pupil by P_e, an index of prices of inputs to education.

e_S = real expenditure per capita by state government for all functions other than aid to public schools. It is obtained by deflating the dollar amount of direct state outlays by P_g, an index of prices of inputs to government services other than education.

t_L, t_S, t_F = real per capita tax collections by local school districts, the state government, and the federal government, respectively. In each case, real values are obtained by deflating the dollar amount of per capita taxes by P_O, an index of prices of private goods (general price index). The use of this deflator means that real taxes are measured in terms of the loss of private consumption or purchasing power.

f_L = real federal aid per pupil to school districts.[*] The deflator is P_e, which means that aid is measured in units of purchasing power for educational services.

f_S = real federal aid per capita to the state government. In this case, the deflator is p_g so that aid will be measured in units of purchasing power for other government services.

θ = real state aid per pupil to local school districts, measured in the same units as federal aid to school districts.

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[*]Federal aid to school districts includes aid that is channeled through the state for redistribution to specific local districts according to federal rules.

In terms of these variables, the point of the analysis is to determine how all the state-local fiscal variables, but especially the school district fiscal variables, e_L and t_L, are likely to respond to changes in the federal aid variables and the level of federal taxes. The significance of using a two-stage model of the state-local sector is that the state and local responses will first be worked out separately. That is, we construct a state government model to determine how e_S, t_S, and s respond to changes in f_S and t_F, than a local government model to determine how the state fiscal variables and the relevant federal variables (f_L and t_F) influence e_L and t_L.

LIMITATIONS

Two limiting characteristics of the analysis are implicit in the flow diagrams and the enumeration of variables, but deserve to be made explicit.

One is that the analysis throughout is highly aggregative. This is so in two senses. It is aggregative, first, in that individual expenditure programs are not considered. Only undifferentiated totals of local expenditure, state aid, and direct state expenditure are included in the models. The principal loss is that important differences in the composition of spending from one state to another are neglected although they could be important in assessing effects of new federal aid. The gain, of course, is that one can work with a relatively manageable two-sector model instead of a multi-sector model for which the analytical and data requirements would be far more severe. The model is also aggregative in that the unit of observation is the state, i.e., either the state government itself or the aggregate of all local governments (school districts) within the state. This eliminates the possibility of making statements about differential effects of federal funding on various classes of school systems (e.g., urban, suburban, rural). The task of expanding the model in that direction does not seem particularly difficult and may be attempted as a follow-on to the initial study.

The other major limitation is that this is a "closed" rather than an "open" model of the state-local public sector. The model is closed in that it does not deal with repercussions of state-local decisions on the rest of the economy. Community characteristics are treated as "given." Possible effects of expenditure and tax shifts on aggregate income and of changes in state-local purchases on prices are neglected. Also, possible effects of state-local fiscal decisions on federal finances (e.g., changes in federal tax revenue because of changes in levels of state or local activity) are ignored. An open model would consider some or all of these linkages explicitly. However, to do so would require both a macroeconomic model of income determination and a model of factor markets to deal with demand-induced price changes. It is clearly not feasible to construct so comprehensive a model and, in any case, the effects are probably second-order compared to those that are included.

THEORETICAL MODELS

Behavior of both states and school districts is analyzed within
the framework of the general economic model of constrained maximizing
behavior. It is assumed that decision-makers at each level have
consistent preferences with respect to different combinations of
expenditures and taxes. These preferences are expressed in terms
of a governmental unit's willingness to "trade off" marginal incre-
ments in expenditures for increments in taxes and, if the unit is a
state, willingness to trade off increments in one category of expen-
ditures for increments in another. Government decisionmakers are
assumed to seek an optimal balance among the fiscal variables they
control, subject to a budget constraint that reflects the availability
of grants-in-aid (utility maximization). By working out the impli-
cations of maximization, one is able to derive a set of empirically
testable expenditure and tax equations.[*]

THE MODEL OF SCHOOL DISTRICT BEHAVIOR

Although we seek a set of equations to represent the behavior
of all school districts within a state, it is convenient to develop
the theory with reference to a single school district. Aggregation
can be handled in a straightforward manner and poses no special
theoretical problems.[**]

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[*]Several writers have used the constrained maximization approach
to analyze responses to grants-in-aid, both of state-local governments
in general and school districts in particular. See, for example,
James A. Wilde, "The Expenditure Effects of Grant-in-Aid Programs,"
National Tax Journal, 21, September 1968, pp. 340-48; and Gail S.R.
Wilensky, "State Aid to Education," Ph.D. dissertation, University
of Michigan, Ann Arbor, Michigan, 1968. Usually, these expositions
use indifference curve diagrams directly analogous to these used in
developing the theory of consumer demand. The assumptions and results
of this analysis differ from those earlier studies in some important
respects. See below, especially p. 14 fn.

[**]See S.M. Barro, *Theoretical Models of School District Expendi-
ture Determination and the Impact of Grants-in-Aid*, The Rand Corpora-
tion, R-867-FF, February 1972, pp. 67-70.

We think of the decisionmakers in a school district as seeking to establish an optimal balance between the educational services they provide and the local property tax burdens they impose on citizens. It is assumed that preferences among alternative combinations of educational services and tax burdens are consistent in a given district at a given time, but that these preferences may be influenced by community characteristics ("taste variables") that vary both over time and among districts. Educational services, as measured by real expenditures per pupil, are valued positively. Tax burdens are valued negatively. The burden of local school taxes is assumed to depend on both the real dollar amount of taxes per capita and on per capita disposable income. A given level of taxes is assumed to be more burdensome when income is lower.

Preferences

The above assumptions are embodied in a school district preference function of the form

$$U = U(e_L, b_L(t_L, y_D), z), \tag{i}$$

where e_L and t_L are real per pupil spending and real per capita school taxes, defined as before; the function b_L represents the burden of local school taxes; $y_D = y - t_S - t_F = $ real per capita disposable income, where t_S and t_F are state and federal taxes; and z represents unspecified taste variables that may enter into the preference function. First derivatives of the local preference function have the following signs:

$$\frac{\partial U}{\partial e_L} > 0, \quad \frac{\partial U}{\partial b_L} < 0, \quad \frac{\partial b_L}{\partial t_L} > 0, \quad \frac{\partial b_L}{\partial y} < 0.$$

It is convenient to work with a quantity analogous to the marginal rate of substitution in consumer demand theory, which we call the *marginal rate of trade-off between expenditures and taxes* and denote by m. The marginal rate of trade-off may be interpreted as the maximum increment in school taxes per capita the district would be willing to impose to obtain one additional dollar of educational spending per pupil. The definition of m in terms of the utility function is

$$m = \frac{dt_L}{de_L}\bigg|_{dU = 0} = -\frac{\partial U}{\partial e_L}\bigg/\frac{\partial U}{\partial t_L} = -\frac{\partial U}{\partial e_L}\bigg/\left(\frac{\partial U}{\partial b_L}\frac{\partial b_L}{\partial t_L}\right).$$

In developing the model, we assume that $\underline{m}$ has the following properties:[*]

(1) $\partial m/\partial e_L < 0$: As educational outlay per pupil, e_L, increases, each marginal increment becomes less urgent than the preceding one and the district becomes willing to impose a progressively smaller tax to obtain it.

(2) $\partial m/\partial t_L < 0$: As the level of the school property tax increases, each marginal increment becomes more burdensome than the previous one and the district requires a greater increment in "education" to justify a one dollar tax increase.

(3) $\partial m/\partial y_D > 0$: As disposable income increases, the burden of a given school tax diminishes, making the district less reluctant to impose additional taxes for a given increment in school spending.

These stipulations about properties of the marginal rate of trade-off function allow unambiguous inferences to be made about the signs of the effects of most exogenous variables on levels of school spending and taxes.

The Budget Constraint

Assuming that no borrowing is allowed and that districts do not accumulate cash balances, and neglecting capital outlays, the budget constraint is that total educational expenditure must equal local tax revenue plus state and federal aid. For simplicity, assume that both state and federal aid take the form of lump-sum grants to districts. An examination of state and federal grant-in-aid formulas

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[*] These properties can be thought of as *assumptions* about m or they can be *derived* from properties of the preference function if appropriate additional assumptions are made about U. For example, the assumption that U is separable in e_L and b_L is sufficient, but not necessary to assure the stated signs of derivatives of m.

show that this is realistic for all districts in all but a handful of states and for most districts in the few remaining states.[*] Having made that assumption, the budget constraint may be written

$$P_e A e_L = P_O N t_L + p_e A(s + f_L),$$

where s and f_L are real state and federal aid per pupil to the school district, as defined earlier; A is the number of pupils in the district; N is the total population; P_e is an index of prices of educational inputs, and P_O is an index of prices of private goods. It is convenient to define two ratio variables, $a = A/N$, the pupil/population ratio, and $p_e = P_e/P_O$, the *relative* price of education. The budget constraint may then be rewritten more compactly as

$$p_e a(e_L - s - f_L) = t_L.$$

Maximization

Conditions for constrained maximization of U are obtained by maximizing the Legragian

$$U(e_L, b_L(t_L, y_D), z) - [p_e a(e_L - s - f_L) - t_L].$$

The first order conditions are

$$\frac{\partial U}{\partial e_L} - \lambda p_e a = 0,$$

$$\frac{\partial U}{\partial b_L} \frac{\partial b_L}{\partial t_L} + \lambda = 0.$$

The two conditions can be combined into a single condition involving the marginal rate of trade-off:

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[*]An analysis of data in Public School Finance Programs, 1968-69, Office of Education, U.S. Department of Health, Education, and Welfare (Washington: 1969) shows that aid is always in the form of lump-sum grants in all but five or six states, which have matching provisions in their aid formulas. Even in these states, ceiling provisions in the aid formulas convert the grants to lump-sum amounts for most districts.

$$- \; \frac{\dfrac{\partial U}{\partial e_L}}{\dfrac{\partial U}{\partial b_L} \dfrac{\partial b_L}{\partial t_L}} \; = \; m(e_L, \; t_L, \; y_D, \; z) \; = \; p_e a.$$

This relationship and the budget constraint equation constitute a pair of equations in e_L and t_L from which responses of those two variables to changes in the exogenous variables, y_D, a, s, f_L, p_e, and z, can be deduced.

Expenditure and Tax Implications

A direct procedure for determining the effects on school district expenditures and taxes of changes in the exogenous variables is to solve the budget constraint equation for t_L, substitute the resulting expression for t_L into the marginal rate of trade-off equation given above, and then differentiate totally and solve for the change in spending, de_L. These computations are shown in the previously cited study.[*] The principal effects on spending, which derive directly from the previously stated properties of the marginal rate of trade-off function, are as follows:

1. _Effect of a change in disposable income:_ The model implies a _positive_ relationship between real per pupil spending and real per capita disposable income $(de_L/dy_D > 0)$. This means a _negative_ relationship between per pupil spending and changes in state or federal personal taxes.

2. _Effect of a change in aid:_ The model implies a positive relationship between real per pupil spending and real state and federal lump-sum aid $[de_L/d(s + f_L) > 0]$. The magnitude of the aid effect must be less than one; i.e., lump-sum aid is always partly additive to local spending, partly substitutive for local school taxes. The aid effect and the income effect are not the same, and under reasonable assumptions about local preferences, it can be shown that the aid effect will be considerable _greater_ than the

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[*]Ibid. pp. 14-17.

income effect.[*] This has the important implication that the positive effect of an increase in aid will not be offset by the negative effect of taxes even when an increment in aid is fully financed, in effect, by taxes on residents of the aid-receiving district.[**]

3. Effects of changes in the pupil/population ratio and the relative price of education: The model implies that spending will depend negatively on the product of the two variables p_e and a $[de_L/d(p_e a) < 0]$. This means that, other things being equal, lower real per pupil spending is expected where a greater fraction of the population attends public school or where prices of inputs to schooling are high in relation to prices in general. The price effect will have a direct bearing on predictions whenever fiscal alternatives involve either matching grants to districts or taxes on private spending (e.g., state sales taxes or a federal value-added tax).

4. Effects of "taste" variables. The response of real per pupil spending to a change in an exogenous taste variable will be positive $[de_L/dz > 0]$ if that variable is positively associated with the district's marginal rate of trade-off between taxing and spending.

Effects of the exogenous variables on real school taxes per capita may be derived in a similar manner. Very briefly, the income effect, dt_L/dy_D, is positive; the aid effect, $dt_L/d(s + f_L)$ is negative, with the decrease in taxes per dollar increase in aid less than one; and the price effect, $dt_L/d(p_e a)$ is of indeterminate sign—positive if the price elasticity of per pupil expenditures is greater than minus one, negative otherwise.

Other Effects on Local Spending

It has been shown elsewhere[***] that effects of several other

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[*]Barro, op.cit., pp. 15-18. This result distinguishes the model used here from one that is more directly analogous to a consumer demand model. The latter would simply treat outside aid as an addition to the income that the community has available to allocate between education and other goods, which would imply equal income and aid effects.

[**]Ibid., pp. 40-41.

[***]Ibid., p.33 ff.

determinants of school spending and taxes can be incorporated into
the foregoing basic model. Some of these (e.g., differences in the
composition of local tax bases and equalization provisions of state
aid formulas) are relevant only to studies of *interdistrict* expen-
diture variations. However, the following two factors are relevant
to both interdistrict and interstate studies and are included in the
empirical models:

(1) Real per pupil expenditures should be negatively associated
with the *rate of enrollment growth* in a district or state. This is
because the property taxes required to pay for new facilities compete
with property taxes for operating expenses.

(2) Per pupil spending will respond more strongly to categorical
than to unrestricted aid if the restrictions attached to categorical
aid force districts to spend more on specific programs or activities
than they would have chosen to spend themselves. Since a greater
portion of federal than state aid is likely to be categorical, this
suggests that the federal aid and state aid coefficients in the model
may be different.

Hypotheses

The results of the theoretical analysis of school district behav-
ior can be stated as a set of hypotheses about relationships between
local spending and taxes and a number of exogenous variables. Taking
the last two propositions stated above into account, we expect to
obtain expenditure and tax equations.

$$e_L = e_L(y_D, s, f_L, p_e a, \Delta A/A, z)$$

and

$$t_L = t_L(y_D, s, f_L, p_e a, \Delta A/A, z)$$

that satisfy the conditions

a. $\dfrac{de_L}{dy_D} > 0$

b. $1 > \dfrac{1}{p_e a} \cdot \dfrac{de_L}{df_L} \geq \dfrac{1}{p_e a} \cdot \dfrac{de_L}{ds} > \dfrac{de_L}{dy_D}$

c. $\dfrac{de_L}{d(p_e a)} < 0$

d. $\dfrac{de_L}{d(\Delta A/A)} < 0$

e. $\dfrac{dt_L}{dy_D} > 0$

f. $-1 < \dfrac{1}{p_e{}^a}\dfrac{dt_L}{d_S} < \dfrac{1}{p_e{}^a}\dfrac{dt_L}{df_L} < 0$

g. $\dfrac{dt_L}{d(p_e{}^a)} = ?$ (sign indeterminate)

h. $\dfrac{dt_L}{d(\Delta A/A)} < 0$ (N.B. t_L is taxes for current expenses only).

Further specification of the hypotheses depends on additional assumptions about the functional form of the expenditure and tax equations. Discussion of that issue is delayed until after presentation of the state-level theoretical model.

THE MODEL OF STATE FISCAL BEHAVIOR

The state model is similar in many respect to the local model. Each state is viewed as seeking an optimal valance between expenditures and state tax burdens subject to a budget constraint that reflects the level and form of outside (federal) aid. Expenditure and tax implications of maximizing behavior are derived, and used to specify a set of empirically testable expenditure and tax equations. But the state model has two features that make it different from and more complicated than the local model. First, there are two categories of state expenditure to consider---aid to local school districts and "other" state spending. Only one category had to be treated in the local model. Second, in order to explain state aid to local governments, it is necessary to assume that the state is responsive to fiscal conditions at the local level. Each of these features leads to the inclusion of arguments in the state preference function that have no counterparts at the local level.

The State Preference Function

Two approaches can be taken in formulating a state government

preference function. In both, state taxes and "other state spending (direct state outlay plus aid to other local governments) appear as arguments. The difference is that in one, state aid to school districts enters directly into the preference function (i.e, aid is assumed to be valued "in itself" by the state), while in the other local school spending and local school taxes enter the state's preference function while state aid enters the model elsewhere as in intermediate variable. The two yield similar results, but only the first approach is followed here.

Assume an additively separable state preference function of the form

$$V = V_1(e_S) + V_2(t_S, t_L^*, y-t_F) + V_3(s, s^*),$$

where t_L^* is the *expected* level of local taxes and s^* is a measure of the "needed" amount of state aid per pupil (see below). State aid, s, and other state spending, e_S, are assumed to be positive goods to the state government. State taxes, t_S, is a negative good. The marginal disutility to the state of a given increment in state taxes is assumed to decrease as income increases, but to increase as the level of expected local taxes increases. The marginal utility of a given level of state aid, s, is assumed to increase as the local "need" for state aid, s^*, increases. Other exogenous variables, which were previously denoted by z, may enter into the preference function, but they have been omitted for simplicity.

Since there are three state fiscal variables in the preference function, we need to define two marginal rates of trade-off among them. The marginal rate of trade-off between other state expenditures and state taxes is defined as

$$M_1(e_S, t_S, t_L^*, y-t_F) = -\frac{\partial V_1}{\partial e_S} \Big/ \frac{\partial V_2}{\partial t_S}$$

The marginal rate of trade-off between state aid to school districts and state taxes is defined as

$$M_2(s, t_S, t_L^*, s^*, y-t_F) = -\frac{\partial V_3}{\partial s} \Big/ \frac{\partial V_2}{\partial t_S}$$

when appropriate assumptions are made about second derivatives of V, it can be shown that the two functions have the following properties:

$$\frac{\partial M_1}{\partial e_S} < 0 \qquad \frac{\partial M_1}{\partial t_S} < 0 \qquad \frac{\partial M_1}{\partial t_L^*} < 0 \qquad \frac{\partial M_1}{\partial (y-t_F)} > 0$$

$$\frac{\partial M_2}{\partial e} < 0 \qquad \frac{\partial M_2}{\partial t_S} < 0 \qquad \frac{\partial M_2}{\partial t_L^*} < 0 \qquad \frac{\partial M_2}{\partial e}^* > 0 \qquad \frac{\partial M_2}{\partial (y-t_F)} > 0.$$

From these it will be possible to deduce the signs of the effects
of exogenous variables on state aid, other state spending, and state
taxes.

The Budget Constraint

The state budget constraint is that total state aid to school
districts plus the total amount of other state expenditures must
equal total state taxes plus federal aid. We will assume that all
federal aid to states consists either of lump-sum grants or fully-
utilized matching grants; i.e., that there is no matching by the fed-
eral government at the margin. This is not completely true, but does
seem to be roughly correct for most federal aid programs for most
states. While it would be possible to allow for matching provisions
in the budget constraint, this introduces serious measurement and
statistical problems. On balance, the simpler assumption seems
preferable.[*] Noting carefully the deflators that were used in
defining each variable (p. 6), we can write the constraint as

$$P_e Ae + P_g Ne_S = P_o Nt_S + P_g Nf_S.$$

Define the relative price of other public services, $\ddot{p}_g = P_g/P_o$.
Then, using the previously defined relative price of education,
p_e, and the pupil/population ratio, a, the budget constraint may
be rewritten:

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[*]For a highly pertinent comment on this issue, see Edward M.
Gramlich, "The Effect of Federal Grants on State-Local Expendi-
tures: A Review of the Econometric Literature," _Proceedings of
the 62nd Annual Conference of the National Tax Association, 1969_,
pp. 581-582.

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$$p_e as + p_g e_S = t_S + p_g f_S.$$

Maximization

The first-order conditions derived from maximizing V, subject to the budget constraint, are

$$\frac{\partial V_1}{\partial e_S} - \lambda p_g = 0$$

$$\frac{\partial V_2}{\partial t_S} + \lambda = 0$$

$$\frac{\partial V_3}{\partial s} - \lambda p_e a = 0.$$

Combining the first with the second and the third with the second, we obtain the two marginal rate of trade-off equations:

$$M_1(e_S, \; t_S, \; t_L^*, \; y-t_F) = p_g$$
$$M_2(s, \; t_S, \; t_L^*, \; s^*, \; y-t_F) = p_e a$$

These two relationships together with the budget constraint are the basic relationships of the state model.

Measurement of t_L^* and s^*

An important part of this model is a set of definitions for expected local taxes, t_L^*, and "needed" state aid, s^*. Consideration of the roles of these variables in the model suggests several possibilities. The purpose of including t_L^* is to allow for the influence of local tax burdens on the state's willingness to impose taxes of its own. The assumption is that the state will be more reluctant to tax when it anticipates a higher level of local taxes. Two possible specifications of t_L^* are as follows:

(a) Let $t_L^* = t_{L, -1}$, the level of real local school taxes per capita in the preceding year. I.e., the state expects local taxes to remain at prior year levels, adjusted for changes in the general price level.

(b) Let t_L^* be the level of school taxes per capita necessary to maintain the prior year's level of real per pupil spending,

assuming no change in state aid, i.e.,

$$t_L^* = p_e a \left[e_{L,-1} - s_{-1} - f_L \right]$$

The role of s^* in the model is to represent the urgency of state
aid to local school districts from the point of view of the state.
It is assumed that, other things being equal, the state prefers to
have local districts as favorably situated as possible in terms of
the combinations of per pupil spending and per capita taxes access-
ible to them. For this reason, the state is willing, to a certain
extent, to impose state taxes or curtail other state spending in
order to provide education grants-in-aid. A reasonable measure
of "need," therefore, is the amount of state aid per pupil required
to prevent any worsening of the existing (prior year) local budget
constraint. We define s^*, accordingly, as the amount of state aid
per pupil needed to maintain *both* the prior year level of real per
pupil spending and the prior year real per capita local tax, or

$$s^* = e_{L,-1} - f_L - (t_{L,-1}/p_e a).$$

State Aid and State Tax Equations

Effects of the various exogenous variables on state aid to
school districts, state taxes, and other state expenditures can be
determine by differentiating the three equations of the model totally
and solving for de_S, dt_S, and ds. Details of these computations
are available elsewhere.[*] As with the local model, the results
can be translated into a set of hypotheses about signs of the effects
of individual exogenous variables on the dependent variables. Focus-
ing on the two fiscal variables of direct interest, state aid and
state taxes, the analysis lends us to expect equations of the general
forms,

$$s = s(y - t_F, f_S, t_L^*, s^*, p_e a, p_g)$$

and

$$t_S = t_S(y - t_F, f_S, t_L^*, s^*, p_e a, p_g)$$

Expected signs of the various effects, are shown in Table 1.

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[*] S.M. Barro, "Models for Estimating the Impact of Revenue Sharing,"
unpublished paper, The Rand Corporation, 1971.

Table 2. Effects of Changes in Exogenous
Variables on State Aid and State Taxes

Variable	Effect on State Aid	Effect on State Taxes
$y - t_F$	$(?)^a$	> 0
f_S	$> 0, < 1$	$< 0, > -1$
t_L^*	$(?)^a$	$\lessgtr 0$
s^*	> 0	> 0
p_e^a	$\lessgtr 0$	$?$
p_g	$?$	$?$

^{a}See text.

Note that the signs of certain effects are ambiguous: The income
effect will be greater than zero provided that neither s nor e_s is
an inferior good and that the positive effect of an increase in income
on the state's willingness to tax outweighs the negative effect of
a reduction in local tax burdens. The effect of t_L^* on s will be
positive under the same conditions. Signs of the ambiguous price
effects depend on whether certain price elasticities are greater or
less than one.

Empirical analyses of the state-level data are aimed at testing
the hypotheses implied by the table, including the general hypothesis,
where the expected sign is uncertain, that the indicated variable
"makes a difference."

EMPIRICAL RESULTS

THE LOCAL MODEL

The local model was tested using cross-section and pooled time-series cross-section data. The principal source of data on expenditures, revenues, and numbers of pupils in each state was the biennial U.S. Office of Education (USOE) report, Statistics of State School Systems.[*] Complete information was obtained for 48 states (Alaska and Hawaii were excluded) for nine alternate school years from 1951-52 to 1967-68. Items obtained from this source include (a) total current expenditure for education, (b) total school district revenues from state sources, (c) total revenues from federal sources, and (d) numbers of pupils in average daily attendance. Data on disposable personal income per capita were obtained from a special tabulation published in the Survey of Current Business.[**] Population data consisted of Census estimates included in the biennial USOE reports.

Price Variables

The only serious measurement problem that arises in testing the local model concerns the two price variables, P_e, the index of prices of inputs to schooling, and P_o, the general price index. The former enters into the model as the deflator of per pupil spending and state and federal aid. The latter enters as the deflator of per capita income. The two also appear as a ratio, the relative price of education (p_e), in the composite price term, $p_e a$.

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[*] U.S. Department of Health, Education, and Welfare, Washington, D.C. (In earlier years, these data were published in the USOE Biennial Survey of Education in the United States.)

[**] R.B. Bretzfelder, Q.F. Dallavalle, and D.A. Hirschberg, "Personal Income, 1968, and Disposable Income, 1929-68, by States and Regions," Survey of Current Business, Office of Business Economics, U.S. Department of Commerce, April 1969.

Ideally, we would want to compute the education price index as a weighted average of prices of individual inputs into education, holding input quality constant across states and from year to year. By far the largest component of the index is professional salaries-- the "prices" paid by school districts for teachers, administrators, and other professional education personnel. Such salaries average about 70 percent of current educational expenditures. Therefore, the possibility of constructing a satisfactory interstate index hinges on the availability of data that would permit identification of the quality component in interstate salary variation.

Such data are not now available. In fact, the concept of "quality" of educational personnel is not well defined, even in principle. Therefore, construction of a "true" interstate price index is not an available option. We must rely, instead, on some more sweeping assumption about quality, recognizing the biases that will inevitably result.

Dresch[*] has suggested the following three alternatives:

(a) Assume constant input quality in all areas (states), thereby assuming that all observed salary differences are price differences. This implies that the salary component of expenditures in each state is to be deflated by the ratio of the salary index in that state to the salary index in the nation.

(b) Assume that all observed salary differences reflect quality rather than price variations. The appropriate deflator then becomes the average salary index in the *nation* (i.e., an identical deflator for all states).

(c) Assume that observed salary differences are price differences only to the extent that they correspond to inter-state variations in all wages and salaries (or in income levels), but that any variation in excess of the general wage and salary

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[*]"Aid, Income, and Patterns of Metropolitan Fiscal Activity," in S.P. Dresch and R.J. Struyk, <u>Grants-in-Aid and Local Fiscal Activity: Intra- and Interstate Analyses of the Local Governments Economy: Three Essays</u>, unpublished manuscript, <u>The National Bureau of Economic Research</u>, 1971, pp. 190-192.

variation is attributable to quality differences. The appropriate
deflator would be the same index as in (a), multiplied by the
ratio of general wages and salaries (or incomes) in the area to
the same magnitude for the nation.

Since all of these assumptions are unrealistic and none seems
clearly preferable a priori, we have estimated our equations according
to assumption (b), which is the simplest to handle. In other words,
we use an education price index that varies from year to year, but not
from state to state. The salary component of this national price index
(base year = 1965) is computed from data on salaries of instructional
staff published in the previously cited USOE biennial reports. This
component is given a weight of 70 percent. Prices of other inputs into
schooling are measured by the services (less rent) component of the
consumer price index and given a weight of 30 percent.

Much the same situation prevails with respect to the general
price index, P_o, except that there are fewer options. Lacking a
consumer price index for each state, we are forced to use the national
index, thereby implicitly assuming that there is variation over time
only, but not across states. Some data are available that could be
used to construct crude state-by-state indexes, but we have not
undertaken that task in this study.[*]

Our incentives to develop interstate indexes have been substan-
tially reduced by the realization that little is to be gained from

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[*]Comparative data on family budgets for different U.S. cities
compiled by the Bureau of Labor Statistics (e.g., in City Worker's
Family Budget for a Moderate Standard of Living, U.S. Department
of Labor, Bulletin 1570-1, Washington, D.C., 1967) could conceivably
be used to develop rough indexes of general price variation across
states. Another possibility would be to use the "iso-prop" indexes
developed by Harold W. Watts ("The Iso-Prop Index: An Approach
to the Determination of Differential Poverty Income Thresholds,"
Journal of Human Resources, Vol. II, No. 1, Winter 1967). Both
types of data, however, are available for only one year and could
only be applied to a single cross-section analysis.

developing either a P_e or a P_o index unless both are developed sim-
ultaneously. This is because the two appear in the model as a *ratio*
in the price term, $p_e\alpha$. While it is unrealistic to assume that P_e
and P_o are constant across states, it is probably much less unreal-
istic to assume that the *relative* price of education, P_e/P_o is constant.
Therefore, using a true interstate index of either P_e or P_o, while
continuing to treat the other as a constant, could actually result
in poorer measurement of the relative price of education.

Cross-sectional Results

Two forms of the model were tested using cross-section data:
(a) a conventional linear form containing all the variables identi-
fied under "hypotheses" (pp. 15 - 16), and (b) an alternative form, linear
in the same variables or ratios thereof, based on the Linear
Expenditure System used in studying consumer demand.[*] In addition,
we tested variants of the models containing additional state charac-
teristics (urbanization, population density, and regional identity)
and forms allowing for interactions among the independent variables.
Both elaborations are compatible with the theoretical model.

The conventional linear form was found to be superior to the
form based on the Linear Expenditure System according to all the
usual measures (R^2, F, and t-values of coefficients) whether or not
additional state characteristics or interaction terms were included.
Therefore, the latter form was eliminated. The linear model, without
either type of elaboration, generally satisfied the expectations re-
garding signs and magnitudes of coefficients, except that no signifi-
cant difference between federal and state aid effects could be detected.

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[*]Recent discussion of the linear expenditure system and its prop-
erties include Robert A. Pollak and Terence J. Wales, "Estimation of the
Linear Expenditure System," *Econometrica* 37, No. 4, October 1969, pp.
611-28; and Richard W. Parks, "Systems of Demand Equations: An Empirical
Comparison of Alternative Functional Forms," *Econometrica* 37, No. 4,
October 1969, pp. 629-50.

In addition, examination of the residuals of the cross-sectional regressions revealed that a South vs. non-South regional difference remained unaccounted for. A series of tests led to the inclusion of both an additive regional (Southern) dummy variable and a term representing a regional influence on the aid coefficient. There was also found to be a significant difference in spending between states with especially low population densities (population per square mile less than 30) and the other states. The final set of cross-section equations, therefore, includes the income, aid, price $(p_e a)$, and enrollment growth terms, the Southern region and low density dummy variables, and the regional-aid interaction term.

The nine cross section equations are shown in Table 2 (t-values in parentheses). Some of the major results are as follows:

(1) The equations are generally consistent from year to year with respect to both "fit" (R^2 and coefficient of variation) and values of coefficients of the major variables. The degree of consistency is greater than has generally been found in comparative cross section studies of state-local expenditures.[*] Experiments with alternative forms of the model showed that this consistency is at least partly attributable to inclusion of the variables p_e and a in the product form called for by the theory. There are, however, perceptible time trends in income and price coefficients. These may reflect either improper measurement of prices or actual structural changes over the 17 year period spanned by the data (see the discussion of pooled results below).

(2) The magnitude of the income coefficient, which varies roughly from .14 to .20, means that 3.5 to 5.0 percent of each marginal dollar of income translates into school spending, holding the level of state aid constant.[**] The figure varies from state to state

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[*]Gramlich, op.cit.

[**]Note that the model relates spending *per pupil* to income and outside aid *per capita*. Since there is roughly one pupil for every four or five persons in the population, the coefficients must be multiplied by a factor on the order of 0.20 to 0.25 to determine the fraction of each dollar increment in income or aid that is used for educational spending.

Table 2   SCHOOL DISTRICT EXPENDITURE

EQUATIONS, LINEAR MODEL: CROSS-SECTIONAL RESULTS

$$e = \beta_0 + \beta_1 R + \beta_2 y + \beta_3 p_e a(s + f_L) + \beta_4 p_e a(s + f_L)R + \beta_5 p_e a + \beta_6 \frac{\Delta A}{A} + \beta_7 LD$$

School Year	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	R^2(adj.)	Coefficient of Variation
51-52	431 (3.9)	-186 (3.6)	.146 (4.6)	1.74 (3.2)	4.18 (2.6)	-2260 (3.3)	-290 (2.0)	34.4 (1.8)	.86	9.0
53-54	450 (4.7)	-196 (3.4)	.163 (4.8)	1.10 (2.5)	4.64 (2.8)	-2380 (4.3)	-440 (2.1)	78.4 (4.1)	.86	8.6
55-56	539 (4.7)	-125 (2.5)	.133 (4.0)	1.60 (3.1)	1.67 (1.4)	-2590 (4.3)	-297 (1.2)	70.8 (3.4)	.85	9.0
57-58	440 (3.9)	-169 (3.0)	.144 (4.5)	1.21 (3.2)	2.52 (2.2)	-1873 (3.5)	-28 (.14)	46.4 (2.4)	.85	8.9
59-60	449 (3.5)	-190 (3.0)	.151 (4.7)	.87 (2.2)	2.71 (2.1)	-1752 (3.2)	-181 (.78)	47.8 (2.3)	.83	9.3
61-62	433 (3.7)	-134 (2.0)	.201 (6.2)	1.08 (3.1)	1.90 (1.6)	-2045 (4.4)	-697 (2.9)	75.4 (3.6)	.86	8.3
63-64	472 (3.7)	-144 (1.7)	.195 (5.8)	1.16 (3.8)	1.59 (1.1)	-2057 (4.3)	-686 (3.0)	64.9 (3.0)	.84	8.9
65-66	503 (3.8)	-63 (.65)	.170 (5.4)	1.43 (5.2)	.026 (.020)	-2094 (4.7)	-459 (1.5)	51.5 (2.4)	.83	8.4
67-68	602 (4.6)	-142 (1.4)	.181 (6.7)	1.76 (5.9)	.91 (.76)	-2575 (5.5)	-641 (1.9)	62.9 (2.9)	.85	7.0

Variables: e = real current educational expenditure per pupil

y = real per capita disposable income

s = real state aid per pupil

f_L = real federal aid per pupil

p_e = relative price of education

a = ratio of pupils to population

R = Southern region dummy variable

$\Delta A/A$ = percentage increase in average daily attendance during a two-year period

LD = dummy variable identifying states with low population densities

in any one year because of interstate variations in the ratio of pupils to population. The state aid coefficient, which averages about 1.3 for non-Southern states (but is less stable over time than the income coefficient) signifies that roughly 25 to 30 cents out of each marginal dollar of outside aid goes for increased school spending. The amount varies according to the pupil/population ratio in each state. The larger fraction of aid funds, therefore, goes for local property tax relief. Note that the aid coefficient is always substantially higher than the income coefficient (by a factor of six to ten), as hypothesized. This means that if the state or federal government chose to provide additional aid and to finance it entirely out of additional taxes on residents of the recipient districts, the increased state or federal taxes would offset only a fraction of the expenditure effect of the increased aid.

(3) Values obtained for the price coefficient translate into estimated price elasticities of per pupil spending ranging from -0.5 to -0.9, depending on the year. This means that somewhat less than half of an increase in costs of education, e.g., teachers salaries, will be compensated for by increased spending, and the remainder by reductions in staffing or other input ratios. The sign of the effect of the price variables on real school taxes per capita, which was left ambiguous by the theory, must therefore be positive; i.e., tax burdens rise with costs of education.

(4) Until about 1961, Southern states appeared to differ from the rest of the nation in two respects: (a) they spent less per pupil, even after effects of lower than average per capita income and other variables had been taken into account; (b) southern districts appeared to be more responsive than districts elsewhere to changes in outside aid. After 1961-62, however, both regional effects become insignificant, suggesting that over time Southern tastes for education have become more like those in the rest of the country.

(5) As to the remaining variables, the enrollment growth effect is negative, as expected, but is significant ($t \geq 2$) in only four of the nine equations. The dummy variable designating states with low population densities has a positive coefficient as hypothesized.

Pooled Results

The equation obtained by simply pooling the nine sets of cross-section data, without allowing for structural changes over time, is the following:

$$e = 218 - 109R + .202y + (1.15 + 1.01R)p_e a(s + f_L)$$
$$\;\;\;\;\;(12.1)\;(-7.1)\;(24.9)\;\;\;(9.7)\;\;\;(4.1)$$

$$-949p_e a - 372\Delta A/A + 34.4LD$$
$$\;\;(-11.5)\;\;\;\;\;(-6.5)\;\;\;\;\;(5.4)$$

$R^2 = .85$, Coeff. of Var. = 9.1 percent

Note that a number of the coefficient values do not seem compatible with those obtained by fitting single cross-sections. For example, the price coefficient is much lower than that obtained in any cross-sectional equation, the income coefficient is just outside the range of values obtained cross-sectionally, and the constant term is much lower than in any of the equations in Table 2. This suggests that some structural changes have taken place over time that are not properly reflected by simple pooling.

We were not able to discover other variables or interaction terms that accounted for structural shifts. Having noticed, however, that there seemed to be trends in values of certain coefficients over the nine years, we tested for systematic time trend effects on coefficients in the pooled model. The following equation, with three significant time trend effects, was obtained (the time trend variable, T, is 0 for school year 1951-52, 1 for 1953-54, and so forth):

$$e = 477 - 163R + (.123 + .0074T)y + (1.35 + 3.09R - .22RT)p_e a(s + f_L)$$
$$\;\;\;\;\;(13.1)\;(-8.1)\;\;\;(9.7)\;\;\;(4.5)\;\;\;\;\;\;\;(5.2)\;\;\;(5.1)\;\;\;\;\;(-3.8)$$

$$-(2218 - 43.2T)p_e a - 241\Delta A/A + 52.1LD$$
$$\;\;(-9.9)\;(1.6)\;\;\;\;\;\;(-4.1)\;\;\;\;\;(8.4)$$

$R^2 = .87$, Coeff. of Var. = 8.5 percent

The values of income, aid, and price coefficients in this equation are all consistent with values obtained in the cross-section regressions. Also, the regional effect on the aid coefficient diminishes over time,

as appeared to be the case in comparing cross-sections. No significant trend term, however, was found to be associated with the additive Southern region dummy variable.

Extrapolation to school year 1971-72 yields the following expenditure equation:

$$e = 477 - 163R + .197y + (1.35 + .89R)p_e a(s + f_L)$$
$$-1786p_e a - 241\Delta A/A + 52.1LD$$

The values of these coefficients imply, for the "average" non-Southern state:

    a. an increase of roughly $0.05 in real school spending for each one dollar increase in real per capita income (or a corresponding decrease in spending per dollar increase in state or federal taxes);

    b. an increase of about $0.33 in real school spending per pupil for each dollar of per pupil state aid;

    c. a price elasticity of per pupil spending of approximately -0.6.

These values (with state to state variations, as called for by the model) will be used to estimate effects at the local level of various federal funding alternatives.

THE STATE MODEL

Data

In testing the state model, it was possible to work with a larger data base, not limited to biennial observations only as was the data base used for the local expenditure analysis. The principal data source was the U.S. Bureau of the Census series, _State Government Finances_.[*] Data for 18 fiscal years, from FY 1953 to FY 1971, were

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[*]U.S. Bureau of the Census, _State Government Finances in 1970_, Series GF70, No. 3, U.S. Government Printing Office, Washington, D.C. 1971, and volumes for earlier years.

obtained on the following variables: (a) state intergovernmental
expenditure for education (used as the measure of state aid to local
schools); (b) total intergovernmental revenue from the federal
government (federal aid to states); (c) total state taxes; and
(d) total state expenditures for all functions. By having annual
data on the fiscal magnitudes, it became possible to calculate
year-to-year increments in variables of interest and to examine time
series for individual states, neither of which was feasible with the
USOE biennial data.

It was necessary to adjust the data in two respects: First,
regarding the data on federal aid to states, it is Census practice
to count federal aid to local governments that is channeled through or
distributed by states as part of federal aid to states.[*] To make
the data compatible with the model, it was necessary, therefore, to
subtract federal aid to local school districts from the federal-to-
state aid totals. Data on aid to school districts was not available
from the Census documents, but was obtained from the previously
cited USOE reports and National Education Association estimates.[**]

The second adjustment pertains to data on numbers of pupils
in public schools in each state. It was necessary to interpolate
figures on average daily attendance for years between the USOE biennial
surveys. These interpolations were based, however, on an annual
series on "opening fall enrollment" in elementary and secondary
schools, also published by USOE.[***] Therefore, the interpolation is
likely to be quite accurate. In addition, the data were adjusted

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[*]Ibid., p. 55.

[**]Estimates of School Statistics, 1970-71, Research Division,
National Education Association, Washington, D.C., 1971, and
volumes for earlier years.

[***]Fall 1968 Statistics of Public Elementary and Secondary Day Schools,
Office of Education, U.S. Department of Health, Education and Welfare,
Washington, D.C., 1969, and volumes for earlier years, some published
under slightly different titles.

to account for students enrolled in locally operated public junior colleges in several states. Such students are not counted in the USOE average daily attendance or fall enrollment figures, but state aid for the schools they attend is included in the Census' intergovernmental expenditure data. The adjustments were based on incomplete data on junior college enrollment published by USOE.

The state model requires one additional price variable that did not appear in the local model, namely, the price index for state-local services other than education (P_g). This is measured by the implicit price deflator for purchases of goods and services by state and local governments.[*] For reasons discussed earlier, we used a price index that varies from year to year, but not across states.

Problems of Structure and Timing

In attempting to model the behavior of state governments in supporting their public schools, one encounters major problems that did not arise or were negligible at the local level. Perhaps the most important is that there are major differences in fiscal structure among states that do not appear amenable to economic or econometric explanation. For reasons that may lie deep in history and tradition, states vary substantially in the scope of their responsibilities vis-a-vis local governments and in the institutional arrangements, e.g., tax structures, for carrying our their fiscal roles.

With respect to public elementary and secondary education, in particular, there are very large interstate differences in the division of responsibility for school support between the state and the localities. A few states, such as North Carolina and Delaware, have assumed dominant roles in school finance, providing 78 and 77 percent, respectively (in 1970-71), of school revenues directly from state funds.

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[*] *Economic Report of the President*, United States Government Printing Office, Washington, D.C., 1972, p. 199.

Several other states, e.g., New Hampshire (10 percent) and South
Dakota (17 percent) leave school finance almost entirely to
localities.[*] The others occupy positions everywhere in between.
There is little apparent connection between either the absolute level
of state support or the relative size of the state contribution
(state support as a fraction of total state plus local support)
and such likely state characteristics as per capita income, indus-
trialization, and urbanization.[**] Therefore, the prospects for an
adequate explanation of interstate variation in state aid to local
schools do not seem high just from preliminary examination of the
data and simple correlation analysis.

Another serious problem is that changes in levels of real state
aid per pupil are highly discontinuous. A typical pattern is that
a state will set a particular level of aid in one year, then more
or less maintain it with "cost of living" increases over the next
few years, perhaps with some erosion in real per pupil amounts,
until finally deciding on a new level several years after the last
major decision. The result is that any cross-sectional analysis
is likely to catch different states at different stages of these
cycles, making comparisions very difficult.[***] Moreover,
analysis of year-to-year increments in state aid--an otherwise pro-
mising approach, given the structural differences that exist--is
virtually ruled out by this pattern of behavior.

Undoubtedly, very similar patterns would have been found at the
local level if individual districts were the units of observation.

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[*] National Education Association, Rankings of the States, 1971,
Washington, D.C., 1971, p. 50.

[**]One exception is that being a Southern state appears to be
strongly associated with a high state share of state plus local
support.

[***]On this point, see Gramlich, op.cit., p. 579.

However, the effect of working with aggregates of local spending in each state is that such discontinuities tend to be smoothed out. Similar aggregation of state observations is not an available option. It is necessary to work directly with the individual decision making units, which means that all the idiosyncracies, cyclical patterns, and singular events affecting state aid are reflected directly in the dependent variables of the equations. Thus, the probability of discovering an equation with high explanatory power is further reduced.

State Aid Equations and Results

Our low expectations regarding ability to explain state aid were borne out by the results. Cross-sectional tests proved to be almost worthless, with most coefficients insignificant and the "significant" ones highly inconsistent from one cross-section to the next. This was true whether the cross-sectional dependent variables was the *level* of state aid in a given year of the state aid *increment*. Only pooled time-series, cross-section estimates showed any promise of yielding a meaningful state aid equation.

A state aid equation obtained from the full pooled sample (18 years x 48 states = 864 observations) and including only the income, aid, and price variables is as follows:

$$s = 189 + .062y_D + .34f_S + 628p_e a - 283p_g$$
$$\quad (3.5) \quad (6.2) \quad (2.2) \quad (3.0) \quad (-2.6)$$

$$R^2 = .10, \quad \text{Coeff. of Var.} = .50$$

This equation obviously accounts for very little of the interstate and intertemporal variance. Values of the income and state aid coefficients seem plausible, however, when it is recalled that both variables are stated in per capita terms, while s is measured in dollars per pupil, and that there are roughly 0.25 pupils per capita. I.e., the equation implies that approximately $0.015 out of each dollar increment in personal income and approximately $0.08 out of each dollar increment in federal aid to states will be used to increase state aid to local schools. If the latter value seems

low, the reason may well be that most federal aid to states in the
past has been categorical aid for state programs other than education.
To the extent that categorical restrictions have been enforced,
shares of such aid going to other than the stipulated purposes
(e.g., to state school aid or state tax reduction) would be smaller
than the corresponding shares under a system of unrestricted grants.
This point is discussed further below in connection with the state.
tax and other state expenditure equations.

The price of education term, $p_e a$, enters positively in the
above equation, contrary to hypothesis. However, since the "expected
tax" and "needed" state aid variables, t_L^* and s^*, are omitted (see
below), this is not ruled out. The reason is that an increase in
the price of education would have the effect of increasing s^*, the
level of aid needed to maintain prior year levels of services and
tax burdens, and t_L^*, the tax level needed to maintain prior year
values of per pupil spending, both of which would tend, according
to the theory, to stimulate more state aid. The two effects could
offset the pure price effect and result in a positive $p_e a$ coefficient.

It appeared, from cursory inspection of the residuals, that
important regional differences were not accounted for by the variables
in the equation. Consequently, we added regional dummy variables
for the Northeast, Midwest, and Southern states, denoted by $R1$, $R2$,
and $R3$, respectively. When the three variables were included addi-
tively in the model the following equation was obtained:

$$s = 8.5 + .095y_D + .52f_S - 549p_e a + 78p_g - 93R1 - 78R2 + 39R3$$
$$(0.2) \quad (9.1) \quad\quad (3.4) \quad (-2.4) \quad\quad (0.6) \quad (-8.0) \quad (-8.5) \quad (4.0)$$
$$R^2 = .29, \quad \text{Coeff. of Var.} = .44$$

In this version considerably more variance is accounted for, albeit
by regional dummies; the income and federal aid coefficients are
larger than before (this reflects systematic regional differences
in both federal aid allotments and per capita incomes); and the price
coefficients have changed signs. The behavior of the $p_e a$ and p_g
coefficients seems to be erratic and not readily interpretable because

of the high colinearity (simple correlation about 0.88) between
the two variables. The Eastern and Midwestern states provide lower
state aid per pupil, while Southern states provide higher aid
(taking differences in per capita income and other variables into
account) than those in the West.

Further tests for effects of urban-rural differences and for
differential effects of federal aid among regions yielded the
following equation (the new variable, URB, is the fraction of a
state's population that is urban):

$$s = 208 + .071y_D + (.35 + 1.72R1 + .71R2 + 1.58R3)f_S$$
$$(3.4) \quad (6.0) \qquad (2.1) \quad (5.4) \quad (2.2) \qquad (5.1)$$

$$+ 159p_e a - 384p_g - 145R1 - 87R2 - 2.6R3 + 233URB$$
$$(0.7) \qquad (-3.0) \quad (-8.8) \quad (-5.2) \quad (-0.2) \quad (7.0)$$

$$R^2 = .34, \quad \text{Coeff. of Var.} = .42$$

This version provides some support for the proposition that states
in different parts of the country respond differentially to federal
aid (or, more generally, that there are important differences in
behavior patterns among states not accounted for by the variables
in the model). Some of the estimates of regional parameters, how-
ever, do not seem reasonable. For example, the Northeastern region's
federal aid coefficient would be more than 2.0, which would imply that
more than 50 percent of all federal aid to that region would go for
increased state aid to schools. Since only about 20 to 22 percent
of state expenditure now goes for education aid, and since some
fraction of federal aid to states would probably be allocated to
state tax relief rather than increased expenditure, a response
coefficient that high seems very unlikely. A more plausible expla-
nation is that the regional dummies in this, and the preceding,
equation are acting as proxies for other state characteristics or
individual state factors that have been omitted from the model and
that the specification of the equation is incomplete.

An additional shortcoming of the empirical models is that the
variables representing expected local tax burdens and "needed" state
aid failed to operate as hypothesized. When entered into a state aid

equation, they typically appeared significant, but with an incorrect
sign. The reason is clear retrospectively. Given the way t_L^* is
measured, for example (see p. 19), a state that finances a relatively
small share of school support from state funds will "expect" a
greater increase in local taxes to maintain prior year expenditure
levels. Other things being equal, however, the level of aid in
such a state will be low. Therefore, a negative, rather than
positive, correlation between t_L^* and s will be observed.

The existence of a state reaction to the local fiscal situation
was confirmed, however, in one respect. It follows from the
role of s^* in the model that state aid to local districts should
decline as direct federal aid to districts increases. The reason
is that, other things being equal, the state will perceive declining
local "need" for state transfers as additional outside funds become
available (note that federal aid, f_L enters negtively in the defini-
tion of s^* on p. 20). To test this effect, a term in f_L was added
to the state aid equation. The result was the following:

$$s = 164 + .073y_D - (.38 + 1.66R1 + .73R2 + 1.64R3)f_S$$
$$(2.6) \quad (6.2) \qquad (2.3) \quad (5.2) \qquad (2.2) \qquad (5.3)$$

$$+ 92p_e a - 304p_g - .41f_L - 151R1 - 95R2 - 7.5R3 + 224URB$$
$$(0.4) \qquad (-2.3) \quad (-2.3) \quad (-9.1) \quad (-5.6) \quad (-0.5) \quad (6.7)$$

$$R^2 = .35, \quad \text{Coeff. of Var.} = .42$$

The f_L coefficient of -.41 represents an offsetting state aid reduction
of 41 percent of the amount of new federal aid to local districts.
Should such a coefficient be confirmed by further analysis it would
have considerable policy significance, since it implies a much smaller
effect of federal aid than would be preducted from analysis of only the
local recipients' behavior. The quality of the present equation is too
low, however, to justify treatment of the offset effect as anything
but a suggestion and a matter for further investigation.

In an effort to eliminate individual state effects from the
pooled equation, we also estimated difference forms of equations
similar to those given above. The results were generally discouraging,

with many of the key coefficients either insignificant or of unreasonable magnitude. It appeared that the difference form not only failed to eliminate individual state effects, but suffered from the previously noted discontinuity in state expenditure changes over time. In hopes of minimizing the latter effect, we reestimated the equations using three-year and five-year differences rather than the single year differences used in the initial version. However, this attempt to measure "average" changes over several years was also unsuccessful.

Given these results our overall conclusion is that the state aid equation has not yet been developed in an acceptable form. The major required improvements seem to be (a) additional provisions to take account of differences in fiscal structure among states (e.g., by including structural dummy variables, or perhaps, incorporating individual state effects in the pooled cross-section, time-series analysis); (b) inclusion of time trend or other structural shift parameters in the equations to allow for changes in "tastes" over time; (c) development of better measures of state perceptions of local "needs" for aid, to avoid the ambiguities inherent in the definitions of the s^* and t_L^* variables, and (d) investigation of the possibility that state responses to federal aid and other variables operate with lags. Work along all these lines is proceeding.

Equations for Other Expenditures and State Taxes

Efforts were made to develop predictive equations for the remaining two state fiscal variables, "other" state expenditure, e_S, and state taxes, t_S. The latter, of course, is needed to provide inputs to the local expenditure model. The former, while not entering directly into the school district model, provides a useful consistency check.

Pooled time-series, cross-section equations for e_S and t_S, containing income, aid, and price terms only, are as follows:

$$e_S = 69.4 + .031y + 1.58f_S - 28p_g$$
$$(5.3) \quad (11.9) \quad (36.7) \quad (-1.5)$$

$$R^2 = .75, \quad \text{S.E.E.} = 30.0$$

$$t_S = -138 + .024y + .40f_S + 212p_g$$
$$(-11.9) \quad (10.5) \quad (10.6) \quad (13.2)$$

$$R^2 = .62, \quad \text{S.E.E.} = 26.7$$

A series of tests for differential regional effects and influences of other state characteristics yielded the following more elaborate forms:

$$e_S = 105 + .048y + (1.54 + .25R1 + .13R3)f_S - 70p_g - 60URB$$
$$(7.3) \quad (11.9) \quad (34.4) \quad (4.1) \quad (1.9) \quad (-3.2) \quad (-5.4)$$

$$R^2 = .76, \quad \text{S.E.E.} = 29.1$$

$$t_S = -121 + (.050 - .21R1 - .014R2)y + (.20 + .64R1 + .21R2)f_S$$
$$(-10.4) \quad (17.7) \quad (-11.9) \quad (-6.7) \quad (5.0) \quad (7.7) \quad (2.3)$$

$$- 152p_g$$
$$(8.5)$$

$$R^2 = .71, \quad \text{S.E.E.} = 23.5$$

Despite the apparently good results in terms of "fit" and variance explained, especially in comparison with the state aid equations, the most important feature of these results is that the values of federal aid coefficients in each equation are in direct conflict with theory. In the equation for e_S, the federal aid coefficient is greater than one. This, if taken literally, would imply that federal lump-sum grants do not substitute at all for states' own revenues, but actually stimulate the state to raise more revenues than would otherwise have been forthcoming. Similarly, the positive coefficient of f_S in the equation for t_S (where a negative value was called for by the theory) reinforces the implication of a net stimulative effect. Such results are very difficult to accept, since they are barred by

relatively weak theoretical assumptions.[*] Moreover, the results, if questionable, also call into question the validity of estimates of the effect of f_S on state aid to local schools. It would clearly be unjustified to use these state equations for predictive purposes unless the conflict with the theory can be resolved.

A possibility that has been discussed in the literature is that federal aid coefficients in equations of this type are likely to be incorrectly estimated because the simultaneity of expenditure and aid determination has not been taken into account.[**] According to Pogue and Sgontz[***], biased estimates of the aid effect will be obtained if either (a) aid payments are a function of expenditures, or (b) aid payments are a function of some of the exogenous determinants of expenditures. It does not seem likely that case (a) is of great importance, although some feedback effects from expenditures to aid undoubtedly exist.[****] Case (b), however, is almost certain to be relevant, since income, urbanization, and possibly other variables omitted from the expenditure equation appear frequently in federal grant-in-aid formulas.

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[*]E.g., the assumption that neither education aid, other state spending nor private spending is an inferior good is sufficient to preclude a net stimulative effect of lump-sum grants-in-aid.

[**]See T.F. Pogue and L.G. Sgontz, "The Effect of Grants-in-Aid on State-Local Spending," National Tax Journal, Vol. XXI, No. 2, June 1968; also Gramlich, op.cit., pp. 581-82.

[***]Op.cit., pp. 198-99.

[****]For a feedback effect to exist, there must be federal matching at the margin. Such is not the case, however, with the major federal aid programs. Aid to education generally does not carry matching requirements. Highway aid takes the form of a fully-utilized matching grant. Welfare (AFDC) payments in most states are above the level at which there is federal matching. However, welfare is an open-ended program in that the federal government will provide additional funds as more welfare recipients (but not higher payment levels) are added. The overall extent of federal matching at the margin has not been determined, but the likelihood that the overall marginal matching ratio (if such a concept can be defined) is sufficiently great to produce the observed stimulative effect seems negligible.

To allow for the second kind of simultaneity, we reestimated
the state equations by two-stage least squares, incorporating such
additional variables as population density and state size in the
first stage equation for f_S. The results, however, did not consti-
tute an improvement. In the case of the state aid equation, several
coefficients changed in a direction contrary to theory and the
federal aid coefficient *increased* to values of 5.0 or more. These
results are clearly not acceptable either from a theoretical or
common sense point of view. In the case of the state tax and other
state spending equations, the federal aid coefficient changed only
slightly, decreasing in some cases and increasing in others. In
no instance did the value become less than one as required by the
theory. Therefore, although it is certainly proper to acknowledge
econometrically that some determinants of state spending also act
as determinants of federal aid, that refinement alone does not resolve
the basic problem.

Another possible explanation of the disturbing empirical results
rests on the *categorical* nature of most federal aid to states. As
a general proposition, the expenditure effect of a categorical
grant will be greater than the effect of an unrestricted grant if
the categorical grant is accompanied by *binding constraints*, (i.e.,
conditions that force the aid recipient to spend more for the aided
function than he would have in their absence). If, in addition,
the aid recipient is required to *match* categorical grants, then
even where matching is not open-ended, it becomes possible for the
aid effect to be greater than one.

The practical problem that makes it difficult to include
categorical aid in the model is that one cannot do so using aggre-
gative data. It is necessary to observe the magnitudes of both
federal categorical grants and required state matching funds. We
are attempting to determine whether the necessary data can be
obtained or approximated from published statistics. Until such
a test is carried out, it remains possible that the interrelated
phenomena of categorical aid and state matching requirements
(plus the relatively few open-ended matching formulas that exist)
may account for the otherwise unacceptable empirical results.

GENERAL AID TO STATES VS. DIRECT AID TO SCHOOL DISTRICTS: SOME ILLUSTRATIVE COMPARISONS

It is convenient to dichotomize the analysis of impacts of federal aid into two parts, the first concerned with the effects of a given type of aid on an "average" state or on the nation as a whole, the second with differential effects among states and their distributional implications. The reason for this separation is that different kinds of information, or analytical results, are needed to address the two questions. To estimate the average impact of a grant-in-aid proposal, one needs to know only (a) the *form* of aid (e.g., whether a lump-sum or matching formula is used and whether there are restrictions on the use of aid funds); (b) the method of *financing* the aid (e.g., out of taxes, by reducing other programs, or by borrowing); and (c) the response characteristics (coefficients) of the "average" state. Analysis of distributional aspects requires much more, including (d) the formula for allocating aid among the states and (e) information on the response characteristics of *each* state and interactions between the response characteristics and the form of aid.

Given the results of the state-level empirical analysis, there is little enough information on which to base even the "average state" projections. Essentially arbitrary assumptions must be made about the magnitudes of state fiscal responses to federal aid and other variables. There is even less information bearing on distributional effects, especially since the regional parameters in the state equations, which provide the main evidence for differential effects among states, do not appear to be reliable. Consequently, in developing illustrative projections of grant-in-aid impacts we have concentrated on the average, or national, effects, focusing on simple aid alternatives with clearcut differences in effects.

ALTERNATIVES

Specifically, we consider two types of aid and two contrasting

assumptions about financing. The aid types are (1) unrestricted
aid to states (revenue sharing) and (2) direct federal aid to
local school districts.[*] The two assumptions about financing are
(1) that aid is financed out of reductions in other federal
programs (i.e., the total federal budget and federal taxes remain
unchanged), and (2) that aid is fully financed out of an increase
in per capita taxes equal to the increase in per capita federal
aid.

In each case it is assumed that aid takes the form of a
lump-sum grant, either to the state or the school district, as the
case may be. There are assumed to be no binding restrictions
on uses of aid funds. Incremental state and federal taxes are
treated as simple subtractions from personal income (i.e., no
price effects are included).

The projections are for the school year 1970-71. Extra-
polated values of the various national price indexes for that year
are as follows: $P_e = 1.26$, $P_g = 1.21$, $P_o = 1.16$ (base year = 1967).
The only other required datum when personal income and prices
are assumed constant is the pupil/population ratio, α, for which we
assume a value of 0.25.

ASSUMPTIONS ABOUT STATE RESPONSES

Since the empirical analysis did not confirm the hypotheses
about state responses to federal grants and did not yield usuable
values of federal aid coefficients, our only recourse is to examine
the consequences of a series of assumptions about the magnitudes
of state responses. The key parameters can be defined in several
different ways. Our choice is to examine alternative assumptions
about the following three response coefficients:

    a. The fraction of an increment in unrestricted federal

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[*]The aid is considered "direct" even if funds are channeled
through the state, provided that a full pass-through is required.

aid that the average state will allocate to state tax reduction.

b. The fraction of aid *not* allocated to tax reduction (i.e., the fraction spent) that will be used to increase state aid to education.

c. The fraction of direct federal aid to local districts that will be offset by an induced reduction in state aid to local districts.

These are designated c_1, c_2, and c_3, respectively.

We have almost no valid information on the probable magnitude of the first parameter. According to the estimated equations (e.g., the second equation on p. 39) the fraction is negative; i.e., the state allocates none of the aid receipts to tax reduction, but rather *increases* state taxes to a higher level than would have obtained in the absence of aid. For reasons already stated, and regardless of the factors that led to those empirical results, it remains unreasonable to assume a net stimulative effect of lump-sum, unrestricted aid. Therefore, we test assumptions spanning the range from zero to one hundred percent allocation of aid receipts to tax reduction, using as intermediate values fractional reductions of 1/3 and 2/3.

Our "base case" assumption about the second parameter, the fraction of incremental state expenditure allocated to public school aid, is that the fraction will be the same as the average ratio of public school aid to total state expenditures in the U.S. in 1970. That ratio is calculated from Census data as 0.22. The sensitivity of fiscal outcomes to that ratio is tested by using values of one-half and twice the base value, 0.11 and 0.44.

As an initial estimate of the state aid offset to direct federal aid to districts, we use the value obtained in the equation on p. 37 , a coefficient of -0.4. The values used to test sensitivity are -0.2 and -0.8. To complete the specification of state behavior in offsetting federal aid, it is necessary to postulate some application of the released funds. We will assume that the "savings" in state aid funds are distributed proportionately between state tax reduction and other state expenditures.

It is also necessary to make assumptions about two other parameters, the responses of state aid and state taxes, respectively, to changes in the level of federal taxes. Since it has been assumed that the state response to a change in taxes is the same as the response to any equivalent change in disposable personal income, we set these parameters at the values of the income effects estimated in the state aid and state tax equations (pp. 37 and 39), i.e., .07 and .024 respectively.

Finally, we add one behavioral element that was not included in the econometric model. This is an assumption that some fraction of "other state expenditures," e_S, is used for aid to local governments other than school districts and ultimately translates, in part, into reductions in non-school local taxes. Arbitrarily, we assume that 0.25 of e_S goes into tax reduction, and that the reduction affects state school aid and state taxes in the same way as an equivalent reduction in federal taxes.

PROJECTION MODEL

The above assumptions are embodied in the following set of equations for projecting changes in local school spending and taxes, Δe and Δt_L, in response to changes in federal aid to states, Δf_S, direct federal aid to districts, Δf_L, and federal taxes, Δt_F.

State-Level Equations

$$\Delta t_S = c_1 p_g \Delta f_S - c_4 \Delta t_F - c_7 c_3 p_e a \Delta f_L$$

$$\Delta e = (1/p_e a)[c_2 (1 - c_1)\Delta f_S - c_6 \Delta t_F] + c_3 \Delta f_L$$

$$\Delta t_o = c_5[(1 - c_2)(1 - c_1)p_g \Delta f_S - c_8 c_3 p_e a \Delta f_L].$$

Local Equations (from extrapolated equation, p. 30)

$$\Delta e = 1.35 p_e a(\Delta e + \Delta f_L) - .197(\Delta t_S + \Delta t_o + \Delta t_F)$$

$$\Delta t_L = p_e a(\Delta e - \Delta e - \Delta f_L).$$

The only variable not previously defined is Δt_o, the change in
taxes by local governments other than school districts, which was
introduced immediately above.

Values of the parameters are as follows (multiple entries
indicate that alternative parameter values are to be tested):

$a_1 = 0, -1/3, -2/3, 1$ $a_5 = .25$

$a_2 = .11, .22, .44$ $a_6 = .07$

$a_3 = -.2, -.4, -.8$ $a_7 = a_1/[a_1 + (1 - a_2)(1 - a_1)]$

$a_4 = .024$ $a_8 = 1 - a_7$

RESULTS

Table 3 displays the effects of each federal aid alternative
on the major state and local fiscal variables under various sets of
assumptions about parameter values. The four alternatives, as pre-
viously identified, are revenue sharing and direct aid to school
districts, each with and without increases in federal taxes. The
effects shown in the table are (a) changes in outside aid per
pupil to local districts (state, federal, and total); (b) changes
in all non-school taxes (state, federal, other local, and total);
and (c) changes in school spending and local school taxes, each
stated in per pupil and per capita terms. All values in the tables
correspond to a federal aid increment of one dollar per capita.

The sensitivity of outcomes to alternative assumptions about
state responses to federal aid is explored most fully in connection
with alternative A, revenue sharing with no change in federal
taxes. Cases A1 - A4 show what happens as the fraction of aid allo-
cated to state tax reduction rises from ... to one hundred percent.
The surprising result is the relative insensitivity of school
spending. Increments in per pupil expenditure fall only from
$0.36 to $0.21 as a_1 varies from zero to one. The amount of local
property tax relief does depend strongly, however, on the extent
to which federal funds are used to relieve state taxes. A distinct
trade-off, or substitution, effect between state and local taxes
is evident. The degree of total (state + local) tax relief rises

Table 3. Effects of a One Dollar Increase in Federal Aid per Capita to an "Average" State Under Various Assumptions About State Behavior

Form of aid and financing	Assumed values of State response parameters				Change in aid to local districts ($/pupil)			Changes in state, federal, and "other local" taxes ($/capita)				Change in school spending		Change in local school taxes	
	Case	c_1	c_2	c_3	State	Fed.	Total	State	Fed.	Other Local	Total	$ per pupil	$ per capita	$ per pupil	$ per capita
A. Revenue sharing, no increase in federal taxes	A1	0	.22	NA	.88	0	.88	0	0	-.20	-.20	.36	.09	-.52	-.13
	A2	-1/3	.22	NA	.59	0	.59	-.33	0	-.13	-.46	.31	.08	-.27	-.07
	A3	-2/3	.22	NA	.29	0	.29	-.67	0	-.07	-.74	.26	.07	-.03	-.01
	A4	-1	.22	NA	0	0	0	-1.00	0	0	-1.00	.21	.05	.21	.05
	A5	-1/3	.44	NA	1.17	0	1.17	-.33	0	-.09	-.42	.52	.13	-.65	-.16
	A6	-1/3	.11	NA	.29	0	.29	-.33	0	-.15	-.48	.21	.05	-.08	-.02
B. Revenue sharing fully financed out of federal taxes	B1	-1/3	.22	NA	.51	0	.51	-.36	1.00	-.13	.51	.08	.02	-.43	-.11
	B2	-2/3	.22	NA	.22	0	.22	-.69	1.00	-.07	.24	.03	.01	-.19	-.05
	B3	-1/3	.44	NA	1.10	0	1.10	-.36	1.00	-.09	.55	.28	.07	-.81	-.20
C. Direct aid to districts, no increase in federal taxes	C1	-1/3	.22	-.4	-1.60	4.00	2.40	-.16	0	-.06	-.22	.92	.23	-1.48	-.37
	C2	-1/3	.22	-.8	-3.20	4.00	.80	-.31	0	-.12	-.43	.39	.10	-.41	-.10
	C3	-1/3	.22	-.2	-.80	4.00	3.20	-.08	0	-.03	-.11	1.19	.30	-2.01	-.50
	C4	-2/3	.22	-.4	-1.60	4.00	2.40	-.29	0	-.03	-.32	.94	.24	-1.46	-.36
	C5	-1/3	.44	-.4	-1.60	4.00	2.40	-.19	0	-.05	-.24	.93	.23	-1.47	-.37
D. Direct aid to districts, fully financed out of federal taxes	D1	-1/3	.22	-.4	-2.03	4.00	1.97	-.18	1.00	-.06	.76	.56	.14	-1.41	-.35
	D2	-1/3	.22	-.8	-3.63	4.00	.37	-.34	1.00	-.12	.54	.02	0	-.35	-.09
	D3	-1/3	.22	-.2	-1.23	4.00	2.77	-.10	1.00	-.03	.87	.83	.21	-1.94	-.49

and the expenditure increment diminishes as the state diverts increasing fractions of aid funds to state tax reduction.

Cases A5 and A6 show effects of variations in the school aid share of state expenditures. An increase in that ratio channels a greater share of federal aid to local districts to be allocated between increased spending and local property tax relief. Note, however, that even in the most favorable case from the local point of view (case A5), only $0.13 out of each dollar of federal aid translates into increased school spending and only $0.16 into local property tax relief.

Cases B1, B2, and B3 illustrate the effects of paying for aid out of increased federal taxes. Comparing B1 and A2, which are identical except for the federal tax, it can be seen that a dollar of aid produces only 25 percent as great an increase in per pupil spending when financed out of taxes as when financed out of resources drawn from other programs. The amount of local tax relief is not very much affected. In fact, more local tax relief is forthcoming in cases B2 and B3 than in the corresponding cases with no federal taxes. Again, the phenomenon of tax substitution is shown to be important. As federal taxes increase, the local response is to generate a partially compensating local tax reduction primarily at the expense of public school outlays.

When aid is channeled directly to school districts rather than to the state, the direct effect, of course, is to make a considerably larger fraction of aid available for school spending and local property tax relief. But the extent of the local gain--or whether there is any gain at all--depends critically on how the state adjusts its own aid apportionments. As can be seen by comparing cases C1, C2, and C3 (or D1, D2, and D3), the results are highly sensitive to the value of the state aid offset parameter, c_3. Assuming $c_1 = -1/3$, $c_2 = .22$, and $c_3 = -.4$, the effect of shifting from revenue sharing to direct aid to districts (cases C1 vs. A2) is to multiply the school spending increment by more than two and one-half and the amount of local property tax relief by five. However, if the state offsets twice as much federal aid (case C2),

then direct aid gives local districts only a very slight advantage.

Finally, in comparing direct aid to districts with and without corresponding tax increases, we see that most of the impact of tax financing falls on school spending, as was the case when we compared alternatives A and B. The amount of local tax relief falls only very slightly, but the increment in school spending falls by a considerable percentage. If we compare alternatives B and D, both of which include federal tax financing, it can be seen that direct aid generates a much greater boost in spending and much more property tax relief when only 40 percent of federal aid is offset by the state, but no increase in spending and only slightly greater property tax relief when federal aid acts primarily as a substitute (case D2) for state assistance.

IMPLICATIONS

Although this rather artificial exercise adds little to our substantive knowledge of fiscal impacts of federal aid, it does have implications for both empirical research and aid formula design.

With respect to the former, the main contribution of sensitivity testing is to identify the critical parameters on which future empirical work must focus. Clearly, the two most important are (a) the marginal ratio of state school aid to total state expenditure and (b) the coefficient of state aid response to a federal aid increase. Additional empirical work should be targeted specifically on estimates of those critical ratios. Our reformulation of the state model for the purpose of these projections suggests an alternative estimation procedure. Instead of estimating an aid equation directly, the alternative would be to develop the estimate in two stages. Stage one would be a total state expenditure equation, fitted to time-series, cross-section data with appropriate allowances for structural factors, categorical aid, and simultaneity of expenditure and aid determination. Stage two would estimate state school aid as a function of total state expenditure and other relevant variables. The point of the two

stage separation is that the second step could be conducted on a state-by-state basis, using time series data for individual states, if that appeared necessary to avoid the problems caused by structural differences across states. At the same time, the interplay between state aid and direct federal aid to school districts could be examined as part of the individual state, time-series studies. A further advantages is that it would be possible to construct different price indexes for the individual states without having to be concerned about the cross-state validity of price level comparisons. Except for the latter improvement, the data are available to carry out the two-stage analysis. That will be our first order of business in attempting to upgrade the empirical results.

The relevance of the projections for aid formula design is that they point to facets of state-local behavior that must be modified or controlled if federal aid is to have its intended effects. This statement is true independent of what the effects are intended to be. Whether the goal is higher per pupil spending, local property tax relief, or reduced state taxes, so long as there is a preferred set of outcomes, state responses are too uncertain to make general, unrestricted aid an effective fiscal instrument.

A way of using the projection model, therefore, is to turn the analytical question around, asking not what state responses are likely to be or what outcomes they imply, but what values or ranges of values are necessary if desired results are to be achieved. This leads naturally to the study of appropriate aid *conditions:* incentive provisions; constraints; effort maintenance, matching, and tax reduction requirements; and other devices for controlling the state response parameters. Many possible conditions can be built in to the projection models and tested under alternative sets of assumptions about the form of federal aid and the underlying pattern of state behavior. Adaptation of the models to this use will be a major objective of our future work.