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ABSTRACT

A linear programing model and procedures for optimal assignment of students to attendance centers are presented. An example of the use of linear programing for the assignment of students to attendance centers in a particular school district is given. (CK)

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A LINEAR PROGRAMMING MODEL FOR ASSIGNING STUDENTS TO ATTENDANCE CENTERS

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A LINEAR PROGRAMMING MODEL
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Introduction

One of the current problems in education, particularly in the urban areas, is the assignment of students to attendance centers in such a way that a balance can be attained on certain student characteristics--particularly racial characteristics currently.

The United States Supreme Court, in its 1954 Brown vs. Board of Education decision, ruled that state imposed segregation is unconstitutional (347 U.S. 483). In subsequent rulings the Supreme Court has also held that, "Federal Courts have... powers including altering attendance zones...and requiring necessary busing...using racially based mathematical ratio of students..." (Swann vs. Charolette-Mecklenburg Board of Education, as reported in The United States Law Week).

On April 20, 1971, the Supreme Court affirmed a Federal District Court decision that ruled a North Carolina anti-busing law unconstitutional (North Carolina State Board of Education vs. Swann, as reported in The United States Law Week).

Present methods of assignment that depend on setting up a plan and checking whether or not the plan meets the criteria are basically inadequate, since there is no way of knowing if a particular plan is optimal. Linear programming offers a method of developing an optimal plan for the assignment of students to attendance centers based on predetermined criteria and policy decisions. This paper presents a linear programming model and procedures for optimal assignment of students to attendance centers as well as an example of the use of linear programming for the assignment of students to attendance centers in a particular school district.

The Linear Programming Model

Space does not permit a detailed discussion of the theory of linear programming here. That information can be found in materials by Hillier (1967), Van Dusseldorp (1971), and Wagner (1969), listed in the Bibliography as well as numerous texts on linear programming.

Briefly, linear programming is a method of expressing an optimization problem as a group of simultaneous linear equations, then solving the model mathematically to arrive at the optimal solution. The model consists of two types of equations, an object function and constraints. The object function is of the type:

$$C_1X_1+C_2X_2+\ldots+C_iX_i+\ldots+C_nX_n=Z$$

Where Z represents the criterion to be optimized (perhaps the total distance traveled to school by all students in a system), the X_i's represents the variables that affect the criterion measure (the assignment of a particular group of students to a particular school, for example) and C_i's represent the contribution of each X_i to the criterion measure (in the case of student assignment C_i may represent the distance from home to school for a particular group of students).

This type of relation is often written $\sum_{i=1}^{n} C_iX_i=Z$.

Constraints are of the type:

$$a_{1j}X_1+a_{2j}X_2+\ldots+a_{ij}X_i+\ldots+a_{nj}X_n\ (\leq,=,\geq)b_j$$

where the X_i's represent the variables as in the object function, the a_{ij}'s represent the contribution of those variables in the constraint relations, and the b_j's represent the exact value of the constraint or the maximum or minimum value of the constraint. For example, b_j may represent the maximum proportion of black students that may be assigned to a school, or may represent the maximum number of total students that may be assigned to a school. The constraint set is usually represented mathematically as:

$$\sum_{i=1}^{n} a_{ij}X_i\ (\leq,=,\geq)\ b_j\ (\text{for } j=1,2,\ldots,m)$$

representing m relations in n unknowns.

Object Function

Assuming that the object of the busing plan is, within certain constraints, to minimize the total distance all students must be bused to their assigned attendance centers. The object becomes:

$$\sum_i \sum_j \sum_g S_{ig}\ D_{ij}\ X_{ij}=Z \qquad (1)$$

where Z is the total distance traveled by all bused students expressed in pupil-miles. The subscripts are defined as:

i represents the residential areas

j represents the schools

g represents the grade levels

Then S_{ig} represents the total number of students living in residential area i and attending grade g. D_{ij} represents the distance from residential area i to school j in miles. X_{ij} indicates whether or not students in residential area i are assigned to school j ($X_{ij}=0$ or $X_{ij}=1$).

Constraints

The limitations of the system must be incorporated into the model. A critical limitation in assigning students to schools is the capacity of the school buildings. In order to compute the number of students in each grade in each building

$$\sum_i S_{ig} X_{ij} = E_{jg} \quad (j=1,2,\ldots,s;g=1,\ldots,h) \tag{2}$$

is used to calculate enrollment of grade g within school j and is represented by E_{jg}. Then the capacity for grade g in school j, represented by C_{jg}, is included in the model by:

$$E_{jg} \leq C_{jg} \quad (j=1,2,\ldots,s;g=1,\ldots,h) \tag{3}$$

where 1 indicates the lowest grade level and h the highest grade level in the organizational level under consideration. The number of residential areas is represented by n, while s represents the number of schools.

An important consideration is that every area must be assigned to one and only one school. This requirement is included by:

$$\sum_j X_{ij} = 1 \quad (i=1,2,\ldots n) \tag{4}$$

For residential area 1 this equation becomes

$$X_{11}+X_{12}+\ldots+X_{ij}+\ldots+X_{1s}=1$$

One of the X_{ij}'s (say X_{12}) should equal one and then the others would equal zero. This would indicate that the students in residential area 1 would attend school 2.

A_{ig} represents the number of specially identified students in residential area i and grade g, a_{jg} is the maximum allowable percentage of specially identified students in grade g of school j, and b_{jg} the minimum allowable percent of specially identified students in grade g of school j. The following two constraints control the number of specially identified students in each school and grade.

$$\sum_i A_{ig} X_{ij} \leq a_{jg} E_{jg} \qquad (j=1,2,\ldots,s;g=1,\ldots,h) \qquad (5)$$

$$\sum_i A_{ig} X_{ij} \geq b_{jg} E_{jg} \qquad (j=1,2,\ldots,s;g=1,\ldots,h) \qquad (6)$$

To add additional criterion variables such as socio-economic status, academic achievement, or academic ability to the model, relations similar to (5) and (6) must be added to the constraint set for each additional criterion variable.

Application of the Model

The model was constructed and tested using mathematically equivalent, but analytically more convenient, relations.

The object was to minimize the total distance traveled by students to their assigned attendance centers. In other words, the object was to minimize Z where

$$Z = \sum_i \sum_j \sum_g S_{ig} D_{ij} X_{ij} \qquad (7)$$

The constraint (2) which calculates enrollment was rewritten as:

$$\sum_i S_{ig} X_{ij} - E_{jg} = 0 \qquad (j=1,2,\ldots,s;g=1,\ldots,h) \qquad (8)$$

Then an upper bound of C_{jg} was placed on the corresponding E_{jg} for all schools (j) and grades (g) under consideration in the model. This incorporated the capacity constraint (3) in the model.

The assignment constraint (4)

$$\sum_j X_{ij}=1 \quad (i=1,2,...,n) \tag{9}$$

was not rewritten.

The percentage constraints (5) and (6) were rewritten as:

$$\sum_i A_{ig}X_{ij}-G_{jg}=0 \quad (j=1,2,...,s;g=1,...,h) \tag{10}$$

$$\sum_g G_{jg}-M_j=0 \quad (j=1,2,...,s) \tag{11}$$

$$\sum_g E_{jg}-T_j=0 \quad (j=1,2,...,s) \tag{12}$$

$$G_{jg}-a_{jg}E_{jg}\leq 0 \quad (j=1,2,...,s;g=1,...,h) \tag{13}$$

$$G_{jg}-b_{jg}E_{jg}\geq 0 \quad (j=1,2,...,s;g=1,...,h) \tag{14}$$

$$M_j=\alpha_j T_j\leq 0 \quad (j=1,2,...,s) \tag{15}$$

$$M_j-\beta_j T_j\geq 0 \quad (j=1,2,...,s) \tag{16}$$

where G_{jg} represents the number of minority students in grade g of school j.

M_j represents the total number of minority students in school j.

T_j represents the total number of students in school j.

α_j represents the maximum percentage of minority students in school j.

β_j represents the minimum percentage of minority students in school j.

Application of the Model

The linear programming model for assignment of students to attendance centers was applied to the assignment of 7th, 8th, and 9th grade students to junior high schools in the Waterloo, Iowa

Community School District. This involved the assignment of 5,420 students, 5.4% of which were black, to six junior high schools. The residential areas used in this study were the same as the enumeration districts as defined by the United States Bureau of the Census. There are 117 of these residential areas in Waterloo. The number of junior high students living in each residential areas ranged from zero to 165. It was deemed desirable to, in as far as was possible, assign all students in a particular residential area to the same school. It was assumed that only those students who were assigned to schools more than one mile distant from the residential areas in which they lived would be transported. Thus, in this application, any distance between a residential area and a school of less than one mile was considered to be zero in computing miles transported.

The purpose of this application of linear programming was to assign pupils by residential area to schools in such a way that the total pupil-miles of transportation would be minimized while at the same time meeting certain criterion of racial balance.

In order to observe the affect of different racial balance criteria, two applications of the model were run. In one case the lower limit for percentage of minority students in each grade in each school and for the total school was set at zero and the upper limit at 10%. In the other case the lower limit was set at 3% and the upper limit at 7%. Thus in one case the constraints designating student racial mix became:

$$G_{jg} - 0.10\ E_{jg} \leq 0 \text{ (for all j and g)}$$
$$G_{jg} - 0.00\ E_{jg} \geq 0 \text{ (for all j and g)}$$
$$M_j - 0.10T_j \leq 0 \text{ (for all j)}$$
$$M_j - 0.00T_j \geq 0 \text{ (for all j)}$$

where G_{jg} is the number of black students assigned to grade g in school j, E_{jg} is the total number of students assigned to grade g in school j, M_j is the total number of black students assigned to school j, and T_j is the total number of students assigned to school j.

In the other case the corresponding constraints were:

$$G_{jg} - 0.07\ E_{jg} \leq 0 \text{ (for all j and g)}$$
$$G_{jg} - 0.03\ E_{jg} \geq 0 \text{ (for all j and g)}$$
$$M_j - 0.07\ T_j \leq 0 \text{ (for all j)}$$
$$M_j - 0.03\ T_j \leq 0 \text{ (for all j)}$$

Input data for solution of the model included the actual values for Waterloo of S_{ig}, D_{ij}, C_{jg}, and A_{ig} as defined above. For solution of the linear programming student assignment model the IBM Mathematical Programming System/360 (360 A-CO-14X) Version 2, Linear and Separable Programming computer program was used and run on the University of Iowa IBM 360 Model 65 computer. Similar programs are available for most large computers. It is not possible with the computer program used to force an integer solution. That is, in a few cases the results indicated a split of a residential area, with some students attending one school and the remaining students attending another. These splits can be eliminated either by hand manipulation after the computer run or by using a computer program which forces an integer solution.

The results of the solution of the linear programming model include the assignment of students in residential areas to schools in such a way that the total number of pupil-miles transported is minimized and all the constraints are satisfied. That is, the X_{ij} values are found.

A brief summary of the results of the application of linear programming to the school assignment of Waterloo students is given in the tables below.

TABLE 1

STUDENT ASSIGNMENTS, CASE ONE

School	Pct. Minority Students	Students Transported: Minority Pct.	Minority Ave. Dist.	Total Pct.	Total Ave. Dist.
A	0.2	100.0	1.4	84.6	3.4
B	9.6	93.0	2.6	85.9	1.9
C	0.2	0.0	0.0	60.4	2.5
D	8.8	76.3	1.2	67.6	1.8
E	9.0	53.2	1.2	67.0	2.3
F	2.9	94.3	4.1	20.4	2.1
Total	5.4	75.3	2.1	60.4	2.3

TABLE 2

STUDENT ASSIGNMENTS, CASE TWO

School	Pct. Minority Students	Students Transported			
		Minority		Total	
		Pct.	Ave. Dist.	Pct.	Ave. Dist.
A	5.2	100.0	4.2	84.9	3.7
B	6.2	89.1	2.9	85.9	1.9
C	3.0	92.0	4.9	60.4	2.8
D	6.5	66.7	1.1	66.6	2.0
E	6.6	39.1	1.2	68.3	2.6
F	5.0	96.8	3.0	20.5	2.1
Total	5.4	76.3	2.8	60.6	2.5

Concluding Remarks

Linear programming offers a logical method for assignment of students to attendance areas according to prescribed criterion. It is fairly easy and inexpensive to use assuming the necessary data on student residence is available. It also offers the opportunity, as illustrated by the two cases described above, for school administrators to test the affect of various policy decisions concerning the assignment of students.

BIBLIOGRAPHY

Adelman, I. "A Linear Programming Model of Educational Planning: A Case Study of Argentina," The Theory and Design of Economic Development. New York: The John Hopkins Press, 1966.

Alkin, Marvin C. "The Use of Quantitative Methods as an Aid to Decision Making in Education." Paper read at the annual meeting of the American Educational Research Association, Los Angeles, February 5-8, 1969. (ERIC No. ED028525).

Allen, James E. Jr. "Integration is Better Education," Integrated Education, 7:30-1, September-October 1969.

Allen, W. R. "Report on Educational Systems Engineering", American School Board Journal, 151:67-70, October, 1965.

Banghart, Frank W. Educational Systems Analysis. New York: Macmillan, 1969.

Belgard, Maria R., and Min, Leo Y. "Optimizing the Teacher-Learning Process through a Linear Programming Model - An Operations Research Approach." Paper read at the annual meeting of the American Educational Research Association, New York, February 4, 1971.

Benard, J. "General Optimization Model for the Economy of Education," Mathematical Models in Educational Planning. Paris: OECD, 1967. (ERIC No. ED024138).

Bern, H. A. "Wanted: Educational Engineers," Phi Delta Kappan, 67:230-6, January, 1967.

Bowles, Samuel. "The Efficient Allocation of Resources in Education," Quarterly Journal of Economics, 81:189-219, May, 1967.

Bruno, James E. "An Alternative to the Fixed Step Salary Schedule," Educational Administration Quarterly, January, 1969.

Bruno, James E. "A Linear Programming Approach to Position-Salary Evaluation in School Personnel Administration," Santa Monica: Rand Corporation, 1968, (Paper No. P-4039).

Bruno, James E. "A Proposed Scheme for Federal Support for Education," Santa Monica: Rand Corporation, 1969, (Paper No. P-4038).

Bruno, James E. "The Use of Mathematical Programming Models to Optimize Various Objective Functions of Foundation Type State Support Programs." Unpublished Doctoral dissertation, University of California, Los Angeles, 1968. (University Microfilms, 68-14,559).

Clarke, S., and Surkis, J. "An Operations Research Approach to Racial Desegregation of School Systems." Socio-Economic Planning Sciences 1:259-272, 1968.

Conant, Eaton H. "A Linear Programming Model to Identify an Optimal Teaching Division of Labor for Teachers and Non-professional Teaching Aids." Paper read at the annual meeting of the American Educational Research Association, New York, February 6, 1971.

Correa, Hector. "More Schools or Better Schools?" Scientia Paedagogica Experimentallis, 3:123-41, 1966.

Correa, Hector. "Optimum Choice Between General and Vocational Education," Kyklos, 18:107-15, 1965.

Correa, Hector. Quantitative Methods of Educational Planning. Scranton, Pennsylvania: International Textbook Company, 1969.

Correa, Hector. "Quantity vs. Quality in Teacher Education," Comparative Education Review, 8:141-6, October 1964.

Gaisford, H. R. and Ritteispach, K. C. "Resource Allocations in Urban Education." Paper read at the annual meeting of the American Educational Association, New York, February 6, 1971.

Galladay, Fredrick L. "A Dynamic Linear Programming Model for Educational Planning with Application to Morocco," Unpublished Doctoral dissertation, Northwestern University, 1968 (University Microfilms, 69-6928).

Heckman, Leila B. and Taylor, Howard M. "School Rezoning to Achieve Racial Balance: A Linear Programming Approach." Socio-Economic Planning Sciences. 3:127-33, 1969.

Hershkowitz, Martin and others. "An Optimal Solution to the Problem of Combining Methods of Reading Instruction." Paper read at the annual meeting of the Operations Research Society of America, Detroit, October 28, 1970.

Hertz, David B. A Comprehensive Bibliography on Operations Research, New York: Wiley and Sons, 1957.

Hillier, Frederick S., and Leiberman, Gerald J. Introduction to Operations Research. San Francisco: Holden-Day, Inc., 1967.

Holtman, A. G. "Linear Programming and the Value of Input to a Local Public School System." 1966, (Mimeographed).

Hopkins, Charles Oliver. "State-Wide Systems of Area Vocational-Technical Training Centers in Oklahoma." Stillwater: Oklahoma Vocational Research Training Unit, 1970. (ERIC No. ED036627).

Igoe, Joseph A. "The Development of Mathematical Models for Allocation of School Funds in Relation to School Quality," Paper read at the Conference on School Finance, Dallas, Texas: March 31, April 1-2, 1968. (ERIC No. ED026728).

IBM, Mathematical Programming System/360 Version 2, Linear and Separable Programming - User's Manual, (H20-0476-2). White Plains, N. Y.: IBM Corporation, 1969.

Knezevich, S. J. "Systems Analysis and Its Relationship to Educational Planning," Paper read at the Western Canada Administrators Conference, Banff, Alberta, October 9-10, 1969. (ERIC No. ED036895).

Markel, G. A. "A Concept for Modeling and Evaluating Information Producing Systems," DDC Publication (DDC, Cameron Station, Alexandria, Virginia: January 26, 1967) (Order No. AD628495).

Matzke, Orville R. "A Linear Programming Model to Optimize Various Objective Functions of a Foundation Type State Support Program." Unpublished Doctoral dissertation, The University of Iowa, Iowa City, 1971.

McNamara, James F. "A Mathematical Programming Approach to State-Local Program Planning in Vocational Education," American Educational Research Journal, 8:335-63, March, 1971.

Meals, A. P. "Heuristic Models in Education." Phi Delta Kappan. 47:201-3, January 1967.

Nixon, Richard M. "School Desegregation: 'A Free and Open Society'; Policy Statement by Richard Nixon, President of the United States." Office of the President, Washington, D. C. 24 March 1970. (ERIC No. ED039311).

OECD. Mathematical Models in Educational Planning. Paris: OECD, 1967, (ERIC No. ED024138).

OECD. Systems Analysis for Educational Planning. Paris: OECD. 1969.

Ontjes, Robert L. "A Linear Programming Model for Assigning Students to Attendance Centers," Unpublished Doctoral dissertation, University of Iowa, Iowa City, 1971.

Passow, A. Harry. "Toward Creating a Model Urban School System: A Study of the Washington, D. C. Public Schools." New York: Teachers College Columbia University, 1968.

Piele, Philip K., Eidell, Terry L. and Smith, Stuart C. Social and Technological Change: Implications for Education, The Center for the Advanced Study of Educational Administration, University of Oregon, Eugene, Oregon, 1970.

Schnittjer, Carl J. "Prediction of Success Through Linear Programming." Unpublished Doctoral dissertation, the University of Iowa, Iowa City, 1971.

Shaw, Peg. "Project Concern...Hartford's Experimental Busing Program." February, 1967. (ERIC No. ED017573).

Sweeny, Robert B. "Business Use of Linear Programming," Management Accounting, 47:41, September, 1965.

Trancz, George S. "A Quantitative Approach to the Design of School Bus Routes." Paper read at the annual meeting of the American Educational Research Association, Minneapolis, March 2, 1970.

United States Bureau of the Census. 1970 Census Users' Guide. U. S. Government Printing Office, Washington, D. C., 1970.

United States Supreme Court. Brown vs. Board of Education, 347 U. S. 483.

United States Supreme Court. The United States Law Week. 39:4437-51. April, 1971.

Uxer, J. E. "An Operations Research Model for Locating Area Vocational Schools." Las Cruces, New Mexico: New Mexico State University, School of Education, 1967.

Vajda, S. Readings in Linear Programming. New York: John Wiley and Sons, 1958.

Van Dusseldorp, Ralph A., Richardson, Duane E., and Foley, Walter J. Educational Decision-Making Through Operations Research. Boston: Allyn and Bacon, Inc., 1971.

Wagner, Harvey M. Principles of Operations Research. Englewood Cliffs: Prentice-Hall, 1969.

Wheat, Thomas Earl. "A Study of the Reading Achievement of Pupils Bussed to Predominantly White School as Compared with the Reading Achievement of Pupils Remaining in Predominantly Negro Central-City Schools." Unpublished Doctoral dissertation, Ball State University, 1970. (70-25,182 University Microfilms).

Wurtele, Zivia S. "Mathematical Models for Educational Planning." Santa Monica: System Development Corporation. sp-3015. November, 1967.